

# Delegated Learning in Asset Management\*

## ABSTRACT

We develop a tractable framework of delegated asset management with flexible information acquisition in a multi-asset economy in which fund managers face moral hazard in portfolio allocation decisions. We explore the features of the optimal affine compensation contract for fund managers, in which benchmarking arises endogenously as part of their optimal compensation. In the equilibrium with delegated learning, asset prices reflect both the acquired private information of fund managers and their desire to hedge their exposure to the benchmark. We show a potential gap between our model-implied measure and several widely-adopted empirical statistics intended to capture managerial ability. In a multi-period extension, we provide a microfoundation for convex flow-performance sensitivity, and propose a new performance measure of fund manager skill. Our delegated learning channel can also help rationalize the excess comovement documented in asset returns.

*Keywords:* Managerial Compensation, Endogenous Benchmark, Moral Hazard, Information Acquisition

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# 1 Introduction

There is growing concern that active fund managers lack the superior ability in garnering higher returns to justify their higher fees compared to their passive counterparts. Consistent with this view, in recent years there has been an accelerating shift in fund flows from active to passive strategies.<sup>1</sup> The existing literature has focused on either improving empirical measures to evaluate the unobservable skill of fund managers, or on developing theories to justify the lack of empirical support for their superior ability.<sup>2</sup> Despite the progress of this fast growing literature, the relationship between fund manager ability and the incentives that they face, in equilibrium, is still not well-understood. In this paper, we ask to what extent such unobservable ability is an outcome of the incentives provided to active managers to acquire information through their compensation contracts.

To investigate this issue, we cast the information acquisition and portfolio allocation decisions of a delegated asset manager as a principal-agent problem between the fund manager and its investors. We refer to this as the delegated learning channel. We study an economy in which asset managers trade on behalf of investors in a multi-asset financial market, similar to that in Admati (1985). In the spirit of Kacperczyk et al. (2016), the subset of managers who have the capacity for skill is able to exert costly effort to learn about the aggregate and asset-specific components of the payoffs of the assets in which they can invest. The inability of investors to observe the effort and portfolio decisions of their delegated asset managers, however, forces investors to offer a contract that is incentive compatible to managers, who seek only to maximize their compensation. In equilibrium, these incentives feed into asset

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<sup>1</sup>Since 2005, actively managed equity and fixed income funds have lost fund flows to passive strategies globally. According to MorningStar, over last decade, actively run U.S. stock funds saw net outflows every year, totaling about \$600 billion, while their indexed counterparts saw net inflows of approximately \$700 billion. See <http://www.marquetteassociates.com/Research/Chart-of-the-Week-Posts/Chart-of-the-Week/ArticleID/743/A-Continued-Shift-from-Active-to-Passive-in-U-S-Equities>.

<sup>2</sup>The existing literature has developed several theories to help explain the lack of empirical support that active managers have superior ability, including that fund performance exhibits decreasing-returns-to-scale (Berk and Green, 2004), that managers choose investments based on their benchmark and flow-performance sensitivity (Brennan (1993), Admati and Pfleiderer (1997), Buffa et al. (2014)), and that skill reflects a choice to acquire information over the business cycle (Kacperczyk et al. (2014), Kacperczyk et al. (2016)).

prices as skilled managers trade on their private information in financial markets, which then feed back into the determinants of the optimal contract in the principal-agent problem between the manager and investors.

The optimal contract for skilled managers that we derive features three components: a constant fee, a performance-based reward that evaluates a fund manager for its performance, and benchmarking relative to the ex-ante mean-variance efficient portfolio from the perspective of investors. Since the performance-based reward influences the aggressiveness with which skilled managers trade on their private information, it feeds into the informativeness of asset prices in equilibrium. Through this channel, the performance-based piece impacts the overall uncertainty that skilled managers face when choosing their portfolio, and consequently their incentives to exert effort to acquire private information. By benchmarking in the contract, the investor effectively endows the skilled manager with a tilted short position in the benchmark portfolio, which leads him to seek insurance from unskilled managers in financial markets to hedge its benchmark risk. This tilt, as we show, impacts the level of risk-sharing between the investor and manager. In contrast to such frameworks as those of Basak and Pavlova (2013) and Buffa et al. (2014), the optimal benchmark we derive in our economy is endogenous and arises as an outcome of the compensation contract.

To illustrate how the optimal contract varies with the asset environment, we perform comparative statics by considering two experiments in a two-asset setting when managers are more risk-averse: 1) altering the overall risk in the economy symmetrically through the ex-ante variance of asset payoffs, and 2) altering the cross-sectional risk in the economy through the correlation of asset payoffs. As the overall level of uncertainty about asset payoffs increases, the optimal contract places less emphasis on the performance-based component, and more weight on benchmark-based incentives. This is optimal because the marginal benefit of exerting effort to learn is higher for skilled managers, even in the absence of incentives, the higher the level of uncertainty in the economy, and the shift toward benchmarking reflects the increased value skilled managers are expected to add over direct investment by the

fund's investors. When the correlation of payoffs increases, in contrast, the optimal contract instead puts more weight on the performance-based component, and less on benchmarking. This occurs because the increased correlation both reduces the cross-section of risk in the economy about which skilled managers can learn, and makes prices more revealing about the aggregate sources of risk. As such, skilled managers reduce the overall effort that they exert to acquire private information, which motivates the need for stronger performance-based incentives and lessens the benefit of benchmarking for sharing risk.

We next explore the implications of our model for identifying skill among fund managers. Given that our framework allows the effort that skilled managers exert to acquire private information to vary across asset environments, we can treat their reduction in uncertainty about asset payoffs from acquiring private information as a measure of skill. Through our comparative statics, we highlight a gap between our model-implied measure and empirical statistics meant to capture asset management ability, such as the active share proposed by Cremers and Petajisto (2009) and the return gap of Kacperczyk et al. (2008). When the overall level of payoff uncertainty increases, skilled managers devote more effort to acquiring private information and take more active positions, when compared to the benchmark portfolio, and this is reflected in our theoretical analogues of the two empirical measures. When instead asset payoffs become more correlated, skilled managers exert less overall effort to learn, but may appear more active because the optimal benchmark, the ex-ante mean-variance efficient portfolio, takes smaller positions in the risky assets because of the diminished benefits from diversification. Consequently, our analysis cautions in the interpretation of these empirical measures as proxies for managerial ability, and also highlights the importance of endogenizing the benchmark for theoretical predictions.

The interaction between incentives and learning also delivers rich cross-sectional implications on asset returns. Since we solve for the optimal contract and the learning decisions of managers, our empirical predictions do not rely on observing their compensation structure or their beliefs. The hedging demand of skilled managers for the benchmark portfolio, for

instance, raises the prices of assets held short in the benchmark portfolio, lowering their risk premium in equilibrium to compensate unskilled managers for providing liquidity. Our model can also help rationalize the excess comovement in asset returns, documented in Pindyck and Rotemberg (1990) and Barberis et al. (2005). As prices serve as an endogenous mechanism for skilled managers to coordinate on which private information to acquire, their correlated decisions are amplified in the payoff variation reflected in prices through their trading.

We then investigate two extensions of our model, one in which trading by managers occurs over multiple periods, and one in which we endow investors with background risk that is correlated with the returns on the assets in the economy. The dynamic extension illustrates that having multiple periods introduces intertemporal incentives for skilled managers to acquire private information and, more importantly, can provide investors with a time-series of past fund behavior to improve monitoring. We show that the historical variance of a fund's return gap, downweighted by the dispersion of asset payoffs, provides a consistent measure of average portfolio selection skill, and argue how investors learning about a fund manager's skill through this channel could help explain the nonlinear relationship between performance and fund flows observed empirically. With background risk, we show that managers with skill are forced to internalize this background risk by the appropriate adjustment of the benchmark against which they are evaluated. Finally, we distinguish our mechanism of learning by managers from the literature on learning about managers.

Our work is related to the literature on delegated asset management under asymmetric information. García and Vanden (2009) and Gârleanu and Pedersen (2015) explore the implications for market efficiency of the formation of mutual funds in the presence of costly information acquisition in a single asset setting. García and Vanden (2009) also consider a principal-agent model of delegated asset management, yet they model management fees as a fixed fraction of assets under management and assume managers pay a fixed fee to become informed. Our work focuses on the optimal affine contract between investors and fund managers in a multi-asset principal-agent setting. Kapur and Timmermann (2005) investi-

gate the impact of relative performance contracts on the equity premium and on portfolio herding. Dybvig et al. (2010) and He and Xiong (2013) consider the market-timing benefits of benchmarking in a partial equilibrium setting. Kyle et al. (2011) investigates the incentives to acquire information under delegated asset management for a large informed fund, in the spirit of Kyle (1985), while Savov (2014) microfoundations delegated asset management as a vehicle for investors to hedge their income risk.

This paper is connected to the growing literature on equilibrium asset pricing with flexible information acquisition. Van Nieuwerburgh and Veldkamp (2009, 2010) and Kacperczyk et al. (2016) study the flexible information acquisition problem faced by investors who have limited attention that they can allocate to learning about risky asset payoffs, the latter of which focuses on business cycle implications. Maćkowiak and Wiederholt (2012) investigate the information acquisition decisions of investors who have limited liability, while Huang et al. (2016) models information acquisition as part of a dynamic reputation game between the fund and its investors. In contrast to these studies, we model the information acquisition of managers as being subject to agency issues within an equilibrium framework.

In addition, our work is also related to the literature on manager incentives and benchmarking in the asset management industry. Basak and Pavlova (2013) and Buffa et al. (2014) investigate the asset pricing implications of benchmarking against an exogenous index in a multi-asset setting, with Buffa et al. (2014) embedding benchmarking in a principal-agent framework. Starks (1987) studies the role of symmetric versus bonus performance-based contracts in incentivizing asset managers. Brennan (1993) examines the CAPM implications of delegated management with both exogenous and optimal benchmarking. Admati and Pfleiderer (1997) analyzes benchmarking and manager incentives in a partial equilibrium framework in which managers have superior information to investors, while van Binsbergen et al. (2008) explores how benchmarking can overcome moral hazard issues that arise with decentralization. Cuoco and Kaniel (2011) study the implications for asset pricing when manager compensation is linked to an exogenous benchmark, and Li and Tiwari (2009)

study nonlinear performance-based contracts in the presence of benchmarking. In our work, we derive the optimal benchmark jointly with the optimal affine contract and equilibrium prices, and study their empirical implications for intermediary holdings and asset returns.

## 2 A Model of Delegated Asset Management

In this section, we present a model of delegated asset management with flexible information acquisition in a multi-asset framework. We first introduce the asset environment, and then discuss the agency friction that skilled managers face in portfolio allocation decisions. Finally, we define the asset market equilibrium.

### 2.1 The Environment

There are three dates  $t = \{0, 1, 2\}$ . Suppose that there are  $N$  assets with risky payoffs  $f_i$   $i \in \{1, 2, \dots, N\}$ , which realize at date 2 that satisfy the following decomposition:

$$f_i = \begin{cases} b_1\theta_1, & i = 1 \\ a_i\theta_i + b_i\theta_1, & i \in \{2, \dots, N\} \end{cases}$$

The common component  $\theta_1$  can be viewed as aggregate payoff risk, with  $b_i$  being the loading on this aggregate payoff risk of the asset, while the  $a_i\theta_i, i \in \{2, \dots, N\}$  are the asset-specific components of the risky asset payoffs. This payoff structure we employ is similar to that in Buffa et al. (2014) and Kacperczyk et al. (2016). For interpretation of  $\theta_1$  as aggregate payoff risk, we assume that  $\theta_1 = 0$ , and that the first asset is a composite asset of the remaining assets in the economy with a payoff that loads only on this aggregate payoff risk.<sup>3</sup> In addition to the  $N$  assets, there is a risk-free asset, which can be viewed as asset 0, in perfectly elastic supply with gross return  $R^f > 1$ . Asset  $i$  has price  $P_i$  at  $t = 0$ , and we

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<sup>3</sup>Kacperczyk et al. (2016) employ a similar assumption for the asset payoff structure. While not essential for our analysis, it helps with exposition by ensuring that the map from risk factors  $\{\theta_0, \{\theta_i\}_{i \in \{1, \dots, N-1\}}\}$  to asset payoffs  $\{f_i\}_{i \in \{1, \dots, N\}}$  is invertible.

stack the  $N$  prices into the  $N \times 1$  vector  $\mathbf{P}$ . In what follows, bold symbols represent vectors. For convenience, we define the vector  $\Theta = \left[ \theta_1 \quad \theta_1 \quad \theta_2 \quad \dots \quad \theta_N \right]'$  such that:

$$\mathbf{f} = F\Theta,$$

for the  $N \times N$  matrix  $F$ , which is invertible since  $F$  is lower triangular. In our setting, aggregate risk arises through the correlation structure of asset payoffs, and is represented by the common fundamental  $\theta_1$ .<sup>4</sup>

There are two types of agents in the market: investors and managers. Investors cannot invest directly in asset markets.<sup>5</sup> Instead, at date 0 they must delegate management of their portfolio to fund managers, a fraction  $\chi$  of which are skilled, and a fraction  $1 - \chi$  who are unskilled. The skilled managers can exert certain level of unobservable effort to obtain private signals about asset fundamentals<sup>6</sup>, while the unskilled managers cannot. This is what we refer to as *delegated learning* channel. Each manager owns and operates one fund. The type of manager and the fraction  $\chi$  are observable public information. Similar to van Binsbergen et al. (2008), we analyze the incentive contract between investors and managers by modeling the one layer delegation problem, i.e., investors directly offer compensation contracts to fund managers. Our approach is different from García and Vanden (2009) and Gârleanu and Pedersen (2015), who investigate the fee setting by asset management companies. Given the relatively stable mutual fund fee structure, we focus on studying the direct incentive provision from the compensation contract to managers. We discuss the problem faced by each of these agents in turn.

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<sup>4</sup>This is in contrast to Kacperczyk et al. (2016), where aggregate risk takes the form of the asset fundamental with a higher supply variance. Our derivations will, in fact, be valid more generally for any arbitrary invertible matrix  $F$ .

<sup>5</sup>In equilibrium, unskilled managers will choose the same portfolio for investors that investors would choose for themselves if they could invest directly. One can consequently view the compensation investors give to unskilled managers as the transaction costs or brokerage fees they face, or the fees that they pay to passive mutual funds and ETFs to benefit from their economies of scale and internal clearing.

<sup>6</sup>Brown and Davies (2016) also studied the moral hazard in the active asset management industry in a partial equilibrium framework. They assume the effort exerted by managers are directly linked to returns, while we focus on the incentives of costly information acquisition.



## 2.2 Unskilled Managers

Given their initial assets under management (AUM) or funds  $W_0$ , unskilled managers choose a portfolio choice  $\omega_1^U$  at date 1 for their fund *after* observing market prices  $\mathbf{P}$ . The final AUM of their fund  $W_2^U$  is then given by:

$$W_2^U = R^f W_0 + \omega_1^{U'} (\mathbf{f} - R^f \mathbf{P}).$$

Investors offer unskilled managers a compensation contract  $C_0^U$  for their services. Similar to Admati and Pfleiderer (1997), we assume managers have CARA preferences over their compensation. We assume that fund managers have CARA preferences over their compensation from investors  $C_0^S$  and cost of effort:

$$u(C_0^U; \omega_1^U) = -\exp(-\gamma_M C_0^U),$$

where  $\gamma_M > 0$  is their coefficient of absolute risk aversion. In addition, we assume that unskilled managers have a minimum reservation utility  $u_0$  that the contract must respect. This gives rise to an individual rationality or participation constraint (IR):

$$E[u(C_0^U, \omega_1^U)] \geq u_0 \quad (IR).$$

Having characterized the problem of unskilled managers, we now turn to skilled managers.

## 2.3 Skilled Managers

Similar to unskilled managers, skilled managers face a portfolio choice problem at date 1. Given their information and initial AUM  $W_0$ , skilled managers choose a portfolio allocation strategy  $\omega_1^S$  at date 1 across the  $N$  assets so that the final AUM  $W_2^S$  is given by:

$$W_2^S = R^f W_0 + \omega_1^{S'} (\mathbf{f} - R^f \mathbf{P}).$$

In addition to a portfolio choice problem, skilled managers also face an information acquisition choice. While asset prices are publicly observable, managers that have skill acquire a vector of noisy private signals  $\mathbf{s}$  about  $\theta_1$  and the asset-specific component of asset payoffs  $\theta_i$   $i \in \{2, \dots, N\}$ . They are able to exert effort  $e = \mathbf{e}'\mathbf{1}_{N \times 1} \geq 0$ , with  $\mathbf{e} \geq \mathbf{0}$  element-by-element, to reduce the variance of these signals  $\Sigma(\mathbf{e})$ . Although investors are matched with skilled managers, the level of effort that skilled managers exert is not observable. We assume skilled managers must exert costly effort at date 0 to acquire information about asset payoffs at date 1.

Skilled manager  $j$  receives a vector of noisy signals  $\mathbf{s}_j$  about  $\Theta$  given the effort level  $\mathbf{e}_j$ :

$$\mathbf{s}_j = \Theta + \Sigma_j(\mathbf{e}_j)^{1/2} \varepsilon_j,$$

where  $\varepsilon_j \sim N(\mathbf{0}_{N \times 1}, Id_N)$  is independent across  $j$  and satisfies the Strong LLN  $\int_{-\infty}^{\infty} \varepsilon_j d\Phi(\varepsilon_j) = \mathbf{0}_{N \times 1}$  for  $\Phi(\cdot)$ , the CDF of the standard normal distribution.

In what follows, we impose the monotonicity condition to ensure that more effort reduces the variance of private signal  $\mathbf{s}_j$ . We assume that  $\Sigma_j(\mathbf{e}_j)$  is a diagonal matrix with entry  $K_{ii}^{-1}(e_{ij})$  that satisfies the monotonicity condition:  $\Sigma_j(\mathbf{e}_j'') - \Sigma_j(\mathbf{e}_j')$  is positive-semi definite (PSD) whenever  $\mathbf{e}_j'' \geq \mathbf{e}_j'$ . We assume that  $\Sigma_j(\mathbf{e}_j)$  is diagonal so that there is a direct link between the effort manager  $j$  exerts to learn about the  $i^{th}$  component of  $\Theta$ ,  $e_{ij}$ , and the precision of the signal manager  $j$  receives about that component,  $s_{ij}$ . The monotonicity condition ensures that a higher level of effort (weakly) implies the manager receives more informative signals. To ensure prices are always informative, we regulate  $\Sigma(\mathbf{e}_j)$  by assuming that  $\sup_i \Sigma(\mathbf{0}_{N \times 1}) \leq M^{-1} < \infty$ , although our results will be valid in the limit that  $M \searrow 0$ . In what follows, we choose the parameterization  $K_{ii}(e_{ij}) = M + e_{ij}$ . One can view this observation of private information by a skilled manager as their security selection or “stock picking ability”.

A skilled manager must exert costly effort at date 0 to acquire information about asset

payoffs at date 1, and we specify this cost as a dollar cost that the manager incurs. We assume that fund managers have CARA preferences over their compensation from investors  $C_0^S$  and cost of effort:

$$u(C_0^S; \omega_1^S, \mathbf{e}) = -\exp(H(\mathbf{e}) - \gamma_M C_0^S),$$

where  $\gamma_M$  is the coefficient of absolute risk aversion and  $H(\cdot)$  is the dollar cost for effort  $\mathbf{e}$ , an increasing and (strictly) convex function in each of its arguments, such that  $\partial_i H(\mathbf{e}) > 0$  and  $\partial_{ii} H(\mathbf{e}) \leq 0$ , and  $H(\mathbf{0}_{N \times 1}) = 0$  as a normalization. We specialize  $H(\mathbf{e})$  to the case that  $H(\mathbf{e}) = \frac{1}{2}h(\mathbf{e}'\mathbf{1}_{N \times 1})$ , where  $h'(\cdot) > 0$  and  $h''(\cdot) \leq 0$ . This functional form induces complementarity in manager learning decisions, and therefore a tradeoff to learning too much about one source of asset-specific risk.

Like unskilled managers, skilled managers also have a minimum reservation utility  $u_0$ , which gives rise to the individual rationality (IR) constraint,

$$E[u(C_0^S, \omega_1^S, \mathbf{e})] \geq u_0 \quad (IR).$$

In addition, since fund manager effort is unobservable, a skilled manager must find it optimal to choose the effort level recommended by the investor, which gives rise to the incentive compatibility (IC) constraint:

$$\mathbf{e} \in \operatorname{argsup}_{\mathbf{e} \in \mathbb{R}_+^N} E \left[ \sup_{\omega \in \mathbb{R}^N} E[u(C_0^S; \omega_1^S, \mathbf{e}') \mid \mathcal{F}_j] \right] \quad (IC), \quad (1)$$

where  $\mathcal{F}_j$  is the skilled manager's information set and the optimization implies a natural timing to their decisions. The skilled manager first determines the effort to exert based on the compensation contract  $C_0^S$  with investors at date 0. At date 1, the skilled manager observes prices and private signals, and makes portfolio allocation choice. The skilled manager's information set is then the sigma algebra generated from observing the vector of prices  $\mathbf{P}$

and its private signals  $\mathbf{s}_j$ ,  $\mathcal{F}_j = \sigma(\mathbf{P}, \mathbf{s}_j(\mathbf{e}_j))$ .

## 2.4 Investors

Investors have CARA preferences over the final AUM at date 2,  $W_2^i$ , where  $i \in \{U, S\}$  indicates whether they have invested with an unskilled or skilled manager, respectively. There are as many investors as there are fund managers. They choose the compensation contract at date 0,  $C_0^i$ , for a manager to maximize their utility subject to incurring the cost of incentivizing the manager:

$$U(W_2^i, C_0^i) = -\exp(-\gamma(W_2^i - C_0^i)),$$

where  $\gamma > 0$  is their coefficient of absolute risk aversion. Since investors only have access to public information, they have what we refer to as the common knowledge or public information set,  $\mathcal{F}^c$ , which is the sigma algebra generated by observing prices  $\mathcal{F}^c = \sigma(\mathbf{P})$ .

The investors solve the optimization problem when investing with unskilled and skilled managers:

$$V_0^i = \sup_{C_0^i} E^{e(C_0^i)} [U(W_2^i, C_0^i)], \quad (2)$$

subject to the IR and IC constraints, where  $E^e[\cdot]$  is understood as the expectation under the probability distribution induced by the recommended effort level  $\mathbf{e}(C_0^i)$ , where  $\mathbf{e}(C_0^i) = \mathbf{0}$  for unskilled managers. Consequently,  $V_0^U$  is the expected value to the investor for investing with an unskilled manager and  $V_0^S$  is the value of investing with a skilled manager.

While we derive the optimal contract for unskilled managers, we restrict the space of contracts offered to skilled managers. Similar to Kapur and Timmermann (2005) and Buffett et al. (2014), we focus on affine contracts between investors and managers.<sup>7</sup> One of the key reasons is to advance the understanding of the incentive problems in the delegated asset

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<sup>7</sup>In practice, investors pay fees to advisory firms who then compensate the managers through the incentive contracts. Since mutual fund fees are relatively stable over time (Pástor and Stambaugh, 2012), we focus on the manager incentive problem directly.

management environment. Previous studies have found negative results on affine incentive contracts for fund managers. Stoughton (1993) and Admati and Pfleiderer (1997) both show that affine contracts provide no incentives for fund manager effort, while regulations restrict the form of compensation contracts to only be symmetric around benchmark returns.<sup>8</sup> We analyze the optimal contract in the general linear setup and show that affine contracts can provide managerial incentives when asset prices that contain private information feed back into the compensation contracts.<sup>9</sup> In addition, since we are solving for noisy rational expectations within the linear paradigm of Grossman and Stiglitz (1980) and Hellwig (1980), such a restriction may be seen as a natural extension of the focus on linear equilibria.<sup>10</sup>

Finally, we assume that investors can freely invest with any fund manager, so that they must, in equilibrium, be indifferent to investing with a skilled or an unskilled manager. This implies that the indirect utility to investing with a skilled manager  $V_0^S$  or an unskilled manager  $V_0^U$  must be the same, or

$$V_0^S = V_0^U.$$

This free-entry assumption is similar to that in Berk and Green (2004), where the “net fees” of funds with skilled versus unskilled managers offers similar returns, while “gross of fees” reflects manager skill. Given that the investment decisions of fund managers will be independent of initial wealth in this CARA-normal setting, we are abstracting from the decreasing returns to scale at the fund level that are observed empirically, the consequences of which are explored, for instance, in Berk and Green (2004) and Pástor and Stambaugh

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<sup>8</sup>The 1970 SEC amendment to the Investment Company Act of 1940 requires that performance-based contracts should not contain the “bonus” performance-based fee and should only be symmetric around the benchmark returns.

<sup>9</sup>Starks (1987) shows that linear contracts will lead to optimal portfolio risk exposure by managers, but an under-provision of effort compared to the first-best. As such, the contracts we characterize may potentially be suboptimal in incentivizing managers to acquire information. As Starks (1987) emphasizes, however, asymmetric contracts embedded with bonus incentives lead to an even lower level of effort than in the symmetric case, as well as suboptimal risk exposure.

<sup>10</sup>Optimal contracts in the literature typically focus a priori on either linear or option-like compensation contracts for delegated asset managers. As a result of the equilibrium noisy REE framework, it is difficult to incorporate option-like payoffs and maintain tractability in learning, or even to solve for nonlinear contracts more generally that can condition on a rich state space with  $N$  securities in a very flexible manner.

(2012). Since investors in our model are indifferent to with whom they invest, the market for intermediation between investors and managers trivially clears.

## 2.5 Asset Markets Clearing

Let  $\omega_1^S(i)$  be the portfolio allocation of the skilled manager  $i \in [0, 1]$ , and similarly with  $\omega_1^U(i)$  for unskilled manager  $i$ . Given that all managers are atomistic, unskilled managers will all follow the same portfolio strategy,  $\omega_1^U(i) = \omega_1^U$ . We assume the supply of the asset is given by the vector  $\mathbf{x}$  for the  $N$  assets. Since there are a fraction  $\chi$  of skilled managers, and a fraction  $1 - \chi$  of unskilled managers, market-clearing requires that:

$$\chi \int_0^1 \omega_1^S(i) di + (1 - \chi) \omega_1^U = \mathbf{x}. \quad (3)$$

As is common in the literature, we assume that asset supply  $\mathbf{x}$  is noisy to prevent beliefs from being degenerate.<sup>11</sup> We assume that, from the perspective of all agents,  $\mathbf{x} \sim \mathcal{N}(\bar{\mathbf{x}}, \tau_x^{-1} Id_N)$  has a multivariate normal distribution, and  $\bar{\mathbf{x}} > \mathbf{0}$  (element-by-element). Since all fund managers are atomistic, they take prices as given and each has negligible impact on the price formation process.

Finally, we assume that all agents have a normal prior over  $\Theta$ , and initially believe that  $\Theta \sim \mathcal{N}(\bar{\Theta}, \tau_\theta^{-1} Id_N)$ , where  $\tau_\theta$  is the common precision of the prior over the hidden factors driving asset payoffs. This assumption on priors is consistent with our designation of  $\theta_1$  as aggregate risk and security  $i$ 's asset-specific risk,  $\theta_i, i \in \{2, 3, \dots, N\}$ . One can view the prior as reflecting all publicly available information about the asset payoffs, such as financial disclosures, earnings announcement, and macroeconomic news that agents have before contracting at date 0.

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<sup>11</sup>In the absence of supply noise, beliefs between the common component of payoffs  $\theta_1$  and the idiosyncratic component  $\theta_i, i \in \{2, 3, \dots, N\}$  would have to be perfectly negatively correlated once prices  $\mathbf{P}(\Theta)$  are observable, as a result of Bayesian updating. Since there are  $N$  hidden states  $\Theta$  and only  $N$  assets, the vector price function  $\mathbf{P}(\Theta, \mathbf{x})$  is a rank-deficient map from  $\mathbb{R}^{2N}$  to  $\mathbb{R}^N$ .

We then solve for the perfect Bayesian noisy rational expectations equilibrium defined as follows:

A perfect Bayesian noisy rational expectations equilibrium in this economy is a list of policy functions  $\mathbf{e}(C_0^S)$ ,  $\omega_1^S(\mathbf{s}_j, \mathbf{P})$ , and  $\omega_1^U(\mathbf{P})$ , contracts  $C_0^S$  and  $C_0^U$ , and prices  $\mathbf{P}$  such that:

- Investor Optimization: Contracts  $C_0^S$  and  $C_0^U$  solve the investor's optimization problem (2) and delivers expected utility  $V_0^S$  and  $V_0^U$ , respectively.
- Unskilled Manager Optimization: Given contract  $C_0^U$ , prices  $\mathbf{P}$ , and information set  $\mathcal{F}^c$ ,  $\omega_1^U(\mathbf{P})$  satisfies each unskilled manager's IR constraint.
- Skilled Manager Optimization: Given contract  $C_0^S$ , prices  $\mathbf{P}$ , and information set  $\mathcal{F}_j$ ,  $\mathbf{e}(C_0^S)$ , and  $\omega_1^S(\mathbf{s}_j, \mathbf{P})$  solve each skilled manager's IR and IC constraints.
- Market Clearing: The asset markets clear through equation (3).
- Consistency: Investors and unskilled managers form their expectations about  $\Theta$  based on their information set  $\mathcal{F}^c$ , while skilled managers form their expectations based on their information set  $\mathcal{F}_j$ , according to Bayes' rule.
- Sequential Rationality: For each realization of prices  $\mathbf{P}$  and private signals  $\mathbf{s}_j$ , unskilled and skilled managers find it optimal at date 1 to follow investment policies  $\omega_1^U(\mathbf{P})$  and  $\omega_1^S(\mathbf{s}_j, \mathbf{P})$ , respectively.

### 3 The Equilibrium

We search for a symmetric linear equilibrium in which we conjecture that asset prices  $\mathbf{P}(\Theta, \mathbf{x})$  take the linear form:

$$\mathbf{P}(\Theta, \mathbf{x}) = \Pi_0 + \Pi_\theta \Theta + \Pi_x \mathbf{x}, \quad (4)$$

where  $Rank(\Pi_\theta), Rank(\Pi_x) = N$ . As is standard in the literature, we also focus on linear contracts.

We first derive the conditional beliefs of investors and both unskilled and skilled managers. We then derive the optimal investment policy for unskilled managers, and then turn to the optimal policies for skilled managers, who face both effort and portfolio choice decisions that must be incentive compatible. Imposing market clearing allows us to solve for equilibrium asset prices. Finally, we solve for the optimal contracts offered by investors to unskilled and skilled managers.

### 3.1 Learning

We begin by deriving the learning process for investors and, since they share the same information set, unskilled managers. Since both have a normal prior, after observing the linear Gaussian signals  $\mathbf{P}(\Theta)$ , they update to a posterior for  $\Theta$  that is also Gaussian  $\Theta | \mathbf{P}(\Theta) \sim \mathcal{N}(\hat{\Theta}, \Omega)$  with conditional mean  $\hat{\Theta}$  and conditional variance  $\Omega$ , given by:

$$\hat{\Theta} = \Omega\tau_\theta\bar{\Theta} + \Omega\tau_x\Pi'_\theta(\Pi_x\Pi'_x)^{-1}(\mathbf{P} - \Pi_0 - \Pi_x\bar{\mathbf{x}}), \quad (5)$$

$$\Omega^{-1} = \tau_\theta Id_N + \tau_x\Pi'_\theta(\Pi_x\Pi'_x)^{-1}\Pi_\theta. \quad (6)$$

To get to the posterior of skilled manager  $j$ , we recognize that we can first have the manager update his beliefs based on the publicly observed prices, and then treat these beliefs as an updated prior for when the manager then observes its vector of private signals  $\mathbf{s}_j$ . After observing the public signals  $\mathbf{P}(\Theta)$ , the new prior of skilled manager  $j$  from above is  $\Theta | \mathbf{P}(\Theta) \sim \mathcal{N}(\hat{\Theta}, \Omega)$ , with  $\hat{\Theta}$  and  $\Omega$  given by equations (5) and (6), respectively.

After observing its vector of private signals, the posterior of skilled manager  $j$  is also Gaussian  $\Theta | \{\mathbf{P}(\Theta), \mathbf{s}_j\} \sim \mathcal{N}(\hat{\Theta}(j), \Omega(j))$  will conditional mean  $\hat{\Theta}(j)$  and the conditional



variance  $\Omega(j)$  summarized by the following two expressions:

$$\hat{\Theta}(j) = \Omega(j) \Omega^{-1} \hat{\Theta} + \Omega(j) \Sigma_j (\mathbf{e}_j)^{-1} \mathbf{s}_j, \quad (7)$$

$$\Omega(j)^{-1} = \Omega^{-1} + \Sigma_j (\mathbf{e}_j)^{-1}. \quad (8)$$

This completes our characterization of learning by investors, as well as unskilled and skilled managers. Having solved for the conditional beliefs of all agents, we next analyze the optimal portfolio investment and effort policies of unskilled and skilled managers.

### 3.2 Optimal Policies of Unskilled Managers

We begin our analysis of optimal policies with unskilled fund managers. As a starting point, we consider the portfolio an investor would choose if she could directly participate in asset markets. Given investors have CARA preferences and payoffs are normally distributed, it follows that we can express the investor's optimization problem as:

$$U_0 = \sup_{\omega^c} R^f W_0 + \omega^c \left( F \hat{\Theta} - R^f \mathbf{P} \right) - \frac{\gamma}{2} \omega^c F \Omega F' \omega^c,$$

given the properties of log-normal distributions and the monotonicity of the utility function in wealth. This optimization has the following straightforward interior solution:

$$\omega^c = \frac{1}{\gamma} (F \Omega F')^{-1} \left( F \hat{\Theta} - R^f \mathbf{P} \right), \quad (9)$$

which is consistent with mean-variance preferences. The superscript  $c$  indicates that this is the first-best investment portfolio given information set  $\mathcal{F}^c$ , for reasons that will become clear when we solve for the optimal contract with the unskilled fund manager.

Given that unskilled managers cannot acquire private signals, they choose  $\mathbf{e} = \mathbf{0}_{N \times 1}$ , since they only derive dis-utility from effort. Furthermore, because investors share the same information set  $\mathcal{F}^c$ , the investor can perform perfect monitoring and we can ignore the

IC constraint for the fund manager in the optimal contract. To see this, we recognize that the unskilled manager is restricted to choosing  $\mathcal{F}^c$ -measurable portfolio strategies,  $\omega_1^U = \omega_1^U(\mathbf{P})$ . As such, any strategy the unskilled fund manager could implement would be invertible to the investor once the investor observes the realized return of her portfolio (up to equivalence sets  $\{\omega(w) : W_2^U(\mathbf{f}, \mathbf{P}; \omega) = w\}$ ), since  $W_2^U = W_2^U(\mathbf{f}, \mathbf{P})$ . Therefore, the investor could design a non-linear contract that pays  $C_0^U$  if  $W_2^U = w$  and 0 otherwise, based on the recommended portfolio  $\omega(w)$ . As such, we need only focus on the IR constraint for the unskilled fund manager.

### 3.3 Optimal Policies of Skilled Managers

In contrast to unskilled managers, skilled managers must be incentivized since they add value to the investor's portfolio through their hidden, costly acquisition of private information. As such, they can no longer be perfectly monitored since they are free to choose  $\mathcal{F}_j$ -measurable portfolio strategies,  $\omega_1^S = \omega_1^S(\mathbf{P}, \mathbf{s}_j)$ , and  $\mathcal{F}^c \subseteq \mathcal{F}_j$ . Consequently, it is not generically possible for the investor to invert the private signals  $\mathbf{s}_j$  the skilled manager received from the realized portfolio excess payoff  $W_2^S - R^f W_0$  to ensure that the manager followed the investor's recommendation contingent on observing signals  $\mathbf{s}_j$ . What is worse is that, even if the investor could observe  $\mathbf{s}_j$  directly, the investor could not ex post verify that the skilled manager exerted the recommended effort level  $\mathbf{e}$  to obtain the desired precision of the signals.<sup>12</sup>

These considerations motivate us to consider compensation schedules  $C_0^S$  that are contingent on outcomes observable to investors at date 2 and, as such, we consider contracts that condition on the realized portfolio return per share of the fund  $W_2^S - R^f W_0$  and the realized excess payoffs of the risky assets  $\mathbf{f} - R^f \mathbf{P}$ ,  $C_0^S = C_0^S(W_2^S - R^f W_0, \mathbf{f} - R^f \mathbf{P})$ .<sup>13</sup> Since we

<sup>12</sup>In part, the assumption that the variance of private signals is regulated from above in the sup norm by  $\frac{1}{M}$  would ensure that there are limits to monitoring low levels of effort by observing very extreme realizations of private signals.

<sup>13</sup>We also considered a version where the contract conditions on the return of unskilled managers is  $W_2^U$ . We did not find the results were qualitatively different, since  $W_2^U$  is based on public information that is exogenous to the choices of any skilled manager.

focus on linear contracts, we conjecture the optimal contract  $C_0^S$  is in the form of

$$C_0^S = \rho_0 + \rho_S (W_2^S - R^f W_0) + \rho'_R (\mathbf{f} - R^f \mathbf{P}). \quad (10)$$

Conditioning compensation on the realized excess payoff of the portfolio potentially helps to align the incentives of the skilled manager and investor by giving the manager an equity stake in the portfolio. This feature is similar to the fixed fraction of assets under management fee that mutual funds charge in practice. In addition, allowing the compensation schedule to vary with observed excess payoffs  $\mathbf{f} - R^f \mathbf{P}$  can also improve incentives by providing flexibility for the contract to take into account realized market conditions through  $\mathbf{f} - R^f \mathbf{P}$ .

One recognizes that the compensation contract for unskilled managers  $(\omega_1^U, \mathbf{0}_{N \times 1}, C_0^U)$  is always feasible for skilled managers  $C_0^S$  who can always choose zero effort and to ignore their private information. Therefore, the participation constraint can be rewritten as:

$$E [u(C_0^S, \omega, \mathbf{e})] \geq -\exp(-\gamma_M C_0^U),$$

with equality if the endowed skill  $M$  is zero.

Since their effort and portfolio choice are unobservable, skilled managers choose incentive compatible portfolios that solve the inner optimization program (1). Conditional on this portfolio choice, which has both a mean-variance component and a hedge against the excess payoff portion of their contract, they choose their optimal effort to minimize the conditional variance of their excess payoff. This is summarized in Proposition 1.

**Proposition 1** *The optimal portfolio of a skilled manager  $\omega^S$  is given by:*

$$\omega_1^S(j) = \frac{1}{\gamma_M \rho_S} (F \Omega(j) F')^{-1} (F \hat{\Theta}(j) - R^f \mathbf{P}) - \frac{1}{\rho_S} \rho_R,$$

and the optimal level of effort  $\mathbf{e}$  satisfies:

$$\text{Diag} \left[ (\Omega^{-1} + M \cdot \text{Id}_N + \text{diag}(\mathbf{e}))^{-1} \right] \leq h'(\mathbf{e}' \mathbf{1}_{N \times 1}) \mathbf{1}_{N \times 1}, \quad (11)$$

where  $\text{Diag}$  is the diagonal operator. If  $F$  is diagonal, then this condition further reduces to

$$\frac{1}{\Omega_{ii}^{-1} + M + \mathbf{e}_i} \leq h'(\mathbf{e}' \mathbf{1}_{N \times 1}) \quad \forall i \in \{1, \dots, N\}. \quad (12)$$

From Proposition 1, the linear contract induces the skilled manager to take the optimal mean-variance portfolio given its beliefs, with effective risk aversion  $\gamma_M \rho_S$ , corrected by a hedging position  $-\frac{1}{\rho_S} \rho_R$  that takes into account that the manager is exposed to payoff risks  $\mathbf{f} - R^f \mathbf{P}$  independent of the return on the portfolio he manages. The optimal level of effort  $\mathbf{e}$  from equation (11) is determined only by the second moments of the conditional excess payoff  $F \hat{\Theta}(j) - R^f \mathbf{P}$ , and seeks to minimize  $\Omega(j)$ , since  $\Omega^{-1} + \Sigma_j(\mathbf{e}_j)^{-1} = \Omega(j)^{-1}$  is the expression within the trace operator in the condition for optimality.

The correlation structure of asset payoffs  $F$  induces complementarity in learning across asset fundamentals  $\Theta$  for skilled managers, in addition to the ex post correlation in beliefs captured in  $\Omega$ . Skilled managers choose their effort recognizing that learning about asset-specific fundamental  $\theta_i, i \in \{2, 3, \dots, N\}$  also reveals information about the aggregate fundamental  $\theta_1$  through prices, which further reveals information about the other asset-specific fundamentals  $\theta_j$  for  $j \neq i$ . In the special case that  $F$  is diagonal, the FOC for the optimal effort from Proposition 1 reduces to equation (12). As one can see from equation (12), the benefit to the skilled manager for increasing effort becomes separable across assets  $\frac{1}{\Omega_{ii}^{-1} + M + \mathbf{e}_i}$ . With (weakly) convex costs to exerting effort, it then makes sense for the skilled manager to allocate all his attention to the asset that reduces the conditional variance of its excess payoff the most. Consequently, one would expect corner solutions to the skilled manager's optimal effort problem when  $F$  is diagonal, and for manager to allocate his attention to fundamentals for which he is able to equate his marginal benefit of learning with the marginal cost.

Having characterized the optimal policies of unskilled and skilled managers, we can now solve for equilibrium asset prices by imposing market clearing. Appendix A.1 shows the solution of equilibrium asset prices.

### 3.4 Optimal Contracts

We begin our analysis of optimal contracting with the unskilled managers. Given that the investor can perfectly monitor an unskilled manager, it can make the IR constraint bind, otherwise the investor could always compensate the manager less in expectation and strictly raise her own utility. As such, the optimal contract will impose that:

$$E [U (C_0^U, \omega)] = u_0.$$

Consequently<sup>14</sup>, we choose  $C_0^U$  such that:

$$C_0^U = -\frac{1}{\gamma_M} \log (-u_0).$$

The unskilled manager earns a fixed fee for his service as an intermediary. Recognizing that the unskilled fund manager has been incentivized, the investor then maximizes her utility by recommending to the manager the portfolio allocation decision that  $\omega = \omega^c$ . Since the unskilled manager does not speculate on information, we refer to  $\omega^c$  as a *passive* strategy.

We can also calculate the expected utility of investors who invest in the unskilled managers' fund,  $V^U$ . By the law of iterated expectations, first conditioning on  $\mathcal{F}^c$ , the expected

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<sup>14</sup>In addition, recognizing that the unskilled manager has CARA preferences, it is always less costly to pay the manager his certainty equivalent (CE) than a risky payoff. To see this, we recognize that:

$$U (E [C_0^U | \mathcal{F}^c], \omega) \geq E [U (C_0^U, \omega) | \mathcal{F}^c],$$

and the manager would always prefer to be taken off of a lottery over his compensation.

utility to investors who invest with unskilled managers,  $V^U$ , is then:

$$V^U = -E \left[ \exp(-\gamma(W_1 - C_0^U)) \right] = -(-u_0)^{-\frac{\gamma}{\gamma_M}} E \left[ \exp \left( -\gamma R^f W_0 - \frac{1}{2} \mathbf{Z}' \Omega^{-1} \mathbf{Z} \right) \right],$$

where  $\mathbf{Z} = \hat{\Theta} - R^f F^{-1} \mathbf{P}$  is an ex ante excess asset return, and  $\mathbf{Z} \sim \mathcal{N}(\mu, \Omega_Z)$ .<sup>15</sup>

We now focus on the optimal linear contract  $C_0^S$  that investors offer to the  $\chi$  fraction of skilled managers. Our first step is to examine how different components of the linear contract  $\rho_0$ ,  $\rho_S$ , and  $\rho_R$  impact the information acquisition choice of skilled managers. Substituting equation (A1) into equation (11) from Proposition 1, we can find the equilibrium level of effort exerted by skilled managers in a symmetric equilibrium:

$$\text{Tr} \left[ \left( \begin{array}{c} (\tau_\theta + M) \cdot Id_N + \tau_x \left( \frac{\chi}{\gamma_M \rho_S} \right)^2 (M \cdot Id_N + \text{diag}(\mathbf{e})) \cdot \\ (F'F)^{-1} (M \cdot Id_N + \text{diag}(\mathbf{e})) + \text{diag}(\mathbf{e}) \end{array} \right)^{-1} \right] \leq h'(\mathbf{e}' \mathbf{1}_{N \times 1}). \quad (13)$$

Importantly, it is the investor's choice of the sensitivity of the skilled manager's compensation to the fund's return  $\rho_S$  that determines how the contract impacts the managerial incentives to acquire private signals, along with the manager's risk aversion  $\gamma_M$  and parameters that characterize the conditional uncertainty of asset payoffs given prices. With this condition characterizing the optimal effort of the skilled manager in equilibrium, we can perform several comparative statics with equation (13) to understand how optimal effort changes with different features of the economic environment, taking into account that changes in effort change the informational content of prices. These comparative statics are summarized in Proposition 2.

**Proposition 2** *The optimal choice of skilled manager effort  $\mathbf{e}$ , in equilibrium, is increasing (element-by-element) in the coefficient of manager risk aversion,  $\gamma_M$ , and the sensitivity of manager compensation to his realized portfolio return,  $\rho_S$ . It is decreasing in the precision of the prior on  $\Theta, \tau_\theta$ , the precision of the prior on the liquidity trading  $\mathbf{x}, \tau_x$ , and the fraction*

<sup>15</sup>See Appendix A.2 for the detailed proof.

of skilled managers,  $\chi$ .

From Proposition 2, in equilibrium, the sensitivity of the manager's compensation,  $\rho_S$ , increases the effort that the skilled manager exerts to learn about the payoffs of risky assets. Intuitively, the more the manager's compensation depends on the fund's excess payoff, the more incentive the manager has to acquire information to improve the fund's performance. Similarly, the more risk-averse the manager (higher  $\gamma_M$ ), the more effort the manager will exert to collect information to reduce the uncertainty of the fund's final AUM. In addition, as one would expect, the more uncertain the economic environment (lower  $\tau_\theta$ ,  $\tau_x$ , and  $\chi$ ), the more beneficial for the manager to exert effort to achieve the performance objectives of the fund.

Having solved for the determinants of optimal skilled manager effort, in equilibrium, we now provide a characterization of the optimal linear contract, which is summarized in Proposition 3.

**Proposition 3** *The optimal contract for a skilled manager is a  $N + 2 \times 1$  vector  $(\rho_0, \rho_S, \rho_R)$  that sets*

$$\rho_R = - \left( \rho_S + \frac{\gamma}{\gamma_M} (1 - \rho_S) \right) \omega^0,$$

where

$$\omega^0 = \frac{1}{\gamma} F'^{-1} \text{Var} (\Theta - R^f F^{-1} \mathbf{P})^{-1} E [\Theta - R^f F^{-1} \mathbf{P}] = \frac{1}{\gamma} F'^{-1} (\Omega_Z + \Omega)^{-1} \mu,$$

is the ex ante mean-variance efficient portfolio, and

$$\rho_0 = \frac{1}{\gamma} \log \frac{V^U}{v^S},$$

where  $v^S$  is given in the Appendix. Furthermore, the optimal sensitivity on the realized excess payoff of the skilled manager's fund  $\rho_S$  satisfies the FONC (A5) given in the Appendix.<sup>16</sup>

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<sup>16</sup>Substituting for  $\Omega$  with equation (A1),  $\mathbf{e}_j$  with equation (13), and  $\rho_R$  from Proposition 3 into the FONC

To help explore the implications of Proposition 3, we rewrite the optimal linear contract for a skilled manager as:

$$\begin{aligned} C_0^S &= \frac{1}{\gamma} \log \frac{V^U}{v^S} + \rho_S \omega_1^S(i)' (\mathbf{f} - R^f \mathbf{P}) - \left( \rho_S + \frac{\gamma}{\gamma_M} (1 - \rho_S) \right) \omega^{0'} (\mathbf{f} - R^f \mathbf{P}) \\ &= \frac{1}{\gamma} \log \frac{V^U}{v^S} + \rho_S (\omega_1^S(i)' - \omega^{0'}) (\mathbf{f} - R^f \mathbf{P}) - \frac{\gamma}{\gamma_M} (1 - \rho_S) \omega^{0'} (\mathbf{f} - R^f \mathbf{P}) \end{aligned}$$

The first piece of the contract is a constant fee that ensures that, net fees, investors are indifferent between investing with skilled and unskilled managers. The second piece is the manager's compensation based on the fund's performance relative to the ex ante mean-variance portfolio  $\omega^0$ . The third adjusts compensation by the performance of an index that tracks the ex ante mean-variance efficient portfolio investors would choose at the time that the contract is signed. Essentially, compensation beyond a fixed fee is offered for the value added by the manager over the investment strategy that investors could achieve through direct investment without acquiring any private information.

Notice that  $\omega^0$  plays the role of a passive benchmark for skilled manager compensation, since it is a portfolio whose holdings are chosen based on only public information. As such, benchmarking is a feature of optimal contracting for delegated asset management with asymmetric information. Under certain conditions, this passive portfolio is also featured as an optimal benchmark in Admati and Pfleiderer (1997).<sup>17</sup> The sensitivity of manager compensation to this benchmark,  $-\frac{\gamma}{\gamma_M} (1 - \rho_S)$ , is intimately linked to the sensitivity of manager compensation to the fund's performance,  $\rho_S$ . The more risk-averse is the investor relative to the manager (higher  $\frac{\gamma}{\gamma_M}$ ), the greater the magnitude of the sensitivity of the manager's compensation to the benchmark portfolio over  $1 - \rho_S$ , since  $\rho_S \in [0, 1]$ . This is a similar feature to the optimal benchmark in van Binsbergen et al. (2008), which features a tilt

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(A5), we can then solve for the fixed point to find the equilibrium value of  $\rho_S$ .

<sup>17</sup>Admati and Pfleiderer (1997) identify the global minimum variance portfolio, tilted by the assets held by investors in separate accounts, as the optimal benchmark in a partial equilibrium setting. We derive benchmarking against the ex ante mean-variance efficient portfolio as a feature of the optimal affine compensation structure for skilled managers with market-clearing. Since prices are determined by market-clearing, the benchmark portfolio is, itself, an equilibrium object that depends on the optimal contract.



that corrects for differences in risk attitudes between the fund manager and the delegating CIO, in addition to the minimum variance portfolio.

To further understand the impact of incentives on a skilled manager's actions, we rewrite the optimal portfolio choice of the skilled manager by substitute for  $\rho_R$ :

$$\omega_1^S(j) = \frac{1}{\gamma_M \rho_S} (F\Omega(j)F')^{-1} (F\hat{\Theta}(j) - R^f \mathbf{P}) + \left(1 + \frac{\gamma}{\gamma_M} \left(\frac{1}{\rho_S} - 1\right)\right) \omega^0. \quad (14)$$

The portfolio of skilled managers essentially have two components: a mean-variance efficient portfolio and a long position in  $\omega^0$ . Since skilled manager's compensation is tied to the benchmark portfolio  $\omega^0$ , they are effectively endowed with a negative exposure to the benchmark portfolio, and they take a long position in  $\omega^0$  to hedge themselves. This benchmark-driven demand causes managers to over-invest in assets that are representative in their benchmark portfolio, and we refer this demand as *hedging* demand. This hedging channel is also a feature in Cuoco and Kaniel (2011), Basak and Pavlova (2013), and Buffa et al. (2014). In contrast to models in which benchmarking is assumed in the preferences of investors, such as in Basak and Pavlova (2013) and Duarte et al. (2015), in our model, the benchmark enters into security selection through the hedging demand of skilled managers. Skilled managers here only care about benchmarking insofar as it affects their compensation, and this leads to the sterilization of the benchmark in the manager's optimal portfolio. In addition, the sensitivity of the contract to fund performance  $\rho_S$  is the transmission channel through which investors influence managerial effort to acquire information, rather than the benchmarking aspects of the compensation contract.

This completes our characterization of the perfect Bayesian noisy rational expectations equilibrium.

## 4 Model Implications

In this section, we discuss several empirical implications of our analysis. We begin by investigating the behavior of intermediaries. We then turn to the asset pricing implications of our framework, with an emphasis on predictions for the cross-section of asset returns.

### 4.1 Implications for Intermediaries

Since both the choice of benchmark portfolios and the skill of fund managers are endogenous and vary with respect to the fundamentals, it allows us to offer empirical predictions without conditioning on actual compensation contracts and observing the managerial effort. By relating characteristics of the asset fundamentals to potentially observable fund outcomes such as their holdings and performance through incentive contracts, our model also provides the theoretical link between the unobservable effort (skill) of the skilled manager and the cross sections of fund behaviors.

We consider a numerical example with two assets to illustrate our predictions. We choose as our baseline specification:

$$F = \begin{bmatrix} 1 & 0 \\ b & \sqrt{1-b^2} \end{bmatrix}, \quad \bar{\Theta} = \frac{1}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \bar{\mathbf{x}} = \frac{1}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

In our discussion in this section, we refer to the asset whose payoff depends only on the aggregate fundamental  $\theta_1$  as Asset 1, and the asset that also has an asset-specific fundamental  $\theta_2$ , with loading  $b$  on  $\theta_1$ , as Asset 2. The  $F$  matrix is set to ensure that the comparative static of  $b$  is implemented keeping the level of uncertainty constant. Finally, we choose the effort function  $h(\cdot)$  to be linear in effort  $\mathbf{e}'\mathbf{1}_{2 \times 1}$ ,  $h(\mathbf{e}'\mathbf{1}_{2 \times 1}) = \mathbf{e}'\mathbf{1}_{2 \times 1}$ , so that the marginal cost of learning is constant. As a result, any complementarity in learning arises from the co-variance structure of asset prices. Although we consider a two-asset example for ease of exposition, we find that our results hold more generically.

### 4.1.1 Optimal Effort and Portfolios

The affine incentive contract has two key features: performance-based incentive  $\rho_s$  and benchmarking.  $\rho_s$  determines how the contract impacts the manager's incentive to acquire private information. Panels (a) and (c) in Figure 1 show that as the ex ante uncertainty of asset payoffs  $\tau_\theta^{-1}$  declines, there are less benefits for the skilled manager to acquire information. The optimal contract then puts more weight on the performance-based component  $\rho_s$  to provide incentives, and  $\rho_s$  increases with respect to  $\tau_\theta$ . Since we allow a general structure for asset payoffs, varying the correlation between asset payoffs also indicates the shift of incentives, as shown in Panels (b) and (d) of Figure 1. As  $b$  increases, the individual asset payoffs are more correlated with the aggregate component and the marginal benefit of learning about asset-specific information is low, while learning about the aggregate component pays off more. Since information about aggregate component becomes more important as two assets are more correlated, the optimal contract provides more incentives to acquire private information by increasing  $\rho_s$ .

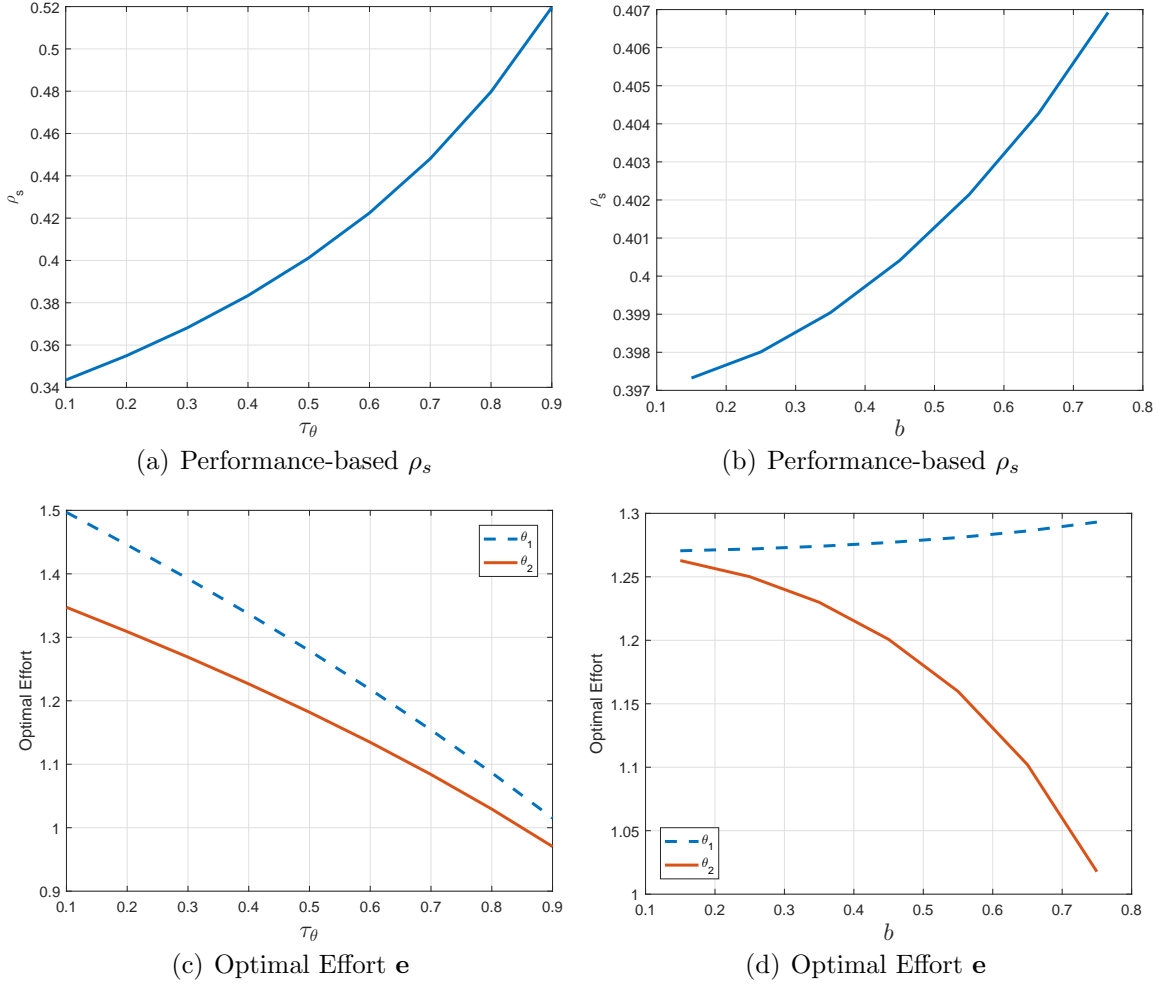
Benchmarking rises endogenously in the optimal contract. To understand the active fund holdings through the hedging channel, we conduct a comparative statics analysis with respect to  $\tau_a$  and  $b$ . Figure 2 and Figure 3 show the holdings of both skilled and unskilled fund managers and the composition of the benchmark portfolio. As  $\tau_a$  increases, the payoffs of risky assets are less uncertain, and the representation of risky assets in the benchmark portfolio increases. This is because the benchmark is determined based on the risk-return trade off of the assets ex ante. Since skilled managers' compensation is benchmarked to  $\omega_0$ , they tend to hedge the benchmark risk by longing<sup>18</sup> the assets that are representative in the benchmark portfolio. This hedging behavior drives up the equilibrium prices of the assets that are representative in the benchmark. The change of correlation structure  $b$  impacts the benchmark portfolio too. As two assets become more correlated, there is less diversification benefit in the ex ante mean-variance portfolio. Hence, as  $b$  increases in Figure 3 Panel (b),

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<sup>18</sup> $\rho_R$  is negative since the managers are compensated relative to the benchmark.

Figure 1: **Performance-based Compensation and Optimal Effort**

Parameters:  $\tau_\theta = 0.5$ ,  $\tau_x = 1$ ,  $b = 0.5$ ,  $\gamma_M = 2$ ,  $M = 0$ ,  $\chi = 0.3$ ,  $\gamma = 1$ ,  $W_0 = 1$ ,  $R^f = 1.02$



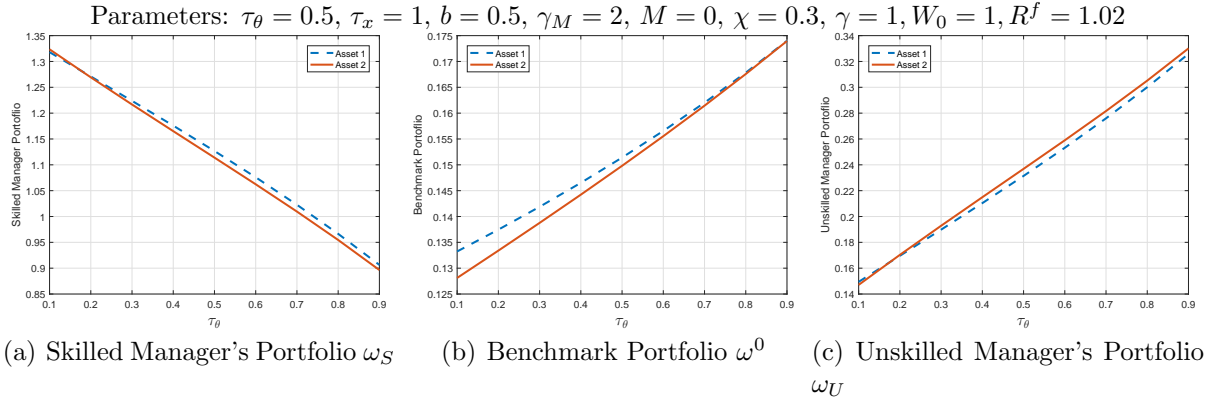
the holdings of both risky Assets 1 and 2 decline in the benchmark portfolio.

Information aggregation through the general equilibrium effect is the key to understanding the implications of benchmarking on the active fund holdings. The expected excess payoffs to the fundamentals  $E[\Theta - R^f F^{-1} \mathbf{P}]$  contain an additional component:

$$-\frac{1}{R^f} \chi \left( 1 + \frac{\gamma}{\gamma_M} \left( \frac{1}{\rho_S} - 1 \right) \right) \left( \left( \frac{1-\chi}{\gamma} + \frac{\chi}{\gamma_M \rho_S} \right) \Omega^{-1} + \frac{\chi}{\gamma_M \rho_S} \Sigma_j (\mathbf{e}_j)^{-1} \right)^{-1} F' \omega^0,$$

that reflects the risk of the skilled manager's benchmark. If the manager pushes up prices because of hedging demand for assets that are more representative in the benchmark port-

Figure 2: **Portfolios and Benchmark:**  $\tau_\theta$



folio, this lowers the expected payoff. This effect helps skilled managers, however, because they are evaluated relative to the performance of the benchmark, which now has a lower expected payoff. This is a general equilibrium effect through which the asset prices that are most inflated by benchmarking are those that are the largest constituents of the benchmark portfolio. As a consequence, skilled managers both hedge themselves in their portfolios against the benchmark, and benefit from the benchmark's lower expected payoff from their aggregate hedging demand, as shown in Panels (a) and (c) in Figure 2.

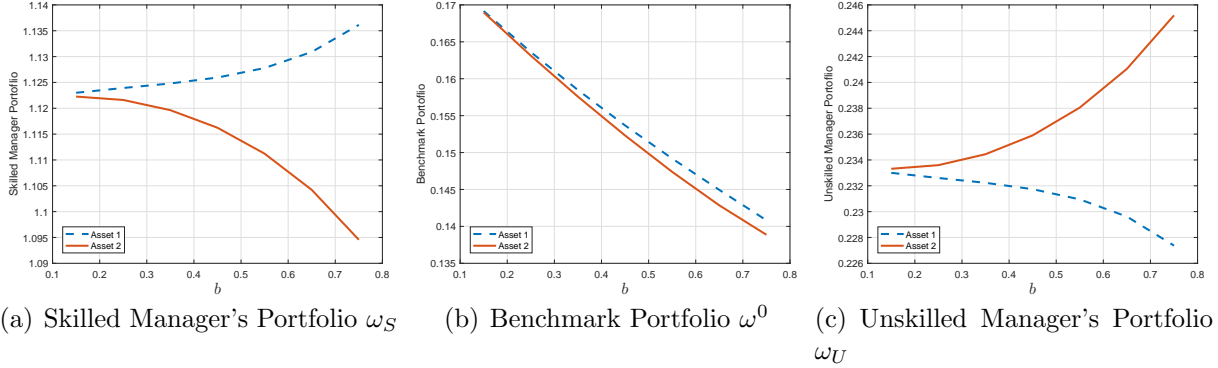
One may wonder what impact benchmarking has on the performance of unskilled managers. Since skilled managers all have a hedging demand for their exposure to the benchmark in their compensation, the liquidity providers for this demand are the unskilled managers. These unskilled managers are compensated by tilting their portfolios away from the benchmark, and toward assets that offer relatively higher excess payoffs. Consequently, unskilled managers benefit from benchmarking in skilled manager compensation, since they earn risk premia for insuring skilled managers against their benchmark exposure.

#### 4.1.2 Performance Evaluation

We next examine the link between our theoretical measure of skills and some popular empirical measures. In our framework, fund manager skill is the optimal effort level given the manager's incentive contract. Given the optimal decision, skilled managers choose how

Figure 3: **Portfolios and Benchmark:  $b$**

Parameters:  $\tau_\theta = 0.5$ ,  $\tau_x = 1$ ,  $b = 0.5$ ,  $\gamma_M = 2$ ,  $M = 0$ ,  $\chi = 0.3$ ,  $\gamma = 1$ ,  $W_0 = 1$ ,  $R^f = 1.02$



much their portfolios deviate from the endogenous benchmark portfolio. To relate the effort exerted by the fund manager to the active share that is first introduced by Cremers and Petajisto (2009), we define our active share as the deviation of a skilled fund manager's portfolio holdings from the benchmark portfolio. The change in fundamental risk impacts the active share not just from shifting the learning incentives, but also from rebalancing the benchmark portfolio ex ante.

We derive an analogous expression for the average active share of a skilled manager in our economy  $AS$ :

$$AS = \frac{1}{2} E [\mathbf{1}' |\omega_1^S(j) - \omega^0|],$$

where  $\omega^0$  is the benchmark for the skilled manager. Substituting for  $\omega_1^S(j)$  with equation (14),  $\omega^0$  with Proposition 3,  $\hat{\Theta}(j)$  with equation (7), and  $\Pi_\theta$  and  $\Pi_x$  with equations (A2) and (A3), respectively, we can employ results for the expectation of a folded normal distribution to arrive at:

$$AS = \frac{1}{2\gamma_M \rho_S} \sum_{i=1}^N \left( \sqrt{\frac{2}{\pi}} \sigma_i e^{-\mu_i^2/2\sigma_i^2} + \mu_i \left( 1 - 2\Phi\left(-\frac{\mu_i}{\sigma_i}\right) \right) \right),$$

where:

$$\begin{aligned}\mu_i &= f'_i (\Omega^{-1} + \Sigma_j (\mathbf{e}_j)^{-1} + (1 - \rho_S) (\Omega_Z + \Omega)^{-1}) \mu, \\ \sigma_i^2 &= f'_i (\Gamma_\theta \tau_\theta^{-1} \Gamma'_\theta + \Gamma_x \tau_x^{-1} \Gamma_x + \Sigma_j (\mathbf{e}_j)^{-1}) f_i,\end{aligned}$$

where  $f_i$  is the  $i^{\text{th}}$  column of  $F^{-1}$ , and

$$\begin{aligned}\Gamma_\theta &= \tau_x \left( \frac{\chi}{\gamma_M \rho_S} \right)^2 \Sigma_j (\mathbf{e}_j)^{-1} (F' F)^{-1} \Sigma_j (\mathbf{e}_j)^{-1} - R^f (\Omega^{-1} + \Sigma_j (\mathbf{e}_j)^{-1}) F^{-1} \Pi_\theta + \Sigma_j (\mathbf{e}_j)^{-1}, \\ \Gamma_x &= \left( \tau_x \Sigma_j (\mathbf{e}_j)^{-1} (F' F)^{-1} \Sigma_j (\mathbf{e}_j)^{-1} - R^f \left( \frac{\gamma_M \rho_S}{\chi} \right)^2 (\Omega^{-1} + \Sigma_j (\mathbf{e}_j)^{-1}) F^{-1} \Pi_\theta \right) \Sigma_j (\mathbf{e}_j) F' .\end{aligned}$$

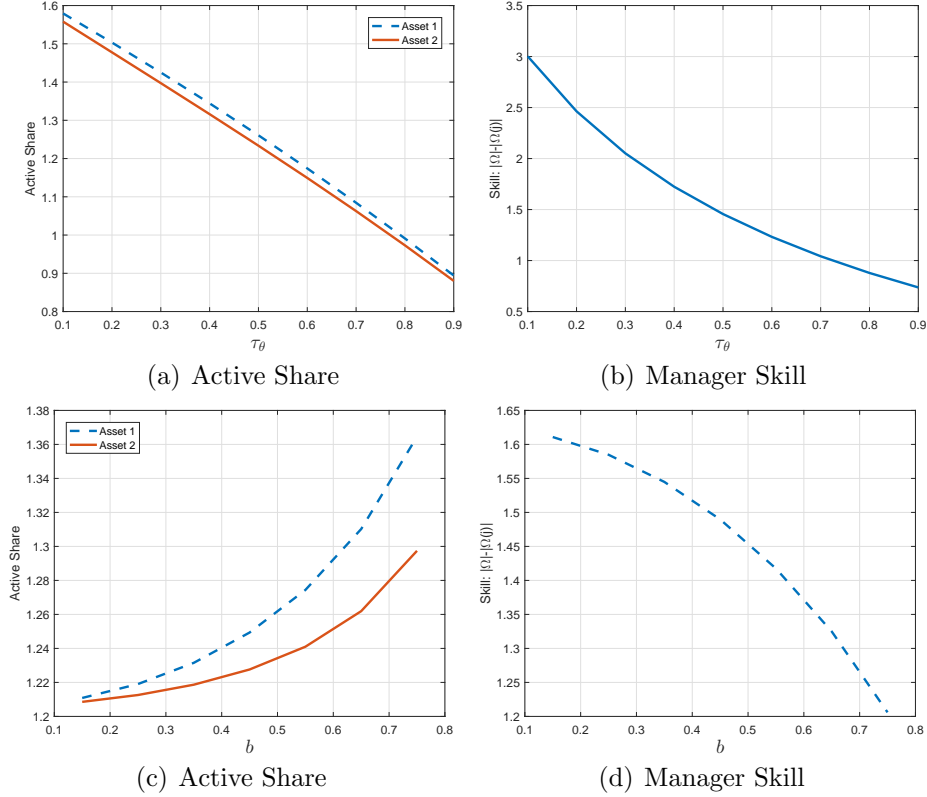
Figure 4 shows the comparative statics of the theoretical active share and the model-implied measure of skill  $|\Omega| - |\Omega(j)|$  with respect to the change of  $\tau_\theta$  and  $b$ . As the uncertainty of asset payoffs goes up (i.e.,  $\tau_\theta$  decreases), the skilled manager exerts more effort to learn and acquires more private information. Hence, both the benefit of learning and the measure of manager skills increases as  $\tau_\theta$  increases, as shown in Figure 4 Panel (b). By acquiring additional private information about asset fundamentals, the skilled manager's portfolio further deviates from their benchmark portfolio,  $\omega^0$ . Similarly, when asset fundamentals are less uncertain (higher  $\tau_\theta$ ), the manager acquires less private information and takes a more passive position in financial markets. This is a potential explanation for the historical decline of active share.

This is a potential explanation to the recent discussion on fund flows shifting away from active funds. Cremers and Petajisto (2009) also found that aggregate-level and fund-level active shares trended downward after the 1980s. During this period of time, the aggregate volatility of asset fundamentals also declined in the U.S. (e.g., Blanchard and Simon (2001), Comin and Mulani (2009)). Our prediction on the time series of active shares is consistent with the empirical findings of Cremers and Petajisto (2009).

We then obtain comparative statics for fund manager skill as the correlation  $b$  between the

Figure 4: **Active Share and Fund Manager Skill:  $\tau_\theta$  &  $b$**

Parameters:  $\tau_\theta = 0.5$ ,  $\tau_x = 1$ ,  $b = 0.5$ ,  $\gamma_M = 2$ ,  $M = 0$ ,  $\chi = 0.3$ ,  $\gamma = 1$ ,  $W_0 = 1$ ,  $R^f = 1.02$



asset payoff increases (Figure 4 Panel (d)) through the incentive channel. The active shares of the two assets, however, go up. This occurs because the change in the correlation between two assets impacts the ex ante choice of the mean-variance portfolio (i.e., the benchmark portfolio). The benchmark portfolio reduces its weight in both Asset 1 and Asset 2 as there is less benefit from diversification when risky assets are more correlated. Although skilled managers invest less in Asset 2, which is the asset with both aggregate and asset-specific components in its payoff, the decline in the weight of Asset 2 in the portfolio is not as much as the reduction in the benchmark portfolio.

Our prediction on the relation between active shares and the benchmark portfolio also aligns with Frazzini et al. (2016). The active share measure can deviate from the underlying level of manager skill, since the incentives for the skilled manager to learn are not strong enough to dominate the changes in the ex ante choices for benchmark portfolio. Our predic-



tion is also consistent with Jiang and Sun (2014), who studied dispersion in fund managers' beliefs about future stock returns based on their active holdings. The degree of information asymmetry is positively correlated with the dispersion of active mutual funds holdings under our delegated learning channel, since the incentives for fund managers to learn rises as the degree of uncertainty of asset payoffs increases, which is proxied by the idiosyncratic volatility of stock return in Jiang and Sun (2014).

Our setting also allows us to explore another empirical measure for unobservable mutual fund actions, return gap ( $RG$ ), employed in Kacperczyk et al. (2008). We rewrite the skilled manager's portfolio as:

$$\omega_1^S(j) = \frac{\gamma}{\gamma_M \rho_S} \omega^U + \left(1 + \frac{\gamma}{\gamma_M} \left(\frac{1}{\rho_S} - 1\right)\right) \omega^0 + \frac{1}{\gamma_M \rho_S} F'^{-1} \Sigma_j (\mathbf{e}_j)^{-1} (\mathbf{s}_j - R^f F^{-1} \mathbf{P}).$$

The first two elements reflect the position a skilled manager without private information would take based on public information and the benchmark portfolio, while the last element captures the speculative bet the skilled manager makes based on his informational advantage after observing its private signals. Consequently, we view the first two elements as the “holdings” portfolio that is publicly observable to investors, and measure the expected return gap between the gross return a skilled manager garners and that of this “holdings” portfolio,  $RG$ .

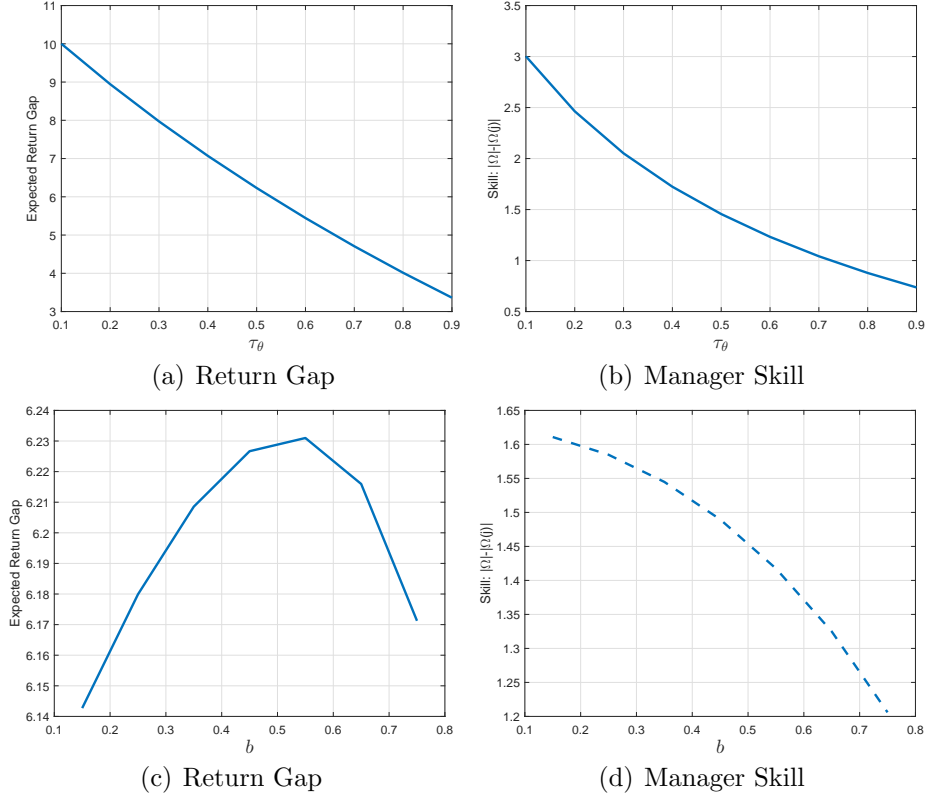
$$E[RG] = \frac{1}{\gamma_M \rho_S} \mu' \Sigma_j (\mathbf{e}_j)^{-1} \mu + \frac{1}{\gamma_M \rho_S} Tr [\Sigma_j (\mathbf{e}_j)^{-1} (\Omega_Z + \Omega)].$$

The expected return gap is driven by skilled managers trading more aggressively to collect the risk premia on assets since they face less risk due to their private information, and from the reduction in overall uncertainty they have when speculating.

Figure 5 shows the comparative statics of the expected return gap and the model-implied measure of fund manager skill. The return gap is moving in the same direction as the skill measure of our model when the uncertainty level of the fundamental  $\tau_\theta^{-1}$  declines. The return gap, however, does not monotonically decrease in the correlation of asset fundamentals. As

Figure 5: **Return Gap and Fund Manager Skill:  $\tau_\theta$  &  $b$**

Parameters:  $\tau_\theta = 0.5$ ,  $\tau_x = 1$ ,  $b = 0.5$ ,  $\gamma_M = 2$ ,  $M = 0$ ,  $\chi = 0.3$ ,  $\gamma = 1$ ,  $W_0 = 1$ ,  $R^f = 1.02$



the correlation between asset fundamentals increases, there is less overall benefit to learning since the asset-specific fundamental is less relevant to returns, and the aggregate fundamental is more revealed by prices to all managers. The expected return gap, in contrast, trades off two competing forces. On the one hand, there is increased risk in asset markets because a higher correlation among asset returns reduces the diversification benefit to holding both assets, increasing the risk premia earned by skilled managers who bear more risk because they face less uncertainty through learning. On the other hand, the increased correlation also reveals more information to unskilled managers about the aggregate asset fundamental, reducing the information asymmetry between skilled and unskilled managers. These two forces contribute to the humped-shaped return gap in Panel (c) in Figure 5.

We can also evaluate fund manager performance by computing expected excess returns in our setting, and compare them across the benchmark portfolio and both skilled and unskilled

managers. Given the benchmark portfolio  $\omega^0$  in Proposition 3, it is straightforward that a fund manager with portfolio that has an initial wealth  $W_0$  who invests in the benchmark portfolio with final wealth  $W_2^0$  will have an expected excess return:

$$E [W_2^0 - R^f W_0] = \frac{1}{\gamma} \mu' (\Omega_Z + \Omega)^{-1} \mu.$$

Making use of properties of chi-squared random variables, and that the trace operator is linear and satisfies  $Tr [ABC] = Tr [BCA]$ , we arrive at the expected excess return for investors that invest with an unskilled manager:

$$E [W_2^U - C_0^U - R^f W_0] = \frac{1}{\gamma} \mu' \Omega^{-1} \mu + \frac{1}{\gamma} Tr [\Omega^{-1} \Omega_Z] + \frac{1}{\gamma_M} \log(-u_0),$$

and similarly with a skilled manager:

$$\begin{aligned} E [W_2^S - C_0^S - R^f W_0] &= E [W_2^0 - R^f W_0] + \frac{1}{\gamma_M} \left( \frac{1}{\rho_S} - 1 \right) \mu' ((\Omega_Z + \Omega)^{-1} + \Omega^{-1} + \Sigma_j (\mathbf{e}_j)^{-1}) \mu \\ &\quad + \frac{1}{\gamma_M} \left( \frac{1}{\rho_S} - 1 \right) Tr [(\Omega^{-1} + \Sigma_j (\mathbf{e}_j)^{-1}) \Omega_Z + \Sigma_j (\mathbf{e}_j)^{-1} \Omega] - \rho_0. \end{aligned}$$

Kacperczyk et al. (2016) highlights that the additional return that a skilled fund manager earns arises from their information acquisition decisions. As volatility falls in our setting, the expected excess return of both unskilled managers and the benchmark portfolio increases, reflecting the decreased uncertainty in investing and the more liquidity that their portfolios provide. In contrast, the expected excess return of skilled managers falls as their superior information degrades. Interestingly, the first piece of skilled fund returns is the benchmark portfolio's return.

## 4.2 Implications for Asset Pricing

Our equilibrium setup allows us to investigate predictions for asset returns. To facilitate our discussion, we first derive prices in the special case in which there are no skilled managers, or

$\chi = 0$ . One can then show, in this “no information” setting, that prices  $\mathbf{p}$  take the following linear form:

$$\mathbf{p} = \frac{1}{R^f} F \bar{\Theta} - \frac{1}{R^f} \gamma \tau_\theta^{-1} F F' \mathbf{x}.$$

Prices in this setting reflect the prior beliefs of investors and unskilled managers about the payoff fundamentals  $\bar{\Theta}$ , and the net asset supply  $\mathbf{x}$ . The realized excess payoffs to the fundamentals  $\Theta$  take the form:

$$\Theta - R^f F^{-1} \mathbf{p} = \Theta - \bar{\Theta} + \gamma \tau_\theta^{-1} F' \mathbf{x}, \quad (15)$$

and the unconditional covariance of excess payoffs is given by:

$$Cov [\Theta - R^f F^{-1} \mathbf{p}] = \tau_\theta^{-1} Id_N + \gamma^2 \tau_\theta^{-2} F' \tau_x^{-1} F. \quad (16)$$

In this setting, asset returns are correlated only insofar as their payoffs load on the common asset fundamental  $\theta_1$ . When there are skilled managers who acquire private information, however, returns become further correlated because conditional beliefs about asset fundamentals become correlated ex-post through learning from prices. We summarize this feature of our multi-asset noisy rational expectations equilibrium in Proposition 4.

**Proposition 4** *The tracking error in the market’s beliefs about the aggregate fundamental  $\theta_1$  is related to its tracking error in the asset-specific fundamental  $\theta_i, i \in [2, 3, \dots, N]$  through:*

$$\theta_1 - \hat{\theta}_1 = \sum_{i=1}^{N-1} b_i (\theta_i - \hat{\theta}_i) + \frac{\gamma_M \rho_S}{\chi} (M + e_{1j})^{-1} (x_1 - \hat{x}_1),$$

where  $x_1$  is the liquidity shock to the asset whose payoff loads only on aggregate risk. It follows that correlation between the asset-specific risky payoff of asset  $i \in [2, 3, \dots, N]$ ,  $\theta_i$ , and the aggregate fundamental  $\theta_1$  depends on  $b_i$ , such that  $b_i Cov(\theta_i, \theta_1 | \mathcal{F}^c) \geq 0$ . The correlation between the asset-specific risky payoffs of assets  $i$  and  $j$ ,  $\theta_i$  and  $\theta_j$ , respectively, depend on

their  $b$ 's, such that  $b_i b_j \text{Cov}(\theta_i, \theta_j | \mathcal{F}^c) \leq 0$ .

Admati (1985) emphasizes that, in a multi-asset setting, asset fundamentals become correlated through the learning channel once investors observe prices. Our setting allows us to refine this insight by highlighting the role that the correlation structure of asset payoffs plays in shaping investor expectations. As a result of the common component of asset payoff risk  $\theta_1$ , the asset-specific fundamentals  $\theta_i, i \in [2, 3, \dots, N]$  are ex post correlated with each other after managers observe prices, and the sign of this correlation depends on the exposure of each asset to  $\theta_1$ . If payoffs of assets  $i$  and  $j$  are positively correlated with  $\theta_1$ ,  $b_i b_j > 0$ , then these two asset specific shocks are negatively correlated with each other through learning, since observing two higher-than-expected prices,  $P_i$  and  $P_j$ , leads investors to revise their expectations of  $\theta_1$  upwards, and their expectations of  $\theta_i$  and  $\theta_j$  downwards. A similar intuition applies to the correlation between  $\theta_i$  and  $\theta_1$ . If  $b_i > 0$ , then a higher-than-expected price  $P_i$  leads investors to attribute the positive surprise to both a higher  $\theta_i$  and a higher  $\theta_1$ . In this sense, prices act as additional signals about unrelated asset-specific fundamentals through their common dependence on  $\theta_1$ .<sup>19</sup> In contrast to Admati (1985), this induced correlation structure through learning also feeds back into the information acquisition decision of skilled managers, and their performance-based incentives.

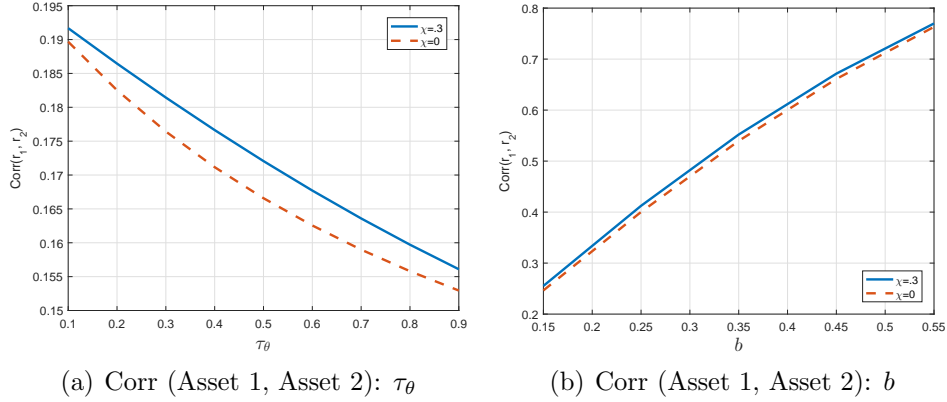
This induced correlation structure from learning allows us to understand the comovement of asset prices. Pindyck and Rotemberg (1990), Pindyck and Rotemberg (1993), and Barberis et al. (2005) find that asset returns appear to comove in excess of the correlation implied by their dependence on common fundamentals. Veldkamp (2006) rationalizes this phenomena in a learning framework in which information markets induce complementarities in learning about a subset of the fundamentals that drive asset prices. Similar to Veldkamp (2006), we view the limit of no information in our setting,  $\chi \searrow 0$ , as a benchmark for the correlation that an econometrician would expect when estimating a model of asset returns given their common fundamentals. We also frame excess comovement in terms of excess correlation,

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<sup>19</sup>We expect that these correlations for  $\theta_i$  are increasing in asset  $i$ 's loading  $b_i$  on the aggregate payoff fundamental  $\theta_1$ .

Figure 6: **Excess Comovement: Three Assets Case**

This plot compares the correlation of asset returns when skilled managers learn from private signals with the correlation of asset returns under no-information benchmark. We consider a three-asset environment, where the payoff of Asset 1 depends only on the aggregate fundamental  $\theta_1$ , and the payoffs of Asset 2 and Asset 3 have both aggregate fundamental and the asset-specific fundamentals. The plots report the correlation of returns between Asset 1 and Asset 2. The Parameter:  $\gamma = 1$ ,  $\gamma_M = 2$ ,  $R^f = 1.02$ ,  $N = 3$ ,  $\bar{\Theta} = .5$ ,  $\chi = .3$ ,  $\tau_x = 1$ .



since varying parameters related to uncertainty in the model will affect the overall level of risk in the market, and therefore the variances of asset prices.

As Panel (a) of Figure 6 illustrates, the correlation between the returns of Assets 1 and 2 can be higher than in the no information benchmark ( $\chi = 0$ ) when uncertainty, measured by the inverse of the precision of the common prior about fundamentals  $\tau_\theta^{-1}$ , is sufficiently high. For low  $\tau_\theta$ , skilled managers have a strong incentive to acquire information about the fundamentals, which become ex post correlated in beliefs after observing prices. The impact of this correlation structure on asset prices through their trading can cause asset returns to have a higher correlation than that implied by the correlation of their payoffs. As such, a prediction of our model is that excess comovement among assets is likely to be higher in periods of high aggregate uncertainty or among assets with higher uncertainty. When altering the exogenous correlation of asset payoffs  $b$  in Panel (b), the learning channel also generates reasonable amount of excess correlation. This occurs because the correlation structure of asset returns impacts the information that skilled managers choose to acquire,

and this feeds back into prices through their portfolio decisions. Since managers learn from prices, prices act as a coordination mechanism for both their information acquisition and portfolio decisions. This can potentially amplify the correlation of asset returns beyond the fundamental correlation of their asset payoffs, which is reflected in the no information benchmark, through their trading. This suggests that generating this phenomenon relies on our endogenous learning channel.

## 5 Extensions

In this section, we discuss three extensions to our delegated learning channel. We first analyze the extension where we allow fund managers to trade over multiple periods. We then discuss the implications from the model in which investors are endowed with some exposure  $\psi$  to wealth correlated with the excess return of the asset fundamentals that they cannot trade. Finally, we introduce the channel in which investors learn about managerial skills to provide a complete picture on understanding our delegated learning channel.

### 5.1 Trading over Multiple Periods

In this section, we discuss a dynamic extension of our model in which fund managers trade over multiple periods. Further details of the model, its derivation, and a more thorough discussion of the results are in the Internet Appendix.

In this extension of the model, we assume that time is discrete and trading occurs between the initial date 0 and some known, final date  $T$ . Assets on this final date pay a single risky dividend:

$$\mathbf{f}_T = F\Theta + \delta,$$

where a priori  $\Theta \sim \mathcal{N}(\mathbf{0}_{N \times 1}, \tau_\theta^{-1} Id_N)$  represents the learnable fundamental components of the dividend,  $F$  is the matrix of loadings on these fundamental components, and  $\delta \sim \mathcal{N}(\mathbf{0}_{N \times 1}, \tau_\delta^{-1} Id_N)$  represents the unlearnable component. In addition, we assume the noisy

supply of the assets  $\mathbf{x}_t$  is mean-reverting over time, and follows a VAR(1) process:

$$\mathbf{x}_{t+1} = \nu \mathbf{x}_t + \tau_x^{-1/2} \varepsilon_{t+1}^x,$$

where  $\nu$  is a  $N \times N$  constant matrix and  $\varepsilon_{t+1}^x \sim iid \mathcal{N}(\mathbf{0}_{N \times 1}, Id_N)$ .

Investors continue to have CARA preferences over their final wealth, and contract with fund managers to manage their portfolios, a fraction  $\chi$  of which are skilled and  $1 - \chi$  are unskilled. Unskilled managers choose a portfolio  $\omega_t^U$  at date  $t$ , while skilled manager  $i$  chooses a portfolio  $\omega_t^S(i)$ . Skilled manager  $i$  can exert unobservable effort  $\mathbf{e}_{t+1}(i)$  at each date  $t$ , for which they incur a disutility  $h(\mathbf{e}'_{t+1}(i) \mathbf{1}_{N \times 1})$ , to acquire a private signal about  $\Theta$  at date  $t + 1$ , the precision of which  $\Sigma(\mathbf{e}_{t+1}(i))^{-1}$  is inversely related to this effort. For simplicity and tractability, we assume that this effort is chosen from a two-point distribution  $\mathbf{e}_{t+1}(i) \in \{\mathbf{0}_{N \times 1}, \bar{e} \mathbf{1}_{N \times 1}\}$ , corresponding to no effort and effort by the skilled fund managers, respectively. Investors again commit at  $t = 0$  to an affine contract with the skilled managers.

We use the multi-period model to generalize several of our insights to a dynamic setting. As is endemic to dynamic portfolio choice problems, the portfolio allocation decisions of managers,  $\omega_t^U$  and  $\omega_t^S(i)$ , not only reflect a speculative component based on the mean and variances of asset payoffs, but also an intertemporal hedging motive to insure against future fluctuations in the payoff environment. Novel to our setting is that the effort choices of skilled managers  $\mathbf{e}_{t+1}(i)$  are now also forward-looking. Whereas in the static setting, skilled managers seek to minimize the conditional variance of their portfolio excess payoff through their information acquisition decision, in the dynamic setting managers take into account the benefits of learning in early versus later periods for the same level of disutility from exerting effort. Investors take into account these intertemporal incentives when choosing the optimal affine contract to offer to skilled managers.

An important point of departure of the multi-period model from its static counterpart is the possibility that investors can observe a time series of of fund manager performance



during the intermediate trading periods. In practice, the SEC 13F filings, which are publicly available, require large mutual funds to report all long positions held at the end of a quarter. This potentially enables investors to improve their monitoring of skilled managers' behavior. Based on the availability of the SEC 13F filings for mutual funds, we allow investors to observe an unbiased but noisy measure of their fund's return gap at each date  $t$ . This noise, which we assume is i.i.d. across assets and dates, can be thought of as portfolio rebalancing driven by nonfundamental, noninformational reasons or measurement error. Let  $R_t^i$  be this noisy observation. Given their observations of past asset and fund-specific returns, investors can form their posterior beliefs about their skilled manager's private information conditional on a given path of effort  $\{\mathbf{e}_t(i)\}_{s=1}^T$ . They can then derive a log-likelihood ratio under the null hypothesis that their manager exerts no effort  $H_0 : \{\mathbf{e}_t(i)\}_{s=1}^T = \{\mathbf{0}_{N \times 1}\}_{s=1}^T$ , which corresponds to no exhibition of ability. We show that the weighted historical variance of this return gap  $S_T$  is the log-likelihood ratio of the null hypothesis of no effort to the alternative hypothesis:

$$S_T = \frac{1}{T} \sum_{t=1}^T \left( \frac{(R_t^i)^2}{\text{Var}_U [R_t^i | \mathcal{F}_t^c]} - \frac{(R_t^i - E [R_t^i | \mathcal{F}_t^I(i)])^2}{\text{Var} [R_t^i | \mathcal{F}_t^I(i)]} \right),$$

which is a consistent estimator for the average level of effort of the skilled manager. Admati and Pfleiderer (1997) demonstrate that the first moment of benchmark-adjusted returns is not sufficient to identify skill. Our analysis suggests that, instead, investors should focus on second moments when evaluating skill. In the above statistic,  $\text{Var}_U [R_t^i | \mathcal{F}_t^c]$  is the conditional variance of the return gap if the manager exerts no effort to learn, or if the manager has no skill,  $E [R_t^i | \mathcal{F}_{t-1}^I(i)]$  is the best predictor of the return gap given all public information, including current realized asset returns, and past fund returns, and  $\text{Var} [R_t^i | \mathcal{F}_t^I(i)]$  is the conditional variance. This test statistic, furthermore, is asymptotically distributed

$\mathcal{N}(s_0, \frac{2}{T}\sigma_0^2)$  under the null hypothesis that the manager is unskilled, where:

$$s_0 = \frac{1}{T} \sum_{t=1}^T \left( 1 - \frac{Var_U [R_t^i | \mathcal{F}_t^c]}{Var [R_t^i | \mathcal{F}_t^I(i)]} \right)^2,$$

$$\sigma_0^2 = \frac{1}{T} \sum_{t=1}^T \left( 1 - \frac{Var_U [R_t^i | \mathcal{F}_t^c]}{Var [R_t^i | \mathcal{F}_t^I(i)]} \right)^4,$$

which relates the noise-to-signal ratio of variances  $\frac{Var_U [R_t^i | \mathcal{F}_t^c]}{Var [R_t^i | \mathcal{F}_t^I(i)]}$  to the confidence interval of the hypothesis test. If, furthermore, the noise in  $R_t^i$  is homoskedastic across assets, then  $Var_U [R_t^i | \mathcal{F}_t^c] \propto \mathbf{R}'_t \mathbf{R}_t$ .

If one believes that the value that skilled managers add to asset management is their acquisition of superior private information, then one should examine the variance of their return gap over time, especially since first moments are highly path-dependent. Intuitively, in any given trading period, skilled managers may appear more or less active because of noise in their information or time variation in expected returns: however, systematically their portfolios should deviate from their more passive counterparts. This motivates examining dynamic measures of skill, such as the empirical analogue of our  $\hat{S}_T^2$  statistic, to measure the fund manager skill in an active management.

Empirically, the greatest challenge is to link conditional estimates of the return gap,  $E [R_t^i | \mathcal{F}_{t-1}^I(i)]$ , to the noise-to-signal ratio  $\frac{Var_U [R_t^i | \mathcal{F}_t^c]}{Var [R_t^i | \mathcal{F}_t^I(i)]}$ . Since the return gap must not be predictable by any public information by construction, it follows that  $E [R_t^i | \mathcal{F}_t^c]$ , which motivates the conditional moment restrictions,  $E \left[ R_t^i \begin{bmatrix} 1 \\ \mathbf{R}_t \end{bmatrix} \middle| \mathcal{F}_t^c \right] = \mathbf{0}_{N+1 \times 1}$ , when specifying a functional form for  $R_t^i$ .

While the  $S_T$  statistic would allow for better monitoring of fund managers, since it could relax their IC constraints, such asymmetric payoff provisions are prohibited by the SEC. Though backward-looking as a measure, if investors allocate capital to funds based on the historical  $S_T$  statistic, then managers would face forward-looking incentives for effort to achieve a higher  $S_T$  statistic, and through this channel it would dynamically enter their

compensation. To see this, suppose that a new generation of investors at date  $T$  cannot distinguish between skilled and unskilled managers, and allocates their capital to funds in proportion to the signal that the fund manager has skill based on the fund's returns, according to the rule  $w(S_T)$ :

$$w(S_T) = (S_T - S_{crit,\alpha}) W_0,$$

where  $\frac{1}{\sqrt{2T}} \left( \frac{S_{crit,\alpha}}{\sigma_0^2} - 1 \right) = \Phi^{-1}(1 - \alpha)$  is the  $\alpha$ -level of confidence under the null hypothesis, and  $\Phi(\cdot)$  is the CDF of the normal distribution. Importantly,  $w(S_T)$  is increasing and convex in the most recent fund return gap  $R_T^i$  through  $S_T$ , leading to a convex flow-performance relation. In addition, if the compensation of skilled managers is based on their final AUM at date  $T$ , so that:

$$C_T^S = \rho_0 + \rho_S \left( W_T^S - (R^f)^T W_0 + w(S_T) \right) + \rho'_R \sum_{t=1}^T \mathbf{R}_t,$$

which is still a symmetric performance contract, then the future flow-performance sensitivity can incentivize forward-looking skilled managers to exert effort before date  $t$  to raise their fund flows at date  $T$ . Such a mechanism suggests that convex flow-performance sensitivity is a reaction to information about manager skill through this  $S_T$  statistic, and nonlinear flow-performance sensitivity could be a tool for completing the contracting space that is restricted in direct compensation by the SEC to linear contracts.

## 5.2 Background Risk

In this section, we detail an extension of the model in which investors are endowed with some exposure  $\psi$  to wealth correlated with the excess return of the asset fundamentals that they cannot trade. This specification is meant to capture the idea that investors face background

risk that is correlated with asset market fluctuations.<sup>20</sup> Specifically, we assume that each investor's final wealth is now given by:

$$w_2^I = W_2^i - C_0^i + \psi (\Theta - R^f F^{-1} \mathbf{P} + \varepsilon_I),$$

where  $\varepsilon_I \sim \mathcal{N}(\mathbf{0}, \tau_I^{-1} Id_N)$  is independent of  $\Theta$  and  $\mathbf{x}$ . and across investors. One can interpret  $\varepsilon_I$  as income risk that is specific to the investor or as idiosyncratic asset payoffs from market incompleteness. Similar arguments to those in deriving the equilibrium in the main setting give rise to the following proposition. For brevity, we only highlight the salient features that distinguish this equilibrium with background risk.

**Proposition 5** *The optimal contract for a skilled manager is a  $N + 2 \times 1$  vector  $(\rho_0, \rho_S, \rho_R)$  that features:*

$$\rho_R = - \left( \rho_S + \frac{\gamma}{\gamma_M} (1 - \rho_S) \right) \omega^\psi,$$

where  $\omega^\psi = \omega^0 - \frac{\rho_S}{\rho_S + \frac{\gamma}{\gamma_M}(1 - \rho_S)} F'^{-1} \psi'$ . The portfolio choice of an unskilled manager is given by:

$$\omega_1^U = \frac{1}{\gamma} (F \Omega F')^{-1} (F \hat{\Theta} - R^f \mathbf{P}) - F'^{-1} \psi,$$

and the constant in the price functional is shifted by the investor loading on background risk:

$$\Pi_0 = \frac{1}{R^f} F \left( \frac{\chi}{\gamma_M \rho_S} \Omega(j)^{-1} + \frac{1 - \chi}{\gamma} \Omega^{-1} \right)^{-1} \left( \begin{array}{c} \left( \frac{1 - \chi}{\gamma} + \frac{\chi}{\gamma_M \rho_S} \right) (\tau_\theta \bar{\Theta} - \tau_x \Pi'_\theta \Pi'_x^{-1} \bar{\mathbf{x}}) \\ + \chi \left( 1 + \frac{\gamma}{\gamma_M} \left( \frac{1}{\rho_S} - 1 \right) \right) F' \omega^0 - \psi \end{array} \right).$$

From Proposition 5, the presence of aggregate investor background risk shifts equilibrium asset prices by a factor  $-\frac{1}{R^f} F \left( \frac{\chi}{\gamma_M \rho_S} \Omega(j)^{-1} + \frac{1 - \chi}{\gamma} \Omega^{-1} \right)^{-1} \psi$ , which reflects the aggregate hedging demand of investors, and changes the benchmark for skilled managers from  $\omega^0$  to  $\omega^\psi$ . Unskilled managers will incorporate aggregate investor background risk into their portfolio choice because they essentially provide a vehicle for investors to invest directly for

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<sup>20</sup>The assumption that investors are endowed with risk correlated with the excess return to the fundamentals  $\Theta - R^f F^{-1} \mathbf{P}$  rather than the fundamentals directly is only for expositional convenience.

a fee, while incentives must be offered to skilled managers to internalize this risk by choosing the appropriate benchmark. This adjusted benchmark also impacts the optimal sensitivity of the skilled managers' performance-based compensation,  $\rho_S$ , and, through this delegated learning channel, their incentives to acquire private information.<sup>21</sup>

Savov (2014) argues that active management can provide superior insurance to investors against their aggregate income risk. Our analysis suggests that, in the presence of moral hazards in delegation, skilled managers internalize such risks through the benchmark against which they are evaluated. Investors in our setting choose funds with benchmarks that reflect their hedging needs. Our framework, consequently, offers insight into differences in benchmarks across funds that invest in similar asset classes, as well as an additional dimension by which the choice of benchmark adds value to investors. Investors in our setting choose funds with benchmarks that reflect their hedging needs.

### 5.3 Learning about Assets vs. Learning about Fund Managers

So far, our framework explores how manager skill can be endogenous to both the economic environment and their compensation contracts. In contrast to this mechanism, a literature, which includes Berk and Green (2004), Pástor and Stambaugh (2012), Berk and van Binsbergen (2015), Barber et al. (2016), and Starks and Sun (2016), investigates how investors try to infer the persistent skill of active managers from past performance, given that managers can influence this perception. Managers in these settings have incentives to signal their skill because fund flows are sensitive to past performance, and their compensation is tied to their fund's AUM. As such, we view both channels as complementary to understanding delegated asset management.

It may be difficult to disentangle measures of manager skill from the assets in which the manager invests and the incentives that the manager faces. A manager whose compensation is not performance-based, for instance, has no incentive to exert costly effort to provide

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<sup>21</sup>This can be seen from the dependence of the FONC (A5) on  $\rho_R$  and  $\bar{\mathbf{Z}}$ .

investors with superior performance. Similarly, a manager who invests in assets with more volatile payoffs has more incentive to exert effort to gain an informational advantage over other managers. In both situations, manager skill, at least to some extent, is an endogenous decision. While studies, such as Starks and Sun (2016), do allow the ability of fund managers to vary exogenously with the investment environment, as differences in their stock-picking and market-timing abilities may be an optimal response to this variation. Since private information is imperfect, measures derived from the level of fund returns may imply that manager ability is transient, and are likely to be noisy predictors of future performance. Our dynamic framework suggests that investors can learn about the characteristics of a fund's manager through observation of the variability of the fund's return gap, and that such a channel can help explain the nonlinear flow performance sensitivity found empirically. Since the SEC only allows fund managers to have symmetric performance-based incentives, the nonlinear flow performance sensitivity may also reflect an attempt by investors to enlarge the space of contractible payoffs.

While asymmetric information about assets and about fund manager ability provides incentives for managers and investors to learn, respectively, their incentives to learn are likely very different. Active managers can often fully exploit their private information in their investment decisions, while investors can only choose whether to invest in a fund. This limited ability for investors to act on negative news about managers suggests that the expected return to managers from exerting costly resources to learn is likely to be higher than that for investors. As such, the learning process is likely to be slower for investors and more dependent on publicly available information, such as realized past performance.

## 6 Conclusion

In this paper, we study an economy in which investors delegate their investment in financial markets to asset managers, and must incentive asset managers with skill to exert costly ef-

fort to acquire private information about asset payoffs. Our framework features an optimal (affine) contract that has both performance-based and benchmarking incentives, and allows us to study the interaction between manager incentives and their learning and trading decisions. Since we solve for the optimal contract and the learning decision of skilled managers as part of the equilibrium, we are able to offer predictions about intermediary asset holdings and asset prices that condition on the economic environment rather than the contract or managers' beliefs, which are easier to test empirically. Our framework cautions the use of existing empirical measures of skill employed in the literature, such as active share and return gap, and offers a new measure theoretically motivated by a dynamic extension of our framework.

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# A Appendix

## A.1 Equilibrium Asset Prices

Given the asset demand of unskilled and skilled managers from equation (9) and Proposition 1, respectively, we are now in a position to derive equilibrium prices. Aggregating the demand of skilled and unskilled managers,  $\omega_1^S(j)$  and  $\omega_1^U = \omega^c$ , respectively, the market-clearing condition reveals that:

$$\chi \frac{1}{\gamma_M \rho_S} (F \Omega(j) F')^{-1} \left( F \int_0^1 \hat{\Theta}(j) di - R^f \mathbf{P} \right) - \chi \frac{1}{\rho_S} \rho_R + (1 - \chi) \frac{1}{\gamma} (F \Omega F')^{-1} (F \hat{\Theta} - R^f \mathbf{P}) = \mathbf{x}.$$

Substituting for  $\hat{\Theta}$  and  $\hat{\Theta}(j)$  with equations (5) and (7), respectively, and imposing the Strong LLN, we find that:

$$\begin{aligned} \mathbf{P} = & \left( \left( \frac{\chi}{\gamma_M \rho_S} \Omega(j)^{-1} + \frac{1 - \chi}{\gamma} \Omega^{-1} \right) R^f F^{-1} - \left( \frac{1 - \chi}{\gamma} + \frac{\chi}{\gamma_M \rho_S} \right) \tau_x \Pi'_\theta (\Pi_x \Pi'_x)^{-1} \right)^{-1} \times \\ & \left( \left( \frac{1 - \chi}{\gamma} + \frac{\chi}{\gamma_M \rho_S} \right) (\tau_\theta \bar{\Theta} - \tau_x \Pi'_\theta (\Pi_x \Pi'_x)^{-1} (\Pi_0 + \Pi_x \bar{\mathbf{x}})) + \frac{\chi}{\gamma_M \rho_S} \Sigma_j (\mathbf{e}_j)^{-1} \Theta - F' \mathbf{x} - \frac{\chi}{\rho_S} F' \rho_R \right). \end{aligned}$$

Matching coefficients with the conjectured form of prices (4), and the imposing equation (6), we find that:

$$\Omega^{-1} = \tau_\theta Id_N + \tau_x \left( \frac{\chi}{\gamma_M \rho_S} \right)^2 \Sigma_j (\mathbf{e}_j)^{-1} (F' F)^{-1} \Sigma_j (\mathbf{e}_j)^{-1}, \quad (\text{A1})$$

and that  $\Pi_\theta$ ,  $\Pi_x$ , and  $\Pi_0$  are given by:

$$\Pi_\theta = \frac{1}{R^f} F \left( \tau_\theta \left( \tau_x \left( \frac{\chi}{\gamma_M \rho_S} \right)^2 \Sigma_j (\mathbf{e}_j)^{-1} (F' F)^{-1} \Sigma_j (\mathbf{e}_j)^{-1} + \left( 1 + \frac{1 - \chi}{\chi} \frac{\gamma_M}{\gamma} \rho_S \right)^{-1} \Sigma_j (\mathbf{e}_j)^{-1} \right)^{-1} + Id_N \right)^{-1} \quad (\text{A2})$$

$$\Pi_x = -\frac{\gamma_M \rho_S}{\chi} \Pi_\theta \Sigma_j (\mathbf{e}_j) F' \quad (\text{A3})$$

$$\Pi_0 = \frac{1}{R^f} F \left( \frac{\chi}{\gamma_M \rho_S} \Omega(j)^{-1} + \frac{1 - \chi}{\gamma} \Omega^{-1} \right)^{-1} \left( \begin{array}{c} \left( \frac{1 - \chi}{\gamma} + \frac{\chi}{\gamma_M \rho_S} \right) (\tau_\theta \bar{\Theta} - \tau_x \Pi'_\theta \Pi_x^{-1} \bar{\mathbf{x}}) \\ -\frac{\chi}{\rho_S} F' \rho_R \end{array} \right), \quad (\text{A4})$$

which confirms the conjectured linear equilibrium.

Several features of the equilibrium are immediately apparent from the price coefficients.

We see, for instance, that if  $\Sigma_j(\mathbf{e}_j)^{-1}$  is zero, so that skilled managers have no private information, then  $\Pi_\theta, \Pi_x \rightarrow 0_{N \times N}$ , and prices reflect only prior information about the risky asset payoffs. In addition, the signal-to-noise ratio of prices as signals about the risky asset payoffs,  $\Pi_x^{-1}\Pi_\theta = -\frac{\chi}{\gamma_M \rho_S} F'^{-1} \Sigma_j(\mathbf{e}_j)^{-1}$ , depends not only on the correlation structure of asset payoffs and the effort exerted by skilled managers, but also negatively on their risk aversion  $\gamma_M$  and the sensitivity of their compensation to the realized return of their fund,  $\rho_S$ . That these latter two features enter as  $\gamma_M \rho_S$  highlights that  $\rho_S$  makes the skilled manager effectively more risk-averse over his fund's performance, and, as a result, more conservative in his investment policies.

## A.2 Expected Utility of Investors with Unskilled Managers

We calculate the expected utility of investors in funds with unskilled managers  $V^U$ . By the law of iterated expectations, first conditioning on  $\mathcal{F}^c$ , the expected utility to investors who invest with unskilled managers  $V^U$  is then:

$$V^U = -E \left[ \exp(-\gamma(W_1 - C_0^U)) \right] = -(-u_0)^{-\frac{\gamma}{\gamma_M}} E \left[ \exp \left( -\gamma R^f W_0 - \frac{1}{2} \mathbf{Z}' \Omega^{-1} \mathbf{Z} \right) \right],$$

where  $\mathbf{Z} = \hat{\Theta} - R^f F^{-1} \mathbf{P}$ . From an ex-ante perspective,  $\mathbf{Z} \sim \mathcal{N}(\mu, \Omega_Z)$ . With some manipulation,

$$\begin{aligned} \mu &= \bar{\Theta} - R^f F^{-1} \bar{\mathbf{P}}, \\ \Omega_Z &= \left( \Omega^{-1} + \left( 1 + \frac{1-\chi}{\chi} \frac{\gamma_M}{\gamma} \rho_S \right) \Omega^{-1} \Sigma_j(\mathbf{e}_j) \Omega^{-1} \right)^{-1} \times \\ &\quad \left( \tau_{\bar{\Theta}}^{-1} Id_N - \Omega \right)^{-1} \left( \Omega^{-1} + \left( 1 + \frac{1-\chi}{\chi} \frac{\gamma_M}{\gamma} \rho_S \right) \Omega^{-1} \Sigma_j(\mathbf{e}_j) \Omega^{-1} \right)^{-1}, \end{aligned}$$

and  $\bar{\mathbf{P}} = \Pi_0 + \Pi_\theta \bar{\Theta} + \Pi_x \bar{\mathbf{x}}$ . Then, by completing the square,

$$V^U = -\frac{\exp \left( -\gamma R^f W_0 - \frac{1}{2} \mu' \left( \Omega_Z^{-1} - \Omega_Z^{-1} (\Omega^{-1} + \Omega_Z^{-1})^{-1} \Omega_Z^{-1} \right) \mu \right)}{(-u_0)^{\frac{\gamma}{\gamma_M}} |Id_N + \Omega_Z \Omega^{-1}|}.$$

### A.3 Proof of Proposition 1

Assuming the linear contract, the IC constraint of the skilled manager, conditional on an effort choice  $\mathbf{e}$ , reduces to the mean-variance optimization problem:

$$\sup_{\omega_1^S(j)} \left\{ \begin{array}{l} \rho_0 + \rho_S \omega_1^S(j)' (F \hat{\Theta}(j) - R^f \mathbf{P}) + \rho'_R (F \hat{\Theta}(j) - R^f \mathbf{P}) \\ - \frac{\gamma_M}{2} (\rho_S \omega_1^S(j) + \rho_R)' F \Omega(j) F' (\rho_S \omega_1^S(j) + \rho_R) \end{array} \right\},$$

given its CARA-normal structure. It then follows from the FOC for  $\omega_1^S$  at interior solution that:

$$\omega_1^S(j) = \frac{1}{\gamma_M \rho_S} (F \Omega(j) F')^{-1} (F \hat{\Theta}(j) - R^f \mathbf{P}) - \frac{1}{\rho_S} \rho_R.$$

Substituting this optimal portfolio choice into the manager's utility, the IC constraint when choosing effort level  $\mathbf{e}$  becomes:

$$\mathbf{e} \in \operatorname{argsup}_{\mathbf{e}_j \in \mathbb{R}_+^N} \left\{ E \left[ - \exp \left( \begin{array}{l} H(\mathbf{e}_j) - \gamma_M \rho_0 \\ - \frac{1}{2} (\hat{\Theta}(j) - R^f F^{-1} \mathbf{P})' \Omega(j)^{-1} (\hat{\Theta}(j) - R^f F^{-1} \mathbf{P}) \end{array} \right) \right] \right\}.$$

To solve for the optimal level of effort for skilled managers, we invoke the law of iterated expectations and first find the expected utility of a skilled manager conditional on having observed market prices. The optimal choice of effort conditional on having observed market prices is independent of the specific realization of prices. As a result, the optimal effort of skilled managers conditional on observing prices is also a measurable strategy for skilled managers before observing prices. Since unconditional strategies cannot improve on strategies that condition on more information, this optimal effort ex-post must also be optimal ex-ante.

Recognizing that  $\mathbf{s}(j) \mid \mathcal{F}_0^c \sim \mathcal{N}(\hat{\Theta}, \Omega + \Sigma_j(\mathbf{e}_j))$ , and that

$$\hat{\Theta}(j) - R^f F^{-1} \mathbf{P} = \hat{\Theta} - R^f F^{-1} \mathbf{P} + \Omega(j) \Sigma_j(\mathbf{e}_j)^{-1} (\mathbf{s}(j) - \hat{\Theta}),$$

where

$$\Omega(j)^{-1} = \Omega^{-1} + \Sigma_j(\mathbf{e}_j)^{-1},$$

by completing the square for normal random variables, the expected utility of skilled manager  $i$  given only the market beliefs and effort  $\mathbf{e}' E [\sup_{\omega \in \mathbb{R}^N} E [u(C_0^S; \omega', \mathbf{e}') \mid \mathcal{F}_j] \mid \mathcal{F}^c]$  is

$$\begin{aligned}
& E \left[ \sup_{\omega \in \mathbb{R}^N} E [u(C_0^S; \omega', \mathbf{e}') \mid \mathcal{F}_j] \mid \mathcal{F}_0^c \right] \\
&= -E \left[ \exp \left( \begin{array}{c} \frac{1}{2} h((\mathbf{e}')' \mathbf{1}_{N \times 1}) - \gamma_M \rho_0 \\ -\frac{1}{2} (\hat{\Theta}(j) - R^f F^{-1} \mathbf{P})' \Omega(j)^{-1} (\hat{\Theta}(j) - R^f F^{-1} \mathbf{P}) \end{array} \right) \mid \mathcal{F}_0^c \right] \\
&= -\frac{(2\pi)^{-\frac{N}{2}}}{|\Omega + \Sigma_j(\mathbf{e}_j)|^{1/2}} \int_{-\infty}^{\infty} \exp \left( \begin{array}{c} \frac{1}{2} h((\mathbf{e}')' \mathbf{1}_{N \times 1}) - \gamma_M \rho_0 \\ -\frac{1}{2} (\hat{\Theta}(j) - R^f F^{-1} \mathbf{P})' \Omega(j)^{-1} (\hat{\Theta}(j) - R^f F^{-1} \mathbf{P}) \\ -\frac{1}{2} (\mathbf{s}(j) - \hat{\Theta})' (\Omega + \Sigma_j(\mathbf{e}_j))^{-1} (\mathbf{s}(j) - \hat{\Theta}) \end{array} \right) d\mathbf{s}(j) \\
&= -\frac{(2\pi)^{-\frac{N}{2}}}{|\Omega + \Sigma_j(\mathbf{e}_j)|^{1/2}} \int_{-\infty}^{\infty} \exp \left( \begin{array}{c} \frac{1}{2} h((\mathbf{e}')' \mathbf{1}_{N \times 1}) - \gamma_M \rho_0 \\ -\frac{1}{2} (\hat{\Theta} - R^f F^{-1} \mathbf{P})' \Omega^{-1} (\hat{\Theta} - R^f F^{-1} \mathbf{P}) \\ -\frac{1}{2} (\hat{\Theta} - R^f F^{-1} \mathbf{P} + \mathbf{s}(j) - \hat{\Theta})' \Sigma_j(\mathbf{e}_j)^{-1} (\hat{\Theta} - R^f F^{-1} \mathbf{P} + \mathbf{s}(j) - \hat{\Theta}) \end{array} \right) d\mathbf{s}(j) \\
&= -\exp \left( \begin{array}{c} \frac{1}{2} h((\mathbf{e}')' \mathbf{1}_{N \times 1}) - \frac{1}{2} \log |Id_N + \Sigma_j(\mathbf{e}')^{-1} \Omega| - \gamma_M \rho_0 \\ -\frac{1}{2} (\hat{\Theta} - R^f F^{-1} \mathbf{P})' \Omega^{-1} (\hat{\Theta} - R^f F^{-1} \mathbf{P}) \end{array} \right).
\end{aligned}$$

A similar result can be found by applying results for the moment-generating function of the non-central chi-square random variables. As one can see, the optimal choice of effort enters the conditional expected utility only through the  $-\frac{1}{2} \log |Id_N + \Sigma_j(\mathbf{e}_j)^{-1} \Omega|$  term. Since skilled managers are price-takers and the conditional variance of market beliefs  $\Omega$  is known ex-ante, we find that:

$$\begin{aligned}
E \left[ \sup_{\omega \in \mathbb{R}^N} E [u(C_0^S; \omega', \mathbf{e}')] \right] &= -\exp \left( \frac{1}{2} h((\mathbf{e}')' \mathbf{1}_{N \times 1}) - \frac{1}{2} \log |Id_N + \Sigma_j(\mathbf{e}')^{-1} \Omega| - \gamma_M \rho_0 \right) \\
&\quad \times E \left[ \exp \left( -\frac{1}{2} (\hat{\Theta} - R^f F^{-1} \mathbf{P})' \Omega^{-1} (\hat{\Theta} - R^f F^{-1} \mathbf{P}) \right) \right].
\end{aligned}$$

The optimization program for the effort of skilled manager is then equivalent to:

$$\mathbf{e} \in \operatorname{argsup}_{\mathbf{e}' \in \mathbb{R}_+^N} \left\{ \log \left| \Omega^{-1} + \Sigma_j(\mathbf{e}')^{-1} \right| - h((\mathbf{e}')' \mathbf{1}_{N \times 1}) \right\}.$$

Recognizing that  $\Sigma_j(\mathbf{e}_j)^{-1} = M \cdot Id_N + \operatorname{diag}(\mathbf{e})$ , and invoking results of the matrix calculus, the FOC for the optimal level of effort  $\mathbf{e}_i$  is:

$$\operatorname{Tr} \left[ (\Omega^{-1} + M \cdot Id_N + \operatorname{diag}(\mathbf{e}))^{-1} J_i \right] - h'(\mathbf{e}' \mathbf{1}_{N \times 1}) \leq 0 \quad (= \text{if } e_i > 0).$$

where  $J_i$  is the  $N \times N$  matrix with entry  $J_i = 1$  and zero otherwise. Since  $\operatorname{Tr}$  is a linear

operator, we can stack all the FOCs to arrive at:

$$Diag \left[ (\Omega^{-1} + M \cdot Id_N + diag(\mathbf{e}))^{-1} \right] - h'(\mathbf{e}'\mathbf{1}_{N \times 1}) \mathbf{1}_{N \times 1} \leq \mathbf{0}_{N \times 1},$$

where  $Diag$  is the operator that stacks the diagonal of a matrix into a vector. Furthermore, the second-order derivative of  $\log |\Omega^{-1} + \Sigma_j(\mathbf{e}')^{-1}|$  is:

$$\partial_{\mathbf{e}_i, \mathbf{e}_i}^2 \log |\Omega^{-1} + \Sigma_j(\mathbf{e}')^{-1}| = -(\Omega^{-1} + M \cdot Id_N + diag(\mathbf{e}))^{-1} J_i (\Omega^{-1} + M \cdot Id_N + diag(\mathbf{e}))^{-1}.$$

Since  $h'(\cdot)$  is a (weakly) convex function, the optimization program is concave in  $\mathbf{e}$ , and therefore the FOC is both necessary and sufficient for the optimal  $\mathbf{e}$ .

If  $F$  is diagonal, so that asset payoffs are independent, then  $\Omega^{-1}$  is also diagonal, and the above condition reduces to:

$$\frac{1}{\Omega_{ii}^{-1} + M + \mathbf{e}_i} \leq h'(\mathbf{e}'\mathbf{1}_{N \times 1}) \quad \forall i \in \{1, \dots, N\}.$$

## A.4 Proof of Proposition 2

Define

$$G = Tr [X^{-1} J_i] - h'(\mathbf{e}'\mathbf{1}_{N \times 1}) = 0,$$

where  $X = ((\tau_\theta + M) \cdot Id_N + k(M \cdot Id_N + diag(\mathbf{e})) (F'F)^{-1} (M \cdot Id_N + diag(\mathbf{e})) + diag(\mathbf{e})) J_{ii}$ , and  $k = \tau_x \left( \frac{\chi}{\gamma_{MPS}} \right)^2$ . By the implicit function theorem,

$$\partial_z \mathbf{e}_i = -\frac{\partial_z G}{\partial_{\mathbf{e}_i} G},$$

for parameter  $z$ . Recognizing that  $\partial(X^{-1}) = X^{-1}(\partial X)X^{-1}$ , taking the derivative under the  $Tr$  operator since the  $Tr$  operator is linear and the trace is bounded, it follows that:

$$\partial_{\mathbf{e}_i} G = Tr [A J_i] = \mathbf{v}'_i A \mathbf{v}_i - h''(\mathbf{e}'\mathbf{1}_{N \times 1}),$$

where  $J_i$  is the  $N \times N$  matrix with entry  $J_{ii} = 1$  and zero otherwise,  $\mathbf{v}_i$  is the Euclidian  $N \times N$  basis vector in the  $i^{th}$  direction, and

$$A = -X^{-1} \left( k(M \cdot Id_N + diag(\mathbf{e})) F^{-1} F'^{-1} + k(F'F)^{-1} (M \cdot Id_N + diag(\mathbf{e})) + Id_N \right) X^{-1} - h''(\mathbf{e}'\mathbf{1}_{N \times 1}) Id_N.$$

Given that  $F$  is a lower triangular matrix with entries of 1 on the diagonal,  $F'F$  is a positive definite (PD) matrix since  $\det(AB) = \det(A)\det(B)$ . Since  $F'F$  is a positive definite (PD) matrix, it follows that  $(F'F)^{-1}$  is a PD matrix, since the eigenvalues of  $(F'F)^{-1}$  are the inverse of the eigenvalues of  $F'F$ . Since  $(F'F)^{-1}$  is a PD matrix,  $X$  is also a PD matrix, and it follows that  $A$  is a negative definite (ND) matrix. Therefore, it follows since  $\mathbf{v}_i$  has non-negative entries and  $h(\mathbf{e}'\mathbf{1}_{N \times 1})$  is convex that:

$$\partial_{\mathbf{e}_i} G = \mathbf{v}'_i A \mathbf{v}_i - h''(\mathbf{e}'\mathbf{1}_{N \times 1}) < 0.$$

Consequently,

$$\partial_z \mathbf{e}_i = \frac{\partial_z G}{|\partial_{\mathbf{e}_i} G|} = \frac{\partial_z \text{Tr}[X^{-1}]}{|\partial_{\mathbf{e}_i} G|},$$

and the sign of  $\frac{\partial \mathbf{e}_i}{\partial z}$  is the same as the sign of  $\partial_z \text{Tr}[X^{-1}]$ . Differentiating under the  $\text{Tr}$  operator again, it follows that:

$$\partial_z G = -\text{Tr} \left[ X^{-1} \partial_z \left( (\tau_\theta + M) \cdot \text{Id}_N + k(M \cdot \text{Id}_N + \text{diag}(\mathbf{e})) (F'F)^{-1} (M \cdot \text{Id}_N + \text{diag}(\mathbf{e})) \right) X^{-1} J_i \right].$$

For  $z = \tau_\theta$ , it is straightforward to verify that  $\partial_{\tau_\theta} G > 0$ , since  $\text{Tr}[X^{-1} \tau_\theta X^{-1} J_i] = \tau_\theta \text{Tr}[X^{-1} X^{-1} J_i]$  and  $X$  is PD, and therefore:

$$\partial_{\tau_\theta} \mathbf{e}_i < 0.$$

Similarly, it follows that:

$$\partial_k \mathbf{e}_i < 0.$$

The results for elements of  $k$  then follow by the chain rule.

## A.5 Proof of Proposition 3

Substituting for  $W_2^S$  and  $C_0^S$ , the utility of investors that invest with skilled managers is

$$V(W_2^S, C_0^S) = -\exp \left( -\gamma \left( \begin{aligned} & R^f W_0 - \rho_0 + \frac{1-\rho_S}{\gamma_M \rho_S} (\mathbf{s}_j - \hat{\Theta})' \Sigma_j (\mathbf{e}_j)^{-1} (\Theta - R^f F^{-1} \mathbf{P}) \\ & + \frac{1}{\rho_S} \left( \frac{1-\rho_S}{\gamma_M} (\hat{\Theta} - R^f F^{-1} \mathbf{P})' (\Omega^{-1} + \Sigma_j (\mathbf{e}_j)^{-1}) - \rho'_R F \right) (\Theta - R^f F^{-1} \mathbf{P}) \end{aligned} \right) \right).$$

Importantly,  $\mathbf{e}_j$  is independent of the realization of  $\Theta$ . To find expected investor utility when investing with skilled managers, we recognize by the law of iterated expectations that  $E[V(W_2^S, C_0^S)] = E[E[V(W_2^S, C_0^S) | \mathcal{F}^c]]$ , and that  $E[V(W_2^S, C_0^S) | \mathcal{F}^c] = E[E[V(W_2^S, C_0^S) | \Theta, x] | \mathcal{F}^c]$ . Taking conditional expectations with respect to the realized shocks, and integrating over the idiosyncratic signal noise of skilled managers, we



find:

$$\begin{aligned}
& E [V (W_2^S, C_0^S) \mid \Theta, x] \\
&= -\exp \left( -\gamma \left( \begin{aligned} & R^f W_0 - \rho_0 + \left( \frac{1}{2} \left( \frac{1-\rho_S}{\gamma_M \rho_S} \right)^2 + \frac{1-\rho_S}{\gamma_M \rho_S} \right) (\Theta - \hat{\Theta})' \Sigma_j (\mathbf{e}_j)^{-1} (\Theta - R^f F^{-1} \mathbf{P}) \\ & + \frac{1}{\rho_S} \left( \frac{1-\rho_S}{\gamma_M} (\hat{\Theta} - R^f F^{-1} \mathbf{P})' (\Omega^{-1} + \left( 1 + \frac{1}{2} \frac{1-\rho_S}{\gamma_M \rho_S} \right) \Sigma_j (\mathbf{e}_j)^{-1}) - \rho'_R F \right) (\Theta - R^f F^{-1} \mathbf{P}) \end{aligned} \right) \right)
\end{aligned}$$

Taking conditional expectations with respect to the market beliefs, we then arrive at:

$$\begin{aligned}
& E [V (W_2^S, C_0^S) \mid \mathcal{F}^c] \\
&= \frac{\exp \left( \begin{aligned} & \gamma \rho_0 - \gamma R^f W_0 - \frac{1}{2} \mathbf{Z}' \Omega^{-1} \mathbf{Z} \\ & + \frac{1}{2} \left( \left( 1 + \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right) \Omega^{-1} \mathbf{Z} - \frac{\gamma}{\rho_S} F' \rho_R \right)' \left( \Omega^{-1} + \left( 1 - \left( 1 - \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right)^2 \right) \Sigma_j (\mathbf{e}_j)^{-1} \right)^{-1} \times \\ & \left( \left( 1 + \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right) \Omega^{-1} \mathbf{Z} - \frac{\gamma}{\rho_S} F' \rho_R \right) \end{aligned} \right)}{\left| Id_N + \left( 1 - \left( 1 - \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right)^2 \right) \Omega \Sigma_j (\mathbf{e}_j)^{-1} \right|^{1/2}},
\end{aligned}$$

where  $\mathbf{Z} = \hat{\Theta} - R^f F^{-1} \mathbf{P}$ . From an ex ante perspective,  $\mathbf{Z} \sim \mathcal{N}(\mu, \Omega_Z)$ . Taking unconditional expectations, we arrive at:

$$\begin{aligned}
& E [V (W_2^S, C_0^S)] \\
&= \frac{\exp \left( \begin{aligned} & -\gamma R^f W_0 + \frac{1}{2} \left( \frac{\gamma}{\rho_S} \right)^2 \rho'_R F \left( \Omega^{-1} + \left( 1 - \left( 1 - \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right)^2 \right) \Sigma_j (\mathbf{e}_j)^{-1} \right)^{-1} F' \rho_R \\ & + \frac{1}{2} G' H^{-1} G - \frac{1}{2} \mu' \Omega_Z^{-1} \mu + \gamma \rho_0 \end{aligned} \right)}{\left| Id_N + \left( 1 - \left( 1 - \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right)^2 \right) \Omega \Sigma_j (\mathbf{e}_j)^{-1} \right|^{1/2} |\Omega_Z H|^{1/2}},
\end{aligned}$$

where

$$\begin{aligned}
G &= \frac{\gamma}{\rho_S} \left( 1 + \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right) \left( Id_N + \left( 1 - \left( 1 - \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right)^2 \right) \Sigma_j (\mathbf{e}_j)^{-1} \Omega \right)^{-1} F' \rho_R + \Omega_Z^{-1} \mu, \\
H &= \Omega^{-1} + \Omega_Z^{-1} - \left( 1 + \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right)^2 \Omega^{-1} \left( \Omega^{-1} + \left( 1 - \left( 1 - \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right)^2 \right) \Sigma_j (\mathbf{e}_j)^{-1} \right)^{-1} \Omega^{-1}.
\end{aligned}$$

Investors in skilled managers are used to solve the optimization problem:

$$\begin{aligned}
V^S &= \sup_{\{\rho_0, \rho_S, \rho_R\}} E [V (W_2^S, C_0^S)] \\
s.t. & : E [V (W_2^S, C_0^S)] = V^U \quad (\text{indifference}), \\
& : Tr \left[ (\Omega^{-1} + M \cdot Id_N + diag(\mathbf{e}_j))^{-1} J_i \right] - h'(\mathbf{e}'_j \mathbf{1}_{N \times 1}) \leq 0 \quad \forall i \in \{1, \dots, N\} \quad (\text{optimal } \mathbf{e}_j).
\end{aligned}$$

Importantly,  $V^U$  and the FOC for the optimal choice of skilled manager effort are independent of the contract from the perspective of investors.

The FOC for  $\rho_R$  can be solved explicitly for  $\rho_R$  such that:

$$\begin{aligned} \rho_R &= - \left( \frac{\rho_S}{\gamma} + \frac{1 - \rho_S}{\gamma_M} \right) F'^{-1} \\ &\times \left( \left( 1 + \frac{\gamma}{\gamma_M} \frac{1 - \rho_S}{\rho_S} \right)^2 \Omega^{-1} H^{-1} \Omega^{-1} \left( \Omega^{-1} + \left( 1 - \left( 1 - \frac{\gamma}{\gamma_M} \frac{1 - \rho_S}{\rho_S} \right)^2 \right) \Sigma_j (\mathbf{e}_j)^{-1} + Id_N \right)^{-1} \right. \\ &\times \left. \Omega^{-1} H^{-1} \Omega_Z^{-1} \mu, \right. \end{aligned}$$

which, with some manipulation, simplifies to:

$$\rho_R = - \left( \frac{\rho_S}{\gamma} + \frac{1 - \rho_S}{\gamma_M} \right) F'^{-1} (\Omega_Z + \Omega)^{-1} \mu.$$

Recognizing that we can rewrite  $F'^{-1} (\Omega_Z + \Omega)^{-1} \mu$  as  $(F' (\Omega_Z + \Omega) F)^{-1} F \mu$ , where  $F \mu$  is the unconditional expected excess return on the risky assets, it follows that  $F'^{-1} (\Omega_Z + \Omega)^{-1} \mu$  is a portfolio allocation chosen before prices are observed that accounts for both the overall uncertainty of excess returns and the uncertainty given common prices  $\Omega$  augmented by the uncertainty over the realization of prices  $\Omega_Z$ .

Defining  $\omega^0 = \frac{1}{\gamma} F'^{-1} (\Omega_Z + \Omega)^{-1} \mu$  to represent this “naive” portfolio, we can express  $\rho_R$  as:

$$\rho_R = - \left( \rho_S + \frac{\gamma}{\gamma_M} (1 - \rho_S) \right) \omega^0.$$

Furthermore, by the law of total variance:

$$\begin{aligned} Var (\Theta - R^f F^{-1} \mathbf{P}) &= E [Var (\Theta - R^f F^{-1} \mathbf{P} \mid \mathcal{F}^c)] + Var (E [\Theta - R^f F^{-1} \mathbf{P} \mid \mathcal{F}^c]) \\ &= E [\Omega] + Var (\hat{\Theta} - R^f F^{-1} \mathbf{P}) \\ &= \Omega + \Omega_Z. \end{aligned}$$

Therefore,  $\omega^0$  can be expressed as:

$$\omega^0 = \frac{1}{\gamma} F'^{-1} Var (\Theta - R^f F^{-1} \mathbf{P})^{-1} E [\Theta - R^f F^{-1} \mathbf{P}],$$

which is the ex-ante mean-variance efficient portfolio.

Since  $\rho_0$  impacts  $V^S$  only through the  $e^{\gamma\rho_0}$  term, we can define  $v^S$ , where:

$$V^S = e^{\gamma\rho_0}v^S,$$

and  $v^S$  is independent of  $\rho_0$  from the perspective of investors. Since the expected utility of investors with unskilled managers  $V^U$  is independent of  $\rho_0$  from the perspective of investors, from the indifference condition it follows that:

$$\rho_0 = \frac{1}{\gamma} \log \frac{V^U}{v^S}.$$

Assuming the program for the investor is concave in  $\rho_U$  and  $\rho_S$ , the optimal choices of  $\rho_S$  satisfies:

$$\rho_S = \arg \sup_{\rho_S} \left\{ \begin{array}{l} -\frac{1}{2} \left( \frac{\gamma}{\rho_S} \right)^2 \rho'_R F \left( \Omega^{-1} + \left( 1 - \left( 1 - \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right)^2 \right) \Sigma_j (\mathbf{e}_j)^{-1} \right)^{-1} F' \rho_R \\ + \frac{1}{2} \log \left| Id_N + \left( 1 - \left( 1 - \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right)^2 \right) \Omega \Sigma_j (\mathbf{e}_j)^{-1} \right| + \frac{1}{2} \log |H| - \frac{1}{2} G' H^{-1} G \end{array} \right\}.$$

Applying matrix calculus, we derive the FONC for the optimal choice of  $\rho_S$ :

$$0 = \left\{ \begin{array}{l} \frac{1}{\rho_S} \left( \frac{\gamma}{\rho_S} \right)^2 \rho'_R F A F' \rho_R \\ - \left( 1 - \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right) \frac{\gamma}{\gamma_M} \left( \frac{\gamma}{\rho_S^2} \right)^2 Tr \left[ A F' \rho_R \rho'_R F A \Sigma_j (\mathbf{e}_j)^{-1} \right] \\ + \frac{1 - \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S}}{1 - \left( 1 - \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right)^2} \frac{\gamma}{\gamma_M \rho_S^2} \left( N - \frac{Tr \left[ \left( \left( 1 - \left( 1 - \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right)^2 \right)^{-1} Id_N + \Omega \Sigma_j (\mathbf{e}_j)^{-1} \right)^{-1} Id_N \right]}{1 - \left( 1 - \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right)^2} \right)' \\ + Tr \left[ \frac{\gamma}{\gamma_M} \frac{1}{\rho_S^2} \left( 1 + \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right) H^{-1} \Omega^{-1} A \Omega^{-1} \left( Id_N - \left( 1 - \left( 1 - \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right)^2 \right) \Omega \Sigma_j (\mathbf{e}_j)^{-1} A \Omega^{-1} \right) \right] \\ + \frac{\gamma \left( 1 + \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right)}{\gamma_M \rho_S^2} Tr \left[ H^{-1} G G' H^{-1} \Omega^{-1} A \Omega^{-1} \left( Id_N - \left( 1 - \left( 1 - \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right)^2 \right) \Omega \Sigma_j (\mathbf{e}_j)^{-1} A \Omega^{-1} \right) \right] \\ + Tr \left[ F' \rho_R G' H^{-1} \left( \frac{\gamma}{\rho_S^2} \frac{1 + \frac{\gamma}{\gamma_M} \frac{2-\rho_S}{\rho_S} - 2 \frac{\gamma}{\gamma_M} \frac{1}{\rho_S} \frac{1 - \left( \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right)^2}{1 - \left( 1 - \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right)^2} B + 2 \frac{\gamma}{\rho_S^3} \frac{\gamma}{\gamma_M} \frac{1 - \left( \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right)^2}{\left( 1 - \left( 1 - \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right)^2 \right)^3} B B \right) \right] \end{array} \right\}, \quad (A5)$$

where:

$$A = \left( \Omega^{-1} + \left( 1 - \left( 1 - \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right)^2 \right) \Sigma_j (\mathbf{e}_j)^{-1} \right)^{-1},$$

$$B = \left( \left( 1 - \left( 1 - \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right)^2 \right)^{-1} Id_N + \Sigma_j (\mathbf{e}_j)^{-1} \Omega \right)^{-1}.$$

## A.6 Proof of Proposition 4

Given the functional form of the price  $\mathbf{P}$ , and that it is publicly observed, we can decompose it as:

$$\Pi_{\theta}^{-1}(\mathbf{P}-\Pi_0) = \Theta - \frac{\gamma_{M\rho S}}{\chi} \Sigma_j(\mathbf{e}_j) F' \mathbf{x} = \hat{\Theta} - \frac{\gamma_{M\rho S}}{\chi} \Sigma_j(\mathbf{e}_j) F' \hat{\mathbf{x}},$$

from which it follows that

$$\Theta - \hat{\Theta} = \frac{\gamma_{M\rho S}}{\chi} \Sigma_j(\mathbf{e}_j) F' (\mathbf{x} - \hat{\mathbf{x}}), \quad (\text{A6})$$

where we have substituted for  $\Pi_{\theta}^{-1}\Pi_x$  with our expression in the main text,  $\Pi_{\theta} = -\Pi_x \frac{\chi}{\gamma_{M\rho S}} F'^{-1} \Sigma_j(\mathbf{e}_j)^{-1}$ . It follows that beliefs across  $\hat{\Theta}$  are correlated, and the innovations to the supply shocks  $\hat{\mathbf{x}} - \mathbf{x}$  are entangled by the payoff matrix  $F'$ . Notice that the first row of  $F'$  is the vector of asset  $b$ 's, and  $F'$  is the bordered identity matrix  $Id_{N-1}$  for rows 2 to  $N$ .

Substituting with the system of equations (A6), and recognizing that  $K_{ii}(e_{ij}) = M + e_{ij}$ , we obtain:

$$\theta_1 - \hat{\theta}_1 = \frac{\gamma_{M\rho S}}{\chi} \sum_{i=2}^N K_{ii}(e_{ij})^{-1} b_i (x_i - \hat{x}_i) = \frac{\gamma_{M\rho S}}{\chi} K_{11}(e_{1j})^{-1} (x_1 - \hat{x}_1) + \sum_{i=1}^{N-1} b_i (\theta_i - \hat{\theta}_i),$$

with the understanding that  $b_1 = 1$ . It then follows that if the market overestimates  $\theta_i$ ,  $\theta_i < \hat{\theta}_i$ ,  $i \in [2, 3, \dots, N]$ , that it marginally overestimates the aggregate risk  $\theta_1$  if  $b_i \geq 0$ ,  $\theta_1 < \hat{\theta}_1$ . Consequently, it follows that:

$$b_i \text{Cov}(\theta_i, \theta_1 \mid \mathcal{F}^c) \geq 0 \quad \forall i \in \{2, \dots, N\}.$$

Therefore, if the market overestimates the asset-specific payoff to asset  $i$ , it will also overestimate the aggregate fundamental  $\theta_1$ , all else equal and all other prices held constant.

Next we fix the market's perception of the aggregate risk  $\hat{\theta}_1$ . A positive shift in the perception of asset-specific payoff to asset  $i$ ,  $\theta_i - \hat{\theta}_i$ ,  $i \in [2, 3, \dots, N]$ , must be offset by a shift in the perception of the remaining asset-specific asset payoffs  $\sum_{2, j \neq i}^N \frac{b_j}{b_i} (\theta_j - \hat{\theta}_j)$  to hold fixed the perception of aggregate risk  $\hat{\theta}_1$ . It then follows that:

$$b_i b_j \text{Cov}(\hat{\theta}_i, \hat{\theta}_j \mid \mathcal{F}^c) \leq 0 \quad \forall (i, j) \quad i \neq j.$$

## A.7 Proof of Proposition 5

We discuss the essential steps of the proof of Proposition 5 since the steps follow closely the derivation of the equilibrium for the main model.

Since investors can perfectly monitor unskilled managers, their asset demands will be the same as if investors could invest directly. Investors would solve for their portfolio allocation  $\omega^c$  that satisfies the mean-variance program:

$$U_0 = \sup_{\omega^c} R^f W_0 + (\omega^c + F'^{-1}\psi)' (F\hat{\Theta} - R^f\mathbf{P}) + \frac{1}{2}\tau_I^{-1}\psi'\psi - \frac{\gamma}{2}\omega^{c'}F\Omega F'\omega^c - \gamma\omega^{c'}F\Omega\psi,$$

with the addition of  $\psi$  to capture background risk. This has the interior solution:

$$\omega = \frac{1}{\gamma} (F\Omega F')^{-1} (F\hat{\Theta} - R^f\mathbf{P}) - F'^{-1}\psi.$$

Thus  $\omega_1^U = \omega$  now incorporates a hedging term for investor background risk. The optimal contract for unskilled managers is again to give them a fixed fee, which we abstract from here for brevity, since deriving the full equilibrium of the extended model is not our focus.

Skilled manager asset demand  $\omega_1^S(j)$  is the same as before, since the functional form of the linear contract is unchanged, as is the condition that characterizes their optimal effort  $\mathbf{e}$ . These are both given in Proposition 1.

Imposing market-clearing, the only modification is that  $\Pi_0$ , the constant in the asset price, now incorporates the hedging term of unskilled managers:

$$\Pi_0 = \frac{1}{R^f} F \left( \frac{\chi}{\gamma_M \rho_S} \Omega(j)^{-1} + \frac{1-\chi}{\gamma} \Omega^{-1} \right)^{-1} \begin{pmatrix} \left( \frac{1-\chi}{\gamma} + \frac{\chi}{\gamma_M \rho_S} \right) (\tau_\theta \bar{\Theta} - \tau_x \Pi'_\theta \Pi_x^{-1} \bar{\mathbf{x}}) \\ -\frac{\chi}{\rho_S} F' \rho_R - (1-\chi)\psi \end{pmatrix}.$$

We recognize that realized investor utility if they invest with a skilled manager is given by:

$$V(W_2^S, C_0^S) = -\exp \left( -\gamma \begin{pmatrix} R^f W_0 - \rho_0 + \psi' (\Theta - R^f F^{-1} \mathbf{P} + \varepsilon_I) \\ + \frac{1-\rho_S}{\gamma_M \rho_S} (\mathbf{s}_j - \hat{\Theta})' \Sigma_j (\mathbf{e}_j)^{-1} (\Theta - R^f F^{-1} \mathbf{P}) \\ + \frac{1}{\rho_S} \left( \frac{1-\rho_S}{\gamma_M} (\hat{\Theta} - R^f F^{-1} \mathbf{P})' (\Omega^{-1} + \Sigma_j (\mathbf{e}_j)^{-1}) - \rho'_R F \right) (\Theta - R^f F^{-1} \mathbf{P}) \end{pmatrix} \right),$$

with the modification for the background risk the investor faces  $\psi'\Theta$ . Taking conditional expectations with respect to the market beliefs, we then arrive at:

$$\begin{aligned} & E [V(W_2^S, C_0^S) \mid \mathcal{F}^c] \\ & \exp \left( +\frac{1}{2} \left( \left( 1 + \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right) \Omega^{-1} \mathbf{Z} + \gamma\psi' - \frac{\gamma}{\rho_S} F' \rho_R \right)' \left( \Omega^{-1} + \left( 1 - \left( 1 - \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right)^2 \right) \Sigma_j (\mathbf{e}_j)^{-1} \right)^{-1} \right. \\ & \quad \left. \times \left( \left( 1 + \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right) \Omega^{-1} \mathbf{Z} + \gamma\psi' - \frac{\gamma}{\rho_S} F' \rho_R \right) \right) \\ = & - \frac{\gamma \rho_0 + \frac{1}{2} \tau_I^{-1} \psi' \psi - \gamma R^f W_0 - \frac{1}{2} \mathbf{Z}' \Omega^{-1} \mathbf{Z}}{\left| Id_N + \left( 1 - \left( 1 - \frac{\gamma}{\gamma_M} \frac{1-\rho_S}{\rho_S} \right)^2 \right) \Omega \Sigma_j (\mathbf{e}_j)^{-1} \right|^{1/2}}, \end{aligned}$$

where  $\mathbf{Z} = \hat{\Theta} - R^f F^{-1} \mathbf{P}$ . Defining  $\tilde{\rho}_R = \rho_R - \rho_S F'^{-1} \psi'$ , similar steps reveal:

$$\tilde{\rho}_R = - \left( \rho_S + \frac{\gamma}{\gamma_M} (1 - \rho_S) \right) \omega^0,$$

and consequently that:

$$\rho_R = \rho_S F'^{-1} \psi' - \left( \rho_S + \frac{\gamma}{\gamma_M} (1 - \rho_S) \right) \omega^0.$$

It follows, given the structure of the investor's problem, that  $\rho_S$  will satisfy the same FONC as in Proposition 3 with  $\rho_R$  suitably modified.