# Slow Recovery in an Economy with Uncertainty Shocks and Optimal Firm Liquidation

Indrajit Mitra*

University of Michigan

February 28, 2017

**Abstract**

I examine optimal dynamic contracting between risk-averse investors and firm insiders in a dynamic general equilibrium model with heterogeneous firms. I quantify the effect of an aggregate increase in the uncertainty of firm-specific productivity. I obtain three results. First, the shock generates a large drop in GDP. Second, recovery is slow due to a large increase in the exit rate of young firms. Third, an increase in firm-specific uncertainty leads to an increase in the dispersion of output growth rates. These features are quantitatively consistent with empirical patterns in the Great Recession.

*Correspondence: Ross School of Business, University of Michigan, 701 Tappan Street, Ann Arbor, MI 48109, e-mail: imitra@umich.edu.

# 1 Introduction

The US economy experienced a weak recovery following the Great Recession of $2007 - 09$. Reinhart and Rogoff (2009) document that this phenomenon is not limited to the most recent crisis – financial crises, especially deep ones, are followed by slow recoveries[1] A second robust feature of recessions is that micro-economic uncertainty increases during these episodes. Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) documents that during the recent crisis, the cross-sectional variance of plant level TFP shocks increased by about 76%, while the cross-sectional variance of output growth increased by 156%[2]. In this paper I show that slow recovery is an equilibrium outcome in an economy with an agency friction which worsens following an increase in the uncertainty of firm-level productivity. I analyze the aggregate dynamics and cross-sectional implications of this agency friction between investors and firm insiders in a general equilibrium model of investment.

My model describes a continuum of firms exposed to aggregate and firm-specific productivity shocks. The Modigliani-Miller theorem does not hold because of an agency friction – investors do not observe firm output and firm insiders could divert it for their private benefit[3]. In equilibrium, investors offer insiders an incentive contract which is subject to constraints induced by the moral hazard friction. Under the optimal contract, firms can sometimes be liquidated. This is necessary to provide ex ante incentives to the insiders, but liquidations are inefficient ex post – investors recover only a fraction of the firm value while insiders receive their outside option. I model the aggregate shock as a permanent increase in the cross-sectional variance of firm-specific productivity of all firms in the economy. I show that

[1]While there is debate on the extent to which financial frictions played a role in the recent crisis, evidence of binding constraints during this period is well documented. Campello, Graham, and Harvey (2009) surveyed chief financial officers of US firms and found that 86% of constrained firms cut back on investing in attractive projects in 2008, compared to only 44% during normal times.

[2]Stock and Watson (2012) find evidence that the Great Recession was characterized by an increase in micro-level uncertainty along with a financial shock.

[3]Insiders in my model are individuals or groups with significantly higher control rights than cash-flow rights.

the agency friction has a first-order quantitative effect on equilibrium dynamics in my model.

I derive two sets of results for aggregate dynamics and one result for the term structure of interest rates. First, output, consumption, and investment experience a large drop upon impact. Second, this drop in aggregate quantities is followed by a slow recovery along the transition path. Output growth stays low and takes a long time to recover. This slow recovery is due to a large decline in the number of young firms in the economy. My model features a risk-averse representative investor whose consumption smoothing motive determines the equilibrium path of interest rates. Immediately after realization of the shock, the slope of the term structure of interest rates decreases.

In contrast to prior studies which typically fixes the type of financial contract (typically single period debt), I allow investors and firm insiders to enter into long-term contracts which are optimal, given the agency friction. This approach provides insight into the effect of aggregate shocks on contract features. I obtain two results on this front. First, the hazard rate of younger firms increase disproportionately more than older firms. Second, the cross-sectional variance of output growth increases as a result of this systematic increase in the dispersion of firm-level productivity.

The large drop in aggregate quantities followed by a slow recovery is the consequence of a worsening of the agency problem. An increase in uncertainty of firm-level productivity makes it harder for the investor to provide incentives to firm insiders to take value maximizing actions. For firms which are close to the liquidation region, incentives are provided using the threat of liquidation. An increase in the variability of firm-level output necessitates an endogenous increase in the liquidation probability. On average, older firms are less exposed to this risk because firms in their cohort with low output realizations were liquidated. This survivorship bias, results in older, surviving firms to have superior past performance than younger firms. As a result, after the shock, the increase in hazard rates for older firms is smaller than that of younger firms. The increase in the exit rate leads to depletion of capital stock in the economy. Output drops and stays low for several periods before recovering. This channel for slow recovery is distinct from a reduction in investment resulting from an increase in the option value of waiting after an increase in uncertainty in models with non-convex adjustment costs.

The increase in dispersion of output growth is the result of two effects. First, the equilibrium investment rate for firms with poor recent performance declines. This is because of an increase in the liquidation probability of these firms as a result of an increase in firm-specific uncertainty. Firms with above average past performance, for which the chance of liquidation is not a concern, experience an increase in investment. This is a general equilibrium phenomenon and is the result of a drop in equilibrium interest rates which increases the value of investing in these firms. In short, the cross-sectional dispersion of output growth increases after the shock.

This paper is the first to use long-term optimal contracts to analyze endogenous firm liquidations due to an increase in firm-level uncertainty in general equilibrium. There are at least two advantages in using an optimal contract. First, my results do not depend on specific assumptions about the borrowing constraints or how they change as a result of the aggregate shock. This is useful because it is a priori unclear how borrowing constraints for a heterogenous cross-section of firms would respond to an unanticipated change in the economic environment such as an increase in uncertainty. A potential worry with an exogenously assumed form of borrowing constraint is that if investors and firms were allowed to optimally respond to the financing friction by using a better contract, a higher rate of firm liquidations and the accompanying slow recovery following an increase in firm-level uncertainty would not be an equilibrium outcome. Second, alternative forms of financing helps us better connect the models to empirical observations, since in reality firms widely use lines of credit, debt of multiple maturity, interest rate derivatives and other forms of state-contingent financial securities[4].

**Related literature**

My paper relates to three existing strands of the literature. The first examines the effect of financial frictions on dynamics of aggregate macro-economic quantities and asset prices. The conventional view is that financial frictions tighten as a result of a shock to the first moment of productivity. This weakens the balance sheet of firms and as a result, their ability to borrow. The influential work of Bernanke and Gertler (1989), Bernanke, Gertler, and Gilchrist (1999), Kiyotaki and Moore (1997), and more recent results by Brunnermeier and Sannikov (2014) examine the impact of a drop in aggregate productivity on the borrowing ability of firms. All of these papers use an assumed form of financing, predominantly single-period debt contracts. The increase in borrowing constraints arise as a result of a drop in collateral value (see also Khan and Thomas (2013), Khan, Senga, and Thomas (2014)) or a drop in the recovery rate upon default (Gomes and Schmid (2009)). The latter considers multi-period debt contracts.

A more recently studied channel focusses on an increase in micro-uncertainty as the underlying driver leading to tighter financial constraints. This is the channel I consider in this paper. Bloom (2009) and Bloom et al. (2012) find that changes in micro-economic uncertainty has large aggregate effects in the US, although these papers abstract away from financial constraints and instead derive these results assuming non-convex cost of adjustment costs. Closer to my paper, Arellano, Bai, and Kehoe (2016), Christiano, Motto, and Rostagno (2014), and Gilchrist, Sim, and Zakrajsek (2014) consider the effect of an increase in firm level uncertainty on the ability of firms to raise financing. In these models firms are restricted to borrow using single-period, non-state contingent debt. In contrast, firms in my model borrow using optimal, long-term contracts. A big advantage of using optimal contracts is that it alleviates the concern that results arise from the assumed form of borrowing constraints and

[4]Sufi (2009) reports that 85% of firms in his sample obtained a line of credit. This included fully equity financed firms which held no debt.

that if firms were allowed to optimally contract, key features of aggregate dynamics such as the higher rate of firm exits would not be an equilibrium outcome. Dyrda (2016) is analyzes a model similar to my paper. However, in his model, firm exits are exogenous. In contrast, since exits play an important role in determining the growth rates of aggregate quantities, I model them as an equilibrium outcome.

The second area of research to which my paper relates is the role of new firms in shaping aggregate dynamics especially growth. Early empirical evidence that young firms have higher growth rates was provided by Evans (1987). Using micro-data in the US, Haltiwanger, Jarmin, and Miranda (2013) provide evidence of the important role played by younger firms in aggregate dynamics. Ibsen and Westergaard-Nielsen (2011) confirm this finding in Danish data. The theoretical literature which attempts to explain differences in productivity with firm age usually assumes ex-ante differences in productivity across firms. An early model is Jovanovic (1982) in which firms learn about their productivity over time. Clementi, Khan, Palazzo, and Thomas (Clementi et al.) consider a similar model and analyze the effect of a drop in mean productivity accompanied by a simultaneous increase in fixed costs. These shocks lower the number of young firms leading to drop in the growth rate of aggregate quantities. In contrast to this paper, I focus on the effect of an uncertainty shock. Moreover, all firms in my model are ex-ante identical.

The third area of research to which my paper relates is optimal dynamic contracting. The financial contracts in my model are related to the discrete-time models of DeMarzo and Fishman (2007a), DeMarzo and Fishman (2007b), and Biais, Mariotti, Plantin, and Rochet (2007) (see also DeMarzo and Sannikov (2006) for a characterization of the contract in a continuous time setting.) DeMarzo, Fishman, He, and Wang (2012), Hoffmann and Pfeil (2010), and Piskorski and Tchistyi (2010) consider the effects of persistent, publicly observable, productivity shocks on the optimal contract in a partial equilibrium setting. The key difference of my paper with these studies is that while these papers examine the optimal contract in a partial equilibrium setting with an exogenously assumed discount rate, I provide a general equilibrium perspective where the discount rate investors use to value future cash flows is an endogenous equilibrium outcome. The latter arises from consumption smoothing motives of the risk-averse representative investor. Prior literature considers optimal contracts for other sources of financing frictions in a general equilibrium setting. Cooley, Marimon, and Quadrini (2004) analyze the implications of limited contract enforcement, while Dow, Gorton, and Krishnamurthy (2005) and Albuquerue and Wang (2008) model the effects of a free cash-flow problem on investment and state prices. Apart from the difference in the underlying source of financing friction, none of these papers consider the impact of an increase in uncertainty, which is the focus of my paper.

The rest of this paper is organized as follows. In Section 2, I describe the model. In Section 3, I discuss features of the optimal contract and provide a partial equilibrium perspective of the consequences of an increase in the uncertainty of firm-level productivity. In Section 4, I quantitatively analyze the aggregate dynamics and the cross-sectional behavior of firms in general equilibrium. Section 5 concludes.

# 2 The Model

The economy is populated by two kinds of agents: a representative investor and a continuum of firm insiders. Each insider has the sole expertise to operate a production technology, but has insufficient wealth to pay for the initial installation cost of the firm. Each insider, therefore, enters into a long term financing arrangement with the representative investor. Borrowing constraints arise endogenously because of a moral hazard problem: investors do not observe firm revenue which insiders could divert for private consumption. The financial contract provides incentives so that insiders do not steal. These incentives take the form of current and future promised payments and also the threat of liquidation. I begin by describing the optimal contract between the representative investor and each firm insider, followed by a description of the representative household's problem. I close the section with a description of the general equilibrium.

## 2.1 Production and Financing

There is a continuum of ex-ante identical firms. Each firm produces a single homogenous good which can be used both for consumption and investment. Each firm is operated by a risk-neutral insider who relies on external financing to operate a decreasing returns to scale technology which produces output $y = Rk_\kappa^\nu$ where $k_\kappa$ is the capital stock of the firm at the beginning of the period and $0 < \nu < 1$ is the returns to scale parameter. The firm-specific shock $R$ takes on two values $R_1 > R_2$. I assume that $R$ is independently distributed in the cross-section and across time. The friction in this economy is an agency problem which arises because the investor does not observe firm output. She has to rely on reports made by the firm insider. The two parties respond to the agency problem by entering into a long-term financial contract and commit to following the terms of the contract in every possible state with no possibility of renegotiation[5].

The insider in this economy has limited liability, does not participate in financial markets, and has an outside option normalized to zero. Since the insider cannot diversify firm-specific risk, his valuation of cash flows will be lower than the representative investor who is able to pool cash flows across all firms and diversify firm-specific risk. To capture this, I assume that the risk-neutral insider's time-preference parameter $\beta_A$ is smaller than the investor's time-preference parameter $\beta_P$.

I use the dynamic programming approach of Spear and Srivastava (1987) and Green (1987) to solve for the optimal contract. In this recursive formulation, the present discounted value of the future payments (or continuation value) to the insider is a sufficient statistic for

[5]While this is a restrictive assumption, it provides a useful benchmark and can be thought of as a limiting case where renegotiation is extremely costly as would be the case if the investors consist of a large dispersed pool of individuals.

the entire past history of the insider's reports of realizations of firm output. I denote the investor's continuation value by $V$. The financial contract specifies the payments made to the investor, the investment or disinvestment rate, and the probability with which the contract will be terminated. These policies are a function of the past history of the insider's reports of firm output (captured by the insider's continuation value) and the prevailing interest rate. Figure 1 divides the problem into sub-periods and outlines the notation used. I use backward induction within each period starting with the investment decision which takes place at the end of that period.

**Investment**

Firm size is discrete in this economy. An individual firm's capital stock $k_\kappa$ can lie on one of $N_k+1$ grid points: $k_\kappa \in k_l\{1, e^{\Delta}, e^{2\Delta}, \cdots, e^{N_k\Delta}\}$. $\Delta$ measures the spacing between grid points[6]. An existing firm can increase its capital stock by choosing the probability $p_1$ of increasing its size to the adjacent higher grid-point $k_{\kappa+1} = k_l e^{(\kappa+1)\Delta}$. This amounts to choosing an expected investment rate $\iota_1(p_1) = p_1(e^{\Delta}-1)$. Firms can also disinvest by choosing the probability $p_2$ of lowering firm size to the immediate lower level resulting in an expected disinvestment rate of $\iota_2(p_2) = p_2(e^{\Delta}-1)$. Investment (disinvestment) costs are quadratic in the expected investment (disinvestment) rate: $c(i_{1,2}) = \theta\iota_{1,2}^2$. A firm with the maximum possible capital stock $k_l e^{N_k\Delta}$ cannot invest. A firm with $k_\kappa = k_l$ can only disinvest by liquidating. The investor chooses the probability to invest and disinvest to maximize the present value of the contract

$$
\begin{aligned}
F_t^I\big(k_\kappa, V_t^I, r_t\big) &= \max_{p_1,p_2}\Big[-\iota_1(p_1)k_\kappa - c(p_1)k_\kappa + p_1F_t^e\big(k_{\kappa+1}, V_t^e, r_t\big) + (1-p_1)F_t^e\big(k_\kappa, V_t^e, r_t\big) \\
&- \iota_2(p_2)k_\kappa - c(p_2)k_\kappa + p_2F_t^e\big(k_{\kappa-1}, V_t^e, r_t\big) + (1-p_2)F_t^e\big(k_\kappa, V_t^e, r_t\big)\Big], \\
\{p_1, p_2\} &\in [0,1]\times[0,1], \qquad (1)
\end{aligned}
$$

[6]The continuous $k$ limit corresponds to $\Delta \to 0$.

where $F_t^e(k_\kappa, V_t^e, r_t)$ is the contract's value at the end of this period $t$. The choice variables $p_{1,2}$ and contract values depend on the current capital stock of the firm $k_\kappa$, the insider's continuation value and the current interest rate $r_t$. Next I turn to the agency problem.

### Agency problem

The agency problem arises because investors do not observe firm output and have to rely on the report of insiders who can potentially lie and divert firm output[7]. Diversion is inefficient. For every unit of output diverted, insiders are left with a fraction $0 \leq \lambda \leq 1$ for their private consumption. In response to this moral hazard problem, the representative investor and the firm insider enter into a long-term contract and agree to abide by its terms in all possible contingencies, with no possibility of renegotiation. In this sub-period, the investor uses the contract to dissuade the insider from stealing by providing incentives which take the form of current and future promised payments and also the threat of liquidation. The investor's problem in this sub-period is

$$
\begin{aligned}
F_t^y\big(k_\kappa, V_t^y, r_t\big) &= \max_{\zeta, d_{1,2}, V_{1,2}^I} \zeta \chi k_\kappa \\
&+ (1-\zeta)\Big[p\big(d_1 + F_t^I(k_\kappa, V_1^I; r_t)\big) + (1-p)\big(d_2 + F_t^I(k_\kappa, V_2^I; r_t)\big)\Big] \\
V_t^y &= (1-\zeta)\left[p\big(R_1 k_\kappa^\nu - d_1 + V_1^I\big) + (1-p)\big(R_2 k_\kappa^\nu - d_2 + V_2^I\big)\right] \\
R_1 k_\kappa^\nu - d_1 + V_1^I &\geq R_2 k_\kappa^\nu - d_2 + V_2^I + \lambda(R_2 - R_1)k_\kappa^\nu\,, \\
d_1 &\leq R_1 k_\kappa^\nu\,, \quad d_2 \leq R_2 k_\kappa^\nu \\
\left\{\zeta, V_{1,2}^I\right\} &\in [0,1] \times \mathbb{R}_+^2\,. \qquad (2)
\end{aligned}
$$

In the above equations, the choice variables are the liquidation probability of the firm $\zeta$, the investor's share of firm output $d_1$ and $d_2$ when the insider reports high output $R_1$ and low output $R_2$ respectively. The insider's continuation value is also adjusted: $V_1^I$ if the insider

[7] Investors cannot observe the consumption of insiders.

reports a high realization of the shock $R_1$, or to $V_2^I$ if he reports that $R_2$ is realized.

The first equation in ( 2) represents the investor's payoffs. The first possibility is liquidation. At the beginning of each period, a public lottery is held, and the firm is liquidated with probability $\zeta$. In this case, the investor receives a fraction $\chi$ of the physical assets of the firm. The insider receives his outside option which is normalized to zero and exits the economy. Conditional on survival, the firm produces either high or low output. If the insider reports high output, the investor is paid $d_1$. If he reports low output, the investor receives $d_2$. The second equation above is the promise-keeping constraint which ensures that the insider's continuation value is the expected value of his future payments. I have implicitly assumed that if the contract is terminated, the insider receives his outside option of zero (his future consumption is set to zero). The third equation is the incentive compatibility constraint and ensures that the insider has no incentive to steal by mis-reporting. In this equation, $\lambda$ represents the efficiency with which the insider can divert firm output. From this equation we see that the severity of the agency problem depends on the product of the parameter $\lambda$ and the volatility of firm output $R_1 - R_2$. Finally, the fourth pair of inequalities in ( 2) represent the insider's limited liability condition.

**Intra-period**

The representative investor discounts future cash flows from each contract using her discount rate process which is her marginal utility process evaluated at the equilibrium aggregate consumption level[8]. Since the representative investor has constant relative risk-aversion $\gamma$, her marginal utility process, $M_{t+1}/M_t = \beta_P \left(C^\star_{t+1}/C^\star_t\right)^{-\gamma}$. The quantity, $C^\star$ is the equilibrium

[8]Since the representative household holds a well-diversified portfolio consisting of all the contracts and a risky-free asset in zero net supply, only systematic shocks are priced. Alternatively, we could picture identical, individual investors each entering into a long-term contract with a single insider. In the latter case, individual investors act as pass-throughs – they collect payments and pass them on to the representative household. Since they belong to the same risk-sharing household, all investors use the same discount rate to value cash flows. Both these pictures have identical pricing implications: the idiosyncratic component of cash flows of individual contracts can be completely diversified away, and are therefore not priced.

aggregate consumption of the investor, and is determined by her consumption and savings decision.

$$
\begin{aligned}
F_t^e(k_\kappa, V_t^e; r^t) &= \frac{1}{1+r_{t+1}}\Big(\pi^d \chi k_\kappa + (1-\pi^d)\big[p^\delta F_{t+1}^y\big(k_{\kappa-1}, V_{t+1}^y, r_{t+1}\big) \\
&+ (1-p^\delta) F_{t+1}^y\big(k_\kappa, V_{t+1}^y, r_{t+1}\big)\big]\Big) \\
V_t^e &= \beta_A (1-\pi^d) V_{t+1}^y \,. \qquad (3)
\end{aligned}
$$

In the above equation, $\pi^d$ is the probability with which the firm might be liquidated for exogenous reasons. This is to model the fact that not exits are related to financial frictions. For simplicity, I assume that this shock is independent of firm profitability and the state of the economy. If a firm is liquidated, investors recover the liquidation value $\chi k_\kappa$ while insiders receive their outside option (normalized to zero). Conditional on survival, the capital stock of the firm $k_\kappa$ depreciates with probability $p^\delta = \delta/(1-e^{-\Delta})$ to the adjacent lower grid-point $k_{\kappa-1} = k_l e^{(\kappa-1)\Delta}$. This corresponds to an expected rate of depreciation of $\delta$. The capital stock of the smallest firms (those with size $k_l$) does not depreciate. These firms can only disinvest by liquidating the firm. The second equation is the promise-keeping constraint and ensures that the insider's continuation value is the expected value of his future payments.

It is important to note from Equation 3 that the contract's value depends on the future path of interest rates. In the steady-state, the interest rate will turn out to be constant. However, for the transition dynamics analysis, the contract value and therefore contract policies will depend on this path. In equilibrium, agents are able to correctly guess the future evolution of the interest rate and the realized path of interest rates will coincide exactly with their conjectured path. Since I consider a permanent shock, agents have perfect foresight and given an initial distribution of firms, they will be able to compute the exact future path of interest rates. The exact computational procedure is outlined in the Appendix.

Timing in this economy is as follows. At the beginning of each period, a public lottery for termination of the contract is held in which firms with low continuation values might exit. Surviving firms produce output using the capital stock installed in the previous period net of any depreciation. Insiders are paid, investment or disinvestments are made, and the investor consumes.

**Firm entry**

Each period, there is a mass of potential firms ready to enter the economy[9]. To begin production, a one-time fixed cost $\tilde{e}k$ has to be paid. This random cost is drawn from a uniform measure $H$, and is revealed at the beginning of each period. To ensure balanced growth, I assume that the mass of potential entrants is proportional to the measure of existing firms: $H = hN_t$, where $N_t = \int d\mu_t$ is the total number of firms in existence at the beginning of period $t$ before any entry and exit in that period. The constant of proportionality $h$ is a measure of the investment opportunity set. A new firm becomes operational if the insider is able to secure financing. The contract pays for the initial installation cost, and in subsequent periods, conditional on continuation, the investor commits to providing capital for investment, and compensating the insider. I assume that the representative investor has all the bargaining power, so she chooses the insider's initial continuation value and initial firm size to maximize the value of her contract

$$V_0 = \arg \max_{k_\kappa, V \in \mathcal{R}_+} \left\{ F(k_\kappa, V, r_t) \right\}. \tag{4}$$

The decreasing returns to scale assumption ensures that firms start out from the smallest possible size. Production begins from the period subsequent to entry. Projects with high entry costs are unable to secure funding and expire worthless.

[9]I borrow this modeling technique from Gomes, Kogan, and Zhang (2003)

**Aggregation**

Each existing firm produces output $y = Rk_\kappa^\nu$ so total output in this economy is given by $Y_t = \sum_j y_t^j$. The summation is over surviving firms and is over the cross-sectional distribution of $k$ and $V$. Firm insiders of these firms are paid $\mathcal{T}_t = \sum_j Rk_\kappa^\nu - d_{j,t}$ in aggregate. In this economy, there are three components of aggregate investment. First, existing firms invest or disinvest resulting in an aggregate cost of $I_t^e = \sum_j \theta\iota_{1,j}^2 k_j + \iota_{1,j}k_j + \theta\iota_{2,j}^2 k_j - \iota_{2,j}k_j$. Second, there is disinvestment from firm liquidation which in aggregate is $I_t^l = \sum_{j\in\text{Liq}} \chi k_{j,t}$. Third, there is investment along the extensive margin from new firm entry $I_t^n = \int_0^{\bar{e}} dH = \frac{h}{2}\bar{e}^2 N_t$, where $N_t$ is the number of existing firms at the beginning of period $t$. Total investment is the sum of these three components $I_t = I_t^e - I_t^l + I_t^n$. The representative investor receives

$$D_t = Y_t - I_t - \mathcal{T}_t \tag{5}$$

in aggregate each period which includes output net of aggregate investment and compensation to firm insiders.

## 2.2 The Representative Household and Aggregation

I assume that the economy is populated by a single representative household. Individual investors are assumed to be members of this single risk-sharing household and therefore, they all share the same stochastic discount factor. This household derives utility from consumption of the single good $C_t$ and has standard time-separable power utility

$$E_0 \sum_{t=0}^{\infty} \beta_P^t \frac{C_t^{1-\gamma_l}}{1-\gamma_l}, \tag{6}$$

where $\gamma_l$ is the household's risk-aversion, and $\beta_P$ is the time preference parameter. The household has access to complete financial markets and derives income from accumulated wealth $W_t$. The household makes consumption and investment decisions to maximize expected lifetime utility subject to her budget constraint

$$E_t\Big[\sum_{s=0}^{\infty} M_{t,t+s}C_{t+s}\Big] \leq W_t\,. \tag{7}$$

Optimality of the household's decision implies the standard expression for the investor's discount rate process $M_{t,t+s}$

$$M_{t,t+s} = \beta_P \left(\frac{C_{t+s}}{C_t}\right)^{-\gamma}. \tag{8}$$

## 2.3 General Equilibrium

Given a sequence of interest rates $\{r_t\}$, the optimal contract policy of a firm with capital $k$ and insider continuation value $V$ at time $t$ are functions of $k$ and $V$[10]. The probability of investment/dis-investment are $p_{1,2}(k, V)$, the liquidation probability is $\zeta(k, V)$, the insider's payments when he reports high/low output are $d_{1,2}(k, V)$, and the adjusted continuation value following reports of high/low output are $V^I_{1,2}(k, V)$. Given the realization of this period's firm-specific shock, next period's continuation values and the capital stock are completely determined.

I denote the joint distribution of capital stock and continuation values in the cross-section by $\psi_t(k, V)$. The optimal contract policies for investment and adjustment of continuation values determines the transition probability for individual insider's states $(k, V)$ for each realization of firm-specific shock. Finally, the law of large numbers ensures that we do not need to keep track of individual realizations of the firm-specific shock , but rather the policies of the optimal contract for investment and adjustment of continuation values determines the distribution $\Psi_{t+1}$, given the period $t$ distribution $\Psi_t$. I can now define the equilibrium.

**Definition 1** *A recursive competitive equilibrium is defined as a sequence of interest rates $\{r_t\}$, a sequence of contract policies $\{p_{1,2}(k, V^I, t), \zeta(k, V^y, t), d_{1,2}(k, V^y, t), V^I_{1,2}(k, V, t)\}$ for*

[10]In this section, I suppress the super-script which labels the sub-periods within period $t$.

*existing firms, the initial capital stock $k_0$ and initial continuation value $V_0$ with which new contracts are initiated, the consumption policies of the representative household $C_t$, and a sequence of distributions for capital and continuation values $\{\Psi_t\}$ such that, given the initial distribution $\Psi_0$:*

*(i) individual contracts are optimal,*

*(ii) the initial state is determined by (Eq. 4),*

*(iii) the representative investor's policies are optimal according to Eq. 1 and Eq. 2 subject to her budget constraint, Eq. 7*

*(iv) the goods market clears $C_t = D_t$*

*(v) the market for contracts clears*

*(vi) contract policies correctly anticipate the law of motion of the future evolution of the distribution $\Psi_{t+1}$.*

A sketch of the numerical approach used to compute both the steady-state equilibrium and the transition dynamics is described in the Appendix.

# 3 Solution Features

In this section I first discuss features of the optimal contract in the steady-state. Next, I provide a partial equilibrium analysis of the consequences of an increase in the uncertainty of firm-specific productivity. This will provide intuition for the general equilibrium results discussed in Section 4.

The economy discussed in Section 2 admits a steady-state equilibrium[11]. To see this, suppose the economy starts from an initial distribution of firm-size and continuation value, $\Psi_0$, in which the initial growth rate of consumption of the representative investor is higher than the constant rate along the balanced growth path. During this period, the equilibrium

[11]The proof is similar to the entry-exit equilibrium of Hopenhayn (1992).

interest rate will be higher than the steady-state value. This will result in lower aggregate investment which will slow down the growth rate of the representative investor's consumption till it equals the steady-state value.

The steady-state equilibrium features constant entry and exit rate of firms. Aggregate output, investment, and consumption grow at a constant rate; the risk-free rate is also constant. Contract policies such as liquidation, insider-payments, and investment rates are time-independent functions of firm-size and the insider's continuation value. The optimal contract in the steady-state is very similar to DeMarzo and Fishman (2007a)[12].

Incentives are provided to the insider either by higher levels of investment in the firm, direct payments, promises of future payments, or the threat of liquidation. Figure 2 plots the policies for a fixed firm size. At very high continuation values, the agency problem disappears because the insider is promised a large portion of future firm output. Since I assume the insider to be more impatient than the outside investor, in this region it becomes costly to delay payments to the insider. As a result, incentives are provided by direct payment. For lower, intermediate continuation values, incentives are provided by a spread in future promises. Panel A shows that reports of high output leads to an increase in the continuation value of the insider (solid, blue line), while low output leads to a reduction (dotted, red line). The continuation value of the insider therefore, summarizes past performance.

At very low continuation values, the insider's limited liability makes it becomes impossible for the investor to provide the spread in continuation values necessary to dissuade the insider from diverting firm output. In this region, incentives are provided using the threat of liquidation. Panel B shows the liquidation policy for a fixed firm size. A firm with very low continuation value has a non-zero liquidation probability which increases as the continuation value drops to zero. Although liquidations are ex-post inefficient, they are necessary to

[12] The investor's value functions $F^I(k, V^I)$, $F^y(k, V^y)$, and $F^e(k, V^e)$ are time-invariant (weakly) concave functions of the insider's continuation value. Since the proof is very similar to DeMarzo and Fishman (2007a), I omit it for brevity.

provide incentives. Since continuation values measure how far the firm is from the liquidation region, it can be interpreted as the financial slack of a firm. Indeed, implementations of the optimal contract using realistic securities, use measures of financial slack such as unused lines of credit (see DeMarzo and Fishman (2007b)) or cash reserves (Biais et al. (2007)) of a firm to capture the insider's continuation value[13].

Panel C shows the investment rate for a fixed firm-size. It is an increasing function of the insider's continuation value except for the region that is close to the liquidation region[14]. With a higher chance of firm liquidation, the investor is reluctant to invest. Following good performance, investors reward insiders by increasing the continuation value which lowers the threat of liquidation. The incentives of the insider are now better aligned with the investor which loosens the borrowing constraint. The agency problem is relaxed and the investment rate increases. Panel D shows the distribution of continuation values for firms of this size in the steady-state.

To provide intuition for the effect of an increase in uncertainty on aggregate quantities, I perform a partial equilibrium analysis and compare firm liquidation rates and investment rates in the baseline economy with another one with higher firm-specific volatility of productivity. In the economy with heightened uncertainty, the firm-specific output volatility parameter $\sigma$ is twice the value in the baseline economy. The two economies are otherwise identical. Figure 3 shows the results. Panel A compares the liquidation policies in the two economies as a function of the insider's continuation value scaled by firm size. Liquidation rates are higher in the economy with higher firm-specific volatility for all continuation values. An increase in firm-specific uncertainty makes the agency problem more severe. To provide incentives in

[13]The only instance for which I will rely on an implementation of the contract is in choosing the value of the product $\lambda\sigma(R)$ which determines the strength of the agency problem. No other result presented in this paper depends on the particular form of implementation. Therefore, I do not discuss implementation of the contract using realistic securities. The interested reader is referred to these papers.

[14]For very low levels of slack, close to the liquidation region, investment is a decreasing function of continuation value because the investor's downside is limited to recovering the liquidation value while investing increases the upside.

the economy with higher $\sigma$, the investor needs to provide a bigger spread in continuation values between reports of high and low output. The critical value below which it becomes impossible to incentivize the insider by a spread in future promised payments is therefore higher in the economy with higher uncertainty. As a consequence, the investor has to resort to the threat of liquidating the firm at a higher continuation value compared to the baseline economy. This higher risk of liquidation depresses the investment rate as seen from Panel B.

While Panels A and B compare policies as a function of the history of past output of firms, it is interesting to compare the difference in hazard rates of firms as a function of firm age in the two economies. The model predicts that younger firms experience a bigger increase in hazard rate when firm-level uncertainty increases. Panel C shows shows the difference in quarterly hazard rates. We see that the hazard rates are higher for firms of all ages in the economy with higher uncertainty. However, younger firms experience the biggest increase. This is due to a survivorship bias. Firms which receive low output realizations are liquidated. Older, surviving firms have had better past realizations of output and therefore, compared to younger firms they have higher continuation values on average. This is shown in Panel D.

# 4 Quantitative Analysis

## 4.1 Parameters and Calibration

I calibrate the model at quarterly frequency. I set some of the parameters to standard values used in the literature and calibrate the remaining parameters to approximately match the risk-free rate and moments of investment and cash-ratios in the cross-sectional. I report all parameters in Table 1 and a comparison of simulation moments along with their data counterparts in Table 2.

The investor's preference parameters are chosen to match the risk-free rate which depends on the investor's time-preference parameter, the steady-state growth rate of consumption, and the investor's risk-aversion. In the baseline model, I choose the investor's coefficient of relative risk aversion $\gamma = 2$. For this value of risk aversion, I choose the investor's time preference parameter $\beta_P = 1.005$ to target the mean risk-free rate. Even though $\beta_P > 1$, since this is an economy with growth, prices are well-defined (see Kocherlakota (1990)). I choose the insider's time preference parameter to be lower than the aggregate investor. This is justified because firm insiders in my model cannot diversify firm-specific risk. Their valuation of the firm is lower than the investor's. Studies by Silber (1991) and Longstaff (1995) estimate this discount to be in the range of 35–40%. I choose a conservative value of 15% for this discount which implies $\beta_A = 0.969$.

I set the decreasing returns-to-scale parameter $\nu = 0.6$ following Cooper and Haltiwanger (2006). The model implied relative firm size as measured by the ratio of the inter-quintile range of the size distribution to the median size is 0.73. For firms in COMPUSTAT, this number is about 0.83. I set the depreciation parameter $\delta = 1.75\%$ which corresponds to an annual depreciation rate of 7%. This is in line with commonly used values (Cooper and Haltiwanger (2006) use an annualized depreciation rate of 6.9%). I choose the parameter $h = 0.20\%$ to match the steady-state growth rate of per-capita consumption in the US. I choose the quadratic adjustment cost parameter $\theta = 7.5$, the mean of the cash-flow process $E(R) = 0.98\%$ and the product $\lambda\sigma(R) = 0.67\%$. These parameters are chosen to target the mean and standard-deviation of investment rate of firms in the cross-section, the mean and standard-deviation of the ratio of cash flow to firm assets, and mean and dispersion of the cash ratio of firms as measured by the inter-quintile range. Note that model predictions depend on the product $\lambda\sigma(R)$, not on the individual values of $\lambda$ and $\sigma(R)$. This product has a first-order effect on the equilibrium distribution of continuation values in the cross-section. I follow Biais et al. (2007) who implement the optimal contract with cash playing the role of $V$ and therefore target the mean and dispersion (as measured by the inter-quintile range) of cash ratios of firms. Although the average cash ratio of COMPUSTAT firms over the entire sample period starting from 1980 is 0.155, I target a higher rate for two reasons. First, Bates, Kahle, and Stulz (2009) document a steady increase in cash ratios of US firms and find the average to be 0.232 in 2006. Since I am interested in quantifying the response of the US economy to an increase in uncertainty at the onset of the recent financial crisis in 2007, a higher value is more appropriate. Second, Bates et al. (2009) report that for firms in the smallest quintile of their sample, the average cash ratio is more than 0.35. Since the biggest effect of an increase in the intensity of the agency friction is felt by smaller, younger firms, I choose $\lambda\sigma(R)$ so that the average cash-ratio is closer to the pre-crisis value in small firms. I choose the recovery rate upon liquidation $\chi = 0.85$. Finally, I choose the exogenous exit rate of firms $\pi^d = 1.0\%$ to target the exit rate of firms in the data Siemer (2012).

## 4.2 The Effect of an Increase in cash-flow volatility

In this section I quantify the effect of a permanent increase in the volatility of firm-specific productivity. Bloom et al. (2012) documents evidence of this shock. The increase in output volatility makes the agency problem more severe. I compute the equilibrium dynamics of aggregate quantities, interest rates, and changes in contract policies along the transition path. Details of the numerical implementation is provided in the Appendix. I use the steady-state distribution of capital and continuation values as the initial distribution for all simulations.

**Slow Recovery**

I consider a one-time permanent doubling of cash-flow volatility $\sigma(R)$. This is within the estimates obtained by Bloom et al. (2012). Panel A of Figure 4 shows that aggregate output growth drops by more than 10% from the steady-state growth rate. Recovery from this low growth phase is extremely slow. Even after 4 years, output growth is still 2.5% below the steady-state value. The reason for the large drop in the growth rate and the subsequent anemic recovery is an increase in the exit rate of firms. Panel B shows that the exit rate increases by 3.2% (annualized) upon impact of the shock and continues to increase. At its peak, the exit rate is about 7.2% higher than the pre-shock level. Siemer (2012) reports using Business Dynamics Statistics data that the 2007-2009 recession was marked by a 15% increase in the exit rate of firms over eight quarters. I obtain a lower exit rate since my model abstracts away from factors such as an increase in risk-premia or drop in average productivity (first-moment shock) which would further increase firm exits.

The higher rate of liquidations immediately upon arrival of the shock is due to the necessity of making the insider's continuation value more volatile in the new regime. This increases the probability of liquidation for firms with low continuation values. Firms which realize low output are liquidated. As time goes by, firms which realize a good sequence of shocks move sufficiently far away from the liquidation region and the aggregate exit rate slowly drops. The time it takes for the exit rate to decline to the steady-state value in the new equilibrium depends on the cross-sectional distribution of continuation values at the time of arrival of the shock. A higher mass of firms near the liquidation boundary results in a significantly stronger propagation of the shock.

Panel C of Figure 4 shows the drop in the one-period interest rate after the arrival of the shock. This arises because of the representative investor's consumption smoothing desire. Since she anticipates a decline in consumption over the next several periods her desire to save increases. This leads to a drop in the equilibrium interest rate. Panel C shows that the drop in interest rate is persistent and in the quantitative exercise, rates continue to drop for about 2 years. Panel D shows the term structure of long term interest rates. We see that the model predicts that the drop in medium term rates is much bigger than the drop in the short end of the curve, so that the term structure of short to medium term rates flattens out.

Figure 5 shows the drop in the level of aggregate quantities as a result of the shock. Panel A shows that aggregate output drops immediately upon impact and continues to drop compared to the level immediately prior to the impact of the shock. At its lowest point, output is about 8% below pre-shock levels. The large decline in output is due to the depletion of capital stock in the economy shown in Panel B. An increase in liquidation rate of firms leads to about an 11% decline in capital. Panel C shows that aggregate investment by existing firms drops by 70% upon impact of the shock. Aggregate consumption increases by about 0.6% immediately upon impact of the shock before declining by a maximum of about 2.8% (annualized) from the pre-shock level. The magnitude of these drops are consistent with those observed during the Great Recession.

**Cross-sectional dispersion of output growth**

Bloom et al. (2012) reports a three-fold increase in the variance of sales growth at the onset of the Great Recession. My model generates this pattern. Figure 6 shows this increase. The maximum increase is about 0.036 which is smaller than the value of 0.079 reported by Bloom et al. (2012) in the data. I conjecture that adding non-convex adjustment costs for investment will increase this dispersion.

The increase in variance of output growth is the result of an increase in the dispersion of investment rates. Figure 7 shows the drop in the investment rate for an average sized firm. Panel A shows the change in investment rate relative to the steady-state equilibrium immediately upon impact of the shock. We see that the largest drops are for firms with low $V$. The intuition behind this is that firms which are close to the liquidation region have a high chance of exiting. Investors are therefore reluctant to invest in such firms. Firms far from the liquidation region, i.e. those with higher continuation value, do not have this concern. These firms, in fact, invest more than in the old, steady-state equilibrium. The latter is a general equilibrium effect. With perfect foresight, the representative investor anticipates a drop in her consumption in the near future due to the higher exit rate following the shock. This increases her desire to save. As a result, the equilibrium interest rate drops. Compared to firms closer to the liquidation region, firms which have experienced good past performance have higher continuation values. These firms increase in value and they experience an increase in investment compared to the steady-state equilibrium. As time passes, the exit rate of firms in the economy starts to decline to the level in the new equilibrium. Interest rates increase and as shown in Panel B, the excess investment by firms with good recent performance almost disappears two years after the original shock.

I conclude by analyzing the composition of firms exiting the economy. Most of the firms which are liquidated immediately when the shock hits are young. This is mainly because most of the firms in this economy are young. In the current calibration, about 40% of the firms are less than five years old. In addition, as Figure 8 shows, younger firms experience the biggest increase in hazard rate.

# 5 Conclusion

In this paper I analyze the effect of an increase in the dispersion of firm-level productivity in the presence of an agency friction, in a general equilibrium framework. In contrast to existing literature, I allow firms and investors to enter into long-term contracts which are optimal given the agency friction. I show that upon impact of the aggregate shock, output and investment drops. This is followed by a very slow recovery due to a large decline in the number of young firms in the economy. These features are in agreement with empirical evidence following the financial crisis of 2007–2009. A key feature of my analysis is that the borrowing constraints are optimal.

There are several avenues for future research. The most promising is to incorporate transitory aggregate shocks to analyse the dynamics of risk-premia. This will however require tracking the cross-sectional distribution of financial slack of firms in the economy. Unlike the results of this paper, incorporation of transitory aggregate shocks will have to be based on approximating the cross-sectional distribution by a lower dimensional proxy. I leave this for future work.

# References


Albuquerue, R. and N. Wang (2008). Agency conflicts, investment, and asset pricing. *Journal of Finance 63*(1), 1 – 40.

Arellano, C., Y. Bai, and P. J. Kehoe (2016). Financial frictions and fluctuations in volatility. Staff Report 466, Minneapolis Fed Research.

Bates, T., K. Kahle, and R. Stulz (2009). Why do u.s. firms hold so much more cash than they used to? *Journal of Finance 64*(5), 19852021.

Bernanke, B. and M. Gertler (1989). Agency costs, net worth, and business fluctuations. *The American Economic Review 79*(1), 14 – 31.

Bernanke, B., M. Gertler, and S. Gilchrist (1999). The financial accelerator in a quantitative business cycle framework. *Handbook of Macroeconomics 1*(C), 1341 – 1393.

Biais, B., T. Mariotti, G. Plantin, and J.-C. Rochet (2007). Dynamic security design: Convergence to continous time and asset pricing implications. *The Review of Economic Studies 74*(2), 345 – 390.

Bloom, N. (2009). The impact of uncertainty shocks. *Econometrica 77*(3), 623–685.

Bloom, N., M. Floetotto, N. Jaimovich, I. Saporta-Eksten, and S. J. Terry (2012). Really uncertain business cycles. Working Paper 18245, National Bureau of Economic Research.

Brunnermeier, M. and Y. Sannikov (2014). A macroeconomic model with a financial sector. *The American Economic Review 104*(2), 379 – 421.

Campbell, J. Y., A. W. Lo, and A. C. MacKinlay. The econometrics of financial markets.

Campello, M., J. Graham, and C. R. Harvey (2009). The real effects of financial constraints: Evidence from a financial crisis. Working Paper 15552, NBER.

Christiano, L. J., R. Motto, and M. Rostagno (2014). Risk shocks. *American Economic Review 104*(1), 27–65.

Clementi, G. L., A. Khan, B. Palazzo, and J. K. Thomas. Entry, exit and the shape of aggregate fluctuations in a general equilibrium model with capital heterogeneity. Technical report.

Cooley, T., R. Marimon, and V. Quadrini (2004). Aggregate consequences of limited contract enforceability. *The Journal of Political Economy 112*(4), 817 – 847.

Cooper, R. W. and J. C. Haltiwanger (2006). On the nature of capital adjustment costs. *Review of Economic Studies 73*, 611633.

DeMarzo, P. and M. Fishman (2007a). Agency and optimal investment dynamics. *Review of Financial Studies 20*(1), 151 – 188.

DeMarzo, P. and M. Fishman (2007b). Optimal long-term financial contracting. *Review of Financial Studies 20*(1), 2079 – 2128.

DeMarzo, P. M., M. J. Fishman, Z. He, and N. Wang (2012). Dynamic agency and the q theory of investment. *The Journal of Finance 67*(6), 2295 – 2340.

DeMarzo, P. M. and Y. Sannikov (2006). Optimal security design and dynamic capital structure in a continuous-time agency model. *The Journal of Finance 61*(6), 2681 – 2724.

Dow, J., G. Gorton, and A. Krishnamurthy (2005). Equilibrium investment and asset prices under imperfect corporate control. *The American Economic Review 95*(3), 659–681.

Dyrda, S. (2016). Fluctuations in uncertainty, efficient borrowing constraints and firm dynamics. Working paper, University of Toronto.

Evans, D. (1987). The relationship between firm growth, size and age: Estimates for 100 manufacturing firms. *Journal of Industrial Economics XXXV*, 567 – 581.

Gilchrist, S., J. W. Sim, and E. Zakrajsek (2014). Uncertainty, financial frictions, and investment dynamics. Working Paper 20038, National Bureau of Economic Research.

Gomes, J., L. Kogan, and L. Zhang (2003). Equilibrium cross section of returns. *Journal of Political Economy 111*(4), 693 – 732.

Gomes, J. F. and L. Schmid (2009). Equilibrium credit spreads and the macroeconomy. Working paper, Duke and Wharton University.

Green, E. (1987). Lending and the smoothing of uninsurable income. Technical report, E. Prescott and N. Wallace (eds).

Haltiwanger, J., R. S. Jarmin, and J. Miranda (2013). Who creates jobs? small versus large versus young. *The Review of Economics and Statistics 95*(2), 347–361.

Hoffmann, F. and S. Pfeil (2010). Reward for luck in a dynamic agency model. *Review of Financial Studies 23*(9), 3329 – 3345.

Hopenhayn, H. (1992). Entry, exit, and firm dynamics in long run equilibrium. *Econometrica 60*(5), 1127–1150.

Ibsen, R. and N. Westergaard-Nielsen (2011). Job creation by firms in denmark. Working Paper 5458, Institute for the Study of Labor.

Jovanovic, B. (1982). Selection and the evolution of industry. *Econometrica 50*(3), 649–670.

Khan, A., T. Senga, and J. K. Thomas (2014). Default risk and aggregate fluctuations in an economy with production heterogeneity. Working paper.

Khan, A. and J. Thomas (2013). Credit shocks and aggregate fluctuations in an economy with production heterogeneity. *The Journal of Political Economy*.

Kiyotaki, N. and J. Moore (1997). Credit cycles. *Journal of Political Economy 105*(2), 211 – 248.

Kocherlakota, N. R. (1990). On tests of representative consumer asset pricing models. *Journal of Monetary Economics 26*, 285–304.

Longstaff, F. A. (1995). How much can marketability affect security values? *The Journal of Finance 50*(5), 1767–1774.

Piskorski, T. and A. Tchistyi (2010). Optimal mortgage design. *Review of Financial Studies 23*, 3098 – 3140.

Reinhart, C. M. and K. S. Rogoff (2009). The aftermath of financial crises. *American Economic Review 2*(99), 466–472.

Siemer, M. (2012). Firm entry and employment dynamics in the great recession. Working Paper 201456, Federal Reserve Board.

Silber, W. L. (1991). Discounts on restricted stock: The impact of illiquidity on stock prices. *Financial Analysts Journal 47*, 60–64.

Spear, S. and S. Srivastava (1987). On repeated moral hazard with discounting. *Review of Economic Studies 54*(4), 599 – 617.

Stock, J. H. and M. W. Watson (2012). Disentangling the channels of the 200709 recession. *Brookings Papers on Economic Activity Spring*.

Sufi, A. (2009). Bank lines of credit in corporate finance: An empirical analysis. *The Review of Financial Studies 22*(2), 1057 – 1088.

# Appendix

## Proof of Goods Market Clearing condition and Discount rate

The representative household takes the contract prices $F_t^i$ as given. The household's Bellman equation is

$$U_t(\vec{b}) = \max_{C_t \geq 0, \vec{b'}} \left[ u(C_t) + E_t\left[\beta_P U_{t+1}(\vec{b'})\right] \right], \tag{A-1}$$

subject to the budget constraint

$$\begin{aligned} \sum_{i \in \text{Continue}} b_{i,t}\big(F^e_{i,t} \; + \; y_{i,t} - \tau_{i,t} - \iota_{1,i,t}k_{j,t} - \theta\iota^2_{1,i,t}k_{i,t} - \iota_{2,i,t}k_{i,t} - \theta\iota^2_{2,i,t}k_{i,t}\big) + \sum_{j \in \text{Exit}} \chi k_{j,t} \\ - \sum_{i \in \text{Entry}} \tilde{e}_i k_{i,t} = C_t + \sum_{i \in \text{Continue}} b_{i,t+1} F^e_{i,t}\,, \end{aligned} \tag{A-2}$$

where $\vec{b}$ is the household's holding of the contracts with individual firms and $F^e_{i,t}$ is the ex-dividend price of contract $i$ at the end of period $t$. $\tau_t^i$ is the payment made to the insider in firm $i$ in period $t$, $\iota_{1,i,t}$ is the investment rate of firm $i$ in period $t$, and $\iota_{2,i,t}$ is the dis-investment rate of firm $i$ in period $t$. The household consumes $C_t$ in period $t$. The summation over "continue" refers to firms which remain in operation after the lottery for firm liquidation. It excludes new entrants. The summation over "exits" refers to firms which are liquidated as a result of the liquidation lottery. A fraction $\chi$ of the physical assets $k_{j,t}$ of the liquidated firm is recovered.

**Market Clearing:** In equilibrium the market for financial contracts clears

$$b_{i,t} = b_{i,t+1} = 1\,, \tag{A-3}$$

for continuing firms $i$. Substituting this market clearing condition into the budget constraint Eq. A-2 results in the contract price $F^e_{i,t}$ dropping out. The representative household's consumption

$$C_t = Y_t - I_t - \mathcal{T}_t,\,, \tag{A-4}$$

where

$$\begin{aligned} Y_t &= \sum_{i \in \text{Continue}} y_{i,t} \\ I_t &= \sum_{i \in \text{Continue}} \big(\iota_{1,i,t}k_{j,t} - \theta\iota^2_{1,i,t}k_{i,t} - \iota_{2,i,t}k_{i,t} - \theta\iota^2_{2,i,t}k_{i,t}\big) - \sum_{j \in \text{Exit}} \chi k_{j,t} + \sum_{i \in \text{Entry}} \tilde{e}_i k_{i,t} \\ \mathcal{T}_t &= \sum_{i \in \text{Continue}} \tau_{i,t}\,, \end{aligned}$$

which is Eq. 5 after setting total investment in existing firms $I_t^e = \sum_{i\in\text{Continue}} \big(\iota_{1,i,t} k_{j,t} - \theta\iota_{1,i,t}^2 k_{i,t} - \iota_{2,i,t} k_{i,t} - \theta\iota_{2,i,t}^2 k_{j,t}\big)$, total dis-investment from liquidated firms $I_t^l = \sum_{j\in\text{Exit}} \chi k_{j,t}$, and total investment in new firms $I_t^n = \sum_{i\in\text{Entry}} \tilde{e}_i k_{i,t}$. From Eq. 5 and Eq. A-4 above, we have

$$C_t = D_t \,, \tag{A-5}$$

which is the goods-market clearing condition.

**Optimality condition:** The ex-dividend price of the financial contract is determined by the household's first-order condition

$$U'(C_t) F_{i,t}^e = \beta_P E\big[U'(C_{t+1})\big(d_{i,t+1} + F_{i,t+1}^e\big)\big] \,,$$

where $d_{i,t}$ is the investor's dividend from financial contract $i$ in period $t$ and equals output of firm $i$, net of the insider's payment and investment expense: $d_{i,t} = y_{i,t} - \tau_{i,t} - \iota_{1,i,t} k_{i,t} - \theta\iota_{1,i,t}^2 k_{i,t} - \iota_{2,i,t} k_{i,t} - \theta\iota_{2,i,t}^2 k_{i,t}$.

## Sketch of the Numerical Solution Approach

### Steady-state equilibrium

In the steady-state, the interest rate is a constant $r_t = r^\star$. To solve for this constant equilibrium interest rate, I first pick a grid for possible values of the interest rate $r$. For each value of $r$ I solve for the optimal contract. Each set of contract policies (for a particular conjectured value of $r$) implies a fixed growth rate of the investor's consumption along the balanced growth path. This growth rate determines the investor's marginal utility process $M_{t,t+1} = \beta_P \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}$ and an implied interest rate. The equilibrium rate is the fixed point – the realized interest rate is equal to the conjectured value.

### Transition dynamics

I model the shock as a permanent shock to which agents assign zero probability. However, since contract policies are allowed to depend on the interest rate, the shock leads to a change in policies from the steady-state. I compute the new policies along the transition path iteratively using the following steps:

1. First, I compute the steady-state equilibrium for the new value of $\sigma(R)$.

2. Next, I choose $T^\star$ to represent the number of periods that it takes for the economy to reach the new steady-state along the transition path. For my quantitative analysis, I choose $T^\star = 100$ and make sure that the resulting dynamics does not change when $T^\star$ is increased.

3. I start with a guess for the path of the interest rates: $\{r_t^{(n)}\}$.

4. I compute contract policies along this path using the conjectured path of interest rates. Note that policies are time-dependent along the transition path.

5. Using these policies for investment and insider payments, I aggregate them over the cross-section of firms and compute the representative investor's consumption along the transition path.

6. The implied path of interest rate, given this consumption path is computed from the investor's marginal utility process $\frac{1}{r_{t+1}} = M_{t,t+1} = \beta_P \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma}$.

7. If the realized path of interest rate is within a pre-specified tolerance (for every period between $t = 1$ and $t = T^{\star}$) then stop. Otherwise, use a weighted average of the realized path of interest rates obtained in Step 6 and $\{r_t^{(n)}\}$. This weighting scheme allows for smooth convergence.

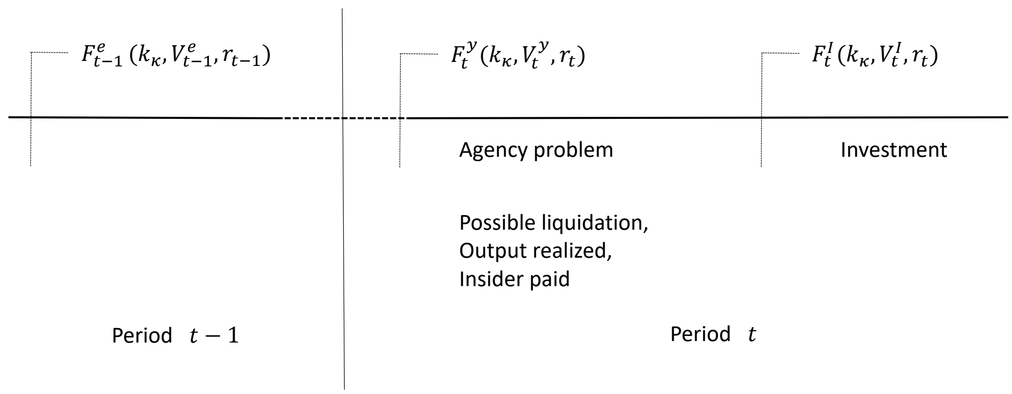


**Figure** 1: Intra-period time-line of the model. $F^y$, $F^I$, and $F^e$ represent the investor's valuation of the contract given promised payoffs to insiders $V^y$, $V^I$, and $V^e$ at various points within a period.

**Table 1**: Parameter values: The model is calibrated at quarterly frequency. The cash flow process has expected value $E(R)$, volatility $\sigma(R)$, with $p$ as the probability of realizing a high cash flow $R_1$. $\nu$ is the decreasing returns-to-scale parameter. $h$ measures the number of potential new firms as a fraction of existing firms. It is a measure of the investment opportunity set in the economy. $\theta$ is the quadratic investment costs for investment. Capital depreciates at rate $\delta$. The investor recovers a fraction $\chi$ of a liquidated firm's capital stock. The investor has time preference parameter of $\beta_P$ and a risk aversion $\gamma$. The insider has a time preference parameter $\beta_A$. The insider can divert firm output with an efficiency $\lambda$. Model predictions depend on the product $\lambda\sigma(R)$, not on the individual values of $\lambda$ and $\sigma(R)$. $\pi^d$ is the exogenous rate of firm exit.

| $p$ | $E(R)$ | $\lambda\sigma(R)$ | $\nu$ | $h$ | $\theta$ | $\delta$ | $\chi$ | $\beta_P$ | $\gamma$ | $\beta_A$ | $\pi_d$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.5 | 0.0098 | 0.0067 | 0.6 | 0.002 | 7.50 | 0.0175 | 0.85 | 1.005 | 2 | 0.969 | 0.010 |

**Table 2**: Calibration Moments: This table compares sample moments in the data to those generated by simulated data using parameters in Table 1. Consumption growth and the risk-free rate in the Data column is from Campbell, Lo, and MacKinlay (Campbell et al.). The exit rate is from Siemer (2012). The average investment rate and the proportion of positive spikes which are defined to be investment rates greater than 0.2 are borrowed from Cooper and Haltiwanger (2006). Relative firm size, statistics for Tobin's Q, and cash ratio are estimated using COMPUSTAT data. I report time series averages of the median and inter-quintile range (IQR).

| Moment | Data | Model |
|---|---|---|
| Aggregate moments | | |
| $E[C_{t+1}/C_t]$ | 0.017 | 0.019 |
| $E[r^f]$ | 0.018 | 0.018 |
| Exit rate | 0.086 | 0.062 |
| | | |
| Cross-sectional moments | | |
| Investment rate (mean) | 0.122 | 0.091 |
| Investment rate (IQR) | 0.159 | 0.184 |
| Relative firm size (IQR) | 0.830 | 0.734 |
| Cash flow/Capital (median) | 0.160 | 0.435 |
| Cash flow/Capital (IQR) | 0.234 | 0.223 |
| Cash ratio (mean) | 0.155 | 0.321 |
| Cash ratio (standard deviation) | 0.220 | 0.242 |

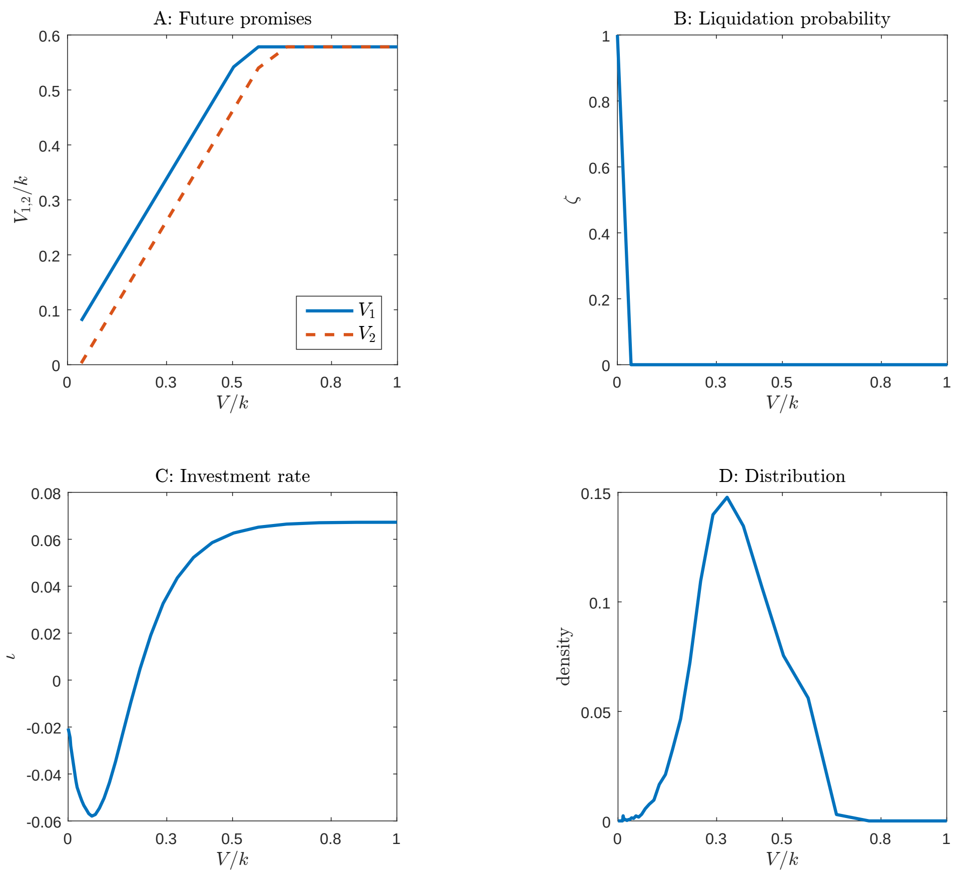


**Figure** 2: Steady-state Policies: Panel A shows the liquidation probability $\zeta$, as a function of insider continuation value for fixed firm size $k$. Panel B shows the adjustment to continuation values for high and low output. The solid blue line is for high output, while the dotted red line shows the insider's continuation value when he reports low output. Panel C shows the investment rate $\iota$ as a function of the insider's continuation value. Panel D shows the steady-state distribution over continuation values for this size.

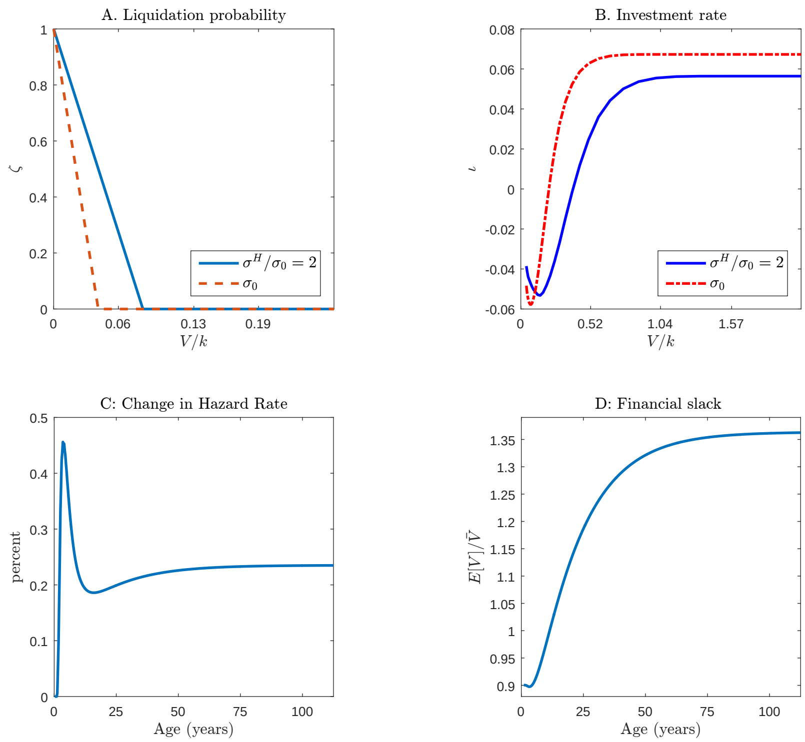


**Figure** 3: Comparative statics: Panel A compares the liquidation probability as a function of the insider's continuation value scaled by firm size while Panel B compares the investment rates for firms in these two economies. The dotted red lines show the policy in the baseline economy. The dotted blue lines show the policy in an economy with firm-specific productivity equal to twice the value in the baseline economy. All policies in Panel A and B are for firms whose size is equal to the mean firm size in the baseline economy. All other parameters are the same for the two economies. Panel C shows the difference in hazard rates as a function of firm age in the two economies. Hazard rates are higher for firms of all ages in the economy with higher uncertainty, with younger firms experiencing the biggest increase. Panel D shows average continuation value (scaled by the cross-sectional mean) as a function of firm age.

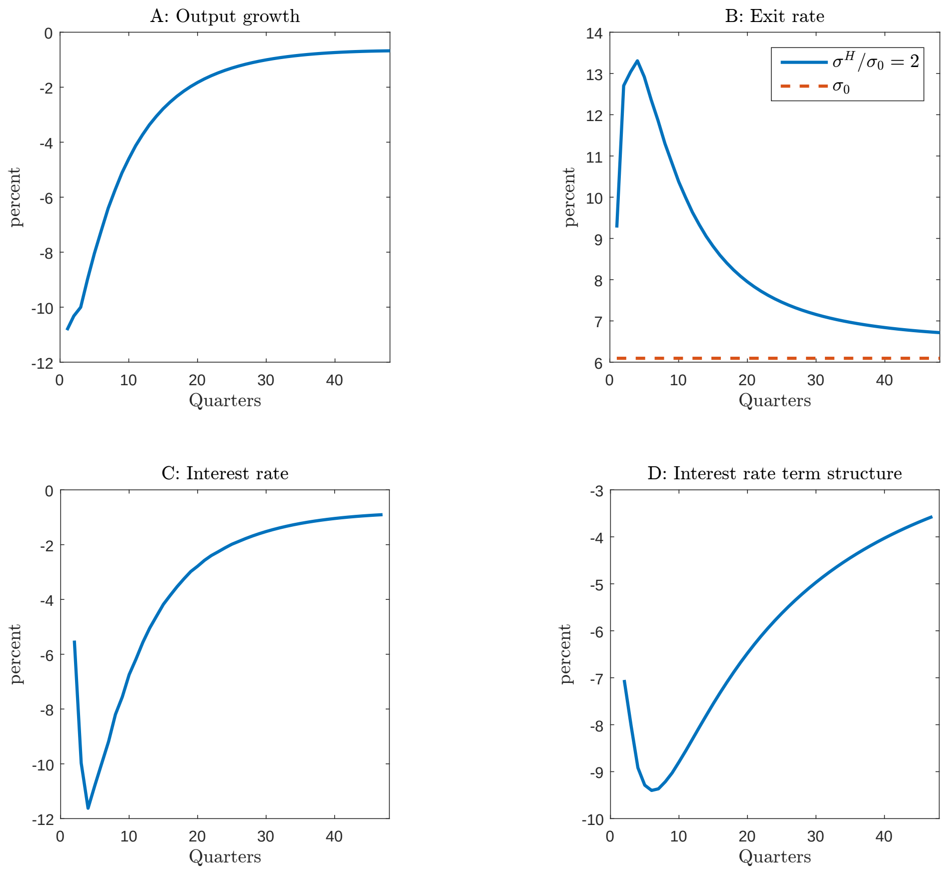


**Figure** 4: Transition path dynamics: Panel A shows the change in growth rate of output $Y_{t+1}/Y_t$ from the initial steady-state equilibrium value along the transition path. Panel B shows the increase in firm exit rate. The solid blue line shows the exit rate along the transition path. The dotted red line is the exit rate in the initial steady-state equilibrium. Panel C shows the change in the one-period interest rate from its value in the initial steady-state equilibrium along the transition path. Panel D shows the change in the term-structure of interest rates. All deviations are measured in percent and are annualized values.

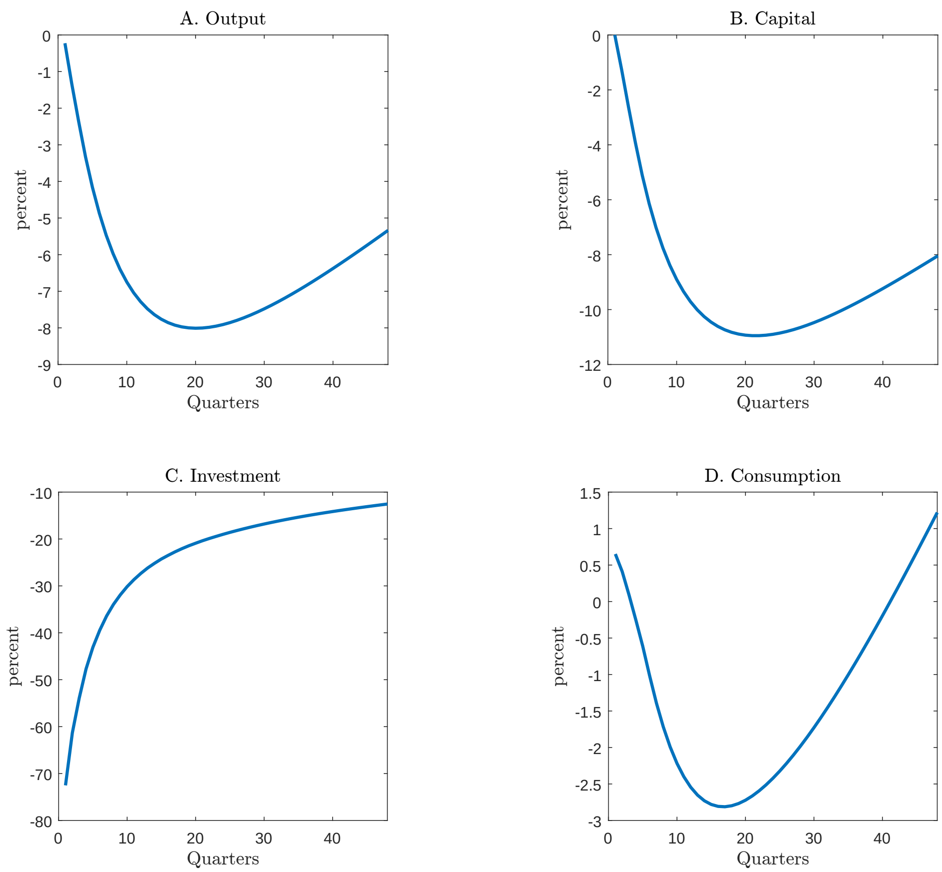


**Figure 5**: Response of aggregate quantities: The figure shows the effect of a one-time permanent doubling of cash-flow volatility of all firms in the economy. All the plots show percentage deviation from the initial steady-state levels. Panel A shows the response of output, panel B the change in total capital stock in the economy. Panel C shows the change in aggregate investment compared to the level in the period prior to the impact of the shock. Panel D shows the change in the representative investor's consumption.

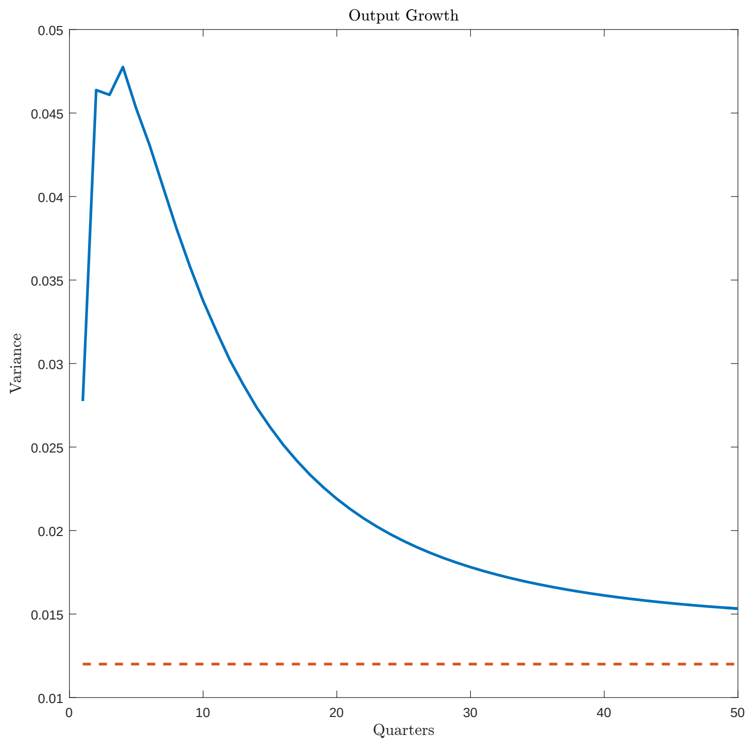


**Figure** 6: Increase in the cross-sectional dispersion of growth rates after an increase in firm-specific uncertainty of all firms.

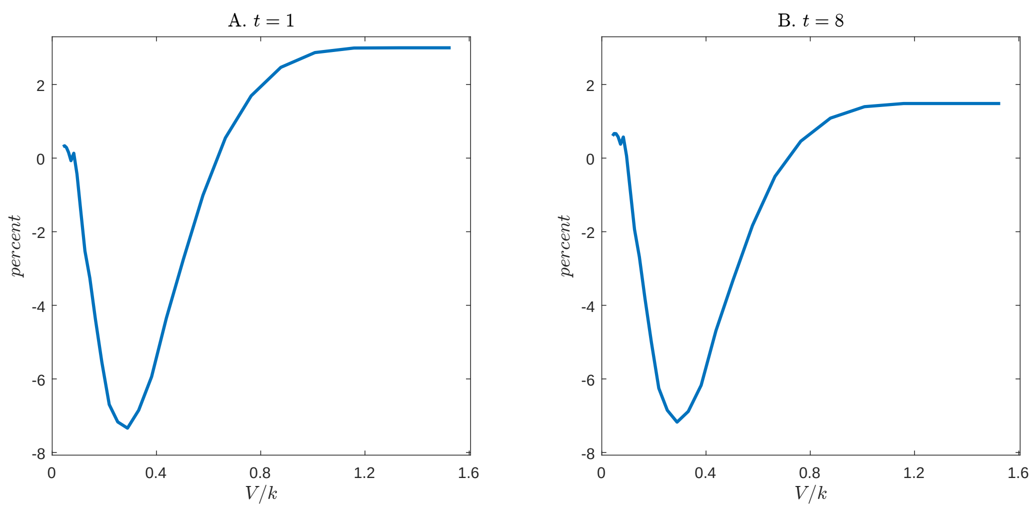


**Figure 7**: Heterogeneous response of investment rates: Panels A and B show the change in investment rate for the median firm in this economy as a function of the insider's continuation value. Panel A shows this change immediately upon impact. Panel B shows the change $t = 8$ quarters after the increase in firm-specific uncertainty.

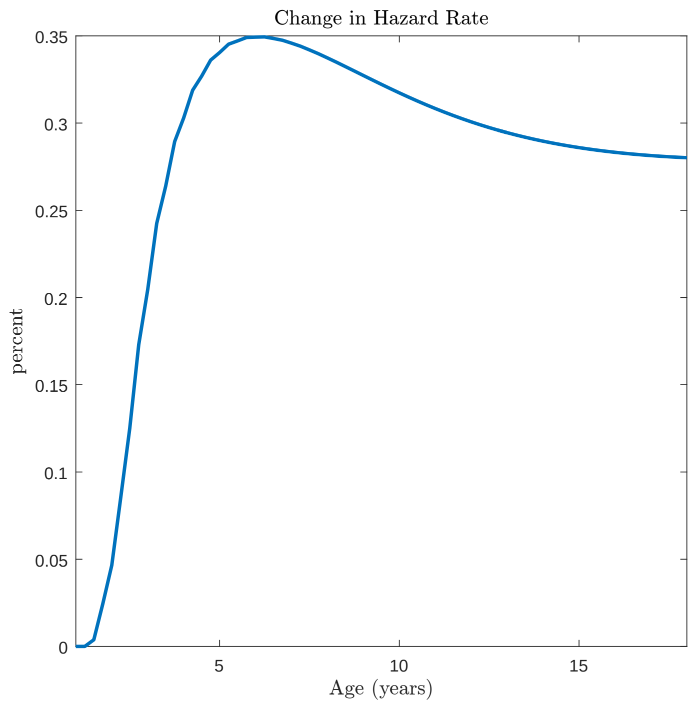


**Figure** 8: Increase in the hazard rate immediately after the impact of the shock.