That is not my dog: why doesn’t the log dividend-price ratio seem to predict future log returns or log dividend growth?

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That is not my dog: why doesn’t the log dividend-price ratio seem to predict future log returns or log dividend growth?  

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Abstract

Campbell and Shiller’s “accounting identity” implies that changes in the log dividend-price ratio must predict either future returns or future log dividend growth. However, neither quantity seems to be predictable – a well-known puzzle in the literature. We examine this puzzle step-by-step from theoretical estimation through empirical testing. Stationarity of the log dividend-price ratio is an important assumption behind the accounting identity, but Campbell and Shiller’s test justifying this assumption does not make sense, and a corrected test does not reject non-stationarity. Nonetheless, a truncated accounting identity works reasonably well in the existing sample, and we find that the log dividend-price ratio predicts log dividend growth, not returns. Unfortunately, this result does not seem to be robust to subsamples. Also, it seems unwise to rely too much on asymptotic properties of estimators when the entire sample includes only five non-overlapping observations.

Key words: return predictability, dividend-price ratio, stationarity test. [JEL G12 G17]
Clouseau: Does your dog bite?
Innkeeper: No.
Clouseau: Nice doggy.
(Clouseau tries to pet the dog on the floor and is bitten)
Clouseau (angry): I thought you said your dog did not bite.
Innkeeper: That is not my dog. ²

Often, like Clouseau, we get into trouble because we ask the wrong question. This paper examines the failure of the log dividend-price ratio (hereafter LDPR) to predict either future log returns or future log dividend growth. Campbell and Shiller’s (1988) “accounting identity” asserts that the current LDPR is approximately equal to a constant plus the sum of present values of future log returns minus the sum of present values of future log dividend growths. This implies that current LDPR should be able to predict future returns, or dividend growth rates, or both; empirically, however, it seems to predict neither. This puzzle was examined by Cochrane (2008), who side-stepped the puzzle by assuming a just-identified model, using the analogy of the “dog that didn’t bark” from Sherlock Holmes, but that is not our dog. gp Since, like Clouseau, we do not know which question to ask, we go step-by-step through the entire argument to uncover where the problem is. We find that:

(1) The log-linear approximation works very well both in a single period and over 30 years.

(2) Campbell and Shiller make a theoretical assumption that the long-term mean LDPR exists. They justify this assumption by an empirical test that unfortunately does not actually test this at all. Our correctly specified test fails to reject the null that the LDPR is not a stationary series.

(3) Even though the long-term mean of the LDPR seems not to exist, the approximation works reasonably well in the sample to date. In particular, discarding the final term has an impact, but the relationship is still strong without it.

(4) Using a test with many lags as in the theory,³ future log dividend growth is significantly predictable but expected returns are not. We show that this result is hard to uncover because the large prediction error introduced by the unpredictable part of future log returns is a common factor in future

³The test uses the appropriate Newey-West adjustment for serially correlated errors and the Stambaugh adjustment for spurious regression bias.
log dividend growth but cancels in the accounting identity.

(5) Although these results are significant, they do not seem to be very robust. In particular, the relationship is reversed significantly in the second half-sample (consistent with Chen (2009)). Also, the estimation asks a lot of the Newey-West adjustment, since there are only about five non-overlapping observations in the whole sample (and even fewer in subperiods). The lack of evidence for stationarity of the LDPR is also troubling for estimation. If the LDPR is not stationary, the approximation in the accounting identity should get worse and worse over time.

The Campbell-Shiller “accounting identity” can be derived by starting with the single period definition of returns as the sum of dividends and capital gains. Using algebra, taking logs, and doing a Taylor series expansion around some typical value for the LDPR, we obtain a one-period approximation formula linking log returns and log dividend growth with beginning- and end-of-period LDPR. By telescoping this approximation over many periods, current LDPR can be linearly approximated by the sum of weighted log returns and the sum of weighted dividend growth rates over future periods plus the LDPR in the final period. Using the annual dividend payments and prices of S&P500 index from 1871 to 2015, and performing a regression of the LDPR on the sums over future 30 years and the final log dividend-price ratio, all coefficients on independent variables are close to the theoretical value one (or minus one) and the $R^2$ is close to 100% (98.91%). The approximation is worse but still acceptable if we drop the final LDPR in year 30 (the $R^2$ dropped to 82.25%). Therefore, the source of the puzzle is the lack of power in previous tests gp rather than any intrinsic problem with the theory, at least in the truncated identity.

We correct an error in the Campbell-Shiller stationarity test (in which both the null and the alternative contain a trend and therefore neither is stationary), and conduct a conventional stationarity test in which the alternative hypothesis is that the underlying series is stationary (without trends). We cannot reject the null of nonstationarity (which may mean the long-term mean does not exist) when we use the entire sample. In principle, this is a big problem for the Campbell-Shiller approximation, which is based on an expansion around the long-term mean, but the truncated version of the approximation still works well in our sample looking 30 years out.

We test the predictability of stock returns and dividend growth rate using an equation similar to

\footnote{Campbell and Shiller expand around the long-term mean, but that is not necessary.}
the equation in the model rather than using one or a few lags as is common in the literature. Our estimation uses statistical corrections for the correlation in error terms and for spurious regression bias. The Campbell-Shiller approximation implies that the current LDPR is able to predict either future returns or dividend growth or both; we find that future log dividend growth is significantly predictable, but future returns are not. The results are robust when we expand the log dividend price ratio around alternative points rather than the sample mean.

As noted by Cochrane (2008), dividends are smooth. He concludes that log dividend growth is not predictable (implying under the model restriction that returns are predictable). However, it is more accurate to assert that the predictability of log dividend growth is spread over many maturities and that nearby dividends are not very predictable because dividends are smooth. What is happening is that there is small predictability of dividend growth spread over many periods, which is buried by noise in conventional simple regression or vector-autoregressive (VAR) estimation. This issue could be exaggerated in tests with one or few lags if the log dividend-price ratio is not stationary.

The limitation inherent in using small lags to search for predictability of dividend growth seems to be a solution of the puzzle of why the theory (based on an accounting identity and an approximation that is not so bad in the current sample) is hard to verify. In general, the predictability of log dividend growth is difficult to find because of the large prediction error introduced by the unpredictable part of future log returns, which is a common factor with future log dividend growth that cancels in the accounting identity.

Although the best evidence (based on our whole sample) suggests that the LDPR predicts log dividend growth but not log returns, this result seems fragile. The result is not robust to subperiods, consistent with Chen (2009) and explaining an apparent inconsistency with a similar regression of Cochrane (2008, Section 7.2). We also worry about the statistical properties of the estimators, both because the whole sample has only about five non-overlapping observations (and subsamples even fewer) and because of the apparent instability over time. If the LDPR is not, in fact, stationary, the problems will become bigger as the sample size gets larger because the Taylor series expansion will become much less accurate as the range of values increases over time. One interesting aspect of the accounting identity is that it is not an economic model since its derivation uses only manipulation of identities and approximations. If we think about the economics, Modigliani-Miller suggests that to
first order dividends are irrelevant, which is consistent with instability of these relationships over time. Therefore, both statistical and economic arguments suggest that asymptotic justification of estimators do not apply.

The rest of this paper is organized as follows. We review the approximation leading to the accounting identity in Section 1 and test the quality of this approximation in Section 2. We propose a model-implied novel approach to test the predictability of returns and dividend growth in Section 3. In Section 4, we analyze whether the failure of the LDPR to predict stock returns is caused by noise in itself or noise introduced by the modeling procedure. Section 5 conducts robustness analysis and Section 6 concludes.

## 1 Dividend-Price Decomposition

We begin by specifying the standard definition relating returns, future prices, and dividend payments. Define gross investment return over one period as:

\[
1 + R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{P_{t+1}}{P_t} \left(1 + \frac{D_{t+1}}{P_{t+1}}\right),
\]

where \(P_t\) and \(P_{t+1}\) denote the stock prices at the begin and the end of the period, respectively, and \(R_{t+1}\) and \(D_{t+1}\) denote respectively the net return over the period and the dividend payment at the end of the period, abstracting from splits and distributions other than dividends.\(^5\) This may seem like a strange way to write the return, since we would normally look at gross capital gains \(P_{t+1}/P_t\) and dividend yield \(D_t/P_t\), with information known at the beginning of the period in the denominator. For our purpose, we simply manipulate accounting identities to express returns in this more unconventional manner because placing \(P_t\) in both denominators would give us a telescoping series in which the final term

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\(^5\) For empirical tests, we might treat all dividend payments during the period as coming at the end of the period. Alternatively, we could try to construct a more accurate return calculation that takes into account the timing of the dividends and the returns within each period. In practice, these approaches are likely to yield very similar results.
does not vanish. Taking logs on both sides, equation (1) becomes:

\[
\log(1 + R_{t+1}) = \log \left( \frac{P_{t+1}}{P_t} \right) + \log(1 + \delta_{t+1}),
\]

where \( \delta_{t+1} \equiv \log(D_{t+1}/P_{t+1}) \). We will approximate equation (2) by a first-order Taylor series expansion around \( \delta_{t+1} = \delta \). Traditionally, \( \delta \) is taken to be the long-term mean of the LDPR \( \log(D_t/P_t) \), but we will take a broader view and think about \( \delta \) being some reasonable value, since we will present some evidence that the long-term average may not exist. Letting \( \rho \equiv 1/(1 + \exp(\delta)) \), then

\[
\frac{d \log(1 + \exp(\delta_{t+1}))}{d \delta_{t+1}} \bigg|_{\delta_{t+1} = \delta} = \frac{\exp(\delta_{t+1})}{1 + \exp(\delta_{t+1})} \bigg|_{\delta_{t+1} = \delta} = 1 - \rho.
\]

Therefore, letting \( \kappa \equiv \log(1 + \exp(\delta)) - (1 - \rho)\delta \), the Taylor approximation is:

\[
\log(1 + R_{t+1}) = \log \left( \frac{P_{t+1}}{P_t} \right) + \log(1 + \exp(\delta_{t+1}))
\approx \log \left( \frac{P_{t+1}}{P_t} \right) + \log(1 + \exp(\delta)) + (1 - \rho)(\delta_{t+1} - \delta)
= \log(P_{t+1}) - \log(P_t) + \log(1 - \exp(\delta))
+ (1 - \rho)(\log(D_{t+1}) - \log(P_{t+1})) - (1 - \rho)\delta
= \kappa + \rho \log(P_{t+1}) + (1 - \rho)\log(D_{t+1}) - \log(P_t).
\]

We follow Campbell and Shiller (1988) in being informal about the sense of the approximation; we will take an empirical approach to determine how well the approximation works. We can rewrite equation (3) as

\[
\log \left( \frac{D_t}{P_t} \right) \approx -\kappa + \log(1 + R_{t+1}) + \rho \log \left( \frac{D_{t+1}}{P_{t+1}} \right) - \Delta \log(D_{t+1}).
\]
Substituting the same for \( t + 1, t + 2, \) and so forth for the \( \log(D_{t+1}/P_{t+1}) \) on the right-hand side telescopes to imply:

\[
(5) \quad \log \left( \frac{D_t}{P_t} \right) \approx -\frac{\kappa}{1 - \rho} \left( 1 - \rho T^{-t} \right) + \sum_{s=t+1}^{T} \rho^{s-t-1} \left( \log((1 + R_s) - \Delta \log(D_s)) + \rho^{T-t} \log \left( \frac{D_T}{P_T} \right) \right).
\]

This is the essential relationship that we will work with. Since \( \rho < 1 \) it is at least plausible to argue (as do Campbell and Shiller) that the final term should vanish as \( T \) increases, and we have the asymptotic expression

\[
(6) \quad \log \left( \frac{D_t}{P_t} \right) \approx -\frac{\kappa}{1 - \rho} + \sum_{s=t+1}^{\infty} \rho^{s-t-1} \left( \log((1 + R_s) - \Delta \log(D_s)) \right),
\]

often referred to in the literature as the accounting identity. This identity says that, subject to the quality of the approximation, today’s log dividend-price ratio \( \log(D_t/P_t) \) is identically equal to a linear combination of future log returns \( \log(1 + R_s) \) and future changes in log dividends \( \Delta \log(D_s) \). This implies that the log dividend-price ratio must predict one or both of these. The puzzle in the literature is that the log dividend-price ratio seems to predict neither future log returns nor future log dividend growth.

### 2 Approximation Test

Before testing whether stock returns are predictable, we test the quality of the LDPR approximation in equation (5) using the annual prices and aggregate dividend payments of the S&P 500 index firms between 1871 and 2015. We focus on annual data because monthly dividend payments are linearly interpolated from annual and quarterly dividend payments, and we do not want to deal with the approximation error this might entail. Over the sample period, the average gross return on the S&P 500 index is 10.56% with a standard deviation of 18.17%, and the average annual log gross return is 8.61% with a standard deviation of 17.33%; the average dividend-price ratio is 4.47% with a standard deviation of 1.52% while the average log dividend price ratio is -3.18 with a standard deviation of 6

\[\text{The data is collected from Robert Shiller’s website at} \ http://www.econ.yale.edu/~shiller/data.htm. \text{ We thank Robert Shiller for making this data available online.} \]
Finally, the average annual log dividend growth rate is 4.37% with a standard deviation of 12.16%. The sample LDPR mean of 4.47% implies a $\rho$ of 0.95.

Campbell and Shiller (1988) suggest a vector autoregression (VAR) approach to test equation (5) without the final term instead of using the conventional predictive regression. They find that the LDPR series is persistent and able to predict both future stock returns and future dividend growth, but the associated $R^2$s in their tests are small. We replicate and confirm their results. Although Campbell and Shiller claim that the VAR procedure is better suited to “detect long-term deviations of stock prices from the ‘fundamental value’ ” than single linear regressions, the VAR approach suffers several shortcomings. First, a VAR procedure with a limited number of lags does not sufficiently capture the long-term relationship among current dividend-price ratio, future returns and future dividend growth rates. Cochrane (2011) shows that VAR estimates can be biased and significantly different from those in the true linear regressions. Second, the calculations required by a VAR procedure are much more complicated for more than two variables even with very limited lags. Finally, this procedure ignores the final term in the equation. To conclude, the analysis in Campbell and Shiller (1988) does not tell us whether (5) holds empirically.

An improved approach that avoids such shortcomings is to conduct a true linear regression of log dividend-price ratio on 2($T - t$) terms of discounted log return and dividend growth plus one final term. This procedure, however, is burdensome and may not be implementable when ($T - t$) is large and the sample period is not sufficiently long. In this study, we propose a parsimonious regression in the form of equation (5), that is, regressing the current LDPR on the sum of weighted future returns, the sum of weighted dividend growth rates and the log dividend-price ratio in the last period:

$$\log\left(\frac{D_t}{P_t}\right) = \alpha + \beta_1 \left( \sum_{s=t+1}^{T} \rho^{s-t-1} \log(1 + R_s) \right) + \beta_2 \left( \sum_{s=t+1}^{T} \rho^{s-t-1} \Delta \log(D_s) \right) + \beta_3 \left( \rho^{T-t} \log\left(\frac{D_T}{P_T}\right) \right) + \epsilon_t.$$  

By construction, this regression overcomes the shortcomings of both conventional linear and VAR estimations and is more parsimonious. If the approximation of equation (5) is effective, we should expect that the estimated $\beta_1$ and $\beta_3$ in equation (7) are close to one and the estimated $\beta_2$ is close to
Table 1: Approximation Test

This table reports the empirical results of whether the approximation of log dividend-price ratio in equation (5) is effective. The results are based on the annual prices of and dividend payments on the S&P 500 index from 1871 to 2015. $(T - t)$ is set to be 30 years. The associated Newey-West standard error with four lags are in parentheses. ***(denotes statistical significance at the 1% level.

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>$-2.96^{***}$</td>
<td>$-3.64^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.96^{***}</td>
<td>0.88^{***}</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>$-0.99^{***}$</td>
<td>$-1.08^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>1.19^{***}</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td></td>
</tr>
<tr>
<td>$N$</td>
<td>115</td>
<td>115</td>
</tr>
<tr>
<td>Adj-$R^2$ (%)</td>
<td>98.91</td>
<td>82.25</td>
</tr>
</tbody>
</table>

minus one, and the corresponding $R^2$ is close to 100%.

We take $(T - t)$ to be 30 years, which is reasonably long and gives us 115 overlapping observations (years) for analysis. The results are reported in the first column of Table 1 and suggest that the LDPR approximation in equation (5) is effective. The coefficient on the sum of discounted returns is positive and close to one, and the coefficient on the sum of discounted dividend growth is approximately equal to minus one. All coefficients are statistically significant at the 1% level and the corresponding $R^2$ is as high as 99%. This regression uses Newey-West standard errors to adjust for serial correlation (including that due to overlapping observations) and heteroscedasticity.\(^7\) The high $R^2$ suggests that current LDPR predicts at least one regressor but does not indicate which one(s). Note that we have not adjusted for spurious regression bias (caused by low frequency series on both sides). For now, it suffices to note that the fit is very good. We will correct for spurious regression bias when we conduct predictive regressions.

\(^7\)We report the Newey-West standard errors with 4 lags. In an untabulated analysis, we find that the Newey-West standard errors based on 10, 20 or 30 lags are similar.
we further test whether the final term in equation (5) is small by repeating the analysis on equation (7) after dropping this term. The results are reported in the second column of Table 1 and suggest that the log dividend-price ratio in the final period (30 years from now) is neither trivial nor particularly large. The coefficients on the sum of discounted returns and dividend growth are still significant but slightly deviate from 1 (or -1), and the $R^2$ drops significantly by 17%, from 99% to 82%, evidence that equation (7) is a good specification for empirically estimating the log dividend-price ratio.

3 Predictability Test

Equation (5) suggests a predictive relationship between current LDPR and cumulative future log returns or cumulative log dividend growth rates (see, for example, Campbell and Shiller, 1988; Cochrane, 2008). Furthermore, equation (5) suggests that the true predictive tests should be conducted by reversing the dependent and independent variables in equation (7) as:

\[ \sum_{s=t+1}^{T} \rho^{s-t-1} \log(1 + R_s) = \alpha + \beta_1 \log \left( \frac{D_t}{P_t} \right) + \mu_T. \]

This type of regression is presented in Cochrane (2008), although it is based a subset of our sample period (1926-2004 rather than 1871-2015); Cochrane’s sample period is similar to the second half of ours, which is described in our subsample results in Section 5.2.

A significant $\beta_1$ and a reasonable $R^2$ would suggest a predictable relationship between the LDPR and the cumulative future returns. This specification applies to the predictability of future log dividend growth rates. The use of cumulative present values of the predicted variable in future periods has advantages over a conventional predictive specification, in which one-period leading predicted variable is mostly used, in that it can capture the total predictability of future returns or dividend growth. In other words, equation (8) captures both short-run and long-run return predictabilities (if any).

Equation (5) implies that the sum of the coefficients on the log dividend-price ratio ($\beta_1 s$) across all three predictive tests (i.e. predictability tests of $\sum_{s=t+1}^{T} \rho^{s-t-1} \log(1 + R_s)$, $\sum_{s=t+1}^{T} \rho^{s-t-1} \Delta \log(D_s)$, and $\rho^{T-t-1} \log(D_t / P_t)$) should be one if our predictive specification is appropriate. However, it is not possible to explore this relationship with three conventional predictive regressions. Granger and
Newbold (1974), Stambaugh (1999), and Ferson, Sarkissian, and Simin (2003) show that linear regressions with lagged stochastic regressors and finite samples may suffer spurious regression bias (SRB). Stambaugh shows that this bias is pronounced in the predictive coefficient but not in the standard error of the predictive coefficient or $R^2$. By assuming $\log \left(D_t / P_t\right)$ a first-order autoregressive process as $\log \left(D_t / P_t\right) = c + \tau \log \left(D_{t-1} / P_{t-1}\right) + \nu_t$, Stambaugh shows that the magnitude of SRB in the predictive coefficient in equation (8) equals $-\frac{\sigma_{\mu \nu}}{\sigma^2 \nu} \left(1 + 3\tau N\right)$, where $\sigma_{\mu \nu}$ is the covariance of $\mu_t$ and $\nu_t$, $\sigma^2 \nu$ the variance of $\nu_t$, and $N$ the number of observations of the sample. Newey and West (1987) propose an adjustment in the standard error to overcome the serial correlation in the error term $\mu_T$. In this section, we follow Newey and West (1987), and Stambaugh (1999) to calculate the Newey-West standard errors, and the SRB-adjusted coefficients of the predictor (i.e. the LDPR), respectively.

**Table 2: Predictability Test**

This table reports the empirical results of whether current log-dividend-price ratio is able to predict the sum of discounted future returns, the sum of discounted future dividend growths, or the discounted log dividend-price 30 years from now. The results are based on the annual data of the S&P 500 index from 1871 to 2015. The spurious regression bias (SRB) is estimated following Stambaugh (1999). The associated Newey-West standard errors with four lags are in parentheses. ***, ** and * denote statistical significance at the 1%, 5% and 10% levels, respectively.

<table>
<thead>
<tr>
<th>Predicted variable</th>
<th>$\alpha$</th>
<th>$\beta_1$</th>
<th>SRB-adjusted $\beta_1$</th>
<th>Adj-$R^2$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sum_{s=t+1}^{T} \rho^{s-t-1} (\log(1 + R_s))$</td>
<td>1.98***</td>
<td>0.19</td>
<td>0.17</td>
<td>0.43</td>
</tr>
<tr>
<td></td>
<td>(0.76)</td>
<td>(0.24)</td>
<td>(0.24)</td>
<td></td>
</tr>
<tr>
<td>$\sum_{s=t+1}^{T} \rho^{s-t-1} \Delta \log(D_s)$</td>
<td>-1.26*</td>
<td>-0.60***</td>
<td>-0.56***</td>
<td>14.79</td>
</tr>
<tr>
<td></td>
<td>(0.66)</td>
<td>(0.20)</td>
<td>(0.20)</td>
<td></td>
</tr>
<tr>
<td>$\rho^{T-t} \log(D_T / P_T)$</td>
<td>-0.19</td>
<td>0.17**</td>
<td>0.17**</td>
<td>17.31</td>
</tr>
<tr>
<td></td>
<td>(0.19)</td>
<td>(0.07)</td>
<td>(0.07)</td>
<td></td>
</tr>
</tbody>
</table>

Table 2 shows that the sum of the LDPR coefficients from the three predictability tests of $\sum_{s=t+1}^{T} \rho^{s-t-1} (\log(1 + R_s))$, $\sum_{s=t+1}^{T} \rho^{s-t-1} \Delta \log(D_s)$ and $\rho^{T-t} \log(D_T / P_T)$ is close to one ($0.19 - (-0.60) + 0.17 = 0.96$) and the corresponding sum of SRB-adjusted predictor coefficients is similar (0.90). This is evidence that our predictive specifications are theoretically appropriate and that spurious regression bias is not severe in our analysis. More interestingly, Table 2 shows that the

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8 We report the Newey-West standard errors with 4 lags in Table 2. The Newey-West standard errors based on 10, 20 or 30 lags are similar.
LDPR (predictor) coefficient is insignificant in the predictability test of cumulative future returns (row 1) while it is significant in the predictability tests of cumulative future dividend growth (row 2) and the discounted final-period log dividend-price ratio (row 3). The magnitude of the coefficient on LDPR in the predictability test of future dividend growth rates dominates its counterparts and represents 63% (0.60/0.96) of the sum of all three predictor coefficients. The patterns remain after we make adjustments for the spurious bias in the predictive coefficients. The predictor coefficient on LDPR in the predictability test of future returns represents 20% of the sum of all three predictor coefficients. Similar patterns hold in the $R^2$s for both coefficients.

These results provide significant evidence to reject the hypothesis that future dividend growth rates are not predictable but not the hypothesis that future returns are not predictable. The main reason for this, as we show below, is that the generating process of stock returns is too noisy to be predictable, consistent with Shiller (1981), and Poterba and Summers (1988) that stock prices are too volatile to be explained by fundamentals. The unpredictability of stock returns supplements the argument by Lanne (2002), Valkanov (2003), and Boudoukh, Richardson and Whitelaw (2008) that conventional analysis of long-term predictability of stock returns is spurious. Moreover, the results from Table 2 suggest that the relationship among the three predictor coefficients (Cochrane, 2008) is not informative about stock return predictability.

It may be surprising that the predictability of the LDPR 30 years out in Table 2 is both economically and statistically significant, leading us to ask what we know now about what will happen 30 years in the future.\footnote{We must, of course, grant that the entire economic system may change over 30 years and the stock market may not exist.} This view is compelling if we take the dividend process as exogenous, but as Modigliani and Miller (1958) point out, dividends are somewhat arbitrary. Although our new information today may be primarily about cash flows in the coming ten years, this cash may go into repurchasing shares rather than paying dividends, with the actual dividend increase spread slowly over decades. The predictability of the LDPR 30 years out only depends on (1) the predictability of cash flows over a short horizon, and (2) firm policies implying that it takes a very long time for these increased cash flows to appear in dividends. All of this is consistent with the smoothness of dividends as noted by Lintner (1956) and others over time.

In the meantime, it is worth further exploring why the coefficient on stock returns in Table 1 is
consistently close to one and statistically significant while the predictor coefficient (on the LDPR) in the stock return predictability test in Table 2 is small and insignificant. Our explanation is that the coefficient of one in the first test is caused by high collinearity between stock returns and dividend growth rather than information innovation in stock return generating process. To illustrate our argument, let start with equation (5) as 

\[
\log\left(\frac{D_t}{P_t}\right) \approx \alpha + \beta_1 (\sum_{t=1}^{T} \rho^{s-t-1}(\log(1+R_s))) + \beta_2 (\sum_{t=1}^{T} \rho^{s-t-1}\Delta \log(D_s)) + \epsilon_t.
\]

Assume that the first term only contains noise (denoted by \(Z_t\)) and the second term contains both information and noise as follows:

\[
\sum_{s=t+1}^{T} \rho^{s-t-1}(\log(1+R_s)) \approx Z_t.
\]

\[
\sum_{s=t+1}^{T} \rho^{s-t-1}\Delta \log(D_s) \approx \log\left(\frac{D_t}{P_t}\right) + Z_t.
\]

Then the covariance matrix between LDPR and sum of discounted future dividend growth rates becomes:

\[
\text{var}\left(\log\left(\frac{D_t}{P_t}\right), \sum_{s=t+1}^{T} \rho^{s-t-1}\Delta \log(D_s)\right) = \begin{pmatrix}
\sigma_\delta^2 & -\sigma_\delta^2 \\
\sigma_\delta^2 & \sigma_\delta^2 + \sigma_Z^2
\end{pmatrix}.
\]

When we run current LDPR on the sum of discounted future dividend growth rates as 

\[
\log\left(\frac{D_t}{P_t}\right) = \alpha + \beta (\sum_{s=t+1}^{T} \rho^{s-t-1}\Delta \log(D_s)) + \mu_t,
\]

the coefficient on the independent variable is given by:

\[
\beta = \frac{-\sigma_\delta^2}{\sigma_\delta^2 + \sigma_Z^2}.
\]

When the noise \(\sigma_Z^2\) in stock returns is large, then the coefficient will be downward biased. Moreover, the correlation between the sum of discounted future returns and the sum of discounted future dividend growth also becomes large.

\[
\text{corr}\left(\sum_{s=t+1}^{T} \rho^{s-t-1}(\log(1+R_s)), \sum_{s=t+1}^{T} \rho^{s-t-1}\Delta \log(D_s)\right) = \frac{\sigma_Y^2}{\sigma_Y^2 + \sigma_\delta^2}.
\]
Equation (9) implies that when we conduct a stock return predictability test as \( \log \left( \frac{D_t}{P_t} \right) = \alpha + \beta (\sum_{s=t+1}^{T} \rho^{s-t-1}(\log(1 + R_s))) + \mu_t \), the \( \beta \) coefficient should be close to zero and our untabulated empirical results confirm this. However, equation (13) suggests that when the noise is large, we may end up with a spurious coefficient significantly different from zero, which suggests that the approximation test of LDPR specified in equation (5) is different from the predictive tests specified in equation (8). Our explanation is consistent with the empirical data. Figure 1 shows that the evolutions of cumulative log returns and dividend growth rates are closely correlated. In fact, the correlation between the log return and the log dividend growth is 0.63, and the correlation between \( (\sum_{s=t+1}^{T} \rho^{s-t-1}(\log(1 + R_s))) \) and \( (\sum_{s=t+1}^{T} \rho^{s-t-1}\Delta \log(D_s)) \) is 0.84. Moreover, the standard deviations of the cumulative returns and log dividend growth rates are respectively 38.1% and 34.3% which implies that the sum of the two terms’ variances is as high as 37.2%, or 61.0% in terms of standard deviation, while the standard deviation of the log dividend-price ratio over the same period is 23.6%. This spurious relationship is also observed by Ferson, Sarkissian and Simin (2003) and Valkanov (2003) with simulated data.

Figure 1: Time Series of Cumulative Discounted Returns and Dividend Growth Rates.

One concern about the results in this section is that our sample includes only about five non-overlapping observations of the weighted average of log differenced dividends. Although it is impressive that the estimates (with Newey-West and Stambaugh corrections) are significant in spite
of this, this puts a lot of demand on the Newey-West adjustment and we are far from its asymptotic justification.

4 Further Analysis

4.1 The Long-Term Mean Log Dividend-Price Ratio

The empirical analyses in the previous section are based on expanding LDPR around its sample mean. The literature typically assumes that the long-term mean exists and that the expansion is around this long-term mean. Unfortunately, Campbell and Shiller do not correctly test for the existence of the long-term mean: the long-term mean does not exist under either their null or their alternative hypothesis because both hypotheses include trends. In a corrected version of their test without a trend, we cannot reject the null of non-stationarity under which the long-term mean LDPR does not exist. This certainly weakens the interpretation of the sample mean as the long-term mean, but it doesn’t necessarily invalidate the analysis using the current data, as we will discuss in the next Section. The time series of the dividend-price ratio of S&P 500 index (solid line) and the corresponding log ratio (dash line) are plotted in Figure 2.

Figure 2: Time Series of $\frac{D}{P}$ and $\log(\frac{D}{P})$.

Both time series show a strong declining trend. For instance, the dividend-price ratio is around 5% over the Campbell-Shiller period (1871-1986), but declines to around 2% over the post Campbell-
Shiller period (1987-2015). Figure 2 suggests that the long-term mean of dividend-price or log dividend-price may not exist. To formally test the existence of the LDPR’s long-term mean, we follow the conventional stationarity test, in which the null is that LDPR is a non-stationary process. The stationarity test is specified as $\log(D_t/P_t) = \alpha + \beta \log(D_{t-1}/P_{t-1}) + \varepsilon_t$ and the Campbell-Shiller trend assumption is removed. The results are reported in Table 3.

**Table 3: Stationarity Tests**

This table reports the empirical results of whether the annual series of log dividend-price ratio of the S&P 500 index is stationary over the whole sample period, the Campbell-Shiller period, and the post Campbell-Shiller period, respectively. The stationarity test is specified as $\log(D_t/P_t) = \alpha + \beta \log(D_{t-1}/P_{t-1}) + \varepsilon_t$. The whole sample period is from 1871 to 2015 and the Campbell-Shiller period is from 1871 to 1986.

<table>
<thead>
<tr>
<th>$\log(D_t/P_t)$</th>
<th>1871 – 2015 (Entire sample)</th>
<th>1871 – 1986 Campbell-Shiller</th>
<th>1987 – 2015 Post Campbell-Shiller</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>-0.16</td>
<td>-0.39</td>
<td>-0.34</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.09)</td>
<td>(0.15)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.89</td>
<td>0.71</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.07)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>Dicky-Fuller stat</td>
<td>-15.78</td>
<td>-32.95</td>
<td>-4.76</td>
</tr>
<tr>
<td>Dicky-Fuller critical</td>
<td>-16.30</td>
<td>-16.30</td>
<td>-14.60</td>
</tr>
<tr>
<td>$N$</td>
<td>139</td>
<td>111</td>
<td>26</td>
</tr>
<tr>
<td>Adj$-R^2$</td>
<td>77.14</td>
<td>49.55</td>
<td>72.37</td>
</tr>
<tr>
<td>Reject unit root</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

Consistent with the declined dividend payments on the S&P 500 index over time depicted in Figure 2, the LDPR time series is stationary over the Campbell-Shiller period but non-stationary over the whole sample period and the post-Campbell-Shiller period. The Dicky-Fuller statistic is -15.8 over the whole sample period, -33.0 over the Campbell-Shiller sample period, and -4.8 over the post Campbell-Shiller period, and the corresponding critical values at the 5% level are -16.3, -16.3 and -14.6, respectively. In short, we fail to reject the hypothesis that the LDPR is a non-stationary series (without a long term mean). If the LDPR series does not have a long-term mean, the sample mean cannot be an estimate of the long-term mean (which does not exist). Practically speaking the failure to reject non-stationarity means that we don’t know whether the long-term mean exists — perhaps it exists but the test does not have sufficient power to discern it or is misspecified, — but even if it
does exist, lack of power in the test suggests we do not have a long enough data series to get a good estimate. It will certainly be a problem over time if the long-term mean does not exist and the LDPR gets more and more dispersion that will make the Taylor approximation worse and worse. However, a Taylor expansion around the sample mean may still be useful with the sample we have, and that is what we test next. In fact, we will find that our main results are not sensitive to the choice of \( \delta \) in a reasonable range. This also implies that our results are not being caused by a look-ahead bias stemming from constructing the covariates using the sample mean of the LDPR for the whole period.

### 4.2 Approximation Error in a Single Period

Previous analyses show that stock return predictability is buried by noise but the LDPR approximation around its sample mean is helpful and appropriate. The seemly controversial results suggest that the noise contained in stock return series is endogenous as illustrated in Section (3) rather than introduced by the LDPR approximation. It is interesting to understand whether and how much noise is introduced in the LDPR series generating process in equation (5). For this purpose, we examine the noise in one-period approximation and then the cumulative noise in equation (5).

Note that equation (3) is a Taylor expansion of log return around the LDPR long-term mean. The expansion around the sample mean in our empirical analyses may introduce a significant amount of noise when the long-term mean does not exist. We examine whether exogenous noise is added to our empirical approximation in equation (3) and define the noise term as:

\[
\xi_t = \kappa + \log\left(\frac{D_t}{P_t}\right) - \rho \log\left(\frac{D_{t+1}}{P_{t+1}}\right) + \log\left(\frac{D_t}{D_{t-1}}\right) - \log(1 + R_t),
\]

where \( \rho \) and \( \kappa \) are derived using the LDPR sample mean.

The time series of the single-period noise (\( \xi_t \)) is plotted in Figure 3.A and seems volatile. Although the mean of this noise term is close to zero (-0.37\%) over the sample period from 1871 throughout 2015, its standard error is as high as 19.25\%. An ARMA test suggests that the \( \xi_t \) series is an AR(1) process. Although the magnitude of this noise is small relative to LDPR, which has a mean of -317.84\% and a standard deviation of 40.44\% over the same period, it is large relative to the log gross return, which has a mean of 8.61\% and a standard deviation of 17.33\%. Moreover, this noise
primarily contaminate the stock return process because the dividend growth process is more persistent. These results provide an explanation for findings in existing studies that short-term returns cannot be predicted by dividend-price ratio. In short, the approximation in equation (3) with the LDPR sample mean introduces a significant amount of noise into the process of returns and stock prices.

Figure 3: Time Series of Approximation Error
4.3 Approximation Error in Multiple Periods

We further examine the cumulative approximation error in equation (5), which is defined as the following:

\[
\zeta_t = -\frac{\kappa}{1-\rho} (1 - \rho^{T-t}) + \sum_{s=t+1}^{T} \rho^{s-t-1} \left( \log((1 + R_s) - \Delta \log(D_s)) + \rho^{T-s} \log \left( \frac{D_T}{P_T} \right) - log \left( \frac{D_t}{P_t} \right) \right),
\]

where \( \rho \) and \( \kappa \) are derived using the LDPR sample mean.

To be consistent with previous analyses, we take \((T-t)\) to be 30 and plot the evolution of \( \zeta_t \) in Figure 3.B, which shows that the evolution of this noise term is smoother than the single-period noise and increases overtime. It has a sample mean of -1.5\%, which is almost four times as large as that of the single-period noise, and a sample standard deviation of 3.5\%, also higher than that of the single-period noise. An ARMA test shows that \( \zeta_t \) is an ARMA(1,1) process. The means of \( \log \left( \frac{D_T}{P_T} \right), \sum_{s=t+1}^{T} \rho^{s-t-1} \left( \log((1 + R_s) - \Delta \log(D_s)) \right) \) are -3.02, 1.33 and 0.53, respectively, and their standard errors are 0.24, 0.38 and 0.34, respectively. Compared with the single-period noise, these numbers suggest that the noise generated by the approximation can be reduced by cumulating short-term returns. However, combining Figure 3 and Table 2 shows that our cumulative approach cannot reduce the endogenous noise in stock return generating process, suggesting that the lack of predictability of stock returns is caused by endogenous noise rather than by exogenously introduced noise.

5 Robustness Analysis

5.1 Alternative Expanding Point

In the log linear approximation in equation 3, we approximate \( \log(1 + \exp(\delta_{t+1})) \) around some value \( \delta \) using a first-order Taylor expansion. Using a 3\textsuperscript{rd} order Taylor expansion, we have that \( \log(1 + \exp(\delta_{t+1})) \approx \log(1 + \exp(\delta)) + (1 - \rho)(\delta_{t+1} - \delta) + (1/2)\rho(1 - \rho)(\delta_{t+1} - \delta)^2 + (1/6)\rho(1 - \rho)(1 - 2\rho)(\delta_{t+1} - \delta)^3 \), where the terms beyond the first two represent approximation error. Intuitively, the approximation error is least severe if \( \delta \) is in the middle of the range of \( \delta_{t+1} \)'s, but how sensitive is
the error to our choice? Figure 2 shows that the sample mean of dividend-price ratio is smaller than
but close to 5% over the whole sample period and decreases to lower than 2% over years in the 21st
century. We first test whether the LDPR approximation in equation (5) is effective when it is expanded
around alternative points. We consider four expanding points to take into account the declining trend
in LDPR: 2%, 3%, 7% and 8% and two cases of approximation: with and without the final-period log
dividend-price ratio. The empirical test is specified in the same way as equation (7) and results are
reported in Table 4.

When the LDPR is expanded around 2%, which is close to the payout ratio over years in the 21st
century, Panel A in Table 4 shows that the precision of the LDPR approximation is slightly impacted
but still reasonably good. The coefficients are close to one (minus one) and the adjusted $R^2$ is around
94%. However, dropping the final term reduces the adjusted $R^2$ by 39% to a level of 55.6%, suggesting
that the final term is important when dividend payments are small, relative to stock price. When the
LDPR is expanded around 3% (Panel B), the coefficients on the sums of discounted future returns and
dividend growth rates are very close to that in Table 1 and the adjusted $R^2$ is slightly reduced by 0.8%.
The adjusted $R^2$ is reduced by about 15% when the final-period log dividend-price ratio in equation
(7) is dropped from the LDPR approximation test. Panels C and D shows that the adjusted $R^2$ and the
coefficients on cumulative discounted log returns, cumulative discounted log dividend growth rate,
and the final term are similar to those in the base case in Table 2 when the LDPR is expanded around
a relatively large point. These findings suggest that the approximation in equation (5) is effective
when firm’s payouts are around a reasonable level, such as 5%, and that the log dividend-price ratio
in the final period should not be dropped. In short, Table 4 suggests that expanding LDPR around its
sample mean is meaningful and effective with current sample even though its long-term mean does
not exist.

We further test whether the findings on the predictability tests of future returns and dividend
growth rates in Section (3) hold for different discounting rates (corresponding to different expanding
points). The empirical results are reported in Table 5 and show that the predictability of future
dividend growth rates and the unpredictability of future stock returns are not impacted by expanding
points. When LDPR is expanded around the point of 0.02 (Panel A), which is close to the level
in 2100s, the coefficient on current LDPR becomes negative and insignificant in the stock return
predictability test suggesting that stock returns are not predictable at all when LDPR becomes small.
Table 4: Approximation Test: Alternative Expanding Point

This table reports the empirical results of the Taylor expansion of \( \log \left( \frac{D_t}{P_t} \right) \) around alternative points. The regression is specified as: \( \log \left( \frac{D_t}{P_t} \right) = \alpha + \beta_1 (\sum_{s=t+1}^{T} \rho^{s-t-1} \log(1 + R_s)) + \beta_2 (\sum_{s=t+1}^{T} \rho^{s-t-1} \Delta \log(D_s)) + \beta_3 (\rho^{T-t} \log(D_T / P_T)) + \epsilon_t \). The results are based on the annual data of the S&P 500 index from 1871 to 2015. \((T - t)\) is set to be 30 years. The associated Newey-West standard errors with four lags are in parentheses. *** denotes statistical significance at the 1% level.

<table>
<thead>
<tr>
<th>Panel</th>
<th>Expanding point: 0.02 (( \rho \approx 0.98 ))</th>
<th>Expanding point: 0.03 (( \rho \approx 0.97 ))</th>
<th>Expanding point: 0.07 (( \rho \approx 0.94 ))</th>
<th>Expanding point: 0.08 (( \rho \approx 0.93 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>( \alpha )</td>
<td>( \beta_1 )</td>
<td>( \beta_2 )</td>
<td>( \beta_3 )</td>
</tr>
<tr>
<td>Panel A</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 1</td>
<td>(-2.71***)</td>
<td>(0.92***)</td>
<td>(-0.87***)</td>
<td>(0.82***)</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.04)</td>
<td>(0.04)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>Model 2</td>
<td>(-3.66***)</td>
<td>(0.62***)</td>
<td>(-0.80***)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.08)</td>
<td>(0.08)</td>
<td></td>
</tr>
<tr>
<td>Panel B</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 1</td>
<td>(-2.82***)</td>
<td>(0.95***)</td>
<td>(-0.93***)</td>
<td>(0.92***)</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>Model 2</td>
<td>(-3.66***)</td>
<td>(0.74***)</td>
<td>(-0.93***)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.07)</td>
<td>(0.08)</td>
<td></td>
</tr>
<tr>
<td>Panel C</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 1</td>
<td>(-3.00***)</td>
<td>(0.95***)</td>
<td>(-1.02***)</td>
<td>(1.53***)</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.04)</td>
<td>(0.04)</td>
<td>(0.14)</td>
</tr>
<tr>
<td>Model 2</td>
<td>(-3.58***)</td>
<td>(0.93***)</td>
<td>(-1.13***)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.09)</td>
<td>(0.08)</td>
<td></td>
</tr>
<tr>
<td>Panel D</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 1</td>
<td>(-3.02***)</td>
<td>(0.95***)</td>
<td>(-1.03***)</td>
<td>(1.80***)</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.05)</td>
<td>(0.05)</td>
<td>(0.22)</td>
</tr>
<tr>
<td>Model 2</td>
<td>(-3.54***)</td>
<td>(0.94***)</td>
<td>(-1.14***)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.09)</td>
<td>(0.08)</td>
<td></td>
</tr>
</tbody>
</table>
The magnitude and significance of the coefficient on LDPR in the predictability test of dividend growth rates are slightly improved, suggesting that dividend growth rate is still predictable even when the LDPR is small. The results are almost unchanged when the expanding point is 0.03 (Panel B). When the value of expanding point increases to 0.07 (Panel C), or 0.08 (Panel D), the coefficient on current LDPR becomes positive but small and statistically insignificant in the predictability test of future returns, and it is large and significant in the predictability test of future dividend growth rates.

5.2 Subsample Period Analysis

Our second robustness test considers whether our findings over the whole sample period (from 1871 to 2015) hold over shorter subsample periods. After computing the 30-year averages, we split the whole sample period into two equal-long subsample periods and repeat our predictability tests for each period. Table 6 presents the predictability results for the two subsample periods, corresponding to the results in Table 2 for the whole sample. In the first subsample from 1871 to 1928 (Panel A), we see that the cumulative discounted dividend growth is significantly predictable by current LDPR but the cumulative discounted returns are not, consistent with the results over the whole sample period. In this subperiod, the coefficient of the LDPR for the predictability of LDPR in 30 years is negative and insignificant, in contrast to the significant positive coefficient in the whole sample. The results on the second subsample from 1929 to 1985 (Panel B) are much different from the results in the whole sample. In this subsample, the sum of discounted dividend growth is not predictable by the LDPR while the cumulative discounted log returns are significantly predictable. We repeat the Newey-West correction substantially from 5 to 30 lags and the standard errors are almost unchanged. This is consistent with a similar test of Cochrane (2008, Section 7.2) on a similar sample period. Unfortunately, the half-periods have even fewer nonoverlapping observations (about 2 1/2 instead of about 5) than the whole sample period regressions presented in Section 3.

It seems unrealistic to think that economic relationships will remain stable over 30 years (Chen, 2009) left alone so on 100+ years, which is a weakness of the whole literature. Certainly dividend policy has changed over time, we saw that on the test of stationarity of the LDPR in Section 4.1 and “disappearing dividends” have also been documented by DeAngelo, DeAngelo and Skinner (2004), and Brav, Gramham, Harvey and Michaely (2005). This could be why the final term is significant
Table 5: Predictability Test: Alternative Expanding Point

This table reports the empirical results of whether current log dividend-price ratio is able to predict sums of discounted future returns or discounted dividend growth rates, or the discounted final-period log dividend-price ratio. The results are based on the annual data of the S&P 500 index from 1871 to 2015. The spurious regression bias (SRB) is estimated following Stambaugh (1999). The associated Newey-West standard error with four lags are in parentheses. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

<table>
<thead>
<tr>
<th>Predicted variable</th>
<th>( \alpha )</th>
<th>( \beta_{1} )</th>
<th>SRB-adjusted ( \beta_{1} )</th>
<th>Adj-( R^{2} ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Expanding point: 0.02 (( \rho \approx 0.98 ))</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sum_{t=1}^{T} \rho^{t-t-1}(\log(1 + R_{s})) )</td>
<td>1.78*</td>
<td>-0.08</td>
<td>-0.07</td>
<td>-0.75</td>
</tr>
<tr>
<td>( \sum_{t=1}^{T} \rho^{t-t-1}\Delta \log(D_{s}) )</td>
<td>-1.48*</td>
<td>-0.76***</td>
<td>-0.70***</td>
<td>14.83</td>
</tr>
<tr>
<td>( \rho^{T-t} \log\left(\frac{D_{T}}{P_{T}}\right) )</td>
<td>-0.48</td>
<td>0.43**</td>
<td>0.41**</td>
<td>17.31</td>
</tr>
<tr>
<td><strong>Panel B: Expanding point: 0.03 (( \rho \approx 0.97 ))</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sum_{t=1}^{T} \rho^{t-t-1}(\log(1 + R_{s})) )</td>
<td>1.87**</td>
<td>-0.08</td>
<td>-0.08</td>
<td>-0.87</td>
</tr>
<tr>
<td>( \sum_{t=1}^{T} \rho^{t-t-1}\Delta \log(D_{s}) )</td>
<td>-1.42*</td>
<td>-0.71***</td>
<td>-0.65***</td>
<td>15.14</td>
</tr>
<tr>
<td>( \rho^{T-t} \log\left(\frac{D_{T}}{P_{T}}\right) )</td>
<td>-0.36</td>
<td>0.32***</td>
<td>0.30***</td>
<td>17.31</td>
</tr>
<tr>
<td><strong>Panel C: Expanding point: 0.07 (( \rho \approx 0.94 ))</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sum_{t=1}^{T} \rho^{t-t-1}(\log(1 + R_{s})) )</td>
<td>2.01*</td>
<td>0.28</td>
<td>0.26</td>
<td>2.36</td>
</tr>
<tr>
<td>( \sum_{t=1}^{T} \rho^{t-t-1}\Delta \log(D_{s}) )</td>
<td>-1.14*</td>
<td>-0.53***</td>
<td>-0.51***</td>
<td>13.80</td>
</tr>
<tr>
<td>( \rho^{T-t} \log\left(\frac{D_{T}}{P_{T}}\right) )</td>
<td>-0.14***</td>
<td>0.10***</td>
<td>0.10***</td>
<td>17.31</td>
</tr>
<tr>
<td><strong>Panel D: Expanding point: 0.08 (( \rho \approx 0.93 ))</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sum_{t=1}^{T} \rho^{t-t-1}(\log(1 + R_{s})) )</td>
<td>2.01***</td>
<td>0.31</td>
<td>0.28</td>
<td>3.48</td>
</tr>
<tr>
<td>( \sum_{t=1}^{T} \rho^{t-t-1}\Delta \log(D_{s}) )</td>
<td>-1.08**</td>
<td>-0.49***</td>
<td>-0.46***</td>
<td>13.14</td>
</tr>
<tr>
<td>( \rho^{T-t} \log\left(\frac{D_{T}}{P_{T}}\right) )</td>
<td>-0.09</td>
<td>0.08***</td>
<td>0.08***</td>
<td>17.31</td>
</tr>
</tbody>
</table>
in the second period but not the first. As Modigliani and Miller (1958) emphasize, in a frictionless world, the mix between dividends and share repurchases would be irrelevant, and probably we should not expect dividend policy to be stable in the actual economy. However, it seems unlikely that markets were very efficient during the unstable times in the first half of the sample but inefficient later. The reversal of the results on the subperiods makes us wonder about the size of all the tests (both on the whole sample and on the subperiods). Of our whole sample of 145 years and discounted weighted sums over 30 years, we only have about five non-overlapping observations in the whole sample and even fewer over subperiods. The lack of robustness to subperiods may also be due to model instability over time.

Table 6: Predictability Test over Subsample Period

We equally split the whole sample period into two non-overlapping periods. This table reports the empirical results of whether current log dividend-price ratio is able to predict sums of discounted future returns or discounted dividend growth rates over future 30 years, or the discounted final-period log dividend-price ratio over each subsample period. The results are based on the annual data of the S&P 500 index from 1871 to 2015. The spurious regression bias (SRB) is estimated following Stambaugh (1999). The associated Newey-West standard errors with four lags are in parentheses. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

\[
\begin{array}{cccccc}
\text{Predicted variable} & \alpha & \beta_1 & \text{SRB-adjusted } \beta_1 & \text{Adj-}R^2 (\%) \\
\hline
\text{Panel A: First Subsample Period (1871-1928)} & & & & \\
\sum_{s=t+1}^{T} \rho^{s-t-1} \left( \log(1+R_s) \right) & 1.64^{*} & 0.18 & 0.17 & 0.66 \\
& (0.86) & (0.29) & (0.29) & \\
\sum_{s=t+1}^{T} \rho^{s-t-1} \Delta \log(D_s) & -2.21^{**} & -0.85^{***} & -0.81^{***} & 33.01 \\
& (0.91) & (0.31) & (0.31) & \\
\rho^{T-t} \log \left( \frac{D_T}{P_T} \right) & -0.74^{***} & -0.03 & -0.03 & 0.33 \\
& (0.16) & (0.05) & (0.05) & \\
\hline
\text{Panel B: The Second Subsample Period (1929-1985)} & & & & \\
\sum_{s=t+1}^{T} \rho^{s-t-1} \left( \log(1+R_s) \right) & 4.00^{***} & 0.75^{***} & 0.69^{***} & 34.15 \\
& (0.39) & (0.13) & (0.13) & \\
\sum_{s=t+1}^{T} \rho^{s-t-1} \Delta \log(D_s) & 0.59^{**} & -0.08 & -0.07 & -1.01 \\
& (0.23) & (0.07) & (0.07) & \\
\rho^{T-t} \log \left( \frac{D_T}{P_T} \right) & -0.25 & 0.17^{**} & 0.16^{**} & 25.09 \\
& (0.20) & (0.07) & (0.07) & \\
\end{array}
\]
6 Conclusion

Whether stock returns are predictable is an important and challenging question for both academia and industry. Campbell and Shiller (1988) argue, based on accounting definitions and some approximations, that the log dividend-price ratio must predict future returns, future log dividend growth, or both. However, in past literature neither prediction has been found to be economically or statistically significant, creating a well-known puzzle. We check each step of Campbell and Shiller’s argument, from the accounting definition through the approximation to the statistical tests. Our findings show that the source of the failure to find a significant relationship arises from a mismatch between the small lags in the traditional tests and the many terms in the theoretical expression. When we conduct a test closer to the theoretical expression, with appropriate correction for serial correlation due to overlapping data, the possible heteroscedasticity, and spurious regression bias, we find that future log dividend growth is significantly predictable but future returns are not, thus resolving the puzzle.

While this is the best conclusion given the data currently available, this result does not seem to be robust for several reasons. For one, there are only a few (about five) non-overlapping observations of the truncated identity for the whole period, so we are asking a lot of the Newey-West adjustment. Also, the results are different in the two half periods, which calls into question any reliance on asymptotic properties of the statistical estimates. Perhaps we should not expect stability of the dividend process over time, since according to Modigliani and Miller dividends are irrelevant. Even if Modigliani-Miller’s arguments should not be taken too literally, they do mean that seemingly small changes in taxes or transaction cost can have a big impact on dividend policy and affect the time series properties of log returns, log dividend growth, and log dividend-price rations. Possible nonstationarity, which we cannot reject for the whole sample or for the second half of the sample, is a serious problem for the theory because the Taylor series approximation worsens as the range of the LDPR increases. For these reasons, it seems that the limitations of this approach may be intrinsic, and the accounting identity may never tell us much about return predictability, even as we collect more and more data.
References


