Falling Behind: Has Rising Inequality Fueled the American Debt Boom 1980–2007?

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Abstract

The household debt boom since 1980 is considered one of the main drivers of the Great Recession of 2007–9. In lockstep with household debt, income inequality has risen to new extremes. We build a model that links rising inequality to the mortgage debt boom. It builds on the old idea that people care about their social status. In an attempt to keep up with ever richer Joneses, the middle class substitutes status-enhancing houses for status-neutral consumption. These houses are mortgage-financed, creating a debt boom across the income distribution.

Our mechanism is consistent with the following stylized facts: (i) Real mortgage debt, (ii) debt-service-to-income ratios and (iii) house sizes (in sqft) have increased since the 1980 across all income quintiles. This happened despite (iv) stagnating real incomes for the bottom 50% since the 1980s.

We build a tractable dynamic network model with housing to illustrate how our mechanism generates these facts. We extend it to a quantitative general equilibrium life-cycle model to show how status concerns and rising inequality amplify previously studied origins of the debt boom: the saving glut, the banking glut and financial innovation. Preliminary results suggest that social comparisons boost the debt boom and the house price boom by about 25%.

Keywords: mortgages, housing boom, social comparisons, consumption networks, keeping up with the Joneses, behavioral macroeconomics

JEL Codes: D14, D31, E21, E44, E70, R21

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1 Introduction

In the decades leading up to the Great Recession, the distribution of national income growth has become ever more skewed. Average post-tax incomes of the top ten percent have grown more than twice as fast as incomes of the “middle forty” (P50-P90), and more than five times as fast as those in the bottom half of the distribution (Piketty et al., 2018a). This divergence in incomes has led to increasing inequality in living standards (Aguiar and Bils, 2015; Bellet, 2018). Relative to the rich, everyone else has fallen behind.

It has been argued (e.g. Rajan, 2010; Frank, 2013) that this was one of the main drivers of the American debt boom – the rise in US household debt across the entire distribution since the 1980s (see section 2 and Kuhn et al., 2017). There is plenty of evidence that people compare themselves with others and suffer from relative deprivation (e.g. Luttmer, 2005; Card et al., 2012). Non-rich households will then attempt to keep up with the living standards set by the rich. To this end, they shift expenditures towards status-enhancing goods, cutting down on both present and future status-neutral consumption (Frank, 2013; Bertrand and Morse, 2016; De Giorgi et al., 2016). Housing is arguably one of the most visible and status-enhancing goods and accounts for the largest share in households’ expenditures. As housing is largely debt-financed, household debt rises with growing housing aspirations.

In this paper we explore the extent to which rising inequality and status concerns have fueled the American debt boom prior to the Great Recession. This question is important for two reasons. First, the secular surge in inequality warrants a deeper understanding of how it may affect the aggregate economy. Second, understanding the determinants of household debt and its distribution is paramount for future policies as they were among the main drivers of the Great Recession (Mian et al., 2013, 2017; Martin and Philippon, 2017). Our preliminary results suggest that in an economy with social comparison motives, both the debt boom and the house price boom are about 25% stronger than in an economy without social preferences.

We first develop a tractable infinite-horizon consumption network model in order to illustrate the mechanism analytically. We then introduce social comparisons into a heterogeneous agents macroeconomic model with housing and heterogeneous income profiles for a quantitative analysis.

Our tractable infinite-horizon network model (in the spirit of Ballester et al., 2006) illustrates how other-regarding preferences can rationalize the afore-mentioned evidence. Agents can spend their lifetime wealth on a non-durable and status-neutral consumption...
good and on durable and status-enhancing housing. Given prices, agents’ optimal houses and debt are linear functions of one’s own income and that of the reference group. If the incomes of households in the reference group rise, these households will upgrade their houses. Hence, reference housing (a weighted average of all houses in the reference group) will increase. In order to keep up with its reference group, the households substitute consumption for housing which requires taking out additional debt. Agents substitute future consumption flows with debt service payments. We provide conditions on the comparison weights of the network, under which rising income inequality raises aggregate debt. This is the case if comparisons are upward-looking and each member of the reference group is equally important.

In the quantitative part of the paper, we analyze an incomplete market model with heterogeneous income profiles and upward-looking comparisons in housing. The rich type-dependent income process allows for tight control over the income distribution and its evolution – a key ingredient to our quantitative analysis. The key difference to the tractable model is the presence of precautionary savings to insure against income risk. Our preliminary results indicate that increasing top incomes lead nonrich households to increase own housing in response to increased houses at the top. To that end, households reduce expenditures on nondurable consumption and take on more debt. House prices increase and interest rates decrease.

Our paper makes three contributions. First, we provide a systematic quantitative investigation of the explanatory power of the popular hypothesis that rising inequality has driven the surge in household debt due to status concerns of non-rich households. Second, while there is a growing literature documenting a steady rise in income and wealth inequality, we know rather little about potential consequences of this distributional shift (Guvenen et al., 2017; Piketty et al., 2018a). Third, psychologists as well as applied microeconomists and experimental economists have amassed extensive evidence on the importance of social comparisons for people’s economic choices (Fehr and Schmidt, 1999; Luttmer, 2005; Bursztyn et al., 2014; De Giorgi et al., 2016; Bursztyn et al., 2017; Bellet, 2018). This evidence has yet to find its way into macroeconomic models leaving us with little knowledge on the aggregate effects of relative income and consumption concerns.

2 The Facts

The American household debt boom 1980–2007 was mainly driven by mortgages. Figure 1a shows that that mortgage debt is the biggest component of household debt and also the component that drives total growth. At the peak of the debt boom in 2007, mortgages accounted for three quarters of total debt. And almost all of the increase in debt came from mortgages.

Income inequality has grown in lockstep with household debt. Figure 1b shows that both the top 10% income share and household debt take off around 1980 and rise until the Great Recession in 2007.

A closer look on debt and inequality reveals a puzzle. Figure 2a shows that the average income in the bottom half of the population has stagnated since 1980. At the same time, figure 2b shows that mortgage debt to income rose across the whole income distribution. These facts suggest that the the bottom half of the population increased their spending in houses despite rising house prices and stagnant incomes, but falling interest rates.

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1 We abstract from the possibility that non-durable goods, such as an expensive drink in a bar, may induce consumption externalities. However, we posit that most typical status goods (cars, clothing) are durable to some degree.

2 The assumption of heterogeneous income types/profiles is both intuitive (different education, innate skill, etc.) and supported by the data (Guvenen, 2007, 2009; Guvenen et al., 2016)
Real average pre-tax income growth from 1962 to 2014 in the US. Data are taken from Piketty et al. (2018b). Growth rates are relative to the base year 1980.

(A) Since 1980 real incomes have stagnated for the bottom 50%.

**Figure 2:** Despite having stagnating incomes, mortgage debt increased for the bottom 50%.

Growth of mean mortgage debt as a fraction of mean income by income quintiles. Use *OECD-modified equivalence scale* for income quintiles. Data from the Surveys of Consumer Finances.

(B) Mortgages rose across the whole income distribution.

**Figure 3:** Interest rates do not seem to drive aggregate mortgages by much.

**Figure 4:** Housing expenditures have risen more strongly than non-housing consumption expenditures.
Consistent with this fact, housing expenditures have risen over the decades to 2007. Figure 4 shows the rise of housing expenditures relative to other goods. From the mid-1980s to 2007 the housing expenditure share grew from about 30% to 34%. This increase is robust to the exact definition of housing expenditures.

Moreover, we see a similar increases in house sizes. Bellet (2018) shows that houses in the top 10% of the housing distribution have more than doubled in size between 1980 and 2007, reflecting the rise in top incomes. But also the houses in the bottom half have grown by a third on average—despite stagnating or declining incomes for half the population and house price growth. These numbers show changes in actual house size (in squarefeet), sidestepping problems of valuation and house price inflation.

So, why did the bottom 50% of the income distribution increase their debt-to-income levels, and their houses sizes?

There is evidence that, in fact, non-rich households adjusted their housing expenditures and debt level because of rising inequality. Bertrand and Morse (2016) show that the bottom 90% adjust their expenditures to rising top incomes of the top 10%. The bottom 90% adjust their consumption expenditures, raising the expenditure shares especially for housing and other visible “status” goods.

Bellet (2018) shows that when a big house is added to a neighbourhood, locals lose satisfaction with their house and increase spending on their houses.

In the following Section 3 we will summarize the classic and recent works on this issue.

**Alternative explanations** Can these facts be explained by rising house prices and falling interest rates? As for house prices, existing homeowners should have substituted away from houses (if anything). If they stayed in their old house, mortgages levels would not have been affected. (Home equity lines of credit are a negligible part of overall debt). For new homeowners to increase housing expenditures by a lot, there need to be some form of status concerns or habits in the preferences.

We can rule out that interest rates have played the most important role. Interest rates increased until 1980 and decreased since then, while mortgage-debt-to-GDP ratio has risen steadily all the way to 2007 (see Figure 3).

### 3 Social Comparisons

> A house may be large or small; as long as the neighboring houses are likewise small, it satisfies all social requirement for a residence. But let there arise next to the little house a palace, and the little house shrinks to a hut. [...] and however high it may shoot up in the course of civilization, if the neighboring palace rises in equal or even in greater measure, the occupant of the relatively little house will always find himself more uncomfortable, more dissatisfied, more cramped within his four walls.

— Karl Marx, 1849

The idea that people care about how our belongings compare to those of our neighbors is certainly not new. Veblen (1899) noticed that the purchase and consumption of conspicuous goods contributes to our social status. Duesenberry (1949) argued that the consumption-savings decision is mainly driven by habits and social comparisons. Following the seminal findings of Easterlin (1974), who first documented that the link between happiness and own income is rather weak, social scientists have amassed extensive evidence on

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3We borrowed this quote from Bellet (2018).
the importance of social comparisons for people’s well-being (e.g. Fehr and Schmidt, 1999; Ferrer-i-Carbonell, 2005; Luttmer, 2005; Clark and Senik, 2010; Card et al., 2012).\footnote{See Clark et al. (2008) for a review and discussion of this literature.}

Besides their importance for individual well-being, there is also correlational and sound (quasi-)experimental evidence that social comparisons matter for economic behavior and that conspicuous consumption is used to enhance one’s social status. Angelucci and De Giorgi (2009) find that cash transfers to eligible households indirectly increase the consumption of ineligible households living in the same villages via increased debt loans and reduced savings. Kuhn et al. (2011) show that the neighbors of lottery winners spend more on cars. In a field experiment with Brazilian bankers, Bursztyn et al. (2014) find substantial peer effects in investment decisions due to keeping-up-motives. In another field experiment, Bursztyn et al. (2017) show that platinum cards are more likely to be used in social contexts, implying social image motivations. In addition, they provide evidence of positional externalities from the consumption of status goods.

On a broader level, De Giorgi et al. (2016) combine matched employer-employee data and Danish wealth data to estimate a sizable elasticity of own with respect to peers’ consumption by exploiting partially overlapping networks.\footnote{While they do not find differential effects between goods of different levels of visibility (Heffetz, 2011), they do not distinguish between durable and non-durable goods. In addition, housing expenditures (rents) are treated as non-visible.} Bertrand and Morse (2016) use state-year variation and detailed expenditure data to document that nonrich households consume a larger share of their current income when exposed to higher top income and consumption levels.\footnote{Frank et al. (2014) find similar evidence.} Competing explanations such as income expectations, wealth effects or upward price pressure cannot account for this pattern. The authors further provide correlational evidence suggesting that households used credit in order to keep up with rising expenditures at the top. This is in line with the findings of Georgarakos et al. (2014) who show that higher incomes of a person’s (richer) peers increases borrowing and the likelihood of personal bankruptcy.

In the context of housing, Bellet (2018) analyzes the effect of a relative downscaling of a person’s house using data from the American Housing Survey in addition to Zillow-data on three million suburban houses built between 1920 and 2009. He finds that relative downscaling due to the construction of bigger houses in the same area leads to lower satisfaction levels with one’s own house. Affected homeowners are also more likely to upgrade their home and take on more debt. Importantly, the effects are highly asymmetric in the sense that only houses at the top of the distribution matter for social comparisons. These findings are consistent with upward-looking comparison behavior.

Interestingly, social comparisons seem to be upward-looking. Households care about what happens above them while paying little attention to relative gains of those below one’s own position. Besides the just-mentioned evidence by Bellet (2018), this asymmetry is also present in the context of self-reported well-being (Ferrer-i-Carbonell, 2005; Card et al., 2012) and born out by direct survey evidence on the strength and direction of comparisons (Clark and Senik, 2010). Bertrand and Morse (2016) also find an asymmetric relationship consistent with upward-looking comparisons: non-rich consumption reacts to top income levels but not to median or low income levels in a state.\footnote{We are not aware of a study where asymmetry was tested for but not discovered.}

The fact that comparisons are upward-looking is highly relevant for the interplay of income inequality and aggregate variables as it may give rise to expenditure and debt cascades. Despite the conclusive evidence on the role of social comparisons in shaping consumption-savings decisions, they have yet to find their way into macroeconomic models leaving us with little knowledge on the aggregate effects of (asymmetric) social comparisons.
4 Relation to the literature

Our paper combines ideas from network economics with methods from continuous time macroeconomics. We build a tractable model that is a two-good, infinite-horizon, general equilibrium version of the static network game by Ballester et al. (2006). While they analyze the Nash equilibrium of a static game with strategic complementarities, we solve for general equilibrium—adding houses as a second good, budget constraints and market clearing conditions.

We formulate our quantitative model in continuous time building on Achdou et al. (2015), who provide a framework to efficiently solve heterogeneous agents models numerically. Our model adds type-dependent income processes (as in Gabaix et al., 2016; Guvenen et al., 2016) and other-regarding preferences. To the best of our knowledge, this is the first incomplete markets model with interdependent preferences such that agents influence each other not only via prices but also directly via their choices.\footnote{Despite the added complexity we can still solve for general equilibrium in a few seconds on a standard laptop computer.}

We provide a specific case where inequality matters for macroeconomic aggregates (see Ahn et al., 2017).

America has seen a boom in household debt since 1980 (see figure 1b). There appear to be three explanations for this debt boom: (i) financial liberalization, (ii) rising demand for safe assets and (iii) rising demand for credit.

Financial innovation might have allowed banks to lower the lending standards for their credit products and offer these products to less credit-worthy households. Livshits et al. (2016) investigate the boom in credit card debt in the US since the 1980s. They argue that financial innovation (that is, better monitoring technologies) allowed banks to issue credit cards to ever lower income households. Indeed, Livshits et al. attribute 20\% of the increase in credit card debt to new, less credit-worthy credit card holders. As we show in figure ??, credit card debt accounts only for a very small fraction of household debt (it is shown as part of “other debt” in the figure). Our focus is on mortgages, by far the most important debt category.

Favilukis et al. (2017) show how hard it is to generate a mortgage debt boom from laxer borrowing constraints in a macroeconomic model. They need the majority of the population to sit at or near the borrowing constraint initially, in order to get a sizable effect on house prices (which are their main focus) and mortgages. Their model has two types, one of which has a very strong bequest motive. Agents of this type are born rich with a bequest and pass it on to their children. The majority of the population is of the other, poor, type who never get far away from the borrowing constraint. Kiyotaki et al. (2011) and Sommer et al. (2013) have documented the same difficulties of generating a debt boom from loser borrowing constraints in standard heterogeneous agent macroeconomic framework.

What is more, the financial innovation doesn’t seem to be a consequence of greater productivity. Philippon (2015) finds constant unit costs of financial intermediation over time—despite large historical variations in the ratio of intermediated assets and GDP.

The debt boom might also be the financial sector’s reaction to an ever bigger demand for safe assets. Gorton (2016) views the production of safe assets as one of the major roles of the financial sector in an economy. Financial innovation allowed intermediaries to create safe assets from bundling and tranching mortgages. To satisfy the high demand for their product, they had to accept ever lower quality inputs—that is, mortgages with lower lending standards. So, where has the demand for safe assets come from? Gorton (2016) argues that the demand was created in emerging markets like China and India. As these
economies have grown richer, they wanted to invest their wealth in safe assets. Since their
immature domestic financial markets could not provide them, investors resorted to the
US.

Not only emerging markets have grown richer, but so have the top income groups in
the US (we show that in figure 2a). Kumhof et al. (2015) build a macroeconomic model
around this fact. In their model the rich have preferences for financial wealth. As they
grow richer, they demand ever more financial assets. This demand needs to be matched
by credit taken out by the rest of the population. So, the surge in inequality causes an
increase debt through higher credit supply.

We add to the literature on household debt by formalizing and testing the demand-
centered theory that inequality has led to rising debt because non-rich households want
to keep up with the living standard set by the rich (Rajan, 2010; Frank, 2013). We also
do not limit our analysis to the period from 2001 to 2007 but recognize that debt started
to grow much earlier.

Alvarez-Cuadrado and Japaridze (2017) analyze this mechanism in a very stylized
setting with one good (no houses!), three income types without idiosyncratic risk and
three periods. Absent durable goods, their model is unable to capture what we believe is
the essential mechanism: substitution from non-durable consumption to durable housing
automatically increases debt.

5 A Model with Mortgage Debt and Social Comparisons

We build our model with two aims in mind. First, we want to illustrate how rising top-
incomes and social comparisons can lead to rising debt levels across the whole income
distribution. And second, we want to quantify the effect of this channel on the increase
in aggregate mortgage debt from 1980 to 2007.

We will use two versions of the same model to achieve each of these tasks. In this
section we describe the basic framework of our model. It is a continuous time version of
the canonical macroeconomic model with housing (Piazzesi and Schneider, 2016), with the
addition of social preferences. With very few changes our model could nest the household
side of the model in Kaplan et al. (2016). A similar model is also briefly described in
Achdou et al. (2015), from which we borrow the solution methods.

In section 6, we present a version of the model that can be solved in closed form. In
order to get closed forms we need to make simplifying assumption on incomes and the
functional form of the social comparison motive.

In section 7, we present a version of the model where the choices of the income process
as well as the nature and magnitude of the social comparison motive is data driven.

5.1 Basic framework

Time is continuous and runs forever. We analyze an endowment economy with a continuum
of households. Households live forever and die at a rate $m$. Households derive utility from
a non-durable consumption good $c$ and the status of houses $s(h, \bar{h})$ which is a function
of the household’s durable house $h$ and the reference measure $\bar{h}$ (a weighted average of
other’s houses). Consumption and status form a composite good, using CES aggregation.
The households’ expected discounted lifetime utility is given by

$$E_0 \int_0^\infty e^{-(\rho+m)t} \left( (1-\xi)c_t^\epsilon + \xi s(h_t, \bar{h}_t)^\epsilon \right)^{1-\gamma} \frac{1-\gamma}{1-\epsilon} dt.$$
Houses depreciate at rate $\delta$ and can be adjusted through (dis-)investment $x$,

$$\dot{h}_t = -\delta h_t + p_t x_t.$$ 

In our baseline specifications, adjustment is frictionless, in some extensions their are adjustment costs.

There is an asset $a$ that serves as mortgage ($a < 0$) and as a savings device ($a > 0$). Households can borrow up to an exogenous collateral constraint $-a_t \geq \omega p h_t$. That is, $\omega \in [0, 1]$ is the maximum admissible loan-to-value (LTV) ratio.\footnote{This specification follows Favilukis et al. (2017) and Kaplan et al. (2017).}

Households earn an exogenous income $y_t$ (to be discussed in the subsequent sections). They earn interest $r^+_{\cdot}$ on their savings, and pay $r^-_{\cdot}$ on their mortgage. This is summarized in the return function $r^a_{\cdot}$. The law of motion for assets $a$ is given by

$$\dot{a}_t = y_t + r^a_t(a_t) - c_t - p_t x_t.$$ 

In the basic version of our model (without adjustment costs), the two states $a$ and $h$ can be collapsed into one state variable $w_t = a_t + p_t h_t$.

Households choose consumption, housing and assets to maximize their expected discounted lifetime utility, subject to the laws of motion and the collateral constraint—taking prices and the reference measure as given.

6 A Tractable Version of the Model

In this section we specialize the framework from section 5 to illustrate how rising top incomes can lead to rising mortgage levels across the whole income distribution. The results depend on the findings that (i) housing expenditures are increasing in incomes, that (ii) the expenditure share of housing is increasing in the reference measure $\bar{h}$ and that (iii) the debt-to-income ratio is increasing in the expenditure share of housing. Prices and interest rate are assumed to be constant.

The crucial assumption to obtain tractability are constant, but type-dependent incomes; that the comparison motive has a simple functional form; and that the interest rate equals the discount rate.

Note: We are in the process of switching the tractable model to continuous time to match with the quantitative model. The intuition and proofs for the results are exactly the same as in the discrete time version.

6.1 Setup

There is a finite number of types of households $i \in \{1, \ldots, N\}$.\footnote{When turning to the general equilibrium analysis, we will assume a continuum of agents for each type, so that no agent’s choices affect prices.} Agents vary by their initial endowments $a_0$ and constant income streams $y$, which are deterministic and constant over time. We assume that agents are ordered by their lifetime income $\gamma^i = ra_0^i + (1+r)y^i$,

$$\gamma^1 \leq \gamma^2 \leq \ldots \leq \gamma^I.$$ 

Agents’ utility depends on reference housing, $\bar{h}$, which is a function of the housing distribution. They maximize life-time utility

$$\sum_{t=0}^{\infty} \beta^t u(c_t, h_t; \bar{h})$$
subject to the period budget constraints
\[ c_0 + p h_0 + a_1 = y + a_0, \]
\[ c_t + p(h_t - (1 - \delta)h_{t-1} + a_{t+1} = y + (1 + r)a_t \text{ for all } t \geq 1, \]
and a life-time budget constraint
\[ c_0 + ph_0 - y + \sum_{t=1}^{\infty} \left( \frac{1}{1+r} \right)^t (c_t + p(h_t - (1 - \delta)h_{t-1} - y) = a_0. \]

We assume that preferences are twice continuously differentiable and strictly concave.

**Assumption 1.** \( \partial_x u > 0, \partial_{xx} u < 0 \) for \( x \in \{c, h\} \)

In addition, we assume that the economy is at its steady state, where all prices are constant over time and the interest rate satisfies \((1 + r)\beta = 1\).

**Assumption 2 (Steady state).** (i) \( p_t = p \) and \( r_t = r \) for all \( t \). (ii) \( (1 + r)\beta = 1 \).

**Simple social preferences** Agents have social status concerns. They value their houses relative to the houses of their reference group. For tractability, we choose the following utility function which results in the above affine relationship between optimal consumption and housing.

**Assumption 3 (Simple social comparisons).** \( s(h, \bar{h}) = h - \phi \bar{h} \) and \( \bar{h}_i = \sum_{j \neq i} \sigma_{ij} h_j \) is a weighted sum of other agent’s consumption. We can write the vector of housing references as
\[ \bar{h} = (\bar{h}^1, \ldots, \bar{h}^N)^T = G \cdot h := (\sigma_{ij}) (h^i). \]

where the matrix \( G \) can be interpreted as the adjacency matrix of the network of types capturing the comparison-links between agents of each type.

We further require the comparisons to satisfy the following condition.

**Assumption 4.** The Leontief inverse \((I - \phi G)^{-1}\) exists and is equal to \( \sum_{i=0}^{\infty} \phi^i G^i. \)

We show that under assumption 3 an agent \( A \)'s debt increases if another agent \( B \)'s lifetime income increases—as long as there is a direct or indirect link from \( A \) to \( B \) (proposition 2 below). That link exists, if agent \( A \) cares about agent \( B \), or if agent \( A \) cares about some agent \( C \) who cares about agent \( B \).

Under the stronger assumption of upward social comparisons, we can link aggregate debt to rising inequality.

**Assumption 3’ (Upward social comparisons).** We assume further that agents are ordered by their lifetime income \( Y^i = ra_0^i + (1 + r)y^i \). Agents compare themselves only with those with a higher income.
\[ \sigma_{ij} = 0 \text{ if } i \text{ richer than } j \text{ (i.e. } i \geq j), \quad \sigma_{ij} \geq 0 \text{ if } i \text{ poorer than } j \text{ (i.e. } i > j) \]

That is, \( G \) is upper triangular with zeros on the diagonal. This is illustrated in figure 5. Note that \( G^N = 0 \in \mathbb{R}^{N \times N} \), so assumption 4 is always satisfied.

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11This assumption is satisfied whenever the sequence \( G^i \to G^\infty \). For example, if \( G \) is a stochastic matrix with a stationary distribution.
6.2 Main results: Debt is increasing in others’ incomes

We first state the results, and explain them below. We begin with providing closed forms analytical solutions for agents’ equilibrium optimal choices for a given house price $p$ and interest rate $r^a = \rho$.

**Proposition 1.** Under assumptions 2 (steady state), 3 (social comparisons) and 4 (existence of Leontief inverse; assumptions 1 and 5 are implied)

$$h = C_2 \sum_{i=0}^{\infty} (C_1 \phi G)^i Y$$

$$a = C_2 ((\alpha_1 + p\delta)a_0 - p(1 - \delta)y - \frac{p(1 - \delta)}{1 + r} \left( \sum_{i=1}^{\infty} (C_1 \phi G)^i \right) Y)$$

where $C_1 = (1 + r)\alpha_1$ and $C_2 = \frac{1}{p(r+\delta)+\alpha_1(1+r)}$.

**Proof.** We have to solve the optimization problems of all agent jointly. For each agent we get an equation of the form

$$h_i = C_2 Y_i + C_1 \phi h_i.$$

These can be stacked into a system of linear equations and solved jointly.

See appendix A.4.

Agent’s choices depend on a weighted average of the lifetime incomes their (direct and indirect) reference groups. The weights are positive, whenever there is a direct or indirect social link between those agents. This is captured by the income-weighted Bonacich centrality, $B = \sum_{i=0}^{\infty} (C_1 \phi G)^i Y$. If the weight $B_{ij}$ is positive, household $j$’s lifetime income affects household $i$’s choices. This is the case whenever $j$ is in $i$’s reference group (there is a direct link $\sigma_{ij} > 0$), or if $j$ is in the reference group of some agent $k$ who is in the reference group of agent $i$ (there is an indirect link of length two, $\sigma_{ik}\sigma_{kj} > 0$) or if there is any other indirect link ($\prod_{n=1}^{N-1} \sigma_{\ell_n},\ell_{n+1} > 0$ where $\ell_1 = i$ and $\ell_{N-1} = j$).

These results are reminiscent of those in Ballester et al. (2006). They showed that the unique Nash equilibrium in a large class of network games is proportional to the (standard) Bonacich centrality.

Importantly, we have just shown that debt is increasing in the incomes of the reference group. In order to get intuition for this result, we rephrase this in a provide an alternative, intuitive proof.

**Proposition 2.** Under assumptions 2 (steady state) and 3 (social comparisons; assumptions 1 and 5 are implied) debt is weakly increasing in the incomes of the reference group.

**Proof.** We show this in a sequence of lemmas.

(i) If the incomes of the reference group rise, the reference measure $\bar{h}_i$ increases. This is because optimal houses are increasing in own lifetime income. See the formula for $h$ in Proposition 1.

(ii) If the reference measure $\bar{h}_i$ increases, household $i$’s optimal house increases. See Lemma 2.

(iii) If initial endowments are sufficiently low (for example $a_0 = 0$) then households will optimally use a mortgage to finance their house. See Lemma 3.

(iv) If agents substitute houses for consumption, debt increases. See Lemma 4.
**Upward social comparisons**  Now, we consider the case where agents only compare themselves with richer peers. We order agents by their lifetime income $y_i + rd_i$. Type 1 is the poorest and type $N$ is the richest. The corresponding graph and adjacency matrix are shown in figure 5.

![Graph](image)

**Figure 5:** The network under upward looking comparisons. Types are ordered by their permanent income. For poorest (1) to richest (N). Each type $i$ has edges only to richer types $j > i$.

**Proposition 3.** With upward social comparisons (assumption $\mathcal{J}$),

(i) rising top incomes lead to rising debt for everybody, and thus to rising aggregate debt.

(ii) debt-service-to-income ratios rise. (because debt rises and income is constant)

**Conjecture 1.** (i) A mean preserving spread in the income distribution leads to rising aggregate debt.

The question will be if the response to the poor is weaker than the response to the rich. And the response is weaker! The “direct effect” will exactly be offset between the poor and the rich. The “indirect effect” will only affect the poor. So, it should be easy to show that total debt increases after redistribution from rich to poor.

6.3 Example: Upward comparisons with three agents

We now illustrate the results for the simple case of three types of agents, poor $P$, middle class $M$, and rich $R$. The poor type compares himself with both other types, the middle type compares himself only with the rich type, and the rich type not at all. Figure 6 shows the corresponding graph and its adjacency matrix.

![Diagram](image)

**Figure 6:** The social network structure with three types, assuming upward comparisons. The network can be represented as a graph and as its adjacency matrix.
As always under upward comparisons, the adjacency matrix is convergent with $G^3 = 0$.

$$
G^2 = M \begin{pmatrix}
0 & 0 & \sigma_{PM} \sigma_{MR} \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix},
$$

The matrix $G^2$ counts the paths of length 2. In our example there is only one such path—from type $P$ to type $R$. Defining $\tilde{\phi} = C_1\phi$, the vector of Bonacich centralities is given by

$$
\sum_{i=0}^{\infty} \alpha^i G^i = I + 2 \sum_{i=1}^{2} \alpha^i G^i = I + \begin{pmatrix}
0 & \alpha \cdot \sigma_{PM} & \alpha \cdot \sigma_{PR} + \alpha^2 \cdot \sigma_{PM} \cdot \sigma_{MR} \\
0 & 0 & \alpha \cdot \sigma_{MR}
\end{pmatrix}
$$

The partial equilibrium choices for housing and assets are now given by

$$
\begin{pmatrix}
h_P \\
h_M \\
h_R
\end{pmatrix} = C_2 \begin{pmatrix}
1 & \tilde{\phi} \cdot \sigma_{PM} & \tilde{\phi} \cdot \sigma_{PR} + \phi^2 \cdot \sigma_{PM} \cdot \sigma_{MR} \\
0 & 1 & \tilde{\phi} \cdot \sigma_{MR} \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
\bar{y}_P \\
\bar{y}_M \\
\bar{y}_R
\end{pmatrix}
$$

An agent’s housing choice increases linearly in own permanent income, $\bar{y} = (1 + r)y + ra_0$, and on the permanent income of agents in the reference group. The poor agent’s consumption increases through the direct links, but also indirect links. These are discounted stronger ($\phi^2$ instead of $\phi$). Agents’ decisions to save or borrow depend on the ratio of initial wealth $a_0$ and income $y$. The higher the income relative to initial wealth, the greater the need to borrow.

### 6.4 Additional results

We will state and prove additional results needed in the proof of Proposition 2. All these results are interesting even in the case without social comparisons.

That is why from now on we work with the following assumption.

**Assumption 5.** $c = \alpha_1 h - \alpha_0$, where $\alpha_1, \alpha_0 \geq 0$.

For example, assumption 5 is satisfied for any utility function $u$ that satisfies $u(c, h, \bar{h}) = f(c^{-\xi}(\phi_1 h - \phi_2 \bar{h})^\xi)$.

(In this case ...)

With this assumption we can explain which households hold debt, and that debt is increasing in the personal preference for housing.

**Lemma 1 (Constant choices).** Under assumptions 1 and 2 the agents’ choices $c, h$ and $a$ are constant over time.

**Proof.** See appendix A.1.
Under these assumptions we can solve the household’s problem for given prices. The solution is summarized in the lemma below. Lemma 2 gives explicit expressions for optimal housing and loans (or savings) which we can use to analyze optimal debt holdings of households.

**Lemma 2.** Under assumptions 1, 2 and 5

\[
h = \frac{Y + (1 + r)\alpha_0}{p(r + \delta) + \alpha_1(1 + r)}, \quad a = \frac{a_0(\alpha_1 + p\delta) - (y + \alpha_0)p(1 - \delta)}{p(r + \delta) + \alpha_1(1 + r)}
\]

where lifetime income \( Y = (1 + r)y + ra_0 \).

*Proof.* See appendix A.2.

**Absent initial wealth, households hold debt** Lifetime wealth consists of two components: initial wealth and income flows. Lemma 3 shows that agents are indebted if their initial wealth is sufficiently low relative to life-time earnings.

**Lemma 3.** Under assumptions 1, 2 and 5 an agent borrows \((a < 0)\) iff

\[
a_0 < (y + \alpha_0)\frac{p(1 - \delta)}{\alpha_1 + p\delta}
\]

*Proof.* This follows directly from the explicit expression for \( a \) in lemma 2.

Since housing is a durable good, it may make sense to use credit in order to buy a big house right away and use future incomes to repay the debt while still enjoying the house. Hence, if agents are poor initially but expect to earn a lot over their lifetime, they will borrow in order to buy an appropriately sized house. This is because the expenditures for the house are relatively high \((ph)\) in the first period and relatively low \((p\delta h)\) in all subsequent periods.

In particular, all agents with zero initial wealth will hold debt. This is illustrated in figure 7.

![Figure 7: Agents without initial wealth hold debt.](image)

Cash flows is not constant over time. In the first period, the household pays

\[
payment_0 = c + ph, \quad payment_t = c + p\delta h \text{ for } t \geq 1.
\]
The full house price is due in the initial period, and only “maintenance costs” to cover depreciation thereafter. In each period $t > 0$, income $y$ will exceed expenditures. There are excess funds

\[ \text{monthly rate} = y - c - \delta ph > 0. \]

(This follows directly from the life-time budget constraint.) The household takes out a loan worth the present discounted value of all these funds

\[ a = \sum_{t=1}^{\infty} \left( \frac{1}{1+r} \right)^t (c + \delta ph - y) = \frac{1}{r} (c + \delta ph - y) < 0, \]

in the initial period. This loan is just so high that it can be repaid until the infinite future. The household will spend the loan and his remaining funds on the house,

\[ ph = a + y - c = \frac{1}{r} (c + \delta ph - y) + y - c. \]

**Debt rises if agents substitute houses for consumption** Assume now two different agents, who take prices as given. They have the same exogenous income stream and initial wealth. One agent values houses more than the other. Given assumption 5 this means that

\[ \alpha_H^1 < \alpha_C^1, \quad \text{or} \quad \alpha_H^0 > \alpha_C^0. \]

Then agent $H$, who values houses more, will be more indebted.

**Lemma 4.** Let assumptions 1, 2 and 5 be given. If agents value houses relatively more ($\alpha_1$ goes up or $\alpha_0$ goes down), borrowing increases (or savings decrease),

\[ \frac{\partial a}{\partial \alpha_1} > 0 \quad \text{and} \quad \frac{\partial a}{\partial \alpha_0} < 0. \]

**Proof.** See appendix A.3.

If for some reason (and given prices) agents substitute consumption for houses their debt will increase. They lower their consumption in every period until the infinite future and need to reshuffle their funds to pay for housing. The biggest chunk has to be paid upfront (downpayment), and only maintenance work has to be paid later to make up for the depreciation. To balance these unequal payment flows they increase their credit.

7 A Quantitative Model

In this section we quantify the effects of rising inequality and social comparisons on aggregate debt in a quantitative incomplete markets model with heterogeneous households and social comparisons. For credibility of our results it is of paramount importance to model the rise of income inequality and nature of social comparisons correctly.

We will discuss the each specification in order. Other than the income processes and the social comparisons, the model is unchanged relative to section 5.

7.1 Modelling Social Comparisons

Agents derive their utility from housing status $s(h, \bar{h})$, their flow utility can be written as $u(c, s(h, \bar{h}))$. In section 6 we assumed the a simple status function $s(h, \bar{h}) = h - \phi \bar{h}$ and a general reference measure $\bar{h} = G \cdot h$, a weighted sum of others’ houses. The choices there were restricted by our desire for analytical tractability. In this section we discuss how can improve on that choice.
7.1.1 The Reference Measure \( \bar{h} \)

The reference measure of housing, \( \bar{h} \), is the social benchmark to which households compare their own house. Moving away from the setting with discrete types we now write the weighted sum as a weighted integral over the housing distribution

\[
\bar{h} = \int x(h(w,y))h(w,y) \, dG(w,y)
\]

(1)

**Candidate 1**  
Evidence from economics and psychology, suggests that comparisons are upward-looking. Households only care about how their house stacks up against bigger/better houses. The asymmetric nature of interpersonal comparisons makes reference housing also a function of own housing, \( \bar{h} = \bar{h}(h,G) \).

\[
\bar{h}(h) = \int x(h, h(w,y))h(w,y) \, dG(w,y)
\]

(2)

A natural assumption is be to define reference housing as the average housing quality of all bigger houses:

\[
\bar{h}(h) = E[h'|h' > h] = \frac{1}{1 - F(h)} \int_{h}^{\infty} h' \, dF(h')
\]

(3)

where \( F_h \) is the marginal distribution of houses in the economy.

**Candidate 2**  
The evidence that is most closely related to our setting comes from Bellet (2018). He uses data on housing satisfaction and house construction to show that the comparison motive is strongest with respect to the 90th percentile of the housing distribution.

Following this evidence, we set the reference measure \( \bar{h} = h_{P90} \) to the 90th percentile of the housing distribution.

7.1.2 The Status Function \( s \)

The status function is supposed to capture how important status concerns are for an agent. Following Bellet (2018) we define the sensitivity to the reference house \( \sigma \) as the ratio of two utility elasticities elasticities.

**Definition 1.** An agent’s sensitivity to the reference house \( \sigma \) is given by

\[
\sigma = -\frac{\partial h_u \cdot \bar{h}}{\partial h u \cdot \bar{h}}
\]

the ratio of utility elasticities with respect to the agent’s house and reference house.

This sensitivity can be interpreted as follows: If the reference house increases by one percent, your own house has to grow by \( \sigma \)% to keep your utility level. We can rewrite the sensitivity in terms of the status function \( s \).

**Lemma 5.** Under the functional form assumption for \( u \), \( \sigma \) is independent of \( u \),

\[
\sigma = -\frac{\partial h_s \bar{h}}{\partial h s \bar{h}}
\]

\[
\sigma = -\frac{\partial h u \cdot \bar{h}}{\partial h u \cdot \bar{h}} = -\frac{\partial u \, \partial s \bar{h}}{\partial s \partial h u} / \frac{\partial u \, \partial s \bar{h}}{\partial s \partial h u}
\]

\[\square\]
Candidate 1  A simple status function that captures this sensitivity is suggested by Bellet (2018). He uses \( s(h, \bar{h}) = \frac{h}{\bar{h}^\phi} \), where the sensitivity is given by the only parameter \( \sigma = \phi \).

This specification, though, does not work well with Cobb-Douglas aggregation. The references measure is a multiplicative constant that factors out of the aggregator.

Candidate 2  For the tractable model in the previous section we used \( s(h, \bar{h}) = h - \phi \bar{h} \). There sensitivity is given by \( \phi \bar{h}/h \). This specification can lead to numerical issues since the the status can become negative, and utility becomes undefined.

The following easy fix is also not suitable. One could use

\[
 s(h, \bar{h}) = (1 - \alpha)h + \alpha \left( h - \frac{\phi}{\alpha} \bar{h} \right) + \begin{cases} 
 (1 - \alpha)h & \text{for } h < \phi \bar{h}, \\
 h - \phi \bar{h} & \text{for } h > \phi \bar{h}.
\end{cases}
\]

This specification reflects the idea that poor households with very small houses do care more about having a shelter, than how their shelter compares to others’ homes. Moving up the housing distribution, the second term at some point becomes positive, so status plays a role.

The sensitivity is \( \sigma = \phi \bar{h}/h \) for \( h > \phi \bar{h} \) and 0 otherwise. Unfortunately this implies that the individual sensitivities are smaller than 1.

Now there is a trade-off. The larger \( \phi \), the more agents will be unable to afford a house bigger than \( \phi \bar{h} \). These agents will have a sensitivity of zero. With a small \( \phi \), more households are in the region with positive sensitivity. But the sensitivity is small, because \( \phi \) is small.

(to be expanded)

7.1.3 Summary

For the main specification we follow evidence by Bellet (2018) and use \( s(h, \bar{h}) = h/\bar{h}^\phi \) and \( \bar{h} = h_{90} \). We set \( \phi = 0.7 \) which is smaller than Bellet’s estimate (close to 1).

We set \( \phi \) smaller because Bellet (2018) estimates a local effect, while want to capture a global effect. These two effects would be identical if the joint distribution of houses and incomes was identical across space in the US—which is not the case.

7.2 Income process

Inequality vs risk  The choice of the income process is central to our model. In particular we need to be careful to distinguish between lifetime inequality and idiosyncratic risk. The total cross-sectional variation is the sum of life-time inequality and idiosyncratic risk. This can be seen from a basic variance decomposition.

\[
 \text{cross-sectional variance} = \text{E}(\text{Var}(y_{it})) + \text{Var}(\text{E}(y_{it}))
\]

Figure 8 illustrates this distinction, showing sample paths of three individuals. The (pooled) cross-sectional variance is very similar, but the contributions from risk and lifet ime inequality are very different. The left panel shows the situation where all individuals incomes follow the same AR(1) process. In this case there is virtually no life-time inequality. The average lifetime incomes are (almost) identical for all agents. The cross-sectional dispersion comes from the fact that all households move from the lowest to the highest region of the stationary distribution over their lifetime.
Our tractable model in section 6 is the complete opposite. Incomes are deterministic and constant for the entire lifetime. There is no risk, all income dispersion is income inequality.

The tradeoff between inequality and risk is important for the optimal choice of debt. Optimal debt levels are increasing in incomes and decreasing in income risk (we show that in a companion project, Drechsel-Grau & Greimel, 20XX). Thus, if we tried to match the increase in the cross-sectional variation from 1980 to 2010 with a higher risk, we would get a decrease in aggregate debt.

We borrow the income process from Kaplan et al. (2016), because it turns out to capture both the risk and the inequality quite well (this is despite the fact that it was calibrated to match risk moments only).

The income process is a mixture of two jump-drift processes

\[
\log y_t = z^1_t + z^2_t \\
\text{d}z^j_t = -\theta_j z^j_t + \text{d}J^j_t(\lambda_j, \sigma_j) \quad \text{for } j = 1, 2
\]

where \(z^1\) jumps frequently (approximately every 3 years) and reverts back quickly. This process broadly captures the idiosyncratic risk. \(z^2\) jumps about once in a lifetime (approximately every 35 years) and is very persistent. This process broadly captures lifetime inequality.

We follow Kaplan et al. (2016) in estimating this process from high-micro data provided by Guvenen et al. (2016) to match earnings risk and cross-sectional variation. As we would expect from our variance decomposition above, lifetime inequality moments are not targeted, but matched quite well.

**Modelling the change in inequality over time** Since we would like to compare steady states in 1980 and 2007 we also need to model the change in the income process. Without doubt, the cross-sectional variation has dramatically increased over that time period. Guvenen et al. (2014) find that the variance of earnings growth shocks has not changed much (that is, idiosyncratic risk has not risen), while Kopczuk et al. (2010) considerly find that the increase in cross-sectional variance since 1970 is due to increase in lifetime inequality. These facts help us generate our results.

**Figure 8:** Distinguishing idiosyncratic risk from lifetime inequality. Simulating two processes for three households.
The Stationary Distribution of log-incomes

![The Stationary Distribution of log-incomes](image)

**Figure 9:** Comparison of the stationary distributions of log incomes in 1980 and 2007

To keep things simple, we keep the income risk process constant over time and adjust the “inequality process” to match the change in the cross P90/P50 ratio, which was roughly 35% higher in 2007.

### 7.3 Choosing the remaining parameters

For now the other parameters are chosen to be sensible, mostly taken from previous literature. The preference parameters $\epsilon$ (intratemporal elasticity of substitution) and $\xi$ (housing share) are chosen to roughly match the aggregate housing-to-income ratio in 1980 (see table 2).

**Table 1:** Parameters (preliminary)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>with</th>
<th>without</th>
<th>internal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>risk aversion</td>
<td>2.0</td>
<td>2.0</td>
<td>no</td>
</tr>
<tr>
<td>$\frac{1}{1-\epsilon}$</td>
<td>elasticity of substitution</td>
<td>0.09</td>
<td>0.25</td>
<td>yes</td>
</tr>
<tr>
<td>$\xi$</td>
<td>(rel.) housing share</td>
<td>0.7159</td>
<td>0.9159</td>
<td>yes</td>
</tr>
<tr>
<td>$\phi$</td>
<td>sensitivity to $h$</td>
<td>0.7</td>
<td>–</td>
<td>no</td>
</tr>
<tr>
<td>$\delta$</td>
<td>depreciation rate</td>
<td>0.02</td>
<td>0.02</td>
<td>no</td>
</tr>
<tr>
<td>$\omega$</td>
<td>maximum debt-to-asset</td>
<td>0.7</td>
<td>0.7</td>
<td>no</td>
</tr>
<tr>
<td>$\rho$</td>
<td>discount rate</td>
<td>0.05</td>
<td>0.05</td>
<td>no</td>
</tr>
<tr>
<td>$\eta$</td>
<td>mortality rate</td>
<td>1/45</td>
<td>1/45</td>
<td>no</td>
</tr>
<tr>
<td>$r$</td>
<td>interest rate</td>
<td>0.05</td>
<td>0.05</td>
<td>no</td>
</tr>
</tbody>
</table>

Note: All rates are annualized.

### 7.4 Solution Method

We solve the HJB- and KF-equations using the finite difference methods proposed in Achdou et al. (2015). Assuming exogenous reference groups enables us to solve the partial equilibrium recursively starting with the highest income type (instead of iteratively finding a reference housing schedule that is consistent with the HJB- and KF-equations).

One such equilibrium—the stationary distribution and policies—is shown in figure 10.
Table 2: Model fit (work in progress)

<table>
<thead>
<tr>
<th></th>
<th>debt-to-asset</th>
<th>debt-to-income</th>
<th>house-to-income</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCF (1983)</td>
<td>0.241</td>
<td>0.387</td>
<td>1.606</td>
</tr>
<tr>
<td>with (1980)</td>
<td>0.12</td>
<td>0.17</td>
<td>1.761</td>
</tr>
<tr>
<td>without (1980)</td>
<td>0.124</td>
<td>0.175</td>
<td>1.758</td>
</tr>
</tbody>
</table>

Figure 10: Policy Functions and Stationary Distribution for Particular Income Type
8 Calibration

8.1 Calibration of preference parameters

\[
\int_0^T \exp(-\rho t) \left( \left( 1 - \xi \right) c_t^\epsilon + \xi s(h_t, \bar{h}_t)^s \right)^{1-\gamma} \left( \frac{1}{1-\gamma} \right) \frac{dt}{\psi (h_T + a_T + b)^{1-\gamma}}
\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>discount factor</td>
<td>debt/income</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>CES elasticity</td>
<td>debt/income</td>
</tr>
<tr>
<td>( \xi )</td>
<td>CES share</td>
<td>house/income</td>
</tr>
<tr>
<td>( \bar{b} )</td>
<td>bequests are luxury good</td>
<td>fraction without bequests</td>
</tr>
<tr>
<td>( \psi )</td>
<td>strength of bequest motive</td>
<td>( % \Delta (60\rightarrow75) ) net worth</td>
</tr>
</tbody>
</table>

Standard value: \( \gamma = 2 \)

- one GE takes 10–30 minutes (without comparison motive) ✔
- after parallelization: < 3 minutes ✔
- first stage: “sophisticated grid search” ✔
- let it run on cluster
- local minimization (BOBYQA and Nelder-Mead)
8.2 Income process

Guvenen et al. (2016)

8.3 Preference parameters

Parameters to estimate:

- $\xi$, “housing share”, CES factor of housing
- $\frac{1}{1-\epsilon}$ elasticity of substitution between houses and consumption
- $\phi$
- $\gamma$ risk aversion / IES

8.4 Technology

- depreciation rate $\delta$
- maximum debt to asset $\omega$

8.5 Global Method

Local-Global optimization from Guvenen. First construct a sequence of starting points using sobol number (pseudo-random numbers that are supposed chosen to yield the best coverage of a high-dimensional statespace).

8.6 Local Method

We use the BOBYQA (Bound Optimization BY Quadratic Approximation) algorithm by Michael Powell. It is a derivative free method, the solve the problem using a trust region method and forms quadratic models by interpolation. It is an the contrained-optimization version of NEWUOA which might stand for NEW Unconstrained Optimization Algorithm, also by Powell.

- April 8
- April 15
- April 22 (Easter Monday)
- April 29
- May 6
- May 13 (Stockholm)

- Make solution of KFE work ✔
- specify initial distribution (using initial $\sigma$)
- discretize over $\alpha$
- Clear asset markets and housing markets
- Try search over two parameters ($\xi$, $\epsilon$) to match housing share in 1980 and what else?

9 Results

- borrowing constraints
- capital inflow
- borrowing spread
- social comparisons and inequality
10 Preliminary Quantitative Results

Below we present some numerical exercises. We use conventional parameter values. In a future version, these will be chosen more carefully. In particular, we want to estimate the parameters of the type-dependent income process.

The results show that in partial equilibrium, agents react to an increase in reference housing through lower consumption, more houses and higher debt.

Further, rising inequality drives up aggregate debt in general equilibrium under social comparisons.

The role of comparisons The intuition from the tractable model carries over to the quantitative model. If reference housing increases, agents substitute housing for consumption. Figure 11 illustrates how agents react (on average) to changes in reference consumption—holding prices constant. When the houses in the reference group increase, agents reduce consumption and increase housing. As a consequence aggregate credit increases.

Note: To produce this figure, we solve the household problem for given prices and varying reference housing (horizontal axis) and the strength of the comparison motive, $\phi_2 \in \{0.1, 0.3, 0.5\}$. 

Figure 11: The Effect of Reference Housing on Household Choices

10.1 Comparison of steady states

We compare two steady states: 1980 and 2007. The only difference between the steady states is the income process. Lifetime inequality is higher in 2007 than in 1980.

Partial equilibrium: Generating 66 percent of the debt boom Figure 12 shows the performance of the model in explaining the debt boom (in partial equilibrium). With the comparison motive, our model can explain much of the increase in the debt-to-income and loan-to-value ratios (counting mortgages only). Switching off comparisons, the debt boom disappears.

This is first evidence that rising top inequality and status concerns are an important channel when it comes to explaining the debt boom. The model without comparison is not able to explain the increase in the housing value or the debt boom.
Note: We are currently finalizing results in GE. In GE, interest rates fall so that even without our mechanism there is a debt boom. With our mechanism we get amplification.

Note: We are currently finalizing results on a horse race between mechanisms: looser borrowing constraints, capital inflow (savings glut) and social comparisons. Social comparisons are an important amplification mechanism of the other two channels.

Outdated (!): Rising top income inequality drives up debt in General Equilibrium

We compare two steady states, that roughly correspond to the years 1980 and 2013 in the US. The models have four types, that loosely correspond to the following income groups: (1) the bottom 20%, (2) P20-P50, (3) P50-P90, and (4) the top 10% of income earners. We solve for a baseline general equilibrium and then double the income of the top-10% (which is close to the observed increase, see figure 2a). We solve these general equilibria with ($\phi = 0.1$) and without ($\phi = 0$) comparisons.

The results are shown in figure 13. The figure shows that changes in aggregate quantities (black boxes) and the contributions of each type (in shaded blue). We see that in the case with comparisons, total debt grows by 15%, around 1/4 more than without comparisons (12%). We can see that the rich buy houses of the poor (the total supply is kept constant). But at the same time all types pay more for their house in the case with comparisons. This is because of the strong increase in house prices.

11 Conclusion

In this paper we showed how inequality can be an important driver of mortgage debt. If households have upward looking social preferences and the status good (here, houses) are durable, then an increase in top incomes will induce the remaining population to shift their expenditures towards the durable status good which increases debt.

In our quantitative macroeconomic model this mechanism can explain (XX%) of the debt boom. Rising inequality and social comparisons where not the only source of the debt boom. But we (will) show that it substantially amplifies the effects of previously studied mechanisms: the saving glut, the banking glut and financial innovation.
Figure 13: Numerical example: Comparison of steady states with and without comparisons.
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Part I
Appendix

A Proofs

A.1 Proof of lemma 1

Under assumption 2, \( \beta (1 + r) = 1 \). The first order conditions are

\[
\lambda_t = \beta (1 + r) \lambda_{t+1} \quad \text{ass 2} =: \lambda \\
\partial_c u(c_t^*) = \lambda_t = \lambda \\
\partial_h u(h_t^*) = \lambda_t p - \beta \lambda_{t+1} p(1 - \delta) = \lambda p(1 - \beta (1 - \delta)),
\]

where \( \lambda_t \) is the Lagrange multiplier on the flow budget constraint. From assumption 2 we get that the multipliers are constant over time. This implies that the marginal utilities of consumption and housing are constants as well. Since the marginal utilities are strictly decreasing functions (assumption 1), the optimal policies must be constant over time,

\[
\partial_c u(x_t^*) = \text{const for all} \quad \text{ass 1} = \Rightarrow x_t^* =: x^* \text{for all} \quad t.
\]

Remains to show that \( a_{t+1} > 0 \) is constant over time. From the flow budget constraint we get that the change in the asset level is constant,

\[
a_{t+1} - a_t (1 + r) = y - c - p \delta h =: \Delta a
\]

We can use this and the period-0 constraint to rewrite the life-time budget constraint

\[
\sum_{t=1}^{\infty} \left( \frac{1}{1 + r} \right)^t \left( c + p \delta h - y \right) = a_0 + y - c_0 - p h_0 \\
- \sum_{t=1}^{\infty} \left( \frac{1}{1 + r} \right)^t \Delta a = - \frac{\Delta a}{r} = a_1.
\]

This implies that

\[
ra_1 = a_t (1 + r) - a_{t+1} \text{ for all } t > 0.
\]

In particular

\[
ra_1 = a_1 (1 + r) - a_2 \implies a_1 = a_2.
\]

By induction, it follows that

\[
a_t = a_{t+1} \text{ for all } t > 0.
\]

A.2 Proof of lemma 2

From the period \( t > 0 \) budget constraint and the assumed form of \( c \) we obtain an expression for \( a \).

\[
a = \frac{\alpha_1 h - \alpha_0 + p \delta h + y}{r} = \frac{(\alpha_1 + p \delta) h - (y + \alpha_0)}{r}
\]

Then we plug \( a \) into the period-0 budget constraint to obtain the desired expression for \( h \).

\[
(y + \alpha_0) + a_0 = (\alpha_1 + p) h + \frac{1}{r} \left( (\alpha_1 + p \delta) h - (y + \alpha_0) \right) =: \tilde{y}
\]

\[
\iff r(\tilde{y} + a_0) + \tilde{y} = (r(\alpha_1 + p) + \alpha_1 + p \delta) h
\]

\[
\iff h = \frac{r(\tilde{y} + a_0) + \tilde{y}}{r(\alpha_1 + p) + \alpha_1 + p \delta} = \frac{(1 + r) \tilde{y} + ra_0}{r(\alpha_1 + p) + \alpha_1 + p \delta} = \frac{(1 + r)(y + \alpha_0) + ra_0}{p(r + \delta) + \alpha_1 (1 + r)}
\]
Plugging in for $h$ in the first equation gives the desired expression for $a$.

\[
ra = (\alpha_1 + p\delta)r \frac{(1 + r)\tilde{y} + ra_0}{r(\alpha_1 + p) + \alpha_1 + p\delta} - \tilde{y}
\]

\[
= \frac{\alpha_1 + p\delta}{p(r + \delta) + \alpha_1(1 + r)}\left((1 + r)\tilde{y} + ra_0\right) - \tilde{y}
\]

\[
= \tilde{y} \left(\frac{\alpha_1 + p\delta}{p(r + \delta) + \alpha_1(1 + r)}(1 + r) - 1\right) + a_0r \frac{\alpha_1 + p\delta}{p(r + \delta) + \alpha_1(1 + r)}
\]

\[
= \tilde{y} \left(\frac{r\delta(\delta - 1)}{p(r + \delta) + \alpha_1(1 + r)} + a_0r \frac{\alpha_1 + p\delta}{r(\alpha_1 + p) + \alpha_1 + p\delta}\right)_{e(0, 1)}
\]

\[
\implies a = \frac{a_0(\alpha_1 + p\delta) - \tilde{y}p(1 - \delta)}{p(r + \delta) + \alpha_1(1 + r)} = \frac{a_0(\alpha_1 + p\delta) - (y + a_0)p(1 - \delta)}{p(r + \delta) + \alpha_1(1 + r)}
\]

### A.3 Proof of proposition ??

Using the explicit solution from lemma 2,

\[
\frac{\partial a}{\partial \alpha_1} = -\tilde{y} \frac{p(\delta - 1)(1 + r)}{(p + \alpha_1(1 + r))} + a_0r \frac{(\alpha + p) + \alpha_1 + p\delta - (\alpha_1 + p\delta)(1 + r)}{(p + \alpha_1(1 + r))^2}
\]

\[
= p(1 - \delta) \frac{(y + a_0)(1 + r) + a_0r}{(p + \alpha_1(1 + r))^2} > 0 \text{ if } y \geq 0 \text{ and } a_0 \geq 0 \text{ (with one >)}.
\]

\[
\frac{\partial a}{\partial a_0} = -\frac{p(1 - \delta)}{p(r + \delta) + \alpha_1(1 + r)} < 0.
\]

### A.4 Proof of lemma ??

We need two more simple lemmas.

**Lemma 6.** With social comparisons (assumption 3)

\[
c = \frac{\xi}{1 - \xi}p(1 - \beta(1 - \delta))(h - \phi\tilde{h}) = \alpha_1h - \alpha_1\phi\tilde{h}
\]

**Proof.** From the first order conditions in the proof of lemma 1 we get that

\[
\frac{\partial h_u(c^*, h^*, \tilde{h})}{\partial c_u(c^*, h^*, \tilde{h})} = p(1 - \beta(1 - \delta)),
\]

From the functional form assumptions we get that

\[
C^* := (c^*)^{1-\xi}(h^* - \phi\tilde{h})^\xi
\]

\[
\frac{\partial h_u(c^*, h^*, \tilde{h})}{\partial c^*} = f'(C^*) \frac{\xi}{h^* - \phi\tilde{h}} C^*.
\]

\[
\frac{\partial h_u(c^*, h^*, \tilde{h})}{c^*} = f'(C^*) \frac{(1 - \xi)}{c^*} C^*
\]

Combining these gives

\[
\frac{1 - \xi}{\xi} \frac{c^*}{h^* - \phi\tilde{h}} = p(1 - \beta(1 - \delta))
\]

Rearranging yields the result.
Lemma 7. If \( \sum_{i=0}^{\infty} a^i W^i \) converges, then
\[
W(I - aW)^{-1} = \frac{1}{a} \left( \sum_{i=0}^{\infty} a^i W^i - I \right) = \frac{1}{a} \left( \sum_{i=1}^{\infty} a^i W^i \right).
\]

Proof. Two lines of algebra that have to be written up.

Denote the lifetime income \( Y = (1 + r)y + ra_0 \).
Using the previous two lemmas we can tackle the proof that we are after. From lemma 6 we know that
\[
c = \alpha_1 h - \alpha_1 \phi h = \overline{a_0}
\]
From lemma 2 we know that
\[
\bar{h} = \frac{\bar{Y} + (1 + r)\alpha_0}{\phi} = \frac{\bar{Y} + (1 + r)\alpha_1 \phi h}{p(r + \delta) + \alpha_1 (1 + r)}.
\]
Stacking these equations for all \( N \) households we get
\[
\bar{h} = \frac{1}{C_2} \bar{Y} + \frac{(1 + r)\alpha_1}{\phi C_1} \bar{h}
\]
using that \( \bar{h} = G\bar{h} \) we get
\[
\bar{h} = C_2 (I - C_1 \phi G)^{-1} \bar{Y}
\]
\[
= C_2 \sum_{i=0}^{\infty} (C_1 \phi G)^i \bar{Y}
\]
Similarly, we stack the expressions for \( a \) (from lemma 2)
\[
a = \frac{1}{p(r + \delta) + \alpha_1 (1 + r)} \left( (\alpha_1 + p\delta) a_0 - p(1 - \delta)(y + \alpha_0) \right)
\]
\[
= C_2 \left( (\alpha_1 + p\delta) a_0 - p(1 - \delta)(y + \alpha_1 \phi G) \right)
\]
\[
= C_2 \left( (\alpha_1 + p\delta) a_0 - p(1 - \delta)(y + \alpha_1 \phi G C_2 \sum_{i=0}^{\infty} (C_1 \phi G)^i \bar{Y}) \right)
\]
using lemma 7 and the definitions of \( C_1 \) and \( C_2 \) yields the result.
\[
= C_2 \left( (\alpha_1 + p\delta) a_0 - p(1 - \delta) y \right)
\]
\[
- p(1 - \delta) \frac{\alpha_1 \phi C_2}{\phi C_1} \sum_{i=1}^{\infty} (C_1 \phi G)^i \bar{Y}.
\]
\[
= C_2 \left( (\alpha_1 + p\delta) a_0 - p(1 - \delta) y \right)
\]
\[
- \frac{p(1 - \delta)}{(1 + r)} \sum_{i=1}^{\infty} (C_1 \phi G)^i \bar{Y}.
\]

B Language of networks

A network is a collection of nodes \( N = \{1, \ldots, N\} \) and edges connecting these nodes. The network can be represented by its adjacency matrix \( G = (\sigma_{ij}) \in \mathbb{R}^{N \times N} \). Each entry \( \sigma_{ij} \) stands for the link from node \( j \) to node \( i \). Node \( j \) is linked to node \( i \) if and only if \( \sigma_{ij} \neq 0 \).

In our setting the network consists of \( N \) consumers (nodes). Consumers are linked if they care about one another. That is, if agent \( i \)'s takes agent \( j \)'s choices into account, then \( \sigma_{ij} \) is positive.

Our network is weighted (consumers care differentially about others) and directed (\( j \) might care about \( i \) while \( i \) does not care about \( j \)).
Paths The adjacency matrix shows which nodes are connected with paths of length one. In addition, agents \( i \) and \( j \) might be connected via a third agent \( k \) (if agent \( i \) cares about \( k \) and \( k \) cares about \( j \)). In this case there is a path of length 2 from \( i \) to \( j \). The weighted number of length-2-paths between to nodes is given by the squared adjacency matrix \( G^2 \). More generally, the \( k \)-th power of \( G \) "count" the weighted paths of length \( k \).

Centrality In order to analyse a network is useful to look at measures of centrality. Agent \( i \)'s Bonacich centrality measures how many paths (of varying length) lead to his node, that is, it measures how much agent \( i \) cares about other agents. Longer path get a smaller weight.

The total (weighted) number of paths is given by the infinite series

\[
\sum_{k=0}^{\infty} v^k G^k = (I - vG)^{-1},
\]

which is called the Leontief inverse (if the series converges). The \((i,j)\) component of this matrix is the discounted weighted number of paths from \( j \) to \( i \).

Using the Leontief inverse we can define Bonacich centrality.

**Definition 2** (Bonacich centrality). For a network with adjacency matrix \( G \) and a scalar \( \nu \in [0,1] \) the vector of Bonacich centralities is given by

\[
b(G, \nu) = (I - \nu G)^{-1}. \]

For an agent \( i \) the Bonacich centrality counts the number of paths of any length from all other agents to \( i \). That is, Bonacich centrality measures how much agent \( i \) cares about others.

**Definition 2'** (Weighted Bonacich centrality). For a an adjacency matrix \( G \in \mathbb{R}^{N \times N} \), a scalar \( \nu > 0 \) and a row vector \( w \in \mathbb{R}^{N} \) define the vector of weighted Bonacich centralities as

\[
b(G, \nu, w) = (I - \nu G)^{-1} w.
\]

The standard Bonacich centrality is obtained by \( b(G, \nu) = b(G, \nu, 1) \).

C Quantitative Model

In this section, we provide the optimal choice of consumption and housing for a given value function \( v(w, y) \). We suppress the \( j \)-superscript for distinct income types. The optimality conditions are:

\[
\frac{(1 - \xi)}{c} \left( \frac{(1 + \phi)(h - \bar{h})}{(1 + \phi)h - \phi h} \right)^{r + \delta} = \partial_w v(w, y) \tag{4}
\]

\[
\frac{\xi (1 + \phi)}{(1 + \phi)h - \phi h} \left( \frac{(1 + \phi)h - \phi h}{(1 + \phi)(h - \phi h)} \right)^{1 - \gamma} = \partial_w v(w, y)(r + \delta)p + \lambda (1 - \psi)p \tag{5}
\]

\[
\lambda (w - (1 - \psi)ph) = 0 \tag{6}
\]

**Case 1: Unconstrained** When the housing choice is unconstrained, i.e. \( \lambda = 0 \), we can express relative housing as a function of consumption

\[
(1 + \phi)h - \phi h = \frac{\xi (1 + \phi)c}{(1 - \xi)(r + \delta)p} \tag{7}
\]

Using this in the first optimality condition gives optimal consumption as

\[
c^*(w, y; v) = \left( \frac{\partial_w v(w, y)}{1 - \xi} \right) \left( \frac{(1 - \xi)(r + \delta)p}{\xi (1 + \phi)} \right)^{(1 - \gamma)} \tag{8}
\]

and thus optimal unconstrained housing as:

\[
h^*(w, y; \bar{h}, v) = \frac{\xi c^*(w, y; v)}{(1 - \xi)(r + \delta)p} + \frac{\phi h}{1 + \phi} \tag{9}
\]
Case 2: Constrained  If the collateral constraint binds, housing is simply given by

\[ h^*(w) = \frac{w}{(1 - \psi)p} \]  \hspace{1cm} (10)

Since the collateral constraint is not a function of consumption, the first optimality condition still holds such that optimal consumption is now:

\[ c^*(w, y; \bar{h}, v) = \left( \frac{\partial_w v(w, y)}{1 - \xi} - (1 + \phi)h^*(w) - \phi \bar{h} \right) \frac{\xi(\gamma - 1)}{1 - \xi} \]  \hspace{1cm} (11)
D Adjustment costs

In this section, we set up a quantitative incomplete markets model with heterogeneous households who value social status derived from relative housing/consumption. The model is cast in continuous time.

D.1 Households

Our economy features a continuum of households who vary in their human capital and face idiosyncratic, income risk. They receive income $y$ which can be used to purchase a non-durable consumption good $c$ at price 1, durable housing $h$ at price $p$ and invest in a safe asset $a$ which pays interest $r$. As we are interested in long-run trends, we assume that housing and assets can be traded frictionlessly. This allows us to work with net wealth, $w = a + ph$, as the only endogenous state variable. The state of the economy is then the joint distribution of wealth and income, $G(w, y)$, with joint density $g(w, y)$.

$$\max_{(c_t,h_t)\geq 0} \mathbb{E}_0 \int_0^\infty \exp(-\rho t)u(c_t, h_t) dt$$

subject to

$$\dot{a}_t = y_t + ra_t - pi_t^h - \chi(i_t^h, h_t) - c_t$$

$$\dot{h}_t = i_t^h - \delta h_t$$

$$dy_t = \cdots$$

$$- a_t \leq \psi p_t h_t$$

and

$$u(c, h) = \frac{(c^{1-\xi}h^\xi)^{1-\gamma}}{1-\gamma}$$

$$\chi(i^h, h) = \begin{cases} \chi_0 + \chi_1 i^h - \delta h & \text{for } i^h > \delta h \\ -\chi_0 + \chi_1 i^h & \text{for } i^h < \delta h \end{cases}$$

The setup is closely related to the model in Kaplan et al. (2016). In contrast to their model, we relabel their illiquid asset into housing, which yields utility. In addition, we let the agents decide endogenously how much to invest in their house.

The collateral constraint is following Favilukis et al. (2017) and Kaplan et al. (2017), the adjustment costs function is following Kaplan et al. (2016).

The can be written in terms of an HJB equation.

$$\rho V(c_t, h_t) = \max_{c_t, h_t} u(c_t, h_t) + \partial_a V \cdot (y_t + ra_t - pi_t^h - \chi(i_t^h, h_t) - c_t)$$

$$+ \partial_h V \cdot (i_t^h - \delta h_t)$$

$$+ \cdots$$

$$+ \lambda(a_t + \psi p_t h_t)$$

The first order conditions are

$$u_c = V_a$$

$$V_h = V_a(p + \chi^h)$$

where

$$\chi^h(i^h, h) = \begin{cases} \chi_0 + \chi_1 i^h \frac{p - h}{h} \chi^2 - 1 & \text{for } i^h > \delta h \\ -\chi_0 + \chi_1 i^h \frac{\delta h - i^h}{h} \chi^2 - 1 & \text{for } i^h < \delta h \end{cases}$$

Thus,

$$V_a = (1 - \xi)h^{\xi(1-\gamma)}c^{(1-\xi)(1-\gamma)-1}$$

$$\implies c = \left( \frac{V_a}{(1 - \xi)h^{\xi(1-\gamma)}} \right)^{\frac{1}{(1-\xi)(1-\gamma)-1}}$$
and

\[
\frac{V_h}{V_a} - p = \chi^i \\
\begin{cases}
\chi_0 + \chi_1 \frac{(i^h - \delta h)}{h} \chi^2 - 1 & \text{for } i^h > \delta h \\
-\chi_0 - \chi_1 \frac{(h - i_h)}{h} \chi^2 - 1 & \text{for } i^h < \delta h
\end{cases}
\]

\[
\Rightarrow i^h = \begin{cases}
h \left( \frac{V_h}{V_a} - p - \chi_0 \right) \frac{1}{\chi_1 \chi^2} + \delta h & \text{for } i^h > \delta h \\
\delta h - h \left( \frac{V_h}{V_a} - p + \chi_0 \right) \frac{1}{\chi_1 \chi^2} & \text{for } i^h < \delta h
\end{cases}
\]

E  Long-term mortgages

We consider long-term, fixed amortization mortgages \( m \). A mortgage that is issued at time \( t_0 \), with constant debt-service flow \( \pi \) must satisfy

\[
m = \int_{t_0}^{\infty} e^{-\tau(t-t_0)} \underbrace{e^{-M(t)}}_{\Pr(\text{survival})} \pi
\]