Term Structures of Credit Spreads with Dynamic Debt Issuance and Incomplete Information

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Abstract

We investigate credit spreads and capital structure dynamics in a model in which management has private information regarding firm value and is able to issue both equity and debt to service existing debt. Rather than choosing to default, managers of investment-grade (IG) firms who receive bad private signals conceal this information by servicing existing debt via new debt issuance. As such, firms with IG-commensurate spreads have zero jump-to-default risk (and hence, command zero jump-to-default premium), at least until their debt capacity is fully utilized and spreads have increased to “fallen angel” status. These predictions match observation well.

JEL Classification Codes: G12; G32; G33
Keywords: Credit spreads; Capital structure; Corporate Default; Jumps to Defaults
1 Introduction

Most empirical studies of corporate bond yields report evidence of a “credit spread puzzle” in that it is difficult to explain observed spreads between corporate bond yields and Treasury yields in terms of expected losses and standard measures of risk. This credit spread puzzle is most striking for short-maturity investment-grade (IG) debt, since historical default rates for these bonds are extremely low. Several explanations for the credit spread puzzle have been suggested in the literature, including: (i) illiquidity premia, (ii) tax asymmetry (i.e., corporate bonds, but not Treasuries, are subject to state taxation) and (iii) jump-to-default (or credit-event) premia. This paper argues both theoretically and empirically that portfolio strategies which hold only investment-grade debt (that is, strategies that sell bonds once their spreads have increased to “fallen-angel” status) are subject to negligible jump-to-default risk, and hence command negligible jump-to-default premia. As such, short-maturity IG spreads must be mostly due to other channels.

The notion of jump-to-default (or credit-event) risk arises naturally in reduced-form models of default (e.g., Duffie and Singleton (1997); Jarrow, Lando, and Turnbull (1997)), in which default is modeled as an unpredictable jump event. If a sufficiently large premium is attributed to jump-to-default risk, then short-maturity spreads can be “explained” through this channel. In their seminal paper, Duffie and Lando (2001) (DL) provide an economic justification for reduced-form models. In particular, they investigate the optimal behavior of a manager of a firm that can issue only equity to service debt in place. They show that if this manager receives a sufficiently bad private signal, then it will be in the best interest of shareholders for the manager to declare default, rather than have them continue to service debt payments. From an outsider’s

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1 Another explanation is provided by Feldhüter and Schaefer (2017), who argue that historical default rates provide a very noisy estimate for ex-ante expected default rates, leaving the possibility that ex-ante default expectations are much higher than ex-post observations. However, they acknowledge that even their explanation cannot explain short-maturity (less than one year) IG spreads.
information set, such a default will appear as an unexpected jump-to-default, which can be characterized by a default intensity process similar to those specified by reduced form models.

In this paper, we build on the insights of Duffie and Lando (2001) by investigating an economy in which a manager with private information can issue new debt to service existing debt, at least until the firm exhausts its debt capacity. In contrast to DL, our framework predicts that IG firms will never jump to default due to a bad private signal, because the manager of an IG firm will maximize shareholders’ value by concealing this bad signal, and issuing new debt to service debt in place. Intuitively, since equity holders receive zero payoff in case of bankruptcy, it is in their best interest to maintain solvency so long as they do not need to invest more funds into the firm. In the absence of jump-to-default risk, there is no compensation for such risk, and short-term spreads cannot therefore be explained by a jump-to-default channel. Only after debt capacity has been used up, and the firm has reached “fallen-angel” status, is a jump to default possible in our framework.

To illustrate the implications of our model, we estimate empirical default rates as a function of both the firm’s spread and its rating. In contrast to DL, which predicts a relatively flat term structure of default probabilities for horizons up to one year, our model better matches empirical observation in that the vast majority of IG firms that default within one year do so near the end of the year – that is, these firms first tend to diffuse toward “fallen angel” status prior to defaulting. As such, a portfolio consisting only of bonds with spreads that are commensurate with IG status is subject to virtually zero default risk and, therefore, should not command a significant jump-to-default premium. Only after a firm drops to fallen angel status does jump-to-default become a possibility through the mechanism proposed by DL. This prediction is consistent with the empirical findings of Davydenko, Strebulaev, and Zhao (2013), who report that the default event of speculative-grade debt is associated with significant losses in firm value, consistent with the notion that the default event was indeed a surprise based on public information.\(^2\)

Our paper contributes to two strands of literature. The first is the extensive body of

\(^2\)Clark and Weinstein (1983), Lang and Stulz (1992) and Warner (1977) also report significant loss of equity and debt value at the time of bankruptcy announcement.
work that studies structural models of capital structure choice and leverage dynamics. Unlike most papers in this literature, we allow for informational asymmetry between the manager and creditors. In our structural model, creditors are aware of the manager’s information advantage, and price it rationally into the firms’ claims.

The second strand of literature to which we contribute investigates credit spreads in reduced-form frameworks. In particular, we build on the body of work that focuses on decomposing credit spreads into components of expected loss, risk premia, liquidity premia and taxes. In this literature, there is considerable disagreement on the magnitude of the jump-to-default premium. For example, in his benchmark model, Driessen (2005) associates 31 bps with this channel. Related, he estimates the ratio of risk-neutral to actual default intensity to be $\lambda^Q/\lambda^P = 2.3$. However, as noted in Bai, Collin-Dufresne, Goldstein, and Helwege (2015), these authors do not estimate jump-to-default premia by assessing how proxies for their pricing kernel covary with bond returns, but rather attribute whatever they cannot explain through other channels to this credit risk channel. Moreover, Bai, Collin-Dufresne, Goldstein, and Helwege (2015) argue that, in a general equilibrium model with contagion risk, the ratio $\lambda^Q/\lambda^P$ has an upper bound of approximately 1.1, hence spreads due to this channel cannot be much above expected losses. In contrast, here we argue that $\lambda^P$ is itself very small for IG firms; in particular, historical one-year default rates significantly exaggerate the jump-to-default risk faced by an investment strategy that holds only corporate bonds with IG-level spreads.

The combination of dynamic debt issuance and the presence of information asymmetry between the manager and creditors provides a framework that nests most of the models studied by the extant literature. We exploit this generality to compare the implications of our model for optimal capital structure choice, credit spreads, and default

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3 An incomplete list of contribution to this literature includes Merton (1974); Leland (1994); Goldstein, Ju, and Leland (2001); Hennessy and Whited (2007); Abel (2016, 2017); DeMarzo and He (2017); Admati, DeMarzo, Hellwig, and Pfeiderer (2017). Our work is also related to the previous literature that models firms’ earnings, or assets, with complete information via jump-diffusion processes (e.g., Zhou (2001); Gorbenko and Strebulaev (2010)). These specifications, however, do not give rise to a stochastic intensity for default unless the only variation in asset levels is through jumps.


5 Similar results are reported by Saita (2006) and Berndt, Douglas, Duffie, Ferguson, and Schranz (2009).

6 Our paper is also related to the vast literature that studies voluntary disclosure of managerial information (e.g., Shin (2003)), and the roll-over of short-maturity debt, market runs, and market freezes (e.g., Diamond and Dybvig (1983), Acharya, Gale, and Yorulmazer (2011), He and Xiong (2012), Schroth, Suarez, and Taylor (2014), Dang, Gorton, and Holmström (2015), and Carré (2016)).
frequencies with this literature. Specifically, by restricting the manager to issue only equity after date-0, our setting reverts back to Duffie and Lando (2001). If we assume that manager and creditors are equally informed, we obtain a model of optimal capital structure dynamics with complete information (e.g., Goldstein, Ju, and Leland (2001); Hennessy and Whited (2007); DeMarzo and He (2017)). By removing both the ability to issue debt and the presence of informational asymmetry, we recover a version of the Leland (1994) model.

Our analysis is most relevant for IG spreads of short-maturity debt where the “credit spread puzzle” is most prevalent. Indeed, a growing literature (e.g., Chen, Collin-Dufresne, and Goldstein (2009), Bhamra, Kuehn, and Streblulaev (2010) and Chen (2010)) argues that IG spreads for maturities greater than a few years can be explained by combining pricing kernels that capture time varying Sharpe ratios over the business cycle with models that match the empirically observed clustering of defaults during recessions. In contrast, we show that incomplete information combined with debt issuance lowers short-term credit spreads of IG firms. Hence, our findings “deepen” the credit spread puzzle for IG firms at short maturities.

The rest of the paper is organized as follows. In Section 2, we present stylized facts on investment grade companies that motivate our analysis. Section 3 builds a model of corporate debt issuance and default decisions in the presence of asymmetric information between the manager and creditors. In Section 4 we present the implications of our model for optimal capital structure decisions and derive model-implied credit spreads and default rates. Section 5 concludes. Proofs are in Appendix A, while further details concerning the empirical analysis and the numerical solution of the model are in an Online Internet Appendix.

2 Stylized facts

Here we discuss a few stylized facts about the corporate bond market that provide empirical foundations for our research.

Fact 1: IG companies dominate the bond market. Figure 1 tracks the percentage of investment-grade (IG) firms over time and compares it to the proportions of higher-
and lower-quality speculative grade companies (labeled B and C).\footnote{Appendix A.1 explains how we classify firms in the three categories, IG, B, and C.} Investment-grade companies are the majority among the firms with bonds monitored by credit rating agencies. The proportion is highest in the early part of the sample period, and fluctuates around 50% starting from the early 1990s. Higher-quality speculative grade companies make up the second largest group, while C-rated companies comprise less than 10% of the market.

Related, a large fraction of IG firms are net issuers of corporate debt. For instance, using the Mergent Fixed Income Security Database (FISD), Greenwood and Hanson (2013) document that on average 68% of all debt issuance in the period 1983–2008 are originated by IG firms. Using Moody’s Bond Surveys, they show that 89% of all debt issuance in the 1926–1982 period originates from IG firms.

**Fact 2: Firms with IG status rarely default.** Table 1 shows average annualized default rates for companies in the IG, B, and C groups. Panels A and B concentrate on firms classified based on credit ratings issued by rating agencies over the periods 1985-2014 and 2001-2014. It is evident that defaults by IG firms are extremely rare. Panel A shows that on average only 0.11% of IG companies file for bankruptcy within a year of being assigned an IG classification. Defaults over the first month are less frequent and are limited to 0.06% of the IG group.

The likelihood of a default for an IG firm is even lower when we use market-based information to rate companies. Agencies do not continuously update their ratings to fully reflect the information available to market participants. Hence, we consider an alternative classification of firms into the same three creditworthiness groups that is based on CDS data (Appendix A.2). At horizons from one to three months, we find IG default rates that are virtually zero, with 0.01% point estimates that are statistically insignificant (Table 1, Panel C).

Overall, this evidence shows that defaults by investment grade firms do not come as a total surprise to market participants. Even for higher-quality speculative grade firms there is limited support for jumps to default. It is mostly the lowest-rated firms that file for bankruptcy unexpectedly to market participants. At the one-month horizon, the average default rate for C companies can exceed 15% (Table 1, Panel A); beyond the first month, default rates for firms in the C group decline progressively but remain
elevated.\textsuperscript{8}

Even if we restrict our attention to firms that held IG status for at least one of the 12 months preceding the default event, we find that the great majority of these companies exhibit a considerable run-up in credit spreads for many months before they default. This provides investors with a signal that the credit worthiness of such companies has deteriorated below IG well before their bankruptcy. Hence, default does not come fully unexpected for these firms. Figure 2 shows the difference between the average CDS premium on those firms and the CDX-IG index. This spread is very small 12 months prior to bankruptcy and then increases in the ensuing months as the firms drop out of the IG group and approach bankruptcy. This evidence suggests that an investment policy that (1) holds bonds issued by firms in the IG category, and (2) unwinds these positions when the firm loses IG status faces virtually zero default risk. Hence, the jump-to-default premium for this portfolio should be negligible.\textsuperscript{9}

3 Model

To explain the stylized facts of the previous section, we develop a model of corporate debt issuance and default decisions in the presence of asymmetric information. As in Duffie and Lando (2001) (DL hereafter) we assume that firms’ creditors have less information than does management. However, two main features distinguish our model from DL. First, we allow firms to issue both debt and equity, a feature motivated by the fact that a large fraction of IG firms are net issuers of corporate debt. Second, we model asymmetric information by assuming that creditors can continuously observe the value of the firm’s assets with a delay. This feature reflects the fact that it takes time for market participants to acquire the accounting information needed to accurately value a firm’s assets.

\textsuperscript{8}It is likely that CDS trading declines considerably when the option component of the contract is deep in the money. In contrast, credit agencies are likely to update their ratings more frequently when conditions for a company deteriorate. Hence, since CDSs are less traded when default risk is high, it is not surprising that empirical default rates for companies assigned a C label based on CDS-implied rating underestimates the default rate obtained when companies are classified based on credit ratings (Table 1, Panel C vs. Panels A and B).

\textsuperscript{9}Note that jump-to-default-risk is conceptually different from the risk that jump intensities increase in reduced-form models.
3.1 Setup

The unlevered firm value $V_t$ follows a geometric Brownian motion under the risk neutral measure $Q$, that is,

$$\frac{dV_t}{V_t} = r \, dt + \sigma \, dB^Q_t,$$

where $r$ and $\sigma$ denote the constant risk-free rate and asset volatility, and $dB^Q_t$ denotes the increment of a standard Brownian motion under $Q$. Defining $v_t \equiv \ln V_t$ and using Itô's lemma, we have

$$dv_t = m \, dt + \sigma \, dB^Q_t, \quad \text{where} \quad m \equiv r - \frac{\sigma^2}{2}.$$  

(2)

Insiders can observe the asset value $V_t$ in real time while creditors can only observe $V_t$ with a time lag $L$. That is, at time $t$, creditors know $V_t-L$. We define the lagged asset value observed by creditor as

$$\hat{V}_t \equiv V_{t-L} \quad \text{and} \quad \hat{v}_t \equiv v_{t-L}.$$  

(3)

Any default is observed immediately by both insiders and outside creditors.

After choosing an initial financing mix of debt and equity at time zero, we assume that firms can issue debt until the time in which they exhaust their “debt issuing capacity” at a future time $t^*$. After this time, the firm is forced to either issue equity to service the debt-in-place, or choose to default. Figure 3 provides an illustration of the model timeline. There are four relevant time regimes:

1. **Regime 1**: $0 \leq t < t^*$. In this regime, firms have the capacity to raise new debt to service debt-in-place. Debt issued at time $t$ promises a perpetual and constant coupon payment, $C_t$, until the firm declares bankruptcy. Because equity holders do not need to infuse money into the firm, there is no reason for them to choose to default during this time regime. Therefore, this interval is characterized by a zero default intensity (and hence, command zero default-risk premia). We refer to firms in this regime as “Investment-Grade” firms.

2. **Regime 2**: $t = t^*$. At this point in time, the firm reaches debt capacity. Because the firm can no longer issue debt, if its asset value (which is known only by the manager) falls below the default threshold (the value of which is publicly
known), then the firm immediately defaults. Rather than a default intensity, this regime/instant of time is associated with a finite probability of default.

3. Regime 3: \( t^* \leq t < t^* + L \). Firms can no longer issue debt and may default. Absent default, creditors infer that the firm’s assets must have been above a certain threshold over the time interval \([t^*, t]\) of length less than \( L \). As we show below, in this regime, the default intensity depends on both lagged asset value \( \hat{v}_t \) and the time since the firm reached debt capacity, \( t - t^* \).

4. Regime 4: \( t \geq t^* + L \). Firms can no longer issue debt and may default. Absent default, creditors infer that the firm’s assets must have been above a certain threshold over the time interval \([t - L, t]\) of length \( L \). As we show below, in this regime, the default intensity depends only on lagged asset value \( \hat{v}_t \), independent of time.

To solve for the value of debt and equity under the information set of creditors, we first need to derive the optimal default boundary chosen by management, and then solve for the value of firm’s securities backward in time, starting from Regime 4.

3.2 Shareholders’ optimal default policy

Recall that, in our framework, it is in the shareholder’s best interest for management to avoid default prior to the firm’s debt capacity being exhausted (i.e., \( t < t^* \)). Hence, to determine the optimal default policy, we consider equity valuation at times after debt capacity is reached (i.e., \( t > t^* \)). In this regime, shareholders are committed to pay a continuous coupon \( C \) to outstanding debt-holders. We assume that coupon payments are tax deductible as long as the firm is solvent and that the tax rate is \( \theta \). The actual coupon level \( C \) will be endogenously determined as a function of the firm’s debt capacity, as illustrated below.

As shown in Leland (1994), in this setting there exists a constant default threshold \( V_B \) such that it is in the shareholders’ bests interests that management default on the debt and declare bankruptcy the first time asset value falls below \( V_B \). Define \( S(V_t) \) as the value of equity when the outstanding debt consists of a perpetual continuous coupon \( C \). To determine the default boundary \( V_B \), note that for asset values \( V_t \) above this default
boundary, the equity claim satisfies:

\[ S(V_t) = -(1 - \theta)C dt + e^{-rt} \mathbb{E}_t^Q [S(V_{t+dt})]. \]  

(4)

Intuitively, shareholders must pay out-of-pocket the after-tax coupon payment \((1 - \theta)C dt\), and own the claim to \(S(V_{t+dt})\). Applying Itô’s lemma to (4), we obtain that the equity value must satisfies the following ordinary differential equation

\[ 0 = -(1 - \theta)C - rS + rV_S + \frac{\sigma^2}{2} V^2_S, \]  

(5)

subject to the boundary conditions

\[ \lim_{V_t \to \infty} S(V_t) = V_t - \frac{(1 - \theta)C}{r}, \]  

(6)

\[ \lim_{V_t \to V_B} S(V_t) = 0, \]  

(7)

\[ \frac{\partial}{\partial V_t} S(V_t) \bigg|_{V_t = V_B} = 0. \]  

(8)

Condition (6) implies that, as asset value goes to infinity, the equity value converges to the default-free value. Condition (7) imposes that, at the default boundary, the equity value goes to zero. Condition (8) is the smooth pasting condition that guarantees the default decision is optimal for shareholders.

The solution of (5) subject to the boundary conditions (6)–(8) is given by

\[ S(V_t) = V_t - \frac{(1 - \theta)C}{r} - \left( \frac{V_t}{V_B} \right)^{-\frac{\sigma^2}{2r}} \left[ V_B - \frac{(1 - \theta)C}{r} \right], \]  

(9)

where

\[ V_B = \frac{C}{\beta}, \]  

with \( \beta \equiv \frac{2(1 - \theta)}{2r + \sigma^2}. \)  

(10)

Because all the parameters are common knowledge to all claim-holders, the default threshold (10) is known to both creditors and shareholders. An important scaling feature that we will use below is that the optimal default boundary \(V_B\) is linear in the size of the coupon \(C\).
3.3 Debt valuation

To determine the value of debt, we proceed by assuming that the default boundary (10) corresponds to a generic value of the cumulative coupon $C$. For convenience, we let $v_B \equiv \ln(V_B)$ and define the processes $y_t$ and $\hat{y}_t$ as:

$$y_t \equiv v_t - v_B, \quad \text{and} \quad \hat{y}_t \equiv y_{t-L} = \hat{v}_t - v_B.$$  

Debt-holders know the location of the default boundary $v_B$, and that equity holders will default the first time $\tau_d > t^*$ in which $v_t$ falls below $v_B$. Formally

$$\tau_d = \inf\{t > t^* : v_t \leq v_b\}.$$  

We label this default time $\tau_d$. In the event of default, debt-holders receive a fraction $(1 - \alpha)$ of the firm’s assets.

Let us denote by $D_t \equiv D(\hat{y}_t, t, C_t, 1_{\{\tau_d > t\}})$ the market value of firm’s debt as assessed by creditors. Debt value for a firm that is not in default at time $t$ depends on the lagged asset value $\hat{y}_t$, the time $t$, and the cumulative coupon level $C_t$. In general, we can express the value $D_t$ recursively as the sum of the coupon payment flow and the discounted expected value of future debt, that is

$$D_t = \begin{cases} 
C_t \ dt + e^{-r dt} \mathbb{E}_t^Q \left[ \left( \frac{C_t}{C_{t+dt}} \right) D_{t+dt} \bigg| \hat{y}_t \right] & \text{if } t < t^* \quad (13a) \\
\mathbb{E}_t^Q \left[ D_{t^*} 1_{\{y_{t^*} > 0\}} + (1 - \alpha)e^{v_B + y_{t^*}} 1_{\{y_{t^*} < 0\}} \bigg| \hat{y}_t \right] & \text{if } t = t^* \quad (13b) \\
C_{t^*} \ dt + e^{-r dt} \mathbb{E}_t^Q \left[ D_{t+dt} + (1 - \alpha)e^{v_B} 1_{\{\tau_d \in (t,t+dt]\}} \bigg| \hat{y}_t \right] & \text{if } t \geq t^* \quad (13c)
\end{cases}$$

where the $1_{\{\tau_d > t\}}$ indicates that default has not occurred by time $t$. Expression (13a) shows that, because the firm issues debt before time $t^*$, existing debt-holders claims are diluted. The quantity $\left( \frac{C_t}{C_{t+dt}} \right) D_{t+dt}$ represents the time-$(t + dt)$ debt value accruing to time-$t$ debt-holders. Note that there is no default before $t^*$. Expression (13b) defines the debt value at time $t^*$, just before the firm reaches debt capacity. Such a value is simply the sum of expected value of debt at time $t^*$, if the firm survives, or the recovery value, otherwise. Expression (13c) differs from (13a) along two dimensions: (i) the firm is no longer issuing debt, and hence existing debt-holders are not diluted; (ii) the firm defaults if $v_t$ reaches $v_B$ within the next $dt$ interval, in which case debt-holders receive the recovery value $(1 - \alpha)e^{v_B}$. 


To solve for the bond price, we work backwards in time, starting with the time interval \( t \geq t^* \). We further break this regime into two separate sub intervals, depending on whether \( t \in [t^*, t^* + L) \) or \( t \in [t^* + L, \infty) \). In the next subsection we formally characterize the solution in each time regime.

### 3.3.1 Debt value in Regime 4, \( t \in [t^* + L, \infty) \)

If default has not yet occurred by time \( t \), debt-holders, aware that the firm has exhausted its debt capacity at time \( t^* \), infer that the value of the firm’s assets in the interval \((t^*, t)\) must have been above the threshold level \( v_B \), that is,

\[
\min_{s \in [t^*, t]} \{ v_s \} > v_B. \tag{14}
\]

Because at time \( t \) debt-holders know the lagged asset value \( \hat{v}_t = v_{t-L} \), and the unlevered firm value process is Markov, it follows that the information inferred from (14) over the interval \((t^*, (t-L))\) is redundant. Hence, the debt-holders’ information set in this regime is

\[
\mathcal{F}_t = \left\{ \hat{v}_t = v_{t-L}; \min_{s \in [t-L,t]} \{ v_s \} > v_B \right\}, \quad t \geq t^* + L. \tag{15}
\]

The following proposition shows that the information set (15) generates a “time invariant” setting in which the price of debt and the default intensity depend only of \( \hat{v}_t = v_{t-L} \), or equivalently, \( \hat{y}_t = \hat{v}_t - v_B \).

**Proposition 1** In the time interval \( t \geq t^* + L \), the price of debt with outstanding cumulative coupon \( C \) is time invariant, \( D_t = D_4(\hat{y}_t, C_t, 1_{\{\tau_d > t\}}) \), and given by the solution of the ODE

\[
0 = C - (r + \lambda d(\hat{y}_t)) D_4 + m \frac{\partial D_4}{\partial \hat{y}} + \frac{\sigma^2}{2} \frac{\partial^2 D_4}{\partial \hat{y}^2} + (1 - \alpha) e^{r_B} \lambda d(\hat{y}_t), \tag{16}
\]

subject to the boundary conditions

\[
\lim_{\hat{y}_t \to \infty} D_4(\hat{y}_t) = \frac{C}{r}; \tag{17}
\]

\[
\lim_{\hat{y}_t \to 0} \frac{\partial}{\partial \hat{y}_t} D_4(\hat{y}_t) = 0. \tag{18}
\]
In equation (16), $\lambda_{4,d}(\hat{y}_t)$ is the default intensity associated with equity-holders choosing to default and is given by

$$
\lambda_{4,d}(\hat{y}_t) = \frac{\sigma^2}{2} \frac{\partial}{\partial \hat{y}_t} \pi_4(y_t | \tau_d > t, \hat{y}_t) \bigg|_{\hat{y}_t = 0} = \frac{\hat{y}_t}{\sqrt{2\pi\sigma^2 L^3}} \left( \frac{e^{-\frac{(\hat{y}_t + mL)^2}{2\sigma^2 L^2}}}{\pi_4(\tau_d > t | \hat{y}_t)} \right) \mathbf{1}_{\{\hat{y}_t > 0\}},
$$

(19)

where the probability of firm survival at time $t$, conditional on bondholders lagged information $\hat{y}_t$, is given by

$$
\pi_4(\tau_d > t | \hat{y}_t) \equiv \pi(\min_{s \in [t-L,t]} \{v_s\} > v_B | \hat{y}_t) = \left[ N\left(\frac{\hat{y}_t + mL}{\sqrt{\sigma^2 L}}\right) - e^{-\frac{2\hat{y}_t mL}{\sigma^2}} N\left(\frac{-\hat{y}_t + mL}{\sqrt{\sigma^2 L}}\right) \right] \mathbf{1}_{\{\hat{y}_t > 0\}},
$$

(20)

with $N(\cdot)$ denoting the cumulative standard normal distribution.

Intuitively, the boundary condition (17) states that as asset value becomes extremely large, the bond can be priced as if it were default-free. The boundary condition (18) follows from a property of the conditional density $\pi$ proven in Lemma 1, Appendix A. To gain intuition for this boundary condition, recall that in this regime bondholders know both $\hat{y}_t$ and that $\min_{s \in [t-L,t]} \{v_s\} > v_B$ for all $s \in (t-L,t)$. The boundary condition implies that as $\hat{y}_t$ approaches zero, conditioning on the exact value for $\hat{y}_t$ becomes immaterial—all that matters is that the firm has survived up to this point (i.e., $\tau_d > t$) in order to predict the value of $y_t$.

### 3.3.2 Debt value in Regime 3, $t \in [t^*, t^* + L)\$

Given the value of debt $D_4(\hat{y}_t, \mathbf{1}_{\{\tau_d > t\}})$ determined in the previous section, we can now determine the value of debt in Regime 3, $t \in [t^*, t^* + L)$. Note that in this regime, when debt-holders do not observe a default at time $t$, they can only infer that the value of the firm’s assets $v_t$ must have been larger than the default boundary $v_B$ from time $t^*$ onwards. Since at time $t$ bondholders observe the asset value with a lag $L$ and $t - L < t^*$, it is possible that there existed dates $s \in (t - L, t^*)$ for which $v_s < v_B$, but, because new debt was being issued to service existing debt, default did not occur under these
circumstances. This implies that the debt-holders information structure in this regime is

\[ \mathcal{F}_t = \left\{ \hat{v}_t = v_{t-L}; \min_{s \in [t^{*}, t]} \{v_s\} > v_B \right\}, \quad t^{*} \leq t < t^{*} + L. \] (21)

Comparing to the information set (15), we note that bondholders have less information in Regime 3 than in Regime 4. The following proposition shows that under the information set (21), the price of debt and the default intensity depend both on \( \hat{y}_t \) and time, in contrast to its time-independent values derived in Proposition 1.

**Proposition 2** In the time interval \( t^{*} \leq t < t^{*} + L \), the price of debt with outstanding cumulative coupon \( C \) is a function of both \( \hat{y}_t \) and \( t \), \( D_t = D_3(\hat{y}_t, t, C_t, 1_{\{\tau_d > t\}}) \), given by the solution to the PDE

\[ 0 = C - (r + \lambda_{3,d}(\hat{y}_t, t)) D_3 + D_3.t + mD_3.y + \frac{\sigma^2}{2}D_3.yy + (1 - \alpha)V_B \lambda_{3,d}(\hat{y}_t, t), \] (22)

subject to the boundary conditions

\[ \lim_{\hat{y}_t \to \infty} D_3(\hat{y}_t, t) = \frac{C}{r}, \] (23)

\[ \lim_{\hat{y}_t \to -\infty} \frac{\partial}{\partial \hat{y}_t} D_3(\hat{y}_t, t) = 0. \] (24)

In equation (22), \( \lambda_{3,d}(\hat{y}_t) \) is the default intensity associated with equity-holders choosing to default and is given by

\[ \lambda_{3,d}(\hat{y}_t) = \frac{e^{-\left(\frac{1}{2\sigma^2(t-t^*)}\right)(y_{t^*} + m(t-t^*))^2}}{\pi_3(\tau_d > t | \hat{y}_t)\sqrt{2\pi\sigma^2(t-t^*)}} \int_0^\infty \frac{y_{t^*}}{\sqrt{2\pi\sigma^2(t^* - (t - L))}} e^{-\left(\frac{1}{2\sigma^2(t^* - (t - L))}\right)(y_{t^*} - \hat{y}_t - m(t^* - (t - L)))^2} dy_{t^*}, \] (25)

where the probability of firm survival at time \( t \), conditional on bondholders lagged information \( \hat{y}_t \), is given by

\[ \pi_3(\tau_d > t | \hat{y}_t) \equiv \pi(\min_{s \in [t^{*}, t]} \{v_s\} > v_B | \hat{y}_t) \]

\[ = \int_0^\infty \frac{1}{\sqrt{2\pi\sigma^2(t^* - (t - L))}} e^{-\left(\frac{1}{2\sigma^2(t^* - (t - L))}\right)(y_{t^*} - \hat{y}_t - m(t^* - (t - L)))^2} dy_{t^*} \]

\[ \times \left\{ N \left[ \frac{y_{t^*} + m(t - t^*)}{\sqrt{\sigma^2(t - t^*)}} \right] - e^{-\frac{2y_{t^*}m}{\sigma^2}} N \left[ \frac{-y_{t^*} + m(t - t^*)}{\sqrt{\sigma^2(t - t^*)}} \right] \right\}. \] (26)
with \( N(\cdot) \) denoting the cumulative standard normal distribution.

The intuition for the boundary condition (23) is that as \( \hat{y}_t \) goes to infinity, the bond becomes risk free. The intuition for the boundary condition (24) is that, as \( \hat{y}_t \to -\infty \) the probability density \( \pi(y_t|\hat{y}_t, \tau_d > t) \) becomes independent of \( \hat{y}_t \), that is

\[
\lim_{\hat{y}_t \to -\infty} \frac{\partial}{\partial \hat{y}_t} \pi(y_t|\hat{y}_t, \tau_d > t) = 0. \tag{27}
\]

Note that there is an important scaling property in both the ODE (16) and PDE (22). Recall that the optimal default boundary \( V_B = \frac{C}{\beta} \) is linear in the coupon level \( C \). Therefore, if we were to scale both \( V_t \) and \( C \) (and thus, also \( V_B \)) by some factor \( \nu \), the value of \( y_t = \log \left( \frac{V_t}{V_B} \right) \) and \( \hat{y}_t = \log \left( \frac{\hat{V}_t}{V_B} \right) \) would be unaffected. Consequently, \( \lambda_4(\hat{y}_t) \), and \( \lambda_{3,d}(\hat{y}_t, t) \) would also be unaffected. Then, it follows from equations (16) and (22) that the values of debt \( D_4(\hat{y}_t, t, C, 1_{\{\tau_d > t\}}) \) and \( D_3(\hat{y}_t, t, C, 1_{\{\tau_d > t\}}) \) would scale by \( \nu \) also. We will further discuss this scaling property in Section 3.5.

### 3.3.3 Debt value in Regime 2, \( t = t^* \)

The instant \( t^* \) at which the firm exhausts its debt capacity is defined as the first time the ratio of the cumulative coupon level \( C_t \) to the lagged unlevered firm value \( \hat{V}_t \) reaches an exogenously specified threshold \( \Psi \). Formally,

\[
t^* = \arg \min_t \left\{ \frac{C_t}{\hat{V}_t} = \Psi \right\}. \tag{28}
\]

For all dates \( t < t^* \), the only information debt-holders receive about the firm is (i) the amount of debt outstanding, which is characterized by the level of coupon \( C_t \), and (ii) the lagged unlevered firm value \( \hat{V}_t = V_{t-L} \), that is,

\[
\mathcal{F}_t = \{ \hat{v}_t = v_{t-L}; C_t \}, \quad t \leq t^*. \tag{29}
\]

Because the unlevered firm dynamics follow a diffusion process, under the information set (29) the event in which debt capacity is exhausted is therefore predictable by both debt-holders and the manager.

Given the process (2) for the unlevered log asset value, at time \( t^* \), the probability density of the current value \( y_{t^*} = v_{t^*} - v_B \), conditional on the lagged value \( \hat{y}_{t^*} = y_{t^*-L} \)
is normal with mean $\hat{y}_{t^*} + mL$ and volatility $\sigma\sqrt{L}$, that is,
\[
\pi (y_{t^*}|\hat{y}_{t^*}) = \frac{1}{\sqrt{2\pi\sigma^2 L}} e^{-\left(\frac{1}{2\sigma^2 L}\right)[y_{t^*}-(\hat{y}_{t^*}+mL)]^2}. \tag{30}
\]

It follows that the probability of a time-$t^*$ default equals the probability that $y_{t^*} < 0$, or
\[
\pi_2 (\tau_d = t^*|\hat{y}_{t^*}) = \int_{-\infty}^{0} \pi (y_{t^*}|\hat{y}_{t^*}) dy_{t^*} = N \left( \frac{-\hat{y}_{t^*} - mL}{\sqrt{\sigma^2 L}} \right), \tag{31}
\]
and the probability of survival is
\[
\pi_2 (\tau_d > t^*|\hat{y}_{t^*}) = N \left( \frac{\hat{y}_{t^*} + mL}{\sqrt{\sigma^2 L}} \right). \tag{32}
\]

From the recursive pricing equation (13b), the debt value at $t^*$ is a sum of two terms: (i) the expected value of debt $D_3(t^*, \hat{y}_{t^*})$ if no default occurs at $t^*$ and (ii) an expectation of recovery in the event of default. Using the above conditional probabilities we can then write the value of debt $D_{t^*} = D_2(\hat{y}_{t^*}, t^*, C_{t^*})$ as follows:
\[
D_2(\hat{y}_{t^*}, t^*, C_{t^*}) = \mathbb{E}_t^Q \left[ D_{t^*} \mathbf{1}_{\{y_{t^*} \geq 0\}} + (1 - \alpha) e^{v_B + \hat{y}_{t^*}} \mathbf{1}_{\{y_{t^*} < 0\}} | \hat{y}_{t^*} \right] = \pi_2 (\tau_d > t^*|\hat{y}_{t^*}) D_3(\hat{y}_{t^*}, t^*, C_{t^*}) + \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi\sigma^2 L}} e^{-\left(\frac{1}{2\sigma^2 L}\right)[y_{t^*}-(\hat{y}_{t^*}+mL)]^2} (1 - \alpha) e^{v_B + \hat{y}_{t^*}} dy_{t^*}
\]
\[
= N \left( \frac{\hat{y}_{t^*} + mL}{\sqrt{\sigma^2 L}} \right) D_3(\hat{y}_{t^*}, t^*, C_{t^*}) + (1 - \alpha) e^{v_B + \hat{y}_{t^*} + (m + \sigma^2 \alpha^2 L)} N \left( -\frac{\hat{y}_{t^*} + (m + \sigma^2 L)}{\sqrt{\sigma^2 L}} \right) \tag{33}
\]
where the debt value $D_3(\hat{y}_{t^*}, t^*, C_{t^*})$ is obtained in Proposition 2 and the last equality uses the expressions from the densities (31) and (32).

### 3.3.4 Debt value in Regime 1, $t \in [0, t^*)$

At time 0, the firm issues a perpetuity with promised cash flows comprised of a continuous coupon $C_0 dt$ until it optimally decides to default at time $\tau_d$, where $C_0$ determines the initial capital structure as discussed in Section 3.6 below.

Because the firm does not generate any intermediate cash flows, we assume it continuously issues new *pari-passu* debt in order to service the debt-in-place until it exhausts its debt capacity at time $t^*$. After exhausting debt capacity, we assume that covenants restrict any future debt issuances, implying that for dates $t > t^*$, the firm has to issue
new equity to service debt-in-place or default. Because it is optimal for shareholders for default to not occur during this regime in which they experience no negative cash flows (i.e., no equity issuances), the default intensity is zero in this regime.

The dynamics $dC_t$ for the size of the coupon payment is determined endogenously by identifying how much future cash flow must be promised to new bondholders in order to entice them to service the current debt due. Define by $D_1(\hat{y}_t, C_t)$ the value of all debt in-place at date $t$ prior to receiving the coupon payment.\footnote{Note that, in continuous-time, the value of debt is the same before and after coupon payment, since the coupon payment is linear in $dt$.} At date $t$, the firm needs to raise $C_t \, dt$. However, due to tax-deductibility of interest on debt, a fraction $\theta C_t \, dt$ is covered by government, where $\theta$ is the effective tax rate. Hence, the firm needs to raise only $(1 - \theta)C_t \, dt$ to service the old debt. Therefore, the present value of the new debt issuance must equal $(1 - \theta)C_t \, dt$. Because all debt is pari passu, the fraction of debt owned by the new owners, determined at date $t$, is

$$\pi_{\text{new}} = \left( \frac{C_{t+dt} - C_t}{C_{t+dt}} \right),$$

whereas the fraction of debt owned by previous owners is

$$\pi_{\text{old}} = \left( \frac{C_t}{C_{t+dt}} \right).$$

It follows that $C_{t+dt}$, and hence, the dynamics $dC_t$, can be determined by equating the value of the new debt claim to the amount new debt-holders pay for this claim:

$$(1 - \theta)C_t \, dt = [D_1(\hat{y}_t, C_t) - C_t \, dt] \left( \frac{C_{t+dt} - C_t}{C_{t+dt}} \right).$$

Using the fact that $C_{t+dt} = C_t + dC_t$, as $dt \to 0$ equation (36) simplifies to

$$dC_t = (1 - \theta) \left( \frac{C_t^2}{D_1(\hat{y}_t, C_t)} \right) \, dt,$$

which specifies the coupon dynamics in terms of the bond price $D_1$.

To determine the bond price $D_1(\hat{y}_t)$ recall that, from the recursive pricing equation (13a), we have

$$D_1(\hat{y}_t, C_t) = C_t \, dt + e^{-r \, dt} \mathbb{E}_t^Q \left[ \left( \frac{C_t}{C_{t+dt}} \right) D_1(\hat{y}_{t+dt}, C_{t+dt}) \right].$$
The above equation states that the present value of the debt-in-place at date $t$ is equal to the value of the coupon, $C_t \, dt$, and the fraction $\pi_{old} = \left( \frac{C_t}{C_{t+dt}} \right)$ of next period’s debt claim, whose date-$t$ present value is determined by risk-neutral discounting. Applying Itô’s lemma to equation (38) we obtain that the value of the debt for $t < t^*$ solves the PDE\(^{11}\)

$$
0 = C_t - \left( r + (1 - \theta) \frac{C_t}{D_1} \right) D_1 + (1 - \theta) \frac{C^2 D_1}{C} \frac{\partial D_1}{\partial C} + m \frac{\partial D_1}{\partial \hat{y}} + \frac{\sigma^2}{2} \frac{\partial^2 D_1}{\partial \hat{y}^2} \quad (39)
$$

subject to the boundary conditions:

$$
\lim_{\hat{y}_t \to \infty} D_1(\hat{y}_t, C_t) = \frac{C_t}{r}, \quad (40)
$$

$$
\lim_{t \to t^*} D_1(\hat{y}_t, C_t) = D_2(\hat{y}_{t^*}, t^*, C_{t^*}), \quad (41)
$$

where $D_2(\hat{y}_{t^*}, t^*, C_{t^*})$ is defined in (33).\(^{17}\)

3.4 Equity valuation

Let $\hat{S}_t \equiv \hat{S}_t(\hat{y}_t, C_t, 1_{\{\tau_d > t\}})$ be the equity value conditional on the creditors’ information set, and $S_t \equiv S_t(y_t, C_t, 1_{\{\tau_d > t\}})$ be the equity value conditional on the manager’s information set. The equity value $\hat{S}_t$ can be expressed recursively as follows:

$$
\hat{S}_t = \begin{cases} 
\mathbb{E}_t^Q \left[ e^{-r(t^*-t)} \hat{S}_{t^*} \right], & \text{if } t < t^* \\
\mathbb{E}_t^Q [S_t], & \text{if } t \geq t^* 
\end{cases} \quad (42a)
$$

where $S_t$ in (42b) is the solution to equations (9)–(10) of the ODE (5) with boundary conditions (6)–(7). The following proposition characterizes the equity value $\hat{S}_t$ for $t \geq t^*$.

\(^{11}\)Note that $C_{t+dt} = C_t + dC_t = C_t + (1 - \theta) \frac{C^2}{D_1} \, dt$ by equation (37). Hence, it follows that $\frac{C_t}{C_{t+dt}} = \frac{1}{1 + (1 - \theta) \frac{C^2}{D_1} \, dt} = \left[ 1 - (1 - \theta) \frac{C_t}{D_1} \, dt \right].$
Proposition 3

The equity value \( \hat{S}_{t^*} \) is

\[
\hat{S}_{t^*} = e^{\hat{y}_{t^*} + v_B} \left\{ e^{mL + \frac{\sigma^2 L}{2}} N \left[ \frac{\log \left( \frac{\beta}{\Psi} \right) + mL + \sigma^2 L}{\sqrt{\sigma^2 L}} \right] - \frac{(1 - \theta) \Psi}{r} N \left[ \frac{\log \left( \frac{\beta}{\Psi} \right) + mL}{\sqrt{\sigma^2 L}} \right] \right\} - \left( \frac{\beta}{\Psi} \right)^{-\frac{2}{2}} \left( \frac{\Psi}{\beta} - \frac{(1 - \theta) \Psi}{r} \right) e^{rL} N \left[ \frac{\log \left( \frac{\beta}{\Psi} \right) + mL - 2rL}{\sqrt{\sigma^2 L}} \right].
\]

(43)

The equity value \( \hat{S}_t \) for \( t > t^* \) is

\[
\hat{S}_t = \begin{cases} 
\int_0^\infty \pi_3(y_t | \tau_d > t, \hat{y}_t) S(y_t, C_{t^*}) dy_t, & \text{if } t^* < t < t^* + L \\
\int_0^\infty \pi_4(y_t | \tau_d > t, \hat{y}_t) S(y_t, C_{t^*}) dy_t, & \text{if } t \geq t^* + L,
\end{cases}
\]

(44a, 44b)

where \( \pi_3(y_t | \tau_d > t, \hat{y}_t) \) is given in equation (A.32), \( \pi_4(y_t | \tau_d > t, \hat{y}_t) \) is given in equation (A.4) and \( S_t(y_t, C_{t^*}) \) is the solution (9)–(10) of the ODE (5) with boundary conditions (6)–(7).

With the value of \( \hat{S}_{t^*} \) obtained in equation (43), we can determine the stock values \( \hat{S}_t \) for \( t < t^* \) by solving the expectation in equation (42a). Intuitively, this expectation captures the fact that there are no cash flows for equity over the interval \( t \in (0, t^*) \) and hence, the equity value is given by its risk-neutral discounted value at \( t = t^* \). To solve this expectation we note that \( e^{-rt} \hat{S}(\hat{y}_t, C_t) = E_t^Q \left[ e^{-r\hat{t}^*} \hat{S}_{t^*} \right] \) is a \( Q \)-martingale. Applying Itô’s lemma and using the coupon dynamic (37) we obtain that \( \hat{S}_t \) satisfies the PDE

\[
0 = -r \hat{S} + m \hat{S}_y + \frac{\sigma^2}{2} \hat{S}_{yy} + \hat{S}_C (1 - \theta) \left( \frac{C^2}{D(V, C)} \right),
\]

(45)

with boundary conditions

\[
\lim_{\hat{y}_t \to \hat{y}_{t^*}} \hat{S}_t(\hat{y}_t, C_t) = \hat{S}_{t^*},
\]

(46)

\[
\lim_{\hat{y}_t \to \infty} \hat{S}_t(\hat{y}_t, C_t) = e^{\hat{y}_t + v_B} - \frac{(1 - \theta) C_t}{r},
\]

(47)

where \( \hat{S}_{t^*} \) is given in equation (43).

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3.5 Scaling property

The valuation expressions for debt and equity that we have obtained above exhibit an important scaling property. From equation (10), the default boundary is \( V_B = C_{t^*}/\beta \). Hence, the state variable \( \hat{y}_t = \log(\hat{V}_t/V_B) \) is unaffected if the lagged asset value \( \hat{V}_t \) and cumulative coupon \( C_{t^*} \) are scaled by a constant factor. From the characterization of debt values for \( t \geq t^* \) in Propositions 1, 2, and equation (13b), we note that the debt values are linear in the coupon level \( C_t \), that is

\[
D_t(\hat{y}_t, t, C) = C \times D_t(\hat{y}_t, t, 1), \quad t \geq t^*. \tag{48}
\]

The following proposition shows that the value of debt and equity at any time \( t \leq t^* \) are homogeneous of degree one in the lagged value of the assets \( \hat{V}_t \). This allows us to reduce the dimensionality of the problem and express debt and equity as a function only of the debt-to-assets ratio \( \Psi_t \equiv C_t/\hat{V}_t \).

**Proposition 4** Let \( \hat{V}_t \) be the lagged asset value (1), \( V_B \) the default boundary (10) for a generic cumulative coupon \( C_{t^*} \) and \( \Psi_t = C_t/\hat{V}_t \) the debt-to-assets ratio. Then the debt value \( D(\hat{V}_t, C_t) \) for \( t < t^* \) is

\[
D(\hat{V}_t, C_t) = \hat{V}_t G(\Psi_t), \tag{49}
\]

where \( G(\Psi_t) \) solves the PDE

\[
\theta \Psi - r G + (1 - \theta)\Psi^2 \frac{G_{\Psi}}{G} + r (G - \Psi G_{\Psi}) + \frac{\sigma^2}{2} \Psi^2 G_{\Psi\Psi} = 0, \tag{50}
\]

subject to the boundary conditions

\[
G(0) = 0 \tag{51}
\]

\[
G(\overline{\Psi}) = \overline{\Psi} D_2 \left( \log \left( \frac{\beta}{\overline{\Psi}} \right), t^*, 1 \right), \tag{52}
\]

where \( \overline{\Psi} = C_{t^*}/\overline{V}_{t^*} \) is the credit constraint in equation (28), and \( D_2(\hat{y}_{t^*}, t^*, C_{t^*}) \) is defined in (33).

Note that the boundary condition (52) implies that the bond value \( G(\Psi_t) \) does not depend on the actual level of coupon \( C_{t^*} \). This is an implication of the scaling property (48) and of the definitions of maximum debt capacity (28), \( \overline{V}_{t^*} = C_{t^*}/\overline{\Psi} \), and the
default boundary (10), \( V_B = \frac{C_{t^*}}{\beta} \). Using these definitions in \( \hat{y}_{t^*} \equiv \log \left( \frac{\hat{V}_{t^*}}{V_B} \right) \), we obtain
\[
\hat{y}_{t^*} = \log(\beta/\Psi),
\]
as in equation (52).

A similar scaling property also holds for the value of equity \( \hat{S}_t \) defined in equations (42a)–(42b) as the following proposition formalizes:

**Proposition 5** Let \( \hat{V}_t \) be the lagged asset value (1), \( V_B \) the default boundary (10) for a generic cumulative coupon \( C_t \) and \( \Psi_t = \frac{C_t}{\hat{V}_t} \) the debt-to-assets ratio. Then the equity value \( \hat{S}(\hat{V}_t, C_t) \) for \( t \leq t^* \) is
\[
\hat{S}(\hat{V}_t, C_t) = \hat{V}_t H(\Psi_t),
\]
where \( H(\Psi_t) \) solves the ODE
\[
-r\Psi H_{\Psi} + \frac{\sigma^2}{2} \Psi^2 H_{\Psi\Psi} + \frac{H_{\Psi}}{G(\Psi)} (1 - \theta)\Psi^2 = 0,
\]
subject to the boundary conditions
\[
H(0) = 1,
\]
where \( \Psi = \frac{C_{t^*}}{\hat{V}_{t^*}} \) is the credit constraint in equation (28), and \( G(\Psi_t) \) is defined in Proposition 4.

The boundary condition (55) follows directly from equation (43) in Proposition 3.

### 3.6 Optimal capital structure and debt dynamics

We use Propositions 4 and 5 to solve for the firm’s optimal capital structure at time 0. For tractability, we assume that the manager does not have an informational advantage at this time and that, like the creditors, she observes only the lagged unlevered asset value \( \hat{V}_0 = V_{-L} \).

To determine the firm’s optimal capital structure, we exploit the scaling property of the problem and, without loss of generality, set \( \hat{V}_0 = 1 \) in Propositions 4 and 5. This
implies $\Psi_0 = C_0$, $D_0 = G(\Psi_0)$ and $\widehat{S}_0 = H(\Psi_0)$. The firm value is then $G(\Psi_0) + H(\Psi_0)$, and therefore the optimal capital structure $\Psi^{\text{opt}}_0$ at time 0 is independent of asset size $\widehat{V}_0$ and is given by

$$\Psi^{\text{opt}}_0 = \arg \max_{\Psi_0} \{G(\Psi_0) + H(\Psi_0)\}. \quad (57)$$

Note that choosing the initial capital structure $\Psi^{\text{opt}}_0$ is equivalent to selecting an initial coupon level $C^{\text{opt}}_0$ for a firm with asset size $\widehat{V}_0$. In general, the initial coupon will be $C^{\text{opt}}_0 = \Psi^{\text{opt}}_0 \widehat{V}_0$.

After choosing its initial capital structure, the firm continuously issues debt at a rate $dC_t$ given by equation (37) until it reaches debt capacity. Applying Itô’s lemma to $\Psi_t \equiv (C_t/\widehat{V}_t)$ we obtain the following dynamics for $\Psi_t$:

$$d\Psi_t = \left[\left(1 - \theta\right) \frac{\Psi_t^2}{G(\Psi_t)} - (r - \sigma^2)\Psi_t\right] dt - \sigma \Psi_t dB_t^Q, \quad \Psi_0 = \Psi^{\text{opt}}_0 = C^{\text{opt}}_0. \quad (58)$$

By construction, at the random time $t^*$, $\Psi_{t^*} = \Psi$, where $\Psi$ is an exogenous parameter representing a firm’s debt capacity. At any point in time $t \leq t^*$ the cumulative coupon $C_t$ for a firm of size $\widehat{V}_0 = 1$ is given by

$$C_t^{(\widehat{V}_0=1)} = \Psi_t \widehat{V}_t^{(\widehat{V}_0=1)}, \quad (59)$$

where $\widehat{V}_t^{(\widehat{V}_0=1)}$ is the lagged value of assets at time $t - L, V_{t-L}$ obtained from the dynamics (1) with initial condition $V_{-L} = 1$. Because of the scaling property discussed in Section 3.5, the cumulative coupon of a firm with initial asset size $\widehat{V}_0 = \nu$ is simply $\nu \times C_t^{(\widehat{V}_0=1)}$.

4 Results

There are two defining features of our model that, taken together, set it apart from previous contributions. First, we allow for information asymmetry between the firm’s manager and creditors, and, second, the firm continues to issue debt until it reaches debt capacity. It is useful to organize the discussion of the model implications along these two elements, so as to more easily draw a comparison with the previous literature.

The key model coefficients associated with information asymmetry and debt issuance are $L$ and $\Psi$. In the baseline case, we assume that it takes creditors six months to learn
the true value of the firm’s assets, i.e., they observe $V$ with a $L = 0.5$ delay. Furthermore, we use $\Psi$ to generate a leverage at time $t^*$ of approximately 75%, in line with leverage values of firms that recently transitioned to “fallen angel” status.

Other special cases are also relevant. For instance, when $L = 0$ and $\Psi = C_0$ manager and creditors share the same information set and the firm is restricted to issues debt at time 0 only. This case is similar to the Leland (1994) setting. Another special case has $L > 0$ and $\Psi = C_0$—this is close to the economy of Duffie and Lando (2001) in which a better-informed manager chooses the optimal mix of debt and equity at time zero, but is prevent to issue more debt afterwards. Finally, the case in which $L = 0$ and $\Psi > C_0$ falls within the literature on optimal capital structure dynamics with complete information (e.g., Goldstein, Ju, and Leland (2001), Hennessy and Whited (2007), DeMarzo and He (2017)).

Table 2 reports the rest of the model coefficients for the baseline calibration. We normalize the total coupon payment at one, $C_{t^*} = 1$. The risk-neutral asset dynamics in equation (1) are governed by a 0.5% riskfree rate and 30% volatility coefficient. In the model, the capital structure choice is driven by the trade-off between debt tax shield and bankruptcy cost. In this respect, we assume that corporate profits are taxed at a $\theta = 25\%$ rate, while the loss in asset value, $\alpha$, in case of default is 40%.

### 4.1 Capital structure

Figure 4 shows the optimal firm capital structure at time $t = 0$ as a function of the information lag $L$. The blue line portrays the solution for our model, computed as in Section 3.6. The red line shows similar results for a firm that is restricted to issue equity at time $t = 0$ only, i.e., $\Psi = C_0$. We label this case as “Duffie-Lando”.

In our $L = 0.5$ baseline calibration, the optimal time-0 leverage is 18%, compared to 28% in the Duffie-Lando case. In our model, the firm continues to borrow after time 0 to service debt in place, and its leverage increases to 73% by the time the credit constraint becomes binding at time $t^*$. Hence, future debt issuance makes the firm riskier compared to a Duffie-Lando company, and initial leverage is therefore considerably lower.

Furthermore, Figure 4 shows that as the information asymmetry between the manager and creditors increases, the firm issues less debt. For instance, when creditors observe the value of the assets with a one-year delay, optimal leverage decreases to 14%
in our model; a similar drop occurs in Duffie-Lando, where time-0 leverage is 24%. On the other extreme, when $L \to 0$, both manager and creditors observe the true value of the assets without a lag. In this case, the Duffie-Lando case collapses into the Leland model, with leverage peaking at 39%. In our model, future debt issuance increases the riskiness of the firm, and initial leverage is lower at 30%.

Next, we illustrate the sensitivity of the firm’s capital structure to the credit constraint coefficient $\psi$. The left panels of Figures 5 show optimal leverage at time 0 (the triangles) and time-$t^*$ leverage (the stars) as a function of $\psi$ when information asymmetry is low ($L = 0.05$, top panel), medium ($L = 0.5$, center), and high ($L = 1$, bottom). These plots highlight the dynamic nature of the firm’s capital structure in our model. In all cases, leverage starts low. After that, the firm continues to issue debt till the debt capacity is reached at time $t^*$. As $\psi$ decreases, the credit constraint becomes more binding and the firm arrives at $t^*$ with a lower leverage. At time 0, the manager anticipates that debt issuance will be more limited in the future, and funds therefore the firm with a larger stock of initial debt. As $\psi$ drops further to approach $C_0$, initial leverage converges to the Duffie-Lando capital structure solution that we discussed in the previous paragraph. In the right panels, we show how the degree of information asymmetry interacts with the credit constraint in determining a firm’s financing decisions. Both at time 0 and $t^*$, a bigger information lag between creditors and the manager increases the cost of debt issuance and therefore reduces leverage.

Figures 5 illustrates a similar exercise in which we vary the coefficient $\sigma$. Firms with higher asset volatility are riskier and reach date $t^*$ with a lower leverage, while at time 0 the manager issues a larger stock of initial debt.

### 4.2 Credit spreads

We use the pricing formulas for perpetual debt, derived in Section 3, to simulate model-implied default times as detailed in the Online Appendix. We then compute the term-structure of defaultable bond spreads as in Duffie and Singleton (1999). Figure 7 shows the model-implied credit spreads curve as a function of the information lag $L$. In the baseline case, we identify a typical investment-grade company with credit spreads of 60 bps at the five-year horizon. We find such firm to have leverage of 47% at time 0; to facilitate comparisons with the other cases, when we change $L$ we also adjust the initial
amount of debt issued to keep leverage at the same level. As the information lag \( L \) increases, debt becomes riskier and credit spreads go up. However, in all cases short-maturity credit spreads remain very low. This is because in our model IG companies are subject to negligible jump-to-default risk; hence, they command little or no jump-to-default premium, consistent with the stylized facts discussed in Section 2.

Figures 8 and 9 document the sensitivity of credit spreads to changes in \( \bar{\psi} \) and \( \sigma \) when keeping initial leverage at the same 47% level. As \( \bar{\psi} \) increases, firms arrive at \( t^* \) with a larger stock of debt. Hence, creditors expect the firm to reach its default boundary sooner than in the baseline case and they value debt less. Even in this case, however, the impact of a higher \( \bar{\psi} \) is mostly visible in longer-dated spreads. In contrast, short-maturity debt largely remains safe, as the IG company can avoid default at short horizons by using up its residual debt capacity. A similar pattern is evident in Figure 9: as asset volatility increases, debt becomes riskier and spreads go up. However, the increase is mostly visible at long-maturity, while short-term IG spreads stay low.

### 4.3 Default rates

Table 3 shows model-implied expected default rates for firms in different credit-rating groups. We simulate a sample of 10,000 firms and track each of them till their eventual default (details on the simulation scheme are in the Online Appendix). For each firm and at each point of its simulated life span, we record the time to the company’s default and use the firm’s leverage as a proxy for credit worthiness. In particular, we assign firms with leverage no higher than 65% to the IG group. Companies with leverage between 65% and 75% are in the B group, while the rest are given a C label. We then compute the proportion of the firms in a rating group that default at various time horizons, and report the annualized default rate in the table.

Model-implied default rates are close to the empirical estimates in Table 1. Just like in the data, IG companies hardly ever go bankrupt; at short horizons default rates are virtually zero and they increase progressively over time. Failures remain infrequent among B firms, though in this case the default rate exceeds 2% at the 9–12 months horizon. Firms in the C category behave instead in a way that is consistent with the possibility of jumps to defaults. At the one-month horizon, the annualized default rate is approximately 15%, a number that matches closely empirical default rates for companies.
that are rated C by the three main rating agencies. Beyond the first month, default rates decline progressively, though they remain elevated, like we have have found in the data.

5 Conclusion

We provide theoretical and empirical support for the notion that IG firms face virtually zero jump-to-default risk, and therefore their short-maturity bonds command virtually zero jump-to-default premium. We develop a model in which the manager has superior information about the value of the firm’s assets relative to creditors, and can access the debt markets if the firm’s debt capacity has not been fully utilized. In this framework, a manager of an IG firm will maximize shareholder value by concealing any bad private signal and servicing existing debt via additional borrowing. This strategy permits IG firms to avoid jumping to default, at least until their debt capacity has been used up and the firm has dropped down to “fallen angel” status with speculative-grade spreads. Creditors are aware of the manager’s information advantage and price it rationally into the firms’ claims. Since firms with IG-level spreads do not face jump-to-default risk, their bond yields do not command a jump-to-default premium.

By allowing for dynamic debt issuance, our model extends the seminal work of Duffie and Lando (2001) in a way that greatly helps to characterize the way a vast portion of corporate bond issuers fund themselves. We acknowledge, however, that our analysis abstracts from some important features of credit markets. For instance, shareholders in our model do not receive any cash dividend payout, and the only reason for issuing debt is to repay debt in place. Adding a dividend payout will accelerate the time in which debt capacity is exhausted but will not qualitatively alter the key insights from our analysis. Further, in our model, firms continue to issue debt until its debt capacity has been fully utilized. In reality, conditions might improve after a firm reaches its debt capacity. For instance, the asset value could grow significantly to bring leverage down back to IG level, thus allowing the firm to tap into the bond market again. We can allow for this possibility in our model, however at significant costs in the computations and exposition.

We note that our framework generates a prediction nearly opposite to DL. While their framework (which precludes firms from issuing debt after date-0) implies that, even if the underlying asset value dynamics follows a diffusion process, firms can jump-to-
default due to asymmetric information, our framework (which permits IG firms to issue debt) implies that, even if the underlying asset value dynamics follows a jump process, IG firms will not jump to default, at least until they lose their IG status and become “fallen angels”. The implication of our model is that the relatively large spreads on short-maturity IG debt over risk-free securities cannot be explained by jump-to-default premia, and therefore implies that other channels (e.g., asymmetric taxes, illiquidity) are needed to explain these large spreads.
A Appendix: Proofs

Lemma 1 Let $t \geq t^* + L$ and $\pi_4(y_t | \tau_d > t, \hat{y}_t)$ be the conditional density of the excess log asset value $y_t = \log(V_t/V_B)$ where $V_t$ is given by (1) and $V_B$ the default boundary in (10). Then

$$\lim_{\hat{y}_t \to 0} \frac{\partial}{\partial \hat{y}_t} \pi_4(y_t | \tau_d > t, \hat{y}_t) = 0. \quad (A.1)$$

Proof. For convenience, we denote by $\{\tau_d > t\}$ the event $\{\min\{v_s\} > v_B \forall s \in (t - L, t)\}$ in the bondholders information set for $t \geq t + L$. It is well known (see, e.g., Proposition 8.1 in Harrison (1985)) that the joint density $\pi_4(y_t, \tau_d > t | \hat{y}_t)$ is characterized by the “free solution” minus an “image solution” whose initial location ($-\hat{y}_t$) is the same distance from the default boundary as is the actual initial location, that is, ($\hat{y}_t$).

$$\pi_4(y_t, \tau_d > t | \hat{y}_t) = \pi_4(y_t, \min\{y_s\} > 0 \forall s \in (t - L, t) | \hat{y}_t)$$

$$= 1_{\{y_t > 0\}} \left\{ \sqrt{\frac{1}{2\pi\sigma^2L}} e^{-\frac{(\sigma y_t | y_t - \hat{y}_t - mL)^2}{2\sigma^2L}} - e^{-\frac{2\hat{y}_t y_t}{\sigma^2}} \sqrt{\frac{1}{2\pi\sigma^2L}} e^{-\frac{(\sigma y_t + \hat{y}_t - mL)^2}{2\sigma^2L}} \right\} \quad (A.2)$$

Integrating $y_t$ from zero to infinity, we find

$$\pi_4(\tau_d > t | \hat{y}_t) = N \left( \frac{\hat{y}_t + mL}{\sqrt{\sigma^2 L}} \right) - e^{-\frac{2\hat{y}_t y_t}{\sigma^2}} N \left( \frac{-\hat{y}_t + mL}{\sqrt{\sigma^2 L}} \right), \quad (A.3)$$

where $N(\cdot)$ is the cumulative standard normal distribution. Combining these last two equations, we have

$$\pi_4(y_t | \tau_d > t, \hat{y}_t) = \frac{\pi_4(y_t, \tau_d > t | \hat{y}_t)}{\pi_4(\tau_d > t | \hat{y}_t)}. \quad (A.4)$$

To simplify the notation, let us define

$$p(\hat{y}_t) \equiv \pi_4(y_t, \tau_d > t | \hat{y}_t) \quad (A.5)$$

$$q(\hat{y}_t) \equiv \pi_4(\tau_d > t | \hat{y}_t) \quad (A.6)$$

$$z(\hat{y}_t) \equiv \pi_4(y_t | \tau_d > t, \hat{y}_t) = \frac{p(\hat{y}_t)}{q(\hat{y}_t)}. \quad (A.7)$$

In what follows we show that

$$\lim_{\hat{y}_t \to 0} z(\hat{y}_t) > 0 \quad (A.8)$$

$$\lim_{\hat{y}_t \to 0} \frac{\partial}{\partial \hat{y}_t} z(\hat{y}_t) = 0. \quad (A.9)$$
where $p_{\hat{y}_t}(\hat{y}_t) = \frac{1}{2\pi \sigma^2 L} \left\{ \left( \frac{1}{\sigma^2 L} \right) (y_t - \hat{y}_t - mL) e^{-\left( \frac{|y_t - \hat{y}_t - mL|^2}{2 \sigma^2 L} \right)} + \left( \frac{2m}{\sigma^2} \right) e^{-\frac{2y_t m}{\sigma^2}} e^{-\left( \frac{|y_t + \hat{y}_t - mL|^2}{2 \sigma^2 L} \right)} + \left( \frac{1}{\sigma^2 L} \right) (y_t + \hat{y}_t - mL) e^{-\frac{2y_t m}{\sigma^2}} e^{-\left( \frac{|y_t + \hat{y}_t - mL|^2}{2 \sigma^2 L} \right)} \right\}$

\[ q_{\hat{y}_t}(\hat{y}_t) = \sqrt{\frac{1}{2\pi \sigma^2 L}} \left[ e^{-\left( \frac{\left( \frac{y_t + mL}{2 \sigma^2 L} \right)^2}{2} \right)} + e^{-\frac{2y_t m}{\sigma^2}} e^{-\left( \frac{\left| \frac{y_t + mL}{2 \sigma^2 L} \right|^2}{2} \right)} \right] + \left( \frac{2m}{\sigma^2} \right) e^{-\frac{2y_t m}{\sigma^2}} N \left( -\frac{\hat{y}_t + mL}{\sqrt{\sigma^2 L}} \right). \]

where $p_{\hat{y}_t}(\hat{y}_t) = \frac{\partial p(\hat{y}_t)}{\partial \hat{y}_t}$ and $q_{\hat{y}_t}(\hat{y}_t) = \frac{\partial q(\hat{y}_t)}{\partial \hat{y}_t}$. Taking the limit as $\hat{y}_t \to 0$, we find

\[ \lim_{\hat{y}_t \to 0} p_{\hat{y}_t}(\hat{y}_t) = \sqrt{\frac{1}{2\pi \sigma^2 L}} \left( \frac{2y_t}{\sigma^2} \right) e^{-\left( \frac{\left( \frac{y_t - mL}{2 \sigma^2 L} \right)^2}{2} \right)} \]

\[ \lim_{\hat{y}_t \to 0} q_{\hat{y}_t}(\hat{y}_t) = 2\sqrt{\frac{1}{2\pi \sigma^2 L}} e^{-\left( \frac{\left( \frac{mL}{2 \sigma^2 L} \right)^2}{2} \right)} + \left( \frac{2m}{\sigma^2} \right) e^{-\frac{2y_t m}{\sigma^2}} N \left( \frac{mL}{\sqrt{\sigma^2 L}} \right). \]

Since both these derivatives are positive, it follows from L'Hôpital's rule that

\[ \lim_{\hat{y}_t \to 0} z(\hat{y}_t) = \pi_1(y_t | \tau_d > t, \hat{y}_t = 0) > 0, \]

thus proving (A.8). To prove (A.9) our second point, we consider

\[ \lim_{\hat{y}_t \to 0} z_{\hat{y}_t}(\hat{y}_t) = \lim_{\hat{y}_t \to 0} \left[ \frac{p_{\hat{y}_t}(\hat{y}_t)}{q(\hat{y}_t)} - \frac{p(\hat{y}_t) q_{\hat{y}_t}(\hat{y}_t)}{q(\hat{y}_t)} \right] = \lim_{\hat{y}_t \to 0} \left[ \frac{p_{\hat{y}_t}(\hat{y}_t) - z(\hat{y}_t) q_{\hat{y}_t}(\hat{y}_t)}{q(\hat{y}_t)} \right]. \]

Since we showed above that

\[ \lim_{\hat{y}_t \to 0} z(\hat{y}_t) = \lim_{\hat{y}_t \to 0} \frac{p_{\hat{y}_t}(\hat{y}_t)}{q_{\hat{y}_t}(\hat{y}_t)}, \]

it follows that both the numerator and denominator of equation (A.13) equal zero at $\hat{y}_t = 0$. Hence, we can apply L'Hôpital's rule again:

\[ \lim_{\hat{y}_t \to 0} z_{\hat{y}_t}(\hat{y}_t) = \lim_{\hat{y}_t \to 0} \left[ \frac{p_{\hat{y}_t}(\hat{y}_t) - z(\hat{y}_t) q_{\hat{y}_t}(\hat{y}_t)}{q(\hat{y}_t)} \right]. \]
Simplifying, we get
\[
2 \lim_{\hat{y}_t \to 0} z_{\hat{y}_t}(\hat{y}_t) = \lim_{\hat{y}_t \to 0} \left[ \frac{p_{\hat{y}\hat{y}}(\hat{y}_t) - z(\hat{y}_t) q_{\hat{y}\hat{y}}(\hat{y}_t)}{q_{\hat{y}}(\hat{y}_t)} \right]
\]
\[
= \lim_{\hat{y}_t \to 0} \left[ \frac{p_{\hat{y}\hat{y}}(\hat{y}_t) q_{\hat{y}}(\hat{y}_t) - p_{\hat{y}}(\hat{y}_t) q_{\hat{y}}(\hat{y}_t)}{q_{\hat{y}}^2(\hat{y}_t)} \right]. \quad (A.16)
\]

Because the denominator is positive, to prove (A.9) we only need to show that the numerator is equal to zero. Direct computation yields
\[
\sqrt{2\pi\sigma^2 L} p_{\hat{y}\hat{y}}(\hat{y}_t) = -\left(\frac{1}{\sigma^2 L}\right) e^{-\left(\frac{|y_t - \hat{y}_t - mL|^2}{2\sigma^2 L}\right)} + \left(\frac{1}{\sigma^2 L}\right)^2 (y_t - \hat{y}_t - mL)^2 e^{-\left(\frac{|y_t - \hat{y}_t - mL|^2}{2\sigma^2 L}\right)}
\]
\[
- \left(\frac{2m}{\sigma^2}\right)^2 \left(\frac{1}{\sigma^2 L}\right) e^{-\left(\frac{|y_t + \hat{y}_t - mL|^2}{2\sigma^2 L}\right)} + \left(\frac{1}{\sigma^2 L}\right)^2 e^{-\left(\frac{|y_t + \hat{y}_t - mL|^2}{2\sigma^2 L}\right)}
\]
\[
- 2\left(\frac{2m}{\sigma^2}\right) \left(\frac{1}{\sigma^2 L}\right) \left(y_t + \hat{y}_t - mL\right)^2 e^{-\left(\frac{|y_t + \hat{y}_t - mL|^2}{2\sigma^2 L}\right)} - \left(\frac{|y_t + \hat{y}_t - mL|^2}{2\sigma^2 L}\right) \right) \quad (A.17)
\]
and
\[
q_{\hat{y}\hat{y}}(\hat{y}_t) = -\left(\frac{1}{\sigma^2 L}\right) (\hat{y}_t + mL) \left(\frac{1}{\sqrt{2\pi\sigma^2 L}}\right) e^{-\left(\frac{|y_t + mL|^2}{2\sigma^2 L}\right)} - \left(\frac{2m}{\sigma^2}\right)^2 e^{-\left(\frac{|y_t + mL|^2}{2\sigma^2 L}\right)} N\left(-\frac{\hat{y}_t + mL}{\sqrt{\sigma^2 L}}\right)
\]
\[
-2\left(\frac{2m}{\sigma^2}\right) \left(\frac{1}{\sqrt{2\pi\sigma^2 L}}\right) e^{-\left(\frac{|y_t + mL|^2}{2\sigma^2 L}\right)}
\]
\[
+ \left(\frac{1}{\sigma^2 L}\right) (-\hat{y}_t + mL) \left(\frac{1}{\sqrt{2\pi\sigma^2 L}}\right) e^{-\left(\frac{|\hat{y}_t + mL|^2}{2\sigma^2 L}\right)} \right) \quad (A.18)
\]
Taking the limit as $\hat{y}_t \to 0$, we find
\[
\lim_{\hat{y}_t \to 0} \sqrt{2\pi\sigma^2 L} p_{\hat{y}\hat{y}}(\hat{y}_t) = -\left(\frac{2y_t}{\sigma^2 L}\right) \left(\frac{2m}{\sigma^2}\right) e^{-\left(\frac{|y_t - mL|^2}{2\sigma^2 L}\right)} \quad (A.19)
\]
\[
\lim_{\hat{y}_t \to 0} q_{\hat{y}\hat{y}}(\hat{y}_t) = -\left(\frac{2m}{\sigma^2}\right) \left[ \frac{2}{\sqrt{2\pi\sigma^2 L}} e^{-\left(\frac{mL^2}{2\sigma^2 L}\right)} + \left(\frac{2m}{\sigma^2}\right) N\left(\frac{mL}{\sqrt{\sigma^2 L}}\right) \right] \quad (A.20)
\]
Combining equations (A.10), (A.11), (A.19) and (A.20), we note that the numerator of equation (A.16) equals zero, which concludes our proof.
Proof of Proposition 1

In the time interval \( t \geq t^* + L \), the price of debt is

\[
D_4(\hat{y}_t, C, 1_{\{\tau_d > t\}}) = 1_{\{\tau_d > t\}} \\
\times \left\{ C dt + e^{-r dt} \mathbb{E}_t^Q \left[ D_4(\hat{y}_{t+d}, 1_{\{\tau_d > t+d\}}) + (1 - \alpha)e^{\nu y} 1_{(\tau_d \in (t, t+d))} \right] \right\}, \quad (A.21)
\]

where \( e^{\nu y} = V_B \) is the default boundary (10). Applying Itô's lemma, this simplifies to

\[
0 = C - (r + \lambda_{4,d}(\hat{y}_t)) D_4 + m D_{4,y} + \frac{\sigma^2}{2} D_{4,yy} + (1 - \alpha)V_B \lambda_{4,d}(\hat{y}_t), \quad (A.22)
\]

where \( \lambda_{4,d}(\hat{y}_t) \) represents the default intensity. As shown in Proposition 2.2 of Duffie and Lando (2001), the default intensity associated with equity holders choosing to default is given by:

\[
\lambda_{4,d}(\hat{y}_t) = \frac{\sigma^2}{2} \frac{\partial}{\partial \hat{y}_t} \pi_4(y_t | \tau_d > t, \hat{y}_t) \bigg|_{y_t=0}, \quad (A.23)
\]

where

\[
\pi_4(y_t | \tau_d > t, \hat{y}_t) = \frac{\pi_4(y_t, \tau_d > t | \hat{y}_t)}{\pi_4(\tau_d > t | \hat{y}_t)},
\]

(A.24)

and, for convenience, we use the notation \( \{\tau_d > t\} \) to express an event in the information set \( \{\min\{y_s\} > 0 \, \forall s \in (t - L, t)\} \). Using (A.2) and (A.3) from Lemma 1 in (A.24), we obtain the following expression for the default intensity

\[
\lambda_{4,d} = \frac{\hat{y}_t}{\sqrt{2\pi\sigma^2L^3}} \left( e^{-\frac{(\hat{y}_t + mL)^2}{2\sigma^2L^2}} \right) \frac{\pi_4(\tau_d > t | \hat{y}_t)}{\pi_4(\tau_d > t | \hat{y}_t)} \, 1_{\{\hat{y}_t > 0\}}. \quad (A.25)
\]

The boundary conditions for the PDE (A.22) are

\[
\lim_{\hat{y}_t \to \infty} D_4(\hat{y}_t) = \frac{C}{r}, \quad (A.26)
\]

\[
\frac{\partial}{\partial \hat{y}_t} D_4(\hat{y}_t) \bigg|_{\hat{y}_t=0} = 0. \quad (A.27)
\]

The upper boundary condition (A.26) implies that, for extremely large values of \( \hat{y}_t \), the bond can be priced as if it were default-free. The lower boundary condition (A.27) can be derived by expressing the bond price as the expectation:

\[
D_4(\hat{y}_t) = \int_0^\infty D_4^*(y_t) \, \pi_4(y_t | \hat{y}_t, \tau_d > t) \, dy_t, \quad (A.28)
\]

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where $D_1^*(y_t)$ equals the value of the corporate bond assuming that bondholders can observe $y_t$ without delay. From Lemma 1, it follows that

$$\frac{\partial}{\partial \hat{y}_t} D_4(y_t) \bigg|_{\hat{y}_t=0} = \int_0^\infty dy_t \frac{\partial}{\partial \hat{y}_t} [\pi_4(y_t|\hat{y}_t, \tau_d > t)]_{\hat{y}_t=0} = 0. \quad (A.29)$$

**Proof of Proposition 2**

In the time interval $t \in (t^*, (t^* + L))$, the price of debt is

$$D_3(\hat{y}_t, t, C, 1_{\{\tau_d > t\}}) = 1_{\{\tau_d > t\}} \times \left\{ C \, dt + e^{-r \, dt} \mathbb{E}_t^Q \left[ D_3(\hat{y}_{t+dt}, t + dt, C, 1_{\{\tau_d > t + dt\}}) \right] + (1 - \alpha) V_B \, 1_{\{\tau_d \in (t,t+dt)\}} \right\}. \quad (A.30)$$

Applying Itô’s lemma, equation (A.30) can be expressed as the PDE:

$$0 = C - (r + \lambda_{3,d}(\hat{y}_t, t)) \, D_3 + D_{3,t} + m D_{3,y} + \frac{\sigma^2}{2} D_{3,yy} + (1 - \alpha) V_B \lambda_{3,d}(\hat{y}_t, t), \quad (A.31)$$

where $\lambda_{3,d}(\hat{y}_t, t)$ is the default intensity. To show why $\lambda_{3,d}(\hat{y}_t, t)$ is a function of both $\hat{y}_t$ and time we need to solve for the conditional probability of $y_t$ given the bondholder’s information set: $\pi(y_t|\hat{y}_t, \min\{y_s\} > 0 \, \forall s \in (t^*, t))$. This is accomplished by first conditioning and integrating over all unobserved values of $y_t^*$:

$$\pi_3(y_t, \tau_d > t | \hat{y}_t) \equiv \pi(y_t, \min\{y_s\} > 0 \, \forall s \in (t^*, t) | \hat{y}_t)$$

$$= \int_{-\infty}^{\infty} \pi(y_t^*, y_t, \min\{y_s\} > 0 \, \forall s \in (t^*, t) | \hat{y}_t) \, dy_t^*$$

$$= \int_{-\infty}^{\infty} \pi(y_t, \min\{y_s\} > 0 \, \forall s \in (t^*, t) | y_t^*) \, \pi(y_t^* | \hat{y}_t) \, dy_t^*$$

$$= 1_{\{y_t > 0\}} \int_0^\infty \frac{1}{\sqrt{2\pi \sigma^2 (t - t^*)}} \times \left[ e^{-\left(\frac{1}{2\sigma^2(t-t^*)}\right) |y_t-y_t^* - m(t-t^*)|^2} - e^{-\frac{2y_t^* m}{\sigma^2} - \frac{1}{2\sigma^2(t-t^*)} |y_t+y_t^*-m(t-t^*)|^2} \right]$$

$$\times \frac{1}{\sqrt{2\pi \sigma^2 (t - (t-L))}} e^{-\left(\frac{1}{2\sigma^2(t-(t-L))}\right) |y_t^*-\hat{y}_t-m(t-(t-L))|^2} \, dy_t^*, \quad (A.32)$$

where the third line holds because, when conditioning on $(y_t^*, \hat{y}_t)$, $y_t^*$ is a sufficient statistic.
By integrating the joint density (A.32) over \( y_t \in (0, \infty) \), we find
\[
\pi_3(\tau_d > t \mid \hat{y}_t) \equiv \pi(\min\{y_s\} > 0 \forall s \in (t^*, t) \mid \hat{y}_t) = \int_0^\infty \frac{1}{\sqrt{2\pi\sigma^2(t^* - (t - L))}} e^{-\frac{(2\sigma^2(t^* - (t - L)))[y_{t^*} - \hat{y}_t - m(t^* - (t - L))]^2}{2\sigma^2}} dy_{t^*}.
\]
Note that, the limit as \( t \to \infty \), from which we derive the default intensity associated with equity holders choosing to default, that is,
\[
\lambda_{3,d}(\hat{y}_t, t) = \frac{\sigma^2}{2} \frac{\partial}{\partial y_t} \pi_3(y_t \mid \tau_d > t, \hat{y}_t)_{y_t=0} = \left( \frac{1}{\pi_3(\tau_d > t \mid \hat{y}_t)} \right) \frac{1}{\sqrt{2\pi\sigma^2(t^* - (t - L))^3}} \int_0^\infty \frac{y_{t^*}}{\sqrt{2\pi\sigma^2(t^* - (t - L))}} e^{-\frac{(2\sigma^2(t^* - (t - L)))[y_{t^*} - \hat{y}_t - m(t^* - (t - L))]^2}{2\sigma^2}} \times e^{-\frac{(2\sigma^2(t - t^*))[y_{t^*} - \hat{y}_t - m(t^* - (t - L))]^2}{2\sigma^2}} dy_{t^*}.
\]
Note that, the limit as \( t \to (t^* + L) \), of the transition density in (A.35) is
\[
\lim_{t \to (t^* + L)} \left[ e^{-\frac{(2\sigma^2(t^* - (t - L)))[y_{t^*} - \hat{y}_t - m(t^* - (t - L))]^2}{2\pi\sigma^2(t^* - (t - L))^2}} \right] = \delta(y_{t^*} - \hat{y}_t)
\]
where \( \delta(\cdot) \) is the Dirac Delta function. Therefore, as \( t \to (t^* + L) \), \( \lambda_{3,d}(\hat{y}_t, t) \) converges to the time-independent default intensity \( \lambda_{4,d}(\hat{y}_t) \) derived in Proposition 1.\(^{12}\)

The boundary conditions for the PDE (A.31) are
\[
\lim_{\hat{y}_t \to \infty} D_3(\hat{y}_t, t) = \frac{C}{r}, \quad (A.37)
\]
\[
\lim_{\hat{y}_t \to -\infty} \frac{\partial}{\partial \hat{y}_t} D_3(\hat{y}_t, t) = 0. \quad (A.38)
\]
\(^{12}\)Note, however, that \( \lambda_{3,d}(\hat{y}_t, t) \) is defined also for values of \( \hat{y}_t < 0 \).
The intuition for (A.37) is that, as \( \hat{y}_t \to \infty \), the bond becomes risk free. The intuition for (A.38) is that, as \( \hat{y}_t \to -\infty \), and therefore a similar argument to the one used to prove the boundary condition (18) in Proposition 1 applies.

**Proof of Proposition 3**

To find \( \hat{S}_t \) for \( t < t^* \), we first determine

\[
\tilde{S}(\hat{y}_t^*, C_{t^*}) = E_{t^*}^Q [S(y_t^*, C_{t^*}) | \hat{y}_t^*, C_{t^*}] .
\]  

(A.39)

Using the fact that, by (8), \( V_B = \frac{C_{t^*}}{\beta} \), and by (28), \( \frac{C_{t^*}}{V_{t^*}} = \overline{\Psi} \), we obtain

\[
\hat{y}_t^* \equiv \log \left( \frac{\hat{V}_{t^*}}{V_B} \right) = \log \left( \frac{\beta}{\overline{\Psi}} \right) .
\]  

(A.40)

Using (9) and recalling the definition of \( y_t^* = \log \left( \frac{V_{t^*}}{V_B} \right) \) and \( \hat{y}_t^* = \log \left( \frac{\hat{V}_{t^*}}{V_B} \right) \), we can re-write the equity value \( S_t \) under the manager’s information set as follows

\[
S(y_t^*, C_{t^*}) = e^{\hat{y}_t^* + v_B} 1_{y_t^* > 0} \left\{ \left( e^{y_t^* - \hat{y}_t^*} - \frac{(1 - \theta)\overline{\Psi}}{r} \right) - e^{-\frac{2\sigma^2}{\beta} (y_t^* - \hat{y}_t^*)} \left( \frac{\beta}{\overline{\Psi}} \right)^{-\frac{2\sigma^2}{\beta}} \left( \frac{1 - \theta}{r} \overline{\Psi} - \frac{(1 - \theta)\overline{\Psi}}{r} \right) \right\} .
\]  

(A.41)

From (2) we have that the distribution of \( y_t^* \) conditional on \( \hat{y}_t^* \), under the risk-neutral measure \( Q \) is

\[
\pi^Q [y_t^* | \hat{y}_t^*] = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y_t^* - \hat{y}_t^* - mL)^2}{2\sigma^2} L} .
\]  

(A.42)

Therefore the equity value \( \tilde{S}_t \) under the creditors’ information set is

\[
\tilde{S}(\hat{y}_t^*, C_{t^*}) = E_{t^*}^Q [S(y_t^*, C_{t^*}) | \hat{y}_t^*] = e^{\hat{y}_t^* + v_B} \left\{ e^{mL + \frac{\sigma^2 L}{2}} N \left[ \frac{\log \left( \frac{\beta}{\overline{\Psi}} \right) + mL + \sigma^2 L}{\sqrt{\sigma^2 L}} \right] - \frac{(1 - \theta)\overline{\Psi}}{r} N \left[ \frac{\log \left( \frac{\beta}{\overline{\Psi}} \right) + mL}{\sqrt{\sigma^2 L}} \right] \right. \\
- \left( \frac{\beta}{\overline{\Psi}} \right)^{-\frac{2\sigma^2}{\beta}} \left( \frac{\overline{\Psi}}{\beta} - \frac{(1 - \theta)\overline{\Psi}}{r} \right) \left. e^{xL} N \left[ \frac{\log \left( \frac{\beta}{\overline{\Psi}} \right) + mL - 2rL}{\sqrt{\sigma^2 L}} \right] \right\} .
\]  

(A.43)
To determine the value of equity \( \hat{S}_t \) for \( t > t^* \), note that, using the change of variable 
\[
e^y = \left( \frac{V_t}{V_B} \right)
\]
we can write the value of equity \( S_t \) in the manager’s information set (9) as
\[
S(y_t) = V_B e^{y_t} - \frac{(1 - \theta)C}{r} e^{-\sigma^2 y_t} \left[ V_B - \frac{(1 - \theta)C}{r} \right], \tag{A.44}
\]
The value of equity in the creditors’ information set for \( t > t^* \) is then obtained by integrating (A.44) over \( y_t \) after using the conditional densities (A.24), for \( t \geq t^* + L \), and (A.34), for \( t^* < t < t^* + L \). This yields
\[
\hat{S}_t = \begin{cases} 
\int_0^\infty \pi(y_t | \tau_d > t, \hat{y}_t) S(y_t, C_{t^*}) \, dy_t, & \text{if } t^* < t < t^* + L \tag{A.45a} \\
\int_0^\infty \pi(y_t | \tau_d > t, \hat{y}_t) S(y_t, C_{t^*}) \, dy_t, & \text{if } t \geq t^* + L \tag{A.45b}
\end{cases}
\]

**Proof of Proposition 4**

Note that if both \( \hat{V}_t \) and \( C \) are scaled by a factor \( \nu \), \( V_B = C/\beta \), would also scale by \( \nu \), and therefore \( \hat{y}_t = \log(\hat{V}_t/V_B) \) would be unaffected by this scaling. This implies that \( D_3(t^*, \hat{y}_{t^*}) \) would scale by the factor \( \nu \). Plugging these scaled factors into equation (33), we see that the bond price \( D_2(t^*, \hat{y}_{t^*}) \) would also scale by the same factor \( \nu \). To emphasize this dependence, we express the debt value as
\[
D_2(\hat{y}_{t^*}, t^*, C_{t^*}) = C_{t^*} \times D_2(\hat{y}_{t^*}, t^*, 1).
\]
Indeed, even though (33) represents the value of debt in Regime 2 for all values of \( \hat{y}_{t^*} \), in fact only one value is relevant, namely, the value for which \( C_{t^*} = \frac{C_{t^*}}{V_{t^*}} \), and thus,
\[
\hat{y}_{t^*} = \log \left( \frac{C_{t^*}}{V_{t^*}} \right) = \log \left( \frac{\beta}{\psi} \right).
\]
That is, the only value we use below is
\[
D_2 \left( \log \left( \frac{\beta}{\psi} \right), t^*, 1 \right). \tag{A.46}
\]

Expressing the bond value for \( t < t^* \) in (13a) as a function of the asset value, \( D(\hat{V}_t, C_t) \), from Itô’s lemma and the law of motion of \( V_t \) in (1) and \( C_t \) in (37), we have that \( D \) satisfies the following PDE:
\[
0 = C_t - \left( r + (1 - \theta) \frac{C_t}{D_1} \right) D + (1 - \theta) \frac{C^2}{D} D_C + r \hat{V} D_{\hat{V}} + \frac{\sigma^2}{2} \hat{V}^2 D_{\hat{V}^2} + D_t \\
= -r D + \theta C + (1 - \theta) \frac{C^2}{D} D_C + r \hat{V} D_{\hat{V}} + \frac{\sigma^2}{2} \hat{V}^2 D_{\hat{V}^2}, \tag{A.47}
\]
subject to the boundary conditions

\[
\lim_{\hat{V} \to \infty} D(\hat{V}, C) = \frac{C}{r} \\
\lim_{C/V \to \Psi} D(\hat{V}, C) = D_2 \left( \log \left( \frac{\beta}{\Psi} \right), t^*, C_t^* \right). \tag{A.48}
\]

Given the scaling property of debt in Regimes 2–4, we guess a solution of the form:

\[
D(\hat{V}_t, C_t) = \hat{V}_t G(\Psi_t), \text{ where } \Psi_t \equiv \frac{C_t}{\hat{V}_t}. \tag{A.49}
\]

The partial derivatives in (A.47) can be expressed as:

\[
D_C = G_\Psi \\
D_{\hat{V}} = G - \Psi G_\Psi \\
D_{\hat{V} \hat{V}} = \left( \frac{\Psi^2}{\hat{V}} \right) G_{\Psi \Psi} \\
D_t = \hat{V} G_t. \tag{A.50}
\]

Substituting into (A.47), and dividing through by \( \hat{V} \), we see that our guess is self-consistent and that the PDE reduces to:

\[
0 = \theta \Psi - r G + (1 - \theta) \Psi^2 G_\Psi \frac{G_{\Psi}}{G} + r (G - \Psi G_\Psi) + \frac{\sigma^2}{2} \Psi^2 G_{\Psi \Psi}. \tag{A.51}
\]

The boundary conditions simplify to:

\[
G(0) = \frac{\Psi_t}{r} \tag{A.52}
\]

\[
G(\Psi) = \Psi D_2 \left( \log \left( \frac{\beta}{\Psi} \right), t^*, 1 \right). \tag{A.53}
\]

\[\square\]

**Proof of Proposition 5**

Expressing the equity value for \( t < t^* \) in (42a) as a function of the asset value, \( \hat{S}(\hat{V}_t, C_t) \), from Itô’s lemma and the law of motion of \( V_t \) in (1) and \( C_t \) in (37), we have that \( \hat{S} \) satisfies the following ODE:

\[
0 = -r \hat{S} + r \hat{V} \hat{S}_\hat{V} + \frac{\sigma^2}{2} \hat{V}^2 \hat{S}_{\hat{V} \hat{V}} + \hat{S}_c (1 - \theta) \left( \frac{C^2}{D(\hat{V}, C)} \right), \tag{A.54}
\]

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subject to the boundary conditions

\[
\lim_{C/V \to \Psi} \hat{S}(\hat{V}, C) = \hat{S}_{t^*}, \quad (A.55)
\]

\[
\lim_{\hat{V} \to 0} D(\hat{V}, C) = 0, \quad (A.56)
\]

where \( \hat{S}_{t^*} \) is given in Equation (43) of Proposition 3. Notice that, from (9), the equity value \( S(V_t, C) \) satisfies the scaling property \( S(V_t, C) = V_t \times S(V_t/C, 1) \). Hence we guess a solution for equation (A.54) of the form

\[
\hat{S}(\hat{V}_t, C_t) \equiv \hat{V}_t H(\Psi_t) \quad \text{where} \quad \Psi_t \equiv \frac{C_t}{\hat{V}_t}. \quad (A.57)
\]

Substituting this guess in (A.54)–(A.56) we find that our scaling assumption is correct, and that \( H \) satisfies the ODE:

\[
0 = -r\Psi H_{\Psi} + \frac{\sigma^2}{2} \Psi^2 H_{\Psi \Psi} + \frac{H_{\Psi}}{G(\Psi)} (1 - \theta) \Psi^2, \quad (A.58)
\]

subject to the boundary conditions

\[
H(\Psi) = e^{mL + \frac{\sigma^2 L}{2}} N \left[ \frac{\log \left( \frac{\beta}{\Psi} \right) + mL + \sigma^2 L}{\sqrt{\sigma^2 L}} \right] - \frac{(1 - \theta)\Psi}{r} N \left[ \frac{\log \left( \frac{\beta}{\Psi} \right) + mL}{\sqrt{\sigma^2 L}} \right] - \left( \beta \right)^{-\frac{2\Psi}{\sigma^2 L}} \frac{\left( \Psi - (1 - \theta)\Psi \right)}{r} \ e^{rL} N \left[ \frac{\log \left( \frac{\beta}{\Psi} \right) + mL - 2rL}{\sqrt{\sigma^2 L}} \right]. \quad (A.59)
\]

\[
H(0) = 1, \quad (A.60)
\]
Table 1: **Empirical Defaults Rates.** Each month, we classify firms as investment grade (IG) higher-quality speculative grade (B), and lower-quality speculative-grade firms (C). In Panels A and B, the classification is based on credit ratings issued by the three main rating agencies (Moody’s, Standard and Poor’s, and Fitch). In Panel C, the classification is implied by the price of CDS contracts written on debt issued by the firms. Panel A shows average annualized default rates from 1985 to 2014 for firms in each rating category that have defaulted in the next 12 months, while Panels B and C show default rates for the 2001-2014 period. Heteroskedasticity- and autocorrelation-robust (Newey-West) standard errors are in parentheses.

<table>
<thead>
<tr>
<th>Rating</th>
<th>Annualized Default Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0-1M</td>
</tr>
<tr>
<td>IG</td>
<td>0.06 (0.02)</td>
</tr>
<tr>
<td>B</td>
<td>0.20 (0.05)</td>
</tr>
<tr>
<td>C</td>
<td>14.46 (1.41)</td>
</tr>
<tr>
<td>IG</td>
<td>0.07 (0.03)</td>
</tr>
<tr>
<td>B</td>
<td>0.21 (0.07)</td>
</tr>
<tr>
<td>C</td>
<td>12.94 (1.64)</td>
</tr>
<tr>
<td>IG</td>
<td>0.01 (0.01)</td>
</tr>
<tr>
<td>B</td>
<td>0.29 (0.11)</td>
</tr>
<tr>
<td>C</td>
<td>3.30 (0.91)</td>
</tr>
</tbody>
</table>
Table 2: **Baseline Model Coefficients.** Below are the values of the model coefficients in the baseline calibration.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative coupon at time ( t^* )</td>
<td>( C_{t^*} )</td>
<td>1</td>
</tr>
<tr>
<td>Annual risk-free rate</td>
<td>( r )</td>
<td>0.5%</td>
</tr>
<tr>
<td>Annual asset volatility</td>
<td>( \sigma )</td>
<td>0.3</td>
</tr>
<tr>
<td>Corporate tax rate</td>
<td>( \theta )</td>
<td>0.25</td>
</tr>
<tr>
<td>Loss given default</td>
<td>( \alpha )</td>
<td>0.4</td>
</tr>
<tr>
<td>Maximum debt capacity</td>
<td>( \Psi )</td>
<td>0.03</td>
</tr>
<tr>
<td>Creditors’ information delay (in years)</td>
<td>( L )</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 3: **Average Model-Implied Default Rates.** We simulate a history of 10,000 firms from our model and track them from inception through their default date. For each firm and at any point in time of the simulations we record the time to default and classify the observation as investment grade (IG) if the firm’s leverage is below 65%. We classify as higher-quality speculative grade (B category) firms with leverage between 65% and 75%. Lower-quality speculative-grade firms (C category) have leverage in excess of 75%. The table shows average default rates across firms in the simulated sample.

<table>
<thead>
<tr>
<th>Average annualized default rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-1M</td>
</tr>
<tr>
<td>IG</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>C</td>
</tr>
</tbody>
</table>
Figure 1: **Percentage of Firms by Credit Ratings.** The plots show the percentage of firms in each of the three rating categories: investment grade (the IG category), higher-quality speculative grade (the B category), and lower-quality speculative grade (the C category). The sample period goes from 1985 to 2014. Source: Mergent database.

Figure 2: **Average CDS Premium on Investment Grade Firms up to Bankruptcy.** Among the firms that went bankrupt from 2001 to 2014, we classify as investment grade those that had CDS contracts trading at a premium no higher than 100 basis points of the CDX Investment Grade Index for at least one of the 12 months preceding the bankruptcy date. The plot shows the average CDS premium, in excess of the CDX Investment Grade Index, on those investment grade firms in the 12 months leading up to their bankruptcy.
Firm issues optimal mix debt/equity
Creditors observe $v_s$
No default, zero default intensity

Firm issues debt to service existing debt
Creditors observe $v_s$
No default, zero default intensity

$t^*$ Firm reaches max debt capacity
Possible "jump to default"

$t^* + L$ Firm can only issue equity
Creditors:
- observe $v_s$
- observe any default immediately
Default intensity depends on $v_s$ and $t^*$

Figure 3: Model Timeline

Figure 4: Optimal Capital Structure. The plots show the optimal time-0 capital structure as a function of the information lag $L$ between creditors and the manager, where $L$ ranges from 0 to 1 year, $0 \leq L \leq 1$. The ‘BBG’ line denotes our baseline model in which the manager can issue debt till borrowing capacity is reached; the ‘Duffie-Lando’ line corresponds to a firm that can only issue equity to service debt in place. Parameter values are in Table 2.
Figure 5: **Optimal Capital Structure and Credit Constraints.** The plots show the optimal leverage at times $t = 0$ and $t = t^*$ as a function of the credit constraint parameter $\psi$ and the information lag $L$. The left panels show optimal leverage as a function $\psi$ for three values of $L$, $L = 0.05, 0.5$ and 1 year. The right panels show optimal leverage as a function of $L$ for three values of $\psi$, $\psi = 0.02, 0.03$ and 0.04. Parameter values are in Table 2.
Figure 6: **Optimal Capital Structure and Assets’ Volatility.** The plots show the optimal leverage at times $t = 0$ and $t = t^*$ as a function of the asset volatility parameter $\sigma$ and the information lag $L$. The left panels show optimal leverage as a function $\sigma$ for three values of $L$, $L = 0.05, 0.5$ and 1 year. The right panels show optimal leverage as a function of $L$ for three values of $\sigma$, $\sigma = 0.25, 0.3$ and 0.35. Parameter values are in Table 2.
Figure 7: **Credit Spreads and Information Asymmetry.** The plots illustrate the sensitivity of the credit spreads curve to the information gap parameter $L$. In all cases, the initial leverage is fixed at 47.5%, so as to match the typical 60 bps five-year spread of an investment-grade firm in the baseline BGG model. Parameter values are in Table 2.

Figure 8: **Credit Spreads and Credit Constraints.** The plots illustrate the sensitivity of the credit spreads curve to the credit constraint parameter $\Psi$. In all cases, the initial leverage is fixed at 47.5%, so as to match the typical 60 bps five-year spread of an investment-grade firm in the baseline BGG model. Parameter values are in Table 2.
Figure 9: **Credit Spreads and Assets’ Volatility.** The plots illustrate the sensitivity of the credit spreads curve to the asset volatility parameter $\sigma$. In all cases, the initial leverage is fixed at 47.5%, so as to match the typical 60 bps five-year spread of an investment-grade firm in the baseline BGG model. Parameter values are in Table 2.
References


