Equilibrium Wealth Share Dynamics

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Abstract

We argue that the first-order movements in sectoral wealth shares are potentially driven by changes in hedging demand and not mean-variance trade-offs, in stark contrast to widely-used models. In the quarterly sample over 1952–2015 we find that on average 60 and 73 percent of financial and real estate wealth, respectively, arises primarily from households’ motive to hedge shocks to demand. We analyze these phenomena with a two-sector model that features endogenous movements in wealth shares, returns, variances, and asset demand. We document two modeling assumptions required to match key features of the data: imperfect goods and demand shocks.

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1 Introduction

Sectoral wealth changes over time and their movements alter the distribution of risks and asset returns: for example, housing contributed 33 percent to households’ total assets in 2005 and six years later had fallen by over three standard deviations to 23 percent. We argue that the first-order movements in wealth shares are potentially driven by changes in hedging demand and not mean-variance tradeoffs, a feature commonly overlooked in the literature. Modeling this feature is crucial to our understanding of investors’ asset allocations and in helping us integrate intertemporal portfolio theory into empirical research and investment practice.

To fix ideas, consider an increase in the capital stock of an asset. Tobin’s $q$ would fall and investors would ideally want to diversify their new wealth across other securities. But not everyone can collectively rebalance. As a result prices and some moments of the asset’s return distribution must adjust. In particular, its return must rise or else its variance fall to satisfy investors’ holding more of the successful assets. These standard predictions of a supply shock, however, do not bear in the data: two key empirical facts we document are that wealth shares negatively relate to risk premia and increase with $q$.

To understand these phenomena, we analyze a model featuring two dividend streams, the simplest and clearest representation of an economy featuring endogenous movements in wealth shares, returns, variances, and hedging demand. The two streams can represent flows from any characteristic-based grouping or broad asset classes like the value of tangible versus intangible capital or the market value of corporations versus residential real estate.

Our model is a two trees production economy to which we add two elements to previous research.1 We first allow the two dividend streams to be imperfect substitutes. A relative price between the two streams then fluctuates endogenously, changing the distribution of risk and return across assets. We then include shocks to demand that are independent of the usual endowment or supply shocks, capturing the idea that agents preferences for these two types of goods fluctuates over time.

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We find that including these two elements are necessary to capture the two empirical facts. First, by calibrating solely to observed fluctuations in supply and using a moderate degree of relative risk aversion, the model cannot easily generate a negative relationship between a sector’s risk premia and wealth share. The basic reason is that as an asset’s wealth shrinks its contribution to aggregate consumption falls, bringing shocks to its cash flows closer to being idiosyncratic, and lowering its required return to the risk-free rate. Second, demand shocks are required to give a positive relationship between a sector’s Tobin’s $q$ and wealth as in the data, simply because an increase in a sector’s supply, all else equal, will decrease $q$. The inclusion of demand shocks, moreover, helps the model capture the commonly studied joint behavior of consumption, investment, and dividends.

Finally, as an application of the model we evaluate its implications by matching one sector to financial wealth, essentially the total market value of debt and equity of productive capital, and the other to real estate. These two assets form the majority of total assets held by households. We then use our model to invert a process for demand shocks between financial and real estate capital by using the model’s cross-equation restrictions on the distributions of capital and wealth. Given the implicit process for demand shocks, we find that hedging demands constitute the majority of a sector’s wealth share dynamics. In our sample we find that on average 60 and 73 percent of financial and real estate wealth, respectively, arises from the demand to hedge.

Relating our results to the large body of work in empirical asset pricing, it seems a lot of work has been done on forming factor-mimicking portfolios that are really projections of state variables onto the space of current returns and then seeing if this factor is priced. But the work does not need to stop here. Investors’ intertemporal hedging demands bid up the prices of assets that do well when future returns are low, raising their prices and lowering their expected returns for a given market beta. Instead, researchers should be focusing on modeling the joint behavior of investment, risk premia, and sectoral wealth, in addition to other factors that would generate hedging demand, such as human or intangible capital or the idiosyncratic risk of running a small business. All of these factors that potentially create hedging demand should in theory forecast future returns or contemporaneously correlate.
with some assets’ wealth shares.

Our work overlaps with several strands of research. First, it builds on the theoretical work that attempts to link wealth shares to expected returns. A partial list is Menzly et al. (2004), Cochrane et al. (2008), Eberly and Wang (2011), Martin (2013). From a modeling perspective, our paper is closest to Eberly and Wang’s (2011). The novelty of our approach is that our setup allows goods to be imperfect and the preference for individual goods to vary over time. We also decompose our wealth shares into mean-variance and hedging demand and examine the implications for portfolio choice.

Our paper relates to both the classic and the evolving literature on intertemporal portfolio allocation: Merton (1971), Campbell (1993), Campbell and Viceira (1999), Campbell and Viceira (2001), Wachter (2002), Wachter (2003), Cochrane (2014). An important distinction of our work is that many of these models are solved in partial equilibrium, often specifying the asset’s return exogenously, while our setup assess the implications in general equilibrium. The latter can be particularly important for longer investment horizons as sectoral adjustments must take place.

The issue of the optimal allocation of wealth across different sectors in the economy is also important from the perspective of production-based multi-sector models. An important margin in these models connects holdings of sectoral wealth and the return distribution. For example, Long and Plosser (1983) and Boldrin, Christiano and Fisher (2001) consider multi-sector models but do not explore the connection between sectoral wealth and returns. Investors in these economies hold sectoral wealth largely along the mean and variance dimension, and thus the empirical puzzle we highlight in this paper provides an important challenge for this literature as well.

In section two, we first examine the puzzle in the data. In the following section we present the setup of our model. In section four, we calibrate and analyze the model, drawing attention to the effects of demand shocks. In the fifth section, we replicate the puzzle in the data and try to infer the level of the demand shock in the data. We then conclude.
2 Data

Quarterly data are from the Federal Reserve’s Flow of Funds and the Bureau of Economic Analysis and cover the period 1952Q4–2015Q4. All nominal variables are deflated using the consumer price index.

Households and nonprofit organizations have about 70 percent of total assets in financial capital and 30 percent in nonfinancial capital, which includes real estate. Because we are interested in modeling households’ portfolio decisions we focus on a subset of total assets that would be more likely held as a result of an active portfolio decision. In particular, we exclude pension entitlements (about 20 percent of total assets), equity in noncorporate business (10 percent), consumer durable goods (5 percent), and other small asset classes, leaving our coverage at about 63 percent of households’ total asset universe as of 2015Q4.

Financial Capital

Financial capital is the sum of debt and equity capital. Debt includes currency and deposits including money market fund shares, debt securities, loans, and shares of bond mutual funds. Equity comprises directly- and indirectly-held securities, such as those held through life insurance companies, pension plans, retirement funds. In our data sample, debt capital contributed about 54 percent to total financial capital on average. It had an contributed up to 70 percent during the decade starting in 1980, when interest rates had peaked and subsequently fell.

We view this sector’s capital stock as the fixed, private, nonresidential capital stock (BEA Table 1.1, Line 4). Thus the “financial” sector produces goods and services that are unrelated to real estate. Our measure of Tobin’s $q$ is therefore household’s financial wealth over the total nonresidential capital stock.

Cash flows from financial capital are the sum of interest earned (BEA Table 2.1, Line 14) and dividends received (BEA Table 2.1, Line 15) by households. This cash flow then subtracts the total change in equity for nonfinancial and financial corporations multiplied by household’s share of total equity to get a dollar amount for total issuance or repurchases absorbed or earned by households.
Real Estate

Household’s real estate is for all owner-occupied real estate, including vacant land and mobile homes at market value (Flow of Funds LM155035015.Q). The stock of real estate is the private, residential fixed assets owned by households (BEA Table 5.1, Line 7).

The flow of rental income (BEA Table 2.1, Line 12) consists of rental income of tenants and the imputed income of owners’ housing services, which is net of “space rent” less expenses such as depreciation, maintenance, property taxes, and mortgage interest.

Summary Statistics

Summary statistics are listed in Table 1. Financial wealth has a larger cash flow yield than does real estate. It also has a higher average excess return. Because the data sample captures the secular decline in interest rates coupled with an initial 70 percent share of debt to financial capital, the average returns over the entire sample period are greater than the stock market’s average return of about 7 percent. Additionally, the low volatility of the debt payout lowers the total volatility of the financial excess return, leading to a historical average Sharpe ratio above one. Surprisingly, real estate’s cash flow growth volatility is over 50 percent higher than financial wealth’s, even when accounting for equity repurchases. The years 1986 until 1992 saw real estate’s real cash flow grow rates within a range of -33 to 40 percent.

Investment in real estate is about half the rate of investment in nonresidential capital, but also depreciates at a lower rate. Average net investment, the average investment rate minus the average depreciation rate, are respectively about 3.5 and 3 percent for financial capital and real estate.

Key Facts

To understand households’ portfolio choice decisions we investigate the relationship between risk premia and wealth shares. We plot both the level and the first
Table 1: Summary Statistics (Annualized)

<table>
<thead>
<tr>
<th></th>
<th>Financial</th>
<th>Real Estate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Means</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash flow yield (%)</td>
<td>8.5</td>
<td>2.0</td>
</tr>
<tr>
<td>Tobin’s q</td>
<td>1.32</td>
<td>1.17</td>
</tr>
<tr>
<td>Investment rate (%)</td>
<td>11</td>
<td>5</td>
</tr>
<tr>
<td>Depreciation rate (%)</td>
<td>7.5</td>
<td>2</td>
</tr>
<tr>
<td>Excess returns (%)</td>
<td>11.3</td>
<td>4.4</td>
</tr>
<tr>
<td><strong>Volatilities</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash flow growth (%)</td>
<td>8</td>
<td>13.5</td>
</tr>
<tr>
<td>Capital stock growth (%)</td>
<td>2.3</td>
<td>2.6</td>
</tr>
<tr>
<td>Excess returns (%)</td>
<td>8.9</td>
<td>5.7</td>
</tr>
</tbody>
</table>

Data are quarterly from 1952Q4 until 2015Q4. Sources: BEA, Federal Reserve Flow of Funds, and BLS for the CPI price deflator. Values are real where applicable.

The difference of sector’s risk premia and Tobin’s $q$ on their wealth shares in Figure 1. Annualized risk premia for each sector are estimated as the fitted values of the regression of future cumulative four-quarter log excess returns on the current cash flow yield

$$\sum_{h=1}^{4} r_{t+h} - r_{t+h}^f = a + b \times \frac{D_t}{P_t} + \sum_{h=1}^{4} \epsilon_{t+h}.$$

Both sectors display the same patterns: Tobin’s $q$ is increasing in wealth share; risk premia, declining. This result holds whether we look at levels or at first differences. These facts are challenges when modeling the nexus of financial markets and the real economy.

In standard models, risk premia generally increase along with wealth shares. As the output share of a sector grows, its output contributes more to aggregate consumption, and therefore the systematic risk of the sector increases, along with its risk premium. In a standard multi-sector production economy, a sector’s Tobin’s $q$ would fall after the sector invests because the marginal product of capital, all else

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2In other work, Bansal, Fang and Yaron (2008) document a similar negative relationship between wealth shares and both expected returns and Sharpe ratios for the 12 Fama-French industry equity portfolios.
Figure 1: Risk premia, Tobin’s $q$, and wealth shares

Data are quarterly from 1952Q4 until 2015Q4. Tobin’s $Q$ divides the household’s invested wealth in the sector by the sector’s current-cost capital stock. Wealth shares are market capitalization shares. Risk premia are estimated by the fitted values of a predictive regression of annual excess returns on today’s cash flow yield. The top two panels report the data in levels. The bottom two panels report the data in first differences. Regression lines are color-coded and overlay the data.

In what follows we set up a two-sector production economy that reconciles the evidence in the data. The key features are that it allows goods across sectors to be imperfect substitutes that allows for variation in relative prices. In addition, we add shocks to demand that we show are necessary for explaining the joint dynamics of Tobin’s $q$, risk premia, and wealth shares that we see in the data.
3 Model

Preferences

The representative investor in the economy has continuation utility

\[ J_t = \mathbb{E}_t \left[ \int_t^\infty f(C_s, J_s)ds \right], \quad (1) \]

where \( C \) denotes aggregate consumption is a CES composite over two sectors’ dividend streams \( D_n, n = 1, 2, 3 \)

\[ C_t = \left( \Omega t^\frac{1}{p} D_{1t}^{\frac{1}{p}} + (1 - \Omega t)^\frac{1}{p} D_{2t}^{\frac{1}{p}} \right)^\frac{1}{\frac{1}{p} - 1}. \quad (2) \]

The aggregator \( f(C, J) \) takes the usual Duffie and Epstein (1992) form

\[ f(C, J) = \frac{\rho}{1 - 1/\psi} \frac{C^{1-1/\psi} - ((1 - \gamma)J)^\theta}{((1 - \gamma)J)^{\psi-1}}, \quad (3) \]

where we interpret \( \rho > 0 \) as the rate of time preference, \( \psi > 0 \) as measuring the intertemporal elasticity of substitution, \( \gamma \geq 1 \) as the coefficient of relative risk aversion, and we define \( \theta = \frac{1-1/\psi}{1-\gamma} \).

The optimal consumption choice for each sector at time \( t \) is given by

\[ D_{1t} = \left( \frac{p_{1t}}{p_t} \right)^{-\epsilon} C_t \Omega_t \quad \text{and} \quad D_{2t} = \left( \frac{p_{2t}}{p_t} \right)^{-\epsilon} C_t (1 - \Omega_t). \quad (4) \]

The intratemporal elasticity of substitution is \( \epsilon \). If \( \epsilon \to \infty \) goods become perfect

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3 For tractability, all investor wealth is assumed to be tradable. Human wealth, the expected discounted value of future labor earnings, could of course affect portfolio choice. In an important paper, Bodie, Merton and Samuelson (1992) show that if labor income is riskless, then human wealth is equivalent to an investment in the riskless asset and the investor should tilt their portfolio towards risky assets holdings relative to an investor who owns only tradable assets. Campbell and Viceira (2002) show that if labor income is idiosyncratically risky, then all investors should tilt their portfolios towards risky assets, no matter how risky. If labor income is positively correlated with the risky asset, however, then investors should tilt their portfolios away from the risky asset. All of these results apply only to one risky asset, and not a choice of how to allocate money across risky assets that is studied here. How assumptions of labor income affect risky asset allocations is left for future work.
substitutes and goods’ prices always equal one. If \( \epsilon \to 1 \), aggregate consumption becomes Cobb-Douglas in both goods. We look at cases where \( \epsilon \in (1, \infty) \).

The good’s price is \( p_n \) and the ideal price index is \( p = \left( p_1^{1-\epsilon} \Omega + p_2^{1-\epsilon} (1 - \Omega) \right)^{\frac{1}{1-\epsilon}} \); with this definition we have \( \sum_n p_n D_n = pC \). We normalize \( p \equiv 1 \) granting each sector’s good a time-varying relative price to the numeraire of aggregate consumption.

We specify \( \Omega_t \) as a stochastic process and interpret it as a (relative) demand shock for sector one’s good. Because we focus on a two-sector economy, we let \( \Omega_t \) take \( M \) possible values in \( 0 < \Omega_1 < \Omega_2 < \ldots < \Omega_M < 1 \). Think of the collection of points as forming a fine grid on the unit interval. The generator matrix is \( \Pi = [\pi_{\omega \omega'}] \) for \( \omega, \omega' \in \{\Omega_m\}_{m=1}^M \), whose elements can simply be thought of as the probability of \( \Omega_t \) moving from state \( \omega \) to \( \omega' \) within time \( \Delta \) is approximately \( \pi_{\omega \omega'} \Delta \).

You can also equivalently represent the demand shock’s process as a sum of Poisson processes:

\[
d\Omega_t = \sum_{\omega' \neq \Omega_t} q_{\omega'}(\Omega_t-)dN_t^{(\Omega_t-\omega')}
\]

where \( q_{\omega'}(\omega) = \omega' - \omega \) and the \( N^{(\omega,\omega')} \) are independent Poisson processes with intensity parameters \( \pi_{\omega \omega'} \). Each jump in \( \Omega_t \) corresponds to a change of state for the Markov chain.

There are two advantages for the process for demand shocks in this way. First, it makes the solution tractable, replacing a high-dimensional partial differential equation with a system of ordinary differential equations. Second, arbitrary autocorrela-

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4The expenditure share

\[
e_{nt} = \frac{p_{nt}D_{nt}}{C_t}
\]

moves with the relative quantity of good-\( i \) consumption (\( \partial e_n / \partial D_n > 0 \)) and against its relative price (\( \partial e_n / \partial p_n < 0 \)) if \( \epsilon > 1 \). Related, the price elasticity of demand varies with the sector’s expenditure share

\[
\frac{\partial \ln D_n}{\partial \ln p_n} = \epsilon + (1 - \epsilon)e_n,
\]

which goes to \( \epsilon \) when \( e_n \) gets small, and goes to one when \( e_n \) gets large.
tions and variances of the demand shock can be set via the generator matrix, allowing us to study how the autocorrelation in demand affects returns and outcomes in the real economy.

The investor owns complete financial market claims to each sector’s dividend.\footnote{To complete markets we assume that the agent can trade a third asset whose return is linearly independent of the other two assets and that is held in zero net supply.} Complete markets allow us to highlight that our results are not generated by financial market imperfections or from some notion of background risk that is untradable. All risks can be completely hedged with portfolios of state contingent claims. All produced goods are either consumed or invested in one or any combination of the other sectors; that is, goods can be reallocated from one sector to the other.

**Technology**

A sector’s firm produces differentiated output with a linear technology possessing constant productivity $A_n > 0$ and capital $K_n$

$$Y_{nt} = A_nK_{nt}. \quad (6)$$

Capital is accumulated via

$$dK_{nt} = \Phi_n(I_{nt}, K_{nt})dt + \sigma_nK_{nt}dB_{nt}, \quad (7)$$

where $\Phi_n(\cdot)$ is the sector’s Penrose-Uzawa installation technology that measures the sector’s efficiency in converting investment goods into capital goods and $\sigma_n > 0$ scales the variability of this efficiency which evolves with a Brownian shock $B_{nt}$. The shocks are correlated and the degree is given by $\varphi \in (-1, 1)$.

The installation technology is homogeneous of degree one in investment and capital

$$\Phi_n(I_{nt}, K_{nt}) = \phi_n(i_{nt})K_{nt}, \quad (8)$$

where $i_n := I_n/K_n$ is the investment rate and $\phi'(\cdot) > 0$ and $\phi''(\cdot) \leq 0$. In particular,
we specify adjustment costs as

$$\phi_n(i_n) = i_n - \delta_n - \frac{\kappa_n}{2}(i_n - \delta_n)^2.$$  \hspace{1cm} (9)

When $\kappa_n > 0$ becomes negligibly small the economy approaches a frictionless economy in the spirit of Cox, Ingersoll and Ross (1985). When $\kappa_n$ gets large capital becomes illiquid and fixed as in the endowment economies of Cochrane et al. (2008) and Martin (2013). In steady state when $i_n = \delta_n$, adjustment costs are zero.

While attaining a maximally diversified economy is ideal to the representative investor, the act of reallocation itself consumes resources. Thus this reallocation cost generates a tradeoff between diversification and growth (Eberly and Wang (2009)). Production also allows our agent to alter the future distribution of capital and risk in the economy, a potentially important consideration for our long-lived investor. In addition, the presence of adjustment costs allows us to relate demand shocks to Tobin’s $q$ which is a feature of the data we explored in Section 2.

Each sector’s firm takes the equilibrium stochastic discount factor as given and maximizes firm value. Each sector’s dividends are $D_n = Y_n - I_n$ and their market values are the product of their capital stock and marginal $q$, which equals Tobin’s $q$ because of linear homogeneity of both output and the installation cost:

$$P_{nt} = q_{nt}K_{nt},$$  \hspace{1cm} (10)

where Tobin’s $q$ is

$$q_{nt} = \frac{1}{\phi_n'(i_{nt})} = \frac{1}{1 - \kappa_n(i_n - \delta_n)}. $$  \hspace{1cm} (11)

Following Hall (2001), we can give the interpretation of $\kappa$ for sector $n$ as a doubling time. If, for example, the sector’s $q$ doubles from one to two, then $i$ exceeds $\delta$ by $1/(2\kappa)$. If this investment rate continues for an interval of time of length $2\kappa$, then in the absence of adjustment costs capital is expected to double.
Solution

The second welfare theorem allows us to solve a Planner’s problem to get allocations and then to decentralize it using prices to examine the behavior of assets. By the principle of optimality, we have the following system of $M$ Hamilton-Jacobi-Bellman equations

$$0 = \max_{I_1, I_2} \left\{ f(C, J(K_1, K_2, \omega)) + \sum_{\omega' \neq \omega} \pi_{\omega\omega'} (J(K_1, K_2, \omega') - J(K_1, K_2, \omega)) + \sum_{n=1}^{2} \left( J_n(K_1, K_2, \omega) \Phi_n(I_n, K_n) + \frac{1}{2} J_{nn}(K_1, K_2, \omega) \sigma_n^2 K_n^2 \right) + J_{12}(K_1, K_2, \omega) \varphi_1 K_1 \sigma_2 K_2 \right\}, \text{ for } \omega \in \{\Omega_m\}_{m=1}^M \quad (12)$$

where $J_n, J_{nn},$ and $J_{12}$ denote the value function’s first, second, and cross-partial derivatives with respect to sector $n$’s capital holding $\omega$ fixed.

It is convenient to write $\eta = (\epsilon - 1)/\epsilon$, which ranges between zero when the goods are Cobb-Douglas substitutes and one when they are perfect substitutes. Define weighted capital as

$$K \equiv (K_1^\eta + K_2^\eta)^{1/\eta} \quad (13)$$

and the first sector’s capital share as a continuous state variable

$$k \equiv \frac{K_1^\eta}{K_1^\eta + K_2^\eta}. \quad (14)$$

Applying Ito’s lemma to (14), we obtain the dynamics for this process

$$dk = k(1 - k)\eta \left[ \phi_1(i_1) + \frac{1}{2} ((\eta - 1) - 2\eta k) \sigma_1^2 - \phi_2(i_2) - \frac{1}{2} ((\eta - 1) - 2\eta(1 - k)) \sigma_2^2 + \eta(2k - 1) \varphi_1 \sigma_2 \right] dt + k(1 - k)\eta \left[ \sigma_1 dB_1 - \sigma_2 dB_2 \right]. \quad (15)$$
If the installation technologies were symmetric and \( \sigma_1 = \sigma_2 \), the drift would be shaped like a sine-curve that equals zero when \( k \) equals 0, 1/2, or 1. When \( 0 < k < 1/2 \) there would be a positive drift; when \( 1/2 < k < 1 \), a negative drift; it therefore would mean-revert to \( k = 1/2 \). The degree to which risk premia are a function of this capital share, then they have the potential to vary along with it. It contrast to endowment economies, it features endogenous growth from investment governed by each sector’s installation technology.

With these definitions we can redefine aggregate consumption in (2) as a product of weighted capital and the consumption-to-capital ratio

\[
C = K \left( k\omega^{1-\eta}(A_1 - i_1)^{\eta} + (1 - k)(1 - \omega)^{1-\eta}(A_2 - i_2)^{\eta} \right)^{1/\eta}.
\]

In contrast to the one-sector iid case, consumption-to-capital now varies over time with the each sector’s capital share and dividend, and the demand for its good. When a sector has a high capital share, the agent prefers to have a high demand for its good, and will choose the sector’s production schedule to maximize consumption of that good.

We conjecture that \( J(\cdot) \) has the homogeneity property in weighted capital

\[
J(K_1, K_2, \omega) = \frac{1}{1-\gamma} \left[ K v(k, \omega) \right]^{1-\gamma},
\]

where \( v(k, \omega) \) is a function to be determined. Using this property to rewrite (12)
gives\textsuperscript{6}

\[ 0 = \max_{i_1, i_2} \left\{ \frac{\rho}{1 - 1/\psi} \left( \frac{c(k, \omega)}{v(k, \omega)} \right)^{1-1/\psi} - 1 \right\} + \sum_{\omega' \neq \omega} \frac{\pi_{\omega' \omega}}{1 - \gamma} \left( \frac{v(k, \omega')}{v(k, \omega)} \right)^{1-\gamma} - 1 \]

\[ + \sum_{n=1}^{2} \left( \phi_n(i_n) I_n + \frac{1}{2} \sigma_n^2 I_{nn} \right) + \varphi_1 \sigma_2 I_{12} \right\}, \text{ for } \omega \in \left\{ \Omega_m \right\}_{m=1}^M. \tag{18} \]

The first term in the equation relates to the agent’s flow utility related to the ratio of consumption to continuation utility. The second term relates to changes in continuation utility that occurs from demand shocks. This setup has demand shocks \( \omega \) and supply shocks \( k \) enter as orthogonal state variables, which will help clarify the model’s mechanisms. The last summation describes changes in supply that affect the agent’s choice of investment and the agent’s value of diversification.

The first-order conditions for \( i_1 \) and \( i_2 \) jointly solve the two-piece nonlinear system of equations

\[ \rho \left( \frac{c(k, \omega)}{v(k, \omega)} \right)^{-1/\psi} p_1 = \frac{\phi_1'(i_1) I_1(k, \omega) v(k, \omega)}{k^{1/\eta}}, \tag{19} \]

\[ \rho \left( \frac{c(k, \omega)}{v(k, \omega)} \right)^{-1/\psi} p_2 = \frac{\phi_2'(i_2) I_2(k, \omega) v(k, \omega)}{(1-k)^{1/\eta}}. \tag{20} \]

Finally, the one sector economy defines the boundaries of the two-sector econ-

\textsuperscript{6}The components of the HJB equation are

\[ I_1(k, \omega) = \left( k + \eta \frac{v'(k, \omega)}{v(k, \omega)} k(1-k) \right), \]

\[ I_2(k, \omega) = \left( (1-k) - \eta \frac{v'(k, \omega)}{v(k, \omega)} k(1-k) \right), \]

\[ I(k, \omega) = \frac{v''(k, \omega)}{v(k, \omega)} \eta^2 k(1-k)^2, \]

\[ I_{11}(k, \omega) = (1-\eta) \left( k^2 - I_1(k, \omega) \right) - \gamma I_1(k, \omega)^2 + I(k, \omega), \]

\[ I_{22}(k, \omega) = (1-\eta) \left( (1-k)^2 - I_2(k, \omega) \right) - \gamma I_2(k, \omega)^2 + I(k, \omega), \]

\[ I_{12}(k, \omega) = (1-\eta) \left( k(1-k) - \eta \frac{v'(k, \omega)}{v(k, \omega)} k(1-k)(2k-1) \right) - \gamma I_1(k, \omega) I_2(k, \omega) - I(k, \omega). \]
omy as one sector becomes negligibly small. The sector’s capital stock $K_n$ is the single state variable. The stochastic growth rates of capital, consumption, investment, and output are equal and because growth rates and returns are iid the ratios of consumption- and investment-, and output-to-capital are constant, as is Tobin’s $q$. The optimal investment rate solves

$$A_n - i_n^* = \frac{1}{\phi'(i_n^*)} \left[ \rho + \left( \frac{1}{\psi} - 1 \right) \left( \phi_n(i_n^*) - \gamma \frac{\sigma_n^2}{2} \right) \right],$$  \hspace{1cm} (21)

and the coefficient that solves the value function is given by

$$b_n = \frac{\rho}{\phi'(i_n^*)} \left[ 1 + \frac{1}{\psi} - 1 \left( \phi_n(i_n^*) - \gamma \frac{\sigma_n^2}{2} \right) \right]^{\frac{1}{\psi-1}}. \hspace{1cm} (22)$$

Using these two coefficients we define the boundaries of $v(0, \omega) = b_1$ and $v(1, \omega) = b_2$ for all $\omega \in \{\Omega_m\}$.

**Asset Pricing**

The stochastic discount factor (SDF), which is the present value of an extra unit of the aggregate consumption good (the numeraire) at time $t$, takes the form given by Duffie and Skiadas (1994):

$$\Lambda_t = \exp \left\{ \int_0^t f_J(C_s, J_s) ds \bigg\} f_C(C_t, J_t) = \exp \left\{ \int_0^t \frac{\rho}{1 - 1/\psi} \left[ \left( \frac{c(k_s, \Omega_s)}{v(k_s, \Omega_s)} \right)^{1 - 1/\psi} - (1/\psi - \gamma) - (1 - \gamma) \right] ds \right\} \rho K_t^{-\gamma} e^{\zeta(k_t, \Omega_t)},$$  \hspace{1cm} (23)

where $\zeta(k, \omega) = (1/\psi - \gamma) \log v(k, \omega) - 1/\psi \log c(k, \omega)$. 


Ito’s lemma then implies that our pricing factor inherits the following dynamics

\[
\frac{d\Lambda}{\Lambda} = -r(k, \Omega)dt - \lambda_1(k, \Omega)\sigma_1 dB_1 - \lambda_2(k, \Omega)\sigma_2 dB_2 + \sum_{\Omega \neq \Omega_0} \left( e^{Z(\Omega_0, \Omega)} - 1 \right) (dN^{(\Omega_0, \Omega)} - \pi_{\Omega_0, \Omega} dt).
\] (24)

The risk-free rate \( r \) in the numeraire’s units is given in the Appendix and responds to the usual features of expected growth and precautionary savings. There are three sources of risk. The first two are diffusive supply shocks:

\[
\lambda_1(k, \omega) = \gamma k - \frac{\partial \zeta(k, \omega)}{\partial k} \eta k(1 - k),
\]
\[
\lambda_2(k, \omega) = \gamma (1 - k) + \frac{\partial \zeta(k, \omega)}{\partial k} \eta k(1 - k)
\] (25)

Looking at the first sector’s price of risk, \( \lambda_1 \sigma_1 \), the first piece increases linearly with \( k \) from 0 to \( \gamma \sigma_1 \) as \( k \) goes from zero to one. The sign of the second piece depends \( \zeta(k, \omega) \) and the degree of substitution \( \eta \) between goods. In our calibration, the first piece dominates and the first sector’s price of risk effectively increases monotonically with \( k \). An analogous phenomenon holds for the second supply shock.

This result of a monotone increasing price of risk lies behind the counterfactual result of standard two-sector economies producing a positive relationship between risk premia and wealth shares (Cochrane et al. (2008)). Calibrations that feature only supply shock volatility between 2 or 3 percent face difficulty in matching negative relationship that is seen in the data.

The third source of risk concerns the demand shocks and equals \( e^{Z(\omega, \omega')} - 1 \), where \( Z(\omega, \omega') = (1/\psi - \gamma) \log \left( \frac{v(k, \omega')}{v(k, \omega)} \right) - 1/\psi \left( \frac{c(k, \omega')}{c(k, \omega)} \right) \). These shocks alter the pricing kernel through their effects on the growth rate of consumption and of continuation utility. Our calibration sets the price of risk on continuation utility growth \( (\gamma - 1/\psi > 0) \) greater than on consumption growth \( (1/\psi) \). An implication is that if consumption growth is expected to increase at the same rate as continuation utility is expected to fall, then the agent’s marginal utility would be expected to rise. Importantly, demand shocks allow the model to feature great variation in marginal
utility, even when the size of relative capital $k$ of a sector is small, helping the model reconcile the pattern of risk premia and wealth in the data.

With our pricing kernel in hand, we can now study the risk and return properties of the assets and distinguish the sources of asset demand.

**Returns and Risk Premia**

The dividend yield for each sector is

$$\frac{D_n}{P_n} = \frac{A_n - i_n}{q_n}, \text{ for } n = 1, 2,$$

and the return on a stock with price in units of the numeraire is

$$dR_n = \frac{d(p_nP_n)}{p_nP_n} + \frac{D_n}{P_n} dt = \frac{dp_n}{p_n} + \frac{dq_n}{q_n} + \frac{dK_n}{K_n} + \frac{D_n}{P_n} dt + \text{second-order terms} + \sum_{\Omega \neq \Omega'} (e^{\mathcal{J}_n(\Omega, \Omega) - 1}) dN^{(\Omega, \Omega)},$$

where $\mathcal{J}_n(\omega, \omega') = \log \left( \frac{p_n(k, \omega')}{p_n(k, \omega)} \right) + \log \left( \frac{q_n(k, \omega')}{q_n(k, \omega)} \right)$, which is the growth rate of Tobin’s $q$ in units of the numeraire. The dividend yield component of expected returns is unit-free, but the expected capital gains component depends on the quantity and value of capital in units of the numeraire. Holding the capital stock of the sector fixed, a demand shock affects a sector’s returns through both the good’s price and marginal $q$.

The risk premium is a function of both the sector’s capital share and the demand for its good. It takes the form

$$\left( \frac{1}{dt} \mathbb{E}_t[dR_n] - r \right) = -\frac{1}{dt} \mathbb{E}_t \left[ \frac{d\Lambda}{\Lambda} dR_n \right]$$

$$= \left( \lambda_1(k, \omega) \sigma_1 dB_1 + \lambda_2(k, \omega) \sigma_2 dB_2 \right) \times \left( \frac{\partial p_n(k, \omega)}{p_n(k, \omega)} + \frac{\partial q_n(k, \omega)}{q_n(k, \omega)} \right) dk + \frac{dK_n}{K_n}$$

$$- \sum_{\Omega' \neq \Omega} \pi_{\omega, \omega'} \left( e^Z(\omega, \omega') - 1 \right) \left( e^{\mathcal{J}_n(\omega, \omega') - 1} \right).$$

If marginal utility rises while the demand for the asset return falls, which occurs
when the good’s price or marginal $q$ drops, the agent will demand a higher risk premium for demand shocks. In particular, if marginal utility strongly covaries with a demand shock when the sector’s share is small, the sector could earn a high risk premium. This feature of demand shocks helps the model generate a high risk premium with a small wealth share, which is consistent with the data.

**Portfolio Choice and Market Returns**

The first sector’s wealth share is

$$w = \frac{p_1 q_1 K_1}{p_1 q_1 K_1 + p_2 q_2 K_2},$$

leaving the other sector with share $1 - w$ of total wealth $W = \sum_n p_n q_n K_n$. We collect these wealth shares in the vector $W = [w, 1 - w]'$. We define the agent’s mean-variance portfolio demand as

$$\text{MV} = \frac{1}{\gamma} \times \left( \begin{array}{cc} \text{var}_t(dR_1) & \text{cov}_t(dR_1, dR_2) \\ \text{cov}_t(dR_1, dR_2) & \text{var}_t(dR_2) \end{array} \right)^{-1} \left[ \frac{1}{\bar{t}} \mathbb{E}_t[dR_1] - r \\ \frac{1}{\bar{t}} \mathbb{E}_t[dR_2] - r \right],$$

and we compute hedging demands as the difference between wealth shares $W$ and the mean-variance demands

$$\text{HD} = W - \text{MV}.$$  

Hedging demand measures how much the agent holds of the sector’s market value in excess of its myopic mean-variance tradeoff. An investor with a long horizon is averse to news that future returns will be lower, because their wealth or consumption will be. The investor will therefore bid up the prices of stocks that do well on such news, hedging this risk. Thus, equilibrium expected returns will depend on partly the covariation of news of future returns, as well as the covariance of the current market return.
Table 2: Calibration (Annual)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Preferences</td>
<td></td>
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<tr>
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<td>$\gamma$</td>
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</tr>
<tr>
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<tr>
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<tr>
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<td>Real Estate</td>
</tr>
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</tr>
<tr>
<td>$\kappa$</td>
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<tr>
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</tr>
<tr>
<td>$\delta$</td>
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</tr>
<tr>
<td>$\varphi$</td>
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</tr>
</tbody>
</table>

4 Calibration and Analysis

Our chosen parameters are in Table 2. We begin by choosing household preferences that are consistent with prior literature in production economies with recursive preferences (Kaltenbrunner and Lochstoer (2010)): Risk aversion is 5 and IES is 1.5. The time rate of preference is set to 0.035. The elasticity across goods is five.\(^7\)

We then calibrate the economy’s technologies, which drive the capital state variables, to fit the data. Specifically, we take observed growth rates of real capital stocks and use them to pin down supply shock volatility and the curvature of the adjustment cost function. The volatilities $\sigma$ are equated to the data’s estimate volatility of the growth of real capital stock: 2.3 and 2.6 percent for financial and real estate capital, respectively. The shocks are left uncorrelated: $\varphi = 0$.\(^8\) The adjustment cost parameter $\kappa$ is set equal to 10 for both sectors, reflecting a doubling-time of about 20 years.\(^9\) Depreciation rates $\delta$ are set to match their empirical counterparts in Table

---

\(^7\) Ogaki and Reinhart (1998) find $\epsilon \in (2.9, 4.0)$ between nondurable and durable goods when using a cointegration approach on long series of annual data (1929-1990). Ravn, Schmitt-Grohe and Uribe (2006) estimate the value of $\epsilon$ in their deep habits model to range between 2.5 and 5.3.

\(^8\) In BEA data, the correlation between the two series is 0.15.

\(^9\) Using overlapping data over from 1925 until 2015 from the BEA’s residential and nonresidential fixed asset tables, the median and mean of doubling times of each sector’s real capital
<table>
<thead>
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<th>$\Omega_m$</th>
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<th>Density</th>
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</tr>
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<td>$\Omega_4$</td>
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</tr>
<tr>
<td>$\Omega_5$</td>
<td>0.500</td>
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<td>$\Omega_6$</td>
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<td>$\Omega_7$</td>
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<td>0.127</td>
</tr>
<tr>
<td>$\Omega_8$</td>
<td>0.821</td>
<td>0.071</td>
</tr>
<tr>
<td>$\Omega_9$</td>
<td>0.951</td>
<td>0.028</td>
</tr>
</tbody>
</table>

1. We calibrate productivity $A$ to match the average investment rates of each sector in their respective one-sector economies via (21). We define $k$ to measure the share of financial sector’s capital stock in the economy, leaving $1 - k$ for real estate’s.

To calibrate the transition matrix and values of the demand shock, we use the quadrature method of Tauchen and Hussey (1991) to approximate an annual AR(1) with a mean of 0.5, an unconditional standard deviation of 0.1, and a persistence of 0.875. We chose the mean of 0.5 to make the distribution symmetric. The unconditional standard deviation was chosen so that $\{\Omega_m\}$ is contained in the unit interval. The choice of persistence pins down the conditional volatility of the shock, $0.1 \times \sqrt{1 - 0.875^2} = 4.84$ percent per year. We then convert the discrete Markov chain to a continuous-time Markov chain using Jarrow, Lando, Turnbull’s approximation (based on assumption of more than one change of state is close to zero within the period (year)). The resulting state space of $\Pi$ and its ergodic distribution is tabulated in Table 3.

**Analysis**

We break the analysis into three parts. In the first part, we discuss the qualitative structure of the model and its intuition, in particular the trade off between growth stock is about 20 years.
and the investor’s desire to achieve a balance in the economy. We next highlight the
effect of demand shocks in the model by contrasting it with a model that has only
supply shocks, the usual benchmark in the macrofinance literature. We finally then
investigate the answer to the question that if asset demand shocks are necessary to
match key features of the data, what are the implications for our understanding of
investor portfolio choice?

We plot several of the model’s policy functions below. Because \( \omega \) is independent
of \( k \) by construction, we plot \( k \) on the horizontal axis. Holding \( \omega \) fixed, therefore,
a shift along a policy function changes only the distribution of capital. Holding \( k \)
fixed, a shift in demand is a shift across policy functions. Of course, an economy
that only has supply shocks will feature different policy functions than the ones
below because it would exclude risk premia attributed to demand shocks.

**Balancedness**

The representative agent faces a trade off between growth and the desire to achieve
a balanced economy.\(^{10}\)

A balanced economy is when a sector has a capital share that is close in size to
the demand for its good; for the first (financial) sector for example, when \( k_t \approx \Omega_t \).
For brevity we’ll refer to this property of the economy simply as *balancedness*. The
economy is unbalanced or features unbalancedness when either a small capital share
produces to meet a high demand, or vice versa, or more generally when there simply
exists a sectoral imbalance of supply and demand. Furthermore, the economy’s
statistics differ when demand is high and supply is low for a sector than from when
demand is low and supply is high, so asymmetries exist.

\(^{10}\)This desire originates from the consumption-to-capital ratio \( c(k, \omega) \) in (16). Loosely speaking,
when \( k_t \) and \( \Omega_t \) are high, then \( i_{1t} \) would be chosen to be low, making the dividend-to-capital ratio,
\( (A_1 - i_{1t}) \), large. (In our two-sector economy this is equivalent to \( 1 - k_t \) and \( 1 - \Omega_t \) being both
small). This choice has two effects. First, \( c(k, \omega) \) would be high because a sector with abundant
capital produces a good in high demand. Second, the volatility of consumption would be low because
the Cobb-Douglas function of \( \Omega_t \) and \( (A_1 - i_{1t}) \) would feature small marginal “products”, reducing
the volatility of these components. These are therefore good times.

In contrast, when \( k_t \) is high but \( \Omega_t \) is low, then a small dividend-to-capital ratio is chosen. A
similar argument would suggest that then \( C/K \) would be low and the volatility of \( dC/C \) would be
high. Analogous arguments hold for the other sector.
When the economy is balanced, the market risk premium and the demand for precautionary savings reach local lows, leading the agent to consume a lot out of wealth. Figure 2 plots these variables on the financial sector’s capital share $k$ for three levels of demand for the sector’s good. Each $(k, \omega)$ pair has a maximal point of balancedness: the three nadirs across $k$ of the market risk premium for each $\omega$ align with these points.

Our agent is long-lived, so the consumption-wealth ratio will depend on the expected discounted value of all future market returns. And because our calibration has $\psi > 1$, the substitution effect dominates the wealth effect, leading the agent to prefer postponing consumption for investment when future market returns are expected to be high. During these high market return periods, then, to a first-order, the consumption-wealth ratio is low.

The agent prefers a balanced economy and can invest to attain this in our production economy. But adjustment is costly as a sector’s output is diverted to investment at an increasing marginal cost. There is therefore a tradeoff between growth and restoring balancedness, which is exemplified by the drift of the financial sector’s capital share, $\mathbb{E}[dk]$.

It is either positive or negative parabola, or a sine curve, depending on the level of demand. The economy is therefore mean-reverting and in the long-run features a nondegenerate ergodic distribution. When demand for the financial good increases, the agent chooses to consume more of it at the expense of investing in production, eventually forcing $\mathbb{E}[dk]$ negative and $k$ towards zero, thereby unbalancing the economy even further. The bottom-right panel shows that when demand increases for the financial sector, its wealth share increases relative to its capital share; a similar pattern holds for real estate.

To preview the asymmetry in the model, consider an increase demand for a good produced by a sector with an abundant supply of capital. Here the agent chooses to consume relatively more out of wealth than before, as demand catches up with supply. By contrast, when the supply of capital is scarce an increase in demand lowers relative consumption, as the agent prefers to invest to restore balance to the economy. These two scenarios differentially affect the agent’s marginal utility and the pricing of risk in the economy, as we’ll soon discuss.
Policy functions from model as a function of $k$ and three levels of $\omega = \{\Omega_1, \Omega_5, \Omega_9\}$. The lightest green line corresponds with the lowest level of demand for the financial sector’s good; increasing darkness increases with demand. $E[dk]$ is the expected drift of the first (financial) sector’s capital share. The variable $w - k$ plots the wealth share of the financial sector in excess of its capital share.

In the next section we distinguish predictions of an economy with demand shocks by contrasting its behavior with an economy that features only supply shocks.

**Demand Shocks**

Demand shocks affect marginal utility by changing the agent’s consumption-savings decision. They affect Tobin’s $q$ (in units of the numeraire) by altering both the relative price a sector’s good and relative value a sector’s capital stock. Of course in
Figure 3: Tobin’s $q$ and Risk Premia

Policy functions from model as a function of $k$ and three levels of $\omega = \{\Omega_1, \Omega_5, \Omega_9\}$. The lightest green line corresponds with the lowest level of demand for the financial sector’s good; increasing darkness increases with demand. Tobin’s $q$ here is in units of the numeraire ($p_n q_n$) for each sector.

In general equilibrium these two considerations are jointly determined with risk premia.

Figure 3 shows that holding $\omega$ fixed, a sector’s Tobin’s $q$ is decreasing in its capital share, regardless of the level of demand. The agent values a marginal unit of capital less and less as the sector’s capital base grows. With only supply shocks, then, there is always a negative relationship between a sector’s $q$ and its wealth share. This last point is at odds with the data: it seems like growing demand jointly produces an upward swing in Tobin’s $q$ and the sector’s share of wealth.

A shift in demand has different effects on $q$ and risk premia depending on the
economy’s distribution of capital. Variation in Tobin’s $q$ will largely reflect movements in risk premia because production functions feature constant returns to scale. When a sector is small, a growth in demand lowers $q$ as the sector’s risk premium rapidly rises; when it’s large, increasing demand lowers the risk premium and raises $q$.

Why the asymmetry? When demand for a small asset increases, the magnitude of its effect on the SDF depends on the current level of demand because the shock is mean-reverting. If $\Omega_t > 0.5$, for example, it is more likely that it would fall than rise in the future. If it fell the small asset’s relative price would decline and there would be less of an incentive to invest in it, because the economy would beneficially have become more balanced. The agent would then substitute consumption to today from tomorrow, ultimately raising the SDF because movements in continuation utility growth command a larger price of risk than does consumption growth in our calibration. The asset’s value would be expected to fall and coincide with an increase in marginal utility, thus creating a risk premium.

When the asset is in abundant supply, on the other hand, an increase in demand is good news for the agent: a larger dividend is consumed out of a sector with a large supply of capital and the economy becomes more balanced, reducing the sector’s (and the market’s) risk premium while inflating $q$. The possibility of this phenomenon importantly allows the model to generate a positive comovement between $q$ and wealth, as in the data.

The potential occurrence of imbalances between demand and supply, moreover, help the model in generating a negative relationship between risk premia and wealth.\footnote{A negative relationship between risk premia and wealth share is difficult prediction for theoretical endowment models to obtain. The reason is that as an asset’s wealth shrinks its contribution to aggregate consumption falls, bringing shocks to its cash flows closer to being idiosyncratic, lowering its required return to the risk-free rate. Martin (2013) is able to break this result in a CRRA endowment economy with mix of high risk aversion and exogenous dividend volatility. There are two problems with this intuition and result holding up in a more general model, however. First, a low IES combined with a reasonable dividend growth volatility induces counterfactually large variation in the risk-free rate. Second, when moving from an endowment to a production economy, a small asset that has a high price-dividend ratio would also have a high Tobin’s $q$, indicating that investment is profitable. If investment were allowed, it would reduce the sector’s valuation, its potential for comovement with the market, and therefore its risk premium.} Looking at the bottom two subfigures, this is possible if there is high de-
mand for a small asset that is subsequently followed by physical investment. Thus when supply chases demand one would see a negative relationship between risk premia and wealth shares in the data.

**Asset Demand**

Demand shocks introduce an additional source of risk faced by the investor over and above a simple exposure to another volatile shock: an unbalanced economy. Now in an economy that can feature unbalancedness, what are the implications for portfolio choice in regards to mean-variance and hedging demand? Upon reflection it should be no surprise that the agent should desire to hold an asset not only for its mean-variance return ability but also to hedge against this risk of unbalancedness. Figure 4 plots each sector’s mean-variance demand and hedging demand as a function of its wealth \( w \).

By construction, the sum of each sector’s mean-variance demand and hedging demand equals the sector’s wealth share. For example, at slightly over a 40 percent wealth share and \( \omega = \Omega_1 \) (so \( k > \omega \)), the level of financial sector wealth is entirely attributed to hedging demand. When the economy is balanced, hedging demand is in general small. The local minima of the hedging demand function occur when the sector’s supply and demand are similar. Moving away from these minima increases hedging demand, but for different reasons depending on the direction of the move.

An ICAPM argument would say that the average investor would bid up the price of an asset that acts as a hedge against adverse realizations of state variables. A long-lived investor will be averse to news that future returns are lower, because their long-term wealth or consumption will be lower.

Fix demand and consider the financial sector’s capital share falling from an initial point of balancedness, so \( k_t < \Omega_t \). The main risk to an investor would be either a positive supply shock to the financial sector (\( k_t \) rises) or a falling demand for its own good (\( \Omega_t \) falls). In the latter case it is necessary that the decline in demand increases \( q \) relatively more than its price falls. Nevertheless, in either case the small asset would experience a positive return and balancedness would be partially restored. This is a risk because when either of these shocks realize, the expected...
Each sector’s mean-variance demand and hedging demand as a function of the sector’s wealth and three levels of $\omega = \{\Omega_1, \Omega_5, \Omega_9\}$. The lightest green line corresponds with the lowest level of demand for the financial sector’s good; increasing darkness increases with demand. The dashed black lines are the 45-degree line and a line at zero.

return on the market declines (see Figure 2), which would be bad news for the long-term investor—future returns are now expected to be lower. Ex ante, then, prices would therefore rise on assets that do well on such news, hedging the reinvestment risk. As an asset becomes small it offers a poorer mean-variance deal while attributing a larger part of its wealth to hedging demand.

On the other hand, when the financial sector’s capital share rises away from a balanced economy again holding demand fixed, so $k_t > \Omega_t$, a similar argument holds. The primary risks have reversed: either an increase in the supply of real
estate or an increase in demand for the financial good produces a positive financial return. Thus holding the financial sector will act as a hedge against a future decline in expected market returns. The average investor will bid up the price of the asset, lowering its myopic demand while raising the proportion of hedging demand.

When the economy is balanced, the need to hold assets for hedging purposes becomes a second-order concern and the primary source of demand for an asset reflects mean-variance considerations. We next look at the implications of demand shocks on the value, risk pricing, and investment of the economy’s productive sectors.

5 Implications of Demand Shocks

To assess the importance of demand shocks we do the following exercise. We first solve the full model with our listed set of parameters. We then simulate it 5,000 times for 480 quarters, burning in the first half and leaving 60 years of quarterly data, a length comparable to our data sample, to compute model statistics. Summary statistics on standard macroeconomic and financial variables are tabulated in Table 4.

Adding demand shocks helps the model generate dynamics that are consistent with the data. Without demand shocks, the volatility of cash flows is basically equal to the volatility of the capital stock in stark contrast to the data. Demand shocks add variation in the price of the good and help generate cash flow volatility while keeping the supply side of the economy unchanged. Adding another shock, of course, raises the average excess return and return volatility of each sector, bringing the model closer to the data.

One tradeoff the model faces when choosing a level of demand shocks is that consumption volatility increases with a more variable demand shock as well. Within the model’s 95 percent confidence interval of (2.0, 5.4), however, the model’s standard deviation of consumption falls in line with the data. If one takes the consumption process facing the agent as the sum of the two dividend streams, then under the assumption that both goods’ relative prices are equal to one the volatility of this consumption stream is 6.9 percent.
Table 4: Summary Statistics

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<th>Both Shocks</th>
<th>Supply Shocks</th>
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<td>Mean</td>
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<td><strong>Aggregate</strong></td>
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<tr>
<td>Consumption (BEA)</td>
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<td>Consumption (cash flow)</td>
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</tbody>
</table>

Data are quarterly from 1952Q4 until 2015Q4. Sources: BEA, Federal Reserve Flow of Funds, and BLS for the CPI price deflator. Values are real where applicable. Consumption (BEA) is the real growth rate of nondurables and services. Consumption (cash flow) is a cash flow-weighted growth rate of the financial and real estate cash flow. Simulation data are calculated with and without a demand shock. Where there are no demand shocks, we fix $\Omega = 0.5$. We simulate each model 5,000 times for 480 quarters, burning in the first half and leaving 60 years of quarterly data. We calculate both the mean and standard deviation of each variable for each simulation. We then average across both of these statistics to report Mean and Stdev for the model.

The next exercise is to see if the model can generate the regression coefficients that characterize the empirical puzzle. We do this by running the two key regressions for each simulation and tabulating the distribution of slope coefficients in Table 5. For regressions of $q$ on wealth, supply shocks alone always generate a negative relation. The reason is simple: a sector has a high $q$ when it is small, reflecting the value of investing in it. Demand shocks are required to produce a positive relationship. The model generates this by having a small level of demand catch up with a large supply of the asset. When a sector has a large capital share, an increase in demand for that good is good news for the agent, lowering macroeconomic risk and the required risk premium. The falling risk premium pushes up $q$. 

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Table 5: Finite Sample Distribution of Regression Slope Coefficients

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<th>Panel A: Tobin’s q on wealth share</th>
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<td>$\hat{b}_n$</td>
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<th>Panel B: Risk premium on wealth share</th>
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Distributions of regression slope coefficients across simulations. In each simulation we run the level regression of $p_{nt}q_{nt} = a_n + b_n \times w_{nt} + \epsilon_{nt}$ in Panel A and the level regression $\frac{1}{dt}E_t[dR_{nt}] - r_t = a_n + b_n \times w_{nt} + \epsilon_{nt}$ in Panel B for each sector $n$; we report the slope coefficients $\hat{b}$ above. We also run the regressions after differencing the regressors and regressand in the subpanels marked Changes. Wealth shares and risk premia are measured in percent. In the data for the level regression, the 1% and 99% form the 99% confidence interval for the slope estimate with Newey-West standard errors with a lag of 8 quarters; the changes regression uses OLS standard errors. The data cover 1952Q4–2015Q4. For clarity and emphasis, positive values are bolded.

Regressions of risk premia on wealth show a similar failing of a model featuring only supply shocks. Because we calibrate to only supply shock and not cash flow volatility, the amount of risk in the economy is small. As a result, when a sector
shrinks its supply shock volatility becomes increasingly idiosyncratic, as it affects aggregate consumption and the pricing kernel less. Therefore a positive relationship between risk premia and wealth occurs. This effect is still present in a model with demand shocks, but demand shocks introduce an additional source of risk to the economy over and above its volatility. In particular, when an asset has a small capital share but a high demand, the economy is unbalanced and the agent requires a large risk premium. As investment occurs to increase the small asset’s capital share, the economy becomes more balanced and the risk premium falls, leading to a negative relationship between risk premia and wealth.

We next contrast the two model’s implications for hedging demand in Figure 5. Because the economy features unbalancedness the level of hedging demand including demand shocks is magnitudes more than in a model that only has supply shocks. Demand shocks generate hedging demand up to 40 percent. As Figure 4 shows, 40 percent could be the sector’s entire wealth, implying that the sector is held only for hedging reasons and not for any mean-variance considerations. In contrast, the model that only has supply shocks generates a small amount of hedging demand on the order of plus-minus 3 percent.

To summarize, demand shocks help match key features of the data, something that previous work has overlooked. They also help a standard two-sector model generate moments that are more closely aligned with the data. Because demand shocks add another layer of risk to the economy, the risk that it becomes unbalanced and becomes even more exposed to a sector’s cash flows, they generate asset demand solely arising from its ability to hedge rather than for typical mean-variance considerations. In the next section, we now try to figure out the level of demand from the data and the associated hedging demand.

**Application: Inferring Demand Shocks from the Data**

As a final exercise, we infer the level of demand $\Omega_t$ from our observed data. We do this in the following steps:

1. We take our data series for both financial sector’s capital share $\{k_t\}$ and its wealth share $\{w_t\}$ over the period 1952Q4 until 2015Q4
Figure 5: Hedging Demand from Both and Only Supply Shocks

Financial Hedging Demand

Real Estate Hedging Demand

Histograms of hedging demand for 5,000 simulations of Both Shocks model and 1,000 simulations for Supply Shocks Only. The Supply Shocks Only simulation fixes $\Omega = 0.5$. Horizontal axis measures hedging demand $HD = w - MV$ in percent.

2. For every quarter in the data we map the data’s $k_t$ into the model directly. Given $k_t$, we also then infer $\Omega_t$ by matching the data’s $w_t$ as close to our model’s policy function $w(k_t, \Omega_t)$ by minimizing the distance between the two measures.

Thus, the model’s choice of $\omega$ is restricted by having it fit $k$ and $w$ simultaneously. The output is depicted in Figure 6.

The top panel depicts the financial sector’s wealth share and the implied level of demand for the financial sector’s good $\Omega_t$. The data $w$ line is what is fed into the model and the implied $w$ line is what the model is restricted to estimate its wealth.
share to be. The time series for the capital share is matched exactly. Because of this restriction the implied level of demand must vary a lot to match wealth shares, and in this calibration it does so to the extremes 0 and 1. Decreasing the elasticity across goods $\epsilon$ would aid in matching the wealth share, but at the cost of increasing consumption volatility above what a typical macrofinance calibration would target. Of course a typical calibration targets aggregate consumption of nondurables and services, whereas if housing services displayed a volatility closer to real estate’s rental income, then aggregate consumption volatility would be much greater.

The bottom panel plots each sector’s implied hedging demand as a percent of the sector’s wealth ($HD_n/w_n * 100$) and filtered to a 12-quarter moving average. The results are striking. Hedging demand on average makes up to about 60 percent
and 73 percent of the financial and real estate sector’s wealth allocated to that sector. At times, real estate hedging demand forms nearly 100 percent of the sector’s wealth. The majority of a sector’s wealth is held for hedging, not for mean-variance considerations.

From where does hedging demand originate? The ICAPM predicts that it arises from investors’ desires to hedge marginal utility against changes in state variables. If an asset’s return is good for bad realizations of the state variable, this raises the desirability and thus overall allocation to the asset. Some of these state variables are omitted here. Investor-specific risk ranges from owning houses to holding a job or running a small business. People will prefer to own stocks if they pay off when housing falls, or else simply as a hedge against rents increasing. Workers in business-cycle sensitive industries will prefer to hold financial assets that do well in recessions.

Empirically it seems a lot of work has been done on forming factor-mimicking portfolios that are really projections of state variables onto the space of current returns and then seeing if this factor is priced. But the work does not need to stop here. Investors’ intertemporal hedging demands bid up the prices of assets that do well when future returns are low, raising their prices and lowering their expected returns for a given market beta. Instead, researchers should be focusing on modeling the joint behavior of investment, risk premia, and sectoral wealth, in addition to other factors that would generate hedging demand, such as human or intangible capital or the idiosyncratic risk of running a small business. All of these factors that potentially create hedging demand should in theory forecast future returns or contemporaneously correlate with some assets’ wealth shares.

Furthermore, our model is constructed free of transaction costs and restrictions on borrowing and short sales. While done for tractability, these or other types of frictions could also play an important role in generating the key facts. In sum, macrofinancial theories either need to be adjusted to reflect larger hedging demands. Microempirical work needs to analyze the role of hedging demand and what sources determine an individual investor’s portfolio choice.
6 Conclusion

In the data, sectors’ wealth shares are negatively related to their risk premia but positively correlated with their Tobin’s $q$. Standard multi-sector models have difficulty with generating these two facts: models that feature only supply shocks tend to have a sector’s risk premia increasing in its wealth share, for the simple reason that larger assets have more systematic risk; and when a sector’s capital stock increases, so does its wealth share, but not its Tobin’s $q$.

We develop a model that reconciles these two facts in the data by appealing extending a standard model to include two features: imperfect goods across sectors, which allow a relative price of each sector’s good to affect the distribution of risk and returns; and demand shocks, which generate variation in financial risk premia and Tobin’s $q$ that is unrelated to supply.

Demand shocks open up the possibly of a mismatch in supply and demand for a sector’s good, leading the average investor to hold large shares of a sector’s wealth for hedging demands. As a result, mean-variance concerns play a second role to hedging demand in the model.

Our empirical analysis suggests that hedging demands also play a large role in practice, contributing to the majority of a sector’s wealth share. In our sample, we find that on average 60 and 73 percent of financial and real estate wealth, respectively, arises from the demand to hedge.

Investors’ intertemporal hedging demands bid up the prices of assets that do well when future returns are low, raising their prices and lowering their expected returns for a given market risk. Instead our study implies that researchers should be focusing on modeling the joint behavior of investment, risk premia, and sectoral wealth, in addition to other factors that would generate hedging demand, such as human or intangible capital. All of these factors that potentially create hedging demand should in theory forecast future returns or contemporaneously correlate with some assets’ wealth shares.
References


Appendix - Derivations

Ito’s lemma shows

\[
\begin{align*}
\frac{d\Lambda}{\Lambda} &= f_J dt - \gamma \frac{dK}{K} + \frac{1}{2} \gamma (1 + \gamma) \left( \frac{dK}{K} \right)^2 - \frac{\gamma dK}{K} \frac{\partial \zeta(k, \Omega_t^-)}{\partial k} dk \\
&+ \frac{\partial \zeta(k, \Omega_t^-)}{\partial k} dk + \frac{1}{2} \left( \left( \frac{\partial \zeta(k, \Omega_t^-)}{\partial k} \right)^2 + \frac{\partial^2 \zeta(k, \Omega_t^-)}{\partial k^2} \right) (dk)^2 \\
&+ \sum_{\Omega_t \neq \Omega_t} (e^{Z(\Omega_t^-, \Omega_t)} - 1) dN_t^{(\Omega_t^-, \Omega_t)},
\end{align*}
\]

(A-1)

where \( Z(\omega, \omega') = (1/\psi - \gamma) \log \left( \frac{v(k, \omega')}{v(k, \omega)} \right) - 1/\psi \left( \frac{c(k, \omega')}{c(k, \omega)} \right) \). The risk-free rate in units of consumption good is therefore

\[
r(k, \omega) = \rho + \rho \left( \frac{1/\psi - \gamma}{1 - 1/\psi} \right) \left( 1 - \left( \frac{c(k, \omega)}{v(k, \omega)} \right)^{1-1/\psi} \right) + \gamma \frac{1}{dt} \mathbb{E}_t \left[ \frac{dK}{K} \right] \\
- \frac{1}{2} \gamma (1 + \gamma) \left( k^2 \sigma_1^2 + (1 - k)^2 \sigma_2^2 + 2k(1 - k)\varphi \sigma_1 \sigma_2 \right) \\
+ \gamma \frac{\partial \zeta(k, \omega)}{\partial k} \eta k(1 - k)(\sigma_1^2 k - \sigma_2^2 (1 - k) + (1 - 2k)\varphi \sigma_1 \sigma_2) - \frac{\partial \zeta(k, \omega)}{\partial k} \frac{1}{dt} \mathbb{E}_t [dk] \\
- \frac{1}{2} \left( \left( \frac{\partial \zeta(k, \omega)}{\partial k} \right)^2 + \frac{\partial^2 \zeta(k, \omega)}{\partial k^2} \right) \eta^2 k^2 (1 - k)^2 (\sigma_1^2 + \sigma_2^2 - 2\varphi \sigma_1 \sigma_2) - \sum_{\omega' \neq \omega} \pi_{\omega\omega'} (e^{Z(\omega, \omega')} - 1).
\]

(A-2)