Venture Capital Contracts *

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Abstract

We develop a dynamic search and matching model to estimate the impact of venture capital contract terms on startup outcomes and the split of value between entrepreneur and investor in the presence of endogenous selection. Using a new data set of over 10,000 first financing rounds of startup companies, we estimate an internally optimal equity split between investor and entrepreneur that maximizes the probability of success. In almost all deals, investors receive more equity than is optimal for the company. In contrast to theoretical predictions, participation rights and investor board seats reduce company value, while shifting more of it to the investors. Eliminating these terms increases startup values through rematching, making entrepreneurs better off and leaving all but the highest quality investors marginally worse off.

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A large body of academic work examines the problem of financial contracting, and frequently uses the context of an entrepreneur negotiating a financing deal with an investor. Start-up firms are key drivers of innovation and employment growth, and the efficient allocation of capital to early-stage firms is crucial to their success (Solow, 1957). Financial contracting plays a key role at this stage, as information asymmetries and agency problems are severe (Hall and Lerner (2010)). The theoretical literature explains the observed complex contracts between entrepreneurs and venture capitalists (VCs) by the improved incentives and information sharing that they engender (Cornelli and Yosha (2003), Repullo and Suarez (2004), Kaplan and Strömberg (2003), Hellmann (2006), Lee and Rajan (2016)). A contrasting view is that investors negotiate for certain complex contract terms not to grow the size of the pie that is divided between the contracting parties, but mainly to change the distribution of the pie in their favor. This is possible because VCs have greater bargaining power, are repeat players in the market, and have a better assessment of the distribution of outcomes, whereas entrepreneurs tend to be over-optimistic and overconfident (e.g., Cooper, Woo, and Dunkelberg (1988), Busenitz and Barney (1997), Camerer and Lovallo (1999) AER, Bernardo and Welch (2001) On the Evolution of Overconfidence and Entrepreneurs, Landier and Thesmar (2009) Financial Contracting with Optimistic Entrepreneurs RFS, Lowe and Ziedonis (2006)), and lawyers do not have strong incentives to correct this imbalance. The resulting contracts are favorable to the VC, even if they reduce value overall, at the expense of the entrepreneur, who experiences poor returns (e.g. Moskowitz and Vissing-Jørgensen (2002) and Hall and Woodward (2010)). As of yet, there is little empirical evidence that quantifies in which direction, let alone how much, various contract terms impact outcomes and the distribution of value. This paper helps fill that gap.

A main empirical problem in addressing this question is that contracts are related to the underlying qualities of the entrepreneur and investor, which are unobserved. To address the resulting omitted variables problem we specify a dynamic search and matching model. In broad strokes, the model works as follows. Penniless entrepreneurs search for investors in their startups, and vice versa. When two potential counterparties meet, the investor offers a contract. The entrepreneur has bargaining power due to the possibility of refusing the contract and resuming the search process in the hopes of meeting a higher quality investor. The model allows for the contract to affect outcomes (the size of the pie) and the split between investor and entrepreneur (the split of the pie). Compared to static matching models, our model is tractable and intuitive despite the addition of dynamics and contracts. Intuitively, the dynamic search feature of the model generates a random component to matches, which helps to identify the impact of contracts on outcomes and value splits, controlling for the qualities of the entrepreneur and the investor.

The second main problem is that startup contracts are private, and data is difficult to find.
To take the model to the data, we collect a new data set that contains more than 10,000 first-round VC financings, of which nearly 5,000 have detailed contract data. This constitutes the largest set of contracts studied in the literature to date, and includes data on both cash flow and control rights. Nearly all contracts are some form of convertible preferred equity. We focus on the investor’s equity share upon conversion, participation rights, and investor seats on the startup’s board. Participation is a cash flow right that gives the investor a preferred equity payout with an additional common equity claim. In contrast, in a convertible preferred security without participation, the investor must ultimately choose between receiving the preferred payout or converting to common equity (Cornelli and Yosha (2003)). Board seats are an important control right that gives the VC direct influence over corporate decisions.

We find the following results. First, there is an internal optimal equity share that maximizes the startup’s probability of a successful exit, consistent with theories of double moral hazard in which both investor and entrepreneur need to effort for the company to succeed (e.g. Schmidt, 2003; Casamatta, 2003; Hellmann, 2006). Second, both participating preferred stock and VC board seats lower the chance of success, while transferring a larger fraction of the startup’s value to the VC. The traditional view that participation makes the entrepreneur exert more effort, may be offset by reduced entrepreneur effort due to the reduction in ownership stake and by asset substitution incentives from the debt-like features of participation rights. The value creation aspects of investor board representation due to improved governance and monitoring may be offset by reduced incentives for the entrepreneur to exert effort because they have less ownership and control over key decisions.

Despite their value-reducing impact, the VC benefits from participation and board seats because the VC receives a larger share of the value, which on balance increases the VC’s expected payoff. The first-best contract that maximizes the startup’s value gives the VC an equity share of 7.1% and no participation or board seats, but due to the other contractual features, the VC actually receives 27% of the startup’s value. In the average observed deal, the startup’s value is only 85% of the first-best value, but the VC gets 44% of the value.

We find important trade-offs between the cash flow and control rights of the contract, as a function of investor and entrepreneur quality. Entrepreneurs (VCs) match with a range of VCs (entrepreneurs) between an upper and lower threshold, and this range is weakly increasing in the entrepreneur’s (VC’s) quality. An entrepreneur who matches with his or her lowest acceptable quality VC negotiates a contract with no participation or VC board seats, and a low investor equity share. If the same entrepreneur encounters and matches with a higher quality VC, the VC’s equity share rises, up to a point where the VC has enough bargaining power to negotiate for board seats. The board seat causes a drop in firm value, but this is mitigated by the higher quality
of the VC (which increases the startup’s value) and a smaller increase in the VCs equity share, leaving the entrepreneur no worse off. If the entrepreneur matches with a higher quality VC, the equity share rises again, to a point where the VC asks for participation rights. For higher quality entrepreneurs, this is offset by the VC giving up its board representation and taking a smaller equity stake (which moves the firm closer to its first-best value). When entrepreneurs match with the very best VCs they can hope to pair up with, the VC gets both participation and board seats.

One limitation of our approach is that we cannot make statements about the impact on value of terms that are always present. However, we can estimate the joint effect of these terms on the value split. Overall, we find that they transfer a larger fraction of the company’s value to the VC. However, since the terms are always present and thus not likely to be very contentious, it is not clear that the entrepreneur is worse off under these terms. They may in fact increase the startup’s value, such that both VC and entrepreneur benefit.

We explore the effects of eliminating the possibility of using participation and board seat contract terms. The immediate effect is that more value shifts towards entrepreneurs, negatively affecting VCs. If we keep matches the same, the effect on firm values is negative but small. The effects become significantly larger if we allow market participants to rematch, and it is most pronounced for low quality entrepreneurs. They are able to match with higher-quality VCs and at higher rates as their bargaining power has increased, because they no longer have to accept participation and investor board seats. In the aggregate, average firm value rises by 2% and, due to higher matching rates, the value of all deals in the market rises by 4.6%. We should note that these effects are all on the intensive margin, because we cannot say what happens on the extensive margin, in terms of how many entrepreneurs and investors would enter or leave the market.

Our paper is related to a few different strands of literature. First, in the empirical literature on selection in venture capital, our paper is related to Sørensen (2007), who estimates the impact of selection (matching) versus entrepreneur and investor characteristics on the firm outcome (specifically, the IPO rate). Sørensen (2007) estimates a static matching model in which the split of total firm value between the entrepreneur and investor is exogenously fixed across matches. Our paper differs in two important ways. First, we model the market for venture capital as a dynamic market, instead of a one-shot market, which is more realistic and more tractable. Second, we allow for the endogenous split of the total firm value between the entrepreneur and investor via negotiated contracts. These modifications affect the estimated impact of selection versus characteristics on the firm value. In addition, endogenous contracting allows us to characterize the impact of various contract terms on outcomes. Our work is also related to Fox, Hsu, and Yang (2015), who study identification in a one-shot matching model with possibly endogenous terms of trade. Their work is mostly theoretical and their application to venture capital does not include contracts.
Second, our paper fits into the empirical literature on VC contracts, surveyed in Da Rin, Hellmann, and Puri (2013). The first paper to study contracts is Kaplan and Strömberg (2003). Based on a sample of 213 investments, they provide evidence that the observed contract terms are consistent with both principal-agent and control-rights theories. Hsu (2004) finds that more reputable VCs invest in startups at more investor-friendly terms, consistent with our results. Cumming (2008) uses a sample of 223 investments in European VC-backed startups and shows that stronger VC control is associated with lower probability of an IPO, also consistent with our results on board seats. Bengtsson and Ravid (2009) find significant regional variation in contracts, which is partially driven by differences in competition among investors. Competition is an important feature of our model. Bengtsson and Bernhardt (2014) show that venture capital firms exhibit “style” in their contracts, recycling them over multiple startups. This result is also consistent with investor quality being a primary determinant of contracts, as in our model, given that quality is likely to be highly persistent. Finally, Bengtsson and Sensoy (2011) find that more experienced VCs obtain weaker downside-protecting contractual cash flow rights than less experienced VCs. Their explanation is that experienced VCs have superior abilities and more frequently join the boards of their portfolio companies, but the result is also consistent with more experienced VCs matching with higher quality entrepreneurs. Bengtsson and Sensoy (2011) and Bengtsson and Bernhardt (2014) use data that is incorporated in our data set, but we significantly expand the number of deals with contracts. They have 1,534 and 4,561 contracts, respectively, across all stages of financing rounds, whereas we have 5,176 deals with some contract data beyond equity shares on first financing rounds alone (across all rounds the data contain over 21,000 contracts).

A recent, complimentary paper by Gornall and Strebulaev (2017) also considers the impact of certain contract terms on valuations, using a contingent claims model in the spirit of Merton (1973). Unlike our paper, they can model terms that are always present and provide valuations in dollars, whereas we can only study sensitivities of valuations to contract terms. However, they cannot determine the impact of control contract terms (such as board seats) on outcomes, or model the role of selection, matching, and the importance of VC and entrepreneur quality. We also do not require a complex option valuation model, which is sensitive, amongst others, to assumptions of a geometric Brownian motion process of the underlying asset, ignoring jumps and time-variation in volatility (Peters (2017)).

The matching model in our paper borrows from the theoretical search-matching literature with endogenous terms of trade. Shimer and Smith (2000) and Smith (2011) establish conditions for existence of a search equilibrium and positive assortative matching in a continuous-time model with a single class of agents encountering each other. In our paper, endogenous contracting implies that generally, conditions for a positive assortative matching do not hold even when entrepreneur
and investor types, given a contract, are complements. Due to the introduction of contracts we do not find that positive assortative matching holds in equilibrium under estimated model parameters. Adachi (2007) models the marriage market with two classes of agents (males and females) and endogenous terms of trade as a discrete-time game and shows that as the frequency of encounters increases, the set of equilibrium matches converges to the set of stable matches in a one-shot problem of matching with contracts of Hatfield and Milgrom (2005). Our model is continuous-time, but the Poisson process for encounters makes it similar to Adachi’s model.

The paper is organized as follows. Section 1 discusses the identification intuition behind our approach. Section 2 introduces the formal model. Section 3 describes our data. Section 4 presents our estimation results, with counterfactuals in section 5. Section 6 discussed robustness and proposes model extensions, and section 7 concludes.

1 Identification

To illustrate the identification problem and the source of variation in the data that the model exploits to identify the impact of contracts on outcomes, consider the following example. Entrepreneurs search for an investor to finance their start-up company, while at the same time investors are searching for entrepreneurs to fund. Due to search frictions, potential counterparties encounter each other randomly. Upon meeting, the parties attempt to negotiate a contract that is acceptable to both sides. For the purpose of this example, a contract, $c$, is the share of common equity in the start-up received by the investor. If successful, the value of the start-up is

$$\pi = i \cdot e \cdot \exp\{-2.5 \cdot c\}. \quad (1)$$

The negative impact of $c$ on the value can be justified by entrepreneurs working less if they retain a smaller share of the start-up (in the estimation, we do not restrict the impact to be negative). Suppose there are three types of investors, characterized by $i = 1, 2, 3$, that an entrepreneur is equally likely to encounter. Similarly, suppose there are three types of entrepreneurs, $e = 1, 2, 3$, that an investor is equally likely to encounter. For example, if an $i = 1$ investor and an $e = 2$ entrepreneur meet and agree on $c = 0.4$, then $\pi = 2 \cdot \exp\{-1\}$, the investor receives shares worth $0.8 \cdot \exp\{-1\}$ and the entrepreneur retains an equity stake worth $1.2 \cdot \exp\{-1\}$.

Let feasible matches be as shown in the table below (these outcomes are presented here as given, but in fact are determined endogenously in the equilibrium of the model for a certain set of parameters). Cells for which a match is feasible, contain the value of the start-up, $\pi$, and contract that is acceptable to both the investor and entrepreneur, $c^*$. Empty cells indicate that
no contract is acceptable to both agents, relative to waiting for another counterparty to come along. For example, an $i = 3$ investor will match an with $e = 2$ or $e = 3$ entrepreneur, whoever is encountered first, but not with an $e = 1$ type, because the value of waiting for one of the higher type entrepreneurs is higher than the value that could be received from making this match.

<table>
<thead>
<tr>
<th>Investor type ($i$)</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$\pi = 4.39$</td>
<td>$\pi = 5.11$</td>
<td>$c^* = 0.13$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Entrepreneur type ($e$)</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$\pi = 2.51$</td>
<td>$\pi = 2.92$</td>
</tr>
</tbody>
</table>

| 1 | $\pi = 0.58$ | $\pi = 0.74$ | $c^* = 0.21$ | $c^* = 0.4$ |

If we could collect a data set of $i$, $e$, $c^*$, and $\pi$ for a number of realized matches from this game, then the regression

$$\log \pi = \beta_1 c^* + \beta_2 i + \beta_3 e + \varepsilon,$$

(2)
is identified and recovers the true coefficients, $\beta_1 = -2.5$, $\beta_2 = 1$, $\beta_3 = 1$, even though matches and contracts are formed endogenously. In practice, in the VC market the researcher has very limited information about most entrepreneurs and infrequent investors. Suppose $e$ is not observed. The regression using remaining observables,

$$\log \pi = b_1 c^* + b_2 i + \varepsilon,$$

(3)
yields the biased estimates $\hat{b}_1 = -4.16$ and $\hat{b}_2 = 2.29$. This is an omitted variables problem, as $e$ is in the residual, and is correlated with $c^*$ and $i$. The bias in $\hat{b}_1$ is negative because higher type entrepreneurs retain a larger share of their companies, so that $e$ and $c^*$ are negatively correlated. The positive bias in $\hat{b}_2$ is due to the positive correlation between $i$ and $e$, as better investors tend to match with better entrepreneurs. Suppose next that both $i$ and $e$ are not observed. A similar regression then yields an even more biased $\hat{b}_1 = 2.04$, which can lead the researcher to incorrectly conclude that $c^*$ improves the company’s value.

To resolve the endogeneity problem, ideally we would have an instrument or natural experiment that generates variation in $i$ and $c$ that is uncorrelated with $e$, or in $e$ and $c$ that is uncorrelated with $i$, but these are very difficult to find. Another alternative would be to include fixed effects into the regression, which would isolate this variation and identify the model, albeit in a less statistically efficient manner compared to including agents’ types, as there are many investors and
entrepreneurs of equal type for whom a separate fixed effect has to be estimated. However, almost all entrepreneurs and some investors only participate in a single start-up in our data set.\footnote{Looking at multiple investment rounds for the same start-up is also not helpful because the start-up’s decision makers and objectives are very different across rounds, implying round-specific fixed effects.}

The final alternative is to exploit the search friction and endogenous match formation. In the example, again suppose $e$ is not observed. Take a given entrepreneur of, say, type $e = 2$. This entrepreneur will match with an investor of type $i = 2$ or $i = 3$, depending on who is encountered first, and sign contract $c^* = 0.19$ or $c^* = 0.29$. A investor of type $i = 2$, in turn, will match with any entrepreneur but only sign contract $c^* = 0.19$ with an entrepreneur of type $e = 2$. Similarly, an investor of type $i = 3$ will match with an entrepreneur of type $e = 2$ or $e = 3$ but only sign contract $c^* = 0.29$ with an entrepreneur of type $e = 2$. Hence, observing $i$ and $c^*$ recovers the entrepreneur’s type. Suppose next that both $i$ and $e$ are not observed. Even then, observing only $c^*$ recovers the investor’s and entrepreneur’s type: for example, $c^* = 0.19$ recovers $i = 2$ and $e = 2$.

In practice, the number of the investor’s and entrepreneur’s types is large, so there can be situations when different combinations of agents sign the same contract. Additionally, the researcher typically does not have a reliable estimate of the value of the start-up $\pi^2$, but instead observes coarse measures of its success (e.g., whether the start-up underwent an initial public offering). These complications mean that the reverse engineering of individual types and the value for each match has to be done simultaneously from contracts and other measures of success, can be imprecise, and is extremely computationally intensive. Instead of reverse-engineering individual $i$, $e$, and $\pi$ for each match, we therefore take a more feasible approach and recover aggregate distributions of $i$, $e$, and $\pi$ across all agents present in the market. We do so by directly matching the aggregate distributions of outcomes across matches produced by the model with the same distributions in the data. Specifically, we use the method of moments to match theoretical and empirical average $c^*$, its variance, its covariance with the IPO rate, etc. Coming back to the example, only the uniform distribution of both investor’s and entrepreneur’s types, and $\beta_1 = -2.5$ would achieve the best fit between theoretical and empirical moments of outcomes.

Among multiple ways to model endogenous match formation, we choose the model of dynamic search and matching. As a point of contrast, the prior approach in the literature has relied on static matching models that lack the search feature (Sørensen (2007)). In these models, all agents immediately see everyone else in the sample and, as a result, each investor type matches with one entrepreneur type (and vice versa). In turn, there is not enough exogenous variation to separately identify the impact of each agent’s type on the contract, and of each agent’s type and the contract on the value. The literature resolves this problem by splitting the sample of

\footnote{In Section 4, we discuss shortcomings of the “post-money valuation” measure sometimes used for this purpose.}
matches into subsamples by time and argues that all agents who match in a given subsample immediately see everyone else in the subsample but not across subsamples. To the extent that subsamples are different, each investor type matches with one, but different, entrepreneur type (and vice versa) across subsamples, thus resolving the problem. Since the model of dynamic search and matching generates random encounters for any given agent’s type, the necessary exogenous variation naturally arises in it. In turn, we can analyze the entire market at once without arbitrarily splitting it. The final advantage of the dynamic search and matching model is that it is more computationally feasible.\(^3\)

2 Model

This section describes the full model, which formalizes the intuition from the previous section. Time is continuous and indexed by \(t \geq 0\). There are two populations of agents in the market, one containing a continuum of investors and the other a continuum of entrepreneurs. Each investor is characterized by a type \(i \in [i_l, i_u]\), distributed according to a c.d.f. \(F_i(i)\) with a continuous and positive density. Similarly, each entrepreneur is characterized by a type \(e \in [e_l, e_u]\), distributed according to a c.d.f. \(F_e(e)\) with a continuous and positive density. Over time, agents cannot switch populations and their types do not change.

Agents arrive to the market unmatched and search for a suitable partner to form a start-up. Search is exogenous: each investor randomly encounters an entrepreneur from the population of entrepreneurs according to a Poisson process with positive intensity \(\lambda_i\). Similarly, each entrepreneur randomly encounters an investor from the population of investors according to a Poisson process with positive intensity \(\lambda_e\).\(^4\) Search is costly because agents discount value from potential future encounters at constant rate \(r\). Upon an encounter, identities of counterparties are instantly revealed to each other, and they may enter contract negotiations.

During negotiations, an investor offers a take-it-or-leave-it contract \(c \in C\) to an encountered entrepreneur, where contract space \(C\) is a set of all possible combinations of contract terms.\(^5\) If

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\(^3\)Because in static matching models, all agents immediately see everyone else, identification proceeds by comparing matches realized in the sample with all unrealized counterfactual matches. The true parameters of the model are obtained when the set of theoretical matches best approximates the set of realized matches in the sample. In the presence of multiple contract terms, the sheer number of counterfactual matches and contracts in them makes this approach infeasible. In contrast, by letting all agents only know the distribution of counterparties’ types and encounter a single agent at a time due to search frictions, the dynamic model of search and matching reduces to a simple comparison of matches realized in the sample with the easily computable agents’ continuation values.

\(^4\)These assumptions imply that the likelihood to encounter a counterparty of a certain type is independent from a searching agent’s type, and independent across agents.

\(^5\)For example, if the counterparties can only negotiate over the fraction of equity that the investor receives, then the contract space is a one-dimensional set of fractions of equity: \(C \equiv [0, 1]\). If the counterparties can additionally negotiate over the liquidation preference, then \(C \equiv [0, 1] \times \{0, 1\}\): the second dimension of the contract space.
the entrepreneur rejects the offer, the agents separate, receive instantaneous payoffs of zero, and resume their search. In a dynamic model, the ability to walk away from an unfavorable offer thus endogenously gives the entrepreneur an entrepreneur type-specific bargaining power, which the investor can internalize in its take-it-or-leave-it offer. If the entrepreneur accepts the offer, the start-up is formed with the instantaneous expected value of

$$\pi(i, e, c) = i \cdot e \cdot h(c).$$

(4)

It is convenient to think of $\pi$ as the expected present value of all future cash flows generated by the start-up, including the exit value. This value is affected by types of counterparties as well as the contract they sign through the continuous and bounded function $h(c)$. The counterparties receive instantaneous payoffs

$$\pi_i(i, e, c) = \alpha(c) \cdot \pi(i, e, c),$$

(5)

$$\pi_e(i, e, c) = (1 - \alpha(c)) \cdot \pi(i, e, c),$$

(6)

where the continuous function $\alpha(c) \in [0, 1]$ is the effective fraction of the expected start-up value that the investor receives. For example, if the counterparties can only negotiate over the fraction of equity that the investor receives, then $\alpha(c) = c$. If the counterparties can additionally negotiate over other contract terms, $\alpha(c)$ can be different from the fraction of equity that the investor receives.

The equilibrium contract $c^* \equiv c^*(i, e)$ offered by investor $i$ to entrepreneur $e$ solves

$$c^*(i, e) = \arg \max_{c \in C: \pi_e(i, e, c) \geq V_i(e)} \pi_i(i, e, c).$$

(7)

Intuitively, the investor offers the contract that maximizes its payoff from the start-up subject to the participation constraint of the entrepreneur, who receives the expected present value $V_i(e)$ if it chooses to walk away. If $\pi_i(i, e, c^*) \geq V_i(i)$, the investor offers $c^*$ and the start-up is formed. Otherwise, the investor does not offer any contract, chooses to walk away, and receives the expected present value $V_i(i)$. Both $V_i(e)$ and $V_i(i)$ are defined below. The counterparties that successfully form a start-up exit the market and are replaced by new unmatched agents in their populations.\(^6\)

\(^6\)This assumption ensures that at any time, populations of unmatched investors and entrepreneurs are characterized by the same density functions. Stationarity of populations implies that since, in equilibrium, measures of encounters by agents from both populations have to be equal, measures of unmatched agents, $m_i$ and $m_e$, have to satisfy $\lambda_i m_i = \lambda_e m_e$. These measures only become relevant again when we examine the present value of all potential deals in the market in Sections 5 and 6.
All unmatched agents maximize their expected present values, \( V_i(i) \) and \( V_e(e) \). Let \( \mu_i(i) \) be the set of types \( e \) of entrepreneurs who are willing to accept offer \( c^*(i, e) \) from investor \( i \). Similarly, let \( \mu_e(e) \) be the set of types \( i \) of investors who are willing to offer \( c^*(i, e) \) to entrepreneur \( e \). Because populations of agents remain stationary over time, the model is stationary, so \( V_i(i) \) and \( V_e(e) \) do not depend on time \( t \). Consider \( V_i(i) \). At any time, three mutually exclusive events can happen over the next small interval of time \( dt \). First, with probability \( \lambda_i dt \int_{e \in \mu_i(i)} dF_e(e) \), investor \( i \) can encounter an entrepreneur with type \( e \in \mu_i(i) \), who is willing to accept the investor’s offer of \( c^*(i, e) \). If \( \pi_i(i, e, c^*) \geq V_i(i) \), the counterparties form a start-up and exit the market, and the investor exchanges its expected present value \( V_i(i) \) for instantaneous payoff \( \pi_i(i, e, c^*) \); otherwise the investor resumes its search and retains \( V_i(i) \). Second, with probability \( \lambda_i dt \left(1 - \int_{e \in \mu_i(i)} dF_e(e)\right) \), investor \( i \) can encounter an entrepreneur with type \( e \notin \mu_i(i) \), who is unwilling to accept the investor’s offer. Third, with probability \( 1 - \lambda_i dt \), the investor may not encounter an entrepreneur at all. In the last two cases, the investor resumes its search and retains \( V_i(i) \). Similarly, there are three mutually exclusive events that can happen to any entrepreneur \( e \) over the next small interval of time \( dt \), which shape \( V_e(e) \). The following proposition (with proof in Appendix A) formalizes the above intuition and presents compact expressions for the agents’ expected present values:

**Proposition 1.** Expected present values admit a discrete-time representation

\[
V_i(i) = \frac{\lambda_i}{r + \lambda_i} \int_{e \in \mu_i(i)} \max \{ 1_{e \in \mu_i(i)} \pi_i(i, e, c^*), V_i(i) \} dF(e),
\]

(8)

\[
V_e(e) = \frac{\lambda_e}{r + \lambda_e} \int_{i \in \mu_e(e)} \max \{ 1_{i \in \mu_e(e)} \pi_e(i, e, c^*), V_e(e) \} dF(i).
\]

(9)

Proposition 1 shows that our model is equivalent to a discrete-time model, in which periods \( t = 1, 2, ... \) capture the number of potential encounters by a given agent. These periods are of random length with expected length equal to \( \frac{\lambda_i}{\lambda_j} \), \( j \in \{ i, e \} \), so that next period’s payoffs are discounted at \( \frac{\lambda_i}{\lambda_i + \lambda_j} \). The discrete-time representation allows us to use the results of Adachi (2003, 2007) to numerically solve the contraction mapping (8) and (9).

The model described above is quite general. Contract terms impact the expected value of a start-up and its split between investor and entrepreneur in a flexible reduced-form way, via functions \( h(c^*) \) and \( \alpha(c^*) \). Since contract terms are generic, they can include the fraction of equity received by the investor, liquidation preferences, the number of investor board seats, and many more. In Section 4, we flexibly parameterize and estimate \( h(c^*) \) and \( \alpha(c^*) \). Importantly, first, we do not explicitly model a multitude of mechanisms, through which contracts can impact
values. By doing so, we do not commit to a specific microeconomic model that can potentially omit or misspecify the important mechanisms. On the contrary, our findings on which contract terms impact values can inform about which mechanisms previously considered in the theoretical literature are likely important in practice. Second, by considering the impact of contracts on expected values and evaluating it from agents' revealed preferences at the time of a start-up formation (agents make rational negotiation decisions to maximize their own payoffs), we avoid the problem of having to derive values of contracts with a multitude of complicated derivative features on an underlying asset. This value is extremely uncertain and most of it is driven by the volatility process of the underlying asset, which is entirely unknown in the VC market.

3 Data

We construct the sample from several sources, starting with U.S.-headquartered start-up company financing rounds between 2002 and 2015, collected from the Dow Jones VentureSource database. Although the sample of financings ends in 2015, we have information on exit events through June of 2017. These additional two years provide time for startups to exit and realize outcomes. We augment the Dow Jones sample with data from VentureEconomics (a well-known venture capital data source), Pitchbook (a relative newcomer in venture capital data, owned by Morningstar), and Correlation Ventures (a quantitative venture capital fund). These additional data significantly supplement and improve the quality and coverage of financing round and outcome information, such as equity stakes, acquisition prices, and failure dates.

A key advantage of Pitchbook over the other data sets is that it contains contract terms beyond the equity share sold to investors, with reasonable coverage going back as far as 2002. We further supplement this sample with contract terms information collected by VC Experts. Both Pitchbook and VC Experts collect articles of incorporation filings from Delaware and California, either electronically or in person, and encode the key venture capital contract terms from prior financing rounds described in those documents.

Our empirical model considers the first-time interaction between an entrepreneur and a profit-
maximizing investor, as the existence of prior investment rounds or alternative objective functions would significantly complicate the contracting game. To best approximate the model setup in the data, we restrict the sample to a start-up’s seed-round or Series A financings in which the lead investor is a venture capital firm.\footnote{Financings rounds greater than $100 are also excluded as they are more likely to be non-VC-backed startups.} Other early-stage investors, such as friends and family, angels, or incubators, may have objectives other than profit-maximization. Although start-ups often raise funds from other investors prior to accepting VC money, such funding is usually small relative to the size of the VC round, and is typically in the form of convertible notes, loans or grants whose terms do not materially affect the VC round contracts. The lead investor is the one who negotiates the contract with the entrepreneur, and is identified by a flag in VentureSource, or if missing, by the largest investor in the round. In the 29% of cases where neither is available, we assume the lead investor is the VC with the most experience by years since first investment by the time of financing. Our final filter limits the sample to rounds that involve the sale of common or preferred equity, the predominant form of VC securities. This filter thus excludes debt financings such as loans and convertible notes that have no immediate impact on equity stakes, or small financings through accelerators or government grants. We lose 11% of first round financings through this exclusion. We apply the above filters after collecting contract data from all articles of incorporation, including restatements filed after later financing rounds, as supplemental first-round contract terms can sometimes be identified from such refilings.

### 3.1 Descriptive Statistics

The sample consists of 10,967 first financing rounds of start-up companies, involving 1,998 unique investors. Table I provides variable definitions, and Table II reports summary statistics. Panel A of Table II reveals that at the time of financing, the average (median) start-up is 1.8 years (1.25 years) old, measured from the date of incorporation. Most start-ups are in the information technology industry (47% of firms), followed by healthcare (19%). To help identify the frequency with which investors and entrepreneurs meet, we compute how much time has passed since the lead VC negotiated its prior deal’s first financing round. The average (median) time between successfully negotiated first financing rounds for a given lead VC is 0.8 (0.3) years. For 1,745 rounds (16% of the sample), the VC is a first-time lead (but may have been a non-lead investor before) and we cannot calculate the time since last lead financing. These deals tend to be smaller, but otherwise do not appear to be systematically different from the deals for which the time since last lead financing is known (results not reported).

In the average (median) round, 2.4 (2) financiers invest $5.2 million ($2.7 million) in the firm
at a post-money valuation of $18.5 million ($10.8 million), where both amounts are in 2009 dollars. Post-money is the valuation proxy of the start-up after the capital infusion, which is calculated in a straightforward manner from the investors’ equity share.\textsuperscript{10} The post-money valuation is usually interpreted as the market value of the firm at the time of financing (π in the model), but it is calculated under the assumption that the entrepreneur (and any other investors) own the same security as the investor in the current round. However, in virtually all cases in our sample (95%), the investor receives preferred equity that is convertible into common stock, whereas the entrepreneur retains common equity (see also Gornall and Strebulaev, 2017). Since we are interested in the impact of contract terms on valuation, the post-money valuation would be a poor metric to use, and we use exit outcomes instead (discussed below). But these valuations are useful to compute the equity share of the company sold to investors from post-money valuation and the total capital invested. One traditional data source used in earlier studies – VentureSource – only contains post-money valuations for 1,938 deals for our sample period, mostly gathered from IPO filings of successful firms. Our additional data collection efforts provide another 4,085 observations, resulting in a more complete and balanced sample of 6,023 equity stakes. Panel B of Table II shows that the average (median) share sold is 35% (32%), ranging from 22% at the first quartile to 46% at the third quartile.

Contract terms beyond the equity share are not reported in the traditional VC data sets, and the empirical literature on contracts is small. Kaplan and Strömberg (2003) analyze 213 contracts from a proprietary data source. Bengtsson and Sensoy (2011) and Bengtsson and Bernhardt (2014) use the VC Experts data and have 1,534 and 4,561 contracts, respectively, across all stages of financing rounds. We are the first to add the Pitchbook data, which contributes more deals and spans a longer time series than VC Experts, and we have 5,176 deals with some contract data beyond equity shares on first financing rounds alone (across all rounds the data contain over 21,000 contracts). We consider two classes of contract terms. The first class involves the cash flow rights of investors. When the start-up is acquired or goes public, the investor can either redeem the preferred security, or convert it into common stock, whichever payoff is higher. In the case of nonconversion, the investor receives a payoff equal to the liquidation preference (or less if funds are insufficient) before common equity receives any payout, similar to a debt security payoff. The liquidation preference is typically equal to the invested amount (referred to as “1X”) in first round

\textsuperscript{10}The investors’ equity share is the share of the company owned by investors upon conversion (assuming no future dilution). For example, suppose the VC invests $2 million by purchasing 1 million convertible preferred shares at $2 per share, with a 1:1 conversion ratio to common stock. The entrepreneur owns 4 million common shares. VCs calculate the post-money valuation to be $10 million (5 million shares at $2 each). The ratio of invested amount to post-money valuation is 20%, which is identical to the ratio of investor shares to total shares upon conversion. Note that this computation does not take into account, e.g., value of convertibility of VC shares.
financings, but in 3% of first rounds the investor receives a higher multiple of invested capital. This provision serves as additional downside protection for the investor, as conversion to common equity is only attractive when the exit valuation is high. Participation is a term, used in 41% of contracts, that allows the investor to take its liquidation preference payout, and then convert its shares to common equity and receive its share of the remaining value. This raises the payoff to the investor in all outcome scenarios.

Other contract features available to preferred shareholders that involve cash flow rights include cumulative dividends, which are set at a fixed rate (e.g., 8%) and cumulate from investment to exit (payable only at liquidation). The investor requests this feature in a fifth of cases. Financings without this term typically have non-cumulative dividends that are only paid if the board declares them. A rarely used full ratchet anti-dilution rights term in our data (1%) acts as another form of downside protection. A financing with these rights would see the conversion price adjusted in step with any future financings with a share price lower than the current price. Some 10% of financings have entrepreneur-friendly pay-to-play requirements. These terms punish investors that do not reinvest in future financings. Finally, 35% of financings have redemption rights. The latter gives the holder of the security the option to call their capital back from the startup after 3-5 years. If a startup is unable to meet this call, then the preferred shareholder is typically given additional control or cash flow rights.

The second class of contract terms involves investor control rights over the start-up. We observe one major investor control right: board seats. Both VentureSource and Pitchbook provide information that allows us to identify whether the lead investor had a board seat at the time of the investment. Table II shows that 62% of lead investors in our sample have a board seat at the time of the first investment.

Panel C of Table II summarizes the exit outcomes. We follow financings through 2009 to allow time for our three exit outcomes: an initial public offering (IPO), acquisition or failure. Some firms have yet to exit by the end of the exit tracking period (June 2017) and are thus still private. The table first shows that 4% of startups exit via an initial public offering.\(^{11}\) Acquisitions are more common at 40%, however, many of these exits are hidden failures (e.g., Puri and Zarutskie (2012)). To separate out high- and low-quality acquisitions, we thus use the reported exit valuations if available. Exit valuations are almost universally available for IPOs and for a subset of typically successful acquisitions. With these data, we create a variable “IPO or Acq. > 2X capital” that equals to one if the start-up had an IPO or had an acquisition at a private at least two times total capital raised. The outcome “Out of business” characterizes whether a startup shut down or went into bankruptcy. It appears to be low at 17%, however, this is because of both the aforementioned

\(^{11}\) The rate falls to 2% if we consider first-financings in the full sample period 2002 to 2015.
hidden failures in acquisitions and the fact that many firms that are still private and in fact failed firms. We find that 16% have either an IPO or successful acquisition. Over one third of start-ups (38%) are still private.

3.1.1 Sample Selection

Revelation of contract terms is non-random. For example, start-ups that eventually achieve a public offering are required to disclose past financing round details, and large, successful start-ups are more likely to reveal their financing valuations while private. Contract data must be actively collected by the data providers Pitchbook and VC Experts, and the data suggests their sampling is non-random. Table III presents summary statistics for the sample of financings with and without contract data. The panel “Deals with contract data” considers the set of financings with at least one, in addition to equity split, observable contract feature discussed above. The panel “All deals” considers the full sample as described in Table II. As before, exit outcomes are only defined for financings before 2010.

There are few differences between financings with contracts and the full sample in terms of firm age, industry or syndicate size. Financings with contract data tend to raise more capital ($7.8 vs. $5.2) and occur earlier in our sample period (2008 vs. 2009). According to the outcome data, financings with contract information also exhibit higher success rates. These financings have lower failure rates (12% vs. 17%) and higher rates of both IPOs (10% vs. 4%) and high quality exits (23% vs. 16%). Overall, the sub-sample of financings with at least some observable contract terms likely represents a positive selection of the underlying population: high-quality startups and high-quality investors. Any resulting bias for the results below is unclear, however, it is important to note that nearly all previous studies using investment-level returns or contracts face similar issues. However, given that our data represents the largest set of both valuation and contracts data, we believe any selection issues are relatively smaller in our sample.

4 Results

We first consider raw correlations and basic regression estimates, and then discuss the search model estimates.

4.1 Correlations

Table IV presents the correlations and covariances for the set of contract, outcome, and VC activity variables. The upper-right triangle of the table first shows that the share of investor
equity is positively correlated with other contract terms and successful outcomes. For example, the use of the participating preference term is positively correlated (28%) with the share of investor equity, while more investor control through board seats exhibits a similar relationship (20%). The correlations between contract terms such as cumulative dividends, liquidation preference, and redemption are also positive. Positive correlations across all contract terms can arise, first, if all terms are value-creating and thus are, optimally, complements in a typical financing. Alternatively, at least some contract terms may not be value-creating and simply transfer value between counter-parties, and may thus be substitutes. However, in the sample of deals, we may still observe positive correlations among such terms because counter-parties select each other non-randomly. Our estimation is designed to differentiate between these two explanations. Finally, all contract terms positively correlate with our two success measures (the last two columns). As before, one has to be careful with interpreting these correlations, as they are insufficient to separate the effects of contract value creation and selection on success.

Table V complements the table of pairwise correlations and covariances by presenting simple OLS regressions of startup outcomes on contract terms. All estimations consider our main outcome variable IPO, regressed on four major contract terms. Regressions outside of columns 1–3 include fixed effects for financing year, startup founding year, industry and startup headquarters state. The results show that higher share of investor equity, pay-to-play and VC board seat (participation preference) correlate with a higher (lower) IPO success rate. Note that controlling for capital raised (column 7), higher share of investor equity implies lower post-money valuations, which contrasts with its regression coefficient estimate. This contrast may be indicative of either the failure of post-money valuations to capture the true start-up value or the presence of selection biasing the regression output.

In sum, simple regression results reveal predictive power for contract terms, however, they do not merit a causal interpretation due to a host of endogeneity concerns (e.g., endogenous selection among counterparties). In the next section, we estimate the impact of contracts on a start-up using our search and matching model.

4.2 Search Model

4.2.1 Empirical Implementation

We assume that $F_i(i)$ and $F_e(e)$ are Beta distributions on $[0,10]$ with parameters $(a_i, b_i)$ and $(a_e, b_e)$, and discretize each of these distributions on a 25 point grid.\footnote{A finer grid delivers very similar outcomes but results in a substantial computational slowdown. The technical role of the normalization is to allow for a sufficiently wide support of qualities so that tails of the Beta distributions disappear at its boundaries. If the support is too narrow so that the density of qualities is positive at its boundaries,} The Beta distribution
is very flexible and can generate hump-shaped, skewed, and even U-shaped distributions. See Appendices B and C for more details on the contraction mapping and the model solution.

We choose flexible functional forms for the impact of contract terms on firm value and its split,

\[
    h(c^*) = \exp \left\{ \beta_1 c_1^* + \beta_2 c_1^2 + \beta_{3:D+1} (1 - c_1^*) c_{2:D}^* \right\},
\]

\[
    1 - \alpha(c^*) = (1 - c_1^*) \exp \left\{ \gamma_{1:D} (1 - c_1^*) (1 - c_{2:D}^*) \right\},
\]

where \( D = \dim\{C\} \) is dimensionality of the contract space. In principle, contact terms entering the functional forms can be generic. However, we pay special attention to the fraction of equity retained by the investor, \( c_1^* \), because of ample theoretical research on its impact on value and also because it serves as a simple benchmark, against which the impact of other terms on the value split can be compared. We also allow \( c_{2:D}^* \) to contain products of any two simple contract terms.

Consider the firm value in equation (10). Theory suggests that there can an internal optimal equity share retained by the investor if there is a double moral hazard problem that requires both the investor and entrepreneur to expend effort (Hellmann (2006)). The linear and quadratic terms, \( \beta_1 c_1^* \) and \( \beta_2 c_1^2 \), in equation (10) allow for that possibility (but we do not enforce an internal optimum in the estimation, allowing for the possibility of a corner solution). \( c_{2:D}^* \) is multiplied by \( 1 - c_1^* \), because other terms become increasingly less meaningful as the investor owns a larger fraction of the company. For example, in the extreme case of 100% equity ownership by the investor, there is no incremental role for investor downside protections and other contract terms such as board seats. Finally, the exponential function prevents valuations from being negative.

Turning to the value split in equation (11), in the case of common equity, the value is split simply according to the equity shares of the investor and entrepreneur (that is, \( \alpha(c^*) = c_1^* \)). The exponential term only appears when there are other contract terms beyond the equity share (when \( D > 1 \)). Similarly to the firm value, \( c_{2:D}^* \) is multiplied by \( 1 - c_1^* \), because the impact of other terms on the value split is more important when the investor owns a smaller fraction of the company, while the value split converges to a common equity split when the investor owns a large fraction. In the example of 100% ownership by the investor, the existence of liquidation preferences or other downside protections for the investor is irrelevant, as the investor owns all of the firm and therefore gets all the value regardless. Most contract terms are downside protections for the investor, such as participation and liquidity preferences, which allocate more value to the investor relative to common equity. To ensure that the value split remains bounded between zero and one, such distribution would be unlikely to be encountered in practice, would indicate that some qualities are not captured by it, and would call for widening of the support. Our results are robust in the presence of wider and slightly narrower supports.

\(^{13}\)In the case of convertible preferred equity, \( c_1^* \) is the share after conversion.
we define any term that is perceived as entrepreneur-friendly in an inverse manner, so that all γ coefficients in equation (11) are less than or equal to zero (but we do not enforce this condition in the estimation). The functional form of equation (11) then ensures that \( \alpha(c^*) \in [c_1^*, 1] \). The intercept, \( \gamma_1 \), captures the value split effect of any terms that we do not have data on, or that are always present. For example, as shown in Panel B of Table II, liquidation preferences are nearly always equal to one in our sample of first-round financings.

Since \( \pi \) is not observed, to take the model to the data we add an outcome equation that captures the probability of an initial public offering. This is the traditional success measure used in the venture capital literature, because true valuations are not observed and cannot be easily recovered from post-money valuations, as explained in Section 4.1. (CITE).\(^{14}\) We use a probit-type specification and define the latent variable

\[
Z(i, e, c^*) = \kappa_0 + \kappa_1 \cdot \pi(i, e, c^*) + \eta,
\]

with \( \eta \sim \mathcal{N}(0, 1) \). A given start-up goes public if \( Z \geq 0 \), which happens with probability

\[
Pr(IPO = 1|i, e, c^*) = \Phi(\kappa_0 + \kappa_1 \cdot \pi(i, e, c^*)),
\]

where \( \Phi(\cdot) \) is the standard normal cumulative distribution function.

We use GMM with the efficient weighting matrix to estimate the main parameters of interest, \( \theta = (\lambda_i, \lambda_e, a_i, b_i, a_e, b_e, \beta, \gamma, \kappa) \). For each \( \theta \) and for each combination of investor and entrepreneur quality, the model produces the set of equilibrium contract terms, \( c^*(i, e; \theta) \), and the probability of an initial public offering, \( Pr(IPO = 1|i, e, c^*; \theta) \). Additionally, for each investor, the model produces the distribution of time since last first-round financing, \( \tau \). We compute all first and second moments of these model outcomes, as well all correlations among them, across all potential deals in equilibrium. For contract terms that only take values of zero and one, the second moment of their distribution across deals does not contain additional, compared to the first moment, information about model parameters, so we do not use it in the estimation. See Appendix D for details on the computation of theoretical moments. We compute the same moments in our final sample and search for \( \hat{\theta} \) that minimizes the difference between theoretical and empirical moments.\(^{15}\)

In this version of the paper, we limit the set of contract terms to the VC equity share and, additionally, one cash flow rights and one control rights term with high variation in the data:

\(^{14}\)In robustness checks, we also use the probability of an IPO or high-quality (> 2X capital) acquisition.

\(^{15}\)Because the GMM objective function is highly non-convex, we use the genetic algorithm to arrive at the neighborhood of a global minimum, then switch to the simplex search algorithm.
participation preference and VC board seats. We thus have 17 moments and 17 parameters to estimate. The model is just identified. The final version of the paper will include all terms.\footnote{The restriction to the first two moments of model outcomes means that at best, in addition to the VC equity share, we can evaluate the impact of no more than three terms. The inclusion of the third moment of the VC equity share, the most variable model outcome, increases this number to four. Table II informs that there is so little variation in the liquidation multiple and full ratchet term that these have to be omitted and captured by $\gamma_1$. Among the remaining terms, the ex-ante least important, despite its frequent occurrence, is redemption. This term appears only relevant if an investor ends up with a start-up whose performance is average but which is unlikely to exit via an IPO or acquisition. In this case, the investor can trigger its redemption rights; however, upon this event, often the entrepreneur does not have the liquidity to buy out the investor. And in case the start-up fails, there is nothing to redeem. So the value of redemption rights is unlikely to be high, and this is the final term we omit.}

### 4.2.2 Estimates

Table VI compares theoretical moments computed at estimated parameter values to empirical moments. The model matches well more informative first moments of contract terms, probability of an IPO, and time since last first-round financing, but generally underestimates second moments of these outcomes. While the test of overidentifying restrictions is not possible in a just identified model, the overall fit appears, visually, to be sensible.

Table VII shows parameter estimates and their standard errors. First, coefficients that capture the concave impact of VC equity share on the total value imply that there is an internal VC equity share, at which the first-best total value of a start-up is realized. Specifically, for any combination of participation preference, $c_2$, and VC board seat, $c_3$, which take values of zero and one, an internal VC equity share that maximizes $\pi(i, e, c)$ has to maximize quadratic equation

$$0.359 \cdot c_1 - 2.522 \cdot c_2^2 + (-0.156 \cdot c_2 - 0.051 \cdot c_2 + 0.020 \cdot c_2 \cdot c_3)(1 - c_1).$$

The first-best VC equity share is the one that maximizes the total value across all combinations of $c_2$ and $c_3$. The contract, at which the first-best value of a start-up is achieved, is thus $c^{FB} = (0.071, 0, 0)$: 7.1\% of VC equity, no participation preference, and no VC board seat.

How far away are equilibrium contracts from the first-best contract? Figure 1 shows equilibrium matches and contracts for each combination of VC and entrepreneur produced by the model using estimated parameter values. VCs and entrepreneurs tend to cluster in blocks (e.g., good VCs usually match with good entrepreneurs), however these blocks are imperfect. We will discuss this result later. The average VC equity share across all possible deals is 30.5\%. Only the best VCs within each block tend to include participation preference and VC board seat, which Table VII suggests are detrimental for the total value but beneficial for VCs. This effect appears to be stronger for participation preference. It is only the best VCs within a block, who enjoy a higher bargaining power and do not risk driving entrepreneurs away, that can afford adding these terms into their contracts. For the same reason, such VCs also retain a higher-than-average VC equity share,
31.5%, which is an unconstrained maximizer of $\pi_i(i, e, c)$, leading to $c^*(\text{high } i, e) = (0.315, 1, 1)$. The distance between $c^*$ and $c^{FB}$ thus appears to be large.

To quantify the difference between the equilibrium and first-best total value for each combination of VC and entrepreneur, in the left panel of Figure 2, we change the VC equity share, participation preference, and VC board seat and show the ratio of the equilibrium to first-best total value for every combination of terms. For example, the deal, in which each contract term is equal to the average of the term across all possible deals, $c^* = (0.305, 0, 1)$, achieves 84.1% of the first-best total value. Deals signed by the best VCs, in which the contract is $c^*(\text{high } i, e) = (0.315, 1, 1)$, perform worse and achieve only 75.7% of the first-best value.

The negative impact of participation preference on the total value is at odds with the theoretical prediction that, by creating convex incentives, participation preference forces entrepreneurs to work harder to achieve an IPO (Hellmann, 2006). It may be that entrepreneurs choose an alternative approach, that of gambling for success, when faced with such incentives. Gambling can increase the likelihood of an IPO by increasing the likelihood of high firm value realizations, but at the same time decrease the average realization of the firm value and, thereby, its expected value. The negative impact of the VC board seat on the total value may be due to entrepreneurs working less, as they have less control over key decisions in the company, and this effect being stronger than positive contributions of VCs to the company, such as better decision monitoring. Overall, our analysis calls into question optimality of an unconstrained contracting environment in the VC market, in which VC-friendly terms may benefit VCs at the cost of a start-up as a whole. In Section 6, we examine, via a counterfactual analysis, the effect of contract regulation.

Next, we quantify the impact of contract terms on the split of value between VC and entrepreneur. In the right panel of Figure 2, we change the VC equity share, participation preference, and VC board seat and show fraction of the total value retained by VCs for every combination of terms. Even in the absence of participation preference and VC board seat, VCs retain a substantially larger fraction than the VC equity share alone would suggest, because contract terms that are always present (such as 1X liquidation multiple) or unavailable in our data are, on average, VC-friendly, as captured by $\gamma_1 = -0.260$. In particular, while 7.1% of VC equity in the first-best contract $c^{FB} = (0.071, 0, 0)$ may appear low, this contract, in fact, leaves the VC with 27% of the total value. The presence of participation preference and VC board seat further increases the VC fraction of the firm. For example, the deal, in which each contract term is equal to the average of the term across all possible deals, $c^* = (0.305, 0, 1)$, leaves the VC with 44.0% of the total value. Deals signed by the best VCs, in which the contract is $c^*(\text{high } i, e) = (0.315, 1, 1)$, leave the VC with 49.4% of the total value.

The substantial difference between the VC equity share and the fraction of the start-up it
retains due to the inclusion of other contract terms confirms that the post-money valuation, calculated under the assumption that the VC equity share is the only relevant term, is a poor metric to evaluate the start-up value. A sensible practical modification is to use the fraction of the start-up retained by the VC instead. For example, because the best VCs choose \( c^*(\text{high }\xi,\epsilon) = (0.315, 1, 1) \), the post-money valuation of their start-up, per dollar of capital invested, would be \( \frac{81}{0.315} = \$3.17 \). In contrast, because the best VCs retain 49.4% of the total value, the modified valuation would instead be \( \frac{81}{0.494} = \$2.02 \), a 36.2% decrease compared to the post-money valuation. In large first-round financings by such VCs, the difference between valuations can reach millions of dollars.

Panel A of Table VIII provides additional detail on the total value and the split of value across deals completed by different quartiles of investor and entrepreneur qualities. Deals completed by top 25% of investors are, on average, twice as large as deals completed by bottom 25% of investors. Bottom 25% of entrepreneurs are effectively driven off the market (although they do sign deals very rarely) and there is substantially more heterogeneity in the total value as a function of entrepreneur quality than investor quality: most high-value deals are signed by top 25% of entrepreneurs. The VC share of the total value increases with its quality and decreases with entrepreneur quality.

Returning to coefficient estimates, frequencies of investor and entrepreneur encounters suggest that an investor meets a entrepreneur of a sufficiently high quality, on average, every \( \frac{1}{777} = 47 \) days, while such an entrepreneur meets an investor, on average, every \( \frac{1}{629} = 55 \) days. Panel B of Table VIII shows that these, combined with less interpretable estimates of quality distributions, result in investors (entrepreneurs) signing deals, on average, every \( \frac{1}{1049} = 348 \) (\( \frac{1}{1089} = 408 \)) days. Lower-quality investors are more active (but less selective) in deal signing: bottom 25% sign a deal, on average, every 257 days, while top 25% – every 426 days. The opposite is true for entrepreneurs: bottom 25% rarely sign a deal, while top 25% sign it, on average, in 114 days.

Panel C of Table VIII converts our estimates of total values and frequencies of encounters into estimates of the expected present value of all deals in the market and its segments. To obtain these, we need to know measures of investors and entrepreneurs in the market. In equilibrium, measures of encounters by the counterparties have to be equal: \( \lambda, m = \lambda, m \). This gives the ratio of measures of entrepreneurs to investors as \( \frac{m_e}{m_i} = \frac{\lambda_e}{\lambda_i} \). On a per-investor basis, then, the present value of all deals in the market is the sum, across all i and e with appropriate probability weights, of \( V_i(i) + \frac{m_e}{m_i}V_e(e) \). Panel C of Table VIII shows that overall, investors create 74.35% of the value in the market. Even bottom 25% of investors are productive and create 10.04% of this value, while top 25% create 26.58%. In contrast, bottom 25% of entrepreneurs create almost no value, while top 25% create value comparable to top 25% of investors: 22.31%. In the next section, we examine the impact of regulation of the contracting environment on these ultimate measures of value in the VC market.
We complete the section by returning to Figure 1 and discussing the tendency of VCs and entrepreneurs to cluster in blocks. Block segregation is a standard result in the search-matching literature in the absence of endogenous contracting (e.g., see Shimer and Smith (2000) and Smith (2011)). The following proposition shows that if the contracts were, instead, exogenous, we would also obtain a clear block segregation and, immediately, positively assortative matching (e.g., good VCs would always match with good entrepreneurs):

**Proposition 2.** Suppose that $c^*(i, e) \equiv \text{const}$ is exogenous. Then, the model solution admits block segregation: for $k \geq 1$, any investor quality $[\hat{i}_k, \hat{i}_{k-1}]$ matches with any entrepreneur quality $[\hat{e}_k, \hat{e}_{k-1}]$, where $(\hat{i}_0, \hat{e}_0) = (\hat{i}, \hat{e})$ and $(\hat{i}_k, \hat{e}_k)$, $k \geq 1$ are endogenous functions of model parameters.

Proof: The result follows from Shimer and Smith (2000) and Smith (2011), because, when $c^*(i, e) \equiv \text{const}$, $\pi(i, e, c^*)$ depends on types $i$ and $e$ multiplicatively.

When contracts are endogenous, there is, in general, no clear block segregation. Moreover, there is no guarantee that the model solution admits positively assortative matching. In particular, Figure 1 shows that this matching pattern does not occur under our parameter estimates. This result calls into question the validity of simply assuming positively assortative matching in settings with contracts (e.g., Cong (2018)). Intuitively, because contracts are chosen endogenously, it pays, for a lower-quality VC who otherwise would have been excluded by the block of best entrepreneurs, to offer a larger fraction of the start-up to such entrepreneurs to make them enter the deal. The lower the VC quality, the higher is the fraction it has to offer to a given entrepreneur, and the higher is the cut-off on the entrepreneur quality, at which this VC benefits.\(^{17}\)

### 5 Counterfactual Analysis

In this section, we examine the effect of a change in various features of the VC market on the value of a start-up, frequency of deals, and the present value of all deals in the market. The particular focus is on regulating the contracting environment, seeing as how the inclusion of certain terms into the contract can benefit VCs at the expense of the total value as compared to the first best.\(^{18}\)

\(^{17}\)Formally, the VC’s payoff is not log-supermodular in the deal, in which an entrepreneur of the highest quality matches with a VC of the lowest quality allowed for such entrepreneur in equilibrium: $\frac{\partial \pi_i(i, e, c^*(i, e))}{\partial i} < 0$ (see Theorem 1 in Smith (2011)).

\(^{18}\)Note that because we do not explicitly model mechanisms, through which terms affect the value, we are unable to examine the effect of including an additional term. For the same reason, we are unable to examine the effect of regulating terms that are always present, such as 1X liquidation preference (although there is variation in this term in later-round financings). We can only examine the effect of regulating the existing terms that vary in the sample, such that regulated terms stay within the set of realized terms in the sample.
Additionally, we examine the effect of lowering search frictions (e.g., via introducing a centralized platform akin to AngelList, where investors and entrepreneurs can easily encounter each other).

5.1 Contract Terms

The naive approach to examine the effect of regulating terms on deal outcomes would be to simply remove terms in each deal, in which they are present, and then re-calculate the total value and its split. For example, if participation preference (VC board seat, both terms) is regulated, this approach would inform that on average, across all possible deals, the total value would increase by 5.70% (1.47%, 7.78%). Unfortunately, the naive approach is incorrect because it is off-equilibrium: in the new market equilibrium with regulated terms, agents would rebalance the remaining terms and match in a different pattern.

Panel A of Table IX shows the equilibrium effect of regulating terms on the total value and its split. We analyze both the sample of all deals and deals done by various quartiles of investor and entrepreneur quality. We also decompose the aggregate equilibrium effect into two partial effects. The first effect, that of rebalancing, occurs when we only allow agents to rebalance unregulated terms to keep each entrepreneur’s expected present value $V_e(e)$ unaffected, so that its participation constraint remains satisfied. At the same time, we do not allow agents to match differently. Note that the first effect alone is off-equilibrium, because at least some investors, whose expected present value $V_i(i)$ would be affected by regulation, would have incentives to rematch. However, this effect helps understand the impact of terms on the firm in autarky, in the absence of market effects. The second effect, that of rematching, occurs when we allow agents to both rebalance the remaining terms and match differently.

Panel A of Table IX shows that the effect of rebalancing on both the total value and its split is uniformly negative and small across deals. For example, if both participation preference and VC board seat are regulated, rebalancing alone is responsible for a 0.27% decrease of the average, across all deals, total value compared to its pre-regulation level. In the absence of market effects, to keep paying the entrepreneur its pre-regulation expected present value $V_e(e)$, the VC replaces regulated terms with additional VC equity. However, this additional equity has a more detrimental effect on the total value and the VC share of it than regulated terms, decreasing both post-regulation. Thus, it would be tempting to interpret the effect of rebalancing as evidence of optimality of terms in question. This interpretation is potentially incorrect. If in any given match the VC received less post-regulation, its expected present value $V_i(i)$ would decrease. This decrease, in the presence of market effects, would mean that the VC would be willing to match with lower-quality entrepreneurs. In turn, entrepreneurs would end up with higher-quality VCs,
so their expected present value $V_e(e)$ would increase. Finally, in response to this increase, the VC would have to offer better terms in any given match than pre-regulation and than the effect of rebalancing alone would suggest (i.e., replace regulated terms with less additional VC equity). The presence of market effects could then result in on average better matches and better total value in a given match.

The effect of rematching is, indeed, typically positive and large. In the same example, the aggregate equilibrium effect is responsible for a 2.01% increase in the average, across all deals, total value, implying that rematching alone is responsible for a 2.28% increase. As for the split of value, in the same example, the aggregate equilibrium effect is responsible for a 1.27% decrease (3.28% increase) of the investor’s (entrepreneur’s) average, across all deals, value computed in units of the pre-regulation total value, implying that rematching alone is responsible for a 1% decrease (3.28% increase). For a complementary view on the magnitude of the aggregate equilibrium effect, note that a 1.27% decrease (3.28% increase) of the investor’s (entrepreneur’s) average value computed in units of the pre-regulation total value corresponds to a 2.78% decrease (6.03% increase) computed in units of the pre-regulation investor’s (entrepreneur’s) value.

While the effect of rebalancing is uniformly negative, the effect of rematching changes across deals. In particular, it is negative (positive) and large for bottom 25% of investors (top 25% of investors and all but bottom 25% of entrepreneurs). For top 25% of investors (entrepreneurs), the aggregate equilibrium effect is responsible for a 4.84% (4.61%) increase in the average, across deals done by such agents, total value compared to its pre-regulation level; more intriguingly, the same effect for 25-50% of entrepreneurs is 97.87%.

The combined intuition behind these results is as follows. When VC-friendly terms are regulated, entrepreneurs’ value in a deal, $\pi_e$, increases either because they end up with a larger fraction of the start-up or because their incentives to increase the start-up value are less distorted. In turn, their expected present value increases and they become more selective in their matching, choosing better investors. This effect appears to be particularly strong for low-quality entrepreneurs, who have to accept more unfavorable terms in the pre-regulation environment. Investors thus end up with on average lower-quality entrepreneurs than before regulation. The drop in the entrepreneur quality is particularly strong for low-quality investors, resulting in a decrease in the total value of deals they make. High-quality investors, who do not suffer a similar drop in the counterparty quality, sign more valuable deals through better contracting. However, this value increase, and then some ends up in entrepreneurs’ pockets. The high magnitude and heterogeneity of the rematching effect across deals suggests that selection of investors and entrepreneurs into deals is a major factor to take into account if one is concerned about regulation of the contracting environment.

Panel B of Table IX shows the effect of regulation on deal frequencies. Consistent with the
above intuition, as entrepreneurs’ market power increases and they become more selective in their matching, more of them match in the post-regulation equilibrium. The effect is particularly strong for low-quality entrepreneurs, who remain unmatched pre-regulation: for example, if both participation preference and VC board seat are regulated, match frequencies of 25-50% of entrepreneurs increases by a factor of 7. Investors, who accommodate more entrepreneurs post-regulation, also match more often, but the effect is more uniform across investor qualities.

Finally, Panel C of Table IX shows the effect of regulation on the expected present value of all deals in the market. The change, relative to the pre-regulation present value of all deals, is positive and ranges from 1.05% when only VC board seat is regulated to 4.55% when both VC board seat and participation preference are regulated. At the same time, the expected present value of investors (entrepreneurs) uniformly decreases (increases) across various ranges of qualities. For example, when both terms are regulated, VCs on average lose 1.14% of the present value computed in units of the pre-regulation present value of all deals, while entrepreneurs gain 5.69%. The bottom 25% (top 25%) of investors (entrepreneurs) is affected the most, losing 0.41% (gaining 3.38%) of the present value computed in units of the pre-regulation present value of all deals. For a complementary view on the magnitude of regulation on present values, note that a 1.14% decrease (5.69% increase) of the expected present value across all investors (entrepreneurs) computed in units of the pre-regulation present value of all deals corresponds to a 1.54% decrease (26.02% increase) computed in units of the pre-regulation present value across all investors (entrepreneurs).

While it may not be surprising that regulation of VC-friendly terms transfers start-up value from investors to entrepreneurs, the substantial asymmetry of the transfer, due to better value creation, suggests that a possibility of regulation of at least the most VC-friendly terms should be seriously considered by policy makers. One concern with our results is that, while we consider the general equilibrium in the VC market, agents in this market have other opportunities outside of it and can leave, or additional agents can enter, following regulation. Because the negative impact on investors is limited but the positive effect on entrepreneurs is rather strong, it is more likely that the combined effect of regulation of the VC contracting environment would add more value in newly participating entrepreneurs than lose value in departing investors.

### 5.2 Search Frictions

How will the present value of all deals in the market change if the counterparties are able to find each other faster? Who benefits from the decrease in search frictions, investors or entrepreneurs? In the presence of endogenous matching, answers to these questions are not immediately clear; at the same time, they can be important for policy makers concerned with centralizing the search
and matching process via, for example, a platform similar to AngelList for angel investors. In this section, we examine the effect of low search frictions on the VC market. Specifically, in separate analyses, we increase both $\lambda_i$ and $\lambda_e$ by a factor of 2, 5, and 10.

Table X shows that low search frictions do not necessarily increase the size of the VC market. A moderate decrease in frictions ($2X$) leads to a 0.37% increase in the expected present value of all deals computed in units of the estimated present value of all deals. Entrepreneurs lose 4.25%, while investors gain 4.62%. Best entrepreneurs (investors) lose (gain) the most. The outcome is worse when a decrease in costs is substantial ($10X$): it leads to a 0.65% decrease in the expected present value of all deals. Entrepreneurs lose 14.63%, while investors gain 13.98%.

The intuition behind this result is as follows. Armed with more bargaining power than entrepreneurs, investors benefit more from the decrease in search costs and hence an increase in their expected present value of deals. In turn, they become more selective in their matching, choosing better entrepreneurs. This effect appears to be particularly strong for top investors, who do not settle on anything but absolute top entrepreneurs. As a result, the expected present value of all, including top 25% of entrepreneurs (most of whom lose top investors), decreases.

Our results suggest that benefits from low-cost search in the VC market are not obvious and, if positive, are likely small. Low search frictions can bring about a less entrepreneur-friendly environment, which can lead to entrepreneurs departing to seek financing elsewhere. Our results thus guard against any immediate action to decrease search frictions in the market.

The exercise in this section is also useful to assess bias if selection were modeled as a static matching model with no search frictions. Adachi (2003, 2007) shows that when $\lambda_i$ and $\lambda_e$ are very high, our model converges to a one-shot static matching model. Direct estimation of our model when $\lambda_i$ and $\lambda_e$ are exogenously set high is difficult. However, since value is split very differently between counterparties in the low-versus high-friction environment, it is likely that the estimates obtained from this model modification would be very different. This insight underlines the importance of modeling search frictions in the VC market.

6 Robustness and Extensions

6.1 Truncated Exit Data

In the main model, we restrict our final sample to 2002–2009 to allow start-ups formed in this period to exit before June 2017, the end of our outcome observation period. While all exits for financings close in 2009–2015 are unlikely to be completed, this period could still contain

19 Technically, the system of Bellman equations underlying the agent’s decisions converges slowly when the expected discount factor applied to the next encounter is close to one.
potentially useful information on other deal outcomes, such as contract terms. It would then be useful to incorporate this information in our estimates. Lynch and Wachter (2013) develop an augmented moments estimator that helps account for the presence of truncated exit data in our case. Out results are robust to using this estimator. [RESULTS TO BE ADDED]

6.2 Exogenous Shocks

Two key results of the main model is that the set of counterparties an investor or entrepreneur matches with is fixed in equilibrium (however, within this set, the agents can match randomly), and that a given combination of agents always signs the same contract. In reality, there can be other covariates that can cause the change in the set of counterparties an agent is interested in, in turn leading to variation in the contract a given combination of agents signs. One clear example often considered in the VC literature is how “entrepreneur-friendly” the market is, measured for example by the overall amount of VC capital raised (Gompers and Lerner (2000)). In a more friendly market, the same entrepreneur can end up with a better investor at the same contract, or with the same investor but a better contract.

To address this concern, we extend the model to include the possibility of a global state change \( x \) (such as the overall amount of VC capital raised by funds), which affects the agents in the market via distributions \( F_{i,x}(i) \) and \( F_{e,x}(e) \) and frequencies of encounters \( \lambda_{i,x} \) and \( \lambda_{e,x} \). In Appendix E we show that this extension can be solved and estimated similarly to the main model, and derive theoretical moments used in the estimation. The empirical proxy for the global state (“light” versus “tight” VC capital constraints) is whether the annual average of the last \( T \) years’ capital raised by early-stage VC funds is above or below the 2002–2009 annual within-sample median of capital raised. In separate estimations, we use \( T = 3 \) and \( T = 1 \). \( T = 3 \) assumes that tight capital constraints can affect VCs with a lag, as at the time of the shock, their current funds have already raised capital which cannot be retracted, so that these funds’ spending is unaffected. At the same time, new funds formed in the future would have to operate under new constraints. \( T = 1 \) assumes that the impact of tight capital constraints on the VC market is more immediate. [FIGURES OF STATE CHANGES AND RESULTS TO BE ADDED]

6.3 Matching Function

The degree of complementarity between the counterparties in a start-up is typically unknown. Therefore a concern can be that the impact of contract terms is estimated incorrectly because the impact of qualities \( i \) and \( e \) on the expected value is not multiplicative. To address this concern, we modify \( \pi \) to flexibly account for the degree of complementarity. Assume that \( \pi \) is constant-
elasticity-of-substitution (CES):

\[
\pi(i, e, c^*) = \left( \sum_{j \in \{i, e\}} \frac{1}{2^j 2^p} \right)^{\frac{1}{p}} h(c^*).
\]  \hspace{1cm} (14)

In particular, when \( \rho \to 0 \), \( \pi \) converges to (4). When \( \rho = 1 \), qualities of the counterparties are perfect substitutes. Finally, when \( \rho \to -\infty \), qualities are perfect complements. We estimate \( \rho \) together with other parameters. [RESULTS TO BE ADDED]

6.4 Multiple Markets

The VC market is segmented, so that entrepreneurs and investors are isolated in different geographical locations and industries. In model terms, this leads to potentially submarket-specific distributions of qualities, \( F_i(i) \) and \( F_e(e) \), an encounter frequencies, \( \lambda_i \) and \( \lambda_e \). It is also reasonable to think of the IPO outcomes as different across industries and, possibly, locations, leading to submarket-specific \( \kappa_0 \) and \( \kappa_1 \). We split the sample into several submarkets based on industry (hi-tech, biotech, healthcare,...) and geographical locations (California, Massachusetts, Illinois, Texas,...) and estimate submarket-specific parameters. At the same time, we keep sensitivities of the total value and its split to contract terms the same across submarkets (it is less likely that given everything else the same, agents’ incentives would be different). [RESULTS TO BE ADDED]

6.5 Bargaining Power

Entrepreneurs’ expected present value \( V_e(e) \) defines the lower bound on their payoff \( \pi_e \) in contract negotiations. In reality, entrepreneurs can have additional bargaining power (e.g., they can raise financing outside of the VC market). The theoretical literature on bargaining when both sides of the process wield bargaining power is extensive, and the results depend crucially on the agents’ information sets, discount factors, sequence of moves, and outside options. Without imposing substantial structure on the bargaining process and potentially misspecifying it, we incorporate entrepreneurs’ additional bargaining power into the model in reduced form. Specifically, the contract that the VC offers, (7), is modified in the following way:

\[
c^*(i, e) = \arg\max_{c \in C: \pi_e(i, e, c) \geq (1+\gamma)V_e(e)} \pi_i(i, e, c),
\]  \hspace{1cm} (15)
where $\gamma \geq 0$ represents entrepreneurs’ additional bargaining power. We compare parameter estimates of the main model with those of the modified model for $\gamma = 0.05$ and $\gamma = 0.1$.  

[RESULTS TO BE ADDED]

6.6 Overconfidence

There is ample evidence that entrepreneurial individuals are overconfident, i.e., assign a higher precision to their information than the data would suggest.  

Our model is easily extendable to allow for overconfidence on the part of agents. Modify (5) and (6) as

\[
\begin{align*}
\pi_i^j(i, e, c^*) &= \alpha(c^*) \cdot \pi^j(i, e, c^*), \\
\pi_e^j(i, e, c^*) &= (1 - \alpha(c^*)) \cdot \pi^j(i, e, c^*),
\end{align*}
\]

where superscript $j \in \{i, e\}$ indicates that investors and entrepreneurs compute the total value and its split using potentially different beliefs. Let counterparty $j \in \{i, e\}$ believe that with probability $p_j$, signal $e$ about entrepreneur quality is correct, and with probability $1 - p_j$, the signal is completely uninformative, so that entrepreneur quality is a random draw from $F_e(e)$. Then, $\pi^j(i, e, c^*) = i \cdot (p_j e + (1 - p_j) \bar{e}) \cdot h(c^*)$. For example, the case of entrepreneurs entirely relying on the signal about their quality but VCs doubting it is $p_e = 1$ and $p_i < 1$. In the presence of the difference in beliefs, the incentive rationality condition of the entrepreneur, (7), becomes

\[
c^*(i, e) = \arg \max_{c \in C: \pi_e^j(i, e, c) \geq \pi_i^j(i, e, c)} \pi_i^j(i, e, c).
\]

Note that even though the investor solves its optimization problem under its own beliefs, it has to provide the entrepreneur with at least its expected present value from continued search under the entrepreneur’s beliefs. We compare parameter estimates of the main model with those of the modified model for $(p_i, p_e) = (0.75, 1)$ and $(p_i, p_e) = (0.5, 1)$.  

[RESULTS TO BE ADDED]

6.7 Investment Amount

In the main model, we do not consider capital raised by an entrepreneur as an endogenous contract term. This assumption is consistent with the view that entrepreneur’s idea requires a fixed amount of capital and determines its quality. An alternative polar case is to think about capital raised as

\footnote{In additional robustness checks, we formally model the outcome of bargaining as a one-shot Nash Bargaining Solution. It is unlikely that this model of bargaining represents well the actual process. However, our estimates are quantitatively unaffected.}

\footnote{Theoretical and empirical research on entrepreneurial overconfidence includes Cooper, Woo, and Dunkelberg (1988) and Bernardo and Welch (2001).}
an entirely endogenous term. This assumption is consistent with the view that it is entrepreneur’s intrinsic quality, regardless of idea, that determines the amount of capital an investor will give it. The reality is somewhere in between the two polar cases: entrepreneurs may be unable to realize their idea at all if the amount of capital is below a certain threshold, while incremental improvements from the amount of capital above their initial estimate may be modest.

In this section, we take an alternative polar view that capital raised is entirely endogenous. Specifically, we modify (10) as

\[ h(c^*) = \exp \left\{ \beta_0 \log c_0^* + \beta_1 c_1^* + \beta_2 c_1^2 + \beta_{3:D+1} (1 - c_1^*) c_{2:D}^* \right\}, \tag{19} \]

and modify (5) as

\[ \pi_i(i, e, c^*) = \phi(c_0^*) \cdot \alpha(c^*) \cdot \pi(i, e, c^*), \tag{20} \]

keeping (6) unchanged. Equation (19) implies that the matching function in the presence of endogenous investment exhibits returns to scale with factor \( \beta_0 \). Equation (20) implies that an investor experiences costs of investment \( 1 - \phi(c_0^*) \) per unit of profit. These include direct costs, such as loss of \( c_0^* \) at the time of financing, and indirect costs, such as time and effort spent monitoring and making decisions on the board of directors. We parameterize \( \phi(c_0^*) = \exp\{\gamma_0 c_0^*\}. \tag{22} \]

In Appendix E we show that this extension can be solved and estimated similarly to the main model. We estimate \( \beta_0 \) and \( \gamma_0 \) together with other parameters. [RESULTS TO BE ADDED]

6.8 Risk Aversion and Discount Factor

A standard assumption in both empirical finance and industrial organization literature is that the researcher knows the discount rate \( r^{23} \), so we set it at 10% in our model. However, it is likely that at least entrepreneurs are more risk-averse and discount future heavier than large firms, for which this assumption is typically made. We change the entrepreneurs’ \( r \) to 20% and compare the estimates to those of our main model. [RESULTS TO BE ADDED]

6.9 Decomposition of Qualities

To estimate the impact of contracts on value creation in the VC market, we did not need to know what comprised investor and entrepreneur qualities, and which individual agent possessed which quality. \( \tag{24} \) While not critical for this project, it may still be interesting to know which

\[ ^{22} \] It is easy to justify the positive relationship between total costs of investment and the investor’s share of the start-up via a simple model. See, e.g., Grossman and Hart (1986).


\[ ^{24} \] Direct inference of individual qualities is difficult, as outlined in Section 2.
observable characteristics of the agents correlate with qualities. Having estimated \( \hat{\theta} \), we can recover sensitivities of qualities to various characteristics as follows. Let qualities be

\[
i = \gamma^i \cdot y^i; \quad e = \gamma^e \cdot y^e,
\]

where \( y = (y^i, y^e) \) are observable investor and entrepreneur characteristics. For any \( \gamma = (\gamma^i, \gamma^e) \) and for each deal \( n \in N \) in the data, compute hypothetical types \( e_n \) and \( i_n \) from (21). For each deal, let \( X_n \) be the vector of realized outcomes: contract terms, whether the deal resulted in an IPO, and time since investor’s last first-round financing. Let \( X \) be the combined, across all deals, vector of realized outcomes. Similarly, for each deal, let \( m_n(i_n, e_n; \hat{\theta}) \equiv m_n(\gamma \cdot y_n, \hat{\theta}) \) be the vector of theoretical outcomes assuming types \( e_n \) and \( i_n \). Let \( m(\gamma \cdot y, \hat{\theta}) \) be the combined, across all deals, vector of theoretical outcomes.

To obtain \( \hat{\gamma} \), we minimize the GMM objective,

\[
\left(X - m(\gamma \cdot y, \hat{\theta})\right)' W^* \left(X - m(\gamma \cdot y, \hat{\theta})\right),
\]

where \( W^* \) is the efficient weighting matrix. Suppose the object of interest is the sensitivity, for an investor with characteristics \( y^i \) (and type \( i = \gamma^i \cdot y^i \)), of the average contract term \( c^k(i, e) \) it signs to characteristic \( y^i_l \). It is computed as

\[
\frac{\partial}{\partial y^i_l} \frac{\sum_{e \in E} m((\gamma^i, y^i, e); \hat{\theta}) c^k((\gamma^i, y^i, e); \hat{\theta})}{\sum_{e \in E} m((\gamma^i, y^i, e); \hat{\theta})}. 
\]

Consequently, sample-average sensitivities of terms to both agent’s characteristics can be computed and analyzed. [RESULTS TO BE ADDED]

6.10 Discussion of Other Assumptions

In practice, investors consider multiple entrepreneurs at once, and entrepreneurs sometimes compare multiple simultaneous offers (competing term sheets) from different investors. Additionally, even upon an encounter, the counterparties (in particular investors) do not completely observe each other’s type, giving rise to asymmetric information concerns. These considerations are important but rather difficult to model in a way that makes the estimation feasible, as they expand the state space of the model into additional dimensions (multiple counterparties’ qualities that an agent has to simultaneously consider in the first case, and true and perceived quality of each agent in the second case). We leave these considerations for future research. Note that in the presence of these, a given combination of counterparties’ qualities will no longer always sign the same contract, leading to higher variance of contract terms across all possible deals and hence a potentially better fit between theoretical and empirical variance moments.

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7 Conclusion

We develop a dynamic search and matching model to estimate the impact of venture capital contract terms on startup outcomes and the split of value between entrepreneur and investor in the presence of endogenous selection. Using a new data set of over 10,000 first financing rounds of startup companies, we estimate an internally optimal equity split between investor and entrepreneur that maximizes the probability of success. In almost all deals, investors receive more equity than is optimal for the company. In contrast to theoretical predictions, participation rights and investor board seats reduce company value, while shifting more of it to the investors. Eliminating these terms increases startup values through rematching, making entrepreneurs better off and leaving all but the highest quality investors marginally worse off. Our results suggest that selection of investors and entrepreneurs into deals is a major factor to take into account in both the empirical and theoretical literature on financial contracting.
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Appendix

A  Proof of Proposition 1

The agents’ expected present values are

\[
V_i(i) = \frac{1}{1 + rd t} \left( \lambda_i dt \left( \max_{e \in \mu_i} \{ \pi_i(i, e, c^*) \}, V_i(i) \} dF(e) + \int_{e \notin \mu_i} V_i(i) dF(e) \right) + (1 - \lambda_i dt) V_i(i) \right), \tag{22}
\]

\[
V_e(e) = \frac{1}{1 + rd t} \left( \lambda_e dt \left( \max_{i \in \mu_e} \{ \pi_e(i, e, c^*) \}, V_e(e) \} dF(i) + \int_{i \notin \mu_e} V_e(e) dF(i) \right) + (1 - \lambda_e dt) V_e(e) \right), \tag{23}
\]

Consider the expression for \( V_i(i) \) (\( V_e(e) \) is symmetric). Multiply both sides by \( 1 + rd t \), cancel out the two terms that contain \( V_i(i) \) but not \( dt \), and divide by \( dt \) to obtain

\[
rV_i(i) = \lambda_i \int_{e \in \mu_i} \max \{ \pi_i(i, e, c^*) \}, V_i(i) \} dF(e) + \lambda_i \int_{e \notin \mu_i} V_i(i) dF(e) - \lambda_i V_i(i).
\]

Move \( \lambda_i V_i(i) \) to the right-hand side and divide everything by \( r + \lambda_i \). Equation (8) follows.

B  Contraction mapping details

The discrete-time representation derived in Proposition 1 allows to to numerically solve the contraction mapping (8) and (9) as the system of interdependent Bellman equations. Specifically,

1. We assume that \( F_i(i) \) and \( F_e(e) \) are flexible Beta distributions. We discretize qualities \( i \sim F_i(i) \) and \( e \sim F_e(e) \) by using a quadrature with 25 points for each distribution, resulting in 625 possible combinations of partner qualities. This gives a very precise solution.

2. For any \( i \) and \( e \), we set the initial guess of continuation values equal to \( V^0 = (V_i^0(i), V_e^0(e)) = (0, 0), \) where \( V \) is sufficiently large. For example, if the only contract term is the fraction of equity that the investor retains, then \( V = v_e(i, \hat{e}, 0) \): the entrepreneur is guessed to retain the entire firm.\(^{25}\) For any \( i \) and \( e \), we set the initial guess of qualities of those agents from the opposite population, who are willing to match, equal to \( (\mu_i^0, \mu_e^0) = (\mu_i^0(i), \mu_e^0(e)) = (1, [\hat{i}, \bar{i}]) \). This choice implies that few agents are initially guessed to match, so the initial update to \( V^0 \), explained below, is smooth.

3. For every \( n \geq 1 \), we obtain \( V^n = (V_i^n(i), V_e^n(e)) \) and \( (\mu_i^n, \mu_e^n) = (\mu_i^n(i), \mu_e^n(e)) \) by inputting \( V^{n-1} \) and \( (\mu_i^{n-1}, \mu_e^{n-1}) \) into the right-hand side of the system of equations (8)–(9) and solving for the left-hand side. Because the system is a contraction mapping, \( V = \lim_{n \to \infty} V^n \) is the equilibrium.\(^{26}\) We stop the process when \( \|V^n - V^{n-1}\| < \varepsilon \), where \( \varepsilon > 0 \) is sufficiently small.

\(^{25}\) The static matching literature shows that this initial guess is consistent with an entrepreneur making an offer to match with a sufficiently good investor, and leads to computation of the so-called “entrepreneur-friendly” equilibrium. This terminology is somewhat confusing in the dynamic setting with contracts, as, once encountered and offered to match, it is an investor who offers the contract to an entrepreneur. The situation where the entrepreneur approaches the investor but is offered a take-it-or-leave-it contract in return is consistent with practice in the venture capital market. Our robustness checks explore the situation when the entrepreneur has extra bargaining power in addition to its threat to walk away from the deal and match with a different investor in the future.

\(^{26}\) We use the value iteration method to make sure the solution does not jump between potential multiple equilibria.
C Additional details of model solution

Our model differs from other models of search and matching in that in any encounter, we allow an investor to choose an optimal contract subject to the participation constraint of an entrepreneur. In this section of the appendix, we explain how this choice proceeds. First, the unconstrained choice of an investor is obtained from

\[
(1 - (1 - c)e^{\gamma_1 c(1 - c)}(1 - c_2 D)) \cdot i \cdot e^{\beta_1 c_1 + \beta_2 c_2 D + \beta_3 D} = \max_c e^{2}.
\]  

(24)

In the data, \(c_2 D\) are discrete contract terms, such as participation preference. Therefore, for any given value of \(c_2 D\), the first-order condition with respect to equity, \(c_1\), is

\[
e^{\gamma_1 c_1 D(1-c_1)(1-c_2 D)} \cdot (1 + (1 - c_1) \left( \gamma_1 + (\gamma_2 D + \beta_3 c_2 D - \beta_1 - 2\beta_2 c_1) \right)) = \beta_3 D + 1 c_2 D - \beta_1 - 2\beta_2 c_1.
\]

(25)

It is possible to show that this equation has at most two solutions, one of which is the maximum. By comparing such maxima, \(c_1^*(c_2 D)\), across all possible combinations of \(c_2 D\), we obtain the unconstrained choice of an investor, \(c_1^*\). For example, when the only contract term is equity, \(\gamma_1 = 1\) the expression holds trivially. For \(\gamma_1 > 1\), we obtain the first derivative of (24) is positive and decreasing below the higher solution, implying that this solution delivers the maximum. Because \(\beta_2 < 0\),

\[
c_1^* = \frac{-\beta_1}{4\beta_2} - \frac{\sqrt{\beta_1^2 - 8\beta_2 \gamma_1}}{4\beta_2}.
\]

(26)

Next, consider the entrepreneur’s participation constraint. If the entrepreneur’s payoff is decreasing in \(c_1\) for a given combination of \(c_2 D\) then the constrained choice of an investor is \(c_1^*(c_2 D)\), if it belongs to \(0, \hat{c}_1(c_2 D)\), and either 0 or \(\hat{c}_1(c_2 D)\) if the unconstrained maximum lies outside of this interval. \(\hat{c}_1(c_2 D)\) here is the fraction of investor equity, at which the entrepreneur’s payoff reaches its outside value \(V_e(e)\):

\[
(1 - \hat{c}_1) e^{\gamma_1 D(1-c_1)(1-c_2 D)} \cdot i \cdot e^{\beta_1 c_1 + \beta_2 c_2 D + \beta_3 D} = V_e(e).
\]

(27)

The entrepreneur’s payoff is decreasing in \(c_1\) if the derivative of the left-hand side of (27),

\[
e^{\gamma_1 D(1-c_1)(1-c_2 D)} e^{\beta_1 c_1 + \beta_2 c_2 D + \beta_3 D} \left( (1 - c_1) (\beta_1 + 2\beta_2 c_1 - (\gamma_2 D + \beta_3 c_2 D - \gamma_1) - 1) \right),
\]

(28)

is negative. Because \(\beta_2 < 0\), the left-hand side, sans exponent terms, is a parabola with the positive coefficient next to the quadratic term. Therefore, it suffices to check whether the above expression is negative for \(c_1 = 1\) and \(c_1 = 0\). For \(c_1 = 1\) the expression holds trivially. For \(c_1 = 0\), the expression holds if and only if

\[
\beta_1 < 1 + \gamma_1 + (\gamma_2 D + \beta_3 c_2 D).
\]

(29)

For example, when the only contract term is equity, (29) simplifies to \(\beta_1 < 1\). Intuitively, an increase in the expected company value with an increase in investor equity should be sufficiently low to not cause an increase in the expected value of the entrepreneur.
If condition (29) does not hold, the entrepreneur’s payoff is hump-shaped, so that there potentially are more than two solutions to (27), \(c_1(c_{2:D})\) and \(c_1(c_{2:D})\), for a given combination of \(c_{2:D}\). In this case, the constrained choice of an investor is \(c_1(c_{2:D})\), if it belongs to \([\max\{0, c_1(c_{2:D})\}, c_1(c_{2:D})]\), and either \(\max\{0, c_1(c_{2:D})\}\) or \(c_1(c_{2:D})\) if the unconstrained maximum lies outside of this interval. Technically, if condition (29) does not hold, the solution is twice as long to obtain numerically.

D Derivation of theoretical moments

Let \(w_e\) be the discretized probability that an investor meets an entrepreneur of quality \(e\); \(w_i\) be the discretized probability that an entrepreneur meets an investor of quality \(i\); and the match indicator \(m(i, e) = 1\) if \(i\) and \(e\) form a start-up, and zero otherwise.

D.1 Contract-related moments

The expected value of contract term \(c_k^*(i, e)\), \(k \in \{1..D\}\) across all deals is

\[
E(c_k^*) = \frac{\sum_i \sum_e w_i w_e m(i, e) c_k^*(i, e)}{\sum_i \sum_e w_i w_e m(i, e)}.
\]

The variance of \(c_k^*(i, e)\) across all deals is

\[
V(c_k^*) = \frac{\sum_i \sum_e w_i w_e m(i, e) (c_k^*(i, e) - E(c_k^*))^2}{\sum_i \sum_e w_i w_e m(i, e)}.
\]

For terms that only take values of zero or one, the variance does not contain additional, compared to the expected value, information, so we do not use it in the estimation. Finally, the covariance between any two contract terms \(c_k^*(i, e)\) and \(c_l^*(i, e)\), \(k, l \in \{1..D\}\) across all deals is

\[
Cov(c_k^*, c_l^*) = \frac{\sum_i \sum_e w_i w_e m(i, e) (c_k^*(i, e) - E(c_k^*)) (c_l^*(i, e) - E(c_l^*))}{\sum_i \sum_e w_i w_e m(i, e)}.
\]

D.2 Moments related to expected time between deals

Recall that after a successful deal, the distribution of the number of new encounters for investor \(i\) is a Poisson random variable with intensity \(\lambda_i\). Each encounter, in equilibrium, results in a deal with probability \(p_i = \sum_e w_e m(i, e)\). The distribution of the number of deals, conditional on \(k\) meetings, is therefore an independent Binomial distribution with number of trials \(k\) and success probability \(p_i\). This implies that the distribution of the number of deals is a Poisson distribution with intensity \(\lambda_i p_i\). Therefore, the time between deals, \(\tau\), for investor \(i\) has mean and variance equal to

\[
E(\tau|i) = \frac{1}{\lambda_i p_i}; \quad V(\tau|i) = \frac{1}{(\lambda_i p_i)^2}.
\]

Across all deals done by investors with different qualities, the expected time between deals is, from the law of iterated expectations,

\[
E(\tau) = E[E(\tau|i)] = \sum_i w_i^* E(\tau|i),
\]
where $w_i^* = \frac{\sum_e w_e m(i,e)}{\sum_i \sum_e w_e m(i,e)}$ is the equilibrium share of deals done by investor $i$ among all deals. This is different from $w_i$, the probability distribution of investors, because some investors match more frequently than others. Inserting $w_i^*$ into the above equation and using (33),

$$E(\tau) = \frac{\sum_i \sum_e w_i w_e m(i,e) \frac{1}{\sum_i w_i}}{\sum_i \sum_e w_i w_e m(i,e)}.$$  

(34)

Because $\tau$ is random for any given deal, its variance is, from the law of total variance,

$$V(\tau) = E[V(\tau|i)] + V[E(\tau|i)].$$  

(35)

Using (33), the first term of (35) is

$$E[V(\tau|i)] = \frac{\sum_i \sum_e w_i w_e m(i,e) \frac{1}{\sum_i w_i}}{\sum_i \sum_e w_i w_e m(i,e)};$$

additionally using (34), the second term is

$$V[E(\tau|i)] = \sum_i w_i^*(E(\tau|i) - E(\tau))^2 = \frac{\sum_i \sum_e w_i w_e m(i,e) \left( \frac{1}{\sum_i w_i} - E(\tau) \right)^2}{\sum_i \sum_e w_i w_e m(i,e)}.$$

The covariances between $\tau$ and contract term $c_k^*(i,e), k \in \{1..D\}$ across all deals can similarly be derived from the law of total covariance,

$$Cov(\tau, c_k^*) = E[Cov(\tau, c_k^*)] + Cov[E(\tau|i), E(c_k^*|i)]$$  

(36)

The first term of (36) is zero, because the time between deals does not vary with contract terms for a given investor. Using (30), (33), (34), and $E(c_k^*|i) = \frac{\sum_e w_e m(i,e) c_k^*(i,e)}{\sum_i \sum_e w_i w_e m(i,e)}$, the second term is

$$Cov[E(\tau|i), E(c_k^*|i)] = \sum_i w_i^*(E(\tau|i) - E(\tau)) \cdot (E(c_k^*|i) - E(c_k^*))$$

$$= \frac{\sum_i \sum_e w_i w_e m(i,e) \left( \frac{1}{\sum_i w_i} - E(\tau) \right) \cdot (c_k^*(i,e) - E(c_k^*))}{\sum_i \sum_e w_i w_e m(i,e)}.$$

D.3 IPO-related moments

Recall that the probability of an IPO for a given deal is

$$Pr(IPO = 1|i,e) = \Phi(\kappa_0 + \kappa_1 \cdot \pi(i,e,c^*(i,e))),$$  

(37)
with \( \Phi \) the standard normal c.d.f. The expected IPO rate across all deals is then

\[
E(IPO) = E[E(IPO = 1|i, e)] = E[Pr(IPO = 1|i, e)] = \frac{\sum_i \sum_e w_i w_e m(i, e) \Phi(\theta_0 + \theta_1 \cdot \pi(i, e, c^*(i, e)))}{\sum_i \sum_e w_i w_e m(i, e)}.
\] (38)

Similarly to (35), because IPO is random for any given deal, its variance is, from the law of total variance,

\[
V(IPO) = E(V(IPO|i, e)) + V(E(IPO|i, e)) = E(Pr(IPO = 1|i, e) \cdot (1 - Pr(IPO = 1|i, e))) + V(Pr(IPO = 1|i, e))
\]

\[
= \sum_i \sum_e w_i w_e m(i, e) \Phi(\theta_0 + \theta_1 \cdot \pi(i, e, c^*(i, e))) \cdot (1 - \Phi(\theta_0 + \theta_1 \cdot \pi(i, e, c^*(i, e)))) + \sum_i \sum_e w_i w_e m(i, e) \Phi(\theta_0 + \theta_1 \cdot \pi(i, e, c^*(i, e))) - E(IPO))^2 \]

\[
= \frac{\sum_i \sum_e w_i w_e m(i, e) \Phi(\theta_0 + \theta_1 \cdot \pi(i, e, c^*(i, e)))}{\sum_i \sum_e w_i w_e m(i, e)}
\] (39)

where we use (37) and (38) to arrive at the final expression.

The covariances between IPO and contract term \( c^*_k(i, e) \), \( k \in \{1..D\} \) across all deals are

\[
Cov(IPO, c^*_k) = E(Cov(IPO, c^*_k|i, e)) + Cov(E(IPO|i, e), E(c^*_k|i, e))
\] (40)

\[
= \sum_i \sum_e w_i w_e m(i, e) \Phi(\theta_0 + \theta_1 \cdot \pi(i, e, c^*(i, e))) - E(IPO) \cdot (c^*_k(i, e) - E(c^*_k)) \]

where \( \Phi(Cov(IPO, c^*_k|i, e)) \) is zero because the contract is deterministic for a given pair of investor and entrepreneur, and therefore does not vary with the start-up’s IPO outcome. To arrive at the final expression, we use (30), (37), and (38).

Finally, the covariance between IPO and \( \tau \) across all deals is

\[
Cov(\tau, IPO) = E[Cov(\tau, IPO|i)] + Cov[E(\tau|i), E(IPO|i)]
\] (41)

\[
= \sum_i w_i[E(\tau|i) - E(\tau)] \cdot [E(IPO|i) - E(IPO)]
\]

\[
= \frac{\sum_i \sum_e w_i w_e m(i, e) \left( \lambda_{pi} - E(\tau) \right) \cdot \Phi(\theta_0 + \theta_1 \cdot \pi(i, e, c^*(i, e))) - E(IPO))}{\sum_i \sum_e w_i w_e m(i, e)}
\]

where \( \Phi(Cov(\tau, IPO|i)) \) is zero because the time between deals does not vary with the start-up’s IPO outcome for a given investor. To arrive at the final expression, we use (33), (34), (37), (38), and \( E(IPO|i) = \frac{\sum_i w_i m(i, e) \Phi(\theta_0 + \theta_1 \cdot \pi(i, e, c^*(i, e)))}{\sum_i \sum_e w_i w_e m(i, e)} \).
Extensions

E.1 Model with exogenous state of the world change

Consider investor $i$ (the case of an entrepreneur is symmetric). Let $\rho_x$ be the frequency with which the state of the economy exits $x \in \{0, 1\}$. This state can affect populations of agents and frequencies of encounters: $\lambda_{j,x}$ and $F_{j,x}(j)$, $j \in \{i, e\}$. The agents’ expected present values, $V_i(i, x)$, now depend on the state of the economy and are

\[
V_i(i, 0) = \frac{1}{1 + \epsilon^2} \left( \lambda_{i,0}dt \left( \int_{e \in \mu_i, (i,0)} \max \{\pi(i, e, c^*), V_i(i, 0)\}dF_0(e) + \int_{e \notin \mu_i, (i,0)} V_i(i, 0)dF_0(e) \right) + \rho_0dtV_i(i, 1) + (1 - (\lambda_{i,0} + \rho_0)dt)V_i(i, 0) \right);
\]
\[
V_i(i, 1) = \frac{1}{1 + \epsilon^2} \left( \lambda_{i,1}dt \left( \int_{e \in \mu_i, (i,1)} \max \{\pi(i, e, c^*), V_i(i, 1)\}dF_1(e) + \int_{e \notin \mu_i, (i,1)} V_i(i, 1)dF_1(e) \right) + \rho_1dtV_i(i, 0) + (1 - (\lambda_{i,1} + \rho_1)dt)V_i(i, 1) \right).
\]

Transformations similar to those in the proof of Proposition 1 lead to

\[
V_i(i, 0) = \frac{\lambda_{i,0}}{r + \lambda_{i,0} + \rho_0} \left( \int_e \max \{1_{e \in \mu_i, (i,0)} \pi(i, e, c^*), V_i(i, 0)\}dF_0(e) + \frac{\rho_0}{\lambda_{i,0}} V_i(i, 1) \right);
\]
\[
V_i(i, 1) = \frac{\lambda_{i,1}}{r + \lambda_{i,1} + \rho_1} \left( \int_e \max \{1_{e \in \mu_i, (i,1)} \pi(i, e, c^*), V_i(i, 1)\}dF_1(e) + \frac{\rho_1}{\lambda_{i,1}} V_i(i, 0) \right).
\]

Solving for $V(i, x)$, $x \in \{0, 1\}$, we obtain a discrete-time representation

\[
V_i(i, 0) = \frac{\lambda_{i,0}(r + \lambda_{i,1} + \rho_1)}{(r + \lambda_{i,0} + \rho_0)(r + \lambda_{i,1} + \rho_1) - \rho_0\rho_1} \int_e \max \{1_{e \in \mu_i, (i,0)} \pi(i, e, c^*), V_i(i, 0)\}dF_0(e) + \frac{\rho_0\lambda_{i,1}}{(r + \lambda_{i,0} + \rho_0)(r + \lambda_{i,1} + \rho_1) - \rho_0\rho_1} \int_e \max \{1_{e \in \mu_i, (i,1)} \pi(i, e, c^*), V_i(i, 1)\}dF_1(e);
\]
\[
V_i(i, 1) = \frac{\lambda_{i,1}(r + \lambda_{i,0} + \rho_0)}{(r + \lambda_{i,0} + \rho_0)(r + \lambda_{i,1} + \rho_1) - \rho_0\rho_1} \int_e \max \{1_{e \in \mu_i, (i,1)} \pi(i, e, c^*), V_i(i, 1)\}dF_1(e) + \frac{\rho_1\lambda_{i,0}}{(r + \lambda_{i,0} + \rho_0)(r + \lambda_{i,1} + \rho_1) - \rho_0\rho_1} \int_e \max \{1_{e \in \mu_i, (i,0)} \pi(i, e, c^*), V_i(i, 0)\}dF_0(e).
\]

so that same solution techniques as in the base model apply.

The expressions for the theoretical moments related to expected time between deals change in the presence of the change in the state of the economy. The easiest way to compute these is via Bellman equations. For $E[\tau|i, x]$,

\[
\lambda_{i,0}p_{i,0}E[\tau|i, 0] = \lambda_{i,0}p_{i,0}E[\tau|i, always 0] + \rho_0 \left( E[\tau|i, 1] - E[\tau|i, 0] \right);
\]
\[
\lambda_{i,1}p_{i,1}E[\tau|i, 1] = \lambda_{i,1}p_{i,1}E[\tau|i, always 1] + \rho_1 \left( E[\tau|i, 0] - E[\tau|i, 1] \right),
\]

where, similarly to (33), expected times under the assumption that the state of the economy never changes are equal to $E[\tau|i, always 0] = \frac{1}{\lambda_{i,0}p_{i,0}}$ and $E[\tau|i, always 1] = \frac{1}{\lambda_{i,1}p_{i,1}}$. Intuitively, the expected time in the presence of the change in the state of the economy is the expected time in the absence of the change (the first term on the right-hand side), corrected for the possibility of the change, at which point the expected time switches to a different value reflecting the change.
Specifically, making the optimal contract substantially easier compared to the case of exogenous investment. The entrepreneur on its participation constraint, scale the start-up value, for any combination of next, consider the entrepreneur’s participation constraint. Due to the ability of investment to change), the expected squared time between deals, $E[\tau^2|i, x]$, is obtained from
\[
\lambda_{i,0} \hat{p}_i,0 E[\tau^2|i, 0] = \lambda_{i,0} \hat{p}_i,0 E[\tau^2|i, always 0] + \rho_0 (E[\tau^2|i, 1] - E[\tau^2|i, 0]); \\
\lambda_{i,1} \hat{p}_i,1 E[\tau^2|i, 1] = \lambda_{i,1} \hat{p}_i,1 E[\tau^2|i, always 1] + \rho_1 (E[\tau^2|i, 0] - E[\tau^2|i, 1]),
\]
where, similarly to (33), $E[\tau^2|i, always 0] = \frac{2}{(\lambda_{i,0} \hat{p}_i,0)^2}$ and $E[\tau|i, always 1] = \frac{2}{(\lambda_{i,0} \hat{p}_i,0)^2}$. The solution is
\[
E[\tau^2|i, 0] = \frac{2 \lambda_{i,0} \hat{p}_i,0 (\lambda_{i,1} \hat{p}_i,1 + \rho_1)}{(\lambda_{i,0} \hat{p}_i,0 + \rho_0)(\lambda_{i,1} \hat{p}_i,1 + \rho_1) - \rho_0 \rho_1}; \\
E[\tau^2|i, 1] = \frac{2 \lambda_{i,1} \hat{p}_i,1 (\lambda_{i,0} \hat{p}_i,0 + \rho_0)}{(\lambda_{i,0} \hat{p}_i,0 + \rho_0)(\lambda_{i,1} \hat{p}_i,1 + \rho_1) - \rho_0 \rho_1}.
\]
The variance of time between deals, $V[\tau^2|i, x]$, is equal to $E[\tau^2|i, x] - (E[\tau|i, x])^2$. Covariances of time between deals and other outcome variables in each state of the world are computed as in Appendix D, taking into account new expressions for $E[\tau|i, x]$. The expressions for the remaining theoretical moments are not affected by the presence of $x$ (however, their values in different states of the world can be different, because discretized probabilities of encounters, $w_{i,x}$ and $w_{e,x}$, can change).

**E.2 Model with endogenous investment**

As in Appendix C, the unconstrained choice of an investor’s contract terms other than investment is obtained from (24) and lead to the same solution $c_{1,D}$. The unconstrained choice of investment is obtained from the first-order condition of its objective function with respect to $c_0$:
\[
c_0^* = -\frac{\beta_0}{\gamma_0}.
\]
Next, consider the entrepreneur’s participation constraint. Due to the ability of investment to scale the start-up value, for any combination of $c_{1,D}$ there always exists some $\tilde{c}_0$ that puts the entrepreneur on its participation constraint,
\[
(1 - \bar{c}_1)e^{\gamma_{1,D}(1-c_1)(1-c_{2,D})} \cdot i \cdot e^{\gamma_0 \tilde{c}_0 + \beta_0 \log \tilde{c}_0 + \beta_1 c_1 + \beta_2 c_1^2 + \beta_3 c_1^3 + (1-c_1)c_{2,D}} = V_e(e).
\]
making the optimal contract substantially easier compared to the case of exogenous investment. Specifically,
\[
\log \tilde{c}_0 = \frac{1}{\beta_0} \left( \log \frac{V_e(e)}{i \cdot e \cdot (1-c_1)} - \beta_1 c_1 - \beta_2 c_1^2 - (\gamma_1 + (\gamma_2 + \gamma_{3,D+1})c_{2,D})(1-c_1) \right).
\]
The final step is to insert $\bar{c}_0$ into the investor’s objective function and maximize it over $c_{1:D}$. The logarithm of the investor’s objective function is

$$\log \left(1 - (1 - c_1)e^{\gamma_1:D(1-c_1)(1 \cdot c_{2:D})} \right) + \log i + \gamma_0 \bar{c}_0(c_{1:D}) + \beta_0 \log \bar{c}_0(c_{1:D}) + \beta_1 c_1 + \beta_2 c_1^2 + \beta_{3:D+1}(1 - c_1)c_{2:D} \rightarrow \max_{c}.$$  \hspace{1cm} (45)

For a given combination of $c_{2:D}$, the constrained choice of equity by an investor solves the first-order condition

$$\frac{e^{\gamma_1:D(1-c_1)(1 \cdot c_{2:D})}}{1 - (1 - c_1)e^{\gamma_1:D(1-c_1)(1 \cdot c_{2:D})}} + \left( \gamma_0 + \frac{1}{\bar{c}_0(c_{1:D})} \right) \frac{\partial \bar{c}_0(c_{1:D})}{\partial c_1} + \beta_1 + 2\beta_2 c_1 - \beta_{3:D+1} c_{2:D} = 0,$$  \hspace{1cm} (46)

where

$$\frac{\partial \bar{c}_0(c_{1:D})}{\partial c_1} = \frac{1}{\beta_0} \bar{c}_0 \left( \frac{1}{1 - c_1} - (\beta_1 + 2\beta_2 c_1 - (\gamma_0' + \beta_{2:D} + \beta_{3:D+1}) c_{2:D}) \right).$$  \hspace{1cm} (47)
Figure 1: **Equilibrium contract terms for estimated model parameters.** For each combination of investor and entrepreneur quality, the left panel shows VC equity share, the center panel shows participation preference, and the right panel shows VC board seat. Combinations that do not form a start-up are shown in black. VC equity share takes values in $[0, 1]$ and is shown in greyscale. In particular, the unconstrained VC-optimal contract, $c^* = (0.315, 1, 1)$, includes 31.5% VC equity. Participation preference and VC board seat take values in $\{0, 1\}$, and their inclusion is shown in white.

Figure 2: **Impact of contract terms on total value and its split.** For reasonable values of VC equity share, $c_1 \in [c^{FB} = 0.071, 0.6]$, and for four combinations of participation preference and VC board seat, the left panel shows the ratio of the total value created in a start-up to the first-best value and the right panel shows the fraction of the total value retained by the VC. Datatips show the impact of the first-best contract, $c^{FB} = (0.071, 0, 0)$, on the split and the unconstrained VC-optimal contract, $c^* = (0.315, 1, 1)$, on the total value and split.
Table I: Variable definitions.

Notes: The table describes the variables used throughout the paper.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm age at financing (yrs)</td>
<td>Years from the startup’s date of incorporation to the date of the first round financing.</td>
</tr>
<tr>
<td>Information technology</td>
<td>An indicator equal to one if the startup’s industry is information technology.</td>
</tr>
<tr>
<td>Healthcare</td>
<td>An indicator equal to one if the startup’s industry is healthcare, which include biotechnology.</td>
</tr>
<tr>
<td>Years since last round (VC)</td>
<td>The number of years since the lead investors last lead investment in a first round financing.</td>
</tr>
<tr>
<td>Syndicate size</td>
<td>The total number of investors in the first round financing.</td>
</tr>
<tr>
<td>Capital raised in round (2009 $m)</td>
<td>Total capital raised (in millions of 2009 dollars) in the startup’s first financing rounds (across all investors).</td>
</tr>
<tr>
<td>Financing year</td>
<td>The year of the financing.</td>
</tr>
<tr>
<td>% equity sold to investors</td>
<td>The fraction of equity (as-if-common) sold to investors in the financing round, calculated as the capital raised in the round divided by the post-money valuation.</td>
</tr>
<tr>
<td>Participating pref.</td>
<td>An indicator variable equal to one if the stock sold in the financing event includes participation (aka “double-dip”).</td>
</tr>
<tr>
<td>Common stock sold</td>
<td>An indicator variable equal to one if the equity issued in the financing is common stock.</td>
</tr>
<tr>
<td>Liquidation mult. &gt; 1</td>
<td>An indicator variable that is equal to one if the liquidation multiple exceeds 1X. The liquidation multiple provides holders 100% of exit proceeds for sales that are less than X times the original investment amount.</td>
</tr>
<tr>
<td>Cumulative dividends</td>
<td>An indicator variable equal to one if the stock sold includes cumulative dividends. Such dividends cumulate each year pre-liquidation and are only paid at liquidation.</td>
</tr>
<tr>
<td>Full ratchet anti-dilution</td>
<td>An indicator variable equal to one if the preferred stock includes full ratchet anti-dilution protection. Such protection results in the original share price to be adjusted 1:1 with any future stock offerings with a lower stock price (through a change in teh conversion price).</td>
</tr>
<tr>
<td>Pay-to-play</td>
<td>An indicator variable equal to one if the preferred stock sold includes pay-to-play provisions. These provisions penalize the holder if they fail to reinvest in future financing rounds.</td>
</tr>
<tr>
<td>Redemption rights</td>
<td>An indicator variable equal to one if the preferred stock sold includes redemption rights. These are types of puts (available after some number of years) that allow the holder to sell back their shares to the startup at a predetermined price.</td>
</tr>
<tr>
<td>VC has board seat</td>
<td>An indicator variable equal to one if the VC investor has a board seat at the time of the first financing.</td>
</tr>
<tr>
<td>IPO</td>
<td>An indicator variable that is equal to one if the startup had an IPO by the end of 2017Q2.</td>
</tr>
<tr>
<td>Acquired</td>
<td>An indicator variable that is equal to one if the startup was acquired the end of 2017Q2.</td>
</tr>
<tr>
<td>IPO or Acq. &gt; 2X capital</td>
<td>An indicator variable that is equal to one if the startup had an IPO or had an acquisition with a purchase price at least two times capital invested across all its financings by the end of 2017Q2.</td>
</tr>
<tr>
<td>Out of business</td>
<td>An indicator variable that is equal to one if the startup had gone out of business by the end of 2017Q2.</td>
</tr>
<tr>
<td>Still private</td>
<td>An indicator variable that is equal to one if the startup had not exited by the end of 2017Q2.</td>
</tr>
</tbody>
</table>
Table II: Summary statistics.

Notes: Summary statistics of start-ups and their first round equity financings for the sample financed from 2002 - 2015 detailed in Section 3. For exit outcomes IPO, acquisitions and related, we only consider start-ups financed before 2010.

<table>
<thead>
<tr>
<th>Panel A: Firm and financing characteristics</th>
<th>Obs</th>
<th>Mean</th>
<th>25th</th>
<th>Median</th>
<th>75th</th>
<th>Std dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm age at financing (yrs)</td>
<td>10,968</td>
<td>1.83</td>
<td>0.55</td>
<td>1.25</td>
<td>2.51</td>
<td>1.82</td>
</tr>
<tr>
<td>Information Tech.</td>
<td>10,968</td>
<td>0.47</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
<td>0.50</td>
</tr>
<tr>
<td>Healthcare</td>
<td>10,968</td>
<td>0.19</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.40</td>
</tr>
<tr>
<td>Years since last round (VC)</td>
<td>9,228</td>
<td>0.79</td>
<td>0.10</td>
<td>0.30</td>
<td>0.86</td>
<td>1.31</td>
</tr>
<tr>
<td>Syndicate size</td>
<td>10,968</td>
<td>2.37</td>
<td>1.00</td>
<td>2.00</td>
<td>3.00</td>
<td>1.53</td>
</tr>
<tr>
<td>Capital raised in round (m, real)</td>
<td>9,959</td>
<td>5.22</td>
<td>1.17</td>
<td>2.73</td>
<td>6.17</td>
<td>7.89</td>
</tr>
<tr>
<td>Post-money valuation (m)</td>
<td>5,982</td>
<td>18.31</td>
<td>5.96</td>
<td>10.75</td>
<td>19.36</td>
<td>34.84</td>
</tr>
<tr>
<td>Financing year</td>
<td>10,968</td>
<td>2009.62</td>
<td>2006.00</td>
<td>2010.00</td>
<td>2013.00</td>
<td>3.91</td>
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<table>
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<tr>
<th>Panel B: Contracts</th>
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</thead>
<tbody>
<tr>
<td>% equity sold to investors</td>
<td>6,034</td>
<td>0.35</td>
<td>0.22</td>
<td>0.32</td>
<td>0.46</td>
<td>0.18</td>
</tr>
<tr>
<td>count mean p25 p50 p75 sd</td>
<td>4,787</td>
<td>0.41</td>
<td>0.05</td>
<td>0.03</td>
<td>0.20</td>
<td>0.01</td>
</tr>
<tr>
<td>Participating pref.</td>
<td>4,785</td>
<td>0.03</td>
<td>0.01</td>
<td>0.10</td>
<td>0.35</td>
<td>0.62</td>
</tr>
<tr>
<td>Common stock sold?</td>
<td>5,002</td>
<td>0.05</td>
<td>0.10</td>
<td>0.35</td>
<td>0.62</td>
<td>0.38</td>
</tr>
<tr>
<td>Liquidation mult. &gt; 1</td>
<td>4,584</td>
<td>0.20</td>
<td>0.01</td>
<td>0.10</td>
<td>0.35</td>
<td>0.62</td>
</tr>
<tr>
<td>Cumulative dividends</td>
<td>3,458</td>
<td>0.01</td>
<td>0.01</td>
<td>0.10</td>
<td>0.35</td>
<td>0.62</td>
</tr>
<tr>
<td>Full ratchet</td>
<td>2,956</td>
<td>0.10</td>
<td>0.01</td>
<td>0.10</td>
<td>0.35</td>
<td>0.62</td>
</tr>
<tr>
<td>Pay to play</td>
<td>3,383</td>
<td>0.35</td>
<td>0.01</td>
<td>0.10</td>
<td>0.35</td>
<td>0.62</td>
</tr>
<tr>
<td>Redemption</td>
<td>10,968</td>
<td>0.62</td>
<td>0.01</td>
<td>0.10</td>
<td>0.35</td>
<td>0.62</td>
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<table>
<thead>
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<th>Panel C: Exit outcomes</th>
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<th>Mean</th>
<th>25th</th>
<th>Median</th>
<th>75th</th>
<th>Std dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>IPO</td>
<td>4,990</td>
<td>0.04</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Acquired</td>
<td>4,990</td>
<td>0.39</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IPO or Acq. &gt; 2X capital</td>
<td>4,990</td>
<td>0.16</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Out of business</td>
<td>4,990</td>
<td>0.17</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Still private</td>
<td>4,990</td>
<td>0.38</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table III: Summary statistics: with and without contracts data.

Notes: Summary statistics of start-ups and their first round equity financings for the sample financed from 2002 - 2015 detailed in Section 3. The panel “Deals with contract data” report the summary statistics for financings that have all the major contract terms available in the data. The second panel “All deals” include all financings regardless of data missingness.

<table>
<thead>
<tr>
<th></th>
<th>Deals with contract data</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>All deals</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Obs</td>
<td>Mean</td>
<td>Median</td>
<td>Std dev</td>
<td>Obs</td>
<td>Mean</td>
<td>Median</td>
<td>Std dev</td>
<td></td>
</tr>
<tr>
<td>Firm age at financing (yrs)</td>
<td>2,185</td>
<td>1.65</td>
<td>1.14</td>
<td>1.69</td>
<td>10,967</td>
<td>1.83</td>
<td>1.25</td>
<td>1.82</td>
<td></td>
</tr>
<tr>
<td>Information Tech.</td>
<td>2,185</td>
<td>0.47</td>
<td>0.00</td>
<td>0.50</td>
<td>10,967</td>
<td>0.47</td>
<td>0.00</td>
<td>0.50</td>
<td></td>
</tr>
<tr>
<td>Healthcare</td>
<td>2,185</td>
<td>0.25</td>
<td>0.00</td>
<td>0.43</td>
<td>10,967</td>
<td>0.19</td>
<td>0.00</td>
<td>0.40</td>
<td></td>
</tr>
<tr>
<td>Years since last round (VC)</td>
<td>2,185</td>
<td>0.67</td>
<td>0.25</td>
<td>1.17</td>
<td>9,227</td>
<td>0.79</td>
<td>0.30</td>
<td>1.32</td>
<td></td>
</tr>
<tr>
<td>Syndicate size</td>
<td>2,185</td>
<td>2.56</td>
<td>2.00</td>
<td>1.46</td>
<td>10,967</td>
<td>2.37</td>
<td>2.00</td>
<td>1.53</td>
<td></td>
</tr>
<tr>
<td>Capital raised in round (m, real)</td>
<td>2,185</td>
<td>7.18</td>
<td>4.94</td>
<td>8.80</td>
<td>10,967</td>
<td>5.22</td>
<td>3.16</td>
<td>7.51</td>
<td></td>
</tr>
<tr>
<td>Financing year</td>
<td>2,185</td>
<td>2008.27</td>
<td>2008.00</td>
<td>3.56</td>
<td>10,967</td>
<td>2009.62</td>
<td>2010.00</td>
<td>3.91</td>
<td></td>
</tr>
<tr>
<td>Out of business</td>
<td>1,351</td>
<td>0.12</td>
<td>0.00</td>
<td>0.33</td>
<td>4,987</td>
<td>0.17</td>
<td>0.00</td>
<td>0.37</td>
<td></td>
</tr>
<tr>
<td>Still private</td>
<td>1,351</td>
<td>0.38</td>
<td>0.00</td>
<td>0.48</td>
<td>4,987</td>
<td>0.38</td>
<td>0.00</td>
<td>0.49</td>
<td></td>
</tr>
<tr>
<td>IPO</td>
<td>1,351</td>
<td>0.10</td>
<td>0.00</td>
<td>0.30</td>
<td>4,987</td>
<td>0.04</td>
<td>0.00</td>
<td>0.21</td>
<td></td>
</tr>
<tr>
<td>IPO or Acq. &gt; 2X capital</td>
<td>1,351</td>
<td>0.23</td>
<td>0.00</td>
<td>0.42</td>
<td>4,987</td>
<td>0.16</td>
<td>0.00</td>
<td>0.37</td>
<td></td>
</tr>
</tbody>
</table>
Table IV: Pairwise correlations and covariances.

Notes: The table reports the correlations (upper right triangle of matrix) and covariances (lower triangle of the matrix) for the contract, deal flow and outcome variables.

<table>
<thead>
<tr>
<th>% equity sold to investors, (1)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquidation multiple &gt; 1?, (2)</td>
<td>0</td>
<td>1</td>
<td>.048</td>
<td>.072</td>
<td>.042</td>
<td>.002</td>
<td>-.007</td>
<td>.033</td>
<td>.015</td>
<td>.011</td>
</tr>
<tr>
<td>Pay-to-play, (3)</td>
<td>.015</td>
<td>.003</td>
<td>1</td>
<td>.095</td>
<td>.16</td>
<td>-.039</td>
<td>.069</td>
<td>.022</td>
<td>.147</td>
<td>.083</td>
</tr>
<tr>
<td>Cumulative dividends, (4)</td>
<td>.006</td>
<td>.005</td>
<td>.011</td>
<td>1</td>
<td>.231</td>
<td>.064</td>
<td>.037</td>
<td>.114</td>
<td>.027</td>
<td>.008</td>
</tr>
<tr>
<td>Participation preference, (5)</td>
<td>.022</td>
<td>.004</td>
<td>.024</td>
<td>.045</td>
<td>1</td>
<td>.065</td>
<td>.086</td>
<td>.096</td>
<td>.041</td>
<td>.039</td>
</tr>
<tr>
<td>Full ratchet, (6)</td>
<td>.001</td>
<td>0</td>
<td>-.001</td>
<td>.003</td>
<td>.003</td>
<td>1</td>
<td>.01</td>
<td>-.008</td>
<td>-.021</td>
<td>.005</td>
</tr>
<tr>
<td>Board seat, (7)</td>
<td>.015</td>
<td>0</td>
<td>.008</td>
<td>.005</td>
<td>.015</td>
<td>0</td>
<td>1</td>
<td>-.007</td>
<td>.101</td>
<td>.125</td>
</tr>
<tr>
<td>Time between rounds, (8)</td>
<td>.007</td>
<td>.007</td>
<td>.009</td>
<td>.054</td>
<td>.058</td>
<td>-.001</td>
<td>-.004</td>
<td>1</td>
<td>-.008</td>
<td>-.026</td>
</tr>
<tr>
<td>IPO, (9)</td>
<td>.007</td>
<td>.001</td>
<td>.011</td>
<td>.002</td>
<td>.004</td>
<td>0</td>
<td>.008</td>
<td>.008</td>
<td>1</td>
<td>.468</td>
</tr>
<tr>
<td>IPO/Acquisition, (10)</td>
<td>.01</td>
<td>.001</td>
<td>.009</td>
<td>.001</td>
<td>.007</td>
<td>0</td>
<td>.019</td>
<td>-.01</td>
<td>.022</td>
<td>1</td>
</tr>
</tbody>
</table>
Table V: Start-up outcomes, initial contract and equity terms.

Notes: The table reports linear probability regression estimation where the dependent variable is an exit outcome for start-up financed before 2010. “IPO” is equal to one if the start-up has a public offering. The sample includes all startups where we can observe the full assortment of contract terms. “% equity sold to investors” is the total (as-if-common) equity stake sold in the startup’s first round financing. “Participating pref.” is a dummy variable equal to one if the preferred stock sold to investors was participating preferred. “VC has board seat” is equal to one if the lead VC had a board seat at the time of the first financing. “Pay to play” is an indicator variable equal to one if the financing terms include pay-to-play provisions. These provisions require reinvestment by the current investors to maintain their control and/or cash flow rights. “Log Raised” is the log of total capital invested in the financing (2009 dollars). “Year FE” are fixed effects for the financing year. “Year founded FE” are fixed effect for the startup’s founding year. “State FE” are fixed effects for the startup’s state and “Industry FE” are fixed effects for industry. Standard errors reported in parentheses, clustered at the VC firm. Significance: * p < 0.10, ** p < 0.05, *** p < 0.01.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>% equity sold to investors</td>
<td>0.216***</td>
<td>0.173***</td>
<td>0.190***</td>
<td>0.159***</td>
<td>0.125***</td>
<td>0.140***</td>
<td>0.107**</td>
</tr>
<tr>
<td></td>
<td>(0.0419)</td>
<td>(0.0334)</td>
<td>(0.0481)</td>
<td>(0.0408)</td>
<td>(0.0321)</td>
<td>(0.0500)</td>
<td>(0.0533)</td>
</tr>
<tr>
<td>Participating pref.</td>
<td>-0.0417**</td>
<td>-0.0420***</td>
<td>-0.0475***</td>
<td>-0.0420***</td>
<td>-0.0475***</td>
<td>-0.0475***</td>
<td>-0.0475***</td>
</tr>
<tr>
<td></td>
<td>(0.0130)</td>
<td>(0.0139)</td>
<td>(0.0179)</td>
<td>(0.0139)</td>
<td>(0.0179)</td>
<td>(0.0179)</td>
<td>(0.0179)</td>
</tr>
<tr>
<td>VC has board seat</td>
<td>0.0309***</td>
<td>0.0327***</td>
<td>0.0422**</td>
<td>0.0309***</td>
<td>0.0327***</td>
<td>0.0422**</td>
<td>0.0422**</td>
</tr>
<tr>
<td></td>
<td>(0.0111)</td>
<td>(0.0108)</td>
<td>(0.0199)</td>
<td>(0.0111)</td>
<td>(0.0108)</td>
<td>(0.0199)</td>
<td>(0.0199)</td>
</tr>
<tr>
<td>Pay to play</td>
<td>0.0634**</td>
<td>0.0458</td>
<td>0.0485</td>
<td>0.0634**</td>
<td>0.0458</td>
<td>0.0485</td>
<td>0.0485</td>
</tr>
<tr>
<td></td>
<td>(0.0297)</td>
<td>(0.0299)</td>
<td>(0.0307)</td>
<td>(0.0297)</td>
<td>(0.0299)</td>
<td>(0.0307)</td>
<td>(0.0307)</td>
</tr>
<tr>
<td>Log raised</td>
<td>0.0137</td>
<td>-0.0270*</td>
<td>0.0120</td>
<td>0.0543</td>
<td>-0.144**</td>
<td>-0.196***</td>
<td>-0.149**</td>
</tr>
<tr>
<td></td>
<td>(0.0180)</td>
<td>(0.0145)</td>
<td>(0.0199)</td>
<td>(0.0532)</td>
<td>(0.0599)</td>
<td>(0.0688)</td>
<td>(0.0596)</td>
</tr>
<tr>
<td>Observations</td>
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<td>2953</td>
<td>1489</td>
<td>2115</td>
<td>2953</td>
<td>1489</td>
<td>1458</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.0213</td>
<td>0.0166</td>
<td>0.0202</td>
<td>0.0307</td>
<td>0.0312</td>
<td>0.0266</td>
<td>0.0349</td>
</tr>
<tr>
<td>Year FE</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Year founded FE</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>State FE</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Industry FE</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
</tbody>
</table>
Table VI: Empirical and theoretical moments.

Notes: The table describes empirical moments and their theoretical counterparts computed at estimated model parameters.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Theoretical</th>
<th>Empirical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. time since last VC financing</td>
<td>0.7870</td>
<td>0.7518</td>
</tr>
<tr>
<td>Var. time since last VC financing</td>
<td>1.7408</td>
<td>0.5973</td>
</tr>
<tr>
<td>Avg. share of VC equity</td>
<td>0.3523</td>
<td>0.3047</td>
</tr>
<tr>
<td>Var. share of VC equity</td>
<td>0.0306</td>
<td>0.0013</td>
</tr>
<tr>
<td>Cov. time since last VC financing and share of VC equity</td>
<td>0.0064</td>
<td>0.0012</td>
</tr>
<tr>
<td>Avg. participation preference</td>
<td>0.4094</td>
<td>0.5258</td>
</tr>
<tr>
<td>Cov. time since last VC financing and participation preference</td>
<td>0.0563</td>
<td>-0.0100</td>
</tr>
<tr>
<td>Cov. share of VC equity and participation preference</td>
<td>0.0222</td>
<td>0.0053</td>
</tr>
<tr>
<td>Avg. VC board seat</td>
<td>0.6187</td>
<td>0.5139</td>
</tr>
<tr>
<td>Cov. time since last VC financing and VC board seat</td>
<td>-0.0058</td>
<td>-0.0035</td>
</tr>
<tr>
<td>Cov. share of VC equity and VC board seat</td>
<td>0.0144</td>
<td>0.0083</td>
</tr>
<tr>
<td>Cov. participation preference and VC board seat</td>
<td>0.0151</td>
<td>0.0563</td>
</tr>
<tr>
<td>Avg. IPO rate</td>
<td>0.0466</td>
<td>0.0164</td>
</tr>
<tr>
<td>Cov. time since last VC financing and IPO rate</td>
<td>-0.0063</td>
<td>0.0006</td>
</tr>
<tr>
<td>Cov. share of VC equity and IPO rate</td>
<td>0.0073</td>
<td>0.0001</td>
</tr>
<tr>
<td>Cov. participation preference and IPO rate</td>
<td>-0.0022</td>
<td>0.0002</td>
</tr>
<tr>
<td>Cov. VC board seat and IPO rate</td>
<td>0.0074</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

Table VII: Parameter estimates.

Notes: The table describes parameter estimates of the model described in Section 5.2. Standard errors are difficult to compute numerically because of issues related to discretization of qualities and TO BE ADDED. Significance: * p < 0.10, ** p < 0.05, *** p < 0.01.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distribution of qualities, $a_i$</td>
<td>3.050</td>
<td>TBA</td>
</tr>
<tr>
<td>Distribution of qualities, $b_i$</td>
<td>1.194</td>
<td>TBA</td>
</tr>
<tr>
<td>Distribution of qualities, $a_e$</td>
<td>5.537</td>
<td>TBA</td>
</tr>
<tr>
<td>Distribution of qualities, $b_e$</td>
<td>7.766</td>
<td>TBA</td>
</tr>
<tr>
<td>Frequency of encounters, $\lambda_i$</td>
<td>7.771</td>
<td>TBA</td>
</tr>
<tr>
<td>Frequency of encounters, $\lambda_e$</td>
<td>6.629</td>
<td>TBA</td>
</tr>
<tr>
<td>Probability of IPO, intercept, $\kappa_0$</td>
<td>-2.686</td>
<td>TBA</td>
</tr>
<tr>
<td>Probability of IPO, total value, $\kappa_1$</td>
<td>0.015</td>
<td>TBA</td>
</tr>
<tr>
<td>Total value, share of VC equity, $\beta_1$</td>
<td>0.358</td>
<td>TBA</td>
</tr>
<tr>
<td>Total value, share of VC equity squared, $\beta_2$</td>
<td>-2.522</td>
<td>TBA</td>
</tr>
<tr>
<td>Total value, participation preference, $\beta_3$</td>
<td>-0.156</td>
<td>TBA</td>
</tr>
<tr>
<td>Total value, VC board seat, $\beta_4$</td>
<td>-0.051</td>
<td>TBA</td>
</tr>
<tr>
<td>Total value, part. pref. $\times$ VC board seat, $\beta_5$</td>
<td>0.020</td>
<td>TBA</td>
</tr>
<tr>
<td>Split of value, intercept, $\gamma_1$</td>
<td>-0.260</td>
<td>TBA</td>
</tr>
<tr>
<td>Split of value, participation preference, $\gamma_2$</td>
<td>-0.151</td>
<td>TBA</td>
</tr>
<tr>
<td>Split of value, VC board seat, $\gamma_3$</td>
<td>-0.050</td>
<td>TBA</td>
</tr>
<tr>
<td>Split of value, part. pref. $\times$ VC board seat, $\gamma_4$</td>
<td>0.020</td>
<td>TBA</td>
</tr>
</tbody>
</table>
Table VIII: Start-up values, deal frequencies, and present values of deals in the VC market at estimated parameters.

Notes: Panel A of the table reports expected total value and split of value across all deals and deals completed by quartiles of investor and entrepreneur qualities. Expected total values in quartile subsamples, \( \pi^*(\text{Sub}) \), are in units of expected total value across all deals, \( \pi^*(\text{All}) \). Expected values of investors and entrepreneurs across all deals and in quartile subsamples, \( \pi_i^*(\text{All}) \) and \( \pi_e^*(\text{Sub}) \), \( j \in \{i, e\} \), are in units of expected total value in the relevant subsample, \( \pi^*(\text{All}) \) and \( \pi^*(\text{Sub}) \) correspondingly. Panel B of the table reports expected deal frequencies across all market participants, \( \Lambda_j(\text{All}) \), and by quartiles of investor and entrepreneur qualities, \( \Lambda_j(\text{Sub}), j \in \{i, e\} \). Panel C of the table reports present values of all deals in the market, \( PV(\text{All}) \), and deals completed by quartiles of investor and entrepreneur qualities, \( PV_j(\text{Sub}), j \in \{i, e\} \), in units of \( PV(\text{All}) \).

### Panel A: Start-up values

<table>
<thead>
<tr>
<th>Subsample (\text{Sub})</th>
<th>All deals (\text{All})</th>
<th>0-25% ( j ) quantile</th>
<th>25-50% ( j ) quantile</th>
<th>50-75% ( j ) quantile</th>
<th>75-100% ( j ) quantile</th>
</tr>
</thead>
<tbody>
<tr>
<td>( j ) = investor</td>
<td>\text{Ratio of expected profits}</td>
<td>( \pi^*(\text{All}) )</td>
<td>( \pi^*(\text{Sub}) )</td>
<td>( \pi^*(\text{Sub}) )</td>
<td>( \pi^*(\text{Sub}) )</td>
</tr>
<tr>
<td>100</td>
<td>45.63</td>
<td>54.37</td>
<td>61.53</td>
<td>58.27</td>
<td>105.18</td>
</tr>
<tr>
<td>( j ) = entrepreneur</td>
<td>100</td>
<td>45.63</td>
<td>54.37</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

### Panel B: Deal frequencies

<table>
<thead>
<tr>
<th>Subsample (\text{Sub})</th>
<th>All deals (\text{All})</th>
<th>0-25% ( j ) quantile</th>
<th>25-50% ( j ) quantile</th>
<th>50-75% ( j ) quantile</th>
<th>75-100% ( j ) quantile</th>
</tr>
</thead>
<tbody>
<tr>
<td>( j ) = investor</td>
<td>( \Lambda_j(\text{All}) )</td>
<td>( \Lambda_j(\text{Sub}) )</td>
<td>( \Lambda_j(\text{Sub}) )</td>
<td>( \Lambda_j(\text{Sub}) )</td>
<td>( \Lambda_j(\text{Sub}) )</td>
</tr>
<tr>
<td>1.049</td>
<td>1.421</td>
<td>1.067</td>
<td>0.857</td>
<td>0.857</td>
<td></td>
</tr>
<tr>
<td>( j ) = entrepreneur</td>
<td>0.895</td>
<td>0</td>
<td>0.002</td>
<td>0.215</td>
<td>3.201</td>
</tr>
</tbody>
</table>

### Panel C: Present values of deals

<table>
<thead>
<tr>
<th>Subsample (\text{Sub})</th>
<th>All deals (\text{All})</th>
<th>0-25% ( j ) quantile</th>
<th>25-50% ( j ) quantile</th>
<th>50-75% ( j ) quantile</th>
<th>75-100% ( j ) quantile</th>
</tr>
</thead>
<tbody>
<tr>
<td>( j ) = investor</td>
<td>( PV_j(\text{All}) )</td>
<td>( PV_j(\text{Sub}) )</td>
<td>( PV_j(\text{Sub}) )</td>
<td>( PV_j(\text{Sub}) )</td>
<td>( PV_j(\text{Sub}) )</td>
</tr>
<tr>
<td>74.35</td>
<td>10.04</td>
<td>17.82</td>
<td>19.91</td>
<td>26.58</td>
<td></td>
</tr>
<tr>
<td>( j ) = entrepreneur</td>
<td>25.65</td>
<td>0</td>
<td>0.03</td>
<td>3.31</td>
<td>22.31</td>
</tr>
</tbody>
</table>
Table IX: Start-up values, deal frequencies, and present values of deals in the VC market in the presence of contract regulation.

Notes: The table examines effects of three counterfactuals, in which investors are restricted from including participation preference, VC board seat, or both terms into the contract. Panel A of the table reports the change, compared to the estimated value, in the expected total value and split of value across all deals and deals completed by quartiles of investor and entrepreneur qualities. The change in expected total values across all deals and in quartile subsamples, \( \Delta \pi^{cf} (All) = \pi^{cf} (All) - \pi^{cf} (Sub) \) and \( \Delta \pi^{cf} (Sub) = \pi^{cf} (Sub) - \pi^{cf} (Sub) \), as well as the change in expected values of investors and entrepreneurs across all deals and in quartile subsamples, \( \Delta \pi^{cf} (All) = \pi^{cf} (All) - \pi^{cf} (Sub) \) and \( \Delta \pi^{cf} (Sub) = \pi^{cf} (Sub) - \pi^{cf} (Sub) \), \( j \in \{i, e\} \), are in units of estimated total expected value in the relevant subsample, \( \pi^{cf} (All) \) and \( \pi^{cf} (Sub) \) correspondingly. Panel B of the table reports the change, compared to the estimated value, in expected deal frequencies across all market participants, \( \Delta \nu^{cf} (All) = \nu^{cf} (All) - \nu^{cf} (Sub) \), and by quartiles of investor and entrepreneur qualities, \( \Delta \nu^{cf} (Sub) = \nu^{cf} (Sub) - \nu^{cf} (Sub) \), \( j \in \{i, e\} \), in units of estimated deal frequency in the relevant subsample, \( \nu^{cf} (All) \) and \( \nu^{cf} (Sub) \). Panel C of the table reports the change, compared to the estimated value, in present values of all deals in the market, \( \Delta PV^{cf} (All) = PV^{cf} (All) - PV^{cf} (Sub) \), and deals completed by quartiles of investor and entrepreneur qualities, \( \Delta PV^{cf} (Sub) = PV^{cf} (Sub) - PV^{cf} (Sub) \), \( j \in \{i, e\} \), in units of estimated present value of deals in the relevant subsample, \( PV^{cf} (All) \) and \( PV^{cf} (Sub) \). Correspondingly.

### Panel A: Start-up values

<table>
<thead>
<tr>
<th>Subsample (Sub)</th>
<th>All deals (All)</th>
<th>0-25% $j$ quantile</th>
<th>25-50% $j$ quantile</th>
<th>50-75% $j$ quantile</th>
<th>75-100% $j$ quantile</th>
<th>All deals (Sub)</th>
<th>0-25% $j$ quantile</th>
<th>25-50% $j$ quantile</th>
<th>50-75% $j$ quantile</th>
<th>75-100% $j$ quantile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rebalanced terms only</td>
<td>-0.18</td>
<td>-0.18</td>
<td>0</td>
<td>-0.12</td>
<td>-0.12</td>
<td>0</td>
<td>-0.05</td>
<td>-0.05</td>
<td>0</td>
<td>-0.20</td>
</tr>
<tr>
<td>Equilibrium</td>
<td>1.96</td>
<td>-0.86</td>
<td>2.81</td>
<td>-2.62</td>
<td>-1.54</td>
<td>-1.08</td>
<td>1.65</td>
<td>-1.19</td>
<td>2.84</td>
<td>4.08</td>
</tr>
<tr>
<td>Rebalanced terms only</td>
<td>-0.18</td>
<td>-0.18</td>
<td>0</td>
<td>-0.30</td>
<td>-0.30</td>
<td>0</td>
<td>-0.28</td>
<td>-0.28</td>
<td>0</td>
<td>-0.18</td>
</tr>
<tr>
<td>Equilibrium</td>
<td>1.96</td>
<td>-0.86</td>
<td>2.81</td>
<td>-71.30</td>
<td>31.57</td>
<td>39.72</td>
<td>22.42</td>
<td>7.96</td>
<td>14.45</td>
<td>3.78</td>
</tr>
</tbody>
</table>

### Panel B: Deal frequencies

<table>
<thead>
<tr>
<th>Agent</th>
<th>Subsample (Sub)</th>
<th>All deals (All)</th>
<th>0-25% $j$ quantile</th>
<th>25-50% $j$ quantile</th>
<th>50-75% $j$ quantile</th>
<th>75-100% $j$ quantile</th>
<th>All deals (Sub)</th>
<th>0-25% $j$ quantile</th>
<th>25-50% $j$ quantile</th>
<th>50-75% $j$ quantile</th>
<th>75-100% $j$ quantile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rebalanced terms only</td>
<td>-0.02</td>
<td>-0.02</td>
<td>0</td>
<td>-0.01</td>
<td>-0.01</td>
<td>0</td>
<td>-0.02</td>
<td>-0.02</td>
<td>0</td>
<td>-0.05</td>
<td>-0.05</td>
</tr>
<tr>
<td>Equilibrium</td>
<td>0.36</td>
<td>-0.14</td>
<td>0.50</td>
<td>-1.92</td>
<td>-0.20</td>
<td>-1.71</td>
<td>1.03</td>
<td>-0.16</td>
<td>1.20</td>
<td>1.0</td>
<td>-1.0</td>
</tr>
<tr>
<td>Rebalanced terms only</td>
<td>-0.02</td>
<td>-0.02</td>
<td>0</td>
<td>-0.01</td>
<td>-0.01</td>
<td>0</td>
<td>-0.01</td>
<td>-0.01</td>
<td>0</td>
<td>-0.02</td>
<td>-0.02</td>
</tr>
<tr>
<td>Equilibrium</td>
<td>0.36</td>
<td>-0.14</td>
<td>0.50</td>
<td>20.32</td>
<td>9.33</td>
<td>10.99</td>
<td>10.52</td>
<td>4.31</td>
<td>6.21</td>
<td>1.14</td>
<td>0.21</td>
</tr>
<tr>
<td>Rebalanced terms only</td>
<td>-0.27</td>
<td>-0.27</td>
<td>0</td>
<td>-0.19</td>
<td>-0.19</td>
<td>0</td>
<td>-0.09</td>
<td>-0.09</td>
<td>0</td>
<td>-0.25</td>
<td>-0.25</td>
</tr>
<tr>
<td>Equilibrium</td>
<td>2.01</td>
<td>-1.27</td>
<td>3.28</td>
<td>-384</td>
<td>-191</td>
<td>-194</td>
<td>253</td>
<td>-143</td>
<td>396</td>
<td>5.01</td>
<td>-0.39</td>
</tr>
<tr>
<td>Rebalanced terms only</td>
<td>-0.27</td>
<td>-0.27</td>
<td>0</td>
<td>-0.52</td>
<td>-0.52</td>
<td>0</td>
<td>-0.47</td>
<td>-0.47</td>
<td>0</td>
<td>-0.27</td>
<td>-0.27</td>
</tr>
<tr>
<td>Equilibrium</td>
<td>2.01</td>
<td>-1.27</td>
<td>3.28</td>
<td>96.87</td>
<td>42.56</td>
<td>54.31</td>
<td>23.75</td>
<td>7.88</td>
<td>15.87</td>
<td>4.61</td>
<td>-0.17</td>
</tr>
</tbody>
</table>

### Panel C: Present values of deals

<table>
<thead>
<tr>
<th>Agent</th>
<th>Combined</th>
<th>Investor</th>
<th>Entrepreneur</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subsample (Sub)</td>
<td>All deals (All)</td>
<td>All investors</td>
<td>All entrepreneurs</td>
</tr>
<tr>
<td>All deals (All)</td>
<td>All deals (Sub)</td>
<td>All deals (Sub)</td>
<td>All deals (Sub)</td>
</tr>
<tr>
<td>All deals (Sub)</td>
<td>All deals (Sub)</td>
<td>All deals (Sub)</td>
<td>All deals (Sub)</td>
</tr>
<tr>
<td>Rebalanced terms only</td>
<td>3.31</td>
<td>1.44</td>
<td>10.96</td>
</tr>
<tr>
<td>Equilibrium</td>
<td>0.15</td>
<td>0.46</td>
<td>0</td>
</tr>
<tr>
<td>Rebalanced terms only</td>
<td>4.30</td>
<td>4.41</td>
<td>10.96</td>
</tr>
<tr>
<td>Rebalanced terms only</td>
<td>3.49</td>
<td>-0.81</td>
<td>-0.32</td>
</tr>
<tr>
<td>Rebalanced terms only</td>
<td>1.05</td>
<td>-0.20</td>
<td>-0.10</td>
</tr>
<tr>
<td>Rebalanced terms only</td>
<td>4.55</td>
<td>-1.14</td>
<td>-0.41</td>
</tr>
</tbody>
</table>
Table X: Start-up values, deal frequencies, and present values of deals in the VC market when search frictions are low.

Notes: The table examines effects of three counterfactuals, in which investors and entrepreneurs encounter each other with 2X, 5X, and 10X frequency compared to the estimated frequency. It reports the change, compared to the estimated value, in present values of all deals in the market, $\Delta PV^{cf}(\text{All}) = PV^{cf}(\text{All}) - PV^*(\text{All})$, and deals completed by quartiles of investor and entrepreneur qualities, $\Delta PV^{cf}(\text{Sub}) = PV^{cf}(\text{Sub}) - PV^*_j(\text{Sub})$, $j \in \{i, e\}$, in units of the estimated present value of deals in the relevant subsample, $PV^*(\text{All})$ and $PV^*_j(\text{Sub})$ correspondingly.

<table>
<thead>
<tr>
<th>Agent Subsample (Sub)</th>
<th>Combined All deals (All):</th>
<th>Investor</th>
<th>Entrepreneur</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All investors 0-25%</td>
<td>25-50%</td>
<td>50-75%</td>
</tr>
<tr>
<td>Ratio of present values</td>
<td>$\Delta PV^{cf}(\text{All})$</td>
<td>$\Delta PV^{cf}(\text{Sub})_{i}$</td>
<td>$\Delta PV^{cf}(\text{Sub})_{e}$</td>
</tr>
<tr>
<td>2X Frequencies of encounters</td>
<td>0.37</td>
<td>4.62</td>
<td>0.63</td>
</tr>
<tr>
<td>5X Frequencies of encounters</td>
<td>-0.47</td>
<td>10.23</td>
<td>1.51</td>
</tr>
<tr>
<td>10X Frequencies of encounters</td>
<td>-0.65</td>
<td>13.98</td>
<td>2.12</td>
</tr>
</tbody>
</table>