Trading Volume and Time Varying Betas

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ABSTRACT

Existing models of trading volume depend on shocks to agent heterogeneity to generate trade. Consistent with the well known fact that betas of securities change dramatically over time, I propose a new mechanism where shocks to the riskiness of traded securities generate trade. This mechanism can operate concurrently with previous agent level heterogeneity mechanism, together amplifying each other. My model of this mechanism makes three novel predictions about trading volume and securities’ time-varying risk exposures: trading volume and changes in risk exposures are positively correlated; decreases in risk exposure are associated with more trading volume than increases in risk exposure; and the trading volume response to changing risk exposure decreases with the level of the risk exposure. Measuring risk exposures as market betas, I find strong empirical support for these predictions. For example a one standard deviation decrease in beta leads to a 13% increase in turnover. This sensitivity has grown along with the increase in trading volume, peaking at 73%. My model also provides alternative explanations for three well documented links between trading volume and (contemporaneous as well as future) price changes.

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Volume is The Great Unsolved Problem of Financial Economics.

In our canonical models ... trading volume is essentially zero...I gather quantum mechanics is off by 10 to the 120th power in the mass of empty space, which determines the fate of the universe. Volume is a puzzle of the same order, and importance, at least within our little universe.

— John Cochrane, Becker-Friedman Institute Conference 10/7/16

Trade, just as price, is central to how financial markets incorporate information, spread risk and facilitate the efficient deployment of real resources. Trade based on information about stocks is hard to achieve in our benchmark model due to the key idea in Milgrom and Stokey (1982) no trade theorem. When I get good information about a stock I want to buy but this causes the price to rise muting any actual trade and in the extreme shutting it down completely. Meeting with this fundamental difficulty of using information about stocks, explanations of trade focused instead on shocks to investor heterogeneity. Essentially these are shocks to preferences though they come in many forms, including differences of private information (Epps, 1975; Jennings, Starks, and Fellingham, 1981; Diamond and Verrecchia, 1981; Blume, Easley, and O’Hara, 1994), differences in opinion or perceptions (De Long, Shleifer, Summers, and Waldmann, 1989, 1990; Harris and Raviv, 1993), difference in ability (Dow and Gorton, 1997), differences in risk aversion (Campbell, Grossman, and Wang, 1993; Tkac, 1999), differences in outside investment opportunities and labor income (Llorente, Michaely, Saar, and Wang, 2002; Lo and Wang, 2006, 2009), or differences in liquidity needs (Shiller, Fischer, and Friedman, 1984; Grossman and Miller, 1988). In these models investor heterogeneity changes, creating mismatches between securities and the holders of these securities, and generating trade when agents exchange securities to eliminate these mismatches.

While shocks to investor heterogeneity successfully in generate trade, the magnitude is too small to account the true volume of trade. The idea of noise traders arose to fill the remaining vacuum of unexplained trade, but this is more a catchall term than an complete explanation. In this paper, I show how the well know fact that securities’ betas, i.e., risk, change dramatically over time, is a new mechanism to generate trade. This mechanism
works in combination with existing shareholder heterogeneity models of trade, with each mechanism amplifying the other.

Shocks to the risk exposures of the securities generate a mismatch between investors and the securities they would like to hold. But importantly these security level shocks do not have offsetting price changes that eliminate the desire to trade. Suppose two investors Adam and Bill have exactly offsetting income shocks each period. They would like to share these income shocks and eliminate their risk. When Adam and Bill have access to a risky stock that is positively correlated with Adam’s income they can accomplish their goal. Adam shorts the risky stock and Bill longs the stock. Now suppose the correlation reverses due to a change in the return generating process not due to a change in the income processes. After the correlation reversal the stock is negatively correlated with Adam’s income. Adam and Bill now want the opposite positions, and they trade to obtain their desired hedges.

This example illustrates how a shock to a stock’s risk exposure generates trade, even though nothing about Adam’s or Bill’s information set, opinion, risk aversion, investment opportunities, labor income or liquidity needs change. The intuition of this example extends to other sources of heterogeneity like differences of opinion. Also, individual investors need not make these rebalancing trades directly because investors can transfer this need to trade to financial institutions through their purchases of financial products. Northern Trust and the Bank of New York Mellon both market and promote customized index portfolios designed to suit individual client’s risk needs and to “maintain beta exposures” in the “midst of changing markets.”\(^1\) Even hedge funds compete on the dimension of maintaining constant risk exposures. Namvar, Phillips, Pukthuanthong, and Rau (2013) show that investors pull money out of funds that provide inconsistent beta exposure and that investors are willing to pay higher fees to funds that provide constant beta exposure.

Shocks to the risk exposures of securities motivate trade, but not all shocks to securities’ characteristics do. Suppose that in the preceding example the average payout increases for

the risky stock available to Adam and Bill. Both Adam and Bill prefer more dividends, so both Adam and Bill try to buy more (Adam shorts less) of the risky stock. But this general increase in demand causes the stock’s price to rise, offsetting their demand to trade (Milgrom and Stokey, 1982; Judd, Kubler, and Schmedders, 2003). With the shock to the stock’s risk exposure, there is no general increase in demand because Adam and Bill care about the risk exposures of the stock differently: Adam wants more and Bill wants less. Thus there is no price increase to offset their demand to trade.

To explore the intuition of this example, I build a model of trade with heterogeneous agents and endogenous price changes for a security that exhibits both changes to its risk exposure and its average dividend level. Consistent with the previous results of Milgrom and Stokey (1982) and Judd et al. (2003), the model shows that endogenous price changes eliminate trading volume when there are shocks to dividend levels. Shocks to the risk exposures of stocks, however, generate trading volume, even with investor heterogeneity held constant.

My model yields three new predictions about the relation between trading volume and changes in stocks’ risk exposures. First, trading volume is positively related to shocks to a stock’s risk exposure. Second, this relationship is asymmetric. Trading volume is higher when the magnitude of the stock’s risk exposure falls because a falling risk exposure requires the trading of more shares to achieve the same rebalancing effect. Third, stocks with larger risk exposures have trading volume that is less sensitive to risk exposure shocks. Again, the intuition is that larger risk exposures require a smaller amount of trade to accomplish a given amount of rebalancing.

The predictions and intuition of my model apply to all types of risks, priced and unpriced, across which agents are heterogeneous. To test the predictions of my model, I focus on stocks’ risk exposure to the market for two reasons. First, agents have substantial heterogeneity in their exposure to market risk (Mankiw and Zeldes, 1991; Heaton and Lucas, 2000a,b; Brav, Constantinides, and Geczy, 2002). Second, the time variation in individual stocks’ betas
(Blume, 1975) and even the betas of broad portfolios of stocks is large (Fama and French, 1997; Lewellen and Nagel, 2006). I find support for all three predictions, using a panel data set of NYSE listed stocks between 1962 and 2011. On average a one standard deviation decrease in a stock’s market beta corresponds to an increase in annual turnover of 13%. The sensitivity of trading volume to changes in betas increases over time with the general increase in observed trading volume (Chordia, Roll, and Subrahmanyam, 2011). In the last five years of the sample, a one standard deviation decrease in a stock’s beta corresponds to an increase in annual turnover of about 73%. My findings are robust to alternative specifications: using beta changes measured at yearly frequencies; using Dimson (1979) betas to control for beta mis-measurement due to either asynchronous trading or opacity in information (Gilbert, Hrdlicka, Kalodimos, and Siegel, 2013); and focusing only on the most heavily traded stocks. I show the generality of the model’s application by extending the empirical results to changes in risk exposures to the Fama and French (1992, 1993, 1996) size and book-to-market factors in addition to the market.

The model also offers explanations of three well-known facts about trading volume and returns. The first is that trading volume is positively related to the absolute level of contemporaneous returns. This result arises in my model because a change in risk exposure simultaneously alters a stock’s price and drives trading volume. A large change in a stock’s risk exposure leads to a large price change and large trading volume. The second fact is that the relation between trading volume and returns is asymmetric: trading volume is higher on price increases than on price decreases of equal magnitude. Karpoff (1987) reviews the previous literature documenting these two facts while Jain and Joh (1988); Chamberlain, Cheung, and Kwan (1991); Gallant, Rossi, and Tauchen (1992); Kandel and Pearson (1995); Lee and Swaminathan (2000) and Long (2007) document these facts in more recent data. This asymmetry arises in my model because a risk exposure decrease causes a price increase and more trading volume than a similarly sized risk exposure increase.

The third fact is that trading volume helps predict returns: price changes accompanied
by more trading volume are more likely to be reversed than price changes accompanied by low trading volume (Campbell et al., 1993; Conrad, Hameed, and Niden, 1994; Lee and Swaminathan, 2000; Gervais, Kaniel, and Mingelgrin, 2001; Llorente et al., 2002). The intuition from my model is that price changes come from either shocks to the average dividend level (a cash flow effect) or shocks to the risk exposure (a discount rate effect). Shocks to average dividend levels generate no trading volume, while shocks to risk exposures generate trading volume. Thus a price decrease accompanied by trading volume signals a higher likelihood that the price decrease is due to an increased risk exposure. This larger risk exposure is then followed by higher expected returns.

Others such as Karpoff (1988); Campbell et al. (1993); Blume et al. (1994) and Lee and Swaminathan (2000) also offer explanations for these connections between trading volume and price changes. It is likely that their effects in addition to the one I model are present in financial markets. My model however provides a link between trading volume and conditional risk exposures of individual securities that has applications for estimating conditional asset pricing model, interpreting the increase in trading and understanding the risk premia associated with time varying risk exposures. I discuss these applications further in the concluding Section IV after presenting my model of this link in Sections I and II and providing empirical support for this mechanism in Section III.

I. Model

The model economy has two periods, two classes of agents and the prices of the risky asset set endogenously in a competitive market, but in the spirit of Lucas (1978) it abstracts from the underlying production process. The main innovation in my model is that the risky security’s exposure to the systematic shock changes randomly over time. Agents fully anticipate this change, and the revelation of the shock to the systematic risk exposure drives trade amongst the fully and symmetrically informed agents.
A. The Economy

Label the dates \( t = 0, 1 \) and 2. At date 2, the terminal period, there is only consumption. At dates 0 and 1 agents may trade and consume.

Agents have exponential utility over period level consumption:

\[
    u(c) = -\exp(-\gamma c). 
\]

At each date agents maximize the discounted sum of expected exponential utility

\[
    E_t[u(c_0) + \delta u(c_1) + \delta^2 u(c_2)],
\]

by choosing consumption and portfolio holdings subject to current prices and their expectations for the future price distributions.

The model’s agents are heterogeneous. There is a continuum of agents evenly split between two classes labeled \( i = 1 \) and 2. The classes differ only by the outside income process they receive each period:

\[
    L_i t = \lambda_i f_t, 
\]

where \( f_t \) is a systematic shock that affects the dividend of the risky asset as well. This outside income could come from labor income or small business income for example or equivalently be considered as a difference in preferences or a difference in beliefs.

Without loss of generality, I restrict attention to the case where the outside income processes exactly offset each other:\(^2\)

\[
    \lambda^1 = -\lambda^2. 
\]

I refer to the magnitude of \( \lambda \) as the amount of heterogeneity.

\(^2\)So long as the exposures are unequal the results of my model go through. For example, both classes of agents could be positively exposed to the systematic shock \( f_t \). The positive average exposure would raise the level of aggregate risk in the economy and therefore raise the risk premium on the \( f_t \) shock but have little other affect on the results. The difference around the mean exposure is what matters for trade.
Agents face incomplete markets; in particular, they cannot directly trade in the systematic shock $f_t$.\footnote{Market incompleteness is not essential, but structure of the available assets for trade is essential. Completing the market with a contract to directly trade income risk would eliminate trade, while completing the market with contracts that provide insurance against the beta and mean shocks (see Equation (5)) could increase trade.} They can trade in a risk-free bond and a single risky asset. The risk-free rate is constant and set to the inverse of the time discount rate, $R = \frac{1}{\delta}$.

There is one unit of the risky asset. At each date the risky asset pays a dividend. The agents know the dividend distribution one period ahead through their knowledge of its mean, $\mu_{t-1}$, and its exposure, $\beta_{t-1}$, to the systematic shock $f_t$. I model cash flow betas here, but as I describe in Section I.E extending the model to return betas yields similar results. The dividend is

$$D_t = \mu_{t-1} + \beta_{t-1} f_t.$$  \hspace{1cm} (5)

Each period the systematic shock $f_t$ is realized as are the shocks $\mu_t$ and $\beta_t$ which determine the distribution of the dividend the following period. The shocks $\mu_t$ capture the fact that dividend levels change over time. The shocks $\beta_t$ capture the fact that equities’ risk exposures change over time. The source of these shifts are taken as exogenous, but the shifts could arise for example as firms change their lines of business or the general economic environment shifts from one based on manufacturing to one based on knowledge production.

All the shocks are iid and have the following normal distributions:

$$\mu_t \sim N(\bar{\mu}, \sigma_\mu^2),$$  \hspace{1cm} (6)

$$\beta_t \sim N(\bar{\beta}, \sigma_\beta^2) \text{ and}$$  \hspace{1cm} (7)

$$f_t \sim N(0, \sigma_f^2).$$  \hspace{1cm} (8)

I denote each agent $i$'s wealth at date $t$ following the payment of the dividend and outside income as $W_i^t$. I denote the shares of the risky asset held by agent $i$ at date $t$ as $s_i^t$. These share holdings are scaled by the class’s weight in the economy for market clearing.
calculations. I denote the endogenous market clearing price of the risky asset in each period as $P_t$.

The state variables at date 0 and 1 are those that describe the dividend and outside income processes:

\[
\theta_0 \equiv [\mu_0, \beta_0] \quad \text{and} \quad (9) \\
\theta_1 \equiv [\mu_1, \beta_1, f_1]. \quad (10)
\]

**B. Equilibrium**

I now define the equilibrium in this economy.

Definition 1: *An equilibrium is given by a price process $P_0(\theta_0)$ and $P_1(\theta_0, \theta_1)$ and series of consumption and portfolio policy functions

\[
\{c^i_0(\theta_0), s^i_0(\theta_0), c^i_1(\theta_0, \theta_1), s^i_1(\theta_0, \theta_1)\} \quad (11)
\]

for each agent class $i$ such that

1. The policy functions solve each agent’s optimization problem

\[
\max \begin{cases} 
    c^i_0(\theta_0), s^i_0(\theta_0), \\
    c^i_1(\theta_0, \theta_1), s^i_1(\theta_0, \theta_1)
\end{cases} \quad \left[ u(c^i_0) + E_0[\delta u(c^i_1) + \delta^2 u(c^i_2)] \right] \quad (12)
\]
where

\[ W_i^i(\theta_0, \theta_1) = (W_0^i - c_i^i(\theta_0) - s_0^i(\theta_0)P_0(\theta_0))R + L_1^i(f_1) + s_0^i(\theta_0)(P_1(\theta_0, \theta_1) + D_1(\theta_0, \theta_1)) \]

\[ c_i^2(\theta_0, \theta_1, f_2) = (W_i^i(\theta_0, \theta_1) - c_i^1(\theta_0, \theta_1) - s_1^i(\theta_0, \theta_1)P_1(\theta_0, \theta_1))R + L_2^i(f_2) + s_1^i(\theta_0, \theta_1)D_2(\theta_1, f_2). \]

2. The risky asset market clears at each state and date by satisfying

\[ 1 = \frac{1}{2}s_0^i(\theta_0) + \frac{1}{2}s_0^j(\theta_0) \quad \text{and} \]

\[ 1 = \frac{1}{2}s_1^i(\theta_0, \theta_1) + \frac{1}{2}s_1^j(\theta_0, \theta_1). \]

I now characterize this equilibrium.

**Theorem 1:** There exists an equilibrium in this model with price process

\[ P_0(\mu_0, \beta_0) = P_0(s_i^* \lambda^i) \]

\[ = \delta \{ \mu_0 - \frac{\gamma \delta \sigma_j^2 (\bar{\beta} + \lambda^i \gamma^2 \sigma_j^2 \sigma_\beta^2)}{-1 + \delta (-1 + \gamma^2 (-1 + 2s_0^i \sigma_j^2 \sigma_\beta^2))} + \gamma s_0^i \sigma^2_{\mu} \]

\[ + \gamma \delta^2 \sigma_j^4 (-\bar{\beta}^2 - (1 + \delta) \sigma_\beta^2 + \gamma^2 \delta (-1 + 2s_0^i) \sigma_j^2 \sigma_\beta^4) \]

\[ + \gamma \delta (\mu - \bar{\mu}) + \gamma \sigma_j^2 \sigma_\beta^2 \} \]

\[ = \frac{\gamma \beta_0 (\lambda^i + s_0^i \beta_0) \sigma_j^2 + \gamma s_0^i \sigma^2_{\mu}}{1 + \delta} \]

where \( s_i^* \) is the equilibrium shareholdings of agent \( i \in \{1, 2\} \), and

\[ P_1(\mu_1, \beta_1) = \delta (\mu_1 - \gamma \sigma_j^2 \beta_1^2). \]

10
And shareholding policy functions

$$s^i_0(\mu_0, \beta_0) = s^i_0$$  \hspace{1cm} (19)

which solves polynomial

$$P_0(s^i_0, \lambda^i) = P_0(2 - s^i_0, -\lambda^i), \text{ and}$$

$$s^i_1(\mu_1, \beta_1) = 1 - \frac{\lambda_i}{\beta_1}. \hspace{1cm} (21)$$

The optimal consumption policies along with a proof of this equilibrium are in Appendix A. There I show how to solve the model recursively through dynamic programing.

Because there is no algebraic formula for the solution to quintic polynomials with arbitrary coefficients, I cannot simplify the exact solution further (Hungerford, 1980).

### C. Approximation of Price and Shareholdings

Despite the lack of an algebraic expression for optimal shareholdings at date 0, I obtain the following closed form approximation to the optimal shareholdings. This approximation is based upon the price that would occur were there no heterogeneity in the economy. This approximation is very accurate around the calibrated parameters (see Appendix B) and is helpful for understanding the dynamics of the model.\(^4\)

When there is no heterogeneity in the economy the date 0 price is

$$P_0^{NH} = \delta E_0[P_1 + D_1] + RP_f + RP_\mu + RP_\beta. \hspace{1cm} (22)$$

where the risk premium for the systematic factor risk, \(f\), risk premium for mean shifts, \(\mu\),

\(^4\)At the calibrated parameters specified in Appendix B the difference between the no heterogeneity price and the heterogeneity price is $3.7 \times 10^{-4}\%$ and the difference between the exact model solution for shareholdings and this approximation error is $0.073\%$. More on the quality of this approximation can be found in the Internet Appendix A.
and risk premium for risk exposure shifts, \( \beta \), are

\[
RP_f = -\frac{\beta^2 \gamma \delta \sigma_f^2}{1 + \delta},
\]

\[
RP_\mu = -\frac{\gamma \delta^3 \sigma_\mu^2}{1 + \delta}
\]

and

\[
RP_\beta = -\frac{\gamma^3 \delta^3 \sigma^4 \sigma^4_\beta}{1 + \delta - \gamma^2 \delta \sigma_f^2 \sigma^2_\beta} + \beta^2 \gamma \delta^2 \sigma_f^2 \left(1 - \frac{(1 + \delta)^2}{(-1 - \delta + \delta \gamma^2 \sigma_f^2 \sigma^2_\beta)^2}\right).
\]

The total risk premium is the sum of these three:

\[
RP_T = RP_f + RP_\mu + RP_\beta.
\]

If agents faced this no heterogeneity price process rather than the true price process they would hold the following amount of shares which approximates their true shareholding:

\[
s^i_0(\mu_0, \beta_0) \approx s^i_{0,\text{NH}}(\mu_0, \beta_0) = 1 - \left(\frac{\lambda^i}{\beta_0}\right) \left(\frac{RP_f}{RP_T}\right).
\]

From this approximation, we see that agents do not fully hedge their exposure to the systematic risk because the more they hedge the more exposed they become to shifts in betas and means. The amount of hedging depends upon the relative importance of the income risk (exposure to \( f \)) to the total risk as measured by the risk premia. Including this dampening effect gives the approximate share holdings at date 0.

D. Trading Volume

Trading volume is the movement between equilibrium share holdings at date 0 and date 1. Since the agents have exactly offsetting heterogeneity (Equation (4)) and the classes are of the same size, changes in one class of agents’ portfolios mirror the portfolio changes of the other class.

Definition 2: \textit{Trading volume is the absolute change in risky asset holdings scaled by the class}
\[ Volume(\mu_0, \beta_0, \mu_1, \beta_1) = \frac{1}{2} |s_0^i - s_1^i|. \]  

(28)

Using the approximation for optimal shareholdings at date 0, Equation (27), trading volume is approximated by

\[
Volume(\mu_0, \beta_0, \mu_1, \beta_1) \approx \frac{1}{2} \left| \left( \frac{\lambda^i}{\beta_0} \right) \left( \frac{R P_f}{R P_T} \right) - \frac{\lambda^i}{\beta_1} \right|.
\]

(29)

We immediately see that trading volume depends on the beta changes between date 0 and date 1 and not on the mean, \( \mu_t \), changes. In addition to beta changes, some trading volume is created by the passage of time and the elimination of the parameter (\( \mu \) and \( \beta \)) change risk. This parameter risk dampens hedging at date 0, showing up in the date 0 term \( \frac{R P_f}{R P_T} < 1 \). At date 1 there is full hedging and this term is absent.

Because only the trading volume driven by beta changes is of interest, I split trading volume into two pieces: the part driven by changes in betas and the part driven by the transition from incomplete hedging at date 0 to compete hedging at date 1. This split is

\[
Volume(\mu_0, \beta_0, \mu_1, \beta_1) \approx \frac{1}{2} \left| \left( K - 1 \right) \frac{\lambda^i}{\beta_0} + K \left( \frac{\lambda^i}{\beta_0} - \frac{\lambda^i}{\beta_1} \right) \right|
\]

(30)

where

\[ K = \frac{R P_f}{R P_T} \leq 1. \]

(31)

I focus only on this second portion of trading volume that is driven by changes in betas:

\[
V(\Delta \beta) = \frac{K}{2} \left| \frac{\lambda^i}{\beta_0} - \frac{\lambda^i}{\beta_0 + \Delta \beta} \right|
\]

(32)

where

\[ \beta_1 = \beta_0 + \Delta \beta. \]

(33)
I use Equation (32) in the following section to show the relation between trading volume, beta changes and price changes.

**E. Model Extensions**

I extend the model separately in the time dimension and cross-section dimension. In both cases the key results of the single asset finite horizon model remain.

I analytically approximate the solution to an infinite horizon version of the model around the no heterogeneity price process. The results of the finite horizon model go through in the infinite horizon setting. The intuition remains unchanged in this infinite horizon setting. The only difference is that in the infinite horizon model trading volume becomes attributable only to beta shocks as the risk premia are essentially constant across dates due to the infinite horizon. This remaining volume in the infinite horizon model is exactly the portion of trading volume I investigate in the finite horizon model with Equation (32).

I numerically solve a version of the finite horizon model with several assets. Within the many asset model, I am able to consider the relation between trading volume and market return betas rather than trading volume and cash flow betas as in the single asset model. The main results and magnitudes of single asset model remain in the multi-asset version of the model. To make clear the small differences between trading volume and cash-flow betas versus trading volume and market betas, I present the details of this extension using the two asset case in Internet Appendix B.

**II. Links Between Trading Volume, Beta Changes and Returns**

In this section I describe the connections between trading volume, beta changes and price changes in this model. I discuss the main predictions of my model in subsections II.A to II.D. In subsection II.E, I discuss the model predictions for the amount of trading volume in
the economy as the model parameters change. All figures and tables throughout this section use the exact model solution with the parameters calibrated as described in Appendix B. All proofs of the model’s implications are in Appendix C.

A. Trading Volume and Beta Changes

Implication 1: Trading volume and absolute beta changes are positively related.

Implication 2: For positive betas, a beta decrease leads to more trading volume than beta increase.

The asymmetry of Implication 2 reverses when considering negative betas.\(^5\)

In Figure 1, I show the trading volume generated by innovations in the systematic factor, \(f\), the average dividend payment, \(\mu\), and the beta of the dividend payment with respect to that systematic factor, \(\beta\). The dashed red line shows trading volume from the first two risk factors. Innovations to the systematic factor and the average dividend payment generate no trading volume beyond that caused by the disappearance of parameter risk in the last period.

The solid blue line shows trading volume due to beta innovations. Beta innovations generate large and asymmetric trading volume. Both an increase and decrease in beta generate trading volume, but the decrease generates more trading volume than the increase.\(^6\)

In Panel A, I show the total trading volume in the model. One sees that trading volume is not minimized at zero beta changes in this panel because of the disappearance of parameter risk in the final period. In Panel B, I show only the portion of trading volume in my model.

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\(^5\)Strictly speaking Implications 1 and 2 hold only in a region around zero beta change because volume can eventually decrease if the beta change is large enough that the beta crosses zero. For practical applications this caveat is unimportant as almost all equity securities have positive betas.

\(^6\)My work contrasts with that of Judd et al. (2003) whose “investor’s portfolio is constant over time and states” in a dynamically complete market. The model in Judd et al. (2003) also has securities with time-varying covariances with investor’s income shocks. Trade does not occur in the economy of Judd et al. because conditioning upon the investor’s income shock realization the payoff structure of the assets never changes. This constancy means assets always provide the same hedging services. A static beta in my model replicates this constancy and also leads to no trading volume. When beta varies in my model, conditioning upon an agent’s income realization does not give a constant asset payoff structure. Because this structure varies agents must alter their holdings to maintain their hedges.
generated by beta (and μ shocks) per the decomposition described in Equation (30). In Panel B one sees that a one standard deviation decrease in beta generates approximately 10% trading volume.\textsuperscript{7} A one standard deviation increase in beta generates about 3% trading volume. I test and verify these two predictions in the empirical section.

\textbf{B. Trading Volume and Beta Size}

Implication 3: \textit{Trading volume is decreasing in the size of the initial beta value.}

Thus the third prediction of my model: smaller beta stocks have trading volume that is more responsive to beta changes.\textsuperscript{8}

As the ability of the traded security to hedge the risks agents care about decreases, trading volume increases. When the security becomes a worse hedge agents must trade more of it to accomplish the same amount of hedging. These larger positions are more sensitive to changes in the security’s beta. I show this effect in Panel A of Figure 2. In the figure I plot the portion of trading volume generated by beta changes for different values of the initial beta and the unconditional beta mean. A decrease in the initial beta from 1 to .75 nearly doubles trading volume.\textsuperscript{9} I test and verify this prediction of increased trading volume for smaller beta securities in the empirical section. An application of this result is that if equities are only moderately exposed to the risks agents want to hedge or bet on, we should expect more trading volume than if equities are strongly exposed.

\textsuperscript{7}One can see that trading volume increases significantly for the larger beta changes, especially as beta nears zero. In untabulated results, I verify by numerical methods that adding idiosyncratic risk to the dividend payment dampens this effect as beta nears zero, driving the hedging term of shareholdings to zero for very small betas.

\textsuperscript{8}Strictly speaking Implication 3 holds only for positive betas and in a region around a beta change of zero, for the same reasons described for Implications 1 and 2. Practically, this caveat has little impact as most equity securities have positive betas that do not become negative.

\textsuperscript{9}In reality we expect trading volume to increase as beta decreases only up to a point. The extreme result of the model that trading volume approaches infinity as beta approaches zero is due to the ability to achieve perfect hedging at date 1 of the date 2 shock. I numerically solve a version of this model with risk idiosyncratic risk added to the dividend payment. This idiosyncratic risk makes agents unwilling to take large positions in the security as the beta nears zero. This unwillingness comes from the extra risk exposure of an asset eventually trumpping its value as a hedging instrument. For beta values away from zero, the models with and without the uncorrelated risk provide similar trading volume predictions.
These predictions about trading volume and changing betas hold for both priced and unpriced factors. The predictions for unpriced factors can be obtained by solving the model with a risky asset in zero net supply. The case of a priced factor that I consider here also connects trading volume to price changes as I discuss in following subsections.

C. Trading Volume and Price Changes

Implication 4: \textit{If the factor across which there is heterogeneity is positively priced and the risky security is positively exposed to the factor, then larger absolute price changes are associated with larger trading volume.}

The intuition of this result is that trading volume and price changes are both driven by underlying changes in betas. Beta increases raise the discount rate. Larger beta changes create both larger price changes and larger amounts of trading volume.

Implication 5: \textit{If the factor across which there is heterogeneity is positively priced and the risky security is positively exposed to the factor, then there exists an asymmetric relation between price changes and trading volume. Price increases are associated with more trading volume than price decreases.}

The intuition of this results is similar to the last except that the asymmetry in trading volume from beta changes drives an asymmetry in the relation between trading volume and price changes.

Both these results can be seen in Figure 3. In Panels A and C, I plot the price change between date 0 and 1 as function of changes in the security’s beta or average dividend level. In Panel A, I plot the total price response including the price drift due to the evolutions of time (i.e. reduction in risk premia as the terminal period approaches). In Panel C, I isolate the price effect due to beta and mean dividend level shocks. Panel C can be thought of as unexpected price changes. In these panels we see that positive shocks to the mean dividend level increases the price while positive shocks to the beta, i.e. risk exposure, decreases the
In Panels B and D, I plot the relation between trading volume and price changes that result from shocks to beta and mean levels. In Panel B, I plot the total relation. In Panel D, I plot the relation isolating the portion of trading volume and price changes that is due to shocks and not the passage of time. We see in these panels that because shocks to the average dividend level do not generate trading volume there is no relation between price changes driven by shocks to the average dividend level and trading volume. In contrast, because shocks to betas generate trading volume and price changes, there is a relationship between price changes driven by beta shocks and trading volume. In Panel D, we see that larger beta shocks lead to both larger price changes and larger amounts of trading volume—giving Implication 4. Moreover in Panel D we see how the asymmetry in trading volume generated by beta shocks leads to an asymmetry in this relationship between price changes and trading volume. Trading volume increases more sharply for price increases than for price decrease.

These two predictions match the fact that stock trading volume and the absolute size of prices changes are positively correlated. Karpoff (1987) provides a review of papers that establish this fact in individual equities, portfolios of equities, equity market aggregates and future markets. Karpoff (1987) also documents the pervasive findings across equity markets of an asymmetry in this relation: price increases are associated with more trading volume than price decreases. Others continue to document the robustness of this positive and asymmetric relation between trading volume and absolute price changes. Jain and Joh (1988) extend the work to relations at the hourly horizon; Gallant et al. (1992) use non-parametric methods to estimate the joint density of trading volume and price changes; Kandel and Pearson (1995) find the asymmetry on announcement days; and Lee and Swaminathan (2000) find this relation in their work investigating momentum.\(^{10}\)

\(^{10}\) Karpoff (1988) suggests an alternative model where the source of the asymmetric relation between trading volume and price changes is driven by high shorting costs. Karpoff’s model and mine could both be contributing to the observed effects. Though Long (2007) finds the asymmetric relation in option markets and Chamberlain et al. (1991) find the asymmetry in index markets. Because these markets have symmetric
D. Trading Volume and Price Reversals (Return Predictability)

In my model the relation between trading volume and price changes helps to identify the source of the price change. The more trading volume accompanying a price change the more likely the price change is driven by a discount rate change, i.e. a beta change, compared to an average cash flow change, i.e. a change in the average level of dividends. These different responses can be seen Panel D of Figure 3 in the distinct patterns between the price and trading volume relationship driven by beta shocks compared to mean dividend level shocks.

A price change due to cash flow news does not reverse. In contrast, price changes due to discount rate shifts reverse in expectation over time through higher or lower expected returns following the price change. Because trading volume helps to distinguish the underlying cause of a price change, trading volume can be used to predict returns. This gives the final prediction of my model.

Implication 6: Price changes without trading volume indicate news about cash flows and should not be expected to be reverse. Price changes accompanied by trading volume indicate discount rate changes and can be expected to reverse over time.

The connection between trading volume and return predictability is well established in the literature. Campbell et al. (1993) establish this reversal pattern in daily index return data. Conrad et al. (1994) extend this work to individual security data and to a horizon of weeks rather than days. Llorente et al. (2002) find similar return reversals for high trading volume they deem to be uninformed.

Lee and Swaminathan (2000) produce a double sort of momentum portfolios on trading volume. They find that the amount of trading volume predicts how long momentum will continue and how quickly it will reverse. Their results provide evidence on longer term reversals and trading volume. Gervais et al. (2001) produce time-series sorts on trading volume rather than cross-sectional sorts. They identify stocks with high and low trading costs of taking long and short positions, they argue that these results contradict the shorting cost hypothesis of Karpoff (1988).
volume relative to themselves over the last 50 days. Gervais et al. find that these high trading volume stocks have higher returns over the next 100 days than the low trading volume stocks. These return difference are concentrated in stocks that have recently lost value. Their results are consistent with high trading volume indicating these price decreases are due to higher discount rates rather than decreased cash flow expectations hence the higher returns in the following periods.

E. Trading Volume and Model Parameters

The amount of beta risk in the economy has two effects on trading volume. A large amount of beta risk distorts the agents’ holdings away from full hedging, resulting in lower trading volume for the same size beta movements. This dampening of trading volume is small. A change in beta risk, $\sigma_\beta$, from .45 to .6 results in almost no change in the amount of trading volume for a given beta shock.

Counteracting this dampening effect, however, is that with more beta risk the size of the typical beta movement is larger. The increased size of beta movements leads to more trading volume overall. This increase in trading volume can be seen in Panel B of Figure 2. There I show the amount of trading volume generated by beta moves standardized by the amount of beta risk with the dashed red line above the solid blue line.

Trading volume increases as the amount of heterogeneity in the economy increases—a result consistent with previous theoretical work. With more heterogeneity, agents must rebalance larger positions when the security’s beta changes. The increase in trading volume from an increase in heterogeneity can be seen in Panel C of Figure 2.

Trading volume from my mechanism is not sensitive to changes in risk aversion. As risk aversion increases, trading volume decreases very slightly. The change in the trading volume under a risk aversion of 3 versus a risk aversion of 5 is almost unnoticeable. This effect comes from agents increased aversion to bearing the risk of parameters shifting in order to hedge their heterogeneity.
III. Empirical Results

In this section, I test the predictions of my model (Implications 1 to 3) that have not previously been documented in the literature. In taking my model to the data, I am testing one case of the broadly applicable point that changes in the risk exposure of assets should drive a need to trade. In the data I choose to focus on equity exposures to the market return: beta. I focus on this risk exposure because it is the most economically important one in equity markets and because measuring high frequency changes in cash flow risk exposures is infeasible. Strictly speaking, the model I present in the previous sections discusses covariances with cash flows rather than covariances with returns. The intuition and results hold just as well for covariances with returns, so I test the implications on return covariances. I present the model with multiple assets as an extension in the Internet Appendix B because no additional intuition is gained relative to the additional complication.

The data are from the daily CRSP files January 1962 to December 2011, limited to share codes 10 and 11 and firms traded on the NYSE. I limit firms to only those trading on the NYSE to have comparable measures of trading volume. I use average daily turnover (number of shares traded divided by the number of shares outstanding) as my measure of trading volume following the recommendation of Lo and Wang (2009). Each calendar month I estimate a market beta for each firm using the market factor from Kenneth French’s website. Almost all firms have a full month of observations. For firms with partial months of observations, I delete firms months with fewer than 10 trading days.

With these estimated betas, I calculate the change in the betas from the prior month to the current month. Because betas are noisily estimated, I winsorize the data at the 1% level, though the result remain virtually unchanged using the original data.

Trading volume has risen substantially over time though not monotonically (see Lo and Wang (2009)). This increase can be seen Panel B of Table I and Figure 4. In the table and figure we see that in addition to the rise in trading volume there is substantial heterogeneity in trading volume across. The goal of my paper is to help understand this heterogeneity
not explain this rise in trading volume over time. I account for the time trend in two ways: I either include a time fixed effect (monthly) in my regressions or I run my regression on five year subperiods of the data. Lo and Wang (2009) argue that subperiod regressions are a robust way of dealing with the time trend in trading volume that does not force one to take a strong parametric stance on the form of the trend. My results are similar using either method. The subperiod methodology shows the increased sensitivity of trading volume to beta changes in more recent periods meaning the effect I document grows in importance over time.

A. Beta Changes

Beta variation is large and the feature of securities on which I focus as an underlying reason to trade. In Panel A of Table I, I show a measure of this beta variation across months at the stock level. Estimation error inflates my estimate of beta variation. This inflated measure is reported the first column of Panel A. This measurement error in my main variable of interest dampens my coefficient estimates, pushing them towards zero. This dampening makes my coefficients under-estimates, suggesting the true relation between beta changes and trading volume is even stronger than what I report.

By assuming that the beta measurement error is independent of the true beta variation, I estimate the true beta variation on average. With this assumption, the observed time series variance of beta estimates from monthly regressions is the sum of the variance of the true beta changes and the the variance of the measurement error:

\[ \sigma^2(\hat{\beta}) = \sigma^2(\beta) + \sigma^2(\text{measurement error}). \] (34)

I estimate the variance of the measurement error by averaging the time series of standard variances (standard errors squared) from the monthly regressions that produce my beta
estimates. This methodology follows that of Fama and French (1997).\textsuperscript{11}

I report the estimates of the true variation in monthly betas in the second column of Panel A. We see that even after accounting for measurement error there is a substantial amount of beta variation month to month. The average standard deviation in beta variation (mean and median) is about .25. There is substantial heterogeneity in the variation of betas across stocks. The third quartile is 0.41 and the first quartile is estimated to be 0.

Because trading volume is not well defined at the portfolio level, I use individual stock data for my measures of trading volume and beta changes. This means unlike with asset pricing tests I am unable to take advantage of the diversification of idiosyncratic risk in portfolio formation that improves beta estimates. Thus I must accept the dampened coefficient estimates in my regressions.

The beta variation I find at the security level is consistent with a large literature that documents betas vary over time. Blume (1975) shows that extreme betas regress toward the mean. Fama and French (1997) study the betas of industry portfolios finding large time-series variation in the beta estimates of these portfolios. They find that the average across industries of the time-series variance of the market betas is .1 and of the HML betas is .17. These average time-series variances are almost as large as the cross-sectional variance across industry portfolios.

Lewellen and Nagel (2006) estimate variation in the market betas of the factor portfolios SMB, HML and momentum. They find that the time-series standard deviation of the quarterly estimated betas to be 0.33, 0.25 and 0.65. Plots of their beta estimates show the variation in betas occur at relatively high frequency. Betas of the factor portfolios often make movements of 1 or more over the course of a single year.

One sees from these results that stock level beta variation is larger than that of portfolios. The substantial variation in betas remaining in portfolios suggest that there is correlation

\textsuperscript{11}In this methodology there is no guarantee that variance of the estimated betas will be greater than the estimate of the measurement error variance. In cases where the implied true variance would be negative, I set the true variance estimate to zero.
in beta changes across stocks. The beta variation remaining in broad portfolios suggest that even when holding portfolios to hedge (or bet) rather than using individual stocks, one would still be forced to trade to maintain constant risk exposures as betas vary.

B. Trading Volume and Beta Changes

The main prediction of my model is that trading volume and beta changes should be positively related. To impose the minimal assumptions necessary to observe this connection, I begin with sort results in Table II. Each month I sort stocks independently into five portfolios in two dimensions. First, I sort stocks on the absolute value of the change in their beta estimate from the previous month to the current month. Second, I sort stocks on turnover in the current month. I determine new break points between the portfolios monthly. These monthly updated breakpoints are like the inclusion of time fixed effects. I report the fraction of firms across all months that fall in the intersection of these portfolios is reported.

If there were no connection between trading volume and beta changes, we would expect to see stocks evenly spread among these portfolios (4% of stocks in each). Instead, in Table II, we see a concentration of stocks along the main diagonal and a monotonic decline as we step down the minor diagonals. Thus we see that stocks that have larger beta changes tend to have more trading volume, and we see that stocks with smaller beta changes tend to have less trading volume.

This non-parametric methodology provides a slight weighting toward current periods because I weight each firm equally and there are more firms in current periods. The results are virtually unchanged when I instead weight equally across years or use yearly beta changes and trading volume measures. Also the results are unchanged when I consider alternative ways of calculating beta changes: a lead beta change or the beta change between the previous period and following period. This robustness to the skipping the beta measured in the period concurrent with the trading volume suggests that any possible trading volume induced beta mis-measurement is not the source of my result.
For the remainder of the empirical results, I focus exclusively on beta changes to between the lagged period and current periods as my measure of beta changes. I focus on these lagged beta changes because I find it the most natural mapping of my model to the data. Agents are trading in a given period in response to changes to current beta from previous betas because current betas are what will expose them to the variation in returns they desire for either hedging or speculating. Also lagged beta changes are the most natural ones to consider when thinking about the my model’s prediction for connections between price changes and trading volume.

To test my model’s three predictions about the connection between beta changes and trading volume I use a panel regression framework.\textsuperscript{12} The first prediction, that there is a positive relation between trading volume and beta change, is tested by regressing a stock’s average daily turnover in a given month on the absolute value of the estimated beta change from the previous month to the current month. The predicted coefficient is positive.

\[
\text{Turnover}_{i,t} = a + b \times \hat{\beta}_{i,t} + c \times |\Delta \hat{\beta}_{i,t}| + \epsilon_{i,t}. \tag{35}
\]

My model’s second prediction is that there should be an asymmetric relation between trading volume and beta changes: more trading volume accompanies beta decreases than beta increases. I test this prediction by including the additional term of the absolute value of the beta change interacted with a dummy variable if the underlying beta change is positive. I indicate this interaction term with a superscript plus sign. The predicted coefficient is negative.

\[
\text{Turnover}_{i,t} = a + b \times \hat{\beta}_{i,t} + c \times |\Delta \hat{\beta}_{i,t}| + d \times |\Delta \hat{\beta}_{i,t}|^+ + \epsilon_{i,t}. \tag{36}
\]

The third prediction of my model is that there should be a smaller relation between trading volume and beta changes for stocks with large betas. I test this prediction by adding a third

\textsuperscript{12}In forming my regression specifications, I focus on the linearized (via Taylor expansion) prediction of my model rather using \( \frac{1}{\beta} \) terms. The intuition of pursing this linearized functional form is to minimize the effects of beta measurement error. Terms with beta in the denominator will exaggerate any measurement error of beta, especially for small betas.
term that is the interaction between the level of a stock’s beta and the beta change. The predicted coefficient is negative.

\[
\text{Turnover}_{i,t} = a + b \times \hat{\beta}_{i,t} + c \times |\Delta \hat{\beta}_{i,t}| + d \times |\Delta \hat{\beta}_{i,t}|^+ + e \times \hat{\beta}_{i,t} \times |\Delta \hat{\beta}_{i,t}| + \epsilon_{i,t}.
\] (37)

In all three regressions I include a term for the level of the stock’s beta. I include this for two reasons. Though not a prediction of my main model. My model extended to consider multiple assets predicts that some of the trading volume driven by beta changes in other stocks will spill over to other stocks. Stocks with higher betas receive more of this spill over trading volume (see Internet Appendix B, Equations (B-17) through (B-19)). Thus the predicted coefficient is positive. I also include the beta level term because I include an interaction term including the level of beta in my third specification. My inclusion of the beta level term and finding of a positive coefficient is consistent with previous literature (see Lo and Wang (2009)).

I do not test Implications 4 through 6 of my model because these results are already well established empirically in the literature. I discussed the empirical support for the those predictions as they were presented in Section II.

There is a substantial trend in the overall level of trading volume over my sample period (see Table I). My goal is to explain the heterogeneity in trading volume not this trend, so I control for the trend in two ways. For my main control I include monthly time fixed effects. These fixed effect allow for there to be a different level of trade each month. As an alternatively method, I control for changing trading volume over time time by using five year sub-samples as advocated by Lo and Wang (2009). I find similar results across both methods.

To account for any unobserved firm level characteristics that one might worry drives my results I include firm level fixed effects. Firm level fixed effect capture any effect due to a firm characteristic that is constant over my sample. To control for the facts that trading...
volume is higher for stocks with higher idiosyncratic volatility I include as a control the variance\textsuperscript{13} of the residuals from the CAPM regression used to estimate each firm’s monthly beta. These estimates of idiosyncratic volatility vary from month to month, so they are not absorbed by the firm fixed effect. The fact that trading volume is higher in periods of high market volatility is already controlled for by my time fixed effects. When I use the alternative subperiod regressions instead of time fixed effects, I include as a control the estimated market volatility each month.

Though I include time and firm fixed effects in my main specification, I also run a specification where I calculate the standard errors with double clustering. More importantly though, I double cluster the standard errors in my alternative subperiods regressions where I omit time fixed effects. Because I am testing against the null that the coefficients on each of my beta or beta change terms is zero, I do not need to adjust my standard errors to correct for the fact that betas are estimated (see Wooldridge (2002) page 141).\textsuperscript{14}

In Table III, I present the results of these regressions. The first three columns omit the firm fixed effect. Columns four through six present my preferred specification that includes a time and firm fixed effect. Columns seven through nine repeat the specifications of columns four through six but with the addition of double clustered standard errors. Columns ten through twelve add the additional control for firm level idiosyncratic risk. Coefficients in this table and all other regression tables are in basis points of average daily turnover.\textsuperscript{15}

We see results consistent with my model in all specifications. The coefficients are both economically and statistically significant. I now focus my discussion on my preferred specification in columns four through six. As we move across these columns we see that each effect is present incrementally: trading volume and beta changes are positively correlated; there

\textsuperscript{13}Results are similar for controlling with the standard deviation.

\textsuperscript{14}In asset pricing tests we need to include a correction correct for standard errors to account for the estimation of beta, because our null is that the coefficient on the beta, e.g. the market premium, is non-zero (see Shanken (1992)).

\textsuperscript{15}One would multiply by 252, the number trading days, to obtain the magnitude of an annual effect.
is an asymmetric relation between trading volume and changes in beta with larger trading
volume occurring with beta decreases; and the effect of beta changes on trading volume is
smaller for larger beta stocks. Column four shows that an average one standard deviation
change beta (.86) leads to an increase in annual turnover of approximately 9%. In column
six we see the effect of the asymmetric relation between trading volume in beta changes.
For a stock with a beta of 1, a one standard deviation beta decrease causes us to expect a
turnover to increase of 13%. This compares to an expected turnover increase of only 4% for
a beta increase. These compare to a 30% annual standard deviation of average turnover.

In Table IV, I restrict my sample to only the most traded half of stocks in each five year
subperiod. I focus on relative trading volume within subperiod to account for the trend
in trading volume over time in a relatively non-parametric way that captures the average
liquidity level of a stock. This focus on the most traded stocks alleviates the concern that
increases in trading volume for thinly traded stocks might cause changes in measured betas. I
find the strength of the effects predicted by my model increases across nearly all specifications
for this sub-sample. The magnitudes of the coefficients increase approximately 30%. Thus,
we see the relation between beta changes and trading volume are present even in the most
liquid stocks.

In Table V I show the results of my alternative method for controlling for the changes
in the level of trading volume across my sample period. I run the regression that includes
all three effects predicted by my model on five year subperiods. I retain the firm level fixed
effects but I omit the time fixed effects. In place of the time fixed effects I add a control
for market volatility each month and continue to calculate the standard errors via double
clustering. The predictions of my model are statistically significant in every subperiod and
the economic significance grows substantially over time. In the last subperiod the coefficients
on the effects of beta changes are roughly eight times as large. Thus for a stock with a beta of
1, a one standard deviation beta decrease would lead us to expect a 73% increase in turnover.

Note that for consistency, we want to use the measure of beta changes including measurement error
because the coefficients are dampened in the estimation process because of the measurement error in beta.
These increasing effects of beta changes over time are consistent with the falling transaction costs allowing people to pursue more of their preferred trading motivated by beta changes that otherwise would have been too expensive.

C. Robustness Checks

C.1. Estimation with Dimson Betas

I use the methodology of Dimson (1979) with two leads and lags to estimate betas. This is an alternative way to alleviate concerns that trading volume might affect observed prices, through such forces as asynchronous trading, and hence measured betas. A problem with the Dimson beta estimation methodology is that the measurement error (e.g. standard errors) of the estimated beta increases. This increase comes from the Dimson beta estimate being a sum of the coefficients on each of the leads and lags as well as the contemporaneous return. The standard variance of this beta estimate is approximately the sum of the standard variances of each of the sub-components.\textsuperscript{17}

I again use the methodology of Fama and French (1997) (see Table I for more details) to decompose the estimated beta variation across months into an estimate of the true beta variation and variation due to measurement error. The mean standard deviation of total estimated beta changes using Dimson betas is 1.92 versus 0.86 for betas estimated with the standard methodology. After subtracting out the estimated measurement error, the mean estimate of standard deviation of true beta changes is 0.27 under the Dimson methodology compared to 0.24 under the standard beta estimation methodology. Thus we see using the Dimson beta methodology gives similar estimates of true beta variation but comes with more than double the estimation error.

In Table VI, I present the results of my monthly panel regressions using these Dimson beta estimates. We see that across all specifications the coefficients are consistent with\textsuperscript{17}The standard variance of this beta estimate would be $\tilde{\boldsymbol{I}}'\tilde{S}\tilde{I}$ where $S$ is the standard error variance covariance matrix of the beta estimates.
the predictions of my model and are statistically significant. Thus it is unlikely that the
effects I document are an artifact of trading changing our ability to measure betas. The
magnitude of the estimated coefficients is about half that obtained when using standard
beta estimation methodology. This reduction in coefficient magnitude is consistent with the
increase measurement error in the betas dampening the coefficient estimates toward zero.

C.2. Yearly Panel Regressions

I estimate the panel regressions for yearly turnover against stocks’ beta changes across
years, as an alternative to my previous specification of estimating panel regressions for
monthly turnover against stocks’ beta changes across months. When moving from the
monthly to yearly measurement frequency, three effects operate. First, some of the beta
variability that occurs at higher frequencies than yearly averages out. Second, the amount
of measurement error changes because of the longer sample period. Third, the statistical
precision in the panel regression diminishes because the number of observations are cut by
nearly a factor of twelve.

In Table VII, I present the results of these yearly panel regressions. Across all spec-
ifications the first and second predictions of my model (those regarding the positive and
asymmetric relation between trading volume and beta changes) are statistically and eco-
nomically significant. The magnitude of the coefficients for these effects doubles from that
of the coefficients estimated from the monthly panels. The effect of reduced measurement
error dominates, reducing the dampening of the coefficients.

For the coefficient testing the third prediction of my model it appears that the reduced
statistical precision dominates. The coefficient estimates on the interaction of the beta level
and the beta change effect are of the predicted sign and of a magnitude consistent with the
estimates from the monthly panel regressions. However, the coefficient is only statistically
significant in the specification without firm fixed effects.
C.3. Changes in Betas on Multiple Factors

I test the prediction of a positive relationship between changes in risk exposures and turnover on the Fama French size and book-to-market factors. I do not test the additional predictions such as the asymmetry of this relation because those relations depend upon most stocks having a positive exposure to the factor. The average market beta is 1 while the average exposure to other factors such as HML and SMB is closer to zero.

In Table VIII, I present the results of monthly panel regressions of average monthly turnover for a stock on the changes in its beta exposures to the three Fama French factors. I find a positive relation between a change in exposure to any of the Fama French factors and increased turnover. The relation is highly statistically significant across all specifications, though the magnitude of the coefficients on each individual factor’s beta change is reduced from the coefficient estimate on the market beta change alone.

IV. Conclusion

I propose and test a new mechanism for generating trade: shocks to the risk exposures of securities. Shocks to risk exposures matter for pricing assets, and I show how they matter for generating trade. My work departs from the existing trading volume literature that focuses on shocks to investor heterogeneity as the source of trading volume. Shocks to securities’ risk exposures and shocks to investor heterogeneity both exist and surely work in concert to generate the total observed trading volume.

My model makes six predictions. Three describe the joint behavior of trading volume and the risk of stocks. And three describe the joint behavior of trading volume and stock prices (returns). I document three new empirical regularities between trading volume and changes in market beta that coincide with the first three predictions of my model. I also show how the three predictions joining trading volume and stock prices provide a new explanation and interpretation of well document facts.
Connecting trading volume to stocks’ risk exposures has several applications. First, this connection provides a new interpretation for the increase in trading volume, especially that in managed portfolios, e.g. mutual funds and hedge funds, that investors tolerate despite being unaccompanied by higher returns (Carhart, 1997; Asness, Krail, and Liew, 2001; Fama and French, 2010). Over time I find that trading volume is more sensitive to changes in stocks’ risk exposures. This increased sensitivity fits with individual investors and institutions being better able to accommodate their desire for constant risk exposures as transaction costs have fallen. This interpretation is consistent with the findings of Chordia et al. (2011) who find that small orders and trade between institutions have increased dramatically even while prices have become more efficient implying the increasing difficulty of this trade to generate alpha. Additionally, if a security’s characteristics proxy for difficult-to-measure covariances with risk factors, trading to maintain constant risk exposures can help explain and rationalize the findings of Nagel (2004) that between 20-25% of NYSE/AMEX trading volume is explained by fixed trading rules based on stock characteristics.

A second application is that with positive transaction costs investors may value stocks with stable risk exposures. In my model I show that securities with uncertainty in their risk exposures have an additional price discount to compensate investors for holding this risk. Combining this risk premium with the additional transactions associated with these risk changes provides another channel for time varying risk exposures to affect equilibrium levels of asset prices and provides a potential solution to the puzzle raised by Gervais et al. (2001) that high volume stocks tend to have higher returns.

A third application is that this mechanism provides a new channel to use trading volume to learn about the conditional risk exposures of stocks. My model implies trading volume’s connection to return predictability is through risk not just behavioral biases (Lee and Swaminathan, 2000) or compensation for liquidity provision (Campbell et al., 1993), so one could use trading volume to help decompose returns into cash flow shocks and discount rate shocks in the spirit of Campbell and Vuolteenaho (2004). Finally, one could add trading volume
as a state variable in conditional asset pricing models\textsuperscript{18} to help proxy for beta changes and improve such a model’s performance.

\textsuperscript{18}See Jensen (1968); Dybvig and Ross (1985); Jagannathan and Wang (1996); Lettau and Ludvigson (2001); Wang (2003); Zhang (2005); Lustig and Van Nieuwerburgh (2005); Petkova and Zhang (2005); Santos and Veronesi (2006); Ang and Chen (2007).
A. Two Period Model Solution

Below, I present the details of my exact analytical solution of my model.

A. Recursive Problem

I rewrite an agent’s problem in terms of a recursive competitive equilibrium problem. I begin by solving an agent’s problem at date 1.

A.1. Date 1 Problem

The problem faced by an agent of class $i$ at date 1 with wealth $W_{1}^{i}$ and given $P_{1}$ for the risky asset is

$$V_{1}^{i}(W_{1}^{i}, \mu_{1}, \beta_{1}, P_{1}) = \max_{\{c_{1}^{i}, s_{1}^{i}\}} u(c_{1}^{i}) + \delta E_{1} u(c_{2}^{i}),$$

where

$$c_{2}^{i} = (W_{1}^{i} - c_{1}^{i} - s_{1}^{i}P_{1})R + L_{2}^{i} + s_{1}^{i}D_{2}.$$  

(A-2)

I substitute the definitions into the objective function and compute the expectation to give

$$\max_{\{c_{1}^{i}, s_{1}^{i}\}} \left\{ -\exp(-\gamma c_{1}^{i}) - \delta \exp \left( \frac{\gamma}{\delta} \left( c_{1}^{i} + s_{1}^{i}P_{1} - (W_{1}^{i} + s_{1}^{i}\delta\mu_{1}) \right) + \frac{\gamma^{2}}{2} \left( s_{1}^{i}\beta_{1} + \lambda_{i} \right)^{2} \sigma_{f}^{2} \right) \right\}.  \quad \text{(A-3)}$$

From this equation I compute the first order conditions with respect to each of the choice variables:

$$[s_{1}^{i}] : \quad 0 = -\exp \left( \frac{\gamma}{\delta} \left( c_{1}^{i} + s_{1}^{i}P_{1} - (W_{1}^{i} + s_{1}^{i}\delta\mu_{1}) \right) + \frac{\gamma^{2}}{2} \left( s_{1}^{i}\beta_{1} + \lambda_{i} \right)^{2} \sigma_{f}^{2} \right) \quad \text{and}$$

$$[c_{1}^{i}] : \quad 0 = \gamma \left\{ -\exp(-\gamma c_{1}^{i}) \right\}$$

$$- \delta \exp \left( \frac{\gamma}{\delta} \left( c_{1}^{i} + s_{1}^{i}P_{1} - (W_{1}^{i} + s_{1}^{i}\delta\mu_{1}) \right) + \frac{\gamma^{2}}{2} \left( s_{1}^{i}\beta_{1} + \lambda_{i} \right)^{2} \sigma_{f}^{2} \right) \}.$$  \quad \text{(A-5)}
I solve these first order conditions for optimal consumption and share holdings as functions of the price. The solution gives the policy functions:

\[ s_1^i(W_1^i, \mu_1, \beta_1, P_1) = \left( \frac{1}{\beta_1^2 \gamma \sigma_f^2} \right) \left( - \frac{P_1}{\delta} + \mu_1 - \beta_1 \gamma \lambda^i \sigma_f^2 \right) \]

and

\[ c_1^i(W_1^i, \mu_1, \beta_1, P_1) = \frac{P_1^2 - 2P_1 \delta (\mu_1 - \beta_1 \gamma \lambda^2 \sigma_f^2) + \delta (2W_1^i \beta_1^2 \gamma \sigma_f^2 + \delta \mu_1 (\mu_1 - 2 \beta_1 \gamma \lambda^2 \sigma_f^2))}{2 \beta_1^2 \gamma \delta (1 + \delta) \sigma_f^2} \]

(A-6)

(A-7)

Following these policy functions yields the solution to the value function \( V_1^i(W_1^i, \mu_1, \beta_1, P_1) \).

I equate the supply and demand of the risky asset from the two classes of agents to find the equilibrium price. Solving the market clearing condition

\[ 1 = \frac{1}{2} s_1^1(W_1^1, \mu_1, \beta_1, P_1) + \frac{1}{2} s_1^2(W_1^2, \mu_1, \beta_1, P_1) \]

yields the equilibrium price:

\[ P_1(\mu_1, \beta_1) = \delta (\mu_1 - \gamma \sigma_f^2 \beta_1^2). \]

Generally the price would be a function of the entire wealth distribution, but with exponential utility the price is a function of only the dividend’s parameters. I rewrite the policy functions in terms of the equilibrium price. We see they are functions of only personal wealth and the current dividend parameters; hence, I drop the wealth distribution as a state variable for shareholding:

\[ c_1^i(W_1^i, \mu_1, \beta_1) = \frac{1}{1 + \delta} \left( W_1^i + \frac{\delta \gamma \sigma_f^2 \beta_1 (\beta_1 - 2 \lambda^i)}{2} \right) \]

and

\[ s_1^i(\mu_1, \beta_1) = 1 - \frac{\lambda^i}{\beta_1}. \]

(A-8)

(A-9)

I apply this price substitution to the value function finding the only state variables that matter are personal wealth and the current dividend parameters:

\[ V_1^i(W_1^i, \mu_1, \beta_1) = -(1 + \delta) \exp \left( - \frac{\gamma \left( W_1^i + \delta \gamma \beta_1 (\beta_1 - 2 \lambda^i) \sigma_f^2 \right)}{2(1 + \delta)} \right). \]

(A-10)
A.2. Date 0 Problem

I use the value function found from the date 1 problem (Equation (A-10)) to write the date 0 problem. For an agent of class $i$ with wealth $W^i_0$ when the dividend payment parameters are $\mu_0$ and $\beta_0$ the problem is

$$\max_{\{c^i_0, s^i_0\}} u(c^i_0) + \delta E_0 V^i_1(W^i_1, \mu_1, \beta_1)$$

where

$$W^i_1 = (W^i_0 - c^i_0 - s^i_0 P_0)R + L^i_1 + s^i_0 (P_1 + D_1).$$

Agents take the price at date 0, $P_0$, as given. They expect the distribution of prices at date 1 to follow

$$P_1(\mu_1, \beta_1) = \delta (\mu_1 - \gamma \sigma_f^2 \beta_1^2).$$

This distribution is realized since it is the equilibrium price function at date 1.

I compute the expectation of the value function given the price distribution in Equation (A-13). $\phi_f$, $\phi_\mu$, and $\phi_\beta$ denote the normal densities of each random variable.

$$E_0 V^i_1(W^i_1, \mu_1, \beta_1) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} V^i_1(W^i_1, \mu_1, \beta_1) \phi_f(f_1) \phi_\beta(\beta_1) \phi_\mu(\mu_1) df_1 d\beta_1 d\mu_1$$

Substituting in the wealth transition equation (Equation (A-12) and taking advantage of the independence of the three random variables, I obtain

$$= -(1 + \delta) \exp \left( \frac{\gamma (c^i_0 + P_0 s^i_0 - W^i_0 - s^i_0 \delta \mu_0)}{\delta (1 + \delta)} \right)$$

$$\star \int_{-\infty}^{\infty} \exp \left( -\frac{\gamma f_1 (s^i_0 \beta_0 + \lambda^i)}{1 + \delta} \right) \phi_f(f_1) df_1$$

$$\star \int_{-\infty}^{\infty} \exp \left( -\frac{\gamma \delta s^i_0 \mu_1}{1 + \delta} \right) \phi_\mu(\mu_1) d\mu_1$$

$$\star \int_{-\infty}^{\infty} \exp \left( \frac{\delta \gamma^2 \beta_1 ((-1 + 2 s^i_0) \beta_1 + 2 \lambda^i) \sigma_f^2}{2(1 + \delta)} \right) \phi_\beta(\beta_1) d\beta_1.$$
The first two integrals are standard. Their computation yields
\[
\int_{-\infty}^{\infty} \exp \left( -\frac{\gamma_f (s_0^i \beta_0 + \lambda^i)}{1 + \delta} \right) \phi_f (f_1) df_1 = \exp \left( \frac{\gamma^2 (s_0^i \beta_0 + \lambda^i)^2 \sigma_f^2}{2(1 + \delta)^2} \right) \quad \text{and} \quad (A-16)
\]
\[
\int_{-\infty}^{\infty} \exp \left( -\frac{\gamma s_0^i \mu_1}{1 + \delta} \right) \phi_\mu (\mu_1) d\mu_1 = \exp \left( \frac{s_0^i \gamma (\lambda_0^i \sigma_\mu^2)}{2(1 + \delta)^2} \right). \quad (A-17)
\]
Computing the beta integral in closed form requires the following restriction (see Gradshteyn and Ryzhik (2007)): \[(2s_0^i - 1) \gamma^2 \sigma_f^2 \sigma_\beta^2 < 1 + \frac{1}{\delta}. \quad \text{(A-18)}\]
This restriction requires there not be a combination of too much heterogeneity, beta uncertainty and risk aversion. The intuition of this restriction is that the tails of the distribution cannot be too fat. If they are too fat (or risk aversion is too high) there is no price low enough to compensate agents for the risk of holding the risky security, hence there is no equilibrium. Applying this restriction, I calculate the last integral
\[
\int_{-\infty}^{\infty} \exp \left( \frac{\delta \gamma^2 \beta_1 (\lambda_0^i \sigma_\beta^2)}{2(1 + \delta)} \right) \phi_\beta (\beta_1) d\beta_1
\]
\[
= \left( \frac{1 + \delta \left( 1 - 2s_0^i \gamma^2 \sigma_f^2 \sigma_\beta^2 \right)}{1 + \delta} \right)^{-\frac{1}{2}}
\]
\[
\times \exp \left( -\frac{\delta \gamma^2 \sigma_f^2 \left( \lambda_0^i \sigma_\beta^2 \right)}{2(1 + \delta) \left( -1 - \delta - \delta (1 - 2s_0^i \gamma^2 \sigma_f^2 \sigma_\beta^2) \right)} \right) \quad (A-19)
\]
Because of their length, many of the equations I reference in the following text are presented in Subsection A.B. The objective function for the agent’s objective, Equation (A-11), is Equation (A-24) after computing the expectation. The first order conditions of this objective function with respect to share holding is (A-25) and with respect to consumption is (A-26).

From these first order conditions, I compute the optimal consumption policy as function of the price and share holding policy (A-27). I find that the optimal share holding policy is the real root of a fifth degree polynomial (A-28). There is no simpler form for the share holding policy function, because there is no general algebraic form for the roots of fifth degree and higher polynomials (see Hungerford (1980)).

To find the equilibrium price, I form the inverse demand function for each agent. This inverse demand function gives the price necessary to make an agent with outside income exposure \( \lambda^i \), content to hold a given number of shares. I denote the inverse demand function as \( P_0(s, \lambda) \). The full form is Equation (17) is in the main text. A key feature of this inverse
demand function is its independence of agent’s wealth.

Using the market clearing condition and the fact that agents must face the same price in equilibrium, I obtain two equations that characterize the market equilibrium:

\[
P_0(s_0^1, \lambda_1) = P_0(s_0^2, -\lambda_1) \quad \text{and} \quad (A-20)
\]

\[
\frac{1}{2}(s_0^1 + s_0^2) = 0. \quad (A-21)
\]

I substitute the market clearing condition into the inverse demand function to obtain the following

\[
P_0(s_0^1, \lambda_1) = P_0(2 - s_0^1, -\lambda_1). \quad (A-22)
\]

The real root of this fifth degree polynomial (A-30) is the equilibrium share holdings of agent type 1. Substituting this equilibrium share holding into agent 1’s inverse demand function gives the equilibrium price (A-31). I denote the equilibrium price and share holding policy as functions of the state variables

\[
P_0(\mu_0, \beta_0) \quad \text{and} \quad s_0^i(\beta_0, \mu_0). \quad (A-23)
\]

I now have the equilibrium decision rules for each agent and the equilibrium price completing the model’s solution. We can see that at each date the relevant state variables for prices and share holdings are the dividend parameters. The wealth of an agent only matters for his or her optimal consumption decisions.
B. Additional Solution Equations

The objective function is

\[-e^{-\gamma c_0} = e^{\frac{2(1+\delta)\left(-W_0 + c_0^i + P_0 s_0^i - \delta \mu_0 \right)}{1+\delta\left(-1+2 s_0^i\right)\sigma_j^2 \sigma_\beta^2} + \frac{\gamma \delta(1+\delta)\sigma_j^2 \sigma_\beta^2}{1+\delta\left(-1+2 s_0^i\right)\sigma_j^2 \sigma_\beta^2} + \delta(1+\delta)\} \]

(A-24)

The first order condition for share holdings is

\[\frac{\gamma \left(2(1+\delta)\left(-W_0 + c_0^i + P_0 s_0^i - \delta \mu_0 \right)\right)}{2(1+\delta)^2} + \frac{\gamma \delta(1+\delta)\sigma_j^2 \sigma_\beta^2}{2(1+\delta)^2} \delta (1+\delta) \]

(A-25)

\[e^{\frac{2(1+\delta)(P_0 - \delta \mu_0)}{\delta} + 2 \gamma \beta_0 \left(\lambda^i + s_0^i \beta_0 \right) \sigma_j^2} + 2 \gamma^3 \delta^2 (1+\delta) \sigma_j^2 \sigma_\beta^2 \left(2 \lambda^i (1+\delta) \beta + (1+\delta) \beta^2 \left(-1+2 s_0^i\right) + (\lambda^i)^2 \gamma^2 \delta \sigma_j^2 \sigma_\beta^2 \right) \]

\[+ \frac{2 \gamma^2 \beta_0 \left(\lambda^i + s_0^i \beta_0 \right) \sigma_j^2}{\left(-1+\delta\left(-1+2 s_0^i\right)\sigma_j^2 \sigma_\beta^2\right)^2} - \frac{2 \gamma \delta(1+\delta)^2 \beta^2 \sigma_j^2}{-1+\delta\left(-1+2 s_0^i\right)\sigma_j^2 \sigma_\beta^2} \]

\[+ \gamma \delta^2 s_0^i \sigma_\mu^2 + \delta \left(-2(1+\delta)\bar{\mu} + \gamma \delta s_0^i \sigma_\mu^2\right)\] = 0
The first order condition for optimal consumption is

\[
e^{-\gamma c_0} - \frac{e^{-\gamma c_0}}{\gamma} = 0 \quad (A-26)
\]

The optimal consumption policy as a function of the price and the share holding policy is

\[
c_0^i = \frac{1}{\gamma + \delta + \sigma^2} \left( \log \left[ \frac{1}{1 + \delta} + \frac{\delta}{1 + \delta} + \frac{\sigma^2}{1 + \delta} - 2\gamma \delta \sigma^2 \right] \right) - \frac{1}{2(1 + \delta)^2} \gamma \left( -\frac{2W_0(1 + \delta)}{\delta} + \frac{2(1 + \delta)P_0s^0_0}{\delta} - 2(1 + \delta)s^0_0 \mu_0 + \gamma \left( \lambda^i + s^0_0 \beta_0 \right)^2 \sigma^2_f \right.
\]

\[
\gamma \delta (1 + \delta) \sigma^2_f \left( 2\lambda^i (1 + \delta) \beta + (1 + \delta) \beta^2 (-1 + 2s^0_0) + (\lambda^i)^2 \gamma \delta \sigma^2_f \sigma^2_\beta \right) + \delta s^0_0 \left( -2(1 + \delta) \bar{\mu} + \gamma \delta s^0_0 \sigma^2_\mu \right) \right)
\]
The optimal share holding policy is the real root of the following polynomial

\[
P_0 = \delta \left\{ \mu_0 - \frac{\gamma^2 \sigma_f^2 (\bar{\beta} + \lambda^i \gamma^2 \sigma_f^2 \sigma_\beta^2)^2}{(-1 + \delta (-1 + \gamma^2 (-1 + 2s_0^i) \sigma_f^2 \sigma_\beta^2))^2} + \gamma s_0^i \sigma^2_{s_\mu} \right. \quad (A-28)
\]

\[
+ \frac{\gamma \delta \sigma_f^2 (-2\bar{\beta}^2 - (1 + \delta + 2\lambda^i \gamma^2 \bar{\beta} \sigma_f^2) \sigma_\beta^2 + \gamma^2 \delta (-1 + 2s_0^i) \sigma_f^2 \sigma_\beta^4)}{(-1 + \delta (-1 + \gamma^2 (-1 + 2s_0^i) \sigma_f^2 \sigma_\beta^2))^2} + \gamma s_0^i \sigma^2_{s_\mu}
\]

\[
+ \delta \left( \bar{\mu} + \frac{\gamma \sigma_f^2 (-\bar{\beta}^2 - (1 + \delta) \sigma_\beta^2 + \gamma^2 \delta (-1 + 2s_0^i) \sigma_f^2 \sigma_\beta^4)}{(-1 + \delta (-1 + \gamma^2 (-1 + 2s_0^i) \sigma_f^2 \sigma_\beta^2))^2} - \gamma s_0^i \sigma^2_{s_\mu} \right)
\]

\[
- \frac{\gamma \beta_0 (\lambda^i + s_0^i \beta_0) \sigma_f^2 + \gamma s_0^i \sigma^2_{s_\mu}}{1 + \delta}
\]

Taking price as a function of share holdings and \( \lambda^i \) forms an inverse demand function:

\[
P_0(s_0^i, \lambda^i) = \delta \left\{ \mu_0 - \frac{\gamma^2 \sigma_f^2 (\bar{\beta} + \lambda^i \gamma^2 \sigma_f^2 \sigma_\beta^2)^2}{(-1 + \delta (-1 + \gamma^2 (-1 + 2s_0^i) \sigma_f^2 \sigma_\beta^2))^2} + \gamma s_0^i \sigma^2_{s_\mu} \right. \quad (A-29)
\]

\[
+ \frac{\gamma \delta \sigma_f^2 (-2\bar{\beta}^2 - (1 + \delta + 2\lambda^i \gamma^2 \bar{\beta} \sigma_f^2) \sigma_\beta^2 + \gamma^2 \delta (-1 + 2s_0^i) \sigma_f^2 \sigma_\beta^4)}{(-1 + \delta (-1 + \gamma^2 (-1 + 2s_0^i) \sigma_f^2 \sigma_\beta^2))^2} + \gamma s_0^i \sigma^2_{s_\mu}
\]

\[
+ \delta \left( \bar{\mu} + \frac{\gamma \sigma_f^2 (-\bar{\beta}^2 - (1 + \delta) \sigma_\beta^2 + \gamma^2 \delta (-1 + 2s_0^i) \sigma_f^2 \sigma_\beta^4)}{(-1 + \delta (-1 + \gamma^2 (-1 + 2s_0^i) \sigma_f^2 \sigma_\beta^2))^2} - \gamma s_0^i \sigma^2_{s_\mu} \right)
\]

\[
- \frac{\gamma \beta_0 (\lambda^i + s_0^i \beta_0) \sigma_f^2 + \gamma s_0^i \sigma^2_{s_\mu}}{1 + \delta}
\]
The equilibrium share holding policy is
\[ s^*_0 = 1 + h \]
where \( h \) is the real root of the following polynomial in \( h \)
\[
\lambda^i \beta_0 \left( -4h^2 \gamma^4 \delta^2 \sigma_f^2 \sigma^2 + \sigma_f (1 + \delta - \gamma^2 \delta \sigma_f^2) \right)^2 + h \beta^2_0 \sigma^2_f \left(-4h^2 \gamma^4 \delta^2 \sigma_f^4 \sigma^4 + (1 + \delta (1 + \gamma^2 \sigma_f^4)) \right)^2 \]
\[ \quad + \delta^2 \left(8h^2 \gamma^6 \delta^2 (1 + \delta)^2 \lambda^i \beta \sigma_f^2 + 2\gamma^2 (1 + \delta)^2 \lambda^i \beta \sigma_f^4 \left(1 + \delta - \gamma^2 \delta \sigma_f^2 \right)^2 \right) \]
\[ \quad + 16h^5 \gamma^8 \delta^4 \sigma_f^4 \sigma^2 \mu + 8h^3 \gamma^4 \delta^2 \sigma_f^4 \sigma^2 \left(-\gamma^2 (1 + \delta)^2 \sigma_f^4 \right) \quad (A-30) \]
\[ \quad + h \left(-1 + \delta \left(1 + \gamma^2 \sigma_f^2 \sigma^2 \right) \right) \left(4\gamma^2 (1 + \delta)^3 \beta^2 \sigma_f^4 \sigma^2 + 2\gamma^2 (1 + \delta) \sigma_f^4 \sigma^2 \left(1 + \delta \right)^2 + \gamma^2 \delta \sigma_f^2 \left(-1 + \delta + 2\gamma^2 \delta \lambda^2 \sigma_f^2 \right) \sigma^2 \right) \]
\[ \quad - h \left(-1 + \delta \left(1 + \gamma^2 \sigma_f^2 \sigma^2 \right) \right) \left(4\gamma^2 (1 + \delta)^3 \beta^2 \sigma_f^4 \sigma^2 + 2\gamma^2 (1 + \delta) \sigma_f^4 \sigma^2 \left(1 + \delta \right)^2 + \gamma^2 \delta \sigma_f^2 \left(-1 + \delta + 2\gamma^2 \delta \lambda^2 \sigma_f^2 \right) \sigma^2 \right) \]
\[ - \left(-1 - \delta + \gamma^2 \delta \sigma_f^2 \sigma^2 \right)^3 \quad \sigma^2 \mu \]

The equilibrium price as a function of the equilibrium share holding policy of agent type 1 is
\[
P_0 = \delta \left[ \mu_0 - \frac{\gamma \delta^2 \sigma_f^2 \left(\beta + \gamma^2 \lambda^i \sigma_f^2 \sigma^2 \right)}{\kappa^2} + \frac{\gamma \delta^2 \sigma_f^2 \left(\beta^2 - (1 + \delta + 2\gamma^2 \lambda^i \sigma_f^2) \sigma^2 - (1 - 2s^*_0) \gamma^2 \delta \sigma_f^2 \sigma^4 \right)}{\kappa^2} \right] \quad (A-31) \]
\[ + s^*_0 \gamma \sigma^2 \mu + \delta \left(\bar{\mu} + \frac{\gamma \sigma_f^2 \left(\beta^2 - (1 + \delta) \sigma^2 - (1 - 2s^*_0) \gamma^2 \delta \sigma_f^2 \sigma^4 \right)}{\kappa^2} - s^*_0 \gamma \sigma^2 \mu \right) - \beta_0 \gamma \left(s^*_0 \beta_0 + \lambda^i \sigma_f^2 + s^*_0 \gamma \sigma^2 \mu \right) \quad \frac{1}{1 + \delta} \]

where
\[ \kappa = -1 - \delta - \delta(1 - 2s^*_0) \gamma^2 \sigma_f^2 \sigma^2. \]
B. Calibration

I present a calibrated example to show the behavior of trading volume in my model. I choose risk aversion, $\gamma$, equal to 5 and a time discount rate, $\delta$, equal to .99 which is consistent with a 1% risk free rate. The model is not sensitive to either choice.

I calibrate the rest of the parameters by matching the aggregate dividend growth from CRSP to the dividend in my model:

\[
D_t = \mu_{t-1} + \beta_{t-1}f_t + \epsilon_t, \quad (B-1)
\]

\[
\epsilon_t \sim N(0, \sigma^2) \quad (B-2)
\]

\[
(B-3)
\]

where $\epsilon_t$ accounts for the portion of the aggregate dividend not correlated with heterogeneity. I measure heterogeneity by matching the annual per employee wage growth for non-durable and durable manufacturing from NIPA to the income of the two classes in my model.

Unfortunately, the data comes from a multi-period world that likely has investors with power utility and growing wealth and consumption, yet I have a model with exponential utility. Exponential utility has trouble with growth because its local curvature changes as wealth changes, but power utility avoids this problem. One way of mapping these together is to take a trend out of the growing wealth and consumption. The idea is to continually remap the wealth and consumption data to where the exponential utility function and power utility function share the same local curvature. Thus, in the data, I take out a very local trend each period, giving me growth rates.

I extract the heterogeneity factor from these wages through a principal component analysis. The first principal component describes wages increasing or decreasing together. Hence, I use the second principal component that accounts for movements of the non-durable and durable wages in opposite directions as the basis for this factor. I scale the second principal component so that the unconditional beta on the factor thus formed is one. This normalization identifies a factor with a standard deviation of $\sigma_f = .05$. In addition to normalizing $\bar{\beta}$ to one, I normalize $\bar{\mu}$ to zero.

To identify the size of the heterogeneity parameter, $\lambda$, I use the fact that $|\lambda f_t|$ should be the realized cross-sectional standard deviation of income each period.\(^{19}\) In calculating this cross-sectional standard deviation, I use only the variation coming from shocks to the second

\[
StdDev(L^i_t) = \sigma_{CS,t} = \left( \frac{\sum_{t=1}^N (L^i_t - \bar{L}_t)^2}{N} \right)^{\frac{1}{2}} = \left( \frac{(\lambda f_t)^2 + (-\lambda f_t)^2}{2} \right)^{\frac{1}{2}} = |\lambda f_t|
\]

\(^{19}\)
principal component. From this series of cross-sectional standard deviation realizations, I calculate the time-series standard deviation. The amount of heterogeneity, $\lambda$, is proportional to this time series standard deviation:

$$\lambda = \text{StdDev}(|\lambda f_t|) \left\{ \left( 1 - \left( \frac{\phi(0)}{\Phi(0)} \right)^2 \right)^{\frac{1}{2}} \sigma_f \right\}^{-1}. \quad (B-4)$$

where $\phi$ and $\Phi$ are the pdf and cdf of the standard normal distribution.\textsuperscript{20} From these calculations I obtain $\lambda = 0.25$.

I identify the amount of beta variation, $\sigma_\beta$, and mean variation, $\sigma_\mu$, from subsample regressions of the dividend on the heterogeneity factor. For these regressions I use five year long non-overlapping sub-samples.

The beta and mean estimates from these regressions vary because of true parameter variation and estimation error. Following the work of Fama and French (1997), I assume these sources of variation are orthogonal. Also following their work, I measure the estimation error by averaging the squared standard errors. Under these assumptions I obtain estimates for the true parameter variation:

$$\sigma(\text{Actual}) = [\sigma^2(\text{Coeff. Est.}) - \text{Ave}[(\text{Std. Err.})^2]]^{\frac{1}{2}}. \quad (B-5)$$

Based on this method I set $\sigma_\beta = 0.45$ and $\sigma_\mu = 0.01$.

\textsuperscript{20} \left( 1 - \left( \frac{\phi(0)}{\Phi(0)} \right)^2 \right)^{\frac{1}{2}} \approx 0.36338

This scale factor is necessary because I am taking the absolute value of a normal random variable.
C. Proofs of Model Implications

Throughout this appendix I assume $\lambda \geq 0$.

A. Implication 1

Implication 1 can be seen from a first order Taylor expansion of Equation 32 around $\Delta \beta = 0$

$$V(\Delta \beta) \approx \frac{K\lambda}{2\beta_0^2} |\Delta \beta|.$$  \hspace{1cm} (C-1)

B. Implication 2

Implication 2 can be seen from a second order Taylor expansion of Equation (32) around $\Delta \beta = 0$

$$V(\Delta \beta) \approx \frac{K\lambda}{2\beta_0^2} |\Delta \beta| - \text{sign}(\Delta \beta) \frac{K\lambda}{\beta_0^3} (\Delta \beta)^2.$$ \hspace{1cm} (C-2)

C. Implication 3

The derivative of Equation (32) around $\beta_0 = 1$ with respect to $\beta_0$ shows Implication 3.

$$\frac{\partial V}{\partial \beta_0}|_{\beta_0=1} = \begin{cases} \frac{K\lambda}{2} \left( -\frac{-\Delta \beta - \Delta \beta^2}{(1+\Delta \beta)^2} \right) < 0, & 1 > \Delta \beta > 0 \\ \frac{K\lambda}{2} \left( \frac{\Delta \beta + \Delta \beta^2}{(1+\Delta \beta)^2} \right) < 0, & -1 < \Delta \beta < 0 \end{cases} \hspace{1cm} (C-3)$$

D. Implications 4 and 5

Implication 4 and Implication 5 can be seen directly in Figure 3 and is discussed in the main text. Here I demonstrate that these implications extend to correlation measures used by econometricians to measure the relationship between price changes and trading volume. By generating one million simulated data points from my model, I calculate the correlation between trading volume and four measures of price changes: (1) raw price changes, (2) absolute price changes, (3) positive price changes, and (4) negative price changes. For both price changes and trading volume, I focus on the portion driven by innovations to either the beta or average dividend level. Price changes are those between date 0 and date 1. Volume is at date 1. To generate the data, I set the beta and mean dividend level at date 0 to their unconditional averages at date 1: $\beta_0 = 1$ and $\mu_0 = 1$. Because trading volume in my model approaches infinity as beta approaches zero (a phenomenon that disappears if I add idiosyncratic risk at the cost of a numerical solution), I bound the value the beta at date 1 a
small distance above zero. For small values of beta variation, $\sigma_\beta$, this bound is not necessary but for larger values of beta variation, $\sigma_\beta$, without this bound the expectation that defines the correlation between price and trading volume does not exist (the integral diverges).

In Table C-1, I show the correlations for a variety of parametrizations of my model and for two values of the positive bound on beta at date 1. In all cases we see the following results which are consistent with the facts: the correlation between trading volume and price changes is positive; the correlation between trading volume and absolute price changes is positive; and the correlation between trading volume and positive price changes is larger than the correlation between trading volume and negative price changes.

**E. Implication 6**

Implication 6 can be seen directly in Panel D of Figure 3 where I completely separate the shocks to beta level and average dividend levels. I now show that the implication extends to regression where the addition of trading volume interacted with current price changes helps predict the portion of the price change that will reverse. I simulate one million observation points from my model. An observation consists of a price change between data 0 and date 1, an amount of trading volume at date 1, and a price change between date 1 and date 2 where the date 2 price is the terminal cash flow from the risky security. As with the previously simulated data, the price change from date 0 to 1 and the trading volume at date 1 include only that part that is due to innovations in beta or mean levels and I bound the minimum beta value in date one slightly away from zero.

On this simulated data, I run the regression

$$\Delta P_{1\to 2} = a + b \times \Delta P_{0\to 1} + c \times \Delta P_{0\to 1} \times Vol_{0\to 1} + \epsilon.$$  \hspace{1cm} (C-4)

In Table C-2, I present the results of this regression for a variety of parametrizations of my model with two different minimum beta bounds. We see across all the parametrization coefficient on the interaction term of price changes and trading volume is negative. This negativity indicates that the presence of more trading volume predicts price reversals—consistent with the facts cited in the main text. The most interesting change in the regression coefficients across different model parametrization is the change that occurs as the variability in average dividend levels, $\sigma_\mu$, increases. In specifications with relatively low variability in average dividend levels, the change in price level itself is strongly predictive of future price reversals—the coefficient is large and negative. This occurs in these cases because almost every price change is driven by discount rate changes and will reverse. Trading volume still helps to distinguish the types of price changes in these cases but is not as useful. When
the amount of variability in average dividend levels increases, more price changes are driven by cash flow effects rather than discount rate effects. The cash flow price changes will not be reversed and current price changes lose their predictive power for future price reversal. And in these cases only current price changes interacted with trading volume help to predict price reversals. Again, this is consistent with the fact that price changes alone do not predict future price changes but price changes interacted with trading volume do.
Table C-1 Correlations Between Trading Volume and Price Changes: This table presents the correlation between trading volume and price changes in simulated data (one million draws) from my model. Data is the portion of trading volume and price changes between periods 0 and 1 due to shocks to dividend means and betas. I present the correlations for four different combinations of trading volume and prices changes: (1) trading volume and raw price change, (2) trading volume and absolute price change, (3) trading volume and positive price changes, and (4) trading volume and negative price changes. The expectation that defines the correlation diverges for sufficiently high values of beta uncertainty $\sigma_\beta$; I truncate the expectation calculation by imposing a minimum value on beta realizations at date 1. I present the correlations for two different levels of this minimum beta realization in Panel A and Panel B. Data is simulated with the beta and mean dividend level starting at date 1. I show the sensitivity of the correlations to parameter variations by varying one parameter at a time as denoted by column headings. I use the following calibrated parameter values unless stated otherwise: $\delta = .99$, $\gamma = 5$, $\lambda = .25$, $\beta_0 = 1, \beta = 1, \mu_0 = 0, \bar{\mu} = 0, \sigma_f = .05, \sigma_\beta = .45, \sigma_\mu = .01$.

Panel A: Correlation for Minimum Beta of 0.05

<table>
<thead>
<tr>
<th></th>
<th>$\gamma = 5$</th>
<th>$\gamma = 10$</th>
<th>$\gamma = 15$</th>
<th>$\gamma = 5$</th>
<th>$\gamma = 10$</th>
<th>$\gamma = 15$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho(V, \Delta P)$</td>
<td>0.222</td>
<td>0.267</td>
<td>0.279</td>
<td>0.247</td>
<td>0.299</td>
<td>0.312</td>
</tr>
<tr>
<td>$\rho(V,</td>
<td>\Delta P</td>
<td>)$</td>
<td>0.057</td>
<td>0.111</td>
<td>0.133</td>
<td>0.073</td>
</tr>
<tr>
<td>$\rho(V, \Delta P</td>
<td>\Delta P&gt;0)$</td>
<td>0.200</td>
<td>0.361</td>
<td>0.465</td>
<td>0.231</td>
<td>0.420</td>
</tr>
<tr>
<td>$\rho(V, \Delta P</td>
<td>\Delta P&lt;0)$</td>
<td>-0.016</td>
<td>-0.486</td>
<td>-0.845</td>
<td>-0.047</td>
<td>-0.540</td>
</tr>
</tbody>
</table>

Panel B: Correlation for Minimum Beta of 0.10

<table>
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<th>$\gamma = 5$</th>
<th>$\gamma = 10$</th>
<th>$\gamma = 15$</th>
</tr>
</thead>
<tbody>
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<td>$\rho(V, \Delta P)$</td>
<td>0.222</td>
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<td>0.222</td>
<td>0.247</td>
<td>0.247</td>
<td>0.247</td>
</tr>
<tr>
<td>$\rho(V,</td>
<td>\Delta P</td>
<td>)$</td>
<td>0.057</td>
<td>0.057</td>
<td>0.057</td>
<td>0.073</td>
</tr>
<tr>
<td>$\rho(V, \Delta P</td>
<td>\Delta P&gt;0)$</td>
<td>0.200</td>
<td>0.200</td>
<td>0.200</td>
<td>0.231</td>
<td>0.231</td>
</tr>
<tr>
<td>$\rho(V, \Delta P</td>
<td>\Delta P&lt;0)$</td>
<td>-0.016</td>
<td>-0.016</td>
<td>-0.016</td>
<td>-0.047</td>
<td>-0.047</td>
</tr>
</tbody>
</table>

$\sigma_\beta = 0.3$, $\sigma_\beta = 0.45$, $\sigma_\beta = 0.6$, $\sigma_\beta = 0.3$, $\sigma_\beta = 0.45$, $\sigma_\beta = 0.6$. $\sigma_\mu = 0$, $\sigma_\mu = 0.01$, $\sigma_\mu = 0.1$. $\sigma_\mu = 0$, $\sigma_\mu = 0.01$, $\sigma_\mu = 0.1$. $\sigma_\mu = 0$, $\sigma_\mu = 0.01$, $\sigma_\mu = 0.1$. $\sigma_\mu = 0$, $\sigma_\mu = 0.01$, $\sigma_\mu = 0.1$. $\sigma_\mu = 0$, $\sigma_\mu = 0.01$, $\sigma_\mu = 0.1$. $\sigma_\mu = 0$, $\sigma_\mu = 0.01$, $\sigma_\mu = 0.1$. $\sigma_\mu = 0$, $\sigma_\mu = 0.01$, $\sigma_\mu = 0.1$. $\sigma_\mu = 0$, $\sigma_\mu = 0.01$, $\sigma_\mu = 0.1$. $\sigma_\mu = 0$, $\sigma_\mu = 0.01$, $\sigma_\mu = 0.1$. $\sigma_\mu = 0$, $\sigma_\mu = 0.01$, $\sigma_\mu = 0.1$. $\sigma_\mu = 0$, $\sigma_\mu = 0.01$, $\sigma_\mu = 0.1$.
Table C-2 Regressions Showing Price Reversals and Trading Volume: This table presents the regression coefficients of future price changes regressed on current price changes and current price changes interacted with trading volume calculated from simulated data (one million draws) from my model. The regression equation is

$$\Delta P_{1 \rightarrow 2} = a + b \cdot \Delta P_{0 \rightarrow 1} + c \cdot \Delta P_{0 \rightarrow 1} \cdot \text{Vol}_{0 \rightarrow 1} + \epsilon.$$  

Trading volume and price changes between periods 0 and 1 are due to shocks to dividend means and betas. Trading volume goes to infinity as beta at date 1 approaches zero; I truncate the data generation process by imposing a minimum value on beta realizations at date 1. I present the correlation for two different levels of this minimum beta realization in Panel A and Panel B. Data is simulated with the beta and mean dividend level starting at their unconditional means in date 0. I show the sensitivity of the correlations to parameter variations by varying one parameter at a time as denoted by column headings. I use the following calibrated parameter values unless stated otherwise: $\delta = .99$, $\gamma = 5$, $\lambda = .25$, $\beta_0 = 1$, $\bar{\beta} = 1$, $\mu_0 = 0$, $\bar{\mu} = 0$, $\sigma_f = .05$, $\sigma_\beta = .45$, $\sigma_\mu = .01$.

<table>
<thead>
<tr>
<th>Panel A: Correlation for Minimum Beta of 0.05</th>
<th>Panel B: Correlation for Minimum Beta of 0.10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma = 5$</td>
<td>$\gamma = 5$</td>
</tr>
<tr>
<td>$\gamma = 10$</td>
<td>$\gamma = 10$</td>
</tr>
<tr>
<td>$\gamma = 15$</td>
<td>$\gamma = 15$</td>
</tr>
<tr>
<td>$\text{Int.} = 0.014$</td>
<td>$\text{Int.} = 0.014$</td>
</tr>
<tr>
<td>$\Delta P_{0 \rightarrow 1} = -0.549$</td>
<td>$\Delta P_{0 \rightarrow 1} = -0.528$</td>
</tr>
<tr>
<td>$\Delta P_{0 \rightarrow 1} \cdot \text{Vol} = -0.300$</td>
<td>$\Delta P_{0 \rightarrow 1} \cdot \text{Vol} = -0.602$</td>
</tr>
<tr>
<td>$\lambda = 0.1$</td>
<td>$\lambda = 0.1$</td>
</tr>
<tr>
<td>$\lambda = 0.25$</td>
<td>$\lambda = 0.25$</td>
</tr>
<tr>
<td>$\lambda = 0.4$</td>
<td>$\lambda = 0.4$</td>
</tr>
<tr>
<td>$\text{Int.} = 0.014$</td>
<td>$\text{Int.} = 0.014$</td>
</tr>
<tr>
<td>$\Delta P_{0 \rightarrow 1} = -0.549$</td>
<td>$\Delta P_{0 \rightarrow 1} = -0.528$</td>
</tr>
<tr>
<td>$\Delta P_{0 \rightarrow 1} \cdot \text{Vol} = -0.749$</td>
<td>$\Delta P_{0 \rightarrow 1} \cdot \text{Vol} = -1.500$</td>
</tr>
<tr>
<td>$\sigma_\beta = 0.3$</td>
<td>$\sigma_\beta = 0.3$</td>
</tr>
<tr>
<td>$\sigma_\beta = 0.45$</td>
<td>$\sigma_\beta = 0.45$</td>
</tr>
<tr>
<td>$\sigma_\beta = 0.6$</td>
<td>$\sigma_\beta = 0.6$</td>
</tr>
<tr>
<td>$\text{Int.} = 0.013$</td>
<td>$\text{Int.} = 0.013$</td>
</tr>
<tr>
<td>$\Delta P_{0 \rightarrow 1} = -0.306$</td>
<td>$\Delta P_{0 \rightarrow 1} = -0.282$</td>
</tr>
<tr>
<td>$\Delta P_{0 \rightarrow 1} \cdot \text{Vol} = -1.170$</td>
<td>$\Delta P_{0 \rightarrow 1} \cdot \text{Vol} = -1.710$</td>
</tr>
<tr>
<td>$\sigma_\mu = 0.01$</td>
<td>$\sigma_\mu = 0.01$</td>
</tr>
<tr>
<td>$\sigma_\mu = 0.1$</td>
<td>$\sigma_\mu = 0.1$</td>
</tr>
<tr>
<td>$\sigma_\mu = 0.25$</td>
<td>$\sigma_\mu = 0.25$</td>
</tr>
<tr>
<td>$\text{Int.} = 0.014$</td>
<td>$\text{Int.} = 0.014$</td>
</tr>
<tr>
<td>$\Delta P_{0 \rightarrow 1} = -0.549$</td>
<td>$\Delta P_{0 \rightarrow 1} = -0.528$</td>
</tr>
<tr>
<td>$\Delta P_{0 \rightarrow 1} \cdot \text{Vol} = -0.300$</td>
<td>$\Delta P_{0 \rightarrow 1} \cdot \text{Vol} = -0.602$</td>
</tr>
</tbody>
</table>

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REFERENCES


Figure 1. Trading Volume v. Shock Types. This figure shows the trading volume resulting from realizations of either the beta shock or mean shock holding the other shock at zero:

$$\text{Volume} = \frac{1}{2}|s_1^1(\beta_0 + \Delta \beta; \mu_0 + \Delta \mu) - s_0^1(\beta_0, \mu_0)|,$$

where $s_1^1$ is the optimal share holding policy at time $t$. The vertical axis measures trading volume with the average agent holding one share. The horizontal axis measures the innovation in the parameter: either beta, mu or the factor. The solid blue line depicts trading volume resulting from beta variations. The dashed red line depicts trading volume result from either mean or factor variations as they are identical. Panel A depicts the total trading volume. Panel B depicts the portion of trading volume due to shocks to beta or mu, that is the portion of trading volume due to changes in risk premia is eliminated. This figure uses the parameters $\delta = .99$, $\gamma = 5$, $\lambda = .25$, $\beta_0 = 1$, $\beta = 1$, $\mu_0 = 0$, $\bar{\mu} = 0$, $\sigma_f = .05$, $\sigma_\beta = .45$, $\sigma_\mu = .01$.

Panel A: Total Trading Volume

Panel B: Trading Volume Due to $\beta$ or $\mu$ Shocks
Figure 2. Sensitivity to Parameter Changes: Initial Beta, Beta Risk and Agent Heterogeneity. This figure shows the trading volume resulting from realizations of the beta shock under different parametrizations for the initial beta, the amount of beta risk (variation) and the amount of agent heterogeneity:

\[ \text{Volume} = \frac{1}{2} |s_t^1(\beta_0 + \Delta \beta) - s_t^0(\beta_0)|. \]

where \( s_t^1 \) is the optimal share holding policy at time \( t \). The vertical axis measures trading volum with the average agent holding one share. The horizontal axis measures the innovation in beta or standardized innovation in beta. I plot only the portion of trading volume due to beta shocks. In Panel A the solid blue line depicts trading volume resulting from beta variations when the initial beta and the unconditional beta mean is .75. The dashed red line depicts trading volume resulting from beta variation when the initial beta and unconditional beta mean is 1. In Panel B the solid blue line depicts trading volume resulting from beta variations when the standard deviation of beta is .45. The dashed red line depicts trading volume result from beta variation when the standard deviation of beta is .6. In Panel C the solid blue line depicts trading volume resulting from beta variations when the heterogeneity parameters, \( \lambda \), is .2. The dashed red line depicts trading volume resulting from beta variation when the heterogeneity parameter, \( \lambda \), is .4. Unless otherwise specified all panels use the parameters \( \delta = .99, \gamma = 5, \lambda = .25, \beta_0 = 1, \bar{\beta} = 1, \sigma_\beta = .45, \sigma_f = .05. \)

Panel A: Variation in Initial Beta

Panel B: Variation in Beta Risk

Panel C: Variation in Heterogeneity
Figure 3. Trading Volume v. Price Change. This figure shows the price change resulting from shocks to the dividend’s beta or mean parameter:

\[
\text{Volume} = \frac{1}{2}|s_1^1(\beta_0 + \Delta \beta, \mu_0 + \Delta \mu) - s_0^1(\beta_0, \mu_0)| \quad \text{and} \\
\Delta P = P_t(\beta_0 + \Delta \beta, \mu_0 + \Delta \mu) - P_0(\beta_0, \mu_0)
\]

where \(s_t^1\) is the optimal share holding policy and \(P_t\) is the equilibrium price at date \(t\). In Panels A and C the vertical axis measures the change in price and the horizontal axis measures the innovation in the parameter: either beta or mu. In Panels B and D the vertical axis measures the amount of trading volume and the horizontal axis measures the change in price. The solid blue line depicts the changes resulting from beta variations. The dashed red line depicts the changes resulting from mean variations. Panels A and B depict the total price changes and total trading volume of the model. Panels C and D depict the portion of price changes and trading volume due only to beta and mu shocks. This figure uses the parameters \(\delta = .99, \gamma = 5, \lambda = .25, \beta_0 = 1, \bar{\beta} = 1, \mu_0 = 0, \bar{\mu} = 0, \sigma_f = .05, \sigma_\beta = .45, \sigma_\mu = .01\).

Panel A: Total Price Change v. Shock

Panel B: Total Price Change v. Total Trading Volume

Panel C: Decomposed Price Change v. Shock

Panel D: Decomposed Price Change v. Decomposed Trading Volume
Figure 4. Trading Volume. This figure shows the annual average trading volume and the first and third quartiles of trading volume of NYSE stocks from 1962 to 2011.
Table I Summary Statistics: This table presents summary statistics of the data. Daily returns from CRSP are used to estimate monthly betas from January 1962 to December 2011 for all NYSE stocks with share codes 10 and 11. CAPM betas are estimated over calendar months. I report differences in beta estimates between months. Turnover is average monthly turnover for a stock, reported in percent. Data is winsorized at the 1% level. Panel A shows the variation in betas across months and turnover for the entire sample period. \( \sigma(\hat{\beta}) \) is the time series standard deviation of the change in beta estimates across months for a firm. This time series includes both true beta variation and measurement error. \( \sigma(\beta) \) is an estimate of the true underlying beta variation. Following the methodology of Fama and French (1997), I calculate this estimate by assuming that the measurement error is orthogonal to the true beta variation and using the time series average of the squared standard errors on the beta estimates as an estimate of the underlying beta measurement error. To maintain comparability with the standard deviation values of beta, turnover statistics reported are cross-sectional statistics of turnover that is first averaged across time at the firm level. Panel B shows subperiod statistics for turnover again reported in percent average monthly turnover.

Panel A: Total Sample Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>( \sigma(\hat{\beta}) )</th>
<th>( \sigma(\beta) )</th>
<th>Turnover</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.86</td>
<td>0.24</td>
<td>8.98</td>
</tr>
<tr>
<td>Median</td>
<td>0.84</td>
<td>0.25</td>
<td>6.09</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.27</td>
<td>0.22</td>
<td>8.74</td>
</tr>
<tr>
<td>Quartile 1</td>
<td>0.66</td>
<td>0.00</td>
<td>3.38</td>
</tr>
<tr>
<td>Quartile 3</td>
<td>1.02</td>
<td>0.41</td>
<td>11.67</td>
</tr>
</tbody>
</table>

Panel B: Average Monthly Turnover in Each Five Year Subperiod (percent per month)

<table>
<thead>
<tr>
<th>Subperiod</th>
<th>Turnover</th>
<th>Turnover</th>
<th>Turnover</th>
<th>Turnover</th>
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<th>Turnover</th>
<th>Turnover</th>
<th>Turnover</th>
<th>Turnover</th>
<th>Turnover</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>2.29</td>
<td>3.28</td>
<td>2.42</td>
<td>3.64</td>
<td>5.58</td>
<td>6.33</td>
<td>6.58</td>
<td>8.89</td>
<td>14.12</td>
<td>24.39</td>
</tr>
<tr>
<td>Median</td>
<td>1.62</td>
<td>2.42</td>
<td>2.01</td>
<td>3.00</td>
<td>4.80</td>
<td>5.45</td>
<td>5.42</td>
<td>7.52</td>
<td>11.87</td>
<td>20.70</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>2.15</td>
<td>2.72</td>
<td>1.61</td>
<td>2.33</td>
<td>3.41</td>
<td>4.13</td>
<td>4.79</td>
<td>6.03</td>
<td>9.56</td>
<td>15.98</td>
</tr>
<tr>
<td>Quartile 1</td>
<td>0.89</td>
<td>1.35</td>
<td>1.34</td>
<td>1.96</td>
<td>3.29</td>
<td>3.42</td>
<td>3.40</td>
<td>4.80</td>
<td>7.83</td>
<td>13.94</td>
</tr>
<tr>
<td>Quartile 3</td>
<td>2.99</td>
<td>4.28</td>
<td>3.00</td>
<td>4.71</td>
<td>6.98</td>
<td>8.34</td>
<td>8.44</td>
<td>11.49</td>
<td>18.00</td>
<td>30.69</td>
</tr>
</tbody>
</table>
Table II Non-Parametric Relation Between Beta Changes and Turnover

Each month stocks are sorted independently into five portfolios in two dimensions. First, they are sorted on the absolute value of the change in their beta estimate from the previous month to the current month. Second, they are sorted on their turnover in the current month. Break points are determined monthly. The fraction of firms across all months that fall in the intersection of these portfolios is reported. The larger concentration along the main diagonal shows the positive relation between changes in beta and trading volume.

<table>
<thead>
<tr>
<th>Turnover</th>
<th>Smallest</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Largest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smallest</td>
<td>4.7%</td>
<td>4.5%</td>
<td>4.2%</td>
<td>3.7%</td>
<td>2.9%</td>
</tr>
<tr>
<td>2</td>
<td>4.5%</td>
<td>4.5%</td>
<td>4.3%</td>
<td>3.8%</td>
<td>2.9%</td>
</tr>
<tr>
<td>3</td>
<td>4.2%</td>
<td>4.2%</td>
<td>4.2%</td>
<td>4.1%</td>
<td>3.4%</td>
</tr>
<tr>
<td>4</td>
<td>3.7%</td>
<td>3.8%</td>
<td>4.0%</td>
<td>4.2%</td>
<td>4.3%</td>
</tr>
<tr>
<td>Largest</td>
<td>3.0%</td>
<td>3.1%</td>
<td>3.5%</td>
<td>4.2%</td>
<td>6.1%</td>
</tr>
</tbody>
</table>
Table III Monthly Regression of Turnover on Beta Changes

This table presents panel regressions where the dependent variable is average daily turnover for a stock in a given month. $|\Delta \beta_{i,t}|$ denotes the absolute change in CAPM beta from the previous month to the current month. $|\Delta \beta_{i,t}|^+$ is the absolute change interacted with a dummy variable indicating if the beta change was positive. The control variable is an estimate of each stock’s monthly idiosyncratic risk: it is the variance of the residuals from that firm’s monthly CAPM regression used to estimate the current beta. Reported coefficients can be thought of as basis points of average daily turnover. T-statistics are in parentheses. All coefficients are significant at the 1% level. For further details about the data see the caption of Table I.

<table>
<thead>
<tr>
<th>Model</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
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</thead>
<tbody>
<tr>
<td>$\beta_{i,t}$</td>
<td>6.95</td>
<td>10.59</td>
<td>12.58</td>
<td>3.18</td>
<td>5.11</td>
<td>5.84</td>
<td>3.18</td>
<td>5.11</td>
<td>5.84</td>
<td>2.71</td>
<td>4.52</td>
<td>5.18</td>
</tr>
<tr>
<td></td>
<td>(144.06)</td>
<td>(176.71)</td>
<td>(160.86)</td>
<td>(93.00)</td>
<td>(82.77)</td>
<td>(13.65)</td>
<td>(14.22)</td>
<td>(13.72)</td>
<td>(13.27)</td>
<td>(14.04)</td>
<td>(13.45)</td>
<td></td>
</tr>
<tr>
<td>$</td>
<td>\Delta \beta_{i,t}</td>
<td>$</td>
<td>6.27</td>
<td>10.76</td>
<td>11.75</td>
<td>4.16</td>
<td>6.29</td>
<td>6.67</td>
<td>4.16</td>
<td>6.29</td>
<td>6.67</td>
<td>2.41</td>
</tr>
<tr>
<td></td>
<td>(100.74)</td>
<td>(141.38)</td>
<td>(146.78)</td>
<td>(78.02)</td>
<td>(95.29)</td>
<td>(18.68)</td>
<td>(18.52)</td>
<td>(18.47)</td>
<td>(15.86)</td>
<td>(17.73)</td>
<td>(17.67)</td>
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</tr>
<tr>
<td>$</td>
<td>\Delta \beta_{i,t}</td>
<td>^+$</td>
<td>-10.52</td>
<td>-8.27</td>
<td>4.16</td>
<td>6.29</td>
<td>6.67</td>
<td>2.41</td>
<td>4.41</td>
<td>4.75</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-101.28)</td>
<td>(-69.91)</td>
<td>(-54.55)</td>
<td>(-12.25)</td>
<td>(-11.43)</td>
<td>(-12.33)</td>
<td>(-11.51)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{i,t} *</td>
<td>\Delta \beta_{i,t}</td>
<td>$</td>
<td>-2.04</td>
<td>-0.71</td>
<td>-0.71</td>
<td>-0.64</td>
<td></td>
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$R^2$ 0.41 0.42 0.42 0.61 0.62 0.62 0.61 0.62 0.62 0.63 0.63 0.63

$N$ 844086 844086 844086 844086 844086 844086 844086 844086 844086 844086 844086 844086
Table IV Monthly Regression of Turnover on Beta Changes: Most Traded Stocks

This table presents panel regressions where the dependent variable is average daily turnover for a stock in a given month. $|\Delta \beta_{i,t}|$ denotes the absolute change in CAPM beta from the previous month to the current month. $|\Delta \beta_{i,t}|^+$ is the absolute change interacted with a dummy variable indicating if the beta change was positive. The control variable is an estimate of each stock’s monthly idiosyncratic risk: it is the variance of the residuals from that firm’s monthly CAPM regression used to estimate the current beta. Reported coefficients can be thought of as basis points of average daily turnover. T-statistics are in parentheses. All coefficients are significant at the 1% level. The sample is restricted to the half of stocks with the highest average turnover in each five year subperiod. For a list of the subperiods see Table V. For further details about the data see the caption of Table I.

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$R^2$ | 0.53 | 0.54 | 0.54 | 0.65 | 0.65 | 0.65 | 0.65 | 0.65 | 0.65 | 0.67 | 0.67 | 0.67 |

$N$  | 403322 | 403322 | 403322 | 403322 | 403322 | 403322 | 403322 | 403322 | 403322 | 403322 | 403322 | 403322 |
This table presents panel regressions where the dependent variable is average daily turnover for a stock in a given month. $|\Delta \beta_{i,t}|$ denotes the absolute change in CAPM beta from the previous month to the current month. $|\Delta \beta_{i,t}|^+$ is the absolute change interacted with a dummy variable indicating if the beta change was positive. The control variables are an estimate of the market volatility in a given month and an estimate of each stock’s monthly idiosyncratic risk. The market variance is the variance of the daily market factor that month. The firm idiosyncratic risk is the variance of the residuals from that firm’s monthly CAPM regression used to estimate the current beta. Reported coefficients can be thought of as basis points of average daily turnover. T-statistics are in parentheses. All coefficients on beta changes are significant at the 1% level. The sample is divided into 10 five year subperiods. Time fixed effects are omitted. For further details about the data see the caption of Table I.

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| Time FE | N | N | N | N | N | N | N | N | N | N |
| Firm FE | Y | Y | Y | Y | Y | Y | Y | Y | Y | Y |
| Controls | Y | Y | Y | Y | Y | Y | Y | Y | Y | Y |
| Double | Y | Y | Y | Y | Y | Y | Y | Y | Y | Y |
| Clustered SE | |

$R^2$ | 0.34 | 0.34 | 0.25 | 0.32 | 0.16 | 0.08 | 0.12 | 0.18 | 0.16 | 0.15 |
| $N$ | 68904 | 73933 | 83830 | 85697 | 82648 | 79394 | 95302 | 105064 | 88269 | 81045 |
Table VI Monthly Regression of Trading Volume on Dimson Beta Changes

This table presents panel regressions where the dependent variable is average daily turnover for a stock in a given month. $|\Delta \beta_{i,t}|$ denotes the absolute change in CAPM beta from the previous month to the current month. $|\Delta \beta_{i,t}|^+$ is the absolute change interacted with a dummy variable indicating if the beta change was positive. The control variable is an estimate of each stock’s monthly idiosyncratic risk: it is the variance of the residuals from that firm’s monthly CAPM regression used to estimate the current beta. Reported coefficients can be thought of as basis points of average daily turnover. T-statistics are in parentheses. All coefficients are significant at the 1% level. Betas are estimate using the methodology Dimson (1979) with two lead and lag terms. For further details about the data see the caption of Table I.

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- Time FE: Y, Y, Y, Y, Y, Y, Y, Y, Y, Y, Y, Y
- Firm FE: N, N, N, N, Y, Y, Y, Y, Y, Y, Y, Y
- Controls: N, N, N, N, N, N, N, N, N, Y, Y, Y
- Double: N, N, N, N, N, N, Y, Y, Y, Y, Y, Y
- Clustered SE

$R^2$: 0.40, 0.41, 0.41, 0.61, 0.61, 0.61, 0.61, 0.61, 0.63, 0.63, 0.63, 0.63

$N$: 844086, 844086, 844086, 844086, 844086, 844086, 844086, 844086, 844086, 844086, 844086, 844086
This table presents panel regressions where the dependent variable is average daily turnover for a stock in a given year. Betas are calculated at the yearly frequency. $|\Delta \beta_{i,t}|$ denotes the absolute change in CAPM beta from the previous year to the current year. $|\Delta \beta_{i,t}|^+$ is the absolute change interacted with a dummy variable indicating if the beta change was positive. The control variable is an estimate of each stock’s year idiosyncratic risk: it is the variance of the residuals from that firm’s yearly CAPM regression used to estimate the current beta. Reported coefficients can be thought of as basis points of average daily turnover. T-statistics are in parentheses. All coefficients are significant at the 1% level. For further details about the data see the caption of Table I.

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| $\beta_{i,t} \times |\Delta \beta_{i,t}|$ | -3.60 | -0.79 | -0.79 | -1.13 | (-3.60) | (-0.79) | (-0.79) | (-1.13) | (-1.16) | (-0.47) | (-0.47) | (-0.73) |

| Time FE | Y | Y | Y | Y | Y | Y | Y | Y | Y | Y | Y | Y |
| Firm FE | N | N | N | Y | Y | Y | Y | Y | Y | Y | Y | Y |
| Controls | N | N | N | N | N | N | N | N | N | Y | Y | Y |
| Double | N | N | N | N | N | N | Y | Y | Y | Y | Y | Y |

Clustered SE

| $R^2$ | 0.49 | 0.49 | 0.49 | 0.70 | 0.71 | 0.71 | 0.70 | 0.71 | 0.71 | 0.71 | 0.71 | 0.71 |
| $N$ | 70900 | 70900 | 70900 | 70900 | 70900 | 70900 | 70900 | 70900 | 70900 | 70900 | 70900 | 70900 |
Table VIII Monthly Regression of Trading Volume on Fama French Factor Beta Changes

This table presents panel regressions where the dependent variable is average daily turnover for a stock in a given month. The three factor betas are estimated using the three factor model. $|\Delta \beta_{i,t}|$ denotes the absolute change in that factor beta from the previous month to the current month. The control variable is an estimate of each stock’s monthly idiosyncratic risk: it is the variance of the residuals from that firm’s monthly Fama French 3 factor model regression used to estimate the current betas. Reported coefficients can be thought of as basis points of average daily turnover. T-statistics are in parentheses. All coefficients are significant at the 1% level. For further details about the data see the caption of Table I.

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<tr>
<td>Mkt $\beta_{i,t}$</td>
<td>3.13</td>
<td>0.81</td>
<td>0.81</td>
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<td></td>
<td>(81.79)</td>
<td>(25.28)</td>
<td>(6.26)</td>
<td>(5.64)</td>
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<td>HML $\beta_{i,t}$</td>
<td>-0.60</td>
<td>-0.02</td>
<td>-0.02</td>
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<td></td>
<td>(-28.14)</td>
<td>(-1.35)</td>
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<td>SMB $\beta_{i,t}$</td>
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<td>0.02</td>
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<td>-0.07</td>
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<td></td>
<td>(-15.33)</td>
<td>(0.80)</td>
<td>(0.27)</td>
<td>(-1.21)</td>
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<td>Mkt $</td>
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<td>$</td>
<td>1.84</td>
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<td>(40.26)</td>
<td>(33.46)</td>
<td>(12.54)</td>
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<tr>
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<td>1.20</td>
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<tr>
<td></td>
<td>(68.80)</td>
<td>(59.40)</td>
<td>(14.99)</td>
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<td>1.79</td>
<td>1.23</td>
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<tr>
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<td>(62.60)</td>
<td>(51.97)</td>
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$R^2$ | 0.41 | 0.62 | 0.62 | 0.63 |
$N$   | 844086 | 844086 | 844086 | 844086 |