Decomposing long bond returns: 
A decentralized modeling approach*

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Abstract

We develop a new, decentralized theory that determines the fair value of the yield to maturity on a bond or bond portfolio based purely on the near-term dynamics of its own yield, without the need to make assumptions on the instantaneous interest rate dynamics, nor the need to know whether and how the yield dynamics will change in the future. The new theory decomposes the yield into three components: near-term expectation, risk premium, and convexity effects. We propose to estimate the convexity effect with its recent time series and determine the expectation from either statistical models or economists’ forecasts, leaving the remaining component of the yield as a risk premium estimate. Empirical analysis on US and UK swap rates shows that this risk premium component can predict future bond excess returns.

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1. Introduction

The literature on interest rate modeling is vast, with different approaches targeting different challenges. The traditional literature on the expectations hypothesis focuses on predicting changes in short-term interest rates with the slope of the interest rate term structure. The shape of the yield curve contains information on not only the expectation of future interest rates, but also on the risk premium and volatility of future interest rates. Nevertheless, the expectation component dominates the short end of the yield curve (Longstaff (2000)) and can hence be used to predict future short interest rate movements. During the past decade, the literature on no-arbitrage dynamic term structure models (DTSMs) has experienced tremendous growth in terms of both their theoretical characterization\(^1\) and empirical analysis.\(^2\) By specifying the instantaneous interest rate dynamics and applying the principle of no dynamic arbitrage, these models generate fair values on the whole yield curve and are thus capable of pricing bonds of all maturities within one centralized view of the short rate dynamics. The centralization has played important roles in practical applications, such as interpolating values between observed maturities and identifying relative valuation opportunities based on the deviations between market observations and model valuations (Bali, Heidari, and Wu (2009)). By contrast, to price interest rate options, the literature (e.g., Heath, Jarrow, and Morton (1992)) often takes the observed yield curve as given and focuses on the modeling the interest rate volatility. This approach highlights the contribution of interest rate volatility to the option valuation, while proposing to delta hedge the yield curve exposure.

All of these existing frameworks, however, are limited in their ability to explain the short-term return behavior of long-dated bonds. The literature on the expectations hypothesis uses long rates to predict short rate changes, not the other way around. Modeling long rates with DTSM’s also stretches the modeler’s

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imagination on how the short rate will move in the very distant future. The starting point, for example, for this literature is often some mean-reverting assumption for the dynamics of the short rate. Yet, mean reversion calibrated to the short end of the yield curve often implies much smaller movements for long rates than actually observed from data (Giglio and Kelly (2017)). Long rates are neither easily predictable, nor converging to a constant in the foreseeable future. They tend to move randomly and with substantial volatility, a behavior that is difficult to be reconciled within existing DTSMs. Furthermore, the centralized approach inherent to DTSMs leads to stability issues for practical implementations. The centralization dictates that adding or removing one security from the estimation or an accidental data error on one security can alter the fair valuations on all other securities, making the approach vulnerable to error contagion.

It is important to realize that investors can choose to hold a very long-term bond for a very short period of time. In this case, the investor’s concern is mainly about the short-term movement of the long-term yield rather than about the long-term movement of the short rate. Indeed, even long-term investors are concerned with daily fluctuations for risk management purposes, such as value at risk calculations.

In this paper, we propose a new, decentralized theory that provides pricing insights for a particular bond or bond portfolio of interest based on the short-term behavior of the yield on that particular bond or bond portfolio. The new theory complements and contrasts with the centralized approach as it determines the fair value of the yield to maturity on a bond (portfolio) based purely on its own near-term dynamics, without the need to make assumptions on how its dynamics will change in the future, or how the instantaneous interest rate or any other bond yields behave.

The new theory starts by performing a short-term profit and loss (P&L) attribution to a bond investment through its yield representation. Taking expectations on the P&L attribution under the risk-neutral measure and setting the expected instantaneous return to the instantaneous interest rate leads to a simple pricing equation on the bond yield. The pricing equation decomposes the fair valuation of the bond yield into three components: near-term expectation, risk premium, and convexity effects. The expectation component is determined by the current level of the drift of the yield’s statistical dynamics, with no reference to how this
drift will change in the future. It thus represents one’s best directional forecast on how the yield changes over the next instant. One can generate the forecast either via statistical models or by directly borrowing from economist’s forecasts, without worrying about how the forecasts are formulated. The convexity effect is determined by the current level of the diffusion component of the yield’s dynamics, again with no reference to its future variation. Accordingly, a simple yield change variance estimator can be used to determine the convexity effect, leaving the remaining component of the yield as a risk premium estimate.

Since the yield on each bond or bond portfolio can be analyzed on its own, there is no contagion effect from one security to another. Since the theory only relies on the yield’s near-term dynamics, one does not need to make assumptions on how the yield dynamics will change in the future. In particular, when pricing a 60-year bond, the focus is to generate the best conditional mean and variance forecasts on the movements of its yield, rather than making 60-year projections on an unobserved instantaneous interest rate. The decomposition can be performed, locally and with equal ease, on the yield of a zero-coupon bond, a coupon bond, or a bond portfolio. While the centralized approach is better suited to perform relative valuation across bonds of different maturities, our decentralized approach can be used to analyze each bond on its own by linking its pricing at a point in time directly to its own conditional risk estimates at that time.

When an investor desires to analyze and compare a selected basket of bonds, the new theory can be used for the comparative analysis by directly comparing the risk behaviors of the different yields. The new theory also allows one to impose common factor structures on the yield changes and generate comparative pricing implications based on the common risk structure. Empirical results from the literature on risk factor analysis of bond yields can thus be readily incorporated into our pricing framework, making them useful not only for risk analysis, but also for fair pricing of the bonds under consideration.

As an application, we consider the pricing of long-dated bonds with the assumption of no directional prediction on its underlying yield. It is extremely difficult to predict the directional movement of long-term yields. In this application, we take this hard-to-predict feature as our starting point, and infer the risk premium component in the long bond yield, while controlling for the convexity component based on
statistical variance estimators on the yield changes. We show that the risk premium extracted from each bond can be used for out-of-sample prediction of the future excess returns on the bond.

The remainder of this paper is organized as follows. Section 2 develops our new pricing theory, links it to the classic DTSM, and considers several practical applications. Section 3 describe the data used for the empirical analysis and its general behaviors. Section 4 discusses the results. Section 5 provides concluding remarks and directions for future research.

2. A decentralized theory of bond yields

We consider an infinite-horizon continuous-time economy. Uncertainty is represented by a filtered probability space \( \{ \Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t)_{t \geq 0} \} \), where \( \mathbb{P} \) is the physical measure. We assume that the usual conditions of right continuity and completeness with respect to the null sets of \( \mathbb{P} \) are satisfied. We further assume the existence of a money market account (MMA) associated with an instantaneous interest rate \( r_t \geq 0 \). Since the value of the MMA is always strictly positive, this MMA is a numeraire. The assumption of no dynamic arbitrage implies the existence of an equivalent martingale measure \( \mathbb{Q} \) associated to this MMA numeraire.

Let \( B_t \) denote the time-\( t \) value of a riskfree bond (or bond portfolio) that pays a stream of \( N \) fixed cash flows \( \{ C_j \}_{j=1}^N \) at times \( \{ t + \tau_j \} \geq t \) for \( j = 1, 2, \ldots, N \), with \( \tau_j \) denoting the time to maturity of the \( j \)th cash flow. Traditional dynamic term structure models start by modeling the dynamics of the instantaneous interest rate \( r_t \) and value the bond via the following expectation operations,

\[
B_t = \sum_{j=1}^N C_j E^P_t [M_{t, t+\tau_j}]
\]

\[
= \sum_{j=1}^N C_j E^P_t \left[ \left( \frac{dQ}{dP} \right) e^{-\int_{t}^{t+\tau_j} r_u du} \right]
\]

\[
= \sum_{j=1}^N C_j E^Q_t \left[ e^{-\int_{t}^{\infty} r_u du} \right].
\]

where \( E^P_t [\cdot] \) and \( E^Q_t [\cdot] \) denote the expectation operator conditional on the time-\( t \) filtration \( \mathcal{F}_t \) under the
physical measure $\mathbb{P}$ and the risk-neutral measure $\mathbb{Q}$, respectively, $M_{t,T}$ denotes the pricing kernel linking value at time $t$ to value at time $T$, and $\frac{d\mathbb{Q}}{d\mathbb{P}}$ defines the Radon Nikodym derivative linking the physical probability measure $\mathbb{P}$ to the risk-neutral probability measure $\mathbb{Q}$. This measure change $\frac{d\mathbb{Q}}{d\mathbb{P}}$ is the martingale component of the pricing kernel that defines the pricing of various risks. The three equations represent the bond valuation with respect to different starting points. Through the expectation operation, bonds with cash flows at all times are linked together through the centralized modeling of the pricing kernel and/or the instantaneous interest rate.

Given the price of a bond $B_t$, its yield to maturity $y_t$ is defined via the following equality,

$$B_t \equiv \sum_{j=1}^{N} \exp(-y_t \tau_j) C_j.$$  \hspace{1cm} (4)

The yield to maturity can be regarded as the continuously compounded internal rate of return on the bond investment.

2.1. Centralized yield decomposition under the classic setting

Before we introduce our new pricing framework, we first decompose the yield of a zero-coupon bond under the classical setting to set a benchmark for later comparisons. Let $B_t(T)$ denote the time-$t$ price of a zero-coupon bond that pays $1$ at its expiry time $T \geq t$. With a single cash flow, its yield to maturity $y_t(T)$ can be explicitly solved from the bond price as,

$$y_t(T) \equiv -\frac{\ln B_t(T)}{T - t}.$$  \hspace{1cm} (5)

Substituting the bond pricing formula (3) into the yield equation in (5) reveals the link between the
T-maturity yield observed at time $t \in [0, T]$ and future short rates $r_u$ realized at times $u \in [t, T]$:

$$y_t(T) \equiv -\frac{1}{T-t} \ln E_Q^t e^{-\int_t^T r_u du}, \quad t \in [0, T]. \quad (6)$$

Furthermore, by adding and subtracting the same term twice, we can decompose the zero-coupon bond yield into three distinct terms,

$$y_t(T) = E_P^t \frac{\int_t^T r_u du}{T-t} + E_P^t \left[ \left( \frac{d\mathbb{Q}}{d\mathbb{P}} - 1 \right) \frac{\int_t^T r_u du}{T-t} \right] - \frac{1}{T-t} \left[ \ln E_Q^t e^{-\int_t^T (r_u - E_Q^t r_u) du} \right]. \quad (7)$$

The first term in the decomposition (7) represents the expectation of the average short rate $\frac{\int_t^T r_u du}{T-t}$ over the life of the bond between the valuation time $t$ and the expiry time $T$.

The second term in (7) is the risk premium as captured by the covariance under $\mathbb{P}$ of this average short rate with the random variable, $\frac{d\mathbb{Q}}{d\mathbb{P}} - 1$, which has zero mean under $\mathbb{P}$. If interest rates are stochastic and if bond returns are thought to have a positive risk premium, then the covariance in the second term is also positive.

The third term represents the convexity effect. As the term $C \equiv \frac{1}{T-t} \left[ \ln E_Q^t e^{-\int_t^T (r_u - E_Q^t r_u) du} \right]$ is non-negative, the convexity effect $C$ can only lower the yield. One can interpret $C$ as a non-standard deviation under $\mathbb{Q}$ of the zero mean random variable $-\int_t^T (r_u - E_Q^t r_u) du$. When compared to the standard deviation, the nonstandard deviation replaces the quadratic function with an exponential function. When the average future short rate $\frac{1}{T-t} \int_t^T r_u du$ is normally distributed under $\mathbb{Q}$ with variance $V$, the convexity term is equal to $C = \frac{1}{2} V (T-t)$, proportional to the variance of the average future short rate.

The relative importance of the three terms in this yield decomposition can differ across maturities. As the maturity date $T$ approaches the current time, $t$, the last two terms vanish, so that the current yield-to-maturity approaches the short rate:

$$\lim_{T \uparrow t} y_T(T) = r_t, \quad t \geq 0. \quad (8)$$
As we raise the time to maturity $\tau = T - t$, the second and the third terms both start affecting the yield, but at different speeds. In the very special example where the instantaneous interest rate follows a random walk under $\mathbb{P}$,

$$dr_t = \sigma dW_t \tag{9}$$

with the market price of the Brownian risk being $\gamma < 0$, the risk premium component increases linearly with the time to maturity as $-\frac{1}{2}\gamma \sigma \tau$, while the convexity effect increases quadratically with maturity $\frac{1}{2}\sigma^2 \tau^2$. Thus, as maturity increases, the convexity term will ultimately dominate and drive the yield to negative territory. Researchers often impose mean reversion in the short rate dynamics, which allows both the risk premium and the convexity terms to asymptote to finite constants.

The decomposition of the yield into expectations, risk premium, and the convexity effect is generic. What is particular about the classical decomposition in (7) is that the three components are linked to the pricing of one single variable, the instantaneous interest rate, and they are all tied to the distribution over the whole life of the bond. Thus, to decompose the yield of a long-dated bond under the classical setting, one needs to make projections far into the future about the instantaneous interest rate. In practice, an investor can invest in very long-dated bonds for a very short period of time. In this case, the investor worries more about the near-term value fluctuation of the bond than about any long-run projections. Even for investors with a long investment horizon, managing the daily P&L fluctuation of their investment is still vitally important. Given these practical considerations, our new pricing framework does not rely on long-run projections of a centralized variable (i.e., the instantaneous interest rate), but rather builds on a decentralized, short-run P&L attribution of the bond investment.

### 2.2. Decentralized short-run P&L attribution of bond investments

To perform a short-run P&L analysis on a bond investment, we focus on the bond value change over the next instant. To decentralize the P&L attribution, we examine how the bond value varies with its own
yield to maturity. First, we follow industry practice by characterizing the risk of the bond by its duration and convexity, which respectively capture the first- and second-order interest rate sensitivity of the bond value. While there are many variations in the definitions of duration and convexity, we take the following particular definitions that measure the sensitivity of the bond price against its own yield to maturity,

\[
\tau \equiv -\frac{\partial B_t}{B_t \partial y_t} = \sum_{j=1}^{N} w_j \tau_j, \tag{10}
\]

\[
\tau^2 \equiv \frac{\partial^2 B_t}{B_t \partial y_t^2} = \sum_{j=1}^{N} w_j \tau^2_j, \tag{11}
\]

where the weights \( w_j \) are given by

\[
w_j = \frac{\exp(-y_t \tau_j) C_j}{\sum_{i=1}^{N} \exp(-y_t \tau_i) C_i}. \tag{12}
\]

According to these definitions, the duration (\( \tau \)) and the convexity (\( \tau^2 \)) are simply the value-weighted average maturity and maturity squared of the cash flows from the bond. The weight on each cash flow is based on its value as a fraction of the total bond worth. For a zero-coupon bond, its duration is simply its time to maturity, and its convexity is the maturity squared.

The industry quotes the yield to maturity of a bond instead of its price. The motivation for quoting yields over prices is that the former offer greater stability over time and greater comparability across different bonds. The duration and convexity measures capture how much the bond value varies when the yield varies. While both the yield to maturity in (4) and the duration/convexity risk measures in (10) and (11) can be compared cross-sectionally across different bonds, they are decentralized measures whose calculation depends only on the particular bond itself.\(^3\)

With the yield to maturity definition and the decentralized risk exposure measures, we can attribute the short-term investment P&L of a bond with respect to the movement of its own yield to maturity via a Taylor

\(^3\)There are also duration/convexity measures that are calculated by shocking the yield curve in a particular way and thus lose the decentralized feature. Our particular choice of duration and convexity not only have particularly simple forms and interpretations, but are also local to the bond itself, without the need of stripping a yield curve.
expansion,

\[ dB_t = \frac{\partial B_t}{\partial t} dt + \frac{\partial B_t}{\partial y} dy + \frac{1}{2} \frac{\partial^2 B_t}{\partial y^2} (dy)^2 + o(dt), \quad (13) \]

where \( o(dt) \) denotes higher-order terms of \( dt \) when yield moves continuously. When the yield can jump, the jump induces additional higher-order terms,

\[ JC = \int \left( B(y_t e^x) - B(y_t) \right) \nu(x, t) dx dt \]

where \( \nu(x, t) \) counts the number of jumps of size \( x \) in the logarithm of the yield at time \( t \). We henceforth assume that the next move for the yield to maturity of the bond is continuous. Accordingly, we can attribute the bond investment P&L solely to time decay and to first and second-order effects from the yield to maturity movement. Since the P&L attribution focuses on the bond value change over the next instant, the continuity assumption is only for the next instant. The results hold even if the yield can jump at any other times.

Compared to the classical centralized approach to bond pricing, equation (13) focuses on the short-term variation of the bond value regardless of the bond maturity. Furthermore, the P&L attribution is decidedly local and is based on the variation of its own yield. Dividing both sides of equation (13) by \( B_t dt \), and substituting in the definition of yield in (4), the definition of duration in (10), and the definition of convexity in (11), we obtain the following attribution of the annualized investment return,

\[ \frac{dB_t}{B_t dt} = y_t - \tau \frac{dy}{dt} + \frac{1}{2} \tau^2 \frac{(dy)^2}{dt}. \quad (14) \]

The first term in (14) denotes the carry — if the bond yield does not change, the instantaneous bond return is simply the yield to maturity. The second term highlights the directional impact of the yield change on the bond return. The negative bond-yield relation dictates that the bond return declines when the yield goes up. The sensitivity is measured by the bond’s duration \( \tau \). Third term captures the convexity of the bond-yield relation. Larger yield moves of either direction increases the bond return due to the convex bond-yield

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relation. The magnitude of this exposure is captured by the bond’s convexity measure $\tau^2$.

Taking expectation on (14) under the statistical measure $P$, we can attribute the expected bond investment return to three sources,

$$E_t^P \left[ \frac{dB_t}{B_t dt} \right] = y_t - \mu_{t,y} \tau + \frac{1}{2} \sigma_{t,y}^2 \tau^2,$$

(15)

where $\mu_{t,y} = E_t^P (dy_t/dt)$ denotes the time-$t$ expected rate of change on the yield, and $\sigma_{t,y}^2 = E_t [(dy)^2/dt]$ denotes the time-$t$ conditional variance rate of the yield. Equation (15) decomposes the expected bond return into three sources. The first term captures the expected return from carry. Bonds with a higher yield generate a higher return on average due to carry. Second, due to the negative bond-yield relation, any expected yield increase reduces the expected bond return. Third, due to the convexity of the bond-yield relation, higher volatility on the yield changes leads to a higher expected bond return. A duration neutral bond portfolio that is long convexity is analogous to a delta-neutral long options position.

The decomposition highlights the key risk and return sources of a bond investment. If an investor has no view on the direction of the yield curve movements, the investor can form duration-neutral bond portfolios with bonds of nearby maturities. Assuming that yields at nearby maturities strongly co-move, the duration-neutral portfolio will have minimal exposure to common directional movements of the bond yields. Then, the long-short positioning of the two bonds will be driven by the difference between the carry and convexity benefits of the two bonds.

To illustrate the contributions from the different components, let us imagine a situation where zero-coupon bond yields at long maturities (e.g., 10, 15, 30 years) are flat and move in parallel by substantial amounts (i.e., $\sigma^2$ is large).\(^4\) In this case, we can form a self-financing and riskless portfolio (under our parallel movement assumption) that makes money from convexity. First, since the yield is the same, a dollar-neutral portfolio would be self-financing. Second, since the yields move in parallel, a duration-

\(^4\)One can perform similar analysis on coupon bonds with specific duration and convexity estimates. Using zero-coupon bonds make the duration and convexity numbers explicit.
neutral portfolio will have no directional exposure. Then, if such a portfolio can be formed with positive convexity, one would expect to earn positive gains. For example, if we are long $300 of a 10-year zero-coupon bond, long $100 of a 30-year zero-coupon bond, and short $400 of a 15-year zero-coupon bond, this butterfly portfolio will cost zero dollars (cost-neutral), have zero duration, and contain a positive convexity of $\tau^2_f = 75$. By cancelling out carry and duration while retaining positive convexity, the instantaneous P&L on the fly is positive and proportional to the yield change squared,

$$dFly_t = \frac{1}{2} \tau^2_f (dy)^2 \geq 0.$$ \hspace{2cm} (16)

Thus, observing a flat, parallel moving yield curve presents an arbitrage opportunity.

2.3. Decentralized no-arbitrage pricing and yield decomposition

The P&L attribution analysis highlights the local risk sources and return opportunities for the bond investment over the next instant. To generate pricing implications, we take expectation under the risk-neutral measure $Q$ on the attribution in (14),

$$\mathbb{E}^Q_t \left[ \frac{dB_t}{B_t dt} \right] = y_t - \mu^Q_{t,y} \tau + \frac{1}{2} \sigma^2_{t,y} \tau^2,$$ \hspace{2cm} (17)

where $\mu^Q_{t,y} = \mathbb{E}^Q_t [dy_t / dt]$ denotes the time-$t$ expected rate of change on the yield under the risk-neutral measure. Given the continuous path assumption over the next instant with the instantaneous volatility $\sigma_{t,y}$, let $\lambda_t$ denote the market price of the bond price’s Brownian risk (i.e., the negative of the yield’s Brownian risk). We can link the expected rate of yield change under the two measures by

$$\mu^Q_{t,y} = \mu_{t,y} + \lambda_t \sigma_{t,y}.$$ \hspace{2cm} (18)

The market price of bond risk $\lambda_t > 0$ if bond returns are thought to contain a positive risk premium.
No dynamic arbitrage dictates that the risk-neutral expected instantaneous rate of return on any investment is equal to the instantaneous interest rate \( r_t \). Applying this no-dynamic-arbitrage condition to the risk-neutral expectation in (17) leads to a simple pricing relation for the bond yield spread over the instantaneous interest rate,

\[
y_t - r_t = \mu_{t,y} \tau + \lambda_t \sigma_{t,y} \tau - \frac{1}{2} \sigma_{t,y}^2 \tau^2.
\]

(19)

The fair value of the yield spread \((y_t - r_t)\) on the bond investment is determined by its expected rate of change forecast \((\mu_{t,y})\), the risk premium \((\lambda_t \sigma_{t,y})\), and its volatility forecast \((\sigma_{t,y})\).

**Theorem 1.** If the yield of a bond is moving continuously over the next instant, no dynamic arbitrage dictates that the fair spread of this yield over the instantaneous interest rate is linked to its expected rate of change \((\mu_{t,y})\), its risk premium \((\lambda_t \sigma_{t,y})\), and its variance rate \((\sigma_{t,y}^2)\) through the bond’s duration and convexity by

\[
y_t - r_t = \mu_{t,y} \tau + \lambda_t \sigma_{t,y} \tau - \frac{1}{2} \sigma_{t,y}^2 \tau^2.
\]

(20)

Compared to classical centralized bond pricing, the new pricing relation in (20) is highly decentralized. The fair valuation of the bond investment in (20) only depends on the behavior of its own yield to maturity, with no direct dependence on the short rate dynamics or the dynamics of any other yields. In fact, the pricing equation does not even rely on the full dynamics of its own yield, but only depend on the conditional expectation estimators of its rate of change, volatility, and market pricing. All three estimates can change over time, as can the dynamics of the yield, but none of these changes enter the pricing of the current yield spread. Thus, in the end, the pricing relation does not depend on any particular dynamical specification, but only relies on three conditional forecasts. One can bring in the forecasts from any outside sources and directly examine the pricing implications under our new theory. These estimates can come from any model assumptions, allowing greater flexibility and cross-field collaboration.

The pricing relation in (20) also provides a decentralized version of the yield decomposition. Similar to the centralized yield decomposition, equation (20) also decomposes the yield into expectation, risk premium,
and convexity. The difference is that the expectation, risk premium, and convexity in (20) are all measured on the yield of this particular bond. Furthermore, they reflect the expected behavior of the yield over the next instant, rather than the behavior of the short rate over the whole life of the bond.

To distinguish our new pricing framework from the classic pricing framework of DTSMs, we henceforth label our new pricing theory as the **Dynamic Duration Convexity Model** (DDCM).

### 2.4. Contrast with DTSMs

Our theory links the time-\(t\) value of the yield spread of a particular bond to the current forecasts of its rate of change, its risk premium, and its variance rate, with no reference to the exact dynamics of the instantaneous interest rate or any other interest rates or even the future dynamics changes of this particular bond, thus making the analysis completely decentralized and short-term. The decentralized feature also dictates that the no-arbitrage relations in (20) only guarantee dynamic no-arbitrage between the particular bond in consideration and the money market account given the assumptions on the bond yield’s rate of change, risk premium, and volatility levels, but with no direct implications on cross-sectional relations across yields on different bonds.

By contrast, DTSMs derive the fair value for the whole yield curve based on the full dynamic specification of a centralized instantaneous interest rate, and thus guarantees dynamic consistency across the whole yield curve. Therefore, while our theory allows us to compare the level of a yield to its own near-term forecasts, without referencing to other parts of the yield curve, DTSMs are built to analyze the yield curve shape and make cross-sectional comparisons of yields across different maturities. The two theories are complementary in focusing on different aspects of dynamic no-arbitrage.

We can always make assumptions on how the drift and diffusion coefficients of yields vary across maturity, from which we can come up with a relation that governs the whole term structure. This effort amounts to centralizing our decentralized model. In practice, investors can be interested in performing compara-
tive analysis on a selected number of bond yields without making inference on the whole yield curve. In this case, we can make common factor assumption on the selected set of bond yields and derive relative valuations among them, without defining the whole yield curve.

Traditional dynamic term structure models derive implications on the term structure based on risk-neutral dynamics assumptions on the instantaneous interest rate. By deriving everything from the dynamics of one process, it guarantees cross-sectional consistency among the yields across different maturities, and thus provides a framework for cross-sectional comparisons and for relative value. In particular, when one performs statistical arbitrage trading based on the relative valuation, i.e., the deviation between market quotes and the fair value, the assumed dynamics do not play any direct role in prediction, but play important roles in forming the hedge to neutralize the assumed factors.

By contrast, our approach focuses on the relation between the value of one particular yield on a bond and its current rate of change and volatility forecasts. The forecasts play a direct role in our assessment of the yield’s fair value. By focusing on the short-term return of a bond instead of the long-term projection of a short rate, our new framework strives to derive a pricing relation from standard risk-return analysis.

2.4.1. Decentralizing DTSM to DDCM

Since DTSMs are derived based on dynamic no arbitrage of all yields relative to one centralized instantaneous interest rate process, the level of each derived yield is naturally consistent with the derived dynamics of this yield. Thus, the level of the derived yield and the levels of the derived drift and volatility of the yield must satisfy our DDCM pricing relation.

As an example, consider the general diffusion-affine dynamic term structure model of Duffie and Kan (1996), who assume that the instantaneous interest rate is an affine function of $K$ factors with affine contin-
uous dynamics under the risk-neutral measure:

\begin{align*}
    r_t &= a_r + b_r^\top X_t, \\
    dX_t &= \kappa (\theta - X_t) dt + \sqrt{\Sigma(X_t)} dZ_t,
\end{align*}

(21)

(22)

with \( \Sigma(X_t)_{ij} = \alpha_i + \beta_i^\top X_t \) and \( \Sigma_{ij} = 0 \) for \( i \neq j \). Under these dynamics assumptions, the yields on all zero-coupon bonds are affine in the factor,

\[
y_t(T) = \frac{a(\tau)}{\tau} + \left[ \frac{b(\tau)}{\tau} \right]^\top X_t,
\]

(23)

for all \( \tau = T - t > 0 \), where the coefficients \((a(\tau), b(\tau))\) are solutions to the following set of ordinary differential equations,

\[
    a'(\tau) = a_r + b(\tau)^\top \kappa \theta - \frac{1}{2} \sum_i b(\tau)^2_i \alpha_i,
\]

\[
    b'(\tau) = b_r - \kappa^\top b(\tau) - \frac{1}{2} \sum_i b(\tau)^2_i \beta_i,
\]

starting at \( a(0) = 0 \) and \( b(0) = 0 \).

Equation (23) centralizes the yields of all maturities by linking them as affine functions of a common set of factors \( X_t \). To decentralize the affine model, we take one particular zero-coupon bond with an expiry \( T \) as an example and derive the risk-neutral dynamics of the yield on this zero-coupon bond. Applying Ito’s lemma to (23) and (22), we have the risk-neutral drift and variance of the yield \( y_t(T) \) as

\[
    \mu_t^Q = - \left[ \frac{a'(\tau)}{\tau} - \frac{a(\tau)}{\tau^2} \right] - \left[ \frac{b'(\tau)}{\tau} - \frac{b(\tau)}{\tau^2} \right]^\top X_t + \left[ \frac{b(\tau)}{\tau} \right]^\top \kappa (\theta - X_t),
\]

\[
    \sigma_t^2 = b(\tau)^\top \Sigma(X) b(\tau) \frac{1}{\tau^2},
\]

starting at \( a(0) = 0 \) and \( b(0) = 0 \).
with which we can write the risk-neutral dynamics of the yield as

\[ d y_t(T) = \mu^Q_t dt + \sigma_t dW_t, \]

where \( dW_t \) denotes the change of a newly constructed Brownian motion that is linked to the factor Brownian shocks by

\[ dW_t = \frac{b(\tau)^\top \sqrt{\Sigma(X_t)}}{\sigma_t} dZ_t. \]

Multiplying the drift by \( \tau \) and the variance by \( \tau^2 \), and collecting terms, we have

\[ \mu^Q_t \tau = -a'(\tau) + \frac{a(\tau)}{\tau} + b(\tau)^\top \kappa \theta - \left[ b'(\tau) - \frac{b(\tau)}{\tau} + b(\tau) \kappa \right]^\top X_t, \]

\[ \sigma_t^2 \tau^2 = b(\tau)^\top \Sigma(X) b(\tau) = \sum_i b(\tau)_i^2 \alpha_i + \sum_i b(\tau)_i^2 \beta_i^\top X_t. \]

Applying the DDCM relation in (20) and then the instantaneous interest rate function in (21),

\[ y_t(T) = r_t + \mu^Q_t \tau - \frac{1}{2} \sigma_t^2 \tau^2 \]

\[ = a_r + b_r^\top X_t - a'(\tau) + \frac{a(\tau)}{\tau} + b(\tau)^\top \kappa \theta - \left[ b'(\tau) - \frac{b(\tau)}{\tau} + b(\tau) \kappa \right]^\top X_t \]

\[ - \frac{1}{2} \sum_i b(\tau)_i^2 \alpha_i - \frac{1}{2} \sum_i b(\tau)_i^2 \beta_i^\top X_t, \]

which is affine in \( X_t \). Applying (23) and collecting terms, we have

\[ \frac{a(\tau)}{\tau} = a_r - a'(\tau) + \frac{a(\tau)}{\tau} + b(\tau)^\top \kappa \theta - \frac{1}{2} \sum_i b(\tau)_i^2 \alpha_i, \]

\[ \frac{b(\tau)}{\tau} = b_r - \left[ b'(\tau) - \frac{b(\tau)}{\tau} + b(\tau) \kappa \right] - \frac{1}{2} \sum_i b(\tau)_i^2 \beta_i. \]
Re-arranging, our DDCM leads to the same ordinary differential equation as the affine model,

\[
d'(\tau) = a_r + b(\tau) \tau \kappa \theta - \frac{1}{2} \sum_i b(\tau)^2 \alpha_i,
\]

\[
b'(\tau) = b_r - b(\tau) \kappa - \frac{1}{2} \sum_i b(\tau)^2 \beta_i.
\]

Our DDCM pricing relation starts with $\mu^Q_t$ and $\sigma^2_t$ and derive their linkage to the yield level via dynamic no-arbitrage arguments, without referring to other parts of the curve. The standard DTSM starts with the instantaneous interest rate dynamics and derives the whole yield curve as well as their dynamics via no arbitrage arguments. The derived yield level and the derived yield dynamics naturally satisfy the no-arbitrage relation that we derive. By deriving everything from a centralized instantaneous interest rate dynamics, DTSM allows one to compare the cross-sectional behavior of whole yield curve. By focusing on the drift and volatility of a particular yield, our DDMC relation allows one to link the level of one particular yield to its own near-term dynamics.

2.4.2. Centralizing DDCM to DTSM

Our DDCM derives the no-arbitrage relation between the level of one particular yield and its near-term dynamics. By imposing a functional linkage on the near-term dynamics across the whole yield curve, we can also centralize the DDCM relation to arrive at something closer to a dynamic term structure model. This centralization process, however, is not always easy because it is not straightforward to simultaneously assume the near-term dynamics of all yields without introducing arbitrages among them. In what follows, we show one particularly simple example that allows us to do the centralization without introducing cross-sectional arbitrage.

Assume that the continuously compounding yield curve goes up and down in parallel under the physical
measure $\mathbb{P}$. If we use $y_t(\tau)$ to denote the yield at a fixed time to maturity $\tau$, we can write its dynamics as,

$$dy_t(\tau) = \sigma dW_t^\mathbb{P},$$

(24)

for all $\tau \geq 0$, where we assume zero drift and the same volatility $\sigma$ for all maturities $\tau$.

Further assume that the time-$t$ market price of the Brownian risk $(-dW_t)$ on the bond is $\lambda$. We can derive the yield dynamics under a risk-neutral measure $\mathbb{Q}$ as,

$$dy_t(\tau) = \lambda \sigma dt + \sigma dW_t.$$

(25)

Now consider a zero-coupon bond with fixed expiry $T$. The parallel shifting yield curve implies that the risk-neutral dynamics for the yield of this zero-coupon bond $y_t(T)$ can be written as

$$dy_t(T) = dy_t(\tau) - y_t'(\tau) dt = \left[\lambda \sigma - y_t'(\tau)\right] dt + \sigma dW_t,$$

(26)

where the $y'(\tau)$ term accounts for the sliding of the yield for this bond along the yield curve.

Starting with the drift and diffusion in (26), we can apply our DDCM pricing relation in (20), and represent the yield with fixed time to maturity as,

$$y_t(\tau) = r_t - \left(\lambda \sigma + y_t'(\tau)\right) \tau - \frac{1}{2} \sigma^2 \tau^2,$$

(27)

for all $\tau$.

Define $z_t(\tau) = y_t(\tau) \tau = -\ln B_t(T)$, equation (27) implies

$$z_t'(\tau) = r_t + \lambda \sigma \tau - \frac{1}{2} \sigma^2 \tau^2,$$

(28)

for all $\tau$. Thus, under this particular parallel shift assumption and taking logs on the zero-coupon bond price
curve, the short rate is simply the negative of the slope of this log price curve at zero maturity, the risk
premium is the curvature of this log price at zero maturity, and the yield variance is just the third derivative
of the log price curve.

We can solve for the whole yield curve by integrating equation (28) over maturity,

\[
y_t(\tau) = \frac{z(\tau)}{\tau} = \frac{1}{\tau} \int_0^\tau \left( r_t - \lambda \sigma u - \frac{1}{2} \sigma_t^2 u^2 \right) du = r_t - \frac{1}{2} \lambda \sigma \tau - \frac{1}{6} \sigma_t^2 \tau^2, \tag{29}
\]

for all \( \tau \). By assuming parallel shifts on the yield curve and by specifying the full dynamics of all yields, equation (29) centralizes the DDCM pricing relation to arrive at a term structure model.

If we instead only assume the near-term dynamics by allowing \( \sigma_t \) and \( \lambda_t \) to vary over time with unknown
dynamics, the local differential equation in (27) remains valid from our DDCM, but we can no longer
perform the integration in (29) without knowing the full path of \( \sigma_t \) and \( \lambda_t \) from \( t \) to \( T \). To derive the full
term structure model necessitates the specification of the full dynamics.

Merton (1973) considers in a footnote a similar model with \( dr_t = \sigma dW_t^P \) and arrives at a similar term
structure that excludes arbitrage. This particular example not only guarantees no arbitrage between the
particular zero-coupon bond and the money market account, but also guarantees that bonds across all finite
maturities do not allow arbitrage. To verify, we can start with \( dr_t = \lambda \sigma dt + \sigma dW_t \). Then the relation in (29)
between \( y_t(\tau) \) and \( r_t \) suggest that \( dy_t(\tau) = dr_t = \lambda \sigma dt + \sigma dW_t \), just as we have assumed to begin with.

3. Data and summary statistics

We perform an empirical analysis using US and UK swap rates. The financing leg of the swap contracts
for both currencies are the 6-month LIBOR rate. We can treat the swap rates as the coupon of a par bond.
We obtain the LIBOR and swap rate data from Bloomberg, daily from January 3, 1995 to December 29,
2017, spanning 5,790 business days. The swap maturities include 2, 3, 4, 5, 7, 10, 20, 30, 40, and 50 years.
Shorter-maturity swaps are available over the whole sample period. Longer-maturity swaps start at a later date. For the US swap rates, 40- and 50-year swap rates become available since November 12, 2004. For UK, the 20- and 30-year swap rates become available since January 19, 1999, and the 40-year and 50-year swap rates become available since August 8, 2003.

Figure 1 plots the time series of the swap rates at selected maturities for US in Panel A and UK in Panel B. Over the sample period, the long-term rates show a downward sloping trend for both economies, reflecting the downward trend in inflation. The term structure variations, on the other hand, reveal the real business cycle.

Based on the daily changes of the swap rates series, we construct a simple volatility rate estimator on each series with a one-year rolling window. Figure 2 plots the time series of the rolling estimates. For each swap rate series, the volatility estimates vary strongly over time, reaching its peak during the 2009 financial crises but having been calming down ever since. Across different series, the graph shows that long-dated swap rates vary as much as, if not more than, short-term rates. Classic mean-reversion specifications on the instantaneous interest rate often have difficulties capturing the large volatility on very long dated interest rate series. By contrast, our new pricing theory allows us to directly take the volatility estimators as inputs without making explicit assumptions on its dynamics. The persistently high volatility at very long dated swap rates suggest that the convexity effects can be large on long rates. Comparing the two economies, although the interest rate levels are comparable, the interest rate volatility looks lower for the UK than for the US overall.
4. Applications

We explore practical applications of our new pricing theory from several angles. First, we propose to predict future bond excess returns while assuming no predictability on long-dated floating interest rate series. Second, we incorporate expectation hypothesis into our pricing framework to enhance the bond return predictability. Third, we re-examine the well-known butterfly trades under our new pricing framework.

4.1. Predicting long bond returns with no rate predictability

It is extremely difficult to predict long-term interest rate movements. Historically, the literature has strived to predict future short-rate changes based on the slope of the year curve, but that literature does not provide much help in predicting long-term rates. As an application of our new pricing theory, in this section, we take no-predictability on long-term rate as as a starting point, and infer bond risk premium from the observed interest rate level and interest rate volatility estimates. We then examine the predictability of the extracted bond risk premium on future bond excess returns.

We start by assuming that the constant-maturity floating yield $y_t(\tau)$ at some long fixed time to maturity $\tau$ moves diffusively like a random walk over the next instant,

$$dy_t(\tau) = \sigma_t(\tau) dW_t^P,$$  \hspace{1cm} (30)

where $\sigma_t(\tau)$ denotes the time-$t$ conditional forecast of the volatility rate of this yield. If we denote the market price of the risk for the corresponding bond ($-dW_t$) as $\lambda_t$,\(^5\) we can derive the risk-neutral dynamics for the yield as

$$dy_t(\tau) = \lambda_t \sigma_t(\tau) dt + \sigma_t(\tau) dW_t.$$  \hspace{1cm} (31)

\(^5\)Given our notation, the market price of the interest rate risk would be $-\lambda_t$. Throughout this paper, even if we start with the yield dynamics, we deliberately model the market price of the Brownian risk on the bond, which is simply the negative of the Brownian risk on the yield. This way, the market price is more in line (in sign) with the bond excess return that our analysis focuses on.
The risk-neutral drift for the fixed-expiry yield $y_t(T)$ is further adjusted by the local shape of the yield curve as it slides along the curve,

$$\mu_{Q}^{Q_{y_t}} = \lambda_t \sigma_t(\tau) - y_t'(\tau). \quad (32)$$

Plug the no-prediction dynamics into the DDCM pricing relation in (20) leads to the following differential equation

$$\frac{\partial (y_t \tau)}{\partial \tau} = r_t + \lambda_t \sigma_t(\tau) - \frac{1}{2} \sigma_t(\tau)^2 \tau^2. \quad (33)$$

For zero-coupon bonds, $\frac{\partial (y_t \tau)}{\partial \tau} = f_t(\tau)$ is the instantaneous forward rate. Equation (33) then links the instantaneous forward rate level to its volatility and market price of risk.

In the general case of coupon bonds, it is convenient to define the instantaneous volatility weighted duration and convexity as

$$d_t(\tau) = \sigma_t(\tau) \tau, \quad c_t(\tau) = \sigma_t(\tau)^2 \tau^2. \quad (34)$$

If we further assume that the market price of bond risk $\lambda_t$ is the same across bond maturities, we can integrate equation (33) and obtain,

$$y_t = r_t + \lambda_t D_t - \frac{1}{2} C_t, \quad (35)$$

where we single out the common market price of bond risk and use $D_t$ and $C_t$ to denote the integrated duration and convexity over maturity $(0, \tau),$

$$D_t \equiv \frac{1}{\tau} \int_0^\tau d_t(s)ds, \quad C_t \equiv \frac{1}{\tau^2} \int_0^\tau c_t(s)ds. \quad (36)$$

Equation (35) shows that in absence of rate prediction, positive bond risk premium drives the yield curve up while convexity drives the curve down. Furthermore, based on volatility estimators on the yield curve $\sigma_t(\tau)$, we can compute the integrated duration and convexity. Combining them with the observed interest rates $y_t$
and the financing cost $r_t$, we can infer the common market price of bond risk as,

$$\lambda_t = y_t - r_t + \frac{1}{2} C_t \frac{D_t}{D_t}.$$  \hfill (37)

Thus, under the no-predictability assumption on long-term floating rates, we can extract the market price of bond risk based on observed interest rate level as well as the volatility estimators on daily changes on the interest rates. We examine whether the ex ante market price of risk estimated from yield spreads and volatility estimates can predict bond excess returns.

Assuming that long-dated swap rates with maturities 10 years and longer are not predictable, we can infer the market price of bond price from these long-term swap rates based on our volatility-weighted convexity and duration estimates according to equation (37). In computing the market price of bond risk, we take the financing rate (6-month LIBOR) as the short rate $r_t$, and construct volatility estimators on daily changes of each swap rate series with a one-year rolling window. To estimate the convexity contribution, we treat the swap rates as par bond yields and use summations to approximate the integral in (36).

Figure 3 plots the time series of the extracted market price of bond risk ($\lambda_t$) from each swap series. In performing the integration in equation (35), we assume that the market price of bond risk is the same across different bonds. The plots in Figure 3 largely confirm this hypothesis: The market price of bond price extracted from different swap rate series are similar in magnitude and move closely together. Over the common sample, the cross-correlation estimates among the different $\lambda_t$ series average 99.68% for the US and 99.23% for the UK. The evidence supports a one-factor structure for the bond risk premium, as found in Cochrane and Piazzesi (2005).

On average, the market price of bond risk estimates are positive for both economies, supporting the hypothesis of positive bond risk premium. Nevertheless, the estimates vary strongly over time. In the US, the market price of risk estimates approached zero in late 1998, in 2000, and again in 2007, but the estimates...
tend to become higher during recessions. In the UK, the market price of bond risk became quite negative in 1998 and again between 2007 and 2008.

To examine whether the ex-ante risk premium estimates predict future bond excess returns, we measure the forecasting correlation between the ex ante risk premium estimates ($\gamma_t, \sigma^m_t$) and ex post excess returns on each par bond. Table 1 reports the forecasting correlation estimates. The last column reports the forecasting correlation between the average risk premium and the average bond excess return over the common sample period. Paradoxically, the assumption of no prediction on long-dated swap rates leads to significant prediction on bond excess returns. The predictors (risk premium) are generated based purely on a variance estimator and the current slope of the yield curve, without estimating predictive regressions.

4.2. Enhancing bond return prediction by formulating rate expectation

The previous section generates significant bond return prediction with the assumption of no predictability on changes of long-dated floating interest rate series. We can potentially enhance the return prediction if we can formulate accurate predictions on interest rate changes.

The literature on expectation hypothesis proposes to predict future short-term interest rate changes with the yield curve slope. Our new pricing theory highlights the insight that the yield curve slope contains not only the expected rate of yield change, but also a risk premium component and a convexity component. While it is difficult to determine separate the risk premium component from the expectation, we can at least enhance the expectation hypothesis by adjusting for the convexity effect. In particular, we propose to take the long end of the swap rate ($y_{t,L}$) to define the slope against the financing rate ($r_t$) while adjusting for the convexity effect as

$$AS_t = y_{t,L} - r_t + \frac{1}{2}C_{t,L}. \tag{38}$$

Furthermore, while the expectation hypothesis says nothing about the prediction of long rates, we propose to predict long rate movements based on anticipated central bank action, which we capture using the
yield curve slope at the very short end of the swap rate curve,

\[ CB_t = y_{t,2} - y_{t,1}, \]  

(39)

where 1, 2 denote the shortest two maturities. Anticipated monetary tightening reins in future inflation, ultimately bringing down long-dated inflation (Rotemberg and Woodford (1996)).

We propose to combine these two components to generate a prediction on the expected rate of change at each maturity as

\[ \mu_t(\tau) = \frac{1 - e^{-\kappa \tau}}{\kappa \tau} (AS_t + CB_t) - CB_t, \]  

(40)

where the coefficient \( \kappa \) controls the weighting between the two components. The prediction is mainly driven by expectation hypothesis at short maturities, but more driven by the anticipated central bank policy for long maturities.

In our empirical implementation, we set \( \kappa = 1 \) with a one-year decay. We take the 30-year swap rate as the long rate to compute \( AS_t \) and interpolate to obtain the short-term slope estimate between one and two-year maturity. Table 2 reports the forecasting correlation of the formulated expected rate of change in (40) on the corresponding swap rate changes over the next six month (Panel A) and one year (Panel B). The table shows that our expected rate of change formulation generates strong predictions on the swap rate changes across all maturities.

Given the strong predictability, we now incorporate the formulated expected rate of change for the floating rate into the risk premium extraction. In this case, the drift of the fixed expiry yield will be adjusted by \( \mu(\tau) - y'(\tau) \), and we estimate the local slope of the yield curve. Table 3 reports the forecasting correlation of the adjusted risk premium on future bond excess returns with horizons of six months (Panel A) and one year (Panel B). By embedding the interest rate change prediction, we generate strong predictions on the bond excess return across the whole spectrum of bond maturities.
5. Concluding remarks

In this paper, we propose a new modeling framework that is particularly suited for analyzing returns on a bond or bond portfolio. The framework does not try to model the full dynamics of an instantaneous short rate, but focus squarely on the behavior of the bond yield in question. It does not even ask for the full dynamics specification of this bond yield, but only needs estimates of its current expectation, risk premium, and volatility. It can readily accommodate results from other models and algorithms.

The model framework decomposes each yield into three components: expectation, risk premium, and volatility. One can estimate the volatility from historical time series, or infer it from the curvature of the yield curve, or interest rate options. We show that we can predict bond excess returns, without running predictive regressions, even by assuming no prediction on interest rates. For future research, separating risk premium from expectation can be a very challenging, but very fruitful endeavor.
References


Table 1
Forecasting correlation of ex ante bond risk premium on future bond excess returns with no rate predictability assumption
Entries report the forecasting correlation between the bond risk premium extracted from each swap rate and the future excess return of the corresponding par bond over the next six months (panel A) and one year (panel B). Each column denotes one swap series. The last column reports the correlation between the average risk premium estimates from the six swap series and the average excess return on the corresponding bonds.

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<th>20</th>
<th>30</th>
<th>40</th>
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Table 2
Forecasting correlation of expected rate of change on future swap rate changes
Entries report the forecasting correlation between our formulated expectation on the rate of change and future changes in the corresponding swap rate.

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Table 3
Forecasting correlation of ex ante bond risk premium on future bond excess returns with enhancing expected rate of change prediction
Entries report the forecasting correlation between the bond risk premium extracted from each swap rate and the future excess return of the corresponding par bond over the next six months (panel A) and one year (panel B). Each column denotes one swap series.

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</tr>
<tr>
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Fig. 1. The time-series variation of US and UK swap rates. Each panel plots the time series of swap rates at four selected maturities: 2-year (solid line), 10-year (dashed line), and 30-year (dash-dotted line). Panel A represents the US swap rates and Panel B the UK swap rates.
Fig. 2. Volatility estimators on US and UK swap rates. Each panel plots the time series of the one-year rolling volatility estimators on the daily changes of the swap rate series at selected maturities: 2-year (solid line), 10-year (dashed line), and 30-year (dash-dotted line). Panel A represent the US swap rates and Panel B the UK swap rates.
Fig. 3. Market price of bond risk. Lines denote the time series of the market price of bond risk extracted from swap rates with maturities 10 years and longer, with Panel A for US swap rates and Panel B for UK swap rates.