The redistributive effects of bank capital regulation

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Abstract

We build a general equilibrium model of banks' optimal capital structure, where investors are reluctant to invest in financial products other than deposits, and where bankruptcy is costly. We first show that banks raise both deposits and equity, and that investors are willing to provide equity only if adequately compensated. We then introduce (binding) capital requirements and show that: (i) it distorts investment away from productive projects toward storage; or (ii) it increases the cost of raising capital for banks, with the bulk of this cost accruing to depositors. These results hold also when we extend the model to incorporate various rationales justifying capital regulation.

Keywords: limited market participation, bank capital structure, capital regulation

JEL-classification: G18, G2, G21

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1 Introduction

The regulation of financial institutions, and banks in particular, has been at the forefront of the policy debate for a number of years. Much of the concern over financial institutions relates to the perceived negative consequences associated with a bank's failure, and with how losses may be distributed across various sectors, such as borrowers, either corporate or individual, creditors, including depositors, government, with the ultimate bearers of the losses being households and shareholders, among others.

A primary tool for bank regulation is the imposition of minimum capital standards, which amount to requirements that banks limit their leverage and issue at least a minimal amount of equity. Capital requirements thus act as a portfolio restriction on the liability rather than the asset side of banks' balance sheets.[1] The primary roles for such capital can be generally described in two ways. First, by creating a junior security held by the ultimate shareholders of the bank, capital (i.e., equity) stands as a first line of defense against losses, with shareholders absorbing losses before those accrue to other bank claimants and thus trigger distress and ultimately bankruptcy. Second, by forcing shareholders to have "skin in the game", capital is seen as a way of controlling moral hazard or asset substitution problems that may otherwise arise as a result of investment decisions that banks take that are usually highly levered. In fact, recent calls, both among regulators, policymakers, and even academics (see, e.g., Admati, DeMarzo, Hellwig, and Pfleiderer (2013) for a discussion of this issue) have been for banks to dramatically increase the amount of capital they issue as way of reducing risk and ultimately increasing social welfare.

[1] Of course, the imposition of *risk weighted* capital requirements serve to place portfolio restrictions on the asset side as well, in addition to constraining how banks must finance themselves.

What is less understood in the discussion related to bank capital requirements is who bears the costs, to the extent that there are any, associated with requiring banks to increase their capitalization beyond their preferred levels. If banks', or bankers', primary goal is to maximize profits and capital structure is chosen taking this objective into account, then the imposition of a leverage constraint, or any other restriction on banking activities, should lead to a reduction in bank profitability and consequently in the return available to bank claimants. The regulatory perspective, of course, is that the possible loss in profitability associated with the constraints is more than compensated by increases in social welfare through other channels, such as the reduction on the social burden associated with government guarantees, implicit or explicit, as well as the internalization of externalities that might arise when a bank fails. Nevertheless, at the bank level, the reduction in profitability should fall on the shoulders of the various claimants against the bank, such as depositors and shareholders, and there has been little study of which party bears the brunt of this burden. Do shareholders, as is commonly argued, primarily bear the costs associated with regulation, or are depositors and other creditors the parties more affected by tightening regulatory requirements? This is an important consideration for understanding the incidence of regulation, and who may require a transfer in order to be made whole. Moreover, if the aim of (capital) regulation is to protect specific agents, such as retail depositors and by extension the deposit insurance fund, a real question is then whether capital regulation is likely to achieve this goal.

In this paper, we present a general equilibrium model of banks' optimal capital structures where all parties are risk neutral, but investors are reluctant to participate in financial markets, prefering to invest only in safe assets, such as deposits. This key friction, well documented in the literature on household finance (see, e.g., Guiso and Sodini (2013)), implies that in order for investors to be willing to hold anything other than a bank deposit, they must be induced through additional compensation. In particular, investors (or households, more generally) are unwilling to become equityholders unless they receive compensation sufficient to overcome their reluctance.

Specifically, in our model banks exist to channel funds from investors, who have limited outside options for storing their savings, into productive but risky investments. Investors are naturally disinclined to invest in anything other than in storage or in "safe" bank deposits, but can be induced to hold equity if they view the equilibrium return to equity investment as being sufficiently high relative to that of holding a deposit. Investors are otherwise risk-neutral, however, so that there is no premium for holding risk and thus effects related purely to leverage changes (as in Modigliani and Miller (1958)) are shut down. Banks can finance themselves with either debt or equity, but using too much debt exposes the bank to default and, hence, bankruptcy risk. Given that bankruptcy is costly, and that banks ultimately are owned by shareholders and thus try to maximize shareholder value, they endogenously limit the amount of debt financing they want to use in order to reduce expected bankruptcy costs. On the other hand, raising equity capital is difficult because investors face costs in becoming equityholders and thus need to be compensated, with greater compensation demanded the larger amount of equity capital the bank wants to raise. In other words, investors' participation in financial markets as suppliers of bank capital (i.e., equity) is endogenous, and will depend on the difference in the equilibrium returns of deposits versus equity. The tradeoff between these two forces leads to an optimal capital structure, with banks always finding it optimal to raise some amount of debt financing (i.e., deposits), the exact amount depending on how profitable their investment projects are, and how variable are their returns.

We also characterize the equilibrium return to equityholders as well as to depositors, and show that, while the marginal capital supplier is indifferent between being an equityholder or a depositor, inframarginal equityholders earn a strictly positive rent as a result of investing in a bank's equity. The bank therefore creates value for investors by channeling funds from storage into real investment projects, allowing investors to earn a return that more than compensates them for the disutility they associate with being an equity market investor (i.e., their unwillingness to participate in financial markets). When the expected return to investment projects is sufficiently high, all available funds are put to productive use, with no funds going into storage. In this scenario, even depositors earn a premium in that their expected return is strictly higher than what they would earn in storage.

Despite the friction of (endogenous) limited market participation, the individually optimal capital structure decisions for banks are actually socially efficient, and the market solution yields no distortions that can be directly improved with capital regulation. This occurs because banks are competitive in our model, and all profits ultimately accrue to the banks' shareholders, who are the residual claimants. The model thus exhibits a benchmark solution that is efficient, and provides us with an ideal laboratory in which to study how the distortion (on bank profits and, thus, social output) affects the claimants of the bank, namely depositors and shareholders who are, ultimately, the represented by the households who are making portfolio decisions on how to allocate their savings.[2]

We identify two sources of inefficiency associated with (binding) minimal capital requirements. One is that, when project returns are relatively low and not all funds are being invested in productive projects but rather being held as storage, requiring banks to hold even greater amounts of capital reduces further the number of projects that are funded. This, in turn, reduces aggregate surplus since investment projects yield a higher surplus than investing in storage, and the capital requirement introduces a distortion away from productive investments toward storage. The second inefficiency arises at the other extreme, once all investment funds are being allocated to productive projects and no funds are being placed in storage. Here, an increased requirement to hold capital raises the overall cost borne by investors since satisfying the capital requirement necessitates that a larger number of investors be induced to become capital suppliers, and thus bear the disutility associated with holding equity. Much of this increased cost is ultimately borne by those investors who remain as depositors, since their equilibrium return goes down and they earn a lower return on their deposits relative to what they could get in the absence of a capital requirement. In other words, much of the incidence of the increased costs associated with satisfying the capital requirement falls on those investors who are least willing to be financial market participants, rather than on investors who, by holding equity, are the residual claimants of the banks.

[2]As described below, we later introduce a friction that can be at least partly resolved through capital regulation, so that indeed there is a rationale for regulation to solve a social problem. We show that even in that context, where capital regulation can increase social welfare, the incidence of the regulation falls differentially on different classes of investors. In particular, for the most part it is not the households that are most inclined to be equity investors that bear the brunt of the regulatory burden.

While the main part of the analysis focuses on the frictionless case where the market solution is socially efficient and there is no rationale for regulation, we then extend our results to consider various market failures that may justify a need to introduce minimum capital requirements. Specifically, we study three cases that represent three different sources of inefficiency: (1) Externalities that arise as a result of "fire sales" when a large number of banks fail, thus depressing recovery values on bank assets. In that setting, recovery values are lower the more banks are unable to meet their repayment obligations, so that a bank's failure has a negative externality on other banks, which ultimately is reflected in decreased overall value. Capital regulation, by reducing the overall number of banks, helps solve the inefficiency by reducing the externality. Nevertheless, the impact of regulation is felt most strongly by households that would normally choose to be depositors rather than shareholders, as above. (2) We then study how deposit insurance, by reducing the interest rate that needs to be promised to depositors, can reduce the threshold for bankruptcy and thus increase social welfare all things equal. However, deposit insurance also introduces a distortion in banks' capital structure decisions since it leads banks to leverage as much as possible, using no equity and overly exposing themselves to bankruptcy risk. Capital regulation can again help rectify the distortion and leads to overall greater surplus, but the primary benefit of the regulation accrues to shareholders rather than to households who otherwise would have chosen to be depositors anyway. (3) Finally, we consider a standard limited-liability induced risk shifting story where capital reduces the moral hazard problem for banks (see, e.g., Holmstrom and Tirole, 1997, for a classic treatment). Again, the conclusion that follows is that capital adequacy standards can increase social welfare, but it is not depositors who primarily benefit, rather it is those investors who would have been inclined to hold equity anyway.

Our work is related to two strands of literature. The first relates to the role of capital at financial institutions and, more specifically, to the incentives institutions have to use leverage but also finance themselves with equity capital. Allen, Carletti, and Marquez (2015) show that bank capital earns a higher expected return in equilibrium than deposits when banks' funding markets are exogenously segmented. In contrast, we study the endogenous degree of participation of investors in bank equity markets, and outline the consequences for the imposition of binding capital requirements. Bertsch and Mariathasan (2015) develop a model of endogenous bank equity prices during large-scale recapitalizations. A sizable literature has also studied the role of bank capital as a buffer that protects from bankruptcy, or confines moral hazard (e.g. Holmström and Tirole (1997), Dell'Ariccia and Marquez (2006), Allen, Carletti, and Marquez (2011), Hellmann, Murdock, and Stiglitz (2000)). Our basic model incorporates the first effect – buffer stock view of bank capital – and we explore the second in an extension, with the focus on understanding who primarily bears the costs associated with capital regulation.

The second strand of literature relates to the well-documented limited degree of participation in financial markets by households. Taking households' unwillingness to participate as given, an important body of literature has focused on the implications of limited participation on the equilibrium pricing of assets. For instance, Allen and Gale (1994) study how limited market participation by potential investors can lead to amplified volatility of asset prices relative to the case where there is complete participation. Vissing-Jørgensen (2002) uses limited market participation by households to help explain part of the equity premium puzzle in financial economics, which argues that the observed differences between average returns in equity markets and debt markets are difficult to explain through risk aversion alone. In our approach, we assume that investors are heterogeneous in their willingness to participate, with some being very reluctant and others more willing. The equilibrium level of participation is then endogenous, and is pinned down at the same time as are equilibrium returns to all investors. To the best of our knowledge, we are the first paper to focus on the capital structure implications for either financial or nonfinancial firms of investors' reluctance to hold risky assets.

The paper proceeds as follows. The next section lays out the model. Section 3 contains the main analysis of the model, and the characterization of equilibrium. Section 4 looks at social welfare, whereas section 5 studies the effects of a binding capital requirement. Section 6 extends the baseline model to include various settings where there is a social efficiency which capital regulation can help address. Finally, Section 7 concludes.

2 The Model

Our model has two dates, $t = 0, 1$, and one time period. In our economy, there exist two investment options: one is a safe *storage* technology which yields in $t = 1$ a return of one on every unit of funds invested at $t = 0$. The other is a *risky* investment which, for every unit of funds invested at $t = 0$, yields in $t = 1$ a risky return of r. We assume that r is uniformly distributed in $[R - S, R]$, where $0 < S \leq R$ and $R > 2$. These two conditions combined ensure that the realized returns of the risky investment project are non-negative, and that $E[r] = \mu \equiv \frac{2R-S}{2} \geq 1$, so that its expected return is at least as high as investing in the storage technology.

There is a continuum of mass M of risk-neutral investors, endowed with one unit of wealth each. Investors may either invest directly in the storage technology, or they can place their wealth in a bank, either as depositors or as equity holders.[3] However, investors are reluctant to hold anything other than a bank deposit, and have to be induced to invest in risky assets. Specifically, investors are heterogeneous with respect to their willingness to hold risky assets, and each investor i incurs a non-pecuniary cost c_i for becoming *sophisticated* and holding risky assets. This cost c_i is specific to each investor, is i.i.d., and is drawn from a uniform distribution over the $[0, C]$ interval, and each investor understands his degree of reluctance to invest in risky assets (i.e., each investor i knows the disutility c_i he faces).[4] As a result, investors will only be willing to hold risky bank equity (i.e., *bank capital*) if their expected return from doing so exceeds their return from investing in either storage of a bank deposit by at least the cost c_i.

Banks in this model are primarily vehicles that provide investors with access to the risky technology: by becoming a shareholder in a bank, an investor can take a (potentially leveraged) position in the risky technology. Each bank requires one unit of total funding and invests in the risky technology. This funding can either come as bank capital (i.e., equity) k or as deposits, $1 - k$, which must make up the remainder. Investors are willing to hold bank deposits, but incur the idiosyncratic disutility c_i if they hold equity. We will denote the equilibrium expected return to bank deposits as u, and that for bank capital as ρ.

Banks are subject to bankruptcy if they are unable to repay their debt obligations, which here are simply deposits, and bankruptcy is costly. Specifically, suppose that deposits are

[3] We obtain identical results if we also allow investors to invest directly in the risky technology, earning r per dollar they invest. Therefore, for ease of exposition we abstract from this possibility.

[4] This cost may stand for the cost of acquiring the necessary skill set and information to trade successfully in equity markets, or it may capture (in reduced form) heterogeneous taste for safe assets in the investor population, and a consequent disutility associated with holding risky assets.

offered a promised return of r_D: if the bank's investment in the risky asset yields a sufficiently high return that the bank can pay its depositors, i.e., $r \geq (1-k)r_D$, then depositors are paid the promised return of r_D per unit of deposits invested, and the residual is paid to bank capital holders. However, if $r < (1-k)r_D$, the bank goes bankrupt since it is unable to repay its depositors, and it incurs bankruptcy costs which destroy value. We make the extreme assumption that in the event of bankruptcy, all the project's return is dissipated, so that the deadweight loss is the full return of the project, r, when it is less than $(1-k)r_D$. As we will see, this extreme assumption on the bankruptcy cost biases the bank against being levered, and reduces the total return associated with the risky technology.

Finally, we assume that the banking sector in our model is perfectly competitive. Free entry reduces profits to zero, and banks behave as price takers both with respect to both the equilibrium expected return on bank capital, ρ, and to the equilibrium return u in the bank deposit market.

3 Optimal capital structure

The equilibrium of the model is pinned down by the following conditions:

1. Investors optimally decide whether to become sophisticated, and invest as to maximize their expected wealth;

2. Banks choose capital k and select a promised deposit rate r_D as to maximize profits;

3. The market for storage and bank deposits clears;

4. The market for bank equity clears;

5. Free entry reduces bank profits to zero.

We start by analyzing the optimal strategy of investors. At time $t = 0$, investors decide whether to become sophisticated based on how the difference in returns between the two investor groups compares to the relevant cost. Whenever the spread between deposits and equity, $\rho - u$, is smaller than C,[5] there exists a marginal investor whose cost $\hat{c} \in (0, C)$ of sophistication makes him indifferent between earning a return of u on deposits and earning a return of ρ on bank capital after incurring his cost. The marginal investor's cost must therefore satisfy $\hat{c} = \rho - u$, and for a given spread $\rho - u$ between equity and debt, there is a mass

$$K_{supply} = \begin{cases} M\frac{\rho-u}{C} & \text{if } 0 \leq \rho - u < C \\ M & \text{if } \rho - u \geq C \end{cases}$$

of investors who optimally choose to become sophisticated and supply bank equity. The remaining $D_{supply} = M - K_{supply}$ investors find it optimal to either deposit their funds at a bank, or invest in storage. The returns ρ and u, however, are endogenous, and depend as well on the solution to the banks' problem, which we turn to next.

Each bank optimally selects its capital stock k and its promised rate r_D such that it maximizes profits subject to the constraint that depositors be willing to participate by holding bank deposits rather than investing in storage:

$$\max_{k, r_D} E\left[\Pi_B\right] = \frac{1}{S}\int_{\max\{r_D(1-k), R-S\}}^{R} (r - (r_D(1-k))dr - \rho k \tag{1}$$

[5]As we show later, this condition is always satisfied in equilibrium.

subject to

$$E\left[U_D\right] = \frac{1}{S}\int_{\max\{r_D(1-k),R-S\}}^{R} r_D\,dr \geq u \tag{2}$$

$$E\left[\Pi_B\right] \geq 0 \tag{3}$$

$$0 \leq k \leq 1. \tag{4}$$

The first term in expression (1) captures the bank's gross profit: it is zero for any realization of r that falls short of the bankruptcy threshold $r_D(1-k)$; for all realizations of r above the bankruptcy threshold, the bank obtains r from its investment and uses that to repays $r_D(1-k)$ to depositors. The second term, $-\rho k$, reflects the opportunity cost to shareholders for holding bank capital. Constraint (2) captures the expected payoff $E[U_D]$ that accrues to depositors: when the project's realized payoff r is above what is owed to them, depositors receive the promised repayment r_D; otherwise the bank goes bankrupt and all project returns are dissipated. Finally, constraints (3) and (4) ensure that the bank is active and that the chosen capital structure lies within the feasible range.

Before solving for equilibrium allocations, we make the following simple observation: Recall that, for a given deposit rate r_D, banks go bankrupt whenever $r < r_D(1-k)$, which is only possible if $R - S < r_D(1-k)$ or, equivalently, if banks have a capital stock which is so low that $k < 1 - \frac{R-S}{r_D}$. Hence, the equilibrium capital level k and the equilibrium rate on deposits r_D that banks promise are key determinants of the extent to which banks incur bankruptcy risk.

We can now characterize the equilibrium. In what follows, denote by N the equilibrium number of banks that are active.

Proposition 1. *The model has a unique equilibrium. Define* $\hat{u} \equiv \frac{R^2}{4S}\left(1+\sqrt{\frac{R^2}{R^2+8CS}}\right)$*. Then, equilibrium allocations are characterized as follows:*

- *if* $\hat{u} \leq 1 < R/2$*, banks choose a risky capital structure. We have:* $k = \frac{4S}{R^2} - 1$, $r_D = \frac{R}{2}$, $\rho = \frac{2S}{4S-R^2}$, $u = 1$, $E\left[\Pi_B\right] = 0$, $E\left[U_D\right] = u$, $N = MR^2\frac{R^2-2S}{C(4S-R^2)^2}$, $K = Nk$, $D = N(1-k) < M - K$*.*

- *if* $1 < \hat{u} < R/2$*, banks choose a risky capital structure. We have:* $k = \frac{R}{\sqrt{R^2+8CS}}$, $r_D = \frac{R}{2}$, $\rho = \frac{R}{4S}\left(R + \frac{R^2+4CS}{\sqrt{R^2+8CS}}\right)$, $u = \hat{u} = \frac{R^2}{4S}\left(1+\frac{R}{\sqrt{R^2+8CS}}\right)$, $E\left[\Pi_B\right] = 0$, $E\left[U_D\right] = u$, $N = M$, $K = Nk$, $D = N(1-k) = M - K$*.*

- *if* $\hat{u} \geq R/2$*, banks choose a safe capital structure. Denote as* u^* *the unique positive solution of* $\frac{2R-S}{2} - u^* - C\left(1-\frac{R-S}{u^*}\right)^2 = 0$*. We have:* $k = 1 - \frac{R-S}{u^*}$, $r_D = u^*$, $\rho = \frac{Su^*}{2(u^*-R+S)}$, $u = u^*$, $E[\Pi_B] = 0$, $E[U_D] = u$, $N = M$, $K = Nk$, $D = N(1-k) = M - K$*.*

In all three cases, $\rho > E\left[r\right] > u$ *holds.*

Proof. See appendix. □

The proposition establishes that banks always find it optimal to raise some amount of debt financing (i.e., deposits), but the exact amount depends on how profitable their investment projects are, and how variable are their returns, with the variable $\hat{u}$ providing a measure of the tradeoffs between maximal project returns, R, dispersion of returns, S, and investor disutility associated with being an equityholder, C. When project returns are very high and the lowest project return is bounded away from zero, bankruptcy is particularly costly because it destroys the return from the project. At some point, losses under bankruptcy become larger than the cost c_i to investors of becoming capital suppliers, and banks find it optimal to employ relatively little leverage, ensuring that they don't have so much that they risk bankruptcy. In other words, when project returns are very high, which roughly corresponds to either a high R or a low S, banks optimally choose a "safe" capital structure that has no risk of bankruptcy, and hence no expected bankruptcy costs. This corresponds to the region where $\hat{u} \geq R/2$ in the proposition, which reflects that maximal returns may be high (i.e., R), or cash flow dispersion is low (i.e., S).

For intermediate values of project expected project returns and cash flow variability, banks optimally choose to raise enough deposits that they risk bankruptcy with some strictly positive probability. This corresponds to the region where $\hat{u} < R/2$, so that project returns are not (unrealistically) too high, or project risk is very high. The reason is that using leverage increases the return to shareholders since deposits earn a lower return, allowing a greater premium to be paid to investors willing to become equityholders. At the same time, increasing the use of capital becomes progressively costly since it requires the bank to induce ever more averse investors to become equityholders, and this cost is ultimately borne by the banks since markets are competitive. Banks can economize on these costs by using deposits, even if it means incurring some bankruptcy risk as a result. As the returns of the projects increases, so that the losses conditional on bankruptcy also grow, the banks find it optimal to reduce their bankruptcy risk by funding themselves with more capital, until at some point, as described above, the bank raises so little debt financing that it is no longer at risk of failure. Similarly, if project risk is very high, eliminating bankruptcy risk through the use of capital becomes prohibitively costly.

The proposition also shows that the equilibrium return for capital suppliers is always higher than that for depositors. Moreover, the return to capital suppliers is higher than the expected return on the project: $\rho > E[r]$. This arises precisely because of the bank's use of leverage in its capital structure, which, by earning a lower expected return, allows for greater surplus to be distributed to bank equityholders. Proposition 1 also establishes a boundary, characterized by the variable $\hat{u}$, for when all available funds from the mass M of investors are invested in the banking sector and, thus, are being directed toward financing the risky projects, and conversely when some funds are instead being invested in storage. We refer to the former case as that of "full inclusion" for investors, and corresponds to the case where $N = M$, whereas the latter case is one of partial inclusion, with $N < M$. The case of partial inclusion is where the return to being a bank depositor, u, is no higher than investing in storage and, in equilibrium, not enough banks form to invest in all risky, high return projects, so that some investors simply store their wealth across dates. By contrast, under full inclusion depositors earn a return $u > 1$, thus earning some rents to holding bank deposits relative to storage. In this region, all funds are being intermediated by financial institutions, so that all investors are funneling their money to the banking sector, which then uses it for investment purposes. This is only possible, however, when the project's expected return is sufficiently high, so that enough compensation can be offered to capital suppliers ($\rho > E[r]$) and to depositors ($u > 1$).

While the results related to why banks may sometimes adopt a "safe" capital structure and other times a "risky" one are both interesting in their own right, the case where a bank eliminates all bankruptcy risk relies on either extremely high project returns, or extremely low risk projects. To economize on the analysis of the various subcases, we will restrict our analysis in what follows to what we feel is the more empirically relevant case where project returns are in a more intermediate range, and banks choose capital structures that optimally expose them to bankruptcy risk. This case is of greater practical relevance since it mirrors real world considerations associated with bank failure and thus allows us to study, later in the paper, issues associated with the social efficiency of the market solution which still allows for banks to fail with (strictly) positive probability.

With this in mind, the equilibrium characterization of Proposition 1, for the region where $\hat{u} < R/2$, also pins down the explicit expression of the equilibrium probability of bankruptcy p_B as follows:

Corollary 1.1. *The equilibrium probability of bankruptcy is given by*

$$p_B = \begin{cases} 1 - \frac{2}{R} & \text{if } \hat{u} \leq 1 \\ 1 - \frac{R}{2S} - \frac{R^2}{2S\sqrt{8CS+R^2}} & \text{if } R/2 > \hat{u} > 1 \\ 0 & \text{if } \hat{u} \geq R/2 \end{cases}$$

For the risky regions, the probability of bankruptcy is increasing in R if $\hat{u} \leq 1$, and is decreasing in R if $R/2 > \hat{u} > 1$.

Proof. Because bankruptcy cost are 100%, we must have $u = (1 - p_B)r_D$ from which the statement follows immediately. □

We next study how the equilibrium changes as a function of the various underlying parameters, and in particular as either the expected return or the variance of the projects changes. The following results follows from differentiation of the equilibrium expressions for ρ, u, and k obtained in Proposition 1.

Corollary 1.2. *Comparative statics of equilibrium allocations and returns are as follows:*

- *An increase in the expected project return $E[r]$ decreases k and increases ρ for $\hat{u} \leq 1$, while it increases k, ρ and u for $\hat{u} > 1$.*

- *Keeping $E[r]$ constant, an increase in the $var(r)$ of project returns increases k and decreases ρ for $\hat{u} \leq 1$, while it reduces k, ρ and u for $\hat{u} > 1$.*

- *An increase in the equilibrium average cost $E[c_i] = \frac{C}{2}$ of becoming equity holder reduces the number of banks N and leaves k and ρ unchanged for $\hat{u} \leq 1$, while it decreases k and u, increases ρ and leaves N unchanged for $\hat{u} > 1$.*

To understand the comparative statics described in Corollary 1.2, it is useful to consider the case of an increase in the mean, $E[r]$, keeping the variance fixed, separately from the case of a mean preserving spread that widens the interval $[R - S, R]$ in a symmetric fashion, thus leaving the mean constant.

When $N < M$, so that some investors are placing their funds in storage rather than as deposits in a bank (equivalently, when $u = 1$), increases in $E[r]$ create a greater wedge between the return to investing in projects and the return to storage. There is thus a greater demand for funds by banks, leading more investors to instead place their funds as bank deposits. In aggregate, the number of banks, N, increases, as does the aggregate amount of capital K employed by the banking sector. This is reflected in an increase in the equilibrium return to capital suppliers, ρ. The amount of capital employed by each individual bank, k, decreases in this region precisely because the market clearing condition for deposits and capital implies that, at the margin, the only source of funding for new banks to form (i.e., for N to increase) is for investors to move away from storage and instead hold bank deposits. This switch is essentially costless, whereas inducing more investors to become equity suppliers is increasingly costly given that it requires investors with ever increasing costs of sophistication, c_i, to become equityholders.

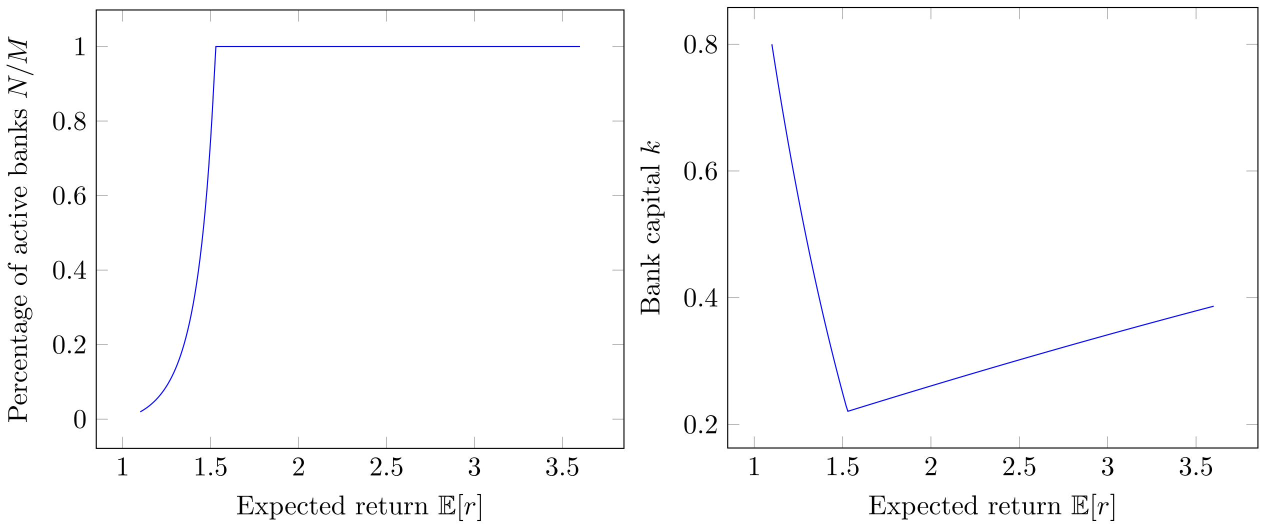

Figure 1: Equilibrium number of banks N and level of bank capital k as a function of the expected return $\mathbb{E}[r]$ of the risky technology. The lowest capitalization is attained exactly at the moment of full inclusion. Parametrization: $C = 8, S = 1.8$

Once $N = M$ (equivalently, when $u > 1$), there is no longer the possibility of allocating more funds to productive projects since all funds are already being used in the banking sector. In this region, for a given bankruptcy probability, increases in $E[r]$ increase the expected value of the deadweight losses associated with bankruptcy since project returns are uniformly higher (i.e., the support of the distribution of payoffs shifts up). Banks therefore find it optimal to reduce their leverage, $1 - k$, even as equilibrium return to depositors, u, increases. The reduced bankruptcy risk, and the increased project returns, lead to an increase in the equilibrium return to capital suppliers, ρ, which, along with the fact that $1 - k$ decreases, implies as well that the aggregate amount of capital being used by the banking sector, K, increases as $E[r]$ rises.

By contrast, a mean preserving spread (MPS) increases the variance of the project's returns without affecting the average, $E[r]$. Ceteris paribus, this can have one of two effects:

if the threshold for bankruptcy, $r_D(1-k)$, is above $E[r]$, then a MPS will reduce bankruptcy risk since it decreases the probability of bankruptcy, which is given by

$$
\begin{aligned}
\Pr\left(r < r_D(1-k)\right) &= \Pr\left(r \leq E[r]\right) + \Pr\left(E[r] < r < r_D(1-k)\right) \\
&= \frac{1}{2} + \frac{1}{S}\int_{E[r]}^{r_D(1-k)} dr,
\end{aligned}
$$

which is decreasing in a mean preserving spread. Conversely, if the threshold for bankruptcy is below $E[r]$, then a MPS will increase bankruptcy risk:

$$
\Pr\left(r < r_D(1-k)\right) = \frac{1}{S}\int_{R-S}^{r_D(1-k)} dr,
$$

which is increasing in a mean preserving spread. We first show that it is the latter that always applies, meaning that $r_D(1-k) < E[r]$. To see this, suppose first that $u = 1$, so that, using the equilibrium expressions for r_D and k, we have

$$
r_D(1-k) < E[r] \Leftrightarrow \frac{1}{R}\left(R^2 - 2S\right) < R - \frac{S}{2} \Leftrightarrow R < 4,
$$

which is always satisfied for the region where $u = 1$. Suppose then instead that $u > 1$. Then, again using the equilibrium expressions for r_D and k, we have

$$
r_D(1-k) < E[r] \Leftrightarrow \frac{R}{2}\left(1 - \sqrt{\frac{R^2}{R^2 + 8CS}}\right) < R - \frac{S}{2},
$$

or, equivalently,

$$
\sqrt{\frac{R^2}{R^2 + 8CS}} > \frac{S}{R} - 1.
$$

Since $S \leq R$, $\frac{S}{R} - 1 \leq 0$, the condition is always satisfied and $r_D(1-k) < E[r]$ whenever $u > 1$. Therefore, keeping r_D and k constant, a MPS always leads to an increase in bankruptcy risk.

The implications of a MPS for the equilibrium allocations can now be seen to depend on whether $N < M$ or $N = M$. Suppose first that $N < M$, so that $u = 1$ and some investors use storage rather than holding bank deposits. Since, ceteris paribus, a MPS increases the probability of bankruptcy and thus reduces the payoff to bank depositors, banks must increase their use of capital to avoid losing all depositors, who would otherwise simply put their savings into the storage technology. However, the increase in inefficiency stemming from the increased bankruptcy risk means that fewer banks operate, and there is less capital employed on aggregate.

When $N = M$, so that $u > 1$, a similar logic applies but now banks can offset part of the increased bankruptcy risk by reducing the equilibrium return to depositors, without risking depositors moving back into storage. With all funds being employed, the marginal adjustment banks can make is to shift their sources of funds, and they find it optimal to reduce the amount of capital being employed. The reason stems from the tradeoff banks contemplate when maximizing their profits: an incremental unit of capital reduces bankruptcy risk and increases the bank's return by a factor of $\frac{1}{S}$, which corresponds to the density of the distribution of project returns around the default boundary (with a uniform distribution, this density is the same everywhere).[6] A MPS increases S, thus reducing the marginal value of holding capital for the bank, and leads banks to reduce their capital holdings. Thus, k falls with a MPS, as

[6] This can be seen directly from the maximization problem for the bank: $\max_{r_D,k} \frac{1}{S}\int_{r_D(1-k)}^{R} (r - r_D(1-k))\, dr - pk$, subject to depositors' participation constraint, $\frac{1}{S}\int_{r_D(1-k)}^{R} r_D dr = u$. Substituting the participation constraint into the bank's maximization problem yields

$$\max_{k} \frac{1}{S}\int_{r_D(1-k)}^{R} rdr - (1-k)\, u - pk.$$

The first order condition is $\frac{1}{S}r_D^2(1-k) - (p-u) = 0$, or $k = 1 - \frac{S(p-u)}{r_D^2}$, which is decreasing in S for given ρ, u, and r_D, meaning that the marginal incentive to hold capital is lower.

do ρ and u, the equilibrium returns to capital suppliers and depositors, respectively.

4 Social efficiency

We now turn to the question how banks' capital structures, determined as part of the market equilibrium, compares to a socially efficient benchmark. To this end, we consider the problem of a social planner who chooses capital k_P such as to maximize a social welfare function that comprises the aggregate surplus of all bank stakeholders net of the participation costs incurred by investors who must be induced to become capital supplier. In particular, the planner solves

$$\max_{k_P} SW = \rho K + uD - \int_0^{\rho - u} M \frac{c}{C} \, dc \tag{5}$$

subject to

$$r_D = \arg\max_{r_D} \frac{1}{S} \int_{r_D(1-k_P)}^{R} (r - (r_D(1-k_P))dr - \rho k \tag{6}$$

$$E\left[U_D\right] = \frac{1}{S} \int_{r_D(1-k_P)}^{\infty} r_D dr \geq u \tag{7}$$

$$E\left[\Pi_B\right] = 0 \tag{8}$$

$$0 \leq k_P \leq 1 \tag{9}$$

Constraint (6) states that, while the planner maximizes total surplus through his choice of capital structure for each bank, he does not control the deposit rate r_D, which is instead chosen by shareholders so as to maximize bank profits. Constraint (7) is the participation constraint for depositors, ensuring that depositors receive a deposit rate that in expectation is equal to the equilibrium expected return on deposits. Finally, constraint (8) demands that all residual profits of banks are paid out to bank shareholders. We can now state the following result:

Proposition 2. *The socially optimal allocation of bank capital coincides with the competitive capital structure:* $k_P = k^*$.

Proof. See appendix. □

Despite the fact that banks behave as price takers in the competitive equilibrium and do not directly consider the impact of their capital structure choices on the equilibrium rate of return on capital, they end up holding a socially optimal level of bank capital. The reason is that there is no pecuniary externality in the bank equity market in our model. Banks maximize profits, to the benefit of bank shareholders, but the competitive nature of the market for bank capital means that they wind up internalizing the costs investors incur in becoming sophisticated and thus willing to hold bank equity. The first welfare theorem thus applies, with the market equilibrium being Pareto optimal and in fact equivalent to the socially optimal solution.

5 The incidence of binding capital requirements

Since, as demonstrated above, the competitive level of capital in our baseline model is efficient, any requirement to hold capital $k > k^*$ must necessarily reduce aggregate surplus. As argued above, this is because there are no externalities in the model, and the competitive banking sector fully internalizes the (social) cost borne by investors in becoming sophisticated and thus willing to hold bank equity. The model therefore provides an efficient benchmark where binding regulation is distortionary and represents a pure cost, reducing aggregate output. More importantly, the model can pin down who primarily bears the burden of regulation, without any offsetting benefit that can be used to generate transfers to the affected parties. Later on, we will study cases where social inefficiencies create a rationale for regulation, and highlight the need to perhaps incorporate transfers into any regulatory rules that might be adopted if the primary goal of the rules is to benefit or protect particular agents, such as retail depositors.

To study this issue, note first that any capital requirement $k \leq k^*$, the market solution, will not be binding since banks will prefer to choose k^* over the regulatory minimum. Therefore, we restrict our analysis to cases of $k > k^*$, or in other words to cases where the capital requirement is binding. Since k^* is an equilibrium value and depends on various parameters, we divide discussion of the incidence of regulation into the same two regions characterized above: (1) the case where $N < M$ and therefore the equilibrium return to depositors is the same as that in storage: $u = 1$; and (2) the case where $N = M$ and therefore depositors earn a premium relative to storage: $u > 1$.

Proposition 3. *For both cases below, suppose that $k > k^*$, and define K^* as the aggregate amount of capital in the market solution: $K^* = N^* k^*$.*

1. *Assume $N < M$, with $u = 1$. Then $\frac{d\rho}{dk} < 0$ and $\frac{dN}{dk} < 0$. This also implies that $K^{reg} < K^*$, where K^{reg} is the aggregate level of capital used in the regulatory solution: $K^{reg} = Nk$.*

2. *Assume $N = M$, with $u > 1$. Then, $\frac{du}{dk} < \frac{d\rho}{dk} < 0$. This then implies that $K^{reg} > K^*$.*

Proof: See appendix.

The proposition establishes that binding capital requirements either lead to fewer banks operating, or to the burden being borne disproportionately by depositors. The first result, which arises when $N < M$, so that not all funds are allocated to the banking sector and some investors use storage, gives rise to an immediate inefficiency since productive projects are funded through banks, so that reducing the total funds that flow to the banking sector, and thus reducing the number of banks in equilibrium, leads to less output being produced. In other words, the deadweight loss associated with capital requirements here is reflected in lower output being produced.

The second result highlights how binding capital requirements affect the different classes of investors. Since the market solution also maximizes aggregate output, a binding capital requirement of necessity leads to less total output being produced. Since the number of projects being run remains constant (at least for local changes in the amount of capital, k), the deadweight loss arises from the increased participation costs borne by the additional investors that need to be induced the hold bank capital, given the capital requirement is larger than the market solution. While the equilibrium return to equityholders, ρ, decreases, the return to depositors, u, decreases *even more* because the difference between them, $\rho - u$, must increase in order for more investors to be willing to be capital suppliers. Thus, while shareholders earn a lower return, reflecting the greater deadweight losses and lower aggregate output, the big losers are depositors, who bear much of the burden of the increased capital requirement. It is thus the investors who in principle should be protected most by larger capital requirements (as indeed they are since bankruptcy risk is reduced) who ultimately pay for this "insurance" through more than commensurate reductions in the return they earn in equilibrium.

Finally, it is worth noting that while we focus on marginal changes in the capital requirement, larger changes may compound both effects. For instance, if $N = M$, a large increase in a capital requirement could cause depositors' return u to hit its lower bound of $u = 1$. At that point, investors are indifferent between holding deposits and putting their savings into storage, so that further increases in the capital requirement would lead to less market participation and further reductions in total output.

6 Extensions

To be completed.

7 Conclusion

This paper presents an analysis of banks' optimal capital structures in a setting where investors may be reluctant to participate in financial markets by holding anything other than safe assets, such as bank deposits, and have to be induced to do so through the promise of higher returns. The equilibrium amount of stock market participation in the banking sector is thus endogenous, and depends on the distribution of returns associated with the investment opportunity set available to banks. We use this framework to study the incidence of capital regulation, and shed light on whether requirements geared toward reducing bank failure and absorbing losses that would otherwise accrue to depositors and by extension the deposit insurance fund affect various classes of investors differently.

In our analysis, we have explicitly sidestepped issues related to the interaction of risk and leverage that are present when systematic risk is priced by assuming risk neutrality. Therefore, the standard results stemming from the work by Modigliani and Miller (1958) are not present, allowing us to isolate the effects stemming from limited market participation and capital regulation. An interesting issue, however, would be to consider how risk aversion, coupled with the existence of systematic risk, interact with the results we obtain here. At present, investors' reluctance to invest in equities implies in our framework that greater demand for capital (i.e., equity) by banks requires that investors earn a higher return in order to induce them to participate. By contrast, the usual logic of risk aversion and systematic risk implies that greater leverage makes equity riskier on a systematic basis, and increases its required return. The study of this issue introduces additional complexities to understand the exact source of households' unwillingness to participate in financial markets, and is left for future research.

A Mathematical Appendix

Proof of Proposition 1:

We consider banks' optimal choice of capital k. Define $\kappa \equiv 1 - \frac{R-S}{r_D}$. There are two fundamentally distinct scenarios regarding the level of bank capital: if banks choose any capital stock $k \geq \kappa$, they are fully protected against bankruptcy because banks owe less to depositors than the lowest possible state, $r_D(1-\kappa) = R - S$. We will turn to this case later. If banks choose $k \in (0, \kappa)$, bankruptcy will occur with positive probability. Solving $EU = u$ for k we see that the bank chooses a capital stock of

$$k = 1 - \frac{R - S(u/r_D)}{r_D} \tag{10}$$

which is smaller than κ if and only if $u < r_D$; hence, the equilibrium which we derive now can hold only as long as $u < r_D$. Back-substituting (10) into the expression for bank profits we find

$$E\left[\Pi_B\right] = \frac{Su^2 - 2Su\rho + 2\rho R r_D - 2r_D^2 \rho}{2r_D^2} \tag{11}$$

Maximization of profits for r_D results in

$$r_D^* = \frac{Su(2\rho - u)}{\rho R} \tag{12}$$

which gives a total maximum profit of

$$E\left[\Pi_B\right] = \frac{\rho\left(\rho R^2 - 2Su(2\rho - u)\right)}{2Su(2\rho - u)} \tag{13}$$

There is free entry, so in equilibrium expected profits in eq. (13) are zero. Solving for ρ yields

$$\rho = \frac{2Su^2}{4Su - R^2} \tag{14}$$

Substituting this expression into eq. (12) and collecting terms yields the remarkable result that the promised depositor rate is always half of the maximal upside of the risky technology, independently of the dispersion S:

$$r_D = \frac{R}{2} \tag{15}$$

Substituting r_D in (10), we obtain

$$k = \frac{4Su}{R^2} - 1 \tag{16}$$

Market clearing in the bank capital market commands that

$$M\frac{\hat{c}}{C} = Nk \tag{17}$$

which can be rewritten as

$$\rho - u = \frac{NCk}{M} \tag{18}$$

Let us for now assume that there is full inclusion, i.e. no storage is used and all investors invest in either equity or deposits: $N = M$. Substituting (16) and (14) in eq. (18) and solving for u, we find two possible roots, $u = \frac{R^2}{4S}\left(1 \pm \sqrt{\frac{R^2}{R^2+8CS}}\right)$. However, back-substituted into (16), one can see that the only economically sensible solution is

$$u = \frac{R^2}{4S}\left(1 + \sqrt{\frac{R^2}{R^2 + 8CS}}\right) \tag{19}$$

because it implies a nonnegative capital stock of

$$k = \sqrt{\frac{R^2}{R^2 + 8CS}} \tag{20}$$

Combining these results with eq. (18), we finally find

$$\rho = \frac{R^2}{4S} + \frac{R\,(4CS + R^2)}{4S\sqrt{8CS + R^2}}$$

If $\hat{u} \equiv u = \frac{R^2}{4S}\left(1 + \sqrt{\frac{R^2}{R^2+8CS}}\right) \geq 1$, this full participation equilibrium also satisfies the depositor participation constraint and is the unique equilibrium of the game. However, in case $\hat{u} < 1$, the full participation equilibrium is not feasible because violates the depositor participation constraint: at full participation returns, depositors would strictly prefer storage over deposits. The unique equilibrium must in this case exhibit incomplete (partial) participation, with some funds going to storage, and the return of deposits matching this storage outside option, thus $u = 1$. We obtain the remaining equilibrium variables

$$k = \frac{4S}{R^2} - 1 \tag{21}$$

$$\rho = \frac{2S}{4S - R^2} \tag{22}$$

$$K = Nk = M\frac{2S - R^2}{C(R^2 - 4S)} \tag{23}$$

directly from equations (14), (16) and (17). This concludes our analysis of the risky bank equilibrium, with the important qualification that $u < r_D$. This means that this risky equilibrium only applies when $\max\left\{\frac{R^2}{4S}\left(1 + \sqrt{\frac{R^2}{R^2+8CS}}\right), 1\right\} < R/2$.

We now turn to the safe equilibrium that is obtained whenever banks choose a safe level of capital, such that

$$r_D(1-k) \geq R - S \Leftrightarrow k \geq 1 - \frac{R-S}{r_D}$$

In this case, the probability of bankruptcy is zero. The expected utility of depositors becomes $EU = r_D = u$, whereas the expected return of shareholders is pinned down by the equation

$$E\Pi = E[r] - (1-k)u - k\rho = E[r] - k(\rho - u) - u = 0$$

which implies that

$$\rho = \frac{E[r] - (1-k)u}{k} = \frac{R - S/2 - (1-k)u}{k}$$

Assume $\rho > u$ (which it must be in equilibrium to attract a nonzero capital stock). Then maximization of profits dictates that banks choose the smallest possible k which still attains safety, $k = 1 - \frac{R-S}{u}$. The return to capital becomes then

$$\rho = \frac{R - S/2 - (R - S)}{1 - \frac{R-S}{u}} = \frac{Su/2}{u - R + S}$$

The equilibrium value of u is determined by market clearing,

$$M\frac{\hat{c}}{C} = Nk$$

with $M = N$ and $\hat{c} = \rho - u$.

Substituting previous results, we find

$$\frac{Su/2}{u - R + S} - u = C\left(1 - \frac{R - S}{u}\right)$$

which can be transformed to read

$$\frac{Su^2 - 2u^2(u - R + S) - 2C(u - R + S)^2}{2(u - R + S)^2} = 0$$

$$\Rightarrow (2R - S - 2u)u^2 - 2C(u - R + S)^2 = 0$$

The latter is the defining equation for the equilibrium $u = u^*$ in the safe regime, and the remaining allocations are obtained by inserting u^* into the previously derived equations. Note that a safe equilibrium can never be optimal when an "interior" risky equilibrium with $u > r_D$ exists: by choosing k arbitrarily close to, and just marginally below, κ, banks can attain any safe equilibrium return in a risky setting. Whenever the risky equilibrium choice does not feature such corner solution, it is thus clear that the benefit from the risky equilibrium must be strictly higher.

In order to show that $\rho > E[r] > u$, we first show that $\rho > E[r]$ implies $u < E[r]$. Consider eq. (1) and eq. (2). The former, combined with the free entry condition, can rewritten as

$$E[\Pi_B] = \frac{1}{S}\int_{-\infty}^{\infty} \max(r - r_D(1-k), 0)\, dr - \rho k = 0$$

whereas the latter, rewritten as strict equality (which it must be in equilibrium) and multiplied on both sides with $(1-k)$, can be bounded as follows: Consider that $\frac{1}{S}\int_{-\infty}^{r_D(1-k)} r\, dr + \frac{1}{S}\int_{r_D(1-k)}^{\infty} r_D(1-k)\, dr = \frac{1}{S}\int_{-\infty}^{\infty} \min(r, r_D(1-k))\, dr$. The first term must be nonnegative since the lower bound of the support of the distribution of r is nonnegative. Hence,

$$\frac{1}{S}\int_{r_D(1-k)}^{\infty} r_D(1-k)\, dr = u(1-k) \leq \frac{1}{S}\int_{-\infty}^{\infty} \min(r, r_D(1-k))\, dr$$

Summing both expressions yields

$$(1-k)u + k\rho \leq \frac{1}{S}\int_{-\infty}^{\infty} r\, dr = E[r]$$

which shows that $E[r]$ is greater or equal to a convex combination of ρ and u. Thus, for any $k \in (0,1)$ having $\rho > E[r]$ implies necessarily that $u < E[r]$.

We conclude the proof by showing that $\rho > E[r]$ for any value of $\hat{u}$.

For $\hat{u} < 1$, we rewrite $\rho - E[r] = \frac{2S}{4S-R^2} - \frac{2R-S}{2} = \frac{(S-2R)\left(4S-R^2\right)+4S}{2(4S-R^2)}$. Since know that $\hat{u} = \frac{R^2}{4S}\Delta < 1$ with $\Delta \equiv 1 + \sqrt{\frac{R^2}{R^2+8CS}} \geq 1$, it must be that $R^2 < 4S$. The denominator of the previous expression is thus always positive and the sign of the expression depends only on the numerator. It is straightforward to show that the numerator has (within the relevant domain of all R, S for which $\hat{u} < 1$ and $0 < S \leq R$) a minimum of zero at $R = 2$, $S = 2$ and is strictly increasing and convex in R for all $R \in (2, 2\sqrt{S})$. Therefore, $\rho > E[r]$ holds as long as $R > 2$, which was part of our initial assumptions.

For $R/2 > \hat{u} \geq 1$, we can calculate

$$\rho - E[r] = \frac{2Su^2}{4Su - R^2} - \frac{2R - S}{2} \tag{24}$$

$$= \frac{2S^2\sqrt{8CS + R^2} + R^2\sqrt{8CS + R^2} - 4RS\sqrt{8CS + R^2} + 4CRS + R^3}{4S\sqrt{8CS + R^2}} \tag{25}$$

which is positive if the numerator is positive. Rewriting the numerator,

$$R(R^2 + 4CS) + \left[R^2 - 4RS + 2S^2\right]\sqrt{R^2 + 8CS} \tag{26}$$

$$\ldots = R(R^2 + 4CS) + \left[2(R - S)^2 - R^2\right]\sqrt{R^2 + 8CS} \tag{27}$$

$$\geq R^2(R^2 + 4CS) - R^2\sqrt{R^2 + 8CS} \tag{28}$$

A lengthy but straightforward analysis yields that for any $x > 0$ the function $R(R^2 + x) - R^2\sqrt{R^2 + 2x}$ is zero at $R = 0$, is strictly increasing in R for $R \in (0, \sqrt{\frac{2}{3}x})$, reaches a maximum value of $\frac{1}{3}\sqrt{\frac{2}{3}x}$ at $R_m = \sqrt{\frac{2}{3}x}$ and is strictly decreasing for $R > R_m$, with $\lim_{R\to\infty} R(R^2 + x) - R^2\sqrt{R^2 + 2x} = 0^+$. Hence, its range for $R \in (0, \infty)$ is strictly positive, and thus the expression in eq. (28) is positive, which establishes that $\rho > E[r]$.

Finally, for $\hat{u} > R/2$ it is immediate from market clearing that $\rho > u$. Since there is no bankruptcy, there are also no bankruptcy cost, and $E[r] = \rho k + u(1 - k)$ holds with equality for a $k \in (0, 1)$, which proves $\rho > E[r] > u$. □

Proof of Proposition 2:

To establish the result, it is useful to start with the problem of bank profit maximization, which can be expressed as

$$\max_k E[\Pi_B] = \frac{1}{S}\int_{r_D(1-k)}^{R} (r - r_D(1 - k))\, dr - \rho k$$

subject to the same constraints (6), (7), and (8) as above, for the social planner's problem. Note that the first order condition implied by (6), for any level of capital k, is always negative, meaning that the bank (and the social planner) always finds it optimal to choose the lowest deposit rate r_D that is consistent with satisfying depositors' participation constraint. Therefore, (7) will always be satisfied with equality, allowing us to substitute the constraint into the bank's maximization problem to obtain

$$\max_k E\left[\Pi_B\right] = \frac{1}{S}\int_{r_D(1-k)}^{R} rdr - u(1-k) - \rho k.$$

The necessary first order condition that must now be satisfied is

$$\frac{1}{S}\left(r_D^2\left(1-k\right) + \frac{\partial r_D}{\partial k}\left(1-k\right)^2 r_D\right) - (\rho - u) = 0,$$

where $\frac{\partial r_D}{\partial k}$ is obtained directly from (7). This first order condition must be satisfied in equilibrium whatever the values for ρ and u, which are obtained from market clearing as in Proposition 1.

Consider now the maximization problem for the social planner, which can alternatively be written as

$$\begin{aligned}\max_{k_P} SW &= N\frac{1}{S}\int_{r_D(1-k)}^{R} rdr - \int_0^{\widehat{c}} M\frac{c}{C}\,dc + M - N \\ &= N\frac{1}{S}\int_{r_D(1-k)}^{R} rdr - \frac{1}{2C}M\widehat{c}^2 + M - N,\end{aligned}$$

reflecting the fact that maximizing the return to all stakeholders is equivalent to maximizing aggregate output, $N\frac{1}{S}\int_{r_D(1-k)}^{R} rdr$, since all output is allocated to either depositors or capital suppliers. The last term, $M-N$, represents the funds that are not invested in the banking sector but rather held as storage, to the extent that M may be strictly greater than N. Recall now the market clearing condition, $M\frac{\widehat{c}}{C} = kN$, which implies that $\widehat{c} = \frac{C}{M}kN$, or that

$\widehat{c}^2 = \left(\frac{C}{M}kN\right)^2$, and which is taken into account by the social planner. We can thus write the maximization problem above as

$$\max_{k_P} SW = N\frac{1}{S}\int_{r_D(1-k)}^{R} rdr - \frac{1}{2C}M\left(\frac{C}{M}kN\right)^2 + M - N$$
$$= N\frac{1}{S}\int_{r_D(1-k)}^{R} rdr - \frac{1}{2}\frac{C}{M}k^2N^2 + M - N.$$

For an interior solution, the necessary first order condition to this problem is

$$N\frac{1}{S}\left(r_D^2(1-k) + \frac{\partial r_D}{\partial k}(1-k)^2 r_D\right) - \frac{C}{M}kN^2 + \frac{\partial N}{\partial k}\frac{1}{S}\int_{r_D(1-k)}^{R} rdr - \frac{C}{M}k^2N\frac{\partial N}{\partial k} - \frac{\partial N}{\partial k} = 0.$$

Grouping terms obtains

$$N\left(\frac{1}{S}\left(r_D^2(1-k) + \frac{\partial r_D}{\partial k}(1-k)^2 r_D\right) - \frac{C}{M}kN\right) + \frac{\partial N}{\partial k}\left(\frac{1}{S}\int_{r_D(1-k)}^{R} rdr - \frac{C}{M}k^2N - 1\right) = 0.$$

Since at equilibrium $\rho - u = \widehat{c} = \frac{C}{M}kN$, we can further rewrite as

$$N\left(\frac{1}{S}\left(r_D^2(1-k) + \frac{\partial r_D}{\partial k}(1-k)^2 r_D\right) - (\rho - u)\right) + \frac{\partial N}{\partial k}\left(\frac{1}{S}\int_{r_D(1-k)}^{R} rdr - k(\rho - u) - 1\right) = 0$$

Now observe that $\frac{\partial N}{\partial k} \neq 0 \Rightarrow u = 1$ since, if $u > 1$, all funds are being used in the banking sector, so a marginal increase in k cannot change $N = M$.

Consider first the case that $u > 1$, so that $\frac{\partial N}{\partial k} = 0$. We are then left with only

$$N\left(\frac{1}{S}\left(r_D^2(1-k) + \frac{\partial r_D}{\partial k}(1-k)^2 r_D\right) - (\rho - u)\right) = 0,$$

which is the same condition as must be satisfied for profit maximization problem.

Alternatively, suppose that $u = 1$, which allows us to express the first order condition for the social planner as

$$N\left(\frac{1}{S}\left(r_D^2(1-k) + \frac{\partial r_D}{\partial k}(1-k)^2 r_D\right) - (\rho - 1)\right) + \frac{\partial N}{\partial k}\left(\frac{1}{S}\int_{r_D(1-k)}^{R} rdr - k\rho - (1-k)\right) = 0.$$

Note now that term in the parentheses of the second line, $\frac{1}{S}\int_{r_D(1-k)}^{R} rdr - k\rho - (1-k)$, is simply $E\left[\Pi_B\right]$ for the case where $u = 1$, which in equilibrium is equal to zero, with all rents going to shareholders through ρ. This leaves exactly the term below, after eliminating the N:

$$\frac{1}{S}\left(r_D^2\left(1-k\right) + \frac{\partial r_D}{\partial k}\left(1-k\right)^2 r_D\right) - \left(\rho - 1\right) = 0,$$

again exactly as in the problem of profit maximization for the case $u = 1$. Therefore, for all cases the necessary condition to be satisfied is identical to that which maximizes profits. Given that the market clearing condition that pins down the equilibrium returns u and ρ is the same across both maximization problems, we can conclude that both problems must have the same solution. □

Proof of Proposition 3:

We first establish a preliminary result, which is that the per-unit return to shareholders ρ is maximized at the market solution k^*. To see this, recall that bank profits are given by

$$E\left[\Pi_B\right] = \frac{1}{S}\int_{\widehat{r}_D(1-k)}^{R}\left(r - \widehat{r}_D(1-k)\right)dr - \rho k,$$

where we use $\widehat{r}_D$ to represent the deposit rate that satisfies depositors' participation constraint, (2) and is a function of the level of capital k. For ease of notation, define $F(k) = \frac{1}{S}\int_{\widehat{r}_D(1-k)}^{R}(r - \widehat{r}_D(1-k))\,dr$, and note that F is increasing and strictly concave in k. This allows us to write bank profits as

$$E\left[\Pi_B\right] = F\left(k\right) - \rho k,$$

where, as always, ρ is chosen such that $E\left[\Pi_B\right] = 0$ in equilibrium. We now want to determine how ρ changes with a change in k: The implicit function theorem tells us that

$$\frac{d\rho}{dk} = -\frac{\frac{\partial E[\Pi_B]}{\partial k}}{\frac{\partial E[\Pi_B]}{\partial \rho}} = -\frac{F'(k) - \rho}{-\rho} = \frac{F'(k)}{\rho} - 1. \tag{29}$$

Consider now the bank's choice of k which, from the first-order condition defining the optimal k^*, satisfies

$$F'(k^*) = \rho. \tag{30}$$

We can now show that returns to equity are maximal at k^*: substituting (30) into (29) to obtain

$$\frac{d\rho}{dk} = \frac{F'(k^*)}{\rho} - 1 = 0.$$

Given concavity of F, which implies that $F'(k) < F'(k^*)$, this means that ρ achieves its maximum at $k = k^*$, and is decreasing in k beyond that. This establishes that $\frac{d\rho}{dk} < 0$ for both cases (1) and (2) in the proposition.

To establish the rest of the result, consider first the case where $N < M$. Market clearing implies

$$\rho - u = Ck\frac{N}{M}.$$

Given the result above that ρ is lower for larger k, and the fact that $u = 1$, the RHS must decrease to satisfy market clearing. Given that we are considering an increase in k, N must fall, and it must fall enough that even kN must be lower. Since $K = kN$, it follows that $K^{reg} < K^*$.

Consider next the case where $N = M$ and therefore $u > 1$. For a marginal increase in k, market clearing simplifies to

$$\rho - u = kC.$$

The RHS must be larger than in the market solution because $k > k^*$. Given that ρ is lower, per the argument above, we must have that u decreases more than proportionally. That is, $0 > \frac{d\rho}{dk} > \frac{du}{dk}$, as desired. Finally, $K^{reg} = Nk > Nk^* = K^*$, which concludes the proof. $\square$

References

Admati, A. R., DeMarzo, P. M., Hellwig, M. F., and Pfleiderer, P. Fallacies, irrelevant facts, and myths in the discussion of capital regulation: Why bank equity is not socially expensive. Working paper, Stanford University, October 2013.

Allen, F. and Gale, D. Limited market participation and volatility of asset prices. *American Economic Review*, 84(4):933–55, 1994.

Allen, F., Carletti, E., and Marquez, R. Credit market competition and capital regulation. *Review of Financial Studies*, 24(4):983–1018, 2011.

Allen, F., Carletti, E., and Marquez, R. Deposits and bank capital structure. *Journal of Financial Economics*, 118(3):601–619, 2015.

Bertsch, C. and Mariathasan, M. Fire sale bank recapitalizations. Working Paper 312, Sveriges Riksbank, September 2015.

Dell'Ariccia, G. and Marquez, R. Competition among regulators and credit market integration. *Journal of Financial Economics*, 79(2):401–430, 2006.

Guiso, L. and Sodini, P. Household finance: An emerging field. In George M. Constantinides, M. H. and Stulz, R. M., editors, *Handbook of the Economics of Finance*, volume 2, Part B of *Handbook of the Economics of Finance*, chapter 21, pages 1397 – 1532. Elsevier, 2013.

Hellmann, T. F., Murdock, K. C., and Stiglitz, J. E. Liberalization, moral hazard in banking, and prudential regulation: Are capital requirements enough? *American Economic Review*, 90(1):147–165, 2000.

Holmström, B. and Tirole, J. Financial intermediation, loanable funds, and the real sector. *The Quarterly Journal of Economics*, 112(3):663–691, 1997.

Modigliani, F. and Miller, M. H. The cost of capital, corporation finance and the theory of investment. *The American Economic Review*, 48(3):261–297, 1958.

Vissing-Jørgensen, A. Limited asset market participation and the elasticity of intertemporal substitution. *Journal of Political Economy*, 110(4):825–853, 2002.