Distortions Caused by Asset Managers
Retaining Securities Lending Income

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Abstract
Using newly-mandated disclosures, we show fund managers often retain a fraction of securities lending income by employing in-house lending agents at above-market rates. This retention incentivizes fund managers to overweight stocks with high lending fees. In a heterogeneous agent model, we show this incentive distorts equilibrium portfolio choices, fund performance, and asset pricing. We confirm our model’s predictions empirically: fee-retaining active mutual funds overweight high lending fee stocks, underperform, and charge lower management fees. Our model also offers a new explanation for the negative relation between lending fees and future fee-inclusive returns.

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State Street is fighting two separate US lawsuits over claims it . . . took an “unreasonably large” 50 per cent share of the net income generated from lending securities owned by the trust, amounting to “fiduciary self-dealing and enrichment in violation of the defendants’ fiduciary obligations.

– Financial Times article “State Street battles two US lawsuits,” 2/10/2013

BlackRock, the world’s largest asset manager, has systematically “looted” securities lending revenues from investors, according to a lawsuit filed by two US pension funds. The suit alleges a number of BlackRock’s US-listed iShares exchange traded funds operated a “grossly excessive” fee model that allowed an affiliate . . . to retain 40 per cent of the revenue generated by lending stock.


Mutual funds and exchange traded funds (ETFs) often lend assets to short sellers, generating a total of just over $1bn in gross lending fee revenue in 2017, equivalent to 7.7% of total management fees. A fraction of lending fee revenues goes to lending agents, rebates to borrowers, and other costs rather than back to investors. As illustrated by the above examples, these costs have proven controversial as some investors argue they are excessive due to ‘self-dealing’ whereby the fund manager uses an affiliated lending agent and pays them above-market rates.

Until recently, such claims were difficult to systematically evaluate because fund managers did not disclose the gross amount of lending revenues they generate, instead only being required to disclose the net amount returned to investors. In 2016, though, the SEC amended Regulation S-X to require mutual fund managers disclose gross lending revenues, a cost breakdown, and the identify of their lending agent starting with fiscal year 2017.

Using the newly-mandated disclosures, we show that self-dealing is common. For all mutual funds and ETFs reporting lending income in 2017, only 76% of gross lending revenues were returned to investors in 2017, with the other 24% going to costs. Furthermore, many large asset management companies used affiliated lending agents, collecting $30mm of lending agent fees, an average of 8.4% of gross lending revenue for mutual funds using an affiliated
lending agent.\textsuperscript{1} We also find that funds give affiliated lending agents larger fractions of lending income than unaffiliated funds using the same lending agent, indicating self-dealing results in above market rates.

Having established asset managers often retain a fraction of securities lending income, we hypothesize this practice results in substantial distortions because it makes mutual funds and ETFS prefer lending fees over capital gains and dividends, as suggested by Blocher and Whaley (2016). We provide a theoretical model and empirical evidence showing this preference for lending fees affects equilibrium portfolio choice, management fees, and after-fee performance for active mutual funds, and drives down equilibrium lending fees.

We formally analyze the role of lending income retention in a model with heterogeneous investors and equilibrium lending fees. Our model features four types of investors: hedge funds, fee-retaining mutual funds, non-fee-retaining mutual funds, and biased individual investors. Hedge funds are unbiased and have no restrictions on their portfolios. Both types of mutual funds are unbiased and cannot engage in short selling. Due to their retention policies, fee-retaining mutual funds value lending fees more than returns, whereas hedge funds and non-fee-retaining mutual funds value them equally to returns.\textsuperscript{2} Finally, biased individual investors misvalue one of the assets and cannot short or lend their shares.

The two frictions in our model are the biased individuals’ misvaluation and the fee-retaining mutual funds’ extra incentive to collect lending fees. Without these frictions, the CAPM holds and all four types of investors hold identical long-only portfolios. The misvaluation by biased individuals who cannot lend shares is what generates non-zero lending fees in equilibrium. When this bias is sufficiently positive for an asset, all of its shares outstanding are held by investors who cannot lend and choose not to sell, the necessary

\textsuperscript{1}BlackRock, BMO, BNY Mellon, Deutsche Bank, Fidelity, Goldman Sachs, State Street, and Vanguard all have affiliated lending agents.

\textsuperscript{2}This a standard assumption in the literature, e.g. Duffie, Garleanu, and Pedersen (2002), because lending fees generate positive cash flow for long shareholders and should therefore be treated like a dividend.
condition in our model and others (e.g. Duffie, Garleanu, and Pedersen (2002), Blocher, Reed, and Van Wesep (2013), and Drechsler and Drechsler (2016)) for positive lending fees. In this case, without fee-retention the asset’s equilibrium expected return is negative but offset by a positive lending fee, making its fee-inclusive expected return exactly enough so that all institutional investors choose to hold zero shares.

The second and most important friction in our model is some funds’ partial retention of lending fees, resulting in them valuing lending fees more than equivalent capital gains or dividends. In equilibrium, these funds buy stocks with non-zero lending fees and lend these shares to hedge funds who open short positions. This side transaction between hedge funds and fee-retaining mutual funds drives down equilibrium lending fees and fee-inclusive expected returns because fee-retaining funds’ incentives increase the supply of lendable shares.

Our model makes four empirical predictions, all of which are supported by empirical evidence presented here and in prior research. The first is that fee-retaining funds overweight stocks with high fees. Consistent with this prediction, we show that funds who use an affiliated lending agent and pay above-median lending agent fees (our empirical proxy for “fee-retaining funds”) choose portfolios with higher average lending fees. Related results from the literature also support our prediction. Prado (2015) finds that institutional investors as a whole increase their ownership when lending fees increase. Evans, Ferreira, and Porras Prado (2017) shows that when a fund manager simultaneously manages a lending fund and a non-lending fund, they respond to increases in lending fees by overweighting the stock in their lending fund relative to the non-lending fund. Blocher and Whaley (2016) shows that ETFs who lend shares tilt their portfolios towards high-fee stocks. Our results complement these existing results by showing the portfolio choice distortion is connected to a direct measure of lending fee retention. We also contribute by modelling the distorted portfolio choice suggested in Blocher and Whaley (2016), and offering it as an alternative explanation for the evidence in Prado (2015) and Evans, Ferreira, and Porras Prado (2017).
Our second prediction is that lending fees negatively predict future returns, even after returns are adjusted for lending fees. Many papers find empirical evidence supporting this prediction, including Jones and Lamont (2002), and Drechsler and Drechsler (2016). Standard models of equilibrium lending fees, such as Duffie, Garleanu, and Pedersen (2002), imply lending fees negatively predict fee-exclusive returns for the same reason stock prices decline on ex-dividend dates: expected returns including dividends and lending fees compensate investors for their risk exposure, meaning prices of stocks with high lending fees must decline or it would be an ‘arbitrage’ to buy and lend them. However, explaining why prices of high lending fee stocks decline by more than the amount of the lending fee (the “lending fee anomaly”) requires an additional mechanism. Drechsler and Drechsler (2016) proposes one such possibility, that there is a systematic component of lending fees that creates risk compensation for shorting high-fee stocks.

We propose another non-mutually exclusive explanation for the lending fee anomaly, that the incentives of fee-retaining institutional investors drive down equilibrium lending fees and fee-inclusive expected returns. This price impact in lending markets is plausible given that self-dealing fund management companies have a combined $477bn TNA in active US equity mutual funds that engage in securities lending, and $10tn total TNA, making them significant players in the share lending market. Unlike Drechsler and Drechsler (2016), our model jointly explains the poor fee-inclusive performance of high lending fee stocks and the puzzle of why institutional investors own these stocks in the first place.

Our third prediction is that fee-retaining mutual funds deliver worse performance to their investors. Consistent with this prediction, we show that fee-retaining funds have lower alphas, net of management fees and including any securities lending income passed back to the fund. Our results support the evidence in Evans, Ferreira, and Porras Prado (2017) that lending mutual funds underperform non-lending mutual funds, and provide nuance by showing fee-retainers underperform other lending funds as well as non-lending funds.
The underperformance of fee retaining mutual funds, especially relative to non-lending mutual funds, may initially be surprising because, holding portfolio positions constant, securities lending strictly increases the performance of a fund, meaning non-lending mutual funds appear to be leaving money on the table. However, this underperformance is a natural consequence of lending funds’ distorted portfolio choices in our model. Securities lending adversely affect the returns for investors in fee-retaining funds both because the funds retain a fraction of the lending fees and more importantly because these funds overweight high-fee stocks that have poor future returns even including the full amount of the lending fee.

The fourth and final prediction of our model is that securities lending income is not benign substitute for management fees. We address this possibility by adding management fees to our model and assuming that mutual fund managers compete away net profits to the point where all funds have the same total revenue (management fees plus retaining lending income) as a fraction of AUM. In this case, high fee retaining fund managers offer lower equilibrium management fees in proportion to their retained fees. We find some evidence this is the case, as there is a negative relation between lending costs and expense ratios for self-dealing funds but not other funds.

Moreover, our model shows that even if fee retainers reduced management fees by the full amount of expected lending fees, as long as funds commit to a fixed management fee before choosing their portfolio, the distorted portfolio choice would still cause the fund to underperform for investors net of all remitted lending income and management fees. As discussed above, we confirm this empirically, finding that high fee retainers have worse bottom-line alphas for their investors than other active mutual funds.

More broadly, our paper’s results show fee retention by asset managers has both direct and indirect costs that have implications for researchers, investors, and policy makers.\footnote{If they instead reduce management fees by the ex-post amount of retained share lending income, this would be equivalent to passing the full amount back to investors and therefore undo the distortion.}
1. Retention of Securities Lending Income by Asset Managers

There is anecdotal evidence, from various news articles and lawsuits, that securities lending is profitable for asset managers that administer their own programs. For instance, two pension funds sued BlackRock, alleging the asset manager was keeping an excessive amount of securities lending revenue, with 35% going to an affiliated lending agent and 5% going to other administrative costs ("US pension funds sue BlackRock" – Financial Times 2/3/2013). State Street was also sued for keeping 50% of securities lending revenue ("State Street battles two US lawsuits" – Financial Times 2/3/2013). These lawsuits pertained to the management of ETFs (as studied in Blocher and Whaley (2016)) and private asset management. For most of our tests, we use active mutual funds as a laboratory for studying the broader effects of self-dealing because unlike private asset management, we have data on their holdings and retention policy, and because active mutual funds have more discretion than most ETFs.

Perhaps in response to these lawsuits or related complaints, in October 2016 the SEC issued amendments to Regulation S-X designed to increase the transparency of mutual fund securities lending activities. These amendments require that funds disclose annual gross securities lending income and expenses in their prospectuses in the form of Figure 1, starting with their prospectus for fiscal year 2017. As the proposal was being considered, several fund management companies and their legal representatives sent protest letters to the SEC opposing increased transparency. For instance, Invesco sent a letter to the SEC stating:

_We also believe the proprietary securities lending information required by the proposed changes to Regulation S-X should also remain non-public. We believe public disclosure of any of this information would be confusing to investors and potentially harmful to funds and the interests of their shareholders due to the complex and proprietary nature of this information. Therefore we believe that public disclosure of these items is neither necessary nor appropriate in the public interest or for the protection of investors._

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4The larger SEC reform was called the Investment Company Reporting Modernization Reform.
Similarly, Fidelity stated:

*Ultimately, we believe shareholders will evaluate the funds results through the total incremental income and return from securities lending activities, not the terms of the fee split. We believe that focusing attention on the terms of the revenue split, which funds negotiate with third-party lending agents, could have the unintended consequence of negatively impacting funds ability to negotiate competitive services and rates.*

Interestingly, we find that Fidelity exclusively uses an affiliated lending agent (Fidelity Services Company) to administer their securities lending program, making it unlikely the rule change would have any effect on their ability to negotiate with themselves.

Our paper takes advantage of the newly available securities lending income data that funds began reporting in their prospectuses for the 2017 calendar year. We collect this data using SEC EDGAR and merge it with the CRSP mutual fund database. We explain the data collection and merge process in more detail in Appendix A.

Our initial sample contains 1,035 open-end mutual funds and exchange traded funds (ETFs), with summary statistics for their disclosures presented in Panel A of Table 1. These funds generated just over $1bn in gross income from securities lending during 2017, while spending $239mm in fees and expenses to administer the programs. The largest category of expenses is rebates, interest paid to the share borrower for their cash collateral, which is $122mm, while $88mm goes to lending agent fees and another $27mm pays for cash collateral management. After these and other smaller costs, the remaining $761mm, 76% of gross income, is returned to the fund. Thus, the fees and expenses associated with securities lending are substantial. These costs are not including in reported management fees.

The only other estimation of the fraction of lending revenue received by investors we know of is in Blocher and Whaley (2016), who estimate that 30% of ETF lending revenue was returned to investors in 2012. Without the benefit of the new Regulation S-X disclosures, their approach estimates gross lending revenues using ETF holdings and stock-level data on
borrowing fees from Markit, and compares this to the net revenues received by fund investors. This approach may overstate actual lending revenues because it assumes ETFs lend out a large portion of shares they own, and because Markit indicative borrowing rates are an upper bound on achievable lending rates. Our approach has the advantage of using SEC-mandated disclosures to measure gross lending revenues, but the disadvantage of only covering the post-rule-change period. Because fund managers had advance notice of the rule change, they may have changed their fee retention policy in 2017 to be more favorable to investors. In this sense, we view our results as a lower bound on the historical extent of fee retention.

A critical assumption in our model, and precursor for our main empirical analysis, is that some management companies retain enough securities lending income to profit from the practice. There are at least three potential channels through with fund managers could benefit from the costs associated with lending programs. The first is by lending shares to an affiliated broker, and paying the broker an above-market rate. The second is by using an affiliated money market fund that charges above-cost management fees. The third is by employing an affiliated lending agent. Because the new Regulation S-X disclosures also list the identity of each fund’s lending agent, but not their cash collateral manager or primary share borrower, our empirical measure of fee retention uses funds we can identify as ‘self-dealers’ because they use an affiliated lending agent. There are likely other funds profiting from securities lending, so we view our results as demonstrating the existence of fee retention, and the problems arising because of it, but not measuring its full extent.

Figure 2 presents the lending agents used in our main sample, which focuses on active US equity mutual funds because they have the most discretion over their portfolio choice, making it easier to measure the impacts of fee retention. Several lending agents are purely used as third party or affiliated agents, while others such as Deutsche Bank, Goldman Sachs, BNY

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5For example, Dimensional Fund Advisors invests cash collateral from share lending in the DFA Short Term Investment Fund, meaning the $1,687,679 that the DFA U.S. Small Cap Fund paid for cash collateral management was revenue for another branch of DFA (DFA Form N-1A).
Mellon, and State Street act as agents for both third parties and affiliated funds. We find that 124 active equity mutual funds, 23% of our sample, employ an affiliated lending agent. Figure 3 shows the frequency of self-dealing among funds with available affiliated lending agents. Some fund management companies always self-deal (e.g., Fidelity and BlackRock), whereas others sometimes use third party agents (e.g., Goldman Sachs and BMO).

In Panel B of Table 1, we present aggregate securities lending income and expense data for self-dealing mutual funds. Self-dealing funds collect $361mm in securities lending income, 36% of the total across all funds, and pay over $30mm to their lending agents.

Table 2 provides summary statistics for the cross-section of how funds allocate fees from securities lending. Panel A includes the 542 active equity mutual funds in our sample. For each fund, we use CRSP holdings data, and borrowing fee rates from Interactive Brokers (see Appendix A for details) to compute the portfolio’s ‘Fee Yield,’ our estimate of the gross yield the fund would receive if it lent all its holdings. We find that the average fee yield in our sample is 13bps, with values above 50bps for the top 5% of funds. We can compare this total to the average fund’s gross income yield reported in their recent disclosures, which is 7bps on average, indicating that due to costs and regulatory requirements, funds do not lend their entire portfolios. In aggregate, the cost of share lending is around 2.2bps of TNA for funds in our sample, with values above 14bps for the top 5% of funds.

We also examine cross-sectional variations in the costs of securities lending as a fraction of the gross lending revenue in Table 2. The average fund pays out 27.6% of its gross lending income in fees and expenses, similar to the aggregate value we find in Table 1. The mean fund-level breakdown of costs between lending agent fees, rebates, and cash collateral fees is also similar to the aggregate we find in Table 1. There are also significant cross-fund differences in cost allocation. Some funds pay lending agents as much as 20% of gross lending revenue, others pay interest rebates to share borrowers as high as 72% of the gross, and others have cash collateral management fees totalling 14% of lending revenues.
Panel B shows the same cross-fund summary statistics for our sub-sample of active equity mutual funds we identify as self-dealing because they use an affiliated lending agent. Self-dealing funds have a larger average Fee Yield (17bps versus 13bps) and larger average lending costs (3bps versus 2bps) than the broader sample of funds. The cost increase is primarily driven by an increase in the rebate rate, which as we argue above may represent a form of fee retention. Self-dealing funds are generally part of large fund families with an average of $1.8tn assets under management.

It is important for our story that self dealing funds pay their affiliated agents above market rates, or at least above costs, in order for funds to truly benefit more from lending fees than capital gains or dividends. Panel B of Table 2 shows that the mean lending agent fee for self-dealing funds is smaller than the overall population of funds. However, this is in part driven by the larger size of self-dealing funds and fund families, also shown in Table 2. Lending agent fees should be proportionally lower for larger funds and fund families to the extent they are driven by fixed costs. More generally, both fund families and lending agents are likely to have different lending agent fees for reasons unrelated to self-dealing, including their size, lending policies, and cost structures. We therefore need an identification strategy that addresses these possibilities.

We identify the effect of self dealing on lending agent fees using fund-level regressions predicting lending agent fees as a proportion of gross lending income. We control for style fixed effects and fund size throughout, and add fund family and lending agent fixed effects to address cross-sectional differences unrelated to self dealing. Our predictor of interest is Self Deal, an indicator for whether the fund uses an affiliated lending agent. When including a fund family fixed effect, the coefficient on Self Deal represents a comparison within fund families of how much they pay affiliated and non-affiliated lending agents. Lending agent fixed effects make the coefficient on Self Deal a comparison within lending agent of how much they receive from affiliated and non-affiliated funds. Figure 2 shows that our sample
includes both fund management companies using a mix of third party and affiliated agents, and lending agents serving a mix of third party and affiliated funds, granting us the variation we need to include these fixed effects.

Using this approach, Table 3 shows that after removing fund family and lending agent fixed effects, affiliated lending agents receive 8.8bps more of the gross lending fee than unaffiliated agents. Without lending agent fixed effects, in Columns (1) and (2) we find a slight negative relation between Self Deal and lending agent fees, consistent with the univariate relation in Table 2. However, Columns (3) and (4) show this relation reverses and is significantly positive when we include lending agent fixed effects. Because lending agents charge affiliated clients more, we conclude self-dealing is profitable for fund management companies. In the next section we will incorporate this feature into an equilibrium asset pricing model to analyze its effects on portfolio choice, fund performance and asset pricing.

2. Model

2.1. Setting

There are two risky assets that trade at time \( t = 0 \) and liquidate at \( t = 1 \) for \( \tilde{V}_1 \) and \( \tilde{V}_2 \), respectively, where:

\[
\begin{bmatrix}
\tilde{V}_1 \\
\tilde{V}_2
\end{bmatrix}
\sim N
\left(
\begin{bmatrix}
\mu_1 \\
\mu_2
\end{bmatrix},
\begin{bmatrix}
\sigma_1^2 & \rho \sigma_1 \sigma_2 \\
\rho \sigma_1 \sigma_2 & \sigma_2^2
\end{bmatrix}
\right).
\] (1)

The two assets have shares outstanding \( Q_1 > 0 \) and \( Q_2 > 0 \), respectively. Trade occurs at equilibrium prices \( p_1 \) and \( p_2 \).

Negative investor demands for shares represent short-selling, which requires borrowing shares from investors with long positions. Short sellers pay non-negative fees to borrow the shares at a rate \( f_1 \) and \( f_2 \), meaning share lenders receive an additional \( f_i p_i \) for each share
they are long and share borrowers pay an additional $f_i p_i$. We assume these fees are paid at $t = 1$, making the cash flows from potential positions as follows:

<table>
<thead>
<tr>
<th>Cash flow at $t = 0$</th>
<th>Cash flow at $t = 1$</th>
<th>Net return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long without lending</td>
<td>$-p_i$</td>
<td>$\tilde{V}_i$</td>
</tr>
<tr>
<td>Long with lending</td>
<td>$-p_i$</td>
<td>$\tilde{V}_i + p_i f_i$</td>
</tr>
<tr>
<td>Short</td>
<td>$p_i$</td>
<td>$-\tilde{V}_i - p_i f_i$</td>
</tr>
</tbody>
</table>

Fees $f$, like prices $p$, are determined in equilibrium at $t = 0$.

Four groups of investors allocate capital between the two risky assets and a risk-free asset paying 0% interest. All groups contain enough investors so that each individual is a price-taker. Each individual has mean-variance preferences over portfolio returns and access to unlimited leverage. We summarize the key features of these groups in Table 4.

The first group of investors are hedge funds, which as a group have $A_h$ assets under management, lend shares when they are long, and are allowed to short. Hedge funds remit the full amount of net lending fees received/paid to the fund, and as a result treat lending fees $f_i$ identically to capital gains $\frac{\tilde{V}_i}{p_i} - 1$. Putting these assumptions together, they choose portfolio weights $w_h$ as a function of prices $p$ using:

$$w_h(p, f) = \arg \max_w w' (\tau(p) + f) - \frac{\gamma}{2} w' \Sigma(p) w,$$  \hspace{1cm} (2)

where $\tau(p)$ and $\Sigma(p)$ are the expected returns and covariance matrix of the two assets given price vector $p$, respectively. Note $w$ need not sum to one because the hedge fund puts weight $1 - w_1 - w_2$ in cash.

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6Our main results hold when hedge funds retain a fraction of lending fees as long as their added incentive to lend is smaller than retaining mutual funds. This is reasonable because hedge funds have higher management fees and direct performance fees, making lending fees comparatively less important. Consistent with this, the evidence in D’Avolio (2002), Boehmer, Jones, and Zhang (2008) Engelberg, Reed, and Ringgenberg (2012) suggests that hedge funds tend to borrow and short special stocks rather than long and lend them.
The second group of investors are fee-retaining mutual funds ("fee retainers") who lend their shares and collectively have $A_{fr}$ assets under management. Like hedge funds, they lend their shares when they are long. Unlike hedge funds, they are not allowed to short, meaning $w_{fr,1} \geq 0$ and $w_{fr,2} \geq 0$.

The most important feature of fee retainers is that keep a fraction $\kappa > 0$ of the fees they receive from lending their shares to the fund, remitting the other $1 - \kappa$ to their investors, and as a result treat lending fees $f_i$ as more desirable per dollar than the capital gains $\frac{V_i}{p_i} - 1$. Furthermore, fund managers prefer additional revenue they keep to additional profits they return to investors, which we parameterize by assuming fund managers’ utility from expected fund return $\tau_{\text{fund}}$ and retained lending yield $f_{\text{retained}}$ is:

$$U = \tau_{\text{fund}} + \psi f_{\text{retained}} - \frac{\gamma}{2} \text{Var} (r_{\text{fund}}), \quad (3)$$

where $\psi > 1$. From the perspective of fee retaining fund managers, the expected returns of the two assets are therefore:

$$\tau_{fr}(p, f) \equiv \begin{bmatrix} \mu_1 + (1 - \kappa)f_1 + \psi \kappa f_1 - 1 \\ \mu_2 + (1 - \kappa)f_2 + \psi \kappa f_2 - 1 \end{bmatrix} = \tau(p) + \lambda f, \quad (4)$$

where $\lambda = 1 + \kappa(\psi - 1)$ summarizes fee retainers extra incentive to lend. This makes fee retainers’ optimal portfolio weights:

$$w_{fr}(p, f) = \arg \max_{w \text{ s.t. } w_1 \geq 0, w_2 \geq 0} w' (\tau(p) + \lambda f) - \frac{\gamma}{2} w' \Sigma(p)w. \quad (5)$$

We refer to fee retainers portfolio choices as distorted because they choose optimal weights $w_{fr}(p, f)$ differently than the weights that would maximize their investor’s expected utility. Assuming lending mutual fund investors also have mean-variance preferences, because they
only receive a fraction $\kappa$ of the lending fees their optimal weights solve:

$$w_{fr}^{\text{inv. opt}}(p, f) = \arg \max_{w} \ w' (\bar{r}(p) + (1 - \kappa)f) - \frac{\gamma}{2} w' \Sigma(p) w.$$  \hspace{1cm} (6)

Comparing Equations (5) and (6) shows immediately that because $1 - \kappa < 1 < \lambda$, lending mutual fund’s portfolio choice will be distorted towards stocks with higher lending fees.

The third group of investors are active mutual funds who do not retain any lending fees (“non retainers”). For simplicity, we assume these funds pass the full amount of lending fee income back to investors. We discuss other possibilities, including a third party lending agent, in Section 2.5. Non retainers collectively have $A_n$ assets under management, can lend their shares, but cannot short, making their optimal portfolio weights:

$$w_n(p, f) = \arg \max_{w} \ w' (\bar{r}(p) + f) - \frac{\gamma}{2} w' \Sigma(p) w.$$  \hspace{1cm} (7)

Given the short-sale constraint, this portfolio choice is optimal from both the fund manager’s and fund investor’s perspective.

The final group is potentially-biased individual investors, which together manage $A_b$ capital. Like non-lending mutual funds, they do not lend their shares and do not short. The unique feature of potentially-biased individual investors is that they believe the mean of $V_1$ is $\mu + b$ rather than $\mu$, making their optimal portfolio weights:

$$w_b(p, f) = \arg \max_{w} \ w' \left( \bar{r}(p) + \begin{bmatrix} b \\ p_1 \\ 0 \end{bmatrix} \right) - \frac{\gamma}{2} w' \Sigma(p) w.$$  \hspace{1cm} (8)

When $b > 0$, the individual investors overvalue asset 1 and bias their portfolio towards asset 1. When $b < 0$, they undervalue it and bias their portfolio away from asset 1.
Each group $g$’s portfolio weights translate into quantities of shares demanded as follows:

$$q_{g,i}(p, f) = \frac{A_g w_{g,i}(p, f)}{p_i},$$  \hspace{1cm} (9)

where $A_g$ is group $g$’s assets under management and $i$ is the asset number.

2.2. Equilibrium

Equilibrium is defined by asset prices $p$ and share lending fees $f$ such that both the primary market and the lending market clear. Primary market clearing requires that the sum of demands for all four groups of investors equals shares outstanding. Lending market clearing requires that, for each asset, either the lending fee $f_i$ equals zero or the sum of negative demands by short sellers equals the sum of positive demands by share lenders. We assume hedge funds mutual funds lend all the shares they own for any positive fee, meaning the total supply in share lending markets when $f_1 > 0$ equals:

$$\text{Lendable supply} = q_{h,1}(q_{h,1} > 0) + q_{fr,1}(q_{fr,1} > 0) + q_{n,1}(q_{n,1} > 0).$$  \hspace{1cm} (10)

Because only hedge funds short, demand for share borrowing equals:

$$\text{Borrowing demand} = -q_{h,1}(q_{h,1} < 0).$$  \hspace{1cm} (11)

Combining Equations (10) and (11), we have that lending supply equals lending demand when the sum of hedge and mutual fund positions equals zero, implying all shares outstanding
are held by the biased individuals. This makes the market clearing conditions:

\[ q_{h,1}(p, f) + q_{fr,1}(p, f) + q_{n,1}(p, f) + q_{b,1}(p, f) = Q_1, \]  
\[ q_{h,2}(p, f) + q_{fr,2}(p, f) + q_{n,2}(p, f) + q_{b,2}(p, f) = Q_2, \]  
\[ q_{b,1}(p, f) = Q_1, \text{ or } f_1 = 0, \]  
\[ q_{b,2}(p, f) = Q_2, \text{ or } f_2 = 0. \]

To simplify notation, we assume \( \mu_1 = \mu_2 = \mu, \sigma_1 = \sigma_2 = \sigma, \) and \( \rho = 0. \) Our main results hold without these assumptions.

### 2.2.1. Frictionless Benchmark

We start by computing the equilibrium when the two main frictions, individual investors’ bias and the retention of lending fees, are shut down. Formally, we set \( b = 0 \) and \( \kappa = 0. \) In this case, our model is a standard mean-variance setting with the following solution. Proofs are in Appendix B.

**Theorem 1** (Frictionless equilibrium). *When \( b = 0 \) and \( \kappa = 1, \) the CAPM holds and equilibrium prices are:*

\[
\begin{align*}
\text{Frictionless} \\
p_1 &= \mu - \gamma \sigma^2 \frac{Q_1}{A^4} \\
p_2 &= \mu - \gamma \sigma^2 \frac{Q_2}{A^4} \\
f_1 &= 0 \\
f_2 &= 0
\end{align*}
\]

*where \( A^4 \equiv A_h + A_{fr} + A_n + A_b. \)

In this setting, because all investors have mean-variance preferences and agree on the assets’ moments, the CAPM holds. All the usual CAPM results therefore apply, with investors
all using the same portfolio of risky assets, the market portfolio, which is the maximum Sharpe Ratio portfolio.

2.2.2. Biased Individual Investors

We next consider the equilibrium when individual investor’s bias is non-zero \((b \neq 0)\) but fee retainers do not retain any lending income \((\kappa = 0)\). In this case, because of the short-sale constraints faced by everyone but hedge funds, a different equilibrium occurs in three distinct regions of \(b\), as described by Theorem 2. The first column of Figure 4 illustrates this equilibrium for a specific parameterization.

**Theorem 2** (Equilibrium with biased individuals). When \(\kappa = 0\), one of three equilibria prevails depending on the value of \(b\):

- **Low \(b\)**: \(b < -\gamma \sigma^2 \frac{Q_1}{A_4 - A_b}\)
  - \(p_1 = \mu - \gamma \sigma^2 \frac{Q_1}{A_4 - A_b}\)
  - \(p_2 = \mu - \gamma \sigma^2 \frac{Q_2}{A_4}\)
  - \(f_1 = 0\)
  - \(f_2 = 0\)
  - \(q_{b,1} = \frac{A_b}{A_4 - A_b} Q_1\)
  - \(q_{fr,1} = \frac{A_{fr}}{A_4 - A_b} Q_1\)
  - \(q_{b,1} = 0\)
  - \(\bar{r}_1 = \frac{1}{A_4 - A_b} \gamma \sigma^2 Q_1 \frac{1}{p_1}\)
  - \(\bar{r}_1 + f_1 = \frac{1}{A_4 - A_b} \gamma \sigma^2 Q_1 \frac{1}{p_1}\)

- **Moderate \(b\)**: \(b \in \left[-\gamma \sigma^2 \frac{Q_1}{A_4 - A_b}, \gamma \sigma^2 \frac{Q_1}{A_b}\right]\)
  - \(p_1 = \mu + b \frac{A_b}{A_4} - \gamma \sigma^2 \frac{Q_1}{A_4}\)
  - \(p_2 = \mu - \gamma \sigma^2 \frac{Q_2}{A_4}\)
  - \(f_1 = 0\)
  - \(f_2 = 0\)
  - \(q_{b,1} = \frac{A_b}{A_4} \left(Q_1 - b \frac{A_b}{\gamma \sigma^2}\right)\)
  - \(q_{fr,1} = \frac{A_{fr}}{A_4} \left(Q_1 - b \frac{A_b}{\gamma \sigma^2}\right)\)
  - \(q_{b,1} = 0\)
  - \(\bar{r}_1 = \frac{1}{A_4} \gamma \sigma^2 Q_1 \frac{1}{p_1} - b \frac{A_b}{p_1} < 0\)
  - \(\bar{r}_1 + f_1 = \frac{1}{A_4} \gamma \sigma^2 Q_1 \frac{1}{p_1} - b \frac{A_b}{p_1} < 0\)

- **High \(b\)**: \(b > \gamma \sigma^2 \frac{Q_1}{A_b}\)
  - \(p_1 = \mu + b \frac{A_b}{A_4} - \gamma \sigma^2 \frac{Q_1}{A_4}\)
  - \(p_2 = \mu - \gamma \sigma^2 \frac{Q_2}{A_4}\)
  - \(f_1 = \frac{p_1 - \mu}{p_1}\)
  - \(f_2 = 0\)
  - \(q_{b,1} = \frac{A_b}{A_4} \left(Q_1 + b \frac{A_4 - A_b}{\gamma \sigma^2}\right)\)
  - \(q_{fr,1} = \frac{A_{fr}}{A_4} \left(Q_1 - b \frac{A_b}{\gamma \sigma^2}\right)\)
  - \(q_{b,1} = 0\)
  - \(\bar{r}_1 = \frac{1}{A_4} \gamma \sigma^2 Q_1 \frac{1}{p_1} - b \frac{A_b}{p_1} < 0\)
  - \(\bar{r}_1 + f_1 = \frac{1}{A_4} \gamma \sigma^2 Q_1 \frac{1}{p_1} - b \frac{A_b}{p_1} < 0\)

For moderate \(b\), all four groups of investors are marginal investors at interior solutions, buying positive quantities of both assets. However, the biased investors still have price
impact that causes their bias $b$ to affect equilibrium prices proportionally to the fraction of overall assets managed by the biased investors, $\frac{Ah}{A_4}$. This price impact is similar to the typical ‘price impact’ of noise traders in microstructure models. This mispricing is not a riskless arbitrage opportunity, and is therefore not fully priced away by risk averse agents, because neither asset 1 nor the biased individuals are infinitesimal in this economy.

As $b$ decreases in the moderate $b$ region, the biased individual investors hold smaller positions until eventually their unconstrained preference would be a negative position in asset 1. Because these investors are not allowed to short, their portfolio choice becomes constrained and the equilibrium switches to one in which only the other three groups are marginal investors in asset 1. As a result, asset 1’s price is lower due to absence of individual investor’s risk-bearing capacity, but is no longer sensitive to changes in $b$.

As $b$ increases in the moderate $b$ region, the biased individual investors hold larger positions, while the other groups of investors hold smaller positions, until eventually the unbiased investors’ unconstrained preference would be a negative position in asset 1. Mutual funds are not allowed to short or lend, so in this region their portfolio has a corner solution of 0 shares in asset 1. If short-sales did not require borrowing shares, or if individual investors lent their shares, hedge funds would short enough shares for the individual investors to keep buying more and more shares, while mutual funds held 0 shares. However, this is not possible in our setting because hedge funds have no one to borrow shares from. Furthermore, with $\kappa = 0$ no short-selling can occur in equilibrium because hedge funds, the only potential short-seller, will have the same expected returns as the two potential share lenders.

The high $b$ equilibrium clears the primary and lending markets by using lending fees $f_1$ as a shadow price that makes hedge funds and mutual funds be at their first order condition when holding zero shares of asset 1. As a result, biased individual investors own all shares of asset 1, whose price equals the equilibrium price that would prevail in a market with only biased investors. This may be initially surprising because hedge funds and mutual funds are
also at their first-order conditions, but remember for these investors the marginal investment in asset 1 comes with the additional return $f_1$, where the equilibrium $f_1$ is such that the expected fee-inclusive return of asset 1 is zero. As a result, only biased investors are marginal in the primary market, while institutional investors are marginal in the lending market.

2.2.3. Fee Retention

Finally, we present the equilibrium in our full model with both biased individual investors ($b \neq 0$) and lending fee retention ($\kappa > 0$). For low and moderate $b$, lending fees are zero and so $\kappa > 0$ has no impact on the equilibrium described above. Theorem 3 present the equilibrium prices and quantities in the high $b$ equilibrium, and the second column of Figure 4 illustrates this equilibrium for a specific parameterization.

**Theorem 3** (Equilibrium with lending fee retention and biased individuals). When $\kappa > 0$, implying $\lambda > 1$, the $\kappa = 0$ equilibrium from Theorem 2 prevails when $b \leq \sigma^2 Q_1 A_b$. When $b > \sigma^2 Q_1 A_b$, a modified High $b$ equilibrium prevails:

<table>
<thead>
<tr>
<th>\text{High $b$}</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1$ = $\mu + b - \gamma \sigma^2 Q_1 A_b$</td>
</tr>
<tr>
<td>$f_1$ = $\frac{p_1 - \mu}{p_1} \frac{A_b + A_{fr}}{A_b + \lambda A_{fr}}$</td>
</tr>
<tr>
<td>$q_{h,1}$ = $-(\lambda - 1) \frac{A_{fr}}{A_b + \lambda A_{fr}} (p_1 - \mu) \frac{A_b}{\gamma \sigma^2}$</td>
</tr>
<tr>
<td>$q_{fr,1}$ = $(\lambda - 1) \frac{A_{fr}}{A_b + \lambda A_{fr}} (p_1 - \mu) \frac{A_b}{\gamma \sigma^2}$</td>
</tr>
<tr>
<td>$q_{n,1}$ = 0</td>
</tr>
<tr>
<td>$q_{b,1}$ = $Q_1$</td>
</tr>
<tr>
<td>$\bar{r}_1$ = $\frac{1}{A_b} \frac{\gamma \sigma^2 Q_1 - b A_b}{p_1}$ &lt; 0</td>
</tr>
<tr>
<td>$\bar{r}<em>1 + f_1$ = $(\lambda - 1) \frac{A</em>{fr}}{A_b + \lambda A_{fr}} \bar{r}_1$ &lt; 0</td>
</tr>
</tbody>
</table>

The high $b$ equilibrium changes when $\kappa > 0$ because fee retainers have an extra incentive to lend shares, and therefore form long positions in asset 1 to lend shares to hedge funds.
Doing so requires price concessions to hedge funds, who choose not to short at the prices prevailing in the $\kappa = 0$ equilibrium. These concessions occur in the lending market rather than the primary market because, as in the $\kappa = 0$ equilibrium, biased investors are the marginal investors in the share market while fee retainers and hedge funds are marginal in the share lending market. As a result, the biggest effects of $\kappa > 0$ are on prices and quantities in the lending market, the portfolio choice of lending mutual funds, and expected fee-inclusive returns, as summarized by Theorem 3.

The ‘disconnect’ in our model between lending fees and share prices, whereby many exogenous parameter changes (e.g. to $\kappa$, $A_h$, or $A_{fr}$) affect the former but not the latter, is consistent with a number of empirical papers showing various exogenous changes to lending markets have little or no impact on equilibrium stock prices. One strand uses regulatory interventions to study the impact of short-sale costs (Diether, Lee, and Werner (2009), Boehmer, Jones, and Zhang (2013), Beber and Pagano (2013), Grullon, Michenaud, and Weston (2015), and Crane et al. (2018)). Another strand uses natural or designed experiments unrelated to regulatory changes to estimate the effects of supply and demand shocks in the shorting market (Cohen, Diether, and Malloy (2007) and Kaplan, Moskowitz, and Sensoy (2013)). Both strands conclude, with the exception of Grullon, Michenaud, and Weston (2015), that shorting market conditions have no impact on returns and volatility.

2.3. Equilibrium Results

We now turn to the main results of our model. Proofs are in Appendix B.

**Result 1** (Lending fee retention distorts portfolio choice). *For values of $b$ such that $f_1 > 0$, fee retainers hold a larger position in asset 1 than non retainers, who hold the fee-inclusive mean-variance optimal portfolio given a short-sale constraint. For lower values of $b$, both
types of funds hold the mean-variance optimal portfolio. Formally:

\[
\begin{align*}
  w_{fr,1} > w_{n,1} = w_{opt,1} = 0 & \quad \text{for } b \text{ s.t. } f_1 > 0, \\
  w_{fr,1} = w_{n,1} = w_{opt,1} = 0 & \quad \text{for } b \text{ s.t. } f_1 = 0,
\end{align*}
\]

(16)

where \( w_{fr,1}, w_{n,1}, \) and \( w_{opt,1} \) are the equilibrium weights chosen by fee retainers, non retainers, and a mean-variance investor who lends their shares but cannot short.

Result 1 shows that in our model fee retainers overweight stocks with non-zero lending fees, which is not surprising given they keep a fraction \( \kappa \) directly as revenue and prefer revenue to dollar-equivalent capital gains for their investors.

**Result 2** (Lending fee anomaly). If \( b \) varies and other parameters are fixed, both the expected return \( \bar{r}_1 \) and fee-adjusted expected return \( \bar{r}_1 + f_1 \) for asset 1 are negative and decreasing in the equilibrium lending fee. Formally, for \( b \) such that \( f_1 > 0 \),

\[
\begin{align*}
  \bar{r}_1 < \bar{r}_1 + f_1 < 0, \\
  \frac{\partial \bar{r}_1}{\partial b} < \frac{\partial (\bar{r}_1 + f_1)}{\partial b} < 0, \\
  \text{and } \frac{\partial f_1}{\partial b} > 0
\end{align*}
\]

(17, 18, 19)

Result 2 offers a potential explanation for the “anomaly” whereby stocks with high lending fees subsequently depreciate by more than the fee, making their fee-inclusive future abnormal returns negative (documented in Jones and Lamont (2002) and Drechsler and Drechsler (2016)). Other models of asset and lending market equilibrium, for example in the appendix model of Blocher, Reed, and Van Wesep (2013) and Weitzner (2016), predict that fee-inclusive returns have no alpha relative to the appropriate benchmark.\(^7\) Similarly, our

\(^7\)The model in Drechsler and Drechsler (2016) provides an alternative ‘arbitrage risk’ explanation for negative fee-inclusive alphas.
model predicts fee-inclusive average returns are zero without lending fee retention. With fee retention, the added incentive to buy and lend shares drives down equilibrium lending fees, resulting in negative fee-inclusive expected returns for high-fee stocks.

The first row of Figure 4 illustrates the impact of fee retention on fee-inclusive expected returns. As described above, when \( \kappa > 0 \), fee retainers bias their portfolios towards asset 1, pushing down \( f_1 \) and causing fee-inclusive expected returns to decrease in \( b \).

**Result 3** (Underperformance of fee retainers). For values of \( b \) such that \( f_1 > 0 \), fee retainers have portfolios with smaller fee-inclusive average returns and Sharpe Ratios than non retainers. Formally, for \( b \) such that \( f_1 > 0 \) we have:

\[
\frac{w'_{fr}(\tau + f)}{\sqrt{w'_{fr} \Sigma(p)w_{fr}}} < \frac{w'_{n}(\tau + f)}{\sqrt{w'_{n} \Sigma(p)w_{n}}}.
\]

Result 3 shows fee retainers’ distorted portfolio choice results in worse performing portfolios than non retainers when including the full amount of the fee in computing returns. This is caused by the long positions fee retainers hold in the overpriced asset 1, which results in poor performance relative to non retainers who put zero weight in asset 1.

The first row of Figure 5 shows each group’s fee-inclusive portfolio expected return as a function of the bias \( b \). With no fee retention, all hedge funds and mutual funds all choose the same long-only portfolio to benefit from the mispricing caused by individual investors, overweighing asset 1 when \( b < 0 \) and underweighting asset 1 when \( b > 0 \). Biased investors earn slightly higher average returns for moderate values of \( b \) because they lever up their portfolio to maximize exposure to asset 1, which still has a positive expected return. In the high \( b \) region, though, this aggressive allocation to asset 1 hurts their portfolio expected return more and more as asset 1’s expected return is negative and decreasing in \( b \).

When some funds retain a fraction of lending fees, the three types of funds no longer have
identical portfolios. When $f_1 > 0$, fee retainers gradually erode their fee-inclusive expected returns by overweighting asset 1. Hedge funds benefit from this mistake, increasing their fee-inclusive expected return by shorting asset 1. Non retainers outperform fee retainers because they keep zero weight in asset 1.

The second row of Figure 5 shows the same pattern holds for fee-inclusive Sharpe Ratios, with hedge funds outperforming non retainers, who outperform fee retainers. Biased individuals have lower Sharpe Ratios despite moderately higher average returns when $b$ is moderately positive because the leveraged positions they take in asset 1 add enough risk to their portfolio to offset the gain in expected returns.

2.4. Fund Investor’s After-Fee Returns

One potential defense of lending fee retention by mutual fund managers is that it allows them to lower management fees, meaning net of all fees their investors may not be worse off.

To assess this possibility, we augment our model to include management fees charged by mutual funds. We model investors returns as differing from the fund portfolio’s returns for two reasons. The first is the fee retainers retains a fraction $\kappa > 0$ of the lending fees they collect. The second is that both types of mutual funds charge fixed management fees that do not depend on portfolio performance or lending fees collected. Fund investor’s after-fee expected returns therefore satisfy:

\begin{align}
\bar{r}_{a.f.}^{fr} &= w_{fr}'(\bar{r} + (1 - \kappa)f) - m_{fr}, \\
\bar{r}_{a.f.}^{n} &= w_{n}'(\bar{r} + f) - m_{n},
\end{align}

where $m_{fr}$ and $m_{n}$ are the management fees charged by fee retainers and non retainers.

We consider two possible scenarios for management fees: equal fees ($m_{fr} = m_{n}$), and a fee discount equal to anticipated retained lending revenue ($m_{fr} = m_{n} - \kappa w_{fr}'f^*$, where
$w^*_r$, and $f^*$ are equilibrium weights and lending fees).\(^8\) Importantly, we assume that fund managers choose their management fee $m_g$ prior to choosing their optimal portfolio and have no mechanism for committing to a portfolio choice. The fixed fee therefore has no impact on their portfolio choice or the equilibrium described above.

**Result 4 (Management fees).** For values of $b$ such that $f_1 > 0$, if management fees are equal ($m_{fr} = m_n$) or mutual funds offer a discount equal to anticipated lending revenue ($m_{fr} = m_n - \kappa w^*_r f^*$), fee retainers have smaller after-fee expected returns than non retainers:

$$r_{af}^{fr} < r_{af}^n.$$ (24)

The first part of Result 4, that lending funds have lower after-fee performance when management fees are the same, follows directly from their smaller portfolio expected return (Result 3), which is only worsened by fund managers keeping a fraction $\kappa$ of lending fees. The second part of Result 4 is perhaps more surprising, because a natural intuition is that investors’ after-fee return will be the same regardless of lending policy if the retained lending income is offset by reduced management fees. However, this intuition only works if lending fee retention has no effect on portfolio choice. In our model, fee retainers’ portfolios have worse fee-inclusive returns, meaning that if they collect the same total fees (management fees plus retained lending fees), the net for investors remains smaller than for non retainers.

2.5. Alternative Fund Structures

In this subsection, we discuss asset managers who treat lending fees differently than described above. The first variation we consider is a fund that lends their shares using a lending agent that charges a fraction $\nu$ of the dollar amount lent. Unlike the fee retainers discussed above, these funds receive no additional revenue from share lending and therefore

\(^8\)For simplicity, we assume mutual fund managers and investors know $b$ along with all other model parameters when computing management fees based on anticipated lending revenue.
have no extra incentive to lend. They will therefore choose the same equilibrium portfolio non retainers, with no shares invested in asset 1 when lending fees are positive, and outperform fee retainers by the same amount as non retainers. The fraction $\nu$ paid to lending agents only serves as an additional deterrent to buying and lending asset 1.

The second variation we consider is a fund who commits not to lend shares at all, which occurs frequently in practice. These funds will also stick with the corner solution of zero shares in asset 1, only differing from the non retainers in our baseline model in how desperately they want to avoid high-fee stocks. By the same reasoning, any alternative fund structure that does not include lending fee retention by fund managers, or some other distorted incentive to own high-fee assets, will behave in equilibrium like a non retainer and hold zero shares of asset 1 when $f_1 > 0$. One can therefore interpret the non retainers in our model as including all mutual funds except those with non-trivial lending fee retention. This interpretation motivates our empirical approach, which compares funds we can clearly identify as fee retainers to all other mutual funds.

3. **Empirical Evidence of Distortions Caused by Fee Retention**

In this section, we present new empirical evidence and discuss existing evidence for the distortions our model predicts are caused by lending fee retention.

3.1. **Data**

We obtain monthly mutual fund returns, quarterly holdings and quarterly fund characteristics from CRSP, and hand-collect Regulation S-X securities lending disclosures from fund prospectuses on Edgar. We obtain lending fee data from Interactive Brokers (IB) for 6/30/2017, 9/29/2017 and 12/31/2017, the last trading day of each quarter, and stock CUSIPs from CRSP. We merge the IB data to CRSP via stock tickers then use the CUSIPs from CRSP to merge the data to the CRSP mutual fund holdings data. Many funds in CRSP
have multiple share classes so we aggregate fund characteristics and returns by weighting each share class within a fund by its TNA. See Appendix A for a detailed description of our data collection and variable construction.

Using the cross-section of fee retention, we define an empirical indicator “High Fee Retainer” designed to identify funds we can cleanly categorize as profiting from fee retention, the key feature of fee retainers in our model. High Fee Retainer equals one if the fund’s lending agent fees, divided by gross lending income, exceeds the median in the sample, 8.8%, and it self deals. The fund management companies that operate these funds are BNY Melon, BlackRock, Fidelity, Goldman Sachs and State Street. Undoubtedly other funds retain and profit from securities lending revenue in and out of our sample, but we focus on this stricter definition to cleanly isolate the effects we study.

3.2. Result 1: Distorted Portfolio Choice

We first test the Result 1 of our model, that fee retainers overweight high lending fee stocks relative to other funds. To do so, we examine whether High Fee Retainer funds choose portfolios with higher Fee Yield, the weighted average lending fee of a fund’s portfolio.

Table 5 shows that, as hypothesized, High Fee Retainer funds choose portfolios with Fee Yields around 8bps higher than other active funds in our sample. This effect is economically substantial relative the median fund’s Fee Yield of 5bps (Panel A of Table 2). Column (1) shows that Self Deal funds as a whole have 3bps higher fee yields than other funds in our sample, though this difference is not statistically significant. Columns (2) through (4) show that this effect is concentrated among High Fee Retainer funds (self-dealers who also pay high lending agent rates). When we include both High Fee Retainer and Self Deal as independent variables (Column 3), High Fee Retainer is still positive and statistically significant while Self Deal appears to have no effect. The relation between High Fee Retainer and Fee Yield is consistent across specification, and holds within fund families as well as across them.

Our findings in Table 5 confirm and extend the evidence in Prado (2015), and Evans,
Ferreira, and Porras Prado (2017). Prado (2015) analyzes quarterly 13-F filings and finds that institutional investors increase their ownership the quarter after a stock becomes special. Our results narrow down the types of institutional investors that are attracted to high lending fee stocks. Evans, Ferreira, and Porras Prado (2017) find that when a fund manager simultaneously manages a fund that is able to lend and a fund that is not, the fund able to lend reduces its holdings less after a stock becomes special. Our results differ from those in Evans, Ferreira, and Porras Prado (2017) because we compare portfolio choices of lending funds within a fund management company, and find that funds that retain more lending income hold higher fee-yield portfolios. We also use indicative lending fee data rather than short interest to measure short-sale costs.

We also differ from Prado (2015) and Evans, Ferreira, and Porras Prado (2017) in our explanation for these findings. Prado (2015) argues institutional investors increase their positions to profit from the lending fees. While these investors should incorporate lending fees in their evaluation of potential investments, as discussed below the evidence is high fee stocks underperform even when including the lending fee. The Prado (2015) story, therefore, does not explain why some institutions seem to value lending fees more than the subsequent underperformance. Our story explains this because fee-retaining funds often keep a fraction of lending fees while passing through the full amount of capital gains to their investors.

Evans, Ferreira, and Porras Prado (2017) argues that funds keep and lend shares in high fee stocks rather than selling them because fund family restrictions prevent them from selling, leading to the underperformance of lending funds relative to non-lending funds (see Section 3.4). They support this argument by showing that the under-performance of lending funds is concentrated among funds with high values of a Manager Restriction Index (from Almazan et al. (2004)). However, it is not clear why overall product strategies would require maintaining positions in high fee stocks when 90% of stocks typically have near-zero lending fees (D’Avolio (2002)). We also show that our results hold after controlling for fund style.
The only other paper we are aware of that discusses lending practices as potentially harming fund investors is Blocher and Whaley (2016), which finds studies the holdings of ETFs who lend their shares. If ETFs were maximizing their performance on behalf of their investors, they would use any discretion they have in portfolio choice to reduce their exposure to under-performing high lending fee stocks. However, Blocher and Whaley (2016) find the opposite: ETFs slant their holding towards high lending fee stocks. We show that this effect is prevalent in active mutual funds that have more discretion in their portfolios and among funds that retain more securities lending income for the fund management company. Blocher and Whaley (2016) also show that ETFs with a proprietary benchmark rather than a third party one, making them less constrained in their portfolio choice, slant their holdings even more towards high lending fee stocks. This behavior also casts doubt on the Evans, Ferreira, and Porras Prado (2017) story because constraining ETFs appears to reduce the tilt and improve fund performance, the opposite of what the Evans, Ferreira, and Porras Prado (2017) story would predict.

Finally, related but distinct evidence in Adams, Mansi, and Nishikawa (2014) indicates that funds with affiliated lending agents earn more lending revenue per dollar TNA, but less lending revenue per dollar lent. Their explanation is a conflict of interest between the board of directors and the fund. However, their study does not consider the portfolio choice distortion. Funds with affiliated agents may have an incentive to buy more high lending fee stocks, thus increasing their security lending revenue through the asset side, but decreasing the fraction of that revenue flowing back to investors, exactly what we find in Table 5.

3.3. Result 2: Lending Fee Anomaly

Result 2 predicts stocks with high lending fees are overpriced relative to frictionless benchmarks, and have abnormally negative future returns relative to benchmarks even on a fee-inclusive basis. There is extensive empirical evidence this is the case, for example in Jones and Lamont (2002), Geczy, Musto, and Reed (2002), Ofek, Richardson, and Whitelaw
(2004), and Drechsler and Drechsler (2016). Models of equilibrium lending fees, for example in Duffie, Garleanu, and Pedersen (2002) or the appendix model of Blocher, Reed, and Van Wesep (2013), predict that lending fees affect equilibrium stock prices in a manner similar to dividends, with asset prices declining in proportion to the lending fee. As a result, the fee-exclusive underperformance of high lending fee stocks is unsurprising. As discussed in Drechsler and Drechsler (2016), however, the fee-inclusive underperformance is more surprising and requires an additional mechanism to explain.

Our model provides a simple answer to explain the fee-inclusive underperformance of high lending fee stocks: fee-retaining funds buy and lend high-fee stocks, driving down equilibrium lending fees.\footnote{Fee retainers buying and lending does not drive up share prices because biased individuals remain the only marginal investor ignoring lending fees, and must own all shares outstanding of high-fee assets.} This explanation is complementary to the systematic short-selling risk hypothesis in Drechsler and Drechsler (2016).

The endogenous portfolio choice of fee-retaining funds also provides an explanation for a related phenomenon: the upward sloping supply curve for share lending documented in Kolasinski, Reed, and Ringgenberg (2013). Barring some institutional friction or lending cost, shareholders with long positions should be willing to lend all their shares for any positive fee, and therefore the lending supply curve should be inelastically equal to the supply of shares held by potential lenders. Kolasinski, Reed, and Ringgenberg (2013) find that is indeed the case when the stock whenever lending fees are small. However, they find that once a stock has non-trivial borrowing fees, the slope of the supply schedule becomes positive and steep. Our model offers a potential explanation for this steepness when lending fees are non-trivial: even if lending institutions inelastically supplies all their shares of special stocks to borrowers, an increase in lending fees results in a larger supply of lendable shares because fee-retaining mutual funds endogenously choose to buy and lend more shares.
3.4. Result 3: Underperformance of Fee Retainers

We next test whether funds that retain more securities lending income underperform. Following Evans, Ferreira, and Porras Prado (2017), we use CRSP return data to calculate monthly Carhart (1997) four-factor alphas by estimating 36-month rolling betas from 2000 to 2017. We begin our sample in 2000 because there is very little CRSP fund-level coverage prior.\textsuperscript{10} We then test if high fee retaining funds underperform as our theory predicts. Because we only have detailed securities lending data in 2017, an implicit assumption of our empirical analysis is that funds lending fee retention policies are persistent. It is reassuring that several funds from two of the fund families that were embroiled in lawsuits prior to 2013, State Street and BlackRock, are in our high fee retention pool. We also find similar point estimates when we limit the sample to more recent years; however we have less power in those tests.

Table 6 shows that fee-retaining funds have lower alphas than other active funds, as predicted by our model. Column (1) shows that self-dealers as a whole have monthly alphas 4.4bps lower than other active funds, and Column (2) shows this effect is stronger among High Fee Retaining funds (5.9bps per month, 71bps per year). In Column (3) we include both High Fee Retainer and Self Deal, and find they both have negative effects on alpha but only High Fee Retainer remains marginally statistically significant while Self Deal is not, which is consistent with the holdings evidence in Table 5. In Column (4), we include fund family fixed effects and find similar results, suggesting that even within the same fund family, High Fee Retainer funds hold higher lending fee portfolios and therefore underperform.

In Columns (5) and (6) of Table 6, we compare funds we identify as high fee retainers to broader sets of funds than our main sample, which requires 2017 Regulation S-X data. In Column (5), we include all funds that lend their shares, according to their NSAR filings, over the 2000–2017 sample period. In Column (6), we include all open-end, equity mutual funds in CRSP. In both specifications, high fee retaining funds exhibit lower alphas than non-high

\textsuperscript{10}Specifically, the fund identifier we use, crsp.cl.grp begins in August 1998.
fee retaining funds, though the effect is smaller and less statistically significant. However, there is a substantial survivorship bias dampening our effect in the last two specifications because High Fee Retaining funds all have survived through 2017 and we are comparing them to many funds that have not survived. Given this survivorship bias, it is potentially even more compelling that High Fee Retainers underperform.

Our model predicts that fee-retaining funds will underperform relative to any non-fee-retaining fund, including funds that do not lend shares at all. Consistent with this prediction, Evans, Ferreira, and Porras Prado (2017) finds that funds who lend their shares have 72bps lower per-year alphas than funds that do not. As discussed above, they attribute this performance gap to fund family objectives preventing lending funds from selling their shares in high lending fee stocks. We argue a performance gap of this magnitude is more consistent with distorted portfolio choices than fund-family objectives. In particular, we find that even after controlling for fund-family characteristics with fund management fixed effects, funds that retain more lending fees still choose higher lending fee portfolios.

Finally, Rizova (2011) hand collects securities lending data for mutual funds and finds that net returns to investors and returns from securities lending are negatively correlated, a confirmation of the Evans, Ferreira, and Porras Prado (2017) result in panel data.

3.5. Result 4: Management Fees

Result 4 predicts that funds that retain securities lending income will use higher lending fees to reduce their expense ratios. In Table 7, we test this possibility using regressions with expense ratio as the dependent variable. In Column (1), we include Self Deal and Lending Agent Fees as independent variables and find no significant relationship between Self Deal and expense ratios. However, when we include fund family fixed effects in Column (2), we find that self-dealing funds have 11bps lower expense ratios than non self-dealing funds and the estimate is statistically significant. In Columns (3) and (4) we interact Self Deal with Lending Agent Fees and find that self-dealing funds appear to pass along the costs of
administering securities lending programs in the form of lower expense ratio, with the effect being stronger with fund family fixed effects.

4. Conclusion

A simple but prevalent feature of institutional investing, the ability to retain a fraction of share lending revenue, leads to a variety of distortions in financial markets. This feature can explain why some lending funds hold more high lending fee stocks and underperform despite lower management fees, and why stocks with high lending fees perform poorly on a fee-inclusive basis. Our model also disputes conventional wisdom regarding the limits to arbitrage and institutional investor performance: as more fee-retaining funds lend their shares, the lending fee anomaly worsens rather than improves, and allowing fee-retaining funds to lend their shares worsens their performance rather than improves it. Going forward, investors and researchers should consider the incentive impact of securities lending fee retention when evaluating or predicting mutual fund performance.

Our paper also has policy implications. Fund management and asset prices would become more efficient on a fee-inclusive basis without the added incentives for fund managers to lend. In our model, this could be achieved by simply requiring funds to remit the full amount of lending fees to investors, and pay any costs associated with share lending out of management fees. In practice, we believe a similar outcome could be is possible via pressure from investors to reduce fee retention. A precursor to such pressure is transparency about the uses of securities lending income. The amendments to Regulation S-X are a step in this direction, however they remain unlikely to be salient to investors because they are buried in regulatory filings. Instead, because lending fees are equivalent to capital gains and dividends from investors’ perspective, we believe gross and net securities lending revenue should be reported alongside portfolio returns and at the same frequency, and that costs of securities lending should be included in reported management fees.
Appendix A. Data Collection and Variable Definitions

Following Evans, Ferreira, and Porras Prado (2017), we gather all NSAR-B filings from the SEC’s Edgar database from 1996–2017. Mutual funds report their financial information at the CIK or “series” level which often contains multiple funds. Each fund in Edgar has a “series ID” associated it. Within each series there are multiple tickers that correspond to each share class within the fund. We create a mapping between CIK, series ID, fund name and ticker by scraping the SEC website and then merging the funds to the CRSP Mutual Fund Database based on ticker. Using the NSAR filings, we identify open-ended funds that lent their shares at any point during the 2017 calendar year. This corresponds to answering “Y” to Q70N02.

We take the CIK from each fund that engaged in securities lending in 2017 and collect all of their prospectuses (forms 485APOS or 485BPOS) that correspond to the 2017 calendar year. In the prospectuses, funds generally report a disclosure similar to Figure 1 which includes gross revenue, lending agent fees, rebate, total fees and net income associated with securities lending. We hand collect each reported item in the prospectus. Within each prospectus there are often multiple funds within a single reporting entity. We collect the fund name and match these by hand with the possible fund names within each CIK. We then use our mapping to find each ticker for each fund and merge these into CRSP.

We find information for and are able to successfully merge securities lending income for 1,035 funds and ETF’s. We exclude 14 funds that have negative net income from securities lending as these funds have non-standard disclosures. We then limit the sample to active open-end, equity mutual funds which reduces our sample size to 542 funds. This compares to 806 open-end, active equity funds that engaged in securities lending in 2017 according to our NSAR sample. We define an equity fund as a fund in which the first letter of the crsp_obj_cd is “E” and an index fund as a the field index_fund_flag is “D”. For our control variables we use the quarterly CRSP data and use the TNA from that quarter to weight them. For the returns analysis we use the monthly returns data with monthly TNA, Family TNA and Flow, but use the quarterly data for our other control variables.

In all of our fund family fixed effects regressions we treat “Goldman Sachs Asset Management LP” and “Goldman Sachs CO/GSAM” as one family and do the same for “BlackRock Inc” and “BlackRock Fund Advisors” so we capture all of the variation for fee retention within Goldman Sachs and BlackRock.

This table identifies the data sources and describes the construction of variables we use in our analysis. Lending fee data is from Interactive Brokers, while mutual fund holdings,
returns and characteristics are from CRSP, NSAR filings and prospectuses are collected from
the SEC’s Edgar database.

\[ \text{Alpha}_t: \text{Realized alpha in month } t \text{ from a Carhart four-factor model, with betas estimated in }
\text{months } t-36 \text{ through } t-1, \text{ from CRSP.} \]

\[ \text{Aggregate Fees/Compensation for Securities Lending: Total fees and compensation associated}
\text{with securities lending in 2017, from Reg S-X disclosure.} \]

\[ \text{Cash Collateral Fees: See “Fees for Cash Collateral Management”.} \]

\[ \text{Cost of Lending: See “Aggregate Fees/Compensation”.} \]

\[ \text{Expense Ratio: Average CRSP expense ratio (exp\_ratio) across funds’ share classes, weighted by}
\text{TNA (crsp\_cl\_grp), winsorized at [1\%, 99\%], from CRSP.} \]

\[ \text{Family TNA: Sum of TNA across all funds with the same management code (mgmt\_cd), win-}
\text{corsized at [1\%, 99\%], from CRSP.} \]

\[ \text{Fee Yield: Average borrowing fee of stocks in funds’ holdings, weighted by dollar value of holding,}
\text{minus 25bp. If borrowing fee missing, set to 25bp. From Interactive Brokers and CRSP} \]

\[ \text{Fees for Cash Collateral Management: Fees paid to manage cash collateral from securities lending}
\text{in 2017, from Reg S-X disclosure.} \]

\[ \text{Fees for Securities Lending Agent: Fees paid to securities lending agent, from Reg S-X disclosure.} \]

\[ \text{Flow: Monthly net fund flow, estimated using } \frac{TNA_t - TNA_{t-1}(1+\text{Return}_{t})}{TNA_{t-1}}, \text{ winsorized [1\%, 99\%],}
\text{from CRSP.} \]

\[ \text{Gross Income from Securities Lending: Gross income from securities lending in 2017, from Reg}
\text{S-X disclosure.} \]

\[ \text{Gross Income Yield: Gross Income from Securities Lending / TNA.} \]

\[ \text{High Fee Retainer: Indicator equal to one when Self Deal equals one and Lending Agent Fees (%}
\text{gross) is higher than the median fund.} \]

\[ \text{Indemnification Fees: Indemnification fees paid for securities lending in 2017, from Reg S-X dis-}
\text{closure.} \]

\[ \text{Lending Agent Fees: See “Fees for Securities Lending Agent”} \]

\[ \text{Net Income to Fund: Net income from securities lending paid to fund in 2017, from Reg S-X}
\text{disclosure.} \]
Net Income Yield: Net Income to Fund / TNA

Other Fees: In Table 1, this includes only fees the fund reports as “Other fees not included in revenue split” in 2017. Expanded in Table 2 to include Administrative Fees and Indemnification Fees.

Rebate: Rebate paid to securities borrowers in 2017, from Reg S-X disclosure.

Return: Average return (mret) across funds’ share classes, weighted by lagged TNA (crsp.cl.grp), from CRSP.

Self Deal: Indicator equal to one when at least one of lending agents share a parent company with the fund management company in 2017, from Reg S-X disclosure.

TNA: Total net assets in fund across all share classes (crsp.cl.grp), winsorized at [1%, 99%], from CRSP.

Turnover: Average turnover ratio (turn_ratio) across funds’ share classes, weighted by TNA (crsp.cl.grp), winsorized at [1%, 99%] from CRSP.

We remove observations with negative values of Lending Agent Fees or Cash Collateral Fees and observations with Lending Agent Fees or Cash Collateral Fees greater than Gross Income from Securities Lending because these values are economically implausible. When Rebate is negative, we add the absolute amount to Gross Lending Income because a negative rebate is economically equivalent to positive lending income, and set Rebate to zero.

Appendix B. Proofs

In this appendix, we prove the Theorems, Lemmas, and Results in Section 2. Throughout, we use the assumption that \( \rho = 0 \), and that investors have access to unlimited leverage, to separately evaluate the unconstrained optimal weights in the two assets for each group of investors using the following:

\[
 w^*_{1,g} = \arg \min_w w \bar{r}_{1,g}(p_1) - \frac{\gamma}{2} w^2 \frac{\sigma^2}{p_1^2} = \bar{r}_{1,g}(p_1) \frac{p_1^2}{\gamma \sigma^2} 
\]

\[
 w^*_{2,g} = \bar{r}_{2,g}(p_2) \frac{p_2^2}{\gamma \sigma^2},
\]

where \( w^*_{i,g} \) is the optimal weight in assets \( i \) for group \( g \), \( \bar{r}_{i,g}(p_i) \) is the expected return for asset \( i \) to group \( g \), including any bias, lending fee, or added lending incentive. Note Equations
(25) and (26) have squared prices in them because \( \sigma^2 \) is the variance of final payoffs, making \( \frac{\sigma^2}{p_1} \) and \( \frac{\sigma^2}{p_2} \) the variances of returns.

**Theorem 1** (Frictionless prices). When \( b = 0 \) and \( \kappa = 1 \), the CAPM holds and equilibrium prices are:

\[
\begin{align*}
p_1 &= \mu - \gamma \sigma^2 \frac{Q_1}{A_4} \\
p_2 &= \mu - \gamma \sigma^2 \frac{Q_2}{A_4} \\
f_1 &= 0 \\
f_2 &= 0
\end{align*}
\]

where \( A_4 \equiv A_h + A_{fr} + A_n + A_b \).

**Proof.** We show that the quantities presented below are for each agent given prices presented in Theorem 3. Because these quantities clear both the share and lending markets, we have an equilibrium.

\[
\begin{align*}
q_h &= \frac{A_h}{A_4} Q_1 \\
q_{fr} &= \frac{A_{fr}}{A_4} Q_1 \\
q_n &= \frac{A_n}{A_4} Q_1 \\
q_b &= \frac{A_b}{A_4} Q_1 \\
r_i &= \frac{\gamma \sigma^2 Q_1}{p_1} \\
r_i &= \frac{\gamma \sigma^2 Q_2}{p_2}
\end{align*}
\]

Given prices in Theorem 1, each agent has the same optimal weights in the two assets:

\[
w_i = \frac{\gamma \sigma^2 Q_1}{p_i} \frac{p_i^2}{\gamma \sigma^2} = p_i \frac{Q_i}{A_4},
\]

This translates into quantities for each group of investors \( g \) satisfying:

\[
q_{g,i} = \frac{A_g w_g}{p_i} = \frac{A_g}{A_4} Q_i.
\]

CAPM holds because each investor chooses market portfolio, which has maximum Sharpe Ratio of all risk asset portfolios.

**Theorem 2** (Equilibrium with biased individuals). When \( \kappa = 0 \), one of three equilibria prevails depending on the value of \( b \):
\[
\begin{array}{cccc}
\text{Low } b & \text{Moderate } b & \text{High } b \\
\begin{array}{c}
b < -\gamma \sigma^2 \frac{Q_1}{A_4 - A_b} \\
\end{array} & \begin{array}{c}
b \in \left[-\gamma \sigma^2 \frac{Q_1}{A_4 - A_b}, \gamma \sigma^2 \frac{Q_1}{A_b}\right] \\
\end{array} & \begin{array}{c}
b > \gamma \sigma^2 \frac{Q_1}{A_b} \\
\end{array} \\
p_1 &= \mu - \gamma \sigma^2 \frac{Q_1}{A_4 - A_b} & p_2 &= \mu + b \frac{A_b}{A_4} - \gamma \sigma^2 \frac{Q_1}{A_4} & p_2 &= \mu - \gamma \sigma^2 \frac{Q_1}{A_4} \\
p_2 &= \mu - \gamma \sigma^2 \frac{Q_2}{A_4} & f_1 &= 0 & \frac{p_1 - \mu}{p_1} &= 0 \\
f_2 &= 0 & f_2 &= 0 & \\
q_{b,1} &= \frac{A_b}{A_4 - A_b} Q_1 & q_{b,1} &= \frac{A_b}{A_4} \left(Q_1 - b \frac{A_b}{\gamma \sigma^2}\right) & q_{b,1} &= \frac{A_b}{A_4} \left(Q_1 - b \frac{A_b}{\gamma \sigma^2}\right) \\
q_{fr,1} &= \frac{A_{fr}}{A_4 - A_b} Q_1 & q_{fr,1} &= \frac{A_{fr}}{A_4} \left(Q_1 - b \frac{A_b}{\gamma \sigma^2}\right) & q_{fr,1} &= \frac{A_{fr}}{A_4} \left(Q_1 - b \frac{A_b}{\gamma \sigma^2}\right) \\
q_n,1 &= \frac{A_n}{A_4 - A_b} Q_1 & q_n,1 &= \frac{A_n}{A_4} \left(Q_1 - b \frac{A_b}{\gamma \sigma^2}\right) & q_n,1 &= \frac{A_n}{A_4} \left(Q_1 - b \frac{A_b}{\gamma \sigma^2}\right) \\
q_{fr,1} &= 0 & q_{fr,1} &= 0 & q_{fr,1} &= 0 \\
\bar{\tau}_1 &= \frac{1}{A_4 - A_b} \frac{\gamma \sigma^2 Q_1}{p_1} & \bar{\tau}_1 &= \frac{1}{A_4} \frac{\gamma \sigma^2 Q_1 - b A_b}{p_1} & \bar{\tau}_1 &= \frac{1}{A_b} \frac{\gamma \sigma^2 Q_1 - b A_b}{p_1} < 0 \\
\bar{\tau}_1 + f_1 &= \frac{1}{A_4 - A_b} \frac{\gamma \sigma^2 Q_1}{p_1} & \bar{\tau}_1 + f_1 &= \frac{1}{A_4} \frac{\gamma \sigma^2 Q_1 - b A_b}{p_1} & \bar{\tau}_1 + f_1 &= 0
\end{array}
\]

Proof. We show that the quantities presented in Theorem 2 are optimal for each agent given prices presented in Theorem 3. Because these quantities clear both the share and lending markets, we have an equilibrium.

We prove the low and moderate \( b \) equilibria quantities are optimal here, and leave the high \( b \) equilibrium to the proof of Theorem 3, of which this is a special case with \( \kappa = 0 \).

Because we assume assets 1 and 2 are uncorrelated, asset 2 prices and quantities remain unchanged as \( b \) changes relative to the frictionless benchmark.

In the low \( b \) region, given the equilibrium prices in Theorem 2, biased individuals’ optimal weight in asset 1 satisfies:

\[
w_{1,b}^* = \left(\frac{1}{A_4 - A_b} \frac{\gamma \sigma^2 Q_1}{p_1} + b \right) \frac{p_1^2}{p_1} = \frac{p_1 Q_1}{A_4 - A_b} + b \frac{p_1}{\gamma \sigma^2} \\
< \frac{p_1 Q_1}{A_4 - A_b} - \left(\frac{\gamma \sigma^2 Q_1}{A_4 - A_b}\right) \frac{p_1}{\gamma \sigma^2} = 0,
\]

meaning the biased individuals would like to short asset 1 but cannot, and instead are at a corner solution of \( w_{1,b}^* = 0 \).
For low $b$, the other groups have optimal weights and quantities in asset 1 satisfying:

$$w_{1,g}^* = \left( \frac{1}{A_4 - A_b} \frac{\gamma \sigma^2 Q_1}{p_1} \right) \frac{p_1^2}{\gamma \sigma^2} = \frac{p_1 Q_1}{A_4 - A_b}$$

$$\Rightarrow q_{1,g} = \frac{A_g}{A_4 - A_b} Q_1.$$  \hspace{1cm} (31)

In the moderate $b$ region, biased individuals’ optimal weight in asset 1 satisfies:

$$w_{1,b}^* = \left( \frac{1}{A_4} \frac{\gamma \sigma^2 Q_1 - b A_b}{p_1} + \frac{b}{p_1} \right) \frac{p_1^2}{\gamma \sigma^2} = \frac{p_1 Q_1}{A_4} + b \left( 1 - \frac{A_b}{A_4} \right) \frac{p_1}{\gamma \sigma^2}$$

$$\Rightarrow q_{1,b} = \frac{A_b}{A_4} \left( Q_1 + b \frac{A_4 - A_b}{\gamma \sigma^2} \right).$$  \hspace{1cm} (33)

For moderate $b$, the other groups’ optimal weight satisfy:

$$w_{1,g}^* = \left( \frac{1}{A_4} \frac{\gamma \sigma^2 Q_1 - b A_b}{p_1} \right) \frac{p_1^2}{\gamma \sigma^2} = \left( \frac{p_1 Q_1}{A_4} - b \frac{A_b}{A_4} \right) \frac{p_1}{\gamma \sigma^2}$$

$$\Rightarrow q_{1,b} = \frac{A_g}{A_4} \left( Q_1 - b \frac{A_4}{\gamma \sigma^2} \right),$$  \hspace{1cm} (35)

as specified by Theorem 2.  \hspace{1cm} \hfill \blacksquare

**Theorem 3** (Equilibrium with lending fee retention and biased individuals). When $\kappa > 0$, implying $\lambda > 1$, the $\kappa = 0$ equilibrium from Theorem 2 prevails when $b \leq \sigma^2 Q_1 A_b$. When $b > \sigma^2 Q_1 A_b$, a modified High $b$ equilibrium prevails:

<table>
<thead>
<tr>
<th>High $b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1 = \mu + b - \gamma \sigma^2 Q_1 A_b$</td>
</tr>
<tr>
<td>$f_1 = \frac{p_1 - \mu}{A_b + \lambda \sigma^2}$</td>
</tr>
<tr>
<td>$q_{h,1} = (\lambda - 1) \frac{A_f}{A_b + \lambda A_f} (p_1 - \mu) \frac{A_b}{\gamma \sigma^2}$</td>
</tr>
<tr>
<td>$q_{f,1} = (\lambda - 1) \frac{A_f}{A_b + \lambda A_f} (p_1 - \mu) \frac{A_b}{\gamma \sigma^2}$</td>
</tr>
<tr>
<td>$q_n,1 = 0$</td>
</tr>
<tr>
<td>$q_b,1 = Q_1$</td>
</tr>
<tr>
<td>$\bar{r}_1 = \frac{1}{A_b} \frac{\gamma \sigma^2 Q_1 - b A_b}{p_1} &lt; 0$</td>
</tr>
<tr>
<td>$\bar{r}_1 + f_1 = (\lambda - 1) \frac{A_f}{A_b + \lambda A_f} \bar{r}_1 &lt; 0$</td>
</tr>
</tbody>
</table>

**Proof.** Below, we show that the quantities presented in Theorem 3 are optimal for each agent given prices presented in Theorem 3. Because these quantities clear both the share and lending markets, we have an equilibrium.
Given equilibrium prices in Theorem 3, biased individuals choose weights:

\[ w_{1,b}^* = \left( \frac{1}{A_b} \frac{\gamma \sigma^2 Q_1 - b A_b}{p_1} + \frac{b}{p_1} \right) \frac{p_1^2}{\gamma \sigma^2} = \frac{p_1 Q_1}{A_b} \]  

(37)

\[ \Rightarrow q_{1,b} = Q_1. \]  

(38)

Before computing hedge funds’ optimal weights, we show that the formula for \( r_1 + f_1 \) given in Theorem 3 follows from the prices in Theorem 3:

\[ r_1 + f_1 = r_1 + \frac{p_1 - \mu}{p_1} A_h + A_{fr} = r_1 - r_1 \frac{A_h + A_{fr}}{A_h + \lambda A_{fr}} \]  

(39)

\[ = (\lambda - 1) \frac{A_{fr}}{A_h + \lambda A_{fr}} r_1, \]  

(40)

which is negative because \( b > \sigma^2 Q_1 / A_b \) assures \( r < 0 \). Using this expression for \( r_1 + f_1 \), we have that hedge funds choose weights:

\[ w_{1,h}^* = \left( r_1 + f_1 \right) \frac{p_1^2}{\gamma \sigma^2} = \left( (\lambda - 1) \frac{A_{fr}}{A_h + \lambda A_{fr}} \frac{\mu - p_1}{p_1} \right) \frac{p_1^2}{\gamma \sigma^2} \]  

(41)

\[ = -(\lambda - 1) \frac{A_{fr}}{A_h + \lambda A_{fr}} (p_1 - \mu) \frac{p_1}{\gamma \sigma^2} \]  

(42)

\[ \Rightarrow q_{1,h} = (\lambda - 1) \frac{A_{fr}}{A_h + \lambda A_{fr}} (p_1 - \mu) \frac{A_h}{\gamma \sigma^2}. \]  

(43)

Fee retainers choose weights:

\[ w_{1,fr}^* = \left( r_1 + \lambda f_1 \right) \frac{p_1^2}{\gamma \sigma^2} = \left( r_1 - \lambda r_1 \frac{A_h + A_{fr}}{A_h + \lambda A_{fr}} \right) \frac{p_1^2}{\gamma \sigma^2} \]  

(44)

\[ = r_1 \frac{1 - \lambda}{A_h - \lambda A_{fr}} \frac{p_1^2}{\gamma \sigma^2} = -(\lambda - 1) \frac{1}{A_h - \lambda A_{fr}} (p_1 - \mu) \frac{p_1 A_h}{\gamma \sigma^2} \]  

(45)

\[ \Rightarrow q_{fr,l} = (\lambda - 1) \frac{A_{fr}}{A_h - \lambda A_{fr}} (p_1 - \mu) \frac{A_h}{\gamma \sigma^2}. \]  

(46)

Finally, non retainers choose \( q_{1,n} = 0 \) because asset 1 has negative expected return and they cannot short, as discussed above.

Results 1 through 3 all follow directly from Theorem 3.

**Result 4.** For values of \( b \) such that \( f_1 > 0 \), if management fees are equal \( (m_{fr} = m_n) \) or fee retainers offer a discount equal to anticipated lending revenue \( (m_{fr} = m_n - \kappa w_{fr}^* f^*) \), fee
retainers have smaller after-fee expected returns than non retainers:

\[ \tau_{a.f.}^{fr} < \tau_{a.f.}^{n} \]  

(47)

Proof. If the two funds charge equal management fees, we have:

\[ \tau_{a.f.}^{fr} = \tau_{fr} - m_{fr} - w_{fr}^{s} f < \tau_{fr} - m_{n} = \tau_{a.f.}^{n}. \]  

(48)

If \( m_{fr} = m_{n} - w_{fr}^{s} f^{*} \), we have:

\[ \tau_{a.f.}^{fr} = \tau_{fr} - m_{fr} - w_{fr}^{s} f^{*} = \tau_{fr} - m_{n} < \tau_{a.f.}^{n}. \]  

(49)

Reducing management fees by the amount of retained lending fees results in an equal total fee payment to the fund managers of fee-retaining and non-fee-retaining funds, however the portfolio choice distortion still results in a worse expected return for lending fund investors.
References


Weitzner, Gregory, 2016, The term structure of short selling costs, Working paper, University of Texas at Austin.
Figure 1: Sample Disclosure from Regulation S-X Amendment

This figure presents the sample disclosure form provided by the SEC to fund managers as part of the Regulation S-X Amendment that requires funds disclose gross income from securities lending activities, the costs associated with those activities, and the net income received by the fund.

<table>
<thead>
<tr>
<th>SECURITIES LENDING ACTIVITIES</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gross income from securities lending activities</strong></td>
</tr>
<tr>
<td><strong>Fees and/or compensation for securities lending activities and related services</strong></td>
</tr>
<tr>
<td>Fees paid to securities lending agent from a revenue split</td>
</tr>
<tr>
<td>Fees paid for any cash collateral management service (including fees deducted from a pooled cash collateral reinvestment vehicle) that are not included in the revenue split</td>
</tr>
<tr>
<td>Administrative fees not included in revenue split</td>
</tr>
<tr>
<td>Indemnification fee not included in revenue split</td>
</tr>
<tr>
<td>Rebate (paid to borrower)</td>
</tr>
<tr>
<td>Other fees not included in revenue split (specify)</td>
</tr>
<tr>
<td><strong>Aggregate fees/compensation for securities lending activities</strong></td>
</tr>
<tr>
<td><strong>Net income from securities lending activities</strong></td>
</tr>
</tbody>
</table>
Figure 2: Market for Lending Agents

The figure shows the number of funds using each lending agent, with third party funds tallied in black and affiliated funds tallied in grey. Our sample includes 542 active US equity mutual funds with 2017 Regulation S-X data. We exclude funds that list multiple lending agents except for cases where one agent is affiliated, in which case we consider the fund as using the affiliated agent.
Figure 3: Frequency of Self-Dealing by Fund Family

This figure shows the frequency of self-dealing across fund families that have affiliated lending agents, with funds using third party agents tallied in black and funds using affiliated agents tallied in grey. Our sample includes 542 active US equity mutual funds with 2017 Regulation S-X data.
This figure shows equilibrium prices and quantities in our model as a function of individual investor’s bias $b$. In the first column of plots, there is no fee retention, meaning $\kappa = 0$ in our model. In the second column of plots, fee retainers keep 20% of lending revenue, meaning $\kappa = 0.20$. The first row of plots shows the expected return, fee-inclusive expected return, and lending fee rate for asset 1. The second row of plots shows equilibrium quantities demanded for each of the four groups of investors. The parameterization we use sets $\mu = 1$, $\sigma = 0.2$, $\gamma = 2$, $\psi = 5$, $Q_1 = 1$, $Q_2 = 10$, and $A_h = A_{fr} = A_n = A_b = 3$. The vertical lines indicate the cutoffs between low, moderate and high $b$ equilibrium regions.
Figure 5: Portfolio Expected Returns and Sharpe Ratios as a Function of Bias

This figure shows equilibrium fee-inclusive portfolio expected returns and Sharpe Ratios for each group of investors, as a function of individual investor’s bias \( b \). In the first column of plots, lending mutual funds do not retain lending revenue, meaning \( \kappa = 0 \) in our model. In the second column of plots, they retain 20\% of lending revenue, meaning \( \kappa = 0.20 \). The first row of plots shows the fee-inclusive average return of each group of investor’s equilibrium portfolio. The second row of plots shows their portfolio’s Sharpe Ratios. For all groups we ignoring payoffs to fund managers or biased expectations. The parameterization we use sets \( \mu = 1, \sigma = 0.2, \gamma = 2, \psi = 5 \), \( Q_1 = 1, Q_2 = 10 \), and \( A_h = A_{fr} = A_m = A_b = 3 \). The vertical lines indicate the cutoffs between low, moderate and high \( b \) equilibrium regions.


Table 1: Aggregate Uses of Lending Income and Self-Dealing

This table contains aggregate values of securities lending revenue, expenses, and net income for mutual funds and ETFs based on Regulation S-X disclosures for 2017. Panel A includes our full sample of 1,035 mutual funds and ETFs, while Panel B focuses on 319 mutual funds and ETFs using affiliated lending agents.

### Panel A: All mutual funds and ETFs (N = 1,035)

<table>
<thead>
<tr>
<th></th>
<th>$</th>
<th>% gross</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Income from Securities Lending</td>
<td>1,000,531,136</td>
<td></td>
</tr>
<tr>
<td>Fees for Securities Lending Agent</td>
<td>88,920,208</td>
<td>8.9</td>
</tr>
<tr>
<td>Fees for Cash Collateral Management</td>
<td>27,454,546</td>
<td>2.7</td>
</tr>
<tr>
<td>Administrative Fees</td>
<td>1,190,396</td>
<td>0.1</td>
</tr>
<tr>
<td>Indemnification Fees</td>
<td>1,112</td>
<td>0.0</td>
</tr>
<tr>
<td>Other Fees</td>
<td>24,462</td>
<td>0.0</td>
</tr>
<tr>
<td>Rebate</td>
<td>121,887,664</td>
<td>12.2</td>
</tr>
<tr>
<td>Aggregate Fees/Compensation for Securities Lending</td>
<td>239,478,400</td>
<td>23.9</td>
</tr>
<tr>
<td>Net Income to Fund</td>
<td>761,052,736</td>
<td>76.1</td>
</tr>
</tbody>
</table>

### Panel B: Self-dealing mutual funds and ETFs (N = 319)

<table>
<thead>
<tr>
<th></th>
<th>$</th>
<th>% gross</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Income from Securities Lending</td>
<td>360,959,936</td>
<td></td>
</tr>
<tr>
<td>Fees for Securities Lending Agent</td>
<td>30,270,252</td>
<td>8.4</td>
</tr>
<tr>
<td>Fees for Cash Collateral Management</td>
<td>3,702,830</td>
<td>1.0</td>
</tr>
<tr>
<td>Administrative Fees</td>
<td>1,190,396</td>
<td>0.3</td>
</tr>
<tr>
<td>Indemnification Fees</td>
<td>1,112</td>
<td>0.0</td>
</tr>
<tr>
<td>Other Fees</td>
<td>0</td>
<td>0.0</td>
</tr>
<tr>
<td>Rebate</td>
<td>47,294,132</td>
<td>13.1</td>
</tr>
<tr>
<td>Aggregate Fees/Compensation for Securities Lending</td>
<td>82,458,720</td>
<td>22.8</td>
</tr>
<tr>
<td>Net Income to Fund</td>
<td>278,501,216</td>
<td>77.2</td>
</tr>
</tbody>
</table>
Table 2: Fund-Level Uses of Lending Fees and Self-Dealing

This table presents fund-level descriptive statistics for the uses of lending fees. Securities lending income data is measured annually, while other mutual fund characteristics correspond to the fourth quarter of 2017. See Appendix A for detailed descriptions of the variables. Our sample for Panel A consists of 542 active US equity mutual funds with 2017 Regulation S-X data. Panel B focuses on the 124 active US equity mutual funds using affiliated lending agents.

### Panel A: Active US equity mutual funds \((N = 542)\)

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>SD</th>
<th>P5</th>
<th>P25</th>
<th>P50</th>
<th>P75</th>
<th>P95</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fee Yield (bps TNA)</td>
<td>12.7</td>
<td>24.1</td>
<td>0.3</td>
<td>2.5</td>
<td>5.0</td>
<td>12.1</td>
<td>51.6</td>
</tr>
<tr>
<td>Gross Income Yield (bps TNA)</td>
<td>6.9</td>
<td>26.9</td>
<td>0.0</td>
<td>0.6</td>
<td>2.3</td>
<td>5.8</td>
<td>22.0</td>
</tr>
<tr>
<td>Cost of Lending (bps of TNA)</td>
<td>2.2</td>
<td>9.4</td>
<td>-1.2</td>
<td>0.1</td>
<td>0.5</td>
<td>1.6</td>
<td>11.2</td>
</tr>
<tr>
<td>Net Income Yield (bps TNA)</td>
<td>4.6</td>
<td>20.4</td>
<td>0.0</td>
<td>0.4</td>
<td>1.5</td>
<td>4.3</td>
<td>13.9</td>
</tr>
<tr>
<td>Cost of Lending (% gross)</td>
<td>27.6</td>
<td>41.3</td>
<td>-42.7</td>
<td>15.0</td>
<td>26.0</td>
<td>50.5</td>
<td>80.8</td>
</tr>
<tr>
<td>Lending Agent Fees (% gross)</td>
<td>9.9</td>
<td>9.4</td>
<td>0.0</td>
<td>5.2</td>
<td>8.8</td>
<td>13.1</td>
<td>20.1</td>
</tr>
<tr>
<td>Rebate (% gross)</td>
<td>12.9</td>
<td>48.5</td>
<td>-68.5</td>
<td>0.9</td>
<td>11.5</td>
<td>38.1</td>
<td>71.8</td>
</tr>
<tr>
<td>Cash Collateral Fees (% gross)</td>
<td>3.6</td>
<td>8.0</td>
<td>0.0</td>
<td>0.0</td>
<td>1.1</td>
<td>4.2</td>
<td>13.9</td>
</tr>
<tr>
<td>Other Fees (% gross)</td>
<td>0.1</td>
<td>0.4</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>1.2</td>
</tr>
<tr>
<td>Expense Ratio (bps TNA)</td>
<td>91.3</td>
<td>36.6</td>
<td>27.0</td>
<td>67.1</td>
<td>97.0</td>
<td>115.0</td>
<td>148.2</td>
</tr>
<tr>
<td>TNA ($mm)</td>
<td>2,684</td>
<td>6,552</td>
<td>15</td>
<td>153</td>
<td>626</td>
<td>2,208</td>
<td>10,896</td>
</tr>
<tr>
<td>Family TNA ($bn)</td>
<td>523</td>
<td>1,081</td>
<td>3</td>
<td>24</td>
<td>100</td>
<td>431</td>
<td>2,283</td>
</tr>
<tr>
<td>Turnover (%)</td>
<td>60.3</td>
<td>48.7</td>
<td>8.0</td>
<td>24.0</td>
<td>48.0</td>
<td>84.0</td>
<td>152.0</td>
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### Panel B: Self-dealing active US equity mutual funds \((N = 124)\)

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<th>Mean</th>
<th>SD</th>
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<th>P50</th>
<th>P75</th>
<th>P95</th>
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</thead>
<tbody>
<tr>
<td>Fee Yield (bps TNA)</td>
<td>16.6</td>
<td>30.9</td>
<td>0.4</td>
<td>3.0</td>
<td>6.4</td>
<td>18.3</td>
<td>64.5</td>
</tr>
<tr>
<td>Gross Income Yield (bps TNA)</td>
<td>7.0</td>
<td>11.9</td>
<td>0.1</td>
<td>0.7</td>
<td>2.8</td>
<td>5.8</td>
<td>38.5</td>
</tr>
<tr>
<td>Cost of Lending (bps of TNA)</td>
<td>3.4</td>
<td>8.4</td>
<td>0.0</td>
<td>0.2</td>
<td>0.6</td>
<td>1.3</td>
<td>29.2</td>
</tr>
<tr>
<td>Net Income Yield (bps TNA)</td>
<td>3.6</td>
<td>5.4</td>
<td>0.1</td>
<td>0.5</td>
<td>2.0</td>
<td>4.5</td>
<td>13.1</td>
</tr>
<tr>
<td>Cost of Lending (% gross)</td>
<td>33.6</td>
<td>22.2</td>
<td>8.6</td>
<td>18.0</td>
<td>24.8</td>
<td>45.5</td>
<td>83.0</td>
</tr>
<tr>
<td>Lending Agent Fees (% gross)</td>
<td>7.5</td>
<td>5.5</td>
<td>0.0</td>
<td>2.9</td>
<td>8.1</td>
<td>9.6</td>
<td>19.4</td>
</tr>
<tr>
<td>Rebate (% gross)</td>
<td>22.0</td>
<td>23.4</td>
<td>0.2</td>
<td>3.6</td>
<td>13.2</td>
<td>34.9</td>
<td>75.7</td>
</tr>
<tr>
<td>Cash Collateral Fees (% gross)</td>
<td>3.3</td>
<td>5.1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.6</td>
<td>5.2</td>
<td>12.7</td>
</tr>
<tr>
<td>Other Fees (% gross)</td>
<td>0.6</td>
<td>0.7</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>1.1</td>
<td>1.8</td>
</tr>
<tr>
<td>Expense Ratio (bps TNA)</td>
<td>79.4</td>
<td>39.5</td>
<td>3.0</td>
<td>45.0</td>
<td>85.9</td>
<td>109.2</td>
<td>138.1</td>
</tr>
<tr>
<td>TNA ($mm)</td>
<td>3,848</td>
<td>9,437</td>
<td>5</td>
<td>177</td>
<td>847</td>
<td>2,351</td>
<td>20,186</td>
</tr>
<tr>
<td>Family TNA ($bn)</td>
<td>1,784</td>
<td>1,665</td>
<td>13</td>
<td>273</td>
<td>1,383</td>
<td>2,283</td>
<td>4,998</td>
</tr>
<tr>
<td>Turnover (%)</td>
<td>66.4</td>
<td>48.9</td>
<td>9.0</td>
<td>31.5</td>
<td>51.5</td>
<td>94.0</td>
<td>161.0</td>
</tr>
</tbody>
</table>
Table 3: Self Dealing and Lending Agent Fees

This table contains results from regressions of fund-level lending agent fees, as a percent of gross lending income, on Self Deal, an indicator for whether a fund uses an affiliated lending agent, and control variables. See Appendix A for detailed descriptions of the variables. Our sample for Panel A consists of 542 active US equity mutual funds with 2017 Regulation S-X data. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

<table>
<thead>
<tr>
<th>Lending Agent Fees (% gross)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self Deal</td>
<td>-0.009</td>
<td>-0.018*</td>
<td>0.019*</td>
<td>0.088***</td>
</tr>
<tr>
<td></td>
<td>(-0.85)</td>
<td>(-1.69)</td>
<td>(1.80)</td>
<td>(3.67)</td>
</tr>
<tr>
<td>Log(TNA)</td>
<td>-0.002</td>
<td>0.006***</td>
<td>0.002</td>
<td>0.005***</td>
</tr>
<tr>
<td></td>
<td>(-0.67)</td>
<td>(4.25)</td>
<td>(1.16)</td>
<td>(3.77)</td>
</tr>
<tr>
<td>Log(Family TNA)</td>
<td>-0.008**</td>
<td>-0.020***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-2.14)</td>
<td>(-4.00)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expense Ratio</td>
<td>0.019</td>
<td>0.029**</td>
<td>0.020*</td>
<td>0.024**</td>
</tr>
<tr>
<td></td>
<td>(1.49)</td>
<td>(2.41)</td>
<td>(1.66)</td>
<td>(2.06)</td>
</tr>
<tr>
<td>Turnover</td>
<td>0.000</td>
<td>0.002</td>
<td>-0.008</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.39)</td>
<td>(-1.21)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>Style FE</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Fund Family FE</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>Lending Agent FE</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>R²</td>
<td>0.137</td>
<td>0.774</td>
<td>0.490</td>
<td>0.810</td>
</tr>
</tbody>
</table>
Table 4: Groups of Investors in our Model

This table summarizes the four groups of investors in our model, indicating whether they are allowed to lend or short-sell shares, whether they are biased, and their subjective expected returns for each of the two assets.

<table>
<thead>
<tr>
<th></th>
<th>Lend?</th>
<th>Short?</th>
<th>Biased?</th>
<th>Expected Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hedge funds (h)</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>$\frac{\mu_1 + f_1 - 1}{p_1} \quad \frac{\mu_2 + f_2 - 1}{p_2}$</td>
</tr>
<tr>
<td>Fee retainers (fr)</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>$\frac{\mu_1 + (1 + \kappa(\psi - 1)) f_1 - 1}{p_1} \quad \frac{\mu_2 + (1 + \kappa(\psi - 1)) f_2 - 1}{p_2}$</td>
</tr>
<tr>
<td>Non retainers (n)</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>$\frac{\mu_1 + f_1 - 1}{p_1} \quad \frac{\mu_2 + f_2 - 1}{p_2}$</td>
</tr>
<tr>
<td>Biased individual investors (b)</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>$\frac{\mu_1 + b - 1}{p_1} \quad \frac{\mu_2 + b - 1}{p_2}$</td>
</tr>
</tbody>
</table>
Table 5: Retention Policy and Portfolio Choice

This table contains results from regressions of Fee Yield, the weighted average lending fee of the fund’s holdings in percentage points, on Self Deal, an indicator for whether the fund uses an affiliated lending agent, and High Fee Retainer, an indicator for whether the fund uses an affiliated lending agent and pays that agent more than the median lending agent fee. The sample includes 1,566 fund-quarter observations from 2017Q2 to 2017Q4. T-statistics calculated using standard errors clustered by fund are in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

<table>
<thead>
<tr>
<th></th>
<th>Fee Yield (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>Self Deal</td>
<td>0.032</td>
</tr>
<tr>
<td>(1.39)</td>
<td></td>
</tr>
<tr>
<td>High Fee Retainer</td>
<td>0.080*</td>
</tr>
<tr>
<td>(1.91)</td>
<td>(1.88)</td>
</tr>
<tr>
<td>Log(TNA)</td>
<td>-0.014*</td>
</tr>
<tr>
<td>(-1.81)</td>
<td>(-1.83)</td>
</tr>
<tr>
<td>Log(Family TNA)</td>
<td>-0.003</td>
</tr>
<tr>
<td>(-0.40)</td>
<td>(-0.52)</td>
</tr>
<tr>
<td>Expense Ratio</td>
<td>-0.020</td>
</tr>
<tr>
<td>(-0.45)</td>
<td>(-0.58)</td>
</tr>
<tr>
<td>Turnover</td>
<td>-0.001</td>
</tr>
<tr>
<td>(-0.01)</td>
<td>(-0.07)</td>
</tr>
<tr>
<td>Style x Time FE</td>
<td>Y</td>
</tr>
<tr>
<td>Fund Family FE</td>
<td>N</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.350</td>
</tr>
</tbody>
</table>
Table 6: Retention Policy and Fund Performance

This table contains results from regressions of \( \text{Alpha}_t \), a fund’s Carhart four-factor alpha in month \( t \) (in percent) on Self Deal, an indicator for whether the fund uses an affiliated lending agent, and High Fee Retainer, an indicator for whether the fund uses an affiliated lending agent and pays that agent more than the median lending agent fee. Columns (1) – (4) include all available months from 2000–2017 for the 542 active US equity mutual funds with 2017 Regulation S-X data. Column (5) expands the set of funds to include all funds that lent their shares over the period according to their NSAR files. Column (6) expands the set of funds further to include all funds in CRSP, including non-lenders. T-statistics calculated using robust standard errors clustered by fund are in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(-0.044***)</td>
<td>-0.028</td>
<td>-0.020</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self Deal</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-2.79)</td>
<td>(-1.50)</td>
<td>(-0.54)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High Fee Retainer</td>
<td>-0.059***</td>
<td>-0.041*</td>
<td>-0.063**</td>
<td>-0.043**</td>
<td>-0.026*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-2.94)</td>
<td>(-1.76)</td>
<td>(-1.97)</td>
<td>(-2.42)</td>
<td>(-1.70)</td>
<td></td>
</tr>
<tr>
<td>Log(TNA)</td>
<td>-0.014***</td>
<td>-0.013***</td>
<td>-0.014***</td>
<td>-0.023***</td>
<td>-0.006*</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>(-2.90)</td>
<td>(-2.84)</td>
<td>(-2.95)</td>
<td>(-4.29)</td>
<td>(-1.81)</td>
<td>(0.78)</td>
</tr>
<tr>
<td>Log(Family TNA)</td>
<td>0.010**</td>
<td>0.008**</td>
<td>0.011**</td>
<td>-0.033***</td>
<td>0.007***</td>
<td>0.003***</td>
</tr>
<tr>
<td></td>
<td>(2.54)</td>
<td>(2.16)</td>
<td>(2.58)</td>
<td>(-2.64)</td>
<td>(2.64)</td>
<td>(2.03)</td>
</tr>
<tr>
<td>Expense Ratio</td>
<td>-0.029</td>
<td>-0.023</td>
<td>-0.024</td>
<td>-0.002</td>
<td>-0.033**</td>
<td>-0.078***</td>
</tr>
<tr>
<td></td>
<td>(-1.57)</td>
<td>(-1.20)</td>
<td>(-1.27)</td>
<td>(-0.07)</td>
<td>(-2.34)</td>
<td>(-6.78)</td>
</tr>
<tr>
<td>Turnover</td>
<td>-0.047***</td>
<td>-0.048***</td>
<td>-0.046***</td>
<td>-0.037***</td>
<td>-0.047***</td>
<td>-0.019*</td>
</tr>
<tr>
<td></td>
<td>(-3.71)</td>
<td>(-3.87)</td>
<td>(-3.60)</td>
<td>(-2.92)</td>
<td>(-6.46)</td>
<td>(-1.85)</td>
</tr>
<tr>
<td>Flow</td>
<td>0.170*</td>
<td>0.174*</td>
<td>0.171*</td>
<td>0.082</td>
<td>0.057</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>(1.82)</td>
<td>(1.86)</td>
<td>(1.84)</td>
<td>(0.89)</td>
<td>(0.89)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>Alpha(_{t-1})</td>
<td>0.034***</td>
<td>0.034***</td>
<td>0.034***</td>
<td>0.030***</td>
<td>0.026***</td>
<td>0.038***</td>
</tr>
<tr>
<td></td>
<td>(5.91)</td>
<td>(5.91)</td>
<td>(5.89)</td>
<td>(5.23)</td>
<td>(4.23)</td>
<td>(6.60)</td>
</tr>
</tbody>
</table>

Sample                  | Reg S-X | Reg S-X | Reg S-X | Reg S-X | Lenders | All   |
<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Style x Time FE</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Fund Family FE</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>N (fund-months)</td>
<td>61801</td>
<td>61801</td>
<td>61801</td>
<td>61801</td>
<td>113633</td>
<td>564762</td>
</tr>
<tr>
<td>R(^2)</td>
<td>0.604</td>
<td>0.604</td>
<td>0.604</td>
<td>0.606</td>
<td>0.657</td>
<td>0.327</td>
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</table>
Table 7: Retention Policy and Expense Ratios

This table contains results from regressions of each fund’s Expense Ratio, the fund’s total management fee scaled by total net assets, on Self Deal, an indicator for whether the fund uses an affiliated lending agent, and High Fee Retainer, an indicator for whether the fund uses an affiliated lending agent and pays that agent more than the median lending agent fee. Our sample includes 542 active US equity mutual funds with 2017 Regulation S-X data. T-statistics are in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
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<tbody>
<tr>
<td><strong>Self Deal</strong></td>
<td>0.034</td>
<td>-0.114*</td>
<td>0.073*</td>
<td>-0.062</td>
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<tr>
<td></td>
<td>(0.84)</td>
<td>(-1.84)</td>
<td>(1.71)</td>
<td>(-1.03)</td>
</tr>
<tr>
<td><strong>Lending Agent Fees (% TNA)</strong></td>
<td>0.193</td>
<td>0.518***</td>
<td>0.270</td>
<td>0.687***</td>
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<tr>
<td></td>
<td>(0.67)</td>
<td>(3.36)</td>
<td>(1.04)</td>
<td>(5.33)</td>
</tr>
<tr>
<td><strong>Self Deal × Lending Agent Fees (% TNA)</strong></td>
<td>-11.649*</td>
<td></td>
<td>-16.073***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-1.75)</td>
<td></td>
<td>(-2.64)</td>
</tr>
<tr>
<td><strong>Log(TNA)</strong></td>
<td>-0.031***</td>
<td></td>
<td>-0.035***</td>
<td>-0.024***</td>
</tr>
<tr>
<td></td>
<td>(-3.43)</td>
<td></td>
<td>(-4.03)</td>
<td>(-3.12)</td>
</tr>
<tr>
<td><strong>Turnover</strong></td>
<td>0.100***</td>
<td>0.099**</td>
<td>0.088***</td>
<td>0.072*</td>
</tr>
<tr>
<td></td>
<td>(2.98)</td>
<td>(2.40)</td>
<td>(2.84)</td>
<td>(1.91)</td>
</tr>
<tr>
<td><strong>Log(Family TNA)</strong></td>
<td>-0.058***</td>
<td></td>
<td>-0.056***</td>
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<tr>
<td></td>
<td>(-6.80)</td>
<td></td>
<td>(-6.51)</td>
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<tr>
<td><strong>Style FE</strong></td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td><strong>Fund Family FE</strong></td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td><strong>R²</strong></td>
<td>0.368</td>
<td>0.737</td>
<td>0.377</td>
<td>0.750</td>
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</table>