Low Inflation:
High Default Risk AND High Equity Valuations*

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Abstract

We develop an asset pricing model with endogenous corporate policies that explains how inflation jointly impacts real asset prices and corporate default risk. Our model accounts for two important sources of nominal rigidity present in the data. First, nominal coupons paid to long-term corporate debt are fixed in the short run, that is, leverage is sticky. Second, in the short run, earning growth is less sensitive to variations in expected inflation than the nominal risk-free rate, that is, firm profitability is sticky. These features combined result in higher real equity prices and credit spreads when inflation falls. An increase in inflation has the opposite effects, but with smaller magnitudes. The relation between equity prices and inflation is thus asymmetric. In the cross-section, the model predicts the negative impact of inflation on real equity values and credit risk is stronger for low leverage firms. We find empirical support for our theoretical predictions.

JEL Classification Numbers: E44, G12, G32, G33

Keywords: low inflation, default risk, equity, leverage, credit spreads

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1 Introduction

Corporate defaults spike during times of low expected inflation, whereas equity valuations also tend to be high. Figure 1 documents these two stylized facts for the U.S. over the period 1970Q2–2016Q4. Panel A illustrates the strong negative relation between expected inflation, and the number of quarterly defaults in the U.S. but Panel B shows a similar negative relation between expected inflation and the price-dividend ratio.

Researchers often rationalize the default clustering during times of low inflation with lower growth in firms’ nominal earnings combined with lower nominal interest rates. Lower nominal rates increase the real debt burden for firms. Lower earnings and higher debt values increases the probability of default.

The negative relation between stock valuation and expected inflation is well documented over different sample periods and using various valuation measures (e.g., Modigliani and Cohn (1979); Feldstein et al. (1980); Ritter and Warr (2002); Sharpe (2002); Campbell and Vuolteenaho (2004)).\(^1\) The relation between valuation ratios and expected inflation contradicts the Fisher (1930) hypothesis: nominal asset returns should move one-for-one with the expected inflation. One common explanation is money illusion: investors discount real cash flows with nominal discount rates.\(^2\)

Higher real debt burdens during times of low inflation can rationalize higher defaults but seem to be inconsistent with higher equity valuation. At the same time, money illusion can rationalize higher equity valuation during times of low inflation and seems inconsistent with a higher default rate. How can shareholders value stocks more favorably when facing greater bankruptcy risk?

We propose a rationale model to jointly explain these two puzzling observations. We model the economy as a dynamic cross-section of firms that issue debt and equity. The theoretical framework endogenizes firms’ financing and default policies. It provides asset pricing predictions from a corporate finance perspective. We consider a representative agent with standard Epstein-Zin-Weil utility.\(^3\) A new ingredient in our model is firms face not only real macroeconomic risk but also the risk that the

\(^1\)Similarly, this result is in line with the negative relationship between stock returns and measures of expected or unexpected inflation (e.g., Lintner (1975), Bodie (1976), Fama and Schwert (1977), Miller, Jeffrey, and Mandelker (1976), Nelson (1976), Fama (1981), Schwert (1981), Geske and Roll (1983), Gultekin (1983), Solnik (1983), Pindyck et al. (1984), Kaul (1987), Pearce and Riley (1988), Kaul and Seyhun (1990), Boudoukh and Richardson (1993), Bekaert and Wang (2010)).

\(^2\)Alternative explanations are the non-neutrality of inflation and the existence of an inflation risk premium. We detail the literature below.

\(^3\)Our work builds on Chen (2010) and Bhamra, Kuehn, and Strebulaev (2010a,b), who analyze firms’ capital and default decisions, as well as levered asset prices, in a consumption-based environment with macroeconomic conditions.
Figure 1: Defaults, stock valuation, and inflation in the U.S.

This figure illustrates the relation between expected inflation, default risk, and stock valuation. Panel A reports the number of quarterly defaults of firms domiciled in the U.S. with debt rated by Moody’s. Panel B displays the price-dividend ratio, computed as the value-weighted CRSP price index in the last month of the quarter divided by the sum of dividends paid in the last twelve months. Defaults are obtained from Moody’s Default and Recovery Database. Expected inflation is the one-year ahead inflation forecast from the Survey of professional Forecaster. The sample spans the period 1970Q2-2016Q4.

economy switches between different inflation regimes via a Markov chain. We refer to the fluctuations in the inflation rate as nominal risk.

Nominal risk matters for real asset prices, because firms do not instantaneously adjust their nominal coupons to changes in inflation. This stickiness of leverage means real coupons are affected by nominal conditions, and so changes in inflation will impact real asset prices.

Changes from high (or moderate) inflation to low inflation affect asset prices through several channels. First, a firm’s default probability increases with lower inflation due to a lower nominal cash flow growth rate. Higher default risk decreases equity valuation through the first channel. Second, lower inflation decreases the nominal risk-free rate, which increases real corporate bond values. Again, the value of equity declines. Equity prices decrease through both channels, which reinforce one another. A third mechanism, however, pushes equity valuation up when inflation decreases: the nominal risk-free rate changes immediately and one-for-one with changes in inflation, whereas cash flows are sticky in the short run due to nominal price rigidities, as documented in Katz, Lustig, and Nielsen (2016).
This incomplete inflation passthrough of firm cash flows arises rationally when firms face costs of price adjustment.\(^4\)

For empirically-plausible values of inflation passthrough, the third mechanism dominates the other two channels, predicting lower equity prices, lower debt prices, and lower default risk with higher inflation rates. Hence, our model provides a rational explanation for the ex-ante puzzling features of the data, rationalizing why both defaults and equity valuation rise when the inflation rate decreases.

The model also predicts an interesting asymmetry consistent with the data: low inflation is not the mere mirror image of high inflation consistent with the data (see Figure 1). As we argue above, low inflation increases the expected future value of fixed nominal coupons, thereby increasing the real value of debt. Hence, it appears natural to assume that an increase in inflation of the same size will result in an equal-sized decrease of real debt values. But, such an analysis is incomplete, because it ignores how shifts in nominal risk-free rates impact levered equity values non-linearly via a cash-flow discounting channel. We obtain this prediction even though default probabilities are convex in the distance-to-default. The convexity implies an increase in default risk depresses the value of equity more than a decrease in default risk of the same size. But we show that this effect is not sufficient to offset the asymmetry arising from the nominal discount rate channel. The asymmetric effect of low inflation on asset prices is important in light of the extremely low inflation levels that have been observed over the recent years.

An interesting implication of our model is that nominal risk has a positive—and not a negative—effect on real asset values. Because lower inflation increases stock prices more than higher inflation depresses them, fluctuations in inflation increases equity prices on average.\(^5\) Our model thus contributes to understanding why the impact of fluctuating inflation rates for equity investors can be economically beneficial.

Besides predictions for the aggregate market, our models also makes cross-sectional predictions. Variations in inflation have a smaller impact on equity prices of firms with higher leverage, because of two offsetting effects.

\(^{4}\) Nominal price rigidities are the leading explanation of the real effects of monetary policy. Menu cost models generate a bound of inaction, rationalizing price non-adjustment to shocks (see, e.g., Mankiw (1985) and Ball and Mankiw (1994)). Ample evidence exists on the stickiness of output prices (see, e.g., Steissson and Nakamura (2008) and Gorodnichenko and Weber (2016)).

\(^{5}\) We compare the model’s prediction with that of an hypothetical economy with an expected growth rate of prices set at its unconditional mean to reach this conclusion.
On one hand, the incomplete inflation pass-through results in a higher present value of cash flows in times of low inflation. On the other hand, the value of debt also increases. The sticky-leverage channel drives this second effect, which is naturally stronger for firms with high leverage. Hence, equity valuation is less sensitive to changes in inflation for high-leverage firms. Therefore, the presence of debt reduces, rather than exacerbates, the sensitivity of stock prices to changes in nominal conditions.

We reach similar conclusions for credit spreads. In the time-series, the effect of inflation on asset valuation is countercyclical with respect to real economic conditions, as recessions correspond to times of greater financial stress. As such, the model predicts that equity prices and credit spreads will be lowest during economic contractions coupled with episodes of high inflation.

We reach these conclusion in a calibration to the U.S. economy. In our calibration, the real states of the economy are characterized by the conditional moments of consumption growth using a two-state Markov-regime switching model on quarterly U.S. consumption data over the period 1970Q2-2016Q4. The aggregate earnings from S&P determine the dynamics of firm real cash flows. We determine the inflation states using the one-year average expected inflation rate from by the Survey of Professional Forecasters. We estimate a three-state Markov-regime switching model assuming that the inflation rate in the moderate inflation regime is equal to its unconditional mean.

1.1 Related Literature

Existing studies, going back to Fama (1981), provide explanations for the negative relation between stock valuations and inflation based on the idea that expected inflation is non-neutral because it has a negative effect on real growth. Agents demand a positive inflation risk premium, which reduces stock prices (e.g. Eraker, Shaliastovich, and Wang (2015)). However, default risk is typically ignored in such asset pricing models. We stress that an inflation risk premium does not drive any of the model predictions. In fact, we intentionally model preferences such that they are completely independent of the nominal state, in line with the view that periods of inflation/deflation can be associated with either good or bad economic conditions. Our model shows that leverage stickiness combined with incomplete inflation pass-through are the only ingredients necessary to generate relations between inflation, equity valuation, and default risk that are consistent with the data.

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6There is no consensus that agents should like inflation and dislike deflation, or vice versa. For example, Piazzesi and Schneider (2006) show inflation predicts consumption growth negatively, while Boons, Duarte, de Roon, and Szymanska (2016) suggest the relation is time-varying. This finding is consistent with the evidence inflation periods do not always reflect a bad state of the economy. See, e.g., Bekaert and Wang (2010), Campbell, Sunderam, and Viceira (2016), and David and Veronesi (2013).
This paper also contributes to the literature exploring theoretically the interaction between inflation and stock returns. These studies include Day (1984), Stulz (1986), Wachter (2006), Gabaix (2008), Hess and Lee (1999), Chen (2010), Bansal and Shaliastovich (2013), and Gomes, Jermann, and Schmid (2016). Chen, Roll, and Ross (1986) and Ang, Briere, and Signori (2012) find inflation risk is priced in the cross section of U.S. stock returns, whereas Boons et al. (2016) show the inflation risk premium varies over time conditional on the relation between inflation and the real economy. We depart from this literature by explaining the relation between inflation and equity valuation without linking inflation to consumption. Our work is closely related to work by Weber (2015), showing how inflation risk impacts equity returns via a sticky-prices channel. We combine the idea of an incomplete pass-through of inflation to profits with a sticky-leverage channel. Finally, Kang and Pflueger (2015), which studies how inflation risk impacts corporate bond prices is another closely-related paper. Our paper differs from this study by offering an explication of the joint relation between inflation, default risk, and equity prices in a unified framework. Furthermore, we contribute to the existing literature by providing empirical evidence and a theoretical explanation for the asymmetric relation between asset prices and inflation.

Our paper makes several contributions. First, we build an asset pricing model of multiple firms which can issue debt and equity with the option to default, where inflation risk impacts firms’ asset prices. We explain the negative relation between stock valuation and expected inflation without any inflation risk premium. Second, our model generates the negative impact of inflation on default risk through variations in real leverage. Third, we show a fall in inflation has a greater impact on equity prices than the risk of higher inflation, which suggests a fundamental asymmetry in the asset-pricing implication of inflation risk. Fourth, we find the risk of inflation is stronger for less levered firms. Hence, our paper allows us to shed light on the role of fluctuating inflation for asset prices and helps us understand the firm characteristics and economic conditions that make equity prices more sensitive to changes in inflation. We are thereby able to analyze aspects of both the cross-sectional and time-series asset-pricing implications of inflation risk.

Section 2 describes a consumption-based asset pricing model with inflation and deflation risk, while Section 3 derives asset prices together with optimal default and capital structure decisions. Section 4 shows how we calibrate the model. Section 5 discusses the model’s theoretical predictions. Section 7 concludes.
2 Model

This section presents a consumption-based corporate finance model with real and nominal risks. We here define aggregate consumption, inflation and derive the real and nominal stochastic discount factors, using an Epstein-Zin-Weil representative agent. We then derive the asset values of firms, which issue nominal debt and equity, and describe their optimal policies.

2.1 Aggregate economic variables

Aggregate consumption at date-\( t \) is denoted by \( C_t \) and its dynamics are given exogenously by

\[
\frac{dC_t}{C_t} = g_t dt + \sigma_{C,t} dZ_t, \tag{1}
\]

where \( Z_t \) is a standard Brownian motion under the physical measure \( \mathbb{P} \). The conditional first and second moments of aggregate consumption growth, \( g_t \) and \( \sigma_{C,t} \), respectively, can take different values, depending on the current state of the real economy, denoted by \( \nu_t \). The real economy is risky and transitions between a recession state, \( \nu_t = R \), and an expansion state, \( \nu_t = E \), vary according to a 2-state Markov chain. The probability under the physical measure of moving from the expansion state to the recession state within an instant \( dt \) is \( \lambda_{ER}^{\text{real}} dt \), where the intensity \( \lambda_{ER}^{\text{real}} \) is constant. Similarly, the probability under the physical measure of moving from the recession state to the expansion state within an instant \( dt \) is \( \lambda_{RE}^{\text{real}} dt \). We have \( g_R < g_E \) and \( \sigma_{C,R} > \sigma_{C,E} \) to ensure that the mean and volatility of consumption growth are cyclical and countercyclical, respectively.

Inflation dynamics are specified exogenously. The date-\( t \) level of the price index is denoted by \( P_t \) and satisfies

\[
\frac{dP_t}{P_t} = \mu_{P,t} dt, \tag{2}
\]

where we neglect inflation volatility stemming from small Brownian shocks, and assume that date-\( t \) conditional expected inflation, \( \mu_{P,t} \), depends on the nominal state \( \epsilon_t \). We assume 3 nominal states: a low inflation state, \( \epsilon_t = L \), a moderate inflation state, \( \epsilon_t = M \), and a state of high inflation, \( \epsilon_t = H \). From the definition of the nominal state, \( \mu_{P,L} < 0 < \mu_{P,M} < \mu_{P,H} \). The physical probability of moving from the nominal state \( l \) to \( k \), within the instant \( dt \), is \( \lambda_{lk}^S dt \) and the probability of moving back within a subsequent instant is \( \lambda_{kl}^S dt \).
For ease of notation we combine the real and nominal states into 6 distinct states, where the current combined state is denoted by $s_t = (\nu_t, \epsilon_t)$. The economy is thus characterized by state 1 (recession with low inflation, RL), state 2 (recession with moderate inflation, RM), state 3 (recession with high inflation, RH), and similarly in expansion. In summary, the different states are

<table>
<thead>
<tr>
<th>State Description</th>
<th>$s_t$</th>
<th>$g_t$</th>
<th>$\sigma_{C,t}$</th>
<th>$\mu_{P,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession &amp; Low Inflation (RL)</td>
<td>1</td>
<td>$\mu_{C,R}$</td>
<td>$\sigma_{C,R}$</td>
<td>$\mu_{P,L}$</td>
</tr>
<tr>
<td>Recession &amp; Moderate Inflation (RM)</td>
<td>2</td>
<td>$\mu_{C,R}$</td>
<td>$\sigma_{C,R}$</td>
<td>$\mu_{P,M}$</td>
</tr>
<tr>
<td>Recession &amp; High Inflation (RH)</td>
<td>3</td>
<td>$\mu_{C,R}$</td>
<td>$\sigma_{C,R}$</td>
<td>$\mu_{P,H}$</td>
</tr>
<tr>
<td>Expansion &amp; Low Inflation (EL)</td>
<td>4</td>
<td>$\mu_{C,E}$</td>
<td>$\sigma_{C,E}$</td>
<td>$\mu_{P,L}$</td>
</tr>
<tr>
<td>Expansion &amp; Moderate Inflation (EM)</td>
<td>5</td>
<td>$\mu_{C,E}$</td>
<td>$\sigma_{C,E}$</td>
<td>$\mu_{P,M}$</td>
</tr>
<tr>
<td>Expansion &amp; High Inflation (EH)</td>
<td>6</td>
<td>$\mu_{C,E}$</td>
<td>$\sigma_{C,E}$</td>
<td>$\mu_{P,H}$</td>
</tr>
</tbody>
</table>

The transitions between combined real and nominal states are given exogenously by a 6-state Markov chain. The probability under the physical measure of moving from state $i$ to state $j \neq i$ within an instant $dt$ is $\lambda_{ij} dt$, where the intensity $\lambda_{ij}$ is constant. Real and nominal regimes are assumed to switch independently over period $dt$, such that the transition intensities for the aggregate state of the economy, $\Lambda$, solves

$$e^{\Lambda dt} = \left(S'^e_{\text{real}} e^{\Lambda^\text{real} dt} S_{\text{real}}\right) \circ \left(S'^e_{\text{s}} e^{\lambda^e dt} S_{\text{s}}\right),$$

where $\circ$ is the Hadamard product and

$$\Lambda^\text{real} = \begin{pmatrix} -\lambda^\text{real}_{RE} & \lambda^\text{real}_{RE} \\ \lambda^\text{real}_{ER} & -\lambda^\text{real}_{ER} \end{pmatrix}, \quad \Lambda^e = \begin{pmatrix} -\lambda^e_{LM} - \lambda^e_{LH} & \lambda^e_{LM} & \lambda^e_{LH} \\ \lambda^e_{ML} & -\lambda^e_{ML} - \lambda^e_{MH} & \lambda^e_{MH} \\ \lambda^e_{HL} & \lambda^e_{HM} & -\lambda^e_{HM} - \lambda^e_{HL} \end{pmatrix},$$

$$S_{\text{real}} = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}, \quad S_e = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}.$$
2.2 Representative agent and stochastic discount factors

The representative agent has the continuous-time analog of Epstein-Zin-Weil preferences. The representative agent’s real stochastic discount factor (SDF) at time-\( t \), \( \pi_t \), depends on the state of the real economy and is given by (see Appendix A.1 for the derivation)

\[
\pi_t = \left( \beta e^{-\beta t} \right)^{1-\frac{1}{\gamma}} C_t^{-\gamma} \left( p_{C,t} e^{\int_0^t p_{C,u}^{-1} du} \right)^{-\frac{\gamma-1}{\psi}},
\]  

(5)

where \( \beta \) is the rate of time preference, \( \gamma \) is the coefficient of relative risk aversion (RRA), and \( \psi \) is the elasticity of intertemporal substitution under certainty (EIS).

The real stochastic discount factor at date-\( t \), \( \pi_t \), evolves as follows

\[
\frac{d\pi_t}{\pi_t} |_{s_{t-1}=i, s_t=j} = -r_i dt - \gamma \sigma_{C,i} dZ_t + (\omega_{ij} - 1) dN_{ij,t},
\]  

(6)

where \( r_i \) is the equilibrium real risk-free interest rate in state \( i \), given by

\[
r_i = \begin{cases} 
    r_R, & i \in \{1,2,3\} \\
    r_E, & i \in \{4,5,6\}
\end{cases}
\]  

(7)

with \( r_E > r_R \) such that the real interest rate evolves in a cyclical fashion with the real economy.

The pricing of risk is determined by the martingales \( Z_t \) and \( N_{ij,t}^P \) introduced in Equation (6). The increment in the standard Brownian motion \( dZ_t \) is the relatively small risk from unexpected consumption growth and \( \gamma \sigma_{C,i} \) is the associated price of risk, which is higher in recessions. The compensated Poisson process \( N_{ij,t}^P \) is given by

\[
N_{ij,t}^P = dN_{ij,t} - \lambda_{ij} dt,
\]  

(8)

where \( N_{ij,t} \) is a Poisson process which jumps up by one when the (combined) state switches from \( i \) to \( j \neq i \). The increment in the compensated Poisson process, \( dN_{ij,t}^P \), represents the risk stemming from a change in the state of the economy. The associated price of risk is \( \omega_{ij} - 1 \).

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7The continuous-time version of the recursive preferences introduced by Epstein and Zin (1989) and Weil (1989) is known as stochastic differential utility, and is derived in Duffie and Epstein (1992). Schroder and Skiadas (1999) provide a proof of existence and uniqueness.
The pricing of securities is based on the risk-neutral switching probabilities per unit of time, $\hat{\lambda}_{ij}$, which are related to the actual switching probabilities per unit of time, $\lambda_{ij}$, such that $\hat{\lambda}_{ij} = \lambda_{ij}\omega_{ij}, i \neq j$. The diagonal elements of $\hat{\Lambda}$ are $\hat{\lambda}_{ii} = -\sum_{j \neq i} \hat{\lambda}_{ij}$, ensuring that $\hat{\Lambda}$ is a proper generator matrix under the risk-neutral measure. Under Epstein-Zin-Weil preferences, the distortion factor is defined by $\omega_{ij} = \frac{p_{C,j}}{p_{C,i}}$, where $p_{C,t}$ is the value of the claim to aggregate consumption per unit consumption, i.e. the price-consumption ratio. The price-consumption ratio depends on the current state of the real economy, but not on the nominal state, that is:

$$p_{C,t} = \begin{cases} 
  p_{C,R}, & \text{if } s_t \in \{1, 2, 3\} \\
  p_{C,E}, & \text{if } s_t \in \{4, 5, 6\}.
\end{cases}$$

Hence, $\omega_{ij} = 1$ if $\nu_i = \nu_j$.

The representative agent cares about future consumption growth and prefers intertemporal risk to be resolved sooner rather than later when $\gamma > 1/\psi$, which implies that $\omega > 1$. Therefore, the risk-neutral probability per unit of time of switching from expansion to recession is higher than the actual probability, as $\lambda(\cdot,E,R) > \lambda(\cdot,E,R)$, which means that the agent values securities as if recessions (expansions) were more (less) likely than in reality. However, because only real risks are priced by the agent, the risk-neutral and the actual probabilities related to a change in the nominal state are identical, that is $\hat{\lambda}_{ij} = \lambda_{ij}$ if $\nu_i = \nu_j$.

Financial securities have nominal prices, which requires us to consider a nominal stochastic discount factor for asset pricing. The date-$t$ nominal SDF, denoted by $\pi_t^S$, is defined as

$$\pi_t^S = \pi_t \frac{P_t}{r_t},$$

whose dynamics satisfy

$$\frac{d\pi_t^S}{\pi_t^S} \big|_{s_t=i, s_{t-1}=j} = -r_t^S dt - \gamma \sigma_{C,i} dZ_t + \sum_{j \neq i} (\omega_{ij} - 1) dN_{ij,t},$$

where $r_t^S$ is the nominal interest rate in state $i$, given by

$$r_t^S = r_i + \mu_{P,i}.\quad (12)$$
The nominal interest rate depends on both real and nominal states and can thus take 6 different values—it changes when the conditional moments of consumption growth change and also when expected inflation changes. The nominal risk-free rate is lowest during the recession/low inflation state and highest during the expansion/high inflation state.

2.3 Firm cash flows

The date-\(t\) level of the real cash-flow of firm \(n\) is denoted by \(Y_{n,t}\), which evolves according to the process

\[
\frac{dY_{n,t}}{Y_{n,t}} = \mu_{Y,t} dt + \sigma_{Y,t}^{id} dZ_{n,t}^{id},
\]

where the real cash flows have a conditional expected growth rate \(\mu_{Y,t}\) and a conditional volatility \(\sigma_{Y,t}^{id}\). Both moments are identical across firms. The standard Brownian motion \(Z_{n,t}^{id}\) represents firm \(n\)’s idiosyncratic risk factor and its changes are independent across firms and unrelated to shocks to consumption growth.\(^8\)

Systematic risk in real cash flows is exclusively associated with low-frequency changes in economic conditions. The expected growth rate is higher in expansions than in recessions, whereas the conditional idiosyncratic volatility is lower in expansions than in recessions. In sum, for all firms, we have \(\mu_{Y,t} = \mu_{Y,R}, \sigma_{Y,t}^{id} = \sigma_{Y,R}^{id}\) in recessions and \(\mu_{Y,t} = \mu_{Y,E}, \sigma_{Y,t}^{id} = \sigma_{Y,E}^{id}\) in expansions, where \(\mu_{Y,R} < \mu_{Y,E}\) and \(\sigma_{Y,R}^{id} > \sigma_{Y,E}^{id}\).

Because firms issue nominal securities and pay nominal taxes, investors care about the dynamics of the nominal cash flows. Firm \(n\)’s nominal date-\(t\) cash flow level is then given by \(X_{n,t}\), where

\[
X_{n,t} = Y_{n,t} P_t^{\sigma},
\]

which satisfies

\[
\frac{dX_{n,t}}{X_{n,t}} = \mu_{X,t} dt + \sigma_{X,t}^{id} dZ_{n,t}^{id},
\]

with \(\mu_{X,t} = \mu_{Y,t} + \varphi \mu_{P,t}\). Because we ignore instantaneous Brownian shocks on the price index, the volatility of the nominal cash flows is given by \(\sigma_{X,t}^{id} = \sigma_{Y,t}^{id}\). The inflation passthrough parameter, \(\varphi\), captures the extent to which changes in inflation expectations are reflected in the firm’s earnings. It

\(^8\)We ignore a common diffusive component in firm real cash flows, because the associated risk premium is negligible as evidenced by the earlier asset pricing literature.
provides a reduced-form way to reflect the fact that firm’s input costs and selling prices may exhibit different levels of nominal rigidity.

Overall, firms exhibit heterogeneity in their cash flows due to idiosyncratic shocks but, at the same time, all firms have moments in their expected growth rate that similarly vary over a combined nominal and real cycle.

3 Asset Prices and Corporate Financing Decisions

In this section, we derive asset prices together with optimal default and capital structure decisions.

Firms pay taxes on nominal cash flows $X_t$ (for parsimony, we omit the subscript $n$) and issue debt to shield profits from such taxes. Each firm has a debt contract that is characterized by a constant and perpetual nominal debt coupon. Leverage is sticky because the coupon is nominal and kept fixed. Hence, when the nominal state changes, the real coupon changes, which affects valuations. Consequently, sticky leverage acts as a nominal rigidity. In other words, firms cannot adjust the nominal quantity of debt to news about the inflation/deflation state. \(^9\) When a firm’s nominal cashflows reach a state-dependent boundary $X_{D,i}$, which is selected by equityholders to maximize equity value, the firm is liquidated. Debtholders recover a fraction of the after-tax unlevered asset value of the firm, while the remaining fraction is lost in bankruptcy costs.

3.1 Liquidation value

The nominal asset value at time of liquidation, denoted by $A_{i,t}$ in state $i \in \{1, ..., 6\}$, corresponds to the present value of the after-tax nominal unlevered cash flows, which equals

$$A_{i,t} = (1 - \eta)X_t \frac{1}{r_{A,i}}, \quad (16)$$

where $\frac{1}{r_{A,i}}$ is defined by

$$\frac{1}{r_{A,i}} = E_t \left[ \int_t^\infty \frac{\pi_u^i}{\pi_t^i} \frac{X_u}{X_t} du | s_t = i \right]. \quad (17)$$

\(^9\) A debt structure with constant real coupons would make firm leverage dependent the real state of the economy but not on nominal prices.
The value of $r_{A,i} = v_{A,i}^{-1}$ is given by the $i$'th element of the vector $V_A = [v_{A,1}, \ldots, v_{A,6}]^\top$, where

$$V_A = (R_A - \hat{\Lambda})^{-1}1_{6\times1}, \quad (18)$$

$1_{6\times1}$ is the 6 by 1 vector of ones and $R_A$ is the following 6 by 6 diagonal matrix

$$R_A = \text{diag}(r_1^\$ - \mu_{X,1}, \ldots, r_6^\$ - \mu_{X,6}) \quad (19)$$

and $\hat{\Lambda}$ is the 6 by 6 risk-neutral generator matrix of the Markov chain characterizing the real and nominal states of the economy, as defined in Section 2.2.

Our assumption that the rate of change of the price is locally risk-free means that there is no inflation risk premium. Hence, the matrix $R_A$, which show how macroeconomic growth rates impact the discounting of future cashflows, is independent of inflation. We can intrepret $r_{A,i}$ as the discount rate for a perpetuity with stochastic expected growth rate $\mu_{X,t}$, which is currently equal to $\mu_{X,i}$. Note that if the economy stays in state $i$ forever, the discount rate reduces to the standard expression $r_{A,i} = r_i^\$ - \mu_{X,i}$. In general, however, the economy can change state, and so the discount rate depends on the risk-neutral generator matrix of the Markov chain governing the economy’s transitions. The presence of the risk-neutral generator matrix as opposed to the physical risk-neutral generator matrix incorporates the pricing of risk.

3.2 Arrow-Debreu default claims

Default risk is key to firm valuation. We thus express the value of a firm’s asset as a function of a set of Arrow-Debreu default claims.

We define an Arrow-Debreu default claim as an asset, which pays out one dollar if default occurs in state $j$ and the current state is $i$. We denote the nominal price of such a security by $q_{D,ij,t}^\$, which satisfies (see Appendix A.4)

$$q_{D,ij,t}^\$ = \mathbb{E}_t \left[ \frac{\pi_{D,t}}{\pi_t} I_{\{s_{\tau_D} = j\}} \bigg| s_t = i \right], \quad (20)$$

where $\tau_D$ is the date at which default occurs, making $I_{\{s_{\tau_D} = j\}}$ the indicator function which equals one, if default occurs in state $j$, and zero elsewhere.
When valuing assets that depend on the level of cash-flows at time of default, \( X_{\tau_D} \), we have to consider additional Arrow-Debreu securities. The reason is that our economy features what we call “deep defaults”, which can occur when the state of the economy jumps from its current state to a worse state. Default boundaries are countercyclical and can suddenly move upward when the economy deteriorates, for example when a deflation period starts. In such a situation, a fraction of firms may immediately default upon a change in state. Consider a firm which has a nominal cash-flow level of say $10 while the default boundary is $8. If the economy suddenly deteriorates by moving into a new state where the default boundary is $11, the firm will immediately default. Clearly all firms with nominal cash flow level below $11 would immediately default, thereby creating a default cluster. More formally we can consider a firm with nominal cash flow level \( X_{\tau_D^-} \), at time \( \tau_D^- \), which is the time just before default, where \( X_{\tau_D^-} \) is below the new state’s default boundary, \( X_{D,j} \). This firm will default as soon as the economy enters the new state, and so \( X_{\tau_D^-} = X_{\tau_D} < X_{D,j} \) (\( X_{\tau_D^-} = X_{\tau_D} \) because \( X \) is a continuous process). Hence, it not necessarily the case that at default, a firm’s cash flow level is at the default boundary. Consequently, to value securities which depend on firm’s cash flows, we need a modified set of Arrow-Debreu default claims. We derive them in Appendix A.5.

This second type of Arrow-Debreu default claim pays out \( \frac{X_{\tau_D}}{X_{D,j}} \) at default if default occurs in state \( j \) and the current state is \( i \). The date-\( t \) nominal price of this security is denoted by \( q_{D,ij,t}^s \), where

\[
q_{D,ij,t}^s = E_t \left[ \frac{\pi_{\tau_D}^s}{\pi_t^s} \frac{X_{\tau_D}}{X_{D,j}} 1_{\{s_{\tau_D} = j\}} \left| s_t = i \right. \right].
\] (21)

Overall, there are 36 Arrow-Debreu default prices for each type, because 6 states characterize the aggregate economy.

### 3.3 Corporate bond value

A firm which issues debt promises to pay a nominal coupon \( c \) per unit time. If the firm defaults, the coupon is no longer paid and instead debt holders receive a fraction of the firm’s liquidation value. This fraction is known as the recovery rate and the state-\( t \) recovery rate is denoted by \( \alpha_t \). Hence, the date-\( t \) nominal value of corporate debt, conditional on the current state being \( i \), is given by

\[
B_{i,t}^s = cE_t \left[ \int_t^{\tau_D} \frac{\pi_{\tau_D}^s}{\pi_t^s} du \right] + E_t \left[ \frac{\pi_{\tau_D}^s}{\pi_t^s} \alpha_{\tau_D} A_{\nu_D}^s (X_{\tau_D}) du \right].
\] (22)
The above expression is simply the present value of future coupon flows up until some random default time, \( \tau_D \), plus the present value of the unlevered firm assets net of bankruptcy costs. We assume that the recovery rate depends solely on the state of the economy, and so \( \alpha_j \) denotes its value in state \( j \).

We can rewrite the above expression as

\[
B_{i,t} = c \left( \frac{1}{r_{P,i}^s} - \sum_{j=1}^{6} q_{D,ij,t}^s \frac{1}{r_{P,j}^s} \right) + \sum_{j=1}^{6} \alpha_j A_j^s(X_{D,j}) q_{D,ij,t}^s,
\]

where \( r_{P,i}^s \) is the nominal discount rate for perpetuity paying a flow of 1 USD, conditional on the current state being \( i \). Observe that

\[
\frac{1}{r_{P,i}^s} = E_t \left[ \int_t^{\infty} \frac{\pi_i^s}{\pi_i} du | s_t = i \right].
\]

To understand the intuition underlying the expression the the corporate bond price given in (23), observe that \( c \frac{1}{r_{P,i}^s} \) is the present value in nominal terms of a bond paying a coupon flow of \( c \) USD in perpetuity with no default risk. The expression \( c \sum_{j=1}^{6} q_{D,1,ij}^s \frac{1}{r_{P,j}^s} \) is the present value of coupons lost because of the possibility of default and \( \sum_{j=1}^{6} \alpha_j A_j^s(X_{D,j}) q_{D,ij,t}^s \) is the present value of the assets recovered.

The nominal discount rate for a constant nominal perpetuity, \( r_{P,i}^s \), is given by \( r_{P,i}^s = v_{B,i}^{-1} \), where \( v_{B,i} \) is the \( i \)'th element of the vector \( V_B = [v_{B,1}, ..., v_{B,6}]' \), given by

\[
V_B = (R^s - \Lambda)^{-1} 1_{6 \times 1},
\]

where \( R^s \) represents the 6 by 6 diagonal matrix such that \( R_{ii}^s = r_{i}^s \). Therefore, \( r_{P,i}^s \) accounts for the possibility that the nominal risk-free rate takes different future values as macroeconomic fundamentals and inflation fluctuate over time.

### 3.4 Equity value

Shareholders are entitled to the firm’s cash flows net of taxes and debt servicing as long as the firm does not default. When the firm is in default, which occurs at some random time \( \tau_D \), shareholders recover nothing and lose their rights to any future cash flows. The nominal value of equity at date-\( t \),
conditional on the current state $i$, is then given by

$$S^s_{i,t} = (1 - \eta)E_t \left[ \int_t^{T_D} \frac{\pi^u_t}{\pi^s_t} (X_u - c)du \bigg| s_t = i \right],$$

which can be rewritten as

$$S^s_{i,t} = A^s_i(X_t) - (1 - \eta) \frac{c}{r^s_{P,i}} - \sum_{j=1}^{6} \left( A^s_j(X_{D,j})q^s_{D,ij,t} - (1 - \eta)q^s_{D,ij,t} \frac{c}{r^s_{P,j}} \right).$$

The first two terms represent the present value of cash flows net of coupon payments when there is no default, whereas the summation term captures the present value of the net cash flows that shareholders lose in the case of default.

### 3.5 Default and capital structure decisions

Shareholders maximize the value of their default option by choosing when to default. There exists a state-contingent endogenous default boundary $X_{D,s}$, that depends on the current real and nominal state of the economy, i.e. $s_t \in \{1, \ldots, 6\}$. Expected inflation matters for default decisions, because a change in the nominal cash flow growth is not offset by a change in the nominal coupon rate, i.e. leverage is sticky. Hence, equityholders are entitled to smaller expected future cash flows in deflation than under high inflation.

The default boundaries satisfy the following four standard smooth-pasting conditions:

$$\left. \frac{\partial S^s_{s,t}(X)}{\partial X} \right|_{X=X_{D,s,t}} = 0, \ s_t \in \{1, \ldots, 6\}.$$ (28)

Shareholders also choose the optimal nominal coupon to maximize firm value at date 0 by balancing marginal tax benefits from debt against marginal expected distress costs. There are two important features to note. First, as is standard in the capital structure literature (Leland, 1994), by maximizing firm value shareholders internalize debtholders’ value at date 0. However, in choosing default times they ignore the considerations of debtholders. This feature creates the usual conflict of interest between equity and debtholders. Second, the optimal coupon depends on the state of the economy at date 0. To make this clear, we denote the date-0 coupon by $c_{s_0,0}$, where $s_0$ is the date-0 state of the economy.
Shareholders choose the coupon to maximize date-0 firm value, $F^s_{s_0,0} = B^s_{s_0,0} + S^s_{s_0,0}$, i.e.

$$c_{s_0,0} = \arg\max F^s_{s_0,0}(c).$$  \hfill (29)

The optimal default and capital structure decisions are numerically obtained by maximizing Equation (29) subject to the conditions stated in Equation (28). As a result, the optimal default boundaries depend on the debt policy, which is determined by the initial financing state. Hence, if the economy is in state $i$, the default boundary for nominal earnings is given by $X_{i}(c_{s_0,0})$, where $i$ denotes the dependence on the current state, the presence of $c_{s_0,0}$ denotes dependence on the coupon and hence on the state of the economy at date 0, $s_0$.

### 3.6 Equity risk premium and equity volatility

The model allows us to compute the state-$i$ equity risk premium, which is equal to (see Appendix A.7)

$$\mu_{R,i} - r^s_i = \sum_{j \neq i} (1 - \omega_{ij}) \sigma_{R,ij}^{P} \lambda_{ij},$$  \hfill (30)

where $\sigma_{R,ij} = \frac{s^j}{s^i} - 1$ is the conditional volatility of equity associated with a change in real states. Recall that only real risk is priced by the representative agent.

The conditional level of equity volatility in state $i$ is given by (see Appendix A.7)

$$\sigma_{R,i} = \sqrt{\left(\frac{\partial \ln S_{i,t}}{\partial \ln X_{t}} \sigma_{X,i}^{id} \right)^2 + \sum_{j \neq i} \lambda_{ij} \left(\sigma_{R,ij}^{P}\right)^2}$$  \hfill (31)

and thus accounts for the firm’s idiosyncratic Brownian shocks multiplied by the sensitivity of equity to cash flow, which is a positive function of financial leverage, and the Poisson shocks stemming from a change in states.

### 4 Calibration

We calibrate the model to investigate how nominal risk affects firm asset prices. The real states of the economy are characterized by the conditional moments of consumption growth. In the spirit of Bhamra et al. (2010a,b), we obtain the probability of transition from one real state to another, $\lambda_{\nu_i \rightarrow \nu_j}^\text{real}$,
using a two-state Markov-regime switching (MS) model. The MS model is estimated using quarterly U.S. real consumption jointly with real aggregate earnings data over the period 1970Q1-2016Q4.

We proxy for global consumption using data on real non-durable goods plus service consumption expenditures from the Bureau of Economic Analysis, and proxy for real cash-flows using quarterly US aggregate earnings from S&P provided by Robert J. Shiller’s website. The personal consumption expenditure chain-type price index is used to deflate nominal earnings. The conditional moments of real consumption and cash flow growth rates are reported in Table 1.10 In particular, the estimates of the actual probabilities of being in expansion and recession are respectively $f_E = f_1 + f_2 + f_3 = 81.8\%$ and $f_R = f_4 + f_5 + f_6 = 18.2\%$.

For the nominal states, following Sharpe (2002), we use the expected inflation from surveys of professional forecasters. We use a three-state MS model calibrated on quarterly data observed between 1970Q1-2016Q4. We impose the condition that the inflation rate in the moderate inflation regime is equal to its unconditional mean. We further discipline the chain by imposing that the stationary probabilities of being in low, moderate or high inflation regimes be respectively $f_L = f_1 + f_4 = 25\%$, $f_M = f_2 + f_5 = 50\%$, and $f_H = f_3 + f_6 = 25\%$. This ensures that asymmetries in asset pricing implications, if any, won’t be arising from asymmetric probabilities of being in the $L$ or $H$ inflation regimes.

For the firm parameters, the corporate tax rate is set to $\eta = 15\%$ and, in the baseline case, the liquidation value in default is $\alpha = 50\%$. Firms with lower (higher) liquidation value will endogenously chose lower (higher) leverage ratios, allowing us to study cross-sectional implications of the model for otherwise equal firms. We normalize the initial value of the cash flow to $X_0 = 1$. Regarding the preference parameters, we fix the risk aversion to $\gamma = 10$, the elasticity of intertemporal substitution (EIS) to $\psi = 2$, and the subjective discount factor to $\beta = 0.03$. Finally, the inflation passthrough is exogenously set at $\varphi = 0.5$.

Based on this calibration, the characteristics of the economy are as follows. The expected inflation rate is 1.96\% in the low inflation state, 3.54\% in moderate inflation, and 5.13\% during times of high inflation. The nominal growth rate of cash-flows varies between -9.91\% in the recession/low inflation state to 7.50\% in the expansion/high inflation state. The real-risk free rate is 2.83\% in recession and 4.00\% in expansion.

Following Bhamra et al. (2010a,b), we account for an additional 22.58\% of idiosyncratic volatility. The total cash flow volatility is thus approximately 25\% for our benchmark firm, which is the average volatility of firms with outstanding rated corporate debt.
5 Theoretical predictions

This section discusses the model predictions and sheds light on the role of inflation for corporate asset prices and default risk.

5.1 Characteristics of the economy

Table 3 reports the characteristics of an economy in which firms face variations in the nominal state. That is, the inflation rate can be low, moderate, or high. This economy features an unconditional equity risk premium of 1.99%, a stock return volatility of 43.3%, and a credit spread of 136.4 bps. As in Bhamra et al. (2010a,b) and Chen (2010), firms issue more debt in expansion and shareholders choose to default more rapidly in recession. Corporate claim values are all countercyclical with respect to real economic conditions. The model also generates asset-pricing moments that are higher in bad economic times, thus complementing other channels proposed in existing asset-pricing theory (Campbell and Cochrane, 1999; Bansal and Yaron, 2004; Gabaix, 2012; Wachter, 2013). The model predicts that the level of debt issuance is lowest in low inflation and highest in the high inflation state. In contrast, the default barriers are very similar across the nominal conditions. We now investigate how asset prices vary with expected inflation in our economy.

5.2 Implications for default risk and debt prices

Default risk is highest in a low-inflation environment. The credit spread increases from 121.4 bps to 132.5 bps from state EH to EL, which indicates that default risk is higher as inflation decreases. The reason is as follows. First, nominal cash flows grow less rapidly in times of low inflation. The deterioration in firm fundamentals increases the risk that firm cash flows reach one of the default boundaries. This is what we call the "cash-flow effect". Second, the nominal risk-free rate decreases as inflation slows down, which increases the present value of debt payments. Financial leverage then increases. This is what we call the "leverage effect". Both channels generate an increase default risk as inflation is reduced.
These effects occur because a firm’s capital structure is fixed and cannot be adjusted in the short term to a change in nominal conditions. As the economy moves into lower inflation, firms would benefit from a decrease in their debt level (e.g. through debt buyback), but are stuck with their existing debt issues. Leverage stickiness is thus key to understand why default risk varies with fluctuating nominal prices.

Turning to the value of debt, there are two opposing effects of inflation. On the one hand, the lower risk-free rate in low inflation increases the present value of the coupons accrued to debtholders, thereby increasing the market debt value for a given capital structure. On the other hand, the firm faces a higher default probability, which reduces the value of the debt claim given the presence of bankruptcy costs. Overall, we find that the first effect dominates. Hence, our model can explain why periods of low inflation are associated with higher debt value and credit risk at the same time.

5.3 Implications for equity valuation

The model predicts that equity prices are higher during low-inflation times compared to periods of high inflation (see Table 3). We now explain the forces driving the beneficial impact of lower inflation for equity investors.

As low inflation periods are associated with higher default risk, it is intuitive to predict lower equity values as well. A higher likelihood of default indeed reduces the present value of the cash-flows accrued to shareholders, as such investors recover nothing in default. However, a third mechanism goes in opposite direction and generates an increase in equity prices when inflation slows down: the nominal risk-free rate is more sensitive than the cash-flow discount rate to changes in inflation. This is due to the incomplete inflation passthrough of firm cash flows. We find that this channel dominates, thus predicting that equity prices decrease with the inflation rate.\(^\text{11}\)

Our model thus explains why increases in inflation are negative news for shareholders, while a decrease in inflation rate is good news. We highlight two central mechanisms of nominal risk: an increase in default risk and a decrease in the nominal discount rate during low inflation times. Both effects go in the opposite direction for debt and equity valuation. Yet the second effect dominates, which leads disinflation to have a positive real effects on financial asset prices. Our results thus complement the analysis of Kang and Pflueger (2015), whose focus is on credit risk.

\(^{11}\)The possibility of being in the high inflation regime affects shareholders through the same channels, but in the reverse direction.
5.4 **Low vs. high inflation: an asymmetric impact**

We now separately analyze the risk of low and high inflation and highlight an asymmetry that is consistent with the data: low inflation is not the mere mirror image of high inflation.

Low and high inflation have sizeable impacts on shareholder wealth, but in opposite directions. Table 3 indicates that, in expansion, the value of equity increases by 9.73% (from 12.63 to 13.86) when the economy switches from the moderate inflation state into low inflation. In contrast, the value of equity is only reduced by 3.32% (from 12.63 to 12.21) when the economy enters the high inflation state. The message is qualitatively similar when considering the recession state. The quantitative predictions are not of equal size. Figure 2 illustrates this non-linear effect clearly, not only for equity value, but also for the value of debt, and the level of credit spreads. Hence, the risk of facing low inflation in the future more severely affects asset prices than the risk of high inflation, although both states are equally likely.

![Figure 2](about here)

The decomposition of the variations in nominal prices into low and high inflation risk thus suggests the presence of a strong asymmetric effect. The risk of facing high inflation reduces the value of equity less than the possibility that low inflation increases it. The reason is that a shift in nominal risk-free rates impact equity and debt values non-linearly via cash-flow and coupon discounting.

5.5 **Impact of nominal risk on asset prices**

We now show how asset prices vary with the presence of nominal risk, which relates to the risk of future low and high inflation. To do so, we compare the results of the full model with the case in which we switch off variations in the nominal state. In this specification, the expected inflation rate is set at its unconditional mean, which corresponds to the “moderate inflation” regime. The firm characteristics in the absence of nominal risk are reported in Table 2. The credit spread is equal to 133.9 bps, the equity risk premium is 1.99%, and the level of stock return volatility is 43.2%. These values are very close to those of the full model. Although nominal risk does not appear to affect these asset pricing moments, on average, it affects the prices of corporate assets.

![Table 2](about here)
We report in Table 4 the differences in asset prices with and without nominal risk. The results indicate that the risk of fluctuating nominal prices increases the value of equity. Based on our calibration to the U.S. economy, the impact associated with nominal risk amounts to 0.62% of shareholder wealth, on average. For a market capitalization of listed companies representing US$19.3 trillion at the NYSE (as of June 2016), the unconditional increase in equity value would represent a gain of US$119.7 billion. The impact of nominal risk for equity investors is thus economically important, on average.

Our theory suggests that nominal risk exerts a positive influence on equity valuation. This arises because high inflation periods are relatively less negative for shareholders than periods of low inflation are positive. Hence, the asymmetric impact of nominal conditions on asset prices is fundamental to understand the asset-pricing implications of nominal risk.

It is worth noting that the impact of nominal risk greatly varies over time. The role of nominal risk is predicted to be strongest during the recession/low inflation state, which corresponds to times of high leverage and lower nominal cash flow growth. In this state, nominal risk raises equity value by up to 8.58%. In contrast, nominal risk reduces equity prices by 4.33% in the expansion/high inflation state. Debt values are also more sensitive to nominal risk in low inflation than in high inflation, and in expansions than in recessions. As such, the model predicts that nominal risk affects asset prices asymmetrically over time. The effect is countercyclical with respect to nominal conditions but procyclical with respect to real economic conditions. Fluctuations in nominal prices thus greatly amplifies the time variation in asset prices.

5.6 Cross-sectional predictions

The model suggests strong firm heterogeneity in the sensitivity of equity prices to inflation. In the cross-section, equity prices of firms with higher leverage are less impacted by variations in inflation. Two effects tend offset each other. On one hand, because of the incomplete inflation passthrough, lower inflation increases the present value of cash flows and thus the firm value. On the other hand, the value of debt also increases. This second effect is driven by the sticky leverage channel, which is naturally stronger for high leverage firms. Hence, for such firms, equity valuation (determined by firm value minus debt value) becomes less sensitive to changes in inflation. Consequently, the presence of debt reduces, rather than exacerbates, the sensitivity of stock prices to changes in nominal conditions.
We reach a similar conclusion for credit spreads. Nominal fluctuations thus have greater asset pricing implications for firms that are less levered (see Figure 3).

Firms vary in their optimal capital choices for two main reasons within the model. On the one hand, a firm facing greater bankruptcy cost will exhibit a lower expected liquidation value. In default, debtholders are entitled to the unlevered firm value net of the liquidation costs. As debtholders recover less in default, debt is priced at a higher credit spread, which decreases the incentives for high debt issuance. Hence, the model predicts that shareholders of firms with lower bankruptcy costs should be less sensitive to nominal risk. On the other hand, firms choose to issue less debt when the capital structure is set in a more uncertain and slowing economy, as characterized by the recession state. Hence, the model predicts that firms financed during an economic expansion should display less sensitivity to nominal prices.

5.7 Discussion

Our model implies that higher inflation is expected to negatively affect equity prices, although such risk is not priced by the representative agent in our environment. Hence, none of our predictions are driven by the presence of an inflation risk premium. In fact, we intentionally model preferences such that they are completely independent of the nominal state. Investors have a preference for early resolution of uncertainty regarding the future (real) consumption regimes but not for the nominal regimes. Hence, only the quantity of risk drives our predictions for equity valuation. Overall, nominal conditions have no effect on the representative agent’s pricing kernel, but nonetheless have a non-trivial influence on asset prices.

The key channel for default risk is the presence of sticky leverage, while the incomplete inflation passthrough determines the negative sensitivity of equity prices to inflation. As Figure 4 shows, high inflation passthrough would indeed revert the relation between equity prices and inflation. To sum up, both the sticky-prices and the sticky-leverage channels are necessary ingredients to generate relations between inflation, equity valuation, and default risk that are consistent with the data.
6 Empirical validation

Under revision...

7 Conclusion

We have built a theoretical model showing how nominal risk impacts real asset prices via the sticky profitability and leverage channels. The key mechanism at play is that long-term nominal debt coupons are fixed, but inflation is risky. This assumption makes expected future real debt coupons dependent on future inflation, ensuring that nominal risk impacts real corporate bond values and hence default risk. In addition, earning growth is less sensitive than the nominal risk-free rate to variations in expected inflation, which induces stickiness in firm profitability. This feature makes equity prices convex in the nominal risk-free rate, which varies with the inflation rate. Our model thus implies that lower inflation simultaneously increases real asset values and default risk. This effect is stronger for firms with less leverage. Importantly, a shift towards lower inflation has a greater negative impact on real equity values than an equal shift towards inflation. The theoretical predictions of our model are supported in the data, lending credence to the idea that the sticky leverage and price channels are important for understanding the economics behind how inflation risk impact real asset values and corporate default risk.

In our modelling framework the stochastic discount factor is exogenous. As such, understanding how default clusters caused by lower inflation impact aggregate consumption and hence the stochastic discount factor, which will in turn impact default decisions is beyond the scope of this paper. Understanding the general equilibrium implications for how inflation risk can amplify default risk and its impact on real asset values would be an interesting topic for future research.
References


APPENDIX

A.1  The Economy

First, we introduce some notation related to jumps in the state of the economy. Suppose that during the small time-interval \([t - \Delta t, t)\) the economy is in state \(i\) and that at time \(t\) the state changes, so that during the next small time interval \([t, t + \Delta t)\) the economy is in state \(j \neq i\). We then define the left-limit of \(\nu\) at time \(t\) as

\[
\lim_{\Delta t \to 0} s_{t-} = s_{t-}\Delta t,
\]

(A.1)

and the right-limit as

\[
\lim_{\Delta t \to 0} s_{t+} = s_{t+}\Delta t.
\]

(A.2)

Therefore \(s_{t-} = i\), whereas \(s_t = j\), so the left- and right limits are not equal. If some function \(E\) depends on the current state of the economy i.e. \(E_t = E(s_t)\), then \(E\) is a jump process which is right continuous with left limits, i.e. RCLL. If a jump from state \(i\) to \(j \neq i\) occurs at date \(t\), then we abuse notation slightly and denote the left limit of \(E\) at time \(t\) by \(E_i\), where \(i\) is the index for the state. i.e. \(E_{t-} = \lim_{s \to t} E_s = E_i\). Similarly \(E_t = \lim_{s \to t} E_s = E_j\). We shall use the same notation for all processes that jump, because of their dependence on the state of the economy.

Using simple algebra we can write the normalized Kreps-Porteus aggregator in the following compact form:

\[
f(c, v) = \beta \left( h^{-1}(v) \right)^{1-\gamma} u \left( c/h^{-1}(v) \right),
\]

(A.3)

where

\[
u(x) = \frac{x^{1-1/\psi} - 1}{1 - 1/\psi}, \psi > 0,
\]

\[
h(x) = \begin{cases} x^{1-\gamma} & \gamma \geq 0, \gamma \neq 1, \\ \ln x, & \gamma = 1. \end{cases}
\]

The representative agent’s value function is given by

\[
J_t = E_t \int_t^\infty f(C_t, J_t) \, dt.
\]

(A.4)

**Proposition A.1** The SDF of a representative agent with the continuous-time version of Epstein-Zin-Weil preferences is given by

\[
\pi_t = \begin{cases} \left( \beta e^{-\beta t} \right)^{1-\psi} C_t^{-\gamma} \left( p_{C,t} e^{\int_0^t \frac{1 - \beta}{\psi} ds} \right)^{-\gamma - 1/\psi}, & \psi \neq 1, \\ \beta e^{-\beta \int_0^t [1 + (\gamma - 1) \ln(V_s^{-1})] ds} C_t^{-\gamma} V_t^{-\gamma - 1}, & \psi = 1. \end{cases}
\]

(A.5)
When $\psi \neq 1$, the price-consumption ratio in state $i$, $p_{C,i}$, satisfies the nonlinear equation system:

$$p_{C,i}^{-1} = r_i + \gamma \sigma_{C,i}^2 - g_i - \left(1 - \frac{1}{\psi}\right) \sum_{j \neq i} \lambda_{ij} \left(\frac{(p_{C,j}/p_{C,i})^{1-\psi} - 1}{1 - \gamma}\right), \quad i \in \{1, \ldots, N\}. \quad (A.6)$$

where

$$r_i = \beta + \frac{1}{\psi} g_i - \frac{1}{2} \gamma \left(1 + \frac{1}{\psi}\right) \sigma_{C,i}^2, \quad i \in \{1, \ldots, N\}. \quad (A.7)$$

When $\psi = 1$, define $V_i$ via

$$J = \ln(CV). \quad (A.8)$$

Then $V_i$ satisfies the nonlinear equation system:

$$\beta \ln V_i = g_i - \frac{\gamma}{2} \sigma_{C,i}^2 + \lambda_i \left(\frac{(V_j/V_i)^{1-\gamma} - 1}{1 - \gamma}\right), \quad i \in \{1, 2\}, \quad j \neq i. \quad (A.9)$$

### A.2 Derivation of the real SDF

In this section, we derive the real SDF shown in Proposition A.1. When we refer to states of the economy within this proof we mean only the real states, $L$, $H$.

Duffie and Skiadas (1994) show that the SDF for a general normalized aggregator $f$ is given by

$$\pi_t = e^{\int_0^t f_v(C_s, J_s) dt} f_c(C_t, J_t), \quad (A.10)$$

where $f_c(\cdot, \cdot)$ and $f_v(\cdot, \cdot)$ are the partial derivatives of $f$ with respect to its first and second arguments, respectively, and $J$ is the value function given in (A.4). The Feynman-Kac Theorem implies

$$f(C_t, J_t)_t|_{\nu_t = i} dt + E_t[dJ_t|_{\nu_t = i}] = 0, \quad i \in \{L, H\}.$$  

Using Ito’s Lemma we rewrite the above equation as

$$0 = f(C, J_i) + C J_i g_i + \frac{1}{2} C^2 J_i CC \sigma_{C,i}^2 + \lambda_i (J_j - J_i), \quad (A.11)$$

for $i, j \in \{1, 2\}, \quad j \neq i$. We guess and verify that $J = h(CV)$, where $V_i$ satisfies the nonlinear equation system

$$0 = \beta u \left(V_i^{-1}\right) + g_i - \frac{1}{2} \gamma \sigma_{C,i}^2 + \lambda_i \left(\frac{(V_j/V_i)^{1-\gamma} - 1}{1 - \gamma}\right), \quad i, j \in \{L, H\}, \quad j \neq i. \quad (A.12)$$

Substituting (A.3) into (A.10) and simplifying gives

$$\pi_t = \beta e^{-\beta \int_0^t \left[1 + \left(\gamma - \frac{1}{\psi}\right) u(V_s^{-1})\right] dt} C_t^{-\gamma} V_t^{-\left(\gamma - \frac{1}{\psi}\right)}. \quad (A.13)$$
When \( \psi = 1 \), the above equation gives the second expression in (A.5). We rewrite (A.12) as

\[
\beta \left[ 1 + \left( \gamma - \frac{1}{\psi} \right) u \left( V_i^{-1} \right) \right] = \tau_i - \left( \gamma - \frac{1}{\psi} \right) \lambda_i \left( \frac{(V_j/V_i)^{1-\gamma} - 1}{1 - \gamma} \right) - \left[ \gamma g_i - \frac{1}{2} \gamma (1 + \gamma) \sigma_{C,i}^2 \right], \quad i, j \in \{L, H\}, j \neq i,
\]

(A.14)

where \( \tau_i \) is given in (A.7). Setting \( \psi = 1 \) in (A.14) gives (A.9). To derive the first expression in (A.5) from (A.13) we prove that

\[
V_i = (\beta p_{C,i})^{\frac{1}{1 + \psi}}, \quad \psi \neq 1.
\]

(A.15)

We proceed by considering the optimization problem for the representative agent. She chooses her optimal consumption, \( C^* \), and risky asset portfolio, \( \varphi \), to maximize her expected utility

\[
J_t^* = \sup_{C^*, \varphi} E_t \int_t^\infty f \left( C_t^*, J_t^* \right) dt.
\]

Observe that \( J^* \) depends on optimal consumption-portfolio choice, whereas the \( J \) defined previously in (A.8) depends on exogenous aggregate consumption. The optimization is carried out subject to the dynamic budget constraint, which we now describe. If the agent consumes at the rate, \( C^* \), invests a proportion, \( \varphi \), of her remaining financial wealth in the claim on aggregate consumption (the risky asset), and puts the remainder in the locally risk-free asset, then her financial wealth, \( W \), evolves according to the dynamic budget constraint:

\[
\frac{dW_t}{W_{t-}} = \varphi_{t-} (dR_{C,t} - r_{t-} dt) + r_{t-} dt - \frac{C_t^*}{W_{t-}} dt,
\]

where \( dR_{C,t} \) is the cumulative return on the claim to aggregate consumption. We define \( N_{i,t} \) as the Poisson process which jumps upward by one whenever the real state of the economy switches from \( i \) to \( j \neq i \). The compensated version of this process is the Poisson martingale

\[
N_{i,t}^P = N_{i,t} - \lambda_{i,t}, \quad i \in \{L, H\}
\]

It follows from applying Ito’s Lemma to \( P = p_{C} C \) that the cumulative return on the claim to aggregate consumption is

\[
dR_{C,t} = \frac{dP_t + C_t dt}{P_{t-}} = \mu_{RC,t-} dt + \sigma_{C,t-} dB_{C,t} + \sigma_{R_{C,t-}}^P dN_{v_{t-},t}^P,
\]

where

\[
\mu_{RC,t-} \bigg|_{v_{t-} = i} = \mu_{RC,i} = g_i + \frac{1}{2} \sigma_{C,i}^2 + \lambda_i \left( \frac{p_{C,j}}{p_{C,i}} - 1 \right) + \frac{1}{p_{C,i}},
\]

\[
\sigma_{C,t-} \bigg|_{v_{t-} = i} = \sigma_{C,i},
\]

\[
\sigma_{R_{C,t-}}^P \bigg|_{v_{t-} = i} = \sigma_{R_{C,i}}^P = \frac{p_{C,j}}{p_{C,i}} - 1,
\]

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for \( i \in \{L, H\}, j \neq i \). The total volatility of the return to holding the consumption claim, when the current state is \( i \), is given by
\[
\sigma_{RC,i} = \sqrt{\sigma_{C,i}^2 + \lambda_i \left( \sigma_{RC,i}^p \right)^2}.
\]

Note that \( C^* \) is the consumption to be chosen by the agent, i.e. it is a control, and at this stage we cannot rule out the possibility that it jumps with the state of the economy. In contrast, \( C \) is aggregate consumption, and since it is continuous, its left and right limits are equal, i.e. \( C_{t-} = C_t \).

The system of Hamilton-Jacobi-Bellman partial differential equations for the agent’s optimization problem is
\[
\sup_{C^*, \varphi} f \left( C^*, J^* \right) \bigg|_{\nu_{t-} = i} \ dt + E_t \left[ dJ^*_t \mid \nu_{t-} = i \right] = 0, \ i \in \{L, H\}.
\]

Applying Ito’s Lemma to \( J^*_t = h(W_t, \nu_t) \) allows us to write the above equation as
\[
0 = \sup_{C^*, \varphi} f \left( C^*, J^* \right) + W_i J^*_t W_i \left( \varphi_i (\mu_{RC,i} - r_i) + r_i - \frac{C^*_i}{W_i} \right) + \frac{1}{2} W_i^2 J^*_t W_i \varphi_i^2 \sigma_{RC,i}^2 + \lambda_i \left( J^*_j - J^*_i \right), \ i \in \{L, H\}, j \neq i.
\]

We guess and verify that \( J^*_t = h(W_t, F_t) \), where \( F_t \) satisfies the nonlinear equation system
\[
0 = \sup_{C^*, \varphi} \beta u \left( \frac{C^*_i}{W_i F_t} \right) + \left( \varphi_i (\mu_{RC,i} - r_i) + r_i - \frac{C^*_i}{W_i} \right) - \frac{1}{2} \gamma \varphi_i^2 \sigma_{RC,i}^2 + \lambda_i \left( \frac{(F^*_j/F^*_i)^{1-\gamma} - 1}{1-\gamma} \right), \ i \in \{L, H\}, j \neq i.
\]

From the first order conditions of the above equations, we obtain the optimal consumption and portfolio policies:
\[
C^*_i = \beta \psi_i F^{-\left(\psi-1\right)}_i W_i, \ i \in \{L, H\},
\]
\[
\varphi_i = \frac{\mu_{RC,i} - r_i}{\gamma_i \sigma_{RC,i}^2}, \ i \in \{L, H\}.
\]

The market for the consumption good must clear, so \( \varphi_i = 1, W_i = P_t, C^*_i = C \) (and thus \( J = J^* \)). Note that this forces the optimal portfolio proportion to be one and the optimal consumption policy to be continuous. Hence
\[
\mu_{RC,i} - r_i = \gamma \sigma_{RC,i}^2,
\]
and
\[
p_{C,i} = \beta^{-\psi} F_1^{1-\psi}. \tag{A.16}
\]

The above equation implies that for \( \psi = 1, p_{C,i} = 1/\beta \). The equality, \( J = J^* \), implies that \( CV_i = WF_i \). Hence, \( F_i = p_{C,i}^{-1} V_i \). Using this equation to eliminate \( F_i \) from (A.16) gives (A.15). Substituting (A.15) into (A.13) and (A.14) gives the expression in (A.5) for \( \psi \neq 1 \) and (A.6).
A.3 Liquidation Value

The abandonment or liquidation value of a firm is just its unlevered value, i.e. the present value of future cashflows, ignoring coupon payments to debtholders and default risk. Small, but frequent shocks to a firm’s real cashflow growth are modelled by changes in the standard Brownian motion $Z^{id}_t$, where we omit the subscript $n$ for ease of notation. Small, but frequent shocks to the real SDF are modelled by changes in the standard Brownian motion $Z_t$. The assumption that $dZ_t dZ^{id}_t = 0$ means that small, but frequent shocks to cashflow growth are not priced. However, changes in the expected real cashflow growth rate are driven by the same Markov chain as those driving jumps in the SDF. Hence, changes in unlevered firm value driven by changes in the expected real cashflow growth rate will be priced.

Suppose the economy is currently in state $i$. Then, the risk-neutral probability of the economy switching into state $j \neq i$ different state during a small time interval $\Delta t$ is $\lambda_{ij} \Delta t$ and the risk-neutral probability of not switching is $1 - \lambda_{ij} \Delta t$. We can therefore write the unlevered real firm value in state $i$ as

$$A_i = (1 - \eta)X\Delta t + e^{-(r_i - \mu_Y i)\Delta t} \left[ (1 - \lambda_{ij} \Delta t) A_i + \sum_{j \neq i} \lambda_{ij} \Delta t A_j \right], \quad i, j \in \{1, 2, 3, 4\}, j \neq i. \quad (A.17)$$

The first term in (A.17) is the after-tax cash flow received in the next instant and the second term is the discounted continuation value. The discount rate is just the standard discount rate for a perpetuity. Observe that the volatility of real cashflow growth does not appear in the discount rate, because $dZ_t dZ^{id}_t = 0$. The continuation value is the average of $A_i$ and $A_j$, weighted by the risk-neutral probabilities of being in states $i$ and $j \neq i$ a small instant $\Delta t$ from now. For example, with risk-neutral probability $\lambda_{ij} \Delta t$ the economy will be in state $j \neq i$ and the unlevered real firm value will be value will be $A_j$. The continuation value is discounted back at a rate reflecting the discount rate $\mu_i$ and the expected earnings growth rate over that instant which is $\theta_i$.

We take the limit of (A.17) as $\Delta t \to 0$, to obtain

$$0 = (1 - \eta)X - (r_i - \mu_Y i)A_i + \sum_{j \neq i} \lambda_{ij} (A_j - A_i), \quad i \in \{1, 2, 3, 4\}, j \neq i.$$ 

To obtain the solution of the above linear equation system, we define

$$v_{A,i} = \frac{1}{(1 - \eta)X} A_i.$$
the before-tax price-earnings ratio in state $i$. Therefore

$$
\begin{pmatrix}
\text{diag} (r_1 - \mu_{Y,1}, \ldots, r_4 - \mu_{Y,4}) - \tilde{\Lambda}
\end{pmatrix}
\begin{pmatrix}
v_{A,1} \\
v_{A,2} \\
v_{A,3} \\
v_{A,4}
\end{pmatrix} =
\begin{pmatrix}
1 \\
1 \\
1 \\
1
\end{pmatrix},
\tag{A.18}
$$

where $\text{diag} (r_1 - \mu_{Y,1}, \ldots, r_4 - \mu_{Y,4})$ is a $4 \times 4$ diagonal matrix, with the quantities $r_1 - \mu_{Y,1}, \ldots, r_4 - \mu_{Y,4}$ along the diagonal and $\tilde{\Lambda}$, defined by $[\tilde{\Lambda}]_{ij} = \hat{\lambda}_{ij}, i, j \in \{1, 2, 3, 4\}$, where

$$
\hat{\lambda}_{ij} = \omega_{ij} \lambda_{ij}, j \neq i
\tag{A.19}
$$

and

$$
\hat{\lambda}_{ii} = -\sum_{j \neq i} \omega_{ij} \lambda_{ij}, j \neq i
\tag{A.20}
$$
is the generator matrix of the Markov chain under the risk-neutral measure. Solving (A.18) gives (18), if $\det \left( \text{diag} (r_1 - \mu_{Y,1}, \ldots, r_4 - \mu_{Y,4}) - \tilde{\Lambda} \right) \neq 0$. We now define $P_i^X = p_i X$, the before-tax value of the claim to the earnings stream $X$ in state $i$. Hence, from the basic asset pricing equation

$$
E_t \left[ \frac{dP_i^X + X dt}{P_i^X} - r dt \middle| \nu_t = i \right] = -E_t \left[ \frac{d\pi}{\pi} \frac{dP_i^X}{P_i^X} \middle| \nu_t = i \right],
$$

we obtain the unlevered risk premium:

$$
E_t \left[ \frac{dP_i^X + X dt}{P_i^X} - r dt \middle| \nu_t = i \right] = \gamma \rho_{XC} \sigma_{X,i} \sigma_{C,j} dt - \left( \hat{\lambda}_i - \lambda_i \right) \left( \frac{p_j}{p_i} - 1 \right) dt, i \in \{1, 2\}, j \neq i.
$$

Applying Ito’s Lemma,

$$
dP_i^X = p_i dX_t + \lambda_i (p_j - p_i) dt + (p_j - p_i) dN_{i,j}^P, i \in \{1, 2\}, j \neq i.
$$

Thus, the unlevered volatility of returns on equity in state $i$ is given by

$$
\sigma_{R,i} = \sqrt{\sigma_{X,i}^2 + \lambda_i \left( \frac{p_j}{p_i} - 1 \right)^2}, j \neq i,
$$

where $\sigma_{X,i} = \sqrt{(\sigma_{X,i}^{d})^2 + (\sigma_{X,i}^{d})^2}$.

A.4 Arrow-Debreu Securities

The Arrow-Debreu default claim denoted by $q_{D,ij,t}$ is the value of a unit of consumption paid if default occurs in state $j$ and the current state is $i$. There are $N^2$ such claims in our economy: $\{q_{D,ij}\}_{i,j \in \{1, \ldots, N\}}$. 

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We assume, without loss of generality, that the regimes are labelled so that the default boundaries respect a monotonic ordering $X_{D,1} > X_{D,2} > X_{D,3} > X_{D,4}$.

We say that a firm’s earnings are in region $k$, $k = 0, \ldots, N - 1$, when they fall in the interval $[X_{D,k+1}, X_{D,k})$, assuming that $X_{D,0} \rightarrow \infty$. Region $N$ is $(\infty, X_{D,N})$.

**Proposition A.2** Let $A_k$ be

$$A_k = \begin{pmatrix} 0_{N-k\times N-k} & -I_{N-k\times N-k} \\ 2S_{x,k}^{-1}(\beta_k - R_k) & 2S_{x,k}^{-1}M_{x,k} \end{pmatrix},$$

where $0_{n\times m} \in \mathbb{R}^{n\times m}$ denotes a matrix of zeros, $I_{n\times n} \in \mathbb{R}^{n\times n}$ denotes the $n$-dimensional identity matrix, $\beta_k$ is the $N - k$ by $N - k$ matrix obtained by removing the first $k$ rows and columns of $\beta$, and

$$S_{x,k} = \text{diag}(\sigma_{x,k+1}^2, \ldots, \sigma_{x,N}^2), \quad R_k = \text{diag}(r_{k+1}, \ldots, r_N), \quad M_{k,\lambda} = \text{diag}(\mu_{y,k+1}, \ldots, \mu_{y,N}),$$

are $N - k$ by $N - k$ diagonal matrices, with $\hat{\mu}_{k,i} = \hat{\mu}_{X,i} - \frac{1}{2} \sigma_{X,i}^2$ and $\sigma_{X,i}$ the drift and diffusion coefficient of $x = \log X$ under the risk-adjusted measure.

Given the integration constants $h_{i,j}(\omega)$, the Arrow-Debreu in region $k$ are given by

**Region 0:**

$$q_{D,0,ij}(x) = \sum_{\ell=1}^{N} h_{ij}(\omega_{0,\ell}) e^{-\omega_{0,\ell} x}, \quad \text{(A.21)}$$

where $\omega_{0,1} > \ldots > \omega_{0,N} > 0$ are the $N$ positive eigenvalues of $A_0$.

**Region $k \in \{1, \ldots, N - 1\}$:**

$$q_{D,k,ij}(x) = \delta_{ij}, \quad \text{if } i \in \{1, \ldots, k\}, j \in \{1, \ldots, N\},$$

$$q_{D,k,ij}(x) = 2(N-k) \sum_{\ell=1}^{2(N-k)} h_{ij}(\omega_{\ell}) e^{-\omega_{\ell} x} - [A_k^{-1}B_k]_{i,k-j}, \quad i \in \{k+1, \ldots, N\}, j \in \{1, \ldots, N\}. \quad \text{(A.22)}$$

where $\omega_{k,\ell}$ are the $2(N-k)$ positive eigenvalues of $A_k$ and

$$B_k = \begin{pmatrix} 0_{N-k\times N-k} & 0_{N-k\times N-k} \\ B_k^N & 0_{N-k\times N-k} \end{pmatrix}, \quad B_k^N = \begin{pmatrix} 2\gamma_{k+1,1} & 2\gamma_{k+1,2} & \ldots & 2\gamma_{k+1,k} \\ \frac{\sigma_{k+1}^2}{\sigma_{k+1}^2} & \frac{\sigma_{k+1}^2}{\sigma_{k+1}^2} & \ldots & \frac{\sigma_{k+1}^2}{\sigma_{k+1}^2} \\ 2\gamma_{k+2,1} & 2\gamma_{k+2,2} & \ldots & 2\gamma_{k+2,k} \\ \frac{\sigma_{k+2}^2}{\sigma_{k+2}^2} & \frac{\sigma_{k+2}^2}{\sigma_{k+2}^2} & \ldots & \frac{\sigma_{k+2}^2}{\sigma_{k+2}^2} \\ \vdots & \vdots & \ddots & \vdots \\ 2\gamma_{N,1} & 2\gamma_{N,2} & \ldots & 2\gamma_{N,k} \\ \frac{\sigma_{N}^2}{\sigma_{N}^2} & \frac{\sigma_{N}^2}{\sigma_{N}^2} & \ldots & \frac{\sigma_{N}^2}{\sigma_{N}^2} \end{pmatrix}. \quad \text{(A.23)}$$
Region N:

\[ q_{D,k,ij}(x) = \delta_{ij}, \quad \forall i, j. \]  \hspace{1cm} (A.23)

In each region, for each \( \omega \), the integration constants \( h_{k+1,\cdot}(\omega) \equiv [h_{k+1,j}(\omega)]_{j=1,...,N} \in \mathbb{R}^{1 \times N} \), are identified by the boundary conditions (Section A.4.3), and the remaining integration constants

\[
\Pi_k(\omega) = \begin{pmatrix}
  h_{k+1,1}(\omega) & \cdots & h_{k+1,N}(\omega) \\
  \vdots & \ddots & \vdots \\
  h_{N,1}(\omega) & \cdots & h_{N,N}(\omega)
\end{pmatrix}
\]

are given by

\[
\Pi_k(\omega) = G_k^{-1}(\omega) g_{k+1,\cdot}(\omega) h_{k+1,\cdot}(\omega)
\]  \hspace{1cm} (A.24)

where \( g_{k+1,k+1}(\omega) \equiv [g_{i,k+1}(\omega)]_{i=k+2,...,N} \in \mathbb{R}^{(N-k-1) \times 1} \) comprises the last \( N - k - 1 \) elements of the first column of

\[
G(\omega) = 2S^{-1}_x(\Lambda - R) - \omega(2S^{-1}_x M_x - \omega I_{N \times N}).
\]  \hspace{1cm} (A.26)

and \( G_k(\omega) \) is the \( N - k - 1 \) by \( N - k - 1 \) matrix obtained by removing the first \( k + 1 \) rows and columns of \( G(\omega) \).

The next two subsections outline the proof of Proposition A.2.

A.4.1 Region 0: \( X_t \geq X_{D,1} \)

We start by analyzing the case where earnings at the current date \( t \) are above the highest default boundary, i.e. \( X_t > X_{D,1} \). Hence, if earnings hit the boundary \( X_{D,j} \) from above for the first time in state \( j \), \{\( q_{D,ij} \)\}_{i,j} \in \{1,...,N\} will pay one unit of consumption; otherwise, the security expires worthless. Since each Arrow-Debreu default claim is effectively a perpetual digital put, their values can be derived by solving a system of ordinary differential equations, derived from the standard equations

\[
E_t^Q[dq_{D,ij} - r_i q_{D,ij} dt] = 0, \quad i, j \in \{1, \ldots, N\}.
\]  \hspace{1cm} (A.27)

Using Ito’s Lemma, the above equation can be rewritten as the following second-order ordinary differential-equation system:\footnote{Note that since the puts are perpetual, \( \frac{\partial q}{\partial t} = 0 \). Hence, \( q \) is solely a function of the stochastic process \( x = \log X \).
}

\[
\frac{1}{2} \sigma^2 \frac{d^2 q_{D,ij}}{dx^2} + \mu \frac{dq_{D,ij}}{dx} + \sum_{k \neq i} \lambda_{ik} (q_{D,kj} - q_{D,ij}) = r_i q_{D,ij}, \quad i, j \in \{1, \ldots, N\},
\]  \hspace{1cm} (A.28)
where $\mu_{x,i} = \tilde{\mu}_{X,i} - \frac{1}{2}\sigma_{X,i}^2$ and $\sigma_{x,i} = \sigma_{X,i}$ are the drift and diffusion coefficient of $x = \log X$ under the risk-adjusted measure.

In order to solve this system of ODEs, define

$$z_{ij} = q_{D,ij}, i, j \in \{1, \ldots, N\} \tag{A.29}$$

$$z_{N+i, j} = \frac{dq_{D,ij}}{dx}, i, j \in \{1, \ldots, N\}. \tag{A.30}$$

Then, we obtain the following first order linear system

$$\frac{dz_{ij}}{dx} - z_{N+i, j} = 0, i, j \in \{1, \ldots, N\}, \tag{A.31}$$

$$\frac{dz_{N+i,j}}{dx} + \frac{2\tilde{\mu}_{x,i}}{\sigma_{x,i}^2}z_{N+i,j} + \sum_{k \neq i} \frac{2\lambda_{ik}}{\sigma_{x,i}^2}(z_{kj} - z_{ij}) - \frac{2r_i}{\sigma_{x,i}^2}z_{ij} = 0, i, j \in \{1, \ldots, N\}.$$

Expressing the above equation system in matrix form gives

$$Z' + A_0 Z = 0_{2N \times N}, \tag{A.32}$$

where the $ij$’th element of the $2N$ by $N$ matrix, $Z$, is

$$[Z]_{ij} = z_{ij}, i \in \{1, \ldots, 2N\}, j \in \{1, \ldots, N\}, \tag{A.33}$$

and $Z' = \frac{dZ}{dx}$.

To solve eq. (A.32), one first finds the eigenvectors and eigenvalues of $A_0$. Their defining equation is

$$A_0 e_i = \omega_i e_i, i \in \{1, \ldots, 2N\}, \tag{A.34}$$

where $\omega_i$ is the $i$’th eigenvalue and $e_i$ is the corresponding eigenvector. Note that $A_0$ has $N$ positive and $N$ negative eigenvalues (Jobert and Rogers, 2006).

It follows from (A.34) that the eigenvalues of $A_0$ are the roots of its characteristic polynomial; that is, any eigenvalue $\omega$ is a solution to the following $2N$’th-order polynomial:

$$\det(A_0 - \omega I) = 0,$$

To simplify the above expression for the characteristic polynomial, we then use the following identity from Silvester: If $F = \begin{pmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{pmatrix}$, where $F_{ij}, i, j \in \{1, 2\}$ are $N$ by $N$ matrices, any two of which commute with each other, then

$$\det F = \det(F_{11}F_{22} - F_{12}F_{21}). \tag{A.35}$$
Since
\[ A_0 - \omega I = \begin{pmatrix} -\omega I & -I \\ 2S_x^{-1}(\tilde{\Lambda} - R) & 2S_x^{-1}M_x - \omega I \end{pmatrix} \]
and diagonal matrices commute with all other matrices of the same size, any pair of the 4 submatrices in \( A_0 \) commute. Therefore, one can apply (A.35) and
\[ 0 = \det(A_0 - \omega I) = \det(\omega I - 2S_x^{-1}M_x) - 2S_x^{-1}(\tilde{\Lambda} - R)). \tag{A.36} \]
When \( N \leq 2 \) the above polynomial is of order 4 or less and can be solved exactly in closed-form. When \( N \geq 3 \), it must be solved numerically. Once the eigenvalues have been obtained, the eigenvectors are obtained by solving (A.34). We then define the \( 2N \) by \( 2N \) matrix of eigenvectors, \( E \), by stacking the eigenvectors as follows
\[ E = (e_1, \ldots, e_{2N}). \]
Hence, the \( ij \)'th component of \( E \) is the \( i \)'th element of the \( j \)'th eigenvector, i.e.
\[ E_{ij} = (e_j)_i. \]
Given the \( E \) matrix, we can define the \( 2N \) by \( N \) matrix \( W \) via
\[ EW = Z. \tag{A.37} \]
We can then rewrite eq. (A.32) as
\[ EW' + A_0 EW = 0_{2N \times N}, \tag{A.38} \]
\[ \Leftrightarrow E^{-1}(EW' + A_0 EW) = W' + E^{-1}A_0 EW = 0_{2N \times N}, \tag{A.39} \]
\[ \Leftrightarrow W' + DW = 0_{2N \times N}. \tag{A.40} \]
where
\[ D = E^{-1}A_0 E = \text{diag}(\omega_1, \ldots, \omega_{2N}). \tag{A.41} \]
The first order differential equation system of eq. (A.40) is similar to that of eq. (A.32), with the notable difference that \( D \) is a diagonal matrix while \( A_0 \) isn’t. Making use of this, we premultiply both sides of eq. (A.40) by the integrating factor \( e^{Dx} \), which yields
\[ e^{Dx}W' + e^{Dx}DW = (e^{Dx}W)' = 0_{2N \times N}. \]
Integrating the above equation gives
\[ e^{Dx}W = K, \]
where \( K \) is a \( 2N \) by \( N \) matrix of constants of integration. Therefore, the general solution of eq. (A.40) is
\[ W = e^{-Dx}K. \]
which, given eq. (A.37), implies
\[ Z = E e^{-Dx} K = e^{-Dx} E K, \]  
(A.42)
given that \( D \) is \( 2N \times 2N \) and diagonal, and that \( E \) is \( 2N \times 2N \).

Thus,
\[ q_{D,ij}(x) = \sum_{l=1}^{2N} h_{ij}(\omega_l) e^{-\omega_l x}, \]  
(A.43)
where the \( h_{ij}(\omega_l) \) are constants of integration that depend on the eigenvalues.

Note that, for any eigenvalue \( \omega \) of \( A_0 \), the particular solution \( q_{D,ij} = h_{ij}(\omega) e^{-\omega x} \), with \( h_{ij}(\omega_l) = 0, \forall \omega_l \neq \omega \), solves (A.43). Indeed, we then have
\[ Z = H(\omega) - \omega H(\omega) \]
\[ = -\omega I_{2N \times 2N} \]
(A.44)
This particular solution is important since it allows us to express \( N(N - 1) \) of the \( N^2 \) integration constants in terms of the first \( N \) ones. Indeed, simplifying (A.44) gives
\[ -\omega I_{N \times N} H(\omega) + I_{N \times N} \omega H(\omega) = 0_{N \times N}, \]
(\(2S_x^{-1}(\hat{A} - R) - \omega(2S_x^{-1}M_x - \omega I_{N \times N})\)\) \(H(\omega) = 0_{N \times N}, \)
where the first equation is trivial. To solve the second equation, we first consider
\[ G(\omega) (h_{1j}(\omega), \ldots, h_{Nj}(\omega))^T = 0_{N \times 1}, \]  
(A.45)
where
\[ G(\omega) = 2S_x^{-1}(\hat{A} - R) - \omega(2S_x^{-1}M_x - \omega I_{N \times N}). \]  

39
We denote the $ij$'th element of $G(\omega)$ by $g_{ij}(\omega)$. We know from (A.36) that $\det G(\omega) = 0$. Thus, the equations

$$\sum_{k=1}^{N} g_{ik}(\omega) h_{kj}(\omega), \ i \in \{1, \ldots, N\}$$

(A.46)

are linearly dependent. However, the system

$$\sum_{k=1}^{N} g_{ik}(\omega) h_{kj}(\omega), \ i \in \{2, \ldots, N\}$$

(A.47)

is linearly independent, allowing us to solve for $h_{kj}(\omega), \ j \in \{1, \ldots, N\}$, that is

$$\Pi_{k}(\omega) = G_{k}^{-1}(\omega) g_{k+1,1}(\omega) h_{k+1,*}(\omega).$$

Solving the above linear equation system gives us $h_{ij}(\omega), \ i \in \{2, \ldots, N\}, \ j \in \{1, \ldots, N\}$ in terms of $h_{1j}(\omega)$, for $j \in \{1, \ldots, N\}$, that is

$$q_{D,0,ij}(x) = \sum_{l=1}^{N} h_{ij}(\omega_{0,l}) e^{-\omega_{0,l}x},$$

(A.48)

where, without loss of generality, $\omega_{0,1} > \ldots > \omega_{0,N} > 0$ are the $N$ positive eigenvalues of $A_0$. Note that (A.48) contains $N^2$ free integration constants $h_{1j}(\omega_{0,l})$, which will be identified by the value and smooth pasting conditions (Section A.4.3).

**A.4.2 Region k: $X_{D,k+1} \leq X < X_{D,k}$**

We now turn to the analysis of Arrow-Debreu securities in region $k$, i.e. when current earnings are above default boundary $k + 1$, but below default boundary $k$. Note that the above analysis in region $k = 0$ can be seen as a special case of the analysis below, with $X_{D,0} \to \infty$.

First, note that $(q_{D,k})_{ij} = \delta_{ij}, \forall i \leq k$. Indeed, if earnings are currently lower than $X_{D,k} < X_{D,k-1} < \ldots$, and if the current state is $i \leq k$, then the firm is in default and the present value of a dollar when the firm defaults in state $j$ is 1 if $i = j$, and 0 otherwise. In particular, this means that, in region $N$, where $X \leq X_{D,N}$, we have

$$q_{D,N,ij} = \delta_{ij}.$$
Applying (A.27) to the unknown \(q_{D,k,ij}, i > k\), yields the following system of ODEs

\[
\frac{dz_{k+1,j}}{dx} - z_{N+k+1,j} = 0,
\]
\[
\frac{dz_{k+2,j}}{dx} - z_{N+k+2,j} = 0,
\]
\[
\vdots
\]
\[
\frac{dz_{N,j}}{dx} - z_{2N,j} = 0,
\]

\[
\frac{dz_{N+k+1,j}}{dx} + \frac{2\tilde{\mu}_{y,k+1}}{\sigma_{y,k+1}^2} z_{N+k+1,j} + \sum_{l=1}^{k} \frac{2\tilde{\lambda}_{k+1,l}}{\sigma_{y,k+1}^2} (\delta_{lj} - z_{k+1,j})
\]
\[
+ \sum_{l=k+2}^{N} \frac{2\tilde{\lambda}_{k+1,l}}{\sigma_{y,k+1}^2} (z_{lj} - z_{k+1,j}) - \frac{2r_{k+1}}{\sigma_{y,k+1}^2} z_{k+1,j} = 0,
\]

\[
\frac{dz_{N+k+2,j}}{dx} + \frac{2\tilde{\mu}_{y,k+2}}{\sigma_{y,k+2}^2} z_{N+k+2,j} + \sum_{l=1}^{k} \frac{2\tilde{\lambda}_{k+2,l}}{\sigma_{y,k+2}^2} (\delta_{lj} - z_{k+2,j})
\]
\[
+ \sum_{l=k+1,l\neq k+2}^{N} \frac{2\tilde{\lambda}_{k+2,l}}{\sigma_{y,k+2}^2} (z_{lj} - z_{k+2,j}) - \frac{2r_{k+2}}{\sigma_{y,k+2}^2} z_{k+2,j} = 0
\]

\[
\vdots
\]

\[
\frac{dz_{2N,j}}{dx} + \frac{2\tilde{\mu}_{y,N}}{\sigma_{y,N}^2} z_{2N,j} + \sum_{l=1}^{k} \frac{2\tilde{\lambda}_{N,l}}{\sigma_{y,N}^2} (\delta_{lj} - z_{N,j}) + \sum_{l=k+1}^{N-1} \frac{2\tilde{\lambda}_{N,l}}{\sigma_{y,N}^2} (z_{lj} - z_{N,j}) - \frac{2r_{N}}{\sigma_{y,N}^2} z_{N,j} = 0,
\]

for \(j = \{1, \ldots, N\}\). Rewriting the above equation system in matrix form, we obtain

\[
Z'_k + A_k Z_k + B_k = Z'_k + A_k (Z_k + A_k^{-1} B_k) = \tilde{Z}'_k + A_k \tilde{Z}_k = 0,
\]

(A.49)

where \(\tilde{Z}_k = (Z_k + A_k^{-1} B_k)\), \(Z_k\) is the following \(2(N-k)\) by \(N\) matrix

\[
Z_k = 
\begin{pmatrix}
  z_{k+1,1} & z_{k+1,2} & \cdots & z_{k+1,N} \\
  z_{k+2,1} & z_{k+2,2} & \cdots & z_{k+2,N} \\
  \vdots & \vdots & \cdots & \vdots \\
  z_{N,1} & z_{N,2} & \cdots & z_{N,N} \\
  z_{N+k+1,1} & z_{N+k+1,2} & \cdots & z_{N+k+1,N} \\
  z_{N+k+2,1} & z_{N+k+2,2} & \cdots & z_{N+k+2,N} \\
  \vdots & \vdots & \cdots & \vdots \\
  z_{2N,1} & z_{2N,2} & \cdots & z_{2N,N}
\end{pmatrix}.
\]
Thereafter, the development made, in region 0, between equations (A.34) and (A.42) can be applied to \( \tilde{Z}_k \) in (A.49) to yield
\[
\tilde{Z}_k = e^{-D_k x} E_k K_k, \tag{A.50}
\]
or, equivalently,
\[
Z_k = e^{-D_k x} E_k K_k - A_k^{-1} B_k. \tag{A.51}
\]
Therefore,
\[
q_{D,k,ij}(x) = \delta_{ij}, i \in \{1, \ldots, k\}, j \in \{1, \ldots, N\}
\]
\[
q_{D,k,i,j}(x) = \sum_{l=1}^{2(N-k)} h_{ij}(\omega_l)e^{-\omega_l x} - [A_k^{-1} B_k]_{i-k,j}, i \in \{k+1, \ldots, N\}, j \in \{1, \ldots, N\}.
\]
Once more, for each eigenvalue \( \omega \) of \( A_k \), the particular solution
\[
Z_k = \left( \begin{array}{c}
H_k(\omega) \\
-\omega H_k(\omega)
\end{array} \right) e^{-\omega y} - A_k^{-1} B_k,
\text{ where } H_k(\omega) = \left( \begin{array}{c}
h_{k+1,1}(\omega) \\
H_k(\omega)
\end{array} \right),
\]
can be used to express the constants in all lines of \( H_k \) but the first, as functions of the \( h_{k+1,1}, \ldots, h_{k+1,N} \) constants. This leaves us with \( N \) free constants for each of the \( 2(N-k) \) eigenvalues.

**A.4.3 Boundary conditions**

Given the above development, we are left with \( N^2 \) free integration constants in region 0, and \( 2(N-k)N \) free constants in region \( k \in \{1, \ldots, N-1\} \). Hence, we still have to solve for the \( N^3 \) constants,
\[
N^2 + \sum_{k=1}^{N-1} 2(N-k)N = N^2 + 2N \sum_{k=1}^{N-1} (N-k) = N^3, \tag{A.52}
\]
that satisfy the \( N^3 \) boundary conditions (value matching & smooth pasting) of the problem at hand. For each default boundary \( X_{D,k}, k \in \{1, \ldots, N\} \), we have that:

(VM) The value of the \( N + (N-k)N \) Arrow-Debreu securities with \( i \geq k \), must be the same on both sides of the default boundary, i.e.
\[
q_{D,k-1,ij}(x_{D,k}) = q_{D,k,ij}(x_{D,k}), \text{ where } , i \in \{k, \ldots, N\}, j \in \{1, \ldots, N\}; \tag{A.53}
\]
(SP) The dynamics of the \((N - k)N\) Arrow-Debreu securities with \(i > k\), must be the same on both side of the default boundary, i.e.

\[
\frac{dq_{D,k-1,ij}}{dx} \bigg|_{x_{D,k}} = \frac{dq_{D,k,ij}}{dx} \bigg|_{x_{D,k}}, \quad \text{where} \quad i \in \{k+1, \ldots, N\}, j \in \{1, \ldots, N\}. \tag{A.54}
\]

A.5 Modified Arrow-Debreu securities

Arrow-Debreu securities provide the expected value of a 1$ cash flow conditional on the state of the world in which they occur. In particular, in region \(k\) at time \(t\), Arrow-Debreu prices

\[
q_{D,k,ij,t}(x) = E_t \left[ \frac{\pi_{\tau_D}}{\pi_t} \mathbbm{1}_{\nu_D = j} | \nu_t = i \right] \tag{A.55}
\]

\[
= E_t^Q \left[ \mathbbm{1}_{\nu_D = j} | \nu_t = i \right] \tag{A.56}
\]

\[
= Q \left[ \nu_D = j | \nu_t = i \right], \tag{A.57}
\]

can be interpreted risk-adjusted probability that default, occurring at unknown time \(\tau_D\), will occur in state \(j\) conditional on the current state of the economy, \(\nu_t\), being \(i\). For cash flows that do not depend on the level of earnings when the firm defaults, \(X_{\tau_D} = e^{x_{\tau_D}}\), these Arrow-Debreu securities yield a straightforward approach to derive the cash flows’ expected values. If a cash flow does depend on \(X_{\tau_D}\), the Arrow-Debreu securities may not be as useful.

In a continuous model, the earnings always approach the default boundary from above and default occur when \(X_{\tau_D} = X_D\); that is, there is no uncertain with respect to the level of earnings upon default and Arrow-Debreu securities can readily be used to compute expected cash flows. In our economy, however, “deep defaults” can occur when the state of the economy jumps from its current state to a worse state.

Recall that we ordered the default boundaries such that \(X_{D,1} > \ldots > X_{D,N}\); hence, state \(N\) is the best state of the economy, state \(1\) is the worst. When the state of the economy jumps toward a better state, the default boundary decreases as growth opportunities improve; hence, if the firm was not in default, it is even further away from default after the jump. However, if the level of earnings is \(X_{D,j+1} < X_{\tau_D} \leq X_{D,j}\) prior to a jump to state \(j\) at time \(\tau_D\), the firm automatically defaults. The level of earnings \(X_{\tau_D}\) is then only a fraction of the default boundary \(X_{D,j}\), and the firm will thus be able to honor its obligations to the debtholders, for instance, only partially.

We thus introduce “modified” Arrow-Debreu securities

\[
\tilde{q}_{D,k,ij,t}(x) = E_t \left[ \frac{\pi_{\tau_D}}{\pi_t} \frac{X_{\tau_D}}{X_{D,j}} \mathbbm{1}_{\nu_D = j} | \nu_t = i \right] \tag{A.58}
\]

\[
= E_t \left[ \frac{\pi_{\tau_D}}{\pi_t} e^{x_{\tau_D} - x_{D,j}} \mathbbm{1}_{\nu_D = j} | \nu_t = i \right] \tag{A.59}
\]

\[
= E_t^Q \left[ e^{x_{\tau_D} - x_{D,j}} \mathbbm{1}_{\nu_D = j} | \nu_t = i \right], \tag{A.60}
\]
to account for the uncertainty surrounding the recovery rate. Note that, as long as shareholders have no bargaining power, they shouldn’t care about the depth of deep defaults.

Technically, the (standard) Arrow-Debreu securities are special cases of their modified counterparts, with \( \frac{X_{np}}{X_{D,j}} = 1 \). Moreover, when in region 0, deep defaults are not a direct concern as the firm would survive even to a jump to the worse state, state 1. Hence, the general solution in (A.48) holds. However, in region \( k > 0 \), applying (A.27) to the unknown \( \tilde{q}_{D,k,ij} \), \( i > k \), accounting for deep defaults, yields the following system of ODEs

\[
\begin{align*}
\frac{dz_{k+1,j}}{dx} &- z_{N+k+1, j} = 0, \\
\frac{dz_{k+2,j}}{dx} &- z_{N+k+2, j} = 0, \\
\vdots & \\
\frac{dz_{N,j}}{dx} &- z_{2N, j} = 0,
\end{align*}
\]

or, equivalently,

\[
Z_k' + A_k Z_k + \overline{B}_k = 0,
\]

where

\[
\overline{B}_k = \begin{pmatrix} 0_{N-k \times k} & 0_{N-k \times N-k} \\ B_k & 0_{N-k \times N-k} \end{pmatrix},
\]
and

$$\bar{B}_k^N = \begin{pmatrix} \frac{2\hat{\lambda}_k}{\sigma_k+1} e^{x-x_{D,1}} & \frac{2\hat{\lambda}_k}{\sigma_k+1} e^{x-x_{D,2}} & \cdots & \frac{2\hat{\lambda}_k}{\sigma_k+1} e^{x-x_{D,k}} \\ \frac{2\hat{\lambda}_{k+1}}{\sigma_{k+2}} e^{x-x_{D,1}} & \frac{2\hat{\lambda}_{k+1}}{\sigma_{k+2}} e^{x-x_{D,2}} & \cdots & \frac{2\hat{\lambda}_{k+1}}{\sigma_{k+2}} e^{x-x_{D,k}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{2\hat{\lambda}_N}{\sigma_N} e^{x-x_{D,1}} & \frac{2\hat{\lambda}_N}{\sigma_N} e^{x-x_{D,2}} & \cdots & \frac{2\hat{\lambda}_N}{\sigma_N} e^{x-x_{D,k}} \end{pmatrix}. $$

Now, $\bar{B}_k$ is not constant with respect to $x$ anymore, but $\bar{B}_k' = \bar{B}_k$. Hence, letting $\bar{Z}_k = Z_k + (A_k + I)^{-1} \bar{B}_k$, we have

$$\bar{Z}_k' + A_k \bar{Z}_k = Z_k' + (A_k + I)^{-1} \bar{B}_k + A_k Z_k + A_k (A_k + I)^{-1} \bar{B}_k = 0. \quad (A.62)$$

Once more, the development made between equations (A.34) and (A.42) can be applied to $\bar{Z}_k$ in (A.63) to yield

$$\bar{Z}_k = e^{-D_k x} E_k K_k, \quad (A.64)$$

or, equivalently,

$$Z_k = e^{-D_k x} E_k K_k - (A_k + I)^{-1} \bar{B}_k. \quad (A.65)$$

Therefore,

$$\bar{q}_{D,k,ij}(x) = \delta_{ij} e^{x-x_{D,j}} = \delta_{ij} \frac{X}{X_{D,i}}, \quad i \in \{1, \ldots, k\}, j \in \{1, \ldots, N\},$$

$$\bar{q}_{D,k,i,j}(x) = 2(N-k) \sum_{l=1}^{2(N-k)} h_{ij}(\omega_l) e^{-\omega_l x} - [(A_k + I)^{-1} \bar{B}_k]_{i-k,j}, \quad i \in \{k+1, \ldots, N\}, j \in \{1, \ldots, N\}.$$
possibilities:

\[ X_{D,1} > X_{D,2} > X_{D,3} > X_{D,4} \]  \hspace{1cm} (A.66)
\[ X_{D,2} > X_{D,1} > X_{D,3} > X_{D,4} \]  \hspace{1cm} (A.67)
\[ X_{D,1} > X_{D,2} > X_{D,4} > X_{D,3} \]  \hspace{1cm} (A.68)
\[ X_{D,2} > X_{D,1} > X_{D,4} > X_{D,3} \]  \hspace{1cm} (A.69)

If deflation is worse than inflation, the overall ordering of states will be 1, 2, 3, 4, going from worst to best. It therefore makes sense, at least at the outset to work with the following ordering of default boundaries:

\[ X_{D,1} > X_{D,2} > X_{D,3} > X_{D,4} \]  \hspace{1cm} (A.70)

In this proof it is not necessary to distinguish between the state of the economy at dates \( t^- \) and \( t \). The central part of our proof consists of proving that

\[
E_t \left[ \int_t^{\tau_D} \frac{\pi_s^T}{\pi_t^T} ds \mid s_t = i \right] = \frac{1}{r_{P,i}^T} - \sum_{j=1}^{4} \frac{q_{D,ij}^T}{r_{P,j}^T},
\]  \hspace{1cm} (A.71)

where \( r_{P,i}^T \), the discount rate for a fixed nominal perpetuity, when the economy is in state \( i \), is given by

\[
r_{P,i}^T = \left( E_t \left[ \int_t^{\tau_D} \frac{\pi_s^T}{\pi_t^T} ds \mid s_t = i \right] \right)^{-1},
\]  \hspace{1cm} (A.72)

and

\[
E_t \left[ \frac{\pi_{\tau_D}^T}{\pi_t^T} \alpha_{\tau_D} A_{\tau_D}^T (X_{\tau_D}) \mid s_t = i \right] = \sum_{j=1}^{4} \alpha_j A_j^T (X_{D,j}) \hat{q}_{D,ij}^T.
\]  \hspace{1cm} (A.73)

To prove (A.71), we note that

\[
E_t \left[ \int_t^{\tau_D} \frac{\pi_s^T}{\pi_t^T} ds \mid s_t = i \right] = E_t \left[ \int_t^{\infty} \frac{\pi_s^T}{\pi_t^T} ds \mid \nu_t = i \right] - E_t \left[ \int_t^{\tau_D} \frac{\pi_s^T}{\pi_t^T} ds \mid \nu_{\tau_D} = i \right],
\]

and conditioning on the event \( \{\nu_{\tau_D} = j\} \), we obtain

\[
E_t \left[ \frac{\pi_{\tau_D}^T}{\pi_t^T} \int_t^{\tau_D} \frac{\pi_s^T}{\pi_{\tau_D}^T} ds \mid s_t = i \right] = \sum_{j=1}^{4} E_t \left[ \operatorname{Pr} (\nu_{\tau_D} = j \mid s_t = i) \frac{\pi_{\tau_D}^T}{\pi_t^T} \int_t^{\tau_D} \frac{\pi_s^T}{\pi_{\tau_D}^T} ds \mid s_t = i \right].
\]
Since consumption is Markovian, so is the state-price density, which implies that

\[
E_t \left[ \Pr \left( s_{\tau_D} = j \mid s_t = i \right) \frac{\pi_{\tau_D}^S}{\pi_t^S} \int_{\tau_D}^{\infty} \frac{\pi_{t}^S}{\pi_{\tau_D}^S} \, ds \right] = E_t \left[ \Pr \left( s_{\tau_D} = j \mid s_t = i \right) \frac{\pi_{\tau_D}^S}{\pi_t^S} \right] E_t \left[ \int_{\tau_D}^{\infty} \frac{\pi_{t}^S}{\pi_{\tau_D}^S} \, ds \right] = \int_{\tau_D}^{\infty} \frac{\pi_{t}^S}{\pi_{\tau_D}^S} \, ds \, \left| s_{\tau_D} = j \right].
\]

Therefore

\[
E_t \left[ \int_{\tau_D}^{\infty} \frac{\pi_{t}^S}{\pi_{\tau_D}^S} \, ds \right] = \int_{\tau_D}^{\infty} \frac{\pi_{t}^S}{\pi_{\tau_D}^S} \, ds \, \left| s_{\tau_D} = j \right].
\]

Conditional on being in state \( i \), the value of a claim which pays one risk-free unit of consumption in perpetuity is \( E_t \left[ \int_{\tau_D}^{\infty} \frac{\pi_{t}^S}{\pi_{\tau_D}^S} \, ds \right] \), so the discount rate for this perpetuity, \( r_{P,i} \), is given by (A.72). Consequently, (A.74) implies

\[
E_t \left[ \int_{\tau_D}^{\infty} \frac{\pi_{t}^S}{\pi_{\tau_D}^S} \, ds \right] = \frac{1}{r_{P,i}} - \sum_{j=1}^{4} E_t \left[ \Pr \left( s_t = i \mid s_{\tau_D} = j \right) \frac{\pi_{\tau_D}^S}{\pi_t^S} \right] E_t \left[ \int_{\tau_D}^{\infty} \frac{\pi_{t}^S}{\pi_{\tau_D}^S} \, ds \right] = \frac{1}{r_{P,i}}.
\]

To obtain (A.71) from the above expression, we note that

\[
q_{D,ij,t}^S = E_t \left[ \Pr \left( s_{\tau_D} = j \mid s_t = i \right) \frac{\pi_{\tau_D}^S}{\pi_t^S} \right].
\]

To prove (A.73), we condition on the event \( \{ s_{\tau_D} = j \} \) to obtain

\[
E_t \left[ \frac{\pi_{\tau_D}^S}{\pi_t^S} A_{\tau_D} X_{\tau_D} \mid s_t = i \right] = \sum_{j=1}^{4} \alpha_j A_j (X_{\tau_D}) E_t \left[ X_{\tau_D} \frac{\pi_{\tau_D}^S}{\pi_t^S} \Pr \left( s_{\tau_D} = j \mid s_t = i \right) \right].
\]

Using (A.76) to simplify the above expression we obtain (23).

### A.7 Equity risk premium and equity volatility

Applying Ito’s Lemma to \( S_{t,\tau_D}^S \) gives

\[
\frac{dS_{t,\tau_D}^S}{S_{t,\tau_D}^S} + (X_t - c) \, dt = \sum_{j=1}^{\infty} \frac{S_{t,\tau_D}^S}{X_t \partial X_t} \, dX_{\tau_D} + \frac{X_t^2 \partial^2 S_{t,\tau_D}^S}{X_t} \left( dX_t \right)^2 + \sum_{j \neq i} \frac{S_{j,\tau_D}^S - S_{i,\tau_D}^S}{2 S_{t,\tau_D}^S} \, dN_{ij,t} + \frac{(X_t - c) \, dt}{S_{t,\tau_D}^S}.
\]
Observe that
\[
\frac{\partial S_{i,t}}{\partial X_t} = (1 - \eta) \frac{1}{r_{A,i}} - \sum_{j=1}^{4} \left( A^j(X_{D,j}) \frac{\partial q^j_{D,ij,t}}{\partial X_t} - (1 - \eta) \frac{\partial^2 q^j_{D,ij,t}}{\partial X_t^2} \frac{c}{r_{P,j}} \right) \tag{A.78}
\]
\[
\frac{\partial^2 S_{i,t}}{\partial X_t^2} = -\sum_{j=1}^{4} \left( A^j(X_{D,j}) \frac{\partial^2 q^j_{D,ij,t}}{\partial X_t^2} - (1 - \eta) \frac{\partial^2 q^j_{D,ij,t}}{\partial X_t^2} \frac{c}{r_{P,j}} \right) \tag{A.79}
\]

Define the date-\(t\) conditional nominal expected return

\[
\mu^{S}_{R,i,t} = E_t \left[ \frac{dS^S_{st-} + (X_t - c)dt}{S^S_{st-}} \bigg| s_t = i \right]. \tag{A.80}
\]

and the date-\(t\) conditional real expected return

\[
\mu_{R,i,t} = E_t \left[ \frac{dS_{st-} + (Y_t - c/P_t)dt}{S_{st-}} \bigg| s_t = i \right]. \tag{A.81}
\]

The basic asset pricing equation is

\[
\mu^{S}_{R,i,t} - r^S_{i,t} = -E_t \left[ \frac{d\pi^S_t}{\pi^S_t} \frac{dS^S_{st-}}{S^S_{st-}} \bigg| s_t = i \right]. \tag{A.82}
\]

The real SDF is given by

\[
\frac{d\pi_t}{\pi_t} \bigg|_{s_t = i} = -r_t dt - \gamma \sigma_{C,i} dZ_t - \sum_{j \neq i} (\omega_{ij} - 1) dN^P_{ij,t}. \tag{A.83}
\]

and the nominal SDF by

\[
\frac{d\pi^S_t}{\pi^S_t} \bigg|_{s_t = i} = \frac{d\pi_t}{\pi_t} \bigg|_{s_t = i} + \mu_{P,i} dt = -r^S_t dt - \gamma \sigma_{C,i} dZ_t - \sum_{j \neq i} (\omega_{ij} - 1) dN^P_{ij,t}. \tag{A.84}
\]

Hence

\[
\mu^{S}_{R,i,t} - r^S_{i,t} = \sum_{j \neq i} (1 - \omega_{ij}) \frac{S^j - S^S_i}{S^S_i} \lambda_{ij} \tag{A.86}
\]

Observe that because

\[
\mu^{S}_{R,i,t} = \mu_{R,i,t} + \mu_{P,i} \tag{A.87}
\]
and
\[ r_{i,t}^S = r_{i,t} + \mu_{P,i}, \] (A.88)
we have
\[ \mu_{R,i,t} - r_{i,t} = \mu_{R,i,t}^S - r_{i,t}^S \] (A.89)

The unexpected stock return in state \( i \) is given by
\[ \sum_{j \neq i} \sigma_{R,ij} dN_{ij,t}, \] (A.90)
where
\[ \sigma_{R,ij} = \frac{S_j}{S_i} - 1 \] (A.91)

The real SDF is given by
\[ \frac{d\pi_t}{\pi_t} \bigg|_{\nu_t=i} = -r_t dt - \gamma \sigma_{C,i} dZ_t - \sum_{j \neq i} (\omega_{ij} - 1) dN_{ij,t}. \] (A.92)

Now the risk premium in state \( i \) is
\[ \mu_{R,i} - r_i = \sum_{j \neq i} (\omega_{ij} - 1) \sigma_{R,ij} \lambda_{ij}. \] (A.93)

First, note that if \( s \equiv \log S = \log f(X) \) and \( x \equiv \log X \), then
\[ \frac{\partial \log S}{\partial \log X} = \frac{\partial f(e^x)}{\partial x} = \frac{1}{f(e^x)} f'(e^x) e^x = \frac{X}{S} f'(X). \] (A.94)

Second, recall that
\[ S_{i,t}^S = A_i^S(X_t) - (1 - \eta)v_{B,i} c - \sum_{j=1}^4 \left( A_j^S(X_{D,j}) q_{D,ij,t}^S(X_t) - (1 - \eta) q_{D,ij,t}^S(X_t) v_{B,i} c \right) \] (A.95)
\[ = (1 - \eta) X_t v_{A,i} - (1 - \eta)v_{B,i} c - \sum_{j=1}^4 \left( A_j^S(X_{D,j}) q_{D,ij,t}^S(X_t) - (1 - \eta) q_{D,ij,t}^S(X_t) v_{B,i} c \right), \] (A.96)
where we made explicit that the Arrow-Debreu prices, \( q_{D,\cdot,ij,t}^S \), depend on the current value of \( X_t \). Hence,
\[ \frac{\partial \log S_{i,t}}{\partial \log X_t} = \frac{X_t}{S_{i,t}} \left[ A_i^S(X_t) \frac{\partial A_i^S}{\partial X_t} - \sum_{j=1}^4 \left( A_j^S(X_{D,j}) \frac{\partial q_{D,ij,t}^S}{\partial X_t} - (1 - \eta) \frac{\partial q_{D,ij,t}^S}{\partial X_t} v_{B,i} c \right) \right], \] (A.97)
Table 1: Model Calibration

This table presents the parameter values of the model. Real consumption data (non-durable goods plus service consumption expenditures) and the expected inflation rates from the Surveys of Professional Forecasters (SPF) are used to calibrate the model to the aggregate U.S. economy, while aggregate earning data from S&P are considered to calibrate the model to a benchmark U.S. firm. We use quarterly data covering the period 1970Q1-2016Q4. We retrieved the consumption data from the Bureau of Economic Analysis, the earning series from Robert J. Shiller’s website, and the SPF data from the website of the Federal Reserve Bank of Philadelphia. The personal consumption expenditure chain-type price index is used to deflate nominal earnings. The real and nominal conditions are determined by the estimation of two separate Markov-regime switching models. All estimates are in percentage points and annualized when applicable. The calibration is detailed in Section 4.

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<th>Economic environment</th>
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<td>State 2</td>
</tr>
<tr>
<td></td>
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<td>Real interest rate</td>
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<td>Nominal interest rate</td>
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<td>Riskless perpet. discount rate</td>
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<td>Risky flow discount rate</td>
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<table>
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<td>Tax rate</td>
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</table>
Table 2: Firm Policies and Asset Prices Without Nominal Risk

This table presents the predictions for an economy without nominal risk. Panel A reports the state-contingent policies of the firms, given by the endogenous debt coupons and default boundaries. Each row represents the state in which these policies have been selected, which we label the Financing State. Panel B reports the unconditional asset pricing quantities for the economy. We consider a weighted average of firms that are identical except in their financing state. For both panels, each column displays the predictions for a specific state of the real economy, which can be in recession (R) or in expansion (E). Given the absence of nominal risk, the inflation rate is set to its unconditional mean over the sample period, which corresponds to the “moderate inflation” state (M). The nominal earnings level is standardized at 1. Market leverage is the ratio of the market value of debt to the sum of the market values of debt and equity. The parameter values of the model are reported in Table 1 and discussed in Section 4.

### Panel A: Decision Variables

<table>
<thead>
<tr>
<th>Financing State</th>
<th>Coupons</th>
<th>State 2</th>
<th>State 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>RM financing</td>
<td>0.4898</td>
<td>0.1638</td>
<td>0.1599</td>
</tr>
<tr>
<td>EM financing</td>
<td>0.7602</td>
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<td>0.2482</td>
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<td>Unconditional</td>
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</table>

### Panel B: Asset Pricing Quantities

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<th>Stationary Probability</th>
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<th>State 5</th>
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<tbody>
<tr>
<td></td>
<td>0.1817</td>
<td>0.8183</td>
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</tbody>
</table>

| Firm Value    | 20.35  | 17.21  | 21.05  |
| Equity Value  | 12.18  | 9.55   | 12.77  |
| Debt Value    | 8.28   | 7.90   | 8.37   |

| Market Leverage (%) | 40.61  | 45.28  | 39.57  |
| Equity Volatility (%) | 43.23  | 65.83  | 38.22  |
| Equity Risk Premium (%) | 1.99   | 5.52   | 1.20   |
| Credit Spreads (bps) | 133.91 | 182.89 | 123.03 |
Table 3: Asset Prices With Nominal Risk

This table presents the predictions for an economy with nominal risk when the debt and default are chosen optimally. Panel A reports the state-contingent debt coupons and default boundaries. Each row represents the state in which these policies have been selected, which we label the Financing State. Panel B reports the unconditional asset pricing quantities for the economy. We consider a weighted average of firms that are identical except in their financing state. For both panels, each column displays the predictions for a specific state of the economy. With nominal risk, the expected inflation rate can be low (L), moderate (M), or high (H), whereas the real economy can be in recession (R) or in expansion (E). The nominal earnings level is standardized at 1. Market leverage is the ratio of the market value of debt to the sum of the market values of debt and equity. The parameter values of the model are reported in Table 1 and discussed in Section 4.

<table>
<thead>
<tr>
<th>Panel A: Decision Variables</th>
<th>Coupons</th>
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<th>State 3</th>
<th>State 4</th>
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<td>0.1631</td>
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<tr>
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<td>0.653</td>
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<td>45.30</td>
<td>44.60</td>
<td>40.46</td>
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<td>66.53</td>
<td>65.96</td>
<td>65.33</td>
<td>38.66</td>
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<td>37.94</td>
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<td>5.53</td>
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<td>1.20</td>
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<td>181.95</td>
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<td>121.41</td>
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</table>
Table 4: Asset Pricing Implications of Nominal Risk

This table presents the impact of nominal risk on asset prices. We report the difference between the asset pricing predictions in economies with and without nominal risk. In the latter case, the inflation rate is constant and set to its unconditional mean (i.e. moderate inflation state), and the model predictions are reported in Table 2. Tables ?? and 3 are used for the results with nominal risk in the case of exogenous and endogenous (debt and default) policies, respectively. Reported differences in asset values are in relative terms, whereas those for equity volatility, risk premium, leverage, and credit spread are in absolute terms. Each column reports model predictions for a different current state of the economy. The expected inflation rate can be low (L), moderate (M), or high (H), whereas the real economy can be in recession (R) or in expansion (E). The parameter values of the model are reported in Table 1 and discussed in Section 4.

<table>
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<tr>
<th>Stationary Probability</th>
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<th>State 2</th>
<th>State 3</th>
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<tr>
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<tr>
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<td>Panel D: Market Leverage (%)</td>
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<td>-0.27</td>
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<tr>
<td>Panel F: Equity Risk Premium (%)</td>
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<tr>
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<td>-0.00</td>
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<td>0.01</td>
<td>-0.00</td>
<td>-0.01</td>
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<td>0.01</td>
<td>-0.05</td>
<td>0.01</td>
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<td>-0.01</td>
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<tr>
<td>Panel G: Credit Spreads (bps)</td>
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<td>Exogenous Policies</td>
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<td>-1.61</td>
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</table>
Figure 2: Asymmetric impact of inflation
This figure illustrates the asset pricing impact of the inflation. Each panel reports the value of the asset pricing quantities for different nominal conditions (on the x-axis): low, moderate, and high inflation. Firms have the debt and default policies presented in Table 2. The parameter values of the model are reported in Table 1 and discussed in Section 4.
Figure 3: Cross-sectional impact of nominal risk

This figure illustrates the asset-pricing implications of nominal risk for different leverage levels. The left panel shows the relative change in equity value when moving from moderate inflation to either low (L) or high (H) inflation. The right panel reproduces the analysis of the left panel but for the credit spread. Low (high) leverage corresponds to an endogenous debt-to-asset ratio of 27% and 65%, respectively.
Figure 4: Asset prices and the sensitivity of earnings to nominal conditions

This figure illustrates the asset pricing impact of imperfect price sensitivity of earnings. Panels in the left (right) column report equity (debt) values for a given level of earnings. Each row reports asset values for a given level of sensitivity. In each panel, the asset values are reported for firms with different leverages.