NETTING*

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Abstract

We present a model to explain why banks hold off-setting debts without netting them out. We find that off-setting debts help a bank to raise liquidity with new debt from a third party, since diluting old debt subsidizes the new debt. Even though a diluted bank is worse off ex post, a network of gross debts is stable ex ante. This is because it provides banks with valuable liquidity co-insurance, since each bank exercises its option to dilute when it needs liquidity most. However, the network harbors systemic risk: since one bank’s liabilities are other banks’ assets, a liquidity shock can transmit through the network in a default cascade.

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1 Introduction

Gross interbank positions are an order of magnitude larger than net interbank positions. For example, Barclay’s gross repo position is 28 billion pounds, almost ten times its net position.\footnote{Barclay’s annual reports are available online here\http://www.home.barclays/barclays-investor-relations/results-and-reports/annual-reports.html}. JPMorgan’s gross derivatives position is 1.5 trillion dollars, almost twenty times its net position.\footnote{Corbi (2012) estimates “the potential impact of grossing up” JPMorgan’s derivatives at $1.485 trillion, relative to the bank’s “reported derivatives” of $80 billion.} These banks are not atypical. In Germany, a quarter of the average bank’s assets and liabilities comprise unnetted interbank exposures of various maturities, with an average maturity longer than a year (Bluhm, Georg, and Krahnen (2016)). These banks choose to hold these gross positions rather than net them out, even when they constitute off-setting debts with the same bank—i.e. a bank is likely to borrow from another bank and simultaneously lend to that same bank. Banks maintain these positions even though practitioners and policy makers alike champion the benefits of netting. Indeed, according to ISDA, “Support for netting is well-nigh universal in the financial industry as well as among policy markers” (Mengle (2010), p. 2). The literature suggests that cross-holdings of demand deposits may function as emergency credit lines (Allen and Gale (2000)). But why do banks choose not to net out fixed-term debts of longer maturities?

In this paper, we develop a model to address this question. We find that off-setting debts help a bank to raise liquidity with new debt from a third party, since diluting old debt subsidizes the new debt. Even though a diluted bank is worse off ex post, a network of gross debts is stable ex ante. This is because it provides banks with valuable liquidity co-insurance, since each bank exercises its option to dilute when it needs liquidity most. However, the network harbors systemic risk: since one bank’s liabilities are other banks’ assets, a liquidity shock can transmit through the network in a default cascade.

**Model preview.** In the model, there are \(N\) banks, \(B_1, \ldots, B_N\), each of which suffers from a maturity mismatch—it has long-term assets in place, but may need short-term liquidity due to a “liquidity shock” before its assets mature. To meet this liquidity need, a bank can borrow in the market from an outside creditor \(C\). However, its ability to do this is impeded by two contracting frictions. First, liquidity shocks are non-contractable
and, second, asset pledgeability is limited.

These frictions make it hard for a bank to avoid having to liquidate its assets to meet a liquidity shock—since liquidity shocks are non-contractable, a bank cannot pay a small upfront premium in exchange for a large payoff in the event of a shock; since asset pledgeability is limited, a bank cannot mortgage its assets to meet its liquidity need in the event of a shock.

**Figure 1: Balance sheet before and after liquidity shock**

<table>
<thead>
<tr>
<th>Gross Positions</th>
<th>Cash from Sale and New Debt</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>assets</strong></td>
<td><strong>liabilities</strong></td>
</tr>
<tr>
<td>long-term assets</td>
<td>risky short-term liabilities</td>
</tr>
<tr>
<td>debt from $B_j$</td>
<td>debt to $B_j$</td>
</tr>
<tr>
<td>equity</td>
<td>cash from debt sale</td>
</tr>
<tr>
<td></td>
<td>debt to $C$</td>
</tr>
<tr>
<td></td>
<td>cash from new debt</td>
</tr>
<tr>
<td></td>
<td>equity</td>
</tr>
</tbody>
</table>

**Results preview.** Our first main result is that entering into off-setting debt positions allows banks to circumvent these frictions and thus to co-insure liquidity risk. To see why, suppose that you take on off-setting debts with bank $B_j$, i.e. you have a debt from $B_j$ on the assets side of your balance sheet and an off-setting debt to $B_j$ on the liabilities side of your balance sheet. Now, if you are hit by a liquidity shock, you have two ways to raise liquidity in the market from $C$. (i) You can sell the debt on the assets side of your balance sheet and (ii) you can take on senior debt on the liabilities side of your balance sheet (see Figure 1). Taking on senior debt from $C$ dilutes your debt to $B_j$: the cost of your default is transferred to $B_j$ and, hence, the new debt to $C$ is subsidized. As a result, $C$ is more willing to lend to you, which allows you to meet your liquidity shock.

Our second main result is that the network of off-setting debts is pairwise stable, i.e. you and $B_j$ both want to enter into off-setting debts ex ante. Why does $B_j$ want to
enter into this position with you, even though it knows you will dilute its debt when you are hit by a liquidity shock? Because the off-setting debts allow B_j to raise liquidity by diluting your debt when it is shocked, just as they allow you to raise liquidity by diluting its debt when you are shocked. In other words, off-setting debts implement valuable liquidity co-insurance for you and B_j, since each of you exercises your option to dilute the other when you need liquidity most (and only when you need liquidity most). Hence, your zero-net debts do not have zero net present value.

This mechanism explains not only why a bank may borrow from one bank and simultaneously lend to that bank—i.e. the absence of bilateral netting—but also why a bank may borrow from one bank and simultaneously lend to another bank—i.e. the absence of multilateral netting, or so-called “novation.” In particular, networks like the ring network (Subsection 5.1), in which banks intermediate between other banks, also implement liquidity insurance. There, banks accept the risk of being diluted by their borrowers in exchange for the option to dilute their creditors. Indeed, since banks rely on new debt to a third party to circumvent the limited pledgeability friction, interbank debt is optimally non-exclusive in our model. Thus, our model suggests a rationale for why there is a lot of borrowing and lending within groups of banks generally and, as such, may cast light on the core-periphery structure of interbank debt networks.

Our third main result is that, although the network of off-setting debts provides banks with liquidity insurance, it harbors systemic risk. Since your liability is B_j’s asset, B_j may be unable to repay its creditors if you default. Specifically, if enough banks are hit by liquidity shocks, then all banks default, including those that are not shocked. In other words, whereas interconnectedness helps the financial system to sustain shocks in normal times, it can propagate shocks in the event of a liquidity crisis. This is in line with results in the networks literature (see below).

Policy. Some policy makers have suggested that “close-out netting,” i.e. netting in the event of default, may be enough to mitigate systemic risk (see, e.g., Mengle (2010)). This is because if you default on B_j, B_j can just cancel out its off-setting debt, so your default does not transmit to B_j’s creditors. Our analysis suggests that close-out netting may not be enough. This is because debts can be traded in the market before your default and, as a result, even if B_j has an off-setting position with you today, it may not have a debt to set off against you if you default. It may now owe money to a third party who bought its debt from you in the market.

Empirical content. Our analysis is motivated by the empirical observation that banks often do not net out off-setting debts, but it may cast light on a number of other stylized facts as well. (i) In our model, banks with off-setting debts accept the costs of

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4See, e.g., Craig and von Peter (2010), in ’t Veld and van Lelyveld (2014), and Peltonen, Scheicher, and Vuillemy (2013).

5For the legal details of close-out netting, see, e.g., Johnson (2015) and Paech (2014).
dilution without asking for increased repayments, since they gain reciprocal liquidity insurance. This is consistent with Afonso, Kovner, and Schoar’s (2014) finding that “borrowers get...lower rates from lenders that they also lend to...consistent with a model where...borrowers and lenders...insure each other against liquidity shocks at favorable rates” (p. 3). (ii) In our model, banks use a mix of junior debt (to one another) and senior debt (to C), which we interpret as a mix of unsecured debt (e.g. bonds and loans) and senior debt (e.g. repos). This casts light on why banks’ capital structure contains a mix of secured and unsecured debt and why banks may lend unsecured despite the risk of dilution. (iii) In our model, banks lend to one another in a network of zero-net positions to co-insure liquidity shocks from outside this core network. This may cast light on the core-periphery structure of banking systems, as touched on above.

**Related literature.** Our first contribution relative to the literature is to provide a micro-founded explanation for banks’ off-setting debts. This is related to Allen and Gale’s (2000) result that cross holdings of demand deposits can implement liquidity insurance when banks cannot borrow in the event of a liquidity shock. In that model, cross-holdings effectively allow shocked banks to take on new debt, even though there is no market to borrow in. However, excessive drawdowns on these deposits can lead to financial contagion. We add to these results in three ways. (i) We show that the absence of a debt market at the interim date is not necessary for off-setting debts to provide liquidity insurance. Indeed, our mechanism relies on a market in which banks can take on new debt, diluting old debt. (ii) We show that cross-holdings of off-setting fixed-term debts, not just demand deposits, can implement liquidity insurance. In contrast, Allen and Gale’s (2000) mechanism is specific to demand deposits, since it relies on the fact that they repay less when redeemed early. (iii) We show that these off-setting debts can generate systemic risk through the assets side of bank balance sheets (banks default because they are not repaid) rather than through the liabilities side (banks default because their deposits are drawn down).

In a 2011 lecture, John Moore also takes up the question of how unnetted debts contribute to systemic risk. He outlines a macro model based on the assumption that banks with larger gross interbank positions can borrow more from households. Our finding that mutual gross debts help shocked banks to raise liquidity from a third party suggests that Moore’s (2011) reduced-from idea that mutual gross positions facilitate new borrowing is right. However, unlike in Moore’s setup, access to new debt does not come for free in our model, since creditors bear the risk of dilution.

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Although a market can provide liquidity efficiently at the interim date, it can also distort investment decisions at the initial date. Thus, in Allen and Gale’s (2000) setup, a mechanism designer may wish to close markets (see their discussion on p. 26 as well as Jacklin (1987) and Allen and Gale (2004)). In our model, we take it as a primitive that debt markets exist and find that a mechanism designer does not want to close them, but rather relies on them.
Our second contribution is to uncover a mechanism by which off-setting non-contingent, non-exclusive debts implement state-contingent liquidity insurance, since they embed the option to dilute. This complements the idea that the option to default can implement valuable contractual contingencies when contracts are incomplete (see Allen and Gale (1998) and Zame (1993), as well as Hart and Moore (1998), in which the option to buy assets plays a similar role).

Our third contribution is to show that non-exclusive contracts can mitigate the inefficiencies arising from limited pledgeability, i.e. the ability to take on new debt to a third party can loosen borrowing constraints. This is the counterpart of Donaldson, Gromb, and Piacentino’s (2016) finding that limited pledgeability can mitigate inefficiencies arising from non-exclusive contracts. Our result also gives a new, positive perspective on non-exclusivity, which the literature has generally considered as a friction.

Finally, we confirm that the “robust-yet-fragile” results in the networks literature apply in our environment with endogenous contracts.

2 Model

In this section, we present the model.

2.1 Dates and Players

There are three dates, \( t \in \{0, 1, 2\} \), and \( N \) symmetric banks, \( B_1, \ldots, B_N \), as well as a creditor \( C \), representing a competitive capital market. Everyone is risk-neutral, consumes only at Date 2, and discounts the future at rate zero. Banks have long-term assets but relatively short-term liabilities. Specifically, each bank has riskless assets \( y \) realized at Date 2, but the possibility of a “liquidity shock” at Date 1. If a bank is hit by a liquidity shock, it must generate liquidity \( L < y \), otherwise it defaults and its assets are destroyed. We assume that exactly \( M \) banks are hit with liquidity shocks at Date 1, each with probability \( \pi := M/N \). Since \( M \) is deterministic, there is no aggregate risk and thus there is the possibility of perfect insurance. \( C \) has enough wealth \( w \) to provide liquidity to all the shocked banks, but not to all the banks, i.e. \( w \in [ML, NL) \).

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2.2 Contracting Frictions

Two contracting frictions limit banks’ ability to insure against liquidity shocks. First, liquidity shocks are non-contractable. Second, asset pledgeability is limited, in the sense that a bank can divert a fraction \(1 - \theta\) of its assets.\(^{10}\)

These frictions are especially important for banks. Liquidity shocks are a major concern for banks since, almost by definition, they invest in long-term assets funded by short-term liabilities. Bank regulations such as Basel III’s LCR expressly target these liquidity risks. This regulation reflects bank’s inability to insure against liquidity shocks via direct insurance contracts—if banks could insure directly, they would not have to insure indirectly by holding securities as “liquidity coverage.” Banks’ complexity contributes to limited pledgeability, making it hard to value assets ex ante and costly to liquidate them ex post (e.g., Shleifer and Vishny (1992)).

2.3 Payoffs

If a bank is liquidated at Date 1, its assets are destroyed and it gets zero. Otherwise, it decides whether to default or to repay its debt. If it defaults, it gets the diversion value of its long-term assets \((1 - \theta)y\). If it repays, it gets \(y\) minus the net repayments from its financial assets, i.e. the repayments it makes to other banks less the repayments it gets from other banks and any cash it may hold:

\[
B_i’s\ payoffs = \begin{cases} 
0 & \text{if liquidated at Date 1,} \\
(1 - \theta)y & \text{if default at Date 2,} \\
y - \text{net repayments} & \text{if no default.}
\end{cases}
\]

(1)

It may be worth emphasizing that the bank cannot divert financial assets, but only the long-term assets \(y\); for example, a real-world bank may be able to “divert” clients or employees in the event of default, but not to steal securities outright. Alternatively, it may need to exert costly effort to produce \(y\); if it decides to default, it does not exert this effort and gets net benefit \((1 - \theta)y\) in saved effort cost.

2.4 Timeline

We now describe the sequence of moves. The capital market is competitive, so whenever we say that a bank borrows from or sells to \(C\), it implies that the bank makes a take-
it-or-leave-it offer.

**Date 0**
Banks contract bilaterally with C or among themselves.

**Date 1**
- M random banks suffer liquidity shocks.
- Banks sell assets to or borrow from C.
- Each shocked bank either pays $L$ or defaults.

**Date 2**
Asset payoffs realize, banks either divert and default or repay their debt.

### 2.5 Equilibrium

Below we consider various Date-0 contractual relationships. Given these contracts, we find the subgame perfect equilibrium. I.e. $B_1, ..., B_N$, and C act to maximize their expected utility given their beliefs about players’ strategies and these beliefs are consistent with players’ strategies.

### 2.6 Assumption: Liquidity Risk

We assume that the mortgage value $\theta_y$ of a bank’s assets is between its expected liquidity need $\pi L = ML/N$ and its realized liquidity need $L$ given a shock,

\[
\frac{M}{N} L < \theta_y < L.
\]

Intuitively, this assumption says that (i) there is enough liquidity to go around at Date 1, since the total mortgage value of banks’ assets $N\theta_y$ exceeds the total need for liquidity $ML$, but that (ii) no shocked bank has enough liquidity to self-insure, since the mortgage value of its own assets $\theta_y$ is less than its liquidity need $L$ if it is shocked.

### 3 Benchmarks

In this section, we consider four benchmark allocations.

#### 3.1 First Best

The first best is the allocation that maximizes the total surplus accrued to $B_1, ..., B_N$, and C. Since $L < y$, liquidation is inefficient. Thus, all liquidity shocks are met in the first best: a bank generates surplus $y - L$ if it is hit by a liquidity shock and $y$ otherwise.

**Lemma 1.** (First best.) *The first-best surplus is $N(y - \pi L)$.***
3.2 Second Best

We define the second-best allocation as the allocation that maximizes banks’ payoffs given limited pledgeability and individual rationality, but not the non-contractability of the liquidity shocks. In this case, a bank \( B_i \) can insure with \( C \) by paying a premium in every state for full insurance in the event of a shock. Specifically, at Date 2, \( B_i \) pays \( C \) the premium \( \pi L \) (which is less than \( \theta y \) by (3)). And, at Date 1, \( C \) pays \( L \) to \( B_i \) whenever \( B_i \) suffers a liquidity shock (which occurs with probability \( \pi \)).

**Lemma 2.** (Second best.) *The second-best surplus equals the first-best surplus \( N(y - \pi L) \) from Lemma 1.*

3.3 Autarky

We now turn to the extreme case in which banks are in autarky, i.e. there are no transactions whatsoever among \( B_1, ..., B_N \), and \( C \). Due to maturity mismatch, a bank’s assets are destroyed whenever it is hit by a liquidity shock. Thus, a bank gets zero with probability \( \pi \) and \( y \) otherwise.

**Lemma 3.** (Autarky.) *If banks are in autarky, the surplus is \( N(1 - \pi)y \).*

3.4 No Interbank Market

We now turn to the case in which there is no interbank market, i.e. banks cannot contract with one another, but only with \( C \) (subject to the contracting frictions). In this setting, a bank cannot contract on the liquidity shock to insure fully, as in the second best (Subsection 3.2).

**Lemma 4.** (No interbank market.) *With no interbank market, full insurance is impossible. The surplus is strictly less than the first best (Lemma 1) and is given by*

\[
N(1 - \pi)y + \pi \left\lfloor \frac{w}{L} \right\rfloor (y - L).
\]  

(2)

Notice this surplus is above the autarky surplus. This is because \( C \) can insure a bank \( B_i \) by transferring it \( L \) at Date 0 as a “liquidity buffer.” This allows \( B_i \) to endure a liquidity shock at Date 1 and hence improves efficiency. However, since \( C \) has limited wealth, \( w \in (ML, NL) \), it can insure at most \( w/L < N \) banks this way, so inefficient liquidation is unavoidable and thus the surplus is less than the first best.
In this section, we consider a network of mutual gross interbank debts. We show it implements the first-best surplus in equilibrium and is stable ex ante. However, it can harbor systemic risk.

**Figure 2: Complete network \((N = 3)\) and \(B_i\)'s balance sheet**

<table>
<thead>
<tr>
<th>assets</th>
<th>liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>long-term assets (y)</td>
<td>risky short-term liabilities (L)</td>
</tr>
<tr>
<td>debt from (B_1)</td>
<td>debt to (B_1)</td>
</tr>
<tr>
<td>(\vdots)</td>
<td>(\vdots)</td>
</tr>
<tr>
<td>debt from (B_{i-1})</td>
<td>debt to (B_{i-1})</td>
</tr>
<tr>
<td>(\vdots)</td>
<td>(\vdots)</td>
</tr>
<tr>
<td>debt from (B_{i+1})</td>
<td>debt to (B_{i+1})</td>
</tr>
<tr>
<td>(\vdots)</td>
<td>(\vdots)</td>
</tr>
<tr>
<td>debt from (B_N)</td>
<td>debt to (B_N)</td>
</tr>
<tr>
<td></td>
<td>equity</td>
</tr>
</tbody>
</table>

4.1 Mutual Gross Debts Implement First Best

Consider the following network of contracts among the banks \(B_1, ..., B_N\). Every pair of banks \(B_i\) and \(B_j\) enters an offsetting position: \(B_i\) and \(B_j\) exchange promises to repay \(F\), as depicted in Figure 2. Suppose that these debts are paid proportionally (pari passu) but a bank \(B_i\) can take on new senior debt to \(C\) at Date 1. Thus, the interbank debts can be interpreted as unsecured, e.g. loans or bonds, and the debt to \(C\) as secured, e.g. repos.

**Proposition 1.** (Equilibrium with mutual gross debts.) *The complete network of gross positions with \(F = (L - \theta y)/(N - M)\) implements the first best.*

*The following constitutes an equilibrium outcome.*
**Date 1**
Shocked banks sell all the debt they hold and borrow $\theta y$ from $C$ via senior debt.

**Date 2**
Shocked banks default and get $(1-\theta)y$. Not-shocked banks do not default and get $y - MF$.

This says that gross positions allow banks to ensure liquidity shocks, since they embed the option to dilute debt with new debt, which implements a transfer from not-shocked banks to shocked banks.

In equilibrium, not-shocked banks’ debt trades at par (they never default) and shocked banks’ debt trades at zero (they always default on their debts to other banks and all their pledgeable assets go to $C$ as the senior creditor). The face value $F = (L - \theta y)/(N - M)$ is determined such that if a shocked bank sells all the debt it holds, it raises exactly $L - \theta y$. Specifically, since each shocked bank holds the debt of all the $N - 1$ other banks, of which $M - 1$ are shocked themselves and do not repay their debt, it can sell its debt for

$$
\left((N - 1) - (M - 1)\right) F = \left(N - M\right) \frac{L - \theta y}{N - M} = L - \theta y.
$$

(3)

This implies that if a shocked bank sells all its debt, it can mortgage all of its pledgeable assets $\theta y$ to generate liquidity of exactly $L$ to meet its liquidity shock.

### 4.2 Mutual Gross Debts Are Stable

Now turn to Date 0. Given the network of mutual gross debts, would a pair of banks $B_i$ and $B_j$ like to alter their bilateral agreement? For example, would $B_i$ and $B_j$ like to change the face value of debt that they owe each other? In other words, is this network of mutual gross debts pairwise stable? Or are covenants or other contractual provisions necessary?

**Proposition 2. (Stability of mutual gross debts.)** The network in which all banks have debts to and from all other banks with face value $F = (L - \theta y)/(N - M)$ is pairwise stable in the sense that no pair of banks can enter into another pair of bilateral (pari passu or prioritized) debts that makes one strictly better off without making the other strictly worse off.

A pair of banks may try to increase its joint surplus at the expense of the other banks, i.e. alter their bilateral contracts to dilute other banks. With debt contracts, they can do this in two ways: (i) they can prioritize each others’ debt, e.g. securing the assets $\theta y$ to ensure seniority, or (ii) they can lower each others’ face values, so they make lower repayments to each other relative to other banks. However, both (i) and
(ii) are undesirable because they undermine the liquidity insurance role of off-setting debts—they lead a bank to default when it is hit by a liquidity shock. This is because both arrangements reduce the amount that a bank can raise in the market from C: prioritizing debt reduces the amount that a bank can borrow from C, since it reduces the value of the assets that a bank pledges to C as security; decreasing face values decreases the amount that a bank can raise from debt sales, since it reduces the market value of the other bank’s debt.

In the N-bank environment, we establish network stability within the class of debt contracts. However, our analysis speaks to the optimality of debt as well, as we show formally for $N = 2$.

**Corollary 1. (Optimality of debt.)** Suppose $N = 2$ and $M = 1$. Off-setting non-prioritized debts with face value $F = L - \theta y$ are globally optimal bilateral contracts between $B_1$ and $B_2$.

The fact that these off-setting debts implement the first best outcome between $B_1$ and $B_2$ (Proposition [1]) implies that the two-bank network in which $B_1$ and $B_2$ take off-setting debts is stable to deviation to any contract, not just debt contracts. Further, $B_1$ and $B_2$ optimally choose not to prioritize their debts. Each accepts the possibility of being diluted with new debt to C in exchange for the option to dilute the other with new debt to C. This works because each bank exercises its option to dilute only when it is shocked, given that dilution is only valuable when default is possible. This mechanism relies on non-exclusive contracts—those that allow banks to enter into a new relationship with C—to help banks to co-insure, suggesting a positive side to non-exclusive contracting which contrasts with the literature. Further, banks use a mix of junior debt (to each other) and senior debt (to C) to implement efficiency. This casts light on the coexistence of unsecured and secured debt in real-world banks’ capital structure.

### 4.3 Mutual Gross Debts Harbor Systemic Risk

Now consider a “large” liquidity need given the network of gross debts above. What happens if $M' > M$ banks need liquidity? Do mutual gross debts still serve as liquidity insurance?

**Proposition 3. (Systemic crises.)** Suppose that all banks have unsecured debt to and from each other with face value $F = (L - \theta y)/(N - M)$ and the number of shocked banks is $M'$, where

$$M' > \frac{(N - M)\theta y}{L - \theta y}.$$  

(4)
There is a “systemic crisis” in which all banks default: the $M'$ shocked banks fail to meet their liquidity shocks at Date 1 and the $N - M'$ not-shocked banks default at Date 2.

Even though mutual gross debts allocate liquidity efficiently whenever there is enough of it to go around, we now see that gross debts can also exacerbate liquidity shortages when there is not enough of it to go around. Indeed, if the number of shocked banks $M'$ is large enough, liquidity shocks transmit through the network of gross debts leading all banks to default. The shock passes from one bank to the next in a default cascade: since each bank’s liabilities are another bank’s assets, a bank’s failure to repay its debt harms not only its creditors but also its creditors’ creditors. This finding echoes results in the networks literature that financial linkages can mitigate the effects of some shocks while amplifying the effects of others (see, e.g., Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013) and Allen and Gale (2000)).

5 Extensions

We now consider extensions of the baseline setup.

5.1 Multilateral Netting and the Ring Network

So far we have focused on why banks hold mutual gross debts, i.e. on the absence of bilateral netting. However, our mechanism also casts light on why banks lend and borrow from many other banks, i.e. on the absence of multilateral netting (“novation”). For example, consider the ring network in which $B_i$ has debt $F$ to $B_{i+1}$ for $i \in \{1, \ldots, N - 1\}$ and $B_N$ has debt to $B_1$, as depicted in Figure 3. This zero-net position among the banks $B_1, \ldots, B_N$ can also implement liquidity insurance, albeit under more restrictive conditions.

Lemma 5. (Ring network.) Suppose $M = 1$ and $L \leq 2\theta y$. The ring network with $F = L - \theta y$ implements the first best.

The ring network implements the first best only for $M = 1$ because it cannot provide insurance if two adjacent banks are shocked: if $B_i$ and $B_{i+1}$ are both shocked, then $B_{i+1}$ cannot raise much in the market from selling $B_i$’s debt. However, only such adjacent shocks lead to liquidation, and thus the ring network still provides all banks with liquidity insurance, even if not enough to implement the first best. More generally, gross exposures within networks provide liquidity insurance via the option to dilute; this mechanism is not specific to the complete network.
5.2 Unobservable Liquidity Shocks

So far, we have assumed that liquidity shocks were observable but non-contractable, following the incomplete contracts paradigm. However, if liquidity shocks are unobservable, gross positions can still implement liquidity insurance. The core mechanism is unchanged: shocked banks liquidate debt in the market and mortgage their assets to meet their liquidity shocks. However, there is one twist on the equilibrium above: not only do shocked banks sell to avoid liquidation, but not-shocked banks also sell to pool with the shocked banks. This is because they sell at a subsidy, since their portfolios are worse on average than not-shocked banks’ portfolios—not-shocked banks hold the debts of $M$ shocked banks and $N - 1 - M$ not-shocked banks, whereas shocked banks hold the debts of $M - 1$ shocked banks and $N - M$ not-shocked banks.

We make one additional assumption to construct the equilibrium: there are two markets at Date 1, first, a resale market for debt in which banks sell debts to C and, second, a market for new loans, in which banks can borrow from C or from other banks. This assumption allows all banks to sell debts at a pooling price in the market as well as for banks to reallocate liquidity among themselves.

**Lemma 6.** Suppose the model is as specified in Section 2 except liquidity shocks are unobservable and there are Date-1 markets as described above.

Let

$$F = \frac{L - \theta y}{(1 - \pi)(N - 1)}. \tag{5}$$

As a long $(1 - \pi)w > N (L - \theta y)$, the complete network of gross positions with face value $F$ implements the first best.
The following constitutes an equilibrium outcome.

**Date 1**  
Both shocked banks and not-shocked banks sell each of the $N-1$ debts they hold at price $(1 - \pi)F$.

**Shocked banks** borrow $\theta y$ in the market via new senior debt.

**Date 2**  
Shocked banks default and get $(1 - \theta)y$. Not-shocked banks do not default and get $y - MF$.

The out-of-equilibrium beliefs are that any bank that deviates is believed to be not-shocked.

6 Conclusion

Why do banks choose to hold off-setting gross positions rather than net them out? Because they embed the option to dilute. Since banks exercise this option when they need liquidity most, unnetted gross positions implement mutually beneficial liquidity insurance. Indeed, as long as there is enough liquidity to go around, these mutual gross positions allocate liquidity efficiently. However, if there is not enough liquidity to go around, they can exacerbate liquidity shortages—with gross positions, a liquidity shortage can even trigger a systemic crisis in which all banks default.
A Proofs

A.1 Proof of Lemma 1

The result follows immediately from the argument in the text given that the probability each of the $N$ banks is shocked is $\pi$. \hfill $\square$

A.2 Proof of Lemma 2

The result follows immediately from the argument in the text. \hfill $\square$

A.3 Proof of Lemma 3

The result follows immediately from the argument in the text. \hfill $\square$

A.4 Proof of Lemma 4

We first show that it is individually rational for $C$ to insure any given bank $B_i$ with a non-contingent contract: $C$ transfers a “liquidity buffer” $L$ to $B_i$ at Date 0 in exchange for the promised repayment $R$ at Date 2. This bilaterally optimal and inefficient liquidation of $B_i$’s assets. However, $C$ has limited wealth $w$ and cannot a make non-contingent transfer of $L$ to more than $w/L$ banks. This leads to inefficient liquidation with some probability.

Bilateral insurance. Suppose $C$ transfers $L$ to $B_i$ at Date 0 and $B_i$ promises to repay $R$ at Date 2. If $B_i$ shocked, it uses $L$ to meet its liquidity need. It has assets $y$ at Date 2 repays $\min\{R, \theta y\}$ to $C$ at Date 2. If $B_i$ is not hit by a liquidity shock, it has assets $y + L$ at Date 2 and repays $\min\{\theta y + L, R\}$ to $C$. Thus, $C$’s break-even condition is

$$L = \pi \min\{R, \theta y\} + (1 - \pi) \min\{\theta y + L, R\}, \quad (6)$$

which has the solution

$$R = \frac{L - \pi \theta y}{1 - \pi}. \quad (7)$$

This is between $\theta y$ and $L + \theta y$ by $\{x\}$, so if $B_i$ shocked $C$ gets less than $L$ from seizing $B_i$’s assets and if $B_i$ is not shocked $C$ gets more than $L$ from repayment. $C$ breaks even and $B_i$ gets $y - \pi L$, as in the first best.

Wealth constraint. $C$’s has wealth $w \in [ML, NL)$. Thus to minimize the number of liquidations and maximize the surplus, $C$ transfers $L$ to $[w/L]$ banks. This gives the expected surplus

$$\text{expected surplus} = N \left((1 - \pi)y + \pi \frac{1}{N} \left\lfloor \frac{w}{L} \right\rfloor (y - L)\right). \quad (8)$$
This is less than the first-best surplus since \( w < NL \).

A.5 Proof of Proposition \[1]\

We prove the result using the one-shot deviation principle as follows. Given the supposed equilibrium, we find the market prices of debt for shocked and not-shocked banks and show (i) that there are no profitable deviations at Date 2 and (ii) that there are no profitable deviations at Date 1.

**Debt prices.** In the proposed equilibrium, the shocked banks repay nothing on their interbank debts, since they must secure all their pledgeable assets \( \theta y \) to \( C \) in order to raise \( L \) to meet their liquidity shocks (see equation (3)). Thus, the Date-1 market price of shocked banks’ debt is zero. In contrast, the not-shocked banks repay the face value of their interbank debts. Thus, the Date-1 market price of not-shocked banks debt is \( F = (L - \theta y)/(N - M) \).

**Date 2.** It follows from the expression for the payoff in equation (1) that a bank defaults if its net repayments exceed the value of its pledgeable assets \( \theta y \).

A shocked bank has total debt \((N - 1)F + \theta y\), comprising the interbank debt it took on with the \((N - 1)\) other banks at Date 0 and the debt to \( C \) it took on at Date 1. It defaults since its total debt exceeds the value of its pledgeable assets:

\[
(N - 1)F + \theta y \geq \theta y. \tag{9}
\]

A not-shocked bank has total debt \((N - 1)F\), comprising the interbank debt it took on with the \((N - 1)\) other banks at Date 0. It has pledgeable assets \((N - 1 - M)F + \theta y\), comprising the debt owed to it from the \((N - 1 - M)F\) other not-shocked banks and the pledgeable value of its long-term assets \( y \). It does not default since its total debt is less than the value of its pledgeable assets:

\[
(N - 1)F \leq (N - 1 - M)F + \theta y \tag{10}
\]

or

\[
\theta y \geq MF = M \frac{L - \theta y}{N - M}. \tag{11}
\]

This can be re-written as \( N \theta y \geq ML \), which is satisfied by (11). The bank gets payoff

\[
y + (N - 1 - M)F - (N - 1)F = y - MF. \tag{12}
\]

**Date 1.** Given the prices above, each shocked bank has interbank debt worth

\[
((N - 1) - (M - 1))F = L - \theta y, \tag{13}
\]
since it holds the debt of all of the $N - 1$ other banks, of which $M - 1$ are shocked. Thus, the only way each shocked bank can raise $L$ is to mortgage its assets entirely. It always prefers to do this, since if it does not raise $L$, it is liquidated and gets zero. Thus it has no incentive to deviate to any other strategy.

A not-shocked bank can sell assets or borrow from the market to raise cash from $C$. There are two cases: (i) the bank stays solvent or (ii) the bank defaults.\(^{11}\)

*Case (i): Not-shocked bank is solvent.* Intuitively: two risk-neutral players—the not-shocked bank and $C$—cannot enter into a profitable trade, so there is no profitable deviation for the bank. More formally: if a not-shocked bank raises $c$ by selling assets (i.e. selling debts from other banks) and stays solvent, the repayment it receives from other banks is decreased by $c$, since it raised $c$ at the fair price. This is not a profitable deviation. If it raises $c$ by borrowing and stays solvent, it must repay $c$ to $C$, since it borrowed at the fair price. This is not a profitable deviation.

*Case (ii): Not-shocked bank defaults.* If a not-shocked becomes insolvent, it diverts and gets $(1 - \theta)y$. This is not a profitable deviation because it is less than the payoff from staying solvent, as established above (equations (11) and (12)).

A.6 Proof of Proposition \(^{2}\)

The main idea behind the proof reflects the fact that if two banks, $B_i$ and $B_j$, deviate to dilute other banks, it will make it impossible for either $B_i$ or $B_j$ to endure a liquidity shock, as discussed in the text. In the formal proof, we must show that the benefits of dilution are always outweighed by the cost of liquidation for any deviation $(F_i, F_j)$, where $F_i$ is the face value of debt $B_i$ owes $B_j$ and $F_j$ is the face value of debt $B_j$ owes $B_i$.

We divide the proof into the following four steps.

1. We argue that we can restrict attention to equilibria in which a shocked bank mortgages its assets to borrow from $C$ at Date 1, as in the equilibrium in Proposition \(^{1}\).

2. We show that any “very asymmetric” deviation—i.e. one in which one bank lowers its face value and the other raises it or in which the values are very different—cannot be profitable.

3. We show that any other deviation—i.e. a “more symmetric” deviation—cannot be profitable.

4. We show that no deviation to prioritized debts can be profitable.

\(^{11}\)Note that there is no case in which a bank defaults with some probability $p \in (0, 1)$, since all uncertainty is resolved at Date 1 and we have not allowed $C$ to sell lotteries (but only to buy debt and lend).
Throughout the proof, $F$ refers to the equilibrium face value $F = (L - \theta y)/(N - M)$.

**Step 1: Always mortgage at Date 1.** We restrict attention to equilibria in which a shocked bank always mortgages its assets with $C$ at Date 1. This is because this can never reduce a shocked banks’ payoff—at most it holds the cash and repays $C$ at Date 2—so it is always incentive compatible. To show the network is stable we need only to show that it is stable for these (consistent) equilibrium beliefs.

**Step 2: No “very asymmetric” deviations.** We divide this step into two parts, showing first that it cannot be that one bank increases the face value and one bank decreases the face value, and then that the difference between face values cannot be too large, regardless of the whether they are bigger or smaller than $F$.

**Step 2, Part I: No deviation with $F_i \leq F \leq F_j$.** If either $F_i < F \leq F_j$ or $F_i \leq F < F_j$, then $B_j$ is worse off: it repays at least as much in total and gets at most as much from from $B_i$. It cannot be a profitable deviation.

**Step 2, Part II: No deviation with $F_i << F_j$.** Specifically, we show that

$$|F_j - F_i| < \theta + MF.$$  

(14)

We show that there is no profitable deviation in which $F_i > F_j + \theta y + MF$, since then $B_i$ would always default and hence be worse off. To see this, suppose $F_i > F_j + \theta y + MF$. In the best case scenario, in which $B_i$ is not shocked and all the debt it holds as assets is repaid at par, $B_i$ still defaults since its

$$\text{net repayment} = \left((N - 2 - M)F + F_j \right) - \left((N - 2)F - F_i \right)$$  

(15)

$$= F_j - F_i - MF$$  

(16)

$$< \theta y,$$  

(17)

by hypothesis. Thus, we conclude that any profitable deviation must satisfy the condition in equation (14).

**Step 3: No other (“more symmetric”) deviations.** Now we rule out deviations in which both banks increase their face values or both decrease it.

From Step 2 above, we can rule out deviations that would lead a bank to default even if it is not shocked, since in that case it would always default. Further, if both banks are shocked they both necessarily default, and the deviation does not affect their payoffs. As a result, we can now focus on the states in which one bank is shocked and the other is not. In particular, a necessary condition for a deviation to be profitable is that it increases the joint surplus when one bank is shocked and the other is not.

We now show that no deviations in which both banks increase their face values or both banks decrease their face values can increase the joint surplus in states in which $B_i$ is shocked and $B_j$ is not.
Step 3, Part I: No deviation in which both banks increase face values.

Since $B_i$ is shocked, it gets

$$\max \{ y + F_j - F_i - MF - L , (1 - \theta)y \} = (1 - \theta)y$$

(18)
since $F_j - F_i < \theta y + MF + L$ by Step 2 above. I.e., the shocked bank always defaults.

After meeting its liquidity shock, $B_i$ may have remaining pledgeable assets. (In the equilibrium in Proposition \textcolor{red}{[1]} it mortgaged all its assets, so other creditors got zero; of course this is not necessarily the case given a deviation.) The value of $B_i$’s pledgeable asset value is at most

$$\theta y + \left( N - 2 - (M - 1) \right) F + F_j - L = (N - M) F + F_j - F + \theta y - L$$

(19)

$$= F_j - F,$$

(20)
since $F = (L - \theta y)/(N - M)$. Thus, if $B_j$ were to get the entire “recovery value” of $B_i$’s assets its payoff would be increased by at most $F_j - F$. However, its payoff is decreased by its increased liability, since it must repay an additional $F_j - F$. Thus, the joint surplus is changed by at most $(F_j - F) + (F_j - F) = 0$. There cannot be a profitable deviation.

Step 3, Part II: No deviation in which both banks decrease face values.

Since $B_i$ is shocked, it sells other banks’ debts and gets

$$\left( (N - 2) - (M - 1) \right) F + F_j < (N - M) F = L - \theta y.$$ 

(21)

Thus, $B_i$ cannot raise $L$ even if it mortgates all its assets for $\theta y$. $B_i$ must liquidate and gets zero. This reduces its payoff by $(1 - \theta)y$, since it gets $(1 - \theta)y$ in equilibrium if it meets its liquidity at Date 1 and defaults at Date 2.

Whereas $B_i$ is worse off, $B_j$ may be better off, since it has less debt to repay at Date 2 given $F_j < F$. Indeed, $B_j$’s payoff is increased by $F - F_j$.

In order for the joint surplus to be increased, it must be that $F - F_j > (1 - \theta)y$, i.e. the benefit of to $B_j$ outweighs the cost to $B_i$. Since $F_j \geq 0$, this implies that $F \geq (1 - \theta)y$ or

$$\frac{L - \theta y}{N - M} \geq (1 - \theta)y,$$

(22)

which can be re-written as

$$L \geq y + (N - M - 1)(1 - \theta)y \geq y,$$

(23)

which is violated since $L < y$ by assumption.

Step 4: No deviation to prioritized debt. Finally, suppose either bank priori-
tizes the other’s debt. In this case, the bank cannot mortgage its assets to raise liquidity from C in the event of a shock, since the equilibrium face value $F$ is determined such that C must have the senior claim on all the pledgeable assets $\theta y$ at Date 1. Thus, it must liquidate. This destroys its assets and both banks get zero. Thus, if banks prioritize their assets, they are both worse off when either is shocked. When neither is shocked they do not improve the payoffs, since they are not diluting other banks: there is no dilution if there is not default; their debts just net out. □

A.7 Proof of Corollary 1
The result follows immediately from Proposition 1. □

A.8 Proof of Proposition 3
We first show that shocked banks fail at Date 1 and then that not-shocked banks default at Date 2.

Shock banks fail at Date 1. A shocked bank can raise $((N-1)-(M'-1))F = (N-M')F$ from selling its debt. Thus, the maximum it can raise is

$$\theta y + (N-M')F = \theta y + (N-M')\frac{L-\theta y}{N-M} < L.$$  (24)

This holds since $M' > M$, following from the hypothesis that $M' > (N-M)\theta y/(L-\theta y)$ and (22).

Not-shocked banks default at Date 2. A not-shocked banks’ payoff payoff from repayment is greater than its payoff $(1-\theta)y$ from default:

$$y + (N-M'-1)F - (N-1)F = y - M'\frac{L-\theta y}{N-M} < (1-\theta)y,$$  (25)

since $M' > (N-M)\theta y/(L-\theta y)$ by hypothesis. □

A.9 Proof of Lemma 5
The equilibrium strategies are exactly as in Proposition 1 and almost the same proof applies. We must only check the corresponding conditions for the ring network, that (i) the single shocked bank can raise enough liquidity at Date 1 and (ii) the not-shocked banks do not default at Date 2.

Shock bank. In the ring network, the shocked bank, say $B_i$, has the debt of one bank, $B_{i-1}$, to sell to raise liquidity, where we suppose $1 < i < N-1$ w.l.o.g. to keep notation simple below. Thus, it raises $F = L-\theta y$ and it can meet its liquidity shock
by mortgaging its assets to raise $\theta y$ form C. It defaults at Date 2 and the value of its unsecured debt is zero at Date 1.

**Not-shocked banks.** No bank defaults at Date 2. To see this, consider $B_{i+1}$, that holds the debt of the shocked bank $B_i$ as an asset. $B_{i+1}$ does not default as long as its pledgeable assets $\theta y$ exceed its debt $F$ to $B_{i+2}$ or

$$\theta y \geq F = L - \theta y$$

which can be re-written as $2\theta y \geq L$, which holds by hypothesis.

$B_{i+1}$ has the same liabilities as all other banks but lower assets. Since $B_{i+1}$ does not default, no other bank defaults either.

\[\square\]

### A.10 Proof of Lemma \[6\]

The proof is analogous to the proof of Proposition \[1\]. We apply the one-shot deviation principle as follows. Given the supposed equilibrium, we find the market price of debt for shocked and not-shocked banks and show (i) that there are no profitable deviations at Date 2 and (ii) that there are no profitable deviations at Date 1.

**Debt price.** In the proposed equilibrium, the shocked banks repay nothing on their interbank debts (since they have secured all their pledgeable assets $\theta y$). Thus, the Date-2 value of shocked banks’ debt is zero. In contrast, the not-shocked banks repay the face value of their interbank debts. Given we have supposed a pooling equilibrium, all debt trades at the average price at Date 1

$$\text{Date-1 debt price } = \frac{M}{N} \times 0 + \frac{N - M}{N} \times F = (1 - \pi)F.$$ \[28\]

**Date 2.** It follows from the expression for the payoff in equation \[10\] that a bank defaults if its net repayments exceed the value of its pledgeable assets $\theta y$.

A shocked bank has total assets $(N - 1)F + \theta y$, comprising the interbank debt it took on with the $N - 1$ other banks at Date 0 and the debt it took on at Date 1. It defaults since its total debt exceeds the value of its pledgeable assets:

$$(N - 1)F + \theta y \geq \theta y.$$ \[29\]

Recall that, unlike in the equilibrium of the baseline model, in this equilibrium a not-shocked bank raises cash $(N - 1)(1 - \pi)F$ at Date 1 as well, which it either holds in cash or lends out senior at the risk-free rate (zero) in the debt market. Thus, at Date 2 it has pledgeable assets $(N - 1)(1 - \pi)F + \theta y$. Still, as in the equilibrium of the baseline
model, it does not default since its debt is less than the value of its pledgeable assets:

\[(N - 1)F \leq (N - 1)(1 - \pi)F + \theta y\]  \hspace{1cm} (30)

or, substituting in for \(F\),

\[(N - 1)\pi \frac{L - \theta y}{(1 - \pi)(N - 1)} \leq \theta y.\]  \hspace{1cm} (31)

This can be re-written as \(\pi L \leq \theta y\), which is satisfied by (x).

**Date 1.** Given the price above, each bank has interbank debt worth

\[(N - 1)(1 - \pi)F = L - \theta y.\]  \hspace{1cm} (32)

Thus, the only way that a shocked bank can raise \(L\) is to mortgage its assets entirely. It always prefers to do this, since if it does not raise \(L\), it is liquidated and gets zero.

A not-shocked bank can sell its assets or not. If it pools with the shocked bank, it sells its assets for \(L - \theta y\). Otherwise, it is believed to be good, given the out-of-equilibrium beliefs. In this case, it can sell it assets at the fair price \((N - 1 - M)F\), since it holds the debt of \(N - 1\) other banks, \(M\) of which are shocked and have debt worth zero. But this is less than the value it can get for its assets if it pools, since it pools with shocked banks that hold the debt of \(N - 1\) other other banks, only \(M - 1\) of which are shocked, i.e.

\[(N - 1 - M)F = (N - 1) \left(1 - \frac{M}{N - 1}\right)F\]  \hspace{1cm} (33)

\[< (N - 1) \left(1 - \frac{M}{N}\right)F\]  \hspace{1cm} (34)

\[= (N - 1)(1 - \pi)F \equiv (N - 1) \times \left(\text{Date-1 debt price}\right).\]  \hspace{1cm} (35)

Thus, a not-shocked bank strictly prefers sell at the pooling price than at the separating price. Further, all other separating actions make it worse off. Thus, it prefers to pool by selling all debt.

We conclude the proof with by checking that \(C\) has enough liquidity to buy all banks’ debt. The total amount of liquidity \(C\) needs to buy all banks’ debt is

\[N(N - 1)F = N(N - 1)\frac{L - \theta y}{(1 - \pi)(N - 1)}\]  \hspace{1cm} (36)

\[= \frac{N(L - \theta y)}{1 - \pi}\]  \hspace{1cm} (37)

\[< w,\]  \hspace{1cm} (38)
by hypothesis.

Shocked banks meet their liquidity needs by borrowing via senior debt in the anonymous market from both C and not-shocked banks with excess cash, who are indifferent between lending and holding cash at rate zero.
## B Table of Notations

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