Interest Rates Under Falling Stars

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Abstract
Structural changes in monetary policy and the economy shift the long-run trends in inflation and the real interest rate, $\pi_t^*$ and $r_t^*$. Absence of arbitrage implies that the term structure of interest rates is closely tied to these macroeconomic trends. However, this important macro-finance link has received little attention in the term structure literature, and in particular the variation in the equilibrium real interest rate, $r_t^*$, has been largely ignored. This paper fills this gap. Using common proxies for $\pi_t^*$ and $r_t^*$ we provide evidence of their tight empirical connection with the U.S. Treasury yield curve. These macro trends capture the persistent movements in long-term interest rates, help improve long-range interest rate forecasts, predict excess bond returns, and account for a substantial share of interest rate risk at low frequencies. We find that for all these results it is important to account not only for changes in $\pi_t^*$ but also for shifts in $r_t^*$, because a substantial portion of the secular decline in long-term rates, and most of this decline since the late 1990s, is attributable to changes in $r_t^*$. Our results imply that for modeling of the yield curve, expectations and bond risk premia, it is crucially important to account for these macroeconomic trends.

Keywords: yield curve, macro-finance, inflation expectations, equilibrium real interest rate, common trends

JEL Classifications: E43, E44, E47

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1 Introduction

The general level of U.S. interest rates has gradually fallen since the early 1980s, as illustrated by the secular decline in the 10-year Treasury yield shown in Figure 1. A key driver of this decline was the reduction in the long-run trend of inflation, or $\pi^*$. However, for the past two decades, $\pi^*$ has been stable at around 2 percent—the Federal Reserve’s official inflation target—while longer-term interest rates have continued to trend lower. This recent divergence is illustrated in Figure 1, which displays a measure of trend inflation based largely on long-horizon inflation projections from a survey of professional forecasters (details are provided in Section 3). Instead of reflecting reduced inflation expectations, much of the recent downtrend in yields appears to be caused by a lower equilibrium real interest rate, or $r^*$, an estimate of which is included in Figure 1. This steady-state real rate is the risk-free real rate of return that would prevail in the absence of transitory disturbances, and it has been pushed down by structural factors including slower productivity growth and an aging population, which affect global saving and investment.\(^1\) According to this measure, the equilibrium short real rate remained little changed in a range between 2 and 3 percent from 1970 to around 2000 when it started to gradually fall to below 1 percent currently. Changes in the level of $r^*$ are important for both financial investors and macroeconomic policymakers (Clarida, 2014; Yellen, 2015). In this paper, we argue that accounting for the earlier downward trend in $\pi^*$ and the recent downtrend in $r^*$—an environment of falling stars—is crucial for understanding the dynamics of interest rates and assessing bond risk and expected returns.

Absence of arbitrage implies that nominal interest rates reflect expectations of future inflation and real short rates, subject to a risk adjustment. Thus, if inflation or real rates have trend components, then these trends will in general also be present in nominal interest rates. We make this argument explicit in the context of a simple affine term structure model that allows for stochastic trends in inflation and the real short rate, which we denote by $\pi_t^*$ and $r_t^*$. While we assume for ease of exposition that these macroeconomic trends follow random walks, the same arguments apply to the case when they are highly persistent stationary variables. Our five-factor model formalizes the intuitive results that $\pi_t^*$ and $r_t^*$ act as level factors for the nominal yield curve, that the cyclical components of inflation and the real rate affect short rates more than long rates, and that a risk-premium factor affects long rates more than short rates. The presence of these two macroeconomic trends leads to testable implications that we take to the data using the observable counterparts to $\pi_t^*$ and $r_t^*$ shown in Figure 1. The

\(^1\)The estimate of $r^*$ shown in Figure 1, further described in Section 3, is the average of four model-based estimates that filter macroeconomic data, from Laubach and Williams (2003), Johannsen and Mertens (2016), Kiley (2015) and Lubik and Matthes (2015). Further discussions of the decline in $r^*$ include Hamilton et al. (2016) and Rachel and Smith (2015) among many others.
central empirical question when testing these implications is not whether trends in inflation and real rates are reflected in the yield curve (of course they are), but whether the variation in these trends is quantitatively important for interest rates. We find that this is clearly the case.

First, we document that time variation in both $\pi_t^*$ and $r_t^*$ is responsible for most of the persistence in yields. While interest rates themselves are extremely persistent, the difference between long-term interest rates and $i_t^* = \pi_t^* + r_t^*$ exhibits quick mean reversion and we strongly reject the null of a unit root. Furthermore, regressions of long-term yields on the macroeconomic trends recover the unit coefficients predicted by our model. Our findings bring a different perspective to the results of Cieslak and Povala (2015), henceforth CP, who regress nominal yields on their proxy for trend inflation. While we confirm their finding that $\pi_t^*$ is an important persistent component of interest rates, we show that it is important to include $r_t^*$ in order to fully capture the trend component. In particular, the difference between long-term yields and $\pi_t^*$ is still very persistent, and a regression of yields on $\pi_t^*$ neither produces a unit coefficient nor results in residuals that appear stationary. The importance of time variation in $r_t^*$ also explains why past research has generally been unable to find a stable cointegrating relation between nominal interest rates and inflation, going back to Rose (1988) and recently summarized by Neely and Rapach (2008).

Second, accounting for the macroeconomic trend components in interest rate forecasts, using shifting endpoints, leads to material improvements in forecast accuracy over the common no-change/random walk benchmark at medium and long forecast horizons. Furthermore, it is insufficient to only consider $\pi_t^*$ for the shifting endpoint of nominal interest rates, because also incorporating $r_t^*$ leads to better forecasts. We document improvements in forecast accuracy over the random walk and “$\pi_t^*$-only” alternatives that are both economically and statistically significant. Our results are consistent with previous evidence, in particular in Dijk et al. (2014), that it is important to allow for time-varying endpoints when forecasting interest rates, but our approach using the endpoint $i_t^* = \pi_t^* + r_t^*$ is both simpler and more accurate. Furthermore, our no-arbitrage model shows that these recently documented forecast improvements are a natural consequence of the time series properties of inflation and the real short rate.

Third, accounting for the persistent components of yields is important for understanding return predictability and estimating bond risk premia. Recently, CP employed the connection between trend inflation and interest rates to decompose the nominal yield curve into risk premiums and the expectations hypothesis (EH) component (i.e., a term that equals the average short-term interest rate that investors expect to prevail during the life of a bond). They present strong evidence for predictive power of the inflation trend for excess bond returns,
which suggests that incorporating the inflation trend is important to understand the risk premium/EH decomposition. Since $r^*_t$ plays the same role of a persistent level factor for nominal interest rates as does $\pi^*_t$, our model predicts that if $\pi^*_t$ has predictive power for bond returns so should $r^*_t$. Hence we expand the analysis of CP by introducing shifts in the equilibrium real interest rate into the analysis of risk premiums, and we find indeed find our prediction confirmed in the data, namely that $r^*$ has strong incremental predictive power for bond returns. This indicates that measures of the bond risk premium based on a shifting $\pi^*$, such as the cycle factor proposed by CP, as well as empirical decompositions of long-term interest rates into expectations and term premium components, can and should be enlarged to include a shifting $r^*$ as well.

Fourth, we estimate directly how important the trend components of inflation and real rates are for the variance of interest rate changes at different frequencies. In other words, we answer the question how much interest rate risk is due to these macroeconomic trends. Duffee (2016) proposes to use the ratio of the variance of inflation news to the variance of innovations in nominal interest rates as a measure of inflation risk. He documents that for one-quarter innovations, this ratio is small for U.S. Treasury yields, using survey data to measure inflation surprises. We propose to consider variance ratios not only for one-period (quarterly) innovations but for $h$-period innovations, since the contributions to interest rate risk may be different at lower frequencies. This allows us to measure, for example, the contribution of unexpected changes in inflation over, say, the next five or ten years to the risk in nominal bonds. We estimate ratios of variances of changes instead of variances of innovations, which is much simpler as it does not require a model for expectations—all that is needed are our empirical proxies of the trend components. Empirically, we provide novel estimates of the interest rate risk, across horizons, that is due to changes in the inflation trend, the real-rate trend, and the overall trend. For quarterly changes, we find the same low contribution of inflation risk, around 10%, that was reported by Duffee. But in line with our theoretical predictions, the variance ratios increase quickly and substantially with the horizon, to a ratio of 30%-40% for ten-year changes. Although confidence intervals are necessarily wide, our estimates suggest that over post-war U.S. history, a large share of the risk for investors with long holding periods was due to changes in long-run inflation expectations and hence to changes in the macroeconomic trend components of nominal yields.

Overall, our paper provides strong empirical evidence of the importance of the macroeconomic trends for the nominal yield curve. This has far-reaching implications for macro-finance modeling of interest rates and bond risk premia. Both structural and reduced-form models of the yield curve need to allow for slow-moving macroeconomic trend components in order
to accurately capture interest rate dynamics. Variation in both the inflation trend and the equilibrium real interest rate is quantitatively important, and ignoring either in macro-finance work would be ill-advised. Our results are important for empirical work on the term premium in long-term interest rates, for practitioners forecasting bond yields and returns, and more generally for researchers trying to understand the drivers and historical evolution of the term structure of interest rates.

Our paper relates to several strands of literature. A large literature documents the importance of a trend component in the inflation rate. Prominent examples are Kozicki and Tinsley (2001) and Stock and Watson (2007); for a survey on inflation forecasting that shows the importance of trends see Faust and Wright (2013); Clark and Doh (2014) compare models of trend inflation. The persistence in the real interest is the topic of a large literature that originated with Rose (1988) and was surveyed by Neely and Rapach (2008), and a related strand of literature investigates the long-run Fisher effect, as in Mishkin (1992). A more recent literature, originated by Laubach and Williams (2003), uses macroeconomic and statistical models to estimate the equilibrium real interest rate and to understand its structural drivers (see the references in footnote 1). Our paper sheds a new light on these related strands of literature by showing that the apparent trends in inflation and real interest rates are not only consistent with the observed behavior of the yield curve, but are in fact crucially important to understand this behavior. Our results imply that there is no long-run Fisher effect. Finally, within the macro-finance literature on the term structure of interest rates, some models have been proposed that allow for changes in trend inflation; see Rudebusch and Wu (2008), Bekaert et al. (2010), and Hördahl et al. (2006). However these models generally assume a constant equilibrium real rate.

The paper is structured as follows: Section 2 introduces a simple no-arbitrage model of the yield curve that allows for trend components in inflation and the real rates, and derives testable implications. Section 3 presents our empirical proxies for \( \pi_t^* \) and \( r_t^* \). In Section 4 we investigate the persistence of interest rates and the evidence for cointegration. In Section 5 we show the improvements in forecast accuracy that are possible when accounting for macroeconomic trends in interest rates. Section 6 documents the evidence for the incremental predictive power of \( \pi_t^* \) and \( r_t^* \) for future excess bond returns. Section 7 presents new estimates, using variance ratios, for the role of inflation risk and macroeconomic trends at different frequencies. Section 8 concludes.
2 No-arbitrage model with macroeconomic trends

Our theoretical framework is an affine term structure model for real and nominal yields. This model demonstrates how, under absence of arbitrage, changes in $\pi^*_{t}$ and $r^*_t$—the inflation trend and the equilibrium real rate—affect interest rates.\(^2\)

We model inflation, $\pi_t$, as the sum of trend, cycle and noise components:

$$
\pi_{t+1} = \pi^*_t + c_t + e_{t+1}, \quad \pi^*_t = \pi^*_{t-1} + \xi_t, \quad c_t = \phi_c c_{t-1} + u_t,
$$

where the innovations $\xi_t$ and $u_t$ and the noise component $e_t$ are mutually independent iid normal random variables with standard deviations $\sigma_\xi$, $\sigma_u$ and $\sigma_e$. Inflation expectations are $E_t \pi_{t+h+1} = \pi^*_t + \phi^h c_t$, and $\lim_{h \to \infty} E_t \pi_{t+h} = \pi^*_t$ is the time-varying inflation endpoint, which can be viewed as the inflation target of the central bank. This specification of the inflation process is similar to the one in Duffee (2016), and in line with the modern empirical macro literature that allows for a trend component in inflation (a prominent example is Stock and Watson, 2007). We assume that the shocks $\xi_t$ and $u_t$ affect only expectations of future inflation but not current inflation, which will slightly simplify the bond pricing formulas.

The one-period real rate, $r_t$, also has a trend component, the equilibrium real rate $r^*_t$, and a stationary component, the cyclical real-rate gap $g_t$:

$$
r_t = r^*_t + g_t, \quad r^*_t = r^*_{t-1} + \eta_t, \quad g_t = \phi_g g_{t-1} + v_t.
$$

The shocks are again mutually uncorrelated and iid normal, with standard deviations $\sigma_\eta$ and $\sigma_v$. Since $\lim_{h \to \infty} E_tr_{t+h} = r^*_t$, the equilibrium real rate, also called the natural rate, can be understood as the real rate that prevails in the economy after all shocks have died out; see, for example, Laubach and Williams (2003).

The final state variable determining interest rates is a risk price factor $x_t$, which follows an independent autoregressive process:

$$
x_t = \mu_x + \phi_x x_{t-1} + w_t,
$$

where $w_t$ is iid normal with standard deviation $\sigma_w$. The state variables can be collected as $Z_t = (\pi^*_t, c_t, r^*_t, g_t, x_t)'$, and their dynamics can be compactly written as a VAR(1)

$$
Z_t = \mu + \phi Z_{t-1} + \Sigma \varepsilon_t,
$$

\(^2\)Our model is related to the one in CP, but generalizes it by allowing for time variation in $r^*$. Another difference is that CP assume that inflation expectations are driven by one persistent but mean-reverting state variable, whereas we use a more common trend-cycle specification for inflation.
where \( \mu = (0, 0, 0, 0, \mu_x)' \), \( \phi = \text{diag}(1, \phi_c, 1, \phi_y, \phi_x) \), \( \Sigma = \text{diag}(\sigma_x, \sigma_u, \sigma_n, \sigma_v, \sigma_w) \), and \( \varepsilon_t \) is a \((5 \times 1)\) iid standard normal vector process.

The model is completed by a specification for the log real stochastic discount factor (SDF), \( m_{t+1}^r \), for which we choose the usual essentially affine form of Duffee (2002):

\[
m_{t+1}^r = -r_t - \frac{1}{2} \lambda_t \lambda_t - \lambda_t \varepsilon_{t+1}, \quad \lambda_t = \Sigma^{-1}(\lambda_0 + \lambda_1 Z_t).
\]

Like CP, we only allow \( x_t \) to affect the price of risk, so that the first four columns of \( \lambda_1 \) are zero. Furthermore, shocks to \( x_t \) are not priced, so that the last element of \( \lambda_0 \) and the last row of \( \lambda_1 \) are zero. The non-zero elements of \( \lambda_0 \) are denoted by \( \lambda_{0x}, \lambda_{0c}, \lambda_{0v} \), and \( \lambda_{0y} \), and those of \( \lambda_1 \) by \( \lambda_{1x}, \lambda_{1c}, \lambda_{1v}, \lambda_{1y} \). The log nominal SDF is \( m_{t+1}^n = m_{t+1}^r - \pi_{t+1} \) and the nominal short-term interest rate is

\[
i_t = -E_t m_{t+1}^n - \frac{1}{2} \text{Var}_t (m_{t+1}^n) = r_t^* + g_t + \pi_t^* + c_t - \frac{1}{2} \sigma_e^2.
\]

Due to our timing assumption for the inflation process, and because the noise shocks \( e_t \) are not priced, there is no inflation risk premium in the nominal short rate.\(^3\) Prices of zero-coupon bonds with maturity \( n \), denoted by \( P_t^{(n)} \), are easily verified to be exponentially affine, i.e., \( \log(P_t^{(n)}) = A_n + B_n^t Z_t \), using the pricing equation \( P_{t+1}^{(n+1)} = E_t (\exp(m_{t+1}^n) P_t^{(n)}) \). The coefficients follow the usual recursions of affine term structure models (e.g., Ang and Piazzesi, 2003):

\[
A_{n+1} = A_n + B_n^t (\mu - \lambda_0) + C_n, \quad C_n := \frac{1}{2} (\sigma_e^2 + B_n^t \Sigma \Sigma' B_n), \quad B_{n+1} = -(1, 1, 1, 1, 0)' + (\phi - \lambda_1)' B_n,
\]

where \( C_n \) captures the convexity in bond prices. The initial conditions are \( A_0 = 0, B_0 = (0, 0, 0, 0, 0)' \). For the individual loadings of bond prices on the risk factors we have

\[
B_{n+1}^{\pi} = B_n^{\pi} - 1, \quad B_{n+1}^c = \phi_c B_n^c - 1, \quad B_{n+1}^{\pi} = B_n^{\pi} - 1, \quad B_{n+1}^g = \phi_g B_n^g - 1,
\]

\[
B_{n+1}^x = -\lambda_{\pi, x} B_n^{\pi} - \lambda_{c, x} B_n^c - \lambda_{r, x} B_n^{\pi} - \lambda_{g, x} B_n^g + \phi_x B_n^x.
\]

\(^3\)That is, \( \text{Cov}_t (m_{t+1}^r, \pi_{t+1}) = 0 \). CP make the same assumption, which is justified because estimates of this short-run inflation risk premium are typically very small.
and the explicit solutions are

\[
\begin{align*}
B_n^\pi &= -n, \\
B_n^c &= \frac{\phi_n^c - 1}{1 - \phi_c}, \\
B_n^r &= -n, \\
B_n^g &= \frac{\phi_n^g - 1}{1 - \phi_g},
\end{align*}
\]

\[
B_n^x = \frac{\lambda_{\pi^x} + \lambda_{r^x}}{1 - \phi_x} \left( n - \frac{1 - \phi_n^x}{1 - \phi_x} \right) + \frac{\lambda_{cx}}{1 - \phi_c} \left( \frac{1 - \phi_n^x}{1 - \phi_x} - \phi_n^x c - \phi_c \right) + \frac{\lambda_{gx}}{1 - \phi_g} \left( \frac{1 - \phi_n^x}{1 - \phi_x} - \phi_n^x g - \phi_g \right).
\]

For nominal bond yields the model implies the following decomposition:

\[
y(t)^{(n)} = -\log(P_t^{(n)})/n = -A_n/n - B_n^x Z_t/n
\]

\[
= \pi_t^* + \frac{1 - \phi_n^c}{n(1 - \phi_c)} c_t + \frac{1 - \phi_n^g}{n(1 - \phi_g)} g_t
\]

\[
\sum_{i=1}^{a} E_t \pi_{t+i}/n
\]

\[
\sum_{i=0}^{a-1} E_t r_{t+i}/n
\]

\[
- A_n/n - B_n^x x_t/n.
\]

Equation (2) clearly shows the role of the trend and cyclical components of inflation and real rates, and of the risk-premium factor, for interest rates at different maturities. The trend components \(\pi_t^*\) and \(r_t^*\) naturally act as level factors by affecting yields of all maturities equally. The cyclical components \(c_t\) and \(g_t\) are slope factors as they affect short-term yields more strongly than long-term yields; their loadings approach zero for large \(n\). The risk-premium factor affects long-term yields more strongly than short-term yields.\(^4\) Thus, long-term interest rates are mostly driven by the trend components \(\pi_t^*\) and \(r_t^*\), as well as by the transitory risk-premium factor \(x_t\). They are affected to a much lesser degree than short-term yields by the cyclical factors \(c_t\) and \(g_t\). Appendix A provides additional details and results for this affine model, including expressions for forward rates. In our empirical analysis we will focus on long-term (five-year and ten-year) yields as well as long-term (five-to-ten-year) forward rates, since these will exhibit the closest relationship with \(\pi_t^*\) and \(r_t^*\).

The model implies several predictions for the relationship between interest rates and the two macroeconomic trends, which we describe below. These predictions will become evident in the data only to the extent that there is materially relevant variation in the trend components. Furthermore, an empirical assessment requires accurate estimates of \(\pi_t^*\) and \(r_t^*\) (in Section 3 we describe our empirical proxies). Thus, our empirical analysis should be viewed as tests of the joint hypothesis that (a) the behavior of the trend components and their links to the yield curve conform to our no-arbitrage model, (b) variation in the trend components is quantitatively important, and (c) our empirical proxies accurately capture the evolution of the trend components.

The first obvious prediction of the model is that for any yield maturity \(n\), \(y(t)^{(n)} - \pi_t^* - r_t^*\)

\(^4\)The loadings of yields on \(x_t\) start at zero and tend to \(\lim \frac{B_n^x}{n} = - (\lambda_{\pi^x} + \lambda_{r^x}x)/(1 - \phi_x)\).
is much less persistent than the yield itself. The difference in persistence is particularly pronounced when we use long-term yields or forward rates, which are highly persistent due to the trend components. If we take the unit roots literally, then \( y_t \) is I(1) while \( y_t - \pi_t^* - r_t^* \) is I(0). Furthermore, a regression of \( y_t \) on \( \pi_t^* \) and \( r_t^* \) recovers unit coefficients, since it is a cointegrating regression and the cointegrating vector of \((y_t, \pi_t^*, r_t^*)\) is \((1, -1, -1)\). On the other hand, \( y_t - \pi_t^* \) exhibits substantial persistence, a regression of \( y_t \) on \( \pi_t^* \) does not recover a unit coefficient, since it is a spurious regression, and the residuals from such a regression remain highly persistent. We will investigate these predictions in Section 4.

The fact that interest rates do not have a constant mean but contain the stochastic trend \( i_t^* = \pi_t^* + r_t^* \) also has important implications for yield forecasts. From (1) and (2) we have

\[
\lim_{h \to \infty} E_t y_{t+h} = \text{constant} + \pi_t^* + r_t^*,
\]

so interest rates mean-revert to a “shifting endpoint” (Kozicki and Tinsley, 2001) that is driven by \( i_t^* \). Note that the constant in principle depends on the maturity \( n \) but in practice is sufficiently small that it can be neglected. The model predicts that long-horizon interest rate forecasts that incorporate knowledge of \( i_t^* \) are more accurate than forecasts that ignore it. In particular, long-range forecasts that account for \( i_t^* \) should improve upon the common random walk benchmark forecasts. We will take up this issue in Section 5.

Excess bond returns are

\[
x(n)_{t+1} = p_{n+1}^{(n-1)} - p_{n+1}^{(1)} - y_t^{(1)} = -\frac{1}{2} B_n' \Sigma \Sigma' B_n + B_n' (\lambda_0 + \lambda_1 t_5) x_t + B_n' \Sigma \epsilon_{t+1},
\]

where \( p_t^{(n)} \) denotes the log-price of a zero-coupon bond with maturity of \( n \) quarters and \( t_5 \) is a \((5 \times 1)\)-vector of ones. As in the term structure model of CP, who analyze excess returns in detail, expected excess returns are driven only by the risk premium factor \( x_t \). CP documented large predictive gains when including their measure of trend inflation in predictive regressions for excess bond returns, which suggests that accounting for the trend component helps tease out the information about expected returns (i.e., the risk-premium factor) contained in observed interest rates. But according to our model the equilibrium real rate drives nominal yields in exactly the same way as the trend component of inflation. In light of CP’s findings, this leads to the prediction that including not only \( \pi_t^* \) but also \( r_t^* \) should improve the ability of predictive regressions to capture variation in bond risk premia. We assess the role of the trend components for return predictability in Section 6.

Finally, the model predicts that the importance of macroeconomic trends should be particularly important for interest rate changes at low frequencies. Since the trend components
are by definition highly persistent, their impact is swamped by other volatility when considering changes at relatively high frequencies (e.g., quarterly changes), but they become the dominant drivers for changes over longer intervals (e.g., over a decade or more). This implies that variance ratios (similar to the one suggested by Duffee, 2016), of changes in the trend components relative to changes in interest rates should exhibit a pronounced tendency to rise with the period of the change. In Section 7 we will use such variance ratios to estimate the importance of the trend components at different frequencies.

Our affine term-structure model is obviously very simplified. It only contains few risk factors, does not allow for stochastic volatility, and puts strong restrictions on the risk pricing. Despite its simplicity the model captures the fundamental aspects of the relationship between the trend in inflation, the equilibrium real rate, and the yield curve. Most no-arbitrage models that include trends in inflation and the real rate will have very similar implications on this dimension as our model, simply because absence of arbitrage implies that these trends are also reflected in long term interest rates.

We assumed an exact unit root in inflation and the real rate, but this assumption is not crucial. Taken literally it is of course implausible, since the forecast error variance of neither inflation nor real rates increases linearly with the forecast horizon—both variables have always remained within certain bounds. However, in finite samples a stationary process can always be approximated arbitrarily well by a unit root process (e.g., Campbell and Perron, 1991). In our context, even if we could measure expectations well, we could obviously never hope to learn from the data about $E_t \pi_{t+h}$ or $E_t r_{t+h}$ for infinitely large $h$. Thus, we may simply view $\pi^*_t$ and $r^*_t$ as highly persistent components of $\pi_t$ and $r_t$, which capture expectations at those long horizons that are relevant in practice, even if infinite-horizon expectations are constant. It is well known that assuming a unit root for a variable that in truth is persistent but stationary can be very beneficial for forecasting. The use of exact unit roots also simplifies the exposition of our model and arguments regarding trend components.\(^5\)

3 Data and trend estimates

We now describe the data we will use in our investigation of the relation between long-term interest rates and macroeconomic trends. Our data set is quarterly and extends from 1971:Q4

\(^5\)Our model also features unit roots under the risk-neutral measure, as evident from the loadings of yields on the trend variables that are constant across maturities. Strictly speaking, this is inconsistent with absence of arbitrage—see Dybvig et al. (1996) and Campbell et al. (1997, p. 433)—because the convexity in $-A_n/n$ diverges to minus infinity. However, as in the case of the real-world measure, our predictions are unaffected if the largest roots under the risk-neutral measure are taken to be very close to but below one.
to 2015:Q4. The interest rate data are the smoothed zero-coupon Treasury yields from Gürkaynak et al. (2007) with maturities from one to 15 years. We complement this data with three- and six-month Treasury bill rates from the Federal Reserve’s H.15 data. We use end-of-quarter interest rates. The ten-year yield is plotted in Figure 1. It exhibits the well-known pattern of a pronounced increase over the 1970s with a spike in the early 1980s, and a more or less continuous decrease over the following three decades. To explain this secular downward trend is an important objective of our paper.

For estimation of long-run inflation expectations a host of different methods is available, which mostly rely on surveys, statistical models, or a combination of the two—see, for example, Stock and Watson (2015) and the references therein. In this paper we will focus on a mostly survey-based measure, namely PTR, the perceived inflation target rate in the Federal Reserve’s FRB/US large-scale macroeconomic model. PTR measures long-run expectations of inflation in the price index of personal consumption expenditure (PCE), and is often used in empirical work—see, for example, the handbook chapter of Clark and McCracken (2013). Since 1979 (i.e., for most of our sample) PTR is based exclusively on survey expectations. Figure 1 shows that from the beginning of our sample to the late 1990s, this estimate mostly mirrored the increase and decrease in the ten-year yield. Since then, however, it has been essentially flat at the level of two percent, which in 2012 was announced as the long-run inflation target of the FOMC. Other survey expectations of inflation over the long run, such as the long-range forecasts in the Blue Chip survey, exhibit a similar pattern.

Estimation of the natural real interest rate is more challenging. Survey-based estimates require long-run expectations of both nominal interest rates and inflation, but the former are not available for a sufficiently long time span. In addition, long-range survey forecasts of nominal interest rates also tend to be more erratic and less accurate than forecast of inflation (see, for example Dijk et al., 2014). Modern approaches for estimation of \( r^*_t \) typically rely on statistical models, often with structural underpinnings. A popular example is the model of Laubach and Williams (2003), in which the unobserved natural rate is inferred from macroeconomic data using the Kalman filter. In Figure 2 we plot the one-sided/filtered Laubach-Williams estimate.

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6 We use a similar start date as CP to keep our analysis comparable to theirs.

7 Before 1979 PTR is based on estimates from the learning model for expected inflation of Kozicki and Tinsley (2001). From 1979 to 1991:Q3 expectations are taken from the Hoey survey, and thereafter from the from the Philadelphia Fed’s Survey of Professional Forecasters. Before 2007 expectations are available only for CPI inflation, and PTR makes an adjustment for the CPI-PCE differential by subtracting 0.4 percentage points from the expectations for CPI inflation. For more details on the construction of PTR, see Flint and Tinsley (1996). PTR can be downloaded at https://www.federalreserve.gov/econresdata/frbus/us-models-package.htm.

8 The longest history is available for the biannual long-range expectations in Blue Chip Financial Forecasts, which start in 1986.
of $r_t^*$. In addition we also consider the estimates of Lubik and Matthes (2015), who employ a time-varying parameter VAR model, Kiley (2015), who augments the Laubach-Williams model with a measure of financial conditions, and Johannsen and Mertens (2016), who use a shadow-rate term structure model to account for the lower bound on nominal interest rates. Figure 2 shows that early in the sample, until the early 1980s, the measures differ markedly, but that subsequently they evolve in broadly similar fashion. We calculate the average of these four estimates, which is a simple but effective way to aggregate the various approaches and information into one measure. This average is the measure of $r_t^*$ which we will use in our analysis in Sections 4–7.

A compelling story emerges from the evolution of long-term nominal interest rates and estimates of the trends in inflation and the real rate since the Volcker disinflation of the 1980s: Both interest rates and inflation exhibited a downward trend until about the turn of the millennium. Around that time long-run inflation expectations became firmly anchored near two percent, but long-term interest rates continued to decrease. The explanation is found in a substantial decline in the natural real interest rate. All four estimates of $r_t^*$ fell more or less continuously over the period from 2000 to 2015, with the decline ranging between 1.6 and 3.2 percentage points, with an average of 2.3. Over the same period the ten-year yield fell by 3.8 percentage points. A decline in the term premium and near-term expectations of real rates and inflation likely contributed to this fall. But according to our estimates it seems clear that $r_t^*$ played a crucial role in explaining the second half of the decline in long-term interest rates in the U.S. over the last 30 years.

4 Persistence in long-term yields from $r_t^*$ and $\pi_t^*$

If the trend components of inflation and the real interest rate play an important role in driving interest rates, then these trends should exhibit a close statistical relationship with long-term interest rates and account for most of their persistence. A natural starting point for assessing this relationship is to consider simple regressions of yields on measures of the trend components. Table 1 reports the results for such regressions with three different dependent variables: bond yields with 5 and 10 years maturity, and the 5-to-10-year forward rate. For each maturity, we estimate two versions of the regressions (with standard errors calculated using the Newey-West estimator with six lags). The first version has only a constant and $\pi_t^*$ as regressors, which is the same regression that CP estimated using their simple moving-

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9This filtered estimate is obtained using only past and contemporaneous information, whereas the two-sided/smoothed estimate incorporates information from the whole sample. Note, however, that both estimates are conditional upon estimates of the model parameters which are obtained using the full sample.
average estimate of the inflation trend (see their table 1). The second version adds $r_t^*$ as a regressor. Our results from the first regression confirm the findings of CP, namely, high $R^2$'s at all maturities and $\pi_t^*$ coefficients that are just above one and highly significant. CP interpret these results as indicating that trend inflation drives the level of yield curve, but our second version of this regression shows that incorporating a trend real rate is also important. With the addition of $r_t^*$ to each of the regressions, both trend coefficients are highly significant, and the regression $R^2$'s increase by between 8 and 13 percentage points.

Taken at face value, these estimates suggest that changes in $r_t^*$ along with changes in $\pi_t^*$, are key sources of variation in long-term interest rates. However, the interpretation of these results is complicated by the fact that all of the variables in the regressions are highly persistent. Conventional asymptotic arguments, which justify inference based on the Newey-West standard errors and $R^2$'s in Table 1, are not valid if some of the variables contain autoregressive roots close to or equal to one. Under the assumptions of the model in Section 2, the static regressions estimate the cointegrating relationships between long-term interest rates, inflation, and real rates. If there is a cointegrating relationship among the variables of such a regression, then the regression provides super-consistent estimates of the cointegrating vector, the $R^2$ converges to one, and conventional hypothesis tests are likely invalid for inference about the coefficients (e.g., Hamilton, 1994, Chapter 19). Consequently, the regression results in Table 1—although suggestive of the joint importance of $r_t^*$ and $\pi_t^*$ for the determination of yields—cannot provide definitive answers.

As a first step to provide more compelling statistical evidence that our measures of $\pi_t^*$ and $r_t^*$ capture the persistent variation in interest rates, we compare the time series properties of long-term interest rates with no detrending to the series with one or both of the trend components subtracted out. For the same three maturities considered above, Table 2 includes both simple differences as the detrended series, that is, assuming unit coefficients on $\pi_t^*$ and $r_t^*$, as well as the residuals from the static regressions in Table 1. For each series, Table 2 reports the standard deviation and two measures of persistence: the estimated first-order autocorrelation coefficient, $\hat{\rho}$, and the half-life, which indicates the number of quarters until half of a given shock has died out and is calculated as $\ln(0.5)/\ln(\hat{\rho})$. Several findings stand out: First, detrending with $r_t^*$ as well as $\pi_t^*$ accounts for noticeably more persistence, typically reducing the half-life by about 40-50%. That is, $\pi_t^*$ is not the only important driver of interest

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10Our estimated coefficients on $\pi_t^*$ are somewhat higher than in CP because our measure of the inflation trend is less variable, though when $r_t^*$ is added, the estimated coefficients for $\pi_t^*$ decrease toward one.

11Much empirical work, for example, King et al. (1991), has documented the substantial persistence in nominal interest rates, inflation, and real interest rates. The main difference between our static regressions and the usual cointegration regressions in this context (as in Rose, 1988, for example) is that we use directly observable proxies for the trend components of $\pi_t$ and $r_t$. 

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rate persistence. Second, the detrended series are substantially less variable and less persistent than the original interest rate series. For example, shocks to the ten-year yield have a half-life of about 5-1/2 years, whereas shocks to the series that is the difference of this yield and the trend components have a half-life of only 1-1/4 years. Finally, there is little difference in the statistical properties between the series that are detrended using unit coefficients and those that are detrended using estimated coefficients, in particular for the forward rate.

Unit root tests provide further evidence supporting the detrending with both $r_t^*$ and $\pi_t^*$. We calculate parametric Augmented Dickey-Fuller (ADF) $t$-statistics and non-parametric Phillips-Perron (PP) $Z_\alpha$ statistics to examine the persistence and integration properties of these series by testing the null hypothesis of a unit autoregressive root. These tests show strong evidence against the unit root null for the series that are detrended with both $\pi_t^*$ and $r_t^*$. By contrast, the unit root null is never rejected at the 10 percent level for the original interest rate series or for series that are detrended by only $\pi_t^*$. This supports the use of both the $r_t^*$ and $\pi_t^*$ trends for accounting for interest rate persistence and for understanding interest rate dynamics. Finally, with the exception of the five-year yield, the rejections are equally strong or stronger for the series that are detrended using unit coefficients as for the residuals from the static regressions.

In sum, macroeconomic trends capture a large share of the persistence in long-term interest rates, and both $\pi_t^*$ and $r_t^*$ are needed to accurately capture the trend. One implication is that when calculating the “interest rate cycles” suggested by CP—that is, differences between long-term yields and their trend components—one needs to account for both trend components. Instead, if the cycle is simply calculated as the difference between an interest rate and trend inflation, this purported cycle will still contain an important trend component due to changes in the equilibrium real interest rate. Another broad implication pertains to cointegration. We strongly rejected the unit root hypothesis for the series $y_t - \pi_t^* - r_t^*$ which is consistent with the prediction of our model that long-term interest rates and the macroeconomic trend components are cointegrated, and that the cointegrating vector equals $(1, -1, -1)'$. A large literature has investigated the question whether interest rates and inflation are cointegrated, and whether the real rate has a unit root or is stationary, with conflicting results. Using our proxies for the trend components in inflation and real rates, we obtain clear, unambiguous results: both inflation and the real rate contain trend components, and our proxies for these

\[12\] For the ADF test, we include a constant and $k$ lagged difference in the test regression, where $k$ is determined using the general-to-specific procedure suggested by Ng and Perron (1995). We start with $k = 4$ quarterly lags and reduce the number of lags until the coefficient on the last lag is significant at the ten percent level. For the PP test, we use a Newey-West estimator of the long-run variance with four lags. When the series under consideration is a residual from an estimated cointegration regression, we don’t include intercepts in the ADF or PP regression equations and use the critical values provided by Phillips and Ouliaris (1990), which depend on the number of regressors in the cointegration equation.
trend components are related to interest rates exactly as standard finance theory predicts.

5 Forecasting interest rates with $r^*_t$ and $\pi^*_t$

The previous section illustrated the strong persistence of interest rates and the close link between that persistence and measures of the inflation trend, $\pi^*_t$, and the equilibrium real rate, $r^*_t$. Here, we provide additional forecast-based evidence supporting that result. Specifically, our long-horizon forecast exercise confirms that interest rates exhibit reversion to a time-varying mean or endpoint, which, in line with the theory of Section 2, equals $i^*_t = \pi^*_t + r^*_t$.\(^{13}\)

Our results also show that accounting only for the variation in $\pi^*_t$ is insufficient, and that information in $r^*_t$ needs to be taken into account.

We forecast three different long-term interest rates, the five-year and ten-year yields, and the five-to-ten-year forward rate. At each point in time $t$, starting at $t = 20$ when five years of data is available, we forecast each interest rate $y_t$ at horizons from $h = 1$ to $h = 40$ quarters. As usual in this literature, the benchmark (method 1) is a driftless random walk model, i.e., the forecast $\hat{y}_{t+h} = y_t$ for all $h$. We consider two alternative forecast methods which assume that interest rates mean-revert to a time-varying endpoint $\hat{i}_t$. Method 2 assumes that this endpoint is only driven by the inflation trend, i.e., that $\hat{i}_t$ equals $\pi^*_t$ plus a constant. We estimate this constant recursively as the mean of $y_\tau - \pi^*_\tau$, using observations from $\tau = 1$ to $\tau = t$.\(^{14}\) Method 3 instead assumes that the endpoint is $\hat{i}_t = i^*_t = \pi^*_t + r^*_t$. For both forecast methods, the speed of mean-reversion is recursively estimated as the first-order autocorrelation coefficient of $y_t - i_t$. Denoting this coefficient as $\hat{\rho}_t$ the forecasts for methods 2 and 3 are constructed as $\hat{y}_{t+h} = \hat{\rho}_t y_t + (1 - \hat{\rho}_t^h) \hat{i}_t$. We note that the exact value/method for $\hat{\rho}_t$ affects only short-horizon forecasts and has no impact on our main results.

Table 3 reports the results of this forecasting exercise in terms of root-mean-squared errors (RMSEs) and mean-absolute errors (MAEs), in percentage points. We also calculate $p$-values for tests of equal finite-sample forecast accuracy using the approach of Diebold and Mariano (1995) (DM), following common use with a rectangular window for the long-run variance and the small-sample adjustment of Harvey et al. (1997).\(^{15}\) We calculate DM $p$-values, using standard normal critical values, for one-sided tests of whether forecasts using the endpoint

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\(^{13}\)The model in Section 2 implies that the endpoint equals a maturity-specific constant plus $i^*_t$, but our results show that it is not necessary to try to estimate and include this constant in the endpoint to obtain accurate forecasts.

\(^{14}\)We have found in additional, unreported results that forecasts which assume that this constant is zero perform much worse than this method.

\(^{15}\)Monte Carlo evidence in Clark and McCracken (2013) indicates that this test has good size in finite samples. However, for very long forecast horizons there are of course only few non-overlapping observations in our sample, so the $p$-values should be taken with a grain of salt.
\( \pi_t^* + r_t^* \) (method 3) improve upon random walk forecasts (method 1) and forecasts that use only \( \pi_t^* \) for the endpoint (method 2). We find that method 3 leads to gains in forecast accuracy at long horizons that are both substantial and highly significant. Such gains are present for both RMSEs and MAEs, but are larger and more strongly significant for absolute-error loss. For example, when forecasting the ten-year yield five years ahead, using the information in \( \pi_t^* + r_t^* \) decreases the RMSE by over 25% relative to the random walk forecast, an improvement that is significant at the five-percent level, while the MAE is decreased by more than 40% with significance at the one percent level. This forecast method also improves upon method 2, which does not use the information in \( r_t^* \), and the improvement is typically large and often significant.

This forecast exercise thus confirms that interest rates exhibit reversion to a time-varying endpoint of \( \pi_t^* + r_t^* \). Earlier authors, such as Dijk et al. (2014), have found that when forecasting interest rates it is beneficial to link long-run projections of interest rates to long-run expectations of inflation. Our new result shows that including \( r_t^* \) along with \( \pi_t^* \) improves on long-horizon interest rate forecasting to an even greater degree. Furthermore, while earlier authors run regressions of the level of interest rates on long-run inflation expectations, we have shown that no scaling or estimation is necessary for accurate long-range forecasts if the endpoint is simply taken as \( \pi_t^* + r_t^* \).

Finally, it is useful to consider our results from the statistical standpoint of integration and cointegration. Our results in Table 2 generally could not reject the unit root hypothesis for interest rates, inflation and real rates. Furthermore, these three series appear to be cointegrated, since interest rates that are detrended using our proxies for the trends in inflation and real rates show fast mean-reversion. We need not claim the presence of exact unit roots, but our results suggest that empirical models and forecasts of the yield curve may perform better in finite samples if one assumes the presence of unit roots and cointegration instead of stationarity.\(^{17}\) A common finding in empirical practice, due to the high persistence of interest rates, is that the assumption of a random walk leads to competitive forecasts that are difficult to beat (Duffee, 2002; Diebold and Li, 2006; Duffee, 2013). But our results show that the added assumption of cointegration and the use of our empirical proxies for the macroeconomic trends inherent in interest rates leads to substantially better long-term forecasts than the use of random walks.

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\(^{16}\)Our approach could easily be extended to jointly forecast the whole yield curve as in Diebold and Li (2006) and Dijk et al. (2014), by simply forecasting the Nelson-Siegel level factor in the same fashion that we have forecast each individual interest rate above.

\(^{17}\)This follows from the well-known argument in empirical finance and macroeconomics, elucidated for example in Campbell and Perron (1991), that while highly persistent and integrated series are indistinguishable in finite samples it can be beneficial for forecasting to assume a unit root for a persistent but stationary series.
6 Predicting bond returns with $r^*_t$ and $\pi^*_t$

We now turn to the role of macroeconomic trends for predicting bond returns. According to CP, proxies for the inflation trend appear to have significant predictive power for annual excess bond returns, above and beyond the information contained in the yield curve itself. Since $r^*_t$ also is an important trend driving long-term interest rates, it is natural to ask what role it plays in such predictive regressions. We revisit CP’s analysis and address this larger question.

We predict excess returns for holding periods of one quarter and four quarters. The excess return for a holding period of $h$ quarters for a bond with maturity $n$ is calculated as

$$rx_{t,t+h}^{(n)} = p_{t+h}^{(n-h)} - p_t^{(n)} - hy_t^{(h)} = -(n-h)y_{t+h}^{(n-h)} + ny_t^{(n)} - hy_t^{(h)}.$$  

Our long-term bond yields are available only at annual maturities, so we calculate one-quarter returns with the usual approximation $y_{t+1}^{(n-1)} \approx y_t^{(n)}$. Our dependent variable is the average excess return for bonds with two to 15 years maturity.\(^{18}\) We estimate three specifications of the predictive regressions: The first includes a constant and the first three principal components (PCs) of yields. This common baseline regression is motivated by work going back to Fama and Bliss (1987) and Campbell and Shiller (1991) which showed that there is information in the yield curve itself, and in particular in its slope (PC2), about expected bond returns. The second specification adds $\pi^*_t$ and is closely related to the specifications including an inflation trend that were estimated by CP. In our third specification we also include $r^*_t$, in order to simultaneously capture the effects of both macroeconomic trends in the regression.

The top panel of Table 4 reports the estimation results for the full sample. We calculate White’s heteroskedasticity-robust standard errors for the case of one-quarter returns and Newey-West standard errors with six lags for the four-quarter returns (since the overlap introduces serial correlations in the error term). Our results replicate CP’s main finding, namely that inclusion of the inflation trend raises the predictive power quite substantially in comparison to a regression that only includes yield-curve information, and that both the inflation trend and the level of yields (PC1) appear highly significant.

Adding $r^*_t$ to the regression leads to impressive further gains in predictive power. For both one-quarter and four-quarter returns, the $R^2$ increases substantially, the coefficients and $t$-statistics for $\pi^*_t$ and PC1 rise, and the coefficient on $r^*_t$ itself is large and highly significant according to Newey-West $t$-statistics. Interestingly, the coefficient on $r^*_t$ is about as large as

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\(^{18}\)CP focused on annual holding periods, using a monthly data set and used a weighted average that down-weights longer-term bonds. Our simple average made no material difference to the results and is more common in this literature. Finally, our sample period is similar to the one considered in CP but extends to 2015.
the coefficient on $\pi_t^*$, which confirms the theoretical prediction that these two trends play roles of similar magnitude in determining interest rates.

It is well known that predictive regressions for bond returns raise some knotty econometric issues. Bauer and Hamilton (2016) show that the small-sample inference is particularly problematic when the predictors are highly persistent, like interest rates and our macro trends. They propose a parametric bootstrap procedure to carry out robust inference in such cases. It tests the null hypothesis that only the information in the yield curve is useful for predicting excess returns, and it can accurately gauge the statistical significance of additional proposed predictors. We follow their recommendation and calculate bootstrap small-sample $p$-values for the coefficients on $\pi_t^*$ and $r_t^*$. Our bootstrap simulates yields from a simple VAR(1) factor model and the additional predictors from a separate VAR(1) model, so that the null hypothesis holds by construction. The predictive regression is estimated in each bootstrap sample and the $t$-statistics for the additional predictors are recorded. With this small-sample distribution of the test statistics in hand, $p$-values are calculated as the fraction of simulated samples in which the $t$-statistic is at least as large (in absolute value) as the $t$-statistic in the actual data. These $p$-values, reported in squared brackets in Table 4, indicate that both of our trends are statistically significant even when we account for small-sample econometric concerns.

According to the small-sample $p$-values, in the subsample starting in 1985, the inflation trend is not statistically significant when included on its own. Only when we add our measure of the equilibrium real rate, do both trends matter for bond risk premia; the coefficients on $\pi_t^*$ and PC1 more than double, the $R^2$ increases substantially, and the coefficients on $\pi_t^*$ and $r_t^*$ are statistically significant. These results confirm our intuition from Figure 1 that the real-rate trend has gained in importance and significance over time as it has exhibited a pronounced decline over the past twenty years while the trend in inflation has been essentially constant. Therefore, empirical analysis of long-term interest rates during that period implies that the trend in the real interest rate is at least as important as, and recently more important than, the trend in inflation.

An important contribution of CP is that they estimated interest rate cycles, defined as the de-trended or cyclical component in interest rates, and showed that these cycles captured the predictive power in interest rates and the inflation trend for future bond returns. They estimated the interest rate cycles as the residuals of regressions of interest rates on their

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19 This mirrors the finding of Bauer and Hamilton (2016) who also revisited CP’s evidence and showed that in a subsample starting in 1985 the inflation trend is only marginally significant according to their bootstrap small-sample inference.

20 In additional, unreported results we have found that the predictive gains from including $r_t^*$ are particularly large during the early 2000s when both $r_t^*$ and long-term interest rates decreased while long-run inflation expectations where anchored close to two percent.
measure of the inflation trend. Our results here and in the previous sections indicate that a better estimate of the cycle is obtained by incorporating measures of both an inflation trend and a real rate trend. Furthermore, our results suggest that a simple difference between a long-term interest rate and \( i_t^* = \pi_t^* + r_t^* \) is likely sufficient; there is no need to run a regression to construct interest rate cycles. Such estimates of interest rate cycles could, for example, be used to construct a return-forecasting factor similar in spirit to the one constructed by CP, which would better capture variation in bond risk premia due to the fact that trend components are more accurately accounted for.

An outstanding issue is why information in \( \pi_t^* \) and \( r_t^* \) is not spanned by the yield curve. Unspanned risks in bond returns are an important current issue in macro-finance; see Duffee (2013), Joslin et al. (2013), Joslin et al. (2014), and Bauer and Hamilton (2016). CP and Bauer and Rudebusch (2016) argue the lack of spanning reflects measurement errors in yields, but there may be other, more structural reasons. The incremental predictive power of macro trends and the apparent failure of the spanning hypothesis may reflect some underlying economic mechanisms related to the unobservable nature of trends.

In sum, accounting for the persistent components of yields is important for understanding return predictability and estimation of bond risk premia. We find that \( r_t^* \) has strong incremental predictive power for bond returns, about on par with the importance of \( \pi_t^* \) as a predictor, suggesting that in this context it is important to account for not only the inflation trend but also estimates of the equilibrium real rate.

### 7 Variance ratios

Using variance ratios for quarterly innovations in bond yields Duffee (2016) documents that the contribution of inflation expectations to interest rate movements appears to be small. Here we show that his findings are consistent with our results and provide a new perspective by considering variance ratios for different frequencies.

The variance ratio used by Duffee is

\[
VR_1^{(n)} = \frac{Var(E_t - E_{t-1}) n^{-1} \sum_{i=1}^{n} \pi_{t+i}}{Var(E_t - E_{t-1}) y_t^{(n)}},
\]

which measures the importance of time-\( t \) news about average inflation over the life of the bond for the innovation to the bond yield. A more general variance ratio considers innovations over
not one but $h$ periods:

$$VR_h^{(n)} = \frac{\text{Var}(E_t - E_{t-h})n^{-1}\sum_{i=1}^{n} \pi_{t+i}}{\text{Var}(E_t - E_{t-h}b_t^{(n)})}.$$ 

Our model implies the following analytical expression for the variance ratio:

$$VR_h^{(n)} = \frac{h\sigma^2_\xi + a_c(n)b_c(h)\sigma^2_u}{h\sigma^2_\xi + a_c(n)b_c(h)\sigma^2_u + h\sigma^2_\eta + a_g(n)b_g(h)\sigma^2_v + \left(\frac{B^n}{n}\right)^2 b_x(h)\sigma^2_w}$$

$$a_i(n) = \left(\frac{1 - \phi_i^n}{n(1 - \phi_i)}\right)^2, \quad b_i(h) = \frac{1 - \phi_i^{2h}}{1 - \phi_i^2}, \quad i = c, g, x.$$ \hspace{1cm} (3)

An important result of Duffee’s analysis is that even for long-term bonds, $VR_1^{(n)}$ appears to be small. This can be understood through the lens of our model by considering

$$\lim_{n \to \infty} VR_1^{(n)} = \frac{\sigma^2_\xi}{\sigma^2_\xi + \sigma^2_\eta + \left(\frac{\lambda_{x^*+x^*+x^*}}{1 - \phi_x}\right)^2 \sigma^2_w}.$$ 

This ratio will be small if shocks to the equilibrium real rate and to the risk-premium factor make a more important contribution than shocks to the inflation trend. One plausible interpretation of Duffee’s finding is that at a quarterly frequency the trend in inflation moves much less than the term premium component of long-term interest rates. Such movements in the term premium, which is a catch-all component for price movements not attributable to changes in real-rate and inflation expectations, could be due to a variety of factors including changes in risk-sentiment and “animal spirits.” This interpretation is consistent with additional evidence in Duffee’s paper on the role of the term premium, and with a large body of evidence on excess volatility of interest rates, going all the way back to Shiller (1979).

Another way to try to understand the role of inflation expectations for bond yields is to consider longer horizons instead of one-period innovations. This appears promising because

$$\lim_{h \to \infty} VR_h^{(n)} = \frac{\sigma^2_\xi}{\sigma^2_\xi + \sigma^2_\eta},$$

so that asymptotically this variance ratio is only affected by changes in the trend components. Changes in term premia become irrelevant at very low frequencies if, as in our model, the risk premium factor $x_t$ is stationary. Of course, looking at variance ratios for large $h$ instead of large $n$ is no magic bullet: we quickly lose degrees of freedom and precision in our estimates when we increase $h$ and hence the overlap of the observations. However, the profile of variance
ratios across horizons should give us additional insights about interest rate dynamics, even if
the maximum horizon is limited in practice by data availability.

Estimation of the variance ratios suggested by Duffee and above requires modeling expec-
tations of inflation and interest rates. Here we instead propose to calculate variance ratios
based on observed changes in the data. For example, using $\pi_t^*$ we can calculate

$$VR_{h}^{(n)} = \frac{Var\Delta_h \pi_t^*}{Var\Delta_h y_t^{(n)}}, \quad \Delta_h z_t = z_t - z_{t-h}.$$ 

For any given $n$ and $h$, $\tilde{VR}_{h}$ differs from $VR_{h}$ because the variances in the former include
movements that were anticipated at $t - h$. However, in practice these differences are likely to
be small for the following reasons. For long-term bonds we have

$$\lim_{n \to \infty} \tilde{VR}_{1}^{(n)} = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_{\eta}^2 + \left(\frac{\lambda_{\pi} + \lambda_{s} \phi_x}{1 - \phi_x}\right)^2 \frac{2}{1 + \phi_x} \sigma_w^2} \approx \lim_{n \to \infty} VR_{1}^{(n)},$$

where the accuracy of the approximation depends on how close $2/(1 + \phi_x)$ is to one, i.e., on
how persistent the risk-premium factor $x_t$ is. If $x_t$ is not that persistent, $\tilde{VR}_{h}$ for a long-
term bond is somewhat smaller than the corresponding $VR_{h}$. Another justification for using
variance ratios of changes is the fact that

$$\lim_{h \to \infty} \tilde{VR}_{h}^{(n)} = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_{\eta}^2} = \lim_{h \to \infty} VR_{h}^{(n)},$$

so that for low-frequency movements the two estimates are asymptotically identical. In sum,
variance ratios of changes are easier to calculate than variance ratios of innovations, but help
us learn about the same features of the data. In addition, we can also calculate them for
the real-rate trend or the overall trend component, by simply using the sample variances of
changes in the observed $r_t^*$ or $i_t^*$ in the numerator.

Figure 3 shows sample variance ratios for the five-year and ten-year yield, as well as for
the 5-to-10-year forward rate. We calculate variance ratios for changes in $\pi_t^*$, $r_t^*$ and $i_t^*$, and
for changes from one quarter to $h = 40$ quarters.

For the inflation trend we find that one-quarter variance ratios are around 0.1. This result
is consistent with Duffee’s findings and indicates that changes in the inflation trend play a
small role for variation in yields at the quarterly frequency. When we consider changes at
lower frequencies, however, we find that movements in the inflation trend play a substantial
role for changes in interest rates. The point estimates of the $\pi_t^*$-variance ratio quickly increase
with the horizon, and for \( h = 40 \) reach a magnitude of around 0.3.

The variance ratio for changes in the real-rate trend, i.e., \( \text{Var} \Delta r^*_t / \text{Var} \Delta y_t \), is much lower than the one for the inflation trend, remaining below 0.1 even at the longest horizon. This is unsurprising in light of Figure 1, which shows that over the full sample the movements in \( r^*_t \) were substantially less pronounced than movements in long-term interest rates and the inflation trend.\(^{21}\) Of course, this perspective of unconditional variances should not be taken to conclude that changes in the equilibrium real rate are unimportant for interest rate dynamics, given the ample evidence in Sections 4–6 of the importance of \( r^*_t \) for modeling and forecasting interest rates. Furthermore, since our evidence suggests that \( r^*_t \) has become a relatively more important driver of changes in long-term interest rates over recent decades, we venture the guess that conditional variance ratios would show a switch from \( \pi^*_t \) to \( r^*_t \) as the dominant driver of low-frequency changes.\(^{22}\)

We now focus on the variance ratios \( \text{Var} \Delta i^*_t / \text{Var} \Delta y_t \) to assess the overall importance of the trend components in interest rates. Confidence intervals are obtained using the asymptotic distribution of the sample variances and the delta method. To account for persistence in conditional variances, Newey-West estimates of long-run variances are used.\(^{23}\) We acknowledge that these confidence intervals may understate the true sampling variability due to the small number of nonoverlapping observations, which decreases the reliability of the asymptotic approximations. With this caveat in mind, Figure 3 shows that while the sampling uncertainty around the variance ratios for changes in \( i^*_t \) is substantial we can be reasonably confident that these variance ratios increase from below 0.15 to a range of around 0.25 to 0.4, depending on the maturity of the interest rate. The highest levels are reached for the 5-to-10-year forward rate—the confidence interval at \( h = 40 \) extends from about 0.3 to 0.4—which is consistent with the notion that distant forward rates are more strongly affected by the trend components.

Our model, like any model with a trend component for interest rates, implies that the variance ratios for \( i^*_t \) approach one for sufficiently large \( h \), but our estimates top out around 0.4. One possible explanation is that we are simply not capturing the trend component with sufficient accuracy. The true trend components might well be more (or less) variable than the proxies we have available. However, we have provided ample evidence that our trend proxies are closely linked to the low-frequency movements in interest rates. A more plausible

\(^{21}\) This finding may be partially due to our use of an average of different estimates for \( r^*_t \)—some of the individual estimates exhibit distinctly more volatility.

\(^{22}\) Due to the paucity of data, conditional variance ratios would need to be obtained using a model with stochastic volatility, in order to impose structure on variances across frequencies and increase the efficiency of the estimates (at the cost of possible misspecification).

\(^{23}\) The choice of lag length is informed by the automatic lag selection procedure of Newey and West; we end up using 12 quarterly lags for all long-run variance estimates.
explanation might be that the largest values of $h$ that we consider are simply not large enough, which is a problem that we cannot solve without more data. At the horizons we do provide estimates for, the estimates are increasing substantially with $h$, but even at $h = 40$ the cyclical/transitory components of interest rates appear to still play a substantial role. This could be due either to the cyclical components in inflation and real rates, or, more likely, to the fact that the risk-premium factor itself is sufficiently persistent/trending that it drives interest rate variation at such low frequencies.

To shed additional light on the respective roles of trends and cycles we report in Table 5 the variances of changes in interest rates, in the trend component $i^*_t$, and in the cycle components $y_t - i^*_t$. The variance of yield changes can be decomposed as follows:

$$Var(\Delta_y t) = Var(\Delta_ii^*_t) + Var(\Delta_y t - \Delta_ii^*_t) + 2Cov(\Delta_ii^*_t, \Delta_y t - \Delta_ii^*_t).$$

Table 5 reports all three components. In the data, the contribution of the covariance is small at short horizons, but substantial at long horizons. The two last columns of Table 5 report, first, the same variance ratio shown in Figure 3, and second, a ratio that also includes the covariance contribution in the numerator. This second ratio rises to over 0.7 with horizon, which in turn implies that the cycle component by itself explains only less than 30% of the variance of interest rate changes at low frequencies. These estimates raise some additional questions, such as how to reconcile the strongly positive covariance with our model, which assumes that shocks to trend and cycle are uncorrelated. But what seems clear from these results is that while changes in the trend components are unimportant for interest rate at high frequencies (such as one-quarter changes), they play a crucially important role for driving low-frequency movements in interest rates.

8 Conclusion

In this paper we have provided new evidence that interest rates are closely related to two macroeconomic trends: the trend in inflation (or the Fed’s inflation target), and the equilibrium (or natural/neutral) real rate of interest. Our empirical approach was to use outside estimates from different sources as proxies for these trend components, and then to investigate the relationship between interest rates and these trend proxies from a variety of perspectives, focusing on long-term interest rates because they are less affected by cyclical factors. We have shown that there is indeed a very close relationship, and that accounting for the trend

\(^{24}\text{Measurement error would have to be of a very specific form to explain this finding by itself, because classical measurement error in our estimate of } i^*_t \text{ would lead to a downward bias in the sample covariance.}\)
components using our proxies can help understand, model, and forecast long-term interest rates and bond returns. Our results confirm the predictions of no-arbitrage theory for the links between macroeconomic trends and the yield curve, and demonstrate that these links are quantitatively important.

Our findings open up several avenues for future research. Most importantly, they demonstrate that macro-finance models for the yield curve need to allow for slow-moving changes in the long-run mean of inflation and the real rate. This applies to both reduced-form no-arbitrage models as well as to more structural, equilibrium models of the yield curve. The purpose of specifying and estimating such models is typically to understand bond risk premia/term premia and risk compensation, and constructing the necessary long-run expectations without accounting for shifts in macroeconomic trends is likely to give misleading results for risk premia.

An additional dimension of the relationship of macroeconomic trends and interest rates is that the relative importance of these trend components changes over time. For example, it is well known in empirical macroeconomics (see, for example, Stock and Watson, 2007) that the trend component of inflation was much more variable in the 70s and 80s than in more recent decades. This raises the question how our (mostly unconditional) results are affected by taking a conditional perspective. Furthermore, this suggests that incorporating not only macroeconomic trends but also stochastic volatility in these trend components will be crucially important for macro-finance term structure models.

References


Appendix

A Additional details for affine term structure model

Here we provide further details and additional results for the affine term structure model of Section 2. First, we consider prices and yields of real (i.e., inflation-indexed) bonds. Just like prices of nominal bonds, prices of real bonds are exponentially affine in the risk factors, log(\(\hat{P}_t^{(n)}\)) = \(\hat{A}_n + \hat{B}_n^t Z_t\). Hats denote variables pertaining to real bonds. The loadings are determined by the recursions

\[
\hat{A}_{n+1} = \hat{A}_n + \hat{B}_n^t (\mu - \lambda_0) + \hat{C}_n, \quad \hat{C}_n := \frac{1}{2} \hat{B}_n^t \Sigma \Sigma' \hat{B}_n, \quad \hat{B}_{n+1} = -(0, 0, 1, 0)' + (\phi - \lambda_1)' \hat{B}_n,
\]

\(\hat{C}_n\) captures the convexity in real bonds, and the initial conditions are \(\hat{A}_0 = 0, \hat{B}_0 = (0, 0, 0, 0, 0)'\). Specifically,

\[
\hat{B}^\pi_n = 0, \quad \hat{B}^c_n = 0, \quad \hat{B}^{\pi^*}_{n+1} = \hat{B}^{\pi^*}_n - 1, \quad \hat{B}^g_n = \phi_g \hat{B}^g_n - 1,
\]

\[
\hat{B}^x_{n+1} = -\lambda_{\pi^*} \hat{B}^{\pi^*}_n - \lambda_x \hat{B}^g_n + \phi_x \hat{B}^x_n.
\]

Real yields, \(\hat{y}_t^{(n)} = -\log(\hat{P}_t^{(n)})/n\), are affine in the risk factors. It is instructive to consider real forward rates for inflation-indexed borrowing from \(n\) to \(n+1\), for which we have

\[
\hat{f}_t^{(n)} = \log(\hat{P}_t^{(n)}) - \log(\hat{P}_t^{(n+1)}) = \hat{A}_n - \hat{A}_{n+1} + (\hat{B}_n - \hat{B}_{n+1})' Z_t
\]

\[
= -\hat{B}^t_n (\mu - \lambda_0) - \hat{C}_n + r_t^* + \phi^n g_t + (\hat{B}^x_n - \hat{B}^x_{n+1}) x_t
\]

\[
= -\hat{C}_n + E_t(r_{t+1}^*) + \hat{f}_t^{(n)}.
\]

Therefore, changes in \(r_t^*\) affect all real forward rates equally and hence act as a level factor. Changes in the real-rate gap \(g_t\) affect short-term real rates more strongly than long-term rates, and therefore affect the slope. The last row clarifies that real forward rates can be decomposed into convexity, an expectations component, \(E_t(r_{t+1}^*) = r_t^* + g^n t\), and a real forward term premium, \(\hat{f}_t^{(n)} = -\hat{B}^x_n \mu + \hat{B}^c_n \lambda_0 + (\hat{B}^x_n - \hat{B}^x_{n+1}) x_t\).

For real yields we have

\[
\hat{y}_t^{(n)} = -\log(\hat{P}_t^{(n)})/n = -\hat{A}_n/n - \hat{B}_n' Z_t/n
\]

\[
= r_t^* + \frac{1}{n(1 - \phi^n)} g_t + \sum_{i=0}^{n-1} E_{t+i} r_{t+i} + \hat{f}_t^{(n)}.
\]

which shows that the equilibrium real rate \(r_t^*\) acts as a level factor for the real yield curve, and that the impact of the real-rate gap \(g_t\) diminishes with the yield maturity.

To understand the real term premium it is helpful to first consider the term premium in
the one-period-ahead real forward rate, which is

\[ ft_{tp}^{(1)} = Cov_t(m_{t+1}^r, r_{t+1}) = -[\lambda_{0r} + \lambda_{0g} + (\lambda_{r+x} + \lambda_{gx})x_t]. \]

If the real SDF positively correlates with the real rate, then real bonds are risky in the sense that their payoffs are low in times of high marginal utility. In this case, the real term premium is positive to compensate investors for this risk. \(^{25}\)

Nominal forward rates from \(n\) to \(n + 1\) are:

\[
f_t^{(n)} = \log(P_t^{(n+1)}) - \log(P_t^{(n+1)}) = A_n - A_{n+1} + (B_n - B_{n+1})'Z_t
\]

\[
= -c_n + \pi_t^* + \phi^*_n c_t + r_t^* + \phi^*_n g_t - B_n^* \mu_x + B_n^* \lambda_0 + (B_n^* - B_{n+1}^*)x_t
\]

Naturally, nominal forward rates reflect expectations of future inflation and real rates. Changes in the trend components \(\pi_t^*\) and \(r_t^*\) parallel-shift the entire path of these expectations, and therefore affect forward rates at all maturities equally. Distant forward rates are, on the other hand, only minimally affected by changes in \(c_t\) and \(g_t\). The loading of forward rates on \(r_t\) can be shown to approach \(-(\lambda_{\pi+x} + \lambda_{r+x})/(1 - \phi_x)\) for large \(n\), meaning that \(x_t\) affects distant forward rates due to its effect on the prices of risk of \(\pi_t^*\) and \(r_t^*\).

In our empirical analysis we will consider the five-to-ten-year forward rate, i.e.,

\[ f_t^{(n_1,n_2)} = (n_2 - n_1)^{-1} \sum_{n=n_1}^{n_2-1} f_t^{(n)}, \quad n_1 = 20, \quad n_2 = 40. \]

Our model implies that this interest rate is even less affected by the cyclical components \(c_t\) and \(g_t\) and should exhibit a particularly close relationship with the trend components \(\pi_t^*\) and \(r_t^*\).

Although our empirical analysis does not focus on term premia, it is worth noting the intuition for term premia, which is particularly simple within our model. The term premium in nominal forward rates, \(ft_{tp}^{(n)} = -B_n^* \mu_x + B_{n}^* \lambda_0 + (B_n^* - B_{n+1}^*)x_t\), is composed of the real forward term premium, \(ft_{tp}^{(n)}\), and a forward inflation risk premium, \(firp_t^{(n)}\). The intuition is again easiest for \(n = 1\):

\[ firp_t^{(1)} = Cov_t(m_{t+1}^r, E_{t+1}(\pi_{t+2})) = -[\lambda_{0r} + \lambda_{0c} + (\lambda_{r+x} + \lambda_{cx})x_t]. \]

If shocks to inflation expectations are positively correlated with the real SDF, then nominal bonds are more risky than real bonds and require a higher risk premium, i.e., a positive inflation risk premium. Like the real term premium, the inflation risk premium in this model is driven only by changes in \(x_t\).

\(^{25}\)Variation in the real term premium is driven by changes in the risk-premium factor \(x_t\), which affects prices of risk; quantities of risk are constant due to homoskedasticity of the state variables.
Table 1: Regressions of long-term interest rates on macroeconomic trends

<table>
<thead>
<tr>
<th></th>
<th>( y_t^{(5y)} ) (1)</th>
<th>( y_t^{(10y)} ) (2)</th>
<th>( f_t^{(5y,10y)} ) (3)</th>
<th>( f_t^{(5y,10y)} ) (4)</th>
<th>( f_t^{(5y,10y)} ) (5)</th>
<th>( f_t^{(5y,10y)} ) (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>-0.14</td>
<td>-2.20</td>
<td>1.03</td>
<td>-0.66</td>
<td>2.20</td>
<td>0.87</td>
</tr>
<tr>
<td>( \pi_t^* )</td>
<td>(0.68)</td>
<td>(0.31)</td>
<td>(0.57)</td>
<td>(0.30)</td>
<td>(0.50)</td>
<td>(0.38)</td>
</tr>
<tr>
<td>( r_t^* )</td>
<td>(0.15)</td>
<td>(0.13)</td>
<td>(0.14)</td>
<td>(0.12)</td>
<td>(0.13)</td>
<td>(0.13)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.79</td>
<td>0.92</td>
<td>0.80</td>
<td>0.91</td>
<td>0.78</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Regressions of long-term Treasury yields and forward rates on measures of long-run inflation expectations and the equilibrium real rate, which are described in the text. Numbers in parentheses are Newey-West standard errors with six lags. The data are quarterly from 1971:Q4 to 2015:Q4.

Table 2: Persistence of interest rates and detrended interest rates

<table>
<thead>
<tr>
<th>Series</th>
<th>SD</th>
<th>( \hat{\rho} )</th>
<th>Half-life</th>
<th>ADF</th>
<th>PP</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_t^{(5y)} )</td>
<td>3.18</td>
<td>0.97</td>
<td>23.1</td>
<td>-1.14</td>
<td>-3.27</td>
</tr>
<tr>
<td>( y_t^{(5y)} - \pi_t^* )</td>
<td>1.90</td>
<td>0.93</td>
<td>9.1</td>
<td>-1.83</td>
<td>-9.56</td>
</tr>
<tr>
<td>( y_t^{(5y)} - \pi_t^* - r_t^* )</td>
<td>1.33</td>
<td>0.87</td>
<td>5.0</td>
<td>-3.43**</td>
<td>-20.80***</td>
</tr>
<tr>
<td>( y_t^{(5y)} - 1.77\pi_t^* )</td>
<td>1.45</td>
<td>0.87</td>
<td>5.0</td>
<td>-2.64</td>
<td>-19.71*</td>
</tr>
<tr>
<td>( y_t^{(5y)} - 1.28\pi_t^* - 1.85r_t^* )</td>
<td>0.92</td>
<td>0.73</td>
<td>2.2</td>
<td>-4.63***</td>
<td>-47.28***</td>
</tr>
<tr>
<td>( y_t^{(10y)} )</td>
<td>2.84</td>
<td>0.97</td>
<td>22.5</td>
<td>-1.05</td>
<td>-3.02</td>
</tr>
<tr>
<td>( y_t^{(10y)} - \pi_t^* )</td>
<td>1.58</td>
<td>0.92</td>
<td>7.9</td>
<td>-2.36</td>
<td>-10.57</td>
</tr>
<tr>
<td>( y_t^{(10y)} - \pi_t^* - r_t^* )</td>
<td>1.06</td>
<td>0.84</td>
<td>4.1</td>
<td>-3.79***</td>
<td>-26.21***</td>
</tr>
<tr>
<td>( y_t^{(10y)} - 1.58\pi_t^* )</td>
<td>1.27</td>
<td>0.87</td>
<td>5.0</td>
<td>-2.66</td>
<td>-19.23*</td>
</tr>
<tr>
<td>( y_t^{(10y)} - 1.18\pi_t^* - 1.52r_t^* )</td>
<td>0.88</td>
<td>0.77</td>
<td>2.7</td>
<td>-4.79***</td>
<td>-40.56***</td>
</tr>
<tr>
<td>( f_t^{(5y,10y)} )</td>
<td>2.55</td>
<td>0.96</td>
<td>19.2</td>
<td>-1.12</td>
<td>-3.47</td>
</tr>
<tr>
<td>( f_t^{(5y,10y)} - \pi_t^* )</td>
<td>1.37</td>
<td>0.90</td>
<td>6.4</td>
<td>-2.62*</td>
<td>-13.79*</td>
</tr>
<tr>
<td>( f_t^{(5y,10y)} - \pi_t^* - r_t^* )</td>
<td>1.00</td>
<td>0.83</td>
<td>3.8</td>
<td>-3.95***</td>
<td>-29.67***</td>
</tr>
<tr>
<td>( f_t^{(5y,10y)} - 1.40\pi_t^* )</td>
<td>1.21</td>
<td>0.87</td>
<td>5.0</td>
<td>-2.87</td>
<td>-19.68*</td>
</tr>
<tr>
<td>( f_t^{(5y,10y)} - 1.09\pi_t^* - 1.19r_t^* )</td>
<td>0.97</td>
<td>0.82</td>
<td>3.6</td>
<td>-4.16**</td>
<td>-32.07**</td>
</tr>
</tbody>
</table>

Summary statistics for the persistence of interest rates and detrended interest rates: standard deviation (SD); first-order autocorrelation coefficient (\( \hat{\rho} \)); half-life, calculated as \( \ln(0.5)/\ln(\hat{\rho}) \); Augmented Dickey-Fuller (ADF) and Phillips-Perron unit root test statistics, with *\(^2\), **\(^2\), and ***\(^2\) indicating significance at 10%, 5%, and 1% level. Detail on the unit root tests are in the main text. The data are quarterly from 1971:Q4 to 2015:Q4.
### Table 3: Forecasting long-term interest rates

<table>
<thead>
<tr>
<th>Horizon $h$ (quarters):</th>
<th>RMSE</th>
<th>MAE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td><strong>Five-year yield</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1) Random walk</td>
<td>1.40</td>
<td>2.07</td>
</tr>
<tr>
<td>(2) Endpoint $\pi_t^* + \hat{\mu}_t$</td>
<td>1.51</td>
<td>2.18</td>
</tr>
<tr>
<td>(3) Endpoint $\pi_t^* + r_t^*$</td>
<td>1.40</td>
<td>1.80</td>
</tr>
<tr>
<td>(3) vs. (1) (0.50)</td>
<td>(0.12)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>(3) vs. (2) (0.12)</td>
<td>(0.07)</td>
<td>(0.05)</td>
</tr>
<tr>
<td><strong>Ten-year yield</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1) Random walk</td>
<td>1.26</td>
<td>1.77</td>
</tr>
<tr>
<td>(2) Endpoint $\pi_t^* + \hat{\mu}_t$</td>
<td>1.33</td>
<td>1.86</td>
</tr>
<tr>
<td>(3) Endpoint $\pi_t^* + r_t^*$</td>
<td>1.30</td>
<td>1.62</td>
</tr>
<tr>
<td>(3) vs. (1) (0.58)</td>
<td>(0.26)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>(3) vs. (2) (0.40)</td>
<td>(0.18)</td>
<td>(0.10)</td>
</tr>
<tr>
<td><strong>5-to-10-year forward rate</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1) Random walk</td>
<td>1.23</td>
<td>1.66</td>
</tr>
<tr>
<td>(2) Endpoint $\pi_t^* + \hat{\mu}_t$</td>
<td>1.24</td>
<td>1.64</td>
</tr>
<tr>
<td>(3) Endpoint $\pi_t^* + r_t^*$</td>
<td>1.34</td>
<td>1.69</td>
</tr>
<tr>
<td>(3) vs. (1) (0.71)</td>
<td>(0.57)</td>
<td>(0.10)</td>
</tr>
<tr>
<td>(3) vs. (2) (0.82)</td>
<td>(0.59)</td>
<td>(0.29)</td>
</tr>
</tbody>
</table>

Accuracy of different forecasts for long-term interest rates over horizons from 4 to 40 quarters, measured by the root-mean-squared error (RMSE) and the mean-absolute error (MAE) in percentage points. Method (1) is a driftless random walk. Methods (2) and (3) predict a smooth path from the current interest rate to the endpoint, using a recursively estimated mean-reversion parameter (the first-order autocorrelation coefficient of the detrended interest rate). For method (2) the endpoint is the sum of $\pi_t^*$ and a recursively estimated mean $\hat{\mu}_t$. For method (3) the endpoint is $\pi_t^* + r_t^*$. The data are quarterly from 1971:Q4 to 2015:Q4. The first forecast is made at $t = 20$ (1976:Q3). The last two rows in each panel report one-sided $p$-values for testing the null hypothesis of equal forecast accuracy against the alternative that method (3) is more accurate, using the method of Diebold and Mariano (1995).
Table 4: Predicting excess returns

<table>
<thead>
<tr>
<th>Holding period:</th>
<th>One quarter</th>
<th>Four quarters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td><strong>Full sample: 1971:Q4–2015:Q4</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PC1</td>
<td>0.02</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>PC2</td>
<td>0.61</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>(0.25)</td>
<td>(0.24)</td>
</tr>
<tr>
<td>PC3</td>
<td>-1.84</td>
<td>-1.21</td>
</tr>
<tr>
<td></td>
<td>(1.05)</td>
<td>(1.01)</td>
</tr>
<tr>
<td>$\pi_t^*$</td>
<td>-2.01</td>
<td>-2.96</td>
</tr>
<tr>
<td></td>
<td>(0.45)</td>
<td>(0.58)</td>
</tr>
<tr>
<td>$r_t^*$</td>
<td>-3.22</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.07)</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.09</td>
<td>0.16</td>
</tr>
</tbody>
</table>

| **Subsample: 1985:Q1–2015:Q4** |
| PC1             | 0.08       | 0.18          | 0.59 | 0.15  | 0.55  | 1.75 |
|                 | (0.04)     | (0.06)        | (0.13) | (0.11) | (0.16) | (0.27) |
| PC2             | 0.50       | 0.67          | 0.69 | 1.71  | 2.44  | 2.50 |
|                 | (0.20)     | (0.21)        | (0.21) | (0.55) | (0.49) | (0.40) |
| PC3             | -0.77      | -0.41         | 0.63 | -2.05 | -0.06 | 2.90 |
|                 | (0.97)     | (0.96)        | (1.07) | (1.89) | (2.09) | (1.81) |
| $\pi_t^*$       | -1.16      | -2.50         | -4.59 | -8.49 |
|                 | (0.73)     | (0.77)        | (1.48) | (1.25) |
| $r_t^*$         | -3.92      |               | -11.62 |
|                 | (1.20)     |               | (2.52) |
| $R^2$           | 0.08       | 0.10          | 0.17 | 0.22  | 0.31  | 0.51 |

Predictive regressions for quarterly and annual excess bond returns, averaged across two- to 15-year maturities. The predictors are three principal components (PCs) of the yield curve and measures of long-run inflation expectations and the equilibrium real rate, which are described in the text. Numbers in parentheses are White standard errors for quarterly (non-overlapping) returns, and Newey-West standard errors with 6 lags for annual (overlapping) returns. Numbers in squared brackets are small-sample p-values obtained with the bootstrap method of Bauer and Hamilton (2016). The data are quarterly from 1971:Q4 to 2015:Q4.
Table 5: Variance ratios

<table>
<thead>
<tr>
<th></th>
<th>Variances and covariances</th>
<th>Variance ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta_h y_t$</td>
<td>$\Delta_h i_t^*$</td>
</tr>
<tr>
<td>$h$</td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>1</td>
<td>0.49</td>
<td>0.04</td>
</tr>
<tr>
<td>5</td>
<td>2.33</td>
<td>0.32</td>
</tr>
<tr>
<td>10</td>
<td>3.89</td>
<td>0.75</td>
</tr>
<tr>
<td>20</td>
<td>7.04</td>
<td>1.61</td>
</tr>
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<td>30</td>
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Five-year yield

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Ten-year yield

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5-to-10-year forward rate

Variance ratios for changes in long-term interest rates and the trend component $i_t^* = \pi_t^* + r_t^*$. The first three columns report sample variances for $h$-quarter changes in the interest rate $y_t$, the trend component $i_t^*$, and the cycle component $y_t - i_t^*$. The fourth column reports twice the covariance between changes in the trend component and the cycle component. The last two columns report two different ratios: The first is the ratio of the variance of changes in the trend component relative to the variance of interest rate changes. The second includes the twice the covariance between changes in the trend and cycle components in the numerator, and equals one minus the variance ratio for the cycle component.
Ten-year Treasury yield and estimates of trend inflation, $\pi^*$ (the mostly survey-based PTR measure from FRB/US), and of the equilibrium real rate, $r^*$ (the average of the estimates from Laubach and Williams (2003), Lubik and Matthes (2015), Kiley (2015) and Johannsen and Mertens (2016)).
Four measures of $r^*$ from Laubach and Williams (2003), Lubik and Matthes (2015), Kiley (2015) and Johannsen and Mertens (2016), as well as the average of these measures.
Variance of $h$-quarter changes in $\pi^*_t$, $r^*_t$, and $i^*_t = \pi^*_t + r^*_t$ relative to variance of $h$-quarter changes in long-term interest rate. The dashed lines show 95%-confidence intervals for the $i^*_t$-variance ratio, constructed as described in the text.