Abstract

Arbitrage with limited capital can have an unintended consequence of determining the cross-section of risks of the assets. This happens when arbitrage turns “alphas” into “betas”: an asset that is initially more mispriced attracts more arbitrage capital and attains a correspondingly large sensitivity to arbitrage capital shocks. I develop this prediction in a simple model of capital-constrained arbitrageurs and test it in the cross-section of equity “anomalies” to find evidence that their funding-liquidity exposures have arisen endogenously through this alphas-into-betas mechanism. This finding reverses the direction of “causality” in intermediary asset pricing: a high expected return due to an alpha can “cause” a high funding-liquidity beta, not just the other way around.
1 Introduction

In a financial market with frictionless arbitrage, the role of arbitrageurs is limited to eliminating mispricings. When arbitrage involves frictions, however, arbitrageurs play a more significant role of determining the cross-section of risks of the assets. They do this by turning “alphas” into “betas”: an asset with a large pre-arbitrage abnormal risk-adjusted return attracts more arbitrage capital and attains a correspondingly large sensitivity to arbitrage capital shocks.\(^1\)

To fix ideas, consider two assets \textit{LowAlpha} (“L”) and \textit{HighAlpha} (“H”) that are claims to risk-free payoffs in the future with a present value of $10. Suppose also that absent arbitrage capital, their prices are driven down by behavioral investors to \(P_L = $8\) and \(P_H = $5\), implying that \(H\) has a higher “pre-arbitrage” alpha: \(\alpha_{pre}^L = 25\% < \alpha_{pre}^H = 100\%\). Now, if arbitrageurs enter with unlimited capital, they immediately drive up the prices of both \(L\) and \(H\) to $10, after which no further price movement occurs.

If, however, arbitrageurs enter with capital that is subject to large shocks, the act of arbitrage itself makes \(H\) endogenously riskier than \(L\). This happens because during arbitrage, a shock that lowers the arbitrage capital to zero lowers \(P_L\) by 20% ($10 to $8) but \(P_H\) by 50% ($10 to $5), assuming that the force that caused the initial underpricing stays. Hence, \(H\) attains a larger “post-arbitrage” beta with respect to arbitrage capital shocks and is endogenously riskier for arbitrageurs who want to hedge their capital shocks. In equilibrium, arbitrageurs would not actually drive up \(P_L\) and \(P_H\) all the way to $10 but only to some \(P_H < P_L < $10\) so that their remaining expected returns compensate the arbitrageurs for the endogenous risks.

My contribution is to flesh out this cross-sectional “alphas-into-betas” prediction in a simple model of capital-constrained arbitrageurs and test it in the cross-section of 40 equity “anomalies.”

The model features arbitrageurs who trade a continuum of different “anomalies” but face a stochastic funding constraint that generates exogenous variation in the capital that they can deploy. Importantly, these anomalies differ in their abnormal returns prior to arbitrage or “pre-

\(^1\)Andrew Lo first used the phrase “alpha is becoming beta” to mean a situation where, as in Shleifer and Vishny (1997), quantitative trading strategies designed to have no systematic exposures become commonly exposed to the risk of “unwinding” by the quantitative hedge funds (e.g., Khandani and Lo, 2011). Although related, my expression “alphas into betas” instead refers to my cross-sectional prediction.
arbitrage alphas.”

In this model, an anomaly’s pre-arbitrage $\alpha$ determines its post-arbitrage $\beta$ with respect to arbitrage capital shocks (Proposition 1). A large pre-arbitrage alpha means that in normal times, more arbitrage capital as a fraction of the market capitalization (“arbitrage interest”) is being deployed in that anomaly to sustain its price at the correct level. This, however, also means that the price of this anomaly drops more when the aggregate arbitrage capital faces an extreme negative shock and drops to zero, implying a large $\beta$ with respect to arbitrage capital. What follows from this explanation is that the cross-section of contemporaneous arbitrage interests in the anomalies should also explain the cross-section of post-arbitrage $\beta$s (Proposition 2).

Furthermore, this endogenous post-arbitrage $\beta$ only arises when arbitrageurs are constrained by limited capital—i.e., if arbitrageurs persistently have sufficient capital, pre-arbitrage $\alpha$s do not turn into post-arbitrage $\beta$s (Proposition 3). Finally, an anomaly’s post-arbitrage $\beta$ explains its expected return that survives in equilibrium with arbitrage—i.e., an “intermediary asset pricing” model based on this post-arbitrage $\beta$ can “price” the cross-section of anomalies (Proposition 4).

I test the model’s predictions using data on 40 equity anomaly portfolios (“anomalies”) representing the “long” and “short” portfolios (top and bottom deciles) of 20 anomaly characteristics. Splitting my sample period 1975-2016 in half, I measure an anomaly’s pre-arbitrage $\alpha$ and post-arbitrage $\beta$, respectively, with its pre-1995 CAPM alpha and post-1995 funding-liquidity beta (“funding beta”) net of market exposure, but using another year in the 1990s as the cutoff year does not affect my results. I use two different measures of arbitrage capital shocks. The primary measure I use is the aggregate funding-liquidity shocks measured by broker-dealer leverage shocks (Adrian, Etula, and Muir, 2014) since arbitrageurs of equity anomalies such as hedge funds rely heavily on funding liquidity in their trades (Brunnermeier and Pedersen, 2009; Aragon and Strahan, 2012; Mitchell and Pulvino, 2012). I repeat my key regressions using the arbitrageur portfolio shocks proxied by the equal-weighted (signed) average return of the anomalies as another measure of arbitrage capital shocks. As some anomalies are similar to each other, I use standard errors that account for cross-anomaly covariances.

My empirical tests support the model’s predictions. In the pre-1995 period with little arbitrage, anomalies generated a cross-section of different CAPM alphas, but their funding betas
Figure 1: Alphas Turning into Betas

Notes: Figure 1a plots the relationship between an anomaly’s CAPM alpha and funding-liquidity beta (net of market exposure) in the same pre-1995 period. The figure illustrates that, in this period with little arbitrage, the anomalies featured negative (for the short side) or positive (for the long side) CAPM alphas but no systematic exposures to funding-liquidity shocks. Figure 1b plots the relationship between an anomaly’s pre-1995 CAPM alpha and post-1995 funding-liquidity beta. The strong positive relationship between the two illustrates how pre-arbitrage alphas turn into post-arbitrage betas with respect to systematic shocks to arbitrage capital.

Clustered around zero for both the short-side (solid circles) and long-side (hollow circles) anomalies (Figure 1a). In contrast, in the post-1995 period with more arbitrage on these anomalies, funding betas became strongly negative for short-side anomalies and moderately positive for long-side anomalies. Importantly, these post-1995 funding betas are strongly predicted by the cross-section of pre-1995 CAPM alphas (Figure 1b), consistent with pre-arbitrage $\alpha$ becoming post-arbitrage $\beta$ through the act of arbitrage (Proposition 1). These results are robust to using a different cutoff year in the 1990s to define the pre- and post-arbitrage periods.

The alphas-into-betas relation in Figure 1b can also arise if pre-1995 CAPM alpha represent an exposure to a risk factor that—for whatever reason—has become correlated with funding-liquidity shocks in the post-1995 period. Empirical facts, however, are inconsistent with this alternative explanation. First, my alphas-into-betas result is robust to controlling for other common factors such as the Fama-French 5 factors (Fama and French, 2016) or market liquidity (Pástor and Stambaugh, 2003), suggesting the latent risk factor is not one of these. Second, the alphas-into-betas relation has a larger slope within short-side anomalies, inconsistent with single missing risk factor driving my result but consistent with funding liquidity playing a more prominent role in arbitrage through shorting. Finally, the anomalies’ post-1995 funding betas are not
fundamental cash-flow exposures, implying that post-1995 funding betas are endogenous risks that occur through the discount-rate channel.

Three additional tests support the arbitrage-based explanation for the post-1995 funding betas. First, the anomalies with large funding-liquidity betas in the post-1995 period are also the ones with more arbitrage capital devoted to the anomaly in the same period as measured by the short interest ratio (Proposition 2). Second, the anomalies’ funding-liquidity exposures in the post-1995 period come from high-VIX quarters in which arbitrage capital is more likely to be constrained (Proposition 3). Third, these funding-liquidity betas carry a risk premium in the post-1995 period, preventing a high cost of equity due to a positive pre-arbitrage $\alpha$ from disappearing completely (Proposition 4). My results are robust to using the arbitrageur returns shocks as the arbitrage capital shocks.

Together, my results suggest that the act of arbitrage plays an important role not just in ensuring the elimination of mispricings, but also in determining the cross-section of risks.

This finding is consistent with the “habitat” view of asset return comovement (e.g., Barberis and Shleifer, 2003; Barberis, Shleifer, and Wurgler, 2005; Coval and Stafford, 2007).\(^2\) When a group of assets is traded primarily by a particular set of investors, these assets attain an endogenous comovement coming from the variation in their discount rate due to time-varying risk aversion or capital, if they are so constrained. Since assets with predicted alphas are primarily traded by institutional arbitrageurs, these assets comove endogenously through their discount rate variation (Kozak, Nagel, and Santosh, 2017). My paper tests a cross-sectional prediction of this view that such a comovement should be larger for an asset that attracts more capital from the habitat investors.\(^3\)

My alphas-into-betas prediction is a natural cross-sectional extension of the limits-of-arbitrage hypothesis on an individual asset (Shleifer and Vishny, 1997). Arbitrage with limited capital on single asset is risky since it causes the return on the arbitrated asset and the level of arbitrage


\(^3\)Using a similar logic, Haddad and Muir (2017) exploit cross-section differences in the return correlation with intermediary capital to identify the presence of intermediary asset pricing. Also related is the observation in Brunnermeier, Nagel, and Pedersen (2009) that more profitable carry trades are subject to higher currency crashes, consistent with arbitrageurs generating crash risks (Stein, 2009).
capital to endogenously comove. If this happens simultaneously for multiple assets exposed to
different degrees of arbitrage, a cross-section of different risks arises. This key insight allows
for cross-sectional tests of the limits-of-arbitrage mechanism that have more power than pure
time-series tests.4

My alphas-into-betas result reverses the direction of “causality” in intermediary asset pricing.
The theory behind this growing literature is that, in the presence of financial frictions, assets
that have high betas with respect to systematic shocks that financial intermediaries face should
command high expected returns (Brunnermeier and Pedersen, 2009; Gertler and Kiyotaki, 2010;
He and Krishnamurthy, 2012, 2013; Kondor and Vayanos, 2017).5 Nevertheless, as I show both
theoretically and empirically, high expected returns due to an alpha can cause high betas with re-
spect to those shocks. Hence, expected returns lining up with the betas does not necessarily imply
that these expected returns are compensating for risks implied by the betas. I show that this can
cause an intermediary asset pricing test to generate a biased price of risk estimate (Proposition 5)
and relate this finding to the empirical work in the literature (Adrian, Etula, and Muir, 2014; He,

Finally, my empirical finding suggests that the equity anomalies are—from arbitrageurs’ point
of view—alphas turned into betas. This is consistent with the arbitrage-driven decay in anomaly
return (Chordia, Subrahmanyam, and Tong, 2014; McLean and Pontiff, 2016) and the evidence
that the anomalies offer a “shorting premium” (Drechsler and Drechsler, 2016). My cross-
sectional result on funding betas is also consistent with anomalies on average becoming more
correlated with hedge fund wealth (Liu, Lu, Sun, and Yan, 2015) and with one another (McLean
and Pontiff, 2016). This view of anomalies, however, does not necessarily imply that the anoma-
lies represent true mispricings. Since arbitrageurs rely on realized alphas implied by their models,
these alphas may still represent a premium for risk to non-arbitrageur investors or a measurement
error.

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4The pure time-series approach focuses on detecting a comovement between the arbitraged asset’s return and
arbitrage capital but does not make use of the cross-sectional differences in this comovement (e.g., Gärleanu
and Pedersen, 2011; Frazzini and Pedersen, 2014; Akbas, Armstrong, Sorescu, and Subrahmanyam, 2015; Chu, Hirsh-
leifer, and Ma, 2017). Relatedly, Baker and Wurgler (2006) is a cross-sectional test of limits of arbitrage that exploits
the cross-sectional differences in the ease of arbitrage. Gromb and Vayanos (2017) offer testable implications for
assets that differ in their volatilities.

5Also, Bernanke and Gertler (1989), Holmstrom and Tirole (1997), and Brunnermeier and Sannikov (2014).
2 Theory

Consider a three-period \((t = 0, 1, 2)\) economy with two types of assets: a risk-free asset and a continuum of risky assets \(i \in [0, 1]\) which I call “anomalies.” The risk-free asset is in infinite supply with zero interest rate. An anomaly, in zero net supply, is a claim to a time-2 cash flow that is uncorrelated with the other anomalies’ cash flows and has an expected value \(v > 0\). Hence, as in Shleifer and Vishny (1997), the anomalies are not exposed to any systematic risks and have a fundamental value of \(v\) in all periods, an assumption that helps elucidate the emergence of endogenous risks. There are two types of investors: “behavioral investors” cause anomalies to be mispriced, and “arbitrageurs” trade against these mispricings subject to exogenous funding shocks.

Behavioral investors cause anomalies to have positive abnormal returns.\(^6\) Their time-\(t\) aggregate demand (in units of wealth) for anomaly \(i\) is

\[
B_{i,t} = \frac{E_t[r_{i,t+1}^e]}{r_{\text{max}}^i} - i,
\]

where \(E_t[r_{i,t+1}^e]\) is the objective conditional expected excess return (same as return in this model) and \(r_{\text{max}}^i > 0\) is a constant representing the maximum abnormal return generated by behavioral investors. This demand curve is a simple way to generate abnormal returns without specifying the underlying reason, which could be behavioral or rational. Implicit in this demand function is that all anomalies have the same size since arbitrage position \(x_{i,t}\) that has to net out \(B_{i,t}\) has the same marginal effect on expected return for all anomalies: \(\partial E_t[r_{i,t+1}^e]/\partial x_{i,t} = \partial (r_{\text{max}}^i (i - x_{i,t}))/\partial x_{i,t} = -r_{\text{max}}^i\).

A continuum of identical, risk-neutral arbitrageurs with mass \(\mu\) trade against mispricings generated by behavioral investors. They trade at times 0 and 1 using their (deployable) capital \(k_t\), which is a sum of their own wealth \(w_t\) and short-term funding \(f_t\):

\[
k_t = w_t + f_t
\]

Their objective is to maximize the expected time-2 wealth, where the evolution of wealth depends

\(^6\)This direction of mispricing is chosen for convenience and does not affect the theory results to be presented.
on their positions on different anomalies \( \{ x_{i,t} \} \) and realized returns on anomalies \( \{ r_{i,t} \} \):

\[
w_t = w_{t-1} + \int_0^1 r_{i,t} x_{i,t-1} di
\]

where

\[
\int_0^1 |x_{i,t}| di \leq k_t
\]

so that arbitrageurs can take long or short positions but face a margin rate of 1 for each trade. Hence, they cannot lever up through a long-short trade and lever up only through short-term funding. By setting the margin constant rather than allowing it to be determined by the conditional volatility and the VaR constraint as in some models (e.g., Brunnermeier and Pedersen, 2009; Gromb and Vayanos, 2017), I emphasize that different endogenous betas can arise without differences in the anomalies’ idiosyncratic volatilities. I normalize the time-0 wealth of an individual arbitrageur to be \( w_0 = 1 \) so that the aggregate arbitrageur wealth at time 0 is \( \mu \).

The short-term funding \( f_t \) is uncollaterized and available at the zero risk-free rate but is capped at a stochastic funding constraint. Since the unused part of the available funding can always be invested at the zero risk-free rate, I assume that arbitrageurs always borrow to the limit, which allows me to treat \( f_t \) as the stochastic funding constraint. I solve the model using the aggregate amount of arbitrage capital (“arbitrage capital”),

\[
K_t = \mu k_t,
\]

as the state variable, as it has a one-to-one (nonlinear) relationship with \( f_t \). I do this to make it clear to the reader that endogenous arbitrage risk can be defined as the beta with respect to any systematic shocks to \( K_t \), not just the funding shocks. For example, I could allow the anomalies to experience noise trader shocks as in SV, in which case \( K_t \) would move around because of noise trader shocks generating shocks to arbitrageur portfolio return \( \int_0^1 r_{i,t} x_{i,t-1} di \) in eq. 3.

**Equilibrium and the notion of arbitrage regime**

I look for a symmetric competitive equilibrium under two different assumptions about \( \mu \): (1) the *pre-arbitrage* regime \( (\mu = 0) \) and (2) the *post-arbitrage* regime \( (\mu > 0) \). The three periods
$t \in \{0, 1, 2\}$ defined earlier are considered different time periods within a regime. The distinction between the pre- vs. post-arbitrage regimes models the increase in $K_t$ over a very long horizon (recent 40-50 years) that has made arbitrage capital now a substantial fraction of the total stock market capitalization. Short-term shocks to arbitrage capital that occur within a regime of 20-25 years are modeled by the variation in $K_t$ over $t \in \{0, 1, 2\}$ because of $f_t$.

2.1 Pre-arbitrage regime

How do anomalies behave when $\mu = 0$? In this case, behavioral investors need to clear the market within themselves so that $B_{i,t} = 0$. Hence, (1) implies that anomaly $i$ earns the excess return of $r_{max}i$ each period and is not exposed to any systematic risk. The expected excess return, therefore, is also the abnormal return in this pre-arbitrage regime (“pre-arbitrage alpha”):

$$\alpha_{i}^{pre} \equiv r_{max}i$$

This way, anomalies generate a cross-section of different pre-arbitrage alphas that increase in $i$.

2.2 Post-arbitrage regime

When arbitrageurs enter the economy ($\mu > 0$), the cross-section of alphas turn into a cross-section of endogenous betas with respect to arbitrage capital.

A. Time-1 equilibrium and endogenous risk

Time 1 is the intermediate period in which the anomaly prices endogenously comove with arbitrage capital $K_1$. As anomaly returns at time 2 are not exposed to any systematic risks, arbitrageurs at time 1 prefer anomalies with higher expected returns. Since the expected return prior to arbitrage—the pre-arbitrage alpha—increases with $i$, there exists a marginal anomaly $i^* \in [0, 1]$ such that arbitrageurs (i) take positive positions on anomalies $(i^*, 1]$ (“exploited anomalies”) to earn an identical expected return $r_{max}i^*$, (ii) do not take positions on anomalies $[0, i^*)$
unexploited anomalies which generate lower expected returns \( \alpha_{i_{\text{pre}}} = r_{\max i} \) for anomaly \( i \), and (iii) are indifferent about taking a position on anomaly \( i^* \) (marginal anomaly) which generates the expected return \( \alpha_{i^*_{\text{pre}}} = r_{\max i^*} \). This and the market clearing condition pin down the equilibrium time-1 prices:

**Lemma 1. (Equilibrium time-1 prices).** The equilibrium price of anomaly \( i \) at time 1 is

\[
p_{i,1} = \begin{cases} 
  v (1 + r_{\max i})^{-1} & \text{if } i \leq i^* \\
  v (1 + r_{\max i^*})^{-1} & \text{if } i \geq i^*
\end{cases}
\]

where \( i^* \) is the marginal anomaly given by

\[
i^* = \begin{cases} 
  1 & \text{if } K_1 < 0 \\
  1 - \sqrt{2K_1} & \text{if } K_1 \in [0, \frac{1}{2}] \\
  0 & \text{if } K_1 > \frac{1}{2}
\end{cases}
\]

**Proof.** In Appendix B.2.

Lemma 1 implies two things. First, the prices of anomalies endogenously covary with \( K_1 \) since more arbitrage capital increases the prices of the anomalies. Second, the price covariance with \( K_1 \) is larger for higher-\( i \) anomalies. This works through the extensive margin. The intensive margin is identical for all assets: when \( K_1 \) increases, arbitrageurs increase their positions in all exploited anomalies to ensure that their prices equal. Hence, the prices of all exploited anomalies move together. The extensive margin, in contrast, applies differently to different anomalies. The larger the \( i \), the lower the \( K_1 \) at which the anomaly gets exploited by arbitrageurs. This makes the higher-\( i \) anomaly comove with \( K_1 \) in a wider region of arbitrageur capital, leading to a larger price covariance with \( K_1 \). Figure 2 illustrates the intuition by plotting the equilibrium time-1 prices of two anomalies, \( i \) and \( i' > i \) (hence \( \alpha_{i'_{\text{pre}}} > \alpha_{i_{\text{pre}}} \)).

The larger price covariance with \( K_1 \) for higher-\( i \) anomalies causes such anomalies to be endogenously riskier. Since the arbitrageurs’ marginal value of wealth at time 1 moves in the opposite direction to \( K_1 \), a larger price covariance with \( K_1 \) implies a smaller price covariance with the marginal value of wealth \( \Lambda_1 \). To see this, note that the arbitrageurs’ marginal value of wealth at
time 1 equals \(1 + r_{\text{max}}i^*_1\), the gross expected return generated by the incremental time-1 wealth. Since \(i^*\) is negatively related with \(K_1\), \(\Lambda_1\) is also negatively related with \(K_1\). One complicating issue is that wealth can become negative, in which case the notion of marginal value of wealth is ill-defined.\(^7\) However, an intuitive restriction on \(\Lambda_1\) for this case resolves the issue:

**Assumption 1. (Arbitrageur’s marginal value of wealth at time 1).** An arbitrageur’s marginal value of wealth at time 1, denoted \(\Lambda_1\), is

\[
\Lambda_1 = \begin{cases} 
1 + c & \text{if } K_1 < 0 \\
1 + r_{\text{max}}i^* & \text{if } K_1 \geq 0 
\end{cases}
\]

where \(i^*\) is the marginal anomaly specified in (8) and \(c \geq r_{\text{max}}\) so that the marginal value of wealth is higher in the default region.

This restriction says that arbitrageurs incur a marginal bankruptcy cost of \(c\) for each additional dollar of default and that an additional unit of wealth more valuable in the default region than in any of the non-default region \((c \geq r_{\text{max}})\).\(^8\) Given this, \(\Lambda_1\) unambiguously falls as \(K_1\) rises, which is sufficient to guarantee that the endogenous risk defined as the price covariance with \(\Lambda_1\) is larger for higher-\(i\) anomalies.

**Lemma 2. (Endogenous risk).** An anomaly is risky as indicated by a negative price covariance with the arbitrageur’s marginal value of wealth:

\[
\text{Cov}_0 (p_{i,1}, \Lambda_1) \leq 0 \quad \forall i
\]

Furthermore,

(i) (Shleifer and Vishny, 1997) This risk is endogenous, arising only if arbitrageurs have a positive mass in the market so as to generate non-negligible price pressure:

\[
\text{Cov} (p_{i,1}, \Lambda_1) |_{\mu=0} = 0 \quad \forall i
\]

\(^7\)See Brunnermeier and Pedersen (2009) for a more detailed discussion of this modeling issue.

\(^8\)Without this assumption of \(c \geq r_{\text{max}}\), the marginal value of wealth is lower in the default region than in some parts of the non-default region. This could make an asset that pays low in the state of default and pays high in the state of non-default (e.g. the most mispriced asset \(i = 1\)) safer than an asset that pays the same return in all states (the least mispriced asset \(i = 0\)).
(ii) (Cross-section of endogenous risks) In the cross-section of anomalies $i \in [0, 1]$, the endogenous risk increases with an asset’s pre-arbitrage alpha:

$$- \frac{\partial \text{Cov} \left( p_{i,1}, \Lambda_1 \right)}{\partial \alpha_{i}^{\text{pre}}} \geq 0$$  \hspace{1cm} (12)

**Proof.** In Appendix B.2.

Part (i) of Lemma 2 restates the result of Shleifer and Vishny (1997) (SV). Although the source of the shock in $K_t$ has a different label (funding shock instead of sentiment shock), the mechanism is essentially identical. Part (ii) of Lemma 2 is the new result. It develops a cross-sectional prediction of the SV insight that arbitrage trading makes an anomaly endogenous risky. To bring this prediction to data, however, I need to restate it in terms of return covariance rather than price covariance. This requires the equilibrium price at time 0.

**B. Time-0 equilibrium and turning alphas into betas**

Time 0 is the ex-ante period in which arbitrageurs price the anomalies, taking into account their endogenous risks at time 1. Since each individual arbitrageur is small and risk-neutral, an arbitrageur’s value function at time 0 is simply its wealth multiplied by the marginal value of wealth: $\Lambda_0 w_0$. Furthermore, with no time-0 consumption or discount, this quantity equals the time-0 expectation of wealth multiplied by the marginal value of wealth at time 1:

$$\Lambda_0 w_0 = E_0 \left[ \Lambda_1 \left( \int_0^1 \frac{p_{i,1}}{p_{i,0}} x_{i,0} di + w_0 - \int_0^1 x_{i,0} di \right) \right]$$  \hspace{1cm} (13)

An arbitrageur then maximizes this value function subject to a capital constraint,

$$\int_0^1 |x_{i,0}| di \leq k_0$$  \hspace{1cm} (14)

From this I derive the equilibrium time-0 price in the unconstrained and constrained cases.

**Lemma 3. (Equilibrium time-0 prices).** The equilibrium price of anomaly $i$ at time 0 is as follows:
(i) If constraint (14) is slack so that arbitrageurs are unconstrained,

\[ p_{i,0} = E_0 \left[ \frac{\Lambda_1}{E_0[\Lambda_1]} p_{i,1} \right] \]  

(15)

(ii) If constraint (14) binds so that arbitrageurs are constrained,

\[ p_{i,0} = \begin{cases} 
E_0 \left[ \frac{\Lambda_1}{\Lambda_0} p_{i,1} \right] & \text{if } i \text{ is exploited} \\
E_0[p_{i,1}] (1 + r_{max,i})^{-1} & \text{if } i \text{ is unexploited} 
\end{cases} \]  

(16)

where

\[ \Lambda_0 = \max_{i \in [0,1]} E_0[\Lambda_1 (1 + r_{i,1})] \]  

(17)

Proof. In the following paragraph.

If arbitrageurs are unconstrained, the fundamental theorem of asset pricing holds so that \( \Lambda_0 = E_0[\Lambda_1] \) implies (15). If arbitrageurs are constrained, however, price is determined by the arbitrageur’s stochastic discount factor (SDF) \( \Lambda_1/\Lambda_0 \) only if the anomaly is exploited by arbitrageurs. Thus, unlike the conventional asset pricing models, arbitrageur-based pricing is expected to work only on a subset of anomaly portfolios that arbitrageurs hold, a point overlooked by the existing empirical work in intermediary asset pricing. I come back to this point after Proposition 4.

The equilibrium prices in Lemma 3 for both the unconstrained and constrained cases imply that the prices of anomalies at time 0 decrease with \( i \). If \( i' < i'' \), anomaly \( i'' \) is not only subject to a larger mispricing in the absence of arbitrageurs, but also exposed to a larger endogenous risk. Thus, anomaly \( i'' \) must be valued less than anomaly \( i' \) (summarized as Lemma 4 in the Appendix). This monotonicity of prices at time 0 makes the endogenous risk results in Lemma 2 hold analogously with returns.

**Proposition 1. (Turning alphas into betas).** In the cross-section of anomalies, an anomaly’s post-arbitrage beta \( \beta_{i}^{\text{post}} \) defined as the negative beta with the arbitrageurs’ stochastic discount

\[ \beta_{i}^{\text{post}} = \frac{E_0[p_{i,1}]}{E_0[r_{i,1}]} \]  

\( \text{(18)} \)
factor (SDF),
\[ \beta_{i}^{\text{post}} \equiv -\frac{\text{Cov}_0 \left( r_{i,1}^e, \frac{\Lambda_1}{\Lambda_0} \right)}{\text{Var}_0 \left( \frac{\Lambda_1}{\Lambda_0} \right)}, \]  
(18)

increases with its pre-arbitrage alpha \( \alpha_i^{\text{pre}} \). That is,
\[ \frac{\partial \beta_{i}^{\text{post}}}{\partial \alpha_i^{\text{pre}}} > 0 \]  
(19)

**Proof.** Suppose \( \alpha_i^{\text{pre}} < \alpha_i^{\text{pre}} \iff i' < i'' \). By Lemma 2, \(-\text{Cov}_0 (p_{i',1}, \Lambda_1) < -\text{Cov}_0 (p_{i''}, \Lambda_1)\). Furthermore, by Lemma 4 in the Appendix, \( p_{i,0}^e \geq p_{i,0}^{e'} \). Thus, \( \beta_{i}^{\text{post}} = -\text{Cov}_0 \left( r_{i',1}^e, \frac{\Lambda_1}{\Lambda_0} \right) /\text{Var}_0 \left( \frac{\Lambda_1}{\Lambda_0} \right) < -\text{Cov}_0 \left( r_{i''}^e, \frac{\Lambda_1}{\Lambda_0} \right) /\text{Var}_0 \left( \frac{\Lambda_1}{\Lambda_0} \right) = \beta_{i}^{\text{post}} \).

Proposition 1 states that an anomaly’s pre-arbitrage alpha \( \alpha_i^{\text{pre}} \) determines its post-arbitrage endogenous risk as measured by the post-arbitrage beta \( \beta_{i}^{\text{post}} \). Since this prediction is cross-sectional in nature, it can be tested in the cross-section of different anomalies.

If post-arbitrage beta \( \beta_{i}^{\text{post}} \) is an endogenous outcome of arbitrage activity, we should see a large \( \beta_{i}^{\text{post}} \) in an anomaly in which arbitrage position as a fraction of the anomaly’s size is large. Since all anomalies are normalized to have the same size here, the cross-section of expected time-1 arbitrage position, \( E_0 [x_{i,1}] \), should explain the cross-section of \( \beta_{i}^{\text{post}} \). Furthermore, \( E_0 [x_{i,1}] \) in turn should be explained by pre-arbitrage alphas \( \alpha_i^{\text{pre}} \) (Proposition 2).

**Proposition 2. (Post-arbitrage beta is explained by contemporaneous arbitrage interest).**

(i) Post-arbitrage beta \( \beta_{i}^{\text{post}} \) increases with the expected aggregate arbitrage position in the asset at time 1:
\[ \frac{\partial \beta_{i}^{\text{post}}}{\partial (\mu E_0 [x_{i,1}])} > 0 \]  
(20)

(ii) Furthermore, this expected aggregate arbitrage position increases with the pre-arbitrage alpha:
\[ \frac{\partial (\mu E [x_{i,1}])}{\partial \alpha_i^{\text{pre}}} > 0 \]  
(21)

**Proof.** (i) Since \( E_0 [x_{i,1}] = i - E_0 [i^*] \) is a one-to-one function of \( i \), the chain rule gives \( \frac{\partial \beta_{i}^{\text{post}}}{\partial (\mu E_0 [x_{i,1}])} = (\partial \beta_{i}^{\text{post}} / \partial i) \times (\partial i / \partial (\mu E_0 [x_{i,1}])) \times \partial \beta_{i}^{\text{post}} / \partial i > 0 \). (ii) \( \partial (\mu E [x_{i,1}]) / \partial \alpha_i^{\text{pre}} = 1 / r_{\text{max}} > 0 \).
Even within the post-arbitrage regime, endogenous betas arise only when arbitrageurs are “constrained” at time 1. This is the case in which \( K_t \in [0, 1/2] \) so that the anomaly assets are mispriced and a variation in arbitrage capital generates price pressure on the anomalies. In particular, if \( K_1 > 1/2 \), all anomalies are already correctly priced so that variation in arbitrage capital no longer generates price pressure. This implies the testable prediction that anomalies on average have zero endogenous risks in times when arbitrage capital is persistently large (Proposition 3).

**Proposition 3.** *(Post-arbitrage beta comes from constrained states).* Endogenous risk arises only in the constrained states of time 1. That is, anomaly return does not comove with \( \Lambda_1/\Lambda_0 \) in the unconstrained states \( (K_1 \geq 1/2) \) but comoves with \( \Lambda_1/\Lambda_0 \) only in the constrained states \( (K_1 < 1/2) \):

\[
- \text{Cov}_0 \left( r_{i,1}^e, \frac{\Lambda_1}{\Lambda_0} | K_1 \geq 1/2 \right) = 0; \\
- \text{Cov}_0 \left( r_{i,1}^e, \frac{\Lambda_1}{\Lambda_0} | K_1 < 1/2 \right) > 0
\]  

**(22)**

**Proof.** Follows trivially from the pricing equation in Lemma 3. In fact, the way I set up the model prevents \( \Lambda_1/\Lambda_0 \) from moving at all in the unconstrained state, let alone comove with \( r_{i,1}^e \) (its beta with respect to \( K_1 \) is zero when \( K_1 \geq 1/2 \) and positive when \( K_1 < 1/2 \)).

Once arbitrageurs generate endogenous risks, they require a compensation for these risks. This implies a form of “intermediary asset pricing”: the cross-section of post-arbitrage betas \( \beta_{i}^{\text{post}} \) explain the cross-section of post-arbitrage anomaly returns that survive in equilibrium (Proposition 4).

**Proposition 4.** *(“Intermediary asset pricing” of anomalies).* \( \beta_{i}^{\text{post}} \) explains anomaly \( i \)’s expected return if and only if it is exploited by arbitrageurs at time 0:

\[
E_0 [r_{i,1}] = \left\{ \begin{array}{ll}
 r_{\max i} & \text{if } i \text{ is not exploited} \\
 \frac{1}{E_0 [\Lambda_1/\Lambda_0]} - 1 + \lambda \beta_{i}^{\text{post}} & \text{if } i \text{ is exploited}
\end{array} \right.
\]  

**(23)**

where

\[
\lambda = \frac{\text{Var}_0 (\Lambda_1/\Lambda_0)}{E_0 [\Lambda_1/\Lambda_0]}
\]  

**(24)**
\[ \beta_{i}^{\text{post}} = -\frac{\text{Cov}_0 (r_{i,1}, \Lambda_1 / \Lambda_0)}{\text{Var}_0 (\Lambda_1 / \Lambda_0)} \]  

with \( E_0 [\Lambda_1 / \Lambda_0] = 1 \) if arbitrageurs are unconstrained \((K_0 \text{ large})\) and \( E_0 [\Lambda_1 / \Lambda_0] > 1 \) if they are constrained \((K_0 \text{ small})\).

**Proof.** In Appendix B.2.

**Proposition 4** shows that the limits of arbitrage due to endogenous risk (SV) can be expressed as cross-sectional asset pricing with post-arbitrage betas in this multi-anomaly setting. **Proposition 4** nonetheless also shows that asset pricing with constrained financial intermediaries differs from the usual asset pricing condition in two regards. First, the “intermediaries” (arbitrageurs) are not the marginal investors of all assets, and the beta \( \beta_{i}^{\text{post}} \) only prices the assets that the intermediaries trade. Relatedly, the beta pricing equation on the assets that the intermediaries do trade has the unconventional feature that the zero-beta rate may be above the risk-free rate (of zero) since the arbitrageurs may face a capital constraint and forego a positive risk-adjusted return opportunity.

This cross-sectional pricing should be done within the post-arbitrage regime. When pre-arbitrage alphas and post-arbitrage betas are linked through the alphas-into-betas relation, a cross-sectional pricing test that includes both the pre and post-arbitrage regimes leads one to overestimate the price of risk associated with the beta since the regression attributes the variation in mean excess return coming from pre-arbitrage alphas to the betas, when in fact the causality flows in the opposite direction from pre-arbitrage alpha to (post-arbitrage) beta. I elaborate more on this point in Section 4.5.
3 Data and methodology

3.1 Data and measurement

A. Test assets

I use 40 equity anomalies as test assets. Although the predictions of the theory apply to any asset class in which systematic arbitrage occurs, I focus on equity anomalies since they offer a rich cross section (Green, Hand, and Zhang, 2017). In particular, I use 40 anomaly portfolios (“anomalies”) constructed using 20 most well-known anomaly characteristics. I arrive at this set by taking the long and short-side portfolios (top and bottom deciles of stocks) of 20 anomaly characteristics that exclude the 5 redundant and 7 high-turnover characteristics from the 32 anomaly characteristics surveyed by Novy-Marx and Velikov (2016). The return on each anomaly is the quarterly value-weighted average return on all domestic, common, major exchange (NYSE, AMEX, and NASDAQ) stocks that belong to the anomaly portfolio and is calculated from 1975Q1 to 2016Q4. An online data appendix to this paper provides additional details on the anomaly construction.

Table 1 lists the 20 anomaly characteristics used in this study. The standard errors used in the empirical tests take into account the possible correlations across different anomalies (see Section 3.2).

10 In contrast, fixed income and currency, for instance, offer a smaller cross section. For instance, Duarte, Longstaff, and Yu (2007) study 4 different fixed-income anomalies that require proprietary data and a separate, nontrivial valuation model for each anomaly asset. Avdjiev, Du, Koch, and Shin (2017) study 10 different currencies.

11 I use only the NYSE stocks when computing the decile cutoffs for each anomaly characteristic. The five redundant characteristics are value profitability, value momentum profitability, value momentum, high-frequency combo, and high-frequency combo with seasonality. The seven high-turnover characteristics are industry momentum, industry relative reversals, short-run reversals, seasonality, short-run reversals low volatility, PEAD(SUE), and PEAD(CAR3), all of which—by definition—get completely rebalanced within a quarter so that their risk cannot be easily assessed using a quarterly factor. The two PEAD (post earnings announcement drift) characteristics are categorized as medium-turnover in Novy-Marx and Velikov (where monthly returns are used) but are high-turnover for the purpose of my analysis.

12 Because of the poor quality of quarterly accounting data before early 1970s, there is some disagreement about the correct year to begin the anomaly data. Most papers seem to agree that anomaly data are of sufficient quality at least by 1975Q1.
B. Systematic shocks to arbitrage capital

I use a measure of aggregate funding-liquidity shocks as the systematic shocks to arbitrage capital. Since arbitrageurs of equity anomalies such as quantitative equity hedge funds extensively use leverage and short selling (e.g., Ang, Gorovyy, and Van Inwegen, 2011; Hanson and Sunderam, 2014), their ability to arbitrage depends heavily on the ability of their prime brokers to provide funding in a stable manner (Brunnermeier and Pedersen, 2009; Aragon and Strahan, 2012; Mitchell and Pulvino, 2012). Adrian, Etula, and Muir (2014) suggest measuring this so-called funding liquidity by taking the quarterly growth rate of the aggregate book leverage of security broker-dealers (the majority of which serve as the prime brokers of hedge funds), which I follow in this paper. The resulting series (standardized to have zero mean and unit standard deviation) is plotted as Figure 3. The three largest negative shocks happened during the Great Recession (2008Q4), the Peso crisis (1994Q4), and the mutual fund scandal of 2003 that involved some major security broker-dealers (2004Q4, which is the quarter when some major court actions occurred). An online data appendix to this paper provides additional details on the construction of this series.

C. Pre vs. post-arbitrage regimes

I use pre-1995 (-1995Q4) and post-1995 (1996Q1-) periods as the pre-arbitrage and post-arbitrage regimes in which there was little and more arbitrage on the equity anomalies, respectively. A similar cutoff (-1993Q4 vs. 1994Q1-) was used by Chordia, Subrahmanyam, and Tong (2014) to determine the periods before and after trading technology and liquidity had sufficiently lowered cost of extensive arbitrage, but I prefer to move my cutoff one year later so that it divides my total sample period into exact halves. A cutoff in 1995 or any other year in the 1990s seems appropriate also because arbitrage capital grew rapidly in the 1990s (Stein, 2009) and some of the most influential papers on equity anomalies were published around then (e.g., Fama and French, 1993,

---

13 Adrian, Etula, and Muir (2014) call this series the “leverage factor” in their final draft, but the paper’s earlier draft by Tobias Adrian and Erkko Etula interpreted the series as funding-liquidity shocks in the spirit of Brunnermeier and Pedersen (2009).

14 In a draft that was previously circulated, I used the 1993 (-1993Q4 vs. 1994Q1-) cutoff, which divided my previous sample period (1972Q1-2015Q4) into halves. I now start the sample period three years later (1975Q1), responding to the concern about the poor quality of Compustat quarterly accounting data in the early 1970s.
1996; Jegadeesh and Titman, 1993; Lakonishok, Shleifer, and Vishny, 1994). Since, however, the year 1995 does not represent a structural break in the amount of arbitrage, I repeat my main alphas-into-betas regression using alternative cutoff years.

Although some of the anomalies I consider were not “discovered” in the academic literature by 1995, I take the perspective that arbitrageurs like hedge funds conduct independent research to discover anomaly characteristics beyond those already found in the academic research. Still, if extensive arbitrage begins after academic publication (McLean and Pontiff, 2016), the post-1995 funding-liquidity beta—the dependent variable of the key cross-sectional regression—would contain an error induced by the different publication years. This would increase the standard errors, making it more difficult to reject the null hypothesis of no alphas-into-betas effect, but it would not bias the coefficient estimate.

D. Benchmark pricing model

I use the capital asset pricing model (CAPM) (Treynor, 1961; Sharpe, 1964; Lintner, 1965) as the benchmark pricing model arbitrageurs use to assess the anomalies’ alphas. Berk and van Binsbergen (2016) and Barber, Huang, and Odean (2016) find that mutual fund flows chase CAPM alpha, so it seems reasonable to expect that arbitrageurs who are delegated money managers seek out stocks that generate CAPM alpha. The baseline analysis therefore uses pre-1995 CAPM alpha and post-1995 funding-liquidity beta net of market as the pre-arbitrage alpha and post-arbitrage beta, respectively. I nonetheless repeat the main regression using Fama and French (1993) 3-factor model, Fama and French (2015) 5-factor model, and a 6-factor model that adds the Pástor and Stambaugh (2003) liquidity factor to the Fama-French 5 factor model.

E. Summary statistics

Table 2 reports the pre-1995 and post-1995 benchmark alphas (“alphas”) and funding-liquidity betas (“funding betas”) of the 40 short and long-side anomalies, using the CAPM (Panel A) or the six-factor model (Panel B) as the benchmark model. The short-side and long-side anomalies respectively have predominantly negative and positive benchmark alphas both with respect to
CAPM and the more stringent six-factor model, justifying their identity as an anomaly. The alphas have not fallen in magnitude from pre to post-1995 periods with respect to the CAPM model but have fallen substantially with respect to the six-factor model for short-side anomalies.

The funding betas in the pre-1995 period were centered around zero for both short and long-side anomalies but became predominantly negative in the post-1995 period for short-side anomalies. This suggests that anomalies with no fundamental exposures to funding-liquidity shocks become endogenously exposed to those shocks through arbitrage. This pattern is true with respect to both benchmarks but somewhat clearer with respect to the six-factor model. Importantly, the anomalies that become strongly exposed to funding-liquidity shocks in the post-1995 period are the ones with larger benchmark alphas in the pre-1995 period. This gives a glimpse into pre-arbitrage benchmark alphas turning into post-arbitrage betas with respect to arbitrage capital shocks. There nonetheless is little action in the long-side anomalies, a pattern I revisit in Section 4.1.

3.2 A two-pass procedure and the standard error correction

Studying the endogeneity of betas requires putting betas on the left-hand side of a regression. I briefly discuss the econometric consequence of such a regression specification and suggest standard errors appropriate in a study of the cross-section of betas.

The model of excess returns on anomaly \( i \) in the post-arbitrage regime is

\[
\begin{align*}
    r_{i,t}^e &= \mu_i + g_t' \beta_{i,g} + \beta_{i,f}^\text{post} f_t + \epsilon_{i,t},
\end{align*}
\]  

(26)

where \( g_t \) is a vector of \( k - 1 \) benchmark factors, \( f_t \) is the arbitrage funding shock, \( \beta_{i,f}^\text{post} \) is the beta with respect to \( f_t \), \( t = 1, ..., T \) are time periods within the post-arbitrage regime, and the assumptions made in Shanken (1992) hold, including that \( \Sigma_c \equiv \text{Var} (\epsilon_{1,t}, ..., \epsilon_{n,t} \mid g_t, f_t) \) is not necessarily diagonal but that \( (\epsilon_{1,t}, ..., \epsilon_{n,t}) \) is i.i.d. over time. The model of the endogenous determination of \( \beta_{i,f}^\text{post} \) is

\[
\beta_{i,f}^\text{post} = x_i' b + u_i,
\]  

(27)

where \( x_i \) is anomaly \( i \)'s pre-arbitrage characteristics and the error term \( u_i \) is heteroskedastic but
uncorrelated. For instance, $x_i = (1, \alpha_i^{pre, \text{bench}})$ in the simple “αs into βs” regression.

Since $\hat{\beta}_{i,f}^{post}$ is unobserved, (27) cannot be estimated directly. Instead, I use $\hat{\beta}_{i,f}^{post}$ estimated from (26) as the dependent variable of the equation, which implies the relation

$$\hat{\beta}_{i,f}^{post} = x'_i b + u_i + e_i, \quad (28)$$

where $e_i$ denotes the measurement error arising from estimating $\beta_{i,f}$. When $T$ is finite, estimating (28) instead of (27) raises the concern that “similar” anomalies end up with correlated measurement errors $e$, increasing the likelihood of obtaining a spuriously large $\hat{b}$.

As shown in Appendix C.1, the conditional variance of the estimator $\hat{b}$ that corrects for the measurement error is

$$V_{\hat{b}} = (X'X)^{-1} X' (\Omega_u + \Omega_e) X (X'X)^{-1} \quad (29)$$

where the $n \times n$ matrix $\Omega_e \equiv Var(e_1, ..., e_n)$ is the variance-covariance matrix of measurement errors. The matrix $\Omega_e$, however, does not simplify to be a function of $\Sigma_{\epsilon}$ and $\Sigma_F \equiv Var((g_t', f_t))$ only, so I estimate it through bootstrapping. The estimated $\Omega_e$ is used to obtain the correct standard errors for the regression model (28).

The resulting standard error correction is substantial. Even though I have 84 different quarters in the post-1995 period, the correct standard error is 2 to 3 times larger than the naive heteroskedasticity-robust White standard errors, which highlights the importance of doing this correction in a study of endogenous betas. As is well-known, the unbiasedness and consistency of the estimator $\hat{b}$ are not affected by the measurement errors.

### 4 Turning alphas into betas in equity anomalies

I now use anomalies in the equity market to test the predictions of Section 2. I first establish the main result that pre-arbitrage benchmark alpha turns into post-arbitrage arbitrage beta in the cross-section of anomalies (Proposition 1) and rule out an alternative explanation that an omitted
risk factor drives the result. Tests of the three remaining propositions also point to the arbitrage-based explanation for alphas turning into betas.

4.1 Turning alphas into betas

To test Proposition 1, I test the ability of pre-1995 CAPM alphas to explain the cross-section of post-1995 funding betas:

$$\beta_{\text{post}, i,f} = b_0 + b_1 \alpha_{\text{pre}, i,CAPM} + u_i \quad (30)$$

Table 3 presents the result. An anomaly’s pre-1995 CAPM alpha alone explains 60% of the cross-sectional variation in post-1995 funding betas, consistent with the pre-arbitrage alpha determining the anomaly’s arbitrage intensity and hence the endogenous funding-liquidity exposure (column 1). This contrasts starkly with poor ability of pre-1995 funding beta to explain the post-1995 funding betas (column 2). The magnitude of this “alphas-into-betas” effect is large: a 1% point difference in pre-1995 CAPM alpha leads to a 0.22% point difference in the return response to a one-standard-deviation shock to aggregate funding liquidity. Hence, an average short-side anomaly and an average long-side anomaly have a 1.66% point difference in that return response.\(^\text{15}\) The intercept being close to zero is also consistent with the theory: an anomaly with zero pre-arbitrage alpha does not attain either a positive or negative post-arbitrage beta.

Using the pre-arbitrage alpha that additionally controls for the exposure to funding-liquidity shocks does not affect the result (column 3). Since the funding shocks are not a portfolio, one cannot obtain an alpha with respect to both the market and funding-liquidity shocks from a time-series regression. However, the alphas-into-betas effect can still be identified as the coefficient on the part of pre-arbitrage mean return unexplained by the market exposure as long as the pre-arbitrage funding beta has been controlled for:

$$\beta_{\text{post}, i,f} = b_0 + b_1 \alpha_{\text{pre}, i,m,f} + u_i$$

$$b_0 + b_1 \left( E[r_{i,\text{pre}}] - E[r_{m,\text{pre}}] \beta_{i,m} \right) - b_1 \lambda_{f,\text{pre}} \beta_{i,f} + u_i \quad (31)$$

(where $\beta_{i,m}$ and $\beta_{i,f}$ are estimated jointly in a time-series regression). Column 3 of Table 3

\(^{15}\)0.215% times the difference between 2.32 and -5.38 taken from Table 2.
shows little change in the estimated alphas-into-betas effect, consistent with pre-1995 funding betas representing noise rather than fundamental funding-liquidity exposures.

Since the anomalies are not volatility-hedged, one may be concerned that they are differently scaled copies of one another. To alleviate this concern, I re-estimate the baseline alphas-into-betas model using generalized least squares with the variance-covariance matrix from the OLS estimation ($\Omega_u + \Omega_e$ in Section 3.2) as the weighting matrix. The estimated effect remains similar to the OLS estimate (Table 3 column 4).

A. Alternative benchmark models

How robust is this result to controlling for additional systematic factors when computing both pre-1995 benchmark alpha and post-1995 funding beta? This is a valid concern since both pre-1995 CAPM alpha and post-1995 funding beta may proxy an exposure to another factor such as the profitability or investment factor known to capture part of the covariances among the anomalies (Fama and French, 2016) or the market liquidity factor of Pástor and Stambaugh (2002) that may have become correlated with funding liquidity in the recent years (Brunnermeier and Pedersen, 2009).

Therefore, I estimate the regression models (30) and (31) using the Fama and French (1993) 3-factor model, the Fama-French 5-factor model, and a 6-factor model that adds the Pástor-Stambaugh to the 5 factors as the benchmark model that arbitrageurs use to identify an alpha.

Surprisingly, controlling for as many as five additional factors has little effect on the alphas-into-betas result (Table 3 columns 5-10). In fact, the $R^2$ improves marginally after controlling for the profitability and investment factors, suggesting that they may indeed summarize cross-anomaly covariances that exist even before arbitrage and are therefore avoided by risk-averse arbitrageurs.
B. Alternative sample periods

Using alternative cutoff years in the 1990s to define pre-arbitrage and post-arbitrage regimes generates similar results (Table A1 columns 1-4). The magnitude of the effect increases slightly when a more recent year is used as a cutoff, suggesting that the growth of arbitrage (growth of the mass of arbitrageurs $\mu$ in the model in Section 2) may be gradual so that the estimated exposures of anomalies to funding-liquidity shocks are larger when a more recent sample period is used as the post-arbitrage regime.

If the alphas-into-betas effect is driven by arbitrage, it should not appear within the pre-arbitrage regime. Hence, I conduct a placebo test in which I repeat the regression using the 1975-1985 (“pre-1985”) and 1986-1995 (“post-1985”) sample periods as the pre and post-arbitrage regimes, respectively. Nevertheless, pre-1985 CAPM alpha does not predict post-1985 funding beta (column 5), lowering the likelihood that the alphas-into-betas relation is a spurious result.

C. Is the alphas-into-betas result driven by an omitted risk factor?

An obvious response to this alphas-into-betas result could be that the pre-1995 alpha is a premium for risk that was latent in the pre-1995 period (and thus does not show up as pre-1995 funding beta) but captured by the post-1995 funding beta. To be fair, in Table 3, I have controlled for the likely suspects such as exposures to the profitability, investment, and market liquidity factors as well as the funding-liquidity factor itself when estimating the alphas-into-betas effect, so the likelihood of finding a previously undiscovered risk factor that the anomalies have always been commonly exposed to seems low. This is especially so since the factor also has to be uncorrelated with funding-liquidity shocks in the pre-1995 period but somehow correlated with the shocks in the post-1995 period. Still, I take this omitted risk hypothesis into consideration.

In particular, I use two additional pieces of evidence to rule out the omitted risk hypothesis: (i) The alphas-into-betas effect is stronger for short-side anomalies than long-side anomalies; and (ii) Post-1995 funding betas are not fundamental cash-flow exposures.

For (i), Table 4 reports the differential alphas-into-betas effect for short-side vs. long-side
anomalies:

$$\beta_{i,f}^{post} = b_0 + b_1 \alpha_{i,CAPM}^{pre} + b_2 \alpha_{i,CAPM}^{pre} \times Short_i + u_i$$  \hspace{1cm} (32)

The magnitude of the alphas-into-betas effect is around 2 to 3 times larger for short-side anomalies than for long-side anomalies, regardless of the benchmark factor model (see Figure 4 for the CAPM benchmark case). This difference in the effect is weakly significant for benchmark models that include profitability and investment factors, which subsume a large fraction of the cross-anomaly covariances that inflate the standard errors, and is statistically significant for the pre-1995 alpha that controls for the pre-1995 funding-liquidity exposure. The strong negative effect of pre-1995 funding beta on post-1995 funding beta for short-side anomalies is related to the strength of $b_1$ since, as evident from from eq. (31), this coefficient equals the difference in the alphas-into-betas effect for short-side anomalies times the difference in the price of risk arbitrageurs assign to short-side anomalies.

This result is inconsistent with an omitted risk factor generating the alphas-into-betas result. Under the null of an omitted risk factor, pre-1995 benchmark alpha represents the premium for risk that is later captured by post-1995 funding beta, so the slope of the alphas-into-betas regression—which roughly represents the inverse of the price of risk associated with the funding beta—should be similar for short-side and long-side anomalies. This is simply not the case.\textsuperscript{16}

This result, however, is consistent with the arbitrage-based explanation for alphas turning into betas. Intuitively, the long-side anomalies can be arbitraged by a variety of different investors including mutual funds, pension funds, exchange-traded funds, and even retail investors that are not exposed to funding-liquidity shocks, whereas the short-side anomalies are primarily arbitrated by highly leveraged investors such as quantitative equity hedge funds that are exposed to funding-liquidity shocks.\textsuperscript{17} Hence, if an anomaly’s funding beta is a byproduct of arbitrage activity by leveraged investors, one would expect a larger alphas-into-betas effect for the short-side anomalies.

\textsuperscript{16}One way out is to tell a more elaborate story that the short and long-side anomalies are exposed to two different risk factors with different post-1995 correlations with the funding-liquidity shocks, but this seems inconsistent with the vast majority of the explanations for anomalies that treat the short and long-sides of an anomaly as the flip sides of the same phenomenon.

\textsuperscript{17}It is possible that the long-side anomalies’ alphas have turned into betas with respect to some other shocks that those long-only investors are exposed. A future study can examine this hypothesis.
Next, consider (ii). Since unexpected return is caused by either cash-flow news or discount-rate news (Campbell and Shiller, 1988), the anomalies’ post-1995 funding betas are caused by either the underlying stocks’ fundamental cash flows covarying with funding-liquidity shocks or the discount rate on those cash flows (negatively) covarying with funding-liquidity shocks. If the anomalies’ funding-liquidity exposures were fundamental cash-flow exposures, it would be inconsistent with arbitrage trade generating those exposures through the discount-rate channel. If instead the exposures were not fundamental cash-flow exposures, the explanation based on an omitted risk factor needs to be supplemented with a story of how the anomalies become endogenously exposed to the risk factor despite having no fundamental cash-flow exposures to it. I therefore study the anomalies’ post-1995 cash-flow funding betas estimated using the method of Cohen, Polk, and Vuolteenaho (2003, 2009) (see Appendix C.2 as well as Campbell, Polk, and Vuolteenaho, 2010, for details on calculating a portfolio’s cash-flow news).

The result illustrated in Figure 5 is strongly against the fundamental risk story. The anomalies’ cash-flow betas tend to be positive for both short-side and long-side anomalies, suggesting that the real sector has a slightly positive exposure to the funding-liquidity shocks in today’s economy in which financial frictions play an important role. Furthermore, it is clear that this fundamental cash-flow component of funding beta is not driving the alphas-into-betas relation. Hence, even if we were to look for omitted risk, we would be looking for an endogenous risk of the type that the endogenous funding-liquidity risk due to arbitrage satisfies.

D. Costs of arbitrage

Having established that the alphas-into-betas result is unlikely to be driven by an omitted risk factor, I explore yet another possible source of spuriousity. One explanation for alphas earned by anomalies is that they represent costs of arbitrage, not an investment opportunity (e.g., Tuckman and Vila, 1992; Knez and Ready, 1996; Mitchell and Pulvino, 2001; Lesmond, Schill, and Zhou, 2001). 18

I prefer to estimate the cash-flow beta rather than discount-rate beta since there is more controversy about the VAR methodology used to obtain the latter (see the concern raised by Chen and Zhao, 2009, and the response in Campbell, Polk, and Vuolteenaho, 2010). I calculate the portfolio roe variable quarterly instead of yearly, which subjects the variable to seasonality, but this matters little for cash-flow betas since I use the discounted sum of roe over a year into the future (and the discounted sum of funding shocks over a year as well) when calculating the cash-flow betas.
2004; Korajczyk and Sadka, 2004; Pontiff, 2006). If this were true for the pre-1995 alphas, the alphas-into-betas effect may be a spurious result driven by post-1995 funding betas somehow proxying for arbitrage costs.

To explore this possibility, I calculate for each anomaly the four different value-weighted decile ranks of the underlying stocks’ costs of arbitrage as measured by size, illiquidity (Amihud, 2002), idiosyncratic volatility, or bid-ask spread (Corwin and Schultz, 2012). Specifically, for each month in the pre-1995 period, I assign all stocks their decile ranks on how they compare to the NYSE stocks in terms of each of the four arbitrage cost measures. I then compute the value-weighted average of these ranks for each anomaly and then take the time-series average to obtain each anomaly’s pre-1995 arbitrage cost measure. This measure is then cross-sectionally standardized for an easier interpretation.

The estimated model augments the simple alphas-into-betas model (eq. 30) with the arbitrage cost variable:

$$\beta_{i,f}^{\text{post}} = b_0 + b_1\alpha_{i,CAPM}^{pre} + b_2\alpha_{i,CAPM}^{pre} \times \text{Arbitrage Cost}_i + b_3 \text{Short}_i \times \text{Arbitrage Cost}_i + b_4 \text{Long}_i \times \text{Arbitrage Cost}_i + u_i$$

This model allows the alphas-into-betas effect to depend on the cost of arbitraging the anomaly and also allows the post-arbitrage beta to depend directly on the arbitrage cost measure (interacted with the short and long-side dummies to sign the direction of effect).

Table A2 shows that the alphas-into-betas effect survives controlling for arbitrage costs. The magnitude of the effect, however, falls by as much as a third, implying that part of the previous estimate may represent a bias caused by anomalies with larger arbitrage costs having both larger pre-1995 CAPM alpha and larger post-1995 funding beta. An anomaly with a larger idiosyncratic volatility, for instance, may attain a larger funding beta if part of the beta is caused by more volatile anomalies facing more volatile margin constraints, a prediction of both Burnnermeier and Pedersen (2009) or Gromb and Vayanos (2017).

Due to large standard errors, the coefficients on the arbitrage cost variables do not carry precise information. First, the interaction between pre-1995 alpha and arbitrage costs does not generate much action; anomalies that are costlier to arbitrage do not see their pre-arbitrage alpha turn into a
smaller post-arbitrage beta, consistent with the argument that arbitrage costs do not significantly hinder arbitrage (Frazzini, Israel, and Moskowitz, 2014). Second, the level effect of arbitrage costs on post-1995 funding beta is large in magnitude but statistically insignificant. The point estimate implies that a one-standard-deviation increase in the cost of arbitrage (a decrease in size or an increase in illiquidity, idiosyncratic volatility, or spread by 1) is associated with a fall in post-1995 funding beta by as much as 0.73 for short-side anomalies and 0.32 for long-side anomalies.

4.2 Contemporaneous arbitrage interest explains the post-arbitrage betas

If post-1995 betas are an outcome of arbitrage activity, a measure of contemporaneous “arbitrage interest” measured by fraction of the anomaly’s shares held by arbitrageurs should explain the anomaly’s post-arbitrage beta (Proposition 2). Since arbitrageurs such as hedge funds are responsible for the majority of short interests (Hanson and Sunderam, 2014), arbitrage interests in the short-side anomalies can be measured by value-weighted short interests in the anomalies (Hirshleifer, Teoh, and Yu, 2011; Hanson and Sunderam, 2014; Hwang, Liu, and Xu, 2017).

I therefore test whether the cross-sectional differences in post-1995 shorting activity explain the differences in short-side anomalies’ post-1995 funding betas. Using data from Compustat, I measure shorting activity on an anomaly each month with the cross-sectionally standardized short interest ratio computed as the value-weighted average ratio of short interest to shares outstanding on the underlying stocks (Hanson and Sunderam, 2014). The resulting monthly short interest ratio is averaged across the post-1995 period to generate post-1995 short interest ratio of an anomaly.

I actually begin by whether pre-arbitrage alphas proxied by pre-1995 CAPM alphas determine the cross-section of arbitrage interests proxied by post-1995 short interest ratios (top of Figure 6). Looking at Figure 6b first, I find that pre-1995 CAPM alpha determines the cross-section of arbitrage interests in the post-1995 period. One may wonder why the high idiosyncratic volatility anomaly (“idivol(S)”) or the high failure probability anomaly (“failprob(S)”) attract much short interest given their idiosyncratic risks, but this is natural given the large capital commanded by distressed securities hedge fund that specialize in this type of anomalies. The $R^2$ in Figure 6a
shows that pre-1995 CAPM alpha also helps determine the cross-section of arbitrage interests in the same pre-1995 period but that the level of arbitrage interests were small. This tells us that a set of arbitrageurs has always known the characteristics of stocks predicting CAPM alphas but that the level of capital they command was small in the pre-1995 period and large in the post-1995 period.

The bottom of Figure 6 plots the relationship between pre-1995 short interest and pre-1995 funding beta (Figure 6c) and post-1995 short interest and post-1995 funding beta (Figure 6d) of the short-anomalies, again on the same scale. Since the pre-1995 featured little arbitrage capital and low levels of short interests in the anomalies, these cross-section of short interests do not generate significant funding-liquidity exposures measured by funding-liquidity betas (Figure 6c). This contrasts sharply with the post-1995 period, in which the cross-section of short interests generate the cross-section of different funding-liquidity exposures (Figure 6d). Together, these figures are consistent with the story I tell in this paper that pre-1995 CAPM alphas determine the cross-section of post-1995 arbitrage interests, which in turn determine the post-1995 funding betas.

Table 5 repeats the evidence in Figure 6d under different specifications. It shows that arbitrage interest explaining the cross-section of funding betas in the post-1995 still holds strongly if I calculate value-weighted short interest ratio using raw—rather than cross-sectionally standardized—short interest ratio (“Short Interest Ratio (Not Detrended)”). Since the shorting activity is a noisy measure of the unobserved arbitrage activity by the leveraged arbitrageurs I try to capture, I repeat this analysis using the two-stage least squares regression that uses differences in the anomalies’ post-1995 shorting induced by differences in pre-1995 CAPM alpha as the regressors. Unsurprisingly, the post-1995 short interest ratio instrumented using pre-1995 CAPM alpha has a larger coefficient estimate, consistent with the short interest ratio measuring the true arbitrage activity with an error.19

19The conventional wisdom in labor economics is to avoid comparing OLS coefficient with the 2SLS coefficient because the subpopulation treated by the instrument may have a slope coefficient different from the whole population’s. This concern does not apply here since pre-1995 alpha acts as an instrument for all anomalies.
4.3 Post-arbitrage beta comes from constrained times

Next, I test the prediction that even within the post-arbitrage regime, the betas with respect to arbitrage capital arise during times in which arbitrageurs are constrained (Proposition 3). Intuitively, if arbitrage capital is sufficiently large, a negative shock to the capital may not arbitrageurs to change their positions in the anomalies and generate negative price pressure on the anomaly stocks.

Carrying out this test empirically requires defining in the data the times in which the level arbitrage capital (equity plus attainable funding) was sufficiently high. Since the arbitrageurs I study here are the highly leveraged ones such as the quantitative equity hedge funds (Ang, Gorovyy, van Inwegen, 2011; Ben-David, Franzoni, and Moussawi, 2012), I focus on identifying the times in which the level of arbitrage funding—rather than the arbitrageur equity—was sufficiently high. Presumably, I could recover the history of the level of arbitrage funding from the funding-liquidity shock series, but this would require a structural model that links the shocks to the level, subjecting myself to a new source of arbitrariness.

I instead use the VIX index, which is stationary by nature and can affect the margin requirements faced by the arbitrageurs (Burnmermeier and Pedersen, 2009; Adrian and Shin, 2013; Bruno and Shin, 2015). Defining the above vs. below-median-VIX times as the proxy for constrained vs. unconstrained times within the post-1995 period leads to a sensible classification (Figure 7). It essentially implies that two subperiods within the post-1995 period are the constrained times: 1996Q4–2003Q3 which begins in the aftermath of the Peso crisis and includes the LTCM, 9/11, and the Iraq War and 2007Q3–2012Q2 which coincides with the beginning and the aftermath of the Great Recession. With this classification, I estimate an excess return process that has different betas with respect to the market and the funding-liquidity shocks during constrained and unconstrained quarters within the post-1995 period.

Table 6 analyzes the anomalies’ post-1995 constrained-time and unconstrained-time funding betas. First, looking at the averages, the short and long-side anomalies respectively have negative and positive cross-sectional average in funding betas during constrained times, although the average is close to zero for long-side anomalies (column 1). In contrast, the unconstrained-time funding betas are close to zero for both short and long-side anomalies (column 4), consis-
tent with funding-liquidity shocks in these unconstrained times generating no price pressure on the anomalies since such shocks do not lead the arbitrageurs to rebalance their portfolios. The anomalies whose constrained-time betas are large in magnitude are the ones that had pre-1995 CAPM alphas that were also large in magnitude (column 2), whereas there is no evidence that the alphas-into-betas effect appears in the unconstrained-time funding betas (column 5). Naturally, the differential alphas-into-betas effect also comes from the constrained-time funding betas.

This finding that the pre-arbitrage alphas turn into post-arbitrage constrained-time betas but not into unconstrained-time betas is consistent with arbitrage trade generating the betas endogenously. This result also helps alleviate the concern that the funding-liquidity shocks I use are actually just a proxy for arbitrageur wealth portfolio. If arbitragurs trade different anomalies, their wealth portfolio would mechanically have exposures to the anomaly returns, and more so for anomalies with a larger pre-arbitrage alpha. Although such a post-arbitrage wealth beta also measures endogenous risk for mean-variance optimizing arbitrageurs, it is different from the story in which arbitrage trade changes the nature of anomaly returns through price pressure. However, if the funding betas represent the mechanical wealth betas, those betas should be large even during the unconstrained times within the post-arbitrage regime: even if arbitrageurs held less positions in the anomalies in unconstrained times, if they still held positions in the anomalies, I would expect to see significant wealth betas. Hence, the lack of funding betas during unconstrained times is evidence against the mechanical wealth beta story.

4.4 Risk or comovement? “Intermediary asset pricing” of anomalies

In theory, once the act of arbitrage generates the endogenous post-arbitrage beta, this risk prevents the initial anomaly returns from disappearing completely. Proposition 4 shows that this limits-of-arbitrage insight of Shleifer and Vishny (1997) implies a cross-sectional pricing of the anomalies using the post-arbitrage betas within the post-arbitrage regime. In this sense, “intermediary asset pricing” with the funding-liquidity factor (Adrian, Etula, andd Muir, 2014) can also be interpreted as a cross-sectional test of limits of arbitrage.

In practice, how well do the post-1995 funding betas explain the cross-section of anomaly returns in the post-1995 period? That is, is there evidence that arbitrageurs take into account the
funding-liquidity risks when pricing the anomalies, or do the betas merely serve as covariances that are not priced in equilibrium? The empirical evidence is mixed.

Table 7 presents the results from a cross-sectional asset pricing regression that restricts the market factor’s risk premium to be its realized mean. The cross-sectional variation in anomaly returns net of the market risk premium in the post-1995 period is well-explained by the cross-sectional variation in funding betas, suggesting that endogenous arbitrage risks may indeed determine the anomaly returns that survive in the post-arbitrage period (column 1). There are reasons not to take this result at face value, however. Unlike the alphas-into-betas result, which is robust to controlling for other factors, this beta pricing result is not robust to doing so—the profitability and investment factors subsume funding-liquidity betas’ ability to explain the cross-sectional returns (columns 5 and 6).

How does the cross-sectional pricing compare across different sample periods? Funding-liquidity beta is estimated to carry a zero price of risk in the pre-1995 period, consistent with arbitrageurs who care about funding-liquidity risk being small. Interestingly, pooling the pre-1995 and post-1995 samples together generate a larger price of risk estimate than in the individual samples. This is despite the $R^2$ of the regression being lower in the pooled sample. In fact, the same problem arises in Adrian, Etula, and Muir (2014), which reports the subsample results in their internet appendix. And more generally, as I show in the next subsection, pooling the pre-arbitrage and post-arbitrage samples leads to a positive bias in the price of risk associated with the arbitrage-driven beta. This is because the alphas-into-betas effect implies a reverse causality in the intermediary asset pricing logic.

4.5 Alphas into betas as intermediary asset pricing in reverse causality

Empirical research in intermediary asset pricing has investigated the ability of intermediary-motivated betas to explain the cross-section of asset returns (Adrian, Etula, and Muir, 2014; He, Kelly, and Manela, 2016; Avdjiev, Du, Koch, and Shin, 2017). Nevertheless, the alphas-

---

20Otherwise, the estimated risk premium on the market is a large negative number.
into-betas result implies that the part of the expected return due to an alpha can generate an intermediary beta, implying a reverse causality. To what extent do these papers suffer from the biased price of risk problem mentioned above? To answer this question in a more systematic way, I use a simple empirical model to formalize this point.

Example 1. (A simple model of intermediary asset pricing with an arbitrage-driven beta). Suppose that the excess return, the expected excess return in the post-arbitrage period, and the alphas-into-betas effect follow

\[
\begin{align*}
    r_{i,t}^e &= \alpha_{i}^{pre}\,1\left(t \in t^{pre}\right) + \left(\mu_{i}^{post} + \beta_{i,f}^{post} f_i\right)\,1\left(t \in t^{post}\right) + \epsilon_{i,t}; \\
    E\left[r_{i,t}^{e,post}\right] &= \alpha_{i}^{post} + \lambda_{f}^* \beta_{i,f}^{post}, \quad \left|\alpha_{i}^{post}\right| < \left|\alpha_{i}^{pre}\right|; \quad \text{and} \\
    \beta_{i,f}^{post} &= b_{1} \alpha_{i}^{pre} + u_{i}
\end{align*}
\]

where \(t^{pre}\) (\(t^{post}\)) denotes the pre-arbitrage (post-arbitrage) regime in which arbitrage capital is negligible (non-negligible) and the distribution of \(f_i\) does not depend on time.\(^{22}\) Hence, in the pre-arbitrage period, when no risk factor exists, asset \(i\) generates an abnormal return of \(\alpha_{i}^{pre}\). In the post-arbitrage period, the abnormal return falls in magnitude, and the beta that arises endogenously carries a risk premium.

This model is essentially a single-factor version of the empirical model considered in Section 3.2. The multifactor generalization is straightforward, but the single-factor version captures the point most clearly. In this example, asset pricing in the pooled sample that includes both the pre and post-arbitrage periods using the arbitrage-driven \(\beta_{i,f}\) generates a biased price of risk estimate.

Proposition 5. (A biased price of risk in intermediary asset pricing using an arbitrage-driven beta). Given the empirical model in Example 1, estimating the conventional intermediary asset pricing model using \(\beta_{i,f}\) from the pooled sample,

\[
E\left[r_{i,t}^{e,\text{pooled}}\right] = \alpha_{i}^{*} + \lambda_{f}^* \beta_{i}^{\text{pooled}} + v_{i},
\]

leads to a bias price of risk estimate \(\lambda_{f}^*\). This bias is a function of the alphas-into-betas coefficient

\(^{22}\)The pre-arbitrage regime times do not have to be before post-arbitrage regime times in the chronological sense. For instance, the recent crisis period may be included in \(t^{pre}\) if arbitrage capital dried up (e.g., due to poor funding liquidity) and did not play an important role during the crisis.
and the number of time periods in the pre-arbitrage regime as a fraction of the pooled sample
\[ \tau \equiv \frac{T^{pre}}{T^{pre} + T^{post}}; \]
\[ \lambda^*_f - \lambda_f = \frac{1}{b_1 (1 - \tau)}. \]  \hfill (36)

The intercept is zero if post-arbitrage alpha is zero: \( \alpha^*_i = (1 - \tau) \alpha^*_i \).

**Proof.** The true pooled sample expected return is
\[
E \left[ r_{i, pooled}^e \right] = \tau \alpha^*_i + (1 - \tau) \left( \alpha^*_i + \lambda_f \beta^*_i,f \right)
\]
Since \( \beta^*_i, pooled = (1 - \tau) \beta^*_i, post \) and \( \alpha^*_i, pre = b_1^{-1} \left( \beta^*_i, post - u_i \right) \), rewriting this in terms of \( \beta^*_i, pooled \) gives
\[
E \left[ r_{i, pooled}^e \right] = (1 - \tau) \alpha^*_i + \left( \frac{1}{b_1 (1 - \tau)} + \lambda_f \right) \beta^*_i, pooled - \frac{1}{b_1} u_i.
\]

This proposition is useful in many ways. First, it shows that the bias is related to the alphas-into-betas effect. Whenever there is an arbitrageur turning alphas into betas, there will be a positive bias in the price of risk in the pooled sample. Second, it rationalizes the reason why the pooled sample regression has a lower \( R^2 \) than the post-arbitrage sample regression despite the former having a larger price of risk estimate. This extra noise in the estimation comes from the error term in (35), which the proof shows equals \( b_1^{-1} u \) and also arises whenever the alphas-into-betas effect arises. Third, to the extent that a positive risk-adjusted return in the post-arbitrage period indicates the events in which arbitrageurs are constrained, it can also serve as the indication of the alphas-into-betas effect and hence the biased price of risk effect (recall the discussion at the end of Section 2.2).

Given this, there are two suggestions on spotting the signs of the price of risk bias of an arbitrage-driven beta:

1. Is the intercept of the regression for positive-beta assets (negative-beta assets) positive (negative)?

2. Divide the sample period into two based on the likely size of arbitrage capital. Are the prices of risk estimated from these two subsamples *smaller* than that estimated from the pooled sample (but the \( R^2 \) of the regression is larger than the combined sample at least for
one of those subsamples)?

A negative answer to at least one of these questions indicate that there may be an underlying alphas-into-betas effect that generates the bias result. In this case, the price of risk estimated from the post-arbitrage period is likely to be closer to the true value. If the pre-arbitrage vs. post-arbitrage periods are not obvious, the subsample with a higher $R^2$ and smaller magnitude of the intercept is likely to be the post-arbitrage period. From an econometric point of view, the estimated price of risk being larger in the pooled sample may be a result of a longer sample period alleviating the errors-in-variables bias. If the subsample periods are short enough such that the errors-in-variables bias can be large, then the reduction in $R^2$ in the pooled sample can be informative.

Among the three empirical papers mentioned above, Adrian, Etula, and Muir (2014) (AEM) and Avdjiev, Du, Koch, and Shin (2017) (ADKS) show signs of the alphas-into-betas effect inflating their price of risk, whereas the results in He, Kelly, and Manela (2016) (HKM) do not. In the subsample analysis of AEM, the price of risk of AEM falls by 1/3 to 2/3 in the subsamples compared to the pooled sample. ADKS is harder to analyze, since they do not formalize estimate the price of risk but plots different countries’ dollar betas with their cross-currency basis (Figure 4). There, they find the intercept of the fitted line to be positive, consistent with their test “assets” (covered interest parity trades) generating positive risk-adjusted returns within the sample whenever arbitrage capital is insufficient.23

Interestingly, this is consistent with the way these papers motivate their factors. AEM motivate their factor using the Brunnermeier and Pedersen (2009) model in which betas arise endogenously through price pressure, and an earlier draft (by Adrian and Etula, before it was merged with Muir’s) interpret the factor to reflect funding liquidity, as I do. ADKS interpret their “dollar beta” as an outcome of arbitrage by investors whose funding liquidity depends negatively on the strength of the dollar. However, HKM motivate their factor using He and Krishnamurthy (2013) in which intermediaries hold all equities and hence would not generate larger price pressure on higher pre-intermediary-alpha assets.

23A negative dollar beta in their paper measure a positive exposure to a favorable shock to arbitrage capital and hence needs to be interpreted as what I call a positive beta.
4.6 Alternative Measure of Arbitrage Capital Shocks

Although funding-liquidity shocks are important systematic shocks to arbitrageurs that I model, arbitrage capital is also exposed to variation coming from arbitrage portfolio return shocks. In the context of my model in Section 2, this is shock to wealth $w$ rather than $f$, where $k = w + f$ is deployable capital owned by each arbitrageur. I measure arbitrage portfolio return shocks as the equal-weighted portfolio of 40 anomalies that buys the long-side anomalies and shorts the short-side anomalies.

Table 8 shows that my key cross-sectional tests show similar results with respect to this measure of arbitrage capital shocks. For instance, column 1 shows that anomalies with a larger pre-1995 alpha with respect to this equal-weighted portfolio attains a larger post-1995 beta, controlling for pre-1995 beta whose effect of post-1995 beta is unity. Other columns show results consistent with the conclusion I draw in this paper.

5 Conclusion

In this paper, I show that the act of arbitrage plays an important role not just in ensuring the elimination of mispricings, but also in determining the cross-section of risks. Theoretically, I derive this prediction from a natural cross-sectional extension of the limits-of-arbitrage hypothesis on an individual asset. I find empirical evidence from equity anomalies whose pre-1995 CAPM alphas have turned into post-1995 funding-liquidity betas. This alphas-into-betas result reverses the direction of causality in intermediary asset pricing.

This alphas-into-betas effect can arise in any asset market in which “habitat” investors play an important role. In such a market, assets offering high “alpha” from the habitat investors’ perspective should subsequently attain a high beta with respect to capital owned by these investors. This key cross-sectional prediction along with a set of other predictions of my model can help identify the cross-section of risks arising endogenously in various asset markets.
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### Table 1: List of Anomalies

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<tr>
<th>No</th>
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<th>Authors</th>
<th>Year</th>
<th>Label</th>
<th>Market Cap Share</th>
<th>Stock Turnover</th>
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<td></td>
<td></td>
<td></td>
<td>Short</td>
<td>Long</td>
<td>Short</td>
</tr>
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<td>1</td>
<td>Beta arbitrage</td>
<td>Black</td>
<td>1972</td>
<td>beta</td>
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<td>0.19</td>
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<td>2</td>
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<td>1977</td>
<td>rome</td>
<td>0.04</td>
<td>0.35</td>
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<td>3</td>
<td>Ohlson's O-score</td>
<td>Ohlson</td>
<td>1980</td>
<td>ohlson</td>
<td>0.01</td>
<td>0.27</td>
</tr>
<tr>
<td>4</td>
<td>Size</td>
<td>Banz</td>
<td>1981</td>
<td>size</td>
<td>0.59</td>
<td>0.03</td>
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<tr>
<td>5</td>
<td>Value</td>
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<td>1985</td>
<td>value</td>
<td>0.21</td>
<td>0.1</td>
</tr>
<tr>
<td>6</td>
<td>Long-run reversals</td>
<td>DeBondt and Thaler</td>
<td>1987</td>
<td>rev60m</td>
<td>0.12</td>
<td>0.25</td>
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<tr>
<td>7</td>
<td>Momentum</td>
<td>Jegadeesh</td>
<td>1990</td>
<td>mom12m</td>
<td>0.04</td>
<td>0.36</td>
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<td>8</td>
<td>Net issuance</td>
<td>Ikenberry, Lakonishok, and Vermaelen</td>
<td>1995</td>
<td>netissue</td>
<td>0.08</td>
<td>0.19</td>
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<td>Net issuance monthly</td>
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<td>netissue_m</td>
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<td>Accruals</td>
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<td>1996</td>
<td>acc</td>
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<td>1996</td>
<td>roe</td>
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<td>0.33</td>
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<td>piotroski</td>
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<td>Idiosyncratic volatility</td>
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<td>Investment</td>
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<td>invest</td>
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<td>0.18</td>
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<td>Asset growth</td>
<td>Cooper, Gulen, and Schill</td>
<td>2008</td>
<td>atgrowth</td>
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<td>0.19</td>
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<td>16</td>
<td>Asset turnover</td>
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<td>2008</td>
<td>ato</td>
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<td>0.07</td>
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<td>17</td>
<td>Failure probability</td>
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<td>Gross margins</td>
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<td>0.09</td>
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<td>19</td>
<td>Gross profitability</td>
<td>Balakrishnan, Bartov, and Faurel</td>
<td>2010</td>
<td>profit</td>
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<td>0.09</td>
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<td>20</td>
<td>Return-on-assets</td>
<td>Chen, Novy-Marx, and Zhang</td>
<td>2010</td>
<td>roa</td>
<td>0.03</td>
<td>0.32</td>
</tr>
</tbody>
</table>
Table 2: Summary Statistics (Benchmark Alpha and Funding-Liquidity Beta)

This table summarizes the 40 short and long-side anomalies' benchmark alphas and funding-liquidity betas. Heteroskedasticity-robust White standard errors are used to compute the significance levels of the alphas and betas. The standard errors for the cross-sectional means are adjusted for cross-anomaly covariances in measurement errors. ***, **, and * indicate 1%, 5%, and 10% significance levels. Positive (negative) values that are significant at the 10% level are highlighted in blue (red). Returns are annualized.

### Panel A: CAPM as the Benchmark

<table>
<thead>
<tr>
<th>No</th>
<th>Anomaly</th>
<th>Short-side Anomalies</th>
<th>Long-side Anomalies</th>
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</thead>
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<td></td>
<td></td>
<td>Benchmark Alpha</td>
<td>Funding-Liquidity Beta</td>
</tr>
<tr>
<td>1</td>
<td>Beta arbitrage</td>
<td>-6.2***</td>
<td>-7.1***</td>
</tr>
<tr>
<td>2</td>
<td>Return-on-market equity</td>
<td>-9.3***</td>
<td>-7.9***</td>
</tr>
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<td>3</td>
<td>Ohlson’s O-score</td>
<td>-7.1***</td>
<td>-5.2***</td>
</tr>
<tr>
<td>4</td>
<td>Size</td>
<td>-0.4</td>
<td>-0.3</td>
</tr>
<tr>
<td>5</td>
<td>Value</td>
<td>-4.3***</td>
<td>-0.6</td>
</tr>
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<td>6</td>
<td>Long-run reversals</td>
<td>-3.9***</td>
<td>0.7</td>
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<td>7</td>
<td>Momentum</td>
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<td>-9.4***</td>
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<td>Net issuance</td>
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<td>9</td>
<td>Net issuance monthly</td>
<td>-2.8***</td>
<td>-5.6***</td>
</tr>
<tr>
<td>10</td>
<td>Accurals</td>
<td>-5.3***</td>
<td>-3.6</td>
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<tr>
<td>11</td>
<td>Return-on-book equity</td>
<td>-8.7***</td>
<td>-10.4***</td>
</tr>
<tr>
<td>12</td>
<td>Piotroski’s f-score</td>
<td>-3.1***</td>
<td>-0.8</td>
</tr>
<tr>
<td>13</td>
<td>Idiosyncratic volatility</td>
<td>-13.6***</td>
<td>-9.7***</td>
</tr>
<tr>
<td>14</td>
<td>Investment</td>
<td>-5.1***</td>
<td>-2.2</td>
</tr>
<tr>
<td>15</td>
<td>Asset growth</td>
<td>-4.2***</td>
<td>-3.3</td>
</tr>
<tr>
<td>16</td>
<td>Asset turnover</td>
<td>0.5</td>
<td>-4.7***</td>
</tr>
<tr>
<td>17</td>
<td>Failure probability</td>
<td>-5.4</td>
<td>-13.6***</td>
</tr>
<tr>
<td>18</td>
<td>Gross margins</td>
<td>-1</td>
<td>-1.2</td>
</tr>
<tr>
<td>19</td>
<td>Gross profitability</td>
<td>-1.3</td>
<td>-1.2</td>
</tr>
<tr>
<td>20</td>
<td>Return-on-assets</td>
<td>-9.3***</td>
<td>-8.1***</td>
</tr>
</tbody>
</table>

Cross-sectional Mean: -5.38*** | -4.99*** | 0.11 | -1.33*** | 2.32*** | 2.76*** | 0.05 | 0.08 |

### Panel B: A Six-Factor (Fama-French 5 + Pastor-Stambaugh Liquidity) Model as the Benchmark

<table>
<thead>
<tr>
<th>No</th>
<th>Anomaly</th>
<th>Short-side Anomalies</th>
<th>Long-side Anomalies</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Benchmark Alpha</td>
<td>Funding-Liquidity Beta</td>
</tr>
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<td>1</td>
<td>Beta arbitrage</td>
<td>-4.4</td>
<td>-3.5</td>
</tr>
<tr>
<td>2</td>
<td>Return-on-market equity</td>
<td>-13.8***</td>
<td>-1.7</td>
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<tr>
<td>3</td>
<td>Ohlson’s O-score</td>
<td>-7.3***</td>
<td>-0.5</td>
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<td>4</td>
<td>Size</td>
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<td>0.2</td>
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<tr>
<td>5</td>
<td>Value</td>
<td>-1.2</td>
<td>1.8*</td>
</tr>
<tr>
<td>6</td>
<td>Long-run reversals</td>
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<td>2.7</td>
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<tr>
<td>7</td>
<td>Momentum</td>
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<td>-5.3</td>
</tr>
<tr>
<td>8</td>
<td>Net issuance</td>
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<td>-3.6***</td>
</tr>
<tr>
<td>9</td>
<td>Net issuance monthly</td>
<td>1</td>
<td>-3.6</td>
</tr>
<tr>
<td>10</td>
<td>Accurals</td>
<td>-3.0</td>
<td>-2.1</td>
</tr>
<tr>
<td>11</td>
<td>Return-on-book equity</td>
<td>-10.3***</td>
<td>-3.3</td>
</tr>
<tr>
<td>12</td>
<td>Piotroski’s f-score</td>
<td>-3.0***</td>
<td>-0.2</td>
</tr>
<tr>
<td>13</td>
<td>Idiosyncratic volatility</td>
<td>-13.3***</td>
<td>-2.3</td>
</tr>
<tr>
<td>14</td>
<td>Investment</td>
<td>-3.1</td>
<td>-3.1</td>
</tr>
<tr>
<td>15</td>
<td>Asset growth</td>
<td>-0.5</td>
<td>-1.1</td>
</tr>
<tr>
<td>16</td>
<td>Asset turnover</td>
<td>2.9</td>
<td>-1.8</td>
</tr>
<tr>
<td>17</td>
<td>Failure probability</td>
<td>-10.7***</td>
<td>-7</td>
</tr>
<tr>
<td>18</td>
<td>Gross margins</td>
<td>-0.7</td>
<td>0.5</td>
</tr>
<tr>
<td>19</td>
<td>Gross profitability</td>
<td>1.6</td>
<td>1.7</td>
</tr>
<tr>
<td>20</td>
<td>Return-on-assets</td>
<td>-9.6***</td>
<td>-0.4</td>
</tr>
</tbody>
</table>

Cross-sectional Mean: -4.58*** | -1.62 | 0.06 | -0.95 | 1.12 | 1.68*** | -0.04 | 0.04 |
Table 3: **Turning Alphas into Betas**

Baseline: $\beta_{\text{post}}^{i,f} = b_0 + b_1 \alpha_{\text{pre}}^{i,CAPM} + u_i$

The dependent variable is post-1995 funding-liquidity beta net of exposure to the benchmark factor(s). In the parentheses are standard errors adjusted for cross-anomaly covariances in measurement errors. ***, **, and * indicate 1%, 5%, and 10% significance levels.

| Benchmark: CAPM | OLS | GLS |  | Alternative Benchmark Model | FF 3 Factors | FF 5 Factors | FF5 + PS Liquidity |
|-----------------|-----|-----|  |                              | (5)          | (6)          | (7)             |
| Pre-1995 Benchmark Alpha | 0.215*** | 0.216*** | 0.178*** | 0.167** | 0.175** | 0.194** | 0.199** | 0.197** | 0.199** |
| (0.073) | (0.074) | (0.031) | (0.078) | (0.083) | (0.088) | (0.091) | (0.093) |
| Pre-1995 Funding-Liquidity Beta | -0.203 | -0.291 |  | 0.458 | 0.360 | 0.416 |  | (0.378) | (0.309) | (0.487) | (0.423) | (0.467) |
| Constant | -0.294 | -0.607* | -0.274 | -0.183* | -0.118 | -0.137 | -0.124 | -0.130 | -0.117 | -0.118 |  | (0.256) | (0.357) | (0.256) | (0.098) | (0.239) | (0.248) | (0.248) | (0.253) | (0.237) | (0.241) |
| Observations | 40 | 40 | 40 | 40 | 40 | 40 | 40 | 40 | 40 | 40 |  |  |
| Adjusted $R^2$ | 0.60 | -0.01 | 0.61 | 0.57 | 0.57 | 0.60 | 0.60 | 0.62 | 0.60 | 0.62 |  |  |
Table 4: Alphas Turn into Larger Betas for Short-side Anomalies

Baseline: $\beta_{i,f}^{post} = b_0 + b_1\alpha_{i,CAPM}^{pre} + b_2\alpha_{i,CAPM}^{pre} \times \text{Short}_i + u_i$

The dependent variable is post-1995 funding-liquidity beta net of exposure to the benchmark factor(s). In the parentheses are standard errors adjusted for cross-anomaly covariances in measurement errors. ***, **, and * indicate 1%, 5%, and 10% significance levels.

<table>
<thead>
<tr>
<th>Benchmark: CAPM</th>
<th>Alternative Benchmark Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>Pre-1995 Benchmark Alpha</td>
<td>0.067</td>
</tr>
<tr>
<td>(0.097)</td>
<td>(0.095)</td>
</tr>
<tr>
<td>Pre-1995 Benchmark Alpha $\times$ Short</td>
<td>0.204</td>
</tr>
<tr>
<td>(0.142)</td>
<td>(0.122)</td>
</tr>
<tr>
<td>Pre-1995 Funding-Liquidity Beta</td>
<td>0.339</td>
</tr>
<tr>
<td>(0.375)</td>
<td></td>
</tr>
<tr>
<td>Pre-1995 Funding-Liquidity Beta $\times$ Short</td>
<td>-0.810**</td>
</tr>
<tr>
<td>(0.367)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.030</td>
</tr>
<tr>
<td>(0.250)</td>
<td>(0.244)</td>
</tr>
<tr>
<td>Observations</td>
<td>40</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.63</td>
</tr>
</tbody>
</table>
Table 5: Contemporaneous Arbitrage Activity Explains the Funding-Liquidity Betas

OLS: $\beta_{i,f}^{post} = b_0 + b_1 \text{ArbitrageActivity}_{i}^{post} + u_i$

2SLS: $\beta_{i,f}^{post} = b_0 + b_1 \hat{\text{ArbitrageActivity}}_{i}^{post} + u_i$

The dependent variable is post-1995 funding-liquidity beta net of exposure to the market factor. Short Interest Ratio (Not Detrended) is multiplied by 100 to be interpreted as percentage. In the parantheses are standard errors adjusted for cross-anomaly covariances in measurement errors. ***, **, and * indicate 1%, 5%, and 10% significance levels.

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<tr>
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<th>2SLS</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Short Interest Ratio</td>
<td>-5.257**</td>
<td>-6.496***</td>
</tr>
<tr>
<td></td>
<td>(2.213)</td>
<td>(2.363)</td>
</tr>
<tr>
<td>Short Interest Ratio (Not Detrended)</td>
<td>-1.085**</td>
<td>-1.345***</td>
</tr>
<tr>
<td></td>
<td>(0.465)</td>
<td>(0.486)</td>
</tr>
<tr>
<td>Constant</td>
<td>-1.355**</td>
<td>2.720*</td>
</tr>
<tr>
<td></td>
<td>(0.586)</td>
<td>(1.395)</td>
</tr>
<tr>
<td>Observations</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.63</td>
<td>0.63</td>
</tr>
</tbody>
</table>
Table 6: Do Post-Arbitrage Funding-Liquidity Betas Arise in Constrained Times?

Return process (post-1995): 
\[ r_{i,t} = (\mu_{i,c} + \beta_{i,m,c} r_{m,t} + \beta_{i,f,c} f_t) 1 (t \in Constrained) + (\mu_{i,u} + \beta_{i,m,u} r_{m,t} + \beta_{i,f,u} f_t) 1 (t \in Unconstrained) + \epsilon_{i,t} \]

The dependent variable is post-1995 constrained-time (columns 1-3) or unconstrained-time (columns 4-6) funding-liquidity beta net of exposure to the market factor. The constrained (unconstrained) times are the post-1995 quarters in which the VIX index was above (below) the post-1995 median value. In the parentheses are standard errors adjusted for cross-anomaly covariances in measurement errors. ***, **, and * indicate 1%, 5%, and 10% significance levels.

<table>
<thead>
<tr>
<th></th>
<th>Constrained-time Funding β</th>
<th>Unconstrained-time Funding β</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) (2) (3)</td>
<td>(4) (5) (6)</td>
</tr>
<tr>
<td>Short</td>
<td>-1.519*</td>
<td>-0.392</td>
</tr>
<tr>
<td></td>
<td>(0.826)</td>
<td>(0.607)</td>
</tr>
<tr>
<td>Long</td>
<td>0.173</td>
<td>-0.083</td>
</tr>
<tr>
<td></td>
<td>(0.274)</td>
<td>(0.302)</td>
</tr>
<tr>
<td>Pre-1995 CAPM Alpha</td>
<td>0.250**</td>
<td>0.058</td>
</tr>
<tr>
<td></td>
<td>(0.099)</td>
<td>(0.071)</td>
</tr>
<tr>
<td>Pre-1995 CAPM Alpha × Short</td>
<td>0.327**</td>
<td>0.057</td>
</tr>
<tr>
<td></td>
<td>(0.129)</td>
<td>(0.096)</td>
</tr>
<tr>
<td>Pre-1995 CAPM Alpha × Long</td>
<td>0.048</td>
<td>0.059</td>
</tr>
<tr>
<td></td>
<td>(0.130)</td>
<td>(0.156)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.290</td>
<td>-0.149</td>
</tr>
<tr>
<td></td>
<td>(0.324)</td>
<td>(0.306)</td>
</tr>
<tr>
<td>Observations</td>
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<td>40</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.34</td>
<td>0.05</td>
</tr>
</tbody>
</table>

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Table 7: Do Funding-Liquidity Betas Limit Arbitrage?

\[
\bar{r}_i^e = \lambda_0 + \lambda_f \beta_{i,f} + \beta_{i,g} \lambda_g + u_i \quad (\lambda_m = \bar{r}_i^e)
\]

This table presents the cross-sectional asset pricing results. The market risk premium is constrained to be the realized mean of the market factor. Adjusted $R^2$ is defined as the variation in mean excess return net of the market exposure explained by the other factor exposures. In the parentheses are Shanken (1992) standard errors. ***, **, and * indicate 1%, 5%, and 10% significance levels.

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<th></th>
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<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>Funding Liquidity</td>
<td>3.06***</td>
<td>0.41</td>
<td>4.87***</td>
<td>2.78**</td>
</tr>
<tr>
<td></td>
<td>(1.17)</td>
<td>(1.18)</td>
<td>(1.18)</td>
<td>(1.11)</td>
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<tr>
<td>Market</td>
<td>7.49</td>
<td>9.31</td>
<td>8.40</td>
<td>7.49</td>
</tr>
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<td>SMB</td>
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<td></td>
<td>(2.67)</td>
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<td>HML</td>
<td>8.04**</td>
<td>1.81</td>
<td>1.72</td>
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<tr>
<td></td>
<td>(3.49)</td>
<td>(3.19)</td>
<td>(3.23)</td>
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<td>RMW</td>
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<td></td>
<td>(2.98)</td>
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<td>CMA</td>
<td>6.84***</td>
<td>7.45***</td>
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<tr>
<td></td>
<td>(2.51)</td>
<td>(2.49)</td>
<td></td>
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</tr>
<tr>
<td>Market Liquidity</td>
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<td>(5.71)</td>
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<tr>
<td>Constant</td>
<td>0.81</td>
<td>-1.54**</td>
<td>-0.03</td>
<td>0.90*</td>
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<td></td>
<td>(0.60)</td>
<td>(0.76)</td>
<td>(0.69)</td>
<td>(0.53)</td>
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<td>Observations</td>
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<td>40</td>
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<tr>
<td>Adjusted $R^2$</td>
<td>0.70</td>
<td>-0.02</td>
<td>0.63</td>
<td>0.66</td>
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</tbody>
</table>
Table 8: Anomaly Portfolio Return as the Alternative Measure of Arbitrage Capital Shocks

This table repeats the key cross-sectional tests using equal-weighted anomaly return as the alternative measure of arbitrage capital shocks. All alphas and betas are with respect to this “meanfactor.” In the parantheses are standard errors adjusted for cross-anomaly covariances in measurement errors. ***, **, and * indicate 1%, 5%, and 10% significance levels.

<table>
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<tr>
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<th>αs into βs</th>
<th>Costs of Arbitrage</th>
<th>Arbitrage Interests</th>
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<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>Pre-1995 Alpha</td>
<td>0.135***</td>
<td>0.132***</td>
<td>0.127***</td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>(0.034)</td>
<td>(0.027)</td>
</tr>
<tr>
<td>Pre-1995 Beta</td>
<td>1.002***</td>
<td>0.986***</td>
<td>0.999***</td>
</tr>
<tr>
<td></td>
<td>(0.072)</td>
<td>(0.150)</td>
<td>(0.088)</td>
</tr>
<tr>
<td>Pre-1995 Beta × Short</td>
<td>-0.002</td>
<td>(0.015)</td>
<td></td>
</tr>
<tr>
<td>Pre-1995 Alpha × Size</td>
<td>0.021</td>
<td>(0.014)</td>
<td></td>
</tr>
<tr>
<td>Short × Size</td>
<td>-0.467</td>
<td>(0.382)</td>
<td></td>
</tr>
<tr>
<td>Long × Size</td>
<td>-0.785*</td>
<td>(0.456)</td>
<td></td>
</tr>
<tr>
<td>Pre-1995 Alpha × IdioVol</td>
<td>-0.017</td>
<td>(0.018)</td>
<td></td>
</tr>
<tr>
<td>Short × IdioVol</td>
<td>0.156</td>
<td>(0.523)</td>
<td></td>
</tr>
<tr>
<td>Long × IdioVol</td>
<td>0.138</td>
<td>(0.549)</td>
<td></td>
</tr>
<tr>
<td>Pre-1995 Alpha × Amihud</td>
<td>-0.021</td>
<td>(0.014)</td>
<td></td>
</tr>
<tr>
<td>Amihud × Short</td>
<td>0.464</td>
<td>(0.393)</td>
<td></td>
</tr>
<tr>
<td>Amihud × Long</td>
<td>0.839*</td>
<td>(0.441)</td>
<td></td>
</tr>
<tr>
<td>Pre-1995 Alpha × Bid-ask Spread</td>
<td>-0.011</td>
<td>(0.017)</td>
<td></td>
</tr>
<tr>
<td>Bid-ask Spread × Short</td>
<td>0.225</td>
<td>(0.635)</td>
<td></td>
</tr>
<tr>
<td>Bid-ask Spread × Long</td>
<td>0.117</td>
<td>(0.622)</td>
<td></td>
</tr>
<tr>
<td>Short Interest Ratio</td>
<td>-5.783***</td>
<td>(0.906)</td>
<td></td>
</tr>
<tr>
<td>Short Interest Ratio (Not Detrended)</td>
<td>-1.203***</td>
<td>(0.183)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-1.403**</td>
<td>-1.382**</td>
<td>-1.095*</td>
</tr>
<tr>
<td></td>
<td>(0.579)</td>
<td>(0.599)</td>
<td>(0.619)</td>
</tr>
<tr>
<td>Observations</td>
<td>40</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.87</td>
<td>0.87</td>
<td>0.88</td>
</tr>
</tbody>
</table>
Figure 2: **Prices of Anomalies and Arbitrage Capital (Time 1)**

The figure plots the time-1 price of anomaly \( i \) \( (p_{i,1}) \) as a function of aggregate arbitrage capital \( (K_1) \). The equilibrium price equals \( p_{i,1} = \max \left\{ \frac{v_i}{1+r_{\text{max}i}}, \frac{v_{i'}}{1+r_{\text{max}i'}} \right\} \), where \( i_1^* = \min \left\{ 0, 1 - \sqrt{2K_1} \right\} \) is the marginal anomaly.

Figure 3: **Funding-Liquidity Shocks Measured by Broker-Dealer Leverage Shocks**
Figure 4: Alphas Turn into Larger Betas for Short-side Anomalies

Figure 5: “Alphas into Betas” Is Not Driven by the Fundamental Cash-flow Component of Anomaly Returns
Figure 6: Short Interests Explain the Cross-Section of Funding-Liquidity Betas

Notes: The top two figures plot the relationship between pre-1995 CAPM alpha and pre-1995 short interest (Figure 6a) and pre-1995 CAPM alphas and post-1995 short interest (Figure 6b) of the short-side anomalies on the same scale. The $R^2$ in both figures tell us that pre-arbitrage $\alpha$s measured by pre-1995 CAPM alphas determine the cross-section of arbitrage interests measured by cross-sectionally standardized short interest ratio. However, the two periods contrast sharply in the extent of arbitrage: the post-1995 period features much greater arbitrage capital counteracting apparent mispricings in the anomalies. The bottom two figures plot the relationship between pre-1995 short interest and pre-1995 funding beta (Figure 6c) and post-1995 short interest and post-1995 funding beta (Figure 6d) of the short-anomalies on the same scale. Since the pre-1995 featured little arbitrage capital and low levels of short interests in the anomalies, these cross-section of short interests do not generate significant funding-liquidity exposures measured by funding-liquidity betas (Figure 6c). This contrasts sharply with the post-1995 period, in which the cross-section of short interests generate the cross-section of different funding-liquidity exposures (Figure 6d).
Figure 7: Constrained vs. Unconstrained Post-1995 Quarters As Determined by the VIX
## A Additional tables

Table A1: **Turning Alphas into Betas: Alternative Cutoff Years**

\[ \beta_{i,f}^{post} = b_0 + b_1 \alpha_{i,CAPM}^{pre} + u_i \]

The dependent variable is post-cutoff funding-liquidity beta net of market exposure for columns 1-4 and 1985-1995 funding-liquidity beta net of market exposure for column 5. In the parantheses are standard errors adjusted for cross-anomaly covariances in measurement errors. 

***, **, and * indicate 1%, 5%, and 10% significance levels.

<table>
<thead>
<tr>
<th>Pre-Cutoff CAPM Alpha</th>
<th>Alternative Cutoff Year</th>
<th>Placebo Cutoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.152**</td>
<td>0.173***</td>
<td>0.208***</td>
</tr>
<tr>
<td>(0.069)</td>
<td>(0.067)</td>
<td>(0.072)</td>
</tr>
<tr>
<td>Constant</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.240</td>
<td>-0.294</td>
<td>-0.197</td>
</tr>
<tr>
<td>(0.245)</td>
<td>(0.261)</td>
<td>(0.238)</td>
</tr>
<tr>
<td>Observations</td>
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<td>40</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.61</td>
<td>0.57</td>
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Table A2: **Alphas into Betas Controlling for Arbitrage Costs**

\[
\beta_{i,f}^{post} = b_0 + b_1 \alpha_{i,CAPM}^{pre} + b_2 \alpha_{i,CAPM}^{pre} \times Arbitrage\ Cost_i + b_3 \text{Short}_i \times Arbitrage\ Cost_i + b_4 \text{Long}_i \times Arbitrage\ Cost_i + u_i
\]

The dependent variable is post-1995 funding-liquidity beta net of exposure to the market factor. In the parentheses are standard errors adjusted for cross-anomaly covariances in measurement errors. ***, **, and * indicate 1%, 5%, and 10% significance levels.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-1995 CAPM Alpha</td>
<td>0.171**</td>
<td>0.178**</td>
<td>0.134**</td>
<td>0.133**</td>
</tr>
<tr>
<td></td>
<td>(0.069)</td>
<td>(0.072)</td>
<td>(0.059)</td>
<td>(0.060)</td>
</tr>
<tr>
<td>Pre-1995 CAPM Alpha × Size</td>
<td>-0.005</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.056)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Short × Size</td>
<td>0.692</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.731)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long × Size</td>
<td>0.057</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>(0.209)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-1995 CAPM Alpha × Amihud</td>
<td>-0.018</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.068)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Amihud × Short</td>
<td>-0.801</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.868)</td>
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<tr>
<td>Amihud × Long</td>
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</tr>
<tr>
<td></td>
<td>(0.225)</td>
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<tr>
<td>Pre-1995 CAPM Alpha × IdioVol</td>
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<tr>
<td></td>
<td>(0.046)</td>
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<tr>
<td>Short × IdioVol</td>
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<tr>
<td></td>
<td>(0.671)</td>
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<td></td>
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<tr>
<td>Long × IdioVol</td>
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<tr>
<td></td>
<td>(0.282)</td>
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<tr>
<td>Pre-1995 CAPM Alpha × Bid-ask Spread</td>
<td>0.031</td>
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<tr>
<td></td>
<td>(0.048)</td>
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<tr>
<td>Bid-ask Spread × Short</td>
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<tr>
<td></td>
<td>(0.582)</td>
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<tr>
<td>Bid-ask Spread × Long</td>
<td>-0.383</td>
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<tr>
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<td>(0.305)</td>
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<tr>
<td>Constant</td>
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<td>-0.336</td>
<td>-0.342</td>
<td>-0.302</td>
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<td></td>
<td>(0.296)</td>
<td>(0.307)</td>
<td>(0.310)</td>
<td>(0.289)</td>
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<tr>
<td>Observations</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.66</td>
<td>0.67</td>
<td>0.71</td>
<td>0.68</td>
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</tbody>
</table>
B Theory appendix

B.1 Lemma 4

Lemma 4. (Monotonicity of prices at time 0). For any \( i' < i'' \) such that \( i', i'' \in [0, 1] \),

\[
p_{i', 0} \geq p_{i'', 0}
\]

Proof. Suppose for a contradiction that \( i' < i'' \) but \( p_{i', 0} < p_{i'', 0} \). Suppose also that \( i'' \) is priced by arbitrageurs so that

\[
p_{i'', 0} = E_0 \left[ \frac{\Lambda_1}{\Lambda_0} p_{i'', 1} \right]
\]

Since \( p_{i', 1} \geq p_{i'', 1} \) in all states of \( t = 1 \), it must be that

\[
p_{i', 0} \geq E_0 \left[ \frac{\Lambda_1}{\Lambda_0} p_{i', 1} \right] \geq E_0 \left[ \frac{\Lambda_1}{\Lambda_0} p_{i'', 1} \right]
\]

which is a contradiction. Now suppose that \( i'' \) is priced by behavioral investors so that

\[
p_{i'', 0} = \frac{1}{1 + r_{\text{max}}'} E_0 [p_{i'', 1}]
\]

Since \( p_{i', 1} \geq p_{i'', 1} \) in all states of \( t = 1 \), it must be that

\[
p_{i', 0} \geq \frac{1}{1 + r_{\text{max}}'} E_0 [p_{i', 1}] \geq \frac{1}{1 + r_{\text{max}}'} E_0 [p_{i'', 1}]
\]

which is also a contradiction.
B.2 Proof of Lemmas 1 and 2 and Proposition 4

Proof of Lemma 1 (Equilibrium price at time 1). Trivially, if $K_1 \leq 0$, behavioral investors price all anomalies to ensure

$$E_1 [r_{i,2}] = \frac{v}{p_{i,1}} = 1 + r_{max} i,$$

which implies

$$p_{i,1} = \frac{v}{1 + r_{max} i}.$$

Next, suppose $K_1 \geq 0$ but arbitrageurs cannot eliminate abnormal returns completely. Then, by Lemma 3, there exists $i^* \in (0, 1)$ such that arbitrageurs exploit anomalies if and only if $i \in (i^*, 1]$ and earn the expected return

$$E_1 r_{i,2} = r_{max} i^*$$

from them. Then, behavioral investor demand for each anomaly $i \in (i^*, 1]$ is

$$B_{i,t} = i - i^*$$

in dollar position. Thus, for the market to clear, arbitrageur’s demand for the anomaly needs to be $x_{i,1} = i^* - i$. Integrating this over $(i^*, 1]$ should equal total arbitrage capital, so that

$$\int_{i^*}^{1} (i - i^*) \, di = K_1$$

This gives\(^{24}\)

$$i^* = 1 - \sqrt{2K_1}$$

and

$$p_{i,1} = \frac{v}{1 + r_{max} i^*}.$$

\(^{24}\)Note that the other root is ruled out since it is always greater than 1, the largest possible value of $i^*_1$. 

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On the other hand, unexploited anomalies are priced by behavioral investors so that for all 
$i \in [0, i^*_1]$, 
\[
p_{i,1} = \frac{v}{1 + r_{max}^i}
\]
Finally, suppose arbitrageurs are unconstrained; that is, $K_1 \geq 1/2$. Then, all anomalies are 
fully exploited so that 
\[
p_{i,1} = v
\]
for all $i \in [0, 1]$.

**Proof of Lemma 2 (Endogenous risk).** The negative covariance follows from the fact that 
\[
\frac{\partial \Lambda_1}{\partial p_{i,1}} = \frac{\partial \Lambda_1}{\partial k_1} = - \frac{v}{1 + r_{max}^i} \leq 0
\]
(i) Suppose $\mu \to 0^+$. Then, $K_1 \to 0^+$ so that $\Lambda_1 = 1 + r_{max}$ and 
$p_{i,1} = v / (1 + r_{max}^i)$.
Since both are deterministic, 
\[
\lim_{\mu \to 0} Cov_0 (p_{i,1}, \Lambda_1) = 0
\]
(ii) Note 
\[
\frac{\partial Cov_0 (p_{i,1}, \Lambda_1)}{\partial (\alpha_i)} = \frac{\partial Cov_0 (p_{i,1}, \Lambda_1)}{\partial (r_{max}^i)} = \frac{\partial Cov_0(p_{i,1},\Lambda_1)}{\partial k_1} \frac{\partial k_1}{\partial (r_{max}^i)} = r_{max}^i \times \frac{\partial Cov_0 (p_{i,1}, \Lambda_1)}{\partial i}
\]
Now, since $Cov_0 (p_{i,1}, \Lambda_1) = E_0 [\Lambda_1 p_{i,1}] - E_0 [\Lambda_1] E_0 [p_{i,1}]$, 
\[
Cov_0 (p_{i,1}, \Lambda_1) = v \int_{-\infty}^{0} \frac{1 + c}{1 + r_{max}^i} dF (K_1) + v \int_{0}^{K_1(i)} \frac{1 + r_{max}^i}{1 + r_{max}^i} dF (K_1) + v \int_{K_1(i)}^{\infty} dF (K_1)
\]
\[
-v E_0 [\Lambda_1] (\int_{-\infty}^{0} \frac{1}{1 + r_{max}^i} dF (K_1) + \int_{0}^{K_1(i)} \frac{1}{1 + r_{max}^i} dF (K_1) + \int_{K_1(i)}^{\infty} dF (K_1))
\]
\[
+v \int_{K_1(i)}^{1/2} \frac{1}{1 + r_{max}^i} dF (K_1) + v \int_{1/2}^{\infty} dF (K_1),
\]
where $K_1 (i)$ denotes the value of $K_1$ that makes $i$ the marginal anomaly, and $F$ is the 
conditional cumulative density function of $K_1$. Thus, the derivative of the covariance with
respect to \(i\) gives

\[
\frac{\partial \text{Cov}_0 (p_{i,1}, \Lambda_1)}{\partial i} = -v \left( \int_{-\infty}^{0} \frac{(1 + c) r_{\text{max}}}{(1 + r_{\text{max}} i)^2} dF (K_1) + \int_{0}^{K_1(i)} \frac{(1 + r_{\text{max}} i_{1}^*) r_{\text{max}}}{(1 + r_{\text{max}} i)^2} dF (K_1) \right)
\]

\[+ E_0 [\Lambda_1] v \left( \int_{-\infty}^{0} \frac{r_{\text{max}}}{(1 + r_{\text{max}} i)^2} dF (K_1) + \int_{0}^{K_1(i)} \frac{r_{\text{max}}}{(1 + r_{\text{max}} i)^2} dF (K_1) \right),\]

where the Leibniz terms cancel out by the fact that \(i_{1}^* (K_1 (i)) = i\). Rearranging the terms gives

\[
\frac{\partial \text{Cov}_0 (p_{i,1}, \Lambda_1)}{\partial i} = -\frac{vr_{\text{max}}}{(1 + r_{\text{max}} i)^2} \left( \int_{-\infty}^{K_1(i)} \Lambda_1 dF (K_1) - E_0 [\Lambda_1] \int_{-\infty}^{K_1(i)} dF (K_1) \right)
\]

\[= -\frac{vr_{\text{max}}}{(1 + r_{\text{max}} i)^2} (E_0 [\Lambda_1 | i \leq i_{1}^*] - E_0 [\Lambda_1]) F (K_1 (i)) < 0\]

since \(E_0 [\Lambda_1 | i \leq i_{1}^*] > E_0 [\Lambda_1]\).

**Proof of Proposition 4** (*"Intermediary asset pricing" of anomalies*). (16) implies that exploited anomalies are priced according to

\[1 = E_0 [\Lambda_1 \Lambda_0^{-1} (1 + r_{i,1})]\]

so that \(1 = E_0 [\Lambda_1 / \Lambda_0] E_0 [1 + r_{i,1}] + \text{Cov}_0 (\Lambda_1 / \Lambda_0, r_{i,1}^e)\). This gives

\[E_0 [r_{i,1}] = \frac{1}{E_0 [\Lambda_1 / \Lambda_0]} - 1 + \lambda \beta_{i}^{\text{post}}\]

If arbitrageurs are unconstrained, \(\Lambda_0 = E_0 [\Lambda_1]\) so that the zero-beta rate becomes the risk-free rate of zero. If arbitrageurs are constrained, \(\Lambda_0 = \max_{i \in [0,1]} E_0 [\Lambda_1 (1 + r_{i,1})] > E_0 [\Lambda_1]\) so that the zero-beta rate is higher than the risk-free rate of zero; otherwise, arbitrageurs are not optimally choosing the set of exploited anomalies.
C Empirical appendix

C.1 Variance-covariance matrix of $\hat{b}$ and bootstrapping

Rewrite (26) as

$$ r^e_i = \mu_i 1_n + F \beta^\text{post}_i + \epsilon_i $$

(37)

where $r^e_i$ is a $T$-vector of excess returns, $F$ is a $T \times k$ matrix of factor realizations, and $\epsilon_i$ is a $T$-vector of unexplained returns. Then, the measurement error induced by the OLS estimation of $\beta^\text{post}_{i,f}$ is

$$ e_i \equiv \hat{\beta}^\text{post}_{i,f} - \beta^\text{post}_{i,f} = (0, 0, ..., 1) \left( F'_d F_d \right)^{-1} F'_d \epsilon_i $$

(38)

where $F_d$ is a matrix of factor realizations demeaned by the realized time-series sample means.

Hence, (28) can be explicitly written as

$$ \hat{\beta}^\text{post}_{i,f} = x'_i b + u_i + (0, 0, ..., 1) \left( F'_d F_d \right)^{-1} F'_d \epsilon_i $$

(39)

It follows from (39) that the finite-sample conditional variance-covariance matrix of $\hat{b}$ is

$$ V_b = (X'X)^{-1} X' \left( \Omega_u + Var \left( \epsilon' F'_d \left( F'_d F_d \right)^{-1} (0, 0, ..., 1)' \right) \right) X' (X'X)^{-1} $$

(40)

where $X \equiv (x_1, ..., x_n)$, $\Omega_u \equiv Var \left( (u_1, ..., u_n) \right)$, and $\epsilon$ is a $T \times n$ matrix of unexplained returns. This shows that the correct standard errors that account for the measurement errors are (i) higher due the second component $\Omega_e$ within the parantheses and are (ii) affected by the cross-sectional covariances $Cov \left( \hat{\beta}^\text{post}_{i,f}, \hat{\beta}^\text{post}_{j,f} | X \right) \neq 0$ between correlated anomalies.

Since $\Omega_e$ cannot be expressed as a function of $\Sigma \epsilon$ and $\Sigma F$, I estimate it through bootstrapping. To do this, I draw the cross-sectional vector $(\hat{\epsilon}_{1,t}, ..., \hat{\epsilon}_{n,t}, \hat{g}_t, \hat{f}_t)$ from a randomly chosen time period within the sample ($t = 1, ..., T$) $T$ times with replacement to obtain a vector of bootstrapped measurement errors $e_s \equiv (e_{1,s}, ..., e_{n,s})$. Then, I repeat this 10,000 times to obtain $e_1, ..., e_{10,000}$ and use this to estimate $\Omega_e$. Finally, I augment usual heteroskedasticity-robust White standard
error estimation with this estimated $\hat{\Omega}_e$, thereby accounting for the increased variance and co-
variances of errors induced by measurement errors. The analogous standard error corrections for
for GLS and 2SLS are straightforward.

C.2 Computing cash-flow betas

I follow the methodology outlined in Campbell, Polk, and Vuolteenaho (2010). To measure the
cash-flow news in anomaly $i$ in month $t$, I use the return on equity (ROE) of the firms that have
earnings announcement at $t$. To do this, I compute clean-surplus earnings that equals the earnings
adjusted for equity offerings for earnings-announcing firms and equals zero for non-announcing
firms:

$$X_t = \begin{cases} 
\left[\frac{(1+r_t)ME_{t-1}-D_t}{ME_t}\right] \times BE_t - BE_{t-1} + D_t & \text{if } BE_t \neq BE_{t-1} \\
0 & \text{if } BE_t = BE_{t-1} 
\end{cases} \quad (41)$$

I then compute the time $t+k$ ROE of anomaly $i$ sorted at $t$ as

$$ROE_{i,t,t+k} = \frac{X_{i,t,t+k}}{BE_{i,t,t+k-1}} \quad (42)$$

where the $X$ and $BE$ here are sums across all firms belonging to anomaly portfolio $i$. Finally,
the cash-flow proxy for anomaly $i$ at $t$ (sorted at $t$) is

$$N_{i,CF,t} = \sum_{k=0}^{K-1} \rho^k roe_{i,t,t+k} \quad (43)$$

where $roe_{i,t,t+k} = \ln (1 + ROE_{i,t,t+k}) - 0.4 \ln (1 + r_{f,t+k})$ is log ROE adjusted for inflation.
Since the factors are quarterly, I take the geometric average of this monthly $roe$ over a quarter before computing the betas. To compute the betas, I also compute the discounted sum of quarterly
factor realizations analogous to (43). I use $K = 24$ months and $\rho = 0.975^{1/12}$ (the result in
Figure 5 is robust to using $K = 36$ or 48).