Financially constrained strategic arbitrage*

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Abstract

We develop an equilibrium model of strategic trading under financial constraints. Investors have access to a fundamentally riskless arbitrage opportunity, but may be forced to fire-sell if their capital does not fully cover their losses. Investors internalize both their price impact and the effect of price movements on the constraints of all market participants, giving rise to a strategic motive for the less exposed investors to induce fire-sales of more exposed ones. Ex ante, the presence of predatory risk leads to lower investment by all traders. We show the implications of strategic trading on price dynamics, returns characteristics, and leverage cross-section and dynamics.

Keywords: limits to arbitrage, liquidity, wealth effects, strategic trading, endogenous risk

JEL Classification: C72, D43, G10

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1 Introduction

Large traders, such as dealers, hedge funds and other financial institutions play an important role in financial markets when exploiting the relative mispricing of assets: through their trading, these arbitrageurs bring prices closer to fundamentals and provide liquidity to other market participants. However, their willingness to provide liquidity can be subject to many factors. For example, institutional investors' trades can have significant price impact as their strategies often involve dealing with large positions in assets held by a relatively few number of investors. Also, wealth constraints and risk management policies crucially affect arbitrageurs' allocation of capital to trading opportunities. Therefore, when a small number of arbitrageurs are present in a market, in addition to internalizing their own price impact when making investment decisions, they also internalize the impact of their trades on the constraints and portfolio decisions of other large traders.

In this paper we consider the interaction between strategic arbitrage trading and financial constraints, and study the effect of this interaction on asset price dynamics. We show that strategic investors have an incentive to exploit other market participants' constraints by manipulating the prices in order to induce liquidations and fire-sales. Whether such predatory trading occurs in equilibrium depends on the heterogeneity in the size of investors' positions relative to their capital: when exposures are similar, investors behave cooperatively and spread their orders over several trading periods to minimize price impact; however, if there is significant difference between exposures, less exposed investors predate on the more exposed ones, forcing them to exit the market. Ex ante, predation risk makes agents reluctant to invest even when investment opportunities are fundamentally risk-free. As a result, investors do not correct deviations of asset prices from fundamentals immediately, exploiting such arbitrage opportunities only shortly before they disappear exogenously.

To analyze the effect of financial constraints on arbitrage trading we build on Gromb and Vayanos (2002) and Kondor (2009). Two assets with identical payoffs are traded in segmented markets at different prices, and arbitrageurs take long-short positions to exploit this mispricing. In the absence of arbitrageurs, the gap between prices would be constant for a finite time horizon, then it would disappear exogenously. Therefore, the arbitrage is fundamentally risk-less. Arbitrageurs, by trading, endogenously determine the size of the gap. If arbitrageurs on aggregate buy more of the cheap asset and short more of the expensive asset, i.e. they short the gap, prices of the assets converge, and the gap shrinks. On the other hand, if arbitrageurs sell the cheap asset and buy the more expensive, i.e. they go long in the gap, prices diverge, and the gap widens. We consider a finite set of large arbitrageurs who invest in this arbitrage opportunity. Arbitrageurs have two important features. First, they are strategic and internalize their price impact

on the gap. Second, they have to fully collateralize for losses and, if their capital is insufficient, they must close their positions and leave the market. This constraint implies that arbitrageurs' capital limits the positions they can take if they do not want to violate the constraint. We show that this constraint can interact with arbitrageurs' strategic motive and provide incentives for some traders to make prices diverge and trigger the insolvency of other market participants.

The main results are obtained in a framework with two arbitrageurs and non-zero starting positions. We show that their behaviour depends on their exposure to the arbitrage opportunity. In particular, if traders have similar proportion of capital invested in the assets, they behave cooperatively, and the equilibrium gap decreases quickly: arbitrageurs compete with each other and rush to the market, causing prices converge while the wealth constraint never binds. However, if there is a significant difference in the proportion of their capital invested in the arbitrage opportunity, the trader with a lower proportion of wealth invested in the gap predates on the trader with a high proportion of wealth invested in the gap: the former (short-)sells the cheap asset and buys the expensive one, causing the prices to diverge. Arbitrageurs suffer losses, but these losses are higher for the arbitrageur who was initially more exposed to the gap. If she violates her wealth constraint, she is forced to close her positions in the following period. This in turn widens the price gap even more, and makes future investment opportunities even better for the sole solvent arbitrageur.¹

Next, we examine traders' optimal initial position in the arbitrage opportunity, given the risk of becoming exposed to predation by other traders described above. It is important to emphasize that the possible future losses are due to predatory behaviour as opposed to unforeseen shocks, and are all subject to more than one arbitrageur being present in the market. As liquidation is costly, the threat of predation by other arbitrageurs implies that strategic traders reduce their initial investments so that liquidation

¹The following quote provides an insight on the recent forced liquidation of Focus Capital, by suggesting that arbitrageurs occasionally decide to withdraw liquidity from markets, making prices diverge from fundamentals and forcing distressed institutions to unwind some of their positions at great losses:

"In a letter to investors, the founders of Focus, Tim O'Brien and Philippe Bubb, said it had been hit by "violent short-selling by other market participants", which accelerated when rumors that it was in trouble circulated. Sharp drops in the value of its investments led its two main banks to force it to sell last Tuesday, according to the letter." (Financial Times, March 4, 2008)

Other famous examples of predatory trading include the near-collapse of Long-Term Capital Management (LTCM) in 1998, when Goldman Sachs and other counterparties strategically traded against LTCM to aggravate its situation. The proposal of UBS Warburg, to take over Enron's traders without taking over its trading positions, was opposed on the same ground - it presented potential predatory risk (AFX News Limited, AFX-Asia, January 18, 2002). See Edwards (1999) and Loewenstein (2000) for detailed analyses on the LTCM crisis, and Table I of Brunnermeier and Pedersen (2005) for an extensive list on examples of predatory trading.

does not happen in equilibrium. However, as long as one arbitrageur has a much higher level of capital than the other, it does not affect the gap path significantly, because the increased investment of the former compensates for the small position taken by the latter. We show that the wealth constraint has its strongest effect on the gap process when arbitrageurs start with similarly low level of capital. In this case arbitrageurs are reluctant to invest much, as shorting one more unit of the gap has a large effect on the proportion of wealth put into the arbitrage opportunity, and exposes the trader to become a prey of the other arbitrageur. Therefore, the gap changes very little initially, and agents only race to close the arbitrage opportunity later.

These results are very much in contrast to the case with a single (monopolistic) arbitrageur. She knows that she faces a one-sided bet: if the trader shorts the gap, prices converge. This implies that her mark-to-market wealth never decreases, and the wealth constraint never binds. In the absence of other arbitrageurs, she gradually provides liquidity to the local markets to minimize her price impact, and her profits are not competed away. Our analysis suggests that as the presence of other arbitrageurs creates predatory risk, increased competition in liquidity provision does not always imply that market segmentation and abnormal profits disappear quickly.

The model presented here is related to several strands of the literature, in addition to that on financial constraints. It is connected to models of limited arbitrage, including Shleifer and Vishny (1997), Xiong (2001), Gromb and Vayanos (2002), and Liu and Longstaff (2004). A large part of this literature focuses on potential losses in convergence trading due to institutional frictions or capital constraints. The common element in these models is that their mechanisms amplify exogenous shocks: arbitrageurs have to liquidate part of their positions after an initial shock to prices which creates further adverse price movements and liquidations. In our model, the amplification mechanism is endogenized and entirely strategic. Arbitrageurs are not fully competitive, and hence some of them can exploit their price impact to force others into distress. This type of strategic interaction, which is missing from the above papers, makes a fundamentally riskless arbitrage opportunity risky.

The two papers closest to our analysis on financially constrained arbitrage are Kondor (2009) and Attari and Mello (2006). Kondor (2009) develops an equilibrium model of convergence trading and its impact on asset prices, where arbitrageurs optimally decide how to allocate their limited capital over time. He shows that prices of identical assets can diverge even if the constraints faced by arbitrageurs are not binding, and that in equilibrium arbitrageurs' activity endogenously generates losses with positive probability, even if the trading opportunity is fundamentally riskless. Whereas he works with one representative arbitrageur and his focus is on the endogenous determination of the price gap, we study the trading behaviour of imperfectly competitive arbitrageurs, who try

to exploit the vulnerability of each other by engaging in predatory trading. Attari and Mello (2006) analyze the trading strategy of a monopolistic arbitrageur who can, to some extent, influence the dynamics of prices on which capital requirements are based. They show that financial constraints are responsible for volatile prices and for time variation in the correlations of prices across markets. In contrast, our model allows for heterogeneity among arbitrageurs and focuses on the strategic interaction among them. Moreover, the lack of uncertainty allows us to provide analytical solution in our setting, while they can only numerically solve their model.

The model also belongs to those on predatory trading (i.e. trading that induces and/or exploits the need of other investors to reduce their positions) and forced liquidation. Brunnermeier and Pedersen (2005) show that if a distressed trader needs to sell for exogenous reasons, others also sell and subsequently buy back the asset. This leads to price overshooting and a reduced liquidation value for the distressed trader. Hence, the market is illiquid when liquidity is most needed. Carlin et al. (2007) analyze how episodic illiquidity can arise from a breakdown in cooperation between market participants. They consider a repeated setting of a predatory stage game and show that while most of the time traders provide apparent liquidity to each other, when the stakes are high, cooperation breaks down, leading to sudden and short-lived illiquidity. In these papers liquidation is exogenously imposed on some agents, as arbitrageurs become distressed due to an adverse shock and have to liquidate, while solvent traders take advantage of them. In contrast, the model presented here endogenizes the solvency of arbitrageurs: as capital requirements depend on observed prices, arbitrageurs might be able to induce the distress of others by manipulating the price, thus giving rise to predatory risk, which discourages investors from investing in the arbitrage opportunity.²

Abreu and Brunnermeier (2002, 2003) also provide a model with limited willingness of arbitrageurs to exploit a mispricing. They consider a setup where arbitrageurs want to invest while other arbitrageurs are investing, but asymmetric information causes a coordination problem. In contrast, in the model of this paper information is symmetric, and arbitrageurs want to invest when others do not. It creates an incentive to drive other investors out from the market, which in turn prevents arbitrageurs with limited capital from investing much in the first place.

The rest of the paper proceeds as follows. Section 2 presents the general model. Section 3 solves the case with a single arbitrageur. Section 4 derives the equilibrium of the model with two strategic arbitrageurs. Section 5 analyzes the effect of predatory threat on the initial investment decisions. Finally, Section 7 concludes.

²See also papers that concentrate on endogenous risk as a result of amplification due to financial constraints, e. g. Bernardo and Welch (2004), Danielsson et al. (2004, 2011), and Morris and Shin (2004).

2 Model

The model is similar to the setups of Gromb and Vayanos (2002) and Kondor (2009). Time is discrete and there are four periods, $t = 0, 1, 2$, and 3. There is a set of arbitrageurs who can invest in two traded assets: a riskfree bond that also acts as numeraire and an arbitrage opportunity that is seemingly riskless. The riskless bond has a constant return, normalized to one. The arbitrage opportunity is called a (price) gap, denoted by g_t in period $t = 0, \dots, 3$. We assume that this gap starts at an initial level of $\bar{g} > 0$ and disappears due to an exogenous shock at date 3, i.e. $g_3 = 0$. We also assume, and then confirm in equilibrium, that it is always non-negative.

The natural interpretation of the gap is the difference between the prices of two risky assets with identical payoffs that are traded in segmented markets by local traders, and only a set of arbitrageurs can trade in both of them.³ The prices can be different due to an initial supply shock to the local traders in one market, which disappears at date 3. In this setting, arbitrageurs can take long-short positions by buying the cheaper asset and shorting the expensive asset. This strategy gives a fundamentally riskless arbitrage opportunity if held until the price difference disappears at date 3, which can also be thought of as the maturity of the gap. Investing more into the arbitrage opportunity, which is essentially betting on the converge of the prices of the two assets, happens by increasing the long position in the cheap asset and increasing the short position (in absolute terms) in the expensive asset. We also refer to this as shorting the gap.⁴

There are a finite number of risk neutral arbitrageurs, indexed by $i = 1, \dots, I$; for simplicity we focus on the case of I being either one or two. Arbitrageurs behave strategically. In particular, when there is a single arbitrageur, $I = 1$, she has monopoly power over providing liquidity in the local markets. We refer to the case with two arbitrageurs, $I = 2$, as a duopoly of strategic traders. Arbitrageurs start with positive capital M^i in the riskfree asset and no initial endowment in the arbitrage opportunity, $x^i = 0$, and maximize expected utility of their date 3 wealth.

Arbitrageurs' activity affects the difference between the prices of local assets. In particular, when arbitrageurs in aggregate short x_t units of the gap, its level is given by

$$g_t = \bar{g} - \lambda x_t, \quad t = 0, 1, 2, \quad (1)$$

³See Gromb and Vayanos (2002) for a microfoundation in this spirit. Not modelling the local markets and using the shortcut of a gap asset means that arbitrageurs are not allowed to take asymmetric positions in the two assets.

⁴As the focus of this analysis is on the strategic interaction among large traders facing an arbitrage opportunity, I take market segmentation for local traders as given. Gromb and Vayanos (2002), Zigrand (2004) and Kondor (2009) use similar assumptions. See Van Nieuwerburgh and Veldkamp (2009, 2010) for an information-based mechanism that results in endogenous market segmentation.

where $\lambda > 0$ is an exogenously given illiquidity parameter that describes the price impact of arbitrage trades.⁵ Equation (1) can also be given in the dynamic form:

$$g_t = g_{t-1} - \lambda (x_t - x_{t-1}) \quad (2)$$

for $t = 0, 1, 2$, and with $g_{-1} \equiv \bar{g}$ and $x_{-1} \equiv 0$. Equation (2) shows that when arbitrageurs increase their long position in the cheaper asset and their short position in the expensive asset by one unit, the price difference decreases, and the gap shrinks by λ .

Moreover, arbitrageurs are subject to wealth constraints. In particular, they are required to have non-negative marked-to-market wealth at all times.⁶ If a trader violates this constraint, i.e. she defaults, she has to close all her positions in the following period. We refer to this as fire-sale or liquidation. Formally, if arbitrageur i has M_{t-1}^i in the riskless bond and a short position of x_{t-1}^i units of the gap after trading at date $t-1$, then her mark-to-market wealth is $M_{t-1}^i - g_{t-1}x_{t-1}^i$, and the constraint can be written as:

$$\text{if } M_{t-1}^i - g_{t-1}x_{t-1}^i < 0, \text{ it must be that } x_t^i = 0.$$

The wealth constraint requires that arbitrageurs can always cover their accumulated losses from their bond positions. As long as they do not default, they do not face any restrictions on their orders in the following trading period. However, when they do default, they must close their positions immediately, i.e. sell all the risky assets that they hold and buy back what they short in the following period.⁷

Arbitrageur i 's optimization problem is as follows:

$$\max_{\{x_t^i\}_{t=0}^2} W_3^i(M^i) = M^i + \sum_{t=0}^2 g_t(x_t^i)(x_t^i - x_{t-1}^i), \quad (3)$$

⁵I assume that increasing the short position in the gap by one unit always has the same price impact (as long as $x_t < \bar{g}/\lambda$). It holds, for example, if local traders having exponential utility and asset payoffs are normally distributed.

⁶The specific wealth constraint considered in this model is just one of many financial constraints that are based on market prices, e.g. margin constraints (Brunnermeier and Pedersen (2009) and Garleanu and Pedersen (2011)), or value at risk (VaR) constraints (Garleanu and Pedersen (2007)). They would lead to qualitatively similar results.

⁷The combination of the wealth constraint and the liquidation can be thought of as a shortcut for the joint effect of two well-known phenomena. On one hand, the relationship between past performance and fund flows has been documented for various asset classes. See, for example, Chevalier and Ellison (1997) and Sirri and Tufano (1998), or Berk and Green (2004), who provide a model of active portfolio management when fund flows rationally respond to past performance. On the other hand, Coval and Stafford (2007) show that funds experiencing large outflows decrease existing positions by engaging in fire-sales, which creates price pressure.

subject to the evolution of the gap:

$$g_t = g_{t-1} - \lambda \sum_{i=1}^I (x_t^i - x_{t-1}^i) \text{ for } t = 0, 1, 2, \quad (4)$$

and the wealth constraint:

$$x_t^i = 0 \text{ if } M_{t-1}^i < g_{t-1} x_{t-1}^i \text{ for } t = 0, 1, 2, \quad (5)$$

where $g_{-1} \equiv \bar{g}$, $M_{-1}^i \equiv M^i$ and $x_{-1}^i \equiv 0$ for $i = 1, \dots, I$. In each trading period $t = 0, 1, 2$, first it is determined whether an arbitrageur is solvent. Second, the risky asset is traded.

The equilibrium of the economy is defined as follows:

Definition 1 *A dynamic Nash-equilibrium of the trading game consists of the gap $\{g_t\}_{t=0}^2$ and the holdings of arbitrageurs $\{x_t^i\}_{t=0}^2$ for $i = 1, \dots, I$, such that $\{x_t^i\}_{t=0}^2$ solve (3) subject to (4) and (5).*

Before proceeding to the solution of the model, we make two observations about the optimization problem and the wealth constraint.

First, it is important to notice that as long as there is a single arbitrageur, i.e. she has monopoly power in providing liquidity, the market price used to evaluate her portfolio only depends on her risky holdings. However, when there are at least two strategic agents, the trade order of one of them influences the market clearing price and hence affects the constraint status of the other arbitrageur. In particular, widening the gap between the prices of the two assets creates losses to someone who is betting on the convergence of prices, and might even trigger her fire-sale. When this distressed trader is forced to close her positions, this further widens the gap, and creates a more profitable opportunity to agents still solvent. Therefore, although it is costly to trade against price convergence, there is also a benefit of having a better investment opportunity later on. Moreover, an arbitrageur close to bankruptcy might not mind violating her constraint at all. When others are betting on divergence and thus are effectively widening the gap, it can be very costly to support the price to ensure that she remains solvent.

Second, there is a natural way to simplify the wealth constraint (5). Since the dynamics of the riskless position can be expressed as

$$M_t^i = M_{t-1}^i + g_t (x_t^i - x_{t-1}^i) \quad (6)$$

for $t = 0, 1, 2$, it is easy to show that requiring non-negative capital at time t , $M_t^i - g_t x_t^i \geq 0$, is equivalent to

$$M_{t-1}^i \geq g_t x_{t-1}^i. \quad (7)$$

If it does not hold, arbitrageur i is forced to liquidate in the following period: $x_{t+1}^i = 0$. However, in this 4-period economy, marking to market is only relevant after period 1. This is because for $t = 0$, condition (7) is equivalent to $M^i \geq g_0 x^i$, which always holds as arbitrageurs start with positive bond positions ($M^i > 0$) and no endowment in risky assets ($x^i = 0$). In addition, violating the constraint at $t = 2$ would mean that an arbitrageur has to liquidate her risky position in period 3, but there is no trading at date 3 as assets already pay off. Therefore the wealth constraint is only relevant after period $t = 1$: if arbitrageur i fails to satisfy

$$M_0^i \geq g_1 x_0^i, \quad (8)$$

she must liquidate at period 2, i.e. have $x_2^i = 0$.

Further simplification of (8) can provide additional intuition regarding the nature of the constraint. In particular, from (6), (8) is equivalent to

$$M^i \geq (g_1 - g_0) x_0^i. \quad (9)$$

The left hand side of this inequality is the mark-to-market wealth of arbitrageur i at date 0, which is positive by assumption, and hence the agent is not distressed at date 0. The right hand side of the inequality represents the loss arbitrageur i makes on her positions between date 0 and 1. Hence, (9) requires the arbitrageur's wealth before trading at date 1 to be enough to cover all the losses suffered on her initial position. However, it might not always hold. In particular, when initially arbitrageur i is shorting the gap, $x_0^i > 0$, but it actually widens, $g_1 > g_0$, the wealth constraint gets tighter and she can become distressed if her starting capital is not sufficiently high. Similarly, arbitrageur i 's wealth constraint gets tighter if she bets on price divergence, $x_0^i < 0$, while the gap shrinks, $g_1 < g_0$. On the other hand, as long as arbitrageur i bets on the convergence (divergence), and prices do converge (diverge), the constraint gets relaxed.

3 Monopoly

In this section we solve for the optimal trades of the unconstrained and the constrained monopolist arbitrageur. Dropping the superscript $i = 1$, her optimization problem can be written as:

$$\max_{\{x_t\}_{t=0}^2} W_3^i(M) = M + \sum_{t=0}^2 g_t(x_t)(x_t - x_{t-1}) \quad (10)$$

subject to market clearing:

$$g_t = g_{t-1} - \lambda(x_t - x_{t-1}) \text{ for } t = 0, 1, 2, \text{ and } g_{-1} \equiv \bar{g},$$

Figure 1: **Equilibrium gap path and the optimal holdings of the monopoly in the arbitrage opportunity over time.** The dashed line shows the evolution of the gap and the solid line shows the evolution of the position of the monopolist arbitrageur as a function of time. The monopoly provides liquidity to local markets by trading at dates 0, 1 and 2, and the gap disappears at date 3 and remains closed thereafter. Parameters are set to $\bar{g} = 10$ and $\lambda = 1$.

and the insolvency constraint:

$$x_2 = 0 \text{ if } M < (g_1 - g_0)x_0. \quad (11)$$

First, we solve the optimization problem without (11). The optimal trades and the gap process in absence of the wealth constraint are summarized in the following result:

Proposition 2 *The unconstrained monopolist arbitrageur gradually provides liquidity in the local markets, i.e. she trades the same amount in every period. Formally,*

$$x_{0,u} = \frac{1}{4\lambda}\bar{g}, \quad x_{1,u} = \frac{1}{2\lambda}\bar{g}, \quad \text{and} \quad x_{2,u} = \frac{3}{4\lambda}\bar{g},$$

and the gap decreases linearly over time:

$$g_{0,u} = \frac{3}{4}\bar{g}, \quad g_{1,u} = \frac{1}{2}\bar{g}, \quad \text{and} \quad g_{2,u} = \frac{1}{4}\bar{g}.$$

Proposition 2 states that in case there is a single strategic trader taking advantage of the mispricing across markets, her early trades only compete with her later trades. As she can commit to a strategy that minimizes her price impact, she smoothes her orders across several dates, and hence trades the same amount in each period. This is illustrated on Figure 1.

Suppose now that the monopolist arbitrageur is subject to wealth constraint (11),

which might prevent her to supply liquidity as in Proposition 2. The main question is whether a trader endowed with positive capital and facing a riskless arbitrage opportunity would ever get to a state where she faces liquidation. The answer is negative:

Proposition 3 *The wealth constraint never binds on the equilibrium gap path. Therefore it does not affect the trading of a monopolist arbitrageur, and does not influence the convergence speed of the two prices.*

The result of Proposition 3 is rather straightforward. It is obvious that the constrained arbitrageur can never be better off than the unconstrained arbitrageur of Proposition 2. However, she can achieve the same terminal wealth. This is because when a single strategic trader shorts the gap, the convergence is purely the effect of her trade. Consequently, she is making profits throughout the whole process, and the gap decreases, $g_1 - g_0 < 0$. The wealth constraint thus never binds, and in fact never affects the equilibrium trading of the arbitrageur.

4 Duopoly

4.1 Benchmark case

Similarly to the monopoly case, we start with characterizing the equilibrium orders and the gap process when there are two strategic arbitrageurs, $I = 2$, and they face no constraints on the positions taken in the gap asset. However they are aware that investing one more unit of capital at a certain date decreases the return on future investments of both arbitrageurs. It has two contrasting implications regarding their trading behaviour. First, they would like to trade slowly to minimize their price impact. Second, both of them would still like to trade faster than the other arbitrageur. Formally, we obtain the following results:

Proposition 4 *The equilibrium holdings of unconstrained duopolist arbitrageurs are given by*

$$x_{0,u}^i = \frac{385}{1299\lambda}\bar{g}, \quad x_{1,u}^i = \frac{182}{433\lambda}\bar{g}, \quad \text{and} \quad x_{2,u}^i = \frac{205}{433\lambda}\bar{g}, \quad \text{for } i = 1, 2,$$

and the gap decreases as

$$g_{0,u} = \frac{529}{1299}\bar{g}, \quad g_{1,u} = \frac{69}{433}\bar{g}, \quad \text{and} \quad g_{2,u} = \frac{23}{433}\bar{g}.$$

Figure 2 illustrates the evolution of the gap and the holdings of the duopolist arbitrageurs, and contrasts the gap processes in the monopoly and duopoly cases. The main message of Proposition 4 is that when there are two strategic traders taking advantage

Figure 2: **Equilibrium gap path and the optimal holdings of the duopoly over time.** The left panel plots the evolution of the gap (dashed line) and the position of an unconstrained duopolist arbitrageur (solid line) as a function of time. The right panel compares the gap when a single arbitrageur (solid line) or two unconstrained arbitrageurs (dashed line) provide liquidity in local markets. The duopoly provides liquidity to local markets by trading at dates 0, 1 and 2, and the gap disappears at date 3 and remains closed thereafter. The model parameters are set to $\bar{g} = 10$ and $\lambda = 1$.

of the mispricing across markets, these competing arbitrageurs race to the market, and the price gap decreases much faster than with a single arbitrageur.

This result is clearly intuitive. As before, illiquidity gives arbitrageurs an incentive to spread trades over time, in order to minimize their price impact. However, now the trade order of an arbitrageur at a certain date not only competes with her later investments, but also with all the present and future investments of the other arbitrageur. As arbitrageurs face a downward sloping demand curve, they both try to trade before the other arbitrageur trades, and the presence of another arbitrageur leads to competition between them. The equilibrium strategy shows that the second effect is stronger than the first. This is why duopolist strategic traders cannot commit to a strategy that minimizes their joint price impact and takes advantage of the mispricing the most efficient way (from the viewpoint of arbitrageurs in aggregate). Instead they both race to the market at date 0. As Figure 2 shows, trading volume is large in the early periods; and the gap converges faster than with a single arbitrageur, and slows down later.

4.2 Constrained case

In the remainder of this section we consider a subgame of the optimization program (3) to study how wealth constraints affect arbitrage activity with two strategic traders. We

assume that some trading at date 0 has already taken place: the price gap is given by g_0 , and arbitrageurs already have short positions x_0^i in the gap asset and bond holdings M_0^i , $i = 1, 2$.⁸ We proceed to the overall solution in Section 5 after discussing the equilibria of the subgame and the notion of predatory threat.

The optimization problem of agent i is the following:

$$\max_{x_1^i, x_2^i} W_3^i(M_0^i, x_0^i, g_0) = M_0^i + g_1(x_1^i)(x_1^i - x_0^i) + g_2(x_2^i)(x_2^i - x_1^i).$$

subject to market clearing:

$$g_t = g_{t-1} - \lambda(x_t^i - x_{t-1}^i + x_t^{-i} - x_{t-1}^{-i}) \text{ for } t = 1, 2 \text{ and } i = 1, 2,$$

where $-i$ denotes the other agent; and the insolvency constraints:

$$x_2^i = 0 \text{ if } M^i < (g_1(x_1^i) - g_0)x_0^i, \text{ and } x_2^{-i} = 0 \text{ if } M^{-i} < (g_1(x_1^i) - g_0)x_0^{-i}.$$

The second wealth constraint indicates that arbitrageur i is aware of the constraint for arbitrageur $-i$, and hence can influence the price to trigger her fire-sale.

To define an equilibrium, we define the states of the world and two notions of value functions as follows:

Definition 5 *At date 1 each arbitrageur can be in one of three states: (i) state n for the constraint being satisfied and not binding at the equilibrium holding and gap, i.e. $M^i > (g_1(x_1^i) - g_0)x_0^i$; (ii) state b for the constraint binding, $M^i = (g_1(x_1^i) - g_0)x_0^i$; or (iii) state v for the constraint being violated, $M^i < (g_1(x_1^i) - g_0)x_0^i$.*

At date 2 each arbitrageur can be in one of two states: (i) state s for solvent (i.e. trade freely), or (ii) state l for liquidated/insolvent (i.e. having to close her risky position).

The dynamics of states are as follows: (i) If arbitrageur i satisfies her wealth constraint, she can freely trade in period 2. Formally, if arbitrageur i is in state n or b at date 1, she gets to state s at date 2; (ii) On the other hand, if the arbitrageur violates the constraint, she must liquidate in period 2. Formally, if agent i is in state v at date 1, she gets to state l at date 2.

Given the definition of states, one can define the state-dependent value functions:

Definition 6 *The state-dependent (or conditional) value function of agent $i = 1, 2$ in period $t = 1, 2$ and arbitrageur states $\{jk\}$ is denoted by $V_{t,jk}(M_t^i, x_t^i, M_t^{-i}, x_t^{-i})$, where*

j and k are the states of arbitrageur i and $-i$, respectively; $j, k \in \{n, b, v\}$ if $t = 1$, and $j, k \in \{s, l\}$ if $t = 2$;

⁸One reason for being endowed with the risky assets before trade starts would be because traders previously enjoyed some (unmodelled) private benefits from holding them.

M_t^i and x_t^i are the after-trade holdings of arbitrageur i ; and
 M_t^{-i} and x_t^{-i} are the after-trade holdings of arbitrageur $-i$.

Based on the state-dependent value functions we define the value function such that the optimization problem is the problem of choosing the optimal demand and the state jointly:

Definition 7 *The value function of agent i at date t is the merger of different conditional value functions from different states of the world given as*

$$V_t^i(M_t^i, x_t^i, M_t^{-i}, x_t^{-i}) = \sum_{j,k} 1_{jk} V_{t,jk}(M_t^i, x_t^i, M_t^{-i}, x_t^{-i})$$

where 1_{jk} is an indicator, and takes the value of 1 if, based on their date 1 mark-to-market portfolio value, arbitrageur i is in state j and arbitrageur $-i$ is in state k , and zero otherwise.

Finally, given the value function, we take the standard definition of a Nash-equilibrium:

Definition 8 *A Nash-equilibrium of the economy is a vector of demands $\{x_t^i\}_{i=1,2;t=1,2}$ such that x_t^i solves the program*

$$\begin{aligned} \max_x V_t^i(M_t^i, x_t^i, M_t^{-i}, x_t^{-i} | x_t^{-i}; M_{t-1}^i, x_{t-1}^i, M_{t-1}^{-i}, x_{t-1}^{-i}) \\ = V_t^i(M_{t-1}^i + g_t(x)(x - x_{t-1}^i), x, M_{t-1}^{-i} - g_t(x)(x_t^{-i} - x_{t-1}^{-i}), x_t^{-i}) \end{aligned}$$

where $g_t(x)$ is the market-clearing gap in period t when agent i submits the demand x , and agent $-i$ submits her equilibrium demand x_t^{-i} .

Before proceeding to the equilibria of this game, we make an observation about the wealth constraint. As described in (8), the wealth constraint can be expressed as $M^i \geq (g_1 - g_0)x_0^i$ for $i = 1, 2$. Thus, if arbitrageur i enters period 1 with a zero position in the risky assets, $x_0^i = 0$, her constraint will never bind. Suppose now that both arbitrageurs have taken non-zero positions at date 0. Then M^1/x_0^1 and M^2/x_0^2 exist, and they describe the inverse of the proportion of wealth invested in the gap. Suppose further that both x_0^1 and x_0^2 are positive (as it is going to be in equilibrium), that is arbitrageurs initially bet on the convergence of prices. It implies that the wealth constraints can be rewritten in the form

$$\frac{M^1}{x_0^1} \geq g_1 - g_0 \text{ and } \frac{M^2}{x_0^2} \geq g_1 - g_0.$$

It is easy to see that as long as the proportion invested in the gap asset is different between agents, for example $M^1/x_0^1 > M^2/x_0^2$, there is a natural order between arbitrageurs.

Figure 3: **The utility of the aggressive arbitrageur as a function of her trade at date 1.** The utility of the aggressive arbitrageur as a function of her trade at date 1, x_1^a , while holding the cautious arbitrageur's date-1 trade constant at $x_1^c = 0$. The figure illustrates that the optimal strategy is to go short in the gap, and in this case both arbitrageurs remain solvent. The parameters are set to $g_0 = 3$, $\lambda = 1$, $M^a = 8$, $M^c = 10$, $x_0^a = 1$, $x_0^c = 3$.

If arbitrageur 2 is solvent, arbitrageur 1 remains solvent too. On the other hand, if arbitrageur 1 is insolvent, arbitrageur 2 has to liquidate too. Moreover, there always exists a gap level g_1 such that arbitrageur 1 remains solvent while arbitrageur 2 goes bankrupt. Therefore the trader with higher M^i/x_0^i ratio, i.e. lower proportion of wealth invested in the arbitrage opportunity, can always be more *aggressive*, while the arbitrageur with higher proportion of wealth invested in the gap must be more *cautious* with her trades. In the characterization of the equilibrium we will refer to them as arbitrageurs a and c .⁹

Before proceeding to the solution of the model, we discuss the methodology of the equilibrium construction. The above problem can be solved backwards. First we solve for the optimal trades at date 2 given the conjectured state arbitrageurs are in (ss , sl , or ll), and obtain value functions representing their continuation utilities. Then we solve for the optimal trades of period 1. The complexity of the solution arises here regarding how to deal with the liquidation constraint. The possibility of forced liquidation implies that the optimization problem of an arbitrageur is globally non-continuous and non-concave, so local conditions for the equilibrium are not sufficient. However, the optimization problem is locally concave almost everywhere. Figures 3 and 4 illustrate the utility of

⁹If $M_0^1/x_0^1 = M_0^2/x_0^2$, the constraint binds for them at the same time. It implies that either both arbitrageurs remain solvent, or they both go bankrupt. Also, when, for example, arbitrageur 1 does not trade in period 0, i.e. $x_0^1 = 0$, the constraint will never bind for her. This case can be thought of as the limit when $M_0^1/x_0^1 \rightarrow \infty$.

Figure 4: **The utility of the aggressive arbitrageur as a function of her trade at date 1.** The utility of the aggressive arbitrageur as a function of her trade at date 1, x_1^a , while holding the cautious arbitrageur's date-1 trade constant at $x_1^c = 3$. The figure illustrates that the optimal strategy is to go long in the gap and force the cautious arbitrageur into distress. The parameters are set to $g_0 = 3$, $\lambda = 1$, $M^a = 8$, $M^c = 10$, $x_0^a = 1$, $x_0^c = 3$.

an arbitrageur as a function of her trade at date 1 while holding the other arbitrageur's date-1 trade constant in two particular cases. It is straightforward that the optimization problem can always be divided into three segments that correspond to the states of the world such that the utility function is concave in each segment.¹⁰ The possible portfolios of an arbitrageur in one segment lead to a different continuation state from portfolios in another segment: if a trader increases her short position sufficiently, the gap shrinks and both arbitrageurs remain solvent. However if an arbitrageur decides to go long in the gap, the gap widens, and can push (at least) one arbitrageur into distress. Consequently, for each portfolio choice of the other trader, an arbitrageur compares the locally optimal investment strategies in the three segments, and picks the one with highest utility.

Because of the local concavity, given the other arbitrageur's investment decision, there is an optimal portfolio within each state of the world. Combining these conditions for the two arbitrageurs gives a set of candidate equilibria, satisfying that none of the traders want to alter their strategies as long as the state of the world remains the same. Therefore, it must be also checked whether these trades are globally optimal too, i.e. whether any arbitrageur would prefer to deviate in such a way that changes the state of the world.

¹⁰The three states from the viewpoint of the aggressive arbitrageur are *ss*, *sl* and *ll*. From the viewpoint of the cautious arbitrageur the possible states are *ss*, *ls* and *ll*. This is because the roles of arbitrageurs imply that it is impossible to have a case when the cautious arbitrageur remains solvent and the aggressive arbitrageur becomes insolvent.

4.2.1 Candidate equilibria

We describe the equilibria of the economy in two steps. First, we provide the set of *candidate equilibria* with the locally optimal portfolios, which also determine the gap path. The derived date-1 gap g_1 , combined with (9), thus provides a straightforward necessary condition on the proportion of wealths invested for such an equilibrium to exist. Then we discuss the actual equilibria of the economy for the three cases when (i) both arbitrageurs remain solvent; (ii) the aggressive arbitrageur remains solvent, but the cautious is insolvent; or (iii) both arbitrageurs go bankrupt. These are different from the candidate equilibria because the globally optimal portfolios must satisfy more requirements than local optimality. For tractability, we only discuss the cases when arbitrageurs initially short the gap asset, i.e. $x_0^a, x_0^c > 0$, which will be the case in equilibrium. All the other cases are described in an internet appendix.

Proposition 9 *When both arbitrageurs remain solvent, the locally optimal strategies and the gap path are given by*

$$x_1^i - x_0^i = \frac{7}{23\lambda}g_0 \text{ and } x_2^i - x_1^i = \frac{3}{23\lambda}g_0 \text{ for } i = a, c. \quad (12)$$

and

$$g_1 = \frac{9}{23}g_0 \text{ and } g_2 = \frac{3}{23}g_0.$$

Such a candidate equilibrium exists for every $0 < M^c/x_0^c < M^a/x_0^a$. Moreover, the wealth constraint is not binding for any arbitrageur.

Suppose that both arbitrageurs remain solvent, and it happens without the constraint binding for the cautious arbitrageur. It implies that the locally optimal strategies are those that would emerge in the equilibrium of the economy with no wealth constraint.¹¹ As before, since arbitrageurs face a downward sloping demand curve, they both try to trade before the other arbitrageur trades. It leads to competition between them: arbitrageurs race to the market, and the gap shrinks quickly. Since the gap decreases, $g_1 < g_0$, arbitrageurs record profits throughout the convergence, thus they indeed remain solvent even if they start with very low capital. Moreover, the constraint of arbitrageur c cannot bind in equilibrium, because that would imply the gap must widen, $g_1 > g_0$. However, in this case both arbitrageurs would be willing to short the gap a little bit more to trade before the other arbitrageur, hence the gap would shrink, and the constraint would not bind any more.

¹¹For this note that substituting the date-0 unconstrained gap, $g_{0,u}$, into g_0 gives the same portfolios that were derived in Proposition 4.

Proposition 10 *When the aggressive arbitrageur remains solvent and the cautious liquidates:*

(i) *There exists a candidate equilibrium where the wealth constraint is not binding for the aggressive arbitrageur. The locally optimal strategies and the gap path are given by*

$$\begin{aligned} x_1^i - x_0^i &= \frac{1}{5\lambda} (g_0 - \lambda x_0^c) \text{ for } i = a, c, \\ x_2^a - x_1^a &= \frac{1}{2\lambda} (g_1 + \lambda x_1^c) \text{ and } x_2^c = 0, \end{aligned}$$

and

$$g_1 = \frac{3}{5}g_0 + \frac{2}{5}\lambda x_0^c \text{ and } g_2 = \frac{2}{5}g_0 + \frac{3}{5}\lambda x_0^c.$$

The candidate equilibrium requires $x_0^c > \frac{1}{\lambda}g_0$ and $0 < M^c/x_0^c < -\frac{2}{5}(g_0 - \lambda x_0^c) \leq M^a/x_0^a$.

(ii) *There exists a candidate equilibrium where the wealth constraint is binding for the aggressive arbitrageur when $0 < M^c/x_0^c < M^a/x_0^a < -\frac{2}{5}(g_0 - \lambda x_0^c)$. As the constraint is binding for the aggressive arbitrageur, the locally optimal strategies and the gap path satisfy $g_1 - g_0 = M^a/x_0^a$. Moreover, there are many possible optimal trades as this case corresponds to a corner solution.*

The proposition states that as long as the constraint does not bind for arbitrageur a , the locally optimal strategies satisfy that arbitrageurs sell the same amount from the cheap asset and buy the same amount from the expensive asset, driving the gap up at date 1. In fact, the cautious trader knows that if the aggressive trader goes long in the gap asset to widen the gap, she does not have enough capital to cover her losses emerging due to the price divergence, and she will be forced to close her position. As arbitrageurs face a downward sloping demand curve, the cautious trader wants to avoid a round-trip transaction (buying and then being forced to sell, or selling and then buying), because it would lead to additional losses. She also wants to minimize her price impact when liquidating. Therefore, she conducts the fire-sale in two periods, and closes part of her positions already at date 1 and the rest at date 2.

In the meantime, at date 1 the aggressive arbitrageur finds it optimal to do exactly the same the cautious arbitrageur does. Notice that the condition $x_0^c > \frac{1}{\lambda}g_0$ implies that the gap widens through time, i.e. $g_2 > g_1 > g_0$. Hence when trader c finishes the fire-sale, trader a will face a better arbitrage opportunity to invest than in the very beginning, as the gap is wider. In fact, the aggressive trader withdraws liquidity instead of providing liquidity exactly when the cautious arbitrageur would need it the most. This is in the spirit of Brunnermeier and Pedersen (2005). However, in this model predation happens endogenously, unlike in Brunnermeier and Pedersen (2005), where the prey is passive. Here arbitrageur c could avoid bankruptcy by taking a sufficiently large long position and ensuring she suffers no losses between periods 0 and 1, but she realizes it would be

too costly for her. The constraints on the proportion of the wealth invested correspond to the fact that the aggressive arbitrageur can indeed cover her losses due to the gap diverging from g_0 to g_1 , while the cautious arbitrageur cannot.

Proposition 10 also states that a qualitatively similar candidate equilibrium (but with different trades) can happen even if arbitrageur a has lower level of capital (or higher proportion of capital invested in the arbitrage opportunity). This is because with $M^c/x_0^c < M^a/x_0^a$ the aggressive arbitrageur can always set the gap such that her losses are still covered by her starting wealth while violating the wealth constraint of the cautious arbitrageur.

Proposition 11 *When both arbitrageurs become insolvent, the locally optimal strategies and the gap path are given by*

$$x_1^i - x_0^i = -\frac{1}{3} (x_0^i + x_0^{-i}) \quad \text{and } x_2^i = 0 \text{ for } i = a, c.$$

and

$$g_1 = g_0 + \frac{2\lambda}{3} (x_0^a + x_0^c) \quad \text{and } g_2 = g_0 + \lambda (x_0^a + x_0^c).$$

Such a candidate equilibrium exists if $0 < M^c/x_0^c < M^a/x_0^a < 2\lambda (x_0^a + x_0^c) / 3$.

When both arbitrageurs violate the constraint and become insolvent, they have to strategically liquidate their positions through two periods. Arbitrageurs know that they are facing a downward sloping demand curve, and want to minimize their price impact while liquidating. To close their positions, they have to buy the expensive asset and sell the cheap asset, hence they both want to buy/sell before the other arbitrageur. Thus they race to the market. In equilibrium, they liquidate the same amount at date 1, namely $2/3$ of their aggregate asset holdings, and at date 2 they liquidate the remaining $1/3$.

4.2.2 Equilibrium characterization

Given the locally optimal strategies, it is possible to analyze under what circumstances they are globally optimal too. Regarding the equilibrium with both arbitrageurs solvent, we obtain the following result:

Proposition 12 *There exists an equilibrium of the trading game with both arbitrageurs remaining solvent (state ss) if and only if*

$$\frac{M^a}{x_0^a} > \frac{M^c}{x_0^c} \geq \Delta_{nn,nv}^1(g_0, x_0^c), \quad (13)$$

where the function $\Delta_{nn,nv}^1(\cdot, \cdot) > 0$ is given in Appendix B.1.

According to Proposition 9, it was possible to have a candidate equilibrium such that both arbitrageurs remain solvent for any proportions of wealth invested in the arbitrage opportunity, because prices converged and arbitrageurs made profits throughout the whole trading process. When looking for an actual equilibrium, turns out this is not the case.

In particular, as the aggressive arbitrageur is aware of the wealth constraint of the cautious agent, arbitrageur a can engage in the manipulation of date-1 prices. Facing a downward sloping demand curve, this manipulation is costly because of the price impact. However, manipulation can be profitable due to two sources of profits. First, if the cautious arbitrageur goes bankrupt, the aggressive arbitrageur has monopoly power in providing liquidity to local traders at date 2. Second, as arbitrageur c has a short position in the gap after period 1, i.e. $x_1^c = x_0^c + \frac{7}{23\lambda}g_0 > 0$, her fire-sale widens the gap and makes forced liquidation even more desirable for the aggressive trader. The cost of manipulation is decreasing in the proportion of arbitrageur 2's wealth invested into the arbitrage opportunity, i.e. increasing in M^c/x_0^c , while the profit of the fire-sale is increasing in x_1^c , i.e. in both the cautious arbitrageur's holding before date 1, x_0^c , and the initial gap g_0 . Combining these observations, there exists a threshold for M^c/x_0^c such that if the proportion of arbitrageur c 's wealth invested into the arbitrage opportunity is low enough, forcing her to liquidate is too costly, and an equilibrium with both agents remaining solvent exists.

Next, we present the conditions under which predation happens.

Proposition 13 *There exists an equilibrium with the aggressive arbitrageur remaining solvent and the cautious arbitrageur becoming insolvent (state sl) if and only if*

$$0 < \frac{M^c}{x_0^c} \leq \Delta_{nv,nn}^c(g_0, x_0^c) \text{ and } \frac{M^a}{x_0^a} \geq \Delta_{nv,vv}^c(g_0, x_0^a, x_0^c), \quad (14)$$

where $\Delta_{nv,nn}^c(\cdot, \cdot), \Delta_{nv,vv}^c(\cdot, \cdot) > 0$ are given in Appendix B.2.1.

Comparing Propositions 10 and 13, the main difference is that the wealth requirements are tighter. For an equilibrium it must be that the locally optimal strategies are globally optimal too. It is apparent that the key is whether the cautious arbitrageur would be better off avoiding liquidation as a result of some costly price manipulation at date 1 that changes the state of the world.

The cautious arbitrageur starts trading with an initial long position in the arbitrage opportunity, but due to her limited capital, she cannot sustain losses caused by the activity of the aggressive arbitrageur in the short run. It is apparent that if arbitrageur c wants to remain solvent, she can always do so. This is because if arbitrageur a widens

the gap, trader c can always engage in exactly the opposite trade that leaves the gap unchanged, and thus leaves the state untouched as well. The question is how costly it is.

In particular, suppose the aggressive arbitrageur's strategy is fixed at buying a very large amount of the gap asset, which makes prices diverge. Arbitrageur c , being subject to the wealth constraint, can do two things. First, she can short enough so that she neutralizes the effect of the aggressive arbitrageur's trades and brings g_1 sufficiently close to the original level g_0 . In this case she remains solvent. As she faces a downward sloping demand curve, shorting a large amount of the gap asset is costly, as it diminishes future returns on the assets she is holding. On the other hand, the benefit of this strategy is that the arbitrageur remains solvent and can invest again at date 2. Alternatively, she can accept that she is pushed to insolvency. In that case the optimal liquidation strategy means shorting less at date 1, which leads to smaller price impact. Moreover, since the other trader is still solvent at date 2, the cautious arbitrageur can liquidate at more favourable prices.

Whether the cautious trader thus finds it optimal to liquidate or not, given the selling pressure of the aggressive trader, depends on the relative costs and gains of these two strategies. In particular, the profit from remaining solvent increases in her initial position x_0^c , and in the gap size g_0 . This implies that the threshold for equilibrium on the proportion of wealth invested by the cautious arbitrageur, $\Delta_{nv,nn}^c(\cdot, \cdot)$, is an increasing function of both x_0^c and g_0 .

Finally, regarding equilibria in which both agents get liquidated we obtain the following result:

Proposition 14 *There exists no equilibrium of the trading game with both arbitrageurs being insolvent.*

This result is rather intuitive. Indeed, liquidation imposes a cost on both agents, because they have to close the positions they previously created to bet on the convergence of prices. Put it differently, arbitrageurs must sell assets at lower prices than they have bought them, or buy back previously shorted assets at prices higher than when they started to short them. Given that the divergence in prices is solely the effect of their own activities, arbitrageurs could avoid these self-imposed costs by not trading at all in period 1. By simply holding on to their existing positions the gap would not change, and the constraint would not get tighter than before. Arbitrageurs would remain solvent and their optimal unconstrained trades in period 2 could not make them worse off than the forced liquidation.

The different regions for the proportions of wealth invested in the arbitrage opportunity described in Propositions 12 and 13 are illustrated on Figure 5.

Figure 5: **Capital thresholds for the different types of equilibria.** The horizontal axis plots the inverse of the proportion of capital invested in the arbitrage opportunity by the aggressive arbitrageur, M^a/x_0^a , and the vertical axis plots the same for the cautious arbitrageur, M^c/x_0^c . When both agents have low proportion of wealth invested in the risky assets, top right region, the wealth constraint does not affect arbitrage trading and the gap path, and an sl equilibrium exists. When arbitrageur a has a much lower proportion of wealth invested in the arbitrage opportunity than arbitrageur c , bottom left region, there exists an sl equilibrium. The aggressive arbitrageur predates on the cautious by widening the gap at date 1, and shorting it after the liquidation, at date 2. For other possible levels of proportion of capital invested in the arbitrage that satisfy $M^a/x_0^a \geq M^c/x_0^c$ there is no equilibrium.

5 Predatory threat and the speed of convergence

So far we have taken the initial positions x_0^i and the gap g_0 as given. In this section we endogenize x_0^i by extending the previous analysis with an investment phase at date 0. Arbitrageurs know that the initial positions they take and hence the gap they face affect which state of the world they get into after date 0. We show that liquidation does not happen in equilibrium, but as long as one arbitrageur has a much higher level of capital than the other, it does not affect the gap path significantly. We show that the wealth constraint has its strongest effect on the gap path when arbitrageurs start with similarly low level of capital. In this case the gap decreases very little in period 0, then both agents rush to the arbitrage opportunity.

When solving the date-0 optimization problem, we restrict the (on- and off-equilibrium) action space of arbitrageurs to trades with which they end up in either an ss or an sl equilibrium. It means that for a given x_0^2 , arbitrageur 1 must choose her position x_0^1 in such a way that arbitrage positions maximize her utility while satisfying either (13) or

(14). Of course when deciding on the initial investment x_0^i , arbitrageur i also realizes that as long as her proportion of wealth invested into the arbitrage opportunity is higher than that of the other trader, i.e. $M^i/x_0^i < M^{-i}/x_0^{-i}$, the wealth constraint is tighter for her, and hence she takes the role of the cautious arbitrageur. Formally, we look for a dynamic equilibrium where arbitrageur i solves the problem

$$x_0^i \in \arg \max_x W_3^i = V_0(x|M^i, M^{-i}, x_0^{-i}),$$

where

$$V_0(x|M^i, M^{-i}, x_0^{-i}) = \begin{cases} V_{0,ss}(M_0^i, x_0^i, M_0^{-i}, x_0^{-i}) & \text{if satisfy conditions for } ss \text{ equilibrium} \\ V_{0,sl}(M_0^i, x_0^i, M_0^{-i}, x_0^{-i}) & \text{if satisfy conditions for } sl \text{ equilibrium} \\ V_{0,ls}(M_0^i, x_0^i, M_0^{-i}, x_0^{-i}) & \text{if satisfy conditions for } ls \text{ equilibrium,} \end{cases}$$

As the optimization programs of arbitrageurs with these constraints become difficult to solve in closed form (it includes solving 4th order equations), we make some simplifying steps and then solve the problem numerically. In particular, first we solve the optimization problems given that both agents remain solvent while satisfying the constraints for an *ss* equilibrium, i.e.

$$\max_{x_0^i} V_{0,ss}(M_0^i, x_0^i, M_0^{-i}, x_0^{-i}) \equiv M_0^i + \frac{72}{23^2\lambda} g_0^2 = M^i + g_0 x_0^i + \frac{72}{23^2\lambda} g_0^2$$

subject to (13), and then we confirm that none of the agents have incentives to deviate to the *sl* state when the other arbitrageur chooses the optimal strategy x_0^{-i} that solves her program.¹² Propositions 15 and 16 describe the equilibrium date-0 trading of strategic arbitrageurs:

Proposition 15 *None of the arbitrageurs are forced to liquidate in equilibrium.*

This result is rather intuitive. It shows that arbitrageurs reduce their initial investments such that liquidation does not happen in equilibrium. This is because liquidation is rather costly. As there is no uncertainty in the model, no strategic agent wants to buy an asset that she has to sell later with certainty, since buying an asset pushes its price up while selling decreases its price, both working against the profit of this kind of round-trip transaction. It implies that liquidation does not happen in equilibrium, but the threat of liquidation is still present on the off-equilibrium path.

Proposition 16 *Based on the initial capital of traders, the effect of the wealth constraint on arbitrageur activity can be divided into four cases.*

¹²The deviations allowed here include those when the arbitrageur goes long in the gap, i.e. $x_0^i < 0$, even though those cases were not discussed in Section 4.

Figure 6: **Capital thresholds in the four different cases when arbitrageurs are subject to wealth constraints.** The horizontal axis plots the starting capital of arbitrageur 1, M^1 , and the vertical axis plots the starting capital of arbitrageur 2, M^2 . When both agents have large initial capital to invest, Region I, the wealth constraint does not affect arbitrage trading and the gap path. When at least one arbitrageur has a low capital level to start with, while the other has relatively more, i.e. Regions II and III, the wealth constraint affects the arbitrage positions but the gap path is close to the gap of the unconstrained case. Finally, when both arbitrageurs have similarly low capital level to start with, Region IV, both arbitrageurs trade very little initially and the gap remains large. The model parameters are set to $\bar{g} = 10$ and $\lambda = 1$, which imply that the threshold for Region I is approximately $\frac{1}{\lambda}\Omega\bar{g}^2 \approx 8.651$.

(I) There exists a constant $\Omega > 0$ such that for $M^1, M^2 \geq \frac{1}{\lambda}\Omega\bar{g}^2$, arbitrageur strategies and the gap path are the same as in the unconstrained case, discussed in Proposition 4.

(II) There exists a function $\Phi(\cdot)$ such that when $0 < M^2 < \frac{1}{\lambda}\Omega\bar{g}^2$ and $M^1 \geq \Phi(M^2)$, arbitrageur 2 is the cautious trader, and the constraint (13) binds for her. As a result, she trades less at date 0 than in the unconstrained case.

(III) Similarly, when $0 < M^1 < \frac{1}{\lambda}\Omega\bar{g}^2$ and $M^2 \geq \Phi(M^1)$, arbitrageur 1 becomes the cautious trader, and the constraint (13) binds for her. She trades less at date 0 than in the unconstrained case.

(IV) When both arbitrageurs have low level of capital, $M^1, M^2 < \frac{1}{\lambda}\Omega\bar{g}^2$, and they are close to each other such that $M^1 < \Phi(M^2)$ and $M^2 < \Phi(M^1)$, both arbitrageurs invest less than in the unconstrained case, and hence the gap remains larger.

The four regions for cases (I)-(IV) are illustrated on Figure 6. Arbitrageurs remain solvent in all cases. Moreover, $\Phi(\cdot)$ is positive, strictly increasing, satisfies $\Phi(x) > x$ for $0 < x < \frac{1}{\lambda}\Omega\bar{g}^2$, and $\Phi(x) = x$ for $x = 0$ or $x = \frac{1}{\lambda}\Omega\bar{g}^2$.

Figure 7: **Equilibrium gap path and the optimal holdings of the duopoly in the arbitrage opportunity over time.** The dotted line shows the evolution of the gap, and the dotted/dashed lines show the evolution of the positions of the constrained duopolist arbitrageurs as a function of time, when their initial capital levels M_1 and M_2 are significantly different from each other. To contrast, the dashed line shows the evolution of the gap and the solid line shows the evolution of the positions when the duopoly is unconstrained, as on Figure 2. Trading happens at dates 0, 1 and 2, and the gap disappears at date 3 and remains closed thereafter. The model parameters are set to $\bar{g} = 10$, $\lambda = 1$, $M_1 > 7.18$ and $M_2 = 5$.

Proposition 16 describes the initial trades as a function of arbitrage capital. First, if both arbitrageurs start with sufficiently high level of capital, the wealth constraint does not affect their trades and hence the dynamic equilibrium of the model is exactly the same as in unconstrained case, described in Proposition 4. As arbitrageurs have a lot of cash on hand that can provide a cushion against very large adverse movements in the gap, they race to the market and take large bets on the convergence of prices. Traders' positions and the evolution of the gap are illustrated on Figure 2.

When at least one arbitrageur has a low capital level to start with, while the other has (relatively) more, e.g. $M^1 \geq \Phi(M^2)$, the wealth constraint affects the date 0 trading through affecting agent 2's willingness to invest. Arbitrageur 2 must short less compared to the case when the constraint is not effective, $x_0^2 < x_{0,u}^i$, because she wants to avoid liquidation later on. In fact, she takes such a small position that it is not worth for arbitrageur 1 to push her to insolvency. On the other hand, arbitrageur 1 can invest more, $x_0^1 > x_{0,u}^i$, as long as she has a lower proportion in the gap asset. This is rather profitable for her, as the threat of potential liquidation restricts the ability of arbitrageur 2 to provide liquidity to local traders, and agent 1 has almost monopoly power in doing

Figure 8: **Equilibrium gap path and the optimal holdings of the duopoly in the arbitrage opportunity over time.** The dotted line shows the evolution of the gap, and the dotted/dashed lines show the evolution of the positions of the constrained duopolist arbitrageurs as a function of time, when their initial capital levels M_1 and M_2 are similar. To contrast, the dashed line shows the evolution of the gap and the solid line shows the evolution of the positions when the duopoly is unconstrained, as on Figure 2. Trading happens at dates 0, 1 and 2, and the gap disappears at date 3 and remains closed thereafter. The model parameters are set to $\bar{g} = 10$, $\lambda = 1$, $M_1 = 1.17$ and $M_2 = 0.9$.

so. The large position that arbitrageur 1 takes compensates for the small holdings by arbitrageur 2 so the date 0 gap is not very different from the case when both arbitrageurs have high level of capital. This is illustrated on Figure 7.

Finally, when both arbitrageurs have low capital level to start with, and they are close to each other so that $M^1 < \Phi(M^2)$ and $M^2 < \Phi(M^1)$, the wealth constraint is important for both arbitrageurs. In particular, suppose that the proportion of wealth invested in the arbitrage opportunity is fixed for both agents, and it is larger for arbitrageur 2, i.e. $M^1/x_0^1 > M^2/x_0^2$. It implies that agent 2 is the cautious arbitrageur and faces a tighter wealth constraint, so she must reduce her holdings if she wants to avoid forced liquidation. However, by investing less she decreases her proportion of wealth in the arbitrage opportunity to below that of arbitrageur 1, that is she makes $M^1/x_0^1 < M^2/x_0^2$. Now arbitrageur 1 becomes the cautious arbitrageur, she is more prone to predatory risk, so she should reduce her initial investment. This drives the M^1/x_0^1 ratio above M^2/x_0^2 , and so on. In the end, both arbitrageurs trade very little at date 0, $x_0^1, x_0^2 \ll x_{0,u}^i$, and the gap level remains high, $g_0 \gg g_{0,u}$. Given that both of them remain solvent, in the next period they both race to the arbitrage opportunity, and the gap quickly shrinks. This is

illustrated on Figure 8.

6 Further implications

In this section, we discuss the equilibrium and analyze the effect of the difference in arbitrage capital agents start with (i.e. arbitrageur size) on the leverage ratios apply, both in the cross section in every period and their evolution over time, and study the effect of size on return characteristics, such as mean, variance, and skewness.

6.1 Leverage ratios

TO BE WRITTEN

6.2 Return characteristics

TO BE WRITTEN

7 Final remarks

This paper presents an equilibrium model of endogenous predation and forced liquidation among strategic arbitrageurs who are subject to capital constraints. Arbitrageurs bet on the convergence of prices of two assets, but when prices actually diverge and their marked-to-market portfolio value becomes negative, traders have to unwind their risky holdings immediately and leave the market. This implies that arbitrageurs' wealth limits the positions they can take as long as they do not want to violate the constraint. Strategic traders may trigger the bankruptcy of 'weaker' agents, which creates predatory risk and implies that even if the investment opportunity is a fundamentally riskless arbitrage, traders might be reluctant to invest in it.

First we study a model when agents are already endowed with positions in the risky assets. We show that when traders have similar proportion of wealth invested in the arbitrage opportunity, they behave cooperatively, and prices converge through time, as in a benchmark model without the constraint. However, if there is a significant difference in their proportion of wealth invested, the arbitrageur with lower proportion invested in the arbitrage opportunity predates on the other trader by manipulating the price and forcing her to unwind her position at a large discount.

Then we examine whether a strategic trader is willing to build up a portfolio if it makes her prone to predation and hence large losses. We show that in the equilibrium of the full model liquidation never happens, but the threat of predation makes arbitrageurs reluctant to invest much in the arbitrage opportunity because of the presence of other arbitrageurs. In particular, the wealth constraint seriously affects the gap between the asset prices when arbitrageurs have similarly low level of capital, and implies that instead of racing to the opportunity arbitrageurs stay out, and the gap decreases gradually.

In the model presented here there is no informational asymmetry about the opportunity among arbitrageurs, and prices and positions are always deterministic. Naturally, this provides an opportunity to extend the framework in several dimensions. For example, it would be interesting to allow for asymmetric positions in the two risky assets and see what effect it would have if some strategic traders only had information about one leg of the trades of other arbitrageurs, as anecdotal evidence recalls about the trading counterparties of LTCM. Moreover, it would be important to evaluate the empirical significance of the presented mechanism and to distinguish it from others that result in similarly slow trading of large traders, e.g. Kyle (1985). These are left for future work.

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A Optimal trading of the monopoly

First we solve the problem without the wealth constraint.

Proof of Proposition 2. The arbitrageur's optimization program is given by

$$\max_{\{x_t\}_{t=0}^2} W_3 = M + \sum_{t=0}^2 g_t(x_t)(x_t - x_{t-1}) \quad (15)$$

where $g_t = g_{t-1} - \lambda(x_t - x_{t-1})$ for $t = 0, 1, 2$ and $g_{-1} \equiv \bar{g}$. Writing it as a dynamic program it becomes

$$\max_{x_2} W_3 = M_1 + g_2(x_2)(x_2 - x_1),$$

and the FOC yields $0 = g_2 + \frac{dg_2}{dx_2}(x_2 - x_1) = g_1 - 2\lambda(x_2 - x_1)$, i.e.

$$x_2 - x_1 = \frac{1}{2\lambda}g_1 \text{ and } g_2 = \frac{1}{2}g_1. \quad (16)$$

Moreover, $W_3 = M_1 + g_2(x_2)(x_2 - x_1) = M_1 + \frac{1}{4\lambda}g_1^2$. Going back one more period the optimization program becomes

$$\begin{aligned} \max_{x_1} W_3 &= M_1 + \frac{1}{4\lambda}g_1^2 = M_0 + g_1(x_1 - x_0) + \frac{1}{4\lambda}g_1^2 \\ &= M_0 + (g_0 - \lambda(x_1 - x_0))(x_1 - x_0) + \frac{1}{4\lambda}(g_0 - \lambda(x_1 - x_0))^2, \end{aligned}$$

and the FOC yields $0 = g_0 - 2\lambda(x_1 - x_0) - \lambda\frac{1}{2\lambda}(g_0 - \lambda(x_1 - x_0))$, or $x_1 - x_0 = \frac{1}{3\lambda}g_0$. Therefore, $g_1 = \frac{2}{3}g_0$ and $W_3 = M_0 + \frac{1}{3\lambda}g_0^2$. Going back to the date 0 optimization it becomes

$$\begin{aligned} \max_{x_0} W_3 &= M_0 + \frac{1}{3\lambda}g_0^2 = M + g_0x_0 + \frac{1}{3\lambda}g_0^2 \\ &= M + (\bar{g} - \lambda x_0)x_0 + \frac{1}{3\lambda}(\bar{g} - \lambda x_0)^2, \end{aligned}$$

so the FOC yields $0 = \bar{g} - 2\lambda x_0 - \frac{2}{3}(\bar{g} - \lambda x_0)$, or $x_0 = \frac{1}{4\lambda}\bar{g}$. Therefore, $g_0 = \frac{3}{4}\bar{g}$, which implies that the monopoly gradually provides liquidity in the local markets:

$$x_{t,u} - x_{t-1,u} = \frac{1}{4\lambda}\bar{g} \text{ for } t = 0, 1 \text{ and } 2,$$

that is

$$x_{0,u} = \frac{1}{4\lambda}\bar{g}, \quad x_{1,u} = \frac{1}{2\lambda}\bar{g}, \text{ and } x_{2,u} = \frac{3}{4\lambda}\bar{g},$$

and the gap decreases linearly over time:

$$g_{0,u} = \frac{3}{4}\bar{g}, \quad g_{1,u} = \frac{1}{2}\bar{g}, \text{ and } g_{2,u} = \frac{1}{4}\bar{g},$$

where the subscript u refers to the arbitrageur being unconstrained. ■

Proof of Proposition 3. For the full consideration of the effect of the constraint on the optimal portfolio choice of the monopolistic arbitrageur, one should analyze the 2-period subgame in which the constraint affects the optimal trades, given the gap g_0 and the positions she has after trading at date 0, M_0 and x_0 , and then consider the portfolio choice problem at date 0. However, on the unconstrained equilibrium path the gap converges to zero, i.e. $g_1 - g_0 < 0$, and (8) is always satisfied. Given that arbitrageur can never achieve higher utility in the constrained portfolio choice problem than in the unconstrained problem, but the unconstrained optimum is feasible when incorporating the constraint, it does not affect the equilibrium holdings and gap for a monopolistic arbitrageur. ■

B Optimal trading of the duopoly

Following the same footsteps as with the monopoly, first we solve the problem without the wealth constraint.

Proof of Proposition 4. Arbitrageur i 's optimization program, $i = 1, 2$, is given by

$$\max_{\{x_t^i\}_{t=0}^2} W_3^i = M^i + \sum_{t=0}^2 g_t (x_t^i) (x_t^i - x_{t-1}^i), \quad (17)$$

where $g_t = g_{t-1} - \lambda (x_t^i - x_{t-1}^i) - \lambda (x_t^{-i} - x_{t-1}^{-i})$ for $t = 0, 1, 2$ and $g_{-1} \equiv \bar{g}$. In period 2 it is given by

$$\max_{x_2^i} W_3^i = M_1^i + g_2 (x_2^i) (x_2^i - x_1^i) = M_1^i + (g_1 - \lambda (x_2^i - x_1^i) - \lambda (x_2^{-i} - x_1^{-i})) (x_2^i - x_1^i),$$

and the FOCs yield

$$x_2^i - x_1^i = x_2^{-i} - x_1^{-i} = \frac{1}{3\lambda} g_1 \text{ and } g_2 = \frac{1}{3} g_1. \quad (18)$$

Moreover, $W_3^i = M_1^i + g_2 (x_2^i - x_1^i) = M_1^i + \frac{1}{9\lambda} g_1^2$. Going back one more period the optimization problem becomes

$$\begin{aligned} \max_{x_1^i} W_3^i &= M_1^i + \frac{1}{9\lambda} g_1^2 \\ &= M_0^i + (g_0 - \lambda (x_1^i - x_0^i) - \lambda (x_1^{-i} - x_0^{-i})) (x_1^i - x_0^i) \\ &\quad + \frac{1}{9\lambda} (g_0 - \lambda (x_1^i - x_0^i) - \lambda (x_1^{-i} - x_0^{-i}))^2, \end{aligned}$$

and the FOC yields $x_1^i - x_0^i = x_1^{-i} - x_0^{-i} = \frac{7}{23\lambda}g_0$. Therefore, $g_1 = \frac{9}{23}g_0$, and $W_3^i = M_0^i + \frac{72}{23^2\lambda}g_0^2$. Going back to the date 0 optimization, it becomes

$$\max_{x_0^i} W_3^i = M_0^i + \frac{72}{23^2\lambda}g_0^2 = M^i + (\bar{g} - \lambda x_0^i - \lambda x_0^{-i})x_0^i + \frac{72}{23^2\lambda}(\bar{g} - \lambda x_0^i - \lambda x_0^{-i})^2,$$

so the FOC yields $x_0^i = x_0^{-i} = \frac{385}{1299\lambda}\bar{g}$. Therefore, $g_0 = \frac{529}{1299}\bar{g}$, which implies that the duopoly gradually provides liquidity in the local markets: $x_0^i = \frac{385}{1299\lambda}\bar{g}$, $x_1^i = \frac{182}{433\lambda}\bar{g}$, and $x_2^i = \frac{205}{433\lambda}\bar{g}$, for $i = 1, 2$, and the gap's evolution is given by $g_0 = \frac{529}{1299}\bar{g}$, $g_1 = \frac{69}{433}\bar{g}$, and $g_2 = \frac{23}{433}\bar{g}$. ■

Next we consider the case with the wealth constraint. First we characterize the optimal trades in period 2 conditional on the status of the two agents, and derive the value functions given the states and positions.

Suppose that the positions before trade happens at date 2 are M_1^i and x_1^i , and M_1^{-i} and x_1^{-i} , for agents i and $-i$ in the riskless and the risky assets, respectively. As a reminder, subscript $\{t, jk\}$ refers to time period t and status of traders i and $-i$, respectively, where the date 2 state can take two values: $j, k \in \{s, l\}$, i.e. solvency or liquidation, and it corresponds to whether the agents satisfied or violated the wealth constraint. For example, $g_{2,sl}(x_1^i, x_1^{-i})$ denotes the gap in period 2 as a function of the position x_1^i of the arbitrageur who remains solvent and the position x_1^{-i} of the arbitrageur who is liquidated.

The following propositions state the optimal trades, equilibrium gaps and value functions in three possible cases at date 2. First, we restate a previous result without proof for the ss state, then we solve for the equilibria of the sl and ll states.

Proposition 17 *In period 2, conditional on both agents being solvent, the first-best trade orders and equilibrium gap are given by*

$$x_{2,ss}^i - x_{1,ss}^i = \frac{1}{3\lambda}g_1 \text{ for } i = 1, 2, \text{ and } g_{2,ss} = \frac{1}{3}g_1. \quad (19)$$

Proof. Straightforward from (18). It also results in a continuation value function of $V_{1,ss}(M_1^i, x_1^i, M_1^{-i}, x_1^{-i}) = M_1^i + \frac{1}{9\lambda}g_1^2$. ■

Proposition 18 *In period 2, conditional on agent i being solvent and $-i$ being liquidated, the first-best trade order and the equilibrium gap are given by*

$$x_{2,sl}^i(x_1^i, x_1^{-i}) = x_1^i + \frac{1}{2\lambda}(g_1 + \lambda x_1^{-i}), \quad x_{2,ls}^i = 0 \text{ and } g_{2,sl}(x_1^i, x_1^{-i}) = \frac{1}{2}(g_1 + \lambda x_1^{-i}). \quad (20)$$

Proof. The optimization problem of agent i is the same as in the ss case as she remains solvent, which yields the same FOC

$$0 = g_1 - 2\lambda(x_2^i - x_1^i) - \lambda(x_2^{-i} - x_1^{-i}). \quad (21)$$

As agent $-i$ has to close her position, $x_{2,ls}^{-i} = 0$. Substituting into (21), it becomes

$$x_{2,sl}^i = x_1^i + \frac{1}{2\lambda} (g_1 + \lambda x_1^{-i}),$$

and the gap is

$$g_{2,sl} (x_1^i, x_1^{-i}) = \frac{1}{2} (g_1 + \lambda x_1^{-i}).$$

The value functions for the two agents are $V_{1,sl} (M_1^i, x_1^i, M_1^{-i}, x_1^{-i}) = M_1^i + \frac{1}{4\lambda} (g_1 + \lambda x_1^{-i})^2$, and $V_{1,ls} (M_1^i, x_1^i, M_1^{-i}, x_1^{-i}) = M_1^i - \frac{1}{2} (g_1 + \lambda x_1^i) x_1^i$. ■

Proposition 19 *In period 2, conditional on both agents being liquidated, the trade orders and the equilibrium gap are given by*

$$x_{2,ul}^i = 0 \text{ and } g_{2,ul} = g_1 + \lambda (x_1^i + x_1^{-i}). \quad (22)$$

The value functions are $V_{1,ul} (M_1^i, x_1^i, M_1^{-i}, x_1^{-i}) = M_1^i - (g_1 + \lambda (x_1^i + x_1^{-i})) x_1^i$, $i = 1, 2$.

Suppose now that $M^a/x_0^a > M^c/x_0^c$, hence we can define

$$\bar{x} \equiv x_0^a + x_0^c - \frac{1}{\lambda} \frac{M^c}{x_0^c} > \underline{x} \equiv x_0^a + x_0^c - \frac{1}{\lambda} \frac{M^a}{x_0^a}$$

as the thresholds on trades at date 1 that change the state of the world for arbitrageurs. It implies that at date 1 arbitrageur i faces the following optimization problem:

$$\begin{aligned} \max_x & M_0^i + g_1(x) (x - x_0^i) \\ & + 1_{\frac{M^i}{x_0^i}, \frac{M^{-i}}{x_0^{-i}} \geq g_1(x) - g_0} \frac{1}{9\lambda} g_1(x)^2 + 1_{\frac{M^i}{x_0^i} \geq g_1(x) - g_0 > \frac{M^{-i}}{x_0^{-i}}} \frac{1}{4\lambda} (g_1(x) + \lambda x_1^{-i})^2 \\ & - 1_{\frac{M^{-i}}{x_0^{-i}} \geq g_1(x) - g_0 > \frac{M^i}{x_0^i}} \frac{1}{2} (g_1(x) + \lambda x) x - 1_{\frac{M^i}{x_0^i}, \frac{M^{-i}}{x_0^{-i}} < g_1(x) - g_0} (g_1(x) + \lambda (x + x_1^{-i})) x, \end{aligned}$$

where we have combined the continuation values for the four states of the world, given above. From here it is easy to show that the FOCs become

$$\begin{aligned} 0 &= g_1 - \lambda (x_1^a - x_0^a) - \frac{2}{9} g_1 \text{ if } x_1^a > \bar{x} - x_1^c, \\ 0 &\geq g_1 - \lambda (x_1^a - x_0^a) - \frac{2}{9} g_1 \text{ if } x_1^a = \bar{x} - x_1^c, \\ 0 &= g_1 - \lambda (x_1^a - x_0^a) - \frac{1}{2} (g_1 + \lambda x_1^c) \text{ if } \bar{x} - x_1^c > x_1^a > \underline{x} - x_1^c, \\ 0 &\geq g_1 - \lambda (x_1^a - x_0^a) - \frac{1}{2} (g_1 + \lambda x_1^c) \text{ if } x_1^a = \underline{x} - x_1^c, \\ 0 &= g_1 - \lambda (x_1^a - x_0^a) - (g_1 + \lambda (x_1^a + x_1^c)) \text{ if } x_1^a < \underline{x} - x_1^c \end{aligned}$$

for the aggressive trader and

$$\begin{aligned}
0 &= g_1 - \lambda(x_1^c - x_0^c) - \frac{2}{9}g_1 \text{ if } x_1^c > \bar{x} - x_1^a, \\
0 &\geq g_1 - \lambda(x_1^c - x_0^c) - \frac{2}{9}g_1 \text{ if } x_1^c = \bar{x} - x_1^a, \\
0 &= g_1 - \lambda(x_1^c - x_0^c) - \frac{1}{2}(g_1 + \lambda x_1^c) \text{ if } \bar{x} - x_1^c > x_1^c > \underline{x} - x_1^a, \\
0 &\geq g_1 - \lambda(x_1^c - x_0^c) - \frac{1}{2}(g_1 + \lambda x_1^c) \text{ if } x_1^c = \underline{x} - x_1^a, \\
0 &= g_1 - \lambda(x_1^c - x_0^c) - (g_1 + \lambda(x_1^c + x_1^a)) \text{ if } x_1^c < \underline{x} - x_1^a
\end{aligned}$$

for the cautious trader. Combining these gives the following cases:

- Suppose $x_1^a + x_1^c > \bar{x}$, then the FOCs yield

$$x_1^a - x_0^a = x_1^c - x_0^c = \frac{7}{23\lambda}g_0.$$

The condition $x_1^a + x_1^c \geq \bar{x}$ is equivalent to

$$M^c \geq -\frac{14}{23}g_0x_0^c,$$

which always holds.

- Suppose $x_1^a + x_1^c = \bar{x}$, hence the FOCs simplify to

$$x_1^a - x_0^a \geq \frac{7}{9\lambda} \left(g_0 + \frac{M^c}{x_0^c} \right) \text{ and } x_1^c - x_0^c \geq \frac{7}{9\lambda} \left(g_0 + \frac{M^c}{x_0^c} \right).$$

It implies that

$$g_1 = g_0 - \lambda[x_1^a + x_1^c - (x_0^a + x_0^c)] \leq -\left(\frac{5}{9}g_0 + \frac{14}{9}\frac{M^c}{x_0^c}\right) < 0$$

which cannot happen if $M^c/x_0^c > 0$.

- Suppose we have $\bar{x} > x_1^a + x_1^c > \underline{x}$, which implies that

$$x_1^a - x_0^a = x_1^c - x_0^c = \frac{1}{5\lambda}(g_0 - \lambda x_0^c),$$

hence

$$g_1 = g_0 - \frac{2}{5}(g_0 - \lambda x_0^c),$$

and it must be that

$$0 < \frac{M^c}{x_0^c} < -\frac{2}{5}(g_0 - \lambda x_0^c) \leq \frac{M^a}{x_0^a}.$$

- Suppose $x_1^a + x_1^c = \underline{x}$, then the FOCs become

$$x_1^a + \frac{1}{2}x_1^c \geq \frac{1}{2\lambda} \left(g_0 + \frac{M^a}{x_0^a} \right) + x_0^a$$

and

$$x_1^c \geq \frac{1}{3\lambda} \left(g_0 + \frac{M^a}{x_0^a} \right) + \frac{2}{3}x_0^c.$$

Also it must be that $x_1^a + x_1^c = \underline{x} = x_0^a + x_0^c - \frac{1}{\lambda} \frac{M^a}{x_0^a}$.

- Finally, suppose $x_1^a + x_1^c < \underline{x}$, then we have

$$x_1^a - x_0^a = x_1^c - x_0^c = -\frac{1}{3}(x_0^a + x_0^c),$$

and it requires $x_1^a + x_1^c < \underline{x}$, i.e.

$$0 < \frac{M^c}{x_0^c} < \frac{M^a}{x_0^a} < \frac{2}{3}\lambda(x_0^a + x_0^c).$$

These conditions describe the locally optimal trades and the trivial constraints on the proportion of wealth invested, presented in Propositions 9-11. What is left is to check whether they are globally optimal too, i.e. whether any arbitrageur wants so deviate while changing the state too.

B.1 Optimal trading conditional on getting to state ss

There is no candidate equilibrium with the constraint binding for arbitrageur c . Besides the above requirements on arbitrageur capital, in an equilibrium with not binding constraints it must also be checked whether arbitrageur a would like to deviate and trigger the bankruptcy of arbitrageur c . This is because agent c 's position in the first best solution is not equivalent to full liquidation, thus it might be profitable for agent a to trigger agent c 's bankruptcy. For this, suppose that arbitrageur a deviates so that she remains solvent but arbitrageur c is liquidated. She is better off if and only if

$$\begin{aligned} V_{1,sl} \left(M_0^a + g_1(x_1^a - x_0^a), x_1^a, M_0^c + g_1(x_{1,nn}^c - x_0^c), x_{1,nn}^c \right) \\ > V_{1,ss} \left(M_0^a + g_{1,nn}(x_{1,nn}^a - x_0^a), x_{1,nn}^a, M_0^c + g_{1,nn}(x_{1,nn}^c - x_0^c), x_{1,nn}^c \right), \end{aligned}$$

that is if her utility from getting into state sl with positions $M_0^a + g_1(x_1^a - x_0^a)$ and x_1^a in the riskless and the risky assets, respectively, is higher than the utility she derives in state ss from holding positions $M_0^a + g_{1,nn}(x_{1,nn}^a - x_0^a)$ and $x_{1,nn}^a$ (which are the locally optimal holdings in that state), while assuming that arbitrageur c stays

with the equilibrium holding $x_{1,nn}^c$. After some algebra, she is better off deviating iff $x_1^a \in (\bar{x}_{nn,nv}^a - \delta_{nn,nv}^a, \bar{x}_{nn,nv}^a + \delta_{nn,nv}^a)$, where

$$\bar{x}_{nn,nv}^a = \frac{3}{23\lambda}g_0 + x_0^a + \frac{1}{3}x_0^c,$$

and

$$\delta_{nn,nv}^a = \frac{2}{3\lambda} \sqrt{\left(\frac{15 - 2\sqrt{6}}{23}g_0 + \lambda x_0^c\right) \left(\frac{15 + 2\sqrt{6}}{23}g_0 + \lambda x_0^c\right)}.$$

Given $g_0 \geq 0$ and the assumption $x_0^c > 0$, the discriminant is non-negative and $\delta_{nn,nv}^a$ exists. As arbitrageur 1 can only push arbitrageur 2 into liquidation by increasing the gap while making sure she does not get liquidated, i.e. by choosing a trade

$$-\frac{1}{\lambda}M^a/x_0^a - (x_{1,nn}^c - x_0^c) \leq x_1^a - x_0^a < -\frac{1}{\lambda}M^c/x_0^c - (x_{1,nn}^c - x_0^c),$$

her deviation can increase her utility if and only if both $-\frac{1}{\lambda}M^a/x_0^a - (x_{1,nn}^c - x_0^c) < \bar{x}_{nn,nv}^a - x_0^a + \delta_{nn,nv}^a$ and $\bar{x}_{nn,nv}^a - x_0^a - \delta_{nn,nv}^a < -\frac{1}{\lambda}M^c/x_0^c - (x_{1,nn}^c - x_0^c)$ hold. Hence a simple reorganization of these inequalities implies that a necessary condition for the existence of the equilibrium is that either $M^a/x_0^a \leq -\frac{10}{23}g_0 + \frac{1}{3}\lambda x_0^c - \lambda\delta_{nn,nv}^a$ or $M^c/x_0^c \geq -\frac{10}{23}g_0 + \frac{1}{3}\lambda x_0^c + \lambda\delta_{nn,nv}^a$. As $M^a/x_0^a > 0$, it must be that $M^c/x_0^c \geq \Delta_{nn,nv}^a \equiv -\frac{10}{23}g_0 + \frac{1}{3}\lambda x_0^c + \lambda\delta_{nn,nv}^a$.

B.2 Optimal trading conditional on getting to state sl

To check whether an equilibrium with arbitrageur a being solvent and arbitrageur c having to liquidate exists, it must be checked whether arbitrageur c would prefer to change her trading speed and remain solvent, whether she would prefer to force arbitrageur 1 to distress, or whether the constrained arbitrageur a would prefer to liquidate.

B.2.1 Equilibrium with non-binding constraint

The possible deviations are when arbitrageur c forces arbitrageur a into distress, or when arbitrageur c rescues herself. As the constraint might not bind in equilibrium when arbitrageurs start with different positions, it must also be checked whether arbitrageur c wants to trigger the distress of arbitrageur a while rescuing herself.

Agent c forces agent a into liquidation. Arbitrageur c is better off forcing the liquidation or arbitrageur a iff

$$\begin{aligned} V_{1,l} (M_0^c + g_1 (x_1^c - x_0^c), x_1^c, M_0^a + g_1 (x_{1,nv}^a - x_0^a), x_{1,nv}^a) \\ > V_{1,l} (M_0^c + g_{1,nv} (x_{1,vn}^c - x_0^c), x_{1,vn}^c, M_0^a + g_1 (x_{1,nv}^a - x_0^a), x_{1,nv}^a), \end{aligned}$$

that is if her utility from getting into state ll with positions $M_0^c + g_1 (x_1^c - x_0^c)$ and x_1^c in the riskless and the risky assets, respectively, is higher than the utility she derives in state ls from holding positions $M_0^c + g_{1,nv} (x_{1,vn}^c - x_0^c)$ and $x_{1,vn}^c$ (which are actually the optimal holdings in that state), while assuming that arbitrageur a stays with the equilibrium holding $x_{1,nv}^a$. After some algebra, she is better off deviating iff $x_1^c \in (\bar{x}_{nv,vv}^c - \delta_{nv,vv}^c, \bar{x}_{nv,vv}^c + \delta_{nv,vv}^c)$, where

$$\bar{x}_{nv,vv}^c = -\frac{1}{2}x_0^a + \frac{3}{5}x_0^c - \frac{1}{10\lambda}g_0,$$

and

$$\delta_{nv,vv}^c = \frac{1}{\sqrt{3\lambda}} \sqrt{\left(\frac{1}{10}g_0 - \frac{1}{2}\lambda x_0^a + \frac{7}{5}\lambda x_0^c\right) \left(-\frac{9}{10}g_0 - \frac{3}{2}\lambda x_0^a - \frac{3}{5}\lambda x_0^c\right)},$$

with the discriminant being negative (hence a deviation cannot increase her utility) iff

$$0 < x_0^a < \frac{1}{5\lambda}g_0 + \frac{14}{5}x_0^c.$$

Suppose that the discriminant is non-negative and hence $\delta_{nv,vv}^c$ exists.

Since $x_0^a > 0$, arbitrageur c can push arbitrageur a into liquidation by increasing the gap, i.e. by choosing a trade $x_1^c - x_0^c < -\frac{1}{\lambda}M^a/x_0^a - (x_{1,nv}^a - x_0^a)$. She can deviate while increasing her utility if and only if $-\frac{1}{\lambda}M^a/x_0^a - (x_{1,nv}^a - x_0^a) > \bar{x}_{nv,vv}^c - x_0^c - \delta_{nv,vv}^c$. Hence a simple reorganization of this inequality implies that in equilibrium it must be that $M^a/x_0^a \geq -\frac{1}{10}g_0 + \frac{1}{2}\lambda x_0^a + \frac{3}{5}\lambda x_0^c + \lambda \delta_{nv,vv}^c$. Notice that as $x_0^c > 0$, arbitrageur c will remain distressed when pushing arbitrageur a into bankruptcy, and hence no other condition is needed.

Agent c rescues herself. Arbitrageur c is better off rescuing herself iff

$$\begin{aligned} V_{1,ss} (M_0^c + g_1 (x_1^c - x_0^c), x_1^c, M_0^a + g_1 (x_{1,nv}^a - x_0^a), x_{1,nv}^a) \\ > V_{1,l} (M_0^c + g_{1,nv} (x_{1,vn}^c - x_0^c), x_{1,vn}^c, M_0^a + g_1 (x_{1,nv}^a - x_0^a), x_{1,nv}^a), \end{aligned}$$

that is if her utility from getting into state ss with positions $M_0^c + g_1 (x_1^c - x_0^c)$ and x_1^c in the riskless and the risky assets, respectively, is higher than the utility she derives

in state ls from holding positions $M_0^c + g_{1,nv}(x_{1,vn}^c - x_0^c)$ and $x_{1,vn}^c$ (which are actually the optimal holdings in that state), while assuming that arbitrageur a stays with the equilibrium holding $x_{1,nv}^a$. After some tedious algebra, she is better off deviating iff $x_1^c \in (\bar{x}_{nv,nn}^c - \delta_{nv,nn}^c, \bar{x}_{nv,nn}^c + \delta_{nv,nn}^c)$ with

$$\bar{x}_{nv,nn}^c = \frac{7}{20\lambda}g_0 + \frac{87}{80}x_0^c \text{ and}$$

$$\delta_{nv,nn}^c = \frac{3\sqrt{457}}{80\lambda} \sqrt{\left(\frac{228 + 20\sqrt{2}}{457}g_0 + \lambda x_0^c\right) \left(\frac{228 - 20\sqrt{2}}{457}g_0 + \lambda x_0^c\right)}.$$

Given $g_0 \geq 0$ and the assumption $x_0^c > 0$, the discriminant is non-negative and $\delta_{nv,nn}^2$ exists.

Since $x_0^a, x_0^c > 0$, arbitrageur c can rescue herself by shrinking the gap, i.e. by choosing a trade $x_1^c - x_0^c \geq -\frac{1}{\lambda}M^c/x_0^c - (x_{1,nv}^a - x_0^a)$. She can deviate while increasing her utility if and only if $-\frac{1}{\lambda}M^c/x_0^c - (x_{1,nv}^a - x_0^a) < \bar{x}_{nv,nn}^c - x_0^c + \delta_{nv,nn}^c$. Hence a simple reorganization of this inequality implies that in equilibrium it must be that $M^c/x_0^c \leq -\frac{11}{20}g_0 + \frac{9}{80}\lambda x_0^c - \lambda\delta_{nv,nn}^c$.

Summary for unconstrained sl equilibrium. Combining the initial constraints that are based on the equilibrium price and the above constraints regarding potential deviations yields that the unconstrained sl equilibrium exists if $0 < M^c/x_0^c \leq \Delta_{nv,nn}^c$ and $M^a/x_0^a \geq \Delta_{nv,vv}^c$, where $\Delta_{nv,nn}^c = -\frac{11}{20}g_0 + \frac{9}{80}\lambda x_0^c - \lambda\delta_{nv,nn}^c$ and

$$\Delta_{nv,vv}^c = \max \left\{ -\frac{2}{5}(g_0 - \lambda x_0^2); -\frac{1}{10}g_0 + \frac{1}{2}\lambda x_0^a + \frac{3}{5}\lambda x_0^c + \lambda\delta_{nv,vv}^c \right\}.$$

B.2.2 Equilibrium with binding constraint

The possible deviations from the equilibrium trades are when arbitrageur c either forces arbitrageur a into distress or rescues herself, and when arbitrageur a decides to liquidate.

Agent c forces agent a into liquidation. Arbitrageur c is better off forcing the liquidation or arbitrageur 1 iff

$$\begin{aligned} V_{1,ul}(M_0^c + g_1(x_1^c - x_0^c), x_1^c, M_0^a + g_1(x_{1,bv}^a - x_0^a), x_{1,bv}^a) \\ > V_{1,ls}(M_0^c + g_{1,bv}(x_{1,vb}^c - x_0^c), x_{1,vb}^c, M_0^a + g_1(x_{1,bv}^a - x_0^a), x_{1,bv}^a), \end{aligned}$$

that is if her utility from getting into state ll with positions $M_0^c + g_1(x_1^c - x_0^c)$ and x_1^c in the riskless and the risky assets, respectively, is higher than the utility she derives in state ls from holding positions $M_0^c + g_{1,bv}(x_{1,vb}^c - x_0^c)$ and $x_{1,vb}^c$ (which are actually the optimal holdings in that state), while assuming that arbitrageur a stays with the equilibrium holding $x_{1,bv}^a$. After some algebra, she is better off deviating iff $x_1^c \in (\bar{x}_{bv,vv}^c - \delta_{bv,vv}^c, \bar{x}_{bv,vv}^c + \delta_{bv,vv}^c)$, where

$$\bar{x}_{bv,vv}^c = \frac{1}{6\lambda}g_0 - \frac{1}{2}x_0^a + \frac{1}{3}x_0^c + \frac{2}{3\lambda}\frac{M^a}{x_0^a}, \text{ and}$$

$$\delta_{bv,vv}^c = \frac{1}{\sqrt{3\lambda}}\sqrt{\left(\frac{M^a}{x_0^a} + \frac{1}{2}g_0 - \frac{1}{2}\lambda x_0^a + \lambda x_0^c\right)\left(\frac{M^a}{x_0^a} - \left(\frac{1}{2}g_0 + \frac{3}{2}\lambda x_0^a + \lambda x_0^c\right)\right)},$$

with the discriminant being negative (hence a deviation cannot increase her utility) iff

$$-\frac{1}{2}g_0 + \frac{1}{2}\lambda x_0^a - \lambda x_0^c < \frac{M^a}{x_0^a} < \frac{1}{2}g_0 + \frac{3}{2}\lambda x_0^a + \lambda x_0^c.$$

Suppose now that the discriminant is non-negative and hence $\delta_{bv,vv}^c$ exists. As $x_0^a, x_0^c > 0$, arbitrageur c can push arbitrageur a into liquidation by increasing the gap, i.e. by choosing a trade $x_1^c < x_{1,vb}^c$. She can deviate while increasing her utility if and only if $x_{1,vb}^c > \bar{x}_{bv,vv}^c - \delta_{bv,vv}^c$. A simple reorganization of this inequality implies that a necessary condition for the equilibrium is $M^a/x_0^a + g_0 + 2\lambda x_0^c \leq 0$, which cannot happen. Therefore the equilibrium can only exist if $-\frac{1}{2}g_0 + \frac{1}{2}\lambda x_0^a - \lambda x_0^c < M^a/x_0^a < \frac{1}{2}g_0 + \frac{3}{2}\lambda x_0^a + \lambda x_0^c$.

Agent a decides to liquidate. She is better off triggering her own liquidation iff

$$\begin{aligned} V_{1,ll}(M_0^a + g_1(x_1^a - x_0^a), x_1^a, M_0^c + g_{1,bv}(x_{1,vb}^c - x_0^c), x_{1,vb}^c) \\ > V_{1,sl}(M_0^a + g_1(x_{1,bv}^a - x_0^a), x_{1,bv}^a, M_0^c + g_{1,bv}(x_{1,vb}^c - x_0^c), x_{1,vb}^c), \end{aligned}$$

that is if her utility from getting into state ll with positions $M_0^a + g_1(x_1^a - x_0^a)$ and x_1^a in the riskless and the risky assets, respectively, is higher than the utility she derives in state ls from holding positions $M_0^a + g_1(x_{1,bv}^a - x_0^a)$ and $x_{1,bv}^a$ (which are the optimal holdings in that state), while assuming that arbitrageur c stays with the equilibrium holding $x_{1,vb}^c$. After some algebra, she is better off deviating iff $x_1^a \in (\bar{x}_{bv,vv}^a - \delta_{bv,vv}^a, \bar{x}_{bv,vv}^a + \delta_{bv,vv}^a)$, where

$$\bar{x}_{bv,vv}^a = -\frac{1}{2}\left(\frac{1}{3\lambda}\frac{M^a}{x_0^a} + \frac{1}{3\lambda}g_0 - x_0^a + \frac{2}{3}x_0^c\right), \text{ and}$$

$$\delta_{bv,vv}^a = \frac{1}{2\lambda}\sqrt{\left(\frac{M^a}{x_0^a} + g_0 + \lambda x_0^a\right)\left(\frac{11}{3}\frac{M^a}{x_0^a} - \frac{1}{3}g_0 - 3\lambda x_0^a - \frac{8}{3}\lambda x_0^c\right)},$$

with the discriminant being negative (hence a deviation cannot increase her utility) iff

$$0 < \frac{M^a}{x_0^a} < \frac{1}{11}g_0 + \frac{9}{11}\lambda x_0^a + \frac{8}{11}\lambda x_0^c.$$

Suppose that the discriminant is non-negative and hence $\delta_{bv,vv}^a$ exists.

As $x_0^a, x_0^c > 0$, arbitrageur a can trigger her own liquidation by increasing the gap, i.e. by choosing a trade $x_1^a < x_{1,bv}^a$. She can deviate while increasing her utility if and only if $x_{1,bv}^a > \bar{x}_{bv,vv}^a - \delta_{bv,vv}^a$. A simple reorganization of this inequality implies that a necessary condition for the equilibrium is thus either $0 < M^a/x_0^a < \frac{1}{11}g_0 + \frac{9}{11}\lambda x_0^a + \frac{8}{11}\lambda x_0^c$ or $\frac{1}{11}g_0 + \frac{9}{11}\lambda x_0^a + \frac{8}{11}\lambda x_0^c \leq M^a/x_0^a \leq -\frac{1}{7}g_0 + \frac{3}{7}\lambda x_0^a + \frac{4}{7}\lambda x_0^c$. As this latter would also imply $3g_0 + 5\lambda x_0^a + 2\lambda x_0^c \leq 0$, which never holds, therefore the equilibrium only exists if $0 < M^a/x_0^a < \frac{1}{11}g_0 + \frac{9}{11}\lambda x_0^a + \frac{8}{11}\lambda x_0^c$.

Agent c rescues herself. Arbitrageur c is better off rescuing herself iff

$$\begin{aligned} V_{1,ss} \left(M_0^c + g_1(x_1^c - x_0^c), x_1^c, M_0^a + g_1(x_{1,bv}^a - x_0^a), x_{1,bv}^a \right) \\ > V_{1,ls} \left(M_0^c + g_{1,bv}(x_{1,vb}^c - x_0^c), x_{1,vb}^c, M_0^a + g_1(x_{1,bv}^a - x_0^a), x_{1,bv}^a \right), \end{aligned}$$

that is if her utility from getting into state ss with positions $M_0^c + g_1(x_1^c - x_0^c)$ and x_1^c in the riskless and the risky assets, respectively, is higher than the utility she derives in state ls from holding positions $M_0^c + g_{1,bv}(x_{1,vb}^c - x_0^c)$ and $x_{1,vb}^c$ (which are actually the optimal holdings in that state), while assuming that arbitrageur a stays with the equilibrium holding $x_{1,bv}^a$. After some tedious algebra, she is better off deviating iff $x_1^c \in (\bar{x}_{bv,nn}^c - \delta_{bv,nn}^c, \bar{x}_{bv,nn}^c + \delta_{bv,nn}^c)$ with

$$\begin{aligned} \bar{x}_{bv,nn}^c &= \frac{7}{12\lambda} \left(g_0 + \frac{M^a}{x_0^a} \right) + \frac{41}{48}x_0^c \text{ and} \\ \delta_{bv,nn}^c &= \frac{\sqrt{7}}{4\lambda} \sqrt{\left(\frac{M^a}{x_0^a} + g_0 + \frac{23 + 3\sqrt{2}}{14}\lambda x_0^c \right) \left(\frac{M^a}{x_0^a} + g_0 + \frac{23 - 3\sqrt{2}}{14}\lambda x_0^c \right)}, \end{aligned}$$

where $x_0^a, x_0^c > 0$ implies that the discriminant is non-negative and hence $\delta_{bv,nn}^c$ exists.

As $x_0^c > 0$, arbitrageur c can rescue herself by shrinking the gap, i.e. by choosing a trade $x_1^c - x_0^c \geq -\frac{1}{\lambda}M^c/x_0^c - (x_{1,bv}^a - x_0^a)$. She can deviate while increasing her utility if and only if $-\frac{1}{\lambda}M^c/x_0^c - (x_{1,bv}^a - x_0^a) < \bar{x}_{bv,nn}^c - x_0^c + \delta_{bv,nn}^c$. A simple reorganization of this inequality implies that a necessary condition for the equilibrium is $M^c/x_0^c \leq \frac{3}{4}M^a/x_0^a - \frac{1}{4}g_0 - \frac{3}{16}\lambda x_0^c - \lambda \delta_{bv,nn}^c$.

However, given $M^c/x_0^c > 0$, it should be that $\lambda \delta_{bv,nn}^c < \frac{3}{4}M^a/x_0^a - \frac{1}{4}g_0 - \frac{3}{16}\lambda x_0^c$. After substituting in for $\delta_{bv,nn}^c$ and using that $M^a/x_0^a > 0$ as well, it can be shown that it

cannot hold. Therefore agent c always rescues herself, and a constrained sl equilibrium thus never happens.

B.3 Optimal trading conditional on getting to state ll

As it is shown below, it is enough to consider the possible deviation when arbitrageur 1 rescues herself, as it already implies there is no equilibrium with both agents becoming distressed.

Arbitrageur a rescues herself. If, for example, it is shown that arbitrageur a deviates, there is no equilibrium with double liquidation at all. She is better off rescuing herself iff

$$\begin{aligned} V_{1,sl} (M_0^a + g_1 (x_1^a - x_0^a), x_1^a, M_0^c + g_1 (x_{1,vv}^c - x_0^c), x_{1,vv}^c) \\ > V_{1,ll} (M_0^a + g_{1,vv} (x_{1,v}^a - x_0^a), x_{1,vv}^a, M_0^c + g_{1,vv} (x_{1,vv}^c - x_0^c), x_{1,vv}^c), \end{aligned}$$

that is if her utility from getting into state sl with positions $M_0^a + g_1 (x_1^a - x_0^a)$ and x_1^a in the riskless and the risky assets, respectively, is higher than the utility she derives in state ll from holding positions $M_0^a + g_{1,vv} (x_{1,v}^a - x_0^a)$ and $x_{1,vv}^a$ (which are actually the optimal holdings in that state), while assuming that arbitrageur 2 stays with the equilibrium holding $x_{1,vv}^c$. After some, she is better off deviating iff $x_1^a \in (\bar{x}_{vv,nv}^a - \delta_{vv,nv}^a, \bar{x}_{vv,nv}^a + \delta_{vv,nv}^a)$ with

$$\bar{x}_{vv,nv}^a = x_0^a + \frac{1}{3\lambda} \left(g_0 + \frac{2}{3}\lambda x_0^a - \frac{1}{3}\lambda x_0^c \right)$$

and

$$\delta_{vv,nv}^a = \frac{2}{3\lambda} \left| g_0 + \frac{5}{3}\lambda x_0^a + \frac{2}{3}\lambda x_0^c \right|.$$

As $x_0^a, x_0^c > 0$, agent a has to decrease the gap, i.e. buy more (or short less) to make sure $M^a/x_0^a \geq g_1 - g_0$, and for this she needs a trade of $x_1^a \geq -\frac{1}{\lambda}M^a/x_0^a - x_{1,vv}^c$. Combining with the other condition yields that she can deviate iff $-\frac{1}{\lambda}M^a/x_0^a - x_{1,vv}^c < \bar{x}_{vv,nv}^a + \delta_{vv,nv}^a$. However, as this constraint is equivalent to $0 < g_0 + \lambda x_0^a + M^a/x_0^a$, which always holds as all three components are non-negative, arbitrageur a always deviates and thus there is no equilibrium with both agents getting liquidated.

C Trading under predatory threat

As mentioned in the main part of the paper, the optimization programs of arbitrageurs with these constraints becomes difficult to solve in closed form (it includes solving 4th order equations). Thus, we provide some preliminary analysis in the following three Lemmas to decrease the potential set of equilibria, and then we solve the remaining problem numerically.

First, it is easy to see that:

Claim 20 *There exists no equilibrium without trading at date 0.*

This Lemma is rather intuitive. If, for example, arbitrageur 1 does not trade in period 0, arbitrageur 2 is better off investing a little into the arbitrage opportunity that staying out completely, as long as her trade satisfies (13). Given the assumption $M^2 > 0$, there exists a sufficiently small x_0^2 such that it is possible.

Similarly:

Claim 21 *There exists no equilibrium with only one arbitrageur trading at date 0.*

If, for example, arbitrageur 1 does trade in period 0, arbitrageur 2 can take an arbitrarily small position such that $M^2/x_0^2 > M^1/x_0^1$. It implies that she becomes the aggressive agent, and as there is no equilibrium in which both arbitrageurs are liquidated, she will never be liquidated. As there is no threat of predation on her, investing is strictly better than staying out, as it is a fundamentally riskless arbitrage opportunity.

Finally:

Claim 22 *There is no equilibrium with any agent having $x_0^i < 0$.*

First, an agent cannot have $x_0^i < 0$ and become liquidated later, because in this case not trading at date 0 would make her better off for two reasons: she can trade freely later; moreover, liquidation means closing positions that one has built up previously. Second, if an agent has $x_0^i < 0$ and she uses it to force the other trader to liquidation, arbitrageur $-i$ can decide to withdraw from trading in the first period, with which she stays solvent, moreover faces a better investment opportunity, because the effective gap for her is even larger than before.

Therefore, it must be that in equilibrium $x_0^1, x_0^2 > 0$. Suppose now that under their trades arbitrageurs end up in an unconstrained ss equilibrium. Going back to the date 0 optimization, arbitrageur i 's optimization problem becomes

$$\max_{x_0^i} W_3^i = M_0^i + \frac{72}{23^2 \lambda} g_0^2 = M^i + (\bar{g} - \lambda x_0^i - \lambda x_0^{-i}) x_0^i + \frac{72}{23^2 \lambda} (\bar{g} - \lambda x_0^i - \lambda x_0^{-i})^2, \quad (23)$$

so the FOC yields $x_0^1 = x_0^2 = (1/\lambda) \alpha \bar{g} > 0$ with $\alpha = \frac{385}{1299}$. Therefore, $g_0 = (1 - 2\alpha) \bar{g} = \frac{529}{1299} \bar{g}$, and the optimal holdings and the equilibrium gap are as in the model with no constrained, in Proposition 4. This yields a utility of $V(M^i) = M^i + g_0 x_0^i + \frac{72}{23^2 \lambda} g_0^2 = M^i + \frac{1}{\lambda} \Delta \bar{g}^2$ with $\Delta = (1 - 2\alpha) \alpha + \frac{72}{23^2} (1 - 2\alpha)^2$.

When is it an equilibrium in presence of the wealth constraint? First, it must satisfy the conditions derived in Appendix B.1. The equilibrium candidate order and gap

$$x_0^2 = \frac{385}{1299\lambda} \bar{g} \text{ and } g_0 = \frac{529}{1299} \bar{g}$$

satisfy

$$x_0^2 > \frac{2(\sqrt{233} - 15)}{23\lambda} g_0,$$

hence it must be that

$$\frac{M^1}{x_0^1}, \frac{M^2}{x_0^2} \geq -\frac{10}{23} g_0 + \frac{1}{3} \lambda x_0^2 + \lambda \delta_{nn,nv}^1,$$

which is equivalent to

$$M^1, M^2 \geq \frac{1}{\lambda} \Omega \bar{g}^2$$

with

$$\begin{aligned} \Omega &\equiv \alpha \left[-\frac{10}{23} + \frac{83}{69} \alpha + \frac{2}{3} \sqrt{\left(\frac{15 - 2\sqrt{6}}{23} (1 - 2\alpha) + \alpha \right) \left(\frac{15 + 2\sqrt{6}}{23} (1 - 2\alpha) + \alpha \right)} \right] \\ &= \frac{385 (4\sqrt{130051} - 305)}{3 * 1299^2} > 0. \end{aligned}$$

Second, it must be that none of the agents want to deviate from this profile and change the state. As having to liquidate puts a constraint on the strategy space of an arbitrageur, none of the arbitrageurs want to change the state in order to trigger her own distress. Therefore the only deviation one has to check is triggering the liquidation of the other arbitrageur. We confirm it numerically that as long as arbitrageurs trade such that they satisfy (13), it would be too costly for any trader to go long the gap and trigger the liquidation of the other arbitrageur. Therefore the equilibrium trades are those obtained from (23), subject to (13). This concludes the solution.