Speed Acquisition*

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Abstract

Speed has become a salient feature of modern financial markets. This paper studies investors’ endogenous speed acquisition, alongside their information acquisition. In equilibrium, speed heterogeneity endogenously arises across investors, temporally fragmenting the price discovery process. A deterioration in the long-run price informativeness ensues. Intra- and inter-temporal competition among investors drive speed and information to be either substitutes or complements. The model cautions the possible dysfunction of price discovery: An advancing information technology might complement speed acquisition, which fragments the price discovery process, thus hurting price informativeness. Novel predictions are discussed regarding investor composition, fund performance, and trading volume.

Keywords: speed, information, technology, price discovery, price informativeness
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1 Introduction

Price discovery is a fundamental function of financial markets. It involves two steps: First, investors acquire information about the asset. Second, via trading, such information is incorporated in price. The first step determines the amount of information that the price can eventually reflect, i.e. the magnitude of price discovery. The second step is about the process of how acquired information aggregates into price. Notably, speed is an intrinsic characteristic underlying the price discovery process: Investors race to be the first to reap the information rent.

To date, the literature has mainly emphasized information acquisition, i.e. the magnitude aspect of price discovery, following the pioneering works by Grossman and Stiglitz (1980) and Verrecchia (1982). This paper complements this canonical perspective with an enriched price discovery process, by studying investors’ speed acquisition alongside their information acquisition.

The notion of speed roots in the course of financial securities trading. Loosely speaking, there are three aspects in terms of trading speed: First, after acquiring information, investors can form a concrete trading idea sooner—from the raw data—by hiring a larger analyst team or buying more computers. Second, before execution, an trading order needs to journey through middle/back offices for risk management, due diligence, and compliance. The tightening regulatory environment in recent years arguably has slowed down this second aspect (see, e.g., Thomson Reuters’s Annual Cost of Compliance Report). Third, from trading desks and onward, the order execution speed depends on technology investments on computer hardware, algorithms, and connection to exchange servers. This last aspect has progressed drastically in the last decade, evidenced by the rise of machines—algorithmic and high-frequency trading technologies.

The above aspects of trading speed raises a set of questions: How much speed technology should which investors acquire? Is speed technology favored over information technology? Which securities attract more fast investors than others? Most importantly, what are the implications for the overall quality of price discovery and information efficiency?
This paper develops a model to address these questions. The model builds on an economy populated by a fixed measure of atomless investors, who first invest in both speed and information technology and then trade a risky asset. The information technology determines an investor’s private signal precision about the asset value, while the speed technology allows him to trade ahead of his peers. To fix the idea, consider a hedge fund for example. Its information acquisition involves investments in, e.g., sending analysts for firm visits or buying various datasets. The fund’s speed acquisition covers a different aspect. Based on the aforementioned three aspects, it can invest in equipment, infrastructure, or staff to speed up processing the acquired (raw) data, to streamline the compliance process, and to expedite order execution by its trading desk.

The rent-seeking investors in the model have incentive to acquire both technologies. The information technology directly adds to one’s information rent. Indirectly via the speed technology, the sooner an investor trades, the less price discovery has already occurred and the more rent he can extract—a “first-mover advantage”. The equilibrium is found where each investor optimally acquires the technologies to maximize his information rent, taking into account the investment costs and the competition from others.

A driving feature of the model is the temporal fragmentation effect of the speed technology. Investors acquiring different speed trade at different times. Accordingly, the price discovery process also splits into parts, e.g., an early fragment with fast investors and a late fragment with the slow. So long the speed technology is affordable, such fragmentation is robust in equilibrium: Though all investors want to acquire speed to enjoy the “first-mover advantage”, not everyone will be equally fast, for otherwise there is no “first-mover” and some will want to stay slow to save the speed acquisition cost. Such speed heterogeneity thus temporally fragments the price discovery process.

The fragmented price discovery process delivers novel insights. First, through temporal fragmentation, the speed technology inflicts a nonmonotonic impact on price informativeness, i.e., the magnitude. With a more advanced (cheaper) speed technology, more investors become fast, increasing the magnitude of price discovery in the early fragment. At the same time, fewer investors
remain slow and the late fragment shrinks. The market’s eventual price informativeness, therefore, can be either improved or hurt, depending on whether the boost (early) overcomes the decay (late). This result holds even when information acquisition is shut down.

Second, speed and information can be either substitutes or complements. Consider, for example, a positive shock in the information technology, upon which all investors acquire more information. How is the demand for speed affected? The answer depends on the relative change between fast and slow investors’ rents. As everyone acquires more information, intratemporal competition intensifies, attenuating the rents for both the fast and the slow investors, as in Grossman and Stiglitz (1980). New in this model, the increased early price discovery intertemporally hurts the slow. (Intuitively, if the fast have done almost all the price discovery, little rent will be left for the slow.) Netting the intra- and intertemporal effects, if the fast (the slow) are hurt more, some of them are better off staying slow (becoming fast) instead, in which case speed substitutes (complements) information. Overall, the model shows that either technology has a nonmonotonic effect on the aggregate demand for the other.

Third, an advancement in information technology might still hurt the long-run price informativeness. The reason is that, due to complementarity, an improving information technology also stimulates demand for speed, furthering the fragmentation of the price discovery process. The fragmentation boosts the fast but shrinks the slow fragment of price discovery magnitude. When the decay in the slow fragment dominates, the overall magnitude of price discovery worsens. The key mechanisms at work, as taught by the model, are the temporal fragmentation by the speed technology and its endogenous complementarity with information.

The last result above cautions the dysfunction of information aggregation in financial markets. The “information technology” in the model can be interpreted broadly. For example, recent years have seen strengthened transparency and disclosure requirements by regulators. Policies like Sarbanes-Oxley, Regulation Fair Disclosure, and Rule 10b5-1 have arguably reduced the cost of information acquisition. In the meantime, there is evidence of speed acquisition complementing the
accessibility of information. Du (2015) finds that high-frequency traders are constantly crawling the website of U.S. SEC in order to trade on the information in latest company filings. Through such a complementarity channel, this paper argues that transparency and disclosure policies might generate unintended negative impact on price informativeness.

Some recent empirical evidence echoes this view. Weller (2016) shows that algorithmic trading has risen at the cost of long-run price discovery. Gider, Schmickler, and Westheide (2016) shows how high-frequency trading hurts the predictability of earnings in the far future. To emphasize, the mechanism put forward in this paper is new. For example, the argument by Weller (2016), and via equilibrium models by Dugast and Foucault (2017) and by Kendall (2017), is that short-run (early) price discovery can crowd out the acquisition of more precise information in the long-run (late)—a substitution effect. In contrast, the current paper emphasizes the endogenous complementarity between information and speed acquisition. As the information technology advances and incentivizes more investors to acquire speed, the price discovery process fragments at the cost of the (long-run) magnitude.

Different financial assets are exposed to different levels of information and speed technology. The model thus also offers cross-sectional predictions of how technology advancement might affect different assets (e.g., stocks) differently. Bai, Philippon, and Savov (2016) finds a rising trend of the price informativeness of S&P 500 nonfinancial firms in a half-century sample period starting from the 1960s. The finding for firms beyond the S&P 500, however, is the opposite. Farboodi, Matray, and Veldkamp (2017) reproduce the patterns and explain these phenomena through investors’ attention-constrained information acquisition. This paper adds to the discussion that the distinction in different technologies—speed v.s. information—is important in determining individual stocks’ respective price informativeness over the years.

The model further yields predictions on investor composition across assets. Some assets have lower information acquisition costs (e.g., higher media exposure and analyst coverage) than others, thus attracting investors of different speed. For example, heavily regulated mutual funds and
pension funds arguably trade more slowly—due to the time spent on compliance, due diligence, etc.—than lightly regulated hedge funds and proprietary trading firms. Thus, depending on the exact degree of substitution or complementarity between the two technologies, the model details which assets will attract more hedge funds (fast) than mutual funds (slow). Specifically, the model predicts that hedge funds’ activity (relative to mutual funds’) in small, medium, and large stocks has a nonmonotonic pattern, matching the empirical finding by Griffin and Xu (2009). These results further shed light on the implication of technology advancements on fund performance.

The rest of the paper is organized as follows. Section 2 discusses the related literature. Section 3 sets up the model and Section 4 derives its equilibrium. Section 5 then explores the model implications on investors’ technology acquisition and on price informativeness. Discussions on model assumptions, robustness, and extensions are collated in Section 6. Section 7 then concludes.

2 Related literature

The current modeling framework builds on a series of papers featuring two trading rounds: Grundy and McNichols (1989), Froot, Scharfstein, and Stein (1992), Hirshleifer, Subrahmanyan, and Titman (1994), Holden and Subrahmanyan (1996), Brunnermeier (2005), Banerjee, Davis, and Gondhi (2017), and Dugast and Foucault (2017), among others. A distinguishing feature in the current model is that investors are explicitly allowed to engage in costly speed acquisition, separately from the conventional information acquisition. Instead, in the above literature, investors either cannot choose their speed at all or do not directly invest in speed per se. Rather, the notion of speed appears as a by-product of, hence not separable from, investors’ choice in different

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1 Hedge funds and proprietary trading firms are also known to invest more in the other two aspects of speed. In terms of information processing, one most salient trend recently is hedge funds’ massive investment in machine learning and artificial intelligence; see, e.g., “The Massive Hedge Fund Betting on AI”, September 27, 2017, Bloomberg. In terms of trading technology, it is well known that many of the high-frequency traders are hedge funds and proprietary trading firms (SEC, 2010, IV.B).
types of information.\textsuperscript{2} Separating speed from information, this paper directly studies how the two technologies affect the market differently. Section 5.2 demonstrates that, even without information acquisition, speed technology alone yields a nonmonotonic impact on price informativeness.

Another common denominator among these papers is some form of “substitution” (in information acquisition) that can hurt long-run price informativeness. For example, in Dugast and Foucault (2017), a cheaper raw signal can induce lower investment in the processed signal; in Banerjee, Davis, and Gondhi (2017), a public disclosure reduces investors’ acquisition about the fundamental, as they switch to learning about others’ beliefs. The current model inherits such substitution (the fast intertemporally crowding out the slows), but it does not drive the key result. In fact, Section 5.1 shows that, when speed acquisition is shut down, a cheaper information technology always improves price informativeness.

Instead, once speed acquisition is switched on, a better information technology can hurt price informativeness (Section 5.3). This result arises from the novel complementarity between speed and information (as opposed to substitution): Through a better information technology, the complementarity drives more investors to become fast, fragmenting the price discovery process and hurting price informativeness. Two more unique predictions follow (Panel c and d in Figure 4): 1) Information technology yields a nonmonotonic impact on the composition of fast and slow investors, depending on whether complementarity or substitution dominates. 2) When complementarity prevails, increasingly more fast investors are seen together with stronger information acquisition by the slow—as if speed “crowds in” information. To emphasize, the above new insights only arise from modeling speed and information separately.

This paper further contributes to three themes of the literature. First, the literature on costly information acquisition largely focuses on the magnitude aspect of price discovery, following the

\textsuperscript{2} In Grundy and McNichols (1989) and Brunnermeier (2005), all investors trade in both rounds, hence no speed. Investors’ speed are exogenously assigned in Froot, Scharfstein, and Stein (1992); Hirshleifer, Subrahmanyam, and Titman (1994); and Banerjee, Davis, and Gondhi (2017). Investors invest in speed indirectly by choosing between the short-horizon v.s. the long-horizon information in Holden and Subrahmanyam (1996); or between raw and/or processed signals in Dugast and Foucault (2017).
seemal works by Grossman and Stiglitz (1980) and Verrecchia (1982). Recent studies explore other dimensions. To name a few, Peress (2004, 2011) studies the wealth effect on information acquisition. Van Nieuwerburgh and Veldkamp (2009, 2010) analyze information acquisition under limited attention. Goldstein and Yang (2015) explore the implication of information diversity. To compare, the above literature assumes that the market always clears with all investors trading at the same time—they have the same speed. With endogenous speed acquisition, this paper allows to study the process of price discovery with investors arriving and trading asynchronously.

Second, the temporal fragmentation (due to speed technology) in this paper differs from the existing literature on spatial market fragmentation. Regarding the focus on price discovery, an important feature of temporal fragmentation is that the information revealed in an early fragment naturally carries over to all later fragments—the market never forgets. Thus, slow investors’ information rent is eroded away by fast investors, giving rise to the intertemporal crowding-out effect. Such natural accumulation of information over time is critical in determining the complementarity or substitution between the two technologies. In a model of multiple venues (spatial fragmentation), there is no naturally directional “flow” of information from one venue to another (more fundamentally, the notion of speed does not apply to a spatial setting). Focusing on speed, this paper studies a novel angle of market fragmentation.

Third, this paper lends equilibrium support to the literature with endogenous bundling of speed and information acquisition. The model predicts that fast investors always acquire more information than the slow. This is because price discovery always accumulates over time and the same piece of information has higher marginal benefit the sooner it is traded. This insight justifies a popular connotation for fast traders that they are also more informed. See, e.g., models by Hoffmann.

For example, Admati (1985), Pasquariello (2007), Boulatov, Hendershott, and Livdan (2013), Goldstein, Li, and Yang (2014), Cespa and Foucault (2014), among many others, study information and cross-market learning of correlated assets. Pagano (1989), Chowdhry and Nanda (1991), and Baruch, Karolyi, and Lemmon (2007) study trading of the same asset on different venues (e.g., dual-listed stocks). More recently, market fragmentation has been theorized in the context of dark v.s. lit trading mechanisms, as in Ye (2011), Zhu (2014), Brolley (2016), and Buti, Rindi, and Werner (2017). Finally, Chao, Yao, and Ye (2017a,b) study the competition among exchanges by zooming in on fee structure and tick size.
(2014), Biais, Foucault, and Moinas (2015), and Budish, Cramton, and Shim (2015); evidence by Brogaard, Hendershott, and Riordan (2014) and Shkilko and Sokolov (2016); and surveys by Biais and Foucault (2014), O’Hara (2015), and Menkveld (2016).

In a different line, investors’ speed choice has been studied in limit order models with discrete prices. Examples include Yao and Ye (2017) and Wang and Ye (2017). The main driving feature is the binding tick size which limits investors’ competition on price and as a result they turn to speed competition. The current paper focuses on investors’ incentive to acquire speed due to the transitory nature of informatin advantage.

3 Model

 Assets. There is a risky asset and a risk-free numéraire. At the end of the game, each unit of the risky asset will pay off a normally distributed random amount $V$ units of the numéraire. Without loss of generality, normalize $EV$ to 0. Denoted by $\tau_0^{-1}$ ($> 0$) the unconditional variance of $V$.

Investors. There is a unity continuum of atomless investors, indexed by $i \in [0, 1]$. They have constant absolute risk-aversion (CARA) preference with the same risk-aversion coefficient $\gamma$ ($> 0$).

Speed technology. An investor $i$ can invest in a speed technology to affect $t_i$, a set of time points when he can trade in the market (see “Timeline” below). Without investing in speed, all investors are slow, trading at $t_i = t_S = \{2\}$ (“S” for slow). One can instead become fast and trade at $t_i = t_F$ (“F” for fast) by paying $1/g_t$ units of the numéraire. The exogenous parameter $g_t$ ($> 0$) measures the level of speed technology. The larger is $g_t$, the more advanced (cheaper) is the technology.

This paper explores two scenarios for fast investors: 1) They can only trade at $t_F = \{1\}$, “pure speed differential”; or 2) they can trade at both dates with $t_F = \{1, 2\}$, “frequent fast trading”. The first scenario applies to situations where, for example, fast active funds buy-and-hold (due to, e.g., transaction costs) some securities for long-term investment purposes. The second scenario speaks to funds that are more flexible in frequent, active trading strategies. The analysis will mainly
focus on the first scenario to articulate the model’s main intuition. The second scenario, studied in Section 6.1, serves as a robustness check for the main results.

**Information technology.** Before trading, each investor $i$ observes a private signal $S_i$ about the payoff $V$. Specifically, $S_i = V + \varepsilon_i$, where $\varepsilon_i$ is independent of $V$, independent of any other $\varepsilon_{j\neq i}$, and normally distributed with zero mean and variance $h_i^{-1} (> 0)$. The investor $i$ can spend $m_i \geq 0$ units of the numéraire on an information technology to improve his private signal precision:

$$h_i = g_h k_h(m_i),$$

where $k_h(\cdot)$ is twice-differentiable, concave, and strictly monotone increasing; and $g_h \geq 0$ is a parameter measuring the marginal productivity of this information technology. Without investing in this technology, the investor gets no signal; i.e. $k_h(0) = 0$.

Due to the monotonicity of $k_h(\cdot)$, an investor’s information acquisition can be referred to as either $h_i$ (the precision) or $m_i$ (the cost) interchangeably: There exists a weakly convex, monotone increasing information acquisition cost function $c(\cdot)$, so that $\forall h_i \geq 0$,

$$m_i = c(h_i) := k_h^{-1}(h_i/g_h).$$

To ensure that there is always some information in the market, let $c(0) = 0$; equivalently, $k_h(0) \to \infty$.

The information technology is assumed to be orthogonal to the speed technology ($g_t$ and $g_h$ are exogenous parameters, independent of each other). This is an intentional modeling choice, so that the comparative static analyses will help isolate the effect of one technology against the other. In reality, the two technologies will likely affect each other. Section 6.2 discusses such interdependence and its implications.

**Timeline.** There are four dates in the model: time $t \in \{0,1,2,3\}$, as illustrated in Figure 1. At $t = 0$, all investors independently invest in technologies $t_i$ and $h_i$. Time $t \in \{1,2\}$ are trading rounds. The set of investors $\{i | t \in t_i\}$ arrive at $t$ together and they independently submit demand schedules $\{x_i(p; \cdot)\}$ to trade the risky asset, based on his information set—private signal $s_i$, his
existing holding of the asset (if any), and the public history of past prices. Specifically, at $t = 1$ only fast investors arrive and trade. At $t = 2$, only slow investors trade under “pure speed differential” (main model, Section 4 and 5), while all investors trade under “frequent fast trading” (robustness check, Section 6.1). Finally, at $t = 3$, the risky asset liquidates at $V$ and all investors consume their terminal wealth.

**Trading.** In each trading round $t \in \{1, 2\}$ there is noise demand $U_t$, which is independent of all other random variables and is i.i.d. normally distributed with zero mean and variance $\tau^{-1}$. (Section 6.3 discusses the robustness to time-varying noise trading.) The aggregate demand at $t$ is

\[
L_t(p) = \int_{i[0,1]} x_t(p; \cdot) \mathbb{1}_{\{t\in t\}} di + U_t.
\]

There is a competitive market maker, who clears the market at all times at the efficient price given all historical public information (as in Kyle, 1985). Thus, the trading price in each round $t$ is

\[
P_t = \mathbb{E}[V \{L(\cdot; r)\}_{r \leq t, Vr \in \{1,2\}}].
\]

This way, the market price is always (semi-strong) efficient and suits the purpose of studying price informativeness. See also Hirshleifer, Subrahmanyam, and Titman (1994); Vives (1995); Holden

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**Figure 1: Timeline of the game.** The model has four dates: $t \in \{0, 1, 2, 3\}$. At $t = 0$, all investors invest in technology; at $t \in \{1, 2\}$, investors arrive in the market at the time(s) according to their speed technology and submit their demand schedules to trade the risky asset; finally, at $t = 3$ the risky asset liquidates and all investors consume their terminal wealth. The figure outlines two scenarios: Under “pure speed differential”, fast investors only trade once at $t = 1$ and there are only slow investors trading at $t = 2$. Under “frequent fast trading”, fast investors can also trade at $t = 2$. 

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and Subrahmanyam (1996); and Cespa (2008). Section 6.4 discusses the robustness of this choice.

**Strategy and equilibrium definition.** To sum up, each investor maximizes his expected utility over the final wealth by optimizing his (cumulative) demand $x_i(\cdot)$ at each $t \in t_i$; and, backwardly, by choosing his technology pair $(t_i, h_i) \in \{t_S, t_F\} \times [0, \infty)$ at $t = 0$.

Denote by $\pi(t_i, h_i)$ the investor $i$’s ex ante certainty equivalent (whose functional form will be derived below). Define $\mathcal{P} := \{(t_i, h_i)\}_{i \in [0,1]}$ as the collection of all investors’ investment policies. A Nash equilibrium is a collection $\mathcal{P}$, such that for any investor $i$, fixing $\mathcal{P}\setminus(t_i, h_i)$, he has $\pi(t_i, h_i) \geq \pi(t, h), \forall (t, h) \in \{t_S, t_F\} \times [0, \infty)$.

## 4 Equilibrium analysis

This section studies the equilibrium under “pure speed differential”, where the speed technology enables fast investors to trade early, but only, once at $t_F = \{1\}$. This scenario has more tractability and allows the analysis to zoom in on the economic mechanisms behind the results. Section 6.1, “frequent fast trading” with $t_F = \{1, 2\}$, later demonstrates the robustness of the results. Since in this section each investor only trades once, rather than a set of $t_i = \{1\}$ or $\{2\}$, the notations for investors’ speed acquisition will be simplified to $t_i = 1$ for fast and $t_i = 2$ for slow.

### 4.1 Optimal trading

The analysis begins with an investor’s optimal trading demand at his trading round $t_i$. Fix all other investors’ strategies and consider an investor $i$ with technology $(t_i, h_i)$. At $t = t_i$, he chooses his demand schedule $x_i$ to maximize his expected utility over final wealth:

$$x_i \in \arg \max_{x_i} \mathbb{E}\left[-e^{-\gamma(V-P_t)x_i} | V + \varepsilon_i = s_i, P_t, P_{t-1}, ... \right]$$

where $P_t$ is given by the market maker’s efficient pricing as in Equation (2). Note that the investor also observes the price history $\{P_{t-1}, ...\}$ (with $P_0 = \mathbb{E}[V] = 0$). Standard conjecture-and-verify
analysis as in Vives (1995) yields the following lemma.

**Lemma 1 (Trading under “pure speed differential”)**. For any technology pair \((t_i, h_i)\), an investor \(i\) with signal \(s_i\) submits the optimal linear demand schedule at \(t = t_i\):

\[
x_i = \frac{h_i}{\gamma} (s_i - p_{t_i}).
\]

His certainty equivalent at the time of technology investment \((t = 0)\) is

\[
\pi(t_i, h_i) = \frac{1}{2\gamma} \ln \left(1 + \frac{h_i}{\tau_t}\right) - c(h_i) - \frac{2 - t_i}{\gamma t},
\]

where the price informativeness \(\tau_t := \text{var}[V | \{L_r(\cdot)\}_{r \leq t}]^{-1}\) satisfies the recursion of

\[
\Delta \tau_t = \tau_t - \tau_{t-1} = \left(\int_{\{t_j = t\}} \frac{h_j}{\gamma} \, dj\right)^2 \tau_t^U
\]

with the initial value \(\tau_0 = \text{var}[V]^{-1}\). The equilibrium price \(P_t\) satisfies the recursion of

\[
\Delta P_t = P_t - P_{t-1} = \frac{\Delta \tau_t}{\tau_t} \left(V + \frac{\gamma U_t}{\int_{\{t_j = t\}} h_j \, dj} - P_{t-1}\right)
\]

with the initial value \(P_0 = \mathbb{E} V (= 0)\).

An investor’s demand \(x_i\) scales with the difference between his private signal and the trading price \((s_i - P_{t_i})\), where the scaling factor \(h_i/\gamma\)—his trading aggressiveness—increases with the precision of his signal and decreases with his risk-aversion. His certainty equivalent has three components: the first term represents the information rent due to his private information, while the second and the third term correspond to the cost of information and speed acquisition, respectively.

Note that slow investors \((t_i = 2)\) do not (directly) trade on the fast round price \(p_1\), thanks to the competitive market maker who sets \(p_2\) while recalling the information from \(t = 1\) trading. As such, from a slow investor’s perspective, observing only \(p_2\) is as good as observing both \(p_1\) and \(p_2\). This feature inherits from Vives (1995) and dates back to Kyle (1985), where the dynamic equilibrium only uses the contemporaneous price \(p_t\) as a state variable, not the entire price history.
4.2 Optimal technology acquisition

The next step is to find investors’ optimal technology investment \((t_i, h_i)\) at \(t = 0\). To proceed, define the population sizes as

\[
\mu_F := \int_{i \in [0,1]} \mathbb{1}_{\{t_i=1\}} \, di \quad \text{and} \quad \mu_S := \int_{i \in [0,1]} \mathbb{1}_{\{t_i=2\}} \, di.
\]

By construction, \(\mu_F + \mu_S = 1\). The search for equilibrium is a fixed-point problem: Given the population sizes \(\mu_F\) and \(\mu_S\), what is each investor’s optimal information acquisition \(h_i\)? Given information acquisition \(\{h_i\}\), what is the “break-even” \(\mu_F\) and \(\mu_S\), so that no individual investor wants to change his speed choice?

Consider first an exogenous fraction \(\mu_F \in [0, 1] \ (\mu_S = 1 - \mu_F)\) of investors who are fast (slow). Since an investor is atomlessly small, his individual information acquisition \(h_i\) does not affect the price informativeness \(\tau_1\) or \(\tau_2\). To maximize his certainty equivalent, an investor takes \(\tau_{t_i}\) as given and chooses his information precision \(h_i\) according to the first-order condition of Equation (3):

\[
\frac{1}{2\gamma} \frac{1}{\tau_{t_i} + h_i} - c(h_i) = 0,
\]

which has a unique solution \(h(\tau_{t_i})\), satisfying the second-order condition, thanks to the convexity of the cost \(c(\cdot)\); see the “Information technology” paragraph in Section 3. By symmetry, therefore, all investors of the same speed \(t_i = t \in \{1, 2\}\) acquire the same amount of information: \(h_F = h(\tau_1)\) for the fast and \(h_S = h(\tau_2)\) for the slow.

The convexity of \(c(\cdot)\) further implies that the unique solution to Equation (6), \(h(\tau)\), is decreasing in \(\tau\). As such, fast investors always acquire more information than the slow:

\[
h_F \geq h_S.
\]

This is because the price discovery process is always cumulative: \(\tau_2 \geq \tau_1\), as the market never forgets whatever has been revealed (\(\Delta \tau_t \geq 0\) by Equation 4). The earlier an investor can trade, the less price discovery the market has seen and the more valuable is his private information. To
take this advantage, fast investors always have stronger incentive to acquire more information. This equilibrium result supports a popular connotation for fast traders that they are also more informed; see Menkveld (2016) for a survey of both theory and evidence.

The second step is to find the equilibrium speed acquisition $\mu_F$ and $\mu_S$, with fixed $h_F$ and $h_S$. The price informativeness $\tau_t$ recursion (Equation 4) can be rewritten as

$$
\Delta \tau_1 = \frac{\tau_U}{\gamma^2} h_F^2 \mu_F^2 \quad \text{and} \quad \Delta \tau_2 = \frac{\tau_U}{\gamma^2} h_S^2 \mu_S^2.
$$

These increments, $\Delta \tau_1$ and $\Delta \tau_2$, are referred to as the “early fragment” and the “late fragment” of price discovery, respectively. In contrast, the cumulative price informativeness, $\tau_1$ and $\tau_2$, are called the “short-run” and the “long-run price informativeness”, respectively. An important observation is that the price discovery $\Delta \tau$ is nonlinear in the population size $\mu$ of the trading round. Under the current parametrization, fixing $h_F$ and $h_S$, $\Delta \tau$ is convexly increasing in $\mu$. Such nonlinearity, inherent from Grossman and Stiglitz (1980) and Verrecchia (1982), underlies “the temporal fragmentation effect” of speed technology and is discussed in detail later in Section 5.2.

By symmetry, investors of the same speed have the same ex ante certainty equivalent:

$$
\pi_F = \frac{1}{2\gamma} \ln \left(1 + \frac{h_F}{\tau_1}\right) - c(h_F) - \frac{1}{g_t}; \\
\pi_S = \frac{1}{2\gamma} \ln \left(1 + \frac{h_S}{\tau_2}\right) - c(h_S).
$$

If $\pi_F > \pi_S$, all investors will acquire speed and become fast, leading to a corner solution of $\mu_F = 1$ and $\mu_S = 0$; and vice versa. In an interior equilibrium, it must be $\pi_F = \pi_S$ so that no investor has incentive to change his speed acquisition. The optimal population mix is determined via the break-even condition $\pi_F = \pi_S$.

The following proposition summarizes the discussion above and states the equilibrium.

**Proposition 1 (Equilibrium under “pure speed differential”).** There exists a unique equilibrium $\mathcal{P}$, depending on the speed technology $g_t$ relative to a threshold $\hat{g}_t (> 0$, see the proof):

**Case 1 (corner).** When $g_t \leq \hat{g}_t$, all investors invest in $(t_i, h_i) = (2, h_S)$, where $h_S$, together
with $\tau_2$, uniquely solves the first-order condition (6) and the recursion (8) with $\mu_F = 0$ and $\mu_S = 1$.

**Case 2 (interior).** When $g_t > \hat{g}_t$, a mass $\mu_F \in (0, 1)$ of investors invest in $(t_i, h_i) = (1, h_F)$, while the rest $\mu_2$ investors invest in $(t_i, h_i) = (2, h_S)$, such that the equilibrium is uniquely solved by $\{h_F, h_S, \mu_F, \mu_S\}$ under the following equation system:

- **Optimal information acquisition:**
  \[
  \frac{1}{2\gamma} \frac{1}{\tau_1 + h_F} - \hat{c}(h_F) = \frac{1}{2\gamma} \frac{1}{\tau_2 + h_S} - \hat{c}(h_S) = 0;
  \]

- **Indifference in speed:**
  $\pi_F = \pi_S$;

- **Population size identity:**
  $\mu_F + \mu_S = 1$;

where the expressions of $\tau$ and $\pi$ are given by Equations (8) and (9).

The equilibrium depends on the level of speed technology: When $g_t \leq \hat{g}_t$, investing in speed is too costly for any investor and nobody acquires speed in equilibrium. Only for sufficiently advanced speed technology ($g_t > \hat{g}_t$) will there be some investors acquiring speed.\(^4\) In fact, this same intuition holds in the other way:

**Corollary 1.** *Fixing the speed technology $g_t$, there exists a threshold $\hat{g}_h$ such that the equilibrium is interior if and only if $g_h \geq \hat{g}_h$. 

That is, when the information technology is too poor, the benefit in information rent of becoming fast is not sufficient to compensate for the cost of acquiring speed. As such, all investors stay slow.

Note that the population size pair $\mu_F$ and $\mu_S$ can be alternatively interpreted as investors’ ex ante probability mix between becoming fast or staying slow. That is, they play a symmetric mixed-strategy in speed acquisition: Each independently chooses to acquire speed, $t_i = 1$ (together with $h_F$) with probability $\mu_F$ or to stay slow, $t_i = 2$ (together with $h_S$) with probability $\mu_S = 1 - \mu_F$.

\(^4\) However, there are always non-zero mass of investors staying slow in equilibrium ($\mu_S > 0$). To see the reason, suppose there is an equilibrium with all investors acquiring speed, i.e., $\mu_F = 1$ and $\mu_S = 0$. In this case there is no price discovery in the late fragment, i.e., $\tau_1 = \tau_2$. Equation (9) then suggests that the marginal fast investor is strictly better off if he instead does not invest in the speed technology, saving the speed acquisition cost $1/\hat{g}_t$. Hence, some fast investors will deviate to staying slow.
4.3 Two constrained equilibria

In order to provide a clear contrast of the results, Section 5 will study two constrained versions of the model, where the acquisition of either one of the two technologies is shut down. The following two corollaries provide the existence and the uniqueness of equilibrium under these two constrained models. As both are special cases of Proposition 1, for brevity, their proofs are omitted.

**Corollary 2 (Constrained equilibrium: exogenous speed).** Fix each investor’s speed $t_i$ with exogenous $\mu_F$ and $\mu_S (= 1 - \mu_F)$. Then there exists a unique equilibrium in which fast and slow investors’ information acquisition, $h_F$ and $h_S$, solve the first-order conditions (6).

When the speed technology is not available, only the interior case of Proposition 1 is relevant. Further, since the investors cannot choose speed, the indifference condition $\pi_F = \pi_S$ becomes irrelevant. Only the “optimal information acquisition” condition remains, yielding Corollary 2 above.

**Corollary 3 (Constrained equilibrium: exogenous information).** Fix fast and slow investors’ information acquisition at $h_F$ and $h_S$, respectively. Then there exists a unique equilibrium, depending on the speed technology $g_t$ relative to a threshold $\hat{g}_t$:

**Case 1 (corner).** When $g_t \leq \hat{g}_t$, all investors stay slow with $\mu_F = 0$ and $\mu_S = 1$.

**Case 2 (interior).** When $g_t > \hat{g}_t$, a mass $\mu_F \in (0, 1)$ of investors acquire speed and become fast, while the rest $\mu_S$ stay slow. The equilibrium population sizes $\{\mu_F, \mu_S\}$ uniquely solve $\pi_F = \pi_S$ and $\mu_F + \mu_S = 1$.

Corollary 3 is also a special case of Proposition 1, where the “optimal information acquisition” condition is dropped in the interior equilibrium as investors’ signal precision are exogenously fixed.

5 Equilibrium properties and implications

This subsection studies investors’ endogenous speed and information acquisition and the effects on market quality. Three issues stand out: How does an advancement in one technology affect 1)
(1) Information acquisition | (2) Speed acquisition | (3) Price discovery

| $h_F$ | $h_S$ | $\int_0^1 h_idi$ | $\mu_F$ | $\mu_S$ | $\int_0^1 \mathbb{1}_{\{t=1\}}di$ | $\tau_1$ | $\tau_2$ |

(a) Exogenous speed and endogenous information

$g_h$: \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow

(b) Exogenous information and endogenous speed

$g_t$: \uparrow \downarrow \downarrow \downarrow \uparrow \uparrow \uparrow \uparrow

(c) Endogenous speed and endogenous information

$g_h$: \uparrow \downarrow \downarrow \downarrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow

$g_t$: \downarrow \uparrow \downarrow \downarrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow

Table 1: Summary of effects of technology shocks. This table summarizes how technology shocks affect different aspects of the market: (1) investors’ information acquisition; (2) speed acquisition; and (3) price discovery. For each of these three aspects, both the short-run ($h_F$, $\mu_F$, and $\tau_1$) and the long-run ($h_S$, $\mu_S$, and $\tau_2$) effects are shown. In addition, investors’ aggregate demand for information ($\int_0^1 h_idi$) and for speed ($\int_0^1 \mathbb{1}_{\{t=1\}}di$) are also tabulated. Three setting are considered: investors (a) have exogenous speed but can endogenously acquire information; (b) have exogenous information but can endogenously acquire speed; and (c) can endogenously acquire both speed and information. Each row represents a positive shock in the respective technology, $g_h$ for information and $g_t$ for speed. A monotone increasing (decreasing) response to the technology shock is indicated by \uparrow (\downarrow), while a hump-shape (U-shape) by \uparrow\downarrow (\downarrow\uparrow).

investors’ investment in it, 2) investors’ investment in the other technology, and ultimately 3) the aggregate market quality—in particular, the price discovery.

In order to isolate the different implications of the speed and the information technology, the analysis begins by exploring two constrained model variants: Section 5.1 switches off speed acquisition and Section 5.2 information. Section 5.3 then studies the interaction of the two technologies. As a preview, the results of these sections are summarized in Table 1. Finally, Section 5.4 discusses other model implications: speed of price discovery, investor composition, fund performance, and trading volume.
5.1 Information acquisition with exogenous speed

This subsection sets benchmark results where investors’ speed is exogenously given and can only acquire information. Specifically, fix a mass $\mu_F \in [0, 1]$ of investors who are fast ($t_i = 1$), and the rest $\mu_S = 1 - \mu_F$ investors stay slow ($t_i = 2$). Thus, all results are with respect to the information technology $g_h$. The equilibrium corresponds to Corollary 2.

**Proposition 2 (Information technology and information acquisition).** Fix the fast and the slow investors’ sizes $\mu_F$ and $\mu_S$. As the information technology $g_h$ increases, both the fast and the slow investors individually acquire more information: $\partial h_i / \partial g_h > 0$ for $i \in \{F, S\}$.

The result is not surprising. As $g_h$ increases, each investor can acquire more precise information at the same expense. That is, information becomes relatively cheaper and all investors, fast or slow, acquire more of it. Panel (a) of Figure 2 illustrates this effect. The red-dashed line also plots the total information acquisition in the economy, $\int_{t_i \in [0,1]} h_i di = \mu_F h_F + \mu_S h_S$.

Intuitively, as all investors acquire more information, the price becomes more efficient as well:

**Corollary 4 (Information technology and price informativeness).** Fix the fast and the slow investors’ sizes $\mu_F$ and $\mu_S$. As the information technology $g_h$ increases, both the short-run and the long-run price informativeness improve. Mathematically, $\partial \tau_1 / \partial g_h > 0$ and $\partial \tau_2 / \partial g_h > 0$.

Recall from Equation (8) that $\Delta \tau = \tau_U h^2 \mu^2 / \gamma^2$. Because the population sizes $\{\mu_F, \mu_S\}$ are exogenously fixed and because the individual information acquisition $h_i$ monotonically increases with $g_h$, so does the price discovery $\Delta \tau$. Panel (b) of Figure 2 graphically illustrates the corollary.

Less trivial, perhaps, is the curvature patterned in Panel (a): Both $h_F$ and $h_S$ are concavely increasing in $g_h$, and $h_S$ more so. The reason is that investors’ individual information acquisition “crowds out” each other (Grossman and Stiglitz, 1980): As the information technology improves, investors individually acquire more information, improving the price informativeness $\tau$, which, in turn, discourages investors’ information acquisition $h_i$. (The optimal $h(\tau)$ is, all else equal, a decreasing function per Equation 6.)
 Investors’ information acquisition, \( h_F \) and \( h_S \)

Price informativeness \( \tau_t \)

Figure 2: Varying information technology with fixed speed. This figure shows how information technology \( g_h \) affects individual investors’ information acquisition \( h_i \) in Panel (a) and the price informativeness \( \tau_t \) in Panel (b). The red-dashed line in Panel (a) plots the aggregate demand for information in the economy, \( \int_{\tau \in [0,1]} h_i d\tau \). The primitive parameters used in this numerical illustration are: \( \tau_0 = 1.0, \tau_U = 4.0, \gamma = 0.1, \) and \( k_b(m) = \sqrt{m} \). The fast investor’s population size is fixed at \( \mu_F = 0.4 \); and, hence, \( \mu_S = 0.6 \).

Notably, such crowding out takes two forms. First, \textit{intra}temporally, all fast investors crowd out each other’s information acquisition at \( t = 1 \); and all slow investors at \( t = 2 \). This yields the concavity of \( h_F \) and \( h_S \) in \( g_h \). Second, \textit{inter}temporally, the fast investors crowd out the slow, because, naturally, the price informativeness cumulatively grows over time (\( \tau_2 \geq \tau_1 \)). It is this intertemporal crowding-out effect that makes \( h_S \) even more concave in \( g_h \), compared to \( h_F \).

This \textbf{inter}temporal crowding-out effect is a novel insight revealed by the model. Its distinction versus the conventional \textbf{intra}temporal crowding-out effect (when all investors trade at the same time) bears great significance. Once both the speed and the information technology are made available to investors, the two forces drive substitution/complementarity between the two technologies in opposite directions (Section 5.3). Before that, Section 5.2 looks at the other constrained equilibrium, where investors can acquire speed but not information.
5.2 Speed acquisition with exogenous information

This subsection exogenizes investors’ information acquisition. Specifically, each investor has an endowed signal with precision fixed at the same level of $h_i = h_o > 0$, \forall i \in [0, 1]$. They cannot acquire additional information but can still acquire speed: Their speed choice $t_i \in \{1, 2\}$ and, consequently, the aggregate population sizes $\{\mu_F, \mu_S\}$ are endogenous.\footnote{More generally, one can bundle the speed and the information technology: When an investor acquires speed, he gets the pair $(t_i, h_i) = (1, h_F)$ and instead if he stays slow, he gets $(2, h_S)$, with $h_S \leq h_F$. The special case of $h_F = h_S = h_o$ simplifies the exposition to highlight the effect of speed acquisition. What matters for this subsection is that both $h_F$ and $h_S$ are fixed and investors cannot acquire more information—the information acquisition channel is shut down.} The equilibrium corresponds to Corollary 3.

In this equilibrium, a better speed technology $g_t$ reduces investors’ cost to become fast. The usual demand effect applies: Demand rises when price drops, as illustrated in Panel (a) of Figure 3 and formally stated in the proposition below.

**Proposition 3 (Speed technology and speed acquisition).** Fix all investors’ signal precision at $h_i = h_o (> 0)$. In the interior equilibrium, as the speed technology $g_t$ advances, more investors acquire speed: $\partial \mu_F / \partial g_t > 0$.

An advancement in the speed technology $g_t$, however, has different implications on the short-run and the long-run price informativeness. See Panel (b) of Figure 3.

**Proposition 4 (Speed technology and price informativeness).** Fix all investors’ signal precision at $h_i = h_o (> 0)$. In the interior equilibrium, as the speed technology $g_t$ advances, the short-run price informativeness $\tau_1$ monotonically increases, while the long-run price informativeness $\tau_2$ first decreases and then increases. Mathematically, $\partial \tau_1 / \partial g_t > 0$; and $\partial \tau_2 / \partial g_t < 0 (> 0)$ for small (large) $g_t$.

The driver of this result is the temporal fragmentation effect of the speed technology—it temporally fragments investors’ participation. When the speed technology is affordable (beyond the threshold $\hat{g_t}$), the unity of investors no longer trade at the same time. A fraction $\mu_F$ of them becomes fast and trade at $t = 1$, while the rest $\mu_S (= 1 - \mu_F)$ still trade slowly at $t = 2$. 
Figure 3: Varying speed technology with fixed information. This figure shows how speed technology $g_t$ affects individual investors’ speed acquisition $t_i$ in Panel (a) and price informativeness $\tau_t$ in Panel (b). The horizontal axis shows the speed technology level $g_t$. To the right of the vertical dashed line, the equilibrium is interior—there are both fast and slow investors. In Panel (a), the vertical axis indicates the population sizes of the fast (shaded area) and the slow investors (white area). The primitive parameters used in this numerical illustration are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, and $k_h(m) = \sqrt{m}$. The common signal precision is fixed at $h_o = 0.1$.

Just like the investors, so is the price discovery process fragmented into an early fragment $\Delta \tau_1$ and a late $\Delta \tau_2$. From Equation (8), it follows that the early fragment increases with $g_t$:

$$\Delta \tau_1 = \frac{\tau_U}{\gamma^2} h_o^2 \mu_F^2,$$

as $\mu_F$ is increasing with $g_t$ (Proposition 3). However, the late fragment drops with $g_t$:

$$\Delta \tau_2 = \frac{\tau_U}{\gamma^2} h_o^2 \mu_S^2 = \frac{\tau_U}{\gamma^2} h_o^2 \cdot (1 - \mu_F)^2.$$

The long-run $\tau_2 = \tau_0 + \Delta \tau_1 + \Delta \tau_2$ is subject to the joint force of both fragments of price discovery and, therefore, exhibits a nonmonotonic trend in the speed technology $g_t$.

Further, Proposition 4 states that the long-run price informativeness $\tau_2$ is U-shape in the speed technology. This U-shape arises from the fact that each fragment of price discovery, $\Delta \tau$, is a convex
function in the population size \( \mu \). As such, the impact of a marginal change in \( \mu \) (due to speed technology) on \( \tau \) depends on the initial level of \( \mu \). For example, when \( g_t \) is close to the threshold of \( \hat{g}_t \), most investors are slow—\( \mu_F \) closer to zero and \( \mu_S \) to one. A small increase in the speed technology \( d g_t \) prompts a small population \( d \mu_F \) to move from slow to fast. The resulting loss in price informativeness in the late fragment \( \Delta \tau_2 \) is much larger than the gain in the early \( \Delta \tau_1 \):

\[
d\tau_2 = \frac{\partial \tau_2}{\partial \mu_F} d \mu_F = \left( \frac{\partial \Delta \tau_1}{\partial \mu_F} + \frac{\partial \Delta \tau_2}{\partial \mu_F} \right) d \mu_F = \frac{\tau_U}{\gamma^2 h_s^2} \left( \frac{\partial \mu_F^2 + \partial \mu_S^2}{\partial \mu_F} \right) d \mu_F < 0.
\]

The reverse holds true when \( \mu_F \) is close to one and \( \mu_S \) close to zero.

Finally, to reinforce the understanding of the temporal fragmentation effect, note that the level of the long-run price informativeness \( \tau_2(g_t) \) is the same at the either extreme of \( g_t \):

\[
\lim_{g_t \to \hat{g}_t} \tau_2(g_t) = \lim_{g_t \to 1} \tau_2(g_t) = \tau_0 + \frac{\tau_U}{\gamma^2 h_o^2}.
\]

This equality should not come as a surprise because in either extreme, the investors are no longer fragmented: That is, the temporal fragmentation of speed only manifests for moderate levels of speed technology, which in turn affects price informativeness nonmonotonically.

### 5.3 Interaction between speed and information technology

In this subsection, both speed and information technologies are made available to investors. The unconstrained equilibrium stated in Proposition 1 holds, together with Corollary 1. Suppose there has been an advancement in one technology. The discussion below focuses three effects: 1) investors’ acquisition in this advancing technology; 2) investors’ acquisition in the other technology; and 3) the price discovery function of the financial market.

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6 The convexity of price discovery \( \Delta \tau \) in population size \( \mu \) is a universal feature in the literature. See, among many others, Grossman and Stiglitz (1980), Hellwig (1980), Verrecchia (1982), for example. The source of such convexity—more specifically, the quadratic term \( \mu^2 \)—is the conventional choice of the price informativeness measure: the reciprocal of the conditional variance, a statistic of the second moment, of the risky asset value. Economically speaking, each marginal informed investor’s trading resolves increasingly more uncertainty (from noise trading).
5.3.1 Own-price effect: Acquisition in the advancing technology

The level of a technology is modeled as the (inverse) marginal acquisition cost. As such, when a technology advances, the first-order effect is essentially its own-price effect: investors’ demand responding to a lower price. Unsurprisingly and consistent with Proposition 2 and 3, demand increases when a technology improves.

**Proposition 2 (continued).** Whether investors’ speed acquisition is exogenous or endogenous, in the interior equilibrium, investors’ information acquisition monotonically increases with the information technology: \[ \frac{\partial h_i}{\partial g_{ih}} > 0 \] for \( i \in \{F, S\} \).

**Proposition 3 (continued).** Whether investors’ information acquisition is exogenous or endogenous, in the interior equilibrium, as the speed technology advances, more investors acquire speed: \[ \frac{\partial \mu_F}{\partial g_t} > 0. \]

For completeness, Panel (a) and (b) of Figure 4 illustrate this intuitive own-price effect of the speed and the information technology. The patterns are comparable with the Panel (a)s of Figure 2 and 3.

5.3.2 Cross-price effect: Are speed and information substitutes or complements?

The cross-price effects are graphed in Panel (c) and (d) in Figure 4. Panel (c) plots investors’ aggregate demand for speed, \( \int_{\{0,1\}} 1 \{t_i = 1\} d_i = \mu_F \), against the information technology \( g_{ih} \). Panel (d) plots three lines: a fast investor’s individual demand for information, \( h_F \); a slow investor’s, \( h_S \); and the aggregate demand, \( \int_{\{0,1\}} h_i d_i = \mu_F h_F + \mu_S h_S \) (the red-dashed line). In both panels, it can be seen that the aggregate demand for one technology, when the other improves, is first increasing but eventually decreasing, cateris paribus. The technologies can be either complements or substitutes.

**Proposition 5 (Complementarity and substitution between speed and information).** In the interior equilibrium, as one technology increases, fixing the other, investors’ aggregate speed and information acquisition are initially complements but eventually substitutes. Mathematically,
Panel (a) and (b): Two technologies’ own-price effects
Panel (c) and (d): Two technologies’ cross-price effects

Figure 4: Technology acquisition. This figure illustrates how investors’ technology acquisition (demand for speed and for information) are affected differently by levels of technologies. Panel (a) and (b) show the technologies’ own-price effect. Panel (c) and (d) show the cross-price effect. The vertical dashed lines indicate the thresholds of the corresponding technology, below which all investors stay slow. The red-dashed lines in Panel (a) and (d) are the aggregate demand for information in the economy, \( \int_{i \in [0,1]} h_i \, di \). The primitive parameters used in this numerical illustration are: \( \tau_0 = 1.0 \), \( \tau_U = 4.0 \), \( \gamma = 0.1 \), and \( k_h(m) = \sqrt{m} \). For Panel (b) and (d), \( g_h = 0.2 \). For Panel (a) and (c), \( g_t = 10.0 \).
\[ \frac{\partial \mu_F}{\partial g_h} > 0 \ (\ < 0) \text{ for small (large) } g_h; \] and \[ \frac{\partial (\mu_F h_F + \mu_S h_S)}{\partial g_t} > 0 \ (\ < 0) \text{ for small (large) } g_t. \]

In addition, \[ \frac{\partial h_F}{\partial g_t} < 0; \] but \[ \frac{\partial h_S}{\partial g_t} > 0 \ (\ < 0) \text{ for small (large) } g_t. \]

To understand such cross-price effects, recall the different “crowding-out effects” discussed in Section 5.1. Consider an advancement in the information technology \( g_h \) (as in Panel c), which stimulates both fast and slow investors to acquire more information (Proposition 2). Three crowding-out effects arise: *intratemporal* crowding-out among the fast at \( t = 1 \), *intratemporal* crowding-out among the slow at \( t = 2 \), and *intertemporal* crowding-out from the fast to the slow. The first effect hurts fast investors’ information rent, making them less willing to acquire speed—reducing demand for speed. The second and the third effects hurt slow investors, incentivizing them to leave \( t = 2 \) and to compete with fast investors at \( t = 1 \) instead—raising demand for speed.

It is these countervailing crowding-out effects that drive the net demand for the speed technology to increase or to decrease. When initially the information technology is low (close to \( \hat{g}_h \)), there are very few fast investors (\( \mu_F \) close to zero; Corollary 1). The slow investors’ *intratemporal* crowding-out effect dominates, stimulating them to acquire speed and move to \( t = 1 \). As more investors have acquired speed, they yield additional *intertemporal* crowding-out effect on the remaining slow ones, further strengthening their incentive to move to \( t = 1 \). These two effects result in complementarity between speed and information. Eventually, however, when there are too many fast investors, the *intratemporal* crowding-out effect at \( t = 1 \) dominates: Each individual fast investor’s rent is hurt too much by further advancement of information. It becomes no longer profitable to acquire speed, information thus substituting speed.\(^7\)

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\(^7\) Panel (d) can be intuitively explained with the three crowding-out effects as well. As the speed technology \( g_t \) increases, more investors acquire speed. The increase in \( \mu_F \) intensifies fast investors’ competition and their *intratemporal* crowding-out at \( t = 1 \) reduces \( h_F \). For the slow, the pattern of \( h_S \) looks different because it is subject to both the *intratemporal* crowding-out at \( t = 2 \) and the *intertemporal* crowding-out by fast investors. Initially, when \( g_t \) is low (close to \( \hat{g}_t \)), most investors are slow (\( \mu_F \) close to zero) and, hence, the dominating effect is the reduction of *intratemporal* crowding-out at \( t = 2 \). As a result, with less crowding-out, the remaining slow investors individually acquire more information. Hence, initially, speed complements information. However, when a lot of investors have become fast, the *intertemporal* crowding-out is no longer negligible. It eventually becomes the dominant effect that drives down slow investors’ information acquisition \( h_S \).
5.3.3 Technology and price discovery

Price discovery is a key function of financial markets. The effects of the technologies on price informativeness $\tau_t$ are illustrated in Figure 5. The patterns shown in Panel (a) are qualitatively similar to those shown in Panel (b) of Figure 3. This suggests that even with endogenous information acquisition, the speed technology’s temporal fragmentation effect dominates. The following result extends Proposition 4.

**Proposition 4 (continued).** Whether investors’ information acquisition is exogenous or endogenous, in the interior equilibrium, as the speed technology $g_t$ advances, the short-run price informativeness $\tau_1$ monotonically increases, while the long-run price informativeness $\tau_2$ initially decreases but eventually increases. Mathematically, $\frac{\partial \tau_1}{\partial g_t} > 0$; and $\frac{\partial \tau_2}{\partial g_t} < 0$ ($>0$) for small (large) $g_t$.

The speed technology’s temporal fragmentation effect is a unique finding of this paper. For example, this mechanism differs from Dugast and Foucault (2017) and Kendall (2017), who show that the acquisition of raw information in the short-run can crowd out processed information, thus hurting the long-run price informativeness. Banerjee, Davis, and Gondhi (2017) show that a public announcement could still worsen price informativeness, because investors would switch to learn about others’ beliefs instead of the fundamental. To compare, the temporal fragmentation channel emphasizes that price discovery can still be hurt even when investors’ information acquisition is exogenous (Section 5.2).

Panel (b) of Figure 5 contrasts Panel (b). While the short-run informativeness $\tau_1$ monotonically increases in both cases, the long-run informativeness $\tau_2$ is not when investors can endogenously acquire speed. Notably, information technology might hurt overall price informativeness:

**Proposition 6 (Information technology and price informativeness).** In the interior equilibrium, advancements in the information technology always improves short-run informativeness $\tau_1$. However, with endogenous speed acquisition, long-run informativeness $\tau_2$ is initially hurt but
Figure 5: Price informativeness. This figure illustrates how the aggregate price informativeness $\tau_t$ is affected differently by different technologies. Panel (a) shows the response to varying speed technology $g_t$ and Panel (b) to information technology $g_h$. To manifest the patterns, only the range with interior equilibrium is shown; i.e. $g_t > \hat{g}_t$ in Panel (a) and $g_h > \hat{g}_h$ in Panel (b). Further, the vertical axis in Panel (b) is split into two ranges, respectively, for the long-run and the short-run price informativeness. The primitive parameters used in this numerical illustration are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, and $k_h(m) = \sqrt{m}$. For Panel (a), $g_h = 0.2$. For Panel (b), $g_t = 10.0$.

Eventually improved. Mathematically, $\partial \tau_1 / \partial g_h > 0$; and $\partial \tau_2 / \partial g_h < 0$ ($> 0$) for small (large) $g_h$.

To understand how information technology might hurt price informativeness, one needs to recall from the discussion above 1) that the two technologies can exhibit complementarity (when $g_h$ is close to the threshold $\hat{g}_h$); and 2) that speed technology temporally fragments price discovery. Combining these two sheds light on the U-shaped $\tau_2$: When $g_h$ improves from $\hat{g}_h$, due to the complementarity, investors acquire both speed and information. Those who have acquired speed fragment the price discovery process. In turn, such fragmentation hurts the long-run price informativeness $\tau_2$ (Proposition 4).

Eventually, as the information technology improvement furthers (large $g_h$), the two technologies become substitutes and the population again concentrates at $t = 2$. The fragmentation effect
diminishes, and the aggregate price informativeness improves.\footnote{It is tempting, looking at Figure 5, to conclude that the potential dysfunction of price discovery is not necessarily relevant: Since the decreases in the long-run informativeness only occurs to a local range of parameters, when the technologies have advanced enough, the informativeness would only monotonically increase. Such a conclusion should be qualified with two notes. First, the panels of Figure 5 only show the effect of one technology, holding the other constant. In reality, both technologies advance over time, and it is the joint force of the two that determines whether price informativeness increases or decreases. This point is further illustrated in Figure 10, a contour plot of \((g_t, g_h)\) (Section 6.2). Second, the importance of Proposition 4 and 6 also lies in the cross-sectional implications on individual assets’ different price informativeness, as discussed below in Section 5.4.2-5.4.4.}

Technology has always been evolving but has price informativeness improved alongside the technology? Proposition 4 and 6 both predict that long-run informativeness has a U-shape in either technology, and there is empirical evidence supporting such nonmonotonicity. For example, Figure 6 of Morck, Yeung, and Yu (2000) shows that the firm-specific component of stock returns exhibits a U-shape trend after World War II. They argue that the firm-specific component of stock return variation reflects the firm-level information. The model predictions can also be interpreted cross-sectionally. Bai, Philippon, and Savov (2016) finds heterogeneous price informativeness trends in large and small stocks. Farboodi, Matray, and Veldkamp (2017) explain these phenomena via investors’ strategic information acquisition choice, facing an attention limit. This paper adds to the discussion that the source of technology advancement—speed and/or information—is important in determining the effect on price informativeness.

5.4 Other implications

While the focus of the model is on price discovery, it is accompanied by a number of other implications. This subsection discusses these unique results in the context of existing and future empirical research.

5.4.1 The speed of price discovery

A noteworthy feature of the model is that it opens up the process of price discovery, thanks to investors’ endogenous speed acquisition. This feature enables researchers to study questions like
“how fast price discovery occurs.” This novel angle of “price discovery speed” differs from the conventional focus on the magnitude and is of great importance for market quality. Compare two market environments, both of which will eventually (in the long-run) lead to the same level of price informativeness, $\tau_2$. All else equal, the one that achieves “faster” price discovery is more efficient than the other: End-users of the financial market can utilize such information more timely for purposes like hedging, real investment, and production.

More concretely, consider an economy with only the speed technology, as studied in Section 5.2. There is a firm who learns from the asset’s price to make real investment decisions. It can decide in the short-run at $t = 1$ (e.g., end of the current trading day) or wait for more information in the long-run at $t = 2$ (e.g., end of the week). If waiting is costly (due to, e.g., time value of money, fleeting opportunity, first-mover advantage, etc.), the firm faces the tradeoff between the speed and the magnitude of price discovery.

The magnitude $\tau_t$ is well-define and has been extensively studied in the literature. This paper proposes to gauge the new aspect of “price discovery speed” as:

$$\theta := \frac{\Delta \tau_1}{\Delta \tau_1 + \Delta \tau_2};$$

i.e., the percentage of price discovery that is achieved in the short-run over the total in the long-run. Such a ratio isolates the magnitude of price discovery on the speed. The higher (lower) is the ratio, the faster (slower) is price discovery.

**Proposition 7 (Speed of price discovery and technology).** In the interior equilibrium, the speed of price discovery increases with the speed technology. However, an improvement in the information technology initially increases, but eventually decreases, the speed of price discovery. Mathematically, $\partial \theta / \partial g_t > 0$; and $\partial \theta / \partial g_h > 0 \ (< 0)$ for small (large) $g_h$.

Figure 6 numerically illustrates the patterns. Panels (a) shows that as the speed technology increases, the speed of price discovery monotonically increases as well. This is unsurprising as the dominating effect of speed technology is to drive up $\mu_F$, which in turn adds to the early price
Figure 6: Speed of price discovery. This figure illustrates how the speed of price discovery is affected differently by different technologies. The level of the speed technology $g_t$ varies in Panel (a), while the level of the information technology $g_h$ varies in Panel (b). The primitive parameters used in this numerical illustration are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, and $k_h(m) = \sqrt{m}$. For Panel (a), $g_h = 0.2$. For Panel (b), $g_t = 10.0$.

This monotone increasing pattern no longer holds with respect to $g_h$, as shown in Panel (b). Initially, when $g_h$ increases from a relatively low level, there is complementarity between investors’ speed and information acquisition: Investors acquire more of both information and speed. The increase in $\mu_F$, drives up $\Delta \tau_1$ more than $\Delta \tau_2$, thus speeding up price discovery. Eventually when the information technology is very advanced, however, the substitution effect kicks in, reducing $\mu_F$; see Panel (c) of Figure 4. As more price discovery occurs at $t = 2$, the process slows down.9

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9 Further analysis of how the price discovery speed would affect real-efficiency (Bond, Edmans, and Goldstein, 2012) will be an interesting extension from the current model. In particular, the analysis will likely reveal new channels of how trading technologies could create (or destroy) real social value. As such extension will require a formal model of the real sector, beyond the current paper’s focus on the financial market, it is left for future research.
5.4.2 Investor composition

In the context of the model, one can interpret the fast investors as hedge funds, who trade more often and possibly have information advantage over slow investors like mutual funds and pension funds. The speed advantage of hedge funds can arise from the fact that they are less regulated than, e.g., mutual funds. While trades by mutual funds must go through a prolonged process of compliance, risk control, and bookkeeping, hedge funds can possibly implement the same trades more quickly. In terms of trading technology (execution speed), high-frequency traders—the fastest market participants—are largely proprietary trading firms or hedge funds (SEC, 2010, IV.B).

Empiricists can examine the model prediction using such rough approximation of fast and slow investors. In the equity market, for example, some stocks have higher analyst coverage and more media exposure than others. Investors’ information acquisition cost in these stocks should be relatively lower. Such cross-sectional heterogeneity allows the reinterpretation of Propositions 3 and 5 in terms of which stocks attract which types of investors, fast (hedge funds) or slow (mutual funds).

An empiricist can sort stocks according to their analyst coverage or media exposure. The proportion of fast investors (hedge funds or proprietary trading firms) should exhibit a hump-shape, similar to the pattern outlined in Panel (c) of Figure 4. To the extent that large stocks have higher analyst coverage and media exposure, the predicted hump-shape exactly matches the empirical finding by Griffin and Xu (2009, Figure 3). Following speed technology boosts (e.g., the democratization of microwave transmission in late 2012; see Shkilko and Sokolov, 2016), one should see holdings by fast investors, by and large, increase as shown in Panel (a) of Figure 4.

5.4.3 Fund performance

Following the interpretation in Section 5.4.2 above, what can the model say about hedge fund (fast) performance, relative to mutual funds (slow)? Specifically, how do technologies affect traders’
Figure 7: Fund performance. This figure illustrates fast and slow funds’ performance. The level of the speed technology \( g_t \) varies in Panel (a), while the level of the information technology \( g_h \) varies in Panel (b). In each panel, the blue-solid (the red-dashed) line shows the expected trading profit for the fast (the slow). The primitive parameters used in this numerical illustration are: \( \tau_0 = 1.0, \tau_U = 4.0, \gamma = 0.1, \) and \( k_h(m) = \sqrt{m}. \) For Panel (a), \( g_h = 0.2. \) For Panel (b), \( g_t = 10.0. \)

profit (information rent) in such a competitive setting?

An investor’s (a fund’s) trading performance can be measured as

\[
\mathbb{E}[(V - P_i)x_i(s_i, P_i)] = \frac{h_i}{\gamma \tau_i}, \text{ if } i \text{ is fast (} t_i = 1); \\
\mathbb{E}[(V - P_2)x_i(s_i, P_2)] = \frac{h_i}{\gamma \tau_2}, \text{ if } i \text{ is slow (} t_i = 2). 
\]

These expressions are consistent with the certainty equivalents derived in Equation (9), before applying the technology acquisition costs. Note that the above performance measures can be equivalently interpreted as the return predictability of funds’ holdings: \( \text{cov}[V - P_t, x_t] = \mathbb{E}[(V - P_t)x_t] \). Intuitively, a fund’s performance, measured in its information rent, is higher if and only if it predicts future return more precisely.

Figure 7 illustrates the patterns. Panel (a) shows that the speed technology monotonically
hurts fast traders’ (e.g., fast trading hedge funds or proprietary firms) performance. (That is, the return predictability of their holdings lowers with the speed technology.) This is because of the intratemporal competition among the fast. Instead, slow investors’ (pension funds and mutual funds) performance has a hump-shape: Initially, when more traders acquire speed and become fast, the remaining slow traders acquire more information (complementarity, less intratemporal competition at $t = 2$) and better predict future returns. Eventually, however, the intertemporal competition from the bulk of fast traders erodes the residual rent for the slow. The empirical finding by Qin and Singal (2017) seems to agree with this narrative: Mutual funds (slow) who trade or hold stocks with heavy high-frequency activity tend to underperform.

When information technology increases, Panel (b) shows that both the fast and the slow traders’ performance exhibit hump-shapes. Due to the initial complementarity, more traders acquire both speed and information. This reduces the intratemporal competition among the slow, raising their performance. In the meantime, the increasing information technology overcomes the mild competition among the fast, also raising their performance. Eventually, however, as all traders acquire more information, prices become very revealing, crowding out everyone’s information rent. Both the fast and the slow funds’ performance worsens.

5.4.4 Trading volume

This subsection discusses the model implications on trading volume, a readily available market quality measure which can be used to examine the theory developed in this paper. The short-run trading volume, excluding noise trading, is given by (following Vives, 1995)

$$\int_{i \in [0,1]} I_{\{i,t=1\}}|x(s_i, p_1)|di = \mu_F b E \left( \frac{h_F}{\gamma} s_i - p_1 \right) = \frac{\mu_F h_F}{\gamma} \sqrt{\frac{2}{\tau_1 + h_F} \pi}.$$  

Similarly, the long-run cumulative trading volume is

$$\sqrt{\frac{2}{\pi}} \left( \frac{\mu_F h_F}{\sqrt{\tau_1 + h_F}} + \frac{\mu_S h_S}{\sqrt{\tau_2 + h_S}} \right).$$
Figure 8: Trading volume and technologies. This figure illustrates how trading volume is affected differently by different technologies. The level of the speed technology $g_t$ varies in Panel (a), while the level of the information technology $g_h$ varies in Panel (b). The short-run line shows informed investors’ aggregate trading volume at $t = 1$. The long-run line shows cumulative trading volume at $t = 1$ and at $t = 2$. (Noise trading is excluded.) The primitive parameters used in this numerical illustration are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, and $k_h(m) = \sqrt{m}$. For Panel (a), $g_h = 0.2$. For Panel (b), $g_t = 10.0$.

Figure 8 below shows how such cumulative trading volume is affected by technologies.

As either technology improves, the short-run trading volume monotonically increases. The main driver is the fast population size $\mu_F$. In the case of an increasing $g_t$, this is a direct effect of cheaper speed technology. In the case of an increasing $g_h$, $\mu_F$ grows initially due to the complementarity. When eventually information starts to substitute speed, $\mu_F$ starts to decrease but this effect is dominated by the increase in fast investors’ trading aggressiveness $h_F/\gamma$. Overall, $\mu_F h_F$ monotone increases with both $g_t$ and $g_h$.

The pattern, however, differs for the long-run cumulative trading volume. For information technology, as shown in Panel (b), long-run volume also monotone increases. Intuitively, this is because all investors, fast or slow, acquire more precise information and trade more aggressively.
For speed technology, a hump-shape is seen. Below the threshold of $\hat{g}_t$, speed is too expensive and there are no fast investors. Starting from $\hat{g}_t$, some small fraction of investors acquire speed to trade early. These investors trade rather aggressively to take advantage of their speed, therefore pushing up the overall trading volume. When the majority investors become fast, most of the trading concentrates at $t = 1$ in the short-run, exploiting most of the information rent. The rest slow investors do not trade much, because there is little residual information advantage. The overall trading volume decreases.

An interesting empirical question is how much of the trading volume dynamics over time can be explained by technology shocks. The question is important for policy and market design, as trading volume is often used as a measure for market liquidity and inspected closely by regulators, market organizers, and participants. Since the introduction of Regulation NMS (National Market System) in 2005, the U.S. equity market has grown increasingly fast in trading. The trading volume over the years has also been increasing, a large contribution of which, arguably, comes from ultra-fast, high-frequency trading firms. Qualitatively consistent with the hump-shape prediction seen in Panel (a) of Figure 8, the U.S. equity trading volume peaked in around 2011 and has been declining ever since.\footnote{The hump-shape can be seen from the data reported by Investment Technology Group: \url{https://www.itg.com/trading-volume/month/}. Note that there are many forces driving trading volume. The theory presented here only offers one possible economic channel and does not claim primacy.} The theory developed in this paper serves as a conceptual framework for future empirical work to explore further along this line.

6 Discussion and robustness

This section discusses some choices in setting up the model, the robustness of the results, and some potential alternative interpretations.
6.1 Frequent fast trading

The model so far has analyzed the scenario of “pure speed differential”, in which the speed technology allows fast investors to trade at \( t = 1 \), sooner than slow investors at \( t = 2 \). Under the alternative setup of “frequent fast trading”, fast investors can trade at \( t_F = \{1, 2\} \). That is, the speed technology in addition allows fast investors to trade more frequently. It turns out that the main results studied in Section 5 qualitatively remain the same when fast investors are given this additional trading opportunity.

To begin with, the following lemma establishes investors’ optimal trading in each round, the dynamics of price informativeness, and investors’ ex ante certainty equivalent.

**Lemma 2 (Trading under “frequent fast trading”).** An investor \( i \)'s cumulative demand in round \( t \) is \( x_{it} = \frac{h_i}{T}(s_i - p_t) \), where \( h_i \) is his information acquisition, \( s_i \) is his private signal, and \( p_t \) is the round \( t \) trading price set by the competitive market maker. The price discovery \( \Delta \tau_t \) and the trading price \( p_t \) satisfy the same recursions (4) and (5) as stated in Lemma 1. At \( t = 0 \), fast and slow investors’ certainty equivalent are given, respectively, by

\[
\pi_F = \frac{1}{2\gamma} \ln \left( 1 + \frac{h_F}{\tau_1} \frac{\Delta \tau_2}{\tau_1} \right) - c(h_F) - \frac{1}{g_t}; \quad \pi_S = \frac{1}{2\gamma} \ln \left( 1 + \frac{h_S}{\tau_2} \right) - c(h_S).
\]

Two observations are worth highlighting when compared to Lemma 1: First, inheriting from Vives (1995), the recursions of price informativeness \( \tau_t \) and of the price \( p_t \) remain exactly the same. The reason is that the competitive investors only acquire information once. While the fast investors trade repeatedly, they do not reveal additional information to the market. To see this, note that a fast investor’s cumulative demand at \( t \) is \( x_{it} = \frac{h_i}{T}(s_i - p_t) \). His net demand in round \( t = 2 \), therefore, is \( x_{i2} - x_{i1} = \frac{h_i}{T}(p_1 - p_2) \), independent of his private signal \( s_i \). (He simply rebalances his position based on the new price \( p_2 \).) As such, a fast investor contributes his private signal to price discovery once and only once, at \( t = 1 \).
Figure 9: Price informativeness under repeated fast trading. This figure replicates the patterns shown in Figure 5 to illustrate how the aggregate price informativeness $\tau_t$ is affected differently by different technologies, under the model extension where fast investors can trade at both $t = 1$ and $t = 2$. The primitive parameters used in this numerical illustration are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, and $k_h(m) = \sqrt{m}$. For Panel (a), $g_h = 0.2$. For Panel (b), $g_t = 10.0$.

Second, a fast investor’s ex-ante certainty equivalent sees an extra term of $h_f^2 \Delta \tau^2 \tau_1$ inside the $\ln(\cdot)$ operator. In fact, this is the only difference of this extension compared to “pure speed differential”. This positive term represents fast investors’ additional information rent from repeated trading. It is increasing in $h_F$ as this extra information rent still relies on the precision of his private information. It decreases in both $\Delta \tau_1$ and $\Delta \tau_2$ because more price discovery in either $t = 1$ or $t = 2$ reduces the advantage of his private signal from repeated trading.

To solve the equilibrium, it remains to pin down investors’ optimal information acquisition $h_F$ and $h_S$, together with the population sizes $\mu_F$ and $\mu_S$ ($= 1 - \mu_F$) so that $\pi_F = \pi_S$. Unfortunately, the additional term $h_f^2 \Delta \tau_2 \tau_1$ in fast investor’s certainty equivalent limits the analytic tractability. Nevertheless, the properties of the equilibrium can be numerically examined. After very extensive numerical exploration, it turns out that the patterns found under “pure speed differential” remain robust. (That is, the frequent trading advantage of the speed technology only provides a secondary
effect.) To demonstrate so, Figure 9 reproduces Figure 5 to show that both speed and information technologies have U-shaped nonmonotonic effect on the long-run informativeness. Other numerical results are omitted for brevity.

It is worth emphasizing that allowing fast investors to trade more frequently does not affect the paper’s two novel economic channels, 1) that speed technology has a temporal fragmentation effect on price discovery; and 2) that different (inter/intratemporal) crowding-out effects drive the speed and the information to be either substitutes or complements. In fact, these two findings only depend on the price discovery process, $\Delta \tau_t$, which, as Lemma 2 shows, remains the same as under both setups. The model’s caveat, that better information environment might still hurt price informativeness, remains the same following the two channels, as shown in Panel (b) of Figure 9.

### 6.2 Dependence between the two technologies

In the current model, the acquisition of one technology does not affect the other’s cost. That is, $g_h$ and $g_t$ are two parameters exogenous of each other. Such independence need not necessarily be the case. On the one hand, the two can complement each other. The complementarity can arise from the common hardware needed, e.g. processing capacity (CPUs), bandwidth (cables and optical fiber), etc. Thus, having invested in such hardware for one technology can reduce the cost for the other (e.g., $g_h$ increases in $g_t$). This feature is often seen in the algorithmic trading and high-frequency trading literature, where an investor’s technology investment gives him a “bundled” advantage in both information and speed (see, e.g., Menkveld, 2016 for a review). In contrast, the current model predicts “endogenous bundling” of the two technologies, as fast investors always acquire more information than slow ones (Equation 7).

On the other hand, there can be substitution. Dugast and Foucault (2017) argue that because information processing is time consuming, the speed of investors with “processed” information is limited and they are slower than those who trade on “raw” information. For example, sending analysts for firm visits is a time-consuming way of acquiring information. That is, investing in one
Figure 10: Price informativeness plotted against both technologies. This contour graph plots how the long-run price informativeness $\tau_2$, in blue-solid line, and the short-run price informativeness $\tau_1$, in red-dashed line, vary with the two technologies, $g_t$ and $g_h$. The two arrows illustrate the different effects of an information technology advancement. The left arrow (green) shows complementarity between the two, while the right arrow (blue) shows substitution. The primitive parameters used in this numerical illustration are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, and $k_h(m) = \sqrt{m}$.

technology might increase the (marginal) cost for the other (e.g., $g_h$ decreases in $g_t$).

Exactly how speed and information technologies interfere with each other is perhaps a question of engineering and computer science. The current model specification sets a benchmark with independent technologies—an agnostic view. The outcomes of the model, therefore, offer a clean set of predictions on investors’ endogenous demand for the two technologies, as opposed to the exogenous, built-in substitution/complementarity.

In fact, with independent technologies, the current model offers a starting point to study built-in substitution or complementarity between the two technologies. Suppose the market quality of interest is price informativeness $\tau_t$. Figure 10 plots $\tau_1$ (blue-solid line) and $\tau_2$ (red-dashed line) on a
When the two technologies exhibit complementarity, the effect of an increase in one technology can be examined by, e.g., the left (green) arrow in the figure ($g_i$ increases from 0.15 to 0.16, while $g_h$ increases from about 10 to 100). If instead the substitution of the technologies dominate, the effect can be shown by, e.g., the right (blue) arrow ($g_h$ mildly increases from 0.125 to 0.135, while $g_t$ drops sharply from about 4,000 to 50). In both examples, note that the long-run price informativeness $\tau_2$ (blue-solid contour lines) drops. Note that the right (blue) arrow is consistent with Dugast and Foucault (2017) and Kendall (2017), who show that when processing information takes time, better information might hurt price informativeness.

### 6.3 The amount of noise trading

Introducing noise trading $U_t$ in each round is a standard practice to avoid a fully revealing equilibrium. The current setup assumes that the magnitude of noise trading in each round is the same: $\text{var}[U_t] = \tau_{U_t}^{-1}$ for all $t \in \{1, 2\}$. This assumption can be relaxed. It is straightforward to extend the model to account for time-varying noise trading by allowing time-dependent $\text{var}[U_t] = \tau_{U_t}^{-1}$. The key economic insights of the model are unaffected by such an extension. First, irrespective of the relative sizes of noise trading, the speed technology creates temporal fragmentation in investors’ participation and in the price discovery process. Adapting the price informativeness recursion (Equation 8) yields

$$\tau_2 = \tau_0 + \frac{\tau_{U_1}}{\gamma^2} \mu_h^2 h_1^2 + \frac{\tau_{U_2}}{\gamma^2} \mu_S^2 h_2^2.$$  

It can be seen that the speed technology still fragments the price discovery process into the early and the late fragments (but with different $\tau_U$ in each fragment). Second, the result that the speed and the information technologies can be either substitutes or complements depends only on the relative strength of intratemporal competition and intertemporal crowding out. Having different

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11 Note the pattern shown is consistent with Figure 5: Moving right on a horizontal cut of Figure 10, the information technology $g_h$ is fixed and as the speed technology $g_t$ improves, the short-run price informativeness $\tau_1$ monotonically increases, while the long-run price informativeness $\tau_2$ first decreases and then increases. Moving upward on a vertical cut, $g_t$ is fixed and as $g_h$ increases, $\tau_1$ monotonically increases but $\tau_2$ first decreases and then increases.
sizes of noise trading would only affect the threshold of when and which effect dominates. Indeed, all analyses in Section 5 qualitatively go through. For example, Figure 11 illustrates that the qualitative predictions of Proposition 4 and 6 remain robust about the long-run informativeness $\tau_2$.

The underlying assumption for such (possibly time-varying) exogenous noise trading is that some investors in the economy (unmodeled) have no flexibility at all in terms of how much and when to trade. Endogenizing such “noise trading”, making such “noise” demand either price-elastic or timing sensitive, will lead to richer predictions.

6.4 The market clearing mechanism

In each trading round $t \in \{1, 2\}$, investors’ demand schedules are cleared by a competitive market maker, who can take any position at the efficient price. The key advantage of having such a market maker is that he helps ensure the trading price $p_t$ is always semi-strong efficient (as in Kyle, 1985). This way, price informativeness in a later round is naturally higher than in earlier rounds, as the market never forgets the information already discovered.

A market maker is not the only way to facilitate trading. An alternative is to determine the price $p_t$ by market clearing, as in Grossman and Stiglitz (1980) and Verrecchia (1982). Under this alternative setup, investors will trade for two reasons: 1) their private information and 2) providing liquidity for the noise demand $U_t$. The liquidity provision motive for trading is not in the current model: The competitive market maker can take any position needed to clear the market and investors only trade for their information advantage. Cespa and Vives (2012, 2015) study these two different motives in details.

The current model is set up intentionally with the competitive market maker, so that all the results clearly go through, and only through, the information channel, uncontaminated by liquidity provision motives. Including the latter will be an interesting extension, beyond the scope of the current paper, to study how speed technology affects liquidity provision in the market. Importantly, irrespective of the market maker, this paper’s two key mechanisms—the temporal fragmentation
(a) Varying \( t = 1 \) noise trading \( \tau_{U1} \) (fixing \( \tau_{U2} = 4.0 \))

(b) Varying \( t = 2 \) noise trading \( \tau_{U2} \) (fixing \( \tau_{U1} = 4.0 \))

Figure 11: Time varying noise trading. This figure illustrates how different amount of noise trading \( \tau_{U1} \) and \( \tau_{U2} \) affects the long-run price discovery, \( \tau_2 \). Three levels of noise trading are illustrated: \( \tau_{Ut} \in \{2.0, 4.0, 6.0\} \). Panel (a) varies \( \tau_{U1} \) while fixing \( \tau_{U2} = 4.0 \). Panel (b) varies \( \tau_{U2} \) while fixing \( \tau_{U1} = 4.0 \). In each panel, the left and the right graph increases the speed and the information technology, respectively. The other primitive parameters used in this numerical illustration are: \( \tau_0 = 1.0 \), \( \gamma = 0.1 \), and \( k_h(m) = \sqrt{m} \).
by the speed technology and the novel intertemporal crowding-out effect—remain robust.

7 Conclusion

There are two aspects of price discovery: the magnitude and the process. The magnitude aspect (investors’ information acquisition) has been a key focus of the extant literature. This paper studies a model with investors’ endogenous speed acquisition (alongside their information acquisition). The focus is turned to the process of price discovery, i.e., the process through which acquired information is incorporated into price.

The analysis reveals that these two aspects of price discovery are intrinsically connected via investors’ competition. There are two key mechanisms at work: First, with heterogeneous speed, the investors participate in the market at different times and the price discovery is accordingly fragmented temporally. Such temporal fragmentation allows the model to differentiate investors’ well-known intratemporal competition (e.g., Grossman and Stiglitz, 1980) from a novel intertemporal crowding-out effect. Second, investors’ information and speed acquisition can be either complements or substitutes of each other, depending on the relative strengths of the intratemporal and the intertemporal competition. Based on the interaction of these two mechanisms, the model generates testable implications on how advancement in technology would affect market quality. Most notably, when either the speed or the information technology improves, the aggregate magnitude of price discovery can be hurt. This provides a cautionary tale of the disruptive effects of how technological advancement, as seen in recent years, might negatively affect aggregate price informativeness in financial markets.
Appendix

A Proofs

For notation simplicity, the proofs will often use $\mu_1 = \mu_F$, $\mu_2 = \mu_S$, $h_1 = h_F$, $h_2 = h_S$, $\pi_1 = \pi_F$, and $\pi_2 = \pi_S$. This way, the subscript $t = 1$ can handily refer to both the time $t = 1$ and the “F”ast investors; and similarly, $t = 2$ refers to both the time $t = 2$ and the “S”low investors.

Lemma 1

Proof. The proof proceeds by conjecture-and-verify (as in Vives, 1995). Conjecture that a fast investor $i$’s demand schedule is $x_i = a_{i,1}s_i - b_{i,1}p_1$ and that a slow investor $i$’s demand schedule is $x_i = a_{i,2}s_i - b_{i,2}p_1 - c_{i,2}p_2$. At $t = 1$, with only the fast investors, the aggregate demand is

$$L_1(p_1) = \int_{i\in[0,1]} x_i(p_1, s_i)\mathbb{I}_{\{t_i=1\}}di + U_1 = \left(\int_{t_i=1} a_{i,1}di\right)V - \left(\int_{t_i=1} b_{i,1}di\right)p_1 + U_1,$$

where the convention $\int \epsilon_i di = 0$ is used. From the market maker’s perspective, the sufficient summary statistic, therefore, is the intercept of the above linear demand, which can be transformed into $z_1 := V + U_1/\left(\int_{t_i=1} a_{i,1}di\right)$. Therefore, using standard property of normal distribution,

$$\tau_1 = \text{var}[V|L_1(\cdot)]^{-1} = \tau_0 + \left(\int_{t_i=1} a_{i,1}di\right)^2 \tau_U. \quad \text{(A.1)}$$

The incremental price discovery is $\Delta \tau_1 = \left(\int_{t_i=1} a_{i,1}di\right)^2 \tau_U$. The maker maker sets the efficient price

$$p_1 = \mathbb{E}[V|L_1(\cdot)] = \mathbb{E}[V|z_1] = \frac{\tau_0}{\tau_1}p_0 + \frac{\Delta \tau_1}{\tau_1}z_1. \quad \text{(A.2)}$$

As such, the trading price $p_1$ is an equivalent statistic of $z_1$. From a fast investor’s perspective, $\text{var}[V|s_i, p_1]^{-1} = \text{var}[V|s_i, z_1]^{-1} = h_i + \tau_1$ and $\mathbb{E}[V|s_i, p_1] = \mathbb{E}[V|s_i, z_1] = (\tau_0p_0 + h_is_i + \Delta \tau_1z_1)/(\tau_1 + h_i)$. Using the above, a CARA fast investor $i$’s optimal demand is

$$x_i = \frac{\mathbb{E}[V|s_i, p_1] - p_1}{\gamma \text{var}[s_i, p_1]} = \frac{1}{\gamma}(h_is_i + \Delta \tau_1z_1 - (\tau_0 + h_i + \Delta \tau_1)p_1) = \frac{h_i}{\gamma}(s_i - p_1).$$

(Recall the normalization $p_0 = 0$.) The conjectured linear demand $x_i = a_{i,1}s_i - b_{i,1}p_1$ for fast investors has thus been verified with coefficients $a_{i,1} = b_{i,1} = h/\gamma$. 

44
At \( t = 2 \), only slow investors trade and the aggregate demand is

\[
L_2(p_2; p_1) = \int_{t=1}^{2} x_i(p_2, s_i; p_1) \, \text{d}t + U_2 = \left( \int_{t=2}^{1} a_{i,2} \, \text{d}t \right) V - \left( \int_{t=2}^{1} b_{i,2} \, \text{d}t \right) p_1 - \left( \int_{t=2}^{1} c_{i,2} \, \text{d}t \right) p_2 + U_2,
\]

Recalling \( p_1 \), the market maker updates his information set to \( \{p_1, z_2\} \), where \( z_2 := V + U_2/\left( \int_{t=2}^{1} a_{i,2} \, \text{d}t \right) \) summarises the new information in \( L_2(\cdot) \). Then,

\[
(A.3) \quad \tau_2 = \text{var}[V|p_1, L_2(\cdot)]^{-1} = \text{var}[V|z_1, z_2]^{-1} = \tau_1 + \left( \int_{t=2}^{1} a_{i,2} \, \text{d}t \right)^2 \tau_U,
\]

where the incremental price discovery \( \Delta \tau_2 = \left( \int_{t=2}^{1} a_{i,2} \, \text{d}t \right)^2 \tau_U \). The market maker then sets the efficient price

\[
(A.4) \quad p_2 = \mathbb{E}[V|p_1, L_2(\cdot)] = \mathbb{E}[V|z_1, z_2] = \frac{\tau_0}{\tau_2} p_0 + \frac{\Delta \tau_1}{\tau_2} z_1 + \frac{\Delta \tau_2}{\tau_2} z_2.
\]

A slow investor updates \( \text{var}[V|s_i, p_1, p_2]^{-1} = \text{var}[V|s_i, z_1, z_2]^{-1} = h_1 + \tau_2 \) and \( \mathbb{E}[V|s_i, p_1, p_2] = \mathbb{E}[V|s_i, z_1, z_2] = (\tau_0 p_0 + \Delta \tau_1 z_1 + \Delta \tau_2 z_2 + h_i s_i)/(\tau_2 + h_i) \). Solving a quadratic optimization problem, a CARA slow investor’s optimal demand is

\[
x_i = \frac{\mathbb{E}[V|s_i, p_1, p_2] - p_2}{\gamma \text{var}[s_i, p_1, p_2]} = \frac{1}{\gamma} (h_i s_i + \Delta \tau_1 z_1 + \Delta \tau_2 z_2 - (\tau_0 + \Delta \tau_1 + \Delta \tau_2 + h_i) p_2) = \frac{h_i}{\gamma} (s_i - p_2).
\]

Thus the conjectured linear demand for slow investors is also verified with coefficients \( a_{i,2} = c_{i,2} = h_i/\gamma \) and \( b_{i,2} = 0 \). That is, the slow investor’s demand is independent of \( p_1 \).

The analysis so far has proved the investors’ optimal demand as stated in the lemma. In the meantime, Equations (A.1) through (A.4) verify the recursion systems of \( p_t \) and \( \Delta \tau_t \). It remains to compute the investors’ ex ante certainty equivalent. Consider a fast investor. Before accounting for the technology acquisition cost, his expected utility at \( t = 0 \) is

\[
-\mathbb{E} \left[ \exp \left[ \frac{-\mathbb{E}[V|s_i, p_1] - p_1^2}{2 \text{var}[V|s_i, p_1]} \right] \right].
\]

The expressions derived earlier yield the following: \( \mathbb{E}[V|s_i, p_1] - p_1 = \frac{h_i}{\tau_1 + h_i} \left( \frac{\tau_0}{\tau_1} V + \epsilon_i - \frac{\Delta \tau_1}{\tau_1} \frac{U_i}{\int_{j=1}^{k} (h_j/\gamma) \, \text{d}j} \right) \) and \( \text{var}[V|s_i, p_1]^{-1} = \tau_1 + h_i \). Plug the above into the \( t = 0 \) expected utility for a fast investor, simplify, and the resulting ex ante certainty equivalent before technology acquisition costs is

\[
\frac{1}{\gamma} \ln \left( 1 + \frac{h_i}{\tau_1} \right).
\]

Subtracting the information acquisition cost and the speed acquisition cost gives the expression stated in the lemma. The calculation for slow investors repeats the above and is omitted. \( \square \)
Lemma 2

Proof. An investor $i$’s terminal wealth is $(p_2 - p_1)x_{i,1} + (V - p_2)x_{i,2}$, where $x_{i,t}$ is his cumulative position by round $t$. In particular, a slow investor has $x_{i,1} = 0$. Thus, at $t = 2$, each investor $i$ solves \( \max_{x_{i,2}} \mathbb{E}\left[ -\exp\{-y \cdot \left[(P_2 - P_1)x_{i,1} + (V - P_2)x_{i,2}\right]\} \mid s_i, p_1, p_2, x_{i,1} \right] \), or, equivalently, \begin{equation}
(A.5) \quad -\exp\{-y \cdot (p_2 - p_1)x_{i,1}\} \max_{x_{i,2}} \mathbb{E}\left[ -\exp\{-y \cdot (V - P_2)x_{i,2}\} \mid s_i, p_1, p_2 \right].
\end{equation}

Hence, the optimization problem reduces to the same one as the one faced by the slow investors in the main model. (The position $x_{i,1}$ is irrelevant for the $t = 2$ optimization.) The same conjecture-and-verify analysis as in Lemma 1 applies and gives the optimal linear cumulative demand, $x_{i,2} = (h_i/y)(s_i - p_2)$, for both the fast and the slow investors.

Now consider fast investors’ optimization at $t = 1$. Substituting his optimal demand $x_{i,2}$ into optimization (A.5), at $t = 1$, a fast investor $i$ solves

\[
\max_{x_{i,1}} \mathbb{E}\left[ -\exp\left\{-y \cdot (P_2 - P_1)x_{i,1} - \frac{h_i^2}{2(\tau_2 + h_i)}(S_i - P_2)^2 \right\} \mid s_i, p_1 \right],
\]

where \( \text{var}[V \mid s_i, p_2]^{-1} = \tau_2 + h_i \). Note that the second term in the exponential does not affect the optimization. Further, due to the competitive market maker, \( \mathbb{E}[V \mid s_i, p_1] = \mathbb{E}[\mathbb{E}[V \mid s_i, p_1, P_2] \mid s_i, p_1] = \mathbb{E}[P_2 \mid s_i, p_1] \); and, similarly, \( \text{var}[V \mid s_i, p_1] = \text{var}[P_2 \mid s_i, p_1] \). Hence, the fast investor equivalently solves \( \max_{x_{i,1}} \mathbb{E}\left[ -\exp\{-y \cdot (V - P_1)x_{i,1}\} \mid s_i, p_1 \right] \). The optimization problem is equivalent to the one for fast investors solved in the proof of Lemma 1 and the same conjecture-and-verify analysis gives $x_{i,1} = (h_i/y)(s_i - p_1)$.

The recursions of $\tau_t$ and $p_t$ can be found using the above optimal demand functions. At $t = 1$, since the fast investors’ optimal demand is the same as shown in Lemma 1, the same results hold: $\Delta \tau_1 = \tau_1 - \tau_0 = \left( \int_{t_j=1} \frac{h_j}{Y} dj \right)^2 \tau_U$ and $p_1 = p_0 + \frac{\Delta \tau_1}{\tau_1} \left( V + \frac{y U_1}{\int_{t_j=1} h_j dj} \right)$. At $t = 2$, the market maker observes the aggregate demand

\[
L_2(p_2) = \int_{t_j=1} (x_{j,2}(s_j, p_2) - x_{j,1}(s_j, p_1)) dj + \int_{t_j=2} x_{j,2}(s_j, p_2) dj + U_2
\]

\[= p_1 \int_{t_j=1} \frac{h_j}{Y} dj - p_2 \int_{t_j=2} \frac{h_j}{Y} dj + V \int_{t_j=2} \frac{h_j}{Y} dj + U_2,
\]

where the second equality follows the optimal demand schedules derived earlier. Observe how the fast investors’ signals are exactly offset, not contributing to the price discovery in the second fragment ($t = 2$). Intuitively, this is because their signals are already reflected in the first fragment (the $t = 1$ trading) and the only new information arises from the slow investors’ signals. Again,
the market maker sets the price exactly the same as in Lemma 1 and the resulting recursions hold:
\[ \Delta r_2 = r_2 - r_1 = \left( \int_{j=2}^{h_i} \frac{h_i}{\gamma} \, dj \right)^2 \tau_i \] and
\[ p_2 = p_1 + \frac{\Delta r_2}{\tau_2} \left( V + \frac{y_{U_2}}{\int_{j=2}^{h_i} \frac{h_i}{\gamma} \, dj} - p_1 \right). \]

Finally, consider investors’ ex ante certainty equivalent. Since slow investors only trade once at \( t = 2 \), they expect the same certainty equivalent \( \pi_s \) as solved in Lemma 1. A fast investor \( i \)'s unconditional expected utility, before paying the technology cost, is
\[
\mathbb{E} \left[ -\exp \left\{ -\gamma \cdot (S_i - P_1) x_{i,1} - \frac{h_i^2}{2(\tau_2 + h_i)} (S_i - P_2)^2 \right\} \right]
\]
\[
= \mathbb{E} \left[ -\exp \left\{ -\gamma \cdot (S_i - P_1) x_{i,1} + \gamma \cdot (S_i - P_2) x_{i,1} - \frac{2h_i^2}{2(\tau_2 + h_i)} (S_i - P_2)^2 \right\} \right]
\]
\[
= \mathbb{E} \left[ -\exp \left\{ -h_i \cdot (S_i - P_1)^2 + h_i \cdot (S_i - P_2) (S_i - P_1) - \frac{2h_i^2}{2(\tau_2 + h_i)} (S_i - P_2)^2 \right\} \right]
\]
where the last equality follows the optimal demand \( x_{i,1}(\cdot) \) derived above. Define \( Y := [S_i - P_1; S_i - P_2] \) as a bivariate normal (column) random vector, with
\[
\mathbb{E}Y = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{and} \quad \text{var}Y = \begin{bmatrix} \tau_1^{-1} + h_i^{-1} & \tau_2^{-1} + h_i^{-1} \\ \tau_2^{-1} + h_i^{-1} & \tau_2^{-1} + h_i^{-1} \end{bmatrix}.
\]
Then the above expected utility can be rewritten as \( \mathbb{E}[-e^{Y^TAY}] \) where the coefficient matrix \( A \) is given by \( A = [-h_i, h_i/2; h_i/2, -h_i^2/(2(\tau_2 + h_i))] \). Evaluating the expectation with the density of the bivariate normal \( Y \) yields the expected utility of \(-\tau_1 \tau_2 / \sqrt{\tau_1 \cdot (h_i + \tau_2) (h_i + \tau_2)}\). Solving the certainty equivalent yields the form of \( \pi_F \) stated in the lemma. \( \square \)

**Proposition 1**

Proof. The proof begins by writing investors’ certainty equivalent \( \pi_1 \) and \( \pi_2 \) as functions of the fast population size \( \mu_1 \) in \([0, 1]\). To do this, first note that from the first-order condition (6), investors’ endogenous choice of \( h_i \) can be written as a monotone function of \( \tau_i \). By Lemma 1, \( \tau_1 = \tau_0 + \Delta \tau_1 \) and \( \tau_2 = \tau_0 + \Delta \tau_2 \), where \( \Delta \tau_i = \tau_i \mu_i^2 \gamma^2 / \gamma^2 \). Hence, \( \tau_1 \) is effectively a function of \( \mu_1 \), while \( \tau_2 \) of both \( \mu_1 \) and \( \mu_2 \). Finally, note that \( \mu_2 = 1 - \mu_1 \). As such, investors’ certainty equivalent \( \pi_i \) are functions of \( \mu_i \). Then, depending on \( \mu_1 \), there are three cases.

Case 1: First, suppose \( \mu_1 = 1 \) and \( \mu_2 = 0 \); i.e. all investors pay the speed technology cost \( 1/g_t \) and become fast. If this is the case, then in equilibrium \( \pi_1 \geq \pi_2 \) must hold. Consider an investor \( i \)'s unilateral deviation to not investing in the speed technology, saving the cost of \( 1/g_t \) and becomes slow. By Equation (4), the price informativeness remains the
same, \( \tau_1 = \tau_2 \), because a single investor’s deviation has zero population measure. Then \( i \)’s optimal technology investment \( h_i \), by the first-order condition (6), remains the same as if he were fast: \( h_i = h(\tau_2) = h(\tau_1) = h_1 \). As a result, his certainty equivalent \( \pi_2 = \pi_1 + 1/g_t > \pi_1 \) and he indeed will deviate. Such a case of \( \mu_1 = 1 \) and \( \mu_2 = 0 \), therefore, can never be an equilibrium.

Case 2: Second, consider the case of \( \mu_1 = 0 \) and \( \mu_2 = 1 \). (This will correspond to the corner equilibrium stated in the proposition.) If this is an equilibrium, it has to be the case that \( \pi_1 \leq \pi_2 \), i.e., all investors stay slow. The argument below shows that fixing all other exogenous parameters, \( \pi_1 \leq \pi_2 \) holds if and only if \( g_t < \hat{g}_t \), for some threshold \( \hat{g}_t \).

At \( \mu_1 = 0 \), \( \tau_1 = \tau_0 < \tau_2 \) and thus a slow investor’s unilateral deviation to fast yields
\[
\pi_1|_{\mu_1=0} = \frac{1}{2} \ln \left(1 + \frac{h_1}{\tau_0}\right) - c(h_1) - \frac{1}{g_t},
\]
where \( h_1 \) is the unique solution implied by the first-order condition (6) with \( \tau_1 = \tau_0 \). By envelope theorem, \( \partial \pi_1/\partial g_t = 1/g_t^2 > 0 \). Therefore, \( \pi_1|_{\mu_1=0} \) is monotone increasing in \( g_t \) with limits \( \lim_{g_t \to 0} \pi_1 = -\infty < \pi_2 < \lim_{g_t \to \infty} \pi_1 \). (Note that \( \pi_2|_{\mu_1=0} \) is a finite number unaffected by \( g_t \).) By continuity, therefore, there exists a unique \( \hat{g}_t \) such that \( \pi_1 = \pi_2 \) when \( \mu_1 = 0 \). As such, \( \pi_1 \leq \pi_2 \), supporting \( \mu_1 = 0 \) and \( \mu_2 = 1 \), if and only if \( g_t < \hat{g}_t \). When instead \( g_t > \hat{g}_t \), this corner equilibrium does not exist.

Case 3: Third, consider the interior case of \( \mu_1 \in (0, 1) \), implying \( \pi_1 = \pi_2 \). The key is to show the following result: The difference \( \pi_1 - \pi_2 \) strictly decreases in \( \mu_1 \). Evaluate the partial derivative of \( \pi_1 - \pi_2 \) with respect to \( \mu_1 \) and after some simplification,
\[
\frac{\partial (\pi_1 - \pi_2)}{\partial \mu_1} - 2\gamma = \frac{h_2/\gamma}{\tau_2 + h_2} - \frac{h_1/\gamma}{\tau_1 + h_1} + \frac{h_2/\gamma}{\tau_2 + h_2} \left( \frac{\partial \tau_2}{\partial \mu_1} \right).
\]

Note that the term in the square-brackets is non-positive, because \( \tau_2 \geq \tau_1 \) by construction and because \( h_t = h(\tau_t) \) decreases in \( \tau_t \) as implied by the first-order condition (6).

One still needs to sign both \( \partial \tau_1/\partial \mu_1 \) and \( \partial \Delta \tau_2/\partial \mu_1 \). To do so, rearrange the first-order condition (6) for \( t = 1 \) as \( (\tau_0 + \Delta \tau_1 + g_h k_h(m_1))/\hat{k}_h(m_1) = g_h/(2\gamma) \) with \( \Delta \tau_1 = \mu_1^2 g_h^2 k_h(m_1)^2 \tau_U/\gamma^2 \) following Equation (4). It can then immediately be concluded that \( m_1 \) strictly decreases in \( \mu_1 \), as otherwise the left-hand side of the above equation is always increasing in \( \mu_1 \), unable to maintain the equality. (Recall that \( k_h(\cdot) \) is concavely increasing.) Similarly, it is also known that \( \tau_1 (= \tau_0 + \Delta \tau_1) \) decreases in \( m_1 \). Hence, \( \tau_1 \) (and \( \Delta \tau_1 \)) increases in \( \mu_1 \). For \( t = 2 \), \( (\tau_0 + \Delta \tau_1 + \Delta \tau_2 + h_2)/\hat{k}_h(m_2) = g_h/(2\gamma) \) with \( \Delta \tau_2 = (1 - \mu_1)^2 g_h^2 k_h(m_2)^2 \tau_U/\gamma^2 \). Note that \( \frac{\partial \Delta \tau_2}{\partial \mu_1} = \left( -2(1 - \mu_1)h_2^2 + 2(2 - \mu_1)^2 h_2 \frac{\partial k_h}{\partial m_1} \right) \frac{\tau_U}{\gamma^2} \). As such, if \( \Delta \tau_2 \) increases in \( \mu_1 \), then it has to be the case that \( \partial h_2/\partial \mu_1 > 0 \). Because \( h_2 = g_h k_h(m_2) \), \( m_2 \) is also increasing in \( \mu_1 \). It then follows that the left-hand side of the above equation strictly increases in \( \mu_1 - \Delta \tau_1, \Delta \tau_2 \), and \( m_2 \) all increase with \( \mu_1 \), invalidating the equality.
Therefore, it must be $\Delta \tau_2$ decreases in $\mu_1$.

As $\tau_1$ increases in $\mu_1$ but $\Delta \tau_2$ decreases in $\mu_1$, one can conclude from the above partial derivative that the difference $\pi_1 - \pi_2$ indeed strictly decreases in $\mu_1$.

To sum up, recall from the first cases that at $\mu_1 = 1$, $\pi_1 < \pi_2$. From the second case, at $\mu_1 = 0$, $\pi_1 > \pi_2$ if and only if $g_t > \hat{g}_t$. Hence, when $g_t \leq \hat{g}_t$, the equilibrium with interior $\mu_1$ does not exist due to the above monotonicity of $\pi_1 - \pi_2$ in $\mu_1$. When $g_t > \hat{g}_t$, there exists a unique $\mu_1 \in (0, 1)$ such that $\pi_1 = \pi_2$, sustaining this equilibrium. This completes the proof of this proposition.

\[\Box\]

**Proposition 2 and Corollary 4**

**Proof.** First, the following shows that $h_t$ is monotonically increasing in $g_h$ for both $t = 1$ and $t = 2$.

The first-order condition (6) can be written as $\dot{k}_h(m_t)/(2\gamma) - k_h(m_t) = \tau_t/g_h$, which uniquely solves $m_t$. Holding $g_h$ (and $\gamma$) constant,

(A.6) \[
\frac{\partial m_t}{\partial \tau_t} = \frac{1}{g_h} \left( \frac{\dot{k}_h(m_t)}{2\gamma} - \dot{k}_h(m_t) \right)^{-1} \leq 0
\]

where the inequality follows the concavity of $k_h(m)$. (Note that this also implies that $m_1 \geq m_2$ because $\tau_2 \geq \tau_1$.) In addition,

(A.7) \[
\frac{\partial m_t}{\partial g_h} = -\frac{\tau_t}{g_h} \left( \frac{\dot{k}_h(m_t)}{2\gamma} - \dot{k}_h(m_t) \right)^{-1} = -\frac{\tau_t}{g_h} \frac{\partial m_t}{\partial \tau_t} \geq 0.
\]

From the definition of $h_t = g_hk_h(m_t)$, $\partial h_t/\partial g_h = k_h(m_t) + g_hk_h(m_t)\partial m_t/\partial g_h \geq 0$. Therefore, in any case, the equilibrium $h_t$ is increasing in the information technology $g_h$.

The rest of this proof only deals with the case of exogenous speed acquisition, i.e., with fixed $\mu_1$ and $\mu_2$. The proof for the case of endogenous $\mu_t$ is deferred to proof of Proposition 6. Consider the short-run of $t = 1$. While $g_h$ increases, $h_1$ increases to satisfy the first-order condition, as shown above. It then follows that $\partial \tau_1 / \partial g_h > 0$ because $\tau_1 = \tau_0 + \tau_1 h_1^2 \mu_1^2 / \gamma^2$ with $\mu_1$ exogenous.

Consider the long-run of $t = 2$ now. Suppose the opposite, $\partial \tau_2 / \partial g_h < 0$, is true. Then $h_2$ should be decreasing with $g_h$ because $\tau_2 = \tau_1 + \tau_2 h_2^2 \mu_2^2 / \gamma^2$ with $\tau_1$ is increasing in $g_h$. However, the transformation of first-order condition 6, $g_h/(2\gamma) = (\tau_2 + h_2)k_h^{-1}(h_2/g_h)$, shows that it is impossible for both $\tau_2$ and $h_2$ to be decreasing with $g_h$ at the same time. Thus, the assumed inequality is wrong and $\tau_2$ increases with $g_h$. \[\Box\]
Proposition 3

Proof. To avoid repetition, the proof only considers the full equilibrium where the information acquisition is available. A similar argument can be constructed for the special case where all investors have the same exogenous information $h_o$. In the interior equilibrium, $\pi_1 - \pi_2 = 0$ and, following the proof of Proposition 1, the equality implies an implicit function of $\mu_1$ in terms of the speed technology $g_t$, which implies: $\frac{d\mu_1}{dg_t} = -\frac{\partial \pi_1/\partial g_t}{\partial (\pi_1 - \pi_2)/\partial \mu_1}$, where the denominator of the fraction is negative as shown in Case 3 of the proof for Proposition 1. The numerator equals $1/g_t^2 > 0$ by envelope theorem. Therefore, $\mu_1$ increases in $g_t$. □

Proposition 4

Proof. This proof deals with two cases. The first case is where all investors’ information precision is exogenously given at $h_o$. The second case is where investors endogenously acquire information.

In the first case, as shown in the proof of Proposition 3, $\mu_1$ is increasing with $g_t$, which directly implies that $\tau_1$ is increasing with $g_t$. For the long-run informativeness $\tau_2$, by the implicit function theorem, $\frac{\partial \tau_2}{\partial \mu_1} = 2\tau U h_o^2 \mu_1/\gamma^2 - 2\tau U h_o^2 \mu_2/\gamma^2$, or $\frac{\partial \tau_2}{\partial g_t} = 2(\tau U h_o^2 \mu_1/\gamma^2 - 2\tau U h_o^2 \mu_2/\gamma^2)(\partial g_t/\partial \mu_1)$.

It is clear that $\frac{\partial \tau_2}{\partial g_t} < 0$ when $\mu_1$ is close to zero and $\mu_2$ close to one (i.e., $g_t$ is small), and $\frac{\partial \tau_2}{\partial g_t} > 0$ when $\mu_1$ is close to one and $\mu_2$ close to zero (i.e., $g_t$ is large).

For the second case, two steps are involved. The first step is to prove that $\frac{\partial \tau_1}{\partial g_t} > 0$. In the interior equilibrium, the first-order condition (6) for $t = 1$, together with $\tau_1 = \tau_0 + \tau U h_o^2 \mu_1/\gamma^2$, implies an implicit function of $h_1 = g_t k_h(m_1)$ and $\mu_1$, from which $\frac{\partial h_1}{\partial \mu_1} = -\frac{2\tau_1 \mu_1 h_o^2/\gamma^2}{2\tau U \mu_1^2 h_1/\gamma^2 + 1 - k_h(m_1)/k_h(m_1)} < 0$, where the inequality follows because $k_h(\cdot)$ is concavely increasing. From the effect of speed technology and population of sizes, $\frac{\partial \mu_1}{\partial g_t} > 0$. Therefore, by chain rule, $\frac{\partial h_1}{\partial g_t} < 0$. The first-order condition (6) also implies that $\tau_1$ decreases with $m_1$ and, hence, also with $h_1$, yielding $\frac{\partial \tau_1}{\partial g_t} > 0$.

The second step is to prove that $\tau_2$ first decreases and then increases with $g_t$. In the interior equilibrium, the first-order condition (6) for $t = 2$ always holds. Recall $\tau_2 = \tau_0 + \tau U \mu_2^2/\gamma^2$, or $\frac{\partial \tau_2}{\partial \mu_2} = \frac{4\tau U \mu_2^2}{\gamma}$. By implicit function theorem, it implies

$$\frac{\partial h_2}{\partial \mu_2} = -\frac{4\tau U \mu_2 h_2^2 - \mu_1 h^2 - \mu_1^2 \tau_1 h_1/\partial \mu_1}{\gamma - k_h(m_2)/k_h(m_2) + 2\gamma + 4\tau U \mu_2^2 \tau_2/\gamma}.$$

As done in the proof of step 1, the idea is to first sign the above partial derivative and then sign $\frac{\partial h_2}{\partial g_t}$ using chain rule: $\frac{\partial h_2}{\partial g_t} = \frac{\partial h_2}{\partial \mu_2} \frac{\partial \mu_2}{\partial \mu_1} \frac{\partial \mu_1}{\partial g_t}$, where $\frac{\partial \mu_2}{\partial \mu_1} = -1$ following the identity $\mu_1 + \mu_2 = 1$ and $\frac{\partial \mu_1}{\partial g_t} > 0$. In particular, consider the limits of $\frac{\partial h_2}{\partial \mu_2}$ as $g_t \uparrow \infty$ and $g_t \downarrow \hat{g}_t$. 50
respectively. To evaluate these limits, one needs to show that \( h_1, h_2, \) and \( \partial h_1/\partial \mu_1 \) are have finite bounds.

The finite bounds for \( h_t \) can be easily established by noting from the first-order condition (6) that \( \tau_t \) in equilibrium is monotone decreasing in \( \tau_t \). From the model setting, it is known that \( \tau_t \) has strictly positive lower bound \( \tau_0 \). Therefore, both \( h_1 \) and \( h_2 \) have finite upper bounds. (They also have lower bounds of zero by construction.) Finally, from the expression of \( \partial h_1/\partial \mu_1 \) derived in the proof of the previous step, it can be seen that \( \mu_1, (\partial h_1/\partial \mu_1) = -\frac{2\tau_1 \mu_1^2 h_1/\gamma^2}{2\tau_1 \mu_1^2 + \tau_1/\gamma^2} \), the lower bound of \( h_1 \) is also bounded.

Now the limits can be evaluated. When speed technology \( g_t \uparrow \infty \), almost all investors become fast and \( \mu_2 \downarrow 0 \) and \( \lim_{\mu_2 \downarrow 0} (\partial h_2/\partial \mu_2) = -\frac{4\tau_1 \mu_1^2 h_2^2 - \mu_1^2 h_2 h_1 \partial h_1/\partial \mu_1}{\gamma - k_\mu(m_2)/k_\mu(m_2) + 2\gamma} > 0 \). Similarly, when speed technology \( g_t \downarrow \hat{g}_h \), almost all investors stay slow, \( \mu_1 \downarrow 0 \), and \( \lim_{\mu_1 \downarrow 0} (\partial h_2/\partial \mu_2) = -\frac{4\tau_1 \mu_1^2 h_2^2 - \mu_1^2 h_2 h_1 \partial h_1/\partial \mu_1}{\gamma - k_\mu(m_2)/k_\mu(m_2) + 2\gamma} < 0 \). As the above shows, for sufficiently large (low) \( g_t, h_2 \) increases (decreases) in \( \mu_2 \) and hence decreases (increases) in \( g_t \). By the chain rule expression above. The first-order condition (6) implies that \( \tau_2 \) decreases with \( \tau_2 \) and the stated results are proved.

\[ \square \]

**Proposition 5**

**Proof.** Fixing \( g_t \), \( g_h \) increases from \( \hat{g}_h \) to \( \infty \). The aggregate demand for speed in the economy is \( \int_{[0,1]} \frac{\partial \mu_1}{\partial g_h} \). From \( \Delta \tau_1 = \tau_1 h_2^2 / \gamma^2 \), by implicit function theorem,

\[ (A.8) \]

\[
\frac{\partial \mu_1}{\partial g_h} = \frac{y^2}{2\tau_1 \mu_1 h_2^2} \left( \frac{\partial \Delta \tau_1}{\partial g_h} - \frac{2\tau_1 \mu_1^2 h_1}{\gamma^2} \frac{\partial h_1}{\partial g_h} \right).
\]

Hence, the sign of \( \partial \mu_1/\partial g_h \) depends on the difference between the two terms in the brackets. Consider first the case of a very small \( g_h \). Corollary 1 establishes the existence of a lower bound \( \hat{g}_h \) for \( g_h \), such that the equilibrium is interior if and only if \( g_h \geq \hat{g}_h \). In particular, when \( g_h \downarrow \hat{g}_h \), the marginal investor is just indifferent between becoming fast or not, implying \( \mu_1 \downarrow 0 \). The first-order condition (6) at this limit gives \( 1/(2(\tau_0 + h_1)\gamma) - \hat{c}(h_1) = 0 \), which has interior solution of \( 0 < h_1 < \infty \), thanks to the assumption of \( \hat{c}(0) = 0 \). By differentiability, therefore, \( \partial h_1/\partial g_h \) is finite in this limit as well. Taken together, the second term in the above brackets has limit zero as \( \mu_1 \downarrow 0 \), when \( g_h \downarrow \hat{g}_h \). The remaining term is \( \partial \Delta \tau_1/\partial g_h \), which is shown by Proposition 6 to be strictly positive. Thus, \( \partial \mu_1/\partial g_h \) is positive in the case of a very small \( g_h \), close to the lower bound of \( \hat{g}_h \).

Consider next the case of a very large \( g_h \), i.e. \( g_h \uparrow \infty \). First, there exists an upper bound for
investors’ expense on information acquisition, \( m_t \). To see this, note from the first-order condition (6):

\[
(A.9) \quad \frac{1}{2\gamma} k_h(m_t) > \frac{1}{2\gamma} \hat{k}_h(m_t) - \frac{1}{g_h} \tau_t = k_h(m_t) \geq k_h(0) + m_t \hat{k}_h(m_t) = m_t k_h(m_t)
\]

where the first inequality holds because \( \tau_t \geq \tau_0 > 0 \) and the last inequality holds by concavity of \( k_h(\cdot) \) and by \( k_h(0) = 0 \). Therefore, for \( t \in \{1, 2\} \), there exists an upper bound for \( m_t \leq 1/(2\gamma) \), an upper bound for \( k_h(m_t) \leq k_h(1/(2\gamma)) \), and a lower bound for \( \hat{k}_h(m_t) \geq \hat{k}_h(1/(2\gamma)) > 0 \). Second, in the limit of \( g_h \uparrow \infty \), the equilibrium is always interior (following Corollary 1). Hence, the limit of the fast investor’s ex ante certainty equivalent \( \lim_{g_h \uparrow \infty} \pi_1 = \frac{1}{2\gamma} \lim_{g_h \uparrow \infty} \ln \left( 1 + \frac{h_t}{\tau_1} \right) - \lim_{g_h \uparrow \infty} m_1 - \frac{1}{g_t} \) exists and must be nonnegative to sustain the interior equilibrium. Since \( m_1 \) is bounded from above, it follows that \( \lim_{g_h \uparrow \infty} (h_1/\tau_1) \) also exists and is strictly positive. That is, there exists some \( a \in (0, \infty) \), such that \( \lim_{g_h \uparrow \infty} (\tau_1/h_1) = a \). Equivalently, as \( \tau_0 \) is a finite constant, \( \lim_{g_h \uparrow \infty} (\Delta \tau_1/h_1) = a \). Further, a fast investor’s first-order condition (6) can be rewritten as \( \frac{1}{2\gamma} \frac{g_t}{\tau_1 + h_1} - c(h_1/g_h) = 0 \). Since the above holds under \( g_h \uparrow \infty \), it follows that \( h_1 \sim g_h \); or \( \lim_{g_h \uparrow \infty} (h_1/g_h) = b \in (0, \infty) \). (If \( h_1 \) is of higher magnitude than \( g_h \), the limit of the first term above falls to zero, while the limit of the second term is strictly positive as \( c(\cdot) \) is strictly convex. If instead \( h_1 \) is of lower magnitude than \( g_h \), the limit of the first term approaches infinity, while the second term falls to zero.) Now consider the limit of the difference in the brackets of Equation (A.8):

\[
\lim_{g_h \uparrow \infty} \left( \frac{\partial \Delta \tau_1}{\partial g_h} - 2 \frac{\tau_t \mu_1^2 h_1}{\gamma^2} \frac{\partial h_1}{\partial g_h} \right) = \lim_{g_h \uparrow \infty} \left( \frac{\partial \Delta \tau_1}{\partial g_h} - 2 \frac{\Delta \tau_1}{h_1} \frac{\partial h_1}{\partial g_h} \right) = (ab - 2ab) < 0
\]

where the last equality follows L'Hôpital’s rule. Therefore, in the limit of \( g_h \uparrow \infty \), \( \partial \mu_1/\partial g_h < 0 \). Finally, consider the value of \( \mu_1 \) in this limit. Note that \( \Delta \tau_1 = \tau_0 + \tau_t \mu_1^2 h_1^2 / \gamma^2 \). Therefore, in order for \( \lim_{g_h \uparrow \infty} (\Delta \tau_1/h_1) = a \in (0, \infty) \) to hold, it must be such that \( \lim_{g_h \uparrow \infty} (\mu_1^2 h_1) = c \in (0, \infty) \), i.e., \( \mu_1 \) in this limit is of magnitude \( h_1^{-1/2} \). As \( h_1 \uparrow \infty \), this also implies that \( \mu_1 \downarrow 0 \) in this limit.

**Fixing \( g_h, g_t \) increases from \( \hat{g}_t \) to \( \infty \).** The aggregate demand for information is \( \tilde{h} := \mu_1 h_1 + \mu_2 h_2 \).

Since \( \mu_1 \) is monotone in \( g_t \) (Proposition 3), it is sufficient to examine the partial derivative of the above aggregate demand with respect to \( \mu_1 \): \( \partial \tilde{h}/\partial \mu_1 = h_1 - h_2 + \mu_1 \cdot (\partial h_1/\partial \mu_1) - \mu_2 \cdot (\partial h_2/\partial \mu_2) \).

At the initial extreme of \( g_t \downarrow \hat{g}_t \), the proof of Proposition 4 has shown that 1) \( \mu_1 \downarrow 0 \), 2) \( \mu_1 \cdot \partial h_1/\partial \mu_1 \) is bounded, and 3) \( \partial h_2/\partial \mu_2 < 0 \). Taking these into the above partial derivative yields \( \partial \tilde{h}/\partial \mu_1 \rightarrow h_1 - h_2 - \mu_2 \cdot (\partial h_2/\partial \mu_2) > 0 \), recalling that \( h_1 \geq h_2 \) from Equation (7). At the eventual extreme of \( g_t \uparrow \infty \), the proof of Proposition 4 has shown that 1) \( \mu_2 \downarrow 0 \), 2) \( \partial h_1/\partial \mu_1 < 0 \), and 3) \( \partial h_2/\partial \mu_2 > 0 \).

In addition, since \( \mu_2 \downarrow 0 \), \( \Delta \tau_2 = \mu_2^2 h_2^2 \tau_t / \gamma^2 \downarrow 0 \) (\( h_2 \) is bounded), implying \( \tau_2 \downarrow \tau_1 \) and 4) \( h_2 \uparrow h_1 \). Taking the above into \( \tilde{h} \) yields \( \partial \tilde{h}/\partial \mu_1 \rightarrow \mu_1 \cdot (\partial h_1/\partial \mu_1) - \mu_2 \cdot (\partial h_2/\partial \mu_2) < 0 \). \( \square \)
Proposition 6

Proof. By construction, \( \tau_1 = \tau_0 + \Delta \tau_1 \) and \( \tau_2 = \tau_0 + \Delta \tau_1 + \Delta \tau_2 \). The first-order condition implicitly has \( m_1 \) and \( m_2 \) as functions of \( m_1(\Delta \tau_1) \) and \( m_2(\Delta \tau_1, \Delta \tau_2) \). Further, \( \Delta \tau_t = \tau_U g_h^2 k_h(m_t)^2 \mu_t^2 / \gamma^2 \), or \( \mu_t = \frac{\gamma}{\sqrt{\tau_U}} g_h k_h(m_t) \). Therefore, the unconstrained equilibrium (with endogenous acquisition of both speed and information) is pinned down by a two-equation, two-unknown system:

\[
\begin{align*}
F &= \begin{bmatrix}
\frac{1}{2\gamma} \ln \left( 1 + \frac{g_h k_h(m_1)}{\tau_1} \right) - m_1 - \frac{1}{\mu_1} \\
\frac{1}{2\gamma} \ln \left( 1 + \frac{g_h k_h(m_2)}{\tau_2} \right) - m_2
\end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},
\end{align*}
\]

where \( \{m_t\}_{t \in \{1,2\}} \) are functions of \( \Delta \tau_1 \) and \( \Delta \tau_2 \) following the first-order condition (6), which can be rewritten as \( \dot{k}_h(m_t)/(2\gamma) - k_h(m_t) = \tau_t / g_h \).

Take total derivatives with respect to \( g_h \) on the equilibrium condition \( F = 0 \) to get

\[
\begin{bmatrix} F_{1g} \\ F_{2g} \end{bmatrix} \, dg_h = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.
\]

One can easily evaluate, using envelope theorem,

\[
F_{1g} = \frac{1}{2\gamma} \frac{k_h(m_1)}{\tau_1 + g_h k_h(m_1)} - \frac{1}{2\gamma} \frac{k_h(m_2)}{\tau_2 + g_h k_h(m_2)} = \frac{1}{g_h} \left( \frac{k_h(m_1)}{\dot{k}_h(m_1)} - \frac{k_h(m_2)}{\dot{k}_h(m_2)} \right) > 0,
\]

where the second equality follows the first-order condition (6), while the last inequality follows the concavity of \( k_h(m) \), knowing that \( m_1 > m_2 \). Also,

\[
F_{2g} = -\frac{\sqrt{\Delta \tau_1}}{k_h(m_1)^2} \dot{k}_h(m_1) \frac{\partial m_1}{\partial g_h} - \frac{\sqrt{\Delta \tau_2}}{k_h(m_2)^2} \dot{k}_h(m_2) \frac{\partial m_2}{\partial g_h} - \frac{\sqrt{\tau_U}}{\gamma}
\]

\[
= -\frac{\sqrt{\tau_U}}{\gamma} \mu_1 g_h \frac{\dot{k}_h(m_1)}{k_h(m_1)} \frac{\partial m_1}{\partial g_h} - \frac{\sqrt{\tau_U}}{\gamma} \mu_2 g_h \frac{\dot{k}_h(m_2)}{k_h(m_2)} \frac{\partial m_2}{\partial g_h} - \frac{\sqrt{\tau_U}}{\gamma} < -\frac{\sqrt{\tau_U}}{\gamma} < 0,
\]

where the equality uses the expression of \( \mu_t \) and the inequality holds because \( \partial m_t / \partial g_h \) is derived earlier to be positive (inequality A.7).

The elements in the Jacobian matrix can also be evaluated. Using envelope theorem,

\[
F_{11} = -\frac{k_h(m_1)}{\dot{k}_h(m_1) \tau_1} + \frac{k_h(m_2)}{\dot{k}_h(m_2) \tau_2} \leq 0
\]

where the inequality holds because \( k_h(m_1)/\dot{k}_h(m_1) \geq k_h(m_2)/\dot{k}_h(m_2) \) (concavity) and \( \tau_1 \leq \tau_2 \).

Similarly,

\[
F_{12} = \frac{k_h(m_2)}{\dot{k}_h(m_2) \tau_2} > 0.
\]
Now consider the partial derivatives with respect to $F_2$:

$$
F_{21} = \frac{1}{2\sqrt{\Delta \tau_1 k_h(m_1)}} - \frac{\sqrt{\Delta \tau_1}}{k_h(m_1)^3} \frac{\partial m_1}{\partial \tau_1} \frac{\partial \tau_1}{\partial \Delta \tau_1} - \frac{\sqrt{\Delta \tau_2}}{k_h(m_2)^3} \frac{\partial m_2}{\partial \tau_2} \frac{\partial \tau_2}{\partial \Delta \tau_1} = \frac{1}{2\sqrt{\Delta \tau_1 k_h(m_1)}} - \frac{\sqrt{\Delta \tau_1}}{\gamma} \frac{\partial m_1}{\partial \tau_1} - \frac{\sqrt{\Delta \tau_2}}{\gamma} \frac{\partial m_2}{\partial \tau_2} > 0
$$

where the equality follows the expression of $\mu_i$ and the inequality holds because $\partial m_i / \partial \tau_i \leq 0$ as shown before (inequality A.6). Similarly,

$$
F_{22} = \frac{1}{2\sqrt{\Delta \tau_2 k_h(m_2)}} - \frac{\sqrt{\Delta \tau_2}}{k_h(m_2)^3} \frac{\partial m_2}{\partial \tau_2} \frac{\partial \tau_2}{\partial \Delta \tau_2} > 0.
$$

By Cramer’s rule,

$$
\frac{\partial \Delta \tau_1}{\partial g_h} = \begin{vmatrix}
-F_{1g} & F_{12} \\
-F_{2g} & F_{22}
\end{vmatrix}
\text{ and } \frac{\partial \Delta \tau_2}{\partial g_h} = \begin{vmatrix}
F_{11} & -F_{1g} \\
F_{21} & -F_{2g}
\end{vmatrix}.
$$

The sign of the denominator is easy to shown: $F_{11} F_{22} - F_{12} F_{21} < 0$. It remains to examine the numerators. For $\tau_1$, it can be seen that $-F_{1g} F_{22} + F_{12} F_{2g} < 0$; hence $\partial \tau_1 / \partial g_h = \partial \Delta \tau_1 / \partial g_h > 0$.

To sign $\partial \tau_2 / \partial g_h$ is equivalent to signing the sum of the numerators of $\partial \Delta \tau_1 / \partial g_h$ and $\partial \Delta \tau_2 / \partial g_h$:

$$
(-F_{1g} F_{22} + F_{12} F_{2g}) + (-F_{11} F_{2g} + F_{1g} F_{21}) = (F_{21} - F_{22}) F_{1g} + (F_{12} - F_{11}) F_{2g}
$$

To prove the statement made in the proposition, the objective is to show that under the limits of $g_h \uparrow \infty$ and of $g_h \downarrow \check{g}_h$, the sign of the above term is negative and positive, respectively (recall that the determinant for the denominator is negative). The proof of Proposition 5 shows that in the upper limit, $\mu_1 \downarrow 0$ and $\mu_2 \uparrow 1$. The proof of Corollary 1 shows that in the lower limit, investors are just indifferent between acquiring the speed or not, implying again $\mu_1 \downarrow 0$ and $\mu_2 \uparrow 1$. Using these limiting values of $\mu_1$ and $\mu_2$, the above simplifies to

(A.11) \[ \left( \frac{1}{2\sqrt{\Delta \tau_1 k_h}} - \frac{1}{2\sqrt{\Delta \tau_2 k_h}} \right) F_{1g} + \frac{k_h}{k_{h_1} \tau_1} F_{2g}, \]

where, simplifying the notation, $k_h(\cdot)$ and $\dot{k}_h(\cdot)$ are replaced by subscripts of $t \in \{1, 2\}$.

Consider the limit of $g_h \uparrow \infty$ first. Equation (A.11) satisfies the following inequality:

$$
\left( \frac{1}{2\sqrt{\Delta \tau_1 k_h}} - \frac{1}{2\sqrt{\Delta \tau_2 k_h}} \right) F_{1g} + \frac{k_h}{k_{h_1} \tau_1} F_{2g} < \frac{F_{1g}}{2\sqrt{\Delta \tau_1 k_h}}
$$

because $F_{1g} > 0$ and $F_{2g} < -\sqrt{\gamma} / \gamma < 0$. The proof of Proposition 5 establishes that $\Delta \tau_1 \to \infty$. 

54
In addition, the inequality (A.9) establishes that in equilibrium, both $m_1$ and $m_2$ have finite upper and lower bounds, implying that both $k_{h1}$ and $F_{ig}$ is also finite (since $k_h(\cdot)$ is twice-differentiable). Therefore, $\lim_{g_h \uparrow \infty} (F_{lg} / (2\sqrt{\Delta \tau_1} k_{h1}) = 0$ and

$$\lim_{g_h \uparrow \infty} \left[ \left( \frac{1}{2\sqrt{\Delta \tau_1} k_{h1}} - \frac{1}{2\sqrt{\Delta \tau_2} k_{h2}} \right) F_{lg} + \frac{k_{h1}}{k_{h1} \tau_1} F_{2g} \right] < \lim_{g_h \uparrow \infty} \frac{F_{lg}}{2\sqrt{\Delta \tau_1} k_{h1}} = 0.$$  

This proves that in this upper limit, $\tau_2$ is increasing with $g_h$.

Finally, consider the limit of $g_h \downarrow \hat{g}_h$. As $g_h \downarrow \hat{g}_h$, clearly $F_{lg}$ and $F_{2g}$ are finite. However, $\mu_1 \downarrow 0$, $\Delta \tau_1 \downarrow 0$, and the first term of Equation (A.11) approaches $+\infty$. The sum of numerators above therefore has a positive sign. Given the negative sign of the denominator, it can be concluded that $\partial \tau_2 / \partial g_h < 0$ in the limit of $g_h \downarrow \hat{g}_h$. □

**Proposition 7**

**Proof.** To prove the proposition, it is equivalent to sign the difference between the partial derivatives of $\Delta \tau_1$ and of $\Delta \tau_2$ with respect to the two technology $g_t$ and $g_h$. For example, if $\Delta \tau_1$ increases faster than $\Delta \tau_2$, then it follows that the speed of price discovery is increasing; and vice versa. This proof proceeds with the two types of technology shocks separately.

**Shocking the speed technology** $g_t$. From the proof of Proposition 6, it can be seen that the equilibrium is characterized by the vector function $F(\Delta \tau_1, \Delta \tau_2; g_t, g_h) = 0$ (Equation A.10). The partial derivatives of $F$ with respect to the speed technology $g_t$ are $F_{1g} = 1/g_t^2$ and $F_{2g} = 0$. As in the proof of Proposition 6, by Cramer’s rule,

$$\frac{\partial \Delta \tau_1}{\partial g_t} = -\text{sign}(-F_{lg} F_{22} + F_{12} F_{2g}) = \text{sign} \left( \frac{F_{22}}{g_t^2} \right) > 0,$$

$$\frac{\partial \Delta \tau_2}{\partial g_t} = -\text{sign}(-F_{11} F_{2g} + F_{1g} F_{21}) = -\text{sign} \left( \frac{F_{21}}{g_t^2} \right) < 0.$$  

Therefore, $\partial \Delta \tau_1 / \partial g_t - \partial \Delta \tau_2 / \partial g_t > 0$, implying that $\Delta \tau_1$ is increasing in the speed technology $g_t$, while $\Delta \tau_2$ is decreasing. The speed of price discovery thus increases with the speed technology.

**Shocking the information technology** $g_h$. The objective is to sign the difference of $\partial \Delta \tau_1 / \partial g_h - \partial \Delta \tau_2 / \partial g_h$ under the limits of $g_h \downarrow \hat{g}_h$ and $g_h \uparrow \infty$ respectively. Using the expressions derived from the proof of proposition 6, the sign of the above difference is the same as the sign of $(F_{21} + F_{22}) F_{1g} - (F_{11} + F_{12}) F_{2g}$.

Consider the limit of $g_h \downarrow \hat{g}_h$ first. Clearly, in this limit, $F_{11}$, $F_{12}$, $F_{22}$, $F_{1g}$, and $F_{2g}$ are all finite. (In particular, $\partial m_t / \partial g_h$ is finite following the expression A.7 and the upper bound of $m_t \leq 1/(2 \gamma)$ established in the proof of Proposition 5.) However, Corollary 1 establishes that $\mu_1 \downarrow 0$ and there
is no price discovery in the short-run, i.e., \( \Delta \tau_1 \downarrow 0 \). Thus, the first term in \( F_{21} \) approaches positive infinity, driving the above difference expression also to positive infinity. In this limit, therefore, \( \Delta \tau_1 \) increases faster than \( \Delta \tau_2 \) and the speed of price discovery is increasing in \( g_h \).

Consider next the limit of \( g_h \uparrow \infty \). Observe that because \( F_{21} > 0 \), \( F_{22} > 0 \), and \( F_{1g} > 0 \), the following inequality always holds: \( (F_{21} + F_{22})F_{1g} - (F_{11} + F_{12})F_{2g} < (F_{11} + F_{12})F_{2g} \), where the last equality simply uses the expressions for \( F_{11} \) and \( F_{22} \). Recall that \( m_t \) is bounded from above by \( 1/(2\gamma) \), hence, \( F_{2g} \) is always finite and so are \( k_{ht} \) and \( k_{ht} \). Yet, \( \tau_t \uparrow \infty \) in the limit of \( g_h \uparrow \infty \). Therefore, \( \lim_{g_h \uparrow \infty} ((F_{21} + F_{22})F_{1g} - (F_{11} + F_{12})F_{2g}) < \lim_{g_h \uparrow \infty} ((-k_{ht}/k_{ht} - 2h_{k_2}^2)F_{2g}) = 0 \). That is, in this upper limit of \( g_h \), \( \Delta \tau_1 \) is growing slower than \( \Delta \tau_2 \), or the speed of price discovery becomes decreasing with the information technology \( g_h \). This completes the proof. 

\( \square \)

**Corollary 1**

**Proof.** Consider the threshold \( \tilde{g}_t \), at which the benefit of investing in speed to trade at \( t = 1 \) is just small enough, so that the marginal investor is just willing to stay slow. Therefore, at this threshold \( \mu_1 = 0 \) and \( \mu_2 = 1 \), implying \( \pi_1 = \frac{1}{2\gamma} \ln \left( 1 + \frac{gh_{k_h(m_1)}}{r_0} \right) - m_1 - \frac{1}{\tilde{g}_t} \) and \( \pi_2 = \frac{1}{2\gamma} \ln \left( 1 + \frac{gh_{k_h(m_2)}}{\tau_2} \right) - m_2 \), where \( \tau_2 = \tau_0 + \tau_1 \tilde{g}_t \). In equilibrium, it has to be such that \( \pi_1 = \pi_2 = \pi^* \), which implies \( \tau_2/(\tau_2 + g_{k_h(m_2)}) > \tau_0/(\tau_0 + g_{k_h(m_1)}) \) because \( m_1 + 1/\tilde{g}_t > m_1 > m_2 \). Subtract by 1 on both sides and rearrange to get \( k_{h_2}(m_2)/\tau_2 + g_{k_h(k_h(m_2))} < k_h(m_1)/(\tau_0 + g_{k_h(k_h(m_1))}) \).

Next, from the expression of \( \pi_1 \), by envelope theorem, \( \frac{\partial \pi^*}{\partial g_h} = \frac{1}{2\gamma} \frac{k_{h_1}(m_1)}{\tau_0 + g_{k_h}(m_1)} + \frac{1}{\tilde{g}_t} \frac{\partial \tilde{g}_t}{\partial g_h} \). Similarly, from the expression of \( \pi_2 \), \( \frac{\partial \pi^*}{\partial g_h} = \frac{1}{2\gamma} \frac{k_{h_2}(m_2)}{\tau_2 + g_{k_h}(m_2)} \left( 1 - \frac{gh_{k_h}(m_2)}{\gamma^2 \tau_2} \right) k_{h_2}(m_2) < \frac{1}{2\gamma} \frac{k_{h_2}(m_2)}{\tau_2 + g_{k_h}(m_2)} \). Therefore, \( \partial \tilde{g}_t/\partial g_h < 0 \).

Further, consider the extremes of \( g_h \downarrow 0 \) and \( g_h \uparrow \infty \). Toward the lower bound 0, from the expression of \( \pi_1 \) it can be seen that the first term in \( \pi_1 \) drops down to zero. Since an investor always has the option not to trade, \( \pi_1 \) is bounded below by zero. This leads to \( m_1 \downarrow 0 \) and \( 1/\tilde{g}_t \downarrow 0 \), implying \( \lim_{g_h \downarrow 0} \tilde{g}_t = \infty \). On the other hand, the first-order condition (6) applied to \( \pi_1 \) implies \( 0 = \frac{k_{h_1}(m_1)}{2\gamma} - \frac{\tau_0}{g_h} \frac{\tau_0}{g_h} < \frac{1}{2\gamma} \frac{\tau_0}{g_h} k_h(m_1) \), where the inequality follows because \( \tau_0/g_h > 0 \) and because \( k_h(m_1) \geq \hat{k}_h(m_1) \) by concavity. Hence, \( m_1 \) is always bounded from above by \( 1/(2\gamma) \). From the first-order condition, with \( \tau_1 \) fixed at \( \tau_0 \), it follows the concavity of \( k_h(\cdot) \) that \( m_1 \) monotone increases in \( g_h \), and so does \( k_h(m_1) \). Taken together, \( \lim_{g_h \uparrow \infty} \pi_1 > \frac{1}{2\gamma} \lim_{g_h \uparrow \infty} \ln \left( 1 + \frac{gh_{k_h(m_1)}}{\tau_0} \right) - \frac{1}{\tilde{g}_t} \lim_{g_h \uparrow \infty} \frac{1}{\tilde{g}_t} \). If \( \lim_{g_h \uparrow \infty} \tilde{g}_t > 0 \), then the above limit of \( \pi_1 \) shoots to infinity. In that case, the assumed equilibrium will not hold, however, because all slow investors will have incentive to acquire speed by paying \( 1/\tilde{g}_t \) to earn infinite profit. Therefore, it has to be the case that \( \lim_{g_h \uparrow \infty} \tilde{g}_t = 0 \).

Finally, the above concludes that \( \tilde{g}_t \) is a strictly decreasing function in \( g_h \), with \( \tilde{g}_t(0) \to \infty \) and
\( \hat{g}_t(\infty) \rightarrow 0 \). As the strict monotonicity implies invertibility, there exists \( \hat{g}_h(g_t) \) for all \( g_t \in (0, \infty) \) such that the equilibrium is interior if and only if \( g_h \geq \hat{g}_h(g_t) \). □

References


