Expected Stock Returns and the Correlation Risk Premium*

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Keywords: correlation risk premium, out-of-sample return predictability, option-implied information, diversification

JEL: G11, G12, G13, G17

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Abstract

We show that the correlation risk premium can predict future market excess returns in-sample and out-of-sample for long horizons and contains information that is non-redundant relative to the variance risk premium. To exploit this predictability, we develop a novel estimation methodology that uses contemporaneous increments of option-implied variables, efficiently removing any lag in estimation of variance and correlation risk betas. The methodology leads to considerable out-of-sample predictability, with an $R^2$ of 7.0% at an annual horizon, and substantial economic gains for investors. The results are supported by a multi-asset general-equilibrium model in which variance and correlation risk are endogenously priced.

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Given their forward-looking nature and close theoretical link to the equity risk premium, option-based variables, like the variance and the correlation risk premium, are natural candidates for predicting future market returns. However, even though empirical evidence suggests that investors are willing to pay a variance and correlation risk premium\(^1\) and that variance and correlation are highly correlated with market returns,\(^2\) the empirical evidence regarding their predictive power is limited. In particular, while there is evidence regarding short-term, in-sample return predictability for the variance risk premium (see, among others, Bollerslev, Tauchen, and Zhou (2009), Carr and Wu (2016), and Bandi and Renò (2016)), the evidence for the correlation risk premium is rather scarce (with Cosemans (2011) as a notable exception). Moreover, there is hardly any evidence for out-of-sample predictability.

In this paper, we demonstrate that the correlation risk premium can consistently predict future market excess returns *out-of-sample* for horizons of up to one year, leading to substantial economic benefits for investors. The information contained in the correlation risk premium is non-redundant relative to the variance risk premium, which predicts future market returns for horizons of up to one quarter, but not for longer. We show that one can only fully exploit this predictive power if one uses timely information to estimate the exposure of the market return with respect to the option-implied variables and propose a novel estimation methodology that does exactly that. The results are supported by a general equilibrium model in which both variance and correlation risk are priced endogenously and contain non-redundant information, thus providing strong theoretical support for our estimation approach and the empirical analysis.


\(^2\)Christie (1982), Roll (1988), Bekaert and Wu (2000), and Longin and Solnik (2001) document a negative correlation between the market return and index variance (equal to \(-0.77\) in our sample). For our sample period, we document a correlation of \(-0.61\) between the market return and expected correlation.
In a first step, we study a simple reduced-form framework designed to illustrate the main economic mechanisms underlying the return predictability by the correlation risk premium and to provide a theoretical foundation for our novel estimation approach. The estimation methodology is motivated by two key insights. First, one can estimate the exposure of the market return with respect to variance and correlation risk using contemporaneous increments at a high frequency. Second, one can estimate the betas (under the physical measure) using increments of option-implied variables, that is, implied variance and implied correlation. In contrast to standard predictive regressions that rely on long-term returns on the left-hand side and regressors lagged by the length of the forecasting horizon, our “contemporaneous betas” use exclusively timely information. Also, the methodology can easily be adapted for use with other predictors with observable risk premiums.

Next, we show empirically that the variance and correlation risk premiums predict the market excess return out-of-sample, with $R^2$s of up to 10.4\% at a quarterly horizon and up to 7.0\% at an annual horizon. While the predictability by the variance risk premium peaks at the quarterly horizon and declines after that, the predictive power of the correlation risk premium is strongest for longer horizons—up to one year. Hence, we provide strong empirical evidence for the existence of two components that can be estimated ex-ante using options data and that contain non-redundant information. We show that these predictability results imply substantial economic benefits for investors, with certainty equivalent gains of 2\% p.a. even at the nine-month horizon. Moreover, we demonstrate that most of this out-of-sample predictability can be attributed to our novel estimation methodology. That is, its predictive power is considerably higher than for the traditional approach that, by design, relies on more “outdated” data at lower frequencies. Also, the results are robust to varying option maturity and including fundamental variables used in the return predictability literature.
To justify the reduced-form model, we then study a dynamic, multi-asset general-equilibrium economy with Epstein-Zin preferences. In the model, a two-component structure for the market variance arises endogenously from the underlying dividend trees, which feature stochastic variance and a stochastic dividend correlation. Moreover, variance and correlation risk are priced and contribute to the equity risk premium. Hence, the model provides direct theoretical support for the reduced-form framework that we use to illustrate the main mechanism and, thus, also supports our new estimation procedure. Though we consider the model to be mostly of qualitative nature, we show that it can, in general, match key quantities of the data.

Finally, we study the economic mechanism underlying the correlation risk premium. Our empirical results support a risk-based explanation in the spirit of the Merton (1973) Intertemporal CAPM. In particular, we demonstrate that expected correlation has strong predictive power for future diversification benefits for up to one year, whereas expected variance has a shorter predictability horizon for future risks, consistent with our results for return predictability.

Our paper is related to several strands of the literature. First, we contribute to the literature on market return predictability, in particular studies focusing on option-implied variables. Bollerslev, Tauchen, and Zhou (2009), Drechsler and Yaron (2011), and Bollerslev, Marrone, Xu, and Zhou (2014) show that the variance risk premium is a robust predictor of market returns for up to one quarter ahead. Several studies document return predictability by expected correlation for a horizon of up to one year (e.g., Driessen, Maenhout, and Vilkov (2005, 2012), and Faria, Kosowski, and Wang (2016) using implied correlation and Pollet and Wilson (2010) using realized correlation), but only Cosemans (2011) finds some in-sample return predictability by the correlation risk premium. Fan, Xiao, and Zhou (2018) decompose the variance risk premium into two components—a premium for variance risk and a premium for higher order

\footnote{See the extensive survey and discussion in Goyal and Welch (2008).}
risks—and show that such a decomposition improves market return predictability both in-sample and out-of-sample. Their second component shares similarities with the correlation risk premium, being especially important for predicting longer term returns. Feunou, Jahan-Parvar, and Okou (2017) define a downside variance risk premium and study its predictive power. Related, Kilic and Shaliastovich (2017) show that decomposing the variance risk premium into a “bad” and a “good” component significantly improves the predictability of market returns.

We contribute to this literature by concentrating on the out-of-sample performance of the correlation risk premium. While previous research has documented in-sample predictability (often for implied correlation instead of the correlation risk premium), we demonstrate that the correlation risk premium can predict future market returns out-of-sample for horizons of up to one year. Moreover, it contains non-redundant information relative to the variance risk premium, whose predictive power peaks at the three-month horizon. Our second contribution is the novel estimation methodology. We directly estimate the betas for the variance and correlation risk premium from daily market returns and contemporaneous increments in option-implied variables. Compared to the traditional approach, the new betas substantially improve the out-of-sample predictability.

Consequently, we also contribute to a growing body of literature that uses option-implied information in forecasting and asset pricing (see, e.g., the extensive literature survey in Christoffersen, Jacobs, and Chang (2013)). Bali and Zhou (2016) demonstrate how exposure to variance risk is compensated in the cross-section of stocks. Bali and Hovakimian (2009), Xing, Zhang, and Zhao (2010), Cremers and Weinbaum (2010), Rehman and Vilkov (2010), and Stilger, Kostakis, and Poon (2017) connect various proxies of the variance risk premium and forward-looking skewness to the cross-section of stock returns. DeMiguel, Plyakha, Uppal, and Vilkov (2013) use their results in a portfolio selection exercise. Chang, Christoffersen, Jacobs, and

Finally, our work is related to the theoretical literature on priced market variance and correlation risk. Bollerslev, Tauchen, and Zhou (2009) introduce an equilibrium model with priced variance risk, arising from stochastic volatility-of-volatility. Buraschi, Trojani, and Vedolin (2014) propose a general-equilibrium model with differences-in-beliefs in which correlation risk is endogenously priced. Driessen, Maenhout, and Vilkov (2009) suggest a risk-based explanation for the correlation risk premium, with the average stock correlation serving as a state variable that has predictive power for future market risks and, thus, is priced. Consistent with this approach, Buraschi, Kosowski, and Trojani (2014) empirically relate correlation risk to a “no-place-to-hide” state variable. Mueller, Stathopoulos, and Vedolin (2017) investigate the correlation risk premium in foreign exchange markets. Buraschi, Porchia, and Trojani (2010) show, in a partial-equilibrium setting, that optimal portfolios include distinct hedging components against both stochastic volatility and correlation risk. Boloorforoosh, Christoffersen, Fournier, and Gourieroux (2017) develop a model in which market variance is driven by two components and “beta risk” is priced, thus affecting expected stock returns.

We contribute to this literature by developing a tractable, dynamic, multi-asset general-equilibrium model, in which both variance and correlation risk are priced and non-redundant. The key elements of the model are stochastic variance of individual dividend trees and a stochastic correlation among them. Effectively, our model can be seen as an extension of the model in Bollerslev, Tauchen, and Zhou (2009) to multiple dividend trees with stochastic correlation. In our model, a two-component structure for the variance of aggregate consumption arises, similar to the model with short- and long-run volatility components in Zhou and Zhu (2015). However,
our model allows for easy interpretation of the long-run component, connecting it to the average
stock return correlation. We provide strong empirical support for a two-component structure.
Another contribution is our analysis of the sources of the correlation risk premium, documenting
that a risk-based explanation can rationalize the observed patterns of return predictability.

The remainder of the paper is organized as follows: Section 1 introduces our novel estimation
approach for contemporaneous betas within the framework of a reduced-form model. Section 2
discusses data preparation procedures. Section 3 is devoted to market return predictability—in-
sample and out-of-sample. Section 4 presents a dynamic, general-equilibrium economy that can
rationalize the choice of our reduced-form model. Section 5 analyzes the economic mechanism
underlying the correlation risk premium and, finally, Section 6 concludes. Many theoretical
derivations are delegated to the Appendix.

1 Estimation Methodology

In this section, we introduce our new estimation methodology for predicting market excess
returns. To motivate the approach, we first introduce a reduced-form stock market framework,
with the modeling assumptions being driven by the desire to have the simplest possible setting.
In Section 4, we demonstrate how one can rationalize the setting, that is, the particular form of
the market dynamics and of the pricing kernel, by a general-equilibrium model with stochastic
variance and correlation.

1.1 Economic Framework

The dynamics of the aggregate market are given by:

\[
\frac{dW_t}{W_t} = \mu_W \, dt + \beta_{\varepsilon,t} \, dB_{\varepsilon,t} + \beta_{\varepsilon,t}^\prime \, dV_{W,t} + \beta_{\rho,t}^\prime \, d\rho_{S,t} + \beta_{\zeta,t} \, dZ_t,
\]  

(1)
where $dB_{c,t}$, $dV_{W,t}$ and $d\rho_{S,t}$ denote shocks to aggregate consumption and instantaneous changes in market variance and the average stock correlation, respectively.\(^4\) $dZ_t$ denotes “residual” sources of risk not modelled explicitly here.

We assume that variance and correlation risk are priced, that is, their expectations under the risk-neutral and the physical probability measures differ. There is substantial empirical evidence that both variance and correlation risk carry a positive risk premium (see, e.g., Carr and Wu (2009) and Driessen, Maenhout, and Vilkov (2009)). Intuitively, one can also think of these two variables serving as proxies for latent (unobservable) state variables and, as a result, bearing a non-zero price of risk.\(^5\)

If variance and correlation risk are priced, the market risk premium can be decomposed as:

$$E^P \left[ \frac{dW_t}{W_t} \right] - r_{f,t} dt = \beta_{c,t} \lambda_c dt + \beta_{\xi,t} \lambda_\xi^2 dt + \beta_{V,t} V_{RP,t} + \beta_{\rho,t} C_{RP,t}. \tag{2}$$

The first component captures the classic risk-return trade-off, resulting from consumption risk and the second component captures compensation for residual risk. Most important for us, the last two components represent compensation for market variance and correlation risk; $V_{RP,t} = E^Q [dV_{W,t}] - E^P [dV_{W,t}]$ and $C_{RP,t} = E^Q [d\rho_{S,t}] - E^P [d\rho_{S,t}]$ denote the variance and correlation risk premium, that is, the expectation under the risk-neutral measure $Q$ minus the expectation under the physical $P$ measure.\(^6\)

Integrating (2) over a short period $\Delta t$, yields an approximate finite-period expression:

$$E_t [r_{t+\Delta t}] - r_{f,t} = \beta_{c,t} \lambda_{c,t+\Delta t} + \beta_{\xi,t} \lambda_{\xi,t+\Delta t}^2 + \beta_{V,t} V_{RP,t+\Delta t} + \beta_{\rho,t} C_{RP,t+\Delta t}. \tag{3}$$

\(^4\)Shocks to variance and correlation are not required to be orthogonal to each other. Actually, they are correlated in the general-equilibrium economy presented in Section 4 and in the data.

\(^5\)For example, Bollerslev, Tauchen, and Zhou (2009) develop a model with (latent) stochastic volatility-of-volatility of the aggregate consumption process, in which market variance is a priced state variable. Zhou and Zhu (2015) extend this work to allow for (latent) short- and long-run volatility components.

\(^6\)As a result, $\beta_{V,t} = -\beta_{V,t}'$, and $\beta_{\rho,t} = -\beta_{\rho,t}'$. 
where \( r_{t+\Delta t} \) denotes the realized market return from \( t \) to \( t + \Delta t \). Here, as an approximation, we assume that the exposure to the risk factors (i.e., the “betas”) is constant over period \( \Delta t \).

### 1.2 Estimation Methodology

In the following, we illustrate how expression (3) can be used for market return predictability, concentrating on the two variables with observable risk premiums, that is, variance and correlation. To that end, we do not attempt to calibrate the model and identify the exposure to variance and correlation betas in the pricing equation directly\(^7\) but, instead, develop a novel estimation methodology.

Traditionally, one would estimate the betas by running a time-series regression, inspired by (3); that is, regress past realized market excess returns on lagged—by the forecasting horizon—variance and correlation risk premiums. The resulting betas are then used, together with the time-\( t \) variance and correlation risk premium, to predict future market excess returns.

While such “predictive regressions” have been shown to work in-sample, they typically do not deliver good out-of-sample results (see, e.g., Goyal and Welch (2008)). In particular, to avoid any look-ahead bias, the predictive variables are lagged by the forecasting horizon. As a result, when predicting, for example, quarterly returns at the end of December, the most recent observation of the predictive variables will be from the end of September (i.e., three months old). Consequently, this approach is susceptible to outliers and to time variation in betas; the resulting betas are literally “outdated” when the return forecast is made.

To avoid these problems, our novel estimation approach relies on two key insights. The first key insight is that one can estimate the betas using contemporaneous data at a high frequency instead of relying on long-term historical regressions. That is, the exposures to the variance

\(^7\)Although, in Section 4, we derive a similar pricing equation with an exact parametric form for the coefficients.
and correlation risk premium in the pricing equation (3) represent essentially the diffusion coefficients of the market dynamics (1) and can be estimated as such. In particular, one can estimate “contemporaneous betas” as the integrated quadratic covariance between shocks to the market and contemporaneous increments in market variance and correlation.

Effectively, this comes down to running the following simple multivariate regression, which is based on a finite-horizon counterpart of (1):

$$r_{t+\Delta t} = \alpha + \beta_{t,V} \Delta V_{W_t} + \beta_{t,\rho} \Delta \rho_{S,t},$$

where $\Delta V_{W,t}$ and $\Delta \rho_{S,t}$ denote contemporaneous changes, over period $\Delta t$, in market variance and correlation, respectively.

However, these increments in market variance and correlation are not directly observable. Our second key insight is, thus, that one can also use increments of option-implied variables. Intuitively, a change of measure—from the physical measure $P$ to the risk-neutral measure $Q$—only affects the drift of a process, but not its diffusion components (see, e.g., (Karatzas and Shreve, 1991, page 190)). As a result, the dynamics of the aggregate market $W_t$ under the risk-neutral measure are given by:

$$\frac{dW^Q_t}{W_t} = \mu'_W dt + \beta_{c,t} dB^Q_{c,t} + \beta'_{V,t} dV^Q_{W,t} + \beta'_{\rho,t} d\rho^Q_{S,t} + \beta_{\zeta,t} d\zeta^Q_t,$$

with the drift, $\mu'_W$, being the actual-measure drift, $\mu_W$, adjusted for risk premiums. Importantly, the beta coefficients in (4) are the same as in (1) and, hence, can be estimated from variables under either the actual or the risk-neutral measure. Actually, one can even use a non-matching probability measure for the dependent variable (i.e., realized market returns) because a change of measure does not affect quadratic covariation.

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8Note that because the predictive variables are correlated, computing the covariation in a univariate way would lead to biased estimates of the coefficient. Instead, a multivariate regression captures the partial covariation.
Consequently, one can estimate the betas in the pricing equation (3) using a simple multivariate regression of short-term market returns on contemporaneous changes in the respective implied variables:

\[ r_{t+\Delta t} = \alpha + \beta_{t,\Delta IV} \Delta IV + \beta_{t,\Delta IC} \Delta IC, \]  

where \( \Delta IV \) and \( \Delta IC \) denote increments, over \( \Delta t \), in the risk-neutral expected integrated variance \( IV \) and correlation \( IC \), which can be obtained from option data (see Section 1.3).

Equation (5) effectively summarizes our novel procedure for estimating the exposure to variance and correlation risk using contemporaneous changes in option-implied variables. In particular, the estimation can be implemented for short sample periods, thereby using only timely information and, by construction, eliminating the time lag of the traditional approach.

However, note that the betas estimated using regression (5), \( \beta_{t,\Delta IV} \) and \( \beta_{t,\Delta IC} \), are not exactly the same as the coefficients \( \beta_{t,V} \), and \( \beta_{t,\rho} \) in (3). Intuitively, they need to be adjusted for the difference in the variability of the regressors used for beta estimation in (5) (i.e., increments in risk-neutral expected variance and correlation) and the variability of the predictors in the pricing equation (3) (i.e., the variance and correlation risk premium). In Appendix A, we derive a simple procedure that does exactly that. The resulting betas are given by:

\[ \beta_{t,V} = \beta_{t,\Delta IV} \times \frac{\text{Vol} (\Delta IV)}{\text{Vol} (\text{VRP})}, \quad \beta_{t,\rho} = \beta_{t,\Delta IC} \times \frac{\text{Vol} (\Delta IC)}{\text{Vol} (\text{CRP})}. \]  

1.3 Risk-neutral Variables

The increments of the option-implied variables in (5) can be approximated from the dynamics of risk-neutral integrated variables. Specifically, on each day, the risk-neutral expected
integrated variance $IV$ and correlation $IC$ can be obtained from options with maturity $T$:

$$IV(t, T) = E_t^Q \left[ \int_t^T V_W(s) ds \right], \quad IC(t, T) = E_t^Q \left[ \int_t^T \rho_S(s) ds \right].$$

Decomposing the implied variance, $IV(t, T)$, as:

$$IV(t, T) = E_t^Q \left[ E_t^Q \left[ \int_t^{t+\Delta t} V_W(s) ds + \int_{t+\Delta t}^T V_W(s) ds \right] \right]$$

$$= E_t^Q \left[ \int_t^{t+\Delta t} V_W(s) ds \right] + E_t^Q \left[ IV(t + \Delta t, T) \right],$$

implies that the increments are given by:

$$\Delta IV(t + \Delta t, T) = IV(t + \Delta t, T) - IV(t, T)$$

$$= IV(t + \Delta t, T) - E_t^Q [IV(t + \Delta t, T)] - E_t^Q \left[ \int_t^{t+\Delta t} V_W(s) ds \right]. \quad (7)$$

Similar computations for the implied correlation, $IC(t, T)$, imply that

$$\Delta IC(t + \Delta t, T) = IC(t + \Delta t, T) - \Delta IC(t, T) - E_t^Q [IC(t + \Delta t, T)] - E_t^Q \left[ \int_t^{t+\Delta t} \rho_S(s) ds \right]. \quad (8)$$

In particular, if the last term in equations (7) and (8)—expected integrated variance and correlation over a short period of time $\Delta t$—is small, risk-neutral expected integrated variance and correlation can be well approximated by a martingale. Accordingly, one can use their short-interval increments as proxies for random shocks to variance and correlation:

$$\Delta IV(t + \Delta t, T) \approx IV(t + \Delta t, T) - E_t [IV(t + \Delta t, T)];$$

$$\Delta IC(t + \Delta t, T) \approx IC(t + \Delta t, T) - E_t [IC(t + \Delta t, T)].$$

Empirical evidence lends support to this approximation. For example, Filipović, Gourier, and Mancini (2016) find that a “martingale model provides relatively accurate forecasts for the one-day horizon variance.” Moreover, integrated expected variance and integrated expected correlation are highly persistent, with first-order auto-correlations between 0.97 and 0.994 for variance and between 0.97 and 0.993 for correlation at various option maturities in our data. Average daily increments are also statistically not different from zero.
2 Data and Preparation of Variables

We now discuss the data sources on which we rely for our empirical analysis as well as the computation of realized (implied) variance and correlation. We also discuss the price of variance and correlation risk for various market indices as well as their constituents.

2.1 Data Sources and Preparation

Our analysis focuses on three major U.S. stock indices and their constituents, namely, the S&P500, the S&P100, and the Dow Jones Industrial Average (DJ30) for a sample period from January 1996 to April 2016. We obtain the composition of each index from Compustat and data on the constituents’ daily returns and market capitalizations from CRSP. We proxy for the daily index weights using the constituents’ relative market capitalization (S&P500 and S&P100) or price (DJ30) from the previous day.

For the option-based variables, we rely on the Surface File from OptionMetrics. We select, for each index and its constituents, options with 30, 91, 182, 273, and 365 days to maturity and an (absolute) delta less than or equal to 0.5. While options data for the S&P500 and the S&P100 are available from January 1996 onward, the data for the DJ30 start in October 1997. On average, option data are available for about 98% of the index constituents; for example, the median number of stocks with option data is 491 for the S&P500.

2.2 Variances and Correlations

We compute option-implied variances (IV) using simple variance swaps, as in Martin (2013, 2017), capturing total quadratic variation. For robustness, we also compute implied vari-

\footnote{We merge the two datasets through the CCM Linking Table using GVKEY and IID to link to PERMNO, following the second best method from Dobelman, Kang, and Park (2014).}

\footnote{Matching the historical data with options is implemented through the historical CUSIP link provided by OptionMetrics. In particular, PERMNO is used as the identifier for single stocks in our merged database.}
ances using log contracts (i.e., model-free implied variance) as in Dumas (1995), Britten-Jones and Neuberger (2000), Bakshi, Kapadia, and Madan (2003), and others.\textsuperscript{12} Realized variances ($RV$) are estimated as the sum of squared daily returns. The \textit{ex-ante} variance risk premium, $VRP(t, T)$, is computed as the difference between the day-$t$ implied variance from options with maturity $T$ and the realized variance for the period $t - T$ to $t$.

We construct correlations as equicorrelations; that is, all pairwise correlations are assumed to be equal.\textsuperscript{13} In particular, we identify the equicorrelation under both objective and risk-neutral measures using the restriction that the variance of an index $I$ must be equal to the variance of the portfolio of its constituents:

$$\sigma^2_I(t) \triangleq \sum_{i=1}^{N} \sum_{j=1}^{N} w_i(t) w_j(t) \sigma_i(t) \sigma_j(t) \rho_{ij}(t).$$

Consequently, given the index variance, $\sigma^2_I(t)$, the variances of its constituents, $\sigma^2_i(t), i = 1 \ldots I$, and the index weights, $w_i(t)$, the equicorrelation $\rho_{ij}(t) = \rho(t)$ is computed as:

$$\rho(t) = \frac{\sigma^2_I(t) - \sum_{i=1}^{I} w_i(t)^2 \sigma^2_i(t)}{\sum_{i=1}^{I} \sum_{j \neq i} w_i(t) w_j(t) \sigma_i(t) \sigma_j(t)}.$$

\textsuperscript{(9)}

Intuitively, when using implied variances in equation (9), we arrive at implied correlation ($IC$), whereas when using realized variances, we obtain realized correlation ($RC$). The \textit{ex-ante} correlation risk premium, $CRP(t, T)$, is computed as the difference between the day-$t$ implied correlation for options with maturity $T$ and the corresponding realized correlation for the period $t - T$ to $t$.

\textsuperscript{12}In earlier versions of the paper, Martin (2013) discusses the issue of estimating implied correlations, suggesting that implied correlations should be estimated using simple variance swaps as opposed to model-free variances.

\textsuperscript{13}This is consistent with the assumption that all pairwise correlations are driven by a single state variable, as used in the general-equilibrium economy in Section 4.
2.3 Price of Variance and Correlation Risk

Tables 1 and 2 provide summary statistics for the variance risk premium of the three indices as well as their constituents for various option maturities $T$. For easier comparison across maturities, all quantities are annualized. For the S&P500 index, the average variance risk premium for individual stocks is typically not significantly different from zero (Table 1). Actually, with the exception of a maturity of 30 days, all point estimates are negative; that is, realized variance is, on average, higher than implied variance for individual stocks. In contrast, the variance risk premium for the S&P500 index is always positive and statistically significant.

Note, however, that variance risk premiums for individual stocks in the S&P500 demonstrate considerable heterogeneity (Table 2). That is, while we fail to reject the null hypothesis of an insignificant variance risk premium for a majority of stocks, there is still a sizable fraction of stocks for which we can reject the null of either a positive or a negative variance risk premium.

The results shown in Table 3 demonstrate that the correlation risk premium for the S&P500 is significantly positive for all maturities; that is, implied correlation is, on average, higher than realized correlation. Moreover, the correlation risk premium monotonically increases with option maturity, driven by an increase in the implied correlation with maturity.

In summary, similar to Driessen, Maenhout, and Vilkov (2005), we find that index variance is priced predominantly due to a priced correlation component. Hence, both correlation and index variance risk premiums potentially contain non-redundant information.

The results for the other two indices—S&P100 and DJ30—confirm these findings. In particular, all variables of interest tend to be strongly correlated across indices, with the average correlation being about 0.97. Qualitatively, the magnitude and statistical significance of the variance risk premium as well the correlation risk premium decrease with the number of index
constituents; that is, they are highest for the DJ30. In what follows, we concentrate on the S&P500 and provide results for the S&P100 and the DJ30 for completeness.

3 Return Predictability

We now examine return predictability empirically, in-sample and out-of-sample, using for the latter the novel estimation strategy for variance and correlation betas developed in Section 1.2. We also compare its performance to the traditional approach.

3.1 In-Sample Tests

In a first step, we analyze the predictability of the market excess return in-sample, using the variance and correlation risk premiums as regressors. In particular, we run the following simple predictive regression:

\[
r_{s \rightarrow s+\tau_r} = a + b VRP(s, s + \tau_r) + c CRP(s, s + \tau_r) + \epsilon,
\]

where \( r_{s \rightarrow s+\tau_r} \) denotes the market excess return from date \( s \) to \( s + \tau_r \). The variance and correlation risk premium are obtained from options with a maturity matching the forecasting horizon. We use returns from the end of each month in our sample period and Newey-West standard errors to correct for auto-correlation introduced by overlapping data.

The results are reported in Table 4—for regressions with a single explanatory variable as well as for multi-variate regressions. When using the variance risk premium as the sole explanatory variable, it is highly statistically significant for horizons of up to one quarter, with a maximum (adjusted) \( R^2 \) of 6.90%.\(^{14}\) However, for longer horizons, the variance risk premium has no ex-
exploratory power and the coefficient $b$ even turns negative. That is, a high variance risk premium at time $t$ predicts a low future market excess return—contrary to theory. The correlation risk premium, when used as the single explanatory variable, is statistically significant for horizons of up to nine months. Its exploratory power is high, even for long horizons of up to one year, and peaks at a horizon of 273 days (with an $R^2$ of 9.87%).

In joint regressions, the variance risk premium dominates at a short horizon of one month, but its coefficient is again negative for longer horizons. In contrast, the correlation risk premium is still highly significant for longer horizons, indicating that there exist *two components that provide non-redundant information*.

### 3.2 Out-of-Sample Tests: Contemporaneous Betas Approach

While many variables have strong predictive power in-sample, there is hardly any evidence for *out-of-sample predictability*, as shown convincingly by Goyal and Welch (2008). Accordingly, we now concentrate on the out-of-sample performance of the variance and correlation risk premium. We are particularly interested in their predictive power at different horizons and whether the two variables provide non-redundant information. For that purpose, we rely on our novel estimation strategy introduced in Section 1.2.

In particular, we first estimate, at the end of each month, the contemporaneous betas, $\beta_{t,\Delta IV}$ and $\beta_{t,\Delta IC}$, from equation (5). That is, we regress daily market excess returns on daily increments in implied variance and/or in implied correlation for options with a maturity matching the forecast horizon—using data from the past year. We then compute “normalized betas” $\beta_{t,V}$ and $\beta_{t,\rho}$, as in (6), using the appropriate scaling factor estimated from the same backward window. Next, we devise the out-of-sample prediction for the market excess return for

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15These findings are comparable to Cosemans (2011). Interestingly, a vast majority of existing studies (e.g., Driessen, Maenhout, and Vilkov (2005, 2012), and Faria, Kosowski, and Wang (2016)) documents return predictability for longer horizons of up to one year by implied correlation, and not by the correlation risk premium.
horizon $\tau_r$, $\hat{r}_{t\rightarrow t+\tau_r}$, by combining the normalized betas with the time-$t$ variance and correlation risk premiums from the same options:

$$\hat{r}_{t\rightarrow t+\tau_r} = \beta_{t,V} VRP(t, t + \tau_r) + \beta_{t,\rho} CRP(t, t + \tau_r).$$

For our empirical analysis, we consider four different forecast models, denoting the predicted future returns by $\hat{r}_{j,t+\tau_r}, j \in \{1, \ldots, 4\}$. The first model ($j = 1$) is based on the historical mean of the market excess return. This forecast serves as the natural benchmark because Goyal and Welch (2008) and Campbell and Thompson (2008) show that almost all predictive variables fail to beat it out-of-sample. The second and third model rely on our novel out-of-sample methodology, but use solely the variance risk premium ($j = 2$) or the correlation risk premium ($j = 3$) as predictors. Finally, the last model ($j = 4$) uses the variance and correlation risk premiums jointly. For each model $j$, each point in time $t$, and each horizon $\tau_r$, we define the forecast error, $e_{j,t+\tau_r}$, as the difference between the predicted and the realized market excess return: $\hat{r}_{j,t+\tau_r} - r_{t+\tau_r}$. For ease of exposition, we denote by $\hat{r}_{j,\tau_r}$ and $e_{j,\tau_r}$ the vectors of predicted returns and rolling out-of-sample forecast errors for horizon $\tau_r$, respectively.

We rely on the following three criteria to evaluate the performance of the different models. First, we use the out-of-sample $R^2$-squared relative to the forecasts from the (benchmark) historical average return model ($j = 1$):

$$R^2_{j,\tau_r} = 1 - \frac{MSE_{j,\tau_r}}{MSE_{1,\tau_r}}, \quad j \in \{2, \ldots, 4\},$$

where $MSE_{j,\tau_r} = \frac{1}{N} (e_{j,\tau_r}^T \times e_{j,\tau_r})$ denotes the mean-squared error of model $j$. Second, we use the Diebold and Mariano (1995) loss function, that is, the average square-error loss relative to
the prediction from the benchmark model:

$$\delta_{j,\tau_r} = MSE_{j,\tau_r} - MSE_{1,\tau_r}.$$  

Third, to measure the economic benefits of a superior return forecast, we compute the certainty equivalent gain of a mean-variance investor (similar to Campbell and Thompson (2008)). That is, at the end of each month $t$, we derive, for each model and forecast horizon, the optimal portfolio composed of a risk-free asset and the market portfolio for a myopic mean-variance investor with investment horizon $\tau_r$ and a risk aversion of one.$^{16}$ Using the resulting time series of realized portfolio returns $r_{j,\tau_r}^{MV}$, we compute the mean-variance certainty equivalent, $CE_{j,\tau_r}$, as well as the gain in the certainty equivalent return relative to the benchmark model:$^{17}$

$$\Delta CE_{j,\tau_r} = CE_{j,\tau_r} - CE_{1,\tau_r},$$  

where $CE_{j,\tau_r} = E[r_{j,\tau_r}^{MV}] - \frac{1}{2}\sigma^2(r_{j,\tau_r}^{MV}),$ and $E[r_{j,\tau_r}^{MV}]$ and $\sigma^2(r_{j,\tau_r}^{MV})$ denote the mean and variance of the time series of portfolio returns.

A particular model, $j > 1$, outperforms the benchmark model based on the average historical return if the out-of-sample $R$-squared, $R^2_{j,\tau_r}$, is significantly positive, if the average square-error loss, $\delta_{j,\tau_r}$, is significantly negative, and if the certainty equivalent gain, $\Delta CE_{j,\tau_r}$, is significantly positive. Because of the availability of option data, our sample period spans less than 20 years. As a consequence, asymptotic standard errors may not be accurate, so that we resort to bootstrapping. Specifically, we use the moving-block bootstrap procedure by Künsch (1989),$^{18}$

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$^{16}$The optimal weight in the market is given by $w_{t,\tau,j} = \hat{r}_{j,t,\tau_r}/\sigma_t^2$, where $\sigma_t^2$ denotes the one-year historical variance (same for all models). Following Campbell and Thompson (2008), we restrict the optimal weights to be in the range of $[0, 1.5]$.  

$^{17}$For robustness, we also compute the certainty equivalent gain relative to the model using the correlation risk premium as the sole predictor; that is, $CE_{j,\tau_r} - CE_{3,\tau_r}$.  

$^{18}$Moving-block bootstrap is shown (e.g., in Lahiri (1999)) to be comparable in performance to other widely used methods like stationary bootstrap by Politis and Romano (1994) and circular block bootstrap from Politis and Romano (1992). However, constant block sizes lead to smaller mean-squared errors than with random block sizes as in a stationary bootstrap. We draw 10,000 random samples of 200 blocks, with blocks of 12 observations (i.e., one-year blocks) to preserve the autocorrelation in the data.
randomly resampling with replacement from the time-series of a model’s forecasts to construct bootstrapped distributions for all performance measures.

The out-of-sample results based on the “contemporaneous betas” approach are collected in Table 5. Panel A, showing the out-of-sample $R^2$ and the square-error loss, demonstrates that using solely the variance risk premium generates significant out-of-sample return predictability. The predictability peaks at the quarterly horizon, but then declines monotonically. Similarly, Panel B shows that predictability by the variance risk premium leads to significant certainty equivalent gains relative to the benchmark model. The maximum gain is 3.5% at the quarterly horizon, but quickly declines for longer horizons.

The correlation risk premium produces significantly better return predictability than the benchmark model for all horizons. The out-of-sample $R^2$-squared reaches its maximum of 7.9% at the nine-month horizon and decreases only slightly to 7.0% for a one-year horizon. The results for the square-error loss are comparable. Also, the correlation risk premium leads to substantial certainty equivalent gains, with a gain of 3.9% at the monthly horizon, gains of more than 2% for up to nine months, and gains of slightly less than 1% for one year. Notably, for horizons of six months or longer, the $R^2$-squared is always higher than the variance risk premium. Moreover, comparing the certainty equivalent gains for the two models directly ($CE_{2,\tau} - CE_{3,\tau}$) confirms that the correlation risk premium performs better than the variance risk premium for long horizons, with incremental gains of 0.5-1%. Finally, using the variance and correlation risk premium jointly only improves predictability for short horizons of up to three months.

In summary, we document substantial out-of-sample predictability using the option-implied variables and our novel estimation procedure. Moreover, comparable to the in-sample analysis, the variance and the correlation risk premium provide non-redundant information for future
market returns, with the predictive power of the correlation risk premium being economically and statistically significant for longer horizons than the variance risk premium.

3.3 Out-of-Sample Tests: Traditional Approach

To highlight the importance of the novel estimation approach (i.e., the timely estimation of the regression coefficients), we now also report the results for the traditional predictive regression procedure. In particular, we regress, at the end of each month $t$, historical market excess returns on lagged regressors using a three-year rolling window of past data:

$$r_{s+s+\tau_r} = \beta_{VRP,t} VRP(s, s + \tau_r) + \beta_{CRP,t} CRP(s, s + \tau_r), \quad s + \tau_r \leq t. \quad (10)$$

Next, we use the resulting betas, $\beta_{VRP,t}$ and $\beta_{CRP,t}$, together with the time-$t$ variance and correlation risk premiums, $VRP(t, t + \tau_r)$ and $CRP(t, t + \tau_r)$, to form a forecast for the market excess return, $\hat{r}_{t\rightarrow t+\tau_r}$, for the forecasting horizon $\tau_r$.

Table 6 reports the results for the same three evaluation criteria used before. Panel A shows that the out-of-sample performance is considerably weaker than for the contemporaneous betas approach. Notably, for both risk premiums—variance and correlation—the $R$-squared is always negative and the square-error loss is significantly positive. That is, the predictions are inferior to predictions based on the mean historical return. Moreover, even though there are some modest certainty equivalent gains relative to the historical mean benchmark, the gains are always considerably weaker than for the contemporaneous betas approach (confer Table 5). In summary, relying on the contemporaneous betas approach is of first-order importance, especially for longer term predictions.

One key difference relative to our novel approach is that the most recent observation used to estimate the betas in the traditional approach (10) is from date $t - \tau_r$, whereas the contempo-
raneous betas approach uses information up to time $t$. As a result, the contemporaneous betas approach is much better suited to capture time variation in betas. To illustrate this difference, Figure 1 contrasts the corresponding betas for the two approaches. The differences in betas can be substantial, particularly for longer horizons. Notably, while the variance and correlation betas are quite volatile in general, the contemporaneous betas are considerably more stable, adding to the stability of the return forecast.

3.4 Robustness Tests

We now discuss the results of a number of robustness tests. For ease of exposition, we keep the discussion brief; all accompanying tables are collected in an Internet Appendix.

Instead of using the variance risk premium from options with a maturity matching the forecasting horizon, the literature has often used options with a maturity of one month for all forecasting horizons (see, among others, Bollerslev, Tauchen, and Zhou (2009) and Bollerslev, Marrone, Xu, and Zhou (2014)). Accordingly, in the following, we summarize the in-sample and out-of-sample predictability results when using the variance risk premium and implied variance (used to estimate the beta $\beta_{t,\Delta IV}$) from options with one month to maturity.

Consistent with the literature, the in-sample predictive power for these short-maturity options is typically slightly better (Table IA1), and the coefficients on the variance risk premium do not turn negative. However, qualitatively, the pattern is the same; that is, the $R^2$ peaks at a quarterly frequency and then declines. A similar picture emerges for the out-of-sample return predictability (Table IA2). In summary, using the variance risk premium from options with a 30-day maturity delivers slightly better results than options with a maturity matching the return horizon, but the main results are unchanged.
We also compare the predictive power of the option-based variables relative to a number of fundamental variables, that have been used extensively in the literature. While there is a myriad of variables (see, among others, Goyal and Welch (2008) and Ferreira and Santa-Clara (2011)), we limit our analysis to five variables that encompass non-redundant economic information and have been shown to be highly significant in-sample. Specifically, following Goyal and Welch (2008), we construct the earnings-price ratio, the term spread, the default-yield spread, the book-to-market ratio and the net equity expansion. While a number of fundamental variables successfully improve the predictive power of the return forecast, none of them affects the sign or significance of the correlation risk premium (Table IA3). In some cases, adding the term or default spread actually improves the significance of the correlation risk premium (e.g., for 9- and 12-month predictions).

4 General-Equilibrium Model

In this section, we propose a dynamic general-equilibrium model that can rationalize the reduced-form framework introduced in Section 1 and, thus, serves as further support for our empirical analysis. The model is set in continuous time and there exists a representative investor. Financial markets are assumed to be complete; among others, there exist multiple traded stocks, modelled as claims to individual dividend trees, and a traded market index, modelled as a claim to aggregate consumption.

19 We are grateful to Amit Goyal for making the data available on his website www.hec.unil.ch/agoyal/.
20 The Internet Appendix contains a more detailed description on how the variables are constructed.
21 Note that it is not our intent to test such a model formally. It is developed for illustration only.
In particular, a large number of individual Lucas (1978) dividend trees, $i \in \{1, \ldots, I\}$ exists, with the dynamics of the first $I-1$ dividend trees being given by:\footnote{We do not explicitly specify the dynamics of the last dividend tree $I$; however, in what follows, we specify the process of aggregate consumption. This modeling device is inspired by Menzly, Santos, and Veronesi (2004) and Basak and Pavlova (2013). It allows us to assume that aggregate consumption follows the dynamics in (11) and (12), which improves the tractability of the model considerably.}

$$\frac{dD_{i,t}}{D_{i,t}} = \mu_{D,i} dt + \sigma_{D,i} \sqrt{V_{i,t}} dB_{i,t} + \sigma_{DC,i} \sqrt{V_t} dB_{c,t},$$

where $\mu_{D,i}$ denotes the dividend drift and $\sigma_{DC,i}$ and $\sigma_{D,i}$ capture the sensitivity to systematic shocks, $B_{c,t}$, and (uncorrelated) idiosyncratic shocks, $B_{i,t}$, respectively. Idiosyncratic variance, $V_{i,t}$, is stochastic and follows a square-root process:

$$dV_{i,t} = \kappa_{1,i} (\bar{V}_i - V_{i,t}) dt + \varsigma_i \sqrt{V_{i,t}} dB_{V,t},$$

where $\kappa_{1,i}$, $\bar{V}_i$, and $\varsigma_i$ denote the speed of mean-reversion, the long-run mean, and the “volatility-of-volatility,” respectively. Aggregate variance, $V_t$, is stochastic as well, as discussed below.

Pairwise correlations between the dividend trees are stochastic and driven by a single state variable. That is, the instantaneous correlation between two trees $i \neq j$; $i, j < I$, is:\footnote{Consistent with the discussion in footnote 22, we do not explicitly specify the correlations between dividend trees $i \leq I-1$ and the last dividend tree $I$. They are implicitly defined by the process of aggregate consumption.}

$$\rho_{i,j,t} dt = \rho \ dt.$$

The dynamics of aggregate consumption in the economy, $C_t = \sum_{i=1}^{I} D_{i,t}$, are given by:

$$\frac{dC_t}{C_t} = \mu_c dt + \delta_c \sqrt{V_t} dB_{c,t}, \tag{11}$$

where $\mu_c$ and $\delta_c$ denote consumption growth and the sensitivity to systematic shocks $dB_{c,t}$.

Aggregate variance, $V_t$, is stochastic and described by the following mean-reverting process:

$$dV_t = \kappa_1 (\bar{V} - V_t) dt + \sigma_1 \sqrt{V_t} dB_{V,t} + \sigma_\rho d\rho_t, \tag{12}$$
where $\kappa_1$ and $\bar{V}$ denote the speed of mean-reversion as well as the long-run mean, and $\sigma_1$ and $\sigma_\rho$ capture the sensitivity to uncorrelated shocks $dB_{V,t}$ and $d\rho_t$, respectively. This two-component variance structure is, technically, quite similar to Zhou and Zhu (2015), but it is explicit about the two components—aggregate variance and correlation—instead of relying on latent variables.

We assume that the correlation state variable, $\rho_t$, follows a mean-reverting process:

$$d\rho_t = \kappa_2 (\bar{\rho} - \rho_t) \, dt + \sigma_\rho (\rho_t) \, dB_{\rho,t}, \tag{13}$$

where $\bar{\rho}$, $\kappa_2$, and $\sigma_\rho$ denote the long-run mean, the speed of mean-reversion, and the diffusion sensitivity, respectively. In particular, we use a square-root process: $\nu(\rho_t) = \sqrt{\rho_t}$, which improves tractability considerably and allows for closed-form solutions of all key quantities.\(^24\)

The representative investor has recursive preferences as in Duffie and Epstein (1992b), with relative risk aversion $\gamma > 0$, intertemporal elasticity of substitution $\psi > 0$, and rate of time preference $\beta$. The investor chooses consumption to maximize lifetime utility.

In equilibrium,\(^25\) the pricing kernel is given by:

$$\frac{d\pi_t}{\pi_t} = -r_f \, dt - \lambda_1 \, dB_{c,t} - \lambda_2 \, dB_{V,t} - \lambda_3 \, dB_{\rho,t}, \tag{14}$$

where $\lambda_1 = \gamma \delta_c \sqrt{V_t}$, $\lambda_2 = -\frac{1 - \gamma \psi}{1 - \gamma} \, A_1 \sigma_1 \sqrt{V_t}$ and $\lambda_3 = -\frac{1 - \gamma \psi}{1 - \gamma} \, (A_1 \sigma_\rho + A_2 \sigma_2) \sqrt{\rho_t}$\(^26\) denote the risk premiums for the three sources of risk in the economy—aggregate consumption, aggregate consumption variance, and dividend correlation—respectively.

Solving for the process for the aggregate market (i.e., the wealth process), we obtain:

$$\frac{dW_t}{W_t} = \zeta_W \, dt + \delta_c \, \sqrt{V_t} \, dB_{c,t} - A_1a \, dV_t - A_2a \, d\rho_t, \tag{15}$$

\(^{24}\)Technically, the correlation could end up being above one. However, in calibrations, one can choose parameters such that the correlation stays effectively bounded.

\(^{25}\)The details of the solution are collected in Appendix B.

\(^{26}\) $A_j$, $j = 1, 2$ denote the coefficients of the state variables $V_t$ and $\rho_t$ in the value function. Their closed-form solutions can be found in (A16) in the Appendix.
where $A_{ia} = \frac{1 - \psi}{1 - \gamma} A_i$, $i = 1, 2$, and $\zeta_W$ denotes a “partial” drift.\footnote{It is a “partial” drift because both $dV_t$ and $d\rho_t$ also contain deterministic terms. One could write the wealth process in terms of original sources of risk $dB_{V,t}$ and $dB_{\rho,t}$, but this expression is more convenient for illustration.} Hence, the aggregate market index is driven by (standard) consumption uncertainty as well as shocks to aggregate consumption variance and dividend correlation. Consequently, the equity risk premium, given by the covariance between the pricing kernel (14) and the aggregate market process (15), is:

$$E_t^P \left[ \frac{dW}{W} \right] - r_{f,t} dt = \lambda_1 \delta_c \sqrt{V_t} dt - A_{1a} \left( E_t^P [dV_t] - E_t^Q [dV_t] \right) - A_{2a} \left( E_t^P [d\rho_t] - E_t^Q [d\rho_t] \right) = \text{VRP}_{C,t}$$ (16)

That is, the equity premium can be decomposed into three components: (i) “standard” compensation for consumption risk, (ii) compensation for aggregate consumption variance risk, $\text{VRP}_{C,t}$, and (iii) compensation for dividend correlation risk, $\text{CRP}_{C,t}$.

The risk premiums on consumption variance and dividend correlation are not readily available from the data. But, one can explicitly derive the processes for the market variance:

$$dV_{W,t} = (\delta_c^2 + A_{1a} \sigma_1^2) dV_t + (A_{1a} \bar{\sigma}_\rho + A_{2a} \sigma_2)^2 d\rho_t,$$ (17)

and, from the prices of the individual dividend claims (“stocks”), the dynamics of the average return-correlation between symmetric stocks:\footnote{Intuitively, when stocks are symmetric, the average correlation is equal to pairwise correlations.} $\text{Cov}_S$ denotes the covariance between stocks.

$$d\rho_{S,t} = \zeta_{\rho_S} dt + \frac{V_S - \text{Cov}_S}{V_S^2} \left[ (\sigma_{DC}^2 + A_{1m} \sigma_1^2) dV_t + (A_{1m} \bar{\sigma}_\rho + A_{2m} \sigma_2)^2 d\rho_t \right] - \frac{1}{V_S^2} \sigma_{D,i}^2 dV_{i,t}, \quad (18)$$

where $\zeta_{\rho_S}$ captures the partial drift, $A_{jm}, j = 1, 2$ denote the coefficients in the equilibrium price-dividend ratio, $V_S$ denotes the variance of individual dividend claims, and $\text{Cov}_S$ denotes the covariance between stocks.\footnote{The expressions for these second moments are provided in equations (A25) and (A24) in the Appendix.}

Hence, one can compute the risk premium on market variance risk, $\text{VRP}_t$, and the risk premium on the stock correlation risk, $\text{CRP}_t$—both being observable—and connect them to the
latent risk premiums for consumption variance risk, $VRP_{C,t}$, and dividend correlation risk:

$$
\begin{bmatrix}
VRP_t \\
CRP_t
\end{bmatrix}
= \begin{bmatrix}
\frac{V_{Sc-Corr}}{\sqrt{\delta_c}} (\sigma^2_{DC} + A_1 \sigma^2_{1}) \\
\frac{V_{Sc-Corr}}{\sqrt{\delta_c}} (A_1 \sigma_{\rho} + A_2 \sigma_2)^2
\end{bmatrix}
\times
\begin{bmatrix}
VRP_{C,t} \\
CRP_{C,t}
\end{bmatrix}
. \tag{19}
$$

Solving the equation system (19) for the risk premiums on aggregate consumption variance risk and dividend correlation and substituting in (16) gives the equity risk premium as:

$$
E^{\mathbb{P}} \left[ \frac{dW}{W} \right] - r_{f,t} dt = \lambda_1 \delta_c \sqrt{V_t} dt + A_{1z} VRP_t + A_{2z} CRP_t. \tag{20}
$$

Compared to Bollerslev, Tauchen, and Zhou (2009), both variance and correlation risk premium endogenously arise as predictors for future excess market returns.

Thus, the model strongly supports the two key assumptions of the reduced-form model: (i) priced variance and correlation risk and (ii) an aggregate stock market process driven by shocks to market variance and stock correlation. As a result, the general-equilibrium model yields a decomposition for the equity risk premium similar to (2). By integration, a finite horizon expression similar to (3) arises, thereby lending direct support to our empirical analysis.

While our general-equilibrium model should be interpreted to provide qualitative support for the reduced-form model, we now briefly demonstrate that it can also produce plausible quantitative results for key quantities. We set relative risk aversion, $\gamma$, to 6, the elasticity of intertemporal substitution, $\psi$, to 2, and the rate of time-preference, $\beta$, to 0.95—in line with the literature. For aggregate consumption, we choose a growth rate $\mu_c = 2\%$ and a sensitivity to aggregate risk $\delta_c = 0.5$. We consider symmetric dividend trees, $i < I$, with dividend growth $\mu_{D,i} = 0.02$, a sensitivity to idiosyncratic shocks $\sigma_{D,i} = 0.3$, and a sensitivity to aggregate

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30 Idiosyncratic variance, $V_i$, is not priced and, hence, affects neither the risk premium for market variance nor the risk premium for correlation risk.

31 $A_{1z}$ and $A_{2z}$ are functions of the stock variance, the average covariance between individual stocks, and other matrix elements in (19), as given in equation (A20) in the Appendix.

32 One can rewrite the process for the aggregate market, $W_t$, in (15) in terms of shocks to market variance, $dV_W$, and to pairwise stock-correlation, $d\rho_{S}$; by substituting aggregate consumption risk and dividend correlation risk by market variance risk and stock correlation risk, similar to the substitutions used to arrive at (20).
shocks $\sigma_{DC,i} = 0.4$. The long-term mean of aggregate consumption variance and idiosyncratic variance, $\bar{V}$ and $\bar{V}_i$, are set to 0.0009 and 0.2, respectively. Finally, the long-term mean of dividend correlation, $\bar{\rho}$, is set to 0.1.\footnote{The remaining parameters of the model are set as follows: $\kappa_1 = 0.05$, $\kappa_2 = 0.001$ and $\sigma_\rho = 0.001$ for the aggregate volatility process $d\bar{V}_t$; $\kappa_2 = 0.10$ and $\sigma_2 = 0.04$ for the correlation process $d\rho_t$.}

As a result, aggregate consumption growth is 2%, with an unconditional volatility of 1.5%. The average (real) interest rate is 0.67% and the average equity risk premium, that is, the premium on a leverage claim (with a leverage factor of 2) on aggregate consumption, is 7.6%. The average market return volatility is of about 13%. Following the decomposition (16), the correlation risk premium, the variance risk premium, and the compensation for consumption risk contribute about 68%, 12%, and 10% to the equity risk premium, respectively. Finally, the average return volatility for individual stocks is about 30% (again, with a leverage factor of 2) and the average correlation between individual stocks is around 0.22. All these moments are largely consistent with the data, confirming the models ability to match key quantities.

5 Sources of Priced Correlation Risk

While for our analysis of the market return predictability the source of the correlation risk premium is secondary, understanding its economic foundation is of importance itself. Accordingly, in this section, we study potential explanations for priced correlation risk.

5.1 The Correlation Risk Premium and Economic Uncertainty

In the general-equilibrium setting of Burasi, Trojani, and Vedolin (2014), investors disagree about future earnings. In particular, higher earnings uncertainty leads agents to expect stocks to behave more like the market in the future. Thus, the expected correlation under the objective measure increases and a correlation risk premium arises. Hence, the correlation
risk premium might simply serve as a proxy for economic uncertainty. Buraschi, Trojani, and Vedolin (2014) provide solid empirical support for this mechanism by showing that the ex-post correlation risk premium is positively related to differences-in-beliefs.

We reconstruct $DIB$, the disagreement proxy used in Buraschi, Trojani, and Vedolin (2014). For the sample period used in Buraschi, Trojani, and Vedolin (2014), January 1996 to July 2007, we document a similar positive correlation between $DIB$ and the ex-post correlation risk premium. The correlation ranges from 0.11 for options with 30-day maturity to 0.06 for options with one-year maturity. However, for our whole sample period until April 2016, the correlation with the 30-day correlation risk premium becomes literally zero (0.008), turning negative for longer maturities, and reaching -0.19 for options with one-year maturity.

Hence, while the link between economic uncertainty (disagreement) and the correlation risk premium is theoretically well founded and appealing, the empirical results for the last years do not fully support it. It is outside the scope of this paper to understand the exact reasons for such changes in the empirical evidence and we leave it as an open topic for future research.

5.2 The Correlation Risk Premium and Future Market Risk

Another potential explanation for the existence of a correlation risk premium is the role of correlation as a state variable in the ICAPM, predicting future investment opportunities.

A simple measure of investment opportunities is the portfolio variance of an equal-weighted portfolio of $N$ stocks. As $N$ becomes large, the variance is driven solely by the average covariance, that is, average correlation as well as average variance. Moreover, in a simple one-factor explanation, the variance is driven solely by the average covariance, that is, average correlation as well as average variance. Moreover, in a simple one-factor

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34Following Diether, Malloy, and Scherbina (2002), we compute, for each firm, a disagreement proxy as the standard deviation of a firm’s earnings-per-share forecasts for the next fiscal year (scaled by the absolute value of the forecasts) using the Unadjusted Summary History file for U.S. firms from I/B/E/S. Market-wide disagreement, $DIB$, is then defined as an equal-weighted average of the firm-specific disagreement proxies.

35For robustness, we also measure economic uncertainty using the measure of economic policy uncertainty (EPU) by Baker, Bloom, and Davis (2016) from www.policyuncertainty.com/. The results are comparable.

36The average covariance also strongly affects the number of stocks needed to form a well-diversified portfolio.
market model for returns, covariances \( \sigma_{ij} = \beta_{M,i} \beta_{M,j} \sigma^2_M, \ i \neq j \), being the product of two stocks’ betas and market variance, are decreasing in the cross-sectional dispersion of betas. Consequently, realized correlation, realized market variance, and the dispersion of market betas should serve as good proxies for investment opportunities.

Table 7 provides the regression results for predicting these risk proxies for various horizons—using separately implied variance or implied correlation. The future dispersion of market betas is best predicted by implied correlation, with \( R^2 \)’s of around 30% at long horizons. Implied correlation also predicts future realized correlation, with high \( R^2 \)’s for all horizons. But, the predictive power of implied correlation for future market variance is limited. In contrast, implied variance predicts future market variance. Its explanatory power is impressive at short horizons (\( R^2 \) of almost 50% for one-month), but drops quickly and is only about 12% for an annual horizon. Consistent with expectation, an increase in either variable—implied correlation or implied market variance—predicts a deterioration in future investment opportunities.

In summary, consistent with our empirical results on return predictability, variance and correlation provide non-redundant information for future investment opportunities. In particular, their predictive power varies with the horizon. While variance predicts shorter term risk, correlation plays an important role in determining longer term risks. This allows the link to Buraschi, Kosowski, and Trojani (2014), with correlation as a “no-place-to-hide” state variable.

6 Conclusion

We show that the correlation risk premium can predict future market excess returns in-sample and out-of-sample at horizons of up to one year. As a comparison, the predictability of the variance risk premium peaks already at the quarterly horizon.
In particular, based on an illustrative reduced-form model, we develop a novel methodology for estimating contemporaneous betas using option-implied variance and correlation. With this approach, we document strong long-term out-of-sample predictability for market returns, for instance, an out-of-sample $R^2$ of 7% at the annual horizon as well as substantial economic gains for investors (e.g., a 2% p.a. certainty equivalent gain). The out-of-sample predictability crucially depends on the use of the contemporaneous betas. That is, if one relies on traditional predictive regressions, the evidence is considerably weaker if not non-existent.

We provide theoretical support for our reduced-form model by means of a dynamic, multi-asset general-equilibrium economy with recursive preference. In the model, variance and correlation risk are endogenously priced and the aggregate market is driven by shocks to market variance and stock correlation, justifying the key assumptions of the reduced-form model.

Finally, we study potential economic mechanisms generating a correlation risk premium. Our results lend support to the interpretation of a “no-place-to-hide” state variable. In particular, while expected variance can predict short-term risks, expected correlation predicts diversification benefits for horizons of up to one year, consistent with the predictability results.

In summary, our results show that option-implied variables can have strong predictive power for future market returns. Intuitively, these results should have important implications for portfolio management. Moreover, our methodology can easily be extended to other settings, for example, to study cross-sectional predictability using option data.
References


The table reports the time-series averages of implied variance (IV) as well as realized variance (RV), each expressed in volatility terms, and the variance risk premium (VRP = IV − RV), together with a p-value indicating its significance—for maturities of 30, 91, 182, 273 and 365 (calendar) days. All numbers are expressed in annual terms. For index constituents, we report equal-weighted cross-sectional averages of the respective quantity. Results are reported for three samples of stocks—S&P500, S&P100, and DJ30—and the sample periods span from January 1996 to April 2016 (S&P500 and S&P100), and from October 1997 to April 2016 (DJ30). Implied variance (IV) is computed, on each day, based on simple variance swaps using out-of-the money options with the respective maturity, and realized variance (RV) is calculated, on each day, based on daily returns over a window matching the option maturity. We use Newey and West (1987) standard errors to adjust for autocorrelation.

<table>
<thead>
<tr>
<th>Days</th>
<th>Index Constituents</th>
<th>Indices</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>√IV</td>
<td>√RV</td>
</tr>
<tr>
<td><strong>SP500 Sample</strong></td>
<td></td>
<td></td>
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<tr>
<td>30</td>
<td>0.398</td>
<td>0.397</td>
</tr>
<tr>
<td>91</td>
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<tr>
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<td>0.392</td>
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<tr>
<td>365</td>
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<td>0.392</td>
</tr>
<tr>
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<td></td>
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<tr>
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<tr>
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<tr>
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<td>0.361</td>
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<tr>
<td><strong>DJ30 Sample</strong></td>
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<td></td>
</tr>
<tr>
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<tr>
<td>365</td>
<td>0.304</td>
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</tr>
</tbody>
</table>
### Table 2 Implied and Realized Variance: Index Constituents

The table reports the number of constituents within each index, for which we can either reject the hypothesis that implied variance is higher than realized variance \((IV - RV \geq 0)\) or the hypothesis that the implied variance is smaller than realized variance \((IV - RV \leq 0)\), plus the number of stocks for which we fail to reject both hypotheses \((IV = RV)\)—for maturities of 30, 91, 182, 273 and 365 (calendar) days. Results are reported for three samples of stocks—S&P500, S&P100, and DJ30—and the sample periods span from January 1996 to April 2016 (S&P500 and S&P100), and from October 1997 to April 2016 (DJ30). Implied variance \((IV)\) is computed, on each day, based on simple variance swaps using out-of-the-money options with the respective maturity, and realized variance \((RV)\) is calculated, on each day, based on daily returns over a window matching the option maturity. We use Newey and West (1987) standard errors to adjust for autocorrelation, with lags equal to the number of overlapping observations.

<table>
<thead>
<tr>
<th>Days</th>
<th>(IV - RV \geq 0)</th>
<th>(IV = RV)</th>
<th>(IV - RV \leq 0)</th>
</tr>
</thead>
<tbody>
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<td><strong>SP500 Sample</strong></td>
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<td></td>
</tr>
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<td>91</td>
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</tr>
<tr>
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<td>12</td>
<td>150</td>
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<td>9</td>
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<tr>
<td>365</td>
<td>12</td>
<td>159</td>
<td>40</td>
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<tr>
<td><strong>DJ30 Sample</strong></td>
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</tr>
<tr>
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</tr>
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<td>37</td>
<td>10</td>
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</tbody>
</table>
Table 3 Implied and Realized Correlation: Summary

The table reports summary statistics (time-series mean, \( p \)-value for the mean, median, and the standard deviation) for implied correlation (\( IC \)), realized correlation (\( RC \)), and for the correlation risk premium (\( CRP = IC - RC \))—for maturities of 30, 91, 182, 273 and 365 (calendar) days. Results are reported for three samples of stocks—S&P500, S&P100, and DJ30—and the sample periods span from January 1996 to April 2016 (S&P500 and S&P100), and from October 1997 to April 2016 (DJ30). \( IC \) (\( RC \)) are calculated from daily observations of implied (realized) variances for the index and for all index components using expression (9). Implied variance (\( IV \)) is computed, on each day, based on simple variance swaps using out-of-the money options with the respective maturity, and realized variance (\( RV \)) is calculated, on each day, based on daily returns over a window matching the option maturity. We use Newey and West (1987) standard errors to adjust for autocorrelation.

<table>
<thead>
<tr>
<th></th>
<th>30</th>
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<th>182</th>
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<th>91</th>
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<tr>
<td>( IC )</td>
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<td></td>
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<td></td>
<td></td>
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</tr>
<tr>
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<td>0.454</td>
<td>0.459</td>
<td>0.327</td>
<td>0.326</td>
<td>0.327</td>
<td>0.328</td>
<td>0.327</td>
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<td>0.298</td>
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</tr>
<tr>
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<td>0.498</td>
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<td>0.357</td>
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<td>0.359</td>
<td>0.358</td>
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<td>0.000</td>
<td>0.000</td>
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<tr>
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<td>0.496</td>
<td>0.506</td>
<td>0.509</td>
<td>0.331</td>
<td>0.344</td>
<td>0.339</td>
<td>0.342</td>
<td>0.341</td>
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<td>0.101</td>
<td>0.152</td>
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<td>0.116</td>
<td>0.114</td>
<td>0.090</td>
<td>0.090</td>
<td>0.094</td>
<td>0.093</td>
</tr>
<tr>
<td>( DJ30 )</td>
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<td></td>
<td></td>
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<tr>
<td>Mean</td>
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<td>0.528</td>
<td>0.371</td>
<td>0.373</td>
<td>0.376</td>
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<td>0.377</td>
<td>0.082</td>
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<td>0.000</td>
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<td>Median</td>
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<td>0.541</td>
<td>0.539</td>
<td>0.352</td>
<td>0.363</td>
<td>0.368</td>
<td>0.359</td>
<td>0.359</td>
<td>0.078</td>
<td>0.102</td>
<td>0.134</td>
<td>0.144</td>
<td>0.141</td>
</tr>
<tr>
<td>StDev</td>
<td>0.148</td>
<td>0.129</td>
<td>0.118</td>
<td>0.113</td>
<td>0.105</td>
<td>0.169</td>
<td>0.148</td>
<td>0.143</td>
<td>0.142</td>
<td>0.141</td>
<td>0.130</td>
<td>0.102</td>
<td>0.095</td>
<td>0.094</td>
<td>0.090</td>
</tr>
</tbody>
</table>
Table 4 In-Sample Market Return Predictability: Correlation and Variance Risk Premiums

The table reports the regression coefficients, corresponding p-values (in brackets), and the adjusted $R^2$ (in percentages) of the in-sample regressions for market return predictability. In particular, we regress overlapping market excess returns for a specific horizon on a constant, the variance risk premium and the correlation risk premium. We consider forecasting horizons of 30, 91, 182, 273, and 365 calendar days. Results are reported for three samples of stocks—S&P500, S&P100, and DJ30—and the sample periods span from January 1996 to April 2016 (S&P500 and S&P100), and from October 1997 to April 2016 (DJ30). The variance risk premium, $VRP$, is defined as the difference between implied and realized variance. Implied variance is computed, on each day, based on simple variance swaps using out-of-the-money options with a maturity matching the forecasting horizon, and realized variance is calculated, on each day, based on daily returns over a window of the same length. Similarly, the correlation risk premium, $CRP$, is defined as the difference between implied and realized correlation. Implied correlation and realized correlation are calculated from daily observations of implied (realized) variances for the index and for all index components using expression (9). We use Newey and West (1987) standard errors to adjust for autocorrelation.

<table>
<thead>
<tr>
<th>Return, 30 days</th>
<th>Return, 91 days</th>
<th>Return, 181 days</th>
<th>Return, 273 days</th>
<th>Return, 365 days</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SP500 Sample</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CRP</td>
<td>0.076 (0.027)</td>
<td>-0.027 (0.362)</td>
<td>0.254 (0.002)</td>
<td>-0.195 (0.027)</td>
</tr>
<tr>
<td>VRP</td>
<td>-0.322 (0.004)</td>
<td>0.289 (0.002)</td>
<td>-0.562 (0.027)</td>
<td>-0.270 (0.175)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>2.48 (0.004)</td>
<td>6.90 (0.007)</td>
<td>6.81 (0.001)</td>
<td>7.26 (0.007)</td>
</tr>
<tr>
<td><strong>SP100 Sample</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CRP</td>
<td>0.051 (0.076)</td>
<td>-0.011 (0.679)</td>
<td>0.334 (0.044)</td>
<td>-0.161 (0.062)</td>
</tr>
<tr>
<td>VRP</td>
<td>-0.333 (0.004)</td>
<td>0.319 (0.006)</td>
<td>-0.652 (0.042)</td>
<td>-0.400 (0.101)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>1.21 (0.000)</td>
<td>6.98 (0.000)</td>
<td>6.90 (0.001)</td>
<td>7.24 (0.008)</td>
</tr>
<tr>
<td><strong>DJ30 Sample</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CRP</td>
<td>0.040 (0.117)</td>
<td>-0.010 (0.675)</td>
<td>0.205 (0.007)</td>
<td>-0.133 (0.120)</td>
</tr>
<tr>
<td>VRP</td>
<td>-0.292 (0.005)</td>
<td>-0.277 (0.005)</td>
<td>-0.679 (0.000)</td>
<td>-0.420 (0.044)</td>
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<tr>
<td>$R^2$</td>
<td>0.90 (0.000)</td>
<td>4.53 (0.101)</td>
<td>4.16 (0.006)</td>
<td>6.27 (0.001)</td>
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</table>
Table 5 Out-of-Sample Predictability: Contemporaneous Beta Approach

The table reports the results of the out-of-sample predictability analysis, using the contemporaneous betas. Panel A shows the out-of-sample $R^2_{j,\tau}$ and the Diebold-Mariano statistic $\delta_{j,\tau}$; Panel B shows the certainty equivalent gain of a mean-variance investor relative to the historical average market return forecast—with p-values in brackets and forecasting horizons of 30, 91, 182, 273, and 365 calendar days. Results are reported for three samples of stocks—S&P500, S&P100, and DJ30—and the sample periods span from January 1996 to April 2016 (S&P500 and S&P100), and from October 1997 to April 2016 (DJ30). The variance risk premium, $\text{VRP}$, is defined as the difference between implied and realized variance. Implied variance is computed, on each day, based on simple variance swaps using out-of-the-money options with a maturity matching the forecasting horizon, and realized variance is calculated, on each day, based on daily returns over a window of the same length. Similarly, the correlation risk premium, $\text{CRP}$, is defined as the difference between implied and realized correlation. Implied correlation and realized correlation are calculated from daily observations of implied (realized) variances for the index and for all index components using expression (9). The contemporaneous betas are computed using daily increments over a 12-month rolling window. The p-values are obtained from a bootstrapped distribution using moving-block bootstrap by Künsch (1989), with 10,000 samples and a block length of 12 months.

**Panel A: Out-of-sample $R^2$ and $\delta$**

<table>
<thead>
<tr>
<th>Days</th>
<th>VRP</th>
<th>CRP</th>
<th>VRP+CRP</th>
<th>VRP</th>
<th>CRP</th>
<th>VRP+CRP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R^2_{j,\tau}$</td>
<td></td>
<td>$\delta_{j,\tau}$</td>
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<tr>
<td>SP500 Sample</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>-0.001</td>
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<tr>
<td></td>
<td>(0.000)</td>
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Table 6 Out-of-Sample Predictability: Traditional Beta Approach

The table reports the results of the out-of-sample predictability analysis, using the traditional betas. Panel A shows the out-of-sample $R^2_{\tau_r}$ and the Diebold-Mariano statistic $\delta_{\tau_r}$; Panel B shows the certainty equivalent gain of a mean-variance investor relative to the historical average market return forecast—with p-values in brackets and forecasting horizons of 30, 91, 182, 273, and 365 calendar days. Results are reported for three samples of stocks—S&P500, S&P100, and DJ30—and the sample periods span from January 1996 to April 2016 (S&P500 and S&P100), and from October 1997 to April 2016 (DJ30). The variance risk premium, $VRP$, is defined as the difference between implied and realized variance. Implied variance is computed, on each day, based on simple variance swaps using out-of-the-money options with a maturity matching the forecasting horizon, and realized variance is calculated, on each day, based on daily returns over a window of the same length. Similarly, the correlation risk premium, $CRP$, is defined as the difference between implied and realized correlation. Implied correlation and realized correlation are calculated from daily observations of implied (realized) variances for the index and for all index components using expression (9). The betas are computed using monthly sampled variables over a 60-month rolling window. The p-values are obtained from a bootstrapped distribution using moving-block bootstrap by Künsch (1989), with 10,000 samples and a block length of 60 months.

**Panel A: Out-of-sample $R^2$ and $\delta$**

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Table 6 continued

Panel B: Certainty Equivalent Gain

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SP100 Sample

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DJ30 Sample

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Table 7 Risk Predictability

The table reports the regression coefficient (with corresponding p-value) and the $R^2$ from regressions of future risk proxies on lagged implied variance ($IV$) or on lagged implied correlation ($IC$)—for a horizon of 30, 91, 182, 273 and 365 (calendar) days. Panel A reports the results for the future cross-sectional variance of market betas of all stocks in an index, $\sigma^2(\beta_M)$; Panel B reports the results for future realized equicorrelation, $RC$; and Panel C reports the results for future realized market variance, $RV$. Implied variance is computed, on each day, based on simple variance swaps using out-of-the-money options with the respective maturity, and realized variance is calculated, on each day, based on daily returns over a window matching the option maturity. Implied correlation and realized correlation are calculated from daily observations of implied (realized) variances for the index and for all index components using expression (9). We use Newey and West (1987) standard errors to adjust for autocorrelation.

**Panel A: Dispersion of Market Betas – $\sigma^2(\beta_M)$**

<table>
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<tr>
<th>$\sigma^2(\beta_M)$, 30 days</th>
<th>$\sigma^2(\beta_M)$, 91 days</th>
<th>$\sigma^2(\beta_M)$, 181 days</th>
<th>$\sigma^2(\beta_M)$, 273 days</th>
<th>$\sigma^2(\beta_M)$, 365 days</th>
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<td>$R^2$</td>
<td>$\beta$</td>
<td>p-value</td>
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<td>$IV$</td>
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<td>1.328</td>
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<tr>
<td>$IC$</td>
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<td>$IV$</td>
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**Panel B: Realized Correlation – $RC$**

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**Panel C: Realized Variance – $RV$**

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43
Figure 1. Contemporaneous Betas vs. “Traditional” Betas

The figure shows the time-series of the variance and correlation betas, estimated using our novel, contemporaneous approach (solid, blue line) or estimated using the traditional, predictive-regression approach (dashed, orange line). Panels (a), (c) and (e) illustrate the results for the variance betas and a horizon (and, thus, also an option maturity) of 30, 182, and 365 days, respectively. Similarly, Panels (b), (d) and (f) illustrate the results for the correlation betas. The contemporaneous betas are based on a 12-month historical window of daily returns, and the “traditional” betas are based on a historical rolling window of 60 months. The results are reported for the S&P500 and a sample period from January 1996 to April 2016; although the betas are only reported from 2001 onward because of the 60-months estimation window for the traditional betas.
Appendix

A “Normalizing” Variance and Correlation Betas

The variance and correlation betas, $\beta_{t,\Delta IV}$ and $\beta_{t,\Delta IC}$, as estimated from regression (5) based on increments of option-implied variables, differ from the exposures $\beta_{t,V}$ and $\beta_{t,\rho}$ in the forecasting equation (3). In particular, they need to be adjusted for the difference in the variability of the regressors used for beta estimation (i.e., increments in risk-neutral expected variance and correlation) and the variability of the predictors in the pricing equation (i.e., the variance and correlation risk premium).

Intuitively, the variance beta in the forecasting equation, $\beta_{t,V}$, can be decomposed into (i) the correlation between the market excess return and the variance risk premium, and (ii) the ratio of their volatilities. Similarly, the variance beta in the estimation equation, $\beta_{t,\Delta IV}$, can be decomposed into (i) the correlation between the market excess return and the increments in implied variance, and (ii) the ratio of their volatilities.

Consequently, if we assume that the correlation between returns and increments in implied variance equals the correlation between returns and the variance risk premium, that is, $\text{Corr} (r_{t+\tau}, VRP(t, t+\tau)) = \text{Corr} (r_{t+\tau}, \Delta IV(t, t+\tau))$, one gets:

$$
\beta_{t,V} = \text{Corr} (r_{t+\tau}, VRP(t, t+\tau)) \times \frac{\text{Vol}(r_{t+\tau})}{\text{Vol}(VRP(t, t+\tau))} \\
= \text{Corr} (r_{t+\tau}, \Delta IV(t, t+\tau)) \times \frac{\text{Vol}(r_{t+\tau})}{\text{Vol}(\Delta IV(t, t+\tau))} \times \frac{\text{Vol}(\Delta IV(t, t+\tau))}{\text{Vol}(VRP(t, t+\tau))} \\
= \beta_{t,\Delta IV} \times \frac{\text{Vol}(\Delta IV(t, t+\tau))}{\text{Vol}(VRP(t, t+\tau))}.
$$

Accordingly, one can simply adjust the variance beta, $\beta_{t,\Delta IV}$, by the ratio of the volatility of the increments in implied variance and the volatility of the variance risk premium. Both variables are observable, so that the ratio can easily be estimated from the data.

Similar computations for the correlation risk premium lead to a comparable adjustment:

$$
\beta_{t,\rho} = \beta_{t,\Delta IC} \times \frac{\text{Vol}(\Delta IC(t, t+\tau))}{\text{Vol}(CRP(t, t+\tau))}.
$$
B  Solution of the General-Equilibrium Model

B.1  Value Function

As in Duffie and Epstein (1992b), the representative investor’s value function \( J_t \) is:

\[
J_t = \max_{C_s} \mathbb{E}_t \left[ \int_t^T f(C_s, J_s) ds \right],
\]

with the normalized aggregator \( f(C_t, J_t) \) being given by:

\[
f(C_t, J_t) = \frac{\beta}{1 - \frac{1}{\psi}} (1 - \gamma) J_t \left[ \left( \frac{C_t}{((1 - \gamma)J_t)^{1/\gamma}} \right)^{1 - \frac{1}{\psi}} - 1 \right] = \theta J_t [\beta G_t - \beta], \tag{A1}
\]

where we defined, for notational convenience, \( G_t \equiv \left( \frac{C_t}{((1 - \gamma)J_t)^{1/\gamma}} \right)^{1 - \frac{1}{\psi}} \) and \( \theta \equiv \frac{1 - \gamma}{1 - \frac{1}{\psi}} \).

Thus, the partial derivative of \( f(C_t, J_t) \) with respect to consumption is:

\[
f_C \equiv \frac{\partial f(C_t, J_t)}{\partial C_t} = \frac{\partial}{\partial C_t} \left[ \frac{\beta}{1 - \frac{1}{\psi}} (1 - \gamma) J_t \left[ \left( \frac{C_t}{((1 - \gamma)J_t)^{1/\gamma}} \right)^{1 - \frac{1}{\psi}} - 1 \right] \right] = \beta \frac{(1 - \gamma)J_t}{((1 - \gamma)J_t)^{1/\gamma}} C_t^{1 - \frac{1}{\psi}}. \tag{A2}
\]

Solving (A2) for \( C_t \) and substituting \( f_C \) by \( J_W \) (from the envelope condition), yields:

\[
C_t = J_W^{-\psi} \beta^{\psi} ((1 - \gamma) J_t)^{\frac{1 - \psi}{1 - \gamma}}. \tag{A3}
\]

We conjecture that the value function, \( J_t \), is of the type:

\[
J(W_t, V_t, \rho_t) = \exp \left( A_0 + A_1 V_t + A_2 \rho_t \right) \frac{W_t^{1-\gamma}}{1 - \gamma}, \tag{A4}
\]

such that the partial derivative of \( J_t \) with respect to wealth, \( W_t \), is simply given by:

\[
J_W \equiv \frac{\partial J(W_t, V_t, \rho_t)}{\partial W_t} = \exp \left( A_0 + A_1 V_t + A_2 \rho_t \right) W_t^{-\gamma}. \tag{A5}
\]

Plugging the conjecture (A4) for \( J_t \) and the derivative \( J_W \) from (A5) into (A3), yields:

\[
C_t = \beta^{\psi} \exp \left( A_0 + A_1 V_t + A_2 \rho_t \right) \frac{1 - \psi}{1 - \gamma} W_t. \tag{A6}
\]
Solving (A6) for wealth, \( W_t \), and substituting in (A4), the value function becomes:

\[
J(C_t, V_t, \rho_t) = \exp \left( A_0 + A_1 V_t + A_2 \rho_t \right) \frac{C_t^{1-\gamma} \beta^{-\psi(1-\gamma)} \exp \left( A_0 + A_1 V_t + A_2 \rho_t \right)^{-(1-\gamma)}}{1-\gamma}
\]

\[
= \exp \left[ \psi \left( A_0 + A_1 V_t + A_2 \rho_t \right) \right] \beta^{-\psi(1-\gamma)} \frac{C_t^{1-\gamma}}{1-\gamma},
\]

(A7)

with derivatives:

\[
J_C = \frac{J(1-\gamma)}{C}; \quad J_V = A_1 \psi J; \quad J_\rho = A_2 \psi J; \quad J_{CC} = -\frac{J(1-\gamma)\gamma}{C^2};
\]

\[
J_{VV} = A_1^2 \psi^2 J; \quad J_{V\rho} = \frac{\partial}{\partial \rho} J_V = A_1 \psi \frac{\partial}{\partial \rho} J = A_2 A_1 \psi^2 J; \quad J_{\rho\rho} = A_2^2 \psi^2 J.
\]

(A8)

Plugging the value function, \( J_t \), from (A7) into the expression for \( f_C \) from (A2), we get:

\[
f_C = \beta^{-\psi \gamma} \exp \left[ (A_0 + A_1 V_t + A_2 \rho_t) \frac{1-\gamma \psi}{1-\gamma} \right] C_t^{-\gamma},
\]

with first-order derivatives:

\[
f_{CC} = -\gamma \beta^{-\psi \gamma} \exp \left[ (A_0 + A_1 V_t + A_2 \rho_t) \frac{1-\gamma \psi}{1-\gamma} \right] C_t^{-\gamma-1} = -\gamma \frac{f_C}{C},
\]

\[
f_{CV} = A_1 \frac{1-\gamma \psi^2}{1-\gamma} \beta^{-\psi \gamma} \exp \left[ (A_0 + A_1 V_t + A_2 \rho_t) \frac{1-\gamma \psi}{1-\gamma} \right] C_t^{-\gamma} = A_1 \frac{1-\gamma \psi}{1-\gamma} f_C;
\]

\[
f_{C\rho} = A_2 \frac{1-\gamma \psi^2}{1-\gamma} \beta^{-\psi \gamma} \exp \left[ (A_0 + A_1 V_t + A_2 \rho_t) \frac{1-\gamma \psi}{1-\gamma} \right] C_t^{-\gamma} = A_2 \frac{1-\gamma \psi}{1-\gamma} f_C;
\]

(A9)

and second-order partials:

\[
f_{CCC} = \frac{\partial}{\partial C} f_{CC} = \gamma(\gamma + 1) \frac{f_C}{C^2}; \quad f_{C\rho \rho} = \frac{\partial}{\partial \rho} f_{C\rho} = A_2 \frac{1-\gamma \psi^2}{(1-\gamma)^2} f_C;
\]

\[
f_{CVV} = \frac{\partial}{\partial V} f_{CV} = A_1^2 \frac{1-\gamma \psi^2}{(1-\gamma)^2} f_C; \quad f_{CV \rho} = \frac{\partial}{\partial \rho} f_{CV} = A_1 A_2 \frac{1-\gamma \psi^2}{(1-\gamma)^2} f_C.
\]

(A10)

Starting from the definition of \( G \) in (A1) and using consumption in (A6) as well as the conjecture (A4), the consumption-wealth ratio, \( C_t/W_t \), can, on the one hand, be written as:

\[
\beta G = \beta \left( \frac{C_t}{((1-\gamma)J_t)^{\gamma/\psi}} \right)^{1-\frac{1}{\psi}} = \beta \left( \frac{W_t \beta^{-\psi \exp \left( A_0 + A_1 V_t + A_2 \rho_t \right)^{1-\gamma}}}{(1-\gamma) \exp \left( A_0 + A_1 V_t + A_2 \rho_t \right)^{1-\gamma}} \right)^{1-\frac{1}{\psi}}
\]

\[
= \beta^{-\psi} \exp \left( (A_0 + A_1 V_t + A_2 \rho_t) \frac{1-\psi}{1-\gamma} \right) \frac{C_t}{W_t};
\]

(A11)
while, on the other hand, the continuous-time log-linear approximation by Chacko and Viceira (2005) yields a consumption-wealth ratio of:

$$\frac{C_t}{W_t} = \exp \left( \log \left( \frac{C_t}{W_t} \right) \right) = \exp(c_t - w_t) \approx g_1 - g_1 \log(g_1) + g_1 \log \left( \frac{C_t}{W_t} \right), \quad (A12)$$

where $g_1$ denotes the long-run mean of the consumption-wealth ratio: $g_1 = \exp(E[c_t - w_t])$.

Combining the expressions for the consumption-wealth ratio from (A11) and (A12), implies:

$$\beta G = \frac{C_t}{W_t} \approx g_1 - g_1 \log(g_1) + g_1 \log(\beta G),$$

so that the aggregator function $f(C_t, J_t)$ from (A1) can be written as:

$$f_t = \theta J_t [\beta G - \beta] \approx \theta J_t [g_1 - g_1 \log(g_1) + g_1 \log(\beta G) - \beta]$$

$$= \theta J_t \left[ g_1 - \beta - g_1 \log(g_1) + g_1 \left( \psi \log(\beta) + (A_0 + A_1 V_t + A_2 \rho_t) \frac{1 - \psi}{1 - \gamma} \right) \right]$$

$$= \theta J_t \left[ \xi + g_1 (A_0 + A_1 V_t + A_2 \rho_t) \frac{1 - \psi}{1 - \gamma} \right], \quad \xi \equiv g_1 - \beta - g_1 \log(g_1) + g_1 \psi \log(\beta). \quad (A13)$$

Also, approximation (A12) allows us to write its derivative with respect to $J_t$, $f_J$, as:

$$f_J = (\theta - 1) \beta G - \beta \theta \approx (\theta - 1) [g_1 - g_1 \log(g_1) + g_1 \log(\beta G)] - \beta \theta$$

$$= \xi_1 - g_1 (A_1 V_t + A_2 \rho_t) \frac{1 - \psi \gamma}{1 - \gamma}, \quad \text{with} \quad \xi_1 \equiv (\theta - 1) \xi - \beta - g_1 A_0 \frac{1 - \psi \gamma}{1 - \gamma}. \quad (A14)$$

### B.2 Hamilton-Jacobi-Bellman Equation

The Hamilton-Jacobi-Bellman (HJB) equation for a conjecture of the type (A4), with consumption substituted for wealth, is given by:

$$\max_C \{ f(C, J) + A J(C, V, \rho) \} = 0,$$

where $A$ denotes the infinitesimal generator and the dynamics of the state variables $C_t$, $V_t$ and $\rho_t$ are given in equations (11), (12) and (13), respectively. This PDE needs to be solved for the parameters $A_0$, $A_1$ and $A_2$, emanating from the value function conjecture (A4).

In particular, we have:

$$A J(C_t, V_t, \rho_t) = C_t \mu_c J_C + [\kappa_1 (\bar{V}_t - V_t) + \bar{\kappa}_2 (\bar{\rho} - \rho_t)] J_V + \kappa_2 (\bar{\rho} - \rho) J_\rho$$

$$+ \frac{1}{2} \left[ C_t^2 V_t \delta_c^2 J_{CC} + [\sigma_c^2 V_t + \bar{\sigma}_c^2 \rho_t] J_{VV} + 2 \sigma_c \bar{\sigma}_c \rho_t J_{V\rho} + \sigma_c^2 \rho_t J_{\rho\rho} \right], \quad (A15)$$

where the respective derivatives of the value function $J_t$ are reported in (A8).
Combining the aggregator $f(C_t, J_t)$ from (A13) and the expansion (A15), plugging in the partial derivatives (A8) and collecting the coefficients, the HJB equation becomes:

$$0 = J_t \left[ \theta \xi - g_1 \psi A_0 + \mu_c (1 - \gamma) + A_1 \psi \kappa_1 \bar{V} + A_1 \psi \kappa_2 \bar{\rho} + \kappa_2 \bar{\rho} A_2 \psi 
+ V_t \left( -A_1 \psi \kappa_1 - g_1 \psi A_1 + \frac{1}{2} \left( -\delta^2 \right) (1 - \gamma) + \sigma^2 A^2_1 \psi^2 \right) 
+ \rho_t \left( -A_1 \psi \kappa_2 - \kappa_2 A_2 \psi - g_1 \psi A_2 + \frac{1}{2} \left( \sigma^2 A^2_1 \psi^2 + 2 \bar{\sigma}_2 \sigma_2 A_2 A_1 \psi^2 + \sigma^2 A^2_2 \psi^2 \right) \right) \right].$$

The right-hand side of the HJB equation can only be zero if the coefficients multiplying $V_t$ and $\rho_t$ as well as the free terms are all zero. The resulting equation system has solutions:

$$A_1 = \frac{-b_1 \pm \sqrt{b_1^2 - a_1 c_1}}{a_1} \quad \text{and} \quad A_2 = \frac{-b_2 \pm \sqrt{b_2^2 - a_2 c_2}}{a_2}, \quad (A16)$$

where $a_1 = \sigma^2_1 \psi^2$, $b_1 = -(g_1 + \kappa_1) \psi$, $c_1 = -\delta^2 \psi$, $a_2 = \sigma^2_2 \psi$, $b_2 = -\kappa_2 + g_1 + \bar{\sigma}_2 \sigma_2 A_1 \psi$ and $c_2 = \bar{\sigma}^2_2 A^2_2 \psi^2 - 2 A_1 \kappa_2$. $A_0$ follows simply from setting the free terms to zero.

### B.3 The Pricing Kernel and the Risk-free Rate

The stochastic differential equation for the pricing kernel is (see Duffie and Epstein (1992a)):

$$\frac{d\pi_t}{\pi_t} = f_f(C, J) dt + \frac{df_C(C, J)}{f_C(C, J)}. \quad (A17)$$

Plugging in $f_f$ from (A14), using the derivatives of $f_C$ in (A9) to get the dynamics $df_C$, collecting the deterministic terms $-r_f dt$, and grouping the random terms, one gets the process for the pricing kernel with risk premiums as in (14).

Accordingly, the risk-free rate, $r_f$, is characterized by the deterministic terms in (A17):

$$-r_f dt = f_f dt + E \left[ \frac{df_C(C, J)}{f_C(C, J)} \right],$$

which, after using the second-order partial derivatives of $f_C(C, J)$ in (A10) to get $E \left[ \frac{df_C(C, J)}{f_C(C, J)} \right]$ and dividing by $dt$, yields a risk-free $r_f = r_0 + r_1 V_t + r_2 \rho_t$:

$$-r_f = \xi_1 - \gamma \mu_c + \frac{1 - \gamma \psi}{1 - \gamma} \left[ A_1 \kappa_1 \bar{V} + A_1 \kappa_2 \bar{\rho} + A_2 \kappa_2 \bar{\rho} \right] 
+ \frac{1 - \gamma \psi}{1 - \gamma} \left[ -\kappa_1 A_1 - g_1 A_1 + \frac{1}{2} \gamma (\gamma + 1) \delta^2 + \frac{1}{2} A^2_1 \frac{1 - \gamma \psi}{(1 - \gamma)} \right] V_t 
+ \frac{1 - \gamma \psi}{1 - \gamma} \left[ -A_2 \kappa_2 - A_1 \kappa_2 - g_1 A_2 + \frac{1 - \gamma \psi}{2(1 - \gamma)} A^2_2 \sigma_2^2 + 2 A_1 A_2 \bar{\sigma}_2 \sigma_2 + A^2_2 \bar{\sigma}_2^2 \right] \rho_t. \quad (A18)$$

49
B.4 Consumption Claim (Aggregate Market)

Employing (A6) and defining $A_i = A_i \frac{1 - \psi}{1 - \gamma}$, we can write wealth, $W_t$, as:

$$W_t = C_t \beta^{-\psi} \exp(-A_0 t - A_1 V_t - A_2 \rho_t),$$

such that, after applying Itô’s Lemma to both sides, one gets:

$$dW_t = dC_t \frac{W_t}{C_t} + C_t \frac{W_t}{C_t} \left[ -A_1 dV_t - A_2 d\rho_t + \frac{1}{2} A_1^2 (dV_t)^2 + \frac{1}{2} A_2^2 (d\rho_t)^2 + A_1 A_2 dV_t d\rho_t \right]$$

$$+ dC_t \frac{W_t}{C_t} [ -A_1 dV_t - A_2 d\rho_t].$$

By assumption $dC_t dV_t = 0$ and $dC_t d\rho_t = 0$; thus, after cancelling some terms, one gets:

$$\frac{dW_t}{W_t} = \frac{dC_t}{C_t} - A_1 dV_t - A_2 d\rho_t + \frac{1}{2} A_1^2 (dV_t)^2 + \frac{1}{2} A_2^2 (d\rho_t)^2 + A_1 A_2 dV_t d\rho_t.$$ 

Collecting the deterministic parts into a generic $dt$ term, that is, a “partially drift” $\zeta_W$, and plugging in consumption dynamics (11) for $dC_t$, the dynamics of the aggregate market are given by (15). Hence, the instantaneous variance of the aggregate market is given by:

$$V_{W,t} = \left( \frac{dW_t}{W_t} \right)^2 / dt = (\delta_c^2 + A_1^2 \sigma_1^2) V_t + (A_1 \sigma_1 + A_2 \sigma_2) \rho_t.$$

which, after applying Itô’s Lemma, yields equation (17).

B.5 Individual Dividend Claims

Denoting the price of a generic dividend claim by $S_t$, we conjecture:

$$\frac{D_t}{S_t} = \exp(A_0 m + A_1 m V_t + A_2 m \rho_t). \quad (A19)$$

Applying Itô’s Lemma in the same way as for the consumption claim, we get

$$\frac{dS_t}{S_t} = \frac{d \exp(-A_0 m - A_1 m V_t - A_2 m \rho_t)}{\exp(-A_0 m - A_1 m V_t - A_2 m \rho_t)} + \frac{dD_t}{D_t}$$

$$= -A_1 m dV_t - A_2 m d\rho_t + \frac{1}{2} \left[ A_1^2 (dV_t)^2 + A_2^2 (d\rho_t)^2 \right] + A_1 m A_2 m dV_t d\rho_t + \frac{dD_t}{D_t}.$$

For simplicity we drop the index $i$, hence, $D_{i,t} = D_t$ and $S_{i,t} = S_t$. 

50
Plugging in the dynamics for $dV_t$ in (12), $d\rho_t$ in (13) and their covarations: $(d\rho_t)^2 = \sigma_2^2 \rho_t dt$, $(dV_t)^2 = \sigma_1^2 V_t dt + \sigma_\rho \rho_t dt$, and $dV_t d\rho_t = \sigma_\rho \sigma_2 \rho_t dt$, one obtains:

$$
\frac{dS_t}{S_t} = \frac{dD_t}{D_t} - \left[ A_{1m}(\kappa_1(\bar{V} - V_t) + \bar{\kappa}_2(\bar{\rho} - \rho_t)) + A_{2m} \bar{\kappa}_2(\bar{\rho} - \rho_t) \\
- \frac{1}{2}(A_{1m}^2 \sigma_1^2 V_t + \sigma_\rho \rho_t) + A_{2m}^2 \sigma_2^2 \rho_t) - A_{1m} A_{2m} \sigma_\rho \sigma_2 \rho_t \right] dt \\
- \left[ A_{1m} \sigma_1 \sqrt{V_t} \right] dB_{V,t} - \left[ A_{1m} \sigma_\rho \sqrt{\rho_t} + A_{2m} \sigma_2 \sqrt{\rho_t} \right] dB_{\rho,t},
$$

(A20)

such that:

$$
\mathbb{E}_t \left[ \frac{dS_t}{S_t} \right] / dt = \mu_D - A_{1m}(\kappa_1(\bar{V} - V_t) + \bar{\kappa}_2(\bar{\rho} - \rho_t)) - A_{2m} \bar{\kappa}_2(\bar{\rho} - \rho_t) \\
+ \frac{1}{2}(A_{1m}^2 \sigma_1^2 V_t + \sigma_\rho \rho_t) + A_{2m}^2 \sigma_2^2 \rho_t) + A_{1m} A_{2m} \sigma_\rho \sigma_2 \rho_t.
$$

(A21)

Next, compute the expected risk premium using pricing kernel (A17) and dynamics (A20):

$$
-\mathbb{E}_t \left[ \frac{d\pi_t}{\pi_t} \frac{dS_t}{S_t} \right] / dt = \lambda_1 \sigma_{DC} \sqrt{V_t} - \lambda_2 [A_{1m} \sigma_1 \sqrt{V_t}] - \lambda_3 [A_{1m} \sigma_\rho \sqrt{\rho_t} + A_{2m} \sigma_2 \sqrt{\rho_t}].
$$

(A22)

Approximating the dividend-price ratio using Chacko and Viceira (2005), gives:

$$
\frac{D_t}{S_t} \approx g_{1m} - g_{1m} \log(g_{1m}) + g_{1m} \log \left( \frac{D_t}{S_t} \right) = g_0m + g_1m (A_{0m} + A_{1m} V_t + A_{2m} \rho_t),
$$

where $g_0m = g_{1m} - g_{1m} \log(g_{1m})$.

In order to obtain the coefficients $A_{im}$, one can use the following pricing relation:

$$
\mathbb{E}_t \left[ \frac{dS_t}{\pi_t} \right] + \frac{D_t}{S_t} dt = r_f dt - \mathbb{E} \left[ \frac{d\pi_t}{\pi} \frac{dS_t}{S_t} \right],
$$

(A23)

with the left-hand side, after employing (A21) and conjecture (A19), being given by:

$$
\mathbb{E}_t \left[ \frac{dS_t}{S_t} \right] / dt + \frac{D_t}{S_t} = \mu_D + g_0m + g_1m A_{0m} - A_{1m}(\kappa_1 \bar{V} + \bar{\kappa}_2 \bar{\rho}) V_t \left( A_{1m} \kappa_1 + \frac{1}{2} A_{1m}^2 \sigma_1^2 + g_{1m} A_{1m} \right) \\
- A_{2m} \kappa_2 \bar{\rho} + \rho_t \left[ A_{1m} \bar{\kappa}_2 + A_{2m} \kappa_2 + \frac{1}{2} (A_{1m}^2 \sigma_\rho + A_{2m}^2 \sigma_2 \rho_t) + A_{1m} A_{2m} \sigma_\rho \sigma_2 + g_{1m} A_{2m} \right];
$$

and the right-hand side, after employing (A18) and (A22), being given by:

$$
r_f - \mathbb{E}_t \left[ \frac{d\pi_t}{\pi_t} \frac{dS_t}{S_t} \right] / dt = r_0 + V_t \left( r_1 + \gamma \delta_{\sigma_{DC}} + \frac{1 - \gamma \psi_1}{1 - \gamma} A_{1m} \sigma_1^2 A_{1m} \right) \\
\rho_t \left[ r_2 + \frac{1 - \gamma \psi_1}{1 - \gamma} (A_{1m} \sigma_\rho A_{1m} \sigma_\rho + A_{1m} \sigma_\rho A_{2m} \sigma_2 + A_{2m} \sigma_2 A_{1m} \sigma_\rho + A_{2m}^2 \sigma_2^2 A_{2m}) \right].
$$
By deducting the left-hand side in (A23) from the right-hand side, one gets an expression that must be equal to zero. But, it can only be zero, if the coefficients multiplying $V_t$ and $\rho_t$ as well as the free terms are all zero. This leads to an equation system that includes:

$$0 = 2r_1 + 2\gamma \delta_c \sigma_{DC} + 2\frac{1-\gamma \psi}{1-\gamma} A_1 \sigma_1^2 A_{1m} - 2A_{1m} \kappa_1 - A_{1m}^2 \sigma_1^2 - 2g_{1m} A_{1m}$$

$$= A_{1m}^2 \sigma_1^2 + 2\left[\frac{1-\gamma \psi}{1-\gamma} A_1 \sigma_1^2 \kappa_1 + g_{1m} A_{1m} - 2r_1 - 2\gamma \delta_c \sigma_{DC}\right],$$

with solution

$$A_{1m} = \frac{-b_{1m} \pm \sqrt{b_{1m}^2 - a_{1m} c_{1m}}}{a_{1m}},$$

where $a_{1m} = \sigma_1^2$, $b_{1m} = [-\frac{1-\gamma \psi}{1-\gamma} A_1 \sigma_1^2 \kappa_1 + g_{1m}]$ and $c_{1m} = -2r_1 - 2\gamma \delta_c \sigma_{DC}$.

Also, the equation system contains the following equation:

$$0 = -A_{2m}^2 \sigma_2^2 + A_{2m} \left[2\kappa_2 - A_{1m} \sigma_2 \sigma_2 - g_{1m} + \frac{1-\gamma \psi}{1-\gamma} (A_1 \sigma_2 \sigma_2 + A_2 \sigma_2^2)\right]$$

$$+ 2\left[\kappa_2 - A_{1m} \bar{\kappa}_2 + \frac{1-\gamma \psi}{1-\gamma} (A_1 \sigma_2 A_{1m} \sigma_2 + A_2 \sigma_2 A_{1m} \sigma_2)\right] - A_{1m}^2 \bar{\sigma}_2,$$

with solution

$$A_{2m} = \frac{-b_{2m} \pm \sqrt{b_{2m}^2 - a_{2m} c_{2m}}}{a_{2m}},$$

where $a_{2m} = \sigma_2^2$, $b_{2m} = [-\kappa_2 - A_{1m} \sigma_2 \sigma_2 - g_{1m} + \frac{1-\gamma \psi}{1-\gamma} (A_1 \sigma_2 \sigma_2 + A_2 \sigma_2^2)]$, and $c_{2m} = -2\kappa_2 - A_{1m} \bar{\kappa}_2 + \frac{1-\gamma \psi}{1-\gamma} (A_1 \sigma_2 A_{1m} \sigma_2 + A_2 \sigma_2 A_{1m} \sigma_2)] + A_{1m} \bar{\sigma}_2$. Finally, the last coefficient, $A_{0m}$, follows from setting the free terms to zero.

Assuming that all dividend trees $i < I$ are symmetric, that is, have the same parameters, the average stock-correlation is equal to the correlation between any two stocks. Hence, using the process for any two dividend claims, $i, j < I$, we can compute the instantaneous covariance:

$$\text{Cov}_S \equiv \frac{dS_{i,t}}{S_{i,t}} \frac{dS_{j,t}}{S_{j,t}}/dt = (\sigma_{DC}^2 + A_{1m}^2 \sigma_1^2) V_t + (A_{1m} \bar{\sigma}_2 + A_{2m} \sigma_2)^2 \rho_t,$$

(A24)

and, therefore, the dynamics for the covariance between dividend claims are given by:

$$d\text{Cov}_S = (\sigma_{DC}^2 + A_{1m}^2 \sigma_1^2) dV_t + (A_{1m} \bar{\sigma}_2 + A_{2m} \sigma_2)^2 d\rho_t.$$

Also, the total variance of a stock, $V_{S,i}$, is:

$$V_{S,i} = \sigma_{D,i}^2 V_{i,t} + (\sigma_{DC,i}^2 + A_{1m}^2 \sigma_1^2) V_t + (A_{1m} \bar{\sigma}_2 + A_{2m} \sigma_2)^2 \rho_t,$$

(A25)
such that its dynamics are given by:

\[ dV_{i,t} = \sigma_{D,t}^2 dV_{i,t} + (\sigma_{DC,t}^2 + A_{1m}^2 \sigma_1^2) dV_t + (A_{m} \bar{\sigma}_\rho + A_2 \sigma_2)^2 \rho_t. \]

Therefore, the instantaneous correlation is given by:\(^{38}\)

\[ \rho_s \equiv \frac{Cov_S}{V_S} = \frac{(\sigma_{DC}^2 + A_{1m}^2 \sigma_1^2) V_t + (A_{m} \bar{\sigma}_\rho + A_2 \sigma_2)^2 \rho_t}{\sigma_{D,t}^2 V_{i,t} + (\sigma_{DC,t}^2 + A_{1m}^2 \sigma_1^2) V_t + (A_{m} \bar{\sigma}_\rho + A_2 \sigma_2)^2 \rho_t}, \]

where \( V_{i,t} \) denotes the idiosyncratic variance to differentiate it from systematic variance \( V_t \). To arrive at process for correlation (18), one can simply apply Itô’s Lemma.

### B.6 Equity, Variance, and Correlation Risk Premiums

Risk premiums, in general, can be computed (i) according to Girsanov theorem, that is, they represent the change in drift when going from from the actual to the risk-neutral measure or; (ii) directly, from the pricing kernel.\(^{39}\) Hence, the risk premium for aggregate consumption variance risk, \( VRP_C \), and the risk premium for dividend-correlation risk, \( CRP_C \), are given by:

\[ VRP_C = (E^Q[dV] - E^P[dV])/dt = \frac{d\pi_t}{\pi_t} dV_t = -\lambda_2 \sigma_1 \sqrt{V_t} - \lambda_3 \bar{\sigma}_\rho \sqrt{\rho_t}; \]

\[ CRP_C = (E^Q[d\rho] - E^P[d\rho])/dt = \frac{d\pi_t}{\pi_t} d\rho_t = -\lambda_3 \sigma_2 \sqrt{\rho_t}. \]

Computing the risk premiums for aggregate market variance, whose dynamics are given in (17), and for the stock-correlation, whose dynamics are given in (18), one can derive equation system (19).

Its solution has the form

\[
\begin{pmatrix}
VRP_C \\
CRP_C
\end{pmatrix} = Z \times \begin{pmatrix}
\frac{V_S - Cov_S}{V_S} (A_{m} \bar{\sigma}_\rho + A_2 \sigma_2)^2 - (A_{1a} \bar{\sigma}_\rho + A_2a \sigma_2)^2 \\
\frac{-V_S - Cov_S}{V_S} (\sigma_{DC}^2 + A_{1m}^2 \sigma_1^2) (\delta_c^2 + A_{1a}^2 \sigma_1^2)
\end{pmatrix} \times \begin{pmatrix}
VRP \\
CRP
\end{pmatrix},
\]

where \( Z \) is the inverse of the determinant of the matrix in (19). Consequently, one can write the equity risk premium as in (16). \( A_{1z} \) and \( A_{2z} \) are functions of the total variance of a dividend claim, matrix elements in (19), and coefficients \( A_{1a}, A_{2a} \):

\[ A_{1z} = Z \frac{V_S - Cov_S}{V_S^2} (A_{1a} (A_{m} \bar{\sigma}_\rho + A_2 \sigma_2)^2 - (A_{1a} \bar{\sigma}_\rho + A_2a \sigma_2)^2 (\sigma_{DC}^2 + A_{1m}^2 \sigma_1^2)) \]

\[ A_{2z} = Z (A_{2a} (\delta_c^2 + A_{1a}^2 \sigma_1^2) - A_{1a} (A_{1a} \bar{\sigma}_\rho + A_2a \sigma_2)^2). \]

\(^{38}\)Because dividend trees are symmetric, the stocks have the same instantaneous variance.

\(^{39}\)Consistent with the literature, we define the variance and correlation risk premiums as the difference between the risk-neutral and actual quantities.