A Life-Cycle Model with Unemployment Traps*

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Abstract

The Great Recession highlighted that long-term unemployment may become a trap with loss of human capital. This paper extends the life-cycle model by allowing for a small risk of long-term unemployment with permanent effects on labour income. Such nonlinear income risk dampens early investment in risky assets, resulting in an optimal equity portfolio share that is relatively flat over the life cycle. This flattening in the life-cycle profile is driven by the resolution of uncertainty as the worker ages. Shifting away from a simple age rule to a flatter investment profile yields average welfare gains that are three times larger than those in models with linear labour income shocks.

Keywords: disaster risk, life-cycle portfolio choice, unemployment risk, human capital depreciation, age rule.

JEL classification: D15, E21, G11

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1 Introduction

Unemployment leads to large and persistent earnings losses that increase with the duration of unemployment because of skill deterioration. The magnitude of this effect varies over time and across industries and demographic groups (Rhum, 1991; Jacobson, Lalond and Sullivan, 1993a; Davis and von Wachter, 2011) as well as countries (Machin and Manning, 1999). Recently, the average length of unemployment spells has remarkably increased in developed economies. For example, in the United States the share of unemployed workers who are jobless for more than one year doubled during the Great Recession episode, reaching 24% of total unemployment in 2014. Krueger, Cramer and Cho (2014) and Kroft, Lange, Notowidigdo and Katz (2016) show that the re-employability of the long-term unemployed progressively declines over time, to the extent that they are more likely to exit the labour force than to become re-employed. The presence of more job openings does not lead to increased employment among individuals who are jobless for more than six months, and this pattern holds across all ages, industries and education levels (Ghayad and Dickens 2012). Overall, these findings indicate that long-term unemployment may become a trap, often unsupported by supplementary income provisions, given that unemployment benefits usually decline rapidly as unemployment continues.

In this paper, we embed the possibility of entering long-term unemployment with permanent consequences on human capital in a life-cycle model of consumption and portfolio choice. We model working life careers as a three-state Markov chain driving the transitions between employment and short-term and long-term unemployment states, as in Bremus and Kuzin (2014). Careers are calibrated to broadly match observed US labour market features. Importantly, we allow for (a small probability of) human capital erosion during unemployment. When unemployed, individuals receive benefits but simultaneously experience a reduction in the permanent component of labour income which translates to diminished future income prospects. Permanent earning losses are subsequently observed due to skill loss during long-term unemployment (Neal, 1995; Arulampalam, 2001; Edin and Gustavsson, 2008; Schmieder, von Wachter and Bender, 2016).

Such potential loss in human capital considerably lowers the optimal portfolio share invested in stocks compared with the case of no unemployment risk. Importantly, optimal stock investment no longer decreases with age but remains remarkably flat over the whole working life, in line with evidence on US portfolios (Ameriks and Zeldes, 2004). In contrast, traditional life-cycle models imply that workers should reduce exposure to risky stocks as they approach retirement (Bodie, Merton and Samuelson, 1992; Viceira 2001; Cocco, Gomes and Maenhout 2005). Human capital provides a hedge against shocks to stock returns, which makes financial risk bearing generally acceptable. Investment
in stocks should therefore be relatively high at the beginning of working careers, when human capital is large relative to accumulated financial wealth. Investment then gradually declines until retirement, as human capital decreases relative to financial wealth. The implication of this model is embodied in the popular "age rule" which advises a gradual decrease in stock exposure with increasing age. In our model with unemployment traps, the effect is instead moderated by the resolution of uncertainty concerning labour and pension income as the worker safely approaches retirement age. Since the risk of long-term unemployment falls along with human capital as retirement approaches, the resolution of uncertainty compensates for the hedge effect and the optimal investment in stocks is relatively flat over the life cycle. Our model reinforces and complements the results in Chang, Hong and Karabarbounis (2017). On the one hand the risk of falling in an unemployment trap is present even for young highly educated workers who may ex-ante be confident about their earnings ability. Thus, augmenting the model in Chang, Hong and Karabarbounis (2017) with unemployment traps could improve its ability to match the conditional stock investing observed in US data. Secondly, our insight implies cross-country variation in the age pattern of stock investing depending on the degree of long-term unemployment insurance. For example, our results may be able to explain the decreasing age profile of conditional stock share in a country such as Norway (see, Fagereng, Gottlieb and Guiso, 2017), where the net replacement rate for the long-term unemployed has been fairly high.

The optimal risky portfolios are highly heterogeneous in models without long-term unemployment traps. In contrast, the small probability of such personal disaster shrinks the heterogeneity of optimal portfolio choices across agents characterized by different employment histories. In the face of possible, albeit rare, human capital depreciation, individuals accumulate substantially more financial wealth during their working life to buffer against possible adverse labour market outcomes. Optimal early consumption consequently falls, becoming higher during both late working years and retirement years. The working-year responses to unemployment risk, including the flat age profile in stock investment, are remarkably robust to changes in preferences on the intertemporal correlation of shocks. Allowing for Epstein-Zin preferences only causes slower wealth decumulation and less risk taking during retirement years. Similarly, an increase in the correlation between stock returns and labour income shocks leaves the flat shape of optimal equity investment during working age unaltered, only increasing the portfolio share allocated to the risk-free asset. Thus, unemployment risk due to uninsured long-term unemployment is the first-order determinant of optimal financial risk taking at different ages.

This paper is not the first to explicitly connect life cycle precautionary savings to social insurance in general (Hubbard, Skinner and Zeldes, 1995) and to insurance against
employment risk in particular (Low, Meghir and Pistaferri, 2010). Our analysis uncovers the link between the share of long-term unemployment risk that is left uninsured and the path of optimal equity risk taking during working years. In this respect, acknowledging the presence of unemployment traps provides relevant consequences for the design of pension fund default investment rules. Absent traps, welfare losses due to the use of simple age rules are below of 1% annual consumption in standard calibrations (Cocco, Gomes and Maenhout, 2005; Love, 2013); however, they are above 3% when allowing for traps that unemployment benefits do not cover. Such losses easily reach 10% of consumption for investment rules, mimicking those embedded in Target Date Funds. These losses are due to both excessive financial risk taking when young workers confront higher uncertainty about their future labour and pension income and by insufficient financial risk taking when this uncertainty is resolved. These suboptimal rules lead to lower consumption during retirement.

The above results are obtained based on US data, where the risk of long-term unemployment is small, but uniform across all education groups (e.g., see Kroft, Lange, Notowidigdo and Katz 2016). The implied unconditional probabilities of being short-run unemployed (3.8%) or long-run unemployed (0.6%) are quite conservative. By comparison, the total US unemployment rate in 2016 was about 5%, while the long-term (more than 52 weeks) unemployment rate was 1.7%. We set the unemployment benefit replacement rate at the average level observed in the United States. As for human capital erosion, we set it equal to 0% and 60% in case of short-term and long-term unemployment spells, respectively, to capture the relatively slow re-employment process experienced by US workers caught in the trap. Importantly, we select the human capital erosion during long-term unemployment considering both the total loss of human capital for the fraction of workers abandoning the labour force, and the partial loss for those who are able to find a job. Results go through even when the erosion parameter is substantially reduced, and when the probability of moving into long-term unemployment from an initial unemployment state is decreased by a third (from 0.15 to 0.10). We also experiment with a stochastic human capital loss conditional on long-term unemployment, to represent the possibility of incurring large losses only in very deep crisis situations rather than in normal business cycle downturns. In this case, our results are confirmed even when the expected erosion is as low as 10%-20% of the permanent labour income component after the second year of unemployment.

Previous life-cycle models with unemployment and self-insurance leave the observed age pattern of stock holding during working life largely unexplained. Some versions of

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1 For example, in 2013, the share of US unemployed workers with a high school (college) education who had been looking for work for two or more years is 12.8% (13.5%) (see, Mayer, 2014).

the life-cycle model account for the risk of being unemployed by introducing a (small) positive probability of zero labour income. In these models, unemployment risk affects income only during the unemployment spell and has no consequences on subsequent earnings ability (Cocco, Gomes and Maenhout, 2005), even when unemployment is persistent (Bremus and Kuzin, 2014). With no permanent consequence on subsequent earnings ability, the stock holding still counterfactually decreases with age till retirement, although the decrease is less on average than what occurs without unemployment risk. Thus, the possibility of unemployment traps - rather than unemployment per se - restrains risk taking by young and middle-aged workers. Therefore, our model draws attention to a scenario opposite that depicted by Bodie, Merton and Samuelson (1992) and Gomes, Kotlikoff and Viceira (2008) in which the worker is able - if employed - to modify labour supply to buffer income shocks. In fact, the flexible labour supply may enhance risk-taking, thereby compressing precautionary saving and reducing consumption after retirement. However, this option is available ex post only to long-term unemployed who find a new job; what drives our results is the ex ante risk of permanently losing human capital.

Several papers already investigate alternative hypotheses that may explain the relatively flat stock profile observed in the data that departs from the pattern implied by traditional life-cycle models. Some of this prior research relates the resolution of uncertainty over working life to the flattening of the age profile of stock investment. Hubener, Maurer and Mitchell (2016) highlight the possibility of changing family status during working age (i.e., marriage, fertility, divorce), which affects consumption both directly and through labour supply. In Bagliano, Fugazza and Nicodano (2014), such flattening depends on the presence of both another risky asset, aside from equities, and a positive correlation between stock returns and permanent labour income shocks. Moreover, the flattening only appears when risk aversion or the variance of labour income shocks are higher than in the baseline calibration of Cocco Gomes and Maenhout (2005). Chang, Hong and Karabarbounis (2017) show that realistic life-cycle profiles of occupational uncertainty and gradual learning about income volatility generate an age-increasing stock investment profile. In our model, the flat profile is robust to an in-depth sensitivity analysis and derives from the possibility of a rare personal labour market disaster, in an otherwise standard baseline setting.

This disaster differs from both the individual stock market disaster modelled in Fagereng, Gottlieb and Guiso (2017) and the aggregate economic collapse explaining asset pricing puzzles in Barro (2006). Both of these circumstances concern financial wealth and may occur during retirement as well. The set-up here instead reflects the rare idiosyncratic disaster in Schmidt (2016) that appears to capture both the magnitude and the dynamics of the equity risk premium. Such rare personal disaster makes
returns to human capital negatively skewed, a feature recently uncovered by Guvenen, Karahan, Ozkan and Song (2015). Moreover, Huggett and Kaplan (2016) show that persistence and negative skewness of earnings shocks reduce the value of human capital well below the level implied by discounting earnings at the risk-free rate and increase its stock component. In this light, our paper documents the large effects of non-normal shocks to labour income on life-cycle savings and investment, following the suggestion in Blundell (2014) of capturing higher moments and nonlinearities in shocks to labour income. Thus, our paper extends the literature on portfolio choice that has so far focused only on non-Gaussian returns to financial assets (e.g., see Guidolin and Timmerman, 2008).

The rest of the paper is organized as follows. Section 2 presents the benchmark life-cycle model and briefly outlines the numerical solution procedure adopted. We detail the model calibration in Section 3 and discuss our main results in Section 4. Section 5 provides a quantitative assessment of the welfare loss entailed by departing from optimal asset allocation profiles and adopting conventional age-related investment rules. Several robustness checks are presented in Section 6. Section 7 concludes the paper.

## 2 The life-cycle model

We model an investor who maximizes the expected discounted utility of consumption over her entire life and wishes to leave a bequest as well. The investor starts working at age $t_0$ and retires with certainty at age $t_0 + K$. The effective length of her life, which lasts at most $T$ periods, is governed by age-dependent life expectancy. At each date $t$, the survival probability of being alive at date $t + 1$ is $p_t$, the conditional survival probability at $t$ (with $p_{t0−1} = 1$). Investor’s preferences at date $t$ are described by a time-separable power utility function:

$$
\frac{C_{it}^{1-\gamma}}{1-\gamma} + E_{t0} \left[ \sum_{j=1}^{T} \beta^j \left( \prod_{k=0}^{j-2} p_{t0+k} \right) \left( p_{t0+j-1} \frac{C_{it0+j}^{1-\gamma}}{1-\gamma} + (1 - p_{t0+j-1}) b \frac{(X_{it0+j}/b)^{1-\gamma}}{1-\gamma} \right) \right]
$$

where $C_{it}$ is the level of consumption at time $t$, $X_{it}$ is the amount of wealth the investor leaves as a bequest to her heirs after her death, $b \geq 0$ is a parameter capturing the strength of the bequest motive, $\beta < 1$ is a utility discount factor, and $\gamma$ is the constant relative risk aversion parameter.

### 2.1 Labour and retirement income

During working life individuals receive exogenous stochastic earnings as compensation for labour supplied inelastically. Working life careers are modelled as a three-state Markov
chain considering employment \((e)\), short-term \((u_1)\) and long-term \((u_2)\) unemployment. Individual labour market dynamics are driven by the following transition matrix:

\[
\Pi_{s_t, s_{t+1}} = \begin{pmatrix}
\pi_{ee} & \pi_{eu_1} & \pi_{eu_2} \\
\pi_{u_1e} & \pi_{u_1u_1} & \pi_{u_1u_2} \\
\pi_{u_2e} & \pi_{u_2u_1} & \pi_{u_2u_2}
\end{pmatrix} = \begin{pmatrix}
\pi_{ee} & 1 - \pi_{ee} & 0 \\
0 & 1 - \pi_{u_1e} & 0 \\
0 & 1 - \pi_{u_2e}
\end{pmatrix}
\]

(2)

where \(\pi_{nm} = \text{Prob}(s_{t+1} = n | s_t = m)\) with \(n, m = e, u_1, u_2\). If the worker is employed at \(t\) \((s_t = e)\), she continues the employment spell at \(t + 1\) \((s_{t+1} = e)\) with probability \(\pi_{ee}\), otherwise she enters short-term unemployment \((s_{t+1} = u_1)\) with probability \(\pi_{eu_1} = 1 - \pi_{ee}\). Since she must experience short-term unemployment prior to becoming long-term unemployed, we set the probability of directly entering long-term unemployment at zero, \(\pi_{eu_2} = 0\). Conditional on being short-term unemployed at \(t\) \((s_t = u_1)\), she exits unemployment \((s_{t+1} = e)\) with probability \(\pi_{u_1e}\) or becomes long-term unemployed \((s_{t+1} = u_2)\) with probability \(\pi_{u_1u_2} = 1 - \pi_{u_1e}\); consequently, we set \(\pi_{u_1u_1} = 0\). Finally, if she is long-term unemployed at \(t\) \((s_t = u_2)\), she is re-employed in the following period \((s_{t+1} = e)\) with probability \(\pi_{u_2e}\) and remains unemployed with probability \(\pi_{u_2u_2} = 1 - \pi_{u_2e}\).

As in Cocco, Gomes and Maenhout (2005), the employed individual receives a stochastic labour income driven by permanent and transitory shocks. In each working period, labour income \(Y_{it}\) is generated by the following process:

\[
Y_{it} = H_{it}U_{it} \quad t_0 \leq t \leq t_0 + K
\]

(3)

where \(H_{it} = F(t, Z_{it})P_{it}\) represents the permanent income component. In particular, \(F(t, Z_{it}) \equiv F_{it}\) denotes the deterministic trend component that depends on age \((t)\) and a vector of individual characteristics \((Z_{it})\) such as gender, marital status, household composition and education. Consistent with the available empirical evidence, the logarithm of the stochastic permanent component is assumed to follow a random walk process:

\[
N_{it} = \log P_{it} = \log P_{it-1} + \omega_{it}
\]

(4)

where \(\omega_{it}\) is distributed as \(N(0, \sigma^2_{\omega})\). \(U_{it}\) denotes the transitory stochastic component and \(\varepsilon_{it} = \log(U_{it})\) is distributed as \(N(0, \sigma^2_{\varepsilon})\) and uncorrelated with \(\omega_{it}\).

In our set-up, which differs from that of Bremus and Kuzin (2014), labour income received by the employed individual at time \(t\) depends on her past working history. In particular, we allow unemployment and its duration to affect the permanent component of labour income, \(H_{it}\). Since the empirical evidence suggests that the longer the unemployment spell the larger is the worker’s human capital depreciation (Schmieder, von Wachter
and Bender, 2016), we let human capital erosion increase with unemployment duration. Thus, after 1-year unemployment the permanent component $H_{it}$ is equal to $H_{it-1}$ eroded by a fraction $\Psi_1$, and after a 2-year unemployment spell the permanent component, $H_{it-1}$, is eroded by a fraction $\Psi_2$, with $\Psi_2 > \Psi_1$. This introduces non-linearity into the expected permanent labour income. In compact form, the permanent component of labour income $H_{it}$ evolves according to

$$H_{it} = \begin{cases} F(t, Z_{it}) P_{it} & \text{if } s_t = e \text{ and } s_{t-1} = e \\ (1 - \Psi_1) H_{it-1} & \text{if } s_t = e \text{ and } s_{t-1} = u_1 \\ (1 - \Psi_2) H_{it-1} & \text{if } s_t = e \text{ and } s_{t-1} = u_2 \text{ or if } s_t = u_2 \text{ and } s_{t-1} = u_2 \end{cases} \quad t = t_0, \ldots, t_0 + K$$

(5)

In the short-term unemployment state ($s_t = u_1$) individuals receive an unemployment benefit as a fixed proportion $\xi_1$ of the previous year permanent income $H_{it-1} = F_{it-1} P_{it-1}$, whereas in the long-term unemployment state ($s_t = u_2$) no benefits are available: $\xi_2 = 0$. Thus, the income received during unemployment is

$$Y_{it} = \begin{cases} \xi_1 H_{it-1} & \text{if } s_t = u_1 \text{ and } s_{t-1} = e \\ 0 & \text{if } s_t = u_2 \end{cases} \quad t = t_0, \ldots, t_0 + K$$

(6)

making the unconditional distribution of labour income no longer log-normal.

Finally, during retirement, income is certain and equal to a fixed proportion $\lambda$ of the permanent component of labour income in the last working year:

$$Y_{it} = \lambda F \left( t, Z_{it(t_0+)} \right) P_{it(t_0+)} \quad t_0 + K < t \leq T$$

(7)

where retirement age is $t_0 + K$, $t_0 + l$ is the last working period and $\lambda$ is level of the replacement rate.

### 2.2 Investment opportunities

We allow savings to be invested in a short-term riskless asset, yielding a constant gross real return $R^f$, and one risky asset, characterized as “stocks” yielding stochastic gross real returns $R_t^s$, for each period. The excess returns of stocks over the riskless asset follows

$$R_t^s - R^f = \mu^s + \nu_t^s$$

(8)

where $\mu^s$ is the expected stock premium and $\nu_t^s$ is a normally distributed innovation, with mean zero and variance $\sigma^2_s$. We do not allow for excess return predictability and
other forms of changing investment opportunities over time, as in Michaelides and Zhang (2017).

At the beginning of each period, financial resources available to the individual for consumption and saving are given by the sum of accumulated financial wealth $W_{it}$ and current labour income $Y_{it}$, which we call cash on hand $X_{it} = W_{it} + Y_{it}$. Given the chosen level of current consumption, $C_{it}$, next period cash on hand is given by

$$X_{it+1} = (X_{it} - C_{it})R_{it}^P + Y_{it+1}$$  \hspace{1cm} (9)

where $R_{it}^P$ is the investor’s portfolio return:

$$R_{it}^P = \alpha_{it}^s R_{it}^s + (1 - \alpha_{it}^s) R_{it}^f$$  \hspace{1cm} (10)

with $\alpha_{it}^s$ and $(1 - \alpha_{it}^s)$ denoting the shares of the investor’s portfolio invested in stocks and in the riskless asset respectively. We do not allow for short sales and we assume that the investor is liquidity constrained. Consequently, the amounts invested in stocks and in the riskless asset are non negative in all periods. All simulation results presented below are derived under the assumption that the investor’s asset menu is the same during working life and retirement.

### 2.3 Solving the life-cycle problem

In this intertemporal optimization framework, the investor maximizes the expected discounted utility over life span, by choosing the consumption and the portfolio rules given uncertain labour income and asset returns. Formally, the optimization problem is written as:

$$\max_{\{C_{it}\}_{t=0}^{T-1}, \{\alpha_{it}^s\}_{t=0}^{T-1}} \left[ \frac{C_{it_0}^{1-\gamma}}{1-\gamma} + E_{t_0} \left[ \sum_{j=1}^{T} \beta^j \left( \prod_{k=0}^{j-2} p_{t_0+k} \right) \left( p_{t_0+j-1} \frac{C_{it_0+j}^{1-\gamma}}{1-\gamma} \right. \right. \right. $$

$$\left. \left. + (1 - p_{t_0+j-1}) b \left( X_{it_0+j}/b \right)^{1-\gamma} \right) \right]$$  \hspace{1cm} (11)

$$s.t. \quad X_{it+1} = (X_{it} - C_{it}) \left( \alpha_{it}^s R_{it}^s + (1 - \alpha_{it}^s) R_{it}^f \right) + Y_{it+1}$$  \hspace{1cm} (12)

with the labour income and retirement processes specified above and the no-short-sales and borrowing constraints imposed. Given its intertemporal nature, the problem can be restated in a recursive form, rewriting the value of the optimization problem at the beginning of period $t$ as a function of the maximized current utility and of the value of
the problem at $t + 1$ (Bellman equation):

$$
V_{it} (X_{it}, P_{it}, s_{it}) = \max \left\{ \frac{C_{it}^{1-\gamma}}{1 - \gamma} + \beta E_t [p_t V_{it+1} (X_{it+1}, P_{it+1}, s_{it+1})]
+ (1 - p_t) b \frac{(X_{it+1}/b)^{1-\gamma}}{1 - \gamma} \right\} \quad (13)
$$

At each time $t$ the value function $V_{it}$ describes the maximized value of the problem as a function of three state variables: cash on hand at the beginning of time $t$ ($X_{it}$), the stochastic permanent component of income at beginning of $t$ ($P_{it}$), and the labour market state $s_{it} (= e, u_1, u_2)$. The Bellman equation can be written by making the expectation over the employment state at $t + 1$ explicit:

$$
V_{it} (X_{it}, P_{it}, s_{it}) = \max \left\{ \frac{C_{it}^{1-\gamma}}{1 - \gamma} + \beta \sum_{s_{it+1}=e, u_1, u_2} p_t \pi (s_{it+1}|s_{it}) E_t \hat{V}_{it+1} (X_{it+1}, P_{it+1}, s_{it+1})
+ (1 - p_t) b \sum_{s_{it+1}=e, u_1, u_2} \pi (s_{it+1}|s_{it}) \frac{(X_{it+1}/b)^{1-\gamma}}{1 - \gamma} \right\} \quad (14)
$$

where $E_t \hat{V}_{it+1}$ denotes the expectation operator taken with respect to the stochastic variables $\omega_{it+1}, \varepsilon_{it+1},$ and $\nu_{it+1}$. The history dependence that we introduce in our set-up by making unemployment affect subsequent labour income prospects prevents having to rely on the standard normalization of the problem with respect to the level of $P_t$. To highlight how the evolution of the permanent component of labour income depends on previous individual labour market dynamics we write the value function at $t$ in each possible state as (dropping the term involving the bequest motive):

$$
V_{it}(X_{it}, P_{it}, c) = u(C_{it}) + \beta p_t
\begin{cases}
V_{it+1}(X_{it+1}, P_{it+1}, c) & \text{with prob. } \pi_{e,e} \\
\text{with } P_{it+1} = P_{it} e^{\omega_{it+1}} & \text{and} \\
X_{it+1} = (X_{it} - C_{it}) R_{it} + F_{it+1} P_{it+1} e^{\varepsilon_{it+1}} \\
V_{it+1}(X_{it+1}, P_{it+1}, u_1) & \text{with prob. } 1 - \pi_{e,e} \\
\text{with } P_{it+1} = (1 - \Psi_1) P_{it} & \text{and} \\
X_{it+1} = (X_{it} - C_{it}) R_{it} + \xi_1 F_{it} P_{it}
\end{cases}
$$

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This problem has no closed form solution; therefore, we obtain the optimal values for consumption and portfolio shares, depending on the values of each state variable at each point in time, by means of numerical techniques. To this aim, we apply a backward induction procedure starting from the last possible period of life $T$ and computing optimal consumption and portfolio share policy rules for each possible value of the continuous state variables ($X_{it}$ and $P_{it}$) by means of the standard grid search method.\(^3\) Going backwards, for every period $t = T - 1, T - 2, \ldots, t_0$, we use the Bellman equation (14) to obtain optimal rules for consumption and portfolio shares.

### 3 Calibration

Parameter calibration concerns investor’s preferences, the features of the labour income process during working life and retirement, and the moments of the risky asset returns. For reference, we initially solve the model by abstracting from the unemployment risk as in Cocco, Gomes and Maenhout (2005). Then, we introduce unemployment risk and consider two scenarios: (i) unemployment spells cause only temporary income losses, as in Bremus and Kuzin (2014), and (ii) unemployment has permanent consequences on the worker’s earnings ability.

Across all scenarios, the agent begins her working life at the age of 20 and works for (a maximum of) 45 periods ($K$) before retiring at the age of 65. After retirement, she can live for a maximum of 35 periods until the age of 100. In each period, we take the conditional probability of being alive in the next period $p_t$ from the life expectancy

\(^3\)The problem is solved over a grid of values covering the space of both the state variables and the controls in order to ensure that the obtained solution is a global optimum.
tables of the US National Center for Health Statistics. With regards to preferences, we set the utility discount factor $\beta = 0.96$, and the parameter capturing the strength of the bequest motive $b = 2.5$ (which bears the interpretation of the number of years of her descendants’ consumption that the investor intends to save for). Finally, the benchmark value for the coefficient of relative risk aversion is $\gamma = 5$. The latter choice is relatively standard in the literature (Gomes and Michaelides 2005; Gomes, Kotlikoff and Viceira 2008) and captures an intermediate degree of risk aversion. However, Cocco, Gomes and Maenhout (2005) and Bremus and Kuzin (2014) choose a value as high as 10 in their benchmark setting. The riskless (constant) interest rate is set at 0.02, with an expected equity premium $\mu_s$ fixed at 0.04. The standard deviation of the return innovations is set at $\sigma_s = 0.157$. Finally, we impose a zero correlation between stock return innovations and aggregate permanent labour income disturbances ($\rho_{sY} = 0$). Table 1 summarizes the benchmark values of relevant parameters as well as their changes considered in our subsequent analysis and robustness checks.

### 3.1 Labour income and unemployment risk

The labour income process is calibrated using the estimated parameters for US households with high school education (but not a college degree) in Cocco, Gomes and Maenhout (2005).

The share of long-term unemployment in total unemployment is fairly similar across all education groups (Kroft, Lange, Notowidigdo and Katz, 2016). For instance, Mayer (2014) observes that the percentage of unemployed workers who have been out of work for 2 years or more with a high school degree (12.8%) and with a bachelor’s degree (13.5%) do not differ statistically. For the high school group, the variances of the permanent and transitory shocks ($\omega_{it}$ and $\varepsilon_{it}$ respectively) are equal to $\sigma^2_\omega = 0.0106$ and $\sigma^2_\varepsilon = 0.0738$. After retirement, income is a constant proportion $\lambda$ of the final (permanent) labour income, with $\lambda = 0.68$\textsuperscript{4}. The parameter values assumed above are maintained across all scenarios.

The resulting labour income process does not capture the evidence in Krueger, Cramer and Cho (2014) that the long-term unemployed experience a progressive declining re-employability over time and are more likely to exit the labour force. We use data from the Current Population Survey (CPS) to calibrate the transition probabilities from employment to unemployment to reflect the risk of entering unemployment along with the observed average unemployment rates at different durations. According to the evidence based on CPS reported in Kroft, Lange, Notowidigdo and Katz (2016), the annual tran-

\textsuperscript{4}For a more realistic Social Security System design and its implications on retirement, consumption and investment decisions see Hubener, Maurer and Mitchell (2016).
The transition probability from employment to unemployment is 4%. Given the duration dependence and the steady decline in the annual outflow rate from unemployment to employment during the first year of unemployment (Kroft, Lange, Notowidigdo and Katz, 2016), we set the probability of leaving unemployment after the first year at 85%. Our calibration appears quite conservative, since the chance of being employed 15 months later for those who had been unemployed 27 weeks or more is only 36% (see the evidence on CPS data in Krueger, Cramer and Cho, 2014).

The annual transition probabilities between labour market states are chosen to match the average annual unemployment rate in the United States:

\[
\Pi_{s_t,s_{t+1}} = \begin{pmatrix}
0.96 & 0.04 & 0 \\
0.85 & 0 & 0.15 \\
0.85 & 0 & 0.15
\end{pmatrix}
\] (16)

The assumed transition matrix (16) yields quite conservative unconditional probabilities of being short-run (3.8%) and long-run unemployed (0.6%), compared to the 2015 overall (5.3%) and long-term (1.7%) unemployment rates.

In our baseline calibration with “unemployment traps” we assume a non-negligible human capital depreciation following a 2-year unemployment spell. While \(\Psi_1\) is kept at 0, \(\Psi_2\) is increased up to 0.6, implying a 60% erosion of the individual permanent labour income component after the second year of unemployment, which captures the long-lasting effects of protracted inactivity on job careers. Well-established empirical evidence on job displacement shows that job losses affect earnings far beyond the unemployment spell, though the range of the estimated effects varies considerably. For example, the estimates for immediate losses following displacement may range from 30% (Couch and Placzek, 2010) to 40% of earnings (Jacobson, Lalonde and Sullivan, 1993b). Earnings losses are shown to be persistent in a range from 15% (Couch and Placzek, 2010) to about 25% (Jacobson, LaLonde and Sullivan, 1993a) of their pre-displacement levels. These estimates abstract from the effect of unemployment duration, while Cooper (2013) finds that earnings losses are larger the longer unemployment lasts. Also, based on administrative data, Jacobson, LaLonde and Sullivan (2005) estimate that average earnings losses for displaced workers amount to 43-66% of their predisplacement wage. This body of evidence, combined with a probability of finding a job after being unemployed for 24 months as low as 40% (Kroft, Lange, Notowidigdo and Katz, 2016), leads us to calibrate a substantial expected drop in human capital following a long term unemployment spell. More precisely, we derive the baseline value for the parameter \(\Psi_2\) by considering the probability of leaving the labour force, and thus losing all human capital, as equal to 0.3 and the probability of finding a new job with a 40% cut in wage as 0.7.
Unemployment benefits are calibrated according to the US unemployment insurance system. In particular, considering that the replacement rate with respect to last labour income is on average low and state benefits are paid for a maximum of 26 weeks, we set \( \xi_1 = 0.3 \) in case of short-term unemployment spells and set a value of \( \xi_2 = 0 \) for the long-term unemployed. No additional weeks of federal benefits are available in any state: the temporary Emergency Unemployment Compensation (EUC) program expired at the end of 2013, and no state currently qualifies to offer more weeks under the permanent Extended Benefits (EB) program.\(^5\)

For comparison, we also consider a calibration of the model without unemployment risk. This “no unemployment risk” scenario corresponds to the standard life-cycle set up with \( \pi_{ee} = 1 \) and all other entries equal to zero in the transition probability matrix (2). In addition, to highlight the effects of permanent consequences of unemployment on future earnings prospects, we consider a third calibration by adding the unemployment risk embedded in the transition probability matrix (16) with no human capital erosion. In this “unemployment with no traps” scenario, unemployment has no permanent consequences on future earnings (i.e. \( \Psi_1 = \Psi_2 = 0 \)) but entails only a cut in current income. This case closely corresponds to the set-up studied by Bremus and Kuzin (2014), who focus only on temporary effects of long-term unemployment.

\section{Results}

\subsection{Optimal policies}

Figure 1 compares investors’ optimal stock shares in the standard case of “no unemployment risk” (panel (a)) and in our preferred scenario with “unemployment traps” (panel (b)). In particular, the figure plots the optimal stock share as a function of cash on hand for an average level of the permanent labour income component of investors at three different ages (20, 40, and 70). In the case with no unemployment risk, standard life-cycle results are obtained. Labour income acts as an implicit risk-free asset and affects the optimal portfolio composition depending on an investor’s age and wealth. For example, at age 20 the sizable implicit holding of the risk-free asset (through human capital) makes it optimal for less-wealthy investors to tilt their portfolio towards the risky financial asset. Indeed, for a wide range of wealth levels, agents optimally choose to be fully invested in stocks. The optimal stock holding decreases with financial wealth because of

\(^5\)Low, Meghir and Pistaferri (2010) acknowledge that layoffs are partially insured by the unemployment insurance system, while individual productivity shocks, other than major observable health shocks, are rarely insured in any formal way. As for other welfare programs, we do not model basic consumption needs and therefore overlook basic consumption insurance.
the relatively lower implicit investment in (risk-free) human capital.

When the model is extended to allow for permanent effects of unemployment spells on labour income prospects at re-employment (“unemployment traps”), with the parameters governing the proportional erosion of permanent labour income set at $\Psi_1 = 0$ after one year of unemployment and at $\Psi_2 = 0.6$ after 2 years, the resulting policy functions are shifted abruptly leftward. The optimal stock share still declines with financial wealth but a 100% share of investment in stocks is optimal only at very low levels of wealth. In this case, long-term unemployment implies the loss of a substantial portion of future labour income which severely reduces the level of human capital and increases its risk at any age. Thus, for almost all levels of financial wealth, stock investment is considerably lower than in the case of no unemployment risk.

4.2 Life-Cycle Profiles

On the basis of the optimal policy functions, we simulate the whole life-cycle consumption and investment decisions for 10,000 agents. Figure 2, panel (a), shows the average optimal stock shares plotted against age when unemployment risk is ignored and when it is accounted for. In the case of no unemployment risk (dotted line), the well-known result on the age profile of optimal stock portfolio shares is obtained. Over the life cycle the proportion of overall wealth implicitly invested in the riskless asset through human capital declines with age. Consequently, at early stages of the life cycle, optimal stock investment is about 100% and decreases with age to reach around 80% at retirement. When unemployment risk without human capital erosion is considered (dashed line), the optimal portfolio share of stocks still declines with age, though being slightly lower at all ages, with a 100% optimal stock share only for very young investors.

However, when long-term unemployment implies a rare but large skill erosion (solid line), the optimal stock investment is sizably reduced at any age and almost flat, at around 55-60%. The risk of permanently losing a substantial portion of future labour income prospects reduces the level of human capital and increases its riskiness. Because this effect is particularly relevant for younger workers, it induces a lower optimal stock investment conditional on financial wealth especially when young. Consequently, the age profile remains remarkably flat over the whole working life. These results highlight that possible long-run consequences of unemployment significantly dampen the incentive to invest in stocks, under standard calibrations, whereas unemployment persistence, with only temporary income losses as in Bremus and Kuzin (2014), has almost no effect on the age profile of optimal portfolio composition.

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6 The relatively low investment in stocks during retirement is due to the presence of a positive bequest motive, common to all parametrizations considered in this paper.
The reduction in the optimal portfolio share allocated to stocks is due to higher wealth accumulation, in turn induced by larger precautionary savings. Panel (b) of Figure 2 displays the average financial wealth accumulated over the life cycle for the three scenarios considered. In the face of possible, albeit rare, human capital depreciation, individuals accumulate substantially more financial wealth during working life to buffer possible disastrous labour market outcomes. Optimal consumption when young consequently falls, but it is much higher during both late working years and retirement years. Figure 3 displays the life-cycle profile of the ratio between savings and total (financial plus labour) income, comparing the case without unemployment risk to the one with unemployment traps. When the worker is 20 years old, the average propensity to save is especially high in the latter case, reaching 0.8 compared with less than 0.2 when unemployment risk is absent. Such propensity monotonically decreases in age, converging to the known pattern when the worker is in her forties. The figure clearly depicts the impact on savings of the resolution of uncertainty as individuals age.

Consistent with these predictions, data on Norwegian households show that they engage in additional saving and in shifting toward safe assets in the years prior to unemployment, as well as in depletion of savings after the job loss (see Basten, Fagereng and Telle, 2016). Importantly, our results imply that labour market institutions targeted to long-term unemployment affect both risk taking in the equity market and precautionary saving. The expectation of a higher benefit may mitigate the adverse impact of long term unemployment on human capital, reducing the need for cautious investing and saving during working life. The variation of institutions across countries may thus generate different life-cycle patterns in equity investing. In this light, the decreasing stock holdings in Norwegian data (appearing in Fagereng, Gottlieb and Guiso, 2017) may be a consequence of higher long-term unemployment benefits with respect to the US.

4.2.1 Heterogeneity

The above results imply that the optimal stock investment is flat in age, even for a moderately risk averse worker. In the face of a very rare but large human capital depreciation, workers on average invest about 55% of their financial wealth in stocks. This average pattern may hide considerable differences across agents. The present section investigates the distribution across agents of both conditional optimal stock share and accumulated wealth.

The case of no unemployment risk is displayed in panels (a) and (b) of Figure 4, which

\[^7\]Love (2006) shows that higher unemployment insurance benefits reduce calibrated contributions to pension funds by the young, suggesting that precautionary savings when young is due to unemployment risk.
show the 25\textsuperscript{th}, 50\textsuperscript{th} and 75\textsuperscript{th} percentiles of the distributions. Both the optimal stock share and the stock of accumulated financial wealth are highly heterogeneous across workers as well as retirees. The exception is young workers as they tilt their entire portfolio towards stocks given the relatively riskless nature of their human capital. Heterogeneity of portfolio shares depends on the shape and movements through age of the policy functions displayed in Figure 1, relating optimal stock shares to the amount of available cash on hand, and on the level of cash on hand itself. Relatively steep policy functions imply that even small differences in the level of accumulated wealth result in remarkably different asset allocation choices. At the early stage of the life cycle, when accumulated financial wealth is modest, it is optimal for everybody to be fully invested in stocks. As investors grow older, different realizations of background risk induce large differences in savings and wealth accumulation. This situation pushes investors on the steeper portion of their policy functions and determines a gradual increase in the heterogeneity of optimal risky portfolio shares during their working life. After retirement, investors decumulate their financial wealth relatively slowly, due to the bequest motive, and still move along the steeper portion of their relevant policy functions; as a consequence, the dispersion of optimal shares tends to persist.

Panels (c) and (d) of Figure 4 display the life-cycle distribution of stock share and financial wealth for the case with unemployment risk and human capital erosion. Compared with the case of no unemployment risk, the distribution of optimal stock shares is much less heterogeneous over the whole life cycle. In particular, heterogeneity shrinks during working life even for young workers, given the high human capital risk they bear at the beginning of their careers. In case of unemployment risk, policy functions are relatively flat (see panel (b) of Figure 1) implying that even large differences in the level of accumulated wealth result in homogenous asset allocation choices. Then, as in the previous case, the shape of heterogeneity of stock shares and accumulated financial wealth over the life cycle is due to different realizations of background risk.

5 Welfare analysis of suboptimal choices

Workers usually delegate the task of managing long-term saving on their behalf to their pension funds. The strategies proposed by portfolio managers often embed the feature of a decreasing age profile of investment in the riskier assets, with a portfolio share in stocks often in excess of 80\% when young. These strategies resemble the ones that are optimal in the absence of unemployment risk. In fact, Cocco, Gomes and Maenhout (2005) find that the representative worker should enjoy only a slightly higher (0.64\%) consumption level to be compensated for the adoption of a suboptimal "age rule" by her pension fund.
Bagliano, Fugazza and Nicodano (2014) find that a compensation of a similar amount is needed when the investor’s asset menu also includes bonds, unless stock returns and permanent income shocks are positively correlated.\footnote{Only in the case of positive correlation is the compensating consumption higher; it may reach 3.9\% for the benchmark risk aversion parameter (\( \gamma = 5 \)).} Love (2013) finds even lower welfare losses when optimizing over the parameters of the rule of thumb.

This section provides a quantitative assessment of the welfare loss associated with the adoption of simple portfolio allocation rules of thumb related to age when there are rare unemployment traps. We also explore an alternative suboptimal situation, in which there are unemployment traps but the worker adopts the utility-maximizing consumption and portfolio allocation that ignores them.\footnote{In all the three suboptimal cases, the underlying labour income process is the one implied by the presence of unemployment traps.} This case is inspired by the scant discussion of long-term unemployment in the United States prior to the recent crisis, which implies underestimation of the problem before 2007. The crisis may have generated a structural break in the economy, aggravating the long-term unemployment problem or it may have enhanced the awareness of the rare occurrence of unemployment traps. The analysis of this case also delivers an upper bound to the welfare gains achievable when workers switch to the optimal asset allocation, taking into account the potential occurrence of long-term unemployment spells with permanent consequences on their earnings prospects.

In particular, we consider two suboptimal asset allocation patterns related to the investor’s age. The first is the typical "age rule" analysed by Cocco, Gomes and Maenhout (2005), with a risky portfolio share set at 100 less the investor’s age.\footnote{In a variant of this “age rule”, the worker starts saving for retirement 40 years before the target retirement date, setting the initial share of stocks at 80\% and letting it fall to 40\% at retirement (Bodie, Treussard and Willen, 2007).} The second rule of thumb, denoted as “target-date fund (TDF) rule”, comes closer to actual strategic asset allocation patterns adopted by Target-Date Funds. As shown in panel (a) of Figure 5, the stock portfolio share is set at 90\% until the age of 40, is gradually decreased over the remaining working life down to 50\% at retirement age (65), and further reduced in the early retirement period to reach a low of 30\% at the age of 72. This TDF rule echoes the one investigated by Bagliano, Fugazza and Nicodano (2014) with the investor’s asset menu also including bonds. In both cases considered above, the worker is aware of unemployment traps and optimally chooses saving and consumption given the rule-of-thumb portfolio allocation. Our welfare analysis concludes with the suboptimal case when the worker maximizes expected utility oblivious of rare personal disasters.

The metric used to perform welfare comparisons is the standard consumption-equivalent variation employed by Cocco, Gomes and Maenhout (2005). The consumption-equivalent variation is obtained by simulating optimal consumption and wealth accumulation choices
conditional on following the optimal asset allocation strategy and each of the alternative (suboptimal) investment rules and by deriving the associated expected discounted lifetime utility levels. By inverting the derived expected discounted lifetime utility, we compute the constant consumption stream needed to compensate the investor for the adoption of suboptimal strategies. We then compute the percentage increase in annual consumption required by the investor to obtain the same level of expected utility warranted by the optimal life-cycle strategy for each suboptimal rule. Throughout our comparisons, we adopt the benchmark calibration parameters reported in Table 1.

5.1 Results

The left-hand side of Table 2 shows the welfare gains associated with switching from the "age rule" to the optimal portfolio choice. Both the mean and the median increases in welfare-equivalent consumption are equal to 3.3%. Welfare gains are three times larger than prior estimates in the literature. Such gains derive from the fact that consumption and savings are distorted by the higher risk taking when the worker faces a large amount of uncertainty about future labour and pension income, as well as by the lower risk taking when uncertainty is resolved. Average consumption (panel (b) of Figure 5) is close to the optimal level during early working years under the "age rule", but it is much lower during retirement. Moreover, as shown in panel (c), while wealth accumulation until age 55 is close to optimal, average financial wealth at retirement and thereafter turns out to be lower under the "age rule". This pattern is due to agents who, having incurred a trap, save less and ultimately - given the quick reallocation towards the riskless asset - are also able to consume and bequeath less. Those workers who do not experience personal disasters are able to set aside more wealth, but gradual conversion into the riskless asset reduces the return on the financial wealth relative to investors adopting optimal portfolio shares. The pattern of welfare gains across income brackets is surprising, however, as revealed by panel (b) of Table 2. Mean welfare gains when income at age 64 is below the 5th percentile of income distribution are lower than for agents with income above the 95th percentile (1.6% versus 2.4%). We tentatively ascribe such a result to the fact that a distorted portfolio rule delivers lower utility losses at the bottom of the income distribution because of lower financial wealth.

The middle column of Table 2 displays welfare gains when the investor adopts the optimal asset allocation pattern instead of the TDF rule, with the stock portfolio share exceeding the one dictated by the "age rule" until age 55 and later falling below it. Given that higher exposure to financial risk is present early in life and lower exposure during retirement, mean and median welfare gains from adopting the optimal portfolio rule are much higher (above 10% of yearly consumption). This pattern emerges despite two
seeming improvements, highlighted in Figure 5. The first is that mean consumption under a TDF portfolio allocation is higher from age 30 until age 80 than consumption under the optimal portfolio rule. The second is that mean financial wealth exceeds the optimal one until the investor is past age 80. The increased consumption during working life and early retirement is thus more than offset by the marked reduction during later retirement years, followed by a lower bequest. Once again, the mean welfare gains are lower for those with income (at age 64) below the 5th percentile of income distribution than for those with income above the 95th percentile (9.5% versus 12.3%). Both experiments suggest that distortions in asset allocation produce larger welfare losses for richer households.

The right-hand column of Table 2 delivers an upper bound on median welfare gains from becoming aware of unemployment traps (more than 200%). Mean welfare gains are even higher (642%). This pattern derives from a much higher average consumption until age 40, leading to lower buffer savings, for the unaware worker who chooses consumption and portfolios composition as in Cocco, Gomes and Maenhout (2005) (see panel (b) in Figure 5). If a long-term unemployment spell occurs, consumption and bequest drop dramatically, in some cases almost to zero. This drop implies a large utility loss for a tiny share of workers, causing a sizable increase in mean welfare loss relative to the median value. Since close-to-zero consumption is likelier at the bottom of the income distribution, mean welfare gains from becoming aware of traps are higher for those with low rather than high income, at age 64 (1024% against 218% as shown in panel (b) of Table 2). Figure 6 confirms the above interpretation, displaying mean income profiles for individuals conditional on welfare gains. Workers with lifetime earnings above the mean obtain welfare gains below the 5th percentiles. In turn, workers enjoying the largest welfare gains are those with lifetime labour income well below the average. Clearly, the earlier the traps occur, the lower the buffer wealth and the worse the consumption and welfare consequences. The latter observation hints at a welfare-improving scheme designed to support the fraction of workers hit by long-term unemployment when young, along the lines suggested by Michelacci and Ruffo (2015). More generally, the size of the welfare losses suggests to move towards schemes that protect against longer-term unemployment (e.g., see Setty 2017).

6 Robustness and Other Challenges

This section sheds additional light on the strength of our results, that radically depart from the accepted wisdom concerning optimal life-cycle behaviour during working years and hold across the entire distribution of investors.

A first robustness check concerns the sensitivity of our results to a lower probability
of personal disaster. In performing such analysis, we also allow for an asymmetric reduction in the probability of long-term unemployment with respect to workers’ age. Recent data from US labour market statistics indeed show that the composition of long-term unemployment is shifting towards the elderly. In 2015 the overall and the long-term unemployment rates in US were about 5.7% and 1.7%, respectively, with the share of long-term unemployment in the overall unemployment rate differing widely among age groups: from 20% among young workers (16-24 years old), to 35% among prime age workers (25-55), and up to 41% among older workers (over 55).

A second check regards the modelling of the link between unemployment risk during working life and retirement income, so as to make sure that our results do not originate exclusively from long-term unemployment occurring during the very last working years, which heavily reduces retirement income.

Further, since the power utility function implies that the worker is indifferent to intertemporal correlation of consumption shocks (e.g., see Bommier, 2007), we adopt Epstein-Zin preferences to investigate whether positive correlation aversion boosts the impact of unemployment traps. A similar motivation leads us to analyse the sensitivity of the equity-investment profile to positive correlation between stock returns and labour income shocks.

Importantly, we experiment with more conservative calibrations and alternative modelling of human capital erosion, so as to represent the possibility of incurring in large losses only in deep crisis situations rather than in normal business cycle downturns.

Finally, we address some logical challenges relating to our analysis, including its general equilibrium implications.

6.1 Age-dependent unemployment risk

In this section, we calibrate our model with unemployment traps, allowing for both a smaller and age-dependent long-term unemployment risk. We change the transition probability from short-term to long-term unemployment, denoted as $\pi_{u1u2}$ in the following transition probabilities matrix:

$$
\Pi_{s_t, s_{t+1}} = \begin{pmatrix}
0.96 & 0.04 & 0 \\
1 - \pi_{u1u2} & 0 & \pi_{u1u2} \\
0.85 & 0 & 0.15
\end{pmatrix}
$$

with respect to the baseline calibration in (16) where $\pi_{u1u2} = 0.15$ irrespective of the worker’s age. In “case 1”, the probability of entering long-term unemployment is reduced by one third (from 0.15 to 0.10) only for workers younger than 50 years old. In
“case 2”, we further reduce the probability of entering long-term unemployment for very young workers, setting $\pi_{u1u2} = 0.075$ for workers less than 30 years old. In all scenarios, transition probabilities are rather conservative implying steady-state long-term unemployment rates lower than the actual one. For reference, in the baseline case, the steady-state long-term unemployment rate is 0.6%, while it is 0.5% and 0.4%, in cases 1 and 2, respectively.

Figure 7 reports the life-cycle profiles for the optimal conditional stock holding and financial wealth accumulation when the long-term unemployment risk is age-dependent. Compared with the baseline case, the age profile of stock investment is only slightly modified. A lower long-term unemployment risk at young ages implies a moderately higher stock share during prime age but it does not significantly alter investors’ behaviour later over the working life and during retirement. In addition, it has virtually no effect on wealth accumulation.

6.2 Unemployment risk and retirement income

In our model, pension benefits are a fixed proportion of the last labour income earned prior to retirement age. Such income is especially sensitive to human capital loss due to the occurrence of long-term unemployment in years just before retirement. Thus, we analyse whether our results are robust to changes in modelling the link between long-term unemployment at old ages and subsequent pension provisions.

To begin with, we assume no human capital loss in the event that unemployment occurs in the years immediately before retirement. Our simulation results show that the flattening of the optimal stock share profile carries over to this setting, suggesting that it is not an artifact of how we model pension income. In a second check, we take the solution of our original model (calibrated in the case of unemployment risk with human capital erosion) and focus on simulated life-cycle profiles for two selected groups of agents. The first group includes workers who have experienced just one long-term unemployment spell of 5 years over the entire working life at the beginning of their job career (i.e., before the age of 35), whereas the other group contains workers who have experienced just one long-term unemployment spell of 5 years over their entire working life but at the end of their career (i.e., after the age of 60). We find that in both cases, average life-cycle stock share profiles exhibit the flattening property. This experiment confirms that the flattening is due to the riskier nature of human capital, together with the resolution of uncertainty during working age, and it is not affected by specific assumptions on the determinants of pension income.
6.3 Boosting the effect of unemployment traps

In this section, we consider two possible avenues that might reinforce our results. The first one is to allow for a positive correlation between stock return innovations and the innovations in permanent labour income ($\rho_{\Delta Y} > 0$), on top of human capital erosion. Results in Bagliano, Fugazza and Nicodano (2014) show that a realistically small correlation has large effects on life-cycle choices when it interacts with a higher variance of the permanent component of labour income shocks. One may therefore expect a similar effect in the presence of unemployment traps. Empirical estimates of the stock return-labour income correlation differ widely, even when we restrict the scope to the US economy. Cocco, Gomes and Maenhout (2005) report estimated values not significantly different from zero across various education groups, in line with Heaton and Lucas (2000), whose estimates range from -0.07 to 0.14. However, Campbell, Cocco, Gomes and Maenhout (2001) find higher values, ranging from 0.33 for households with no high school education to 0.52 for college graduates. In the simulations below, we adopt an intermediate positive value of $\rho_{\Delta Y} = 0.2$.

Figure 8 shows optimal portfolio shares of stocks and the pattern of financial wealth accumulation with no correlation and with a positive correlation between labour income shocks and stock returns. While the shape of life-cycle profiles is relatively unaffected, the average stock share is lower at all ages. In case of positive correlation, labour income is closer to an implicit holding of stocks, reducing the incentive to invest in stocks at all ages. More specifically, in comparison with the case of no correlation, such investors are relatively more exposed to stock market risk and will prefer to offset such risk by holding a lower fraction of their financial portfolio in stocks. The stock share remains substantially flat over the whole working life, displaying limited variability around a level of about 50%. At the retirement age of 65, human capital becomes riskless since pension income is certain and therefore uncorrelated with stock return innovations. Thus investors rebalance their portfolio towards stocks: during retirement, the level and time profile of the stock share are very close to the case of no correlation. Further, the relative increase in human capital risk due to a positive correlation does not substantially alter the pattern of financial wealth accumulation.

The second experiment implements a change in preferences that allows for intertemporal correlation aversion (Bommier, 2007). With a power utility function, the worker is indifferent to positive or negative intertemporal correlation of consumption (shocks). With Epstein-Zin preferences, the worker is averse to positive correlation when the coefficient of relative risk aversion is greater than the inverse of the elasticity of intertemporal substitution (EIS). Adopting a recursive (Epstein-Zin) formulation for preferences and keeping the risk aversion parameter constant ($\rho_{\Delta Y} = 0.2$), we simulate the model with
positive (EIS=0.5) and negative (EIS=0.1) correlation aversion, comparing the results with our baseline case of indifference (i.e., power utility, EIS=0.2). Figure 9 shows that aversion to positive correlation has a negligible effect during working years, while it causes a slower wealth decumulation and less risk taking during the retirement period, especially as death approaches. This finding is consistent with the known property that higher mortality risk magnifies the effects of intertemporal correlation aversion (Bommier, 2013).

Overall, the preceding experiments confirm the robustness of the flattening of the life-cycle profile to changes in both hedging opportunities in the stock market and to the intertemporal elasticity of substitution, pointing to the dominance of the personal disaster effect.

6.4 Sensitivity to human capital erosion

We now check the sensitivity of life-cycle profiles with respect to the magnitude of the human capital erosion effect due to long-term unemployment, captured by the parameter $\Psi_2$ (set equal to 0.6 in our baseline calibration). Since, as discussed in Section 3, available estimates of earnings losses due to long-term unemployment are as low as 15% of the level of pre-displacement earnings, Figure 10 shows the results of an experiment with the human capital erosion parameter $\Psi_2$ in the range $0.2 - 0.6$. Once this parameter falls to 0.4, the hump shape in the portfolio share of stocks appears again over working life. The life-cycle profiles of both the optimal risky portfolio share and financial wealth accumulation track closely in the case of no unemployment when the potential human capital loss is equal to 20%.

So far, the human capital erosion parameters $\Psi_1$ and $\Psi_2$ are set equal to constants. However, they could more appropriately be modelled as stochastic variables. After all, the tragic human capital consequences of the unemployment trap emerged in the United States during the Great Recession, but were less evident in the past. This outcome may correspond to high (low) realizations of $\Psi_2$ for a large number of workers in the Great Recession (business cycle expansions) and an intermediate value in ordinary business cycle contractions.

The Beta distribution can be usefully employed to model the behaviour of random variables that take values limited to finite intervals. In the current case, random variables $\Psi_j$ ($j = 1, 2$) are fractions limited to the interval [0, 1]. We therefore assume that $\Psi_j$ follow Beta distributions with support [0, 1] and shape parameters ($a_j$, $b_j$); thus, $\Psi_j \sim \text{Beta}(a_j, b_j)$. The flexibility of the distribution allows us to concentrate nearly all the probability mass towards the most probable values of proportional human capital erosion, leaving the possibility of extremely unlikely realizations open. The most revealing case is displayed in Figure 11. While $\Psi_1$ is kept at 0, $\Psi_2$ follows the Beta distribution.
with expected value ranging between 0.10 and 0.20 and standard deviation between 0.21 and 0.32, implying an expected 10% – 20% erosion of the individual permanent labour income component after the second year of unemployment. More precisely, the calibrated distribution for $\Psi_2$ implies a median value for the proportional human capital erosion lower than 1%, while the 75\textsuperscript{th} percentile ranges between 6% and 25%. It turns out that under all distributional assumptions, life cycle profiles are very similar to the case of the benchmark value of $\Psi_2 = 0.6$.

This outcome provides a final indication that extremely rare but potentially disastrous labour income shocks may be relevant to understand cautiousness by young workers and their limited risk taking in the stock market.

### 6.5 Additional challenges

Positive work on life-cycle consumption and saving shows how to calibrate all the key parameters to match the empirical evidence. On the one hand, considering a sufficiently high subjective discount factor prevents households from accumulating an excessive amount of wealth. It also reduces the stock market participation cost needed to match the observed non-participation patterns at a realistic level (e.g., see Fagereng, Gottlieb and Guiso, 2017). On the other hand, the introduction of subsistence consumption levels avoids counterfactually high saving rates, especially early in life when individuals are more likely to be liquidity constrained (see e.g. Hubbard, Skinner and Zeldes, 1995; DeNardi, French and Jones, 2010). While these patterns obtain in our model in unreported calibrations, this paper focuses, instead, on the normative implications of long-term unemployment risk for conditional optimal stock holdings.

Even though we do not attempt to explicitly compare our results to data on portfolio choice, our set-up generates a more empirically plausible life-cycle profile of portfolio choice with respect to models that abstract from possible permanent losses following long-term unemployment. In our calibrations, we focus on income profiles of workers with a high school education. Our results also apply to college-educated workers given that the long-term unemployment risk uniformly affects both high school and college-educated individuals (e.g., see Mayer, 2014).

The paper focuses on long-term unemployment risk as a source of rare personal disaster during working life. However, individuals face various additional sources of potential disaster (illness, accidents, personal bankruptcy, etc.) that may lead to substantial human capital loss, which would reinforce our results on optimal risk taking.

Finally, our model obtains the optimal reduction in stock holdings over working life in partial equilibrium. The combination of individuals’ life cycle saving behaviour and population age structure in an overlapping generation framework may reinforce our results
through asset pricing (e.g., see Poterba, 2001). A relatively large middle-age cohort depresses expected stock returns, strengthening the reduction in optimal risk taking by the younger generation in response to long-term unemployment risk.

7 Conclusions

As the recent Great Recession episode has highlighted, long-term unemployment spells may persistently damage workers’ human capital. Against this backdrop, this paper investigates the effects of unemployment traps on life-cycle savings and portfolio choice, introducing higher moments and non-linearities in the labour income process. This methodological innovation enables new insights. Even a small probability of experiencing human capital erosion due to long-term unemployment can generate optimal conditional stock shares more in line with those observed in the data. Because of the possibility of human capital loss, young workers face higher uncertainty concerning future income and social security pension levels than older workers. At the same time, young workers with continuous careers have larger human capital than older workers. When a highly unlikely unemployment spell may potentially lead to considerable human capital erosion, the first effect offsets the second and the optimal investment in stocks is relatively flat over the life cycle. This result departs from the implications of previous models, with linear income shocks highlighting the importance of unemployment traps for the life-cycle portfolios.

Our calibrations also suggest an alternative, more balanced design for target-date investment funds. Such modified design implies an average 3%-10% increase in welfare-equivalent annual consumption, depending on the benchmark "age rule". This more balanced design fits different kinds of workers, given the limited heterogeneity in life-cycle investments induced by the threat of personal disasters. More generally, our analysis indicates that the pattern of risk taking at different ages in Target Date Funds should be related to the share of uninsured long-term employment risk.

Our analysis implies that changes should be observed in the life-cycle profile of household portfolios, both across cohorts and across countries, in response to the coverage of long-term unemployment. Clearly there are other sources of human capital erosion, as well as other partial insurance vehicles. The optimal flat asset allocation will extend to such scenarios to the degree that such additional shocks remain partially uninsured by additional hedges and that they have worse consequences the earlier they hit the worker.
References


Table 1: Calibration parameters
This table reports benchmark values of relevant parameters.

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working life (max)</td>
<td>$T$</td>
<td>20 - 65</td>
</tr>
<tr>
<td>Retirement (max)</td>
<td>$t_0 + K$</td>
<td>65 - 100</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$(\beta)$</td>
<td>0.96</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>$(\gamma)$</td>
<td>5</td>
</tr>
<tr>
<td>Replacement ratio</td>
<td>$(\lambda)$</td>
<td>0.68</td>
</tr>
<tr>
<td>Variance of permanent shocks to labour income</td>
<td>$(\sigma^2_\omega)$</td>
<td>0.0106</td>
</tr>
<tr>
<td>Variance of transitory shocks to labour income</td>
<td>$(\sigma^2_\epsilon)$</td>
<td>0.0738</td>
</tr>
<tr>
<td>Riskless rate</td>
<td>$(r)$</td>
<td>2%</td>
</tr>
<tr>
<td>Excess returns on stocks</td>
<td>$(\mu^s)$</td>
<td>4%</td>
</tr>
<tr>
<td>Variance of stock returns innovations</td>
<td>$(\sigma^2_s)$</td>
<td>0.025</td>
</tr>
<tr>
<td>Stock ret./permanent lab. income shock correlation</td>
<td>$(\rho_{sY})$</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2: Welfare Gains
This table reports welfare gains (in percentage points) due to the adoption of optimal choices instead of suboptimal ones. The first and second columns display gains from abandoning suboptimal “Age Rule” and “Target Date Fund Rule” for asset allocation. The third column refers to gains deriving from taking account of unemployment traps in the optimization process with respect to being unaware of traps. Panel (a) displays the distribution of welfare gains. Panel (b) compares mean gains for workers, at age 64, with income below the 5th percentile and above the 95th percentile of income distribution.

<table>
<thead>
<tr>
<th>Unemployment benefits</th>
<th>No unemployment risk</th>
<th>Unemployment with no traps</th>
<th>Unemployment traps</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Short-term unemployed ($\xi_1$)</td>
<td>-</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Long-term unemployed ($\xi_2$)</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Human capital erosion</th>
<th>No unemployment risk</th>
<th>Unemployment with no traps</th>
<th>Unemployment traps</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Short-term unemployed ($\Psi_1$)</td>
<td>-</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>Long-term unemployed ($\Psi_2$)</td>
<td>-</td>
<td>-</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Table 2: Welfare Gains
This table reports welfare gains (in percentage points) due to the adoption of optimal choices instead of suboptimal ones. The first and second columns display gains from abandoning suboptimal “Age Rule” and “Target Date Fund Rule” for asset allocation. The third column refers to gains deriving from taking account of unemployment traps in the optimization process with respect to being unaware of traps. Panel (a) displays the distribution of welfare gains. Panel (b) compares mean gains for workers, at age 64, with income below the 5th percentile and above the 95th percentile of income distribution.

<table>
<thead>
<tr>
<th>Age Rule</th>
<th>TDF rule</th>
<th>Unaware of Traps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>3.3</td>
<td>12.0</td>
</tr>
<tr>
<td>Median</td>
<td>3.3</td>
<td>11.8</td>
</tr>
<tr>
<td>5th</td>
<td>1.5</td>
<td>8.0</td>
</tr>
<tr>
<td>95th</td>
<td>5.4</td>
<td>17.0</td>
</tr>
</tbody>
</table>

(b) Welfare gains conditional on income at age 64 ( % points)

<table>
<thead>
<tr>
<th>Below 5th percentile</th>
<th>Above 95th percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1.6</td>
</tr>
<tr>
<td>Median</td>
<td>9.5</td>
</tr>
<tr>
<td>5th</td>
<td>-40.5</td>
</tr>
<tr>
<td>95th</td>
<td>573.6</td>
</tr>
</tbody>
</table>
Figure 1: Policy functions
This figure shows the portfolio rules for stocks as a function of cash on hand for an average level of the stochastic permanent labor income component. The policies refer to selected ages: 20, 40, and 70. Panel (a) and (b) refer respectively to the cases with no unemployment risk and with unemployment traps. In the latter case, the parameters governing the human capital erosion during short-term and long-term unemployment spells are $\Psi_1 = 0$ and $\Psi_2 = 0.6$. Cash on hand is expressed in ten thousands of U.S. dollars.

(a) No unemployment risk

(b) Unemployment traps
Figure 2: Life-cycle average profiles
This figure displays the mean simulated stock investment and financial wealth accumulation life-cycle profiles. Age ranges from 20 to 100. The three cases correspond to no unemployment risk (dotted line); unemployment risk with no traps (dashed line); unemployment risk with traps (solid line). In the latter case, the parameters governing the human capital erosion during short-term and long-term unemployment spells are $\Psi_1 = 0$ and $\Psi_2 = 0.6$. Financial wealth is expressed in ten thousands of U.S. dollars.

Figure 3: Life-cycle profiles of savings rate
This figure displays the savings dynamics for individuals of age 20 to 100, relative to total income (i.e. labor income plus financial income). The two cases correspond to no unemployment risk (dotted line) and unemployment risk with traps with human capital erosion: $\Psi_1 = 0$ and $\Psi_2 = 0.6$ (solid line).
Figure 4: Life-cycle percentile profiles
This figure displays the distribution of simulated stock investment and financial wealth accumulation life-cycle profiles for individuals of age 20 to 100 in the case of unemployment risk (panels (a) and (b)) and unemployment traps (panels (c) and (d)). The parameters governing the human capital erosion during short-term and long-term unemployment spells are $\Psi_1 = 0$ and $\Psi_2 = 0.6$. Financial wealth is expressed in ten thousands of U.S. dollars.

No unemployment traps

Unemployment traps
Figure 5: Optimal and suboptimal life-cycle profiles
This figure contrasts the optimal (solid line) and suboptimal life cycle profiles (dotted line: “Age Rule”; dashed-dotted line: “Target Date Fund rule”; dashed line: “unaware of traps”, i.e. optimization without taking into account the existence of unemployment traps). Financial wealth and consumption are expressed in ten thousands of U.S. dollars.

(a) Equity portfolio share

(b) Consumption

(c) Financial wealth
Figure 6: Income profiles conditional on welfare gains
This figure displays mean income profiles conditional on welfare gains from becoming aware of traps. The dotted line represents the mean income profile for individuals with welfare gain below the 5-th percentile. The solid and dash-dotted lines refer to individuals with welfare gains respectively at the average and above the 95th percentile.

Figure 7: Life-cycle profiles with unemployment traps: age-dependent long-term unemployment risk
This figure displays the average simulated stock investment and financial wealth accumulation life-cycle profiles for individuals of age 20 to 100. The probability of entering long-term unemployment for an unemployed worker is set to 0.15 in the baseline case, to 0.10 only for workers younger than 50 in Case 1, to 0.10 for all workers in Case 2. Human capital erosion: $\Psi_1 = 0$ and $\Psi_2 = 0.6$. Financial wealth is expressed in ten thousands of U.S. dollars.
Figure 8: Life-cycle profiles with unemployment traps: positive correlation between labor income and stock returns
This figure displays the average simulated stock investment and financial wealth accumulation life-cycle profiles for individuals of age 20 to 100. Positive correlation between labor income shocks and innovation to stock returns: $\rho_{sY} = 0.2$. Human capital erosion: $\Psi_1 = 0$ and $\Psi_2 = 0.6$. Financial wealth is expressed in ten thousands of U.S. dollars.

Figure 9: Life-cycle profiles with unemployment traps: recursive preferences
This figure displays the average simulated stock investment and financial wealth accumulation life-cycle profiles for individuals of age 20 to 100. Preferences over consumption are recursive, represented by an Epstein Zin utility function. Elasticity of intertemporal substitution varies from 0.1 to 0.5. Human capital erosion: $\Psi_1 = 0$ and $\Psi_2 = 0.6$. Financial wealth is expressed in ten thousands of U.S. dollars.
Figure 10: Life-cycle profiles with unemployment traps: sensitivity to human capital erosion
This figure displays the average simulated stock investment and financial wealth accumulation life-cycle profiles for individuals of age 20 to 100. Various cases are considered: no unemployment risk; unemployment traps with alternative values of human capital erosion, i.e. with $\Psi_2$ decreasing from 0.6 to 0.2 (in all cases $\Psi_1 = 0$). Financial wealth is expressed in ten thousands of U.S. dollars.

(a) Average stock share  
(b) Average financial wealth

Figure 11: Life-cycle profiles with unemployment traps: stochastic human capital erosion
The figure displays the average simulated stock investment and financial wealth accumulation life-cycle profiles for individuals of age 20 to 100. Various cases are considered: no unemployment risk; unemployment traps with deterministic human capital erosion ($\Psi_1 = 0$ and $\Psi_2 = 0.6$), unemployment traps with stochastic human capital erosion. In the latter case, $\Psi_2$ follows a beta distribution with shape parameters $a$ and $b$. In particular, $a=0.1$ and $b=0.4, 0.7$ and $0.9$, implying an expected value for $\Psi_2$ equal to $0.20, 0.12$ and $0.10$, respectively. Financial wealth is expressed in ten thousands of U.S. dollars.

(a) Average stock share  
(b) Average financial wealth