Default Risk and the Pricing of U.S. Sovereign Bonds

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Abstract

United States Treasury securities are viewed in academics and practice as being free of default risk. In principle, nominal outstanding Treasury debt can be inflated away by issuing fiat currency. The same does not hold true, however, for inflation-indexed debt. We examine the relative pricing of nominal and inflation-indexed debt in the presence of risk of default. We show empirically that the breakeven inflation rate between nominal Treasury securities and TIPS is statistically significantly related to the premium paid on U.S. credit default swaps, controlling for measures of liquidity and slow-moving capital. This evidence motivates us to model the prices of nominal and inflation-protected securities in the presence of default risk. Our model shows that breakeven inflation is related to perceptions of differing rates of recovery in the two markets. Results from estimating our model suggest that we can simultaneously capture variation in breakeven inflation rates and United States Treasury credit default swap spreads.

JEL classification: E4, E6, G12.

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1 Introduction

In both academic literature and practice, United States Treasury securities are often viewed as risk-free securities, in the sense that their nominal payoffs are certain. At least part of the logic behind this treatment is the fact that the United States Treasury can inflate away its debt by issuing fiat currency. As a result, there is no a priori reason that the Treasury should default on its obligations denominated in current dollars. Recent history, however, has called this assumption in doubt. Starting in late 2007, as shown in Figure 1, the premium paid to insure United States sovereign debt increased dramatically from 1-2 basis points to nearly 100 basis points. While the spread has since declined, it has remained elevated relative to pre-crisis levels. Moreover, repeated political conflict over the debt ceiling has prompted concern about the possibility of a U.S. Treasury default. In 2011, a debt ceiling crisis led to the downgrading of United States sovereign debt by Standard & Poor’s. These crises repeated in 2013 and again, most recently, in the Fall of 2017. Public reaction to these crises suggest that there is a perception that there is a non-trivial risk of Treasury default.

While concerns about default have received greater attention in the past decade, the question of whether or not United States Treasury CDS spreads reflect actual fears of default remains a subject of debate, and the Treasury is still able to issue fiat currency to inflate the debt away.\textsuperscript{1,2} However, the Treasury cannot inflate away inflation-protected debt. As of December 31, 2017, approximately $1.3 trillion in Treasury Inflation Protected Securities (TIPS) were held by the public.\textsuperscript{3} While the total outstanding represents only 9\% of public debt, it is still an economically significant amount; for example according to the Bank for International Settlements, it is more than Germany’s outstanding debt at the end of 2016. Not only are these securities not free of default, but a failure to make payment on inflation-protected securities would also trigger default on nominal Treasury securities. As a result, by issuing inflation protected securities, the Treasury has made all of its debt subject to risk of default.

\textsuperscript{1}In general, Arora, Gandhi and Longstaff (2012) show that counterparty credit risk is priced in the CDS market using data covering the height of the 2008 crisis, but the magnitude is trivial resulting from the full collateralization of CDS liabilities.

\textsuperscript{2}This point is not without contention, Hilscher, Raviv and Reis (2014) document that increased inflation is unlikely to substantially lower the real liability of the U.S. as a fraction of GDP due to the fact that Treasury debt held by the public is typically short-dated.

\textsuperscript{3}Monthly Statement of the Public Debt of the United States, see \url{www.treasurydirect.gov}.
In this paper, we investigate the degree to which this credit risk plays a role in the pricing of U.S. sovereign debt securities. More specifically, we examine the effects of credit risk on the relative pricing of nominal and inflation-protected securities. We first document that breakeven inflation rates implied in the yields of nominal Treasuries and TIPS are significantly related to premia paid on U.S. Treasury credit default swaps. Over a sample period from late 2007 through 2015, a one standard deviation increase in the Euro-denominated U.S. CDS spread is associated with an approximately 9 basis point increase in hedged breakeven inflation. This is not a phenomenon dominated by the financial crisis; in the sample period from 2010 onward the relation continues to hold. In this latter period, the relation is robust to controlling for variables meant to capture illiquidity and slow-moving capital in the Treasury market. While we cannot completely rule out alternative explanations, our evidence is suggestive of the fact that breakeven inflation co-moves positively with sovereign credit risk.

Motivated by this evidence, we derive a model of nominal and inflation-protected sovereign debt subject to risk of default. Our modeling strategy follows Monfort et al. (2017a) in modeling bond yields as affine functions of standard state variables, as well as a default risk variable. The approach departs from the standard Duffie and Singleton (1999) Gaussian credit risk framework in modeling credit events as Gamma-zero distributed. The advantage to our approach is that it produces closed-form pricing formulas and allows us to avoid the specification of inflation dynamics. We show in this setting that the spread between inflation-linked swaps and breakeven inflation rates (ILS-BEI) is related to the relative rate of recovery anticipated on nominal and inflation-protected securities.

We estimate the parameters of the model using the extended Kalman filter, utilizing the five-year U.S. CDS spreads and five maturities of the ILS-BEI spread as observables. We force the estimation to perfectly match CDS spreads, but capture variation in the ILS-BEI spreads with error. Our results indicate that the model is able to simultaneously capture variation in credit default swap spreads and breakeven rates of inflation. Specifically, while the CDS spread is matched by construction, credit risk factors are able to capture approximately 20% of the total ILS-BEI variation. Further, our estimates indicate that the market perception of the recovery rate on TIPS is about 12 percentage points lower than that of nominal bonds. The results support our conjecture that the pricing of nominal and inflation-protected securities are affected by default risk.

Our paper contributes to at least three broad strands of the fixed income literature. The
first is the literature investigating the role of default risk in the pricing of sovereign securities. On the empirical front, Ang and Longstaff (2013) estimate an affine multi-factor model of U.S. and European state and country credit default swaps and conclude that systemic sovereign risk is strongly linked to financial market variables. The authors observe that the estimated U.S. systemic credit risk-neutral default intensities spiked at the beginning of 2009, immediately after the onset of the financial crisis. Similarly, Chernov, Schmid and Schneider (2016) note that CDS spreads of sovereign debt securities in major developed markets rose during the financial crisis and remained elevated in subsequent years. The authors construct a macrofinance model in which CDS premia reflect default probabilities. The authors calibrate their model to the United States macroeconomy, and show that it is able to generate the high premium paid to insure U.S. sovereign debt. Their results suggest that high CDS spreads can arise in an equilibrium framework with default risk. Additionally, Filipovic and Trolle (2013), Monfort et al. (2017a), and Augustin, Chernov and Song (2018) investigate the joint pricing of Treasuries and CDS. Our point of departure from this literature is in investigating the role that default risk plays in the differential pricing of nominal and inflation-protected securities.

The second area to which we contribute is the relative pricing of nominal and inflation-protected securities. Fleckenstein, Longstaff and Lustig (2014) document apparent no-arbitrage violations in the pricing of nominal and inflation-protected securities. Specifically, they show that an arbitrage strategy using nominal treasuries, TIPS and inflation swaps generated large arbitrage profits during the financial crisis, and that these profits were present before and after the crisis period. Their empirical investigation suggests that the arbitrage arises due to slow-moving capital; a lack of arbitrage activity in the Treasury market allows the profits to persist. The ILS-BEI spread utilized in our empirical analysis is closely related to the mispricing that they document. Our results suggest that part of this mispricing is related to credit risk.\footnote{The authors note that inflation-protected securities are not necessarily default risk free, but suggest that since CDS do not distinguish between nominal and inflation-protected debt, default risk is unlikely to explain the arbitrage profits.} Other papers investigating the term structure of nominal bonds and TIPS include Buraschi and Jiltsov (2005), Ang, Bekaert and Wei (2008), Chernov and Mueller (2012), Christensen, Lopez and Rudebusch (2012), Haubrich, Pennacchi and Ritchken (2012), Hordahl and Tristani (2012), Abrahams, Adrian, Crump and Moench (2016), Campbell, Sunderam and Viceira (2016), Christensen, Lopez and Rudebusch (2014), Roussellet (2017), and Fleckenstein, Longstaff and Lustig (2017). We differ
from this literature in explicitly considering the role of default risk in the pricing of these bonds.

A third strand of literature estimates the liquidity premium embedded in TIPS with respect to nominal bonds. Grishchenko and Huang (2012) construct inflation risk premium employing only TIPS yields and controlling for the liquidity premium between TIPS and nominal bonds. Pflueger and Viceira (2016) suggest that there is a large and economically significant liquidity premium that affects the relative pricing of nominal and real bonds. Their evidence includes both U.S. and U.K. nominal and inflation-indexed bond prices. In the same vein, Abrahams, Adrian, Crump, Moench and Yu (2016) decompose real and nominal yields into liquidity, inflation, and real interest rate risk components in an affine term structure model. They conclude that forward breakeven inflation is primarily driven by risk and liquidity premia. D’Amico, Kim and Wei (2018) again propose a substantial liquidity premium as the primary factor driving the wedge between TIPS yields and real riskfree rates, thus causing distortions in the term structure of breakeven inflation. Andreasen, Christensen and Ridell (2017) identify liquidity risk in TIPS with the average deviation across bonds from what a no-arbitrage pricing model would predict. Recent studies also use the ILS-BEI spread as a proxy for liquidity risk only (see e.g. Christensen and Gillan (2018) or Moench and Vladu (2018)). This is because very high liquidity is usually attributed to the swap market in the U.S. (see for instance Driessen, Nijman and Simon (2017) or Camba-Mendez and Werner (2017)). Our evidence suggests that considering the probability of sovereign default further contributes to the understanding of the breakeven spread.

2 Empirical Analysis

In this section we investigate the extent that default risk may influence the relative pricing of nominal and inflation-protected sovereign obligations. Specifically, we test whether CDS spreads are related to the difference between inflation swap rates (ILS) and breakeven inflation (BEI) as represented by nominal less real yields of U.S. sovereign debt. We examine variation in these quantities over the full sample period and a subperiod that does not include the financial crisis of 2007-2009.
2.1 Credit Risk in Breakeven Inflation Rates

Fleckenstein, Longstaff and Lustig (2014) show that the cash flows of a nominal Treasury bond can be replicated by a portfolio of TIPS, U.S. Treasury STRIPS, and inflation swaps, and that nominal Treasuries trade at a premium to this replicating portfolio. We investigate a measure that is related to their approach, but not subject to exact timing of cash flows: the difference in the inflation level swap rate and breakeven inflation (ILS-BEI). A zero-coupon inflation swap pays cumulative inflation in exchange for a fixed rate determined at initiation of the contract. In its construction, including the reference index of CPI, an inflation swap is comparable to the breakeven inflation rate as defined by equivalent maturity zero-coupon nominal Treasuries less zero-coupon TIPS (also indexed to CPI). Both the ILS and BEI reflect inflation expectations as well as the inflation risk premium. As seen in Figure 2, the two track rather closely, with a consistent premium attributed to the breakeven rate proxied by the inflation swap derivative contract. Campbell, Shiller and Viceira (2009) suggest the premium is related to the cost of supplying inflation protection and is typical under normal market conditions. ILS-BEI averages 36 basis points over the sample, yet peaks at 210 basis points in late 2008. Both the average and peak spread are roughly comparable to the basis point mispricing between the nominal Treasury bond and the replicating portfolios detailed by Fleckenstein, Longstaff and Lustig (2014).

A divergence in ILS-BEI may be attributable to a series of factors in addition to the average typical cost of supplying inflation protection. We argue that sovereign credit risk contributes to the differential as market participants recognize the non-zero probability of a U.S. sovereign default. Inflation swaps, Treasuries, and TIPS all trade over-the-counter and may be subject to varying liquidity risk. In addition, inflation swaps, although collateral-backed, may incorporate counterparty risk. We attempt to mitigate these potential confounding factors in the analysis.

\footnote{Note that we neglect the deflation floor which is embedded in standard TIPS bonds but not present in inflation-linked swaps. If anything, this will simply reduce the ILS-BEI spread since it will decrease the TIPS yield, which will in turn increase the size of the BEI.}
2.2 Swaps, Breakevens and CDS Spreads

We use Gurkaynak, Sack and Wright (2006) and Gurkaynak, Sack and Wright (2010) for nominal and inflation-protected zero coupon bonds respectively. We collect inflation swap data from Bloomberg and EUR-denominated CDS spread data from Markit. While we use EUR denominated U.S. Treasury CDS in the main analysis, our results are qualitatively the same when using USD contracts. Our focus is on the five-year maturity of each security since the five-year maturity is the most liquid CDS tenor. Our data are sampled from January 1, 2008 through September 30, 2015 (full sample). While data on CDS are available prior to this sample period, U.S. Treasury CDS exhibit virtually no variation and volume in the pre-sample period. The quotes are often unchanged for weeks at a time and average between one and two basis points.

We depict the time series of U.S. sovereign credit default swap spreads and ILS-BEI differential in Figure 1. As shown in the figure, CDS spreads are essentially zero until late 2007 and, as documented in Chernov, Schmid and Schneider (2016), soar to 100 basis points in the wake of the Lehman Brothers bankruptcy, timing that is similar to that of the large increase in ILS-BEI. Our conjecture is that this event, and the crisis that followed caused investors to reprice the probability of a U.S. sovereign default and the recovery on Treasury and TIPS in a default scenario. The spread is volatile in 2010-2013 before becoming quiescent from about 2014 onward. Notably, the spread spikes to more than 40 basis points in the days prior to the resolution of the the budget showdown of 2013, which threatened to lead to a U.S. sovereign default.

Summary statistics for these data are provided in Table 1. As shown in the table, over the full sample period, both the ILS-BEI and U.S. CDS spread averaged over 30 basis points (36 and 33 basis points respectively). The ILS-BEI is approximately twice as volatile as the CDS spread, ranging from -1 to 210 basis points. In contrast to the CDS spread, the ILS-BEI declines both on average and in volatility in the post-crisis period, which we define as January 1, 2010 and beyond. Thus, even in the post-crisis period, the Treasury CDS spread averages 34 basis points, considerably greater than its pre-crisis levels.

In addition to possible fears of default risk, the crisis generated considerable fear of counterparty credit risk, a lack of liquidity, increases in perceived quantities and prices of risk, and a deterioration in arbitrage capital available to deploy in financial markets. In order
to investigate these other possibilities, we also examine the role of the following variables:

- **LIBOR-OIS**, the spread between LIBOR and the overnight indexed swap rate. As shown in Table 1, this spread, which averages 35 basis points over our sample, rose to 364 basis points during the crisis. This rise has been attributed to an increase in perceived counterparty credit risk in financial markets. Fleckenstein, Longstaff and Lustig (2014) suggest that their arbitrage profits could arise due to counterparty credit risk, especially if nominal Treasuries are viewed as safe haven assets. However, the authors suggest it is an unlikely explanation for their findings due to the collateralization of swap contracts (Arora, Gandhi and Longstaff (2012)).

- **HPW Noise**, the measure of arbitrage capital availability proposed in Hu, Pan and Wang (2013). This measure is constructed as the root mean squared error in the observed yields of Treasury securities relative to those implied by a Nelson-Siegel-Svensson zero coupon curve across the term structure.\(^6\) The measure takes into account the close relationship between availability of arbitrage capital and liquidity. Fleckenstein, Longstaff and Lustig (2014) posit that the inability of arbitrageurs to immediately eliminate arbitrage may have resulted in the divergence between nominal and inflation-protected securities markets. They suggest that this slow-moving capital hypothesis (Mitchell, Pedersen and Pulvino (2007) and Duffie (2010)) may allow arbitrage profits to persist. Again, the root mean square error, which averages 3.52 basis points, rises to 20.47 basis points during the financial crisis.

- **VIX**, the CBOE volatility index. The VIX is often viewed as a measure of the market’s perception of the quantity and/or price of risk in equity markets specifically, and financial markets as a whole. However, Nagel (2012) suggests that an increase in the VIX is associated with a higher premium for liquidity provision, and therefore a reduction in the amount of liquidity in the financial system. The VIX averages 22% over our sample period, with an increase to nearly 81% during the financial crisis.

- **Repo Fails**. Failures occur when a primary dealer fails to either receive or deliver a Treasury security as part of a repurchase agreement. Fleckenstein, Longstaff and Lustig (2014) include this variable to capture liquidity effects in the fixed income market. A material drop in liquidity in either the TIPS or Treasury markets would be expected.

\(^6\)These data are obtained from Jun Pan’s webpage, [http://www.mit.edu/~junpan/](http://www.mit.edu/~junpan/)
to lead to higher repo fails. In line with expectations, fails peak during the height of
the financial crisis (5,311,279) and are magnitudes higher than what is observed under
normal market conditions, with an average of 218,738.

- RelValHF, the relative value hedge fund index reported by Bloomberg to measure
  available arbitrage capital (RelValHF). Fleckenstein, Longstaff and Lustig (2014)
  use a similar measure to capture arbitrage capital availability and the slow-moving
capital hypothesis. We use the relative value index rather than the entire hedge fund
universe, as the latter may encompass a number of strategies unrelated to arbitrage in
these markets.

2.3 Empirical Results

We begin by regressing the ILS-BEI differential on U.S. CDS spreads over the full sample
January 1, 2008 through September 30, 2015. Results are shown in column(1) of Table 2.
As shown in the table, the point estimate of 0.562 suggests that a one basis point increase
in U.S. CDS spreads translates into an approximately 0.6 basis point increase in the ILS-
BEI differential, and that this effect is statistically significantly different than zero based on
a Newey-West-corrected $t$-statistic of 2.70. The magnitude of this effect, which translates
into a 9 basis point increase in the differential for a unit standard deviation increase in CDS
spreads, represents approximately 25% of the mean ILS-BEI differential. This result suggests
that the results are economically, in addition to statistically significant.

In column (2), we include covariates to assess whether the result arises from the comove-
ment of U.S. CDS spreads with liquidity and slow-moving capital variables. Because repur-
chase agreement fails and the relative value hedge fund index are available at the monthly
frequency, we do not control for these variables in this regression. The result suggests that
in the full sample, the effect of U.S. CDS on the ILS-BEI differential is absorbed by these
variables. The LIBOR-OIS spread, VIX, and HPW Noise coefficients are all positive and
statistically different than zero at the 1% level. These results suggest that in the full sam-
ple, CDS spreads are related to breakeven inflation as a proxy for illiquidity or slow-moving
capital.

As noted above, these alternative influences on the spread are especially pronounced
during the financial crisis. We examine the degree to which the crisis influences our con-
clusions by separating the sample into a crisis period, which we specify as January, 2008 through December, 2009, and a post-crisis period from January, 2010 onward. Results for the crisis period are presented in columns (3) and (4). As shown in the table, there is an even larger relation between CDS spreads and the ILS-BEI differential during the crisis; the coefficient of 0.954 is statistically different than zero at the 1% significance level. However, controlling for other variables eliminates this effect. Results in column (4) indicate that statistical significance is absorbed by the VIX, and that the coefficient on the CDS spread is indistinguishable from zero.

In the post-crisis period, the results depicted in columns (5) and (6) again suggest a statistically significant impact of the U.S. CDS swap spread on the ILS-BEI differential. The point estimate of 0.43 is again statistically significantly different than zero at the 1% level. In contrast to the crisis, results in column (6) indicate that the CDS spread retains statistically significant explanatory power for the ILS-BEI differential. The point estimate of 0.30 (t-statistic 4.35) indicates that a one standard deviation increase in CDS spreads translates into an approximately 4 basis point increase in the ILS-BEI differential. Of the remaining covariates, the HPW noise measure is statistically significant, while the LIBOR-OIS and VIX are only marginally statistically significant.

In order to consider the additional covariates, we aggregate our data to the monthly level by taking averages within each month. Results are presented in Table 2. The results are qualitatively similar. In univariate regressions, there is a positive relationship between U.S. CDS spreads and the ILS-BEI differential for the full sample, the crisis sample, and the post-crisis sample, although the full sample coefficient is not statistically different than zero. During the crisis, other variables, most notably the VIX, absorb most of the explanatory power of the other covariates. However, the relation between the CDS spread and the ILS-BEI differential is positive and statistically significant in the post-crisis period, even controlling for other explanations of the differential.

Our interpretation of these results is that determinants of the U.S. CDS spread comove strongly with the difference in ILS and BEI. CDS spreads may be driven by a number of different factors, including actual default risk and liquidity effects. To the extent that the VIX captures liquidity, our results suggest that liquidity effects dominate in the crisis, but not subsequent to the crisis. However, the VIX also represents a measure of aggregate risk and risk premia and may simply absorb any aggregate default risk premia reflected in
CDS pricing during the crisis. Regardless, we view the evidence as sufficiently suggestive to indicate that the IES-BEI differential is influenced by credit risk, and propose a formal model of sovereign nominal and inflation-protected debt and credit default swaps written on these securities.

3 Modeling Nominal and Inflation-Protected Debt with Default Risk

In this section, we discuss the pricing of nominal and inflation-protected sovereign bonds, assuming that there is a possibility of a credit event interrupting the promised payments of the securities. Of particular interest is the spread between inflation-linked swaps and the breakeven inflation rate, the risk neutral inflation rate that equates the prices of the nominal and inflation-protected securities.

3.1 Economic Setting

Our modeling framework follows that of Monfort et al. (2017a) in modeling risky debt in discrete time. In this framework, defaults of any kind are represented by the first jump of a non-negative credit-event variable denoted by $\delta_t$. More formally, if $\tau$ is the default date of the sovereign, we have:

$$\tau = \min\{ t \mid \delta_t > 0 \}. \quad (1)$$

The dynamics of default events are driven by a $k \times 1$ vector of state variables, $y_t$, which follows a vector autoregressive gamma process under the physical probability measure.

$$y_t | y_{t-1} \sim \Gamma_{\nu} (\alpha_y + \Phi y_{t-1}; 1) \quad (2)$$

Using the same notations as in Monfort et al. (2017b), $\nu \in \mathbb{R}_+^k$ denotes the degrees of freedom of each gamma variable, $\alpha_y \in \mathbb{R}_+^k$ and $\Phi$ are respectively the intercept vector and the $k$ by $k$ autoregressive matrix with positive components entering the Poisson intensity, and the vector of scale parameters is normalized to 1 for identification purposes.

Conditionally on the state variables $y_t$, we assume that the credit event variable is
Gamma-zero distributed,
\[ \delta_t \mid (y_t, \delta_{t-1}) \sim \Gamma_0 \left( \alpha + \gamma^T y_t; \mu \right) \]  
where \( \alpha \) is a positive parameter, \( \gamma \) is a \( k \)-vector of positive parameters and \( \mu \) is the scaling parameter of the process. Monfort et al. (2017a) show that Gamma-zero processes are efficient in representing credit events since they can stay at the value of zero for extended periods of time (no default state) and jump to any positive value upon default. Combining the dynamics given by Equations (2) and (3), it can easily be shown that the joint process \((y_t^T, \delta_t)^T\) is affine which has two main implications. First, its conditional moment generating function can be written:

\[ \varphi_{t-1}(u, v) = \mathbb{E}_{t-1} \left[ \exp \left( u^T y_t + v \delta_t \right) \right] = \exp \left[ A(u, v) + B(u, v)^T y_{t-1} \right], \]  
for any value \( v < \frac{1}{\mu} \) and any \( k \)-dimensional vector \( u < 1 - \frac{v \mu}{1 - v \gamma} \) such that the expectation exists. The coefficients \( A(u, v) \) and \( B(u, v) \) are easily computed as:

\[ A(u, v) = \frac{v \mu}{1 - v \mu} \alpha + \alpha^T \left( u + \frac{v \mu}{1 - v \mu} \gamma \right) - \nu^T \log \left[ 1 - \left( u + \frac{v \mu}{1 - v \mu} \gamma \right) \right], \]  
\[ B(u, v) = \Phi^T \left( u + \frac{v \mu}{1 - v \mu} \gamma \right) \left( 1 - \left( u + \frac{v \mu}{1 - v \mu} \gamma \right) \right)^{-1}, \]

where the ratio stands for an element-by-element ratio by notation abuse. The second consequence of the affine property is that our variables admit a semi-strong VAR representation and we can write:

\[ \begin{pmatrix} y_t \\ \delta_t \end{pmatrix} = \kappa_0 + \kappa \begin{pmatrix} y_{t-1} \\ \delta_{t-1} \end{pmatrix} + \Sigma(y_{t-1})^{1/2} \xi_t, \]  
where \( \xi_t \) is a martingale difference with zero mean and unit variance. The formulas for \( \kappa_0 \), \( \kappa \) and \( \Sigma(y_{t-1}) \) are detailed in Appendix A.1. We assert the existence of a strictly positive stochastic discount factor between \( t - 1 \) and \( t \), \( M_t \), which is an exponential-affine function of the state variables,

\[ \log(M_t) = -r_{t-1} - \kappa_{t-1} + \Lambda^T y_t + \lambda \delta_t + \theta r_t, \]  
where \( r_{t-1} \) is the risk-free nominal rate of return between \( t - 1 \) and \( t \), \( \Lambda \), \( \lambda \) and \( \theta \) are prices of risk, and \( \kappa_{t-1} \) is the convexity adjustment such that the conditional expectation of the
SDF is equal to $e^{-rt}$. We assume that the risk-free rate has independent dynamics such that:

$$ r_t | r_{t-1} \sim \Gamma_0(\alpha_r + \beta r_{t-1} ; \varsigma) . $$

Given our state dynamics and the SDF, Monfort et al. (2017a) show the risk-neutral dynamics are given by the same conditional distributions, only with shifted parameters. Therefore we have:

$$ y_t | y_{t-1} \sim \Gamma_\nu(\alpha^Q_y + \Phi^Q_y y_{t-1} ; \mu^Q) $$

$$ \delta_t | (y_t, \delta_{t-1}) \sim \Gamma_0(\alpha^Q_r + \gamma^Q r_{t-1} ; \mu^Q) $$

$$ r_t | r_{t-1} \sim \Gamma_0(\alpha^Q_r + \beta^Q r_{t-1} ; \varsigma^Q) , $$

where the risk-neutral parameters are given by:

$$ \alpha^Q = \frac{\alpha}{1 - \lambda \mu} , \quad \gamma^Q = \frac{1}{1 - \lambda \mu} \gamma , \quad \mu^Q = \frac{\mu}{1 - \lambda \mu} , $$

$$ \nu^Q = \text{diag} \left( \frac{1}{1 - \Lambda - \frac{\lambda \mu}{1 - \lambda \mu} \gamma} \right) 1 , \quad \alpha^Q_y = \text{diag}(\nu^Q) \alpha_y \quad \Phi^Q = \text{diag}(\nu^Q) \Phi , $$

$$ \alpha^Q_r = \frac{\alpha_r}{1 - \theta \varsigma} , \quad \beta^Q = \frac{\beta}{1 - \theta \varsigma} , \quad \varsigma^Q = \frac{\varsigma}{1 - \theta \varsigma} . $$

A direct consequence is that the conditional risk-neutral moment generating function of the credit event variable and the credit factors has the same form than Equation (4) only replacing all physical parameters by the risk-neutral ones and can be written:

$$ \varphi^Q_{t-1}(u, v) := \mathbb{E}^Q_{t-1}[\exp(u^T y_t + v \delta_t)] = \exp[A^Q(u, v) + B^Q(u, v)^T y_{t-1}] . $$

### 3.2 Asset Prices

Consider a sovereign state, which issues both nominal and inflation-protected debt with maturity $h$. With some probability, the bond defaults prior to maturity $h$. Default happens when, at any time $\tau \leq h$ where the credit-event variable $\delta_t$ jump from zero to a positive value (see Equation (1)). Investors have also access to risk-free nominal and inflation-protected debt.
securities that are not affected by the default of the sovereign.

In the case of a credit event, bondholders get an uncertain recovery payment on their investment in the sovereign bond, $P_t^{(h)}$. Monfort et al. (2017a) show that the price of a nominal zero-coupon bond is given by:

$$B(t, h) = \sum_{i=1}^{h} \mathbb{E}_t^Q \left[ \exp \left( - \sum_{j=0}^{i-1} r_{t+j} \right) P_{t+i}^{(h)} \times \left( 1 \left\{ \sum_{j=0}^{i-1} \delta_{t+j} = 0 \right\} - 1 \left\{ \sum_{j=0}^{i} \delta_{t+j} = 0 \right\} \right) \right]$$

$$+ \mathbb{E}_t^Q \left[ \exp \left( - \sum_{j=0}^{h-1} r_{t+j} \right) 1 \left\{ \sum_{j=0}^{h} \delta_{t+j} = 0 \right\} \right]. \tag{13}$$

Equation (13) simply states that the price of the nominal bond is the sum of discounted recovery payments if default happens between $t + i - 1$ and $t + 1$, and the discounted principal if no default occurs during the lifespan of the bond. An inflation-indexed bond is priced similarly, with the difference that its payoff is indexed to a reference inflation index, denoted by $\pi_t$. We assume that the bond’s recovery payment upon default may differ from that of the nominal bond, and designate this recovery payment $P_{t+i}^{(h)^*}$. Then the price of the inflation-indexed bond is given by:

$$B^*(t, h) = \sum_{i=1}^{h} \mathbb{E}_t^Q \left[ \exp \left( - \sum_{j=0}^{i-1} r_{t+j} \right) P_{t+i}^{(h)^*} \times \left( 1 \left\{ \sum_{j=0}^{i-1} \delta_{t+j} = 0 \right\} - 1 \left\{ \sum_{j=0}^{i} \delta_{t+j} = 0 \right\} \right) \right]$$

$$+ \mathbb{E}_t^Q \left[ \exp \left( - \sum_{j=0}^{h-1} r_{t+j} - \pi_{t+i+1} \right) 1 \left\{ \sum_{j=0}^{h} \delta_{t+j} = 0 \right\} \right]. \tag{14}$$

One possible reason that recovery may differ between nominal and inflation-protected bonds is devaluation of the issuing currency. That is, the sovereign may not be able to repay the full real face value even if it is able to fully repay nominal indebtedness.

Following Duffie and Singleton (1999), we assume the recovery of market value assumption (RMV), which states that in case of default, recovery payments are proportional to the price of the bond that would have been observed without default, by a factor equal to the recovery rate. Following Monfort et al. (2017a), we assume a stochastic recovery rate for both types of bonds, of $\exp(-\delta_{t+i})$ and $\rho^* \exp(-\delta_{t+i})$ respectively in case of default at date
The total recovery payments write:

$$P_{t+i}^{(h)} = \exp(-\delta_{t+i}) \times \mathbb{E}_t^{Q} \left[ \exp \left( - \sum_{j=0}^{h-1} r_{t+j} + \delta_{t+j+1} \right) \right]$$

$$P_{t+i}^{(h)*} = \rho^* \exp(-\delta_{t+i}) \times \mathbb{E}_t^{Q} \left[ \sum_{k=1}^{i} \pi_{t+k} \right] \mathbb{E}_t^{Q} \left[ \exp \left( - \sum_{j=0}^{h-1} r_{t+j} - \pi_{t+j+1} + \delta_{t+j+1} \right) \right]$$

We show in the Appendix that under these assumptions, using equation (13)-(14), the nominal bond’s price at time $t$ is given by:

$$B(t, h) = \mathbb{E}_t^{Q} \left[ \exp \left( - \sum_{j=0}^{h-1} r_{t+j} + \delta_{t+j+1} \right) \right]$$

$$B^*(t, h) = (1 - \rho^*) \mathbb{E}_t^{Q} \left[ \exp \left( - \sum_{j=0}^{h-1} r_{t+j} - \pi_{t+j+1} + \delta_{t+j+1} \right) \right] 1 \left\{ \sum_{j=0}^{h-1} \delta_{t+j+1} = 0 \right\}$$

$$+ \rho^* \cdot \mathbb{E}_t^{Q} \left[ \exp \left( - \sum_{j=0}^{h-1} r_{t+j} - \pi_{t+j+1} + \delta_{t+j+1} \right) \right],$$

assuming that the bond has survived to time $t$. Equation (15) is the exact same as the formula in Duffie and Singleton (1999) while Equation (16) is the equivalent for inflation-protected securities issued by the sovereign. An immediate object of interest is the breakeven inflation rate $\text{BEI}(t, h)$, that is the spread between nominal and TIPS yields. Assuming that the inflation rate is independent from the default process, we can write:

$$\text{BEI}(t, h) = \text{ILS}(t, h) - \frac{1}{h} \log \left( \rho^* + (1 - \rho^*) \frac{\mathbb{Q}(t, h)}{\mathbb{S}(t, h)} \right)$$

where $\text{ILS}(t, h)$ is the inflation-linked swap rate, a (virtually) risk-free equivalent of the breakeven inflation rate, and:

$$\mathbb{Q}(t, h) := \mathbb{Q}_t \left[ \sum_{j=1}^{h} \delta_{t+j} = 0 \right], \quad \mathbb{S}(t, h) := \mathbb{E}_t^{Q} \left[ \exp \left( - \sum_{j=1}^{h} \delta_{t+j} \right) \right]$$

are the risk neutral survival probability and the (price) spread between defaultable and risk-free nominal bonds, respectively. Equation (17) has two main implications in terms of modeling. First, the spread between breakevens and inflation-linked swaps depend only
on the credit-event dynamics, and both dynamics of risk-free interest rates and inflation are irrelevant. Second, if both recovery rates are equal ($\rho^\ast = 1$), the ILS-BEI spread is consistently null across maturities. Using the factor dynamics from Section 3.1, the quantities from Equation (18) can be expressed in closed-form. Specifically,

$$Q(t, h) = \exp (q_h + Q_h^T y_t)$$

$$S(t, h) = \exp (s_h + S_h^T y_t)$$

where the coefficients $q_h$ and $s_h$ as well as vectors $Q_h$ and $S_h$ follow closed-form recursions that are provided in Appendix A.2. We can use this result to simplify the formula in Equation (17):

$$\text{ILS}(t, h) - \text{BEI}(t, h) = \frac{1}{h} \log \left[ \rho^\ast + (1 - \rho^\ast) \exp \left( q_h - s_h + (Q_h - S_h)^T y_t \right) \right] ,$$

and, for close coefficients $q_h, s_h$ and $Q_h, S_h$ we can approximate this formula by:

$$\text{ILS}(t, h) - \text{BEI}(t, h) = \frac{1 - \rho^\ast}{h} [q_h - s_h + (Q_h - S_h)^T y_t] ,$$

which is simply an affine function of the credit factors $y_t$.

### 3.3 CDS pricing

We assume that a buyer of protection makes periodic payments from time $t$ to maturity $h$ to protect against default on the underlying nominal sovereign bond. The cash flow payment at time $t+i$ conditional on no default is designated as $\sigma(t, h)$. The present value of the stream of cash flows paid by the protection buyer is:

$$\text{PB}(t, h) = \sigma(t, h) \sum_{i=1}^{h} \mathbb{E}_t^Q \left[ \exp \left( - \sum_{j=0}^{i-1} r_{t+j} \right) 1 \left\{ \sum_{j=0}^{i} \delta_{t+j} = 0 \right\} \right]$$

If the sovereign defaults at time $t+i$, we assume that the protection seller pays the buyer a payment of $1 - \exp(-\delta_{t+i})$ (RFV convention). The present value of the protection sold is:

$$\text{PS}(t, h) = \sum_{i=1}^{h} \mathbb{E}_t^Q \left[ \exp \left( - \sum_{j=0}^{i-1} r_{t+j} \right) (1 - e^{-\delta_{t+i}}) 1 \left\{ \sum_{j=0}^{i} \delta_{t+j} = 0 \right\} - 1 \left\{ \sum_{j=0}^{i} \delta_{t+j} = 0 \right\} \right].$$
No arbitrage pricing requires that the present value of the protection bought is equal to the present value of the protection sold. Equating both legs at inception, and solving for the swap spread yields:

\[
\sigma(t, h) = \frac{\sum_{i=1}^{h} \mathbb{E}_{t}^{Q} \left[ \exp \left(- \sum_{j=0}^{i-1} r_{t+j} \right) (1 - e^{-\delta_{t+i}}) \left( \mathbb{1} \left\{ \sum_{j=0}^{i-1} \delta_{t+j} = 0 \right\} - \mathbb{1} \left\{ \sum_{j=0}^{i} \delta_{t+j} = 0 \right\} \right) \right]}{\sum_{i=1}^{h} \mathbb{E}_{t}^{Q} \left[ \exp \left(- \sum_{j=0}^{i-1} r_{t+j} \right) \mathbb{1} \left\{ \sum_{j=0}^{i} \delta_{t+j} = 0 \right\} \right].}
\]

Using the physical dynamics of Section 3.1 along with our notation for survival probabilities (see Equation (18)), we can rewrite the previous expression as:

\[
\sigma(t, h) = \frac{\sum_{i=1}^{h} B_{f}(t, i) \left[ Q(t, i - 1) - D(t, i) \right]}{\sum_{i=1}^{h} B_{f}(t, i) Q(t, i)}, \quad (23)
\]

where \(B_{f}(t, h)\) is the price of the risk-free nominal bond of residual maturity \(h\) and:

\[
D(t, i) := \mathbb{E}_{t}^{Q} \left[ \exp \left(- \delta_{t+i} \right) \mathbb{1} \left\{ \sum_{j=0}^{i-1} \delta_{t+j} = 0 \right\} \right] = \exp \left( d_{i} + D_{i}^{T} y_{t} \right).
\]

where \(d_{i}\) and \(D_{i}\) can be computed through the closed-form recursions presented in Appendix A.2 Note that our short-rate risk-neutral dynamics of Equation (11) also implies that:

\[
B_{f}(t, h) = \exp \left( b_{h} + B_{h} r_{t} \right), \quad b_{h} = b_{h-1} + \frac{B_{h-1}^{Q} \alpha^{Q}}{1 - B_{h-1}^{Q} \rho^{Q}}, \quad B_{h} = \frac{B_{h-1}^{Q} \beta^{Q}}{1 - B_{h-1}^{Q} \rho^{Q}} - 1, \quad (24)
\]

starting from initial values \(b_{0} = 0\) and \(B_{0} = 0\). Therefore, the swap rate can be expressed:

\[
\sigma(t, h) = \frac{\sum_{i=1}^{h} \exp \left( b_{h} + B_{h} r_{t} \right) \left[ \exp \left( q_{i-1} + Q_{i-1}^{T} y_{t} \right) - \exp \left( d_{i} + D_{i}^{T} y_{t} \right) \right]}{\sum_{i=1}^{h} \exp \left( b_{i} + q_{i} + B_{i} r_{t} - Q_{i}^{T} y_{t} \right)}. \quad (25)
\]
as usual, even if the model is affine, the swap rate ends up a closed-form non-affine function of the state variables, which will eventually motivate the use of a non-linear filter. Notice that contrary to the ILS-BEI spreads, the risk-free rate dynamics can influence the size of the swap rate.

4 Model Estimation

4.1 Data and estimation method

We consider different types of observable variables at the monthly frequency. First, the spreads between breakevens and inflation-linked swaps are constructed in two steps. We use Gurkaynak, Sack and Wright (2006) and Gurkaynak, Sack and Wright (2010) for nominal and inflation-protected zero coupon bonds respectively. The BEI variable is the difference between the former and the latter. On the other hand, zero coupon inflation-linked swaps are directly available from Bloomberg. ILS-BEI spreads are calculated for 2-, 3-, 5-, 7-, and 10-years to maturity. We also consider the 5-year U.S. sovereign credit default swap spread from Markit.\footnote{We are well-aware of potential data issues with USD denominated CDS spreads (see for instance Cher- nov, Schmid and Schneider (2016)). However, using directly USD denominated spreads allows us to avoid the modeling of the exchange rate if we were to consider EUR denominated CDS.} Last, we proxy the risk-free rate by the H.15 3-months sovereign interest rate available on the Fed Board of governors website. The input sample period for the model estimation spans July of 2004 to December of 2015. Because of its very little variability prior to the financial crisis, we suppress the first 43 observations of the CDS spread and only use CDS data starting from January of 2008.

All these observable variables are gathered into a vector for each date to form the measurement equations. More precisely, they combine Equations (22), (24) and (25). We complete the state-space model formulation using the semi-strong VAR representation of our risk factors $y_t$ (see Equation (7)) and the risk-free rate $r_t$. Both conditional mean and conditional variances are linear in the lagged factors.\footnote{Note that because $\delta_t$ is null throughout the sample (no default) and that it does not Granger-cause the credit factors $y_t$, we can drop it from the estimation and we do not need to filter it from the data.} Our benchmark model considers 2 credit factors $y_t$. Last, because we want to see how much of the variance of the ILS-BEI spreads can be explained by credit risk factors, we add i.i.d. Gaussian measurement errors on the ILS-BEI...
with the same standard deviation across maturity but we impose that the CDS spread is measured with no error. Once we form our complete state-space representation, we estimate the model by approximate maximum likelihood using the extended Kalman filter.

4.2 Estimation results

Table 5 presents the parameter estimates of the model. Both factors $y_t$ are highly persistent under both measures, with autoregressive parameters above .99 and .94 respectively. As expected these factors are more persistent under the risk-neutral measure, meaning investors fear that states of the world where expected recovery payments are low and default probabilities are high are lasting. Under the physical measure, the scaling parameter of the default process $\delta_t$ is around 13, which, combined with the other parameter values, gives an upper bound to the recovery rate of the nominal bonds to 0.988. For the TIPS, the recovery rate upper bound is lower and given by 0.866. This allows the model to partially fit the ILS-BEI spreads with an average standard deviation of measurement errors of 21bps.

The three latent factors filtered by our model are plotted in Figure 3. The third factor can be seen as a normalization of the 3-month riskfree rate, while the first two factors jointly determine the ILS-BEI and sovereign CDS spreads. Factor 1 tracks the 5-year CDS spread closely and can be thought of as the factor that drives default risk and associated recovery rates. The large spike in factor 1 between 2008 - 2009 coincides with the heightened level of sovereign default risk of U.S. Treasury securities during the financial crisis. Factor 1 also has a smaller jump around 2010 that can be attributed to the Greek debt crisis, which threatened the global financial market. Factor 2, on the other hand, is much more volatile in our sample, and it captures the movements in both spreads outside of the crisis period.

Figure 4 plots the factor loadings of the ILS-BEI spreads on factor 1 and factor 2 across the five maturities we employed as observables. The key takeaway here is that the loadings of ILS-BEI spreads on the two factors are similar in magnitude, which suggests that both factors are important in fitting the model to the data. Moreover, both factor loadings are declining as maturity increases, but the decline is more severe for the first factor. This is
consistent with the fact that factor 1 is closely related to the 5-year CDS spread, which insures default risk up to the 60-month horizon.

Next, we plot the fit of the model in Figure 5 for the 5-year CDS spread and the 2-year, 3-year, 5-year, 7-year, and 10-year ILS-BEI spreads over our estimation period. The three-factor model produces almost perfect fit of the CDS spread since the inception of the 2008 financial crisis, mainly by construction. On the breakeven inflation front, the model does a reasonable job in capturing the ILS-BEI spreads across the maturity spectrum. However, the ILS-BEI spreads actually widened slightly prior to the spike in the CDS spread in the data, and this causes the model to struggle to capture the peaks of the ILS-BEI spreads. In line with Figures 1 and 2, simple breakeven inflations (Treasury yields minus TIPS yields) turned negative by September of 2008, which forced the ILS-BEI spreads to widen. This happened before the rise in the CDS spread at the beginning of 2009. The spike in the estimated factor 1 helps the model to capture the peak in the CDS spread but is insufficient to simultaneously fit the ILS-BEI spreads perfectly.

[Insert Table 4 about here.]

To further assess the fitting quality of the model for the ILS-BEI spreads term structure, we report R-squared measures on Table 4. Unsurprisingly, the model does a nearly perfect job in fitting both the riskless 3m yield and the 5y CDS spread, and this result is consistent with the constraints imposed during estimation. More interestingly, we see that the explanatory power of the default factors on the ILS-BEI spreads is about 20% for all maturities, ranging from 11% for the 10y maturity and can go as high as 31% for the 3y maturity. We thus see that default is a significant risk factor in the TIPS-treasury mispricing and can explain up to a third of the mispricing depending on the maturity.

The model is also able to generate hypothetical yields and spreads under the risk neutral assumption, which allows us to study the risk premium component of the spreads. Figure 6 overlay the risk-neutral spreads on top of Figure 5. As before, the black line is the observed data, the red line is the fitted series. The dashed blue line is the expectation hypothesis component of the spread, that is the spread that would be observed if the representative agent is risk-neutral. The predicted risk premium of the model is the difference between the red and blue lines. Risk premium is positive and grows with maturity of the ILS-BEI spread,
as expected. This risk premium is only linked to default risk, since that is the only priced risk in the model.

5 Conclusion

In this paper, we explore the relative pricing of nominal and real U.S. sovereign securities in the presence of credit risk. In fact, we argue that while most of the previous studies attribute the mispricing of TIPS to liquidity factors or slowly moving capital, credit risk can also represent a significant driver of deviations oftentimes interpreted as violations of no-arbitrage. Our study shows that in the presence of credit risk, the spreads between inflation-linked swaps and breakeven inflation rates can reflect differences in propensity of the sovereign to reimburse nominal and real bonds in case of default, that is a difference in recovery rates. Our first empirical approach that U.S. CDS spreads are positively correlated with the ILS-BEI spreads after the financial crisis, even controlling for liquidity and potential alternative explanations. We then conduct more formal empirical analysis through an intensity-based affine asset pricing model. We show that credit risk factors extracted from the CDS are able to explain 20% of the ILS-BEI yield curve and up to 30% for some maturities. Our model estimates confirms the existence of a lower recovery rate for TIPS than for nominal bonds by about 13 percentage points.
References


Table 1: Summary Statistics

Table 1 provides summary statistics for the variables used in the regression analysis. Panel A includes the full sample period from January 1, 2008 through September 30, 2015. Panel B is the post crisis subsample from January 1, 2010 through September 30, 2015. ILS – BEI is the difference in the 5-year inflation swap rate and the 5-year breakeven inflation rate (Treasury-TIPS). Both Tsy ZC Yield and TIPS ZC Yield are for the 5-year maturity. 5-year US CDS spreads are denominated in EUR. LIBOR – OIS is the difference in the London Inter-bank Offered Rate and the overnight indexed swap rate. HPW Noise follows Hu, Pan and Wang (2013). VIX denotes the CBOE Volatility Index. RepoFails is the total of weekly failed deliveries and receipts.

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Table 2: Breakevens and Credit Risk - Daily

Table III displays the regression results of ILS-BEI on US CDS spreads and control variables using daily observations. \( ILS - BEI \) is the difference in the 5-year inflation swap rate and the 5-year breakeven inflation rate (Treasury-TIPS). 5-year \( US\ CDS \) spreads are denominated in EUR. \( LIBOR - OIS \) is the difference in the London Inter-bank Offered Rate and the overnight indexed swap rate. \( HPW\ Noise \) follows Hu, Pan and Wang (2013). \( VIX \) denotes the CBOE Volatility Index. Newey-West \( t \)-statistics are reported in parentheses with 20 day lags. Superscripts ***, **, and * denote statistical significance at the 1%, 5%, and 10% level respectively.

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*, **, *** denote 10,5,1% significance levels
Table 3: Breakevens and Credit Risk - Monthly Averages

Table II displays the regression results of ILS-BEI on US CDS spreads and control variables utilizing monthly average observations. $ILS - BEI$ is the difference in the 5-year inflation swap rate and the 5-year breakeven inflation rate (Treasury-TIPS). 5-year $US\ CDS$ spreads are denominated in EUR. $LIBOR - OIS$ is the difference in the London Inter-bank Offered Rate and the overnight indexed swap rate. $HPW\ Noise$ follows Hu, Pan and Wang (2013). $VIX$ denotes the CBOE Volatility Index. $RepoFails$ is total of failed deliveries and receipts. $RelVal\ HF$ is the return on the HFR relative value hedge fund index. Newey-West $t$-statistics are reported in parentheses with 3-month lags. Superscripts ***, **, and * denote statistical significance at the 1%, 5%, and 10% level respectively.

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<th>Full Sample</th>
<th>Crisis</th>
<th>Post Crisis</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
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<tr>
<td>US CDS</td>
<td>0.60</td>
<td>−0.09</td>
<td>1.01***</td>
</tr>
<tr>
<td></td>
<td>1.56</td>
<td>−0.60</td>
<td>6.01</td>
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<tr>
<td>LIBOR-OIS</td>
<td>18.81*</td>
<td></td>
<td>14.10</td>
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<tr>
<td></td>
<td>1.95</td>
<td></td>
<td>1.34</td>
</tr>
<tr>
<td>HPW Noise</td>
<td>4.19***</td>
<td></td>
<td>0.05</td>
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<tr>
<td></td>
<td>3.13</td>
<td></td>
<td>0.04</td>
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<tr>
<td>VIX</td>
<td>1.05**</td>
<td></td>
<td>2.98***</td>
</tr>
<tr>
<td></td>
<td>2.21</td>
<td></td>
<td>3.17</td>
</tr>
<tr>
<td>Repo Fails</td>
<td>−1.21</td>
<td></td>
<td>−3.05**</td>
</tr>
<tr>
<td></td>
<td>−1.26</td>
<td></td>
<td>−2.18</td>
</tr>
<tr>
<td>RelVal HF</td>
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<td></td>
<td>−1.88</td>
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<tr>
<td></td>
<td>0.93</td>
<td></td>
<td>−0.14</td>
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<tr>
<td>Intercept</td>
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<td>−4.32</td>
<td>44.02***</td>
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<tr>
<td></td>
<td>1.39</td>
<td>−0.90</td>
<td>5.68</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
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<td>0.90</td>
<td>0.35</td>
</tr>
<tr>
<td>Observations</td>
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<td>24</td>
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## Table 4: R-squared values from term structure model

<table>
<thead>
<tr>
<th>Maturity</th>
<th>3m</th>
<th>5y</th>
<th>2y</th>
<th>3y</th>
<th>5y</th>
<th>7y</th>
<th>10y</th>
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<tbody>
<tr>
<td>R²</td>
<td>1</td>
<td>0.998</td>
<td>0.236</td>
<td>0.311</td>
<td>0.22</td>
<td>0.225</td>
<td>0.105</td>
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</table>

## Table 5: Parameter Estimates

<table>
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<tr>
<th>P</th>
<th>α</th>
<th>γ₁</th>
<th>γ₂</th>
<th>μ</th>
<th>λ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.012834</td>
<td>1.18E-44</td>
<td>0.057347</td>
<td>13.02403</td>
<td>-10.034</td>
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<tr>
<td>Q</td>
<td>9.75E-05</td>
<td>8.98E-47</td>
<td>0.000435</td>
<td>0.098904</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>P</th>
<th>αᵣ</th>
<th>Φᵣ</th>
<th>ζ</th>
<th>θ</th>
<th>σ_{ils-bei}</th>
<th>ρ*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.408104</td>
<td>0.989793</td>
<td>3.13E-05</td>
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<td>0.2148</td>
<td>0.876198</td>
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<tr>
<td>Q</td>
<td>0.346562</td>
<td>0.840533</td>
<td>2.66E-05</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 1: Figure 1 shows the time series of differential in ILS and BEI rates (LHS) and US CDS spreads (RHS)
Figure 2: Figure 2 displays the time series of 5-year inflation swap rates (ILS) and breakeven inflation rates (BEI) as denoted by the difference in the 5-year zero coupon Treasury yield and the 5-year zero coupon TIPS yield.
Figure 3: Filtered factors. Factors 1 and 2 correspond to $y_t$ while factor 3 is a rescaled version of the risk-free short rate.

Figure 4: ILS-BEI spreads factor loadings $Q_h - S_h$. The loadings are normalized by the in-sample filtered factors standard deviations to give them comparable values.
Figure 5: Observable variables fitted values. Black solid lines represent the observable variables while the red solid lines are model-implied. All values are in bps.
Figure 6: Risk premia estimates. Black solid lines represent the observable variables while the red solid lines are model-implied. Blue dashed lines represent the expected hypothesis component, i.e. the implied observables when all prices of risk are set to zero. All values are in bps.
A Appendix

A.1 Conditional moments of Gamma processes

This Appendix presents the computations of first two conditional moments of our processes. This allows us to write the process in semi-strong VAR form.

Even if the factors dynamics are highly nonlinear, the affine property allows to derive closed-form first and second order conditional moments. In essence, we can write:

\[
\begin{pmatrix} y_t \\ \delta_t \end{pmatrix} = K_0 + K \begin{pmatrix} y_{t-1} \\ \delta_{t-1} \end{pmatrix} + \Sigma(y_{t-1})^{1/2} \xi_t
\]

where \( \xi_t \) is a martingale difference with 0 mean and unit conditional variance. To obtain the closed-form formulas of these parameters, we can use the properties of the gamma distribution:

\[
\mathbb{E}(y_t|y_{t-1}, \delta_{t-1}) = \nu + \alpha y + \Phi y_{t-1}
\]

\[
\mathbb{V}(y_t|y_{t-1}, \delta_{t-1}) = \text{diag}(\nu + 2\alpha y + 2\Phi y_{t-1})
\]

and

\[
\mathbb{E}(\delta_t|y_t, \delta_{t-1}) = \mu (\alpha + \gamma^T y_t)
\]

\[
\mathbb{V}(\delta_t|y_t, \delta_{t-1}) = 2\mu^2 (\alpha + \gamma^T y_t)
\]

Thus, using the law of iterated expectations and the law of total variance, we have:

\[
\mathbb{E}(\delta_t|y_{t-1}, \delta_{t-1}) = \mu (\alpha + \gamma^T (\nu + \alpha y + \Phi y_{t-1}))
\]

\[
\mathbb{V}(\delta_{t+1}|y_{t-1}, \delta_{t-1}) = 2\mu^2 (\alpha + \gamma^T (\nu + \alpha y + \Phi y_{t-1})) + \mu^2 \gamma^T \text{diag}(\nu + 2\alpha y + 2\Phi y_{t-1}) \gamma
\]

The last thing we need to compute is the covariance

\[
\text{Cov}(y_t, \delta_t|y_{t-1}, \delta_{t-1}) = \text{Cov}(y_t, \mathbb{E}(\delta_t|y_t, \delta_{t-1})|y_{t-1}, \delta_{t-1})
\]

\[
= \text{Cov}(y_t, \mu (\alpha + \gamma^T y_t)|y_{t-1}, \delta_{t-1})
\]

\[
= \mu \gamma^T \text{diag}(\nu + 2\alpha y + 2\Phi y_{t-1})
\]
So, in the end:

\[ \mathcal{K}_0 = \left( \begin{array}{c} \nu + \alpha_y \\ \mu \left( \alpha + \gamma^T \nu + \alpha_y \right) \end{array} \right), \quad \mathcal{K} = \left( \begin{array}{cc} \Phi & 0 \\ \mu \gamma^T \Phi & 0 \end{array} \right) \]  

(27)

and the variance parameters can be constructed from the different variance blocks. Importantly, both quantities are affine functions of the factors.

A.2 Recursive formulas

We hereby provide the recursions to calculate the quantities in the following conditional expectations:

\[ Q(t, h) := Q_t \left[ \sum_{j=1}^h \delta_{t+j} = 0 \right] = \exp (q_h + Q_h^T y_t) \]

\[ S(t, h) := E_t^Q \left[ \exp \left( - \sum_{j=1}^h \delta_{t+j} \right) \right] = \exp (s_h + S_h^T y_t) \]

\[ D(t, h) := E_t^Q \left[ \exp ( - \delta_{t+h} I \left\{ \sum_{j=0}^{h-1} \delta_{t+j} = 0 \right\} ) \right] = \exp (d_h + D_h^T y_t) \]

These three quantities are related to the one-period risk-neutral conditional moment generating function that we have denoted by \( \phi_{t-1}^Q(u, v) \) in Equation (12). Remember that:

\[ \phi_{t-1}^Q(u, v) := E_{t-1}^Q \left[ \exp (u^T y_t + v \delta_t) \right] = \exp \left[ A^Q(u, v) + B^Q(u, v)^T y_{t-1} \right], \]  

(28)

with:

\[ A^Q(u, v) = \frac{v \mu^Q}{1 - v \mu^Q} \alpha^Q + \alpha_y^Q \frac{\text{diag} \left( \nu \right) \left( u + \frac{v \mu^Q}{1 - v \mu^Q} \gamma^Q \right)}{1 - \text{diag} \left( \nu \right) \left( u + \frac{v \mu^Q}{1 - v \mu^Q} \gamma^Q \right)} \]

\[ - \nu^T \log \left[ 1 - \text{diag} \left( \nu \right) \left( u + \frac{v \mu^Q}{1 - v \mu^Q} \gamma^Q \right) \right] \]

\[ B^Q(u, v) = \Phi^Q \frac{\text{diag} \left( \nu \right) \left( u + \frac{v \mu^Q}{1 - v \mu^Q} \gamma^Q \right)}{1 - \text{diag} \left( \nu \right) \left( u + \frac{v \mu^Q}{1 - v \mu^Q} \gamma^Q \right)}. \]

For this form of conditional MGF, it is easy to show that the multi-period conditional MGF can be expressed as an exponential affine function of the factors, where the loadings are
available through closed-form recursions. More specifically:

\[
\mathbb{E}_t^Q \left[ \exp \left( \sum_{i=1}^{H} u_i^T y_{t+i} + v \delta_{t+i} \right) \right] = \exp \left( A_H^Q(u, v) + B_H^Q(u, v)^T y_t \right),
\]

where

\[
A_H^Q(u, v) = A_{H-1}^Q(u, v) + A^Q \left[ B_{H-1}^Q(u, v), v \right]
\]

\[
B_H^Q(u, v) = B^Q \left[ B_{H-1}^Q(u, v), v \right]
\]

Last, we just have to notice that both \( Q(t, h) \) and \( S(t, h) \) are risk-neutral conditional MGFs applied in different arguments. For \( S(t, h) \), we trivially have:

\[
S(t, h) = \exp \left( A_h^Q(0, -1) + B_h^Q(0, -1)^T y_t \right)
\]

Hence:

\[
s_h = s_{h-1} + A^Q \left[ S_{h-1}^Q(0, -1), -1 \right]
\]

\[
S_h = B^Q \left[ S_{h-1}^Q(0, -1), -1 \right]
\]

In the same fashion, using the results from the Gamma-zero distribution (see e.g. Monfort et al. (2017b)) we have:

\[
Q(t, h) = \lim_{v \rightarrow -\infty} \exp \left( A_h^Q(0, v) + B_h^Q(0, v)^T y_t \right) = \exp \left( A_h^Q(0, -\infty) + B_h^Q(0, -\infty)^T y_t \right),
\]

by abuse of notation. Again the recursions write:

\[
q_h = q_{h-1} + A^Q \left[ Q_{h-1}^Q(0, -\infty), -\infty \right]
\]

\[
Q_h = B^Q \left[ Q_{h-1}^Q(0, -\infty), -\infty \right].
\]

Note that in that case the \( A^Q \) and \( B^Q \) functions transform:

\[
A^Q(u, -\infty) = -\alpha^Q + \alpha_y^Q \frac{\text{diag} (\nu^Q) (u - \gamma^Q)}{1 - \text{diag} (\nu^Q) (u - \gamma^Q)} - \nu^T \log [1 - \text{diag} (\nu^Q) (u - \gamma^Q)]
\]

\[
B^Q(u, -\infty) = \Phi^Q \frac{\text{diag} (\nu^Q) (u - \gamma^Q)}{1 - \text{diag} (\nu^Q) (u - \gamma^Q)}.
\]

Last, for \( D(t, h) \), we need a couple of intermediate steps. We then use a lemma in Monfort et al. (2017a) allowing us to apply the following formula:

\[
D(t, h) = \lim_{v \rightarrow -\infty} \mathbb{E}_t^Q \left[ \exp \left( -\delta_{t+h} + \sum_{i=1}^{h-1} v \delta_{t+i} \right) \right]
\]
Adjusting slightly the notations as:

\[
E^Q_t \left[ \exp \left( -\delta_{t+H} + \sum_{i=1}^{H-1} v\delta_{t+i} \right) \right] = \exp \left( A^Q_H(0, v, -1) + B^Q_H(0, v, -1)^T y_t \right),
\]

where \( A^Q_1(0, v, -1) = A^Q(0, -1) \) and \( B^Q_1(0, v, -1) = B^Q(0, -1) \), and for \( H > 1 \):

\[
A^Q_H(0, v, -1) = \begin{cases} A^Q_{H-1}(0, v, -1) + A^Q \left[ B^Q_{H-1} (0, v, -1), v \right] \end{cases}
\]

\[
B^Q_H(0, v, -1) = \begin{cases} B^Q \left[ B^Q_{H-1} (0, v, -1), v \right] \end{cases}
\]

we easily obtain that

\[
D(t, h) = \exp \left[ A^Q_h (0, -\infty, -1) + B^Q_h (0, -\infty, -1)^T y_t \right]
\]

which again, directly identifies coefficients \( d_h \) and \( D_h \).