Redemption Fees and Information-Based Runs*

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Abstract

We study how the imposition of a redemption fee affects runs on financial institutions when investors are asymmetrically informed about fundamentals. Although the fee eliminates the first-mover advantage and, therefore, discourages runs by informed investors, it also influences learning by uninformed investors and may, thereby, either increase or decrease overall run potential. Additionally, the fee may create a last-mover advantage for the informed, resulting in a wealth transfer from uninformed to informed investors. These effects render the welfare consequences of the fee ambiguous. The fee's impact on preemptive runs is also shown to be ambiguous.

Keywords: Runs; Redemption fees; Learning; Payoff externality; Last-mover advantage

JEL: D8, G2

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1 Introduction

Runs on financial institutions have plagued markets for centuries. Due to the negative welfare consequences associated with runs, banking regulators have devised several mechanisms over time in an attempt to prevent runs on banks, including deposit insurance and suspension of convertibility. While these mechanisms have been relatively successful at preventing runs on traditional banks during recent history, they leave many other types of systemically important "shadow-banking" institutions, such as money market mutual funds ("MMMFs"), which are not subject to banking regulations, vulnerable to runs. As discussed by Schmidt et al. (2016), this vulnerability was exploited by investors during the 2008 financial crisis, resulting in many MMMFs experiencing heavy, run-like redemptions.

In response to the heavy redemptions that occurred during the financial crisis, the U.S. Securities and Exchange Commission ("SEC") adopted new regulations designed to stabilize MMMFs and reduce run potential in 2014.[1] These new regulations embrace a novel approach to mitigating run potential by enabling MMMFs to impose a redemption fee on withdrawals during periods of stress. Specifically, under the new regulations, a MMMF is permitted to impose a fee of up to 2% on redemptions in situations wherein the MMMF's liquidity level is sufficiently low.[2] Many believe that a redemption fee should diminish the potential for runs by forcing withdrawing investors to (at least partially) internalize the liquidity costs that they generate.[3] However, to the best of our knowledge, there are no formal economic analyses that rigorously evaluate the impact of a redemption fee. We attempt to fill this gap.

We analyze a multi-stage deposit withdrawal game to study the effects of a redemption fee on the potential for runs and investor welfare. In our model, investors are asymmetrically

[1]Hanson et al. (2015) evaluate various policy proposals. The SEC also adopted regulations designed to reduce risk and increase transparency of MMMFs in 2010, but these are not pertinent to our analysis.

[2]SEC Rule 2a-7 allows a MMMF to impose a fee of up to 2% of the value of shares redeemed if its weekly liquid assets fall below 30% of total assets. Furthermore, a MMMF must impose a fee if its weekly liquid assets drop below 10% of total assets unless the board of directors determines that imposing a fee is not in the best interests of the fund. The 2014 amendments to Rule 2a-7 also enable a MMMF to impose a gate (i.e., a suspension of convertibility) under similar conditions. Rule 2a-7 still permits investors in retail (but not institutional) MMMFs to buy and sell MMMF shares at a stable net asset value (NAV) of $1.00 per share.

[3]See, e.g., Zeng (2016) and "Money Market Fund Reform; Amendments to Form PF," 79 Federal Register 157 (2014).

informed about fundamentals and may face liquidity shocks, so uncertainty and learning play a key role in shaping the dynamics of runs. A single fund invests all of its deposits in a risky asset at $t = 0$. The investment matures at $t = 2$, when the asset generates a random payoff $\tilde{v}$. Premature liquidation at $t = 1$ is costly. There is a continuum of risk-neutral investors who are identical ex ante. As time progresses, investors are differentiated into three groups based on whether they obtain private information about $\tilde{v}$ and/or experience a liquidity shock.

The first group of investors encounters a liquidity shock at $t = 1$ and must withdraw their deposits before the investment matures, regardless whether they are informed about $\tilde{v}$. Withdrawals are dynamic and can occur at two stages at $t = 1$. This first group withdraws immediately at the first stage. The second group of investors receives an identical and private informative (but noisy) signal, s, on $\tilde{v}$ at the first stage. They have no urgent liquidity needs but may also withdraw at the first stage if s indicates a low $\tilde{v}$ (i.e., if s is less than some endogenously determined signal threshold $\hat{s}_1$). The third group consists of investors who are uninformed and do not experience a liquidity shock. They observe the aggregate first-stage withdrawals by the other two groups (i.e., withdrawals for either liquidity or information reasons), update their beliefs about $\tilde{v}$ accordingly, and may withdraw at the second stage if their posterior beliefs are sufficiently low. The second group of investors also may withdraw at the second stage if they did not previously withdraw at the first stage, and such a possibility is factored into the determination of their first-stage withdrawal threshold $\hat{s}_1$.[4]

As a benchmark against which to evaluate the effects of a redemption fee, we first analyze a setting in which investors may withdraw early (at either stage of $t = 1$) without paying a fee. Because the fund must liquidate a portion of its investment to satisfy the redemption requests, early withdrawers generate liquidation costs that are borne by late withdrawers. This creates a payoff externality and, hence, a first-mover advantage among investors. To avoid potential losses due to this externality, investors without liquidity needs may withdraw too early relative to the first-best outcome, thereby leading to socially inefficient allocations.

[4]This two-stage setting is meant to capture the dynamic nature of runs in reality (e.g., the two runs on Washington Mutual in July and September 2008 as documented by He and Manela (2016)). As discussed below, this setting also permits us to evaluate the impact of a redemption fee on preemptive runs.

When a redemption fee is imposed at both stages of $t = 1$,[5] it is deducted from the amount distributed to early withdrawers and retained by the fund. Thus, the fee reduces the amount received by investors who withdraw prematurely and enables the fund to liquidate a smaller portion (compared to the benchmark setting) of its investment to meet early-withdrawal demands. This alters the tradeoffs faced by investors when making their withdrawal decisions by (at least partially) removing the first-mover advantage. More subtly, but importantly, it also influences how uninformed investors interpret a given first-stage withdrawal size and update their beliefs about the risky asset's return, which is the main novelty of our model.

We focus on a setting wherein the redemption fee is set to exactly offset the liquidation cost, so liquidation costs generated by early withdrawers are borne entirely by themselves rather than being imposed on late withdrawers. The payoff externality is thus fully internalized, and the first-mover advantage vanishes. As a result, investors without liquidity needs make withdrawal decisions based solely on their beliefs about the risky asset's return. This completely shuts down the externality channel and sharply contrasts with the no-fee benchmark setting wherein the (expected) liquidation costs generated by others influence an individual's withdrawal decision. Because the fee in this setting fully eliminates the payoff externality, any undesirable effects of the fee must stem from its impact on information and learning. Thus, this setting allows us to streamline the analysis and focus on the novel learning channel. The qualitative results and intuition are robust to alternative settings with an arbitrary fee size.

The redemption fee affects deposit withdrawals in a number of ways. Compared to the no-fee benchmark, the lowest signal for which informed investors without liquidity needs do not withdraw at the first stage of $t = 1$, $\hat{s}_1$, is lower when they must pay a fee to withdraw. Informed depositors are thus less likely to withdraw early. This influences learning by uninformed investors in two distinct ways. On the one hand, for a given first-stage withdrawal size, the posterior likelihood that informed investors remain invested increases simply because, as explained above, the presence of a fee reduces the informed investors' tendency to withdraw. This causes uninformed investors to become more optimistic about the asset return

[5] We also examine an extension of the model in which only second-stage withdrawals are subject to a fee.

$\tilde{v}$ because, *ceteris paribus*, their expectations are higher conditional on informed investors maintaining their deposits. We call this the "likelihood effect." On the other hand, the lower signal threshold causes uninformed investors to become more pessimistic about $\tilde{v}$, regardless of their beliefs about whether or not informed investors withdraw at the first stage: (i) a non-withdrawal by informed investors in the presence of a fee conveys less optimism about the fundamental because the informed investors' lower signal threshold reduces the average expected asset return for which they maintain their deposits, i.e., $\mathbb{E}[\tilde{v}|s \geq \hat{s}_1]$ decreases as $\hat{s}_1$ decreases; and (ii) an early withdrawal by informed investors conveys greater pessimism because the lower signal threshold reduces the average expected return given a withdrawal, i.e., $\mathbb{E}[\tilde{v}|s < \hat{s}_1]$ also decreases as $\hat{s}_1$ decreases. We call this the "distribution effect." Depending on whether the distribution effect or the likelihood effect dominates, a fee may either raise or lower the tendency of uninformed investors to withdraw, thereby either increasing or decreasing the occurrence of large premature withdrawals (i.e., runs).

Although the fee eliminates the informed investors' first-mover advantage by removing payoff externalities, it simultaneously creates a last-mover advantage by enabling informed investors to maintain their deposits when they would otherwise withdraw absent a fee. Specifically, when the informed investors' signal indicates a high asset return, the fee may effectively create a wealth transfer from early withdrawers (which may include uniformed investors without liquidity needs) to informed investors for two reasons. First, informed investors are not forced to abandon profitable investments (as they sometimes are in the absence of a fee due to payoff externalities) when early withdrawers bear their own liquidation costs. Second, fees paid by early withdrawers enable the fund to maintain a larger investment in the high-return asset, which benefits informed investors who maintain their deposits. Notably, this last-mover advantage, like the first-mover advantage, stems from the informed investors' information advantage. Hence, the informed investors' information advantage is not eliminated by the fee but is merely manifested in a different way.

By altering the investors' withdrawal decisions, the fee also affects social welfare, which is measured as the net surplus generated by fund investment. The fee has an amplifying effect on welfare when it reduces early withdrawals and, thereby, increases aggregate investment in the risky asset. This reduces (improves) welfare in states wherein the asset's expected payoff is low (high). In contrast, the fee may improve welfare in states wherein the asset's expected payoff is low only if it increases the potential for runs. Thus, in periods of economic stress when the risky asset's payoff is expected to be low, a fee may provide a social benefit when it destabilizes MMMFs but harm investors when it stabilizes MMMFs.

Our two-stage setting of the withdrawal game allows us to also evaluate the impact of a redemption fee on preemptive runs. While our main analysis (discussed above) assumes that the fee applies to withdrawals at both the first and second stages at $t = 1$, in an extension we assume that only second-stage withdrawals are subject to a fee. This captures the practical notion that fees may be imposed after realized withdrawals reduce a fund's liquidity reserves. One might expect this to precipitate preemptive runs at the first stage (e.g., Cipriani et al. (2014)), but our analysis reveals that, quite the contrary, this may actually strengthen the informed investors' incentives to remain invested until the fund's investment matures at $t = 2$. The reason is that informed investors enjoy a last-mover advantage when their private signal indicates a high asset return, and second-stage withdrawals by uninformed investors generate a wealth transfer to informed investors through the redemption fees paid by the former.

At a broad level, our analysis shows that regulations may interfere with information structure and influence learning by economic agents, which may render well-intentioned regulations less effective. This point is echoed by Cong et al. (2016), who show that government liquidity injections aimed at mitigating coordination failures may generate information externalities.

Our model is closely related to the literature on the role of information in bank runs, though, to the best of our knowledge, there are no existing models that evaluate the impact of a redemption fee when investors are asymmetrically informed.[6] Chari and Jagannathan

[6]We discuss only a few studies that are closely related to our paper. For other important contributions, see, e.g., Gorton (1985), Jacklin and Bhattacharya (1988), Alonso (1996), Allen and Gale (1998), Chen (1999), and Ennis and Keister (2009). Another strand of the bank-run literature focuses on panic runs in the classic Diamond and Dybvig (1983) setting wherein depositors are symmetrically informed about bank fundamentals. An essential element of the theory is a sequential service constraint, which generates payoff externalities among

(1988) consider a simultaneous-move game in which uninformed depositors may misinterpret large liquidity withdrawals as being caused by adverse information about bank assets, which may trigger a panic run even when no one has any adverse information about future returns. They show that suspension of convertibility can prevent panic runs and improve social welfare. In a modified version of the classic Diamond and Dybvig (1983) model, Gu (2011) analyzes a multi-stage game in which depositors make withdrawal decisions sequentially, and depositors at later stages learn from observed earlier-stage withdrawals. She finds that imperfect learning may lead to herding, in which case a long sequence of withdrawals persuade informed depositors at later stages to the join the withdrawal queue even if they receive a positive private signal. He and Manela (2016) study the timing of runs in a dynamic model with endogenous information acquisition. They show that rumors of illiquidity can motivate depositors to acquire information and lead to runs even when depositors receive neutral signals because there is a possibility that others may receive more negative signals and withdraw before those who receive a neutral signal. Goldstein and Pauzner (2005) assume that depositors receive private noisy signals about bank fundamentals. Using a global game approach, they derive a unique equilibrium in which the occurrence of runs is determined by fundamentals.

Runs on mutual funds have also been studied by others. Parlatore (2016) examines the effects of sponsor support on MMMF stability and the underlying asset market liquidity. Zeng (2016) analyzes runs on mutual funds when all investors are symmetrically informed. Although his main focus is on floating NAV, the analysis also touches upon redemption fees. In contrast to our results, he finds that redemption fees reduce runs, but there is no learning by investors in his model. Cipriani et al. (2014) study the potential for redemption fees to lead to preemptive runs, but there is no learning in their model, either.

Empirically, Kacperczyk and Schnabl (2013) demonstrate that MMMFs hold risky assets and are, therefore, vulnerable to runs. Schmidt et al. (2016) document run-like behaviors in MMMFs during the 2008 financial crisis. Chernenko and Sunderam (2014) show that

depositors. Green and Lin (2003) and Peck and Shell (2003), among others, further explore the sequential service constraint and study the dynamic nature of runs within the Diamond and Dybvig (1983) setting.

MMMFs are systemically important and that runs on MMMFs can have spillover effects on firms' abilities to raise capital. More generally, Chen et al. (2010) and Goldstein et al. (2016) show, respectively, that equity and corporate-bond mutual funds that hold less liquid assets experience greater outflows in response to poor performance, which can precipitate runs.

The remainder of the article is organized as follows. We describe the model in Section 2, analyze a no-fee benchmark setting in Section 3, and examine how a redemption fee affects deposit withdrawals in Section 4. Next, we evaluate welfare in Section 5 and study preemptive runs in Section 6. Finally, Section 7 concludes. Proofs are in the Appendix.

2 Model

We consider an economy comprising a single fund and a continuum of risk neutral investors of mass one. There are three dates indexed by $t \in \{0, 1, 2\}$; date 1 is divided into two stages.

2.1 The Environment

***Preferences*:** Each investor has one unit of account on deposit at the fund at $t = 0$. Investors derive utility from consuming wealth. A random fraction $\tilde{\lambda}$ of investors are impatient, whereas the rest are patient. Impatient investors derive utility only from consumption at $t = 1$. Patient investors consume at $t = 2$; if a patient investor withdraws her deposit at $t = 1$, she can costlessly store what is received from the fund and consume at $t = 2$. No individual investor knows her own consumption type at $t = 0$, but each investor privately learns whether she is patient or impatient at the first stage of date 1. At that stage, the mass of impatient investors λ (i.e., the realized value of $\tilde{\lambda}$) is also determined but is not directly observable to anyone. We stipulate that λ is drawn from some distribution that admits a continuous density function $g(\lambda)$ with support $[\underline{\lambda}, \bar{\lambda}] \subseteq [0, 1)$. Without loss of generality, we normalize $\underline{\lambda}$ to 0.

***Technology*:** The fund invests all deposits in an infinitely divisible risky asset. Each unit invested at $t = 0$ returns either a random amount $\tilde{v} \in \{H, L\}$ at $t = 2$, where $H > 1 > L > 0$, or $(1+\gamma)^{-1} \in (L, 1)$ at $t = 1$, where $\gamma \in (0, L^{-1} - 1)$ represents a liquidation cost. The common prior belief at $t = 0$ is that $\Pr(\tilde{v} = H) = \pi \in (0, 1)$ and $\Pr(\tilde{v} = L) = 1 - \pi$, which satisfies $\pi H + (1 - \pi)L > 1$. Thus, fund investment offers a higher expected long-run return than storage but is illiquid in the short run.

***Information*:** At the first stage of date 1, a fraction α of investors privately receive an identical and informative signal $s \in [0, 1]$ about the prospective date-2 asset return $\tilde{v}$, where α is a constant and common knowledge. We refer to investors who receive s as informed and to those who do not as uninformed. No individual at $t = 0$ knows whether she will become informed. The signal s is realized according to some payoff-dependent distribution functions $F_v(s)$ with corresponding continuous densities $f_v(s)$. We assume that F_H dominates F_L in the Monotone Likelihood Ratio order, so $\frac{f_H(s)}{f_L(s)}$ is strictly increasing in s. The unconditional distribution function of s is denoted by $F(s)$ with density $f(s) = \pi f_H(s) + (1 - \pi) f_L(s)$.

After observing s, informed investors update their beliefs about $\tilde{v}$ (using Bayes' rule) to

$$\pi(s) \equiv \Pr(\tilde{v} = H|s) = \frac{\pi f_H(s)}{\pi f_H(s) + (1 - \pi) f_L(s)}. \tag{1}$$

Clearly, $\partial \pi(s)/\partial s > 0$. We further assume that $f_H(1) > 0$, $f_L(0) > 0$, and $f_H(0) = f_L(1) = 0$, so the signal is fully revealing at the boundaries, i.e., $\pi(0) = 0$ and $\pi(1) = 1$. Informed investors' conditional expectations about the risky asset's return are then given by

$$V(s) \equiv \mathbb{E}[\tilde{v}|s] = \pi(s)H + (1 - \pi(s))L. \tag{2}$$

Note that $V(s)$ is continuous and strictly increasing on $[0, 1]$, with $V(0) = L$ and $V(1) = H$. Together, these imply the existence of a unique interior cutoff $s^\gamma \in (0, 1)$ satisfying

$$V(s^\gamma) = \frac{1}{1 + \gamma}, \tag{3}$$

such that it is ex post efficient to liquidate the asset at $t = 1$ if and only if $s \in [0, s^\gamma)$. There exists another unique interior cutoff $s^* \in (s^\gamma, 1)$ such that $V(s^*) = 1$.

***Redemption fee*:** The fund may impose a redemption fee on early withdrawals at date 1. Absent a fee, an investor who requests an early withdrawal receives her entire deposit, 1, provided that the fund has enough resources (from asset liquidation) to satisfy all withdrawal requests. If fund resources are insufficient to meet all withdrawal demands, then the fund is liquidated, and the liquidation value is equally distributed among all withdrawers, who each receive less than 1. When a fee $\phi \in [0, 1)$ is imposed, an early withdrawer receives only $1 - \phi$, instead of 1. The fund must disperse $1 - \phi$ for each unit of requested withdrawal. Because the fund invests all deposits in the risky asset at date 0 and each unit of the asset returns only $(1 + \gamma)^{-1}$ if liquidated at date 1, the fund must liquidate $(1 - \phi)(1 + \gamma)$ units of its investment for each unit requested to be withdrawn early. When imposed, the redemption fee applies to withdrawals at both the first and second stages at date 1 but not to distributions made at date 2. Section 6 considers an extension in which the fee applies only to second-stage withdrawals.

Our analysis considers two distinct regulatory regimes. Under the first regime, investors may withdraw without paying a redemption fee (i.e., $\phi = 0$). This setting serves as a benchmark for evaluating the impact of a redemption fee. Under the second regime, investors must pay the fee (i.e., $\phi > 0$) if they make an early withdrawal at date 1.

***Withdrawal game at date* 1:** As stated above, date 1 is divided into two stages. We model a two-stage game because runs tend to evolve over time and have important feedback effects (e.g., He and Manela (2016) and Schmidt et al. (2016)). At each stage, investors independently decide whether to withdraw their deposits, so withdrawals may occur at the first stage, the second stage, or both stages of date 1. For simplicity, we assume that investors generally do not make a withdrawal if they are indifferent between withdrawing and not withdrawing. However, in cases wherein investors know with certainty at the first stage that they will withdraw before date 2 but are indifferent between withdrawing at the first or second stage of date 1, we assume that they withdraw at the first stage rather than waiting until the second. Figure 1 illustrates the sequence of events at date 1, which we describe below.

First stage: At the beginning of the first stage, each investor privately learns her consumption type (patient or impatient). A fraction α of investors also become privately informed; they receive the signal s and update their beliefs about the date-2 fund return to $V(s)$. The other $1-\alpha$ fraction remain uninformed; they do not observe s and are unaware, at this first stage, that some other investors have become informed. As explained below, this unawareness assumption simplifies the analysis but does not affect the qualitative results. All investors then decide simultaneously and independently whether to withdraw. Impatient investors (mass λ), regardless of being informed or uninformed, must withdraw at date 1. They withdraw at the first stage rather than the second because waiting until the second stage may result in a lower payment if additional investors withdraw at the second stage and, thereby, prevent the fund from being able to fully satisfy all of the second-stage withdrawal requests due to additional liquidation costs. In contrast, no uninformed patient ("UP") investors (mass $(1-\alpha)(1-\lambda)$) withdraw at the first stage. Being unaware that others have become informed, a UP investor's available information at this stage is the same as that at date 0. Thus, if a UP investor were to withdraw at this stage, she would not have invested in the fund in the first place at date 0.

Informed patient ("IP") investors (mass $\alpha(1-\lambda)$) make their decisions based on the signal s. Formally, each IP investor chooses $w \in \{0,1\}$, where $w=1$ if she withdraws at the first stage and $w=0$ otherwise. An IP investor who does not withdraw at the first stage may still withdraw at the second stage or keep her deposit until date 2.

After all investors have made their decisions, the aggregate first-stage withdrawal

$$n = \lambda + \alpha(1-\lambda)w \tag{4}$$

is realized and observed by all. Liquidity withdrawals λ and individual IP investors' choices of w, however, are not directly observable. Note that $n \in [0, \bar{\lambda} + \alpha(1-\bar{\lambda})]$.

Second stage: After observing n, all remaining investors infer that the amount of deposits remaining at the fund is $1-n(1-\phi)(1+\gamma)$. They again decide simultaneously and independently whether to withdraw at the second stage or wait until date 2. UP investors, who were unaware at the first stage that a fraction α of investors became informed, now realize this fact. The realization could be due to the circulation of rumors, as in He and Manela (2016). Based on n, UP investors (who did not withdraw at the first stage) update their beliefs about the decisions made by IP investors at the first stage (i.e., w), which in turn affects their beliefs about the signal s and the prospective asset return $\tilde{v}$. Then, they decide whether to withdraw.

IP investors make their decisions at the second stage based on the signal s and the realized first-stage withdrawal n (assuming that they did not already withdraw at the first stage, i.e., $w = 0$). These types of investors infer that the first-stage withdrawals were all made by impatient investors (i.e., $n = \lambda$) because they know that $w = 0$.

Assuming that UP investors are unaware at the first stage that some others have become informed ensures that they do not withdraw at the first stage and, thereby, trivialize the second-stage game. Instead, if UP investors realized their information disadvantage at the first stage, then they could choose to make a first-stage withdrawal under certain parameterizations even though ex ante they were willing to invest at date 0. Under such parameterizations, the signal extraction problem faced by UP investors at the second stage and the impact of a redemption fee on such inference, which are the cornerstones of our analysis, would be rendered moot: the two-stage dynamic withdrawal game essentially would degenerate into a one-stage game with simultaneous moves. The unawareness assumption is stronger than required because, depending on parameters, UP investors could choose to maintain their deposits even if they realized their information disadvantage at the first stage. However, determining the parameterizations under which UP investors would (not) withdraw at the first stage would require us to compute a UP investor's expected payoff at the first stage conditional on her beliefs about the strategies played by others at the first *and* second stages. Such an analysis would be tremendously complex without providing much additional insight — after all, we only require that UP investors do not withdraw at the first stage, and this can be achieved by imposing an unawareness assumption. Similar assumptions are employed in Abreu and Brunnermeier (2003) and He and Manela (2016).[7]

[7]In their analysis of rumor-based bank runs, He and Manela (2016) also assume that uninformed investors

2.2 Parametric Restrictions

We make the following two assumptions to ensure that the signal extraction problem faced by UP investors at the second stage is non-trivial and informative.

Assumption 1. *There exist possible realizations of the aggregate first-stage withdrawal* n *upon which UP investors face a non-trivial inference problem at the second stage:*

$$\alpha < \bar{\lambda} < \min\left\{1 - \frac{\gamma}{(1-\alpha)(1+\gamma)}\,,\ 1 - \frac{\gamma H}{\alpha[(1+\gamma)H - 1]}\right\}. \tag{5}$$

As stated above, all possible realizations of n lie in the region $[0, \bar{\lambda} + \alpha(1-\bar{\lambda})]$. If $n \in (\bar{\lambda}, \bar{\lambda} + \alpha(1-\bar{\lambda})]$, then UP investors infer unambiguously that IP investors already withdrew at the first stage (i.e., $w = 1$) because the largest possible first-stage withdrawal by impatient investors is $\bar{\lambda}$. If $n \in [0, \alpha)$, then UP investors know for sure that IP investors did not withdraw at the first stage (i.e., $w = 0$); if they had withdrawn, then the smallest possible first-stage withdrawal would be α. The lower bound on $\bar{\lambda}$ in (5) ensures the existence of a "confounding region" wherein $n \in [\alpha, \bar{\lambda}]$. In this region, UP investors face a non-trivial inference problem because they cannot ascertain whether $w = 0$ or $w = 1$ based on n.

To streamline the analysis and focus on the role of information, we also assume that $\bar{\lambda}$ is bounded from above as in (5). The first upper bound, $1 - \frac{\gamma}{(1-\alpha)(1+\gamma)}$, ensures that the combined first-stage withdrawals by impatient and IP investors (if they choose $w = 1$) do not exhaust fund resources at the first stage. This ensures that UP investors are not fully exploited by first-stage withdrawers and, hence, have a meaningful second-stage game to play. The second upper bound, $1 - \frac{\gamma H}{\alpha[(1+\gamma)H-1]}$, excludes the possibility of second-stage "panic runs" in which the combined liquidation costs generated by withdrawals made by impatient investors (at the first stage) and UP investors (at the second stage) are so large that IP investors who do not make a first-stage withdrawal always withdraw at the second stage regardless of their signal s.

realize with a delay (which they refer to as an "awarenes window") that others may be informed and remain fully deposited before they become aware of their information disadvantage. While they provide parametric restrictions under which such outcome emerges in equilibrium, the parametric restrictions required in our framework, if we were to revoke the unawareness assumption, would be much more complex.

As we make clear in Section 3.1.1 (see footnote 8), this allows us to focus on information-based runs at the second stage. Permitting panic runs by IP investors at the second stage merely complicates the analysis without yielding additional insights.

Assumption 2. *In the confounding region wherein the realized first-stage withdrawal* $n \in [\alpha, \bar{\lambda}]$*, UP investors infer a greater likelihood that IP investors withdrew at the first stage when* n *is larger, but the marginal change in the conditional likelihood is relatively modest:*

$$0 < \frac{\partial \log\left(\frac{g(\frac{n-\alpha}{1-\alpha})}{g(n)}\right)}{\partial n} \leq \frac{1}{1-n} \qquad \forall n \in [\alpha, \bar{\lambda}]. \tag{6}$$

The first inequality in (6) ensures that the first-stage withdrawal $n \in [\alpha, \bar{\lambda}]$ is informative to UP investors: a larger n implies a higher probability that IP investors made a first-stage withdrawal (i.e., $w = 1$). Because $n = \lambda + \alpha(1-\lambda)w$, the likelihood of observing a first-stage withdrawal of size n conditional on $w = 1$ is $g(\frac{n-\alpha}{1-\alpha})$, whereas the likelihood of observing n conditional on $w = 0$ is $g(n)$. The inequality, which states that the ratio $g(\frac{n-\alpha}{1-\alpha})/g(n)$ is strictly increasing in n, means that a larger realization of n assigns a strictly greater probability mass on $w = 1$ than on $w = 0$ in the sense of the Monotone Likelihood Ratio order.

The second inequality states that the likelihood ratio $g(\frac{n-\alpha}{1-\alpha})/g(n)$ does not increase too fast in n. That is, although the likelihood that $w = 0$ (relative to $w = 1$) decreases in n, it does not decrease too much at the margin. The implication of this assumption is as follows. At the second stage, UP investors not only face the aforementioned signal extraction problem but may also suffer from a payoff externality generated by IP investors who did not withdraw at the first stage (if $w = 0$) but who might withdraw at the second stage. Obviously, the externality does not exist if $w = 1$. The assumption here ensures that the likelihood of such an externality being imposed on UP investors at the second stage does not diminish too fast at the margin. This condition is stronger than necessary, but, as we make clear in Section 3.1.2, it provides a tractable and interpretable restriction on the parameter space that guarantees the existence of an equilibrium while maintaining the generality of distribution assumptions.

3 The Benchmark Case without a Redemption Fee

We first analyze the benchmark case without a redemption fee (i.e., $\phi = 0$). This setting serves as a basis for evaluating the effects of a fee, which we examine in Section 4.

***Equilibrium concept*:** We restrict attention to symmetric pure-strategy Perfect Bayesian Equilibria ("PBE"), where the term "symmetric" means that investors of the same type choose the same equilibrium strategies. A PBE of the two-stage game at date 1 consists of IP and UP investors' withdrawal strategies at each stage and their beliefs, with the following specifics.

1. There exists a signal threshold $\hat{s}_1 \in (0, 1)$ such that IP investors withdraw at the first stage (i.e., $w = 1$) if and only if $s < \hat{s}_1$. This strategy maximizes an IP investor's expected payoff conditional on the information available to her at the first stage (i.e., the density $g(\lambda)$ and the signal s), and her beliefs about other IP investors' first-stage strategies and the strategies played by investors at the second-stage subgame (including herself if she chooses not to withdraw at the first stage, i.e., $w = 0$).

2. There exists a signal threshold $\hat{s}_2(n)$, which is a function of the realized first-stage withdrawal n, such that IP investors who chose $w = 0$ withdraw at the second stage if and only if $s < \hat{s}_2(n)$. Such a strategy maximizes the expected payoff for a remaining IP investor at the second-stage subgame given the information available to her at that stage (i.e., n and s), and her beliefs about the strategies played by UP investors (i.e., $\bar{n}$, as described below) and other remaining IP investors at the second stage.

3. There exists a withdrawal threshold $\bar{n}$ such that UP investors withdraw at the second stage if and only if $n > \bar{n}$. This strategy maximizes a UP investor's expected payoff given the information available to her at the second stage (i.e., n), and her beliefs about other UP investors' strategies and the strategies played by IP investors at the first stage (i.e., $\hat{s}_1$) and the second stage (i.e., $\hat{s}_2(n)$) if she believes $w = 0$.

4. Investors' beliefs are updated (whenever possible) according to Bayes' rule, taking others' strategies as given, and they are consistent with those strategies in equilibrium.

The equilibrium definition does not specify any beliefs in response to out-of-equilibrium moves from the first stage. This is because there are no detectable out-of-equilibrium moves from the first stage given that deviation by a single atomistic investor has no consequence on the realization of the first-stage withdrawal n. A PBE is denoted by a triplet $\{\hat{s}_1, \hat{s}_2(n), \bar{n}\}$.

Although panic runs, in which all investors withdraw at the first stage of date 1, can occur in our model, we do not consider them. In our model, IP investors may withdraw at either stage based on a combination of their private signal and the payoff externality generated by withdrawing impatient investors (first stage) and possibly UP investors (second stage). UP investors may withdraw at the second stage based on the information they extract from the realized first-stage withdrawal along with the payoff externality generated by withdrawing impatient investors (first stage) and possibly IP investors (either stage).

We use backward induction to characterize the equilibrium. We first determine the second-stage strategies played by those investors who do not withdraw at the first stage in Section 3.1. In Section 3.2, we characterize the investors' first-stage strategies.

3.1 Second-Stage Subgame Equilibrium

At the beginning of the second stage, the remaining investors consist of IP investors if they did not withdraw at the first stage (i.e., if $w = 0$) and UP investors. After observing the realized first-stage withdrawal n, these remaining investors infer that the amount of deposits remaining in the fund is $1 - n(1 + \gamma)$. We first characterize the IP investors' second-stage strategy $\hat{s}_2(n)$ in Section 3.1.1 (assuming $w = 0$), taking the UP investors' second-stage strategy $\bar{n}$ as given. We then characterize $\bar{n}$ in Section 3.1.2, taking as given $\hat{s}_2(n)$. We show that the pair $\{\hat{s}_2(n), \bar{n}\}$ that characterizes the second-stage subgame equilibrium is unique.

3.1.1 IP Investors

Provided that an IP investor did not already withdraw at the first stage, she withdraws at the second stage when the expected payoff from doing so exceeds that from keeping her deposit until date 2. The latter payoff depends on whether UP investors also keep their deposits until date 2, which in turn depends on the first-stage withdrawal n.

In the remainder of this subsection, we focus on the case wherein $n \in [0, \bar{\lambda}]$ because, as discussed in Section 2.2, there are no IP investors remaining at the second stage if $n > \bar{\lambda}$. We conjecture (and verify in Lemma 2 in Section 3.1.2) that UP investors always withdraw at the second stage if they know for sure that IP investors already withdrew at the first stage (i.e., if $n > \bar{\lambda}$). Thus, the UP investors' second-stage decision threshold satisfies $\bar{n} \in [0, \bar{\lambda}]$.

If $n \leq \bar{n}$, then UP investors do not withdraw at the second stage in the conjectured equilibrium. As a result, each individual IP investor also does not withdraw if and only if

$$\frac{1 - n(1+\gamma)}{1-n} V(s) \geq 1. \tag{7}$$

Equation (7) can be understood as follows. Given that UP investors do not withdraw at the second stage, IP investors know that if they also do not withdraw, then the amount of deposits remaining at the fund until date 2 is $1 - n(1+\gamma)$, resulting in a date-2 fund value of $[1 - n(1+\gamma)]V(s)$. This value is equally distributed among all remaining investors at date 2, the mass of which is $1 - n$. Instead, if an atomistic IP investor deviates from the equilibrium strategy and withdraws at the second stage, then she receives her initial deposit 1.

IP investors face a slightly different tradeoff if $n > \bar{n}$ because UP investors withdraw at the second stage. IP investors also withdraw in this case unless

$$\frac{1 - [n + (1-\alpha)(1-n)](1+\gamma)}{\alpha(1-n)} V(s) \geq 1. \tag{8}$$

Equation (8) can be interpreted similarly as (7), with one difference. When $n > \bar{n}$, IP investors know that if they do not withdraw at the second stage, then the amount of remaining deposits is $1 - [n + (1-\alpha)(1-n)](1+\gamma)$, given that UP investors (mass $(1-\alpha)(1-n)$) withdraw their deposits. The expected date-2 fund value is thus $(1 - [n + (1-\alpha)(1-n)](1+\gamma))V(s)$, which is equally divided among IP investors who keep their deposits until date 2 (mass $\alpha(1-n)$).

IP investors' second-stage strategies depend on their signal s and are characterized by (7) and (8). Because $V(s)$ is continuous and monotonically increasing in s and the liquidation cost borne by non-withdrawing IP investors is monotonically increasing in cumulative withdrawals by others (i.e., the coefficients on $V(s)$ in (7) and (8) are decreasing in n), the signal threshold $\hat{s}_2(n)$ is uniquely determined for each given n. The following theorem characterizes $\hat{s}_2(n)$.

Theorem 1. *For any realized first-stage withdrawal $n \in [0, \bar{\lambda}]$ and the UP investors' second-stage withdrawal threshold $\bar{n} \in [0, \bar{\lambda}]$, there exists a unique signal threshold $\hat{s}_2(n) \in [s^*, 1]$, characterized by*

$$V(\hat{s}_2(n)) = \begin{cases} \dfrac{1-n}{1-n(1+\gamma)} & \text{if } n \in [0, \bar{n}] \\ \dfrac{\alpha(1-n)}{1-[1-\alpha(1-n)](1+\gamma)} & \text{if } n \in (\bar{n}, \bar{\lambda}], \end{cases} \tag{9}$$

such that IP investors who do not withdraw at the first stage withdraw at the second stage if and only if $s < \hat{s}_2(n)$. Furthermore, $\hat{s}_2(n)$ is strictly increasing in n.

Figure 2, which plots n against $V(s)$, demonstrates two main results in Theorem 1. First, IP investors who did not make a first-stage withdrawal withdraw at the second stage if and only if their signal is below some threshold $\hat{s}_2(n)$, which is depicted by the curves in the two regions, $n \leq \bar{n}$ and $n > \bar{n}$. The threshold is strictly increasing in n within each region because IP investors demand a greater expected asset return (equivalently, higher s) to maintain their deposits when a larger mass of investors withdraw at the first stage and, thereby, impose greater liquidation costs on those who remain invested. Second, $\hat{s}_2(n)$ jumps upward as n crosses the UP investors' withdrawal threshold $\bar{n}$. This is because IP investors bear only the liquidation costs imposed by impatient investors' first-stage withdrawals when $n \leq \bar{n}$, in which case UP investors do not withdraw at the second stage, but must absorb the *additional* liquidation costs caused by UP investors' second-stage withdrawal when $n > \bar{n}$. Consequently, IP investors, *ceteris paribus*, require a higher s to maintain their deposits when $n > \bar{n}$.[8]

[8] The assumption that $\bar{\lambda} < 1 - \frac{\gamma H}{\alpha[(1+\gamma)H-1]}$ made in (5) ensures that $\hat{s}_2(\bar{\lambda}) < 1$. Thus, if $s \in [\hat{s}_2(\bar{\lambda}), 1]$, then IP investors remain invested until date 2 (if $w = 0$) even if impatient and UP investors withdraw at date 1.

3.1.2 UP Investors

UP investors face more uncertainty than the remaining IP investors at the second stage because the former do not observe the signal s received by the latter. However, UP investors can update their beliefs about s, and hence the asset return $\tilde{v}$, based on the first-stage withdrawal n. Note that n is correlated with s because IP investors withdraw at the first stage in the conjectured equilibrium only when $s < \hat{s}_1$.

As discussed in Section 2.2, if $n \in [0, \alpha)$, then UP investors know with certainty that IP investors did not make a first-stage withdrawal (i.e., $w = 0$) and, consequently, infer that $s \geq \hat{s}_1$. Conversely, if $n \in (\bar{\lambda}, \bar{\lambda} + \alpha(1 - \bar{\lambda})]$, then UP investors infer unambiguously that IP investors withdrew at the first stage (i.e., $w = 1$) and, hence, $s < \hat{s}_1$. In the confounding region wherein $n \in [\alpha, \bar{\lambda}]$, UP investors cannot disentangle whether the first-stage withdrawal is entirely due to impatient investors withdrawing for liquidity reasons or partially due to IP investors withdrawing for information reasons. In this case, UP investors are uncertain about w, but they can infer the conditional likelihood that $w = 1$ based on n and update their beliefs about s accordingly. Specifically, because $w = 1$ only when $s < \hat{s}_1$ and the density of liquidity-driven withdrawals $g(\cdot)$ satisfies the Monotone Likelihood Ratio Property (see Assumption 2), the probability that $s < \hat{s}_1$ conditional on n, denoted by $F(\hat{s}_1|n)$, is increasing in n. Consequently, UP investors' beliefs about $\tilde{v}$ are more pessimistic when n is larger. The following lemma characterizes $F(\hat{s}_1|n)$ along with UP investors' conditional expectations of $\tilde{v}$.

Lemma 1. *Conditional on a first-stage withdrawal n, UP investors' posterior beliefs about the likelihood that IP investors already withdrew at the first stage (hence $s < \hat{s}_1$) are given by*

$$F(\hat{s}_1|n) = \begin{cases} 0 & \text{if } n \in [0, \alpha) \\ \dfrac{F(\hat{s}_1)g\left(\frac{n-\alpha}{1-\alpha}\right)}{F(\hat{s}_1)g\left(\frac{n-\alpha}{1-\alpha}\right) + (1 - F(\hat{s}_1))g(n)} & \text{if } n \in [\alpha, \bar{\lambda}] \\ 1 & \text{if } n \in (\bar{\lambda}, \bar{\lambda} + \alpha(1 - \bar{\lambda})], \end{cases} \tag{10}$$

and their conditional expectations of $\tilde{v}$ are given by

$$\mathbb{E}[\tilde{v}|n] = \begin{cases} \mathbb{E}[\tilde{v}|s \geq \hat{s}_1] & \textit{if } n \in [0, \alpha) \\ \dfrac{F(\hat{s}_1) g\left(\frac{n-\alpha}{1-\alpha}\right) \mathbb{E}[\tilde{v}|s < \hat{s}_1] + (1 - F(\hat{s}_1)) g(n) \mathbb{E}[\tilde{v}|s \geq \hat{s}_1]}{F(\hat{s}_1) g\left(\frac{n-\alpha}{1-\alpha}\right) + (1 - F(\hat{s}_1)) g(n)} & \textit{if } n \in [\alpha, \bar{\lambda}] \\ \mathbb{E}[\tilde{v}|s < \hat{s}_1] & \textit{if } n \in (\bar{\lambda}, \bar{\lambda} + \alpha(1 - \bar{\lambda})]. \end{cases} \tag{11}$$

Furthermore, $F(\hat{s}_1|n)$ is non-decreasing in n and monotonically increasing in n on $[\alpha, \bar{\lambda}]$. Conversely, $\mathbb{E}[\tilde{v}|n]$ is non-increasing in n and monotonically decreasing in n on $[\alpha, \bar{\lambda}]$.

After updating their beliefs, UP investors decide whether to withdraw. A UP investor makes a second-stage withdrawal when the expected payoff from withdrawing exceeds that from maintaining her deposit until date 2. This decision is complicated by the fact that the payoff from maintaining the deposit also depends on whether the remaining IP investors (if any) withdraw their deposits at the second stage because that will impose additional liquidation costs (beyond those by any first-stage withdrawal) on non-withdrawing UP investors.

Further complicating the matter, UP investors observe only the first-stage withdrawal n but not the signal s. As a result, they generally cannot ascertain: (i) whether IP investors already withdrew at the first stage (when $n \in [\alpha, \bar{\lambda}]$); and (ii) whether IP investors who did not make a first-stage withdrawal will withdraw at the second stage (when $n \in [0, \bar{\lambda}]$). Nevertheless, UP investors can infer the likelihood of (i) based on n, as indicated by Lemma 1. Moreover, as we discuss below, UP investors can also use n to infer the likelihood of (ii).

Before moving forward, we first verify an earlier conjecture (made in Section 3.1.1) that UP investors always withdraw at the second stage if they can ascertain that IP investors already withdrew at the first stage (i.e., if $n \in (\bar{\lambda}, \bar{\lambda} + \alpha(1 - \bar{\lambda})]$).

Lemma 2. *If $n \in (\bar{\lambda}, \bar{\lambda} + \alpha(1 - \bar{\lambda})]$, then UP investors withdraw at the second stage.*

We now characterize the UP investors' withdrawal threshold $\bar{n}$. Lemma 2 implies that $\bar{n} \in [0, \bar{\lambda}]$. To compute the expected payoff to a UP investor who does not withdraw at the second stage, we consider three cases, depending on whether IP investors (i) withdraw at the first stage, (ii) withdraw at the second stage, or (iii) do not withdraw at either stage.

First, suppose IP investors already withdrew at the first stage (i.e., $s < \hat{s}_1$). Conditional on n, UP investors believe that this outcome occurs with probability $F(\hat{s}_1|n)$. In this case, a UP investor's expected payoff from maintaining her deposit until date 2 is

$$\frac{1 - n(1+\gamma)}{1-n}\mathbb{E}[\tilde{v}|s < \hat{s}_1]. \tag{12}$$

The numerator in (12), $1-n(1+\gamma)$, represents the remaining investment in the risky asset after the first-stage withdrawal n, and the denominator, $1-n$, represents the mass of remaining investors at date 2. In this case, no additional liquidation costs beyond those generated by the first-stage withdrawal n are imposed on UP investors who maintain their deposits at the second stage because all IP investors already withdrew at the first stage.

Second, if IP investors did not withdraw at the first stage (i.e., $s \geq \hat{s}_1$), then they may make a second-stage withdrawal, which will impose additional liquidation costs on UP investors who do not withdraw at the second stage. According to Theorem 1, this happens if $s \in [\hat{s}_1, \hat{s}_2(n))$, which, from the UP investors' perspectives, occurs with probability $(1 - F(\hat{s}_1|n))\frac{\max\{F(\hat{s}_2(n))-F(\hat{s}_1),0\}}{1-F(\hat{s}_1)}$. Here, $1 - F(\hat{s}_1|n)$ is the likelihood that $s \geq \hat{s}_1$ conditional on n, and $\frac{\max\{F(\hat{s}_2(n))-F(\hat{s}_1),0\}}{1-F(\hat{s}_1)}$ is the likelihood that $\hat{s}_1 \leq s < \hat{s}_2(n)$ conditional on both $s \geq \hat{s}_1$ and n. In this case, the expected payoff to a non-withdrawing UP investor is

$$\frac{1 - [n + \alpha(1-n)](1+\gamma)}{(1-\alpha)(1-n)}\mathbb{E}[\tilde{v}|\hat{s}_1 \leq s < \hat{s}_2(n)]. \tag{13}$$

In (13), the term $\alpha(1-n)(1+\gamma)$ in the numerator reflects the additional liquidation costs imposed by IP investors who withdraw at the second stage, and the denominator represents the mass of UP investors who keep their deposits until date 2.

The third case occurs when IP investors do not withdraw at either stage (i.e., when $s \geq \max\{\hat{s}_1, \hat{s}_2(n)\}$). From a UP investor's perspective, this occurs with probability $(1 -$

$F(\hat{s}_1|n))\frac{1-\max\{F(\hat{s}_2(n)),F(\hat{s}_1)\}}{1-F(\hat{s}_1)}$. Similar to the second case, $1 - F(\hat{s}_1|n)$ is the likelihood that $s \geq \hat{s}_1$ conditional on n, and $\frac{1-\max\{F(\hat{s}_2(n)),F(\hat{s}_1)\}}{1-F(\hat{s}_1)}$ is the likelihood that $s \geq \max\{\hat{s}_1, \hat{s}_2(n)\}$ conditional on both $s \geq \hat{s}_1$ and n. Here, a non-withdrawing UP investor's expected payoff is

$$\frac{1-n(1+\gamma)}{1-n}\mathbb{E}[\tilde{v}|s \geq \max\{\hat{s}_1, \hat{s}_2(n)\}]. \tag{14}$$

Like the first case (in which IP investors withdrew at the first stage), non-withdrawing UP investors are not exposed to further liquidation costs when the remaining IP investors do not withdraw at the second stage, so the coefficient in (14), $\frac{1-n(1+\gamma)}{1-n}$, is the same as that in (12).

The UP investors' withdrawal threshold $\bar{n}$ is determined by aggregating these three cases. Theorems 2 characterizes $\bar{n}$ and shows it to be unique. Note that $\mathbb{1}_{\{\cdot\}}$ is an indicator function.

Theorem 2. *Given the IP investors' respective first-stage and second-stage signal thresholds $\hat{s}_1$ and $\hat{s}_2(n)$, there exists a unique withdrawal threshold $\bar{n}$, characterized by*

$$\bar{n} = \sup\Bigg\{n \in [0, \bar{\lambda}] : 1 \leq \frac{1-n(1+\gamma)}{1-n}\mathbb{E}[\tilde{v}|n] - \mathbb{1}_{\{\hat{s}_1 < \hat{s}_2(n)\}}(1 - F(\hat{s}_1|n))\frac{F(\hat{s}_2(n)) - F(\hat{s}_1)}{1 - F(\hat{s}_1)}\frac{\alpha\gamma\mathbb{E}[\tilde{v}|\hat{s}_1 \leq s < \hat{s}_2(n)]}{(1-\alpha)(1-n)}\Bigg\}, \tag{15}$$

such that UP investors do not withdraw at the second stage if and only if $n \leq \bar{n}$.

The left-hand side (LHS) of the condition in (15), 1, represents an atomistic individual UP investor's payoff from deviating from the equilibrium strategy by withdrawing while others do not. The right-hand side (RHS) of the condition is a probability-weighted expectation of a UP investor's payoff from not withdrawing at the second stage. The first term, $\frac{1-n(1+\gamma)}{1-n}\mathbb{E}[\tilde{v}|n]$, represents a UP investor's expected payoff from keeping her deposit until date 2, given that IP investors do not withdraw at the second stage. As mentioned above, the lack of second-stage withdrawals by IP investors could arise either because they already withdrew at the first stage or because their signal s is high enough that they choose to keep their deposits until date 2. Given that IP investors do not make a second-stage withdrawal, UP investors know that if they also do not withdraw, then the amount of deposits remaining invested until date 2 is $1 - n(1+\gamma)$. Thus, conditional on n, the expected date-2 fund value is $[1 - n(1+\gamma)]\mathbb{E}[\tilde{v}|n]$, which will be equally divided among all remaining investors at date 2 (mass $1 - n$).

The second term captures the potential additional liquidation costs imposed on UP investors by IP investors who may withdraw at the second stage. This occurs when $s \in [\hat{s}_1, \hat{s}_2(n))$, the probability of which is determined as follows. Conditional on a first-stage withdrawal n, the probability that IP investors did not withdraw at the first stage is $1 - F(\hat{s}_1|n)$; and given that, the probability that IP investors withdraw at the second stage is $\mathbb{1}_{\{\hat{s}_1 < \hat{s}_2(n)\}} \frac{F(\hat{s}_2(n)) - F(\hat{s}_1)}{1 - F(\hat{s}_1)}$. If IP investors make a second-stage withdrawal (i.e., if $s \in [\hat{s}_1, \hat{s}_2(n))$), then non-withdrawing UP investors must bear additional liquidation costs whose expected value is $\frac{\alpha\gamma\mathbb{E}[\tilde{v}|\hat{s}_1 \le s < \hat{s}_2(n)]}{(1-\alpha)(1-n)}$, which is the difference between the expected payoffs to non-withdrawing UP investors when IP investors do not make a second-stage withdrawal and when they do: $\left[\frac{1-n(1+\gamma)}{1-n} - \frac{1-[n+\alpha(1-n)](1+\gamma)}{(1-\alpha)(1-n)}\right]\mathbb{E}[\tilde{v}|\hat{s}_1 \le s < \hat{s}_2(n)]$.

Figure 3 illustrates $\bar{n}$. The figure decomposes a non-withdrawing UP investor's expected payoff into two components: the expected payoff conditional on there being no additional withdrawals at the second stage (i.e., the first term on the RHS of the condition in (15)) and the expected loss due to potential second-stage withdrawals by IP investors (i.e., the second term on the RHS of the condition in (15)). The potential for additional losses kicks in when the signal threshold $\hat{s}_2(n)$ used by the remaining IP investors (if any) in making their second-stage withdrawal decisions exceeds their first-stage signal threshold $\hat{s}_1$. Because $\hat{s}_2(n)$ is strictly increasing in the realized first-stage withdrawal n, these additional losses become a possibility for UP investors when n exceeds some threshold denoted by $\hat{n}(\hat{s}_1)$.[9]

According to Theorems 1 and 2, $\hat{s}_2(n)$ is uniquely determined for a given $\bar{n}$, and vice versa. However, this does not necessarily imply uniqueness of the pair, which requires that there are

[9] We define the threshold $\hat{n}(\hat{s}_1)$ below in Lemma 3. If $n > \hat{n}(\hat{s}_1)$, then $\hat{s}_2(n) > \hat{s}_1$, which means that there is a possibility that IP investors make a second-stage withdrawal. Figure 3 depicts $\hat{n}(\hat{s}_1)$ at a point smaller than $\bar{n}$ but larger than α, though this need not be the case that emerges in equilibrium. The relations between the magnitudes of α, $\hat{n}(\hat{s}_1)$, and $\bar{n}$ depend on the values of the underlying parameters. The curve jumps downward at $n = \alpha$ because UP investors infer that $w = 0$ for certain if $n < \alpha$ but are uncertain about w if $n \in [\alpha, \bar{\lambda}]$, leading to a jump in beliefs about s around $n = \alpha$.

not multiple pairs of $\bar{n}$ and $\hat{s}_2(\bar{n})$ that are consistent with each other for a given $\hat{s}_1$. The following corollary states that the pair is indeed unique.

Corollary 1. *Given the IP investors' first-stage signal threshold $\hat{s}_1$, the pair of second-stage strategies characterized in Theorems 1 and 2, $\{\hat{s}_2(n), \bar{n}\}$, is unique.*

As $\bar{n}$ increases, UP investors withdraw less frequently, which makes IP investors less likely to withdraw *on average* at the second stage. At the margin (i.e., when $n = \bar{n}$), however, IP investors require a higher signal to keep their deposits when $\bar{n}$ is higher because a larger first-stage withdrawal imposes greater liquidation costs on IP investors if they do not withdraw. Thus, $\hat{s}_2(\bar{n})$ is strictly increasing in $\bar{n}$, as Theorem 1 implies. Consequently, a higher $\bar{n}$ increases the likelihood that UP investors will maintain their deposits until date 2 while IP investors withdraw at the second stage. This increases the expected liquidation costs borne by non-withdrawing UP investors, as the second term of the condition in (15) is more likely to kick in with a higher $\hat{s}_2(\bar{n})$. Therefore, $\bar{n}$ is non-increasing in $\hat{s}_2(\bar{n})$. Hence, the pair must be unique.

3.2 First-Stage Equilibrium

We now examine the IP investors' first-stage problem. IP investors decide whether to withdraw at the first stage based on their signal s. When making this decision, they understand the second-stage equilibrium strategies, $\hat{s}_2(n)$ and $\bar{n}$, and withdraw at the first stage (i.e., $w = 1$) if and only if the payoff from withdrawing, 1, exceeds the expected payoff from keeping their deposits at the first stage (i.e., $w = 0$). Unlike at the second stage, IP investors do not know the size of the liquidity withdrawals λ when making their first-stage decisions. Because λ affects the realization of the first-stage withdrawal n, which in turn affects the second-stage actions of both IP and UP investors, IP investors must take into account their uncertainty about λ when determining their first-stage strategies.

To compute an IP investor's expected payoff from maintaining her deposit at the first stage, we examine various scenarios that a non-withdrawing IP investor may encounter at the second stage for a given signal s. If $s < s^*$, then a non-withdrawing IP investor's highest possible payoff is $V(s) < V(s^*) = 1$. Therefore, IP investors always withdraw if $s < s^*$. The following analysis is predicated on IP investors not withdrawing at the first stage (i.e., $s \geq s^*$).

If UP investors do not withdraw at the second stage (i.e., if $n \leq \bar{n}$), then it follows from (7) that IP investors do not withdraw at the second stage if and only if

$$n \leq \frac{V(s) - 1}{(1+\gamma)V(s) - 1}. \tag{16}$$

Similarly, (8) implies that if UP investors make a second-stage withdrawal (i.e., if $n > \bar{n}$), then IP investors do not withdraw at the second stage if and only if

$$n \leq 1 - \frac{\gamma V(s)}{\alpha[(1+\gamma)V(s) - 1]}. \tag{17}$$

Like $V(s)$, the RHSs of (16) and (17) are continuous and monotonically increasing in s. Consequently, there exists a unique threshold $\hat{n}(s)$ such that IP investors withdraw at the second stage if and only if $n > \hat{n}(s)$. The following lemma characterizes $\hat{n}(s)$.

Lemma 3. *For any realized signal $s \in [s^*, 1]$ and the UP investors' second-stage withdrawal threshold $\bar{n}$, there exists a unique threshold $\hat{n}(s) \in [0, \bar{\lambda}]$, characterized by*

$$\hat{n}(s) = \begin{cases} \dfrac{V(s) - 1}{(1+\gamma)V(s) - 1} & \text{if } s \in [s^*, \hat{s}_2(\bar{n})] \\ \bar{n} & \text{if } s \in (\hat{s}_2(\bar{n}), \check{s}_2(\bar{n})] \\ 1 - \dfrac{\gamma V(s)}{\alpha[(1+\gamma)V(s) - 1]} & \text{if } s \in (\check{s}_2(\bar{n}), \check{s}_2(\bar{\lambda})] \\ \bar{\lambda} & \text{if } s \in (\check{s}_2(\bar{\lambda}), 1], \end{cases} \tag{18}$$

where $\check{s}_2(n)$ satisfies

$$V(\check{s}_2(n)) = \frac{\alpha(1-n)}{1 - [1 - \alpha(1-n)](1+\gamma)}, \tag{19}$$

such that IP investors who do not withdraw at the first stage withdraw at the second stage if *and only if* $n > \hat{n}(s)$*. Furthermore,* $\hat{n}(s)$ *is continuous and non-decreasing in* s*.*

Figure 4, which depicts $\hat{n}(s)$, shows that IP investors who maintain their deposits at the first stage withdraw at the second stage if and only if the realized first-stage withdrawal n (equivalently, λ, if only impatient investors withdraw at the first stage) exceeds $\hat{n}(s)$. The threshold, illustrated by the curves in the four regions of the signal s, is weakly increasing in s because the remaining IP investors at the second stage are willing to absorb greater liquidation costs imposed by a larger first-stage withdrawal when s (and, hence, $V(s)$) is higher. Depending on s, $\hat{n}(s)$ may be less than, equal to, or greater than the UP investors' withdrawal threshold $\bar{n}$. Consequently, an IP investor who does not withdraw at the first stage may encounter one of four possible scenarios at the second stage.

1. *Neither IP nor UP investors withdraw at the second stage.* In this scenario, there are no additional liquidation costs beyond those imposed by the first-stage liquidity withdrawals λ. Thus, an IP investor's expected payoff from not withdrawing is $\frac{1-\lambda(1+\gamma)}{1-\lambda}V(s)$.
2. *IP investors withdraw at the second stage but UP investors do not.* In this case, IP investors receive their initial deposit, 1, from withdrawing at the second stage.
3. *UP investors withdraw at the second stage but IP investors do not.* In this case, IP investors must bear the additional liquidation costs imposed by withdrawing UP investors (mass $(1-\alpha)(1-\lambda)$). Hence, an IP investor's expected payoff from not withdrawing is $\frac{1-[\lambda+(1-\alpha)(1-\lambda)](1+\gamma)}{\alpha(1-\lambda)}V(s)$.
4. *Both IP and UP investors withdraw at the second stage.* If this happens, then the fund must liquidate all of its remaining investment after the first-stage liquidity withdrawals, $1-\lambda(1+\gamma)$, and evenly distribute the liquidation proceeds among the IP and UP investors (mass $1-\lambda$), who each receive $\frac{1-\lambda(1+\gamma)}{(1-\lambda)(1+\gamma)}$.

We now compute the ex ante expected payoff to a non-withdrawing IP investor at the first stage, accounting for these four possible second-stage scenarios. At the first stage, IP investors understand that the likelihood of each scenario occurring depends on s. As illustrated by Figure 4 and outlined in Lemma 3, there are four regions of s to consider.

First, if $s \in [s^*, \hat{s}_2(\bar{n})]$, then $\hat{n}(s) < \bar{n}$. In this region, s is so low that IP investors may withdraw at the second stage even though UP investors maintain their deposits until date 2 and, thus, do not impose any additional liquidation costs on IP investors. Hence, scenario 3 is impossible, and an IP investor withdraws at the first stage unless

$$1 \leq \int_0^{\hat{n}(s)} \frac{1-\lambda(1+\gamma)}{1-\lambda} V(s) g(\lambda) \, \mathrm{d}\lambda + \int_{\hat{n}(s)}^{\bar{n}} g(\lambda) \, \mathrm{d}\lambda + \int_{\bar{n}}^{\bar{\lambda}} \frac{1-\lambda(1+\gamma)}{(1-\lambda)(1+\gamma)} g(\lambda) \, \mathrm{d}\lambda. \tag{20}$$

Second, if $s \in (\hat{s}_2(\bar{n}), \check{s}_2(\bar{n})]$, then $\hat{n}(s) = \bar{n}$. There is complete strategic complementarity among the UP and remaining IP investors in this region: IP investors completely ignore their signal s and always take the same action at the second stage as UP investors. This occurs because s is sufficiently high for IP investors to bear the liquidation costs imposed by the first-stage liquidity withdrawals but not high enough to also bear the additional liquidation costs that arise if UP investors make a second-stage withdrawal. Thus, neither scenario 2 nor 3 is possible, and an IP investor makes a first-stage withdrawal unless

$$1 \leq \int_0^{\bar{n}} \frac{1-\lambda(1+\gamma)}{1-\lambda} V(s) g(\lambda) \, \mathrm{d}\lambda + \int_{\bar{n}}^{\bar{\lambda}} \frac{1-\lambda(1+\gamma)}{(1-\lambda)(1+\gamma)} g(\lambda) \, \mathrm{d}\lambda. \tag{21}$$

Third, if $s \in (\check{s}_2(\bar{n}), \check{s}_2(\bar{\lambda})]$, then $\hat{n}(s) > \bar{n}$. Here, s is high enough that when λ is not too large, IP investors are willing to bear not only the liquidation costs generated by the first-stage liquidity withdrawals but also the additional costs generated by UP investors' second-stage withdrawals. Scenario 2 is not possible, and an IP investor withdraws at the first stage unless

$$\begin{aligned} 1 \leq & \int_0^{\bar{n}} \frac{1-\lambda(1+\gamma)}{1-\lambda} V(s) g(\lambda) \, \mathrm{d}\lambda \\ & + \int_{\bar{n}}^{\hat{n}(s)} \frac{1-[\lambda+(1-\alpha)(1-\lambda)](1+\gamma)}{\alpha(1-\lambda)} V(s) g(\lambda) \, \mathrm{d}\lambda + \int_{\hat{n}(s)}^{\bar{\lambda}} \frac{1-\lambda(1+\gamma)}{(1-\lambda)(1+\gamma)} g(\lambda) \, \mathrm{d}\lambda. \end{aligned} \tag{22}$$

Last, if $s \in (\check{s}_2(\bar{\lambda}), 1]$, then $\hat{n}(s) = \bar{\lambda}$, which is bigger than $\bar{n}$. In this region, which is a limiting case of the third scenario described above, s is so high that IP investors are willing to bear the maximum possible liquidation costs that may be imposed by *all* other investors, including those generated by impatient investors at the first stage and UP investors at the second stage. The condition in (22) reduces to

$$1 \leq \int_0^{\bar{n}} \frac{1-\lambda(1+\gamma)}{1-\lambda} V(s) g(\lambda)\, \mathrm{d}\lambda + \int_{\bar{n}}^{\bar{\lambda}} \frac{1-[\lambda+(1-\alpha)(1-\lambda)](1+\gamma)}{\alpha(1-\lambda)} V(s) g(\lambda)\, \mathrm{d}\lambda. \quad (23)$$

The LHSs (RHSs) of (20) – (23) represent an IP investor's payoff from withdrawing (not withdrawing) at the first stage. Theorem 3 characterizes the IP investors' first-stage decisions.

Theorem 3. *Given the UP investors' second-stage withdrawal threshold $\bar{n}$, there exists a unique signal threshold $\hat{s}_1 \in (s^*, 1)$ such that IP investors withdraw at the first stage if and only if $s < \hat{s}_1$. Furthermore, $\hat{s}_1$ is decreasing in $\bar{n}$.*

Equation (A9) in Appendix A characterizes $\hat{s}_1$, which is illustrated in Figure 5. The four regions of $V(s)$ in the figure correspond to the four regions in Figure 4, and $\hat{s}_1$ is the point at which the curve, which represents the ex ante expected payoff to a non-withdrawing IP investor at the first stage, intersects 1. The signal threshold $\hat{s}_1$ is strictly greater than s^* due to the expected liquidation costs imposed on a non-withdrawing IP investor by other investors who may withdraw before date 2. Additionally, $\hat{s}_1$ is decreasing in $\bar{n}$. As $\bar{n}$ increases, UP investors become less likely to make a second-stage withdrawal, which reduces the expected liquidation costs imposed on non-withdrawing IP investors. Anticipating this, IP investors do not require as high of a signal to maintain their deposits at the first stage, so $\hat{s}_1$ decreases.

While Theorem 2 indicates that $\bar{n}$ is unique given $\hat{s}_1$ and Theorem 3 indicates that $\hat{s}_1$ is unique given $\bar{n}$, we are unable to ascertain whether the pair of strategies is unique (i.e., whether there are multiple pairs of $\hat{s}_1$ and $\bar{n}$ that are consistent with one another) without making additional parametric assumptions, which we refrain from doing to maintain generality. Thus, there may be multiple triplets, $\{\hat{s}_1, \hat{s}_2(n), \bar{n}\}$, each constituting a PBE for the overall two-stage game. Multiple equilibria are possible due to the *endogenous* relations between $\hat{s}_1$ and $\bar{n}$.

Nevertheless, the number of equilibria is not essential to our analysis. Whenever there are multiple equilibria, they can be Pareto ranked. Because $\hat{s}_1 > s^*$ in any equilibrium

(Theorem 3), we simply select the "best" equilibrium, i.e., the one with the lowest $\hat{s}_1$ and, correspondingly, the highest $\bar{n}$. As we show below in Section 4, there is a unique first-stage signal threshold for IP investors that is less than s^* when investors must pay a redemption fee to withdraw before date 2. Thus, regardless of the potential for multiple equilibria in the absence of a fee, a redemption fee always reduces the IP investors' first-stage signal threshold.

4 Equilibrium Analysis with a Redemption Fee

We now consider an environment in which investors must pay a redemption fee, ϕ, if they withdraw prior to date 2 (at either stage of date 1). To streamline the analysis, we focus on the case wherein the fee exactly offsets the liquidation costs generated by early withdrawals. An investor who requests an early withdrawal in the presence of a fee receives $1-\phi$, so the fund must liquidate $(1-\phi)(1+\gamma)$ units of its asset for each unit of deposit withdrawn at date 1. When

$$\phi = \frac{\gamma}{1+\gamma}, \tag{24}$$

the fund liquidates exactly 1 unit of the asset for each unit of deposit withdrawn early (recall that absent a fee, the fund must liquidate $1+\gamma$ units for each unit withdrawn early). In this case, the liquidation costs generated by early withdrawers are borne entirely by those investors themselves instead of being imposed on late withdrawers, as transpires without a fee. That is, the fee set as in (24) fully internalizes the liquidation costs generated by early withdrawers.

This setting completely eliminates the payoff externality, and hence the first-mover advantage, that would otherwise exist among investors. This allows us to focus on studying how the fee affects learning by UP investors, which is a main novelty of our model. Although we explicitly analyze only this specific case, the intuition developed here should be robust to any $\phi \in \left(0, \frac{\gamma}{1+\gamma}\right]$. In particular, it is straightforward to show that as ϕ increases, both IP and UP investors place more emphasis on their beliefs about the asset payoff and less on the potential payoff externality. A fee set as in (24) is the limiting case wherein the payoff externality vanishes and investors base their decisions solely on their beliefs about fundamentals.

We conjecture and verify the existence of a unique equilibrium with a fee analogous to the one defined in Section 3. We use a superscript "ϕ" to denote the equilibrium strategies of the IP and UP investors, $\{\hat{s}_1^\phi, \hat{s}_2^\phi, \bar{n}^\phi\}$, when they must pay a fee if they withdraw before date 2.

4.1 IP Investors' Equilibrium Strategies with a Fee

The tradeoffs faced by IP investors at the second stage (provided, of course, that they did not withdraw at the first stage, i.e., $w = 0$) are identical to those faced by them at the first stage. Unlike in the no-fee benchmark case, the IP investors' uncertainty about the mass of impatient investors λ is irrelevant to their first-stage decisions when a fee is imposed. This is because the redemption fees paid by impatient investors fully compensate the fund for the liquidation costs generated by their withdrawals. Similarly, an IP investor's second-stage strategy (if she chooses $w = 0$) is independent of both the first-stage withdrawal n and the UP investors' second-stage actions because the fees paid by withdrawing investors fully internalize the liquidation costs generated by those withdrawers. As a result, regardless of others' withdrawal decisions, the expected payoff to an IP investor who keeps her deposit until date 2 is always $V(s)$, and her payoff from withdrawing at either stage of date 1 is always $1 - \phi = (1 + \gamma)^{-1}$. Therefore, IP investors' first-stage decisions are identical to their second-stage decisions (i.e., $\hat{s}_1^\phi = \hat{s}_2^\phi$).

This is in sharp contrast to the results obtained in the no-fee benchmark. There, the IP investors' first-stage and second-stage signal thresholds, $\hat{s}_1$ and $\hat{s}_2(n)$, generally differ: $\hat{s}_1$ may be either greater or less than $\hat{s}_2(n)$, depending on n. The difference between the two thresholds arises from the fact that without a fee, IP investors' withdrawal decisions are based on not only their information but also the potential payoff externality. While the former remains constant from the first to the second stage, the latter generally does not. At the first stage, IP investors face uncertainty regarding both the size of the first-stage liquidity withdrawals λ and the potential for second-stage withdrawals by UP investors. This uncertainty disappears at the second stage because IP investors can infer both λ and whether UP investors will make a second-stage withdrawal given any realized first-stage withdrawal size n.

Recall, our model assumes that if an investor knows for sure that she will withdraw before date 2 but is indifferent between withdrawing at the first or second stage of date 1, then she withdraws at the first stage rather than waiting until the second. Therefore, if IP investors withdraw at date 1, then they will withdraw at the first stage; if they do not withdraw at the first stage, then they will remain invested until date 2. Hence, IP investors make no second-stage withdrawals in equilibrium. This leads to the following theorem.

Theorem 4. *There exists a unique signal threshold $\hat{s}_1^\phi$, characterized by*

$$V(\hat{s}_1^\phi) = \frac{1}{1+\gamma} \iff \hat{s}_1^\phi = s^\gamma, \tag{25}$$

such that IP investors withdraw at the first stage of date 1 if and only if $s < \hat{s}_1^\phi$; if $s \geq \hat{s}_1^\phi$, then they keep their deposits until date 2.

IP investors' withdrawal decisions are purely information-based. As discussed above, this is because the fee removes the payoff externality that would otherwise exist among investors. Not only does this allow IP investors to focus solely on their signal and ignore the actions of others, but it also eliminates the first-mover advantage by forcing IP investors to bear their own liquidation costs. Consequently, IP investors' decisions are socially optimal: they withdraw at date 1 if and only if their signal indicates that the asset's expected payoff $V(s)$ is below its liquidation value $(1+\gamma)^{-1}$. In other words, their signal threshold $\hat{s}_1^\phi$ equals s^γ, which is the signal cutoff that defines the ex post efficient liquidation threshold in (3).

In contrast, the IP investors' withdrawal decisions are socially suboptimal in the benchmark case without a fee. There, both the IP investors' first stage and second-stage signal thresholds, $\hat{s}_1$ and $\hat{s}_2(n)$, are always greater than s^γ (Theorems 1 and 3), so they withdraw more frequently than is socially optimal. This inefficiency arises because: (i) IP investors who do not withdraw must bear the liquidation costs generated by others who withdraw if there is no fee to internalize the payoff externality; and (ii) an IP investor who withdraws without paying a fee does not bear any of the liquidation costs generated by her own withdrawal.

4.2 UP Investors' Equilibrium Strategies with a Fee

Although the redemption fee removes the payoff externality generated by early withdrawers, it does not eliminate information asymmetry between IP and UP investors. However, the fee alters the UP investors' inference problem because, as shown by Theorem 4, it changes the IP investors' first-stage signal threshold.

Like it does for IP investors, the removal of the payoff externality also enables UP investors to base their second-stage decisions solely on their conditional expectations of the asset payoff. Any first-stage withdrawals are relevant only to the extent that they alter the UP investors' beliefs about s. Liquidation costs generated by prior (first-stage) or contemporaneous (second-stage) withdrawals are not pertinent to the UP investors' decisions because those costs are fully internalized by the investors who generate them.

After observing the first-stage withdrawal size n, UP investors update their beliefs about s according to Lemma 1 (specifically, equation (10)), with $\hat{s}_1^{\phi}$ $(= s^{\gamma})$ in place of $\hat{s}_1$, and then compute their conditional expectations of the risky asset's payoff:[10]

$$\mathbb{E}[\tilde{v}|n, \phi] = F(s^{\gamma}|n)\mathbb{E}[\tilde{v}|s < s^{\gamma}] + (1 - F(s^{\gamma}|n))\mathbb{E}[\tilde{v}|s \geq s^{\gamma}]. \tag{26}$$

A UP investor withdraws when her payoff from doing so, $1 - \phi$, exceeds $\mathbb{E}[\tilde{v}|n, \phi]$.

Theorem 5. *There exists a unique withdrawal threshold* $\bar{n}^{\phi} \in [0, \bar{\lambda}]$, *characterized by*

$$\bar{n}^{\phi} = \sup\left\{n \in [0, \bar{\lambda}] : (1 + \gamma)^{-1} \leq \mathbb{E}[\tilde{v}|n, \phi]\right\}, \tag{27}$$

such that UP investors do not withdraw at the second stage if and only if $n \leq \bar{n}^{\phi}$. *Additionally,*

[10]Note that $\mathbb{E}[\tilde{v}|n, \phi]$ is similarly defined as $\mathbb{E}[\tilde{v}|n]$ in (11), with s^{γ} in replace of $\hat{s}_1$. The "ϕ" in $\mathbb{E}[\tilde{v}|n, \phi]$ is meant to capture the notion that the conditional expectation here is computed in the presence of a fee.

a redemption fee decreases the UP investors' withdrawal threshold (i.e., $\bar{n}^{\phi} < \bar{n}$*) if and only if*

$$\mathbb{E}[\tilde{v}|\bar{n}, \phi] < \frac{1}{1+\gamma}. \tag{28}$$

Theorem 5 shows that the UP investors' withdrawal threshold with a fee, $\bar{n}^{\phi}$, may be either higher or lower than the analogous threshold without a fee, $\bar{n}$. The idea is as follows. In the no-fee benchmark case, for a given n and a given first-stage signal threshold $\hat{s}_1$ for IP investors, a UP investor's conditional expectation about the date-2 asset return is (Lemma 1)

$$\mathbb{E}[\tilde{v}|n] = F(\hat{s}_1|n)\mathbb{E}[\tilde{v}|s < \hat{s}_1] + (1 - F(\hat{s}_1|n))\mathbb{E}[\tilde{v}|s \geq \hat{s}_1].$$

By lowering the IP investors' first-stage signal threshold from $\hat{s}_1$ to s^{γ}, the fee affects $\mathbb{E}[\tilde{v}|n]$ in two distinct ways. On the one hand, for a given n, a lower threshold decreases the posterior likelihood (from $F(\hat{s}_1|n)$ to $F(s^{\gamma}|n)$) that IP investors made a first-stage withdrawal, which in turn elevates $\mathbb{E}[\tilde{v}|n]$, holding the conditional expectations $\mathbb{E}[\tilde{v}|s < \hat{s}_1]$ and $\mathbb{E}[\tilde{v}|s \geq \hat{s}_1]$ fixed. We refer to this as a "likelihood effect." On the other hand, a lower threshold also causes UP investors to be more pessimistic about $\tilde{v}$, *regardless* of their beliefs about whether IP investors withdrew at the first stage, as $\mathbb{E}[\tilde{v}|s < s^{\gamma}] < \mathbb{E}[\tilde{v}|s < \hat{s}_1]$ and $\mathbb{E}[\tilde{v}|s \geq s^{\gamma}] < \mathbb{E}[\tilde{v}|s \geq \hat{s}_1]$. This lowers $\mathbb{E}[\tilde{v}|n]$ for a given n, holding $F(\hat{s}_1|n)$ fixed. We refer to this as a "distribution effect."

The likelihood effect captures the intuitive notion that a lower signal threshold shifts the UP investors' posterior beliefs about s toward the higher distribution (e.g., $s \geq \hat{s}_1$) and, thus, raises $\mathbb{E}[\tilde{v}|n]$. The distribution effect captures the perhaps more subtle consequence that the UP investors' posterior beliefs about $\tilde{v}$, conditional on being either less than or greater than the threshold, decrease and, thus, $\mathbb{E}[\tilde{v}|n]$ declines. Thus, by lowering the signal threshold from $\hat{s}_1$ to s^{γ}, for a given n a redemption fee may lead to $\mathbb{E}[\tilde{v}|n, \phi] > \mathbb{E}[\tilde{v}|n]$ (if the likelihood effect dominates) or $\mathbb{E}[\tilde{v}|n, \phi] < \mathbb{E}[\tilde{v}|n]$ (if the distribution effect dominates).

Theorem 5 (specifically, (28)) implies that the distribution effect dominates and UP investors are more prone to withdraw in the presence of a fee (i.e., $\bar{n}^{\phi} < \bar{n}$) if the corresponding threshold without a fee, $\bar{n}$, is sufficiently high (note that $\mathbb{E}[\tilde{v}|n, \phi]$, like $\mathbb{E}[\tilde{v}|n]$, is non-increasing in n; see Lemma 1). The intuition is as follows. Suppose $\bar{n}$ is high enough so that the inequality in (28) holds. When $n = \bar{n}$, without a fee UP investors are just indifferent between withdrawing and remaining invested (based on the definition of $\bar{n}$). Given such a large first-stage withdrawal n, UP investors assign a very high likelihood to the possibility that IP investors already withdrew at the first stage, and a decrease in the signal threshold does not substantially alter that belief. Consequently, the distribution effect dominates, UP investors strictly prefer to withdraw in the presence of a fee and, therefore, $\bar{n}^{\phi} < \bar{n}$. Conversely, suppose $\bar{n}$ is smaller so that the inequality in (28) does not hold. When $n = \bar{n}$, UP investors' posterior beliefs about whether IP investors already withdrew are not as strong given that n is smaller. As a result, a decrease in the signal threshold can significantly lower the probability that they assign to a first-stage withdrawal by IP investors, so the likelihood effect dominates. In this case, while UP investors are indifferent between withdrawing and staying absent a fee, they strictly prefer to stay invested in the presence of a fee and, therefore, $\bar{n}^{\phi} > \bar{n}$.

5 Impact of the Redemption Fee: A Welfare Analysis

As discussed in Section 4, while a redemption fee leads to socially optimal actions by IP investors, it influences learning by UP investors and may lead to either an improvement or a deterioration in outcomes. In this section, we examine the effects of the fee on both social welfare and the welfare of individual investors. Note first that impatient investors, who always withdraw at the first stage of date 1, suffer a welfare loss in the presence of a fee because they receive only $1 - \phi$ instead of 1. The analysis, therefore, focuses on IP and UP investors.

5.1 IP Investors' Welfare and the Last-Mover Advantage

The impact of the fee on IP investors' welfare depends on the signal s. Recall, the IP investors' first-stage withdrawal threshold is s^{γ} with a fee and $\hat{s}_1$ without a fee, and $s^{\gamma} < s^* <$

$\hat{s}_1$, where $V(s^*) = 1$. If $s < s^*$, then IP investors withdraw at the first stage absent a fee and receive 1, but they receive strictly less than 1 when a fee is imposed. With a fee, they receive $1 - \phi$ if they withdraw or $V(s) < 1$ (given $s < s^*$) if they remain invested until date 2. Conversely, if $s \geq s^*$, then IP investors stay invested until date 2 with a fee and receive $V(s) \geq 1$, but they receive (weakly) less than $V(s)$ in the absence of a fee. Without a fee, if IP investors withdraw at (either stage of) date 1, then they receive at most $1 = V(s^*) \leq V(s)$; if they instead keep their deposits until date 2, then they receive less than $V(s)$ due to the liquidation costs generated by others who withdraw at date 1 (e.g., impatient investors, as $\lambda > 0$ almost surely). The following proposition summarizes these results.

Proposition 1. *Compared to the no-fee case, imposing a redemption fee on early withdrawers decreases IP investors' welfare when $s < s^*$ but increases their welfare when $s > s^*$. The fee has no impact on IP investors' welfare when $s = s^*$.*

Proposition 1 shows that the redemption fee causes IP investors to suffer a loss when the asset's expected payoff is less than 1 (i.e., when $s < s^*$) but enjoy a gain when the asset's expected payoff is greater than 1 (i.e., when $s > s^*$). This occurs because the fee removes the payoff externality, so an IP investor's payoff hinges solely on the asset's fundamental value.

Notably, the fee changes the nature of the IP investors' competitive advantage over UP investors. Without a fee, IP investors possess a first-mover advantage: upon receiving a low signal they withdraw before UP investors and impose the associated liquidation costs on UP investors. The disappearance of this first-mover advantage in the presence of a fee explains the IP investors' welfare loss when $s < s^*$. In contrast, with a fee, IP investors gain a *last-mover advantage*: they remain invested until date 2 upon receiving a high signal and benefit from the fees paid by early withdrawers (including UP investors in some cases, as described below in Section 5.2). The rise of this last-mover advantage explains the IP investors' welfare gain with a fee when $s > s^*$, in which case the fee essentially creates a wealth transfer from early withdrawers to (non-withdrawing) IP investors. Thus, importantly, while the fee diminishes the IP investors' information advantage in bad states (i.e., low s) through the removal of the first-mover advantage, it creates a last-mover advantage for them in good states (i.e., high s).

5.2 UP Investors' Welfare and a Wealth Transfer

The effect of the fee on UP investors' welfare depends on the signal s, the realized first-stage withdrawal n, and whether the UP investors' withdrawal threshold with a fee, $\bar{n}^{\phi}$, is greater or less than the corresponding threshold without a fee, $\bar{n}$. There are four cases.[11]

First, if $n \leq \min\{\bar{n}, \bar{n}^{\phi}\}$, then UP investors keep their deposits until date 2 irrespective of the fee. Because the fees paid by early withdrawers offset the associated liquidation costs, (non-withdrawing) UP investors benefit from the fee in this case.

Second, if $\bar{n}^{\phi} < \bar{n}$ and $n \in (\bar{n}^{\phi}, \bar{n}]$, then the fee converts UP investors from non-withdrawers to withdrawers. With a fee, UP investors withdraw and receive $1 - \phi$. Without a fee, they keep their deposits until date 2, and their payoffs depend on the actions of IP investors, which in turn depend on the signal s. As shown in Appendix A, UP investors benefit from the fee in this case when s is sufficiently low, but suffer a loss when s is sufficiently high. The reason is that if s is sufficiently low, by converting from non-withdrawers to withdrawers, UP investors avoid holding a low-valued asset; whereas if s is higher, then UP investors suffer a detriment because they pay a fee to rid themselves of a high-valued asset.

Third, if $\bar{n} < \bar{n}^{\phi}$ and $n \in (\bar{n}, \bar{n}^{\phi}]$, then, in contrast to the second case, the fee converts UP investors from withdrawers to non-withdrawers. UP investors remain invested until date 2 with a fee and expect to receive $V(s)$, but they withdraw absent a fee and receive an amount that depends on the IP investors' actions. We show in Appendix A that, different from the second case, by converting from withdrawers to non-withdrawers, UP investors benefit from the fee if the signal s is sufficiently high because they retain a high-valued asset but are harmed when s is sufficiently low because they hold a low-valued asset.

Fourth, if $n > \max\{\bar{n}, \bar{n}^{\phi}\}$, then UP investors withdraw regardless of the fee arrangement. With a fee, a UP investor receives $1 - \phi$. Absent a fee, what she receives from her withdrawal

[11] We organize the analysis in this section according to the endogenous variable n because UP investors base their decisions on n. Alternatively, the analysis could be organized based on the exogenous variable λ.

depends on the IP investors' actions. The fee is beneficial to UP investors in this case when s is sufficiently low because they benefit from fees paid by withdrawing IP investors. However, the fee is detrimental when s is sufficiently high because, like in the second case discussed above, they pay a fee to rid themselves of a high-valued asset. The following proposition summarizes the analysis.

Proposition 2. *Compared to the no-fee case, imposing a redemption fee on early withdrawers increases (decreases) the UP investors' welfare when: (i) UP investors do not withdraw regardless of the fee arrangement, i.e.,* $n \leq \min\{\bar{n}, \bar{n}^{\phi}\}$*; (ii) the fee converts UP investors from non-withdrawers to withdrawers, i.e.,* $n \in (\bar{n}^{\phi}, \bar{n}]$*, and the signal is sufficiently low (high); (iii) the fee converts UP investors from withdrawers to non-withdrawers, i.e.,* $n \in (\bar{n}, \bar{n}^{\phi}]$*, and the signal is sufficiently high (low); or (iv) UP investors withdraw regardless of the fee arrangement, i.e.,* $n > \max\{\bar{n}, \bar{n}^{\phi}\}$*, and the signal is sufficiently low (high).*

As alluded to above, the fee generates a potential wealth transfer from UP to IP investors. Specifically, in the second and fourth cases described above, when s is sufficiently high UP investors withdraw and pay a fee while IP investors maintain their deposits. In these cases, fees paid by (withdrawing) UP investors effectuate a wealth transfer to (non-withdrawing) IP investors. That is, as discussed in Section 5.1, the fee transforms IP investors' first-mover advantage into a last-mover advantage, enabling them to extract wealth from UP investors.

5.3 Social Welfare

We now analyze the impact of a redemption fee on social welfare, measured as the aggregate amount received by all investors from the fund over both dates 1 and 2. We denote social welfare in the absence and presence of a fee by Ω and Ω^{ϕ}, respectively.

Investment efficiency drives social welfare. Recall that it is socially efficient to liquidate the fund's investment in the risky asset at date 1 if and only if $s < s^{\gamma}$. Thus, conditional on s, each unit remaining invested in the risky asset until date 2 generates a social welfare gain of $V(s) - V(s^{\gamma})$ if $s > s^{\gamma}$ but a social welfare loss of $V(s^{\gamma}) - V(s)$ if $s < s^{\gamma}$. Therefore, the change in social welfare resulting from the redemption fee is given by

$$\Omega^{\phi} - \Omega = \Theta(V(s) - V(s^{\gamma})), \tag{29}$$

where Θ denotes the change in aggregate investment in the risky asset caused by the imposition of the fee. A positive (negative) Θ indicates that the fee decreases (increases) early liquidation.

Impatient investors withdraw at the first stage regardless of the fee, so we focus on IP and UP investors. Because the impact of the fee on the actions of IP and UP investors depends on n and s, there are a total of twelve distinct scenarios to consider.[12]

Table 1 lists the twelve scenarios and the corresponding Θ for each scenario, along with the sign of $\Omega^{\phi} - \Omega$ (i.e., [+] or [−]). The impact of the fee on UP investors' actions depends on n, which varies across the four rows, whereas the effect of the fee on IP investors' behaviors depends on s, which varies across the three columns. We derive Θ for each scenario in Appendix B. Here, we discuss the intuition behind the results.

As shown in the first row of the table, when UP investors maintain their deposits until date 2 irrespective of the presence of a fee, the fee increases aggregate investment in the risky asset (i.e., $\Theta > 0$) regardless of the signal s. This occurs for two reasons. First, fees paid by impatient and IP investors reduce early liquidation caused by their withdrawals. Second, the fee induces IP investors to make socially efficient withdrawal decisions, whereas they make socially inefficient decisions in the absence of a fee by withdrawing when $s \in (s^{\gamma}, \max\{\hat{s}_1, \hat{s}_2(n)\})$. Because increasing investment is socially beneficial only when $s > s^{\gamma}$, the fee decreases social welfare when $s < s^{\gamma}$ but increases social welfare when $s > s^{\gamma}$. Thus, the fee amplifies the impact of the risky asset on social welfare (by increasing investment in the asset), lowering social welfare in bad states and raising social welfare in good states.

When the fee converts UP investors from non-withdrawers to withdrawers, aggregate investment may either rise or fall, as shown in the second row of Table 1. If the asset's liquidation value is greater than its expected date-2 payoff (i.e., if $s < s^{\gamma}$), then all investors withdraw.

[12] Again, we organize the analysis around n rather than λ because UP investors base their decisions on n.

Obviously, this decreases investment in the risky asset (i.e., $\Theta < 0$), so social welfare rises. Conversely, if the asset's expected payoff is greater than the liquidation value (i.e., if $s > s^{\gamma}$), then IP investors keep their deposits until date 2. If the fee converts IP investors from withdrawers to non-withdrawers, i.e., if $s \in [s^{\gamma}, \max\{\hat{s}_1, \hat{s}_2(n)\})$, then the net effect on investment depends on the relative masses of IP investors versus UP investors. When α is small, the increase in early withdrawals by UP investors outweighs the reduction in early withdrawals by IP investors (i.e., $\Theta < 0$), so social welfare declines. Social welfare rises, however, when α is large because the fee increases investment (i.e., $\Theta > 0$). Alternatively, if the fee does not affect IP investors' actions, i.e., if $s \geq \max\{\hat{s}_1, \hat{s}_2(n)\}$, in which case IP investors remain invested until date 2 irrespective of the fee, then the fee decreases social welfare because the reduction in investment stemming from withdrawals made by UP investors outweighs the fees paid by impatient investors.

The third row indicates that the redemption fee increases aggregate investment (i.e., $\Theta > 0$) when the fee converts UP investors from withdrawers to non-withdrawers. Similar to the scenarios described in the first row, social welfare declines when the risky asset's expected payoff is less than its liquidation value (i.e., when $s < s^{\gamma}$) but rises when the opposite holds (i.e., when $s > s^{\gamma}$). In this case, the fee lowers social welfare in bad states but raises social welfare in good states, thereby amplifying the effect of the risky asset on social welfare.

Finally, the last row shows that aggregate investment either increases (i.e., $\Theta > 0$) or does not change (i.e., $\Theta = 0$) as a result of imposing the fee when UP investors withdraw regardless of the fee. If s is sufficiently low (i.e., if $s < s^{\gamma}$), then all investors withdraw and the fund is liquidated regardless of the fee arrangement, so the fee has no impact on social welfare. Conversely, if s is higher (i.e., if $s > s^{\gamma}$), then fees paid by withdrawing impatient and UP investors increase investment. Furthermore, the fee converts IP investors from withdrawers to non-withdrawers when $s \in [s^{\gamma}, \max\{\hat{s}_1, \hat{s}_2(n)\})$. Thus, the fee raises social welfare when $s > s^{\gamma}$ because it increases investment when the asset's expected payoff is greater than its liquidation value. Here, the fee raises social welfare in good states but is irrelevant in bad states. The following proposition summarizes the welfare effects of a redemption fee.

Proposition 3. *Provided that the redemption fee does not convert UP investors from non-withdrawers to withdrawers, the fee increases aggregate investment in the risky asset, which raises social welfare whenever such investment is socially efficient (i.e., $s > s^\gamma$) but lowers social welfare whenever such investment is socially inefficient (i.e., $s < s^\gamma$). Conversely, if the fee converts UP investors from non-withdrawers to withdrawers, then the fee: decreases aggregate investment whenever such investment is socially inefficient (i.e., $s < s^\gamma$), which raises social welfare; may either increase or decrease aggregate investment whenever such investment is socially efficient but not highly valuable (i.e., $s \in (s^\gamma, \max\{\hat{s}_1, \hat{s}_2(n)\})$), which either lowers or raises social welfare; and decreases aggregate investment whenever such investment is socially efficient and highly valuable (i.e., $s \geq \max\{\hat{s}_1, \hat{s}_2(n)\}$), which lowers social welfare. The fee has no effect on social welfare whenever $s = s^\gamma$.*

Proposition 3 highlights two noteworthy effects of the redemption fee on social welfare. First, as long as it does not convert UP investors from non-withdrawers to withdrawers, the fee amplifies the effects of the risky asset on social welfare, raising social welfare in states wherein it is ex post efficient to liquidate the asset at date 1 (i.e., $s > s^\gamma$) but lowering social welfare in states wherein early liquidation is socially inefficient (i.e., $s < s^\gamma$). This occurs because the fee reduces the potential for runs and forces withdrawing investors to pay for the liquidation costs that they generate, which increases aggregate investment. Second, the fee raises social welfare in states wherein early liquidation is socially inefficient (i.e., $s < s^\gamma$) only when it converts UP investors from non-withdrawers to withdrawers. In other words, the redemption fee enhances social welfare in bad states only if it increases the potential for runs.

6 Preemptive Runs

In this section, we examine whether the prospect of a redemption fee being levied in the future causes investors to withdraw preemptively before the fee is imposed. Much of the analysis here mirrors that in the benchmark setting, so we place the derivations in Appendix C and only discuss the intuition underlying the results.

We assume that only second-stage withdrawals at date 1 are subject to a fee, set as in (24). Investors know at the first stage that a fee will be imposed at the second stage. Although this setup is (perhaps overly) simplistic, it nonetheless captures many of the tradeoffs associated with an impending fee without any loss of tractability. Again, we conjecture and verify the existence of an equilibrium like the ones characterized in Sections 3 and 4. Here, we use a superscript "φ" to denote the investors' equilibrium strategies, $\{\hat{s}_1^\varphi, \hat{s}_2^\varphi(n), \bar{n}^\varphi\}$, when they must pay a fee if they withdraw at the second (but not the first) stage.

A second-stage fee internalizes the liquidation costs generated by second-stage withdrawers but *not* those by first-stage withdrawers, and the absence of a fee at the first stage allows first-stage withdrawers to impose the liquidation costs (generated by their withdrawals) entirely on second-stage investors. Thus, although second-stage withdrawals do not produce any *additional* liquidation costs that must be borne by investors who maintain their deposits until date 2, a payoff externality still exists among second-stage investors because their second-stage actions determine how liquidation costs generated by first-stage withdrawals are shared among them. Second-stage investors share these costs if they all remain invested until date 2, but one group of investors must bear the entire burden if they keep their deposits at the second stage while the other group withdraws.

The IP and UP investors' second-stage strategies, as characterized by Theorems C.1 and C.2 in Appendix C, are similar to their corresponding strategies in the no-fee benchmark setting, though the precise thresholds $\hat{s}_2^\varphi(n)$ and $\bar{n}^\varphi$ generally differ from $\hat{s}_2(n)$ and $\bar{n}$ because the tradeoffs are different with a second-stage fee. Importantly, the presence of a second-stage fee affects the IP investors' first-stage withdrawal decisions (as discussed below), which alters the UP investors' inference problem and, therefore, may either increase or decrease the UP investors' withdrawal threshold for the same reasons discussed in Section 4.2.

IP investors' first-stage strategies are characterized in Theorem C.3 in Appendix C. With a second-stage fee, an IP investor's uncertainty regarding the mass of impatient investors λ affects her first-stage strategy because she may end up bearing the liquidation costs generated by impatient investors if she does not withdraw at the first stage. In contrast, when a fee is present at both stages, IP investors' first-stage decisions are based solely on their signal because the fee fully internalizes the liquidation costs at both stages. This leads to socially optimal decisions and a lower likelihood of a first-stage withdrawal (i.e., $\hat{s}_1^{\phi} = s^{\gamma} < \hat{s}_1$) compared to the no-fee benchmark case, as discussed in Section 4.1. Conversely, when a fee is present only at the second stage, IP investors must account for the premature withdrawals by impatient (and possibly) UP investors. While this results in socially suboptimal decisions (i.e., $\hat{s}_1^{\varphi} > s^{\gamma}$), as the following proposition shows, the likelihood of IP investors making a first-stage withdrawal may either rise or fall relative to the benchmark case without a fee at either stage, depending on the underlying parameters.

Proposition 4. *Compared to the no-fee benchmark case, imposing a redemption fee only on second-stage withdrawals may either increase or decrease the likelihood of IP investors withdrawing at the first stage.*

Proposition 4 states that preemptive runs (i.e., withdrawals by IP investors at the first stage) may become either more or less likely than in the no-fee benchmark case when a second-stage redemption fee is imposed. On the one hand, a second-stage fee raises the inclination for IP investors to make a first-stage withdrawal because they receive their entire deposit, 1, if they withdraw at the first stage but only $1 - \phi$ if they withdraw at the second stage. On the other hand, a second-stage fee lowers the incentive for IP investors to make a first-stage withdrawal because, with a second-stage fee, any potential second-stage withdrawals made by UP investors generate a wealth transfer from UP to IP investors if s is sufficiently high and IP investors keep their deposits until date 2. The reason is that UP investors who withdraw at the second stage must pay a fee to rid themselves a high-valued asset, which benefits those who remain invested until date 2. If the latter effect dominates, then $\hat{s}_1^{\varphi} < \hat{s}_1$, in which case the second-stage fee prevents a preemptive run for $s \in [\hat{s}_1^{\varphi}, \hat{s}_1)$. The fact that a redemption fee may lower the probability of a preemptive run contrasts with claims by Hanson et al. (2015), Cipriani et al. (2014), and others that redemption fees will precipitate preemptive runs.

7 Conclusion

We study how redemption fees affect runs on financial institutions when investors are asymmetrically informed about fundamentals. By reducing the payoff externality, fees mitigate the first-mover advantage of informed investors and, thus, reduce their tendency to run. The essence of our model is that the decline in the informed investors' inclination to run influences learning by uninformed investors, which may cause them to become more pessimistic about fundamentals and, hence, more likely to run. Therefore, the fee's net effect on runs is generally ambiguous. Our welfare analysis reveals that redemption fees may induce a wealth transfer from uninformed to informed investors. Rather than leveling the playing field, the fee transforms the informed investors' first-mover advantage (in the absence of a fee) to a last-mover advantage. Furthermore, social welfare falls in good states (i.e., high fundamentals) when the fee increases the overall run potential.

Our analysis presents many interesting avenues for future research. For instance, because current regulations permit MMMFs to impose a redemption fee only during periods of stress, a fee could affect fund investments, possibly leading funds to invest in less risky assets. Additionally, redemption fees may affect the potential for financial contagion across funds. A direct extension of our model would be to consider multiple funds holding correlated assets. The initiation of a redemption fee in one fund would not only influence learning by uninformed investors in that particular fund, but also those in other funds. This could trigger a contagion through the learning channel across different funds. We leave these for future explorations.

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Appendix A: Proofs

Proof of Theorem 1. We consider the relevant case in which IP investors did not withdraw at the first stage. When $n \leq \bar{n}$, IP investors do not withdraw at the second stage if and only if

$$V(s) \geq \frac{1-n}{1-n(1+\gamma)}, \tag{A1}$$

which follows from (7). Similarly, (8) implies that IP investors do not withdraw at the second stage when $n > \bar{n}$ if and only if

$$V(s) \geq \frac{\alpha(1-n)}{1-[1-\alpha(1-n)](1+\gamma)}. \tag{A2}$$

The results follow from the following facts: (i) $V(s^*) = 1$ and $V(s)$ is continuous and monotonically increasing in s; (ii) the RHSs of both (A1) and (A2) are greater than 1 (if $n \neq 0$) and are increasing in n; and (iii) the RHS of (A1) is strictly less than the RHS of (A2) for a given n. $\square$

Proof of Lemma 1. The results for the cases wherein $n \notin [\alpha, \bar{\lambda}]$ follow immediately from the fact that the first-stage withdrawal $n = \lambda + \alpha(1-\lambda)w$ fully reveals whether $w = 0$ or $w = 1$ in these cases. When $n \in [\alpha, \bar{\lambda}]$, n does not fully reveal w. If $w = 1$, then $n = \lambda + \alpha(1-\lambda)$, and the likelihood of observing n given $w = 1$ is $g\left(\frac{n-\alpha}{1-\alpha}\right)$. If $w = 0$, then $n = \lambda$, and the probability of observing n conditional on $w = 0$ is $g(n)$. Thus, $F(\hat{s}_1|n)$, according to Bayes' rule, is given by (10). Note that

$$\mathbb{E}[\tilde{v}|n] = F(\hat{s}_1|n)\mathbb{E}[\tilde{v}|s < \hat{s}_1] + (1 - F(\hat{s}_1|n))\mathbb{E}[\tilde{v}|s \geq \hat{s}_1]. \tag{A3}$$

Substituting (10) into (A3) yields (11).

Next, provided that $n \in [\alpha, \bar{\lambda}]$, differentiating (10) with respect to n gives

$$\frac{\partial F(\hat{s}_1|n)}{\partial n} = \frac{F(\hat{s}_1)(1-F(\hat{s}_1))\left[g(n)\frac{\partial g\left(\frac{n-\alpha}{1-\alpha}\right)}{\partial n} - g\left(\frac{n-\alpha}{1-\alpha}\right)\frac{\partial g(n)}{\partial n}\right]}{\left[F(\hat{s}_1)g\left(\frac{n-\alpha}{1-\alpha}\right) + \left(1-F(\hat{s}_1)\right)g(n)\right]^2} \tag{A4}$$
$$\propto \left[g(n)\frac{\partial g\left(\frac{n-\alpha}{1-\alpha}\right)}{\partial n} - g\left(\frac{n-\alpha}{1-\alpha}\right)\frac{\partial g(n)}{\partial n}\right]$$
$$> 0,$$

where the inequality follows from Assumption 2. It follows that

$$\frac{\partial \mathbb{E}[\tilde{v}|n]}{\partial n} = \left(\mathbb{E}[\tilde{v}|s < \hat{s}_1] - \mathbb{E}[\tilde{v}|s \geq \hat{s}_1]\right)\frac{\partial F(\hat{s}_1|n)}{\partial n} < 0,$$

given $\mathbb{E}[\tilde{v}|s < \hat{s}_1] < \mathbb{E}[\tilde{v}|s \geq \hat{s}_1]$. If $n \notin [\alpha, \bar{\lambda}]$, then both $F(\hat{s}_1|n)$ and $\mathbb{E}[\tilde{v}|n]$ are constants and, therefore, unaffected by changes in n. □.

Proof of Lemma 2. Proof is by contradiction. Suppose that UP investors do not always withdraw at the second stage when $n > \bar{\lambda}$. This supposition implies that UP investors never withdraw when $n \in [0, \bar{\lambda}]$. Given this strategy by UP investors, IP investors understand that if they do not withdraw at the first stage, then $n \in [0, \bar{\lambda}]$, which means that UP investors will never withdraw at the second stage. Consequently, an IP investor's expected payoff at the second stage is bounded from below by 1, which is the amount she receives if she withdraws at the second stage and UP investors keep their deposits until date 2. Hence, an IP investor will make a first-stage withdrawal if and only if the expected return on the risky asset is less than 1 or, equivalently, $s < s^*$. As a result, if $n > \bar{\lambda}$, then UP investors infer that $s < s^*$ because the IP investors' actions are fully revealed. Conditional on $n > \bar{\lambda}$, the expected payoff to a UP investor at the second stage is $\frac{1-n(1+\gamma)}{1-n}\mathbb{E}[\tilde{v}|s < s^*]$, which is strictly less than 1 because $\frac{1-n(1+\gamma)}{1-n} < 1$ and $V(s^*) = 1$. Therefore, UP investors always make a second-stage withdrawal when $n > \bar{\lambda}$, which is a contradiction. □

Proof of Theorem 2. A UP investor does not withdraw if and only if

$$
\begin{aligned}
1 \leq F(\hat{s}_1|n)\frac{1-n(1+\gamma)}{1-n}\mathbb{E}[\tilde{v}|s<\hat{s}_1] + (1-F(\hat{s}_1|n)) \\
\times\Bigg(\frac{\max\{F(\hat{s}_2(n))-F(\hat{s}_1),0\}}{1-F(\hat{s}_1)}\frac{1-[n+\alpha(1-n)](1+\gamma)}{(1-\alpha)(1-n)}\mathbb{E}[\tilde{v}|\hat{s}_1\leq s<\hat{s}_2(n)] \\
+\frac{1-\max\{F(\hat{s}_1),F(\hat{s}_2(n))\}}{1-F(\hat{s}_1)}\frac{1-n(1+\gamma)}{1-n}\mathbb{E}[\tilde{v}|s\geq\max\{\hat{s}_1,\hat{s}_2(n)\}]\Bigg). \quad \text{(A5)}
\end{aligned}
$$

The LHS, 1, is an individual atomistic UP investor's payoff from deviating from the equilibrium strategy by withdrawing while others do not. The RHS, obtained by aggregating (12), (13), and (14), is a probability-weighted expectation of the payoff from not withdrawing. The fact that

$$
\begin{aligned}
\mathbb{E}[\tilde{v}|n] = F(\hat{s}_1|n)\mathbb{E}[\tilde{v}|s<\hat{s}_1] + \frac{1-F(\hat{s}_1|n)}{1-F(\hat{s}_1)} \\
\times\Big(\max\{F(\hat{s}_2(n))-F(\hat{s}_1),0\}\mathbb{E}[\tilde{v}|\hat{s}_1<s<\hat{s}_2(n)] \\
+(1-\max\{F(\hat{s}_1),F(\hat{s}_2(n))\})\mathbb{E}[\tilde{v}|s>\max\{\hat{s}_1,\hat{s}_2(n)\}]\Big) \quad \text{(A6)}
\end{aligned}
$$

allows (A5) to be rewritten as

$$
1 \leq \frac{1-n(1+\gamma)}{1-n}\mathbb{E}[\tilde{v}|n] - \mathbb{1}_{\{\hat{s}_1<\hat{s}_2(n)\}}(1-F(\hat{s}_1|n))\frac{F(\hat{s}_2(n))-F(\hat{s}_1)}{1-F(\hat{s}_1)}\frac{\alpha\gamma\mathbb{E}[\tilde{v}|\hat{s}_1\leq s<\hat{s}_2(n)]}{(1-\alpha)(1-n)}. \quad \text{(A7)}
$$

We first show that the RHS of (A7) is monotonically decreasing in n. If $\hat{s}_1 \geq \hat{s}_2(n)$, then the second term on the RHS is zero. Differentiating the first term with respect to n yields

$$
\frac{1-n(1+\gamma)}{1-n}\frac{\partial\mathbb{E}[\tilde{v}|n]}{\partial n} - \frac{\gamma\mathbb{E}[\tilde{v}|n]}{(1-n)^2},
$$

which is negative because $\partial\mathbb{E}[\tilde{v}|n]/\partial n \leq 0$ (Lemma 1). If $\hat{s}_1 < \hat{s}_2(n)$, then differentiating the second term with respect to n yields

$$-\frac{\alpha\gamma}{(1-\alpha)(1-n)(1-F(\hat{s}_1))}\Bigg[(1-F(\hat{s}_1|n))\Bigg(\mathbb{E}[\tilde{v}|\hat{s}_1 \le s < \hat{s}_2(n)]\frac{\partial F(\hat{s}_2(n))}{\partial n} + (F(\hat{s}_2(n))-F(\hat{s}_1))\frac{\partial \mathbb{E}[\tilde{v}|\hat{s}_1 \le s < \hat{s}_2(n)]}{\partial n} + \frac{(F(\hat{s}_2(n))-F(\hat{s}_1))\mathbb{E}[\tilde{v}|\hat{s}_1 \le s < \hat{s}_2(n)]}{1-n}\Bigg) - (F(\hat{s}_2(n))-F(\hat{s}_1))\mathbb{E}[\tilde{v}|\hat{s}_1 \le s < \hat{s}_2(n)]\frac{\partial F(\hat{s}_1|n)}{\partial n}\Bigg],$$

which is also negative because $\partial F(\hat{s}_2(n))/\partial n > 0$ (Theorem 1), $\partial \mathbb{E}[\tilde{v}|\hat{s}_1 \le s < \hat{s}_2(n)]/\partial n \ge 0$,[13] and

$$\begin{aligned}
\frac{\partial F(\hat{s}_1|n)}{\partial n} &= \frac{F(\hat{s}_1)(1-F(\hat{s}_1))\Big[g(n)\frac{\partial g(\frac{n-\alpha}{1-\alpha})}{\partial n} - g\big(\frac{n-\alpha}{1-\alpha}\big)\frac{\partial g(n)}{\partial n}\Big]}{\big[F(\hat{s}_1)g\big(\frac{n-\alpha}{1-\alpha}\big)+(1-F(\hat{s}_1))g(n)\big]^2} \\
&= \frac{F(\hat{s}_1)(1-F(\hat{s}_1|n))\Big[g(n)\frac{\partial g(\frac{n-\alpha}{1-\alpha})}{\partial n} - g\big(\frac{n-\alpha}{1-\alpha}\big)\frac{\partial g(n)}{\partial n}\Big]}{g(n)\big[F(\hat{s}_1)g\big(\frac{n-\alpha}{1-\alpha}\big)+(1-F(\hat{s}_1))g(n)\big]} \\
&= \frac{(1-F(\hat{s}_1|n))\Big[\frac{\partial g(\frac{n-\alpha}{1-\alpha})/\partial n}{g(\frac{n-\alpha}{1-\alpha})} - \frac{\partial g(n)/\partial n}{g(n)}\Big]}{1+\frac{(1-F(\hat{s}_1))g(n)}{F(\hat{s}_1)g(\frac{n-\alpha}{1-\alpha})}} \\
&< (1-F(\hat{s}_1|n))\frac{\partial \log\left(\frac{g(\frac{n-\alpha}{1-\alpha})}{g(n)}\right)}{\partial n} \\
&\le \frac{1-F(\hat{s}_1|n)}{1-n},
\end{aligned}$$

where the first equality follows from (A4), the second equality follows from (10), the third equality and the first inequality follow from algebra, and the second inequality follows from Assumption 2.

[13]The conditional expectation can be written as

$$\mathbb{E}[\tilde{v}|\hat{s}_1 \le s < \hat{s}_2(n)] = \int_{\hat{s}_1}^{\hat{s}_2(n)} \frac{V(s)f(s)}{F(\hat{s}_2(n))-F(\hat{s}_1)}\,\mathrm{d}s.$$

Differentiating this expression with respect to $\hat{s}_2(n)$ yields

$$\begin{aligned}
\frac{\partial \mathbb{E}[\tilde{v}|\hat{s}_1 \le s < \hat{s}_2(n)]}{\partial \hat{s}_2(n)} &= \frac{V(\hat{s}_2(n))f(\hat{s}_2(n))}{F(\hat{s}_2(n))-F(\hat{s}_1)} - \int_{\hat{s}_1}^{\hat{s}_2(n)} \frac{V(s)f(s)f(\hat{s}_2(n))}{[F(\hat{s}_2(n))-F(\hat{s}_1)]^2}\,\mathrm{d}s \\
&\propto V(\hat{s}_2(n)) - \int_{\hat{s}_1}^{\hat{s}_2(n)} \frac{V(s)f(s)}{F(\hat{s}_2(n))-F(\hat{s}_1)}\,\mathrm{d}s \\
&= V(\hat{s}_2(n)) - \mathbb{E}[\tilde{v}|\hat{s}_1 \le s < \hat{s}_2(n)] \\
&> 0,
\end{aligned}$$

where the inequality follows from the fact that $\partial V(s)/\partial s > 0$. Then, $\partial \mathbb{E}[\tilde{v}|\hat{s}_1 \le s < \hat{s}_2(n)]/\partial n \ge 0$ follows from the fact that n enters the conditional expectation only through $\hat{s}_2(n)$ and, according to Theorem 1, $\hat{s}_2(n)$ is strictly increasing in n.

Additionally, $\mathbb{1}_{\{\hat{s}_1<\hat{s}_2(n)\}}$ is non-decreasing in n because $\partial\hat{s}_2(n)/\partial n > 0$ (Theorem 1). Together, these imply that the RHS of (A7) is strictly decreasing in n.

Next, we show that the RHS of (A7) crosses the LHS at some unique $n \in [0, \bar{\lambda}]$. Note that $\lim_{n\to 0^+} V(\hat{s}_2(n)) = 1$ (see (9)), which implies that $\lim_{n\to 0^+} \hat{s}_2(n) \leq \hat{s}_1$ because $V(\hat{s}_1) \geq 1$.[14] Thus, the second term on the RHS approaches zero as $n \to 0^+$. Additionally, $\lim_{n\to 0^+} \frac{1-n(1+\gamma)}{1-n} = 1$ and $\mathbb{E}[\tilde{v}|n] = \mathbb{E}[\tilde{v}|s \geq \hat{s}_1] > V(\hat{s}_1) \geq 1$ when $\alpha \geq n \to 0^+$ (Lemma 1), so the RHS approaches a value strictly greater than 1 as $n \to 0^+$. If $n > \bar{\lambda}$, then Lemma 1 implies that the RHS reduces to

$$\frac{1 - n(1+\gamma)}{1-n}\mathbb{E}[\tilde{v}|s < \hat{s}_1],$$

which must be strictly less than 1 for the reason discussed in the proof of Lemma 2. Therefore, the RHS must cross the LHS (i.e., 1) at some $n \in [0, \bar{\lambda}]$. This argument holds as long as the RHS is continuous in n. However, Lemma 1 implies that $\lim_{n\to\alpha^-} \frac{1-n(1+\gamma)}{1-n}\mathbb{E}[\tilde{v}|n] > \lim_{n\to\alpha^+} \frac{1-n(1+\gamma)}{1-n}\mathbb{E}[\tilde{v}|n]$. Thus, if $\lim_{n\to\alpha^+} \frac{1-n(1+\gamma)}{1-n}\mathbb{E}[\tilde{v}|n] < 1 \leq \lim_{n\to\alpha^-} \frac{1-n(1+\gamma)}{1-n}\mathbb{E}[\tilde{v}|n]$, then the withdrawal threshold will be at α. Taking this discontinuity into account, the threshold $\bar{n}$ is defined as in (15). Uniqueness of $\bar{n}$ follows from the fact that the RHS of (A7) is monotonically decreasing in n. □

Proof of Corollary 1. According to Theorems 1 and 2, $\hat{s}_2(n)$ is uniquely determined for a given $\bar{n}$, and vice versa. Theorem 1 also implies that $\hat{s}_2(\bar{n})$ is strictly increasing in $\bar{n}$. However, $\bar{n}$ is non-increasing in $\hat{s}_2(\bar{n})$ (given $\hat{s}_1$), as implied by the RHS of the condition in (15). Thus, conditional on $\hat{s}_1$, the pair of second-stage strategies must be unique.

Proof of Lemma 3. Note that the RHS of (16) is greater than the RHS of (17) for a given s. Additionally, the RHSs of both (16) and (17) are increasing in s. First, suppose $s \in [s^*, \hat{s}_2(\bar{n})]$. In this case, the RHSs of both (16) and (17) are less than or equal to $\bar{n}$. Because UP investors do not make a second-stage withdrawal in this case (Theorem 2), $\hat{n}(s)$ is given by (16). Second, suppose $s \in (\hat{s}_2(\bar{n}), \check{s}_2(\bar{n})]$. In this case, $1 - \frac{\gamma V(s)}{\alpha[(1+\gamma)V(s)-1]} \leq \bar{n} < \frac{V(s)-1}{(1+\gamma)V(s)-1}$. Consequently, neither (16) nor (17) defines $\hat{n}(s)$ because $1 - \frac{\gamma V(s)}{\alpha[(1+\gamma)V(s)-1]}$ is an appropriate threshold only if UP investors make a second-stage withdrawal (i.e., if $n > \bar{n}$), and $\frac{V(s)-1}{(1+\gamma)V(s)-1}$ is an appropriate threshold only if UP

[14] Although we explicitly determine the IP investors' first-stage signal threshold $\hat{s}_1$ in Theorem 3, we note here that $V(\hat{s}_1) \geq 1$ because an IP investor's expected payoff from not withdrawing at the first stage when $s = \hat{s}_1$ equals her expected payoff from withdrawing, 1.

investors do not make a second-stage withdrawal (i.e., if $n \leq \bar{n}$). Instead, $\hat{n}(s)$ equals $\bar{n}$. On the one hand, if $n > \bar{n}$ in this region, then n is greater than the RHS of (17), and IP investors withdraw. On the other hand, if $n \leq \bar{n}$, then n is less than or equal to the RHS of (16), and IP investors do not withdraw. Thus, $\hat{n}(s) = \bar{n}$. Third, suppose $s \in (\check{s}_2(\bar{n}), \check{s}_2(\bar{\lambda})]$. In this case, the RHSs of both (16) and (17) are greater than $\bar{n}$. Because UP investors withdraw at the second stage in this case, $\hat{n}(s)$ is given by (17). Finally, suppose $s \in (\check{s}_2(\bar{\lambda}), 1]$. In this case, IP investors never withdraw at the second stage because the signal s is high enough to offset the largest possible liquidation costs imposed by the aggregate withdrawals of impatient and UP investors. Thus, $\hat{n}(s) = \bar{\lambda}$. Note that $\hat{n}(\hat{s}_2(\bar{n})) = \bar{n}$, $\hat{n}(\check{s}_2(\bar{n})) = \bar{n}$, and $\hat{n}(\check{s}_2(\bar{\lambda})) = \bar{\lambda}$. Hence, $\hat{n}(s)$ is continuous in s. Furthermore, because the RHSs of (16) and (17) are increasing in s, $\hat{n}(s)$ is non-decreasing in s. □

Proof of Theorem 3. The equations that describe the IP investors' first-stage decisions, (20) – (22), can be combined into a single condition

$$\begin{aligned} 1 \leq \int_0^{\min\{\hat{n}(s),\bar{n}\}} \frac{1-\lambda(1+\gamma)}{1-\lambda} V(s) g(\lambda)\,\mathrm{d}\lambda + \int_{\min\{\hat{n}(s),\bar{n}\}}^{\bar{n}} g(\lambda)\,\mathrm{d}\lambda \\ + \int_{\bar{n}}^{\max\{\hat{n}(s),\bar{n}\}} \frac{1-[\lambda+(1-\alpha)(1-\lambda)](1+\gamma)}{\alpha(1-\lambda)} V(s) g(\lambda)\,\mathrm{d}\lambda \\ + \int_{\max\{\hat{n}(s),\bar{n}\}}^{\bar{\lambda}} \frac{1-\lambda(1+\gamma)}{(1-\lambda)(1+\gamma)} g(\lambda)\,\mathrm{d}\lambda. \end{aligned} \tag{A8}$$

IP investors do not withdraw at the first stage if and only if (A8) is satisfied.

First, we show that the RHS of (A8) is continuous and monotonically increasing in s. Note that $\frac{1-[\lambda+(1-\alpha)(1-\lambda)](1+\gamma)}{\alpha(1-\lambda)} < \frac{1-\lambda(1+\gamma)}{1-\lambda} \leq 1$ and $\frac{1-\lambda(1+\gamma)}{(1-\lambda)(1+\gamma)} < \frac{1-\lambda(1+\gamma)}{1-\lambda} \leq 1$ (with strict inequality when $\lambda > 0$). It follows that the RHS is less than 1 unless $V(s) > 1$. Hence, $\hat{s}_1 > s^*$. Additionally, Lemma 3 implies: (i) $\frac{1-\lambda(1+\gamma)}{1-\lambda} V(s) \geq 1$ for $\lambda \leq \hat{n}(s) = \frac{V(s)-1}{(1+\gamma)V(s)-1}$ when $s \in [s^*, \hat{s}_2(\bar{n})]$; (ii) $\frac{1-\lambda(1+\gamma)}{1-\lambda} V(s) \geq 1$ for $\lambda \leq \bar{n}$ when $s \in (\hat{s}_2(\bar{n}), 1]$; and (iii) $\frac{1-[\lambda+(1-\alpha)(1-\lambda)](1+\gamma)}{\alpha(1-\lambda)} V(s) \geq 1$ for $\lambda \leq \hat{n}(s) = 1 - \frac{\gamma V(s)}{\alpha[(1+\gamma)V(s)-1]}$ when $s \in (\check{s}_2(\bar{n}), 1]$. Thus, because $V(s)$ is continuous and increasing in s and $\hat{n}(s)$ is continuous and non-decreasing in s, the RHS is continuous and monotonically increasing in s.

Next, we show that the RHS of (A8) crosses the LHS at some $s \in (s^*, 1)$. As stated above, the RHS is strictly less than 1 if $s = s^*$. Conversely, if $s = 1$, then $V(s) = H > \frac{\alpha(1-\bar{\lambda})}{1-[\bar{\lambda}+(1-\alpha)(1-\bar{\lambda})](1+\gamma)}$ (Assumption 1) and $\hat{n}(s) = \bar{\lambda}$ (Lemma 3), so the integrands in the first and third terms on the RHS are strictly greater than 1, whereas the second and fourth terms are zero. Hence, the RHS is strictly greater than 1. Therefore, there exists a unique $\hat{s}_1 \in (s^*, 1)$ such that IP investors make a first-stage withdrawal if and only if $s < \hat{s}_1$.

It is straightforward to show that: the RHS of (20) characterizes $\hat{s}_1$ if $V(\hat{s}_2(\bar{n})) \geq \Psi$, where

$$\Psi \equiv \frac{1 - \int_{\bar{n}}^{\bar{\lambda}} \frac{1-\lambda(1+\gamma)}{(1-\lambda)(1+\gamma)} g(\lambda)\, \mathrm{d}\lambda}{\int_0^{\bar{n}} \frac{1-\lambda(1+\gamma)}{1-\lambda} g(\lambda)\, \mathrm{d}\lambda};$$

the RHS of (21) characterizes $\hat{s}_1$ if $V(\hat{s}_2(\bar{n})) < \Psi \leq V(\check{s}_2(\bar{n}))$; and the RHS of (22) characterizes $\hat{s}_1$ if $V(\check{s}_2(\bar{n})) < \Psi$. In general, then, $\hat{s}_1$ is characterized by

$$V(\hat{s}_1) = \begin{cases} \dfrac{1 - \int_{\hat{n}(\hat{s}_1)}^{\bar{n}} g(\lambda)\, \mathrm{d}\lambda - \int_{\bar{n}}^{\bar{\lambda}} \frac{1-\lambda(1+\gamma)}{(1-\lambda)(1+\gamma)} g(\lambda)\, \mathrm{d}\lambda}{\int_0^{\hat{n}(\hat{s}_1)} \frac{1-\lambda(1+\gamma)}{1-\lambda} g(\lambda)\, \mathrm{d}\lambda} & \text{if } \Psi \leq V(\hat{s}_2(\bar{n})) \\ \dfrac{1 - \int_{\bar{n}}^{\bar{\lambda}} \frac{1-\lambda(1+\gamma)}{(1-\lambda)(1+\gamma)} g(\lambda)\, \mathrm{d}\lambda}{\int_0^{\bar{n}} \frac{1-\lambda(1+\gamma)}{1-\lambda} g(\lambda)\, \mathrm{d}\lambda} & \text{if } V(\hat{s}_2(\bar{n})) < \Psi \leq V(\check{s}_2(\bar{n})) \\ \dfrac{1 - \int_{\hat{n}(\hat{s}_1)}^{\bar{\lambda}} \frac{1-\lambda(1+\gamma)}{(1-\lambda)(1+\gamma)} g(\lambda)\, \mathrm{d}\lambda}{\int_0^{\hat{n}(\hat{s}_1)} \frac{1-\lambda(1+\gamma)}{1-\lambda} g(\lambda)\, \mathrm{d}\lambda - \int_{\bar{n}}^{\hat{n}(\hat{s}_1)} \frac{(1-\alpha)\gamma}{\alpha(1-\lambda)} g(\lambda)\, \mathrm{d}\lambda} & \text{if } \Psi > V(\check{s}_2(\bar{n})). \end{cases} \tag{A9}$$

Because $\frac{1-\lambda(1+\gamma)}{(1-\lambda)(1+\gamma)} < 1$, the top equation in (A9) is decreasing in $\bar{n}$. Similarly, the middle equation (and hence Ψ) is decreasing in $\bar{n}$ because $\frac{1-\lambda(1+\gamma)}{(1-\lambda)(1+\gamma)} < \frac{1-\lambda(1+\gamma)}{1-\lambda}$. The bottom equation in (A9) is obviously decreasing in $\bar{n}$. Finally, because the RHS of (20) is weakly less than the RHS of (21), which is weakly less than the RHS of (22), and Theorem 1 implies that both $V(\hat{s}(\bar{n}))$ and $V(\check{s}(\bar{n}))$ are increasing in $\bar{n}$, $\hat{s}_1$ is monotonically decreasing in $\bar{n}$. □

Proof of Theorem 5. The determination of the UP investors' withdrawal threshold $\bar{n}^\phi$ in (27) is obvious. Note that $\frac{\partial \mathbb{E}[\tilde{v}|n]}{\partial n} < 0$ for $n \in [\alpha, \bar{\lambda}]$ (Lemma 1) and $\mathbb{E}[\tilde{v}|\bar{n}^\phi, \phi] \geq \frac{1}{1+\gamma}$. Therefore, $\mathbb{E}[\tilde{v}|\bar{n}, \phi] < \frac{1}{1+\gamma} \leq \mathbb{E}[\tilde{v}|\bar{n}^\phi, \phi] \iff \bar{n} > \bar{n}^\phi$. □

Proof of Proposition 2. The first case wherein $n \leq \{\bar{n}, \bar{n}^\phi\}$ is described in the text. For the second case wherein $n \in (\bar{n}^\phi, \bar{n}]$, the effect of the fee on UP investors' welfare depends on s.

- If $s < \max\{\hat{s}_1, \hat{s}_2(n)\}$, then IP investors withdraw at date 1 without a fee, and the expected payoff to a non-withdrawing UP investor is $\frac{1-[\lambda+\alpha(1-\lambda)](1+\gamma)}{(1-\alpha)(1-\lambda)} V(s)$. Setting this expression equal to $(1+\gamma)^{-1}$ defines a threshold $\acute{s}(\lambda) \in (s^{\gamma}, 1]$, characterized by

$$V(\acute{s}(\lambda)) = \frac{(1-\alpha)(1-\lambda)}{1+\gamma-[\lambda+\alpha(1-\lambda)](1+\gamma)^2}, \tag{A10}$$

such that $\frac{1-[\lambda+\alpha(1-\lambda)](1+\gamma)}{(1-\alpha)(1-\lambda)}V(s)$ is less than, equal to, and greater than $1-\phi$ for $s < \acute{s}(\lambda)$, $s = \acute{s}(\lambda)$, and $s > \acute{s}(\lambda)$, respectively.

- If $s \geq \max\{\hat{s}_1, \hat{s}_2(n)\}$, then IP investors also do not withdraw absent a fee, and a UP investor's expected payoff is $\frac{1-\lambda(1+\gamma)}{1-\lambda}V(s)$. This payoff is greater than $1-\phi$, which follows from the definition of $\hat{s}_1$ and the fact that $s \geq \hat{s}_1$. Hence, the fee hurts UP investors.

Thus, the fee benefits UP investors when s is sufficiently low, i.e., $s < \min\{\max\{\hat{s}_1, \hat{s}_2(n)\}, \acute{s}(\lambda)\}$, but harms them when s is sufficiently high, i.e., $s > \min\{\max\{\hat{s}_1, \hat{s}_2(n)\}, \acute{s}(\lambda)\}$. For the third case wherein $n \in (\bar{n}, \bar{n}^{\phi}]$, the effect of the fee on UP investors' welfare again depends on s.

- If $s < \hat{s}_1$, then IP investors withdraw at the first stage without a fee and the fund liquidates when UP investors withdraw at the second stage. Thus, UP investors receive $\frac{1-[\lambda+\alpha(1-\lambda)](1+\gamma)}{(1-\alpha)(1-\lambda)(1+\gamma)}$ because they must bear the liquidation costs generated by impatient and IP investors in addition to those generated by themselves. There exists a threshold $\grave{s}(\lambda) \in (0, s^{\gamma})$, defined by

$$V(\grave{s}(\lambda)) = \frac{1-[\lambda+\alpha(1-\lambda)](1+\gamma)}{(1-\alpha)(1-\lambda)(1+\gamma)}, \tag{A11}$$

  such that a UP investor's payoff from not withdrawing in the presence of a fee, $V(s)$, is less than, equal to, and greater than $\frac{1-[\lambda+\alpha(1-\lambda)](1+\gamma)}{(1-\alpha)(1-\lambda)(1+\gamma)}$ for $s < \grave{s}(\lambda)$, $s = \grave{s}(\lambda)$, and $s > \grave{s}(\lambda)$, respectively. Note that $\grave{s}(\lambda) < \hat{s}_1$.

- If $s \geq \hat{s}_1$, then IP investors do not withdraw at the first stage without a fee and UP investors (who withdraw at the second stage) receive either 1 if IP investors do not withdraw at the second stage or $\frac{1-\lambda(1+\gamma)}{(1-\lambda)(1+\gamma)} < 1$ if IP investors withdraw at the second stage. Because $V(s) > V(s^*) = 1$ given $s \geq \hat{s}_1$, UP investors benefit from the fee.

Hence, the fee benefits UP investors in this case when s is sufficiently high, i.e., $s > \grave{s}(\lambda)$, but harms them when s is sufficiently low, i.e., $s < \grave{s}(\lambda)$. For the fourth case wherein $n > \max\{\bar{n}, \bar{n}^{\phi}\}$, UP investors' welfare depends on s.

- If $s < \max\{\hat{s}_1, \hat{s}_2(n)\}$, then IP investors withdraw at date 1 and the fund is liquidated. If IP investors withdraw at the first stage (i.e., if $s < \hat{s}_1$), then a UP investor receives $\frac{1-[\lambda+\alpha(1-\lambda)](1+\gamma)}{(1-\alpha)(1-\lambda)(1+\gamma)}$, which is equal to each UP investor's pro rata share of the remaining deposits after both impatient and IP investors withdraw, discounted by the liquidation costs. Conversely, if IP investors withdraw at the second stage (i.e., if $s \in [\hat{s}_1, \hat{s}_2(n))$), then each UP investor receives her discounted pro rata share of the remaining deposits after impatient investors withdraw, $\frac{1-\lambda(1+\gamma)}{(1-\lambda)(1+\gamma)}$. Because $1-\phi$ is greater than both $\frac{1-[\lambda+\alpha(1-\lambda)](1+\gamma)}{(1-\alpha)(1-\lambda)(1+\gamma)}$ and $\frac{1-\lambda(1+\gamma)}{(1-\lambda)(1+\gamma)}$, UP investors benefit from the fee in both cases.
- If $s \geq \max\{\hat{s}_1, \hat{s}_2(n)\}$, then IP investors do not withdraw and the fund is not liquidated when UP investors make a second-stage withdrawal. Consequently, UP investors receive 1 in the absence of a fee but only $1-\phi$ in the presence of a fee.

Therefore, the fee is beneficial to UP investors in this case when s is sufficiently low, i.e., $s < \max\{\hat{s}_1, \hat{s}_2(n)\}$, but is detrimental when s is sufficiently high, i.e., $s \geq \max\{\hat{s}_1, \hat{s}_2(n)\}$. The following proposition summarizes the analysis.

Appendix B: Derivation of Social Welfare Effects

In this appendix, we derive the Θs in Table 1 and prove Proposition 3. The derivations are organized based on the realized first-stage withdrawal n and signal s.

Case 1: $n \leq \min\{\bar{n}, \bar{n}^{\phi}\}$. UP investors do not withdraw regardless of the fee arrangement. However, the fee may affect IP investors' actions. Depending on s, there are three possible outcomes.

1.1 $s < s^{\gamma}$. IP investors withdraw at the first stage regardless of the fee. In this case,

$$\Omega = \lambda + \alpha(1-\lambda) + \big(1 - [\lambda + \alpha(1-\lambda)](1+\gamma)\big)V(s), \tag{B1}$$

$$\Omega^{\phi} = \lambda(1-\phi) + \alpha(1-\lambda)(1-\phi) + (1-\alpha)(1-\lambda)V(s). \tag{B2}$$

The first, second, and third terms in each expression represents the respective amounts received by impatient, IP, and UP investors. Subtracting (B1) from (B2) yields,

$$\Omega^{\phi} - \Omega = \gamma[\lambda + \alpha(1-\lambda)](V(s) - V(s^{\gamma})) < 0. \tag{B3}$$

1.2 $s \in [s^\gamma, \max\{\hat{s}_1, \hat{s}_2(n)\})$. IP investors withdraw in the absence of a fee but do not withdraw in the presence of a fee. Here, Ω is given by (B1), and

$$\Omega^\phi = \lambda(1-\phi) + (1-\lambda)V(s). \tag{B4}$$

The first term represents the amount received by impatient investors and the second term represents the amount received by IP and UP investors. Subtracting (B1) from (B4) yields

$$\Omega^\phi - \Omega = (\gamma[\lambda + \alpha(1-\lambda)] + \alpha(1-\lambda))(V(s) - V(s^\gamma)) \geq 0, \tag{B5}$$

which holds with strict inequality when $s \neq s^\gamma$.

1.3 $s \geq \max\{\hat{s}_1, \hat{s}_2(n)\}$. IP investors do not withdraw regardless of the fee arrangement. In this case, Ω^ϕ is given by (B4) and

$$\Omega = \lambda + [1 - \lambda(1+\gamma)]V(s) \tag{B6}$$

because only impatient investors withdraw. Subtracting (B6) from (B4) gives

$$\Omega^\phi - \Omega = \gamma\lambda(V(s) - V(s^\gamma)) > 0. \tag{B7}$$

Case 2: $\bar{n}^\phi < \bar{n}$ and $n \in (\bar{n}^\phi, \bar{n}]$. UP investors withdraw with a fee but not absent a fee. IP investors may also alter their actions, depending on the signal s. There are three possible outcomes.

2.1 $s < s^\gamma$. IP investors withdraw at the first stage regardless of whether they must pay a fee. Here, Ω is given by (B1) but

$$\Omega^\phi = 1 - \phi \tag{B8}$$

because all investors withdraw before date 2 and receive $1-\phi$. Subtracting (B1) from (B8) yields

$$\Omega^{\phi} - \Omega = ([\lambda + \alpha(1-\lambda)](1+\gamma) - 1)(V(s) - V(s^{\gamma})) > 0, \quad \text{(B9)}$$

where the inequality follows from Assumption 1 (which implies $1 - [\lambda + \alpha(1-\lambda)](1+\gamma) < 0$) and the fact that $V(s) < V(s^{\gamma})$.

2.2 $s \in [s^{\gamma}, \max\{\hat{s}_1, \hat{s}_2(n)\})$. IP investors convert from withdrawers in the absence of a fee to non-withdrawers in the presence of a fee. In this case, Ω is again given by (B1), and

$$\Omega^{\phi} = \lambda(1-\phi) + (1-\alpha)(1-\lambda)(1-\phi) + \alpha(1-\lambda)V(s). \quad \text{(B10)}$$

The first, second, and third terms in this expression represent the respective amounts received by impatient, UP, and IP investors. Subtracting (B1) from (B10) gives

$$\Omega^{\phi} - \Omega = (\gamma[\lambda + \alpha(1-\lambda)] + \alpha(1-\lambda) - (1-\alpha)(1-\lambda))(V(s) - V(s^{\gamma})). \quad \text{(B11)}$$

Clearly, $\Omega^{\phi} - \Omega > 0$ if and only if $\alpha > \frac{1-\lambda(1+\gamma)}{(1-\lambda)(2+\gamma)}$.

2.3 $s \geq \max\{\hat{s}_1, \hat{s}_2(n)\}$. IP investors do not withdraw regardless of whether they must pay a fee. In this case, Ω^{ϕ} is again given by (B10), but Ω is given by (B6) because only impatient investors withdraw without a fee. Subtracting (B6) from (B10) yields

$$\Omega^{\phi} - \Omega = [\gamma\lambda - (1-\alpha)(1-\lambda)](V(s) - V(s^{\gamma})) < 0, \quad \text{(B12)}$$

where the inequality follows from Assumption 1 and the fact that $V(s) > V(s^{\gamma})$.

Case 3: $\bar{n}^{\phi} > \bar{n}$ and $n \in (\bar{n}, \bar{n}^{\phi}]$. The fee converts UP investors from withdrawers to non-withdrawers. Depending on s, IP investors' behaviors may also change. Again, there are three possible outcomes.

3.1 $s < s^{\gamma}$. IP investors withdraw at the first stage regardless of the existence of a fee. Because all investors withdraw in the absence of a fee,

$$\Omega = \frac{1}{1+\gamma}. \tag{B13}$$

In the presence of a fee, only UP investors maintain their deposits until date 2, so Ω^{ϕ} is given by (B2). The effect of the fee on social welfare, which is derived by subtracting (B13) from (B2), is

$$\Omega^{\phi} - \Omega = (1-\alpha)(1-\lambda)(V(s) - V(s^{\gamma})) < 0. \tag{B14}$$

3.2 $s \in [s^{\gamma}, \max\{\hat{s}_1, \hat{s}_2(n)\})$. IP investors withdraw at the first stage absent a fee but do not withdraw with a fee. Here, Ω and Ω^{ϕ} are given by (B13) and (B4), respectively. Then,

$$\Omega^{\phi} - \Omega = (1-\lambda)(V(s) - V(s^{\gamma})) \geq 0, \tag{B15}$$

which holds with strict inequality when $s \neq s^{\gamma}$.

3.3 $s \geq \max\{\hat{s}_1, \hat{s}_2(n)\}$. IP investors do not withdraw. In this case, Ω^{ϕ} is again given by (B4), but

$$\Omega = \lambda + (1-\alpha)(1-\lambda) + (1 - [\lambda + (1-\alpha)(1-\lambda)](1+\gamma))V(s). \tag{B16}$$

The first, second, and third terms in this expression represent the amounts received by impatient, UP, and IP investors, respectively. Subtracting (B16) from (B4) yields

$$\Omega^{\phi} - \Omega = [\gamma\lambda + (1-\alpha)(1-\lambda)(1+\gamma))(V(s) - V(s^{\gamma})) > 0. \tag{B17}$$

Case 4: $n > \max\{\bar{n}, \bar{n}^{\phi}\}$. UP investors withdraw regardless of the fee, but the fee may alter IP investors' behaviors. There are three possible outcomes, depending on s.

4.1 $s < s^{\gamma}$. IP investors withdraw irrespective of the fee. Here, Ω and Ω^{ϕ} are given by (B13) and (B8), respectively, because all investors withdraw regardless of the fee. Therefore, $\Omega^{\phi} - \Omega = 0$.

4.2 $s \in [s^{\gamma}, \max\{\hat{s}_1, \hat{s}_2(n)\})$. IP investors convert from withdrawers to non-withdrawers with the imposition of a fee. In this case, Ω is given by (B13), and

$$\Omega^{\phi} = [\lambda + (1-\alpha)(1-\lambda)](1-\phi) + \alpha(1-\lambda)V(s). \tag{B18}$$

Subtracting (B13) from (B18) gives

$$\Omega^{\phi} - \Omega = \alpha(1-\lambda)(V(s) - V(s^{\gamma})) \geq 0, \tag{B19}$$

which holds with strict inequality when $s \neq s^{\gamma}$.

4.3 $s \geq \max\{\hat{s}_1, \hat{s}_2(n)\}$. IP investors maintain their deposits until date 2. Here, Ω and Ω^{ϕ} are given by (B16) and (B18), respectively. Subtracting (B16) from (B18) yields

$$\Omega^{\phi} - \Omega = \gamma[\lambda + (1-\alpha)(1-\lambda)](V(s) - V(s^{\gamma})) > 0. \tag{B20}$$

Appendix C: Derivations for Preemptive Runs

In this appendix, we derive the investors' strategies when a fee is present only at the second stage. We omit formal proofs, which are analogous to those in the benchmark setting with no fee.

At the beginning of the second stage, IP investors who chose $w = 0$ and all UP investors remain invested in the fund. These remaining investors observe the first-stage withdrawal n and infer that the amount of deposits remaining at the fund at the beginning of the second stage is $1 - n(1+\gamma)$.

If $n \leq \bar{n}^{\varphi}$, then UP investors do not withdraw, and IP investors also do not withdraw at the second stage if and only if

$$\frac{1 - n(1+\gamma)}{1-n} V(s) \geq 1 - \phi = \frac{1}{1+\gamma}, \tag{C1}$$

which is identical to (7) except for the RHS, which now accounts for the second-stage fee. Conversely, if $n > \bar{n}^{\varphi}$, then UP investors withdraw. IP investors also withdraw in this case unless

$$\frac{1 - n(1+\gamma) - (1-\alpha)(1-n)}{\alpha(1-n)} V(s) \geq 1 - \phi = \frac{1}{1+\gamma}. \tag{C2}$$

This expression is similar to (8), except that (i) the fees paid by withdrawing UP investors enable the fund to liquidate only $(1-\alpha)(1-n)$ units of its investment, instead of $(1-\alpha)(1-n)(1+\gamma)$ as in (8), to meet UP investors' withdrawal demand, and (ii) the RHS captures the fact that IP investors must pay a fee if they withdraw at the second stage. The following theorem characterizes the IP investors' second-stage signal threshold $\hat{s}_2^{\varphi}(n)$.

Theorem C.1. *For any realized first-stage withdrawal $n \in [0, \bar{\lambda}]$ and the UP investors' second-stage withdrawal threshold $\bar{n}^{\varphi} \in [0, \bar{\lambda}]$, there exists a unique signal threshold $\hat{s}_2^{\varphi}(n) \in [s^*, 1]$, characterized by*

$$V(\hat{s}_2^{\varphi}(n)) = \begin{cases} \dfrac{1-n}{[1-n(1+\gamma)](1+\gamma)} & \text{if } n \in [0, \bar{n}^{\varphi}] \\ \dfrac{\alpha(1-n)}{[\alpha(1-n)-\gamma n](1+\gamma)} & \text{if } n \in (\bar{n}^{\varphi}, \bar{\lambda}], \end{cases} \tag{C3}$$

such that IP investors who do not withdraw at the first stage withdraw at the second stage if and only if $s < \hat{s}_2^{\varphi}(n)$. Furthermore, $\hat{s}_2^{\varphi}(n)$ is strictly increasing in n.

UP investors update their beliefs based on n according to Lemma 1, with $\hat{s}_1^{\varphi}$ in place of $\hat{s}_1$. Like in the benchmark case, if IP investors do not make a second-stage withdrawal (either because they already withdrew at the first stage or because they maintain their deposits until date 2), then a non-withdrawing UP investor's expected payoff is

$$\frac{1-n(1+\gamma)}{1-n}\mathbb{E}[\tilde{v}|n, \varphi]. \tag{C4}$$

If, however, IP investors withdraw at the second-stage, then a non-withdrawing UP investor's expected payoff is

$$\frac{1-n(1+\gamma)-\alpha(1-n)}{(1-\alpha)(1-n)}\mathbb{E}[\tilde{v}|\hat{s}_1^{\varphi} \leq s < \hat{s}_2^{\varphi}(n)]. \tag{C5}$$

The following theorem characterizes the UP investors' withdrawal threshold $\bar{n}^{\varphi}$.

Theorem C.2. *Given the IP investors' respective first-stage and second-stage signal thresholds $\hat{s}_1^{\varphi}$ and $\hat{s}_2^{\varphi}(n)$, there exists a unique withdrawal threshold $\bar{n}^{\varphi}$, characterized by*

$$\bar{n}^{\varphi} = \sup\Bigg\{n \in [0, \bar{\lambda}] : \frac{1}{1+\gamma} \leq \frac{1-n(1+\gamma)}{1-n}\mathbb{E}[\tilde{v}|n, \varphi] - \mathbb{1}_{\{\hat{s}_1^{\varphi} < \hat{s}_2^{\varphi}(n)\}}(1-F(\hat{s}_1^{\varphi}|n))\frac{F(\hat{s}_2^{\varphi}(n))-F(\hat{s}_1^{\varphi})}{1-F(\hat{s}_1^{\varphi})}\frac{\alpha\gamma n\mathbb{E}[\tilde{v}|\hat{s}_1^{\varphi} \leq s < \hat{s}_2^{\varphi}(n)]}{(1-\alpha)(1-n)}\Bigg\}, \tag{C6}$$

such that UP investors do not withdraw at the second stage if and only if $n \leq \bar{n}^{\varphi}$.

As discussed in Section 4.2, imposing a redemption fee changes the UP investors' posterior expectation of the risky asset's payoff. Consequently, a second-stage fee may either increase or decrease the UP investors' withdrawal threshold.

At the first stage, (C3) can be inverted to find a second-stage withdrawal threshold for IP investors that depends on the realized first-stage withdrawal n.

Lemma 4. *For any realized signal* $s \in [s^*, 1]$ *and the UP investors' second-stage withdrawal threshold* $\bar{n}^{\varphi}$*, there exists a unique threshold* $\hat{n}^{\varphi}(s) \in [0, \bar{\lambda}]$*, characterized by*

$$\hat{n}^{\varphi}(s) = \begin{cases} \dfrac{(1+\gamma)V(s)-1}{(1+\gamma)^2 V(s)-1} & \text{if } s \in [s^*, \hat{s}_2^{\varphi}(\bar{n}^{\varphi})] \\ \bar{n}^{\varphi} & \text{if } s \in (\hat{s}_2^{\varphi}(\bar{n}^{\varphi}), \check{s}_2^{\varphi}(\bar{n}^{\varphi})] \\ 1 - \dfrac{\gamma(1+\gamma)V(s)}{\alpha[(1+\gamma)V(s)-1]+\gamma(1+\gamma)V(s)} & \text{if } s \in (\check{s}_2^{\varphi}(\bar{n}^{\varphi}), \check{s}_2^{\varphi}(\bar{\lambda})] \\ \bar{\lambda} & \text{if } s \in (\check{s}_2^{\varphi}(\bar{\lambda}), 1], \end{cases} \tag{C7}$$

where $\check{s}_2^{\varphi}(n)$ *satisfies*

$$V(\check{s}_2^{\varphi}(n)) = \frac{\alpha(1-n)}{[\alpha(1-n)-\gamma n](1+\gamma)}, \tag{C8}$$

such that IP investors who do not withdraw at the first stage withdraw at the second stage if and only if $n > \hat{n}^{\varphi}(s)$*. Furthermore,* $\hat{n}^{\varphi}(s)$ *is continuous and non-decreasing in* s.

There are four relevant scenarios for IP investors at the second stage. First, if both IP and UP investors maintain their deposits until date 2, then an IP investor's expected payoff is $\frac{1-\lambda(1+\gamma)}{1-\lambda}V(s)$. Second, if IP investors withdraw and UP investors do not, then an IP investor receives $(1+\gamma)^{-1}$. Third, if UP investors withdraw and IP investors do not, then an IP investor's expected payoff is $\left(1-\frac{\gamma n}{\alpha(1-n)}\right)V(s)$ because the liquidation costs generated by impatient investors are borne entirely by IP investors rather than being shared with UP investors. Fourth, if both IP and UP investors make a second-stage withdrawal, then the fund is liquidated and an IP investor receives $\frac{1-\lambda(1+\gamma)}{(1-\lambda)(1+\gamma)}$.

The ex ante expected payoff to a non-withdrawing IP investor at the first stage depends on s. If $s \in [s^*, \hat{s}_2^{\varphi}(\bar{n}^{\varphi})]$, then $\hat{n}^{\varphi}(s) < \bar{n}^{\varphi}$, and IP investors are more likely to withdraw at the second stage than UP investors. In this case, an IP investor withdraws at the first stage unless

$$1 \leq \int_0^{\hat{n}^\varphi(s)} \frac{1-\lambda(1+\gamma)}{1-\lambda} V(s) g(\lambda)\,\mathrm{d}\lambda + \int_{\hat{n}^\varphi(s)}^{\bar{n}^\varphi} \frac{1}{1+\gamma} g(\lambda)\,\mathrm{d}\lambda + \int_{\bar{n}^\varphi}^{\bar{\lambda}} \frac{1-\lambda(1+\gamma)}{(1-\lambda)(1+\gamma)} g(\lambda)\,\mathrm{d}\lambda. \tag{C9}$$

If $s \in (\hat{s}_2^\varphi(\bar{n}^\varphi), \check{s}_2^\varphi(\bar{n}^\varphi)]$, then $\hat{n}^\varphi(s) = \bar{n}^\varphi$. An IP investor makes a first-stage withdrawal unless (21), with $\bar{n}^\varphi$ in place of $\bar{n}$, is satisfied. If $s \in (\check{s}_2^\varphi(\bar{n}^\varphi), \check{s}_2^\varphi(\bar{\lambda})]$, then $\hat{n}^\varphi(s) > \bar{n}^\varphi$. In this case, IP investors are less likely to make a second-stage withdrawal than UP investors, and an IP investor withdraws at the first stage unless

$$\begin{aligned} 1 \leq \int_0^{\bar{n}^\varphi} \frac{1-\lambda(1+\gamma)}{1-\lambda} V(s) g(\lambda)\,\mathrm{d}\lambda \\ + \int_{\bar{n}^\varphi}^{\hat{n}^\varphi(s)} \left(1 - \frac{\gamma\lambda}{\alpha(1-\lambda)}\right) V(s) g(\lambda)\,\mathrm{d}\lambda + \int_{\hat{n}^\varphi(s)}^{\bar{\lambda}} \frac{1-\lambda(1+\gamma)}{(1-\lambda)(1+\gamma)} g(\lambda)\,\mathrm{d}\lambda. \end{aligned} \tag{C10}$$

Finally, if $s > \check{s}_2^\varphi(\bar{\lambda})$, then $\hat{n}^\varphi(s) = \bar{\lambda}$, and an IP investor withdraws at the first stage unless

$$1 \leq \int_0^{\bar{n}^\varphi} \frac{1-\lambda(1+\gamma)}{1-\lambda} V(s) g(\lambda)\,\mathrm{d}\lambda + \int_{\bar{n}^\varphi}^{\bar{\lambda}} \left(1 - \frac{\gamma\lambda}{\alpha(1-\lambda)}\right) V(s) g(\lambda)\,\mathrm{d}\lambda. \tag{C11}$$

The following theorem characterizes the IP investors' first-stage withdrawal decisions when a fee is present only at the second stage.

Theorem C.3. *Given the UP investors' second-stage withdrawal threshold $\bar{n}^\varphi$, there exists a unique signal threshold $\hat{s}_1^\varphi \in (s^*, 1)$ such that IP investors withdraw at the first stage if and only if $s < \hat{s}_1^\varphi$. Furthermore, $\hat{s}_1^\varphi$ is decreasing in $\bar{n}^\varphi$.*

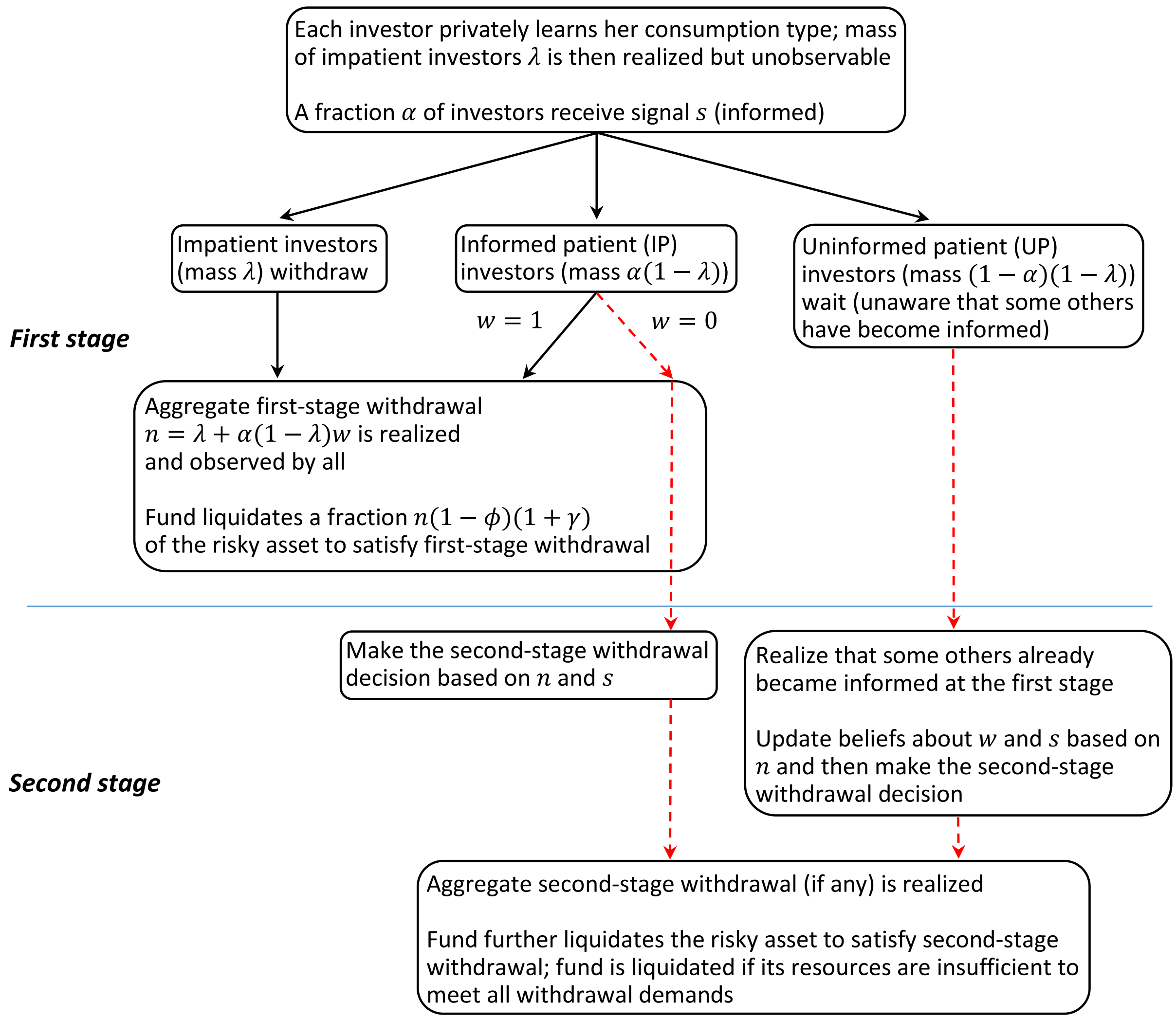

Figure 1: Timing of the withdrawal game at date 1.

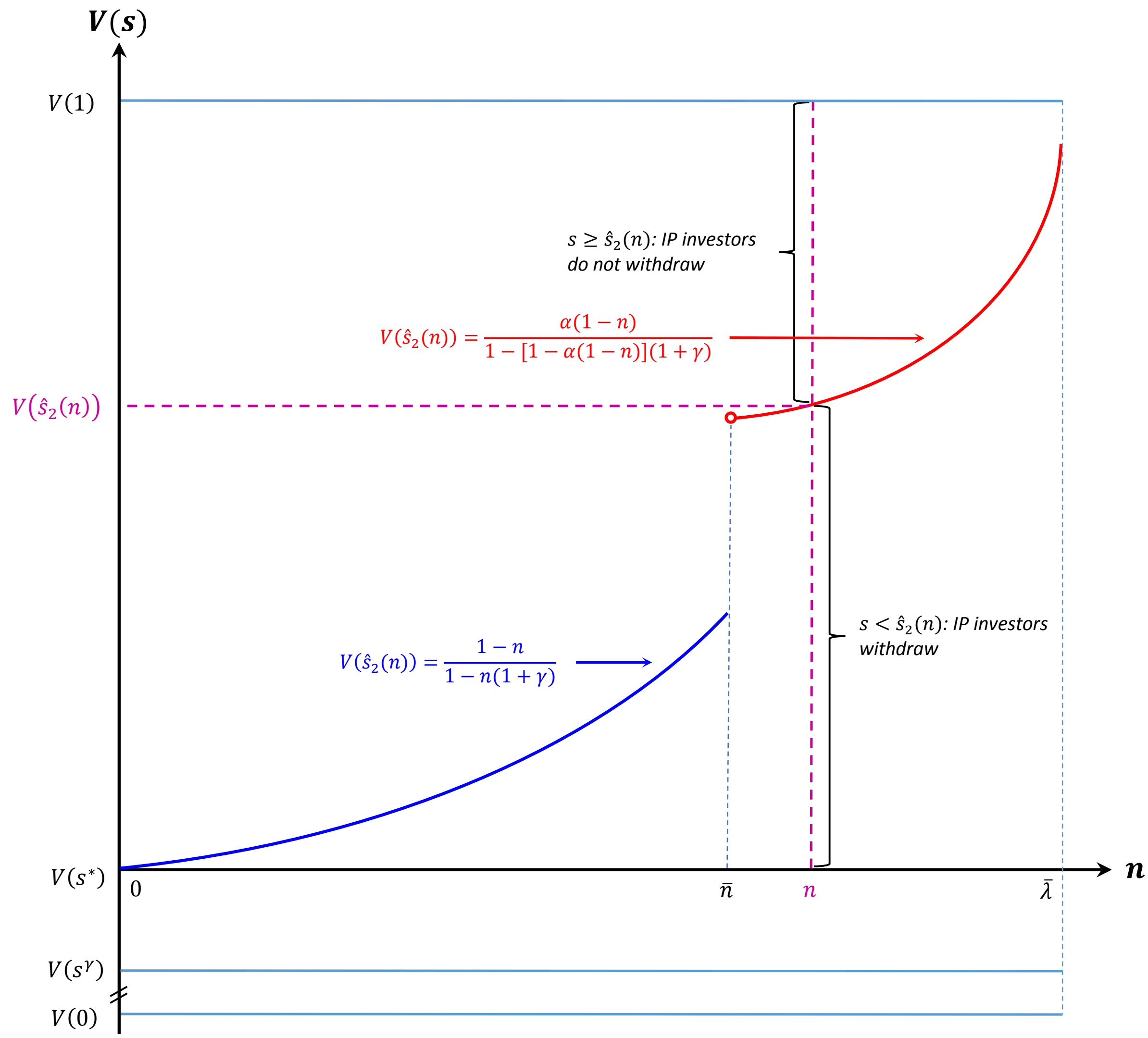

Figure 2: IP investors' second-stage signal threshold $\hat{\mathbf{s}}_2(\mathbf{n})$. The (remaining) IP investors' second-stage signal threshold $\hat{s}_2(n)$ (and, hence, s because $V(s)$ is monotonic) is plotted as a function of the first-stage withdrawal n. The blue curve represents the threshold if UP investors do not withdraw (i.e., if $n \leq \bar{n}$) at the second stage, whereas the red curve represents the threshold if UP investors withdraw (i.e., if $n > \bar{n}$).

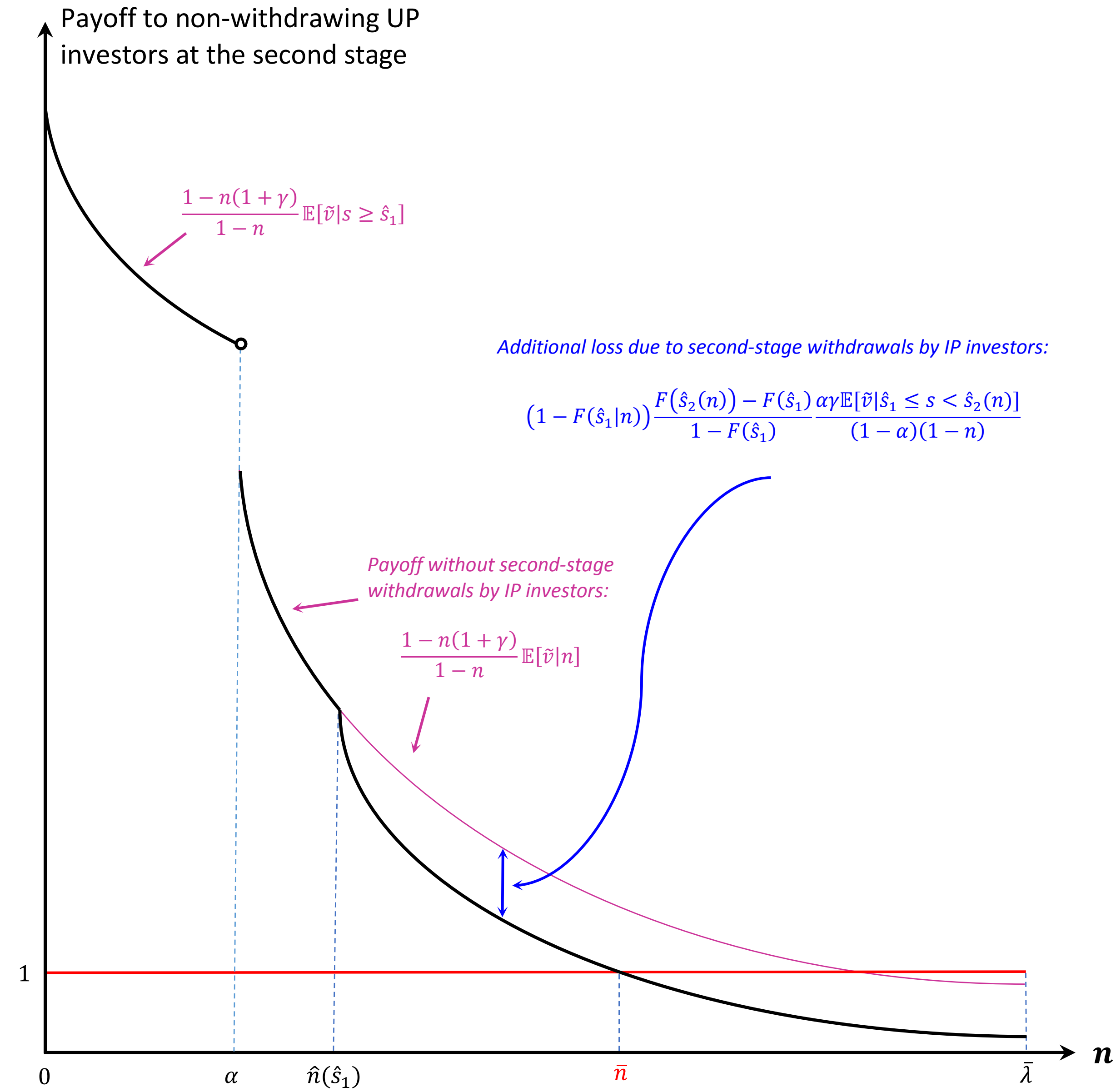

Figure 3: UP investors' second-stage withdrawal threshold $\bar{\mathbf{n}}$. The UP investors' expected payoff from maintaining their deposits until date 2 (black line) is plotted as a function of n. The UP investors' withdrawal threshold $\bar{n}$ is the point at which the curve crosses 1 (red line). The expected additional loss due to potential second-stage withdrawals by IP investors is captured by the distance between the black line and the purple line, which represents the expected payoff to non-withdrawing UP investors if IP investors do not withdraw at the second stage.

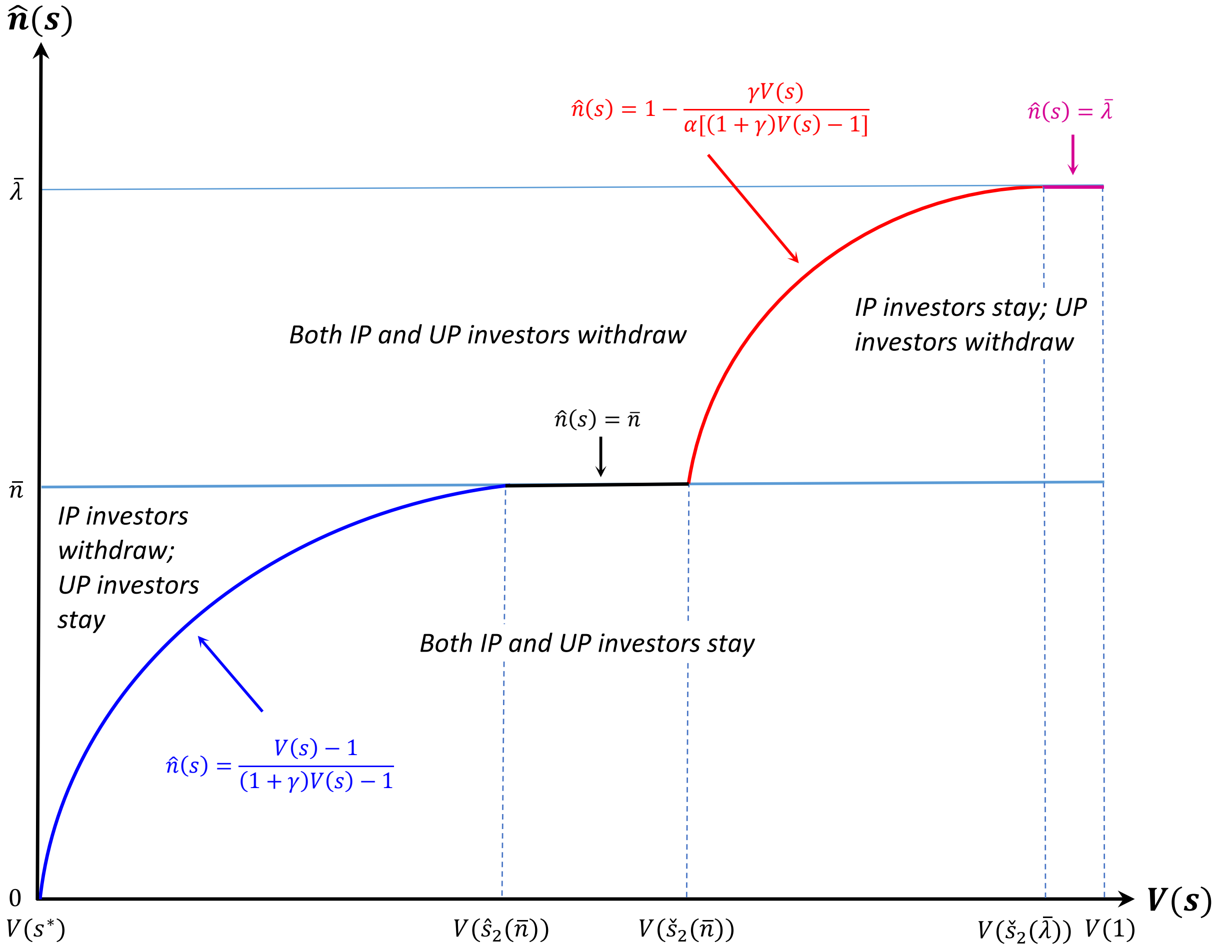

Figure 4: IP investors' second-stage withdrawal threshold $\hat{\mathbf{n}}(\mathbf{s})$. The (remaining) IP investors' second-stage withdrawal threshold $\hat{n}(s)$ is plotted as a function of $V(s)$ (and, hence, s because $V(s)$ is monotonic). The blue curve represents the thresholds if $s \in [s^*, \hat{s}_2(\bar{n})]$ wherein IP investors are more likely to withdraw than UP investors (i.e., $\hat{n}(s) < \bar{n}$), the black curve represents the threshold if $s \in (\hat{s}_2(\bar{n}), \check{s}_2(\bar{n})]$ wherein IP and UP investors' withdrawal decisions are identical (i.e., $\hat{n}(s) = \bar{n}$), and the red curve represents the threshold if $s \in (\check{s}_2(\bar{n}), \check{s}_2(\bar{\lambda})]$ wherein IP investors are less likely to withdraw than UP investors (i.e., $\hat{n}(s) > \bar{n}$). In the region with the pink curve wherein $s > \check{s}_2(\bar{\lambda})$, IP investors do not withdraw at the second-stage regardless of the first-stage withdrawal size n.

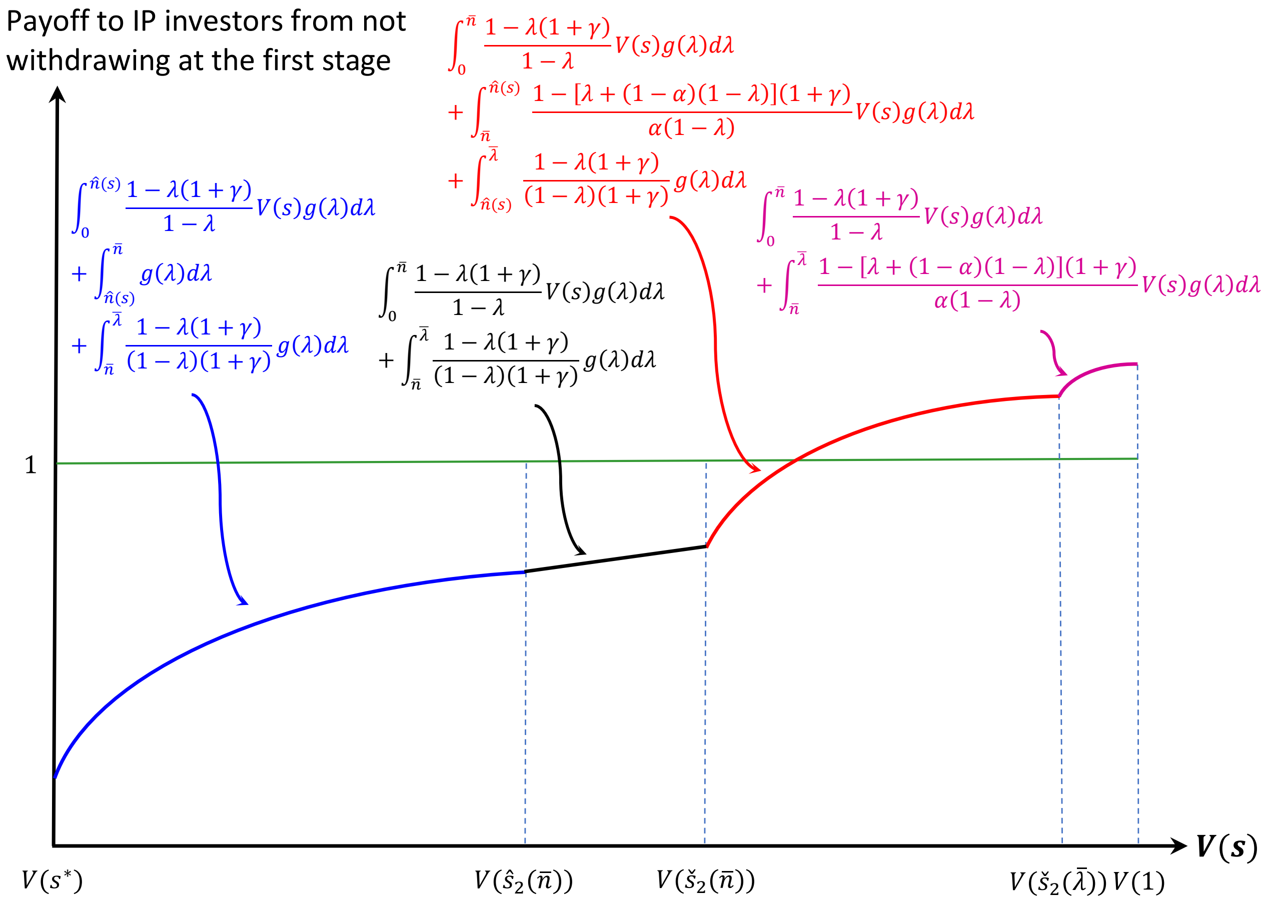

Figure 5: IP investors' ex ante expected payoff from not withdrawing at the first stage. The IP investors' expected payoff before the realized liquidity withdrawal λ is observed is plotted as a function of $V(s)$ (and, hence, s because $V(s)$ is monotonic). The blue curve represents the payoff in scenarios wherein IP investors are more likely to withdraw at the second stage than UP investors, the black curve represents the payoff in scenarios wherein IP and UP investors are equally likely to withdraw at the second stage, the red curve represents the payoff in scenarios wherein UP investors are more likely to withdraw at the second stage than IP investors, and the pink curve represents the payoff in scenarios wherein IP investors do not withdraw regardless of the first-stage withdrawal size n.

Table 1: Social welfare. The change in aggregate investment Θ along with the sign of the change in social welfare (in brackets) are listed for each scenario. The rows represent changes in UP investors' actions: when $n \leq \min\{\bar{n}, \bar{n}^{\phi}\}$, UP investors do not withdraw regardless of the fee arrangement (1st row); when $\bar{n}^{\phi} < \bar{n}$ and $n \in (\bar{n}^{\phi}, \bar{n}]$, the fee converts UP investors from non-withdrawers to withdrawers (2nd row); when $\bar{n}^{\phi} > \bar{n}$ and $n \in (\bar{n}, \bar{n}^{\phi}]$, the fee converts UP investors from withdrawers to non-withdrawers (3rd row); and when $n > \max\{\bar{n}, \bar{n}^{\phi}\}$, UP investors withdraw at the second stage irrespective of the fee (4th row). The columns represent changes in IP investors' actions: when $s < s^{\gamma}$, IP investors withdraw at the first stage regardless of the fee (1st column); when $s \in [s^{\gamma}, \max\{\hat{s}_1, \hat{s}_2(n)\})$, the fee converts IP investors from withdrawers to non-withdrawers (2nd column); and when $s \geq \max\{\hat{s}_1, \hat{s}_2(n)\}$, IP investors do not withdraw irrespective of the fee (3rd column).

	$V(s) - V(s^{\gamma}) < 0$	$V(s) - V(s^{\gamma}) \geq 0$	
	$s < s^{\gamma}$	$s \in [s^{\gamma}, \max\{\hat{s}_1, \hat{s}_2(n)\})$	$s \geq \max\{\hat{s}_1, \hat{s}_2(n)\}$
$n \leq \min\{\bar{n}, \bar{n}^{\phi}\}$	$[\lambda + \alpha(1-\lambda)]\gamma$ [−]	$\lambda\gamma + \alpha(1-\lambda)(1+\gamma)$ [+]	$\lambda\gamma$ [+]
$n \in (\bar{n}^{\phi}, \bar{n}]$	$\gamma - (1-\alpha)(1-\lambda)(1+\gamma)$ [+]	$\lambda\gamma + \alpha(1-\lambda)(1+\gamma) - (1-\alpha)(1-\lambda)$ [+/− if α big/small]	$\lambda\gamma - (1-\alpha)(1-\lambda)$ [−]
$n \in (\bar{n}, \bar{n}^{\phi}]$	$(1-\alpha)(1-\lambda)$ [−]	$1-\lambda$ [+]	$\lambda\gamma + (1-\alpha)(1-\lambda)(1+\gamma)$ [+]
$n > \max\{\bar{n}, \bar{n}^{\phi}\}$	0 [no change]	$\alpha(1-\lambda)$ [+]	$[\lambda + (1-\alpha)(1-\lambda)]\gamma$ [+]