Conceal to Coordinate

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Abstract

How informative is communication when players have an incentive to coordinate, but cannot commit to disclosing their private information? We study one-sided cheap talk in a two player investment game, where each player has noisy private information about fundamentals and the investment decision exhibits complementarity. Despite incentives to coordinate, we find that informative cheap-talk is fragile. Even when payoffs are symmetric, the sender must *conceal* information: when she chooses to invest, she only reveals that she will invest. We then ask whether the ability to commit to full disclosure is valuable. Surprisingly, we find that both the sender and the receiver may prefer partially informative cheap-talk to an equilibrium in which the sender commits to disclosing her information perfectly.

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In many economic settings, players have an incentive to coordinate their actions but cannot commit to communicate with each other. For instance, activist fund managers in a "wolf-pack" try to implicitly coordinate their acquisition of a target, without formally filing as a group for regulatory purposes — see Briggs (2007), Brav, Dasgupta, and Mathews (2014), and Coffee and Palia (2015). While the informal nature of their coordination allows them to delay legal disclosure requirements (e.g., filing a Schedule 13D with the SEC) and avoid defensive measures (e.g., poison pills), it may restrict their ability to communicate with each other. Similarly, firms that are deciding whether to adopt a new industry standard have incentives to coordinate, but may be unable to share their private information about the new technology due to legal restrictions (e.g., anti-trust concerns) or competitive pressures.

When players have an incentive to coordinate, but cannot commit to a disclosure policy, how informative is communication? We study one-sided cheap talk in a two player investment game, where players have noisy private signals about project fundamentals. When incentives are better aligned, one might expect that communication should be more informative, and that this should lead to higher welfare. We find that these conclusions do not hold generally.

We show that informative cheap-talk equilibria always feature *concealment* — when she chooses to invest, the sender only reveals that she will invest. This is true even when payoffs are perfectly symmetric across players (i.e., the sender is not biased in favor of, or against, investment). We then study whether the ability to commit to disclosing more information is valuable given the incentives to coordinate. Surprisingly, we find that expected utility for both players can be higher in the cheap talk equilibria than in the equilibrium where the sender commits to disclosing her information perfectly.

Consider the following setting. Sam (the sender) and Roy (the receiver) are deciding whether or not to invest in a risky project. The payoff from investing depends on the project's fundamentals, which may be good or bad, and players have an incentive to coordinate — Sam's payoff from investing is higher if Roy also invests (and vice versa). Each player observes a private signal about fundamentals, and before investing, Sam can send a message to Roy about her information. We also allow their incentives for coordination to be misaligned by letting Sam's payoff differ from Roy's by a known bias when both invest.[1] Notably, our setting differs from standard sender-receiver games because both players take actions.

We compare outcomes under three scenarios, which allow us to characterize the role of

[1]Both one-sided communication and the asymmetry in payoffs can arise naturally in settings where one player is "closer" to the project. For instance, the "leader" fund in an activist wolf-pack generally has the largest stake, and so is more likely to be better informed and more affected by the outcome of the campaign.

commitment in communication. The benchmark is the no-communication (NC) scenario, where Sam is not allowed to communicate with Roy. In the forced-communication (FC) scenario, Sam commits to disclose her private information to Roy perfectly. Finally, in the strategic-communication (SC) scenario, Sam cannot commit to disclosing information before observing her signal, but strategically chooses to send a cheap-talk message about her information to Roy before they decide whether to invest.

We show that equilibria in the SC scenario feature *concealment* — for signal realizations when Sam chooses to invest, her message to Roy only reveals that she will invest. Moreover, this result holds even when Sam's bias is zero.[2] To see why, note that Sam's message only affects her payoffs through changing the likelihood that Roy invests. When Sam is optimistic enough about fundamentals, she chooses to invest. In this case, however, she always has an incentive to distort her message to increase the chance that Roy invests, irrespective of her signal. In other words, even when their incentives are symmetric ex-ante (i.e., Sam's bias is zero), *conditional* on Sam's decision to invest, her incentive to convince Roy to invest is too strong to credibly convey any additional information.

Given the strong incentives to coordinate, we then ask whether Sam would commit to perfectly disclose her information if she could. In standard cheap-talk settings (e.g., Crawford and Sobel (1982)), commitment to perfect disclosure is generally Pareto superior to cheap-talk equilibria. In contrast, we find that both Sam and Roy can have higher expected utility in the SC scenario than in the FC scenario. Standard intuition suggests that Roy's expected utility should be lower under SC since Sam conceals information in this case. However, unlike pure sender-receiver games, the likelihood that Sam invests also changes across scenarios. In fact, we show that she is more likely to invest under SC, which increases Roy's expected utility. The overall effect on expected utility across the two scenarios depends on the relative impact of these offsetting effects. When payoffs are more symmetric (i.e., the bias is small), the second effect dominates the first, and expected utility is higher with SC. However, when the bias is very positive or very negative, the informational cost of concealment with SC is large, and so Roy's expected utility can be higher in the FC scenario.

The next section briefly discusses the related literature. Section 2 introduces the model, and Section 3 describes the equilibria under the NC and FC benchmarks. Section 4 characterizes the cheap-talk equilibria of our model, and discusses an extension to the case of spillovers. Section 5 studies welfare in the three scenarios, and Section 6 concludes. Proofs and additional results are in the Appendix.

[2]This is in contrast to standard cheap-talk models, where communication can be fully informative when the (analogous) bias is zero.

1 Related Literature

Our paper is related to the large literature on cheap talk initiated by Crawford and Sobel (1982) (see Sobel (2013) for a recent survey), and more specifically, models which introduce a cheap talk stage before a game with strategic complementarities. The most closely related papers include Baliga and Morris (2002), Alonso, Dessein, and Matouschek (2008), Rantakari (2008), and Hagenbach and Koessler (2010). Baliga and Morris (2002) study how adding cheap talk stage before play affects a two player, one-sided incomplete information game with strategic complementarities and positive spillovers, and characterize sufficient conditions for full communication and no communication. Alonso et al. (2008) and Rantakari (2008) compare how centralization affects coordination and communication between divisions who wish to adapt their action to local (independent) conditions, but also coordinate with other divisions. Our setting differs from these in that it features two-sided incomplete information about a common fundamental that affects both players' payoffs.[3]

Hagenbach and Koessler (2010) consider a setting in which the players in a beauty contest game strategically choose to communicate their information about a common fundamental with each other. Although complementary, the focus of their analysis is different from ours. They study the question of *who* players choose to communicate with, and characterize the equilibrium strategic communication network that arises in a setting where players either disclose their information fully or not at all. In contrast, our analysis highlights how partially informative communication can arise when players choose *what* information to communicate, and how this affects welfare.

Our welfare implications also distinguish us from standard cheap-talk models and the recent literature on Bayesian persuasion models. In cheap talk models, commitment to full disclosure Pareto dominates partially informative cheap-talk equilibria — the receiver is usually better off with more informative communication. Similarly, in standard models of Bayesian persuasion (e.g., Rayo and Segal (2010) and Kamenica and Gentzkow (2011)), while the sender may prefer to commit to partially informative communication, the receiver usually prefers more informative signals. In contrast, we find that *both* the sender and the receiver may prefer the less informative cheap talk equilibrium to a fully informative communication. The key distinction from standard sender-receiver games is that in our model, both the sender and the receiver take actions. As a result, welfare depends not only on the informativeness of the sender's messages but also on her actions.

[3]Since ours is a model with two-sided incomplete information and correlated types, it combines the distinguishing features of Examples 2 and 3 in Baliga and Morris (2002).

Our paper is also related to the literature on global games (e.g., Carlsson and Van Damme (1993) and Morris and Shin (2003)), and in particular, papers that study the effect of public information in the presence of strategic complementarities. To the extent that any information communicated in our two-player game is effectively public, our model identifies another source of endogenous public information. However, the nature of the public information in our setting is distinct. First, the earlier literature has focused on the choice of a policymaker (or social planner) who commits to a disclosure policy (e.g., Morris and Shin (2002), Angeletos and Pavan (2007)).[4] The central focus of our analysis is to study choice of communication in the absence of commitment, and whether the ability to commit to a disclosure policy is valuable. Second, while much of the earlier literature focuses on public information that is either directly about fundamentals (e.g., Morris and Shin (2002), Angeletos and Pavan (2007), Angeletos and Werning (2006), Ozdenoren and Yuan (2008)) or reflects the action of other players (e.g., Angeletos, Hellwig, and Pavan (2006), Corsetti, Dasgupta, Morris, and Shin (2004), Angeletos and Pavan (2013)), messages in communication equilibria convey information about both. As such, the Morris and Shin (2002) tradeoff between greater informational efficiency and better coordination can have different implications on welfare in our setting.

2 Model

The payoff to investment depends on fundamentals $\theta \in \{\theta_H, \theta_L\}$, which are high with prior probability $p_0 \equiv \Pr(\theta = \theta_H)$. There are two players: a sender (S, "she") and receiver (R, "he"), who receive private signals of the form $x_i = \theta + \varepsilon_i$ (for $i \in \{S, R\}$), where ε_i are independent and normally distributed with mean-zero and variance σ_i^2 (i.e., $\varepsilon_i \sim N(0, \sigma_i^2)$). Each player must decide whether to invest ($a_i = 1$) or not ($a_i = 0$), and conditional on θ, the payoffs are given by the following table:

$S \setminus R$	Invest ($a_R = 1$)	Not Invest ($a_R = 0$)
Invest ($a_S = 1$)	$\theta + b$, θ	$\theta - 1$,0
Not Invest ($a_S = 0$)	0, $\theta - 1$	0,0

Note that when both players invest, S receives $\theta + b$ while R receives θ. The parameter b captures a potential conflict of interest. We maintain the following assumptions on the parameters:

[4] Banerjee and Liu (2014) characterize optimal precision of public information in the Morris and Shin (2002) setting when the policymaker cannot commit to a disclosure policy, but is restricted to linear strategies.

(A1) $\theta_L < 0 < 1 < \theta_H$: This ensures that it is efficient to each player to invest in the "good" state when fundamentals are high (i.e., $\theta = \theta_H$) and to not invest in the "bad" state when fundamentals are low (i.e., $\theta = \theta_L$).

(A2) $b > -1$: This ensures that S is not too biased *against* coordinated investment, so that S and R both have an incentive to coordinate on investment when fundamentals are good (i.e., $\theta = \theta_H$).[5]

(A3) $b < -\theta_L$: This ensures that S never finds it optimal to invest if she knows $\theta = \theta_L$ i.e., she is not too biased *in favor* of coordinated investment.

We allow for communication between the players — player S can send a message $m(x_S)$ about her signal to player R before they decide whether or not to invest. Specifically, we assume that a messaging rule $\mu : \Re \to M$ is a function that takes a signal realization x_S to an element (or *message*) $m = \mu(x_S) \in M$, where M is the Borel algebra on the reals $\Re$. We consider three scenarios: (i) no communication (NC), (ii) forced communication (FC), and (iii) strategic communication (SC). The no communication scenario serves as a benchmark when the players are not allowed to communicate (i.e., $\mu(x_S) = \Re$). The forced communication scenario assumes that S commits to perfectly disclosing her signal to R before they decide whether to invest (i.e., $\mu(x_S) = x_S$). Finally, in the strategic communication scenario, S can send an arbitrary message $\mu(x_S)$ about her signal x_S to R after they each observe their signals, but before they decide whether to invest.

Remark 1. The assumption that players can take one of a finite number of actions is without loss of generality. For instance, suppose players are risk-neutral, can choose an investment level $a_i \in [0, 1]$, and the payoff to player i is given by $a_i(\theta - 1 + a_j)$. In this case, a player's optimal investment decision is characterized by the same cutoff strategy as in our benchmark specification, since each player chooses the maximum investment level ($a_i = 1$) if she chooses to invest at all.[6]

We explore alternative payoff specifications in supplementary analysis. Specifically, for the proofs of our main results, we consider the case where both S and R can have biased payoffs i.e., if both invest, the payoffs are $(\theta + b_S, \theta + b_R)$. This does not qualitatively change the characterization of equilibria in the three scenarios. Similarly, as we show in Appendix B, the equilibria do not qualitatively change when the cost to investing alone for S can be different i.e., if S invests but R does not, her payoffs are $(\theta - c, 0)$. Finally, Section 4.1 considers a

[5]When $b \leq -1$, S has a higher payoff from investing when R does not invest, and the resulting investment game is one of strategic substitutability.

[6]We assume that if the player is indifferent between investing and not, she chooses $a_i = 1$.

specification with spillovers i.e., S receives an incremental payoff ν when R invests, irrespective of whether she invests.

We restrict attention to a finite number of fundamental states due to tractability. In particular, updating beliefs conditional on private information and messages takes a log-linear form in our setting, as the next result highlights.[7]

Lemma 1. *Conditional on a signal $x_i = x$, and a message $m \in M$, posterior beliefs about θ are given by $p(x,m) \equiv \Pr(\theta = \theta_H | x_i = x, x_j \in m)$, where*

$$\log\left(\frac{p(x,m)}{1-p(x,m)}\right) = \log\left(\frac{p_0}{1-p_0}\right) + \frac{1}{\sigma_i^2}\left(\theta_H - \theta_L\right)\left(x - \frac{\theta_H+\theta_L}{2}\right) + \log\left(\frac{\Pr(x_j \in m|\theta_H)}{\Pr(x_j \in m|\theta_L)}\right). \tag{1}$$

Also, note that $\log\left(\frac{\Pr(x_j=x|\theta_H)}{\Pr(x_j=x|\theta_L)}\right) = \frac{1}{\sigma_j^2}\left(\theta_H - \theta_L\right)\left(x - \frac{\theta_H+\theta_L}{2}\right)$, *and*

$$\log\left(\frac{\Pr(c_1 < x_j \leq c_2|\theta_H)}{\Pr(c_1 < x_j \leq c_2|\theta_L)}\right) = \log\left(\frac{\Phi\left(\frac{c_2-\theta_H}{\sigma_j}\right) - \Phi\left(\frac{c_1-\theta_H}{\sigma_j}\right)}{\Phi\left(\frac{c_2-\theta_L}{\sigma_j}\right) - \Phi\left(\frac{c_1-\theta_L}{\sigma_j}\right)}\right).$$

Conditional on receiving a signal x_S, and sending a message m, player S's optimization problem is given by

$$\Pi_S(x_S, m) = \max_{a_S \in \{0,1\}} a_S\, \pi_S(x_S, m), \tag{2}$$

where π_S is the marginal benefit of investing:

$$\pi_S(x_S, m) = \begin{array}{c} p(x_S, \Re)\left[(\theta_H + b) - (1+b)\Pr(a_R = 0|\theta_H, m)\right] \\ +\left(1 - p(x_S, \Re)\right)\left[(\theta_L + b) - (1+b)\Pr(a_R = 0|\theta_L, m)\right] \end{array}. \tag{3}$$

Similarly, conditional on receiving a signal x_R and a receiving a message m, player R's optimization problem is given by:

$$\Pi_R(x_R, m) = \max_{a_R \in \{0,1\}} a_R\, \pi_R(x_R, m), \tag{4}$$

where π_R is given by

$$\pi_R(x_R, m) = \begin{array}{c} p(x_R, m)\left[\theta_H - \Pr(a_S = 0|\theta_H, m)\right] \\ +\left(1 - p(x_R, m)\right)\left[\theta_L - \Pr(a_S = 0|\theta_L, m)\right] \end{array}. \tag{5}$$

[7]With a continuum of states and standard distributional assumptions (e.g., normal or uniform priors), updating beliefs about fundamentals using both a private signal and general messages in equilibrium is less analytically tractable.

Given a message m, since π_i is increasing in the posterior probability $p(\cdot)$, and $p(\cdot)$ is increasing in the private signal x, each player chooses to follow a cutoff strategy: player i only invests when her signal is greater than, or equal to, a cutoff $k_i(m)$, i.e., if $x_i \geq k_i(m)$, but not otherwise. The cutoff is characterized by the first order condition:

$$\pi_i(k_i(m), m) = 0. \tag{6}$$

We focus on pure strategy, Perfect Bayesian equilibria.[8] In particular, an equilibrium of the game with SC is characterized by a messaging rule $\mu : \Re \to M$ and cutoff strategies $\{k_S(m), k_R(m)\}$, such that: (i) the messaging rule μ is truthful (i.e., for all x_S, $x_S \in \mu(x_S)$), (ii) the messaging rule μ is optimal for player S (i.e., $\Pi_S(x_S, \mu(x_S)) \geq \Pi_S(x_S, m)$ for $m \neq \mu(x_S)$), (iii) given a message m, it is optimal for player i to only invest when $x_i \geq k_i(m)$ (i.e., expression (6) holds), and (iv) players beliefs satisfy Bayes' rule wherever it is well-defined. In particular, the restriction to pure-strategy, truth-telling equilibria implies that given a messaging rule μ, each possible signal realization x_S maps into only one message $\mu(x_S)$. For the games with NC and FC, an equilibrium is characterized by conditions (iii) and (iv) above, since the messaging rule is exogenously specified ($\mu(x_S) = \Re$ and $\mu(x_S) = x_S$, respectively).

3 Benchmarks

This section characterizes the equilibria in natural benchmark scenarios. These are useful in developing intuition, and also for the welfare comparisons we make in Section 5.

3.1 No Communication

The no communication benchmark recovers a standard result from the global games literature.

Proposition 1. *Let the function $K(k; b, \sigma_i, \sigma_j)$ be defined as:*

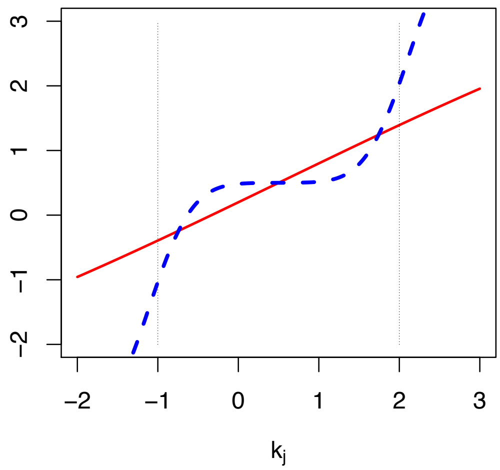
$$K(k; b, \sigma_i, \sigma_j) \equiv \frac{\theta_H + \theta_L}{2} + \frac{\sigma_i^2}{\theta_H - \theta_L}\left\{\log\left(\frac{(1+b)\Phi\left(\frac{k-\theta_L}{\sigma_j}\right) - (\theta_L + b)}{(\theta_H + b) - (1+b)\Phi\left(\frac{k-\theta_H}{\sigma_j}\right)}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right\}, \tag{7}$$

where $\Phi(\cdot)$ is the CDF of the normal distribution. Suppose S cannot communicate with R i.e.,

[8] Since the sender's type (her signal) is continuous and unbounded, restriction to pure strategies is without loss of generality.

Figure 1: Best response function $K_i(k_j)$ in the No Communication Scenario

The figure plots the best response function $K_i(k_j, 0, \sigma_i, \sigma_j)$ in equation (7) when $\sigma_i = \sigma_j = 4$ (solid) and when $\sigma_j = \frac{\sigma_i}{10} = 0.4$ (dashed). The slope for the best response function is much steeper for the latter case when k_j is close to θ_H or θ_L (marked by dotted vertical lines). The other parameters are set to $p_0 = 0.5$, $b = 0$, $\theta_H = 2$, and $\theta_L = -1$.



for all x_S, we have $\mu(x_S) = \Re$. Then there exist equilibria characterized by cutoffs $k_{S,NC}$ and $k_{R,NC}$, which solve the system: $k_{S,NC} = K(k_{R,NC}; b, \sigma_S, \sigma_R)$ and $k_{R,NC} = K(k_{S,NC}; 0, \sigma_R, \sigma_S)$. For given b, θ_H and θ_L, there exist cutoffs $\bar{a}(b, \theta_H, \theta_L)$ and $\underline{a}(b, \theta_H, \theta_L)$ so that if $\frac{\sigma_S^2}{\sigma_R} < \bar{a}$ and $\frac{\sigma_R^2}{\sigma_S} < \underline{a}$, the equilibrium is unique. When $\sigma_S = \sigma_R = \sigma$, there exists a cutoff $a(b, \theta_H, \theta_L)$ such that if $\sigma < a$, the equilibrium is unique.

The best response function (7) is increasing in the other player's cutoff. This is intuitive — player j is less likely to invest when her cutoff is higher, which leads player i to respond by increasing her own cutoff. However, increasing best response functions imply that there may be multiple equilibria. As we discuss in the proof for Proposition 1, a sufficient condition for uniqueness is that the slope of the best response function (7) is less than one (i.e., $\frac{\partial K_i}{\partial k_j} < 1$). In the special case when the signals are symmetrically distributed (i.e., $\sigma = \sigma_R = \sigma_S$), the sufficient condition for uniqueness mirrors those in the earlier literature which require that private signals are sufficiently accurate (see Morris and Shin (2001), Frankel, Morris, and Pauzner (2003), and Morris and Shin (2003) for extensive discussions).

In the general case, the sufficient conditions require not only that each player's private signal is sufficiently precise, but also that neither player's signal is too precise relative to the other's signal. If player j's signal is too precise relative to player i's, then player i's best response changes very quickly when k_j is close to either θ_H or θ_L — Figure 1 presents an example of this. This can lead to multiple solutions for the system of equations in Proposition 1, and consequently, multiple equilibria. In contrast, as we show in the next subsection, there always exists a unique equilibrium when S can commit to revealing her information perfectly.

3.2 Forced Communication

Suppose S is forced to reveal her information to R perfectly i.e., $\mu(x_S) = x_S$ for all x_S. Then, conditional on x_S and her own signal x_R, player R's posterior beliefs about $\theta = \theta_H$ are given by

$$\log\left(\tfrac{p}{1-p}\right) = \log\left(\tfrac{p_0}{1-p_0}\right) + \tfrac{1}{\sigma_S^2}\left(\theta_H - \theta_L\right)\left(x_S - \tfrac{\theta_H+\theta_L}{2}\right) + \tfrac{1}{\sigma_R^2}\left(\theta_H - \theta_L\right)\left(x_R - \tfrac{\theta_H+\theta_L}{2}\right). \quad (8)$$

Since R can perfectly observe x_S, there is no uncertainty about whether R will invest. This implies that if S reveals a signal x_S and uses a cutoff k_S, player R's best response is to invest only if $x_R \geq K_{FC}(x_S, k_S)$, where

$$K_{FC}(x,k) \equiv \begin{cases} \frac{\theta_H+\theta_L}{2} + \frac{\sigma_R^2}{\sigma_S^2}\left(\frac{\theta_H+\theta_L}{2} - x\right) + \frac{\sigma_R^2}{\theta_H-\theta_L}\left(\log\left(-\frac{\theta_L}{\theta_H}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right) & \text{if } x \geq k \\ \frac{\theta_H+\theta_L}{2} + \frac{\sigma_R^2}{\sigma_S^2}\left(\frac{\theta_H+\theta_L}{2} - x\right) + \frac{\sigma_R^2}{\theta_H-\theta_L}\left(\log\left(\frac{1-\theta_L}{\theta_H-1}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right) & \text{if } x < k \end{cases}. \quad (9)$$

Intuitively, the receiver's best response is decreasing in the sender's signal — a higher signal implies that the higher state ($\theta = \theta_H$) is more likely, and this leads R to lower his cutoff. The next result characterizes the equilibrium in this scenario in terms of the above best response function.

Proposition 2. *Let $k_{S,FC}$ be the (unique) fixed point of $x = K(K_{FC}(x,x), b, \sigma_S, \sigma_R)$, where $K(\cdot)$ is defined by equation (7), and $K_{FC}(\cdot)$ is defined by (9). If S is forced to reveal her information x_S to R perfectly (i.e., $\mu(x_S) = x_S$ for all x_S), then the unique equilibrium is characterized by the cutoff $k_{S,FC}$ for player S and the cutoff (function) $K_{FC}(x_S, k_{S,FC})$ for player R.*

The result highlights how S's ability to communicate changes the nature of the coordination game: unlike the NC scenario, there always exists a unique equilibrium with forced communication. The equilibrium is characterized by player S's cutoff (i.e., k_S) given player R's cutoff *conditional* on the information that player S's signal is equal to her cutoff (i.e., $x_S = k_S$). In contrast to the NC scenario, R faces no uncertainty about whether S invests. This implies that conditional on S's signal being equal to her cutoff (i.e., $x_S = k_S$), a higher cutoff is good news about fundamentals and so R's best response decreases in k_S.[9] As we show in the proof, this

[9] This is analogous to the effect of observing the action of an earlier player in a sequential move global game

ensures that there always exists a unique solution to the fixed point problem in Proposition 2 that characterizes the equilibrium.

4 Strategic Communication

We now turn to the case where S can send an arbitrary message to R after observing her signal. As is common in cheap talk models, there exist multiple equilibria. However, as the following proposition describes, they are all characterized by a common feature: the sender S conceals information about the realization of her signal when she invests.[10]

Proposition 3. *Let $k_{S,SC}$ be the fixed point of $x = K(K_R(x), b, \sigma_S, \sigma_R)$, where K is defined by (7), and $K_R(x)$ is given by:*

$$K_R(x) = \frac{\theta_H+\theta_L}{2} + \frac{\sigma_R^2}{\theta_H-\theta_L}\left\{\log\left(\frac{-\theta_L}{\theta_H}\right) - \log\left(\frac{p_0}{1-p_0}\right) - \log\left(\frac{1-\Phi\left(\frac{x-\theta_H}{\sigma_S}\right)}{1-\Phi\left(\frac{x-\theta_L}{\sigma_S}\right)}\right)\right\}. \tag{10}$$

In any sender-optimal strategic equilibrium, (i) S invests if and only if $x_S \geq k_{S,SC}$ and (ii) the messaging rule is equivalent to $\mu(\cdot)$, where for any signal $x_S \geq k_{S,SC}$, the optimal message is $\mu(x_S) = [k_{S,SC}, \infty)$.

Instead of detailing the proof of the above result, we try to provide some intuition for this result. First, note that the sender's message affects her payoff only through the likelihood that the receiver invests. For signal realizations where S chooses to invest, she always has an incentive to report that her signal is higher than it actually is since this increases the likelihood that R invests, but does not affect her payoff otherwise. But this implies that she cannot convey *any* additional information credibly when she chooses to invest.[11] The restriction to sender-optimal equilibria rules out equilibria in which the sender either babbles, or pools some low (no-invest) signals with high (invest) signals.[12]

(e.g., Corsetti et al. (2004)).

[10]Note that the nature of cheap talk equilibrium is not an immediate consequence of the fact that R has a binary action space. For instance, Chakraborty and Yilmaz (2016) consider a setting in which detailed cheap talk communication arises even though the receiver has a binary action.

[11]As we discuss in the proof, there is some indeterminacy. We show that while S can send other messages when $x_S \geq k_{S,SC}$, they must be equivalent to the message $x_S \in [k_{S,SC}, \infty)$ in terms of their impact on R's posterior beliefs. As a result, for all economically relevant implications, the messaging rules are equivalent to the one stated in the Proposition when $x_S \geq k_{S,SC}$.

[12]In either case, a sender with a high signal realization should strictly prefer to separate herself from these no-investment, low types.

Second, as we argue in the proof, it is natural that the messaging rule and the investment decisions are determined by the same cutoff. Intuitively, the message m equivalent to "S will invest" should include all signal realizations such that S chooses to invest having sent message m, but should exclude any signal realizations such that S optimally chooses not to invest having sent that message. Also, note that if S chooses to invest at a signal realization x_S, having sent message m, then she must necessarily choose to invest for all signal realizations $x > x_S$, conditional on sending message m. This, in turn, ensures that the investment and messaging rule intervals are half-lines.

Finally, the unique cutoff $k_{S,SC}$ is the solution to a fixed point problem: player S's cutoff (i.e., k_S) is her best response to player R's cutoff *conditional* on the information that player S's signal is equal to, or greater than, her cutoff (i.e., $x_S \geq k_S$). As in the FC scenario, the existence and uniqueness of this cutoff is guaranteed by the fact that the receiver's best response (10) is decreasing in the sender's cutoff, conditional on her message. However, unlike the FC scenario, uniqueness of the cutoff $k_{S,SC}$ does not imply uniqueness of equilibria. This is because, for signal realizations where S does not invest (i.e., $x_S < k_{S,SC}$), she is indifferent to various messaging rules. This naturally gives rise to two extreme equilibria, which can be characterized by how informative S is about her signal in this region. We describe these in the following result.

Proposition 4. *(i) The least informative strategic equilibrium is characterized by the messaging rule:*

$$\mu\left(x_S\right) = \begin{cases} \left(-\infty, k_{S,SC}\right) & \text{if } x_S < k_{S,SC} \\ \left[k_{S,SC}, \infty\right) & \text{if } x_S \geq k_{S,SC} \end{cases}, \tag{11}$$

and the cutoffs $k_{S,SC}$ for player S and the cutoff function $K_{SC}\left(m\right)$ for player R, where

$$K_{SC}\left(\left(-\infty, k_{S,SC}\right)\right) = \frac{\theta_H+\theta_L}{2} + \frac{\sigma_R^2}{\theta_H-\theta_L}\left\{\log\left(\frac{1-\theta_L}{\theta_H-1}\right) - \log\left(\frac{p_0}{1-p_0}\right) - \log\left(\frac{\Phi\left(\frac{k_{S,SC}-\theta_H}{\sigma_S}\right)}{\Phi\left(\frac{k_{S,SC}-\theta_L}{\sigma_S}\right)}\right)\right\}, \tag{12}$$

$$K_{SC}\left(\left[k_{S,SC}, \infty\right)\right) = K_R\left(k_{S,SC}\right). \tag{13}$$

(ii) The most informative strategic equilibrium is characterized by the messaging rule:

$$\mu\left(x_S\right) = \begin{cases} x_S & \text{if } x_S < k_{S,SC} \\ \left[k_{S,SC}, \infty\right) & \text{if } x_S \geq k_{S,SC} \end{cases}, \tag{14}$$

and the cutoffs $k_{S,SC}$ for player S and the cutoff function $K_{SC}(m)$ for player R, where

$$K_{SC}(x_S) = \tfrac{\theta_H+\theta_L}{2} + \tfrac{\sigma_R^2}{\sigma_S^2}\left(\tfrac{\theta_H+\theta_L}{2} - x_S\right) + \tfrac{\sigma_R^2}{\theta_H-\theta_L}\left(\log\left(\tfrac{1-\theta_L}{\theta_H-1}\right) - \log\left(\tfrac{p_0}{1-p_0}\right)\right), \tag{15}$$

$$K_{SC}([k_{S,SC},\infty)) = K_R(k_{S,SC}). \tag{16}$$

In the least informative equilibrium, S sends one of two possible messages, which correspond to whether or not she invests. In the most informative equilibrium, S reveals her signal perfectly when she chooses not to invest, but conceals the realization of her signal in the investment region. The sender is indifferent between these equilibria because her payoff from not investing is unaffected by the receiver's action. As we discuss in the next subsection, this indifference plays an important role in ensuring one-sided cheap talk is partially informative in our setting.

4.1 Spillovers

We consider an alternative specification of payoffs in this section to establish the robustness of our results, and to highlight the role of the sender's indifference when not investing in generating informative communication. Suppose the payoffs are given by the following table:

$S \setminus R$	Invest $(a_R = 1)$	Not Invest $(a_R = 0)$
Invest $(a_S = 1)$	$\theta + \nu,\ \theta$	$\theta - 1$,0
Not Invest $(a_S = 0)$	$\nu,\ \theta - 1$	0,0

In this case, R's decision to invest has a spillover on S's payoffs i.e., irrespective of whether S invests, she receives an incremental payoff of ν when R invests. To ensure we are in the interesting region of the parameter range, the assumptions (A1)-(A3) generalize to the following: (i) $\theta_L < 0 < 1 < \theta_H$, (ii) $\nu > -1$, and (iii) $\nu < -\theta_L$. While R's incremental payoff from investing π_R remains the same as in the benchmark model, S's optimal decision is given by

$$\Pi_S(x_S,m) = \max_{a_S\in\{0,1\}} a_S\pi_S^1(x_S,m) + (1-a_S)\pi_S^0(x_S,m), \tag{17}$$

where $\pi_S^a(x_S,m)$ is the payoff for action a i.e.,

$$\pi_S^1(x_S,m) = \begin{array}{c} p(x_S,\Re)\left[\theta_H + \nu\left[1 - \Pr(a_R = 0|\theta_H,m)\right] - \Pr(a_R = 0|\theta_H,m)\right] \\ +(1-p(x_S,\Re))\left[\theta_L + \nu\left[1-\Pr(a_R = 0|\theta_L,m)\right] - \Pr(a_R = 0|\theta_L,m)\right] \end{array}. \tag{18}$$

$$\pi_S^0(x_S,m) = \nu\left\{p(x_S,\Re)\left[1-\Pr(a_R = 0|\theta_H,m)\right] + (1-p(x_S,\Re))\left[1-\Pr(a_R = 0|\theta_L,m)\right]\right\} \tag{19}$$

Player S's incremental payoff from investing is independent of ν, since

$$\begin{aligned}\pi_S(x_S, m) &= \pi_S^1(x_S, m) - \pi_S^0(x_S, m) \\ &= p(x_S, \Re)\left[\theta_H - \Pr(a_R = 0|\theta_H, m)\right] + (1 - p(x_S, \Re))\left[\theta_L - \Pr(a_R = 0|\theta_L, m)\right].\end{aligned}$$

As a result, the NC and FC equilibria with a spillover are identical to the corresponding equilibria in the benchmark model with $b = 0$. However, as the following result establishes, with strategic communication, there is no equilibrium in which S can communicate any information about x_S to R when there is a positive spillover. When there is a negative spillover, only a partially informative equilibrium analogous to the least-informative equilibrium survives.

Proposition 5. *If R's investment decision generates a positive spillover for S, i.e., $\nu > 0$, there can (effectively) be no communication in any strategic equilibrium. If R's investment decision generates a negative spillover for S, i.e., $\nu < 0$, any strategic equilibrium is equivalent to the least informative equilibrium described in Proposition 4 (with $b = 0$).*

As in our benchmark specification, informative cheap talk is difficult to sustain. In fact, if the spillover is positive, even if arbitrarily small, no information can be communicated in a one-sided cheap talk equilibrium. When the spillover is negative, only a partially informative cheap-talk equilibrium analogous to the least-informative SC equilibrium above survives. This is because, even if S decides not to invest, she has an incentive to increase (decrease) the likelihood that R invests when the spillover ν is positive (negative, respectively). As a result, when the spillover is positive, the sender always has an incentive to distort her message upwards, and so cannot communicate any information via cheap talk. When the spillover is negative, she has an incentive to distort her message upwards (downwards) when she chooses to invest (not invest, respectively), and so cannot convey any additional information.

The above result is also related to Baliga and Morris (2002), who establish that in special cases of their one-sided, incomplete information model, no communication is possible when there are positive spillovers. Morris and Shin (2003) informally discuss a two-player, investment game (similar to ours) where both players impose spillovers. They suggest that an argument similar to Baliga and Morris (2002) implies that fully informative cheap talk is possible when spillovers are negative, but not when spillovers are positive. Our analysis suggests that the assumption of symmetric payoffs is important for these conclusions: with one-sided spillovers, we show one-sided cheap talk cannot be informative at all with positive spillovers and is, at best, partially informative when spillovers are negative.

5 Welfare

We explore whether the ability to commit to communicate (or not) is valuable in our setting. For instance, when payoffs are completely aligned (i.e., $b = 0$), one might expect that S and R are indifferent between strategic communication and forced communication, and perhaps prefer the latter, since there is more information shared in equilibrium. Similarly, when payoffs are not aligned (i.e., $b \neq 0$), one might expect S and R to strictly prefer forced communication since it allows them to commit around the difference in payoffs. We begin with the following observation.

Proposition 6. *Player S is most likely to invest under strategic communication than under the no-communication or forced communication scenarios i.e.,*

$$k_{S,NC} \geq k_{S,SC} \text{ and } k_{S,FC} \geq k_{S,SC}. \tag{20}$$

Player S's cutoff in each scenario is her best response (given by (7)) to player R's cutoff, conditional on his information set in equilibrium. Since R faces uncertainty about whether S invests in the NC scenario, but no such uncertainty in the SC scenario, he is less likely to invest in the NC equilibrium i.e., his best response to S's cutoff is always higher. This in turn implies that in equilibrium, S's cutoff is higher under NC than under SC.

In contrast, R faces no uncertainty about whether S invests in either the FC or SC scenarios. However, in the SC scenario, R conditions on the information that player S's signal is higher than her cutoff, while in the FC scenario, he conditions on the realization of the signal itself. For any cutoff k chosen by player S, the information that her signal is greater than or equal to her cutoff (i.e., $x_S \geq k$) makes R more optimistic about fundamentals than the information that her signal is equal to her cutoff (i.e., $x_S = k$) — this is because the distribution of the signal, parameterized by θ, satisfies the monotone likelihood ratio property. As we show in the proof of Proposition 6, this implies that, for any cutoff k chosen by player S, player R's best response to $x_S = k$ in the FC scenario is always higher than his response to $x_S \geq k$ in the SC scenario, which, in turn, implies $k_{S,FC} \geq k_{S,SC}$.

We compute the expected utility for player $i \in \{S, R\}$, U_i, as the unconditional expectation of $\Pi_i(x_i, \cdot)$ over realizations of x_i.

$$U_i = \mathbb{E}\left[\Pi_i(x_i, m)\right] = \mathbb{E}\left[a_i(x_i)\left(a_j(\theta + b_i) + (1 - a_j)(\theta - 1)\right)\right], \tag{21}$$

where $b_R = 0$ and $b_S = b$. As a baseline, we first characterize the investment decision rule which maximizes welfare (i.e., the sum $U_S + U_R$).

Proposition 7. *Conditional on signals x_S and x_R, the investment rule that maximizes $U_S + U_R$ is given by: both S and R invest if and only if*

$$\sigma_R^2 x_S + \sigma_S^2 x_R \geq \left(\sigma_S^2 + \sigma_R^2\right) \frac{\theta_H + \theta_L}{2} + \frac{\sigma_R^2 \sigma_S^2}{\theta_H - \theta_L} \left(\log\left(-\frac{b+2\theta_L}{b+2\theta_H}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right) \equiv K_M. \tag{22}$$

The above decision rule, which we refer to as the welfare-maximizing investment decision, represents the recommendation of a social planner who maximizes welfare conditional on both private signals. Relative to the NC, FC and SC scenarios, the decision rule is different for two reasons. First, it is informationally more efficient: both players' decisions are determined by the optimal use of both signals.[13] Second, it accounts for the externality that each player's action has on the other player's payoffs. Since it provides an upper bound on the welfare that may be achieved in our setting, it serves as a natural benchmark for comparison.[14]

In general, the welfare outcomes in the NC, FC and SC scenarios are worse than under the above investment rule. However, as the sender's private signal becomes infinitely precise, welfare under the communication scenarios (i.e., FC and SC) approaches this benchmark, while welfare under the no communication benchmark is strictly lower. This is summarized by the following result.

Proposition 8. *With forced and strategic communication (i.e., in the FC and SC equilibria), welfare is maximized when the sender's private signal becomes infinitely precise (i.e., when $\sigma_S \to 0$) irrespective of the bias b. In the no communication equilibrium (i.e., NC equilibrium), welfare may not be maximized even when the sender's signal is infinitely precise.*

Moreover, when the sender's signal is noisy, welfare is still higher in the cheap-talk equilibrium than in the no communication equilibrium.

Proposition 9. *Expected utility for both players is higher under the least-informative strategic communication equilibrium than it is under no communication.*

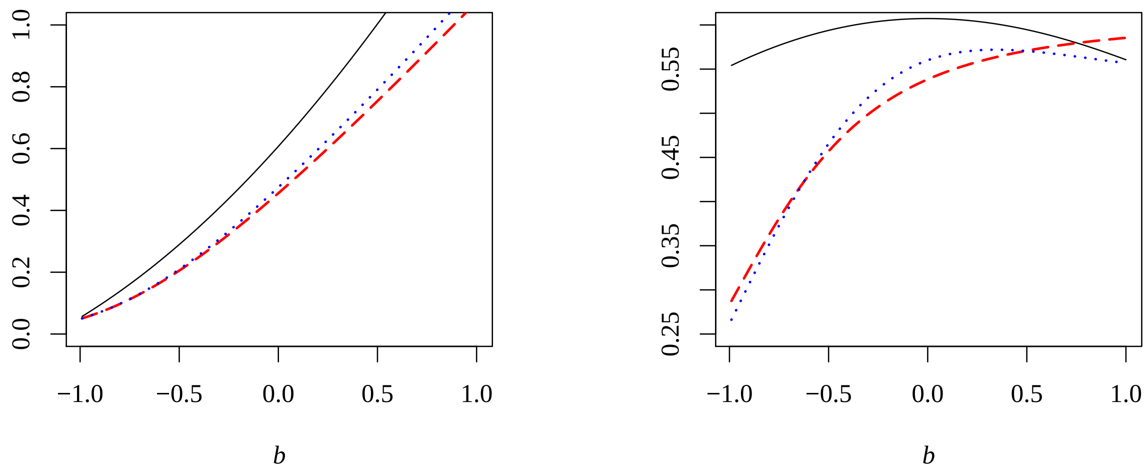
Since more information is communicated to R under SC than under NC, the intuition from standard strategic communication games suggests that the above result may be immediate.

[13] In contrast, even in the most informative of the other three scenarios, while R conditions on both signals, S can only condition on her own signal.

[14] We do not claim this outcome is achievable using an optimally designed mechanism. Our analysis is concerned with situations in which players have no commitment power, and as such, cannot commit to using an optimal mechanism.

Figure 2: Expected utility as a function of b

The figure plots expected utility U_S for player S and expected utility U_R for player R as a function of the bias parameter b for the forced communication equilibrium (dashed), the least informative strategic communication equilibrium (dotted), and the welfare maximizing investment decision (solid). The benchmark parameter levels are set to: $p_0 = 0.5$, $\theta_H = 2$, $\theta_L = -1$, and $\sigma_S = \sigma_R = 4$.



(a) U_S vs. b (b) U_R vs. b

However, an important difference from pure sender-receiver games is that the sender's investment strategy is also different across the two scenarios — specifically, as Proposition 6 suggests, S is more likely to invest under SC than under NC. These effects reinforce each other when comparing the NC and SC equilibria, and as a result, expected utility is higher under SC for both players. However, the two effects offset each other when comparing the FC and SC scenarios: R receives more information under FC, but S is more likely to invest under SC. This implies that, in contrast to standard cheap-talk models, expected utility need not always be higher under commitment to full disclosure.

Unfortunately, analytically characterizing the players' expected utility under forced communication is not tractable. Instead, we numerically compute the expected utility in the FC and SC equilibria for various ranges of parameter values. While we have explored the robustness of these results for other parameter values, we report the results based on a benchmark parametrization, where the values are set to the following unless otherwise specified: $p_0 = 0.5$, $\theta_H = 2$, $\theta_L = -1$, $\sigma_S = \sigma_R = 4$. Figure 2 plots the expected utility for S and R (i.e., U_S and U_R, respectively) as a function of b for this parametrization.

The plots suggest that, somewhat surprisingly, expected utility for both players can be higher with strategic communication than with forced communication. Specifically, for the parameter regions plotted, we find that this is always true for S, and true for R when the bias b is close to zero. In other words, when the incentives to coordinate are better aligned (b is close to zero), neither player prefers to commit to a FC equilibrium in which S perfectly discloses her information to R. However, when incentives are not well aligned (i.e., b is very positive or very negative), the expected utility for R may be higher with FC than with SC.

In interpreting these results, recall the two offsetting effects on R's expected utility: (i) more information is communicated to the receiver with FC, but (ii) the sender is more likely to invest with SC. The receiver's expected utility is higher with SC if the informational disadvantage is smaller than the benefit from more investment. Conditional on knowing whether S will invest, a message is more valuable to R when it is more informative about fundamentals around his cutoff (see Yang (2015) for a discussion of this in the context of flexible information acquisition). When the players' incentives are aligned, their cutoffs are close, and so the message with SC is quite valuable to R. In this case, even though the FC equilibrium is more informative, the information advantage over SC is not very large. As a result, the second effect dominates, and expected utility tends to be higher for SC.

However, if the bias is very positive, the sender's cutoff is much lower than the receiver's, and so the message in SC is not very valuable to R. In this case, the informational advantage of FC dominates, and expected utility is higher for FC. Similarly, in the least-informative SC equilibrium, signals below the investment cutoff are also concealed, and so expected utility can be lower than in the FC (and the most-informative SC equilibrium) when the bias is extremely negative (and consequently, S's investment cutoff is very high).

Comparing the FC and SC plots to the welfare-maximizing investment decision suggests that the largest loss in R's utility is when S is biased against coordinated investment (i.e., when b is negative). Intuitively, by allowing S's investment decision to depend on x_R, the welfare-maximizing decision reduces under-investment by S when her bias is very negative, which improves welfare. This suggests that commitment to an optimal mechanism may be most valuable when S is biased against investment.

6 Conclusions

In many economic environments, players have incentives to coordinate, but cannot commit to disclosing the private information they have. We study how informative one-sided cheap talk is in the benchmark, two-player investment game where each player has noisy private information about fundamentals and the investment decision exhibits complementarity. We find that informative communication is difficult to sustain without commitment. In any cheap-talk equilibrium of the benchmark model, the sender conceals information: for signal realizations where she chooses to invest, the sender can only reveal that she will invest. Moreover, in the presence of positive spillovers, no information can be conveyed via cheap talk. We also find that the ability to commit to full disclosure may not be valuable. In contrast to standard sender-receiver games, we show that both the sender and the receiver may prefer the less informative cheap-talk equilibrium to the more informative equilibrium in which the sender commits to fully reveal her information.

Although stylized, our analysis is based on a widely used, and economically relevant, benchmark model of coordination. Our results suggest that allowing for cheap-talk communication in such settings can lead to different conclusions than in a standard sender-receiver setting (in which only the receiver takes an action). Studying how two-sided strategic communication, either simultaneous or sequential, affects our conclusions would be a natural next step. Moreover, our analysis lends itself to the large literature on global games with strategic complementarity, which has delivered insights into optimal policy decisions when economic agents have incentives to coordinate. It would be interesting to study whether, and if so, how, these conclusions change with the introduction of cheap talk across agents, and from the policymaker to agents. We hope to explore this in future work.

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Appendix

A Proofs of main results

Proof of Proposition 1. Since there is no communication, we have for $i \in \{S, R\}$,

$$\log\left(\frac{p_i}{1-p_i}\right) = \log\left(\frac{p_0}{1-p_0}\right) + \frac{1}{\sigma_i^2}\left(\theta_H - \theta_L\right)\left(x_i - \frac{\theta_H+\theta_L}{2}\right), \tag{23}$$

which implies that player i's best response is to invest only when $x_i \geq K\left(k_j; b_i, \sigma_i, \sigma_j\right)$. Note that

$$\lim_{k_j \to -\infty} K\left(k_j\right) = \frac{\theta_H+\theta_L}{2} + \frac{\sigma_i^2}{\theta_H-\theta_L}\left\{\log\left(\frac{-(\theta_L+b_i)}{(\theta_H+b_i)}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right\} \equiv \underline{k}, \tag{24}$$

$$\lim_{k_j \to \infty} K\left(k_j\right) = \frac{\theta_H+\theta_L}{2} + \frac{\sigma_i^2}{\theta_H-\theta_L}\left\{\log\left(\frac{1-\theta_L}{\theta_H-1}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right\} \equiv \bar{k}, \tag{25}$$

and for $b_i > -1$, we have $\frac{\partial}{\partial k}K > 0$. Since $-\theta_H < -1 < b_i < -\theta_L$, $\underline{k}$ and $\bar{k}$ are well defined and finite. The equilibrium is characterized by the fixed point of $x = H\left(x\right)$, where

$$H\left(x\right) \equiv K\left(K\left(x; b_R, \sigma_R, \sigma_S\right); b_S, \sigma_S, \sigma_R\right). \tag{26}$$

Since K is (strictly) increasing, so is H. Also, $H\left(-\infty\right) > -\infty$ and $H\left(\infty\right) < \infty$, which implies a fixed point exists. To ensure uniqueness, we require H is a contraction, or equivalently,

$$\frac{\partial}{\partial x}H\left(x\right) < 1. \tag{27}$$

A sufficient condition for this to be true is that the best response function for each player has a slope less than one i.e., $\frac{\partial}{\partial k}K < 1$. Note that

$$\frac{\partial}{\partial k}K = \frac{\sigma_i^2}{\theta_H-\theta_L}\left\{\frac{(1+b)\phi\left(\frac{k-\theta_H}{\sigma_j}\right)}{\sigma_j\left((\theta_H+b)-(1+b)\Phi\left(\frac{k-\theta_H}{\sigma_j}\right)\right)} - \frac{(1+b)\phi\left(\frac{k-\theta_L}{\sigma_j}\right)}{\sigma_j\left((\theta_L+b)-(1+b)\Phi\left(\frac{k-\theta_L}{\sigma_j}\right)\right)}\right\} \tag{28}$$

We need to bound the above. Let

$$g\left(x, \theta, b\right) = \frac{(1+b)\phi(x)}{\sigma_j((\theta+b)-(1+b)\Phi(x))} \tag{29}$$

$$\Rightarrow g_x = \frac{(1+b)\phi'(x)((\theta+b)-(1+b)\Phi(x))+(1+b)^2(\phi(x))^2}{\sigma_j((\theta+b)-(1+b)\Phi(x))^2} \tag{30}$$

$$= \frac{(1+b)\phi(x)[-x((\theta+b)-(1+b)\Phi(x))+(1+b)\phi(x)]}{\sigma_j((\theta+b)-(1+b)\Phi(x))^2} \tag{31}$$

A necessary condition for the extremum of $g\left(x, \theta, b\right)$ is that $g_x = 0$, or equivalently,

$$\frac{\theta+b}{1+b} = \left[\frac{\phi(x)}{x} + \Phi\left(x\right)\right]. \tag{32}$$

Recall that $b > -1$, $\theta_L + b < 0$ and $\theta_H > 1$. This implies $g(x, \theta_H, b) > 0$ and $g(x, \theta_L, b) < 0$. Moreover, this implies there is a solution $x_L^*(b, \theta_L) < 0$ for $\theta = \theta_L$ and a solution $x_H^*(b, \theta_H) > 0$ for $\theta = \theta_H$. Finally, the first order condition also implies that

$$g(x^*, \theta, b) = \frac{(1+b)\phi(x)}{\sigma_j((\theta+b)-(1+b)\Phi(x))} = \frac{x^*}{\sigma_j}, \tag{33}$$

that is, $g(x, \theta_H, b)$ is maximized at $\frac{x_H^*(b,\theta_H)}{\sigma_j}$ and $g(x, \theta_L, b)$ is minimized at $\frac{x_L^*(b,\theta_L)}{\sigma_j}$. But this implies that

$$\frac{\partial}{\partial k} K = \frac{\sigma_i^2}{\theta_H - \theta_L}\left\{g\left(\frac{k-\theta_H}{\sigma_j}, \theta_H, b\right) - g\left(\frac{k-\theta_L}{\sigma_j}, \theta_L, b\right)\right\} \le \frac{\sigma_i^2}{\theta_H-\theta_L}\left\{\frac{x_H^*(b,\theta_H)-x_L^*(b,\theta_L)}{\sigma_j}\right\} \tag{34}$$

Given b, θ_H and θ_L and σ_j, one can always pick σ_i^2 small enough so that $\frac{\partial}{\partial k}K_i < 1$. In particular,

$$\frac{\sigma_S^2}{\sigma_R} < \frac{\theta_H-\theta_L}{x_H^*(b_S,\theta_H)-x_L^*(b_S,\theta_L)} \equiv \bar{a}, \quad \frac{\sigma_R^2}{\sigma_S} < \frac{\theta_H-\theta_L}{x_H^*(b_R,\theta_H)-x_L^*(b_R,\theta_L)} \equiv \underline{a}. \tag{35}$$

ensures that there is a unique equilibrium. □

Proof of Proposition 2. Given the belief updating in equation (8), player R's best response cutoff is given by

$$K_{FC}(x,k) = \frac{1}{2}(\theta_H+\theta_L) + \frac{\sigma_R^2}{\theta_H-\theta_L}\left(\log\left(\frac{(1+b_R)1_{\{x\le k\}}-(\theta_L+b_R)}{(\theta_H+b_R)-(1+b_R)1_{\{x\le k\}}}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right) + \frac{\sigma_R^2}{\sigma_S^2}\left(\frac{\theta_H+\theta_L}{2} - x\right). \tag{36}$$

If the signal x coincides with the cutoff k, the above best response simplifies to

$$K_{FC}(k,k) = \frac{1}{2}(\theta_H+\theta_L) + \frac{\sigma_R^2}{\theta_H-\theta_L}\left(\log\left(\frac{-(\theta_L+b_R)}{(\theta_H+b_R)}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right) + \frac{\sigma_R^2}{\sigma_S^2}\left(\frac{\theta_H+\theta_L}{2} - k\right) \equiv g(k), \tag{37}$$

and note that $g(k)$ is decreasing in k. Moreover, note that S should only invest when $x_S \ge k_S$, where

$$k_S = \frac{\theta_H+\theta_L}{2} + \frac{\sigma_S^2}{\theta_H-\theta_L}\left\{\log\left(\frac{(1+b_S)\Phi\left(\frac{g(k_S)-\theta_L}{\sigma_R}\right)-(\theta_L+b_S)}{(\theta_H+b_S)-(1+b_S)\Phi\left(\frac{g(k_S)-\theta_H}{\sigma_R}\right)}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right\} \equiv H(k_S) \tag{38}$$

The equilibrium cutoff k_S is given by the solution to the fixed point $x = H(x)$. Since

$$\lim_{x\to-\infty} H(x) = \frac{\theta_H+\theta_L}{2} + \frac{\sigma_S^2}{\theta_H-\theta_L}\left\{\log\left(\frac{1-\theta_L}{\theta_H-1}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right\} > -\infty, \tag{39}$$

$$\lim_{x\to\infty} H(x) = \frac{\theta_H+\theta_L}{2} + \frac{\sigma_S^2}{\theta_H-\theta_L}\left\{\log\left(-\frac{\theta_L+b_S}{\theta_H+b_S}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right\} < \infty, \tag{40}$$

and $H_x < 0$, we have that a fixed point exists and is unique. □

Proof of Proposition 3. Since we focus on pure strategy messaging rules with truth telling, and consider sender optimal equilibria, the image $\mathcal{M}(\mu) = \{\mu(x_S) : x_S \in \Re\}$ for messaging rule

μ is a partition or $\Re$.[15] Given a message $m \in \mathcal{M}(\mu)$ and cutoff k, player R's best response is a cutoff $K_{R,SC}(m,k)$ given by

$$K_{R,SC}(m,k) = \tfrac{\theta_H+\theta_L}{2}+\tfrac{\sigma_R^2}{\theta_H-\theta_L}\left\{\log\left(\tfrac{\Pr(x_S<k|x_S\in m,\theta_L)(1+b_R)-(\theta_L+b_R)}{\theta_H+b_R-\Pr(x_S<k|x_S\in m,\theta_H)(1+b_R)}\right)-\log\left(\tfrac{p_0}{1-p_0}\right)-\log\left(\tfrac{\Pr(x_S\in m|\theta_H)}{\Pr(x_S\in m|\theta_L)}\right)\right\}. \tag{41}$$

Given this best response, each message m corresponds to a cutoff $k(m)$ for player S, given by:

$$k(m) = \tfrac{\theta_H+\theta_L}{2}+\tfrac{\sigma_S^2}{\theta_H-\theta_L}\left\{\log\left(\frac{(1+b)\Phi\left(\frac{K_{R,SC}(m,k(m))-\theta_L}{\sigma_R}\right)-(\theta_L+b)}{(\theta_H+b)-(1+b)\Phi\left(\frac{K_{R,SC}(m,k(m))-\theta_H}{\sigma_R}\right)}\right)-\log\left(\tfrac{p_0}{1-p_0}\right)\right\}. \tag{42}$$

First we show that for any message m in equilibrium, we must have $k(m) \notin \operatorname{int}(m)$, where $\operatorname{int}(m)$ denotes the interior of m. Suppose otherwise, and let $m_1 = m \cap \{x < k(m)\}$and $m_2 = m \backslash m_1$. Then, it must be that m_1 is to the left of m_2 i.e., $\limsup m_1 < \liminf m_2$ — denote this as $m_1 \prec m_2$. This implies that $K_{R,SC}(m_1, k(m)) > K_{R,SC}(m, k(m)) > K_{R,SC}(m_2, k(m))$, which in turn implies $k(m_2) < k(m) < k(m_1)$, which implies S does not invest for $x_S \in m_1$ and always invests for $x_S \in m_2$, and is strictly (weakly) better off for $x_S \in m_2$ ($x_S \in m_1$, respectively) using messages $\{m_1, m_2\}$ instead of m.

This implies that for any candidate equilibrium messaging rule μ, the corresponding messages must be such that $k(m) \notin \operatorname{int}(m)$. This implies that we can partition the image of μ, $\mathcal{M}(\mu)$, into two subsets $\bar{F}(\mu) = \{m \in \mathcal{M}(\mu) : k(m) \le \liminf(m)\}$ and $\underline{F}(\mu) = \mathcal{M}(\mu) \backslash \bar{F}(\mu)$. Note that $\bar{F}(\mu)$ is the set of all messages $m \in \mathcal{M}(\mu)$ where S invests. If we define $c_1(\mu) \equiv \sup\left\{x : \bar{F}(\mu) \ge x\right\}$ and $c_2(\mu) \equiv \inf\left\{x : \underline{F}(\mu) \le x\right\}$. It is immediate to see $c_1(\mu) \le c_2(\mu)$; otherwise, the interval $(c_2(\mu), c_1(\mu))$ does not exist in $\mathcal{M}(\mu)$. Suppose $c_1(\mu) < c_2(\mu)$. Then, in $(c_1(\mu), c_2(\mu))$, there exists a real number y such that a left neighborhood of y belongs to $\bar{F}(\mu)$ and a right neighborhood of y belongs to $\underline{F}(\mu)$. It implies that $\lim_{z\uparrow y} U_S(z, m(z)) \ge 0$ and $\lim_{z\downarrow y} U_S(z, m(z)) \le 0$, and one of the inequalities is strict. As a result, the indifference requirement at y is violated. Hence, we have $c_1(\mu) = c_2(\mu) \equiv c(\mu)$, and $\limsup \underline{F}(\mu) \le c(\mu)$ and $\liminf \bar{F}(\mu) \ge c(\mu)$.

Unless $\bar{F}(\mu)$ consists of a single interval, for any $m \in \bar{F}(\mu)$, there exists a $\tilde{m} \in \bar{F}(\mu)$, such that $cl(m) \cap cl(\tilde{m}) \ne \varnothing$, where $cl(m)$ denotes the closure of m. The optimality of μ requires that for any $x_S \in cl(m) \cap cl(\tilde{m})$, S is indifferent between sending the message m and $\tilde{m}$, but this implies $K_{R,SC}(m, k(m)) = K_{R,SC}(\tilde{m}, k(\tilde{m}))$. Since $m, \tilde{m} \in \bar{F}(\mu)$ (i.e., for any signals x_S, S invests given message m, $\tilde{m}$ and so $\Pr(x_S < k(m) | x_S \in m, \theta) = \Pr(x_S < k(\tilde{m}) | x_S \in \tilde{m}, \theta) = 0$) this in turn must imply

$$\frac{\Pr(x_S \in m|\theta_H)}{\Pr(x_S \in m|\theta_L)} = \frac{\Pr(x_S \in \tilde{m}|\theta_H)}{\Pr(x_S \in \tilde{m}|\theta_L)}. \tag{43}$$

But $\bar{F}(\mu)$ is a partition of the half-line $[c(\mu), \infty)$, and so for all $m \in \bar{F}(\mu)$, $\frac{\Pr(x_S\in m|\theta_H)}{\Pr(x_S\in m|\theta_L)} = t$ for

[15]Specifically, sender optimality rules out some sender types sending a message $m = \Re$, since it is not optimal for senders with high signals to be pooled with senders with low signals.

some constant. This implies that

$$t=\frac{\sum_{m\in\bar{F}}\Pr\left(x_S\in m|\theta_H\right)}{\sum_{m\in\bar{F}}\Pr\left(x_S\in m|\theta_L\right)}=\frac{\Pr\left(x_S\in\cup_{m\in\bar{F}}m|\theta_H\right)}{\Pr\left(x_S\in\cup_{m\in\bar{F}}m|\theta_L\right)}=\frac{\Pr\left(x_S\in\bar{F}|\theta_H\right)}{\Pr\left(x_S\in\bar{F}|\theta_L\right)}. \tag{44}$$

This implies any candidate messaging rule μ is equivalent to a messaging rule $\tilde{\mu}$, where

$$\tilde{\mu}\left(x\right)=\begin{cases}\mu\left(x\right) & \text{if } x<k\left(\mu\right)\\ \left[k\left(\mu\right),\infty\right) & \text{if } x\geq k\left(\mu\right)\end{cases} \tag{45}$$

and S invests if and only if $x_S\geq k\left(\mu\right)$. An optimal messaging rule must satisfy player S's first order condition:

$$\pi_S\left(k,\left[k,\infty\right)\right)=0, \tag{46}$$

This is characterized by the solution k_{SC} to the fixed point problem $k=K\left(K_R\left(k\right),b_S,\sigma_S,\sigma_R\right)$, where

$$\begin{aligned} K_R\left(k\right) &\equiv K_{R,SC}\left(k,\left[k,\infty\right)\right) && (47)\\ &=\tfrac{\theta_H+\theta_L}{2}+\tfrac{\sigma_R^2}{\theta_H-\theta_L}\left\{\log\left(\tfrac{-(\theta_L+b_R)}{\theta_H+b_R}\right)-\log\left(\tfrac{p_0}{1-p_0}\right)-\log\left(\tfrac{1-\Phi\left(\frac{k-\theta_H}{\sigma_S}\right)}{1-\Phi\left(\frac{k-\theta_L}{\sigma_S}\right)}\right)\right\} && (48)\end{aligned}$$

since for $x\in[k,\infty]$, R knows that S invests (i.e., $\Pr\left(a_S=0|x\in[k,\infty],\theta\right)=0$). Note that $\lim_{x\to\infty}K_R\left(x\right)=-\infty$ and $\lim_{x\to-\infty}K_R\left(x\right)=\frac{\theta_H+\theta_L}{2}+\frac{\sigma_R^2}{\theta_H-\theta_L}\left\{\log\left(\frac{-(\theta_L+b_R)}{\theta_H+b_R}\right)-\log\left(\frac{p_0}{1-p_0}\right)\right\}\equiv \underline{k}<\infty$, and K_R is decreasing in x. But this implies

$$\begin{aligned} \lim_{x\to\infty}H\left(x\right) &=\tfrac{\theta_H+\theta_L}{2}+\tfrac{\sigma_S^2}{\theta_H-\theta_L}\left\{\log\left(-\tfrac{\theta_L+b_S}{\theta_H+b_S}\right)-\log\left(\tfrac{p_0}{1-p_0}\right)\right\}\leq\infty && (49)\\ \lim_{x\to-\infty}H\left(x\right) &=\tfrac{\theta_H+\theta_L}{2}+\tfrac{\sigma_S^2}{\theta_H-\theta_L}\left\{\log\left(\tfrac{(1+b_S)\Phi\left(\frac{\underline{k}-\theta_L}{\sigma_R}\right)-(\theta_L+b_S)}{(\theta_H+b_S)-(1+b_S)\Phi\left(\frac{\underline{k}-\theta_H}{\sigma_R}\right)}\right)-\log\left(\tfrac{p_0}{1-p_0}\right)\right\}\geq-\infty && (50)\end{aligned}$$

and H is decreasing in x, a fixed point exists. Note that for $x_S\leq k_{SC}$, S does not invest and so is indifferent between different messaging rules μ that differ in this region. □

Proof of Proposition 5. First consider the positive spillover case i.e., $\nu>0$. Suppose there is an equilibrium in which S can communicate some information about x_S to R. Then there are messages m and $\tilde{m}$ such that (i) $cl\left(m\right)\cap cl\left(\tilde{m}\right)\neq\varnothing$, (ii) fixing the cutoff strategy k_S for S, $\Pr\left(a_R=0|\theta,m\right)\neq\Pr\left(a_R=0|\theta,\tilde{m}\right)$, and (iii) for $x_S\in cl\left(m\right)\cap cl\left(\tilde{m}\right)$, $\Pi_S\left(x_S,m\right)=\Pi_S\left(x_S,\tilde{m}\right)$. Without loss of generality, suppose $\Pr\left(a_R=0|\theta,m\right)>\Pr\left(a_R=0|\theta,\tilde{m}\right)$. This implies that given a signal x_S, one of the following cases must arise:

(i) S invests for m and $\tilde{m}$: But in this case,

$$\begin{aligned} \Pi_S\left(x_S,\tilde{m}\right)-\Pi_S\left(x_S,m\right) &=\pi_S^1\left(x_S,\tilde{m}\right)-\pi_S^1\left(x_S,m\right) && (51)\\ &=-\left(1+\nu\right)\left(\Pr\left(a_R=0|\tilde{m}\right)-\Pr\left(a_R=0|m\right)\right)\neq0 && (52)\end{aligned}$$

and so we have a contradiction.

(ii) S does not invest for m and $\tilde{m}$:

$$\Pi_S(x_S, \tilde{m}) - \Pi_S(x_S, m) = \pi_S^0(x_S, \tilde{m}) - \pi_S^0(x_S, m) \tag{53}$$
$$= -\nu\left(\Pr(a_R = 0|\tilde{m}) - \Pr(a_R = 0|m)\right) \neq 0 \tag{54}$$

and so we have a contradiction.

(iii) S invests for m but not for $\tilde{m}$: Since $\Pr(a_R = 0|\theta, m) > \Pr(a_R = 0|\theta, \tilde{m})$, we have $\pi_S^1(x, \tilde{m}) > \pi_S^1(x, m)$. But since S is indifferent at x_S, we have $\pi_S^0(x_S, \tilde{m}) = \pi_S^1(x_S, m)$, which implies $\pi_S^1(x_S, \tilde{m}) > \pi_S^0(x_S, \tilde{m})$, i.e., it cannot be optimal to not invest at x_S with message $\tilde{m}$, and so we have a contradiction.

(iv) S invests for $\tilde{m}$ but not for m: Since $\Pr(a_R = 0|\theta, m) > \Pr(a_R = 0|\theta, \tilde{m})$, we have $\pi_S^0(x, \tilde{m}) > \pi_S^0(x, m)$. But since S is indifferent at x_S, we have $\pi_S^1(x_S, \tilde{m}) = \pi_S^0(x_S, m)$. But this implies $\pi_S^0(x_S, \tilde{m}) > \pi_S^1(x_S, \tilde{m})$, i.e., it cannot be optimal to invest at x_S with message $\tilde{m}$, and so we have a contradiction.

This implies that in any strategic equilibrium, S effectively cannot communicate any information to R.

When the spillover is negative i.e., $\nu < 0$, analogous arguments establish for $\Pr(a_R = 0|\theta, m) > \Pr(a_R = 0|\theta, \tilde{m})$ and $x_S \in cl(m) \cap cl(\tilde{m})$, we can only have case (iv) i.e., S invests for $\tilde{m}$ but does not invest for m. Moreover, since cases (i) and (ii) are not possible, the message m must be equivalent to $m = \{a_S = 0\}$ and the message $\tilde{m}$ must be equivalent to $\tilde{m} = \{a_S = 1\}$. Since the incremental payoff to investing $\pi_S(x_S, m)$ is independent of ν, the equilibrium in this case is equivalent to the least informative equilibrium in our main model, when $b = 0$. □

Proof of Proposition 6. Denote the (equilibrium) best response functions for the receiver R in each of the three scenarios as:

$$k_{R,NC}(x) = \frac{\theta_H+\theta_L}{2} + \frac{\sigma_R^2}{\theta_H-\theta_L}\left\{\log\left(\frac{(1+b_R)\Phi\left(\frac{x-\theta_L}{\sigma_S}\right)-(\theta_L+b_R)}{(\theta_H+b_R)-(1+b)\Phi\left(\frac{x-\theta_H}{\sigma_S}\right)}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right\} \tag{55}$$

$$k_{R,FC}(x) = \frac{\theta_H+\theta_L}{2} + \frac{\sigma_R^2}{\theta_H-\theta_L}\left(\log\left(-\frac{\theta_L+b_R}{\theta_H+b_R}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right) + \frac{\sigma_R^2}{\sigma_S^2}\left(\frac{\theta_H+\theta_L}{2} - x\right) \tag{56}$$

$$k_{R,SC}(x) = \frac{\theta_H+\theta_L}{2} + \frac{\sigma_R^2}{\theta_H-\theta_L}\left\{\log\left(-\frac{\theta_L+b_R}{\theta_H+b_R}\right) - \log\left(\frac{p_0}{1-p_0}\right) - \log\left(\frac{1-\Phi\left(\frac{x-\theta_H}{\sigma_S}\right)}{1-\Phi\left(\frac{x-\theta_L}{\sigma_S}\right)}\right)\right\} \tag{57}$$

$$k_S(x) = \frac{\theta_H+\theta_L}{2} + \frac{\sigma_S^2}{\theta_H-\theta_L}\left\{\log\left(\frac{(1+b_S)\Phi\left(\frac{x-\theta_L}{\sigma_R}\right)-(\theta_L+b_S)}{(\theta_H+b_S)-(1+b_S)\Phi\left(\frac{x-\theta_H}{\sigma_R}\right)}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right\}, \tag{58}$$

Note that

$$k_{R,NC}\left(x\right)-k_{R,SC}\left(x\right)$$

$$=\frac{\sigma_R^2}{\theta_H-\theta_L}\left\{\underbrace{\log\left(\frac{(1+b_R)\Phi\left(\frac{x-\theta_L}{\sigma_S}\right)-(\theta_L+b_R)}{(\theta_H+b_R)-(1+b)\Phi\left(\frac{x-\theta_H}{\sigma_S}\right)}\right)-\log\left(-\frac{\theta_L+b_R}{\theta_H+b_R}\right)}_{\geq 0}+\underbrace{\log\left(\frac{1-\Phi\left(\frac{x-\theta_H}{\sigma_S}\right)}{1-\Phi\left(\frac{x-\theta_L}{\sigma_S}\right)}\right)}_{\geq 0}\right\}\geq 0 \tag{59}$$

and

$$k_{R,FC}\left(x\right)-k_{R,SC}\left(x\right)=\frac{\sigma_R^2}{\sigma_S^2}\left(\frac{\theta_H+\theta_L}{2}-x\right)+\frac{\sigma_R^2}{\theta_H-\theta_L}\log\left(\frac{1-\Phi\left(\frac{x-\theta_H}{\sigma_S}\right)}{1-\Phi\left(\frac{x-\theta_L}{\sigma_S}\right)}\right) \tag{60}$$

$$=\frac{\sigma_R^2}{\sigma_S^2}\frac{\sigma_S^2}{\theta_H-\theta_L}\log\left[e^{\frac{\theta_H-\theta_L}{\sigma_S^2}\left(\frac{\theta_H+\theta_L}{2}-x\right)}\left(\frac{1-\Phi\left(\frac{x-\theta_H}{\sigma_S}\right)}{1-\Phi\left(\frac{x-\theta_L}{\sigma_S}\right)}\right)\right] \tag{61}$$

$$=\frac{\sigma_R^2}{\theta_H-\theta_L}\left\{\log\left(\frac{\phi\left(\frac{x-\theta_L}{\sigma_S}\right)}{1-\Phi\left(\frac{x-\theta_L}{\sigma_S}\right)}\right)-\log\left(\frac{\phi\left(\frac{x-\theta_H}{\sigma_S}\right)}{1-\Phi\left(\frac{x-\theta_H}{\sigma_S}\right)}\right)\right\}\geq 0 \tag{62}$$

Since $k_{S,NC}=k_S\left(k_{R,NC}\left(k_{S,NC}\right)\right)$, $k_{S,FC}=k_S\left(k_{R,FC}\left(k_{S,FC}\right)\right)$, and $k_{S,SC}=k_S\left(k_{R,SC}\left(k_{S,SC}\right)\right)$, we must have $k_{S,NC}\geq k_{S,SC}$ and $k_{S,FC}\geq k_{S,SC}$. □

Proof of Proposition 7. First, note that it is never optimal to have only one of the two players invests. If this was the preferred outcome, the total payoff must be higher than if both players invest and if both players do not invest i.e.,

$$\theta-1\geq\max\left\{2\theta+b,\ 0\right\} \tag{63}$$

Note that $\theta-1\geq 2\theta+b$ is equivalent to $\theta\leq -b-1<0$, since $b>-1$, but this contradicts $\theta-1\geq 0$. The cutoff for investment is then characterized by the probability p^* such that

$$b+2\left(p^*\theta_H+\left(1-p^*\right)\theta_L\right)=0. \tag{64}$$

This implies that M recommends investment, conditional on observing x_S and x_R, when

$$\log\left(\frac{p_0}{1-p_0}\right)+\frac{1}{\sigma_S^2}\left(\theta_H-\theta_L\right)\left(x_S-\frac{\theta_H+\theta_L}{2}\right)+\frac{1}{\sigma_R^2}\left(\theta_H-\theta_L\right)\left(x_R-\frac{\theta_H+\theta_L}{2}\right)\geq\log\left(-\frac{b+2\theta_L}{b+2\theta_H}\right), \tag{65}$$

or equivalently,

$$\sigma_R^2x_S+\sigma_S^2x_R\geq\left(\sigma_S^2+\sigma_R^2\right)\frac{\theta_H+\theta_L}{2}+\frac{\sigma_R^2\sigma_S^2}{\theta_H-\theta_L}\left(\log\left(-\frac{b+2\theta_L}{b+2\theta_H}\right)-\log\left(\frac{p_0}{1-p_0}\right)\right)=K_M, \tag{66}$$

which gives us the result. □

Proof of Proposition 8. Given the expressions in equations (7), (9), (10), Proposition 4, and equation (22) we can show the following:

(i) For the NC, FC and SC equilibria, the sender's cutoff in the limit is given by

$$\lim_{\sigma_S\to 0} k_S = \tfrac{\theta_H+\theta_L}{2} \equiv k_S^0. \tag{67}$$

(ii) In the NC equilibrium,

$$\lim_{\sigma_S\to 0} k_{R,NC} = \lim_{\sigma_S\to 0} \tfrac{\theta_H+\theta_L}{2} + \tfrac{\sigma_R^2}{\theta_H-\theta_L}\left\{\log\left(\tfrac{(1+b_R)\Phi\left(\frac{k_S-\theta_L}{\sigma_S}\right)-(\theta_L+b_R)}{(\theta_H+b_R)-(1+b_R)\Phi\left(\frac{k_S-\theta_H}{\sigma_S}\right)}\right)-\log\left(\tfrac{p_0}{1-p_0}\right)\right\} \tag{68}$$

$$= \tfrac{\theta_H+\theta_L}{2} + \tfrac{\sigma_R^2}{\theta_H-\theta_L}\left\{\log\left(\tfrac{1-\theta_L}{\theta_H+b_R}\right)-\log\left(\tfrac{p_0}{1-p_0}\right)\right\} \equiv k_{R,NC}^0. \tag{69}$$

(iii) In the FC equilibrium,

$$k_{R,FC}^0(x_S) \equiv \lim_{\sigma_S\to 0} K_{FC}(x_S,k_S) = \begin{cases} +\infty & \text{if } x_S < \frac{\theta_H+\theta_L}{2} \\ 0 & \text{if } x_S = \frac{\theta_H+\theta_L}{2} \\ -\infty & \text{if } x_S > \frac{\theta_H+\theta_L}{2} \end{cases}. \tag{70}$$

(iv) In the least informative SC equilibrium,

$$k_{R,SC}^0(m_S) \equiv \lim_{\sigma_S\to 0} K_{SC}(m_S) = \begin{cases} -\infty & \text{if } m_S = [k_{S,SC},\infty), \\ +\infty & \text{if } m_S = (-\infty, k_{S,SC}) \end{cases} \tag{71}$$

and in the most informative SC equilibrium,

$$k_{R,SC}^0(m_S) \equiv \lim_{\sigma_S\to 0} K_{SC}(m_S) = \begin{cases} -\infty & \text{if } m_S = [k_{S,SC},\infty), \\ +\infty & \text{if } m_S < k_{S,SC}, \text{ and } m_S = x_S < \frac{\theta_H+\theta_L}{2} \\ 0 & \text{if } m_S < k_{S,SC}, \text{ and } m_S = x_S = \frac{\theta_H+\theta_L}{2} \\ -\infty & \text{if } m_S < k_{S,SC}, \text{ and } m_S = x_S > \frac{\theta_H+\theta_L}{2} \end{cases}. \tag{72}$$

(v) For the welfare maximizing investment decision, both S and R invest if and only if

$$\lim_{\sigma_S\to 0} \sigma_R^2 x_S + \sigma_S^2 x_R \geq \lim_{\sigma_S\to 0} K_M, \tag{73}$$

or equivalently, $x_S \geq \frac{\theta_H+\theta_L}{2}$.

This implies that, as the precision of the sender's signal becomes infinite, the states of the world in which there is investment in the SC and FC equilibria coincide with those in which there is investment under the welfare maximizing investment decision. However, for the NC equilibrium, this is not the case and consequently, welfare is lower. □

Proof of Proposition 9. First consider player S's utility under SC vs. NC. The equilibrium expected utility is given by

$$U_S(k_R) = \max_k \mathbb{E}^\theta \left[(\theta - 1)\Pr(x_S \geq k) + (b_S + 1)\Pr(x_S \geq k)\Pr(x_R \geq k_R)\right]. \tag{74}$$

By the envelope theorem,

$$\frac{\partial}{\partial k_R} U_S = \mathbb{E}^\theta \left[-\frac{1}{\sigma_R}(b_A + 1)\Pr(x_S \geq k)\,\phi\left(\frac{k_R - \theta}{\sigma_R}\right)\right] \leq 0, \tag{75}$$

which implies $U_{S,SC} = U_S(k_{R,SC}) \geq U_S(k_{R,NC}) = U_{S,NC}$. Next consider player R's utility under SC vs. NC. Let

$$V(k_S) = \max_k \mathbb{E}\left[(\theta - 1)\Pr(x_R \geq k, x_S < k_S) + (\theta + b_R)\Pr(x_S \geq k_S, x_R \geq k)\right] \tag{76}$$

and note that $U_{R,NC} = V(k_{S,NC})$. Moreover, since $k_{S,NC} > k_{S,SC}$, the envelope theorem implies $V(k_{S,NC}) < V(k_{S,SC})$. Finally, note that

$$\begin{aligned} U_{R,SC} &= \max_{k_0,k_1} \mathbb{E}\left[(\theta - 1)\Pr(x_R \geq k_0, x_S < k_{S,SC}) + (\theta + b_R)\Pr(x_S \geq k_{S,SC}, x_R \geq k_1)\right] && (77)\\ &\geq \max_k \mathbb{E}\left[(\theta - 1)\Pr(x_R \geq k, x_S < k_{S,SC}) + (\theta + b_R)\Pr(x_S \geq k_{S,SC}, x_R \geq k)\right] && (78)\\ &= V(k_{S,SC}) > U_{R,NC}. && (79) \end{aligned}$$

Hence, both R and S prefer the SC equilibrium to the NC equilibrium. □

B Misaligned cost of investment

In this section, we consider an alternative specification which introduces an asymmetric cost of failing to coordinate. Suppose the payoffs are given by the following table:

$S \setminus R$	Invest ($a_R = 1$)	Not Invest ($a_R = 0$)
Invest ($a_S = 1$)	$\theta + b$, θ	$\theta - c$,0
Not Invest ($a_S = 0$)	0, $\theta - 1$	0,0

As before, when both players invest, S receives $\theta + b$ while R receives θ. Moreover, in this case, when S invests but R does not, S receives $\theta - c$. Assumptions (A1)-(A3) generalize to the following: (i) $\theta_L < 0 < c < \theta_H$, (ii) $b > -c$, and (iii) $b < -\theta_L$. While R's incremental payoff from investing π_R is still given by 5, the incremental payoff to S from investing is now given by

$$\pi_S(x_S, m) = \begin{array}{c} p(x_S, \Re)\left[(\theta_H + b) - (b + c)\Pr(a_R = 0|\theta_H, m)\right] \\ +(1 - p(x_S, \Re))\left[(\theta_L + b) - (b + c)\Pr(a_R = 0|\theta_L, m)\right] \end{array}. \tag{80}$$

The above payoffs imply that player S's best response function is given by

$$K_S(k) = \frac{\theta_H + \theta_L}{2} + \frac{\sigma_S^2}{\theta_H - \theta_L} \left\{ \log \left(\frac{(b_S + c)\Phi\left(\frac{k - \theta_L}{\sigma_R}\right) - (\theta_L + b_S)}{(\theta_H + b_S) - (b_S + c)\Phi\left(\frac{k - \theta_H}{\sigma_R}\right)} \right) - \log \left(\frac{p_0}{1 - p_0} \right) \right\}. \tag{81}$$

The parameter restrictions imply that $\lim_{k\to\infty} K_S(k) < \infty$ and $\lim_{k\to-\infty} K_S(k) > -\infty$ and $\frac{\partial}{\partial k} K_S > 0$. This ensures that the proofs of Propositions 1, 2, and 3 apply immediately to this case, since the fixed point conditions (i.e., $x = H(x)$) that characterize the equilibria inherit analogous existence and uniqueness properties.