The Two-Pillar Policy for the RMB

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Abstract

In this paper we document stylized empirical facts about recent exchange rate (RMB) policies in China. The formation mechanism of the central parity is based on the “two-pillar approach” that balances RMB index “stability” and exchange rate “flexibility.” We then develop a tractable no-arbitrage model of the RMB under the “two-pillar approach.” Using the model we estimate the fundamental exchange rate, the probability of continuation of the two-pillar approach in the future, and the weights put on both pillars. Our model is able to predict both the end of the two-pillar policy in May 2017 when an additional unspecified “countercyclical factor” was introduced for the first time, and the return to the two-pillar policy in January 2018 when the countercyclical factor was suspended. The estimated model not only fits the spot rate and option prices well in the sample, but also forecast future central parity and spot rates out of the sample.

Keywords: Chinese currency, Renminbi, RMB, Central Parity, Two-Pillar Approach.
1 Introduction

The Chinese currency, the Renminbi (RMB), plays an increasingly important role in international trade and the global financial markets. Since August 2015, the People’s Bank of China (PBoC) has implemented new currency reforms to improve its mechanism of setting the official central parity of the RMB against the US dollar.

In this paper we document the recent policy, specifically, the procedure for setting daily central parity rates according to a “two-pillar approach.” According to the PBOC’s Monetary Policy Report in the first quarter of 2016, the formation mechanism of the central parity depends on two key factors or pillars: the first pillar refers to “the closing rates of the previous business day to reflect changes in market demand and supply conditions”, while the second pillar refers to changes in the currency basket, “as a means to maintain the overall stability of the RMB to the currency basket.” Empirically, we find that these two pillars together explain as high as 75% to 80% of the variations in the central parity. Furthermore, we also present empirical evidence that both pillars receive roughly equal weights in the formation mechanism of the central parity, especially since the first quarter of 2016.

Based on the stylized empirical facts, we develop a tractable no-arbitrage model of the RMB under the two-pillar approach. Using the model we estimate the level and the volatility of the fundamental exchange rate, as well as the probability of continuing the two-pillar approach in the future. We find that the RMB is valued on average about 3% higher than its fundamental value, which is consistent with recent expectations of further depreciation in the RMB in the sample period. Interestingly, when the closing rate approached closer and closer to 7 in the end of 2017, our model suggests that the exchange rate could depreciate to 7.1 in the case of one-time revaluation.

Our estimation results indicate that markets attached an average probability of 79% to the policy still being in place three months later. Following highly likely rate hikes by the Federal Reserve (e.g., December 2015 and 2016), the probabilities dramatically decreased. Although the credibility of the policy increased in the beginning of 2016, it gradually decreased until August 2016. Since then, the probability had remained at a relatively low level within a narrow range between 70% and 80% until the PBOC unexpectedly changed the central parity rule again in May 2017 by introducing a new “counter-cyclical factor.” The introduction of the new factor essentially turned the policy into a “three-pillar policy” with the third pillar being the newly introduced counter-cyclical factor. Despite the fact that the policy change was largely a surprise to the market, our model is able to predict the change in the sense that our model-implied probability of policy continuation dropped to the lowest level around 15% in the week preceding the PBOC’s confirmation of the change on May 26,
Since the introduction of the counter-cyclical factor in May 2017, the average probability of policy continuation had stayed low around 36% in the last quarter of 2017. On January 9, 2018, Bloomberg reports that the PBOC has decided to effectively remove the counter-cyclical factor, resulting in a return to the two-pillar policy.\(^2\) Such policy change is also predicted by our model to some extent as the model-implied probability of policy continuation has increased to about 50% on average in January 2018.

Lastly, the estimated model not only fits the spot rate and option prices well in the sample, but also forecast well future central parity and spot rates out of the sample. In particular, it can outperform the random-walk model in forecasting both the central parity and closing rates. As a result, our model can serve as a very important tool to understand and forecast the current exchange rate regime.

This paper is related to the literature on exchange rate target-zones (e.g., Krugman (1991), Bertola and Caballero (1992), Bertola and Svensson (1993), etc.). Our no-arbitrage modeling approach distinguishes our paper from traditional target-zone models that are mostly based on uncovered interest parity. Furthermore, our paper is also related to the literature on the Chinese currency and monetary policy such as Frankel and Wei (2007) and Chang, Liu, and Spiegel (2015). To our best knowledge, this paper is the first one that explicitly models the two-pillar approach and incorporates it into a no-arbitrage model of the RMB. The estimates of the fundamental exchange rate and the probability of continuing the two-pillar approach are novel in the literature.

In the rest of the paper, we review the recent reforms on China’s exchange rate policy and present empirical facts in Section 2. Section 3 presents our model. Section 4 contains the estimation results and section 5 model implications.

## 2 Stylized Facts

We start by describing official policies for the RMB and document the two-pillar approach followed since 2015.

### 2.1 Managed Floating RMB Regime

During the last three decades, China’s transition into a market-based economy and the integration into the global market has been remarkable. Yet China only gradually introduced

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\(^1\)See the statement on the CFETS website: http://www.chinamoney.com.cn/fe/Info/38244066.
\(^2\)See the article “China Changes the Way It Manages Yuan After Currency’s Jump” on Bloomberg News on January 9, 2018.
market forces into the design of its foreign exchange policy. Over the years, China conducted several reforms and experiments on managing its exchange rate.3

Starting 1994, the renminbi was pegged to the dollar until July 21, 2005. Since then, the RMB regime has moved to a “managed floating” regime. In this regime, the RMB exchange rate has been allowed to float within a narrow band around the central parity published by the People’s Bank of China (PBoC). Figure 1 Panel A displays the RMB central parity and closing rates since 2004.

To add flexibility to the exchange rate and strengthen the role of demand and supply forces, China has gradually widened the trading band around the daily US dollar fixing from an initial 0.3% to 0.5% on May 21, 2007, to 1% on April 16, 2012, and 2% on March 17, 2014. Figure 1 Panel B plots the deviation between the central parity and the close since the reform on July 21, 2005. It shows that as the trading band widened, the deviations have become more volatile, reflecting more flexibility of the RMB.

Note that the effective width of the trading band can be much smaller than the announced width. For example, during the financial crisis between mid 2008 and mid 2010, the RMB was essentially re-pegged to the US dollar. As another example, since August 11, 2015, the band around the central parity has effectively been limited to 0.5%, with an exception of three dates only (i.e., Feb. 4-5 and Jun. 24, 2016).

[Insert Figure 1 Here.]

2.2 Two-Pillar Approach to Setting the Central Parity

The reforms introduced on August 11, 2015 were aimed at making the procedure for setting the daily central parity more transparent and more market driven. According to the PBOC’s Monetary Policy Report in 2016Q1, “[o]n August 11, 2015, it was stressed that quotes of the central parity of the RMB to the USD should refer to the closing rates of the previous business day to reflect changes in market demand and supply conditions.”

On December 11, 2015, China Foreign Exchange Trade System (CFETS), a subsidiary of the PBOC, introduced the CFETS index as another measure of the RMB’s performance against a basket of 13 currencies with weighting based mainly on international trade. The basket was unveiled as a way of shifting focus away from the RMB’s moves against the dollar following China’s unexpected devaluation in August that year. In the CFETS index, the dollar has the largest weight of 26.4 percent, followed by the euro and the yen with 21.4 percent and 14.7 percent, respectively. On December 29, 2016, the PBOC decided to expand

the CFETS basket by adding 11 new currencies and at the same time reduce the dollar’s weight to 22.4 percent, effective in 2017. Besides the CFETS index, the PBoC also started publishing two other trade-weighted RMB indices based on the IMF’s Special Drawing Rights (SDR) and a Bank for International Settlement (BIS) baskets since December 2015. We refer to these two RMB indices simply as the SDR and BIS indices throughout the paper. All three indices have the same base level of 100 in the end of 2014 and are published regularly.

The PBOC’s Monetary Policy Report in 2016Q1 provides more details in clarifying the formation mechanism of the central parity. It states that

“a formation mechanism for the RMB to the USD central parity rate [consisting] of ‘the previous closing rate plus changes in the currency basket’ has been preliminarily in place. The ‘previous closing rate plus changes in the currency basket’ formation mechanism means that market makers must consider both factors when quoting the central parity of the RMB to the USD, namely the ‘previous closing rate’ and the ‘changes in the currency basket.’”

As such, the formation mechanism of the central parity can be characterized by the following “two-pillar” approach whereby the central parity is a weighted average of the basket target and last day’s close:

\[
S_{t+1}^{CP} = (\bar{S}_{t+1})^w (S_{t}^{CL})^{(1-w)},
\]

where \(\bar{S}_{t+1}\) denotes the hypothetical rate that achieves "stability" of the basket, and \(S_{t}^{CL}\) the spot exchange rate (RMB per dollar) at the close of day \(t\). These two components are the two pillars of the central parity. As explained in the same report, the former is referred to as “the amount of the adjustment in the exchange rate of the RMB to the USD, as a means to maintain the overall stability of the RMB to the currency basket”, while the latter reflects the “market demand and supply situation.”

To explicitly represent the pillar associated with the currency basket \(\bar{S}_{t+1}\), we turn to the construction of the RMB indices in the next subsection.

2.2.1 Temporary new “counter-cyclical factor” in the second half of 2017

The two-pillar policy came to an abrupt end on May 25, 2017 when Bloomberg first reported and China later confirmed that it would introduce a new “counter-cyclical factor”, a move which “would give authorities more control over the fixing and restrain the influence of market
pricing.” The introduction of the new factor essentially turned the policy into a “three-pillar policy” with the third pillar being the newly introduced counter-cyclial factor. The policy change is perceived by many market participants as a tool to address RMB depreciation without draining foreign reserves. However, it undermines earlier efforts to make RMB more market driven.

The counter-cyclical factor was then subsequently removed as reported by Bloomberg on January 9, 2018. It signals the return to the previous two-pillar policy. Such policy shift resulted from the RMB’s strength over the past year as well as the dollar’s protracted decline.

### 2.3 RMB Indices

Consider an RMB index (e.g., CFETS) as a geometric average of a basket of currencies:

\[
B_t = C_B \left( S_{t, CP; USD/CNY}^{CP; USD/CNY} \right)^{w_{USD}} \left( S_{t, CP; EUR/CNY}^{CP; EUR/CNY} \right)^{w_{EUR}} \left( S_{t, CP; JPY/CNY}^{CP; JPY/CNY} \right)^{w_{JPY}} \cdots
\]  

(2)

where \( S_{t, CP; USD/CNY}^{CP; USD/CNY}, S_{t, CP; EUR/CNY}^{CP; EUR/CNY}, \) etc., are central parity rates in terms of the CNY for the currencies in the basket, and \( C_B \) is a scaling constant.

The RMB index can be written in terms of the implied USD/CNY rate and a USD index of all the non-RMB currencies. We show below that this USD index is closely related to the ICE (formerly NYBOT) U.S. Dollar Index and this will allow us to use option price data on that index.

Specifically, we can factor out \( S_{t, CP; USD/CNY}^{CP; USD/CNY} \) such that

\[
B_t = C_B \cdot S_{t, CP; USD/CNY}^{CP; USD/CNY} \cdot \hat{X}_{t, CP; USD/CNY}^{CP; USD/CNY}
\]

where

\[
\hat{X}_{t} = \left( \frac{S_{t, CP; EUR/CNY}^{CP; EUR/CNY}}{S_{t, CP; USD/CNY}^{CP; USD/CNY}} \right)^{w_{EUR}} \left( \frac{S_{t, CP; JPY/CNY}^{CP; JPY/CNY}}{S_{t, CP; USD/CNY}^{CP; USD/CNY}} \right)^{w_{JPY}} \cdots
\]

\[
\equiv \left( S_{t, CP; EUR/USD}^{CP; EUR/USD} \right)^{w_{EUR}} \left( S_{t, CP; JPY/USD}^{CP; JPY/USD} \right)^{w_{JPY}} \cdots
\]

and \( S_{t, CP; EUR/USD} \equiv \frac{S_{t, CP; EUR/CNY}}{S_{t, CP; USD/CNY}} \), etc. is defined as the cross-currency exchange rate between a non-RMB currency and the USD implied by the corresponding central parity rates.

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4See the article "China Considers Changing Yuan Fixing Formula to Curb Swings" on Bloomberg News on May 25, 2017.
To make \( \tilde{X}_t \) comparable to the well-known USD index, we scale \( \tilde{X}_t \) by a factor \( C_X \) such that the scaled value \( X_t \equiv C_X \tilde{X}_t \) coincides with the USD index on 12/31/2014. As a result, the RMB index can be re-expressed as

\[
B_t = \frac{C_B}{C_X^{1-w_{USD}}} \frac{X_t^{1-w_{USD}}}{S_t^{CP}}.
\]

The pillar associated with the currency basket \( \tilde{S}_{t+1} \) is determined so as to achieve "stability" of the basket.\(^5\) In particular,

\[
B_t = \frac{C_B}{C_X^{1-w_{USD}}} \frac{X_t^{1-w_{USD}}}{S_t^{CP}} = \frac{C_B}{C_X^{1-w_{USD}}} \frac{X_t^{1-w_{USD}}}{S_t^{CP}} = \frac{C_B}{C_X^{1-w_{USD}}} \frac{X_t^{1-w_{USD}}}{S_t^{CP}}.
\]

or

\[
\tilde{S}_{t+1} = S^{CP}_t \left( \frac{X_{t+1}}{X_t} \right)^{1-w_{USD}}.
\] (3)

### 2.4 Empirical Evidence

We document here that the RMB central parity has closely tracked our equation (1) summarizing the official policy statements. We find strong empirical support for a central parity rule that gives equal weights to each of the two pillars.

There are three official indices: CFETS, BIS, and SDR. All of them have the benchmark level of 100 on December 31, 2014, and are published roughly on a weekly basis. We use daily central parity rates to reconstruct the indices on a daily basis as stated in Appendix A. For each of the three indices, the coefficient \( C_B \) is determined such that the reconstructed index has the level of 100 on December 31, 2014. In particular, it has the values of 379.37, 133.14, and 681.46 for the indices of CFETS, BIS, and SDR, respectively.

Based on the estimated coefficients \( C_B \), we reconstruct and plot the reconstructed indices in the figure below (blue lines), as well as the official indices (red circles). The figure shows almost perfect fit of our reconstructed indices with the official ones.

[Insert Figure 2 Here.]

Next, we construct the index-implied basket \( X_t \) that is directly comparable to the ICE U.S. Dollar Index \( DXY_t \) (or more precisely, \( DXY_{t-1} \) due to the time difference). Recall

\(^{5}\)For expositional purpose, the weight of the US dollar \( w_{USD} \) in the RMB index is assumed to be fixed here. However, the dollar’s weights in both CFETS and SDR indices have been adjusted in the end of 2016. In Appendix A, we provide detailed explanation about how to account for the change in \( w_{USD} \) in our analysis.
that \( X_t \equiv C_X \tilde{X}_t \) and \( C_X \) is set such that the resulting \( X_t \) coincides with the dollar index on 12/31/2014. That is, \( C_X = \frac{DXY_{12/31/2014}}{X_{1/1/2015}} \). Based on the estimated coefficients \( C_X \),
we construct and plot the index-implied USD basket \( X_t \) in the figure below, as well as the USDX index (blue solid line). The figure shows in the period between 12/31/2014 and 11/16/2016, the USD basket \( X_{t}^{SDR} \) is most correlated with the USDX index with a correlation of 0.94 (versus 0.72 and 0.69 for the other two USD baskets implied by the CFETS and BIS index, respectively). For the extended period (1/1/2014-11/16/2016), the SDR-implied basket \( X_{t}^{SDR} \) is almost perfectly correlated with the USDX with a correlation of 0.997.

[Insert Figure 3 Here.]

Based on the constructed \( \{X_t\} \) process, we can empirically examine the value of \( w \). The report has an example that seems to suggest that the two-pillar approach places equal weights on “the previous close” \( \bar{S}_t^{CL} \) and “the changes in the currency basket” \( \bar{S}_{t+1} \); that is, \( w = 1/2 \).

We find supportive empirical evidence for \( w = 1/2 \) for the period following December 11, 2015 when the RMB indices were announced for the first time. First, we run the following regression for the period between December 11, 2015 and December 31, 2016:

\[
\log \left( \frac{S_{t+1}^{CP}}{S_t^{CP}} \right) = \alpha \cdot (1 - w_{USD}) \log \left( \frac{X_{t+1}}{X_t} \right) + \beta \cdot \log \left( \frac{S_{t+1}^{CL}}{S_t^{CP}} \right) + \epsilon_{t+1}.
\]

Under the Two-Pillar approach in (1), the coefficients \( \alpha \) and \( \beta \) correspond to \( w \) and \( 1 - w \), respectively. The regression results, reported in Column I in Table 1, supports that \( w = 1/2 \). The R-squared ranges from 0.75 to 0.81. On the other hand, in the sample period between August 11, 2015 and December 10, 2015 before the RMB indices were announced, the regression results, reported in Column II, shows that \( w \) is close to zero as measured by the coefficient \( \alpha \). At the same time, the R-squared is very high, around 0.95. This finding suggests that since the reform announced on August 11, 2015 but before the introduction of the RMB indices on December 11, 2015, the formation mechanism of the central parity indeed has started incorporating information from the close, and to be more precise, it is almost completely determined by the close because the dollar baskets implied in the RMB indices have very little weights.

For each of the three indices, the coefficient \( C_X \) is determined below:

\[
C_X^{CFETS} = \frac{90.269}{1.9147} = 47.1452; C_X^{BIS} = \frac{90.269}{6.3944} = 14.1169; C_X^{SDR} = \frac{90.269}{0.8308} = 108.6492
\]
Instead of running the above regression for a fixed period, we use 60-day rolling windows to run regressions, starting from 60 business days after 8/11/2015, or 11/13/2015 and ending at December 30, 2016. The results, plotted in Figure 4 below, show that the index stability gains more weight in the run-up to December 11 2015 when the RMB indices were announced, and then stabilized around 0.5 since then until the introduction of the counter-cyclicial factor in the end of May 2017. During the period of the effectively three-pillar policy, the weight decreased initially and then reverted back to 0.5 when the counter-cyclicial factor was removed. This finding is very intuitive as the newly introduced factor takes away some weights from the first two pillars.

[Insert Table 1 Here.]
[Insert Figure 4 Here.]

2.5 Does this Policy Restrict Exchange Rate Movements?

It is not a priori obvious that the two-pillar central parity rule represented in equation (1) combined with a daily trading band of 2% or 0.5% does restrict exchange rate dynamics. We provide here some evidence that suggests that even for a wide band of 2%, this policy appears to have some effect.

Consider $V_t$ as the CNY/USD exchange rate without the policy with $v_t \equiv \log (V_t)$, $x_t \equiv \log (X_t)$ and

\[ v_{t+1} = v_t + \epsilon_{v,t+1} \]
\[ x_{t+1} = x_t + \epsilon_{x,t+1} \]

where $\epsilon_{v,t+1} \sim N (-\frac{1}{2} \sigma_v^2 \Delta t, \sigma_v^2 \Delta t)$ and $\epsilon_{x,t+1} \sim N (-\frac{1}{2} \sigma_X^2 \Delta t, \sigma_X^2 \Delta t)$ are i.i.d. normal shocks. Note that $E_t \left[ \frac{X_{t+1}}{X_t} \right] = E_t \left[ \frac{V_{t+1}}{V_t} \right] = 1$.

Suppose under the two-pillar policy, the $b\%$ band never binds, then exchange rate at the close would equal $V_t$. Under the two-pillar policy,

\[ s^{CP}_{t+1} = w s^{CP}_t + w (1 - w_{USD}) \epsilon_{x,t+1} + (1 - w) v_t \]

and the resulting deviation from the central parity is $v_{t+1} - s^{CP}_{t+1}$. If the policy does not restrict exchange rate movements, this deviation should always be bounded within the target range

\[-b \leq v_t - s^{CP}_t \leq b \text{ for any } t.\]

To check how often the constraint would bind we simulate the processes under the cali-
brated parameters: $b = 0.02$, $w = 0.5$, $w_{USD} = 0.264$, $\sigma_X = 0.09$, $\sigma_V = 0.10$, $\Delta t = \frac{1}{365}$, and $T = 91$. The simulation results are reported in Table 2 below.

[Insert Table 2 Here.]

Under all assumptions there are at least some observations outside the target band. Our benchmark is very conservative. As we have seen, while the PBOC announces $b = 2\%$, the observed deviation is mostly below 0.5\%, so the case with $b = 0.5\%$ where the band is binding 45\% of the time is very relevant. In the next section we present a parsimonious model for the equilibrium exchange rate when the target band around the central parity is occasionally binding.

3 A Model for the RMB

3.1 Setup

Building on Jermann (2017), we assume that there is a probability $p$ that the two-pillar policy regime continues and $(1 - p)$ that it ends tomorrow. If it ends, the exchange rate equals the fundamental rate $V_t$. The interpretation of $V_t$ can be quite broad; for example, we can interpret it as the exchange rate that prevails when the RMB becomes freely floating.

The equilibrium exchange rate $\tilde{S}_t$ is given by

$$\tilde{S}_t = \frac{1 + r^S}{1 + r^C} \left[ p E_t^Q H \left( \tilde{S}_{t+1}, S_{t+1}^{CP}, b \right) + (1 - p) E_t^Q V_{t+1} \right]$$

(4)

$$= \frac{1 + r^S}{1 + r^C} p E_t^Q H \left( \tilde{S}_{t+1}, S_{t+1}^{CP}, b \right) + (1 - p) V_t,$$

where $b$ denotes the width of the trading band around the central parity,

$$H \left( \tilde{S}_{t+1}, S_{t+1}^{CP}, b \right) = \max \left( \min \left( \tilde{S}_{t+1}, S_{t+1}^{CP} (1 + b) \right), S_{t}^{CP} (1 - b) \right),$$

(5)

and $E_t^Q [\cdot]$ refers to expectations under the RMB risk-neutral measure, and $r^S$ and $r^C$ are per-period interest rates in the US and China, respectively. Intuitively, the current spot rate is the expected value of the exchange rate in the two regimes, appropriately adjusted for the yields. This follows the same approach as in Jermann (2017), in particular, the second line is derived by assuming that $V_t$ is arbitrage-free itself; that is, $\frac{1 + r^S}{1 + r^C} E_t^Q [V_{t+1}] = V_t$. Note that in the case where the bandwidth $b$ is so large that the band is never binding, $H \left( \tilde{S}_{t+1}, S_{t+1}^{CP}, b \right)$
is always equal to $\tilde{S}_{t+1}$ and the solution to Equation (4) is $\tilde{S}_t = V_t$. In practice, the band with $b = 2\%$ is still occasionally binding as we argue in the end of the last section.

If the equilibrium exchange rate $\tilde{S}_t$ falls within the band around the central parity, it is equal to the observed spot exchange rate at the close. Otherwise, the spot exchange rate is equal to $\tilde{S}_t$ truncated at the (lower or upper) boundary of the band. Therefore, the model-implied spot exchange rate at the close then equals

$$S_{t}^{CL} = H\left(\tilde{S}_t, S_t^{CP}; b\right).$$

Equation (4) implies that the equilibrium exchange rate $\tilde{S}_t = \tilde{S}(S_t^{CP}, V_t)$ is a function of $S_t^{CP}$ and $V_t$. Because the function $H(x, y; b)$ defined in Equation (5) is homogeneous of degree one in the first two arguments, the equilibrium exchange rate $\tilde{S}_t = \tilde{S}(S_t^{CP}, V_t)$ is separable such that

$$\tilde{S}(V_t, S_t^{CP}) = S_t^{CP} \tilde{S} \left(\frac{V_t}{S_t^{CP}}\right) \equiv S_t^{CP} \tilde{S} \left(\frac{\tilde{V}_t}{S_t^{CP}}\right),$$

where

$$\tilde{V}_t \equiv \frac{V_t}{S_t^{CP}}.$$ 

In other words, we can scale the equilibrium exchange rate $\tilde{S}$ and the fundamental rate $V_t$ by the central parity $S_t^{CP}$ and obtain $\tilde{S} \equiv \tilde{S}/S_t^{CP}$ and $\tilde{V} \equiv V_t/S_t^{CP}$. As a result, determination of the equilibrium exchange rate $\tilde{S}_t$ boils down to determination of the univariate function $\tilde{S}(\tilde{V}_t)$. Based on this, we can simplify Equation (4) to

$$\tilde{S}(\tilde{V}_t) = \frac{\tilde{S}(V_t, S_t^{CP})}{S_t^{CP}} = \frac{1 + r^S}{1 + r^C p E^Q_t} \left[H \left(\frac{\tilde{S}(V_{t+1}, S_{t+1}^{CP})}{S_t^{CP}}, \frac{S_t^{CP}}{S_t^{CP}}; b\right)\right] + (1 - p) \tilde{V}_t = \frac{1 + r^S}{1 + r^C p E^Q_t} \left[S_t^{CP} H \left(\tilde{S}(\tilde{V}_{t+1}), 1; b\right)\right] + (1 - p) \tilde{V}_t.$$ 

For ease of notation from now on we simply write $H(\tilde{S}(\tilde{V}_{t+1}), 1; b)$ as $H(\tilde{S}(\tilde{V}_{t+1}))$.

Next, we turn to the formation mechanism of the central parity $S_t^{CP}$, which plays an important role in determining the scaled equilibrium exchange rate $\tilde{S}(\tilde{V}_t)$ as shown in Equation (9). Interpreted in the narrow sense, the mechanism described in the PBOC’s Monetary
Policy Report in the first quarter of 2016 implies the following “two-pillar rule”:

\[
S_{t+1}^{CP} = \left[ S_t^{CP} \left( \frac{X_{t+1}}{X_t} \right)^{(1-w_{USD})} \right]^w \left[ H \left( \tilde{S}_t, S_t^{CP}; b \right) \right]^{1-w} \]

\[
= S_t^{CP} \left( \frac{X_{t+1}}{X_t} \right)^{w(1-w_{USD})} H \left( \tilde{S} \left( \tilde{V}_t \right) \right)^{1-w} .
\]

Recall that \( X_t \) denotes the dollar basket implied in the RMB index. In the above rule, the term in the first square brackets, \( S_t^{CP} \left( \frac{X_{t+1}}{X_t} \right)^{(1-w_{USD})} \), is the first pillar to maintain the stability of the RMB index. On the other hand, the term in the second square brackets, \( H \left( \tilde{S}_t, S_t^{CP}; b \right) \), is the previous close or the second pillar to account for the “market demand and supply situation”.

Under the two-pillar rule in Equation (10), the scaled equilibrium exchange rate \( \tilde{S} \left( \tilde{V}_t \right) \) is determined as follows:

\[
\tilde{S} \left( \tilde{V}_t \right) = \frac{1 + r^S}{1 + r^C} p H \left( \tilde{S} \left( \tilde{V}_t \right) \right)^{1-w} E^Q \left[ \left( \frac{X_{t+1}}{X_t} \right)^{w(1-w_{USD})} H \left( \tilde{S} \left( \tilde{V}_{t+1} \right) \right) \right] + (1-p) \tilde{V}_t .
\]

Furthermore, the evolution of the state variable \( \tilde{V}_t \) is given by:

\[
\frac{\tilde{V}_{t+1}}{\tilde{V}_t} = \frac{V_{t+1}}{V_t} \left( \frac{S_{t+1}^{CP}}{S_t^{CP}} \right)^{-1} = \frac{V_{t+1}}{V_t} \left( \frac{X_{t+1}}{X_t} \right)^{-w(1-w_{USD})} H \left( \tilde{S} \left( \tilde{V}_t \right) \right)^{-(1-w)}. \]

Typically, Equation (11) is solved using value function iterations to obtain \( \tilde{S} \left( \tilde{V}_t \right) \). That is, the right-hand side of Equation (11) can be considered as an operator \( T \left( \tilde{S} \left( \tilde{V}_t \right) \right) \equiv \frac{1 + r^S}{1 + r^C} p H \left( \tilde{S} \left( \tilde{V}_t \right) \right)^{1-w} E^Q \left[ \left( \frac{X_{t+1}}{X_t} \right)^{w(1-w_{USD})} H \left( \tilde{S} \left( \tilde{V}_{t+1} \right) \right) \right] + (1-p) \tilde{V}_t \). If there is a unique fixed point for this operator, we can find such unique fixed point solution by applying the operator repeatedly (see Stokey, Lucas, and Prescott (1989)).

However, despite the simple appearance of Equations (11)-(12), our model is much more challenging to solve for the following reasons. First, the term \( H \left( \tilde{S} \left( \tilde{V}_t \right) \right)^{1-w} \) at the right-hand side of Equation (11) involves \( \tilde{S} \left( \tilde{V}_t \right) \) that is to be solved by iterations. The presence of this term usually causes some numerical instability issue. Second, the reciprocal of the same term shows up in Equation (12), implying that the dynamics of \( \tilde{V}_t \) is further influenced by the function \( \tilde{S} \left( \tilde{V}_t \right) \) we aim to solve. Therefore, it is quite numerically challenging to solve the model under the two-pillar rule in Equation (10).

As a result, we turn to the slightly modified two-pillar approach below, which is consistent with the formation mechanism of the central parity in the PBoC’s report in the broad sense.
Specifically, we keep the first pillar unchanged, but model the second pillar as $S_{t+1}^{CP} \left( \frac{V_{t+1}}{V_t} \right)^\gamma$ where $0 \leq \gamma \leq 1$. That is, the new two-pillar rule is the following:

$$S_{t+1}^{CP} = S_t^{CP} \left( \frac{X_{t+1}}{X_t} \right)^{(1-w_{USD})} \left[ S_t^{CP} \left( \frac{V_{t+1}}{V_t} \right)^\gamma \right]^{1-w}$$

$$\equiv S_t^{CP} \left( \frac{X_{t+1}}{X_t} \right)^\alpha \left( \frac{V_{t+1}}{V_t} \right)^\beta,$$

where $\alpha \equiv (1-w_{USD})w$ and $\beta \equiv \gamma (1-w)$ are some constants bounded between 0 and 1. Furthermore, under the new two-pillar rule, both $\hat{S}(\hat{V}_t)$ and $\hat{V}_t$ are determined as follows:

$$\hat{S}(\hat{V}_t) = \frac{1 + r^SP_{EQ}}{1 + r^{CP}} \left[ \left( \frac{X_{t+1}}{X_t} \right)^\alpha \left( \frac{V_{t+1}}{V_t} \right)^\beta H(\hat{S}(\hat{V}_{t+1})) \right] + (1-p)\hat{V}_t,$$

$$\frac{\hat{V}_{t+1}}{V_t} = \left( \frac{X_{t+1}}{X_t} \right)^{-\alpha} \left( \frac{V_{t+1}}{V_t} \right)^{1-\beta}.$$

The modified two-pillar approach is broadly consistent with the PBoC’s report for the following reasons. First, the second pillar in the modified rule takes into account the “market demand and supply situation” captured by the term $(V_{t+1}/V_t)^\gamma$. Second, in the case with a non-binding band (i.e., $b$ is sufficiently large), the equilibrium exchange rate $\hat{S}$ and the close $S_t^{CL}$ are always equal to $V_t$. If the PBoC had exclusively focused on the second pillar and ignored the first one, it would set $S_{t+1}^{CP} = S_t^{CL} = V_t$ or $S_{t+1}^{CP} = S_t^{CP} V_t/V_{t-1}$, which is almost identical to the modified second pillar $S_t^{CP} \left( \frac{V_{t+1}}{V_t} \right)^\gamma$ if we set $\gamma = 1$ and ignore the one-period lag. Because in our estimation one period is taken as one day, the lag of one period makes essentially no difference. In addition, it is symmetric to use $V_{t+1}/V_t$ in modeling the second pillar as we use $X_{t+1}/X_t$ in modeling the first pillar. The symmetry makes the model very tractable, especially in continuous time. Third, with a possibly binding band (i.e., $b$ is not too large), the second pillar in the original rule in Equation (10) is bounded between $(1-b)$ and $(1+b)$. That is, not all market-driven forces are considered in forming the central parity in Equation (10). To capture this feature, we set $\gamma < 1$ in the new second pillar $S_t^{CP} \left( \frac{V_{t+1}}{V_t} \right)^\gamma$, because imposing the power $\gamma < 1$ in the term $\left( \frac{V_{t+1}}{V_t} \right)^\gamma$ forces it to tilt toward one, similar to imposing the limit $b$ on deviations from the central parity. Later on, we calibrate this parameter $\gamma$ such that the term $\left( \frac{V_{t+1}}{V_t} \right)^\gamma$ is roughly bounded within the limits $(1-b)$ and $(1+b)$ for reasonable fluctuations in the fundamental rate $V_t$. In summary, the modified two-pillar approach in Equation (13) seems to be a very realistic way to model the formation mechanism of the central parity; in particular, the term $(V_{t+1}/V_t)^\gamma$ reflects the market-driven forces and its power $\gamma < 1$ captures the fact that using “the previous close” as the second
pillar may lose some of market-driven forces. In the extreme cases with extremely large or small \(b\), \(\gamma\) should be close to 1 or 0, respectively.

Furthermore, the modified two-pillar approach not only implies a unique solution, but also is much more tractable. In fact, if we cast the model in continuous time, we are able to derive the equilibrium exchange rate \(\hat{S}(\hat{V}_t)\) in closed form. In the special case with \(b = 0\), we are also able to derive option prices in closed form.

### 3.2 Model Solution in Continuous Time

We describe the model setup in discrete time in the last subsection where each period lasts for \(\Delta t\) in years. The model is particularly tractable in continuous time once we let \(\Delta t\) tend to zero. Under the RMB risk-neutral measure \(Q\),

\[
\frac{dV_t}{V_t} = (r_{CNY} - r_{USD}) \, dt + \sigma_V dW_{V,t} \\
\equiv \mu_V \, dt + \sigma_V dW_{V,t},
\]

\[
\frac{dX_t}{X_t} = (r_{DXY} - r_{USD} - \rho \sigma_X \sigma_V + \sigma^2_X) \, dt + \sigma_X \left( \rho dW_{V,t} + \sqrt{1 - \rho^2} dW_{X,t} \right) \\
\equiv \mu_X \, dt + \sigma_X \left( \rho dW_{V,t} + \sqrt{1 - \rho^2} dW_{X,t} \right),
\]

where \(W_{V,t}\) and \(W_{X,t}\) are independent Brownian motions under the measure \(Q\), and \(r_{CNY}\), \(r_{USD}\), \(r_{DXY}\) are instantaneous interest rates for the RMB, the dollar, and the currency basket in the dollar index \(DXY\), respectively. That is, the per-period interest rates in the preceding discrete-time setup satisfy: \(r^S = \exp (r_{USD} \Delta t)\) and \(r^C = \exp (r_{CNY} \Delta t)\). Similarly, the per-period probability \(p = 1 - \lambda \Delta t\) while we assume that the current managed floating regime will be abandoned upon arrival of a Poisson process with intensity \(\lambda\). The processes \(\{X_t\}\) and \(\{V_t\}\) are assumed to have a correlation \(\rho\). Their drifts are specified in the above equations so as to exclude any arbitrage opportunities.

Below we use lowercase variables to denote the logarithm of the corresponding uppercase variables. For example, \(v_t \equiv \log V_t\), \(s^CP_t \equiv \log S^{CP}_t\), \(\hat{v}_t \equiv \log \hat{V}_t\), etc. Under the two-pillar policy in Equation (13), by Ito’s lemma the dynamics of \(S^{CP}_t\) is given by:

\[
\frac{dS^{CP}_t}{S^{CP}_t} \equiv \mu_{CP} dt + (\alpha \rho \sigma_X + \beta \sigma_V) dW_{V,t} + \alpha \sqrt{1 - \rho^2} \sigma_X dW_{X,t},
\]

where \(\mu_{CP} \equiv \alpha (\mu_X - 1/2 \sigma^2_X) + \beta (\mu_V - 1/2 \sigma^2_V) + \frac{1}{2} (\alpha \rho \sigma_X + \beta \sigma_V)^2 + \frac{1}{2} \alpha^2 (1 - \rho^2) \sigma^2_X\) denotes the expected growth rate of the central parity; that is, \(E^Q_t [S^{CP}_{t+\tau}] = S^{CP}_t \exp (\mu_{CP} \tau)\).
Similarly, we derive the dynamics of $\hat{V}_t$ as follows:

$$\frac{d\hat{V}_t}{\hat{V}_t} = \mu_{\hat{V}} dt + ((1 - \beta) \sigma_V - \alpha \sigma_X \rho) dW_{V,t}^{CNY} - \alpha \sigma_X \sqrt{1 - \rho^2} dW_{X,t}^{CNY},$$

where $\mu_{\hat{V}} \equiv -\alpha \mu_x + (1 - \beta) \mu_v + \frac{1}{2} \sigma^2_{\hat{V}}$ denotes the expected growth rate of the scaled fundamental rate and $\sigma_{\hat{V}} \equiv \sqrt{((1 - \beta) \sigma_V - \alpha \sigma_X \rho)^2 + \alpha \sigma_X \sqrt{1 - \rho^2}}$.

We are now ready to solve the (scaled) equilibrium exchange rate $\hat{S}(\hat{V}_t)$. It is straightforward to prove that $\hat{S}(\hat{V}_t)$ is monotonically increasing. Define $\hat{V}_s$ and $\hat{V}^*$ such that $\hat{S}(\hat{V}_s) = 1 - b$ and $\hat{S}(\hat{V}^*) = 1 + b$. As the length of the period $\Delta t$ converges to zero, with probability one $\hat{V}_{t+\Delta t} > \hat{V}^*$ (or $\hat{V}_{t+\Delta t} < \hat{V}_s$) if $\hat{V}_t > \hat{V}^*$ (or $\hat{V}_t < \hat{V}_s$). Therefore, from Equation (14), it must be true that: $\hat{S}(\hat{V}_t) = 1 - b$ if $\hat{V} < \hat{V}_s$, and $1 + b$ if $\hat{V} > \hat{V}^*$.

If $\hat{V} \in (\hat{V}_s, \hat{V}^*)$, it is straightforward to show that $\hat{S}(\hat{V}_t)$ must satisfy the following equation based on Equation (14):

$$\frac{\hat{S}'}{(\hat{V}_t)} \hat{V}_t \mu_{\hat{V}} + \frac{1}{2} \hat{S}''(\hat{V}_t) \hat{V}_t^2 \sigma^2_{\hat{V}} + (\mu_{CP} - \mu_V - \lambda) \hat{S}(\hat{V}_t) + \lambda \hat{V}_t = 0. \quad (19)$$

The solution to this ordinary differential equation is: $\hat{S}(\hat{V}_t) = C_0 \hat{V} + C_1 \hat{V}^{\eta_1} + C_2 \hat{V}^{\eta_2}$, where $\eta_1$ and $\eta_2$ are the two roots of the quadratic equation:

$$\frac{1}{2} \sigma^2_{\hat{V}} \eta^2 + \left( \mu_{\hat{V}} - \frac{1}{2} \sigma^2_{\hat{V}} \right) \eta + (\mu_{CP} - \mu_V - \lambda) = 0, \quad (20)$$

and the coefficient $C_0$ is given by

$$C_0 = \frac{\lambda}{\lambda + \mu_V - \mu_{\hat{V}} - \mu_{CP}}, \quad (21)$$

and the coefficients $C_1$, $C_2$ and the thresholds $\hat{V}_s$, $\hat{V}^*$ are determined from the value-matching and smooth-pasting conditions:

$$C_0 \hat{V}_s + C_1 (\hat{V}_s)^{\eta_1} + C_2 (\hat{V}_s)^{\eta_2} = 1 - b, \quad (22)$$

$$C_0 \hat{V}^* + C_1 (\hat{V}^*)^{\eta_1} + C_2 (\hat{V}^*)^{\eta_2} = 1 + b, \quad (23)$$

$$C_0 + \eta_1 C_1 (\hat{V}_s)^{\eta_1 - 1} + \eta_2 C_2 (\hat{V}_s)^{\eta_2 - 1} = 0, \quad (24)$$

$$C_0 + \eta_1 C_1 (\hat{V}^*)^{\eta_1 - 1} + \eta_2 C_2 (\hat{V}^*)^{\eta_2 - 1} = 0. \quad (25)$$

We summarize the above result in the proposition below.

**Proposition 1** In the continuous-time model, the scaled equilibrium exchange rate $\hat{S}(\hat{V}_t)$ is
determined as follows:

\[
\hat{S}(\hat{V}_t) = \begin{cases} 
1 - b, & \text{if } \hat{V} \leq \hat{V}_s; \\
C_0\hat{V} + C_1\hat{V}^{\eta_1} + C_2\hat{V}^{\eta_2}, & \text{if } \hat{V}_s < \hat{V} < \hat{V}^*; \\
1 + b, & \text{if } \hat{V} \geq \hat{V}^*,
\end{cases}
\]

where \(\eta_1\) and \(\eta_2\) are the roots of Equation (20), the coefficients \(C_0\) through \(C_2\) and the thresholds \(\hat{V}_s\) and \(\hat{V}^*\) can be solved from the system of equations (21)-(25).

Based on our estimation results, we find that \(\hat{S}(\hat{V}_t)\) has the S shape in the middle range when \(\hat{V}\) falls within the interval between \(\hat{V}_s\) and \(\hat{V}^*\). Figure 5 plots the equilibrium exchange rate \(\hat{S}(\hat{V}_t)\). Mathematically, \(\eta_1 > 0 > \eta_2\) and \(C_1 < 0 < C_2\). Intuitively, as the fundamental rate deviates from the central parity in the positive direction, \(\hat{V}_t\) increases, so does the term \(\hat{V}^{\eta_1}\). When \(\hat{V}\) gets closer to \(\hat{V}^*\), the shape of \(\hat{S}\) gets flatter because of expected interventions in the near future.

[Insert Figure 5 Here.]

We estimate the model to match the close and four RMB options. Under this model, the price of a call option with maturity \(\tau\) and strike \(K\) is given by

\[
C(K; \tau) = e^{-r_{CNY}\tau} \left( p^{\tau} E^{Q} \left[ \max \left( \hat{S}_{t+\tau}\left( \hat{S}_{CNY}^{CP}, b \right), K \right) \right] + (1 - p^{\tau}) E^{Q} \left[ \max \left( V_{t+\tau}, K \right) \right] \right).
\]

(27)

The price of a put option can be represented in a similar way.

Even though we are able to determine \(\hat{S}(\hat{V}_t)\) in closed form, we rely on simulation-based method to compute options prices because they do not have closed form solutions in most cases. However, in a special case with \(b = 0\), we are able to derive analytical expressions for option prices. This case is also interesting because it provides some insights into how we can identify certain parameters, particularly \(p\). We turn to this special case next before we present our estimation results in the next section.

3.3 Special case: \(b = 0\)

In the special case with \(b = 0\), by construction, the model-implied spot rate always coincides with the central parity rate \(S_{t}^{CP}\). We can also show that the scaled equilibrium exchange rate \(\hat{S}(\hat{V}_t)\) is always equal to one. That is, the actual and equilibrium exchange rate in this special case are identical and both equal to \(S_{t}^{CP}\).

In the proposition below we derive option prices in closed form.
Proposition 2  In the special case where \( b = 0 \), CNY call and put option prices with maturity \( \tau \) and strike \( K \) are given by:

\[
C(K) = p^\tau C^S(K) + (1 - p^\tau) C^V(K),
\]
\[
P(K) = p^\tau P^S(K) + (1 - p^\tau) P^V(K),
\]

where

\[
C^S(K) = e^{-r_{CNY} \tau} E_t^Q \max \left[ S^C_{t+\tau} - K, 0 \right] = e^{-r_{CNY} \tau} \left( S^C_{t} e^{\mu_{C} \tau} \Phi(d_{1,X}) - K \Phi(d_{2,X}) \right)
\]
\[
P^S(K) = e^{-r_{CNY} \tau} E_t^Q \max \left[ K - S^C_{t+\tau}, 0 \right] = e^{-r_{CNY} \tau} \left( K \Phi(-d_{2,X}) - S^C_{t} e^{\mu_{C} \tau} \Phi(-d_{1,X}) \right)
\]

and

\[
C^V(K) = e^{-r_{CNY} \tau} E_t^Q \max \left[ V_{t+\tau} - K, 0 \right] = e^{-r_{USD} \tau} V_t \Phi(d_{1,V}) - e^{-r_{CNY} \tau} K \Phi(d_{2,V})
\]
\[
P^V(K) = e^{-r_{CNY} \tau} E_t^Q \max \left[ K - V_{t+\tau}, 0 \right] = -V_t e^{-r_{USD} \tau} N(-d_{1,V}) + K e^{-r_{CNY} \tau} N(-d_{2,V})
\]

where the coefficients \( d_{1,X}, d_{2,X}, d_{1,V}, \) and \( d_{2,V} \) are given in the proof.

**Proof.** See Appendix. ■

Proposition 2 shows that in the special case with \( b = 0 \), the option prices can be expressed as the weighted average of the prices that would prevail if the current policy would last or be abandoned for ever. The weights are the corresponding probabilities, \( p^\tau \) and \( (1 - p^\tau) \), respectively. Therefore, fitting the model-implied option prices to the data pins down the probability \( p \). In fact, the estimated model in this special case can fit the option prices in the data pretty well. However, the model-implied close in this special case is always tied to the central parity due to the assumption of \( b = 0 \). As a result, it cannot be used to fit the close in the data. In the next section we report the estimation results for the model with a non-zero \( b \) that is used to fit the close as well as option prices.

4 Estimation Results

Based on the model, we estimate \((V, p, \sigma_V)\) for each day \( t \) in the sample period between December 11, 2015 and May 23, 2017 to fit the close and four option prices in the data. During this period, the limit on the trading band is officially ±2%. However, the effective width is much smaller, around 0.5%. As a result, we choose \( b = 0.5\% \) in estimating the model. The results for the case of \( b = 2\% \), unreported here, are similar and available upon request.

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\(^7\)The estimation results of this special case are not reported, but available upon request.
request. We then choose $\gamma = 1/4$ so that the second pillar, $S_{t+1}^{CP}(V_{t+1}/V_{t})^\gamma$, fluctuates around $S_{t}^{CP}$ within the limit $b = 0.5\%$ most of the time. To simplify the estimation, we fix $\rho = 0$.

No closed-form option pricing formula exist in the model with a nonzero $b$. Therefore, we simulate the model 20,000 times and for each simulation we simulate the paths of the fundamental rate $V_t$, the central parity $S_{t}^{CP}$, the resulting scaled rate $\tilde{V}_{t}$, and the implied equilibrium exchange rate $\tilde{S}_{t}$ based on Equation (26). In the end we can obtain 20,000 simulated option payoffs at maturity and by properly discounting and averaging we compute the model-implied option prices based on these simulations.

In the rest of this section, we first describe the data and then describe our estimation results.

4.1 Data

Our sample period is chosen from December 11, 2015 to January 31, 2018. The beginning date of the sample period is chosen as December 11, 2015 because the RMB indices were introduced on that date for the first time. The ending date is chosen as January 31, 2018 largely because it includes the period where the counter-cyclical factor was introduced and then removed in early January 2018. Note that starting from January 1, 2017 the PBoC has changed the weight of the USD in the CFETS index from 0.2640 to 0.2240. We adjust the value of the parameter $w_{USD}$ accordingly when estimating the model for the period after January 1, 2017.

Main sources of our data are the CFETS website and Bloomberg. The former source is used to retrieve the historical data of the central parity rates and the RMB indices. The latter source is used to obtain the rest of the data.

Besides the exchange rate data and the interest rate data, we also collect data from the derivatives markets, particularly the data of the forwards/futures and options written on the RMB and the dollar index.

The RMB option data consist of implied volatility quotes for at-the-money options (“ATM”), risk reversals (“RR”), and butterfly spreads (“BY”), with a maturity of 3 months. The RR and BY quotes are available for strike prices corresponding to 25% and 10% delta (labeled as “25∆” and “10∆” respectively). These quotes can then be used to infer implied volatilities of 25∆ or 10∆ options by the standard approach (e.g., see Jermann (2017) or Bisesti et al. (2015) for more details). For 25∆ calls and puts, for instance, implied volatilities
are computed as

\[ \sigma_{25C} = \sigma_{ATM} + \sigma_{25BY} + \frac{1}{2} \sigma_{25RR}, \]

\[ \sigma_{25P} = \sigma_{ATM} + \sigma_{25BY} - \frac{1}{2} \sigma_{25RR}. \]

Following the quoting convention of the RMB options markets, the Black-Scholes formula is then used to compute the prices and strike prices corresponding to 25Δ and 10Δ options. We use daily 3-month SHIBOR and LIBOR rates for the RMB and the dollar. In the end, we obtain four option price series: two for puts and two for calls. Table 3 below reports average strike prices, option prices, and implied volatility quotes for these four options during the sample period.

[Insert Table 3 Here.]

The futures and options of the dollar index (DXY) are needed to infer the drift and diffusion terms of the \( \{X_t\} \) process. First, based on Equation (17) and Girsanov’s theorem, we can price the dollar index futures with maturity \( \tau \) under the dollar risk-neutral measure as \( X_t \exp \left[ (r_{DXY} - r_{USD} + \sigma_X^2) \tau \right] \). Once we estimate \( \sigma_X \), the futures data are used to back out the drift term \( (r_{DXY} - r_{USD} - \rho \sigma_X \sigma_V + \sigma_X^2) \) of the process \( \{X_t\} \) based on Equation (17).

The volatility \( \sigma_X \) can be separately estimated from implied volatilities of the dollar index options. The dollar index options are written on the dollar index futures and traded on the ICE electronic platform. Furthermore they are quoted in dollar, rather than in terms of implied volatility as in the case of the RMB options. To measure the implied volatility, we obtain the data of the dollar index options with strike prices ranging from 50 to 115 from Bloomberg. We then construct the implied volatility measure by following a model-free method similarly as how the CBOE constructs the VIX index (see, also, Demeterfi, et al., (1999)).

4.2 Results

Figure 6 displays the main estimation results. As shown in the first panel, the fundamental rate \( V_t \) is estimated to be always greater than both the central parity and the spot rates, which is consistent with the expectations of further depreciation in the RMB. Implied from the estimate of \( V_t \), the RMB is valued on average about 3% higher than its fundamental value. The gap is particular elevated between February and May 2016, and has since then stabilized around 2%. Interesting, when the close approached closer and closer to 7 in the
end of 2017, our model suggests that the exchange rate could depreciate to 7.1 in the case of one-time revaluation.

The second panel in Figure 6 plots the probability that the current policy would still be in place three months later. It suggests that markets attached an average probability of 79% to the policy still being in place three months later. Following highly likely rate hikes by the Federal Reserve (e.g., December 2015 and 2016), the probabilities dramatically decreased. Although the credibility of the policy increased in the beginning of 2016, it gradually decreased until August 2016. Since then, the probability remained at a relatively low level within a narrow range between 70% and 80% until May 2017.

A particularly interesting finding is that the model-implied probability of policy continuation dropped precipitately to as low as 37% around May 15, 2017 and afterwards it continued to decrease and dropped to the lowest level around 15% on May 23, 2017. An article on the Wall Street Journal published on May 25, 2017 quoted a study by Brooks and Ma (2017) who argue that “the central bank has moved away from a rule-based method of fixing the yuan and is exerting more ‘discretion’ over the exchange rate.”

Then on May 26, 2017, the PBOC confirmed the change in the central parity’s formation mechanism and stated in a report that a “countercyclical factor” has been introduced to the formation mechanism, although no detailed information has been disclosed about how the countercyclical factor is constructed. The introduction of the countercyclical factor signals the end of the two-pillar policy. It is very interesting that our model is able to predict it.

Since the introduction of the counter-cyclical factor in May 2017, the average probability of policy continuation had stayed low around 36% in the last quarter of 2017. On January 9, 2018, Bloomberg reports that the PBOC has decided to effectively remove the countercyclical factor, resulting in a return to the two-pillar policy. Such policy change is also predicted by our model to some extent as the model-implied probability of policy continuation has increased to about 50% on average in January 2018.

The implied volatility of the fundamental exchange rate shown in the third panel displays a relatively stable pattern in that the implied volatility fluctuates around its average value 11.7% during the sample period. In the end of 2016 following the rate hike by the Federal Reserve, the implied volatility has picked up, hovering around 14%. It is worthwhile to point out that since the U.S. election, the implied volatility has steadily decreased from around 14% in mid-December of 2016 to only 4% in the end of May, 2017. It suggests “damping currency volatility against the dollar [is] now a bigger priority” as in the aforementioned

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8 See the Wall Street Journal article on May 25, 2017 titled “China hitches Yuan to the Dollar, Buying Rare Calm” by Linglin Wei and Saumya Vaishampayan.

9 See the article “China Changes the Way It Manages Yuan After Currency’s Jump” on Bloomberg News on January 9, 2018.
Next, let us examine the goodness of fit for the baseline model estimation. Figure 7 below plots the root-mean-square errors (RMSEs) of the model that tells us how the model on average can fit the close and the four option prices. The results suggest that the model in general does a good job at fitting the data. In fact, the average fitting error is only about 4%. On the other hand, the first five months following the announcement of the two-pillar approach on December 11, 2016 post a challenge to the model as the average fitting error is much higher (around 7%) and the fitting error remains elevated around 12% in most of February 2016. Since May 2016 and throughout the second half of 2016, however, the model does a much better job and the fitting error drops to around 2%. Since the beginning of 2017, the model’s fitting error has continued to climb up from 1% in early January to about 30% in May. The dramatic increase in the fitting error occurs largely in the month of May, consistent with the structural break in the central parity’s formation mechanism predicted by the model. Figure 8 plots model-implied and actual spot exchange rates as well as the four option prices.

5 Model Implication

We explore a few policy implications in this section. First, we examine whether the modified model does a good job explaining the dynamics of the central parity rate in the data. Next, we examine whether the model can be used to forecast the central parity and the spot rates. We find very positive results for both, validating the model as a very important tool to understand and forecast the current regime.

5.1 Can the modified two-pillar policy explain the dynamics of the central parity in the data?

For tractability, we have modified the two-pillar policy. Under the modified policy, instead of using the previous closing rate as the second pillar, we replace it by $S_{t}^{CP} (V_{t+1}/V_{t})$. One question remains whether there is empirical evidence supporting this modified policy. Or put differently, can the modified two-pillar policy explain the dynamics of the central parity in the data?
To answer this question, we run the following regression based on the modified two-pillar policy:

$$\log \frac{S_{t+k}^{CP}}{S_{t+k}^{EP}} = \alpha \cdot \left(1 - w_{USD}\right) \log \frac{X_{t+k}}{X_t} + \beta \cdot \log \frac{V_{t+k}}{V_t} + \epsilon_{t+k}.$$ 

Note that the second regressor $\log \frac{V_{t+k}}{V_t}$ is based on the estimation results of the model. Under the official two-pillar policy, the second regressor should be replaced by $\sum_{i=0}^{k-1} \log \frac{S_{t+i}^{CL}}{S_{t+i}^{CP}}$. In our earlier empirical results in Section 2, we have studied such regression for a one-day horizon (i.e., $k = 1$) under the official two-pillar policy. The earlier results suggest the weight $w$ is about 50% and the R-square is pretty high around 80%, indicating empirical support for the official two-pillar policy.

It is important to note that different from the previous regression, the second regressor in the current regression (i.e., $\log V_{t+k}/V_t$) is an estimated variable. Consequently, there is a measurement-error-in-variable problem in the current regression, which is absent for the previous regression. The measurement-error problem is most severe for the shortest 1-day horizon when $k = 1$ and the regressor is the growth rate of the fundamental value over one day: $\log (V_{t+1}/V_t)$. Time aggregation helps mitigate the measurement-error problem if the second regressor is measured as the growth rate over a longer $k$-day horizon with $k > 1$. Therefore, we run the current regression for various values of $k$.

Table 4 reports the regression results for the modified policy. The results find similarly strong supportive empirical evidence for the modified policy as well. First, the coefficient estimates for $k = 1$ are close to zero and insignificant. This finding is consistent with the fact that the measurement-error problem tends to bias regression coefficients toward zero. Second, as $k$ gets larger, the estimates of the regression coefficients get stabilized. In particular, for $k$ equal to 15 or larger, the coefficient $\alpha$ is stabilized around 0.5, which is evidence for $w = 0.5$ as we have found in Table 1 for the official policy.

Second, R-square of the regressions are stabilized around 60-74% for $k$ is 15 or larger. This result suggests that the modified policy has a similarly strong explanatory power in explaining the dynamics of the central parity rate in the data as the official policy.

Lastly, note that the regression coefficients $\alpha$ and $\beta$ correspond to $w$ and $(1 - w) \gamma$, respectively. One by-product from running these regressions is that we can estimate the free parameter $\gamma$ based on the relationship: $\gamma = \beta / (1 - \alpha)$. It is worthwhile to point out that in estimating the model, the central parity rate is not used as one of the target moments. Furthermore, the fundamental value $V_t$ is estimated for each day. So the current time-series regression provide a way to further incorporate information from the central parity data, which can be used to estimate the free model parameter $\gamma$. The regression results suggest that $\gamma$ is around 0.5.
5.2 Forecastability of the central parity and the spot exchange rate

For each day in the period between 12/11/2015 and 12/31/2016, we use the estimation results for that date and make \( \tau \)-day-ahead forecast of the central parity and the spot exchange rate, i.e., \( S_{t+\tau}^{CP} \) and \( S_{t+\tau}^{CL} \). The following two graphs plots the forecasting RMSEs compared to the random-walk benchmark for horizons ranging from 1 day to 91 business days.

The results suggest that under the model with \( \gamma = 1/2 \), both CP and CL can be forecastable, especially for the horizon of 50 or 60 business days or longer. This is consistent with our use of 3-month options data.

[Insert Figure 9 Here.]

References


Appendix A: Reconstructing RMB Indices

We use daily central parity rates to reconstruct the indices on a daily basis as follows. Let \( t_0 \) denote December 31, 2014, the date on which the RMB indices are set to the benchmark level of 100. Furthermore, the composition of both CFETS and SDR indices has been revised and the new composition became effective starting January 1, 2017. Below we let \( t_1 \) denote December 31, 2016 and let \( w_{USD}, w_{EUR}, \cdots (w'_{USD}, w'_{EUR}, \cdots) \) denote the weight of the dollar, the Euro, \( \cdots \), prior to and including (after) \( t_1 \).

1. For each non-USD currency \( i \) (i.e., \( i \neq USD \)) in a given RMB index, we construct the implied central parity rate of this currency against the USD:

\[
S_{t_i}^{CP, i/USD} = \frac{S_{t_i}^{CP, i/CNY}}{S_{t_i}^{CP, USD/CNY}}
\]

(28)

2. We then collect all non-USD currencies in the index into a basket for the USD without the RMB. Specifically, for \( t \leq t_1 \), the index-implied USD basket \( \tilde{X}_t \) is constructed as follows:

\[
B_t = C_B \left( S_{t}^{CP, USD/CNY} \right) \left[ \left( S_{t}^{CP, EUR/USD} \right)^{w_{EUR}} \left( S_{t}^{CP, JPY/USD} \right)^{w_{JPY}} \cdots \right]^{1-w_{USD}}
\]

\[
\equiv C_B \left( S_{t}^{CP, USD/CNY} \right) \tilde{X}_t^{1-w_{USD}}
\]

where \( \tilde{X}_t \equiv \left( S_{t}^{CP, EUR/USD} \right)^{w_{EUR}} \left( S_{t}^{CP, JPY/USD} \right)^{w_{JPY}} \cdots \) and the weights in the basket \( \tilde{X}_t \) are determined by the weights from the index. For \( t > t_1 \), new weights are used to calculate the basket, denoted by \( \tilde{X}'_t \).

3. We estimate the scaling constant \( C_B \) by matching the index level of 100 on December 31, 2014 (i.e., date \( t_0 \)). That is,

\[
C_B = \frac{100 \cdot S_{t_0}^{CP,CNY/USD}}{\tilde{X}_t^{1-w_{USD}}}
\]

(30)

\[10\]There are a couple of currencies in which the CNY central parity rates only become available in the recent years. For example, \( S_{t}^{CP, CNY/CHF} \) is available only since 11/10/2015 and \( S_{t}^{CP, THB/CNY} \) is not available. For these currencies, say \( CHF \), in the period when their CNY central parity rates are not available, we use the market cross rates in the previous day between these currencies and the USD to approximate \( S_{t}^{CP, CHF/USD} \).
For each of the three indices, the coefficient $C_B$ is determined as

\[
C_{B,CFETS} = \frac{100 \cdot 6.1190}{1.9147^{1-0.2640}} = 379.3657, \\
C_{B,BIS} = \frac{100 \cdot 6.1190}{6.3944^{1-0.1780}} = 133.1417, \\
C_{B,SDR} = \frac{100 \cdot 6.1190}{0.8308^{1-0.4190}} = 681.4643.
\]

4. We then adjust the USD basket $\hat{X}_t$ by a scaling factor $C_X$ to make it comparable to the US dollar index (DXY):

\[
X_t = C_X \cdot \hat{X}_t.
\]

To be precise, the scaling factor $C_X$ is chosen such that $X_t$ coincides with $DXY_t$ at date $t_0 = 12/31/2014$. That is,

\[
C_X = \frac{DXY_{t_0}}{\hat{X}_{t_0}}.
\]

For each of the three indices, the coefficient $C_X$ is determined as

\[
C_{X,CFETS} = \frac{90.269}{1.91502} = 47.1374, \\
C_{X,BIS} = \frac{90.269}{6.38983} = 14.1270, \\
C_{X,SDR} = \frac{90.269}{0.83083} = 108.6492.
\]

5. For $t > t_1$ when the new weighting scheme applies, the RMB index is similarly constructed:

\[
B_t = C'_B \cdot S_t^{CP,USD/CNY} \cdot \left( \hat{X}_t' \right)^{1-u'_USD},
\]

where the scaling factor $C'_B$ is chosen such that the level of the index is the same at date $t_1$ no matter whether the index is under the old or new weighting scheme. That is,

\[
C'_B = C_B \frac{\hat{X}_{t_1}^{1-u_{USD}}}{\left( \hat{X}_{t_1}' \right)^{1-u'_USD}}.
\]
For each of the three indices, the coefficient $C'_B$ is determined as

\[
C'_{B,CFETS} = \frac{379.3158 \times 2.11099^{(1-0.2640)}}{5.82815^{(1-0.2240)}} = 167.4060,
\]

\[
C'_{B,BIS} = C_{B,BIS},
\]

\[
C'_{B,SDR} = \frac{681.4655 \times 0.95258^{(1-0.4190)}}{0.95878^{(1-0.4685)}} = 677.4886.
\]

6. For $t > t_1$, we adjust the USD basket $\hat{X}'_t$ by a scaling factor $C'_X$ to make it consistent with $X_t$ and comparable with the DXY index. To be precise, the scaling factor $C'_X$ is chosen such that the adjusted USD basket coincides on December 31, 2016 (i.e., $t_1$). That is,

\[
X'_t = C'_X \cdot \hat{X}'_t,
\]

\[
C_X \hat{X}_{t_1} = C'_X \hat{X}'_{t_1}.
\]

For each of the three indices, the coefficient $C'_X$ is determined as

\[
C'_{X,CFETS} = 47.1374 \times \frac{2.11099}{5.82815} = 17.0734,
\]

\[
C'_{X,BIS} = C_{X,BIS},
\]

\[
C'_{X,SDR} = 108.6492 \times \frac{0.95258}{0.95878} = 107.9466.
\]

After some tedious algebra, we can show that the first pillar $\overline{S}_{t+1}$ at date $(t_1 + 1)$ is given by

\[
\overline{S}_{t+1} = S^\text{CP} \frac{X'_{t+1}^{1-w_{USD}}}{X_t^{1-w_{USD}}} \left( \frac{X'_{t+1}}{X_t} \right)^{1-w_{USD}}
\]

\[
= S^\text{CP} \frac{X_{t_1}^{1-w_{USD}} \left( X'_{t+1} \right)^{1-w_{USD}}}{X_{t_1}^{1-w_{USD}} \left( X_t \right)^{1-w_{USD}}}
\]

\[
= S^\text{CP} \left( \frac{X'_{t+1}}{X_t} \right)^{1-w_{USD}},
\]
where \( \chi \equiv \frac{C_B}{C_X} \) and \( \chi' \equiv \frac{C_B'}{C_X} \). Because \( X_{t_i} = X'_{t_i} \), it implies that \( S'_{t_{i+1}} = S_{t_i}^{CP} \left( \frac{X_{t_{i+1}}}{X_{t_i}} \right)^{1-w_{USD}} \). As a result,

\[
S_{t_{i+1}} = \begin{cases} 
S_t^{CP} \left( \frac{X_{t+1}/X_t}{X_{t_i}} \right)^{1-w_{USD}}, & \text{if } t < t_i; \\
S_t^{CP} \left( \frac{X'_{t+1}/X_t}{X_{t_i}} \right)^{1-w'_{USD}}, & \text{if } t \geq t_i.
\end{cases}
\]

**Appendix B: Proofs**

**Proof of Proposition 2.** In the special case with \( b = 0 \), the function \( H(\hat{S}(\hat{V}_t)) \) is always equal to one, implying:

\[
\frac{S_{t+1}^{CP}}{S_t^{CP}} = \left( \frac{X_{t+1}}{X_t} \right)^{w(1-w_{USD})},
\]

and

\[
\hat{S}(\hat{V}_t) = p \kappa + (1 - p) \hat{V}_t,
\]

where \( \kappa \equiv \frac{1 + \tau^2}{1 + 
\kappa} E^Q \left[ \left( \frac{X_{t+1}}{X_t} \right)^{w(1-w_{USD})} \right] \) is a constant. That is, the equilibrium exchange rate is a weighted average of the scaled central parity rate \( \kappa S_t^{CP} \) and the fundamental value \( V_t \), with the weights being the probabilities \( p \) and \( 1 - p \). By construction, the model-implied spot rate in this special case always coincides with the central parity rate \( S_t^{CP} \) because \( b = 0 \).

The option prices can be pinned down based on a Black-Scholes type of formula. Note that when \( b = 0 \), we have \( H \left( \frac{\hat{S}_{t+\tau}}{S_{t+\tau}^{CP}}, 0 \right) = S_{t+\tau}^{CP} \). Below we derive \( C^S(K) \equiv e^{-r_{CNY} \tau} E^Q \left[ \max \left( S_{t+\tau}^{CP} - K, 0 \right) \right] \).

Recall that from Equation (18) we can rewrite the dynamics of \( S_t^{CP} \) below:

\[
\frac{dS_t^{CP}}{S_t^{CP}} = \mu_{CP} dt + \sigma_{CP} dW_{CP,t},
\]

where

\[
W_{CP,t} \equiv \frac{1}{\sigma_{CP}} \left( (\alpha \rho \sigma_X + \beta \sigma_V) dW_{V,t} + \alpha \sqrt{1-\rho^2} \sigma_X dW_{X,t} \right),
\]

and

\[
\sigma_{CP} \equiv \sqrt{(\alpha \rho \sigma_X + \beta \sigma_V)^2 + \alpha^2 (1-\rho^2) \sigma_X^2}.
\]

Therefore,

\[
S_{t+\tau}^{CP} = S_t^{CP} \exp \left\{ \left( \mu_{CP} - \frac{\sigma_{CP}^2}{2} \right) \tau + \sigma_{CP} (W_{CP,t+\tau} - W_{CP,t}) \right\}
\] 

\[
\equiv S_t^{CP} \exp \left\{ \left( \mu_{CP} - \frac{\sigma_{CP}^2}{2} \right) \tau + \sigma_{CP} \sqrt{\tau} z_{CP} \right\},
\]
where we define $z_{CP} \equiv \tau^{-1/2}(W_{CP,t+\tau} - W_{CP,t})$ and by definition $z_{CP}$ follows the standard normal distribution with mean zero and variance one.

As a result, $S_{t+\tau}^{CP} > K$ is equivalent to

$$z_{CP} > -\frac{\log \left( \frac{S_t^{CP}}{K} \right) + (\mu_{CP} - \sigma_{CP}^2/2) \tau}{\sigma_{CP}\sqrt{\tau}} \equiv -d_{2,X}. $$

It follows that:

$$C_I (K; \tau) \equiv e^{-r_{CNY}\tau} E^Q \left[ \max \left( S_{t+\tau}^{CP} - K, 0 \right) \right]$$

$$= e^{-r_{CNY}\tau} \int_{-d_{2,X}}^{\infty} \left[ S_t^{CP} \exp \left\{ \left( \mu_{CP} - \sigma_{CP}^2/2 \right) \tau + \sigma_{CP} \sqrt{\tau} z_{CP} \right\} - K \right] \phi (z_{CP}) d z_{CP}$$

$$= e^{-r_{CNY}\tau} S_t^{CP} e^{(\mu_{CP} - \sigma_{CP}^2/2)\tau} \int_{-d_{2,X}}^{\infty} e^{\sigma_{CP} \sqrt{\tau} z_{CP}} \phi (z_{CP}) d z_{CP} - e^{-r_{CNY}\tau} K \Phi (d_{2,X})$$

$$= S_t^{CP} e^{(\mu_{CP} - r_{CNY})\tau} \left( 1 - \Phi (-d_{2,X} - \sigma_{CP} \sqrt{\tau}) \right) - e^{-r_{CNY}\tau} K \Phi (d_{2,X})$$

$$= S_t^{CP} e^{(\mu_{CP} - r_{CNY})\tau} \Phi (d_{1,X}) - e^{-r_{CNY}\tau} K \Phi (d_{2,X})$$

where $d_{1,X} \equiv d_{2,X} + \sigma_{CP}\sqrt{\tau}$. That is,

$$d_{1,X} = d_{2,X} + \sigma_{CP}\sqrt{\tau} = \frac{\log \left( \frac{S_t^{CP}}{K} \right) + (\mu_{CP} + \frac{1}{2}\sigma_{CP}^2) \tau}{\sigma_{CP}\sqrt{\tau}}$$

$$d_{2,X} = \frac{\log \left( \frac{S_t^{CP}}{K} \right) + (\mu_{CP} - \frac{1}{2}\sigma_{CP}^2) \tau}{\sigma_{CP}\sqrt{\tau}}$$

The expression for the put option $P^S (K)$ can be similarly derived.

Lastly, the derivation is almost the same for $C^V (K)$ and $P^V (K)$, if we replace $S_t^{CP}$ by $V_t$ and its drift and diffusion terms accordingly. In particular,

$$d_{1,V} = d_{2,V} + \sigma_V \sqrt{\tau} = \frac{\log \left( \frac{V_t}{K} \right) + (\mu_V + \frac{1}{2}\sigma_V^2) \tau}{\sigma_V \sqrt{\tau}}$$

$$d_{2,V} = \frac{\log \left( \frac{V_t}{K} \right) + (\mu_V - \frac{1}{2}\sigma_V^2) \tau}{\sigma_V \sqrt{\tau}}$$

■
Table 1: Empirical Evidence for the Two-Pillar Approach

<table>
<thead>
<tr>
<th>Regression</th>
<th>log $\frac{S_{t+1}}{S_t} = \alpha (1 - w_{USD}) \log \frac{X_{t+1}}{X_t} + \beta \log \frac{S_{t+1}}{S_t} + \epsilon_{t+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(I) 12/11/2015-12/31/2016</td>
</tr>
<tr>
<td></td>
<td>$\alpha$</td>
</tr>
<tr>
<td>CFETS</td>
<td>0.5199</td>
</tr>
<tr>
<td>BIS</td>
<td>0.4788</td>
</tr>
<tr>
<td>SDR</td>
<td>0.4826</td>
</tr>
</tbody>
</table>

Table 2: Restriction on the exchange rate movements

<table>
<thead>
<tr>
<th>outside the band (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>benchmark</td>
</tr>
<tr>
<td>$b = 0.01$</td>
</tr>
<tr>
<td>$b = 0.005$</td>
</tr>
<tr>
<td>$\sigma_V = 0.015$</td>
</tr>
</tbody>
</table>

Table 3: Summary Statistics for 3-month RMB options during 12/11/2015-1/31/2018

<table>
<thead>
<tr>
<th>Option</th>
<th>avg. strike price</th>
<th>avg. option price</th>
<th>avg. imp. vol.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Put 10Δ</td>
<td>6.520</td>
<td>0.008</td>
<td>4.98</td>
</tr>
<tr>
<td>Put 25Δ</td>
<td>6.623</td>
<td>0.025</td>
<td>4.81</td>
</tr>
<tr>
<td>Call 10Δ</td>
<td>7.059</td>
<td>0.011</td>
<td>7.35</td>
</tr>
<tr>
<td>Call 25Δ</td>
<td>6.870</td>
<td>0.029</td>
<td>6.02</td>
</tr>
</tbody>
</table>
Table 4: Empirical Evidence for the Modified Two-Pillar Approach

This table reports regression results for the following regression
\[
\log \frac{S_{t+k}^{CP}}{S_t} = \alpha (1 - w_{USD}) \log \frac{X_{t+k}}{X_t} + \beta \log \frac{V_{t+k}}{V_t} + \epsilon_{t+k},
\]
based on the modified two-pillar policy. The regression coefficients \( \alpha \) and \( \beta \) correspond to \( w \) and \( (1 - w) \gamma \), which can thus be used to estimate \( \gamma = \beta / (1 - \alpha) \).

<table>
<thead>
<tr>
<th>( k )</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( \gamma )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0814</td>
<td>-0.0130</td>
<td>-0.0142</td>
<td>0.0129</td>
</tr>
<tr>
<td>2</td>
<td>0.2906</td>
<td>0.0271</td>
<td>0.0382</td>
<td>0.2116</td>
</tr>
<tr>
<td>3</td>
<td>0.3198</td>
<td>0.0914</td>
<td>0.1343</td>
<td>0.3387</td>
</tr>
<tr>
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<td>0.3365</td>
<td>0.1077</td>
<td>0.1624</td>
<td>0.3927</td>
</tr>
<tr>
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<td>0.1241</td>
<td>0.1903</td>
<td>0.4172</td>
</tr>
<tr>
<td>10</td>
<td>0.4065</td>
<td>0.1642</td>
<td>0.2766</td>
<td>0.5188</td>
</tr>
<tr>
<td>15</td>
<td>0.4774</td>
<td>0.1831</td>
<td>0.3503</td>
<td>0.6043</td>
</tr>
<tr>
<td>20</td>
<td>0.5238</td>
<td>0.2068</td>
<td>0.4342</td>
<td>0.6702</td>
</tr>
<tr>
<td>25</td>
<td>0.5397</td>
<td>0.2237</td>
<td>0.4861</td>
<td>0.7048</td>
</tr>
<tr>
<td>30</td>
<td>0.5254</td>
<td>0.2428</td>
<td>0.5117</td>
<td>0.7120</td>
</tr>
<tr>
<td>35</td>
<td>0.5248</td>
<td>0.2533</td>
<td>0.5330</td>
<td>0.7297</td>
</tr>
<tr>
<td>40</td>
<td>0.5426</td>
<td>0.2390</td>
<td>0.5225</td>
<td>0.7399</td>
</tr>
</tbody>
</table>
Panel A of this figure plots historical central parity rate (blue solid line) and closing rate (red dashed line) between 2004 and 2016. Panel B of this figure plots in blue solid line the difference between the logarithms of the central parity and closing rates, and in red solid lines the bounds imposed by the PBoC.
This figure plots the actual periodic RMB indices (red circle) vs. the daily indices we reconstruct (blue solid lines). The three panels correspond to the three RMB indices: CFETS, BIS, and SDR.
Figure 3: Index-implied USD Basket vs. DXY

In this figure we plot the historical dollar index (blue solid line) together with the dollar baskets implied in three indices. The dollar basket implied in the CFETS (respectively, BIS or SDR) index is plotted using red dashed line (respectively, pink dotted or black dash-dotted lines).
Figure 4: Empirical Evidence for the Two-Pillar Approach

In this figure we report results from running 60-day rolling-window regressions: 
\[ \log \left( \frac{S_{CP,t+1}}{S_{CP,t}} \right) = \alpha \cdot \left( 1 - w_{USD}^{ind} \right) \log \left( \frac{X_{t+1}^{ind}}{X_t^{ind}} \right) + \beta \cdot \log \left( \frac{S_{CL,t}}{S_{CP,t}} \right) + \epsilon_{t+1}, \]
where superscript “\text{ind}” indicates one of the three indices "CFETS", "BIS", and "SDR". For each index, we regress the logged growth rate of the central parity on \( (1 - w_{USD}) \) \( \log (X_{t+1}/X_t) \) and the logged ratio between the close and the central parity. The regression coefficient \( \alpha \), which corresponds to the weight \( w \), is plotted in the figure (blue solid line for CFETS, red solid line for BIS, and yellow solid line for SDR indices).
This figure plots in blue solid line the scaled equilibrium exchange rate $\hat{S}(\hat{V})$ as a function of $\hat{V}$. 

\[ (1 + b) \]

$\hat{V}_*$

\[ (1 - b) \]

$\hat{V}^*$
Figure 6: Baseline Parameter Estimates

This figure reports the results of the baseline estimation where we fix $w$ to 0.5. In the top panel, we plot $V_t$, the estimated fundamental exchange rate, in blue solid line, the historical central parity rate in red dashed line, as well as the close in black dashed line. In the middle panel, we plot $p_t$, the probability of the two-pillar approach still being in place three months late. In the bottom panel, we plot $\sigma_V$, the estimated annualized volatility of the fundamental rate process, in blue solid line, and the average implied volatilities of 10-delta options (red dashed line) and 25-delta options (black dashed line) in the data.
Figure 7: Fitting Errors of the Baseline Estimation

This figure plots the fitting error in the baseline estimation. The fitting error is measure by the root-mean-square errors, defined as

$$\sqrt{\frac{1}{5} \left( e_{CL,t}^2 + e_{10P,t}^2 + e_{10C,t}^2 + e_{25P,t}^2 + e_{25C,t}^2 \right)}$$

where

$$e_{CL} \equiv \left( S_{t,model}^{CL} - S_{t,data}^{CL} \right) / S_{t,data}^{CL}$$

$$e_{10P,t} \equiv \left( p_{t,10\Delta,model}^{10\Delta} - p_{t,10\Delta,data}^{10\Delta} \right) / p_{t,10\Delta,data}^{10\Delta}$$

$$e_{10C,t} \equiv \left( c_{t,10\Delta,model}^{10\Delta} - c_{t,10\Delta,data}^{10\Delta} \right) / c_{t,10\Delta,data}^{10\Delta}$$

and $$p_{t,10\Delta,model}^{10\Delta}$$ (or $$c_{t,10\Delta,model}^{10\Delta}$$) denotes the model-implied 10-delta put (or call) option price, etc.
Figure 8: Model-implied (blue cross) vs. Actual (red circle) Option Prices and Spot Rate

The top four panels plot the model-implied option prices (blue cross) against the actual option prices (red circle). For example, the left upper panel plots the model-implied 10-delta option prices $P^{10\Delta}_{t,\text{model}}$ using blue crosses and the actual 10-delta option prices in the data $P^{10\Delta}_{t,\text{data}}$ using red circles. The bottom panel plots the model-implied close $S^{CL}_{t,\text{model}}$ (blue cross) against the actual close $S^{CL}_{t,\text{data}}$ in the data (red circle).
Figure 9: Forecasting RMSEs

This figure plots the RMSEs of forecasting the central parity in the top panel and the close in the bottom panel based on predictive regressions using forecast horizon from 1 to 90 days within the sample period between December 11, 2015 and December 31, 2016. In both panels, the blue line plots the forecasting RMSEs based on the model while the red line plots the forecasting RMSEs based on the random-walk benchmark.