Jumps and the Correlation Risk Premium: Evidence from Equity Options

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Abstract

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Keywords: Correlation Risk · Option-Implied Information · Variance Risk Premium

JEL: G12 · G13

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Abstract

This paper breaks the correlation risk premium down into two components: a premium related to the correlation of continuous stock price movements and a premium for bearing the risk of co-jumps. We propose a novel way to identify both premiums based on a dispersion trading strategy that goes long an index option portfolio and short a basket of option portfolios on the constituents. The option portfolios are constructed to only load on either diffusive volatility or jump risk. We document that both risk premiums are economically and statistically significant for the S&P 100 index. In particular, selling insurance against states of high jump correlation generates a sizable annualized Sharpe ratio of 0.85.

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1 Introduction

Investors fear stock market turbulence. They want to hedge against states of high market volatility and are willing to pay a large premium for insurance, known as the market variance risk premium (VRP). When markets are tumultuous, investors reduce their exposure and flee risky stocks. This collective behavior causes stocks to fall in harmony and losses to spiral. In other words, stock correlations go up and diversification benefits vanish when needed most. In order to eliminate the risk of high correlations, investors pay the so-called correlation risk premium (CRP). It turns out that there is a close theoretical link between the two risk premiums: The market VRP can be expressed as the sum of the CRP and the VRPs of individual stocks.

Previous research has focussed on assessing the size and predictive power of the risk premiums. As summarized by Zhou (2018), empirical studies agree on the notion that the market VRP is economically and statistically significant and predicts future market returns at few-month horizons. Driessen et al. (2009) are the first to provide evidence that the CRP for the S&P 100 index is sizable. In addition, Buss et al. (2018) document that the CRP predicts future market returns at horizons of up to one year. Individual VRPs are examined by Carr and Wu (2009). They show that VRPs of individual stocks have a large cross-sectional variation and, in particular, find that only few stocks generate significant VRPs. Taken together, previous research hence implies that the market VRP is mainly determined by the CRP.

Despite the importance of the CRP, little is known about its drivers so far. In general,
stocks may be correlated because they continuously move in the same direction or because they experience co-jumps, i.e. common discontinuous movements on rare occasions. Depending on the origin, investors may be willing to pay very different premiums to hedge against states of high correlations. Conceivably, co-jumps may pose a greater threat to investors and therefore carry a higher premium. This paper fills the gap and breaks the CRP down into two components: a premium related to the correlation of continuous stock price movements and a premium for bearing the risk of co-jumps. Dissecting the CRP as the key determinant of the market VRP is natural given that Bollerslev and Todorov (2011) find that a large fraction of the market VRP is actually attributable to the compensation for market jump risks rather than diffusive volatility risks.

Our analysis relies on equity options. The basic idea is that options contain rich information about investors’ ex ante assessments of various risks. In particular, excess returns of options provide direct evidence on the risk premiums associated with continuous stock price movements, time-varying volatility, and jumps. In order to isolate volatility and jump risk premiums, we follow Cremers et al. (2015) and set up delta-gamma-neutral or delta-vega-neutral option portfolios which are only exposed to either volatility or jump risks. We construct these option portfolios for the index as well as for its constituents. In order to extract the CRPs, we implement dispersion trades similar to Driessen et al. (2009). More precisely, we go long the index option portfolio and short the basket of option portfolios on the constituents, such that the strategy only loads on the correlation of continuous stock price movements or the correlation of jumps, respectively. The resulting excess returns allow us to assess the pricing of different types of correlation risk. Focussing on option returns
has two crucial advantages. First, it does not involve estimation of the physical and risk-neutral expectations, but instead identifies the premium, i.e. the difference between the two expectations, directly. Second and related, it does not draw on high frequency data, which may be cumbersome to use for a large cross section of stocks, but requires daily data on four options for each underlying only.

The contribution of this paper is both theoretical and empirical. In the theoretical part, we decompose the index VRP and show that it comprises the individual VRPs of all constituents and premiums for their continuous and jump covariations. If the individual VRPs are on average equal to zero, the index VRP arises solely from the CRP. In that case, investors pay a premium to hedge against states of high market volatility because they fear high correlations among stocks and want to eliminate the risk of worsening diversification in the market. We show that specifically constructed option portfolios provide a simple way to separate and identify the various risk premiums and apply this methodology to the S&P 100 index and its constituents in the empirical part. First, we document that the volatility risk premium is economically and statistically significant for the index. Second, we find large cross-sectional differences in the individual volatility risk premiums of the constituents. While the volatility risk premium of the index is negative, those of the constituents are positive on average. Third, we show that the jump risk premium is very large for the index, while the constituents have much smaller ones on average. Most importantly, we find that both types of correlation risk carry significant premiums. The premium for the correlation of jumps, however, is much larger than the premium for the correlation of continuous stock price movements. In particular, selling insurance against states of high jump correlation
(volatility correlation) generates a sizable Sharpe ratio of 0.85 (0.45) per year. Overall our results document the importance of correlation as a priced risk factor and highlight that the market VRP is primarily paid to eliminate the risk of co-jumps.

The remainder of this paper is organized as follows. Section 2 reviews related literature. Section 3 decomposes the index VRP and shows how to construct option portfolios that allow to trade the different types of correlation risk. Section 4 presents the empirical methodology, the data and evidence on the pricing of correlation risk for the S&P 100 index. Section 5 concludes. All proofs are in the Appendix.

2 Related Literature

This paper is related to four strands of literature.

Variance Risk Premium The first strand explores the compensation that investors demand for bearing variance risk.\(^1\) Bakshi and Kapadia (2003a) study the S&P 500 index and find a negative market VRP. In related work, Bakshi and Kapadia (2003b) document that the VRPs for 25 individual stocks are substantially smaller compared to the index. Carr and Wu (2009) confirm the evidence of a negative market VRP for several major U.S. indices and furthermore study 35 individual stocks. They show that individual VRPs are cross-sectionally dispersed and, in particular, find that only few stocks generate significantly

\(^1\)The variance risk of a stock stems from two different sources: diffusive volatility and jumps. For better readability, we refer to the premium associated with diffusive volatility as volatility risk premium (VolRP), the premium for jumps as jump risk premium (JRP), and the premium related to the total variance risk as variance risk premium (VRP) throughout the paper.
negative VRPs. Hollstein and Simen (2018) decompose the index VRP and show that its magnitude is attributable to the CRP, while its time-variation mainly comes from the individual VRPs. However, the CRP is only measured indirectly. We contribute to this strand of literature by examining the volatility risk premiums of the S&P 100 index and its constituents. More importantly, we propose a way to directly quantify the premiums associated with different types of correlation risk.

**Correlation Risk Premium** The second strand of literature focuses on the pricing of correlation risk. Driessen et al. (2009) are the first to provide direct evidence of the CRP, using a dispersion trading strategy that only loads on correlation risk. In a recent study, Faria et al. (2018) find significant CRPs for several major European and U.S. indices and document their co-movement, supporting the idea of a global CRP. While these studies investigate the pricing of correlation risk in the cross section of option returns, several other studies complement the findings for different test assets. Krishnan et al. (2009) show that correlation risk is a priced factor in the cross section of stock returns. Pollet and Wilson (2010) find that the average correlation between daily stock returns explains future market excess returns for horizons of up to 30 months. In a similar vein, Buss et al. (2018) document that the CRP significantly predicts future market excess returns for horizons of up to 12 months, even out-of-sample. In addition, Buraschi et al. (2014) report that correlation risk constitutes a priced factor in the cross section of hedge fund returns. Moreover, they find that funds which sell insurance against states of high correlations typically have large maximum drawdowns, indicating that correlation risk is in some way related to tail risk. Our paper contributes to this discussion by analyzing the drivers of the CRP and paying special attention to the
role of tail risk. More precisely, we isolate and separately quantify the premium related to the correlation of continuous stock price movements and the premium for bearing the risk of co-jumps.

**(Co-)Jumps** The third strand examines the role of jumps in creating tail risk for investors. Bollerslev and Todorov (2011) uncover that more than half of the market VRP is actually attributable to the compensation for market jump risks rather than diffusive volatility risks. Drechsler and Yaron (2011) argue that including jumps in a long-run risks model is necessary in order to match the empirical moments of the market VRP. At the individual stock level, Bollerslev et al. (2008) present evidence for significant co-jumps that predominantly materialize around macroeconomic news announcements. In related work, Bollerslev et al. (2013) reveal that the S&P 500 index and 50 individual stocks exhibit tail dependence which is mostly attributable to co-jumps. Aït-Sahalia and Xiu (2016) decompose the pairwise covariations between different asset classes into continuous and jump components. They report that the global financial crisis did not result in a change of the relative contributions of the components, although covariations increased during this period. In contrast to existing studies, our paper analyzes the premium that is associated with co-jumps. That is, it takes the risk-neutral expectation of co-jumps into account, whereas previous research examined realized co-jumps only. Moreover, while much of this strand of literature is based on high frequency data, which may be cumbersome to use for a large cross section of stocks, our approach relies on daily option returns that can be easily computed from four options for each underlying.
Option Returns  Finally, the fourth strand of literature examines option returns to learn about the moments of the underlying’s return distribution. Coval and Shumway (2001) propose the use of zero-beta, at-the-money straddles to study the pricing of time-varying volatility and provide evidence that it is indeed priced in the returns of index options. Cremers et al. (2015) go one step further and pave the way to disentangle market volatility and jump risks. They use portfolios of index straddles that are constructed to be either delta-gamma-neutral or delta-vega-neutral and show that investors pay premiums to hedge both types of risks. Middelhoff (2019) refines their method by constraining the portfolios to have constant option sensitivities over time, making option returns comparable across time and underlyings. While much of the literature has focussed on index options, little is known about the differential pricing of index and basket options. One notable exception is Kelly et al. (2016) who document that out-of-the-money put options on the financial sector were extraordinarily cheap relative to the basket of individual banks during the global financial crisis and ascribe their finding to an implicit sector-wide government guarantee. Our paper contributes to this literature by applying the idea of Cremers et al. (2015) to the constituents of the S&P 100 index. We then study the differential pricing of the index and basket option portfolios in dispersion trades that isolate the premiums related to continuous correlations and co-jumps.

3 Theoretical Framework

This section lays the theoretical foundation for empirically assessing the pricing of different types of correlation risk in Section 4. We first decompose the index VRP into the individual VRPs of all constituents and premiums for their continuous and jump covariations. In
a general jump-diffusion setting, we then show that these components can be traded using
delta-gamma-neutral and delta-vega-neutral option portfolios. In order to extract the CRPs,
we implement dispersion trades that go long the index option portfolios and short baskets of
the option portfolios on the constituents. We demonstrate that the strategies only load on the
correlation of continuous stock price movements or the correlation of jumps, respectively, and
show that their excess returns provide direct evidence on the corresponding risk premiums.

3.1 Index Variance Risk Premium

Suppose that the index of interest comprises constituents $i = 1, \ldots, N$ with relative shares $\omega_{i,t}$.
The quadratic variation of the index is given by

$$ QV_{I,[t,t+\tau]} = \sum_{i=1}^{N} \omega_{i,t}^2 QV_{i,[t,t+\tau]} + \sum_{i=1}^{N} \sum_{j=1}^{N} \omega_{i,t} \omega_{j,t} QV_{ij,[t,t+\tau]}, \quad (1) $$

where $QV_{ij,[t,t+\tau]}$ denotes the quadratic covariation between the stock price processes of
constituents $i$ and $j$ over the time interval $[t, t + \tau]$. In addition, suppose that the con-
stituents’ stock prices follow jump-diffusions. In that case, we can decompose their quadratic
(co)variations into continuous and jumps parts, i.e. $QV_{ij,[t,t+\tau]} = CV_{ij,[t,t+\tau]} + JV_{ij,[t,t+\tau]}$, and
rewrite Equation (1) as

$$ QV_{I,[t,t+\tau]} = \sum_{i=1}^{N} \omega_{i,t}^2 CV_{i,[t,t+\tau]} + \sum_{i=1}^{N} \omega_{i,t}^2 JV_{i,[t,t+\tau]}
+ \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{j \neq i} \omega_{i,t} \omega_{j,t} CV_{ij,[t,t+\tau]} + \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{j \neq i} \omega_{i,t} \omega_{j,t} JV_{ij,[t,t+\tau]}, \quad (2) $$
That is, the quadratic variation of the index is attributable to the continuous and jump variation of each constituent, in addition to the continuous and jump covariations between each pair of constituents. From this expression, we can derive the index VRP as the difference between the physical (\(\mathbb{P}\)) and risk-neutral (\(\mathbb{Q}\)) expectation of the index quadratic variation (see e.g. Bollerslev et al. (2015))

\[
VRP_{t,t} = E_t^{\mathbb{P}}[QV_{I,[t,t+\tau]}] - E_t^{\mathbb{Q}}[QV_{I,[t,t+\tau]}]
\]

\[
= \sum_{i=1}^{N} \omega_{i,t}^2 \left( E_t^{\mathbb{P}}[CV_{i,[t,t+\tau]}] - E_t^{\mathbb{Q}}[CV_{i,[t,t+\tau]}] \right) + \sum_{i=1}^{N} \omega_{i,t}^2 \left( E_t^{\mathbb{P}}[JV_{i,[t,t+\tau]}] - E_t^{\mathbb{Q}}[JV_{i,[t,t+\tau]}] \right)
\]

\[
+ \sum_{i=1}^{N} \sum_{j=1}^{N} \omega_{i,t} \omega_{j,t} \left( E_t^{\mathbb{P}}[CV_{ij,[t,t+\tau]}] - E_t^{\mathbb{Q}}[CV_{ij,[t,t+\tau]}] \right) + \sum_{i=1}^{N} \sum_{j=1}^{N} \omega_{i,t} \omega_{j,t} \left( E_t^{\mathbb{P}}[JV_{ij,[t,t+\tau]}] - E_t^{\mathbb{Q}}[JV_{ij,[t,t+\tau]}] \right)
\]

\[
= \text{VolRP}_{t,t} + \text{JRP}_{t,t} + \text{CovRP}_{VOL,t} + \text{CovRP}_{JUMP,t}
\]

(3)

The index VRP comprises several risk premiums, namely (i) \(N\) premiums for the continuous variation of each constituent (denoted as \(VolRP\)), (ii) \(N\) premiums for their jump variations \((JRP)\), (iii) \(N(N-1)\) premiums for the pairwise continuous covariations of constituents, which aggregate to the continuous covariance risk premium \((CovRP_{VOL})\), and (iv) \(N(N-1)\) premiums for pairwise jump covariations, which constitute the jump covariance risk premium \((CovRP_{JUMP}).^2\)

The decomposition highlights that investors pay a premium to insure themselves against

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^2Note that quadratic covariations can be normalized and alternatively expressed in terms of correlations as \(QV_{ij,[t,t+\tau]}/\sqrt{QV_{i,[t,t+\tau]} \sqrt{QV_{j,[t,t+\tau]}}\), such that there is a one-to-one correspondence between the covariance risk premiums \(CovRP_{VOL}\) and \(CovRP_{JUMP}\) presented here and the correlation risk premiums \(CRP_{VOL}\) and \(CRP_{JUMP}\) discussed later.
states of high index variance for potentially very different reasons. However, separating the risk premiums from each other presents a great empirical challenge. In the following sections, we show that option returns contain rich information about several risk premiums and argue that specifically constructed option portfolios provide a simple and legitimate way to solve the identification problem.

3.2 Option Returns

Suppose that the processes for the stock price $S_{i,t}$ and variance $V_{i,t}$ of each constituent $i$ at time $t$ are given by the jump-diffusions

$$
\frac{dS_{i,t}}{S_{i,t-}} = \mu_{S_i} dt + \sqrt{V_{i,t}} dW_{t}^{S_i} + \frac{\Delta S_{i,t}}{S_{i,t-}}
$$

(4)

$$
dV_{i,t} = \mu_{V_i} dt + \sigma_{V_i} \sqrt{V_{i,t}} dW_{t}^{V_i} + \Delta V_{i,t},
$$

(5)

where $dW_{t}^{S_i}$ and $dW_{t}^{V_i}$ are Brownian motions, and $\Delta S_{i,t}$ and $\Delta V_{i,t}$ represent jumps. The index is computed as the sum of constituents’ stock prices, weighted by their relative shares in the index. The level of the index $S_{I,t}$ and its local variance $V_{I,t}$ are equal to

$$
S_{I,t} = \sum_{i=1}^{N} \omega_{i,t} S_{i,t}
$$

(6)

$$
V_{I,t} = \sum_{i=1}^{N} \omega_{i,t}^2 V_{i,t} + \sum_{i=1}^{N} \sum_{\substack{j=1 \atop j \neq i}}^{N} \omega_{i,t} \omega_{j,t} \sqrt{V_{i,t}} \sqrt{V_{j,t}} \rho_{ij,t},
$$

(7)

where $\rho_{ij,t}$ denotes the pairwise local correlation between the Brownian motions of constituents $i$ and $j$. We make the simplifying assumption that all pairwise correlations are
driven by a single state variable, which is denoted as equi-correlation $\rho_t$ and follows a diffusion process. For ease of exposition, we abstract from any discontinuities in variances.\(^3\)

The price of any option $O_{i,t} = O_i(t, S_{i,t}, V_{i,t})$ on constituent $i$ then follows

$$dO_{i,t} - rO_{i,t}dt = \frac{\partial O_i}{\partial S_i}(dS_{i,t} - E_t^Q[dS_{i,t}]) + \frac{\partial O_i}{\partial V_i}(dV_{i,t} - E_t^Q[dV_{i,t}]) + \Delta O_{i,t} - E_t^Q[\Delta O_{i,t}]$$

with $\Delta O_{i,t} = O_i(t, S_{i,t} + \Delta S_{i,t}, V_{i,t}) - O_i(t, S_{i,t}, V_{i,t}), \quad (8)$

where $dS^c_{i,t}$ denotes the continuous variation of the stock price. Approximating the jump component $\Delta O_{i,t}$ by a second-order Taylor series allows us to express the excess return of the option as

$$dO_{i,t} - rO_{i,t}dt = \frac{\partial O_i}{\partial S_i}(dS_{i,t} - E_t^Q[dS_{i,t}]) + \frac{\partial O_i}{\partial V_i}(dV_{i,t} - E_t^Q[dV_{i,t}]) + 
\frac{1}{2} \frac{\partial^2 O_i}{\partial S_i^2} \left((\Delta S_{i,t})^2 - E_t^Q[(\Delta S_{i,t})^2]\right) + \xi_{O_{i,t}} - E_t^Q[\xi_{O_{i,t}}], \quad (9)$$

where $\xi_{O_{i,t}}$ denotes the remainder term from the Taylor series approximation and is neglected henceforth. It can be seen that the excess return of the option depends on the risk premiums associated with continuous stock price movements (multiplied by delta), volatility (multiplied by vega), and jumps (multiplied by gamma).\(^4\) At this point, we note that option returns prove to be very useful for measuring risk premiums directly, without having to estimate the

\(^3\)A detailed derivation of the following equations is provided in Appendix A, which also discusses the more general case with discontinuities in the variances and equi-correlation.

\(^4\)To be more precise, the terms inside the parentheses represent instantaneous payoffs from bearing stock price, volatility, and jump risks. In order to interpret them as risk premiums, we need to make the additional assumption that $dS^c_{i,t}$, $dV_{i,t}$, and $(\Delta S_{i,t})^2$ are martingales under the physical measure. It then holds that $E_t^P[dS_{i,t} = dS_{i,t,-}, E_t^P[dV_{i,t} = dV_{i,t,-}, and E_t^P[(\Delta S_{i,t})^2] = (\Delta S_{i,t,-})^2$. 

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physical and risk-neutral expectations separately. Moreover, they accommodate several risk premiums, which can be taken apart by designing suitable option trading strategies.

In general, the excess return of any option $O_{I,t}$ on the index has the same form. However, the excess return of the index option depends on the index level $S_{I,t}$ and variance $V_{I,t}$, which in turn are functions of constituents’ stock prices and variances as well as the equicorrelation as shown in Equations (6) and (7). As a consequence, index options collect each constituent’s premium associated with continuous stock price movements, VolRP, and JRP. On top of that and in contrast to options on the constituents, index options also carry the CRP because they are exposed to correlation risk.

### 3.3 Correlation Risk Premium

In order to extract the CRP, we implement dispersion trades similar to Driessen et al. (2009). That is, we go long the index option and short the basket of options on the constituents. This strategy eliminates individual risks stemming from the constituents and only loads on the correlation between them. As mentioned earlier, there are two types of correlation risk: the risk that stocks continuously move in the same direction and the risk that they experience co-jumps. Our setting allows us to disentangle the premiums associated with the two different types of correlation risk in a simple way. The basic idea is to construct portfolios of options that enter the dispersion trade which are only exposed to either volatility or jump risks. This can be easily accomplished by forming delta-gamma-neutral or delta-vega-neutral option portfolios in the fashion of Cremers et al. (2015) for the index and all constituents.

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5Note that Driessen et al. (2009) rely on a diffusion setting and thus abstract from stock price jumps.
**Volatility CRP**  In order to extract the risk premium for continuous correlation, we form delta-gamma-neutral option portfolios, denoted as $VOL$ portfolios, for all constituents as well as the index. That is, we set

$$\frac{\partial VOL_i}{\partial S_i} = \frac{\partial VOL_I}{\partial S_I} = 0 \quad \text{and} \quad \frac{\partial^2 VOL_i}{\partial S_i^2} = \frac{\partial^2 VOL_I}{\partial S_I^2} = 0 \quad \forall \ i \quad (10)$$

in Equation (9). The excess return of the $VOL$ portfolio for constituent $i$ then becomes

$$dVOL_{i,t} - r VOL_{i,t} dt = \frac{\partial VOL_i}{\partial V_i} (dV_{i,t} - E^Q_t [dV_{i,t}]). \quad (11)$$

The $VOL$ portfolio allows to trade volatility risk. Its return reflects the VolRP of the constituent scaled by the exposure to volatility risk, which is given by the constituent’s vega. For the index, the excess return of the $VOL$ portfolio is

$$dVOL_{I,t} - r VOL_{I,t} dt = \frac{\partial VOL_I}{\partial V_I} \left( \sum_{i=1}^N \frac{\partial V_I}{\partial V_i} (dV_{i,t} - E^Q_t [dV_{i,t}]) \right) + \frac{\partial V_I}{\partial \rho} \left( d\rho_t - E^Q_t [d\rho_t] \right), \quad (12)$$

i.e. it is driven by the sum of all constituents’ individual VolRPs and the risk premium for continuous correlation, scaled by the index vega. In order to isolate the correlation risk premium, we implement a dispersion trade that goes long the $VOL$ portfolio of the index and short the $VOL$ portfolios of all constituents. The size of each short position then is

$$y_{i,t} = \frac{\partial VOL_I}{\partial V_I} \frac{\partial V_i}{\partial V_i}. \quad (13)$$
When practically implementing the strategy, we restrict the vegas of all \( VOL \) portfolios to a constant, i.e. we set \( \frac{\partial VOL_I}{\partial V_I} = \frac{\partial VOL_i}{\partial V_i} = \text{const.} \forall i \) following Middelhoff (2019). On the one hand, this makes \( VOL \) returns comparable across time and underlyings. On the other hand, it allows us to simplify the short position to \( y_{i,t} = \frac{\partial V_I}{\partial V_i} \). Finally, the excess return of the resulting portfolio, denoted as \( CRP_{VOL} \), becomes

\[
dCRP_{VOL,t} - r CRP_{VOL,t} dt = \frac{\partial VOL_I}{\partial V_I} \frac{\partial V_I}{\partial \rho} (d\rho_t - E_t^Q[d\rho_t])
\]

and depends on the risk premium paid for continuous correlation, multiplied by the index vega and the exposure of the index variance to the equi-correlation. The portfolio hence pays the volatility CRP.

**Jump CRP** Similarly, we form delta-vega-neutral \( JUMP \) portfolios in order to extract the risk premium for co-jumps by setting

\[
\frac{\partial JUMP_i}{\partial S_i} = \frac{\partial JUMP_I}{\partial S_I} = 0 \quad \text{and} \quad \frac{\partial JUMP_i}{\partial V_i} = \frac{\partial JUMP_I}{\partial V_I} = 0 \quad \forall i
\]

in Equation (9). The \( JUMP \) portfolio for constituent \( i \) allows to trade jump risk and offers the excess return

\[
dJUMP_{i,t} - r JUMP_{i,t} dt = \frac{1}{2} \frac{\partial^2 JUMP_i}{\partial S_i^2} ((\Delta S_{i,t})^2 - E_t^Q[(\Delta S_{i,t})^2])
\]

which is proportional to the JRP of the constituent, scaled by the exposure to jump risk, as measured by the constituent’s gamma. The excess return of the \( JUMP \) portfolio for the
index is

\[ dJUMP_{I,t} - r JUMP_{I,t} dt = \frac{1}{2} \frac{\partial^2 JUMP_I}{\partial S_I^2} \left( (\Delta S_{I,t})^2 - E_t^Q[(\Delta S_{I,t})^2] \right) \]

\[
= \frac{1}{2} \frac{\partial^2 JUMP_I}{\partial S_I^2} \left( \sum_{i=1}^N \omega_{i,t}^2 \left( (\Delta S_{i,t})^2 - E_t^Q[(\Delta S_{i,t})^2] \right) + \sum_{i=1}^N \sum_{j=1}^N \omega_{i,t} \omega_{j,t} \left( \Delta S_{i,t} \Delta S_{j,t} - E_t^Q[(\Delta S_{i,t} \Delta S_{j,t})] \right) \right). \tag{17} \]

It is proportional to the sum of all constituents’ individual JRPs and the risk premium for co-jumps, scaled by the index gamma. In order to isolate the correlation risk premium, we implement a dispersion trade that goes long the \textit{JUMP} portfolio of the index and short the \textit{JUMP} portfolios of all constituents, where the size of each short position amounts to

\[ y_{i,t} = \frac{\partial^2 JUMP_I}{\partial S_I^2} \omega_{i,t}^2. \tag{18} \]

We follow Middelhoff (2019) and restrict the gammas of all \textit{JUMP} portfolios to a constant, i.e. we set \( \frac{\partial^2 JUMP_I}{\partial S_I^2} = \frac{\partial^2 JUMP_i}{\partial S_i^2} = \text{const.} \) \( \forall i \), such that the short position simplifies to \( y_{i,t} = \omega_{i,t}^2 \). Finally, the excess return of the resulting portfolio, denoted as \( CRP_{JUMP} \), becomes

\[ dCRP_{JUMP,t} - r CRP_{JUMP,t} dt = \frac{1}{2} \frac{\partial^2 JUMP_I}{\partial S_I^2} \sum_{i=1}^N \sum_{j=1}^N \omega_{i,t} \omega_{j,t} \left( \Delta S_{i,t} \Delta S_{j,t} - E_t^Q[(\Delta S_{i,t} \Delta S_{j,t})] \right) \]

\[ = \frac{1}{2} \frac{\partial^2 JUMP_I}{\partial S_I^2} \sum_{i=1}^N \sum_{j=1}^N \omega_{i,t} \omega_{j,t} \left( \Delta S_{i,t} \Delta S_{j,t} - E_t^Q[(\Delta S_{i,t} \Delta S_{j,t})] \right). \tag{19} \]
and depends on the weighted sum of all pairwise risk premiums paid for co-jumps, multiplied by the index gamma. In other words, this portfolio pays the jump CRP.

4 Empirical Evidence

4.1 Methodology

Our goal is to break the correlation risk premium down into two components: a premium related to the correlation of continuous stock price movements ($CRP_{VOL}$) and a premium for bearing the risk of co-jumps ($CRP_{JUMP}$). We follow the methodology outlined in Section 3 to construct portfolios which are only exposed to changes in the correlation of diffusive stock price movements and to co-jumps, respectively.

We build on Cremers et al. (2015) to construct the $VOL$ and $JUMP$ portfolios introduced in Section 3 for the index and all constituents. On each day and for each underlying, we group all available liquid options according to their remaining time to maturity. Within every maturity bucket, we select the call and put options with same strike prices that are nearest-the-money and simultaneously take a position in them, i.e. we set up straddles. If this selection criterium results in more than two straddles with different time to maturity, we choose the two straddles with the shortest and longest time to maturity because this results in the maximum dispersion of options’ sensitivities and makes the later numerical optimization easier.\footnote{As noted in Section 4.2, we limit our attention to options with 14 to 365 days to expiration.} In order to make the returns on the $VOL$ and $JUMP$ portfolios comparable over time and to simplify the initiation of the dispersion trade, we follow Middelhoff

\footnote{As noted in Section 4.2, we limit our attention to options with 14 to 365 days to expiration.}
(2019) and restrict the \textit{VOL} and \textit{JUMP} portfolios to have constant vegas and gammas, respectively. Furthermore, we aim for a balanced allocation of wealth across the four options in order to minimize the impact of any outliers and potential data noise, e.g. stemming from bid-ask spreads. Thus, the optimization problem for the \textit{VOL} portfolios can be written as

\begin{equation}
\min_{\omega} \left\| \frac{\omega \circ O}{\text{abs}(\omega^\top)O} \right\|_2^2
\end{equation}

s.t. $\omega^\top [\Delta, \mathcal{V}, \Gamma] = [0, 200, 0]$ \quad (20)

\begin{equation}
\omega \circ [-1, -1, 1, 1]^\top \geq 0,
\end{equation}

where $\omega = [\omega_{\text{call}, T_1}, \omega_{\text{put}, T_1}, \omega_{\text{call}, T_2}, \omega_{\text{put}, T_2}]^\top$ denotes a $4 \times 1$ vector stacking the positions in call and put options with different times to maturity $T_1 < T_2$. $O$ is the corresponding $4 \times 1$ vector of option prices, $\Delta$, $\mathcal{V}$, and $\Gamma$ are the corresponding $4 \times 1$ vectors of the option sensitivities delta, vega, and gamma, respectively. Equation (20) minimizes the squared Euclidean norm of all option positions relative to total wealth invested, which is defined as the sum of the absolute number of option contracts multiplied by their respective prices.

The \textit{VOL} portfolios are constructed to be delta-gamma-neutral and to always have a vega of 200. They short the straddle with time to maturity $T_1$ and go long the straddle with time to maturity $T_2$, because vega increases with time to maturity.

Analogously, when setting up the \textit{JUMP} portfolios, we optimize Equation (20)

\begin{equation}
\text{s.t. } \omega^\top [\Delta, \mathcal{V}, \Gamma] = [0, 0, 0.01]
\end{equation}

(23)
\[ \omega \circ [1, 1, -1, -1]^T \geq 0, \quad (24) \]
such that they are delta-vega-neutral and always have a gamma of 0.01. We go long the straddle with time to maturity \( T_1 \) and short the straddle with time to maturity \( T_2 \), because gamma decreases with time to maturity. We hold the optimal positions for one trading day and calculate the excess returns from the recorded closing prices of the following day. If we cannot recover an option, we interpolate its implied volatility using the kernel smoothing technique of OptionMetrics.\(^7\)

The resulting \( VOL \) and \( JUMP \) portfolio for constituent \( i \) (denoted as \( VOL_i \) and \( JUMP_i \)) are only exposed to changes in the individual variance and to individual jumps, respectively. In contrast, the \( VOL_I \) of the index portfolio is exposed to the correlation associated with continuous stock price movements (\( CRP_{VOL} \)) and on all individual variances. Analogously, the index \( JUMP_I \) portfolio is exposed to co-jumps (\( CRP_{JUMP} \)) and to individual jumps in the stocks. In order to isolate \( CRP_{VOL} \) and \( CRP_{JUMP} \), we hedge against the individual risk factors by taking appropriate positions when setting up the dispersion trade similar to Driessen et al. (2009).

4.2 Data

The sample period is from January 1996 to December 2017. Our analysis focuses on the S&P 100 index and its constituents. Data on the composition of the S&P 100 index is taken

\(^7\)More precisely, we interpolate across log time to maturity, moneyness defined as stock price divided by strike price, and a call-put identifier. We follow OptionMetrics and set the bandwidth parameters to \( h_1 = 0.05 \), \( h_2 = 0.005 \), and \( h_3 = 0.001 \).
from Compustat, the level of the S&P 100 index is provided by OptionMetrics. We obtain daily stock price data on all constituents from the Center for Research in Security Prices (CRSP) and compute market capitalizations in order to approximate constituents’ relative weights in the index. Daily option prices are taken from OptionMetrics IvyDB US. We apply several filters in the fashion of Goyal and Saretto (2009). First, we exclude options with non-standard settlement, missing implied volatility, and zero open interest. Second, we only keep options whose bid quotes are positive and strictly smaller than their ask quotes. Third, we compute midprices as the average of bid and ask quotes and discard options whose midprices violate standard arbitrage bounds as in Cao and Han (2013). We use the zero-coupon interest rate curve provided by OptionMetrics and linearly interpolate across time to maturity if necessary. When setting up the \( \text{VOL} \) and \( \text{JUMP} \) portfolios as described in Section 4.1, we limit our attention to options with 14 to 365 days to expiration.

Options on the S&P 100 index and its constituents are American-style. Their recorded prices hence include an early exercise premium that distorts option returns. Since the accurate measurement of option returns is central to our analysis, we strip off the early exercise feature as follows. Given the recorded implied volatilities, we reprice all American options in binomial trees of Cox et al. (1979)-type with 1,000 time steps. We explicitly account for expected dividends using data on the S&P 100 dividend yield from OptionMetrics and data on discrete dividends paid by the constituents from OptionMetrics and CRSP. We eliminate options whose recorded prices deviate by more than 1% from the prices implied by the binomial trees. For the remaining options, we compute European prices using the same binomial trees and calculate option sensitivities as Black-Scholes greeks. We exclude options whose
European prices are zero.

4.3 Results

Having described the construction of option portfolios that isolate volatility and jump risk premiums as well as the identification of the corresponding correlation risk premiums, we now present the empirical findings for the S&P 100 index and its constituents.

4.3.1 Volatility Risk Premium

The $VOL_l$ portfolios are only exposed to changes in the variance of the diffusive movements of the underlying as shown in Section 3. In particular, they experience positive returns when the realized change in variance exceeds the risk-neutral expectation (under $Q$).

Index VolRP Table 1 reports summary statistics of the returns on the $VOL_l$ portfolio of the S&P 100 index. It shows that investors pay a premium to insure against volatility risk in the index. The annualized average return of the $VOL_l$ portfolio is $-2.77\%$ with a standard deviation of $23.61\%$ (first column).

Figure 1 shows the time series of $VOL_l$ returns (dots). The daily returns generally fluctuate around zero but occasionally are of great magnitude, both positive and negative. Notably, the time series exhibits an almost zero autocorrelation of $-0.01$ ($p$-value=0.28), which stands in stark contrast to the volatility risk premium that is often calculated as realized volatility minus implied volatility and thus reflects the level of the volatility risk premium. The $VOL_l$ returns, however, load on changes in said premium and may be interpreted as the payoff of a
variance swap. The top three returns were earned on February 27, 2007 (plunge in Chinese stock market, drop in orders for durable goods in the U.S.), September 17, 2008 (global financial crisis, rescue of A.I.G.), and October 27, 1997 (economic crisis in Asia, sell-off in Hong Kong).

**Individual VolRP** Turning to the S&P 100 constituents, Table 1 reports time-series averages of the properties of the cross-sectional distribution of $VOL_i$ returns across the S&P 100 constituents. On average, we are able to construct $VOL$ portfolios for 94 constituents on each day (third column). Furthermore, we find that investors *command* a premium to insure against individual volatility risk. The annualized average return of the $VOL_i$ portfolios amounts to 4.94%, compared to $-2.77\%$ for the index. Stated differently, an equal-weighted investment in the $VOL$ portfolios of all constituents yields positive returns on average, while an investment in the $VOL$ portfolio of the index yields negative returns over the same period. This points towards a negative $CRP_{VOL}$, i.e. to a correlation which is larger under the risk-neutral measure than under the physical measure.

The cross-sectional dispersion in $VOL_i$ is considerable and manifests in an annualized standard deviation of 24.07%. More importantly, the cross-sectional distribution is on average positively skewed. To shed light on the evolution of the cross-sectional dispersion, Figure 1 plots the 5% and 95% percentiles of the cross-sectional distribution of $VOL_i$ returns over time. The deviation between the two percentiles is surprisingly small on most days. On some days, the two percentiles are so close to each other that they no longer cover the $VOL_i$ return of the index, which as a result lies above or below the interval. Once in a while,
the deviation between the two percentiles widens considerably. These days, however, do not necessarily coincide with extreme $VOL_t$ returns. Taken together, the previous findings suggest that the overall positive average return is driven by the right tail of the distribution.

### 4.3.2 Jump Risk Premium

The $JUMP$ portfolios only load on jump risk as shown in Section 3. They exhibit positive returns when the realized jump in the underlying’s price exceeds the risk-neutral expectation (under $Q$).

**Index JRP** Table 1 shows that investors pay a large premium to insure against jump risk in the S&P 100 index. The premium amounts to economically meaningful $-32.81\%$ per year and fluctuates substantially over time as indicated by the annualized standard deviation of $36.91\%$ (second column). Figure 1 plots the time series of $JUMP_t$ returns (dots). The time series shows frequent spikes which are positive in most cases. These extreme positive returns are most likely the result of realized jumps in the index and corroborate the view that the $JUMP$ portfolios are indeed exposed to jump risk. The top three returns were earned on February 27, 2007 (see above), August 8, 2011 (U.S. credit rating downgrade), and October 27, 1997 (see above). While two of these dates happen to coincide with the dates of the top three $VOL_t$ returns, Spearman’s rho over the entire sample period indicates that the rank correlation is actually significantly negative at $-0.17$ ($p$-value=$0.00$) as reported in Panel B of Table 4.

**Individual JRP** With regard to the individual jump risk premiums, Table 1 reports time-series averages of the properties of the cross-sectional distribution of $JUMP_i$ returns across
the S&P 100 constituents. In line with intuition, we find that investors pay a premium to insure against individual jump risk. The annualized average return of the $JUMP_i$ portfolios is $-8.32\%$ (fourth column), compared to $-32.81\%$ for the index. Put differently, an equal-weighted investment in the $JUMP$ portfolios of all constituents yields much smaller negative returns on average than an investment in the $JUMP$ portfolio of the index. This points towards a negative $CRP_{JUMP}$. Yet, the above-mentioned cross-sectional distribution is substantially dispersed with an annualized standard deviation of $47.54\%$ and positively skewed.

Figure 1 shows that the cross-sectional dispersion is not only large but also subject to substantial variation over time. The 5\% and 95\% percentiles are far apart most of the time, and the $JUMP_I$ return of the index typically lies inside the interval. The deviation between the two percentiles widens frequently. Most, but not all, of these days are accompanied by extreme $JUMP_I$ returns. Every now and then, individual jump risk premiums experience strong upward movements, while the index jump risk premium does not change materially.

4.3.3 Correlation Risk Premium

The above discussion has highlighted that the basket of constituents behaves very differently from the index. While the volatility risk premium on the S&P 100 index is negative, the individual volatility risk premiums on the constituents are positive on average. Similarly, the jump risk premium on the S&P 100 index is very large and negative, whereas the individual jump risk premiums on the constituents are negative on average, but much smaller. These two findings provide first indirect and preliminary evidence for the existence of economically meaningful correlation risk premiums $CRP_{VOL}$ and $CRP_{JUMP}$. 
**Volatility CRP**  Table 3 reports summary statistics for the returns of the dispersion trade that is exposed to the correlation of diffusive changes in the stock price and thus collects $CRP_{VOL}$. Investors pay a premium of 13.21% per year to insure against states with a high correlation of the diffusive stock price components in which everyday changes of the stocks tend to have the same direction. This premium is economically meaningful and statistically significant at the 1% level (t-statistic=-3.0183). Note, however, that the returns of the $VOL$ portfolios are scaled by their vega as shown in Section 3, so that the absolute level of the excess returns does not necessarily coincide with the correlation risk premium in index options. We thus report the Sharpe Ratio which is independent of the scaling of the portfolios, since the scaling factor cancels out. Selling insurance against states of high diffusive correlation yields a Sharpe Ratio of 0.6046 per year, which is in the ballpark of the Sharpe Ratio of the S&P 100 index itself.

The second panel of Figure 4 shows the time series of $CRP_{VOL}$ returns. Overall, the returns are often close to zero, negative on 53% of all days within the sample period, and subject to relatively large positive and negative spikes from time to time. Interestingly, periods of more volatile returns of $CRP_{VOL}$ seem only loosely connected to returns of the S&P 100 index. For example, during the burst of the dot-com bubble in 2000 and the global financial crisis in 2008, index returns are the most extreme and volatile, whereas the returns of the $CRP_{VOL}$ portfolio are surprisingly stable and close to zero with almost no swings at all. This suggests that there are no large sudden changes in the correlation or the correlation risk premium for diffusive stock price movements during these prominent crash periods.
The upper graph of Figure 5 gives the histogram for the returns of $CRP_{VOL}$ over the sample period. The time-series distribution of $CRP_{VOL}$ returns is symmetrically centered around zero and close to normally distributed. However, as previously discussed, it exhibits a few extreme returns on both sides of the distribution.

From a theoretical point of view, it is not required that the dispersion trading strategy is easy to implement in practice to identify the $CRP_{VOL}$ premium. However, empirically, it is beneficial if all four options have a similar weight in the trading strategy to minimize any potential data noise evoking from single options. Therefore, Table 2 reports the average number of each option invested in the dispersion trade. Thereby, the first four columns of Panel A report time series quantities of the number of each kind of index option invested in for the long position. As required, short term options ($T_1$) are short and long term options ($T_2$) are long for $VOL_I$. The average number of options is fairly balanced and short term options are on average 0.93 to 0.89 times short and the portfolio consist on average of 1.46 to 1.51 long term options. The time series of these weights is skewed and shows excess kurtosis. Still, no more than 6.33 to 8.05 short term options are being shorted in the most extreme case and no more than 7.28 to 7.44 long term options are held in the maximum. Thus, the portfolio weights are rather homogeneous for the long leg.

The first four columns of Panel B display cross-sectional quantities of the time series averages of each constituents option weights in our basket of firms. Since, we short the basket, short term options are held long and long term potions are shorted. The average amount of options on single stock level is much smaller in magnitude than for the index. On average
we invest in 0.052 to 0.059 short term options and short 0.086 to 0.087 long term options. The cross-sectional distribution is skewed and shows excess kurtosis, which is slightly higher than for the time series of the index. However, this observation mainly emerges from the minimum numbers of options being close to zero. Nevertheless, the amount invested in single options is fairly balanced in the basket.

Figure 6 plots the time series of the long and short leg of the dispersion trade separately and hence allows insights into the sources of the profitability of trading insurance against states of high diffusive correlation. First of all, we observe that both legs offer returns that are of similar magnitude. Yet, the long leg, i.e. the position in the VOL portfolio of the S&P 100 index, varies more than the position in the basket of VOL portfolios of the constituents. This suggests that the return on the volatility portfolio of the index is mostly driven by the exposure to the correlation of continuous stock price movements, whereas the exposures to the individual variances are less relevant. This presumption is confirmed by the rank correlation of 0.85 (p-value=0.00) between \( CRP_{VOL} \) and \( VOL_{I} \) returns as reported in Panel B of Table 4.

The second panel of Figure 7 shows the cumulative log return of \( CRP_{VOL} \). It is rather small and moves around zero in the first half of our sample until 2008, but then starts to steadily drop over time. The premium for the correlation between diffusive stock price changes is thus mainly earned (or paid) after 2008.

To gain a better understanding during which periods the premiums of the \( CRP_{VOL} \) strategy are paid, Figure 8 plots detrended cumulative log returns. In contrast to Figure 7,
we subtract the average log $CRP_{VOL}$ return from the cumulative log returns. Figure 8 is therefore scaled such that the cumulate log returns start and end at zero by construction. A horizontal line of the detrended cumulate log returns thus implies that the strategy earned the average premium during that period. This can be observed for example at the index during 2002 until 2007, as indicated by the graph in the first panel. In contrast, returns of the $CRP_{VOL}$ portfolio show more variation. While returns are higher than average from the beginning of the sample up to mid 2003, they earn below average returns until 2006. Then, the premium is above average until 2008 again and from 2008 onwards below average.

The dashed vertical lines in Figure 8 mark seven events which likely have an influence on the return of the $CRP$ portfolios. While the 1997 mini-crash (1), the bust of the 2007 chínese stock market bubble (3), and the downgraded of EU countries by Standard and Poor’s in 2010 (5) resulted in large positive $CRP_{VOL}$ returns, the default of Lehman Brothers in 2008 (4), fears of contagion of the European sovereign debt crisis to Spain and Italy in 2011 (6), and the 2015 Chinese stock market crash lead to large negative $CRP_{VOL}$ returns. This implies that the first events lead to a payout of the hedge against diffusive volatility risk, since diffusive correlation did go up. In contrast, the other events imply that diffusive correlation did not increase and these events rather resolved uncertainty about the future diffusive correlation. Different to the other events, the beginning of the bust of the dot-com bubble in 2000 (2), goes along with a slight increase of $CRP_{VOL}$ returns on that day only. However, during the whole dot com crash until October 2002, there is a continuous increase in the detrended cumulative log return of $CRP_{VOL}$. This highlights that the price of diffusive correlation risk did tend to go up during the whole crisis period and did not manifest at one specific day.
only, as for the other events.

**Jump CRP**  The premium associated with the risk of co-jumps amounts to $-30.67\%$ per year as shown in Table 3. In other words, investors are willing to pay an economically large premium to eliminate their exposure to correlated jumps among S&P 100 constituents. The premium has an annualized standard deviation of $36.17\%$ and is statistically significant at the $1\%$ level ($p$-value=$0.00$). Hence, selling insurance against states of high jump correlation provides a Sharpe Ratio of $0.85$. Compared to the premium for the correlation of continuous stock price movements, the premium for the correlation of jumps is larger both in terms of average return and Sharpe Ratio.

The lower panel of Figure 4 gives the time series of returns on $CRP_{JUMP}$. An investor who is long the correlation risk from sudden stock price changes occasionally earns very high positive returns (and an investor offering this insurance suffer from large losses). These extreme $CRP_{JUMP}$ returns regularly coincide with extreme index returns. During calm periods when index returns are low and experience little volatility, e.g. between 2004 and 2007, we find stable $CRP_{JUMP}$ returns. In contrast, during the global financial crisis when index returns were the most extreme, $CRP_{JUMP}$ returns spiked more frequently. This suggests that the risk premium stemming from the correlation of jumps is indeed related to stocks’ sudden comovements and investors’ increasing fear of co-jumps during crash periods.

In contrast, there is no obvious relation to extreme $CRP_{VOL}$ returns. In fact, the rank correlation between the two correlation risk premiums over the entire sample period is neg-
ative at −0.25 (p-value=0.00) as reported in Panel B of Table 4.

Figure 5 displays the time-series distribution of $CRP_{JUMP}$ returns and confirms that they are centered below zero and not normally distributed due to their extreme returns and positive skewness. Despite the rare extremely positive returns, $CRP_{JUMP}$ returns are negative on 61% of all days within the sample period. Moreover, the last four columns of Table 2 shows that $CRP_{JUMP}$ invests equally to all option. While the long leg consits on average of 0.6192 to 0.6359 short term options and shorts 0.3792 to 0.3955 long term options, the basket of single stocks shorts on average 0.0012 to 0.0025 short term options and invests in 0.0010 to 0.0018 long term options for a single firm. Again, the smaller number of equity options is by construction to some extent, which increases the cross-sectional kurtosis compared to the long leg. Nevertheless, the single position tend to be small and should be easy to invest.

Looking at the long and short leg of the dispersion trade separately, Figure 6 shows that the $CRP_{JUMP}$ returns are overall very similar to the returns of the long leg. In contrast, returns of the short leg are much smaller in magnitude and show less frequent spikes. This result is not driven by the individual JRPs themselves which were found to be substantial in Section 4.3.2. It is rather the result of individual JRPs that cancel each other out due to diversification as well as the weighting of the short leg when initiating the dispersion trade. Our results are therefore economically plausible and document that market jumps are almost exclusively driven by co-jumps among S&P 100 constituents.

The lower panel of Figure 7 shows the cumulative log return of $CRP_{JUMP}$. It steadily
falls over the whole sample. The negative premium on co-jumps in the stocks is thus not characteristic of one particular time period, but exists over the whole sample period starting in 1996.

The last panel of Figure 8 displays the detrended cumulative log returns of the $CRP_{JUMP}$ portfolios. In contrast to $CRP_{VOL}$, $CRP_{JUMP}$ earned over the majority of the sample the average premium, but experienced large positive spikes from time to time. Most of these spikes go along with one of the seven events, as indicated by the dotted vertical lines. The largest increase in $CRP_{JUMP}$ is observed during the default of Lehman Brothers in 2008 (4). This suggests that during all these events the insurance against co-jumps did pay a positive amount, since co-jumps occurred. In contrast, the only event that did not cause a spike in $CRP_{JUMP}$ is the burst of the dot-com bubble in 2000 (2). During the whole dot-com crash, $CRP_{JUMP}$ shows little to no spikes. This suggest that, only the diffusive correlation did go up, but little to no co-jumps occurred.

5 Conclusion

This paper analyzes the drivers of the correlation risk premium. More precisely, it breaks the correlation risk premium down into two components: a premium related to the correlation of continuous stock price movements and a premium for bearing the risk of co-jumps.

In order to empirically identify the risk premiums, we construct option portfolios for the index as well as for its constituents. These portfolios directly load on changes in the volatil-
ity risk premium and in the jump risk premium, respectively. We find an economically and statistically significant volatility risk premium and jump risk premium for the S&P 100 index. Investors on average pay 2.77% per year in order to hedge market volatility risk and 32.79% to hedge against market jump risk. In contrast, these premiums are much closer to zero for the constituents. Investors on average pay 7.24% per year to hedge against jumps in the constituents. In addition, investors even demand 1.60% per year to hedge out volatility risk of the constituents. The large differences in index and constituents risk premiums are driven by significant correlation risk premiums for both diffusive movements and co-jumps.

By setting up dispersion trades, we identify the correlation risk premium of diffusive movements and co-jumps, separately. While investors are on average willing to pay a premium to hedge both risks, the annualized premium paid to hedge co-jumps is much higher in magnitude (31.26% as opposed to 10.16%). Volatile and extreme market returns primarily go along with changes in the jump correlation risk premium and do not align with the volatility correlation risk premium. In addition, the market volatility and jump risk premiums are almost exclusively explained by the corresponding correlation risk premiums, with a rank correlation of 0.8508 for volatility and 0.9996 for jumps. Overall our results document the importance of correlation as a priced risk factor and highlight that the risk associated with co-jumps is of much greater importance for investors than the risk of correlated diffusive stock price movements.
Figure 1: VOL and JUMP: Index and Cross-Sectional Distribution

This figure shows the time series of daily returns on the VOL and JUMP portfolios of the S&P 100 index (from top to bottom). Data is taken from OptionMetrics within the sample period from January 1996 to December 2017. Shaded areas represent the distance between the 5% and 95% percentiles of the cross-sectional distribution of VOL and JUMP returns across all constituents of the S&P 100 index.
**Figure 2: Quantile-Quantile Plots**

This figure plots the quantile values of VOL and JUMP returns (from top to bottom) on the S&P 100 index and its constituents (from left to right) against standard normal quantiles. Plots for the index are based on the time series of returns within the sample period from January 1996 to December 2017, while plots for the constituents are based on the pooled sample across stocks and time. Solid lines represent theoretical quantile values of fitted normal distributions.
Figure 3: Cross-Sectional Distributions of Individual Risk Premiums

This figure shows the cross-sectional distributions of individual volatility and jump risk premiums (from top to bottom) across the constituents of the S&P 100 index. Individual volatility risk premiums (VolRP) are defined as the annualized time-series averages of the $VOL$ returns on each constituent. Analogously, individual jump risk premiums (JRP) are defined as the annualized time-series averages of $JUMP$ returns. The sample is restricted to constituents for which data is available over a period of at least half a year.
Figure 4: Index Returns and Correlation Risk Premiums

This figure shows the time series of daily excess returns on the S&P 100 index, the correlation risk premium for continuous stock price movements (\(CRP_{VOL}\)), and the correlation risk premium for co-jumps (\(CRP_{JUMP}\)) (from top to bottom). \(CRP_{VOL}\) and \(CRP_{JUMP}\) are based on dispersion trades that go long the \(VOL\) and \(JUMP\) portfolios of the S&P 100 index, respectively, and short a basket of the corresponding portfolios of the constituents. The sample period is from January 1996 to December 2017.
Figure 5: Time-Series Distributions of Correlation Risk Premiums

This figure shows the time-series distributions of correlation risk premiums (CRP) over the sample period from January 1996 to December 2017. CRP_{VOL} and CRP_{JUMP} are based on dispersion trades that go long the VOL and JUMP portfolios of the S&P 100 index, respectively, and short a basket of the corresponding portfolios of the constituents. Solid lines represent fitted normal distributions.
Figure 6: Dispersion Trade Components

This figure shows the time series of the long and short components of the dispersion trades for $CRP_{VOL}$ and $CRP_{JUMP}$. The long leg represents a position in the $VOL$ and $JUMP$ portfolios of the S&P 100 index, respectively, whereas the short leg represents a basket of positions in the corresponding portfolios of the constituents. The sample period is from January 1996 to December 2017.
Figure 7: Cumulative Excess Returns

This figure shows the cumulative log excess returns on the S&P 100 index, the correlation risk premium for continuous stock price movements ($CRP_{VOL}$), and the correlation risk premium for co-jumps ($CRP_{JUMP}$) (from top to bottom). $CRP_{VOL}$ and $CRP_{JUMP}$ are based on dispersion trades that go long the $VOL$ and $JUMP$ portfolios of the S&P 100 index, respectively, and short a basket of the corresponding portfolios of the constituents. The sample period is from January 1996 to December 2017.
Figure 8: Detrended Cumulative Excess Returns

This figure shows detrended cumulative log excess returns on the S&P 100 index, the correlation risk premium for continuous stock price movements ($CRP_{VOL}$), and the correlation risk premium for co-jumps ($CRP_{JUMP}$) (from top to bottom). The detrending procedure adjusts for the average premium of each time-series, such that the graphs start and end at zero. The sample period is from January 1996 to December 2017. Vertical lines indicate the following events: 1) mini-crash (10/27/1997); 2) bust dot-com bubble (3/11/2000); 3) bust chinese stock bubble (2/27/2007); 4) default of Lehman Brothers (9/15/2008); 5) downgrade of EU countries by S&P (4/27/2010); 6) fears of contagion of the European sovereign debt crisis to Spain and Italy (8/4/2011); Chinese stock market crash (8/12/2015)
Table 1: *VOL* and *JUMP*: Summary Statistics

This table reports summary statistics of daily returns on the *VOL* and *JUMP* portfolios of the S&P 100 index and its constituents (from left to right). Data is taken from OptionMetrics. Statistics for the index refer to the properties of the time-series distribution of returns over the sample period from January 1996 to December 2017, while statistics for the constituents refer to time-series averages of the properties of the cross-sectional distribution. Mean and standard deviation are reported at an annual level.
### Panel A: Long Leg

<table>
<thead>
<tr>
<th></th>
<th>(\omega_{\text{call}, T_1})</th>
<th>(\omega_{\text{put}, T_1})</th>
<th>(\omega_{\text{call}, T_2})</th>
<th>(\omega_{\text{put}, T_2})</th>
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<td>-0.7304</td>
<td>1.3601</td>
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<tr>
<td>Skewness</td>
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<td>-1.8512</td>
<td>1.1448</td>
<td>2.1818</td>
</tr>
<tr>
<td>Kurtosis</td>
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<td>10.0563</td>
<td>6.5644</td>
<td>12.7109</td>
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<tr>
<td>Min</td>
<td>-6.3302</td>
<td>-8.0487</td>
<td>0.2458</td>
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<tr>
<td>Max</td>
<td>-0.0466</td>
<td>-0.0057</td>
<td>7.4395</td>
<td>4.4250</td>
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</table>

### Panel B: Short Leg

<table>
<thead>
<tr>
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<th>(\omega_{\text{call}, T_1})</th>
<th>(\omega_{\text{put}, T_1})</th>
<th>(\omega_{\text{call}, T_2})</th>
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<tbody>
<tr>
<td>Mean</td>
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<td>0.0524</td>
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<td>-11.6241</td>
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<td>0.0006</td>
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<tr>
<td>Max</td>
<td>0.6071</td>
<td>0.5891</td>
<td>-0.0017</td>
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</table>

**Table 2: CRP\textsubscript{VOL} and CRP\textsubscript{JUMP}: Option Portfolio Weights**

This table reports summary statistics of the number of options (\(\omega\)) invested in the CRP\textsubscript{VOL} and CRP\textsubscript{JUMP} strategy for the sample period from January 1996 to December 2017. \(T\) stats the options maturity and \(T_1 < T_2\). Panel A reports the quantities for the time-series of the strategies long legs (index) and Panel B reports the quantities for the cross-sectional distribution of time-series averages of the short legs (basket of individual options).
<table>
<thead>
<tr>
<th></th>
<th>( CRP_{VOL} )</th>
<th>( CRP_{JUMP} )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-0.3067</td>
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<tr>
<td>Standard Deviation</td>
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<td>Sharpe Ratio</td>
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<tr>
<td>Kurtosis</td>
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</tbody>
</table>

**Table 3: \( CRP_{VOL} \) and \( CRP_{JUMP} \): Summary Statistics**

This table reports summary statistics of the correlation risk premium for continuous stock price movements (\( CRP_{VOL} \)) and the correlation risk premium for co-jumps (\( CRP_{JUMP} \)) (from left to right). \( CRP_{VOL} \) and \( CRP_{JUMP} \) are based on dispersion trades that go long the VOL and JUMP portfolios of the S&P 100 index, respectively, and short a basket of the corresponding portfolios of the constituents. The sample period is from January 1996 to December 2017. Mean, standard deviation, and the Sharpe ratio are reported at an annual level.
### Table 4: Correlation Coefficients

This table reports pairwise correlation coefficients of daily returns on the S&P 100 index (Index), VOL and JUMP portfolios of the S&P 100 index (VOL\textsubscript{I} and JUMP\textsubscript{I}), the basket of VOL and JUMP portfolios of the constituents (VOL\textsubscript{Short} and JUMP\textsubscript{Short}), and the correlation risk premiums for continuous stock price movements (CRP\textsubscript{VOL}) and co-jumps (CRP\textsubscript{JUMP}). CRP\textsubscript{VOL} and CRP\textsubscript{JUMP} are based on dispersion trades that go long the VOL and JUMP portfolios of the S&P 100 index, respectively, and short a basket of the corresponding portfolios of the constituents. The sample period is from January 1996 to December 2017. Panel A reports Pearson correlation coefficients and Panel B Spearman’s rank correlation coefficients.

#### Panel A: Pearson

<table>
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<tr>
<th></th>
<th>Index</th>
<th>VOL\textsubscript{I}</th>
<th>JUMP\textsubscript{I}</th>
<th>VOL\textsubscript{Short}</th>
<th>JUMP\textsubscript{Short}</th>
<th>CRP\textsubscript{VOL}</th>
<th>CRP\textsubscript{JUMP}</th>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>VOL\textsubscript{I}</td>
<td>-0.2757</td>
<td>1</td>
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<td></td>
</tr>
<tr>
<td>JUMP\textsubscript{I}</td>
<td>-0.2152</td>
<td>0.0036</td>
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<tr>
<td>VOL\textsubscript{Short}</td>
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<td>0.3595</td>
<td>0.3276</td>
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<tr>
<td>JUMP\textsubscript{Short}</td>
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<td>0.1297</td>
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<tr>
<td>CRP\textsubscript{VOL}</td>
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<tr>
<td>CRP\textsubscript{JUMP}</td>
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#### Panel B: Spearman

<table>
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<th>JUMP\textsubscript{I}</th>
<th>VOL\textsubscript{Short}</th>
<th>JUMP\textsubscript{Short}</th>
<th>CRP\textsubscript{VOL}</th>
<th>CRP\textsubscript{JUMP}</th>
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<td>-0.2132</td>
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<td>CRP\textsubscript{VOL}</td>
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<td>CRP\textsubscript{JUMP}</td>
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<td>0.1168</td>
<td>0.4061</td>
<td>-0.2422</td>
<td>1</td>
</tr>
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Appendix A  Proofs

The dynamics of stock $i$ and its diffusive variance $V_i$ are given by

$$\frac{dS_{i,t}}{S_{i,t-}} = \mu_i dt + \sqrt{V_{i,t}} dW^S_{i,t} + \frac{\Delta S_{i,t}}{S_{i,t-}},$$

$$dV_{i,t} = \mu_{V_i} dt + \sigma_{V_i} \sqrt{V_{i,t}} dW^V_{i,t} + \Delta V_{i,t},$$

where $\Delta S_i$ and $\Delta V_i$ denote the jump components of the stock and variance dynamics. We assume
that the jumps are driven by Poisson processes. The jump intensities are either constant or depend
on the local diffusive variance.

The index $S_I$ and its local diffusive variance $V_I$ are equal to

$$S_{I,t} = \sum_{i=1}^{N} w_i S_{i,t}$$

$$V_{I,t} = \sum_{i=1}^{N} w_i^2 V_{i,t} + \sum_{i=1}^{N} \sum_{j=1,j\neq i}^{N} w_i w_j \sqrt{V_{i,t} V_{j,t}} \rho_{ij,t},$$

where $\rho_{ij}$ is correlation of stock $i$ and $j$, and $w_i$ denotes the number of stocks $i$ in the index. In the
following, we will not use the pairwise correlations $\rho_{ij}$ between the stocks, but the equi-correlation
$\rho$ which results in the same variance of the index as do the individual correlations:

$$V_{I,t} = \sum_{i=1}^{N} w_i^2 V_{i,t} + \sum_{i=1}^{N} \sum_{j=1,j\neq i}^{N} w_i w_j \sqrt{V_{i,t} V_{j,t}} \rho_{t}.$$

For the dynamics of the derivative price $C_i = C_i(t, S_{i,t}, V_{i,t})$, it holds that

$$dC_{i,t} = \frac{\partial C_i}{\partial t} dt + \frac{\partial C_i}{\partial S_{i,t}} dS^c_{i,t} + \frac{\partial C_i}{\partial V_{i,t}} dV^c_{i,t} + \frac{1}{2} \frac{\partial^2 C_i}{\partial S_{i,t}^2} (dS^c_{i,t})^2 + \frac{1}{2} \frac{\partial^2 C_i}{\partial V_{i,t}^2} (dV^c_{i,t})^2 + \frac{\partial^2 C_i}{\partial S_{i,t} \partial V_{i,t}} dS^c_{i,t} dV^c_{i,t}

+ C_i(t, S_{i,t-} + \Delta S_{i,t}, V_{i,t-} + \Delta V_{i,t}) - C(t, S_{i,t-}, V_{i,t-}),$$
where the superscript \( c \) denotes the continuous parts of the changes \( dS_i \) and \( dV_i \). The fundamental partial differential equation for \( C \) is given by

\[
\begin{align*}
    rC_{i,t}dt &= \frac{\partial C_i}{\partial t} dt + \frac{\partial C_i}{\partial S_i} E_t^Q [dS_{i,t}^c] + \frac{\partial C_i}{\partial V_i} E_t^Q [dV_{i,t}^c] + \frac{1}{2} \frac{\partial^2 C_i}{\partial S_i^2} E_t^Q [(dS_{i,t}^c)^2] + \frac{1}{2} \frac{\partial^2 C_i}{\partial V_i^2} E_t^Q [(dV_{i,t}^c)^2] \\
    &+ \frac{\partial^2 C_i}{\partial S_i \partial V_i} E_t^Q [dS_{i,t}^c dV_{i,t}^c] + E_t^Q \left[ C_i(t, S_{i,t-} + \Delta S_{i,t}, V_{i,t-} + \Delta V_{i,t}) - C(t, S_{i,t-}, V_{i,t-}) \right].
\end{align*}
\]

Subtracting the two equations from each other gives

\[
\begin{align*}
dC_{i,t} - rC_{i,t} &= \frac{\partial C_i}{\partial S_i} \left( dS_{i,t}^c - E_t^Q [dS_{i,t}^c] \right) + \frac{\partial C_i}{\partial V_i} \left( dV_{i,t}^c - E_t^Q [dV_{i,t}^c] \right) \\
    &+ C_i(t, S_{i,t-} + \Delta S_{i,t}, V_{i,t-} + \Delta V_{i,t}) - C(t, S_{i,t-}, V_{i,t-}) \\
    &- E_t^Q \left[ C_i(t, S_{i,t-} + \Delta S_{i,t}, V_{i,t-} + \Delta V_{i,t}) - C(t, S_{i,t-}, V_{i,t-}) \right],
\end{align*}
\]

where we have used that \((dS_{i,t}^c)^2 = S_{i,t}^2 V_{i,t} dt\), so that \((dS_{i,t}^c)^2 = E_t^Q [(dS_{i,t}^c)^2]\), and that similar relations hold true for \((dV_{i,t}^c)^2\) and \(dS_{i,t}^c dV_{i,t}^c\). The expected excess return of \( C \) thus depends on the risk premia for diffusive stock price risk (scaled by delta), for diffusive variance risk (scaled by vega), and on the premium for jump risk in the stock and in its variance.

For the following calculations, we approximate the jump in the price of the derivative by a second order Taylor-series. The return of \( C \) becomes

\[
\begin{align*}
dC_{i,t} &= C_{i,t} dt + \frac{\partial C_i}{\partial S_i} (dS_{i,t} - r S_{i,t-} dt) + \frac{\partial C_i}{\partial V_i} (dV_{i,t} - E_t^Q [dV_{i,t}]) \\
    &+ \frac{1}{2} \frac{\partial^2 C_i}{\partial S_i^2} \left( (\Delta S_{i,t})^2 - E_t^Q [(\Delta S_{i,t})^2] \right) + \frac{1}{2} \frac{\partial^2 C_i}{\partial V_i^2} \left( (\Delta V_{i,t})^2 - E_t^Q [(\Delta V_{i,t})^2] \right) \\
    &+ \frac{\partial^2 C_i}{\partial S_i \partial V_i} \left( \Delta S_{i,t} \Delta V_{i,t} - E_t^Q [\Delta S_{i,t} \Delta V_{i,t}] \right) + \xi_{C_{i,t}} - E_t^Q [\xi_{C_{i,t}}],
\end{align*}
\]

where \(\xi_{C_i}\) is the remainder term of the second-order Taylor approximation. The expected excess
The price of a derivative on the index depends on all stock price levels and on all local variances. In the following, we make the simplifying assumption that it depends on the level \( S_I \) and the variance \( V_I \) of the index only. The SDE for the derivative \( C_I \) then has the same form as the SDE for the derivatives \( C_i \), and the same holds true for its excess return.

**Variance portfolio** We now consider special portfolios (or derivatives). The first portfolio \( VOL \) has no exposure to stock price risk and jumps in the stock price (up to order two), but is only exposed to variance risk. We do not focus on the construction of such a portfolio, but are interested in its dynamics.

The variance-portfolio \( VOL \) is delta-gamma-neutral:

\[
\frac{\partial VOL_I}{\partial S_I} = \frac{\partial VOL_i}{\partial S_i} = 0 \quad \text{and} \quad \frac{\partial^2 VOL_I}{\partial S_I^2} = \frac{\partial^2 VOL_i}{\partial S_i^2} = 0.
\]

We furthermore assume that their exposures to variance risk differ from zero.

The dynamics of the variance portfolio for stock \( i \) follow from Equation (27) and are given by

\[
dVOL_{i,t} = rVOL_{i,t}dt + \frac{\partial VOL_i}{\partial V_i} \left( dV_{i,t} - E^Q_t[dV_{i,t}] \right) + \frac{1}{2} \frac{\partial^2 VOL_i}{\partial V_i^2} \left( (\Delta V_{i,t})^2 - E^Q_t[(\Delta V_{i,t})^2] \right) \\
+ \frac{\partial^2 VOL_i}{\partial S_i \partial V_i} \left( \Delta S_{i,t} \Delta V_{i,t} - E^Q_t[\Delta S_{i,t} \Delta V_{i,t}] \right) + \xi_{VOL_{i,t}} - E^Q_t[\xi_{VOL_{i,t}}].
\]

This portfolio allows to trade changes in the variance \( V_i \) of the stock. Its exposure to changes in
the diffusive variance of the stock is given by its vega. Without jumps in variance, the additional terms vanish.

Its expected excess return is the premium paid on its exposure to variance risk. In case there are variance jumps, it also pays a premium on the variance of variance \( (E_t^P[(\Delta V_{i,t})^2] - E_t^Q[(\Delta V_{i,t})^2]) \) and on the covariance of jumps in the stock and its variance \( (E_t^P[\Delta S_{i,t}\Delta V_{i,t}] - E_t^Q[\Delta S_{i,t}\Delta V_{i,t}]) \).

The dynamics of the \( VOL \)-portfolio for the index are

\[
\begin{align*}
  dVOL_{I,t} &= rVOL_{I,t}dt + \left( \frac{\partial VOL_I}{\partial V_I} dV_{I,t} - E_t^Q[dV_{I,t}] \right) + \frac{1}{2} \frac{\partial^2 VOL_I}{\partial V_I^2} \left( (\Delta V_{I,t})^2 - E_t^Q[(\Delta V_{I,t})^2] \right) \\
  &\quad + \frac{\partial^2 VOL_I}{\partial S_I\partial V_I} \left( \Delta S_{I,t}\Delta V_{I,t} - E_t^Q[\Delta S_{I,t}\Delta V_{I,t}] \right) + \xi_{VOL_I,t} - E_t^Q[\xi_{VOL_I,t}].
\end{align*}
\]

The diffusive variance of the index depends on the diffusive variances of the stocks and their correlations. Instead of the pairwise correlations \( \rho_{ij} \) between the stocks, we consider their equicorrelation \( \rho \) which results in the same variance of the index as do the individual correlations. With \( V_I = V_I(V_1, \ldots, V_N, \rho) \), the dynamics of the index variance are

\[
\begin{align*}
  dV_I &= \sum_{i=1}^{N} \frac{\partial V_I}{\partial V_i} dV^c_i + \frac{\partial V_I}{\partial \rho} d\rho^c + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{\partial^2 V_I}{\partial V_i \partial V_j} dV^c_i dV^c_j + \frac{1}{2} \frac{\partial^2 V_I}{\partial \rho^2} (d\rho^c)^2 + \sum_{i=1}^{N} \frac{\partial^2 V_I}{\partial V_i \partial \rho} dV^c_i d\rho^c \\
  &\quad + V_I(V_{1,t-} + \Delta V_{1,t}, \ldots, V_{N,t-} + \Delta V_{N,t}, \rho_{t-} + \Delta \rho_t) - V_I(V_{1,t-}, \ldots, V_{N,t-}, \rho_{t-}).
\end{align*}
\]

Again, a Taylor-series expansion of the last term gives

\[
\begin{align*}
  dV_I &= \sum_{i=1}^{N} \frac{\partial V_I}{\partial V_i} dV_i + \frac{\partial V_I}{\partial \rho} d\rho + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{\partial^2 V_I}{\partial V_i \partial V_j} dV_i dV_j + \frac{1}{2} \frac{\partial^2 V_I}{\partial \rho^2} (d\rho)^2 + \sum_{i=1}^{N} \frac{\partial^2 V_I}{\partial V_i \partial \rho} dV_i d\rho + \xi_{VI,t}.
\end{align*}
\]

Given the \( VOL \)-portfolios for the single stocks and the index, the next step is to construct a portfolio with zero exposure to the individual variances. To hedge the exposure of \( VOL_I \) against changes in the variances \( V_i \) of the stocks, we add a short position in the individual \( VOL \)-portfolios.
$VOL_i$, where the size of the position in $VOL_i$ is

$$\frac{\partial VOL_i}{\partial V_I} \frac{\partial V_I}{\partial V_i}.$$

The dynamics of the resulting portfolio $CRP_{VOL}$ are

$$dCRP_{VOL,t} = r \left( VOL_{I,t} - \sum_{i=1}^{N} \frac{\partial VOL_i}{\partial V_I} \frac{\partial V_I}{\partial V_i} VOL_{i,t} \right) dt + \frac{\partial VOL_I}{\partial V_I} \frac{\partial V_I}{\partial \rho} \left( d\rho_t - E_t^Q[d\rho_t] \right) + \frac{1}{2} \frac{\partial VOL_I}{\partial V_I} \left( (\Delta \rho_t)^2 - E_t^Q[(\Delta \rho_t)^2] \right)$$

$$+ \frac{1}{2} \frac{\partial VOL_I}{\partial V_I} \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{\partial^2 V_I}{\partial V_i \partial V_j} \left( \Delta V_{i,t} \Delta V_{j,t} - E_t^Q[\Delta V_{i,t} \Delta V_{j,t}] \right)$$

$$+ \frac{\partial VOL_I}{\partial V_I} \sum_{i=1}^{N} \frac{\partial^2 V_I}{\partial V_i \partial \rho} \left( \Delta V_{i,t} \Delta \rho_t - E_t^Q[\Delta V_{i,t} \Delta \rho_t] \right)$$

$$+ \frac{1}{2} \frac{\partial^2 C_I}{\partial S_I \partial V_I} \left[ (\Delta S_{I,t})^2 - E_t^Q[(\Delta S_{I,t})^2] \right] - \sum_{i=1}^{N} \frac{\partial VOL_i}{\partial V_I} \frac{\partial^2 VOL_i}{\partial VOL_I \partial V_I} \frac{\partial V_I}{\partial V_i} \left( (\Delta V_{i,t})^2 - E_t^Q[(\Delta V_{i,t})^2] \right)$$

$$+ \frac{\partial^2 VOL_I}{\partial S_I \partial V_I} \left( \Delta S_{I,t} \Delta V_{I,t} - E_t^Q[\Delta S_{I,t} \Delta V_{I,t}] \right) - \sum_{i=1}^{N} \frac{\partial VOL_i}{\partial V_I} \frac{\partial^2 VOL_i}{\partial S_I \partial V_I} \frac{\partial V_I}{\partial V_i} \left( \Delta S_{i,t} \Delta V_{i,t} - E_t^Q[\Delta S_{i,t} \Delta V_{i,t}] \right)$$

$$+ \xi_{CRP_{VOL,t}} - E_t^Q[\xi_{CRP_{VOL,t}}].$$

In the following, we furthermore assume that the vegas of the index and stock variance portfolios coincide:

$$\frac{\partial VOL_I}{\partial V_I} = \frac{\partial VOL_i}{\partial V_i} = \text{constant}$$

The return of the portfolio $CRP_{VOL}$ then simplifies to

$$dCRP_{VOL,t}$$
\[
= r \left( \text{VOL}_{t} - \sum_{i=1}^{N} \frac{\partial V}{\partial V_i} \text{VOL}_{i,t} \right) \, dt \\
+ \frac{\partial \text{VOL}_I}{\partial V_I} \frac{\partial V_I}{\partial \rho} \left( d \rho_t - E_{t}^{Q}[d \rho_t] \right) \\
+ \frac{1}{2} \frac{\partial \text{VOL}_I}{\partial V_I} \left( (\Delta \rho_t)^2 - E_{t}^{Q}[\Delta \rho_t]^2 \right) \\
+ \frac{1}{2} \frac{\partial \text{VOL}_I}{\partial V_I} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \frac{\partial^2 V_I}{\partial V_i \partial V_j} \left( \Delta V_{i,t} \Delta V_{j,t} - E_{t}^{Q}[\Delta V_{i,t} \Delta V_{j,t}] \right) \\
+ \frac{\partial^2 \text{VOL}_I}{\partial S_I \partial V_I} \left( \Delta S_{I,t} \Delta V_{I,t} - E_{t}^{Q}[\Delta S_{I,t} \Delta V_{I,t}] \right) \\
+ \frac{\partial^2 \text{VOL}_I}{\partial V_I^2} \left( \Delta V_{I,t} \Delta V_{I,t} - E_{t}^{Q}[\Delta V_{I,t}]^2 \right) \\
+ \frac{\partial^2 C_I}{\partial S_I \partial V_I} \left( \Delta S_{I,t} \Delta V_{I,t} - E_{t}^{Q}[\Delta S_{I,t} \Delta V_{I,t}] \right) \\
+ \xi_{\text{CRP}_{\text{VOL}_I}} - E_{t}^{Q}[\xi_{\text{CRP}_{\text{VOL}_I}}].
\]

If there are no jumps in variances and in the correlation, the excess return of this portfolio is driven by the changes in the equi-correlation:

\[
d\text{CRP}_{\text{VOL}_I} - r \text{CRP}_{\text{VOL}_I} dt = \frac{\partial \text{VOL}_I}{\partial V_I} \frac{\partial V_I}{\partial \rho} \left( d \rho_t - E_{t}^{Q}[d \rho_t] \right) + \xi_{\text{CRP}_{\text{VOL}_I}} - E_{t}^{Q}[\xi_{\text{CRP}_{\text{VOL}_I}}]
\]

The expected excess return depends on the premium paid for the exposure to this equi-correlation, scaled by the vega, and the exposure of the index’ variance to the equi-correlation.

**Jump portfolio** The second type of portfolio we look at are portfolios with no exposure to linear stock price risk and variance risk, but an exposure to jumps only. For the jump-portfolio \( JUMP \), we consider individual derivatives (portfolios) which are delta-vega-neutral:

\[
\frac{\partial JUMP_I}{\partial S_I} = \frac{\partial JUMP_I}{\partial S_i} = 0 \quad \text{and} \quad \frac{\partial JUMP_I}{\partial V_I} = \frac{\partial JUMP_I}{\partial V_i} = 0.
\]
The dynamics of the resulting jump portfolio for stock $i$ are

\[
dJUMP_{i,t} = r_{JUMP_{i,t}}dt + \frac{1}{2} \frac{\partial^2 JUMP_{i}}{\partial S_i^2} \left( (\Delta S_{i,t})^2 - E_t^Q[(\Delta S_{i,t})^2] \right) \\
+ \frac{1}{2} \frac{\partial^2 JUMP_{i}}{\partial V_i^2} \left( (\Delta V_{i,t})^2 - E_t^Q[(\Delta V_{i,t})^2] \right) \\
+ \frac{\partial^2 JUMP_{i}}{\partial S_i \partial V_i} \left( \Delta S_{i,t} \Delta V_{i,t} - E_t^Q[\Delta S_{i,t} \Delta V_{i,t}] \right) + \xi_{JUMP_{i,t}} - E_t^Q[\xi_{JUMP_{i,t}}].
\]

The dynamics of the index jump portfolio are

\[
dJUMP_{I,t} = r_{JUMP_{I,t}}dt + \frac{1}{2} \frac{\partial^2 JUMP_{I}}{\partial S_I^2} \left( (\Delta S_{I,t})^2 - E_t^Q[(\Delta S_{I,t})^2] \right) \\
+ \frac{1}{2} \frac{\partial^2 JUMP_{I}}{\partial V_I^2} \left( (\Delta V_{I,t})^2 - E_t^Q[(\Delta V_{I,t})^2] \right) \\
+ \frac{\partial^2 JUMP_{I}}{\partial S_I \partial V_I} \left( \Delta S_{I,t} \Delta V_{I,t} - E_t^Q[\Delta S_{I,t} \Delta V_{I,t}] \right) + \xi_{JUMP_{I,t}} - E_t^Q[\xi_{JUMP_{I,t}}].
\]

With Equation (25), the squared jump in the index is

\[
(\Delta S_{I,t})^2 = \sum_{i=1}^{N} w_i^2 (\Delta S_{i,t})^2 + \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} w_i w_j \Delta S_{i,t} \Delta S_{j,t}.
\]

The next step is to set up a portfolio which is no longer exposed to individual squared jumps, but only to joint jumps. To partly hedge against the JUMP-portfolio of the index against individual jumps, we add a short position in the individual JUMP-portfolios, where the size of the position in $JUMP_{i}$ is

\[
\frac{\partial^2 JUMP_{I}}{\partial S_I^2} w_i^2.
\]
The dynamics of the resulting jump portfolio $CRP_{JUMP}$ are

$$dCRP_{JUMP,t} = r \left( JUMP_{t} - \sum_{i=1}^{N} \frac{\partial^2 JUMP_i}{\partial S^2_i} w_i^2 JUMP_{i,t} \right) dt$$

$$+ \frac{1}{2} \frac{\partial^2 JUMP_I}{\partial S^2_I} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} w_i w_j \left( \Delta S_{i,t} \Delta S_{j,t} - E^Q_t [\Delta S_{i,t} \Delta S_{j,t}] \right)$$

$$+ \frac{1}{2} \frac{\partial^2 JUMP_I}{\partial V^2_I} \left( (\Delta V_{I,t})^2 - E^Q_t [(\Delta V_{I,t})^2] \right)$$

$$+ \frac{\partial^2 JUMP_I}{\partial S_I \partial V_I} \left( \Delta S_{I,t} \Delta V_{I,t} - E^Q_t [\Delta S_{I,t} \Delta V_{I,t}] \right)$$

$$+ \xi_{CRP_{JUMP},t} - E^Q_t [\xi_{CRP_{JUMP},t}].$$

In the following, we furthermore assume that

$$\frac{\partial^2 JUMP_I}{\partial S^2_I} = \frac{\partial^2 JUMP_i}{\partial S^2_i} = \text{constant.}$$

The return of the portfolio $CRP_{JUMP}$ then simplifies to

$$dCRP_{JUMP,t}$$

$$= r CRP_{JUMP,t} dt + \frac{1}{2} \frac{\partial^2 JUMP_I}{\partial S^2_I} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} w_i w_j \left( \Delta S_{i,t} \Delta S_{j,t} - E^Q_t [\Delta S_{i,t} \Delta S_{j,t}] \right)$$

$$+ \frac{1}{2} \frac{\partial^2 JUMP_I}{\partial V^2_I} \left( (\Delta V_{I,t})^2 - E^Q_t [(\Delta V_{I,t})^2] \right)$$

$$+ \frac{\partial^2 JUMP_I}{\partial S_I \partial V_I} \left( \Delta S_{I,t} \Delta V_{I,t} - E^Q_t [\Delta S_{I,t} \Delta V_{I,t}] \right)$$

$$+ \xi_{CRP_{JUMP},t} - E^Q_t [\xi_{CRP_{JUMP},t}].$$
It earns the premium on joint stock price jumps (or, equivalently, on the covariance of stock price jumps), on the variance of variance jumps in the stock and in the index, and on the covariance of stock price jumps and variance jumps in the stocks and in the index.

If there are no jumps in variances and in the correlation, the excess return of this portfolio is driven by the joint jumps in stock prices:

\[
dCRP_{\text{JUMP},t} - rCRP_{\text{JUMP},t} dt = \frac{1}{2} \frac{\partial^2 JUMP_I}{\partial S_i^2} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} w_i w_j \left( \Delta S_{i,t} \Delta S_{j,t} - E_t^Q [\Delta S_{i,t} \Delta S_{j,t}] \right) + \xi_{CRP_{\text{JUMP},t}} - E_t^Q [\xi_{CRP_{\text{JUMP},t}}].
\]

The expected excess return depends on the premium paid for the exposure to joint jumps, scaled by the gamma of the JUMP-portfolio of the index. This is equal to the premium on the covariance of stock price jumps.

**Appendix B  Dividend Forecasts**

We explicitly account for expected dividends when repricing the American options in binomial trees of Cox et al. (1979)-type as discussed in Section 4.2 and Appendix C. When repricing index options, we use the S&P 100 dividend yield provided by OptionMetrics. For options on the constituents, we collect discrete dividends from OptionMetrics and complement them with any non-redundant dividend data from CRSP. In contrast to the index dividend yield, the discrete dividends of the constituents require extensive data cleansing. We exclude any distributions that are neither regularly-scheduled nor special dividends (such as rights offerings, spin-offs, etc.) and eliminate cancelled and liquidating distributions. We keep only cash dividends denominated in U.S. dollars. We collect each dividend’s declaration date, ex-dividend date, amount, and frequency and check the consistency of the entries. In particular, we discard a dividend if the reported declaration
date lies after its ex-dividend date and if its ex-dividend date does not occur within the declared dividend frequency. We aggregate multiple dividends that are announced on the same declaration date. The dividend forecast is formed as follows. Once a dividend has been declared, we use the announced amount and ex-dividend date as forecast. After the ex-dividend date, we forecast the next dividend based on the last known dividend frequency and amount. This forecast is revised as soon as the next dividend is declared. Throughout the data cleansing and forecasting process, we distinguish between regular and special dividends and treat them differently when repricing the American options as discussed in Appendix C.

**Appendix C   European Option Prices**

To calculate synthetic European option prices, we rely on the implied volatilities provided by OptionMetrics. OptionMetrics uses binomial trees following Cox et al. (1979) to estimate implied volatilities. We employ the same method to calculate European and American option prices.

At every day in the sample and for all traded options on that day, we construct a binomial tree following Cox et al. (1979) using the quoted implied volatility and 1000 time steps. As the starting stock price in the tree, we use the close price of the stock on that day and subtract the present value of future dividend payments, which are paid up to the option’s time-to-maturity. In a second step, we add the present value of the cumulated future dividend payments at each knot in the tree until the dividend is paid. Given a tree, we recalculate the America option price and exclude any tree if the recalculated American option price divides more than 1% from the observed midprice of the option. Afterwards, we use the remaining trees to calculate synthetic European option prices.
References


