The Time Variation in Risk Appetite and Uncertainty

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Columbia and NBER

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Federal Reserve Board

Nancy Xu
Columbia

The views expressed in this document do not necessarily reflect those of the Federal Reserve System, its Board of Governors, or staff.
Three Recent Mood Swings

- European debt crisis:
  “Financial markets have recently been as emotional as Kate Winslet at an awards ceremony, alternating between “risk on” (when markets are in buoyant mood) and “risk off” (when they are anxious) with bewildering rapidity.”

Three Recent Mood Swings

- Deep low 2017:

  “A low VIX reading is usually seen as a sign of investor complacency. The previous two occasions on which the index fell below ten were in 1993 and early 2007. One preceded the bond market sell-off of 1994 and the other occurred just before the first stages of the credit crisis.”

  – The Economist, “The markets are quiet. Too quiet?”, May 2017
Three Recent Mood Swings

- Fear from inflationary pressure:

  “February has shown that the market is still vulnerable. The immediate trigger seems to have been the fear that inflationary pressures would cause bond yields to rise and central banks to push up interest rates. This translates into the sharp jump in the VIX in early February.”

  – The Economist, “The markets still have plenty to fret about”, Feb 2018
Motivation

- **Changes in risk appetites**: Important determinants of asset prices
  - Behavior finance (sentiment)
Motivation

- **Changes in risk appetites**: Important determinants of asset prices
  - Behavior finance (sentiment)
  - Structural dynamic asset pricing (habit formation)
Introduction

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- Changes in risk appetites: Important determinants of asset prices
  - Behavior finance (sentiment)
  - Structural dynamic asset pricing (habit formation)
  - Reduced-form asset pricing (option prices)

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Risk Appetite & Uncertainty
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  - Do not control for the economic environment (e.g. sentiment index)
  - Do not impose consistency across markets (e.g. VIX)
Our Paper Develops A Measure of Time-Varying Risk Aversion

(1) Goal: Control for macroeconomic uncertainty
Methodology: No-arbitrage dynamic asset pricing theory
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(5) **Goal:** Implementable & tractable over time  
**Methodology:** Spanned by observable financial information
Main Results - Indices

- **Two indices:**
  - **Risk aversion (latent)**: \( ra^{BEX} \)
  - **Economic uncertainty**: \( unc^{BEX} \)

Financial proxies to

\[
\begin{align*}
\text{Risk aversion (latent)} & \quad ra^{BEX} \\
\text{Economic uncertainty} & \quad unc^{BEX}
\end{align*}
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Main Results - Indices

- **Two indices:**
  - Financial proxies to
    - Risk aversion (latent) \( ra^{BEX} \)
    - Economic uncertainty \( unc^{BEX} \)
  - Most informative determinants of the two indices:
    - \( ra^{BEX} \): Variance risk premium
    - \( unc^{BEX} \): Credit spread, corporate bond physical volatility
### Model-implied asset moment decomposition:

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<thead>
<tr>
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<th>Risk Aversion ($\text{ra}^{BEX}$)</th>
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<tr>
<td>Equity</td>
<td>RP 12%</td>
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- Predicting excess returns: our RP beats standard instrument sets
- Predicting output growth: our uncertainty index beats VIX or volatility
- Market integration test: The pricing kernel derived from risky asset markets cannot generate observed Treasury bond return moments
Main Results - Asset Pricing

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Section 1: Model

- Modeling risk aversion
- Dynamic asset pricing model

Section 2: The Identification and Estimation of Risk Aversion

Section 3: Risk Aversion, Uncertainty and Asset Prices
### I. Model

#### Modeling Risk Aversion - Introduction

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  - Define **risk aversion** as the RRA of the marginal investor
  - Model RA state variable to be **imperfectly** correlated with fundamentals
Modeling Risk Aversion - Step 1, Control for Macro Environment

- **Why?** Avoid conflating risk with risk aversion
Modeling Risk Aversion - Step 1, Control for Macro Environment

- **Why?** Avoid conflating risk with risk aversion
- **Feature 1**: time-varying expected growth and macroeconomic uncertainty
- **Feature 2**: real upside and downside uncertainties
Problem: Avoid conflating risk with risk aversion

Feature 1: time-varying expected growth and macroeconomic uncertainty

Feature 2: real upside and downside uncertainties

Feature 3: filtering from monthly industrial production growth, $\theta_t$, adding to the literature (e.g. Jurado et al., 2015):
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Modeling Risk Aversion - Step 1, Control for Macro Environment

- **Why?** Avoid conflating risk with risk aversion
- **Feature 1:** time-varying expected growth and macroeconomic uncertainty
- **Feature 2:** real upside and downside uncertainties
- **Feature 3:** filtering from monthly industrial production growth, $\theta_t$, adding to the literature (e.g. Jurado et al., 2015):

$$
\theta_{t+1} = \bar{\theta} + \rho_\theta (\theta_t - \bar{\theta}) + m_p (p_t - \bar{p}) + m_n (n_t - \bar{n}) + u^\theta_{t+1}, \quad (1)
$$

conditional mean
growth shock

$$
u^\theta_{t+1} = \sigma_\theta p \omega_{p, t+1} - \sigma_\theta n \omega_{n, t+1}, \quad (2)
$$

$$
\omega_{p, t+1} \sim \tilde{\Gamma} (p_t, 1), \quad (3)
$$

$$
\omega_{n, t+1} \sim \tilde{\Gamma} (n_t, 1), \quad (4)
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Digression on Gamma Distribution

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\[
\begin{align*}
\omega_{p,t+1} & \sim \tilde{\Gamma}(p_t, 1) \\
\text{Var}_t & = \omega_{p,t+1} - \omega_{n,t+1} \\
\text{unscaled. Skew}_t & = \frac{\delta_{\theta p} \omega_{p,t+1} - \delta_{\theta n} \omega_{n,t+1}}{\delta_{\theta p}^2 p_t + \delta_{\theta n}^2 n_t} \\
\text{scaled. Skew}_t & = \frac{2 \delta_{\theta p}^3 p_t - 2 \delta_{\theta n}^3 n_t}{2 / \sqrt{p_t}}
\end{align*}
\]

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Risk Appetite & Uncertainty 8
I. Model

Digression on Gamma Distribution

Minus Downside Uncertainty Shock
Governing Left-Tail (negative) Skewness

Upside Uncertainty Shock
Governing Right-Tail (positive) Skewness

\[ \omega_{p,t+1} \sim \tilde{\Gamma}(p_t, 1) \]
\[ \delta_{\theta p} \omega_{p,t+1} - \delta_{\theta n} \omega_{n,t+1} \]
\[ \delta_{\theta p}^2 p_t + \delta_{\theta n}^2 n_t \]
\[ 2\delta_{\theta p}^3 p_t - 2\delta_{\theta n}^3 n_t \]

\[ p_{t+1} = \bar{p} + \rho_p(p_t - \bar{p}) + \delta_p \omega_{p,t+1} \]

\[ n_{t+1} = \bar{n} + \rho_n(n_t - \bar{n}) + \delta_n \omega_{n,t+1} \]

\[ Var_t \]
unscaled. Skew
scaled. Skew

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Risk Appetite & Uncertainty 8
Digression on Capturing Non-Gaussianity in Fundamentals

- **Macro shocks**: likely to be non-Gaussian and asymmetric, with time-varying volatilities (and higher moments)

  Hamilton (1990); Fagiolo, Napoletano & Roventini (2008); Gambetti, Pappa & Canova (2008)
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- **Other state-of-the-art modeling choices**:
  
  Segal, Shaliastovich, & Yaron (2015, *JFE*) good and bad volatilities
  Colacito, Ghysels, Meng, & Siwasarit (2016, *RFS*) skew-normal shocks
  Tsai & Wachter (2015, *RFS*) Poisson
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- **BEGE / Realistic**: Fits some financial and macro economic data well

- **BEGE / Tractable**: Fits in the affine class of asset pricing models
  MGF, Gamma
Consider a period utility function in the HARA class:

\[
    U \left( \frac{C_t}{Q_t} \right) = \left( \frac{C_t}{Q_t} \right)^{1-\gamma} \frac{1}{1 - \gamma}
\]  

(7)

\( C_t \): consumption;
\( Q_t \): driving time variation in risk aversion
Consider a period utility function in the HARA class:

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For the general HARA class of utility functions:

\[ Q_t = \left( \frac{a}{\gamma} - \frac{b_t}{C_t} \right)^{-1} = f(C_t) \]  

(8)

\( a, \gamma \): positive parameters;  
\( b_t \): benchmark process that does not depend on consumption
I. Model

Modeling Risk Aversion - Step 2, A Utility Framework

- Consider a period utility function in the HARA class:

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- The coefficient of relative risk aversion:

\[ RRA_t = -\frac{C_t U''(C_t)}{U'(C_t)} = aQ_t \]  

(9)
Modeling Risk Aversion - Step 3, Dynamic Process

- Define $q_t = \ln Q_t$

- Reduced-form process:

$$q_{t+1} = q_0 + \rho_{qq} q_t + \rho_{qp} p_t + \rho_{qn} n_t + \sigma_{qp} \omega_{p,t+1} + \sigma_{qn} \omega_{n,t+1} + u_{t+1}^q$$  \hspace{1cm} (10)
I. Model

Modeling Risk Aversion - Step 3, Dynamic Process

- Define $q_t = \ln Q_t$
  - Time-varying
  - Imperfectly correlated with macroeconomic shocks

- Reduced-form process:

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  - Persistent conditional mean
  - Disturbance

- Disturbance
  - Exposure to the upside uncertainty shock
  - Exposure to the downside uncertainty shock
  - An orthogonal preference shock

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Risk Appetite & Uncertainty
Modeling Risk Aversion - Step 3, Dynamic Process

- Define $q_t = \ln Q_t$
  - Time-varying
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  - Non-Gaussian

- Reduced-form process:

  $$ q_{t+1} = q_0 + \rho_{qq} q_t + \rho_{qp} p_t + \rho_{qn} n_t + \sigma_{qp} \omega_{p,t+1} + \sigma_{qn} \omega_{n,t+1} + u_{t+1}^q $$  \hspace{1cm} (10)

  - Persistent conditional mean
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  $$ u_{t+1}^q = \sigma_{qq} \omega_{q,t+1} $$ \hspace{1cm} (11)

  $$ \omega_{q,t+1} \sim \tilde{\Gamma}(q_t, 1) $$ \hspace{1cm} (12)
Dynamic Asset Pricing Model - Cash Flow State Variables

Focus on two CF variables:

- Corporate defaults (Gilchrist & Zakrajšek, 2012)
- Earnings growth (Longstaff and Piazzesi, 2004)
Dynamic Asset Pricing Model - Cash Flows State Variables

- Assuming constant loss-given-default rate, time variation in the log default rate depends on the time variation in the log loss rate

- Total CF uncertainty = \textbf{Macroeconomic} + \textbf{Financial} uncertainties:

\[
I_{t+1} = \text{Conditional Mean} + \sigma_{lp} \omega_{p,t+1} + \sigma_{ln} \omega_{n,t+1} + u_{t+1}^l \tag{13}
\]

\[
u_{t+1}^l = \sigma_{llp} \omega_{lp,t+1} - \sigma_{lln} \omega_{ln,t+1} \tag{14}
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\[ u_{t+1}^{l} = \sigma_{llp} \omega_{lp,t+1} - \sigma_{lln} \omega_{ln,t+1} \]  
\[ \omega_{lp,t+1} \sim \tilde{\Gamma} (\gamma_{t}, 1) \Rightarrow \text{s.v. capturing financial cash flow uncertainty} \]  
\[ \omega_{ln,t+1} \sim \tilde{\Gamma} (\gamma_{n}, 1) \]
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\omega_{lp,t+1} \sim \tilde{\Gamma}(v_t, 1) \Rightarrow \text{s.v. capturing financial cash flow uncertainty} \tag{15}
\]

\[
\omega_{ln,t+1} \sim \tilde{\Gamma}(v_n, 1) \tag{16}
\]

- Log earnings growth $g_{t+1}$ disturbance:

\[
\begin{cases}
\text{Macro shocks } \omega_{p,t+1}, \omega_{n,t+1} \\
\text{Financial shock } \omega_{l,t+1} \\
\text{An orthogonal Gaussian earnings growth shock } \omega_{g,t+1}
\end{cases}
\]
Dynamic Asset Pricing Model - State Variable Summary

- Shock exposures to the 4 heteroskedastic gamma shocks:

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<th>Y</th>
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<th>$\omega_n$</th>
<th>$\omega_l$</th>
<th>$\omega_q$</th>
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<td>$g$</td>
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<td>Log DE $\eta$</td>
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<td>✓</td>
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<tr>
<td>$v$</td>
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<td></td>
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</tr>
<tr>
<td>$q$</td>
<td>✓</td>
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</tbody>
</table>

- Matrix representation:

\[ Y_{t+1} = \mu + AY_t + \Sigma \omega_{t+1}, \]  \hspace{1cm} (17)

- Asset returns are exposed to
I. Model

Dynamic Asset Pricing Model - Asset Prices

- No-arbitrage condition

- Zero-coupon nominally defaultable corporate bond price-coupon ratio:

\[ PC_{t,n} = \exp(b_{0,n} + b'_{1,n} Y_t) \]  
(18)

- Equity price-dividend ratio:

\[ PD_{t} \approx \exp(e_{0} + e'_{1} Y_t) \]  
(19)

- **Closed-form solution**: Time variation in risk premium, physical conditional variance, risk-neutral conditional variance are determined by

\[
\begin{align*}
\text{Upside uncertainty } p_t & (\text{ - Price of risk}) \\
\text{Downside uncertainty } n_t & (\text{ + Price of risk}) \\
\text{Financial uncertainty } v_t & (\text{ + Price of risk}) \\
\text{Risk Aversion } q_t & (\text{ + Price of risk})
\end{align*}
\]
Section 1: Model

Section 2: The Identification and Estimation of Risk Aversion
  ▶ Estimation Philosophy
  ▶ Estimation Results

Section 3: Risk Aversion, Uncertainty and Asset Prices
II. The Identification and Estimation of Risk Aversion

Estimation Methodology - Philosophy

- **Step 1**: Pre-Filter macroeconomic and financial uncertainties, $\{p_t, n_t, v_t\}$

  Bates estimation

- **Step 2**: Identify risk aversion, $q_t$

  $\Rightarrow$ Asset moments = exact functions of economic uncertainties (pre-determined) + risk aversion in affine models

  $\Rightarrow$ $q_t$ can be spanned by asset variables + risk variables

  $\Rightarrow$ Unknown: spanning parameters in $q_t$
II. The Identification and Estimation of Risk Aversion

Estimation Methodology - Empirical

- **How Is the Estimation Done:** \([\text{eq}]=\text{equity}; \ [\text{cb}]=\text{corporate bond}\]
  - Asset moments to be matched: risk premium (eq, cb), physical variance (eq, cb), risk-neutral variance (eq)
  - Spanning instruments: term spread, credit spread, earnings-price ratio, realized variances (eq, cb), risk-neutral variances (eq)
  - Estimation: GMM; 1986/06–2015/02

- **Advantages:**
  - It can be calculated at high frequency
  - It is disentangled from uncertainty
  - Fast convergence
Estimation Results - Step 1, Macroeconomic Uncertainties

II. The Identification and Estimation of Risk Aversion

Bekaert, Engstrom, Xu
II. The Identification and Estimation of Risk Aversion

Estimation Results - Financial Cash Flow Uncertainty

Model-implied conditional mean of the loss rate, $E_t[l_{t+1}]$

Model-implied conditional volatility of the loss rate, $Vol_t[l_{t+1}]$

Model-implied conditional skewness of the loss rate, $Skew_t[l_{t+1}]$

Model-implied loss rate upside shock shape parameter, $v_t$

details
II. The Identification and Estimation of Risk Aversion

Estimation Results - Financial Cash Flow Uncertainty

- Two loss shocks: 90%
- Dominate shock during recessions: downside uncertainty
Estimation Results - Step 2, Risk Aversion

$q_t$ implied from risky asset markets

< October 1987 Crash
August 1998 Crash >
Russia or LTCM Collapse
Asian Crash >
Dot Com >
Lehman Brother Aftermath >
Bear Stearns >
< TMT Bull Market
< Euro Debt Crisis

194701 195504 196308 197112 198004 198808 199612 200504
0.5 1 1.5

Bekaert, Engstrom, Xu
Risk Appetite & Uncertainty 21
II. The Identification and Estimation of Risk Aversion

Estimation Results - Step 2, Risk Aversion

\[ q_t \text{ implied from risky asset markets} \]

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<thead>
<tr>
<th></th>
<th>constant</th>
<th>( \chi_{tsprd} )</th>
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Estimation Results - Step 2, Risk Aversion

$q_t$ implied from risky asset markets

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<td>-24%</td>
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<td>2%</td>
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<tr>
<td>Corr</td>
<td>35%</td>
<td>74%</td>
<td>50%</td>
<td>66%</td>
<td>90%</td>
<td>58%</td>
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< October 1987 Crash
August 1998 Crash >
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2exp(Inverse Surplus Consumption Ratio), $\rho = 0.26$

Baker and Wurgler (2006), $\rho = -0.24$

Michigan Consumer Sentiment Index, $\rho = -0.40$

Credit Suisse Risk Appetite Index, $\rho = -0.48$
Section 1: Model

Section 2: The Identification and Estimation of Risk Aversion

Section 3: Model Fit, Indices and Asset Prices

- Model Fit
- Our Indices
- Predictive Results
- Market Integration Tests
### Model Fit - Unconditional

<table>
<thead>
<tr>
<th>Moment</th>
<th>Model</th>
<th>Empirical Average</th>
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<tbody>
<tr>
<td>Mom 1</td>
<td>Equity Risk Premium</td>
<td>0.0080</td>
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<td>Mom 2</td>
<td>Equity Physical Variance</td>
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<tr>
<td>Mom 3</td>
<td>Equity Risk-neutral Variance</td>
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<td>Mom 4</td>
<td>Corporate Bond Risk Premium</td>
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<td>Mom 5</td>
<td>Corporate Bond Physical Variance</td>
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<td>Mom 6</td>
<td>$\omega_q$ Variance</td>
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<tr>
<td>Mom 7</td>
<td>$\omega_q$ Unscaled Skewness</td>
<td>0.0015</td>
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</table>
How important is risk aversion state variable \((q_t)\) for asset prices?

⇒ Model-implied coefficients of moments on time-varying state variables

⇒ Variance decomposition
### Model Fit - Conditional

How important is risk aversion state variable ($q_t$) for asset prices?

- Model-implied coefficients of moments on time-varying state variables
- Variance decomposition

<table>
<thead>
<tr>
<th>Asset</th>
<th>Moment</th>
<th>$p_t$</th>
<th>$n_t$</th>
<th>$v_t$</th>
<th>$q_t$</th>
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</thead>
<tbody>
<tr>
<td>Mom 1</td>
<td>Equity</td>
<td>RP</td>
<td>0.3476</td>
<td>8.8898</td>
<td>0.3196</td>
</tr>
<tr>
<td>VARC</td>
<td>-0.071%</td>
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- Model-implied coefficients of moments on time-varying state variables
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<tr>
<td>Mom 2</td>
<td>Var-P</td>
<td>0.0690</td>
<td>5.4052</td>
<td>0.0889</td>
<td>6.8075</td>
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<td>Mom 3 (qvareq fit)</td>
<td>Var-Q</td>
<td>0.0692</td>
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<td>Mom 4 Corporate Bond</td>
<td>0.1614</td>
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<tr>
<td>Mom 5</td>
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Our Indices

- $r_{BEX}^t = 2 \exp(q_t) = 2 \exp(\chi^{ra} z_t)$
  - Key determinant: Variance risk premium

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III. Risk Aversion, Uncertainty and Asset Prices

Our Indices

- \(ra_t^{BEX} = 2 \exp(q_t) = 2 \exp(\chi^{ra} z_t)\)
  - Key determinant: Variance risk premium

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- \(unc_t^{BEX} = \chi^{unc} z_t\)
  - Key determinant: Credit spread

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<th>(\chi_{qvareq})</th>
<th>(\chi_{rvarcb})</th>
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<tr>
<td>Est</td>
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<td>-0.552</td>
<td>2.163</td>
<td>0.224</td>
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<td>73%</td>
<td>9%</td>
<td>-3%</td>
<td>11%</td>
<td>13%</td>
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Our Indices

Correlation = 77.6%
Predictive Results - Risky Asset Returns

- The relative predictive significance of our RP vs. standard instruments

- Mod
  1. Model-implied risk premium
  2. Out-of-sample estimates of model-implied risk premium

- Emp Mod
  1. Detrended EY
  2. Detrended EY + Term spread + Credit spread
  3. Physical variance + VRP

\[ \hat{r}_{t+1} - rf_t = a \times \text{Mod}(t, i) + (1 - a) \times \text{Emp Mod}(t, j) + e_{t+1} \]

<table>
<thead>
<tr>
<th></th>
<th>Equity:</th>
<th>Corporate Bond:</th>
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<tr>
<td></td>
<td>Mod (1)</td>
<td>Mod (2)</td>
</tr>
<tr>
<td>Emp Mod (1)</td>
<td>0.8284</td>
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<tr>
<td></td>
<td>(0.1079)</td>
<td>(0.0963)</td>
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<td>Emp Mod (2)</td>
<td>0.9299</td>
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<tr>
<td></td>
<td>(0.0942)</td>
<td>(0.0778)</td>
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<tr>
<td>Emp Mod (3)</td>
<td>0.8098</td>
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</tr>
<tr>
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<td>(0.0799)</td>
<td>(0.0739)</td>
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</table>
\[ \theta_{t+k} = a + b' x_t + \epsilon_{t+k} \]

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<tr>
<th></th>
<th>(ra^{BEX})</th>
<th>(unc^{BEX})</th>
<th>(QVAR)</th>
<th>(unc^{true})</th>
<th>(R^2)</th>
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<td>1m</td>
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<td>0.000</td>
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<td>1q</td>
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<td>-0.004</td>
<td>33.3%</td>
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<tr>
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<td>(0.002)</td>
<td>(0.001)</td>
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<td>-0.012</td>
<td>-0.002</td>
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<td>(0.006)</td>
<td>(0.003)</td>
<td></td>
</tr>
</tbody>
</table>
Ⅲ. Risk Aversion, Uncertainty and Asset Prices

Predictive Results - Economic Growth

\[ \theta_{t+k} = a + b' x_t + \epsilon_{t+k} \]

<table>
<thead>
<tr>
<th></th>
<th>$ra^{BEX}$</th>
<th>$unc^{BEX}$</th>
<th>QVAR</th>
<th>$unc^{true}$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1m</td>
<td>0.000</td>
<td><strong>-0.002</strong></td>
<td>0.000</td>
<td>-0.001</td>
<td>17.1%</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.000)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td></td>
</tr>
<tr>
<td>1q</td>
<td>0.001</td>
<td><strong>-0.005</strong></td>
<td>-0.001</td>
<td><strong>-0.004</strong></td>
<td>33.3%</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.001)</td>
<td></td>
</tr>
<tr>
<td>4q</td>
<td><strong>0.021</strong></td>
<td><strong>-0.020</strong></td>
<td><strong>-0.012</strong></td>
<td>-0.002</td>
<td>15.5%</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.005)</td>
<td>(0.006)</td>
<td>(0.003)</td>
<td></td>
</tr>
</tbody>
</table>
III. Risk Aversion, Uncertainty and Asset Prices

Market Integration Test

- Risk aversion index: filtered from risky asset classes
- Null of the theory: the pricing kernel should price all asset returns
- **Does this kernel price 10-year Treasury Bond returns and variance swap?**

<table>
<thead>
<tr>
<th>Moment</th>
<th>Model</th>
<th>Model</th>
<th>Empirical Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mom 1</td>
<td>TB Risk Premium</td>
<td>-0.00367</td>
<td>0.00285</td>
</tr>
<tr>
<td>Mom 2</td>
<td>TB Physical Variance</td>
<td>0.00123</td>
<td>0.00035</td>
</tr>
<tr>
<td>Mom 3</td>
<td>TB Risk-neutral Variance</td>
<td>0.00124</td>
<td>0.00043</td>
</tr>
</tbody>
</table>

- We reject the market integration hypothesis
Conclusion

- Develop a new measure of time-varying risk aversion filtered from economic and financial data, featuring:
  - Not confound risk with risk aversion
  - Distinguish sources of time variation
  - Account for non-Gaussianity
  - Incorporate a large information set
  - Implementable & tractable over time at higher frequency

- Support the close relationship between VRP and risk aversion (as suggested in the literature)

- Propose a financial proxy to economic uncertainty, which beats physical variance and VIX in predicting future economic growth

- Risk aversion state variable dominates uncertainty state variables in explaining the time variation in equity first and second moments
Thank You!
For a centered heteroskedastic Gamma shock $\omega_{y,t+1} \sim \Gamma(s_t, \kappa) - s_t$:

\[
MGF(\nu) = E_t[\exp(\nu \omega_{y,t+1})] = \exp \left[-\nu \kappa - \ln(1 - \nu \kappa)\right] s_t \tag{20}
\]

\[
E_t(\omega_{y,t+1}) = 0, \tag{21}
\]

\[
Var_t(\omega_{y,t+1}) = \kappa^2 s_t, \tag{22}
\]

unscaled. Skew$_t(\omega_{y,t+1}) = 2\kappa^3 s_t \tag{23}$
<table>
<thead>
<tr>
<th></th>
<th>$\Delta e$</th>
<th>$\Delta ce$</th>
<th>$\Delta de$</th>
<th>$\Delta c$</th>
<th>$\Delta d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coeff.</td>
<td>0.044</td>
<td>0.044</td>
<td>0.051</td>
<td>0.058</td>
<td>0.103</td>
</tr>
<tr>
<td>SE</td>
<td>(0.054)</td>
<td>(0.054)</td>
<td>(0.054)</td>
<td>(0.054)</td>
<td>(0.054)</td>
</tr>
</tbody>
</table>
For the output growth process, the joint conditional characteristic function for $Y_\theta,t = [\theta_t \ \theta u_t \ \theta d_t]$ is “exponential-affine”

$$E_t [\Phi Y_{t+1}] = \exp (B Y_{\theta,t})$$  \hspace{1cm} (24)


⇒ Filtering routine for non-Gaussian systems

⇒ Exploits exponential affine structure

⇒ Provides time series of the latent variables (upside and downside uncertainties) and filtered shocks, given the observed single-series of $\theta_{t+1}$ (in this case)
This tables reports parameter estimates from the model below using the monthly log growth rate of U.S. industrial production from January 1947 to February 2015. This system involves latent processes (good shape parameter governing positive skewness $p_t$ and bad shape parameter governing negative skewness $n_t$) and is estimated using the MLE-filtration methodology described in Bates (2006).

$$\theta_{t+1} = \bar{\theta} + \rho_{\theta}(\theta_t - \bar{\theta}) + m_p(p_t - 500) + m_n(n_t - \bar{n}) + u^\theta_{t+1}$$

$$p_{t+1} = 500 + \rho_p(p_t - 500) + \sigma_{pp}\omega_{p,t+1}, n_{t+1} = \bar{n} + \rho_n(n_t - \bar{n}) + \sigma_{nn}\omega_{n,t+1}$$

where

$$u^\theta_{t+1} = \sigma_{\theta p}\omega_{p,t+1} - \sigma_{\theta n}\omega_{n,t+1}, \omega_{p,t+1} - \sigma_{xn}\omega_{n,t+1}, \omega_{p,t+1} \sim \tilde{\gamma}(p_t, 1), \omega_{n,t+1} \sim \tilde{\gamma}(n_t, 1).$$

<table>
<thead>
<tr>
<th></th>
<th>$\theta_t$</th>
<th>$p_t$</th>
<th>$n_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.00002</td>
<td>500</td>
<td>16.14206</td>
</tr>
<tr>
<td></td>
<td>(0.000045)</td>
<td>(fix)</td>
<td>(2.14529)</td>
</tr>
<tr>
<td>AR</td>
<td>0.13100</td>
<td>0.99968</td>
<td>0.91081</td>
</tr>
<tr>
<td></td>
<td>(0.03094)</td>
<td>(0.01918)</td>
<td>(0.01350)</td>
</tr>
<tr>
<td>$m_p$</td>
<td>0.00001</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00034)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_n$</td>
<td>-0.00020</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00002)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_{p,t}$ loading</td>
<td>0.00011</td>
<td>0.55277</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00001)</td>
<td>(0.07073)</td>
<td></td>
</tr>
<tr>
<td>$\omega_{n,t}$ loading</td>
<td>-0.00174</td>
<td></td>
<td>2.17755</td>
</tr>
<tr>
<td></td>
<td>(0.00014)</td>
<td></td>
<td>(0.15027)</td>
</tr>
<tr>
<td>LL</td>
<td>2861.30797</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
This tables reports parameter estimates for the corporate loss rate model using monthly data from January 1982 to February 2015. The mean equation of the loss rate is as follows,

\[ l_{t+1} = l_0 + \rho_{ll} l_t + \rho_{lp} p_t + \rho_{ln} n_t + \rho_{lv} (v_t - \tilde{v}) + \sigma_{lp} \omega_{p,t+1} + \sigma_{ln} \omega_{n,t+1} + u_{t+1}^l \]

\[ u_{t+1}^l = \sigma_{llp} \omega_{lp,t+1} - \sigma_{lln} \omega_{ln,t+1} \]

\[ \omega_{lp,t+1} \sim \tilde{f}(v_t, 1), \omega_{ln,t+1} \sim \tilde{f}(v_n, 1), \]

where the variance equation is,

\[ v_{t+1} = \tilde{v} + \rho_{vv} (v_t - \tilde{v}) + \sigma_{vl} \omega_{lp,t+1}. \]

The mean equation is estimated by projection, the variance equation by MLE. Standard errors are in parentheses.

### A. Mean Equation

<table>
<thead>
<tr>
<th></th>
<th>( l_0 )</th>
<th>( \rho_{ll} )</th>
<th>( \rho_{lp} )</th>
<th>( \rho_{ln} )</th>
<th>( \rho_{lv} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_0 )</td>
<td>-0.3463</td>
<td>0.8779</td>
<td>0.0001</td>
<td>0.0047</td>
<td>0.0093</td>
</tr>
<tr>
<td></td>
<td>(0.0911)</td>
<td>(0.0209)</td>
<td>(0.0002)</td>
<td>(0.0010)</td>
<td>(0.0014)</td>
</tr>
</tbody>
</table>

### B. Variance Equation

#### Shock Loadings:

<table>
<thead>
<tr>
<th>( \sigma_{lp} )</th>
<th>( \sigma_{ln} )</th>
<th>( \sigma_{llp} )</th>
<th>( \sigma_{lln} )</th>
<th>( \tilde{v} )</th>
<th>( \rho_{vv} )</th>
<th>( \sigma_{vl} )</th>
<th>( v_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.0004</td>
<td>0.0177</td>
<td>0.0368</td>
<td>0.0785</td>
<td>11.1790</td>
<td>0.9522</td>
<td>1.2085</td>
<td>2.2153</td>
</tr>
<tr>
<td>(0.0004)</td>
<td>(0.0039)</td>
<td>(0.0060)</td>
<td>(0.0041)</td>
<td>(0.7210)</td>
<td>(0.0585)</td>
<td>(0.1197)</td>
<td>(0.2219)</td>
</tr>
</tbody>
</table>

#### Shape Parameter Dynamics:
[APPENDIX] The Fit of Risk-Neutral Conditional Equity Variance

The diagram shows the fit of empirical and model-based methods for risk-neutral conditional equity variance. The x-axis represents years, and the y-axis represents variance values ranging from 0 to 0.03. The data points are marked at specific years: 1986, 1990, 1994, 1998, 2003, 2007, 2011, and 2015. The red dashed line represents the empirical variance, while the black solid line represents the model variance. The blue vertical bars indicate significant events or periods of interest.