The Threat of Intervention

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Abstract

We develop a model in which an activist shareholder can discipline management through intervention and through the threat of intervention. We show that the ex ante threat and ex post intervention can act as complements or substitutes. A weaker disciplinary role played by the intervention mechanism leads to lower firm value and more frequent ex post interventions. Thus, more frequent ex post interventions are not necessarily a sign of enhanced economic efficiency. Factors enhancing the power of the intervention may be less effective at improving economic efficiency than factors enhancing the ex ante threat. Because we endogenize the activist’s choice of toehold, we also show that the effect of liquidity trading on firm efficiency depends on the timing of liquidity trading.
One of the fundamental issues in modern corporate finance is the problem of separation of firm ownership from control. The gap between management and shareholders is potentially wide and the danger is great for agency problems to divert a widely-held firm’s resources from their efficient use. Therefore it is important to understand what mechanisms are available for reconciling these interests, to what extent they are used, and to what extent they are effective.

If a shareholder decides he does not like what a firm’s management is doing, he has two alternatives: He can intervene or he can exit—that is, he can work directly on changing the behavior of the firm’s management or he can sell his shares. Intervention, sometimes referred to as “voice,” includes a variety of possible actions to compel changes in managerial behavior: replacement of boards of directors, support for takeover bids, and proxy initiatives to limit management discretion or to affect management compensation.

However, shareholder activity can also have indirect effects, because the foreknowledge by managers of the possible reactions of dissatisfied shareholders can alter managerial behavior. Thus we are not only interested in exit and intervention as behaviors by the blockholder, we are also interested in how they affect managerial behavior. That is, we are also interested in the ex ante incentive effects on managers of the threats of shareholder exit or intervention. Beginning with Admati and Pfleiderer (2009) and Edmans (2009), a series of recent articles has shown that, provided management compensation is tied in the short run to share price, the threat of exit and the resultant reduction of share price can serve as a disciplinary device. On the other hand, despite empirical investigations of intervention, surprisingly little
theoretical attention has been paid to the role of the threat of intervention.

In this paper we focus on the dual aspect of the intervention mechanism: Intervention can improve the firm ex post (through direct action by the activist) or ex ante (through the threat to management). We ask the following research questions: In equilibrium when does the threat of intervention affect managerial behavior? Under what circumstances does intervention play a stronger disciplinary role? Under what circumstances does intervention play a stronger correction role ex post?

To address these questions, we provide a model in which an activist shareholder can accumulate a toehold of shares. After observing the activist’s toehold size, the manager decides whether to consume private benefits at the expense of shareholders. Once the managerial action is taken, the activist decides whether to extend the toehold and intervene, or to sell shares. Thus the process can improve firm value through two channels: the direct intervention itself, and the effect of the threat of intervention on managerial behavior.

In the model, an important role is played by the market’s revelation through prices of the activist’s response to managerial behavior. If the market fully reveals the activist’s private information, the activist has no incentive to accumulate the toehold. For this reason it is important to consider the effect of liquidity trading. The presence of liquidity trades enables the activist to a certain degree to hide his information. In this paper we consider liquidity trading during toehold accumulation period as well as during the intervention decision period. As we will show, liquidity trading in different phases of activist activity will have different impacts on the relation between market liquidity and economic efficiency.
Our research framework is relevant in modern financial markets, since most publicly traded firms can be subject to governance though the threat of intervention. Recent empirical evidence shows that the threat of intervention is likely to play strong disciplinary role. Fos (2016) shows that when the likelihood of a proxy contest increases, firms take actions to increase firm value and therefore reduce the chances of intervention. Gantchev et al. (2016) show that when an activist hedge funds target a firm, other firms in the target’s industry are more likely to take actions with the intention to increase firm value and therefore reduce the chances of intervention.

The model reveals several key results.

The model shows how ex ante threat and ex post intervention interact and how they are related to economic efficiency. For instance, in the model, more frequent ex post interventions are not necessarily a sign of enhanced economic efficiency. A weaker disciplinary role played by the intervention mechanism leads to lower firm value (because the manager is not disciplined ex ante), which can lead to more frequent ex post interventions, which are costly (both to the activist and the manager) but only partially recover the damage made to the firm value. Thus in this case more frequent ex post interventions are a sign of worsening corporate governance.

Therefore it is important to understand the circumstances in which observed levels of intervention actually correlate with improvement in firm value. Because we endogenize both the activist’ decision to engage in activism and the manager’s decision to take the bad action, we are better able to track the relation between the effectiveness of intervention and apparent empirical measures of that effectiveness. While the interventions themselves improve
firm value, to the extent that they are substituting for the more efficient alternative of an ex ante threat disciplining the manager, the observation of interventions should correlate with decreasing firm value.

We provide conditions under which we predict a positive or negative correlation between firm value and observed degree of intervention. The sign of the correlation will depend not only on the source of the variation but also on the degree to which the two channels act as complements or substitutes. For instance if variation is due to differences in the effectiveness of activists in punishing firm management—as would be the case in business cycle downturns, when loss of a job would have more dire consequences—then the two mechanisms act as substitutes. When the variation is due to differences in the activist’s ex post liquidity needs—activists flush with cash will be unlikely to need to sell for liquidity purposes—then the two mechanisms will be complementary. In the latter case, we should observe increases in intervention correlating with increases in firm value.

The model reveals that it is important to distinguish between sources of liquidity trading. Previous work has emphasized the dual nature of liquidity trading: that it makes it easier for activists to accumulate holdings, but also makes it harder to commit not to dissipate those holdings. Liquidity trading that does not interact with the activist’s actions has a positive effect on market liquidity and on the activist’s trading profits. It therefore leads to larger toehold accumulated by the activist and consequently increases chances of an equilibrium in which the activist intervenes. In contrast, liquidity trading that interacts with activist’s actions leads to wider bid-ask spreads and weaker disciplinary role played by the intervention. This result has impor-
tant implications for the literature that studies the role of market liquidity in corporate governance. To the best of our knowledge, this is the first paper to contrast two phases of liquidity trading and to show their differential effects on economic outcomes.

Because we endogenize the activist’s choice of toehold we can examine the relation between observed block holdings and the use of intervention as a threat. One of key implications of Shleifer and Vishny (1986) is that the presence of a large blockholder increases chances of blockholder governance through voice. The intuition is that a large block allows the blockholder to capture a larger portion of value creation and therefore to cover the cost of exercising voice—that is, the presence of a large blockholder provides a partial solution to the free-rider problem (Grossman and Hart, 1980). To the best of our knowledge, our model is the first to show that the presence of a large blockholder will not necessarily lead to more incidents of blockholder governance through ex post intervention to the extent that the intervention form of governance is very effective ex ante. Instead, the mere presence of a large blockholder who can exercise voice affects the manager’s incentives. In the extreme, one would not observe any intervention events if the threat of intervention were so powerful as to prevent the manager from taking the bad action in any state of the world.\footnote{In this extreme case the situation bears a similarity to the theory of contestible markets, where potential competition, even though unobserved, manages to provide market discipline against temptations toward inefficient behavior (Baumol et al., 1988).} Thus, our model argues that the absence of action by large blockholders can in fact be a sign of well-functioning corporate governance.
We investigate the extent to which increases in the toehold are indicative of improved governance. In particular we ask, when can we expect that size of toehold to be a better/worse indicator than frequency of intervention of the effectiveness of the intervention mechanism? How do changes in the liquidity of the asset market at various points in the activist’s cycle of activity affect activist behavior and managerial response?

**Related Literature**

This paper is related to several strands of the corporate governance literature that studies the role of blockholders in reducing agency costs.\(^2\)

First, the paper contributes to the strand of literature that studies how shareholder intervention can increase firm value ex post (e.g., Shleifer and Vishny, 1986; Kyle and Vila, 1991; Admati et al., 1994; Maug, 1998; Bolton and von Thadden, 1998; Kahn and Winton, 1998; Noe, 2002; Faure-Grimaud and Gromb, 2004; Brav and Mathews, 2011; Back et al., 2016). For example, in their classic paper Shleifer and Vishny (1986) show that the presence of a large minority shareholder provides a partial solution to the free-rider problem and therefore reduces the agency costs.

In this strand of literature, intervention does not play a disciplinary role. Intervention occurs in the absence of managerial action; more effective monitoring does not change the manager’s incentives and therefore is beneficial for shareholders only because it increases firm value *ex post*. Thus, our contribution is to introduce the disciplinary role of intervention.

Second, the paper contributes to the literature that studies how cor-

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\(^2\)Edmans (2013) surveys theoretical and empirical literature on the role of blockholders in corporate governance.
porate governance can affect management’s incentives. Grossman and Hart (1980) were the first to argue that managers face trade offs between a high profit action with an associated low chance of being raided and a low profit (but high managerial-utility) action which leads to a successful takeover bid. In their model managers are more reluctant to take self-serving actions that lower firm value and increase the probability of a takeover. Scharfstein (1988) explicitly models the source of contractual inefficiencies which was not studied by Grossman and Hart (1980). He explores the conditions under which the takeover threat plays a genuine role (beyond incentive contracts) in disciplining management.³

The literature has also studied the governance role of exit and showed that a large shareholder can alleviate conflicts of interest between managers and shareholders through the credible threat of exit on the basis of private information (e.g., Admati and Pfleiderer, 2009; Edmans, 2009; Dasgupta and Piacentino, 2014). Our paper contributes to the corporate governance/management incentive literature by studying the interaction between ex ante and ex post corrections in the intervention equilibrium. Moreover, because we endogenize the activist’s choice of toehold, we can study consequences of liquidity trading during toehold-acquiring phase and the activism phase of the blockholder’s activity.

³While the above papers show that takeover plays a positive disciplinary role, several other papers have highlighted some negative aspects of the threat of intervention (e.g., Stein, 1988; Zwiebel, 1996; Burkart et al., 1997). For example, Stein (1988) develops a model in which takeover pressure can be damaging because it leads managers to sacrifice long-term interests in order to boost current profit.
1. Setup

In the basic model there are three dates 0, 1, and 2 and three types of agents: the manager, whom we denote by $\mathcal{M}$, an activist shareholder, $\mathcal{A}$, and a continuum of uninformed traders (the “market makers”). Markets for shares in the firm occur at each date.

At date 0, $\mathcal{A}$ acquires the initial holding $\varphi$ of shares of the firm (the “toehold”). The choice of optimal $\varphi$, which is analyzed in section 2.2, will take into account the impact of $\varphi$ on $\mathcal{M}$’s incentives (and therefore firm value), the cost of holding $\varphi$ shares, and trading profits.\footnote{We will assume that the ability to amass large initial positions is limited by disclosure requirements, modeled on current regulatory requirements. Details are specified in the toehold analysis in section 2.2.} Trading profits can be positive because participants in the period 0 market expect that with probability $\nu$ the purchase order comes from a non-activist shareholder who cannot change firm value and with probability $(1 - \nu)$ the purchase order comes from an informed $\mathcal{A}$. Higher values $\nu$ correspond to a more liquid period 0 market; $\mathcal{A}$’s purchase can be hidden more effectively in the sea of non-monitor purchases.

After date 0 trading occurs, market participants, including $\mathcal{M}$, observe $\varphi$—that is, they learn the size of $\mathcal{A}$’s holding. Then $\mathcal{M}$ decides whether or not to take a particular action. An agency problem arises because $\mathcal{M}$ and the shareholders have conflicting preferences with respect to the action. Specifically, we assume the action is “bad” in the sense that it reduces the value of the firm, but provides a private benefit to $\mathcal{M}$. The benefit has the positive value $\beta$, known with certainty by all participants.\footnote{Fos and Jiang (2016) document evidence consistent with a manager’s value of private} The cost
of the damage to the firm is \( \delta \), a random value which \( \mathcal{M} \) learns privately immediately before making his decision.\(^6\) Let the decision be denoted \( a \) (either zero or one); then the value of the firm in period 2 will be \( v - a\tilde{\delta} \), in the absence of intervention by the activist. The value \( v \) is common knowledge. All agents know that the value \( \tilde{\delta} \) is drawn from a continuous distribution \( F(.) \) with density \( f(.) \) and support \([0, \bar{\delta}]\), where \( \bar{\delta} \) is sufficiently large. When illustrating some results we will further assume that the distribution of \( \delta \) is exponential with \( F(\delta) = 1 - e^{-\delta \lambda} \).

\( \mathcal{M} \)'s strategy can be described by defining the set \( \Delta \subseteq [0, \bar{\delta}] \), such that \( a = 1 \) if and only if \( \delta \) is in the set \( \Delta \). Let \( \Phi = Pr\{\delta \in \Delta\} \), the ex ante probability that \( \mathcal{M} \) chooses \( a = 1 \).

\( \mathcal{A} \) privately observes the action taken by \( \mathcal{M} \). Given \( \mathcal{M} \)'s strategy, observing \( \mathcal{M} \)'s actions provides \( \mathcal{A} \) with a noisy signal of firm value. Given this private information \( \mathcal{A} \) must decide whether to buy, sell, or hold his shares at the date 1 market. If \( \mathcal{A} \) buys sufficient shares, he can intervene, reducing the benefit to \( \mathcal{M} \) of taking the bad action, and reducing the damage of the action to the firm. Specifically, if \( \mathcal{A} \) intervenes then the benefit to \( \mathcal{M} \) is reduced to \( \beta \gamma \) and the value of the firm is restored to \( v - a\tilde{\delta}\kappa \), where \( 0 < (1 - \gamma) < 1 \) measures the effectiveness of \( \mathcal{A} \) in reducing the private benefits of control and \( 0 < (1 - \kappa) < 1 \) measures his effectiveness in restoring firm value. We further assume that the intervention involves a fixed cost \( \eta \)

\(^6\)In a supplement to this paper we also consider the case where \( \mathcal{M} \)'s action is “good” in that it increases the firm’s value at a private cost to the manager. For the most part that \( G \) version of the model (to use the terminology of Admati and Pfleiderer) provides results parallel to the \( B \) version adopted here.
to \( \mathcal{A} \). Let \( b \in \{0, 1\} \) represent the decision to intervene. Then the ultimate value of the firm is \( \nu - a \tilde{\delta}(\kappa b + 1 - b) \). We assume this value is publicly revealed before the date 2 market, so that trade in the final market occurs at this price.\(^7\)

As we will see, both \( \mathcal{A} \)'s ability to reduce the benefit to \( \mathcal{M} \) of taking the bad action, \((1 - \gamma)\), and \( \mathcal{A} \)'s ability to reduce the damage of the action to the firm, \((1 - \kappa)\), will play important and distinct roles in the model. Whereas \( \mathcal{A} \)'s ability to increase firm value ex post, \((1 - \kappa)\), will be one of key parameters to determine the existence of the intervention equilibrium, \((1 - \gamma)\) will determine the degree of discipline imposed on \( \mathcal{M} \) in that equilibrium. Consistent with \((1 - \gamma) > 0\), Fos and Tsoutsoura (2014) show that activist shareholders are able to impose a significant career cost on directors of companies targeted in proxy contests. Directors of companies that experience a proxy contest lose seats not only on boards of targeted companies, but also on boards of other companies. Several pieces of evidence motivate \((1 - \kappa) > 0\). For example, Brav et al. (2008) show that firm value increases upon intervention by activist hedge funds.

We next characterize \( \mathcal{A} \)'s trading in period 1. Similarly to Maug (1998), we assume \( \mathcal{A} \) can intervene if he controls a fraction \( \alpha \) of the shares in the firm, where \( \alpha \) is fixed and publicly known.\(^8\) As we will see, in any equilibrium

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\(^7\)In our formulation, the power of the blockholder to punish managers or to repair damage to the firm are simply taken as parametric. One formulation which has endogenized the ability of outsiders to punish management is that of Fluck (1999). In her account, the size of outsider holdings affects the likelihood of being able to remove the manager. Since she assumes that the firm is less valuable when managers are removed from control, such a threat is not credible except in an infinitely repeated game. She examines credible threats in that framework.

\(^8\)Several factors could affect \( \alpha \). For example, a larger \( \alpha \) could correspond to cases when
in which $A$ decides to sell shares, it is optimal to sell the entire position $\varphi$. Similarly, in any equilibrium in which $A$ decides to buy shares, it is optimal to buy shares until the block size reaches $\alpha$. Therefore, we assume that if $A$ decides to sell shares, he sells the entire position $\varphi$. If $A$ decides to buy shares, he buys $\alpha - \varphi$ shares. Thus all participants in trading at date 1 know $\alpha$ and $\varphi$.

We follow Admati and Pfleiderer (2009) and assume that $A$ in period 1 will with probability $\theta$ suffer a liquidity shock which requires him to divest himself of any holdings of firm shares and which prevents him from purchasing any shares of the firm. If he does not suffer a liquidity shock, then his purchases and sales will be based on his information and his strategy for future intervention. Thus $A$’s trades in period 1 may reveal information both about $M$’s actions and about $A$’s own intentions for future actions. If $A$ decides to purchase shares, it could indicate either that the shares are undamaged by $M$’s action or that $A$ intends to intervene to repair $M$’s action. Sales on the other hand could be due either to a bad choice by $M$ or to $A$’s liquidity needs. Other participants in the market are unable to observe the liquidity shock of $A$, and so the price prevailing will take into account their expectation of the relative likelihood of the shock.

$M$’s compensation is assumed to be $\omega_2(v - a\delta)$, where $\omega_2$ is a positive coefficient representing the dependence of the compensation on firm value. $M$ chooses whether to take the action or not to maximize his expected utility for every realization of $\delta$. When $A$ is not present, $M$’s preferred cutoff point, denoted $\delta_{BM}$, is equal to $\beta/\omega_2$. That is, $M$ takes the action when

$A$ needs more voting power to make the intervention effective.
\( \tilde{\delta} \leq \delta_{BM} = \beta/\omega_2 \). Let \( p_{BM} \) denote the expected value of the firm when \( A \) is not present.

Next consider the case when \( A \) is present. If \( M \) does not take the action, then \( M \)'s utility is simply his compensation, \( \omega_2 v \). If he takes the action, then his utility depends on \( A \)'s intervention in period 2. If intervention does not occur, \( M \)'s utility is equal to the sum of his compensation and the private benefit \( \omega_2 (v - \tilde{\delta}) + \beta \). If intervention occurs, \( M \)'s utility is equal to the sum of his compensation and the private benefit \( \omega_2 (v - \tilde{\delta}) + \beta \gamma \).\(^9\)

To summarize, the potential impact of \( A \) on \( M \)'s decision to take the action comes about through his impact on private benefits of control. We will describe an equilibrium as **disciplinary** if equilibrium cutoff point is lower than \( \delta_{BM} \).

We assume that prices are set by risk-neutral, competitive market makers and therefore reflect all of the information publicly available. This means, as noted before, that \( P_2 \) equals \( v - a\delta(\kappa b +1 -b) \). The date 1 price, \( P_1 \) reflects the information contained in \( A \)'s trading decision. The timing of events is given in Figure 1.

### 2. Solving the Model

The model is solved backwards. First, we assume \( A \) holds \( \varphi \) shares and characterize equilibrium prices and \( A \)'s trading decisions at date 1, and \( M \)'s actions. Then we endogenize \( A \)'s choice of the initial holding, \( \varphi \).

\(^9\)Note that \( M \) does not benefit from value creation induced by \( A \)'s action. Alternatively, a more productive \( A \) would effectively create an incentive for \( M \) to take the bad action.
2.1. Date 1 Equilibria and \( M \)'s Incentives

Suppose \( A \) holds \( \phi \) shares. We assume \( A \) is restricted to three actions \( T \in \{B, H, S\} \) in the date 1 market: buy enough to get the level to the required amount for intervention \((B)\); sell all holdings \((S)\); or keep holdings unchanged \((H\) for “hold”). As we are going to show later in this section, in any equilibrium in which \( A \) decides to sell shares, it is optimal to sell the entire position. Similarly, in any equilibrium in which \( A \) decides to buy shares, it is optimal to buy shares until the block size reaches \( \alpha \).

It is useful to introduce notation for the prices that would occur if uninformed agents observed \( M \)'s action (denote these as \( p^a_T \)). If \( a = 0 \), \( p^0_T = \upsilon \) for any \( T \). If \( a = 1 \), \( p^1_H = p^1_S = \upsilon - \Lambda \), where \( \Phi \equiv Pr(\delta \in \Delta) \) is the probability of \( M \) taking the action and \( \Lambda \equiv \Phi^{-1} E[1_{\delta \in \Delta} \delta] \) is the expected damage to firm value, conditional on \( M \) taking the action. Note that the price if held is the same as the price if sold, because without having enough of a holding to intervene, \( A \) adds no value to the asset. Finally, \( p^1_B = \upsilon - \kappa \Lambda > p^1_H = p^1_S \), reflecting the benefit from intervention.$^{10}$

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$^{10}$We know that \( \Phi > 0 \) (and so \( \Lambda > 0 \)) because for any fixed value of \( \omega_2 \), for \( \delta \) sufficiently
We next consider the value of $\mathcal{A}$’s position $\pi^T_a$. The value from holding is $\pi^H_a = \varphi v$ if $a = 0$ and $\pi^H_1 = \varphi (v - \Lambda)$ if $a = 1$. The value from selling the lot is $\pi^S_a = \varphi p_S$ (note this does not actually depend on $a$). The value from buying is $\pi^B_0 = \alpha v - (\alpha - \varphi) p_B$ or $\pi^B_1 = \alpha (v - \kappa \Lambda) - \eta - (\alpha - \varphi) p_B$ if $a = 1$.

Hereafter, we will refer to the value net of the undamaged value of the initial holding, $\varphi v$, as “$\mathcal{A}$’s profits.”

A market equilibrium for period 1 specifies the probability mixture for $\mathcal{A}$ between buy, hold, and sell $(\sigma^B_a, \sigma^H_a, \sigma^S_a)$, for $a = 0$ or 1, conditional on no liquidity shock and market prices $p_B, p_S$, such that the probabilities are maximizing choices given prices, and prices are consistent with the probabilities:

$$\sum_{T=B,H,S} \sigma^T_a \pi^T_a \geq \pi^T'_{a}$$(all $T' \in \{B, H, S\}$, for $a = 0, 1$.

$$p^1_T \leq p_T \leq p^0_T$$, for $T \in B, S$

$$p_S = \frac{(1 - \theta)[p^1_S \Phi \sigma^S_1 + p^0_S (1 - \Phi) {\sigma}^S_0] + \theta[p^1_S \Phi + p^0_S (1 - \Phi)]} {(1 - \theta)[\Phi \sigma^S_1 + (1 - \Phi) {\sigma}^S_0] + \theta}$$

$$= v - \Lambda \frac{\Phi \sigma^S_1 + \theta \Phi}{\Phi \sigma^S_1 + (1 - \Phi) {\sigma}^S_0 + \theta}$$

$$p_B = \frac{(1 - \theta)[p^1_B \Phi \sigma^B_1 + p^0_B (1 - \Phi) {\sigma}^B_0]} { (1 - \theta)[\Phi \sigma^B_1 + (1 - \Phi) {\sigma}^B_0]} = v - \kappa \Lambda \frac{\Phi \sigma^B_1} {\Phi \sigma^B_1 + (1 - \Phi) {\sigma}^B_0},$$

where $\bar{\theta} \equiv \theta/(1 - \theta)$ and $p_B$ is defined when the denominator is non-zero. Without loss of generality we can also specify $p_B$ when the denominator is zero: If there is zero probability of buying, we can set $p_B = p^0_B$. To see this,
Table 1: \( A \)'s Profits. This table describes profits of \( A \), as measured by \( \pi - \varphi \nu \), adjusted to results of Lemma 1. The left column reports profits if \( M \) does not take the action and the right column reports profits if \( M \) takes the action. In the case of each bracketed expression, the second term is to be used in case the denominator of the first term is zero.

<table>
<thead>
<tr>
<th></th>
<th>( a = 0 ) (no damage)</th>
<th>( a = 1 ) (damage)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy</td>
<td>((\alpha - \varphi)\kappa \Lambda \frac{\Phi \sigma^H}{(1 - \Phi) \sigma^S} , 0)</td>
<td>(-\alpha \kappa \Lambda - \eta + (\alpha - \varphi) \kappa \Lambda \frac{\Phi \sigma^H}{(1 - \Phi) \sigma^S} , 0)</td>
</tr>
<tr>
<td>Hold</td>
<td>0</td>
<td>(-\varphi \Lambda)</td>
</tr>
<tr>
<td>Sell</td>
<td>(-\varphi \Lambda \frac{\Phi (1 - \sigma^H) + \hat{\delta} \Phi}{\Phi (1 - \sigma^H) + \theta})</td>
<td>(-\varphi \Lambda \frac{\Phi (1 - \sigma^H) + \hat{\delta} \Phi}{\Phi (1 - \sigma^H) + \theta})</td>
</tr>
</tbody>
</table>

note that if \( p_B < p_B^0 \) in an equilibrium with no buying, then higher buying prices also yield an equilibrium with the same allocation; moreover \( p_B \) cannot exceed \( p_B^0 \) in equilibrium. The following Lemma shows that we can put more structure on equilibrium beliefs.

Lemma 1. In equilibrium, \( \sigma^S_0 = 0 \) and \( \sigma^H_1 = 0 \).\(^{11}\)

In other words, if he observes the bad action was taken, \( A \) will definitely trade at date 1 (although it is possible that he may buy or sell). If he observes that the bad action was not taken, he will not sell voluntarily (although he may be forced to sell for liquidity reasons). \( A \)'s profits, as measured by \( \pi - \varphi \nu \), are presented in Table 1.

We next characterize intervention equilibria, those in which \( A \) intervenes with certainty (as long as he is not hit by a liquidity shock), i.e., those in which \( \sigma^B_1 = 1 \). Let \( \Phi_I, \Lambda_I, \delta_I \) denote the equilibrium values of \( \Phi, \Lambda, \delta \).

\(^{11}\) All proofs are in the appendix.
Proposition 1. Suppose $\mathcal{A}$ holds $\varphi$ shares. There exists an intervention equilibrium if and only if

$$\varphi > \varphi_I \equiv \frac{1}{(1 - \kappa)\Phi_I}\{\alpha \kappa (1 - \Phi_I) + \frac{\eta}{\Lambda_I}\},$$

where $\delta_I = \frac{\partial}{\partial \omega} (\theta + (1 - \theta)\gamma)$, $\Phi_I = F(\delta_I)$, and $\Lambda_I = \Phi_I^{-1}E[1_{\delta < \delta_I}]$. Equilibrium beliefs are $(\sigma^B_0 = 1, \sigma^H_0 = 0, \sigma^S_0 = 0; \sigma^B_1 = 1, \sigma^H_1 = 0, \sigma^S_1 = 0)$. Equilibrium prices are $p_B = \upsilon - \kappa \Lambda_I \Phi_I$ and $p_S = \upsilon - \Lambda_I \Phi_I$. This equilibrium is always disciplinary, $\delta_I < \delta_{BM}$.

The impact of the intervention on $\mathcal{M}$’s incentives is illustrated in Figure 2. When $\delta > \delta_{BM}$, $\mathcal{M}$ would not take the bad action even if $\mathcal{A}$ were not present. In this case the damage to firm value is so large that $\mathcal{M}$ prefers to forego the private benefit $\beta$. In the intermediate region $\delta_{BM} > \delta > \delta_I$, $\mathcal{A}$’s presence prevents $\mathcal{M}$ from taking the bad action. This is the disciplinary role of the intervention: the mere presence of $\mathcal{A}$ causes $\mathcal{M}$ to not take the bad action. Finally, when $\delta < \delta_I$, $\mathcal{M}$ takes the bad action and ex post intervention takes place. Only in this region will market participants observe incidents of intervention.

The disciplinary role of intervention is increasing in $\mathcal{A}$’s effectiveness in reducing $\mathcal{M}$’s private benefits of control, $(1 - \gamma)$. A higher probability of the sell-side liquidity shock, $\theta$, shifts $\delta_I$ toward $\delta_{BM}$ and therefore decreases the impact of intervention on $\mathcal{M}$’s incentives. Thus, a higher $\theta$ leads to a less disciplinary equilibria and higher frequency of ex post interventions. It implies that period 1 liquidity generates a tension between ex ante and ex post efficiency of intervention. We discuss this tension in details in Section
The intervention equilibrium exists when \( A \)'s initial toehold \( \varphi \) (which we endogenize in Section 2.2) is larger than \( \varphi_I \). Equation (1) shows that \( \varphi_I \) depends on several parameters. \( \varphi_I \) is smaller when \((1 - \kappa)\) is closer to one \((A \text{ is effective in restoring the damage})\) and as \( \Phi_I \) increases, that is, when \( M \) is more likely to take the bad action. Among other considerations, this happens when \( \beta \) is large \((\text{the agency problem is severe})\), \((1 - \gamma)\) is small \((A \text{ is less effective in reducing } M \text{'s private benefits of control})\), and when \( A \) needs a small toehold to intervene, i.e., \( \alpha \) is small.

The presence of liquidity trades enables \( A \) to a certain degree to hide his information. The equilibrium sell price, \( p_S = v - \Lambda_I \Phi_I \), reflects equilibrium beliefs that \( A \) sells shares only if he experiences a liquidity shock. Specifically, if \( A \)'s sale is caused by the liquidity shock, the probability that \( M \) took the bad action remains equal to its unconditional value \( \Phi_I \). In the absence of

Figure 2: The disciplinary role of Intervention.
the liquidity shock, the sell price would be fully revealing and therefore lower
\( p_S(\theta = 0) = v - \Lambda_I \). Thus, the presence of the liquidity shock increases the
profits from selling the block from \( \varphi(v - \Lambda_I) \) to \( \varphi(v - \Lambda_I\Phi_I) \). The increase in
profitability of selling the block due to the possibility of the liquidity shock
therefore reduces the chances of the intervention equilibrium. Since \( \mathcal{A} \) never
voluntarily sells shares in the intervention equilibrium, it is not reflected in
his expected profits. Instead, it affects the chances that condition (1) holds.

Finally, note that in the intervention equilibrium \( \mathcal{A} \) does not want to
deviate from buying \( \alpha - \varphi \) shares. If \( a = 1 \), \( \mathcal{A} \) loses money on buying shares
because \( p_B > v - \kappa\Lambda_I \). Therefore, he will not buy more than necessary. i.e.,
(\( \alpha - \varphi \)). If \( a = 0 \), an activist \( \mathcal{A} \) makes money on buying shares. However, if
he buys more shares than (\( \alpha - \varphi \)), he reveals that \( a = 0 \) and therefore drives
profits to zero.

Next we construct equilibria in which \( \mathcal{A} \) does not intervene when \( \mathcal{M} \)
takes the bad action (i.e., \( \sigma^B_1 = 0 \)). Again, \( \Phi_E, \Lambda_E, \delta_E \) represent values of the
endogenous variables in the equilibrium.

**Proposition 2.** Suppose \( \mathcal{A} \) holds \( \varphi \) shares. There exists a non-disciplinary
equilibrium with \( \sigma^B_1 = 0 \) if and only if

\[
\varphi < \varphi_E \equiv (\alpha\kappa + \frac{\eta}{\Lambda_{BM}}) \frac{\Phi_{BM} + \bar{\theta}}{\Phi_{BM} + \Phi_{BM}\bar{\theta}},
\]

where \( \delta_{BM} = \beta/\omega_2 \), \( \Phi_{BM} = F(\delta_{BM}) \), \( \Lambda_{BM} = \Phi_{BM}^{-1}E[1_{\delta<\delta_{BM}}\delta] \) and equilib-
rium prices are \( p_B = v \) nd \( p_S = v - \Lambda_{BM}\Phi_{BM}\frac{1+\bar{\theta}}{\Phi_{BM}+\bar{\theta}} \). Equilibrium beliefs are
(\( \sigma^B_0 = 0, \sigma^H_0 \geq 0, \sigma^S_0 = 0; \sigma^B_1 = 0, \sigma^H_1 = 0, \sigma^S_1 = 1 \)).

An equilibrium with \( \sigma^B_1 = 0 \) exists when \( \varphi \) is smaller than \( \varphi_E \). Equation
shows that $\varphi_E$ depends on several parameters. $\varphi_E$ is larger when $(1-\kappa)$ is close to zero ($A$ is not efficient in restoring the damage), when $\Phi_{BM}$ decreases, that is, when $M$ is less likely to take the bad action, and when $A$ needs a large toehold to intervene, i.e., $\alpha$ is large. Moreover, the existence of this equilibrium is positively affected by $\theta$ because when $\theta$ increases, $\varphi_E$ increases and condition (2) is more likely to hold. The equilibrium sell price, $p_S = v - \Lambda_{BM}\Phi_{BM}\frac{1+\delta}{\Phi_{BM}+\theta}$, reflects equilibrium beliefs that $A$ will sell shares if he experiences a liquidity shock. In the absence of the liquidity shock, the sell price would be fully revealing and therefore lower $p_S(\theta = 0) = v - \Lambda_{BM}$. Thus, the presence of the liquidity shock increases the profits from selling the block and therefore increases the chances of this equilibrium.\textsuperscript{12}

We conclude this section by establishing conditions for period 1 equilibria in which $A$ intervenes with a positive probability $0 < \sigma_1^B < 1$.

**Proposition 3.** Suppose $A$ holds $\varphi$ shares and $\varphi_E < \varphi_I$. There is a unique mixed strategy disciplinary equilibrium if both conditions (1) and (2) are violated, i.e., $\varphi_E < \varphi < \varphi_I$. Equilibrium beliefs are $(\sigma_0^B = 1, \sigma_0^H = 0, \sigma_0^S = 0; \sigma_1^B > 0, \sigma_1^H = 0, \sigma_1^S > 1)$. In equilibrium, $p_B = v - \kappa\Lambda_M\Phi_M\frac{\sigma_0^B}{\Phi_M\sigma_1^B+(1-\Phi_M)}$ and $p_S = v - \Lambda_M\Phi_M\frac{(1-\sigma_1^B)^{+}\theta}{\Phi_M(1-\sigma_1^B)+\theta}$, where $\delta_{BM} < \delta_M = (\beta/\omega_2)(1-(1-\theta)\sigma_1^B(1-\gamma)) < \delta_I$, $\Phi_M = F(\delta_M)$, and $\Lambda_M = \Phi^{-1}E[1_{\delta<\delta_M}]$.

Note that when $\varphi_I < \varphi < \varphi_E$, multiple equilibria are possible (both

\textsuperscript{12}Proposition 2 shows that an equilibrium with $\sigma_1^B = 0$ is non-disciplinary. Fos and Kahn (2016) analyze a similar model in which they allow $M$’s compensation to depend on the realized market price of the firm in period 1 and show that in this case there can exist an equilibrium in which $A$ sells when $M$ takes the bad action and that this sell plays a disciplinary role.
conditions (1) and (2) are satisfied). As we are going to see in the next section, however, \( A \) will never purchase toehold \( \varphi \) with an intention to be in a non-disciplinary exit equilibrium. Thus, when (1) holds, we can restrict attention to the intervention equilibria described in Proposition 1, even when \( \varphi_I < \varphi < \varphi_E \).

### 2.2. Initial toehold

We next analyze the formation of the initial toehold, \( \varphi \). The initial toehold plays an important role in the model because it determines the type of period 1 equilibrium and therefore \( A \)'s and \( M \)'s actions. For the reasons noted above we focus on the case when \( \varphi_I < \varphi_E \); analysis of the case where \( \varphi_I > \varphi_E \) is available in the Internet Appendix.

First, we describe disclosure requirements. After the initial trade in period 0 occurs, \( A \)'s toehold \( \varphi \) becomes common knowledge. This assumption is motivated by the fact that market participants can use Schedule 13F filings to infer changes in stock ownership. If \( A \) purchases a toehold smaller than \( \alpha \), no additional disclosure of the position or trade is necessary until period 0. In this case, \( A \) can potentially benefit from hiding behind liquidity trades, which we introduce in the next paragraph. To capture the role of ownership disclosure requirements that are linked to \( A \)'s position size—The Hart–Scott–Rodino Act disclosure requirement (HSR)—we assume that if \( A \) intents to purchase toehold larger than \( \alpha \), market participants become immediately aware of trader’s identity and intention and set prices equal to \( p_I^1 \). As we’re going to see, this assumption implies that \( A \) will not purchase more than \( \alpha \) shares in period 0.

Second, we characterize the source of liquidity trading during toehold-
accumulation period. Participants in the market expect that with probability \( \nu \) the purchase order comes from an uniformed agent (either from a non-activist shareholder who cannot change firm value or from an uninformed activist), and with probability \( (1 - \nu) \) the purchase order comes from an informed \( A \). Similarly to the role of \( \theta \) in period 1 markets, higher values \( \nu \) correspond to a more liquid period 0 market; \( A \)’s purchase can be hidden more effectively in the sea of non-monitor purchases.

Next, we describe the relation between \( A \)’s toehold size and firm value. If \( A \) is not present, the value of the firm is \( p_{BM} = v - \Lambda_B \Phi_B \). If \( A \) is present and purchases \( \varphi < \varphi_I \), Proposition 2 applies. \( A \) always sells shares if the bad action is taken. Since the equilibrium is non-disciplinary and there is no ex post intervention, the value of the firm is still \( p_{BM} \). If \( A \) is present and purchases \( \varphi \geq \varphi_I \) which is sufficient to maintain the intervention equilibrium, Proposition 1 applies. The value of the firm is \( p_I = (1 - \theta)p_B + \theta p_S \), where \( p_B = v - \kappa \Lambda_I \Phi_I \) and \( p_S = v - \Lambda_I \Phi_I \).

The actual price on the market in period 0, denoted \( p_0(\varphi) \), depends on the size of the purchase order and reflects the market’s expectation that \( A \) is participating as well as the activism role played by \( A \) if he is present. If market makers receive an order to purchase \( \varphi < \varphi_I \) shares, they set period 0 price to \( p_0(\varphi < \varphi_I) = p_{BM} \). If market makers receive an order to purchase \( \varphi \geq \alpha \) shares, they set period 0 price to \( p_0(\varphi \geq \alpha) = p_I \) (i.e., the HSR disclosure rule is binding). In this two case prices are fully revealing and \( A \) cannot profit from trading. If market makers receive an order to purchase \( \varphi_I \leq \varphi < \alpha \) shares, they set period 0 price to \( p_0(\varphi_I \leq \varphi < \alpha) = (1 - \nu)p_I + \nu p_{BM} \). In this case more liquid markets (i.e., higher \( \nu \)) lead to higher trading.
profits of \( A \).

In period 0 \( A \) takes the market price function \( p_0(\varphi) \) as given and decides how many shares \( \varphi \) to buy. Holding \( \varphi \) shares between periods 0 and 1 involves private cost \( C(\varphi) = \frac{\varphi^2}{2} \) for \( A \). For example, this cost could correspond to lower diversification of \( A \)'s portfolio or binding capital constraints faced by \( A \). This assumption implies that keeping everything else constant, \( A \) prefers to purchase shares later rather than earlier.\(^{13}\)

\( A \) maximizes expected profits from purchasing \( \varphi \) shares:

\[
\max_{\varphi_{I} \leq \varphi < \alpha} \pi(\varphi; p_I) = \varphi \nu (p_I - p_{BM}) - \frac{\varphi^2}{2}.
\]

Note that when \( \varphi < \varphi_{I} \) and when \( \varphi \geq \alpha \), prices are fully revealing and \( A \)'s expected profit is \( -\varphi \nu (p_I - p_{BM}) - \frac{\varphi^2}{2} < 0 \). Consequently, \( A \) will prefer \( \varphi = 0 \) to purchasing fewer than \( \varphi_{I} \) shares or more than \( \alpha \) shares. Note that as long as \( \nu = 0 \), \( A \) prefers \( \varphi = 0 \). In other words, if \( A \) needs to purchase \( \varphi \) shares in the open market at a price that reflects \( A \)'s impact of firm value, privately-optimal initial stake size will be zero in the absence of liquidity trading. When \( \nu > 0 \), then, since \( p_I > p_{BM} \), \( A \) profits from liquidity trading because prices do not fully reflect \( A \)'s impact of firm value. The following propositions characterize \( A \)'s optimal \( \varphi \).

**Proposition 4.** Let \( \varphi^I_A = \frac{\nu(p_I - p_{BM})}{\varphi} \) be the value of \( \varphi \) that maximizes \( \pi(\varphi; p_I) \) and \( \varphi^I_0 \) be such that \( \pi(\varphi^I_0; p_I) = 0 \).

i. If \( \varphi_I \leq \varphi^I_0 \), \( A \) will choose \( \varphi = \max(\varphi^I_A, \varphi_I) \). In this case condition (1) holds.

\(^{13}\)The analysis can be extended to consider the possibility that higher \( \varphi \) increases the likelihood that \( A \) faces a liquidity shock in period 1.
holds and the market is in disciplinary intervention equilibrium.

ii. If \( \varphi_I > \varphi_A^I \), \( A \) will choose \( \varphi = 0 \).

The intuition behind Proposition 4 is presented in Figure 3. When \( \varphi_I < \varphi_A^I \), \( A \) finds it optimal to choose \( \varphi = \varphi_A^I \). This is because \( \varphi_A^I \) is sufficient to maintain the intervention equilibrium and the corresponding price level, \( p_I \). This case corresponds to the point B on the Figure. When \( \varphi_I \in (\varphi_A^I, \varphi_0^I) \), \( \varphi_A^I \) is not large enough to satisfy condition (1) and therefore maintain the intervention equilibrium and the corresponding price level, \( p_I \). In this case, \( A \) finds it optimal to choose \( \varphi = \varphi_I \) such that condition (1) holds. This case corresponds to a point on the B-D segment. When \( \varphi_I > \varphi_0^I \), \( A \) will choose \( \varphi = 0 \).
3. Discussion and Empirical Implications

3.1. Stock liquidity and economic efficiency

In this section we analyze the relation between the endogenously determined prices and measures of stock liquidity in the disciplinary intervention equilibrium. We start with period 1 prices and liquidity. In the intervention equilibrium, the period 1 bid-ask spread is 

\[ p_B - p_S = (1 - \kappa)\Lambda_I\Phi_I \]

and the expected price 

\[ (1 - \theta)p_B + \theta p_S = v - (\theta + (1 - \theta)\kappa)\Lambda_I\Phi_I. \]

Corollary 1. Consider period 1 prices and liquidity in the intervention equilibrium.

i Higher likelihood of liquidity shocks leads to lower stock liquidity and lower firm value, implying a positive correlation between these two endogenously determined values.

ii An increase in A’s effectiveness in restoring the damage leads to lower stock liquidity and higher firm value, implying a negative correlation between these two endogenously determined values.

Figure 4 plots equilibrium prices as function of the liquidity shock parameter, \( \theta \). We see that both bid and ask prices, as well as expected price, decrease when A is more likely to experience a liquidity shock. This is because \( \theta \) reduces the disciplinary role of the intervention and therefore increases expected damage to firm value. The ask price, however, is less affected by \( \theta \) than the bid price because if A purchases shares of damaged firm, he intervenes and restores part of the damage. Consequently, the bid-ask spread is wider when A is more likely to experience a liquidity shock. Thus, higher
Figure 4: **The effect of $\theta$ on prices in the Intervention equilibrium.** The black line plots period 1 buy price, $p_B = v - \kappa I \Phi I$. The grey line plots period 1 sell price, $p_S = v - \Lambda I \Phi I$. The dashed line plots the expected period 1 price, $p_I^1 = (1 - \theta)p_B + \theta p_S$. $\delta I$, $\Lambda I$, and $\Phi I$ are defined in Proposition 1. We assume $v=100$, $\beta=25$, $\omega_2=2$, $\gamma=0.3$; $\kappa=0.3$, $\varphi=4$, $\alpha=5$, $\eta=0.1$, $f[x] = \lambda \exp(-\lambda x)$, and $\lambda=0.1$.

The bid-ask spread is positive as long as $\mathcal{A}$ is effective in restoring the damage, $(1 - \kappa) > 0$. Only $\mathcal{A}$ knows if there is damage to be restored; thus, $\mathcal{A}$’s activism skill is a source of information asymmetry, even in the absence of liquidity shocks.\(^{14}\) (As Figure 4 shows, this component of bid-ask spread is positive even when $\theta = 0$.)

To understand how $\mathcal{A}$’s effectiveness in restoring the damage affects equilibrium outcomes, note that it does not affect $p_S$ and the disciplinary

\(^{14}\)See also Back et al. (2016).
role of the intervention equilibrium (i.e., $\delta_I$ does not depend on $\kappa$). It does, however, have a positive impact on $p_B$ because a more effective activist recovers a larger fraction of the damage. Thus, an increase in $A$’s effectiveness in restoring the damage leads to lower measured stock liquidity (wider bid-ask spread) and higher firm value.

An important empirical implication of Corollary 1 is that the relation between firm value and stock liquidity depends on the nature of variation in these variables. If changes in liquidity are caused by changes in the likelihood of liquidity shocks, an improvement in stock liquidity will lead to an increase in firm value. In contrast, if changes in liquidity are caused by changes in $A$’s effectiveness in restoring the damage, an improvement in liquidity will lead to a decrease in firm value. Thus, the causal effect of liquidity on firm value depends on the source of variation in liquidity.

So far, we discussed the role of liquidity parameters that affect period 1 equilibrium. Next, we consider the role of stock liquidity in block formation. In period 0, market participants expect that with probability $(1-\nu)$, an order to purchase $\varphi$ shares will be submitted by $A$, and with probability $\nu$ the order will be submitted by an uninformed agent who will not engage in corporate governance. For example, $\nu$ could correspond to the probability that $A$ experiences a liquidity shock and therefore cannot engage in shareholder activism.

Proposition 6 shows that if period 0 market is completely illiquid ($\nu = 0$), $A$ will choose $\varphi = 0$ and the disciplinary intervention equilibrium will not exist. Higher $\nu$ has a positive impact on period 0 liquidity because it reduces price impact of $A$’s trade and therefore allows $A$ to profit from
trading. Therefore, higher \( \nu \) increases chances that \( A \) will accumulate a toehold sufficient to cover costs of holding \( \varphi \) shares as well as direct costs of activism, \( \eta \). Thus, the model predicts a positive relation between initial period stock liquidity and block formation by activist shareholders.

**Corollary 2.** *Consider period 0 prices and liquidity in the intervention equilibrium. More liquid period 0 markets lead to a higher initial toehold and therefore increase chances of the disciplinary intervention equilibrium.*

Corollaries 1 and 2 suggest that the timing of a liquidity shock plays an important role. Whereas higher chances of period 0 liquidity shocks make stock markets more liquid and increase chances of the disciplinary intervention equilibrium, higher chances of period 1 liquidity shocks make stock markets less liquid and reduce the disciplinary role of the intervention.

### 3.2. Ex ante and ex post correction effects

The model reveals an important feature of the intervention equilibrium: Intervention has ex ante and ex post correction effects. In this section we discuss how variations in stock liquidity parameters affect \( A \)'s and \( M \)'s actions and create a tension between ex ante and ex post correction effects.

Relations between stock liquidity and \( A \)'s and \( M \)'s actions depend on the source of variation in stock liquidity. When the activist's liquidity needs increase (measured by \( \theta \)), \( \delta_I \) decreases and \( M \) is more likely to take bad action. As Figure 4 indicates, higher \( \theta \) leads to lower firm value. In the intervention equilibrium, however, \( A \) always intervenes if \( M \) takes the bad action and there is no liquidity shock. Thus, the overall effect of \( \theta \) on the probability of ex post in intervention can be either positive or negative.
Specifically, the probability of intervention $Pr(b = 1) = (1 - \theta)\Phi_I$ and its derivative with respect to $\theta$ is:

$$\frac{\partial Pr(b = 1)}{\partial \theta} = -\Phi_I + f_I(1 - \theta)(1 - \gamma) \frac{\beta}{\omega_2}. \quad (4)$$

Figure 5 show that the effect of $\theta$ of the probability of intervention can be non-monotonic. When $\theta$ is small, an increase in $\theta$ weakens the ex ante correction effect but strengthens the ex post correction effect. In this case, widening bid-ask spreads induced by higher $\theta$ will be associate with lower firm value (weaker ex ante effect) and more frequent ex post interventions (stronger ex post effects). In contrast, when $\theta$ is large, an increase in $\theta$ weakens both the ex ante and ex post correction effects. Thus, widening bid-ask spreads induced by higher $\theta$ will be associate with higher firm value (stronger ex ante effect) and more frequent ex post interventions (stronger ex post effects).

**Corollary 3.** *Ex ante and ex post correction effects of intervention can be either substitutes or complements.*

Next consider $A$’s effectiveness in restoring the damage. In equilibrium, it does not change *ex-ante* effects of $A$’s presence on corporate governance. In contrast, when $A$ is more effective in restoring the damage, *ex post* interventions will create more value. In this case stock liquidity interacts with *ex-post* effects of $A$’s presence on corporate governance.
Figure 5: **The effect of $\theta$ on the probability of intervention.** The dark line plots the probability of intervention specified in equation (4). $\delta_I, \Lambda_I,$ and $\Phi_I$ are defined in Proposition 1. We assume $\nu=100, \beta=25, \omega_2=2, \gamma=0.3; \kappa=0.3, \varphi=4, \alpha=5, \eta=0.1, f[x] = \lambda \exp(-\lambda x),$ and $\lambda=0.1.$

4. Conclusion

In this paper we have developed a model to study two governance roles played by shareholder interventions: the disciplinary role of intervention and the ex post correction role played by interventions. We have derived predictions as to when one or the other is more likely to be available and more likely to be effective in disciplining the manager.

The model suggests that enhancing the disciplinary role of intervention (i.e., ex ante correction effects) can be more effective than enhancing the ex post correction effects in improving economic efficiency. For instance, the ex ante correction effect reduces the manager’s incentive to damage firm value. In contrast, ex post intervention not only recovers only a fraction of the damage, but also requires the activist to bear the cost of intervention.
The model reveals what economic factors can help achieving this goal. For example, increasing personal cost borne by the manager in case of ex post intervention would enhance the disciplinary role of intervention. Similarly, reducing chances of the liquidity shock that forces the activist to sell her toehold would enhance the disciplinary role of intervention.

The model also reveals that ex ante and ex post correction effects can be either complements or substitutes, depending on economic forces that drive variations in these effects. For instance, if the activist is more effective in punish the manager, these two mechanisms will exhibit substitution: the threat of intervention will be stronger and ex post interventions less frequent. In contrast, if the activist is less likely to experience a liquidity shock that forces him to liquidate her position, ex ante and ex post correction effects can be complements. The ex ante correction effect is stronger because the activist is less likely to be disturbed by the liquidity shock. The ex post correction effect can also stronger because the frequency of ex post intervention can increase following a decrease in the likelihood of liquidity shock.

Finally, because we endogenize the activist’s choice of toehold, we can show that the timing of liquidity trading matters. Whereas liquidity trading during the toehold-acquiring phase generally enhances economic efficiency, liquidity shocks experienced during the activism phase of the blockholder’s activity generally lead to worsening of economic efficiency.
References


Appendix for

“The Threat of Intervention”
Proof of Lemma 1.

$\mathcal{A}$’s profits, as measured by $\pi - \varphi v$, are as follows:

<table>
<thead>
<tr>
<th>Action</th>
<th>$a = 0$ (no damage)</th>
<th>$a = 1$ (damage)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy</td>
<td>$(\alpha - \varphi)\kappa\Lambda{\frac{\Phi \sigma_1^B}{\Phi \sigma_1^B + (1 - \Phi) \sigma_0^B}, 0}$</td>
<td>$-\alpha \kappa \Lambda - \eta + (\alpha - \varphi)\kappa\Lambda{\frac{\Phi \sigma_1^B}{\Phi \sigma_1^B + (1 - \Phi) \sigma_0^B}, 0}$</td>
</tr>
<tr>
<td>Hold</td>
<td>0</td>
<td>$-\varphi \Lambda$</td>
</tr>
<tr>
<td>Sell</td>
<td>$-\varphi \Lambda\frac{\Phi \sigma_1^S + \delta \Phi}{\Phi \sigma_1^S + (1 - \Phi) \sigma_0^S + \delta}$</td>
<td>$-\varphi \Lambda\frac{\Phi \sigma_1^S + \delta \Phi}{\Phi \sigma_1^S + (1 - \Phi) \sigma_0^S + \delta}$</td>
</tr>
</tbody>
</table>

where in the case of each bracketed expression, the second term is to be used in case the denominator of the first term is zero.

Since $\Phi > 0$, $\pi_0^B - \varphi v > \pi_0^S - \varphi v$ implies $\sigma_0^S = 0$. Similarly, $\pi_1^S - \varphi v > \pi_1^H - \varphi v$ implies $\sigma_1^H = 0$ if $\Phi < 1$. A sufficient condition for $\Phi < 1$ is that the support of the distribution include sufficiently high values of $\delta$ such that the manager is uninterested in taking the action. For example, it is sufficient to assume $F(\beta/\omega_2) < 1$.

Proof of Proposition 1.

In the case when $\sigma_1^B > 0$, $\sigma_0^B = 1$, $\sigma_0^H = 0$, $\sigma_0^S = 0$, and $\sigma_1^H = 0$, $\mathcal{A}$’s profits can be simplified as follows:

<table>
<thead>
<tr>
<th>Action</th>
<th>$a = 0$ (no damage)</th>
<th>$a = 1$ (damage)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy</td>
<td>$(\alpha - \varphi)\kappa\Lambda\frac{\Phi \sigma_1^B}{\Phi \sigma_1^B + (1 - \Phi)}$</td>
<td>$-\alpha \kappa \Lambda - \eta + (\alpha - \varphi)\kappa\Lambda\frac{\Phi \sigma_1^B}{\Phi \sigma_1^B + (1 - \Phi)}$</td>
</tr>
<tr>
<td>Hold</td>
<td>0</td>
<td>$-\varphi \Lambda$</td>
</tr>
<tr>
<td>Sell</td>
<td>$-\varphi \Lambda\frac{\Phi \sigma_1^S + \delta \Phi}{\Phi \sigma_1^S + \delta}$</td>
<td>$-\varphi \Lambda\frac{\Phi \sigma_1^S + \delta \Phi}{\Phi \sigma_1^S + \delta}$</td>
</tr>
</tbody>
</table>
Condition 1 follows from comparing $\pi_B^1$ and $\pi_S^1$ when $\sigma_B^1 = 1$.

Given the beliefs, $\mathcal{M}$ expects $A$ to intervene as long as the action is taken and there is no liquidity shock. In equilibrium $\mathcal{M}$ consumes private benefits $\beta(\theta + (1 - \theta)\gamma)$ if $a = 1$. If $\mathcal{M}$ does not take the action, his expected utility is $\omega_2v$. If $\mathcal{M}$ takes the action, his expected utility is $\omega_2(v - \tilde{\delta}) + \beta(\theta + (1 - \theta)\gamma)$. The cutoff point is therefore $\delta_I = \beta/\omega_2(\theta + (1 - \theta)\gamma)$. $\gamma < 1$ implies that the equilibrium is always disciplinary.

**Proof of Proposition 2.**

If $\sigma_B^1 = 0$, $\sigma_S^1 = 1$ and $A$’s profits can be simplified as follows:

\[
\begin{array}{|c|c|c|}
\hline
 & a = 0 \text{ (no damage)} & a = 1 \text{ (damage)} \\
\hline
\text{Buy} & 0 & -\alpha \kappa \Lambda - \eta \\
\text{Hold} & 0 & -\varphi \Lambda \\
\text{Sell} & -\varphi \Lambda \frac{\Phi + \bar{\delta}}{\Phi + \bar{\sigma}} & -\varphi \Lambda \frac{\Phi + \bar{\delta}}{\Phi + \bar{\sigma}} \\
\hline
\end{array}
\]

Condition 2 follows from comparing $\pi_B^1$ and $\pi_S^1$.

If $\mathcal{M}$ does not take the action, his expected utility is $\omega_2v$. If $\mathcal{M}$ takes the action, his expected utility is $\omega_2(v - \tilde{\delta}) + \beta$. The cutoff point is therefore $\delta_E = \beta/\omega_2 = \delta_{BM}$ and the equilibrium is non-disciplinary. Period 1 prices are $p_B = v$ and $p_S = v - \Lambda \Phi \frac{1 + \bar{\delta}}{\Phi + \bar{\rho}}$.

**Proof of Proposition 3.**

We want to construct an equilibrium with $\sigma_B^1 > 0$ and $\sigma_S^1 > 0$. When $\sigma_B^1 > 0$, $\sigma_S^1 = 1$. Consider $G(\sigma_B^1) \equiv \pi_B^1 - \pi_S^1$ when $\sigma_B^1 \in (0, 1)$:

\[
G(\sigma_B^1) = -\alpha \kappa \Lambda - \eta + (\alpha - \varphi) \kappa \Phi \Lambda \frac{\sigma_B^1}{\Phi \sigma_B^1 + (1 - \Phi)} + \varphi \Phi \Lambda \frac{(1 - \sigma_B^1) + \bar{\delta}}{\Phi (1 - \sigma_B^1) + \bar{\delta}}.
\]
For there to be an equilibrium with $\sigma_1^B \in (0,1)$ for a given $\Phi$, it is necessary and sufficient that $G(\sigma_1^B) = 0$ at some $\sigma_1^B > 0$. By conjecture, \[ \lim_{\sigma_1^B \to 0} > 0 \quad \text{and} \quad \lim_{\sigma_1^B \to 1} < 0. \] Since the function is quadratic there is exactly one crossing $\in (0,1)$.

The values of $p_B$ and $p_S$ are immediate from their definitions. Given the beliefs, $\mathcal{M}$ expects $P_1(a = 0) = (1 - \theta)p_B + \theta p_S$ and $P_1(a = 1) = (1-\theta) \left[ \sigma_1^B p_B + (1 - \sigma_1^B) p_S \right] + \theta p_S$. Moreover, in equilibrium $\mathcal{M}$ is expected to consume private benefits $\beta \left[ 1 - (1 - \theta)\sigma_1^B (1 - \gamma) \right]$. Thus, if $\mathcal{M}$ does not take the action, his expected utility is $\omega_2v$. If $\mathcal{M}$ takes the action, his expected utility is $\omega_2 (v - \tilde{\delta}) + \beta \left[ 1 - (1 - \theta)\sigma_1^B (1 - \gamma) \right]$. The cutoff point is therefore $\delta_M = (\beta/\omega_2) \left( 1 - (1 - \theta)\sigma_1^B (1 - \gamma) \right)$.

**Appendix B. Alternative Equilibria**

Multiple equilibria are also possible. One possibility is that both condition (1) and condition (2) are satisfied for their respective values of $\Phi$, in which case we will have one pure strategy equilibrium of each sort. The next proposition provides sufficient conditions for this to occur.

**Lemma 2. If the following conditions are satisfied:**

\[ \alpha > \varphi > \frac{1}{(1 - \kappa)\Phi E} (\alpha \kappa (1 - \Phi_E) + \frac{\eta}{\Lambda_E}) \]

\[ \varphi < (\alpha \kappa + \frac{\eta}{\Lambda_E}) \]

**then there exists $\theta^* < 1$ such that for all $\theta$ in the open interval $(\theta^*, 1)$ both an intervention equilibrium and a non-disciplinary equilibrium exist.**
Proof. As $\theta$ approaches 1, $\Phi_I$ and $\Lambda_I$ approach $\Phi_E$ and $\Lambda_E$ and so $\varphi_I$ and $\varphi_E$ in conditions (1) and condition (2) continuously approach the right sides of the two inequalities above. ■

This leads immediately to the following proposition:

**Proposition 5.** Provided

$$\min\{\alpha, \alpha \kappa + \frac{\eta}{\Lambda_E}\} > \frac{1}{(1 - \kappa)\Phi_E}(\alpha \kappa (1 - \Phi_E) + \frac{\eta}{\Lambda_E})$$

there is an open interval of values of $\varphi$ such that for large enough values of $\theta$ both an intervention equilibrium and a non-disciplinary equilibrium exist.

It is not difficult to find parameter values for $(\alpha, \kappa, \eta, \beta$ and $\omega_2)$ such that this condition holds. For example, as $\frac{\beta}{\omega_2}$ grows $\Phi_E$ approaches 1, $\Lambda_E$ approaches $E[\delta]$, the unconditional expectation of $\delta$, and the condition simplifies to

$$(1 - \kappa)\alpha E[\delta] > \eta.$$

**Appendix C. Section 2.2: $\varphi_E < \varphi_I$ Case**

In this section we consider the case when $\varphi_E < \varphi_I$ and mixed-strategy equilibrium is possible.

We begin by describing the relation between $\mathcal{A}$’s toehold size and firm value. If $\mathcal{A}$ is not present, the value of the firm is $p_{BM} = v - \Lambda_{BM}\Phi_{BM}$. If $\mathcal{A}$ is present and purchases $\varphi < \varphi_E$, Proposition 2 applies. $\mathcal{A}$ always sells shares if bad action is taken. Since the equilibrium is non-disciplinary and there is no ex post intervention, the value of the firm is still $p_{BM}$. If
A is present and purchases $\varphi \in (\varphi_E, \varphi_I)$ which is sufficient to maintain the mixed-strategy equilibrium, Proposition 3 applies. A intervenes with positive probability if bad action is taken. In equilibrium, firm value is higher than in the benchmark case, $p_M > p_{BM}$. If $A$ is present and purchases $\varphi \geq \varphi_I$ which is sufficient to maintain the intervention equilibrium, Proposition 1 applies. The value of the firm is $p_I = (1 - \theta)p_B + \theta p_S$, where $p_B = v - \kappa \Lambda_I \Phi_I$ and $p_S = v - \Lambda_I \Phi_I$. Note that it follows from Propositions 1 and 3 that the mixed strategy equilibrium is less disciplinary than the intervention equilibrium. Thus, $p_I > p_M > p_{BM}$, implying that period 0 firm value is weakly increase in A’s toehold size.

Second, we describe disclosure requirements. After the initial trade in period 1 occurs, A’s toehold $\varphi$ becomes common knowledge. This assumption is motivated by the fact that market participants can use Schedule 13F filings to infer changes in stock ownership. If $A$ purchases a toehold smaller than $\alpha$, no additional disclosure of the position or trade is necessary until period 0. In this case, $A$ can potentially benefit from hiding behind liquidity trades, which we introduce in the next paragraph. To capture the role of ownership disclosure requirements that are linked to A’s position size—The Hartâ€”Scottâ€”Rodino Act disclosure requirement (HSR)—we assume that if $A$ purchases toehold larger than $\alpha$, market participants become immediately aware of trader’s identity and intention and set prices equal to $p_I^1$. As we’re going to see, this assumption implies that $A$ will not purchase more than $\alpha$ shares in period 0.

Next, we characterize the source of liquidity trading during toehold-accumulation period. Participants in the market expect that with probability
\( \nu \) the purchase order comes from an uniformed agent (either from a non-activist shareholder who cannot change firm value or from an uninformed activist), and with probability \((1 - \nu)\) the purchase order comes from an informed \( \mathcal{A} \). Similarly to the role of \( \theta \) in period 1 markets, higher values \( \nu \) correspond to a more liquid period 0 market; \( \mathcal{A} \)'s purchase can be hidden more effectively in the sea of non-monitor purchases.

The actual price on the market in period 0, denoted \( p_0(\varphi) \), depends on the size of the purchase order and reflects the market’s expectation that \( \mathcal{A} \) is participating as well as the activism role played by \( \mathcal{A} \) if he is present. If market makers receive an order to purchase \( \varphi < \varphi_E \) shares, they set period 0 price to \( p_0(\varphi < \varphi_E) = p_{BM} \). If market makers receive an order to purchase \( \varphi \geq \alpha \) shares, they set period 0 price to \( p_0(\varphi \geq \alpha) = p_I \) (i.e., the HSR disclosure rule is binding). In this two case prices are fully revealing and \( \mathcal{A} \) cannot profit from trading.

If market makers receive an order to purchase \( \varphi_E \leq \varphi < \varphi_I \) shares, they set period 0 price to \( p_0(\varphi_E \leq \varphi < \varphi_I) = (1 - \nu)p_M + \nu p_{BM} \). If market makers receive an order to purchase \( \varphi_I \leq \varphi < \alpha \) shares, they set period 0 price to \( p_0(\varphi_I \leq \varphi < \alpha) = (1 - \nu)p_I + \nu p_{BM} \). In these two cases more liquid markets (i.e., higher \( \nu \)) lead to higher trading profits of \( \mathcal{A} \).

In period 0 \( \mathcal{A} \) takes the market price function \( p_0(\varphi) \) as given and decides how many shares \( \varphi \) to buy. Holding \( \varphi \) shares between periods 0 and 1 involves private cost \( C(\varphi) = \phi \frac{\varphi^2}{2} \) for \( \mathcal{A} \). For example, this cost could correspond to lower diversification of \( \mathcal{A} \)'s portfolio or binding capital constraints faced by \( \mathcal{A} \). This assumption implies that keeping everything else constant, \( \mathcal{A} \) prefers
to purchase shares later rather than earlier.\textsuperscript{15}

Let $\pi(\varphi; p) = \varphi \nu (p - p_{BM}) - \phi \frac{\varphi^2}{2}$ be $A$’s expected profit given period 0 price $p$. $A$ maximizes expected profits from purchasing $\varphi$ shares:

$$\max_{\varphi_E \leq \varphi < \alpha} \pi(\varphi) = \begin{cases} 
\pi(\varphi; p_I), & \text{if } \varphi_E \leq \varphi < \varphi_I, \\
\pi(\varphi; p_M), & \text{if } \varphi_I \leq \varphi < \alpha.
\end{cases} \quad (C.1)$$

Note that when $\varphi < \varphi_E$ and when $\varphi \geq \alpha$, prices are fully revealing and $A$’s expected profit is $-\phi \frac{\varphi^2}{2} < 0$. Consequently, $A$ will prefer $\varphi = 0$ to purchasing fewer than $\varphi_E$ shares or more than $\alpha$ shares. Note that as long as $\nu = 0$, $A$ prefers $\varphi = 0$. In other words, if $A$ needs to purchase $\varphi$ shares in the open market at a price that reflects $A$’s impact of firm value, privately-optimal initial stake size will be zero in the absence of liquidity trading. When $\nu > 0$, then, provided $p_I(\varphi) \neq p_{BM}$, $A$ profits from liquidity trading because prices do not fully reflect $A$’s impact of firm value. The following propositions characterize $A$’s optimal $\varphi$.

**Proposition 6.** Let $\varphi_I^A = \frac{\nu(p_I - p_{BM})}{\varphi}$ be the value of $\varphi$ that maximizes $\pi(\varphi; p_I)$ and $\varphi_M^A = \frac{\nu(p_M - p_{BM})}{\varphi}$ be the value of $\varphi$ that maximizes $\pi(\varphi; p_M)$. Further, let $\varphi^*_0$ be such that $\pi(\varphi^*_0; p_I) = 0$, $\varphi^*_M$ be such that $\pi(\varphi^*_M; p_M) = 0$, and $\overline{\varphi}_I$ such that $\pi(\overline{\varphi}_I; p_I) = \pi(\varphi_M^A; p_M)$.

1. If $\varphi_I \leq \overline{\varphi}_I$, $A$ will choose $\varphi = \max(\varphi_I^A, \varphi_I)$. In this case condition (1) holds and the market is in disciplinary intervention equilibrium.

2. If $\varphi_I \in (\overline{\varphi}_I, \varphi^*_0)$, $A$ will choose either $\varphi = \varphi_I$ or $\varphi = \max(\varphi_M^A, \varphi_M)$:

\textsuperscript{15}The analysis can be extended to consider the possibility that higher $\varphi$ increases the likelihood that $A$ faces a liquidity shock in period 1.
\textit{ii(a) If }\pi(\varphi_I; p_I) \geq \pi(\max(\varphi^M_A, \varphi_M); p_M)\text{, }A \text{ will choose } \varphi = \varphi_I. \text{ Condition (1) holds and the market is in disciplinary intervention equilibrium.}

\textit{ii(b) If }\pi(\varphi_I; p_I) < \pi(\max(\varphi^M_A, \varphi_M); p_M)\text{, }A \text{ will choose } \varphi = \max(\varphi^M_A, \varphi_M). \text{ Conditions (1) and (2) are violated and the market is in disciplinary mixed-strategy equilibrium.}

\textit{iii. If }\varphi_I > \varphi^I_0 \text{ and } \max(\varphi^M_A, \varphi_M) \leq \varphi^M_0\text{, }A \text{ will choose } \varphi = \max(\varphi^M_A, \varphi_M). \text{ In this case conditions (1) and (2) are violated and the market is in disciplinary mixed-strategy equilibrium.}

\textit{iv. If }\varphi_I > \varphi^I_0 \text{ and } \varphi_M > \varphi^M_0\text{, }A \text{ will choose } \varphi = 0.

The intuition behind Proposition 6 is presented in Figure A1. When \(\varphi_I < \varphi^I_A\), A finds it optimal to choose \(\varphi = \varphi^I_A\). This case corresponds to the point B on the Figure. When \(\varphi_I \in (\varphi^I_A, \varphi^I_T)\), \(\varphi^I_A\) is not large enough to satisfy condition (1) and therefore maintain the intervention equilibrium and the corresponding price level, \(p_I\). In this case, A finds it optimal to choose \(\varphi = \varphi_I\) such that condition (1) holds. This case corresponds to a point on the B-D segment.

If \(\varphi_I \in (\varphi_T, \varphi^I_0)\), A will choose either \(\varphi = \varphi_I\) and will on the D-F segment or \(\varphi = \max(\varphi^M_A, \varphi_M)\) and will be on the C-G segment. If \(\varphi_I > \varphi^I_0\) and \(\max(\varphi^M_A, \varphi_M) \leq \varphi^M_0\), A will choose \(\varphi = \max(\varphi^M_A, \varphi_M)\). This case corresponds to a point on the C-G segment. Finally, when \(\varphi_I > \varphi^I_0\) and \(\varphi_M > \varphi^M_0\), A will choose \(\varphi = 0\).
Figure A1: $\mathcal{A}$'s choice of $\varphi$. 

If $\varphi_i < \varphi_i^L$, A will choose $\varphi_i^L$. 
If $\varphi_i \in (\varphi_i^L, \varphi_i^U)$, A will choose $\varphi_i$. 
If $\varphi_i \in (\varphi_i^U, \varphi_M)$, A will choose either $\varphi_i$ or $\max(\varphi_i^L, \varphi_M)$. 
If $\varphi_i > \varphi_M$, A will choose either $\max(\varphi_i^L, \varphi_M)$ or $\varphi = 0$. 

$\pi(\varphi; p)$