Financial Innovation and Asset Prices*

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Abstract

Our objective is to understand the effects of financial innovation on asset prices. To account for the feedback effect from prices on the asset-allocation decisions of investors in an internally-consistent fashion, we develop a dynamic general-equilibrium framework in which asset-allocation decisions and the resulting asset returns are determined endogenously. Our model has three assets—a risk-free bond, a traditional risky asset, and a new "alternative" asset—and two types of investors—experienced and inexperienced. The new asset is illiquid and inexperienced investors are uncertain about its expected dividend growth, but rationally learn about it. We use this model to study how the the flow of capital into the new asset affects the prices of traditional assets, the returns of the new asset, and the comovement with the returns of traditional assets, how shocks to the cash flows of the new asset get transmitted to traditional assets, and how the *dynamics* of the return moments of the new and traditional assets and the asset-allocation decisions of the experienced and inexperienced investors change as investors learn about the cash flows of the new asset. The model yields several interesting predictions that are supported empirically.

Keywords: parameter uncertainty, heterogeneous beliefs, illiquidity, spillover effects, recursive utility, excess correlation.
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1 Introduction

Financial markets have been transformed by financial innovation, which has resulted in new asset classes being introduced at an ever-increasing pace.[1] At the same time, more and more investors are moving capital into the new asset classes. This is because, in the current economic environment, returns on traditional asset classes have fallen, and correlations between them have increased, leading to lower portfolio return and higher portfolio volatility. However, while the new asset classes offer superior diversification and potentially higher returns, they are also relatively illiquid[2] and have cashflows that are more uncertain.[3]

Our objective is to understand the effects of financial innovation on asset prices. Because of the feedback effect from prices, the asset-allocation decisions of investors themselves depend on the dynamics of equilibrium asset prices. Therefore, in order to have an internally-consistent framework, one needs a general-equilibrium framework in which the asset-allocation decisions of investors and the resulting moments of asset returns are all determined endogenously.

We develop a general equilibrium model that explicitly accounts for the diversification benefits that a new asset class provides, while also incorporating uncertainty regarding its cash-flow dynamics and transaction costs. This allows us to identify the economic mechanisms through which the movement of capital into a new asset class influences its own return moments, and through spillover effects, the return moments of traditional assets. We show that the model yields many interesting empirical predictions regarding

[1] These new asset classes include hedge funds, venture capital and private equity, emerging-market equity and debt, mezzanine and distressed debt, real estate, infrastructure, natural resources, commodities, and precious metals.

[2] The substantial costs for trading alternative asset classes is well documented in the literature. Collett, Lizieri, and Ward (2003) estimate transaction costs of over 3% for institutional real estate, Beber and Pagano (2013) find that bid-ask spreads for small-cap firms can be as high as 10%, and costs of private-equity transactions can exceed even 10% (Preqin Special Report (2013)).

[3] The difficulties encountered in assessing alternative assets are discussed by: Phalippou (2009), Phalippou and Gottschalg (2009), and Ang and Sorensen (2013) for private equity; by Dhar and Goetzmann (2005) for real estate; and by Ang, Ayala, and Goetzmann (2014) for hedge funds and private equity.

the portfolio decisions of investors and the return dynamics of asset classes that are supported by the data. We now discuss in greater detail the model, its predictions, and the empirical support for these predictions.

Specifically, we develop a model of a dynamic general-equilibrium economy with three asset classes and two groups of investors. The three asset classes are: a risk-free bond; a risky, fully-liquid traditional asset; and a new asset, which is risky and not fully liquid because trading this asset entails a transaction cost. The two groups of investors have identical recursive preferences but one group—"experienced investors"— is assumed to know the cash-flow dynamics of the new asset class, while the other group—"inexperienced investors"—is uncertain about the expected growth rate of the asset's cash flows, and learns about it only from realizations of the cash-flow shocks.[4] In summary, the key features of our model are: (1) multiple risky assets; (2) parameter uncertainty, combined with fully rational Bayesian learning and recursive preferences; (3) heterogeneity in beliefs about the cash flows of the new asset; and (4) illiquidity of the new asset class—each of the last three characteristics being a realistic feature of alternative assets.[5]

We use this model to address the following questions. How do the prices of traditional assets react to the flow of capital into the new asset? How do shocks to the cash flows of the new asset get transmitted to traditional assets? How does the movement of capital into the new asset class affect its own returns as well as the comovement with the returns on traditional assets? How do investors allocate wealth across the risk-free asset, traditional risky assets (for example, publicly-traded equity), and the new asset class, which offers diversification but has higher uncertainty regarding its cash-flow dynamics and transaction costs? How are the *dynamics* of the return moments of the new and traditional assets and the asset-allocation decisions of the experienced and inexperienced investors affected as investors learn about the cash flows of the new asset?

[4]For evidence on how lack of familiarity with alternative assets affects the investment behavior of inexperienced investors, see Blackstone (2016, p. 11).

[5]For example, new assets, by definition, do not have long return histories; therefore, the returns from investing in these assets are less transparent than the returns from traditional assets such as public equity. New assets are also costly to trade, as described in footnote 2.

To isolate the effects of parameter learning and illiquidity, we undertake our analysis in two steps. In the first step, we study the economy in the presence of parameter uncertainty about the expected growth rate of dividends for the new asset class but assume that the new asset is fully liquid. Only in the second step, we introduce illiquidity of the alternative asset class. The results and economic intuition for the model with just parameter uncertainty and learning can be summarized as follows. First, inexperienced investors over-weight the risk-free bond, driven by their desire for *precautionary savings* and tilt their portfolio away from the new asset class because of a *negative intertemporal hedging demand*. In particular, a negative hedging position in the new asset allows investors to protect against downward revisions in the perceived growth rate of the new asset class because the returns of the new asset class and its perceived growth rate are positively correlated. Only over time, as the posterior beliefs of the inexperienced investors becomes more precise with *learning*, their negative hedging demand diminishes and they gradually allocate more capital to the new asset. In contrast, experienced investors over-weight the new asset class in their portfolio, which is a consequence of market clearing. These predictions are supported by empirical data on the portfolio composition of investors.[6]

Second, these dynamics of investors' portfolios imply that the risk-adjusted portfolio return of inexperienced investors is initially much lower than that of experienced investors, which is consistent with the findings of empirical research.[7]

Third, the inexperienced investors' rational learning *amplifies* the effects arising from the fluctuations in the underlying dividend process of the new asset class, thus leading to considerably higher return volatility. In particular, positive cash-flow news leads to

[6]For example, while "experienced investors" such as Yale, Princeton, and Harvard invest about 30% of their endowments in private equity and 20% in hedge funds (Goetzmann and Oster (2012)), the average university endowment invests less than 2% in private equity and hedge funds (Brown, Garlappi, and Tiu (2010)). Evidence on the gradual increase—over several *decades*—in the allocation to new asset classes is provided by Lerner, Schoar, and Wang (2008) and Cejnek, Franz, Randl, and Stoughton (2014) for private endowments and for pension funds by Towers Watson (2011).

[7]Lerner, Schoar, and Wang (2008) report that funds that entered earlier into new assets achieved higher returns. Andonov (2014) and Dyck and Pomorski (2014) find that investors with substantial private equity investments perform significantly better than investors with small investments: a one standard deviation increase in private-equity holdings is associated with 4% greater returns per year.

a higher price-dividend ratio if investors have a preference for early resolution of uncertainty, because it implies an upward revision in the perceived dividend growth rate, and vice versa for negative cash-flow news. This increase in return volatility, together with a higher volatility of the stochastic discount factor (induced by uncertainty about the perceived future growth rate), leads to an additional risk premium for the new asset class in its early years. Both quantities—return volatility and the risk premium—decline over time as the inexperienced investors' estimate of the dividend growth becomes more precise. These predictions about the asset-return dynamics are consistent with the findings of several empirical studies.[8]

In the second step of our analysis, we incorporate the other major characteristic of new asset classes: substantial transaction costs. In contrast to the setting with rational learning but *without* transaction costs, in the presence of transaction costs inexperienced investors rebalance their holdings of the new asset class only infrequently, consistent with the data.[9] Moreover, in the presence of transaction costs, the model predicts a sizable *liquidity premium* for the new asset class in the early years, consistent with the results of empirical studies of returns for alternative assets.[10]

Finally, when the transaction cost makes it optimal not to trade the new asset class, inexperienced investors use the liquid traditional asset as a substitute, which has intriguing consequences. First, the inexperienced investors' demand for the traditional asset strengthens after positive cash-flow news about the new asset class, thus creating *excess correlation* between the returns of the two asset classes. Second, the spillover of shocks from the new asset class to the traditional asset leads to an increase in the traditional

[8]A downward trend in hedge-fund returns over 1985–2013 is reported in Ineichen Research and Management (2013, their Figure 4); a decrease in private-equity returns is reported in Harris, Jenkinson, and Kaplan (2014). A substantial decline in volatility of hedge-fund returns is reported in Adrian (2007) and of emerging-market returns in Dimson, Marsh, and Staunton (2014, their Table 4).

[9]Ang, Papanikolaou, and Westerfield (2014, their Table 1) report that trading in many asset classes is infrequent; for instance, typical holding periods for venture capital and private equity portfolios are 3 to 10 years.

[10]Franzoni, Nowak, and Phalippou (2012) and Harris, Jenkinson, and Kaplan (2014) find substantial liquidity premia for private equity.

asset's risk premium. Thus, our model provides new insights on how shocks in illiquid markets are transmitted to liquid markets.[11]

We now discuss how our work is related to the existing literature. The paper that is closest to our work is Collin-Dufresne, Johannes, and Lochstoer (2016a), who study parameter uncertainty and rational learning in general equilibrium with a representative investor and recursive preferences.[12] The comprehensive analysis in that paper shows, among other things, that learning about the expected growth rate of aggregate dividends strongly amplifies the impact of shocks when the investor has a preference for early resolution of uncertainty. While one key feature of our model—parameter uncertainty with Bayesian learning and recursive preferences—is shared with their model, the other three features are distinct. That is, in contrast to their model, we consider an economy with *multiple risky assets*—a traditional asset and a new asset class—with parameter uncertainty only about the new asset's expected dividend growth. Moreover, in our model there are *two groups of investors with heterogeneous beliefs*: experienced investors, who are assumed to know the new asset's true dividend growth rate; and inexperienced investors, who do not know this and learn about it over time. Finally, in our model there is a *transaction cost* for trading the new asset class.

These modeling differences lead to several new insights. First, heterogeneity in beliefs allows us to demonstrate the substantial differences in the asset-allocation strategies of experienced and inexperienced investors. Second, even though learning in our model is only for a fraction of aggregate consumption, represented by the new asset class, we find that it still has substantial effects on asset prices, which strengthens the findings in Collin-Dufresne, Johannes, and Lochstoer (2016a). Third, even though parameter learning is present only for one asset, it creates spillover effects on *all* the assets in the economy, which again highlights the importance of this mechanism. Fourth, illiquidity

[11] For another model with contagion—an increase in return volatility and correlation—see Kyle and Xiong (2001).

[12] Earlier papers on parameter learning in single-agent settings with time-separable utility include: Detemple (1986), David (1997), Veronesi (2000), David and Veronesi (2002, 2013), and Pástor and Veronesi (2003, 2012). For a comprehensive review of this literature, see Pástor and Veronesi (2009).

strengthens the impact of parameter learning on the other assets because inexperienced investors use the liquid traditional asset as a substitute for trading the illiquid new asset.

In contrast to Collin-Dufresne, Johannes, and Lochstoer (2016a), in which investors use Bayesian (rational) learning, Collin-Dufresne, Johannes, and Lochstoer (2016b), and Ehling, Graniero, and Heyerdahl-Larsen (2017) study parameter uncertainty with *biased learning* in overlapping-generation models, in which the newly born generation has less precise priors than the dying older generation. While Collin-Dufresne, Johannes, and Lochstoer (2016b) document a high risk premium and persistent mis-valuation, Ehling, Graniero, and Heyerdahl-Larsen (2017) focus on the difference in the portfolios of the young and old generations and report substantial welfare costs because of experience-biased learning. Both papers study the stationary dynamics of an economy with a single risky asset, differing from our analysis of the transitional dynamics of a new asset class. Moreover, in our model learning is fully rational and trading is costly.

Our paper is related also to the literature on *heterogeneous beliefs*; see, for instance, Harrison and Kreps (1978), Detemple and Murthy (1994), Zapatero (1998), Scheinkman and Xiong (2003), Dumas, Kurshev, and Uppal (2009), and Xiong and Yan (2010). In contrast to these papers, in which investors have risk-neutral or time-additive preferences, we incorporate recursive preferences, in the presence of which parameter learning leads to an additional risk premium and excess volatility, and focus on the changes in the dynamics of the assets' returns over time, the spillover of shocks across asset markets, and the interaction between heterogeneous beliefs and illiquidity. Related to this last topic are papers that study the interaction between heterogeneous beliefs and portfolio constraints, which include Scheinkman and Xiong (2003), Hong, Scheinkman, and Xiong (2006), and Chabakauri (2015).

Closely related are also the papers with full information, but *multiple trees* by Cochrane, Longstaff, and Santa Clara (2008), who study asset prices in a representative investor setting with log-utility, and Chabakauri (2013), who studies asset prices and portfolio choice with heterogeneous investors and portfolio constraints. In contrast, our model features parameter uncertainty, recursive preferences, and illiquidity, which allows us to study the impact of parameter learning, the transitional dynamics of the new asset, and the interplay between illiquidity and rational learning.

Finally, our work is related to papers that study the impact of *illiquidity* on asset allocation and equilibrium asset prices.[13] Particularly, Vayanos (1998) and Acharya and Pedersen (2005) consider illiquidity in the form of transaction costs in models in which the investors' trading decisions are for life-cycle reasons. Longstaff (2009) shows that an exogenously specified blackout period, in which an asset cannot be traded at all, has a substantial impact on portfolio choice and asset prices. Buss and Dumas (2016) study an equilibrium model with trading fees, highlighting the implications for asset prices of the slow movement of capital induced by trading frictions. In contrast to these papers, rational learning constitutes the trading motive in our model, leading to changes in dynamics of asset returns and portfolios. Moreover, the joint modeling of illiquidity and parameter uncertainty allows us to study the spillover effects from the new asset to the traditional asset.

The rest of the paper is organized as follows. In Section 2, we describe the general model of the economy we study. In Section 3, we characterize the equilibrium in this economy and describe our solution approach. In Section 4, we analyze the effect of parameter uncertainty and rational learning on portfolio positions, consumption policies, and return moments. Finally, in Section 5 we discuss the impact of illiquidity on these quantities. Section 6 summarizes the insights from the various exercises we undertake to test the robustness of our results. Section 7 concludes. Technical results are relegated to an online appendix.

[13]For excellent surveys of the literature on portfolio choice and asset pricing with illiquidity, see the review papers by Amihud, Mendelson, and Pedersen (2005) and Vayanos and Wang (2011, 2012, 2013).

2 The Model

In this section, we describe the general-equilibrium model we use for our analysis. The model is set in discrete time at the frequency Δ_t and has a finite number of periods: $t \in \{0, ..., T\}$. The economy is populated by two groups of investors who have homogenous preferences and can trade three assets: a risk-free bond, the broad "market" portfolio consisting of publicly traded equities, and the new asset class. The new asset has two distinct characteristics in our model: (1) it is *less well understood* by a subset of investors, that is, its expected dividend growth rate is uncertain, with investors rationally learning about it, and (2) it is *illiquid*.

2.1 Uncertainty

Uncertainty in the economy is generated by *two* Lucas (1978) trees, indexed by $n \in \{1, 2\}$, each generating a dividend stream $D_{n,t}$. Specifically, we assume that, for each tree, log dividend growth $\Delta d_{n,t+1} \equiv \ln\left[D_{n,t+1}/D_{n,t}\right]$ can be described by an *i.i.d.* normal growth model with dividend-growth rate μ_n and dividend-growth volatility σ_n,

$$\Delta d_{n,t+1} = \mu_n + \sigma_n \, \varepsilon_{n,t+1}, \tag{1}$$

where $\varepsilon_{n,t+1} \sim \mathcal{N}(0, 1)$, and $\varepsilon_{1,t+1}$ and $\varepsilon_{2,t+1}$ are assumed to be uncorrelated.[14]

2.2 Financial Assets

Investors can trade three financial assets. The first asset is a risk-free, one-period discount bond in zero net supply, indexed by $n = 0$. In addition, there exist two risky assets, indexed by $n \in \{1, 2\}$, each modeled as a claim to the dividends of the corresponding tree. We interpret the first risky asset as a broad "market" portfolio, while the second risky asset represents the new asset class.

[14]Even though the dividends are uncorrelated, the asset returns will be correlated in equilibrium.

2.3 Preferences

The two groups of investors, indexed by $k \in \{1,2\}$, are assumed to have identical Epstein and Zin (1989) and Weil (1990) preferences over consumption $C_{k,t}$ of the single consumption good.[15] Specifically, lifetime utility $V_{k,t}$ is defined recursively as

$$V_{k,t} = \left[(1-\beta)\, C_{k,t}^{1-\frac{1}{\psi}} + \beta E_t^k \left[V_{k,t+1}^{1-\gamma} \right]^{\frac{1}{\phi}} \right]^{\frac{\phi}{1-\gamma}}, \tag{2}$$

where E_t^k denotes the time-t conditional expectation under an investor's subjective probability measure, β is the rate of time preference, $\gamma > 0$ is the coefficient of relative risk aversion, $\psi > 0$ is the elasticity of intertemporal substitution (EIS), and $\phi = \frac{1-\gamma}{1-1/\psi}$.

In this setting, the stochastic discount factor (SDF) of investors in group k, $M_{k,t+1}$, is given by

$$M_{k,t+1} = \beta\, \delta_t \, \exp\big((-1/\psi)\, \Delta c_{k,t+1} - (\gamma - 1/\psi)\, v_{k,t+1} \big), \tag{3}$$

where $\delta_t = E_t^k \left[\exp\big((1-\gamma)\, v_{k,t+1} \big) \right]^{(\gamma-1/\psi)/(1-\gamma)}$, $c_{k,t+1}$ denotes log consumption, and $v_{k,t+1}$ denotes the log of lifetime continuation utility. Expressing the SDF in this way makes clear that, if relative risk aversion is not equal to the reciprocal of EIS ($\gamma \neq 1/\psi$), then variations in the future (log) continuation utility, $v_{k,t+1}$, are a source of priced risk—in addition to variations in log consumption growth, $\Delta c_{k,t+1}$.

2.4 Beliefs

One of the key characteristics of the new asset class is that its dynamics are not well understood by a subset of investors.[16] We model this lack of knowledge through parameter uncertainty combined with rational learning.

Particularly, we assume that the second group of investors ($k = 2$), labeled "inexperienced investors," faces parameter uncertainty and has to learn about the dynamics of

[15] It would be straightforward to allow for heterogeneity in investors' preferences. However, our objective is to focus on the dynamic effects of the new asset class, rather than preference heterogeneity.

[16] See footnote 3.

the new asset class. In contrast, the first group of investors ($k = 1$), labeled "experienced investors," is assumed to have perfect knowledge of the asset's dividend dynamics. Moreover, we assume that both groups fully understand the dynamics of the dividends of the market portfolio.[17]

Specifically, we assume that the inexperienced investors are uncertain about the expected dividend growth rate of the new asset class, μ_2, but know its dividend-growth volatility, σ_2. Starting from an conjugate prior $\mu_2 \sim \mathcal{N}(\mu_{2,0}, A_0\,\sigma_2^2)$, inexperienced investors rely on realized dividend growth rates to update their beliefs about the expected growth rate using Bayes' rule. Note that $A_0\sigma_2^2$ represents the reciprocal of the prior precision, so that the prior density converges to a uniform distribution as A_0 approaches infinity and converges to a single value as A_0 approaches zero.

This prior, combined with the dividend dynamics in equation (1), implies a time-t posterior density function $p(\mu_2|d_2^t) = \mathcal{N}(\mu_{2,t}, A_t\,\sigma_2^2)$, where d_2^t denotes the history of all observed dividend growth realizations up to time t: $d_2^t = \{\Delta d_{2,s}\}, s \in \{1, \ldots, t\}$, $t \geq 1$. The dynamics of the parameters $\mu_{2,t}$ and A_t are described by

$$\mu_{2,t+1} = \mu_{2,t} + \left(\Delta d_{2,t+1} - \mu_{2,t}\right) \frac{A_t}{1 + A_t}, \tag{4}$$

$$A_{t+1} = \frac{1}{1/A_t + 1}. \tag{5}$$

Accordingly, even though the dividend dynamics of the new asset are described by an *i.i.d.* model with constant parameters, from the inexperienced investors' perspective the expected dividend growth rate is time-varying. Specifically, as equation (4) shows, dividend-growth shocks lead to permanent shifts in the perceived mean dividend growth because $\mu_{2,t+1}$ is a martingale. Note, however, that the posterior variance $A_t\,\sigma_2^2$ converges deterministically to zero, so that learning converges in the long run—though not neces-

[17] One might argue that, even though the market portfolio is much better understood than a new asset, there remains some parameter uncertainty as well; similarly, one could argue that even the second group of investors may not know perfectly the dynamics of the new asset. For simplicity, we abstract from these generalizations. The model can be extended in a straightforward way to incorporate them.

sarily to the true value μ_2. This can also be seen from equation (4), in which the impact of the realized dividend growth $\Delta d_{2,t+1}$ declines as A_t declines.

The stochastic nature of $\mu_{2,t}$ implies that the difference in the beliefs of experienced and inexperienced investors is fluctuating over time, with inexperienced investors sometimes being more optimistic and sometimes being more pessimistic than the experienced investors. However, these fluctuations also diminish over time, because the updates in the beliefs of inexperienced investors become smaller as their precision increases.

2.5 Illiquidity

A second key characteristic that is associated with the new asset class is illiquidity. To model illiquidity, we assume that trading in the second asset, $n = 2$, entails proportional transaction costs.[18] In contrast, the discount bond and the first risky asset, the market portfolio, are assumed to be perfectly liquid; that is, these two assets can be traded without incurring costs, which proxies for their near-zero transaction costs in reality. Specifically, we assume that the transaction cost $\tau(\cdot)$ that each investor has to pay for trading the new asset class is given by a constant fraction κ of the (dollar) value of the trade.[19] If κ equals zero, then the asset can be traded at no cost and is perfectly liquid.

3 Investor Optimization and Equilibrium

We now first describe the optimization problems of the investors. We then impose market clearing and explain how one can solve for the equilibrium.

[18] See footnote 2.

[19] In our general-equilibrium setting, rather than assuming that the transaction cost is a deadweight loss to society, we assume that the transaction cost is added back to the consumption of investors *after* they have made their consumption and portfolio decisions, thereby eliminating any wealth effects arising from transaction costs. The transaction cost paid by the group buying shares is equal to that paid by the group selling shares, so the total transaction cost is redistributed equally between the two groups of investors.

3.1 Investors' Optimization Problem

The objective of investors in group k is to maximize their expected lifetime utility given in equation (2), in which the expectation is computed with respect to the investors' beliefs, by choosing consumption, $C_{k,t}$, and holdings in the three financial assets, $\theta_{n,k,t}$. This optimization is subject to the budget equation

$$\begin{aligned} C_{k,t} + \theta_{k,0,t}\, S_{0,t} + \sum_{n=1}^{2} \Delta\theta_{k,n,t}\, S_{n,t} + \tau\big(\Delta\theta_{k,2,t}, S_{2,t}\big) \\ \leq \theta_{k,0,t-1} + \sum_{n=1}^{2} \theta_{k,n,t-1}\, D_{n,t} + \xi_{k,t}, \end{aligned} \tag{6}$$

where $\Delta\theta_{k,n,t}$ denotes the change in the shares hold of asset n and $S_{n,t}$ denotes the price of asset n. Transaction costs $\tau(\cdot)$ are given by κ times the (dollar) value of the trade in the second asset

$$\tau\big(\Delta\theta_{k,2,t}, S_{2,t}\big) \equiv \kappa\,\big|\Delta\theta_{k,2,t}\big|\, S_{2,t}.$$

The left-hand side of budget equation (6) describes the amount allocated to consumption, the purchase or sale of the (newly issued) short-term bond, and changes in the portfolio positions in the two risky long-term assets, along with the transaction cost $\tau(\cdot)$ incurred. The right-hand side reflects the available funds, stemming from the unit payoff of the (old) short-term bond as well as the payoffs from the holdings of the two risky assets, and the lump-sum redistribution of the transaction costs, $\xi_{k,t}$.[20]

Buss and Dumas (2016) show that one can transform this optimization problem into a corresponding "dual" problem by augmenting the investors' portfolio positions with non-negative decision variables associated with asset purchases and sales. The resulting set of first-order conditions can then be summarized as follows.[21] First, the budget equation in (6) equates the uses and sources of funds. Second, the kernel condition for each asset equates the price of that asset to the expected payoff from holding it, incorporating

[20] See footnote 19.

[21] An online appendix contains the detailed derivations of the first-order conditions.

potential transaction costs at time t and $t+1$. Finally, we have the complementary slackness conditions associated with the non-negative purchasing and selling decision variables, and the corresponding inequality conditions.

3.2 Equilibrium

Equilibrium in the economy is defined as a set of consumption and asset-allocation policies, along with the resulting price processes for the financial assets such that the consumption policies of all investors maximize their lifetime utility, that these consumption policies are financed by the asset-allocation policies, and that markets for the financial assets and the consumption good clear.

3.3 Solving for the Equilibrium

We solve for the equilibrium numerically, extending the algorithm proposed by Buss and Dumas (2016) to the case of parameter uncertainty with rational learning. In economies with incomplete financial markets, one must solve simultaneously for the consumption and investment policies of the two groups of investors along with asset prices, leading to an equation system that requires backward and forward iteration in time, instead of a purely recursive system. Dumas and Lyasoff (2012) show how the backward-forward system of equations can be reformulated to obtain a purely recursive system, using a "time shift" for a subset of equations. Buss and Dumas (2016) extend this idea, using a dual formulation, to the case of transaction costs. Below, we provide a short overview of our approach, while a detailed description is relegated to the online appendix.

Our algorithm solves for the equilibrium recursively, starting at date $t = T - 1$. At each date t, we solve the "shifted" equation system, consisting of the date-t kernel and market-clearing conditions, as well as the date $t+1$ budget constraints, portfolio flow equations, complementary slackness, and inequality conditions over a grid of all state variables. In the next step, when solving the equation system for date $t-1$, we interpolate over the grid the date-t variables required by the equation system, that is, the optimal date-t portfolio positions and corresponding security prices.

4 Effects of Parameter Uncertainty and Rational Learning

In this section, we illustrate the impact of parameter uncertainty and rational learning on the dynamics of the asset allocation decisions of the experienced and inexperienced investors, as well as on the dynamics of asset prices and returns. For ease of exposition, in this section we focus on the case without transaction costs, postponing the discussion of illiquidity to the next section.

The results reported below are based on $50,000$ simulations of our economy. We solve the model at a quarterly frequency, $\Delta_t = 1/4$ years, for $T = 600$ periods – in total 150 years. The long horizon is chosen to minimize any effects resulting from the finite horizon. We drop the initial years in which the posterior belief of the inexperienced investors is very dispersed and, instead, start our analysis at year 11, implying that the inexperienced investors have had access to 40 realized dividend growth observations. We stop reporting the quantities of interest after year 60 because most quantities have leveled off by this time.

In a first step, we study the average quantities across the $50,000$ simulations. Particularly, for moments of returns and consumption growth, we report the average conditional moments. In a second step, we then study the quantities of interest for a particular simulation ("sample path"). Because the changes in the posterior mean cancel out to a large extent, the averages across simulations mostly capture the impact of changes in *posterior variance*.[22] In contrast, studying a particular sample path allows us to illustrate the impact of changes in the *posterior mean*.

[22]Note that although the changes in the posterior mean cancel out when we average across paths, nonlinear responses to the these changes will not cancel out perfectly.

Finally, to highlight the importance of parameter uncertainty with rational learning as well as heterogeneity across the two groups of investors, we report results for two additional cases. The first case is for an economy with known mean ("Known μ_2"), that is, an economy in which there is neither parameter uncertainty nor heterogeneity. The second case is for an economy in which *both* groups of investors are uncertain about the mean ("Parameter Unc.: Inv. 1&2"), that is, with parameter uncertainty but no heterogeneity across the two groups of investors.

4.1 Parameter Values

The parameter values used in our numerical illustrations are summarized in Table 1. We use the same parameter values as in Collin-Dufresne, Johannes, and Lochstoer (2016a), who study a similar economy, but for a model with a single representative agent and one risky asset.

For the preference parameters, we use a (quarterly) rate of time-preference $\beta = 0.994$, a coefficient of relative risk-aversion, γ, equal to 10, and an elasticity of intertemporal substitution $\psi = 2.0$—common choices in the literature.

For both trees, $n \in \{1, 2\}$, we set the expected dividend growth rate, $\mu_n = 0.45\%$, and the corresponding volatility, $\sigma_n = 1.64\%$, with the correlation between the dividends equal to zero. Together with an initial dividend share of the first tree, $\delta_{1,0}$, of 0.80, this implies an expected growth rate of quarterly aggregate consumption of 0.45% and an average aggregate consumption growth volatility of 1.35%—equivalent to the values used by Collin-Dufresne, Johannes, and Lochstoer (2016a).[23] When computing the returns of the claims to the dividend trees we apply a leverage factor of $\lambda = 2.5$ to accommodate the fact that most assets are implicitly levered—a common assumption in the literature.

The symmetric choices for the mean and volatility of the two dividend trees guarantee a stable mean dividend share over time, which is important because a drift in the

[23]The annual time-averaged mean and volatility are 1.8% and 2.2%, respectively, matching U.S. per capita consumption growth from 1929 to 2011.

Table 1: Model Parameters

This table reports the parameter values used for our numerical illustrations. The choice of these parameter values is explained in Section 4.1.

Variable	Description	Base Case
Δ_t	Trading (and observation) interval	1/4 year
T	Total number of trading dates (quarters)	600
β	Rate of time-preference per quarter	0.994
γ	Relative risk-aversion	10
ψ	Elasticity of intertemporal substitution	2.00
$w_{2,0}$	Initial wealth share of the inexperienced investors	2/3
μ_n	Expected dividend growth per quarter	0.45%
σ_n	Dividend growth volatility per quarter	1.64%
ρ	Correlation between dividend growth rates	0
$\delta_{1,0}$	First asset's share of total initial dividends	0.80
λ	Leverage factor	2.5
$\mu_{2,0}$	Initial mean of inexperienced investor's prior distribution	0.45%
A_0	Initial precision of inexperienced investor's prior distribution	1
$\underline{\mu}_2, \bar{\mu}_2$	Truncation boundaries for beliefs of inexperienced investors	$[-0.21\%, 1.11\%]$

mean dividend share would already have strong implications for the dynamics of the two assets' prices and returns, as shown by Cochrane, Longstaff, and Santa Clara (2008). We abstract from these effects here. The market capitalizations implied by a long-term mean dividend share for the new asset class of 20% are realistic for a variety of asset classes.[24]

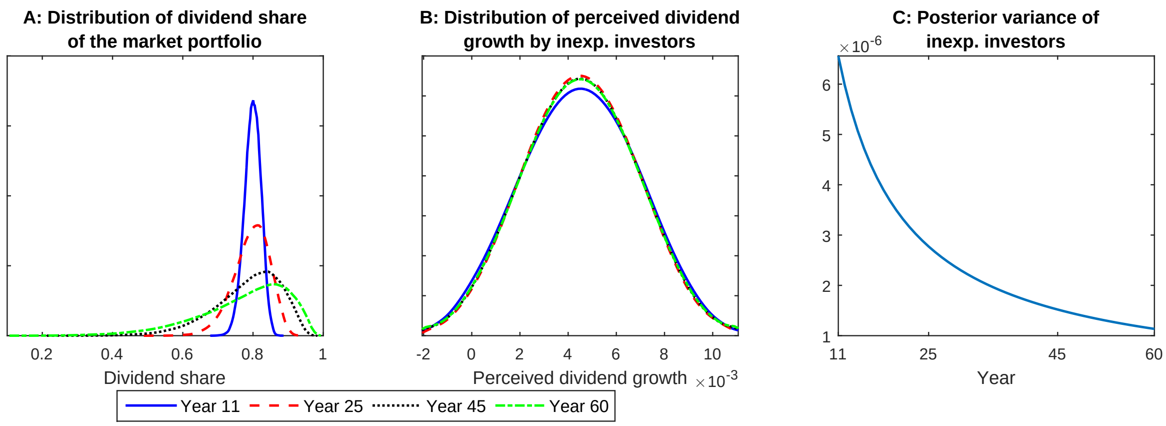
We set the mean of the inexperienced investors' prior distribution, $\mu_{2,0}$, equal to the true expected dividend growth rate $\mu_2 = 0.45\%$, leading to, on average, unbiased beliefs. The parameter governing the initial precision, A_0, is set to one. Again, both choices correspond to the ones in Collin-Dufresne, Johannes, and Lochstoer (2016a). We truncate the inexperienced investors' beliefs at $\underline{\mu}_2 = -0.21\%$ and $\bar{\mu}_2 = 1.11\%$.[25]

[24] For example, assets under management for private equity and hedge funds are about $3.5 and $1.7 trillion, respectively—relative to $18 trillion for U.S. stock-market capitalization.

[25] As discussed in the online appendix of Collin-Dufresne, Johannes, and Lochstoer (2016a), truncation is needed for an elasticity of intertemporal substitution that differs from one, to guarantee the existence of equilibrium. Similar to their findings, our results reported in Section 6 document that the effect of the truncation bounds is negligible, that is, our results are not driven by extreme values of the parameter about which the investors are uncertain.

Figure 1: State Variables

This figure depicts the dynamics of three state variables based on the parameter values described in Section 4.1. Panels A and B show the density functions for the dividend share of the market portfolio, $\delta_{1,t}$, and the dividend growth rate of the new asset as perceived by the inexperienced investors, $\mu_{2,t}$, respectively; in both cases for different years. Panel C shows the (deterministic) variance of the inexperienced investors' posterior, $A_t\sigma_2^2$, over time.



Finally, we assume that at time 0 the group of inexperienced investors is endowed with 2/3 of the total wealth, as it seems reasonable that initially a majority of investors is not well informed about the dynamics of a new asset class. We assume that the wealth of the inexperienced investors is fully concentrated in the market portfolio, that is, they do not carry any debt and are not endowed with any holdings in the new asset class.

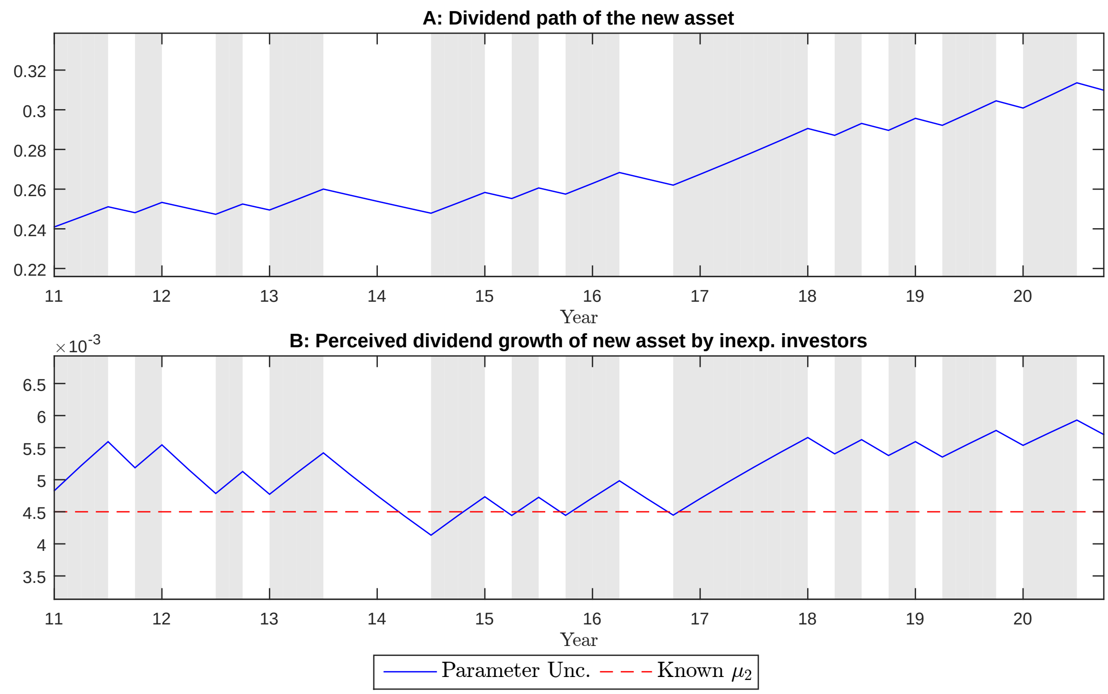
4.2 Dynamics of the Dividend Share and the Inexperienced Investors' Perceived Dividend Growth Rate

As a starting point, Figure 1 depicts the dynamics of the distributions of three state variables. Panel A shows how the density function of the first tree's dividend share, $\delta_{1,t}$, changes over time. As expected, the distribution becomes more dispersed over time, because changes in the individual dividends allow the dividend share of the first asset to drift away from its time-0 value of 0.80. As desired, the mean dividend share is pretty stable, being equal to 0.794 after 60 years.[26]

[26]Note, however, that in the long-run one approaches a bimodal distribution with dividend shares of zero and one. This is a standard result for such models; see, for example, Cochrane, Longstaff, and

Figure 2: A Simulated Path of the Economy

This figure shows a particular simulated path of the economy, based on the parameter values described in Section 4.1. Panel A shows the dividend of the new asset class and Panel B the dividend growth rate for the new asset as perceived by the inexperienced investors. The shaded gray areas indicate periods of positive dividend realizations for the new asset, which lead to an upward revision in its perceived growth rate (unless its dividend growth rate is known, in which case there is no change in the perceived dividend growth rate).



Panel B of Figure 1 shows the density of the new asset's expected dividend growth rate as perceived by the inexperienced investors, $\mu_{2,t}$. One can already observe substantial variation in the investors' beliefs at the start of year 11, that is, after 40 observations. The distribution then quickly converges, being very similar after 25, 45, and 60 years, and always nicely centered around the true mean, $\mu_2 = 0.45\%$. The convergence is driven by the (deterministic) decline in posterior variance over time, shown in Panel C, which implies ever smaller rational updates to the perceived growth rate, as can be seen in (4).

Santa Clara (2008). While this might pose some problems for a long-term analysis and implies that there exists no stationary distribution, it is less important for our analysis, because our focus is on the "transitional dynamics" for the initial years. Moreover, even after 100 years, the distribution is still well behaved. Cochrane, Longstaff, and Santa Clara (2008) argue that for average prices and returns only the expected share matters and show that the expected share over a century "appears indistinguishable from that which would be generated by a mean-reverting process." Moreover, in Section 6 we study an economy with a stationary distribution of the dividend share and find similar results.

Figure 2 shows the dynamics of the new asset's dividend and the corresponding perceived dividend growth rate of the inexperienced investors for the sample path. One can nicely see that positive dividend growth shocks (Panel A), highlighted in shaded gray,[27] lead to an upward revision of the posterior mean (Panel B), and negative shocks lead to a downward revision. However, as time proceeds and the precision increases, the revisions become smaller.

4.3 Asset Allocation

We start by studying the asset allocation decisions of the two groups of investors. Figure 3 shows the dynamics of the inexperienced investors' portfolio positions.[28]

Note that when both investors have the same beliefs—either because there is no parameter uncertainty, or if *both* investors face parameter uncertainty and start with the same prior—the asset allocations are trivial, because investors have identical preferences. In that case, investors hold shares in the two risky assets corresponding to their time-0 wealth share and have zero holdings in the bond.

However, in our main economy in which only inexperienced investors face parameter uncertainty, the portfolio positions of the two groups of investors differ substantially. As shown in Panel A of Figure 3, parameter uncertainty leads to a strong precautionary-savings motive, resulting in large, positive holdings in the risk-free bond for the inexperienced investors. As the investors' posterior variance declines over time, so does their precautionary-savings motive and, thus, their holdings in the bond. Note that the decline in bond holdings is more pronounced in the early years, mimicking the pattern observed for the posterior variance (see Panel C of Figure 1).

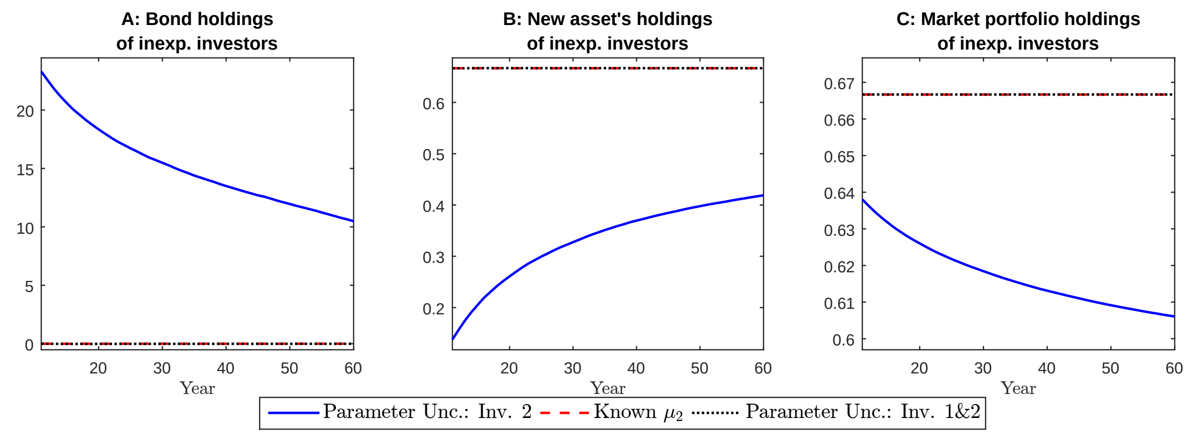
Focusing now on the inexperienced investors' holdings in the new asset class (Panel B of Figure 3), we can see that they tilt their portfolio away from the new asset class,

[27]For ease of exposition, we reproduce the shaded gray areas in all sample-path figures that follow, facilitating their interpretation.

[28]Because of market clearing, the portfolio positions of the experienced investors are given by an asset's aggregate supply minus the holdings of the inexperienced investors.

Figure 3: Asset Allocation

The figure shows the evolution of the inexperienced investors' portfolio positions over time, based on the parameter values described in Section 4.1. Panels A to C show the average number of shares of the bond, the new asset, and the market portfolio held by the inexperienced investors, respectively.



holding barely any shares in the early years. Intuitively, this result is driven by a *negative* intertemporal hedging demand for the new asset class in the presence of parameter uncertainty and rational learning. That is, inexperienced investors wish to form a portfolio that performs well when marginal utility is high or, equivalently, the new asset's perceived growth rate is low. This is achieved by a negative hedging position in the new asset because its return is positively correlated with the perceived growth rate. In contrast, experienced investors do not face parameter uncertainty and therefore, because of market clearing, tilt their portfolio toward the new asset class.

Panel B of Figure 3 also shows that inexperienced investors gradually increase their holdings in the new asset class. Particularly, with the decline in posterior variance over time, the shocks to the perceived dividend growth rate also decline, and so does the magnitude of the negative hedging demand. Again, due to the faster decline in posterior variance in the early years, the holdings in the new asset class initially increase faster.

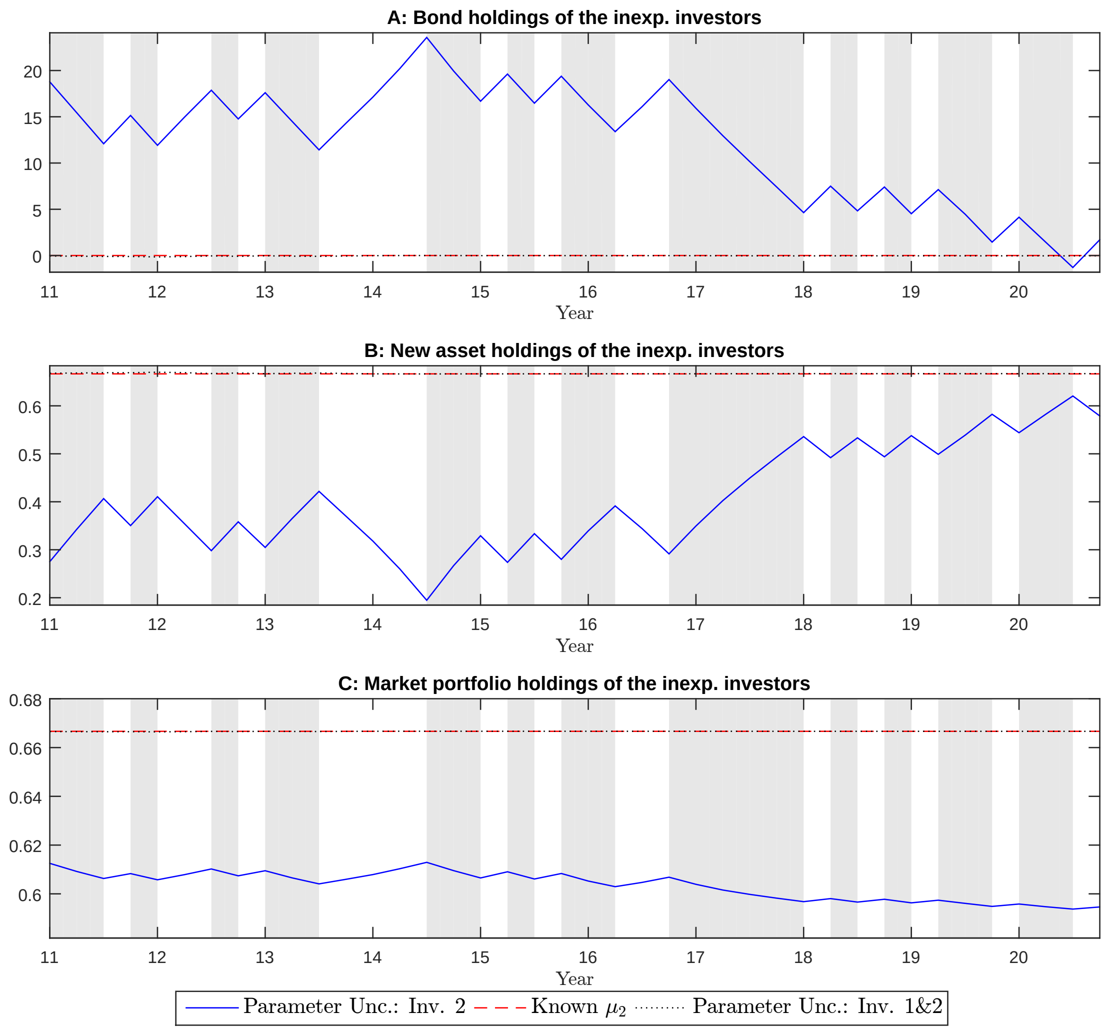
Finally, Panel C of Figure 3 shows the holdings of inexperienced investors in the market portfolio. Inexperienced investors tilt their holdings away from the market portfolio—though to a much smaller extent. This result is driven by the positive correlation between the market portfolio and the new asset class, discussed in greater detail below, which implies a positive, but low, correlation between shocks to the market portfolio and the investors' perceived growth rate. Similar to the new asset, this creates a negative intertemporal hedging demand in the market portfolio; however, this hedging demand is much smaller in magnitude than that for the new asset class due to the lower correlation. Interestingly, the holdings of experienced investors in the market portfolio *decline* over time, which is driven by a decline in their share of wealth, as discussed below.[29]

These shifts in the portfolio compositions have a direct impact on the expected returns and volatilities of investors' portfolios. Particularly, because experienced investors overweight the new asset class, they earn the positive risk premium associated with the new asset, and, thus, earn higher expected portfolio returns. However, at the same time, the experienced investors' portfolio volatility is substantially higher, as their portfolio is levered, whereas the inexperienced investors allocate substantial capital to the risk-free bond. Over time, as the inexperienced investors increases their investment in the new asset class, the difference in portfolio expected returns and volatilities narrows.

These results are consistent with the empirically observed asset holdings of endowment funds; see, for instance, Lerner, Schoar, and Wang (2008) and Goetzmann and Oster (2012), who find that experienced funds strongly overweight alternative assets relative to their peers. These results are also consistent with the empirical findings of Brown and Tiu (2013), who find that, over the period of 1989–2011, as universities gained experience investing in alternative assets, their allocations to the traditional fixed-income assets declined substantially, while their allocation to alternative assets, such as non-U.S. equities and hedge funds, increased. Also, the result that the experienced investors reap the benefits of the higher (risk-adjusted) portfolio returns offered by the new asset class is consistent with the empirical findings documented in Lerner, Schoar, and Wang (2008), Andonov (2014), and Dyck and Pomorski (2014).

[29]These results are consistent with those in the partial-equilibrium models in Brennan (1998), Barberis (2000), and Brandt, Goyal, Santa-Clara, and Stroud (2005), who study portfolio optimization for an individual investor who is uncertain about the mean return of a risky asset, but who learns about it over time.

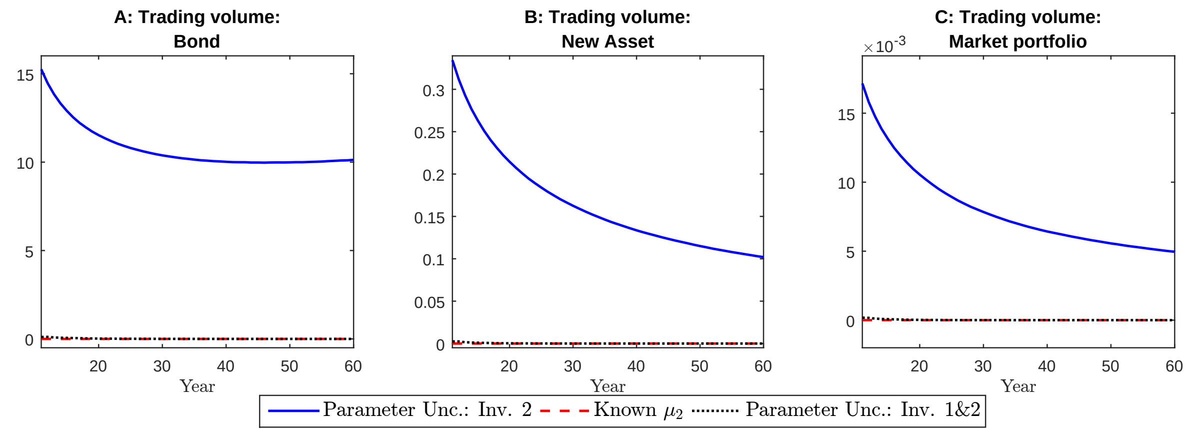
Figure 4: Asset Allocation over a Simulated Path
The figure shows the dynamics of the asset allocation of the inexperienced investors for a particular simulated path of the economy, which is illustrated in Figure 2. Panel A shows the holdings in the bond, Panel B the holdings in the new asset, and Panel C the holdings in the market portfolio. The shaded gray areas indicate periods of positive dividend realizations for the new asset, leading to an upward revision in its perceived growth rate.



While the findings discussed above were mostly driven by changes in precision, Figure 4, which depicts the inexperienced investors' portfolio positions for a particular sample path, allows us to illustrate the impact of changes in the *posterior mean.* Panels A and B show that inexperienced investors react to an upward revision in their posterior mean, highlighted in shaded gray, by reducing their position in the short-term bond and

Figure 5: Turnover

The figure shows the evolution of the assets' turnover over time, based on the parameter values described in Section 4.1. Panels A to C show the total number of shares of the bond, the new asset and the market portfolio traded, on average, within a year, respectively.



moving these funds into the new asset class about which they are now more optimistic. The increase in precision over time implies that the magnitude of these portfolio changes declines over time. Moreover, keep in mind that the increase in precision over time, naturally leads to a decrease (increase) in the holdings in the bond (new asset class). As one might expect, the variations in the market portfolio holdings (Panel C) are much smaller, because its dynamics are known to all investors, and thus, trade in the market portfolio occurs only due to fluctuations in wealth and changes in the hedging demand, which is small in magnitude.

The effects that we have just described for the sample path can be directly linked to the dynamics of the average annual trading volume in the three assets, shown in Figure 5. That is, in the case of heterogenous beliefs trading volumes for the bond and new asset class are substantial, especially in the early years, driven by the updates in the posterior mean of inexperienced investors and the corresponding portfolio changes, as described above.[30] As the posterior variance declines, and with it the magnitude of updates in the posterior mean decrease, trading volumes decline as well. In contrast, the turnover in the

[30] In the model, about 92% of the trading volume can be attributed to changes in the posterior mean, with the remainder being due to changes in the posterior variance.

market portfolio is considerably smaller, because it is driven mostly by slow-frequency changes in posterior variance and wealth share, and is not affected much by changes in the posterior mean. Note, in the case of homogeneous beliefs, i.e., identical investors, there are no changes in portfolio holdings, implying that the trading volume is zero.

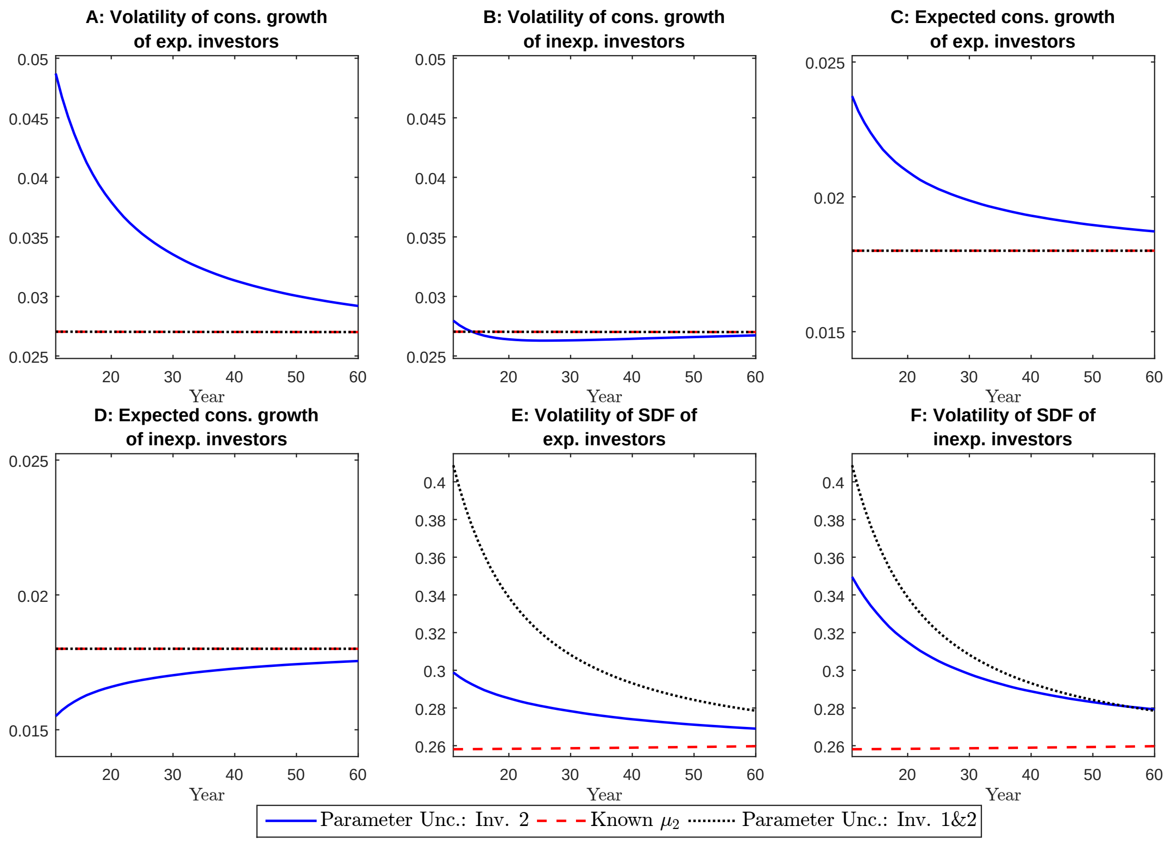
4.4 Consumption Policies and the Stochastic Discount Factor

We now study the optimal consumption choices and the resulting SDFs of the two groups of investors. As before, in the absence of parameter uncertainty, the two investors are identical, and hence both investors consume a constant fraction, equal to their initial wealth share, of aggregate consumption. Therefore, in this case their consumption policies inherit the dynamics of aggregate consumption growth. However, if only one group of investors faces parameter uncertainty, then the consumption-growth patterns differ substantially. In particular, precautionary savings allow the inexperienced investors to better smooth consumption, who therefore achieve a consumption policy that has considerably lower volatility than that of the experienced investors, as shown in Panels A and B of Figure 6. These results are directly related to the observation that, because of leverage, the experienced investors' portfolio volatility is considerably higher than that of the inexperienced investors.

Of course, this risk-sharing arrangement, in which the experienced investors offer "insurance" to inexperienced investors, needs to be paid for. That is, as shown in Panels C and D of Figure 6, the expected consumption growth of the experienced investors is higher than that of the inexperienced investors. Again, we can relate this to the portfolios that finance the consumption plans. Specifically, because the experienced investors tilt their portfolio toward risky assets, which earn a positive risk premium, they earn to a higher expected portfolio return, thus allowing them to increase their consumption faster. As the posterior variance of the beliefs of the inexperienced investors declines over time, they invest more into risky assets, so that the difference in expected growth rates between the two groups of investors declines over time. These differences in expected consumption

Figure 6: Consumption Policies and Stochastic Discount Factors

The figure shows the dynamics of the consumption policies and the stochastic discount factors of the two groups of investors over time, based on the parameter values described in Section 4.1. Panels A and B show the average consumption growth volatility, Panels C and D depict the average expected consumption growth, and Panels E and F show the average volatility of the stochastic discount factor, in each case for the two groups of investors.



growth lead to a gradual, but small, decline in the inexperienced investors' consumption share—from about 63.8% to 60.1% over the 50-year horizon that we study.[31]

Finally, Panels E and F show that the volatilities of the SDFs are considerably higher in the presence of parameter uncertainty, but decline over time. To understand these

[31]The risk-sharing arrangement between the experienced and inexperienced investors is captured well by Brian Tracy's quote: "When a man with money meets up with a man with experience, the man with the experience is going to end up with the money and the man with money is going to end up with the experience." We are grateful to Francis Longstaff for bringing this quote to our attention.

effects, write the shocks to the log SDF, $m_{k,t+1} \equiv \log(M_{k,t+1})$, as:

$$\begin{aligned} m_{k,t+1} - E_t[m_{k,t+1}] = &- (1/\psi) \left(\Delta c_{k,t+1} - E_t[\Delta c_{k,t+1}]\right) \\ &- (\gamma - 1/\psi) \left(v_{k,t+1} - E_t[v_{k,t+1}]\right), \end{aligned} \tag{7}$$

where $v_{k,t+1}$ denotes, as in (3), the log of the continuation utility. In the presence of recursive preferences, the increase in the volatility of the SDF can be attributed to the second component in (7). That is, the continuation utility is highly sensitive to shocks to the perceived dividend growth rate of the new asset, because these shocks are permanent. Even though this sensitivity remains high over time, the volatility of the SDF gradually declines because changes in the perceived dividend growth rate itself diminish. In contrast, in a setting with a constant relative risk averse utility function ($\gamma = 1/\psi$), the SDF is driven exclusively by shocks to consumption growth, so that parameter uncertainty has virtually no effect on the volatility of the SDF.

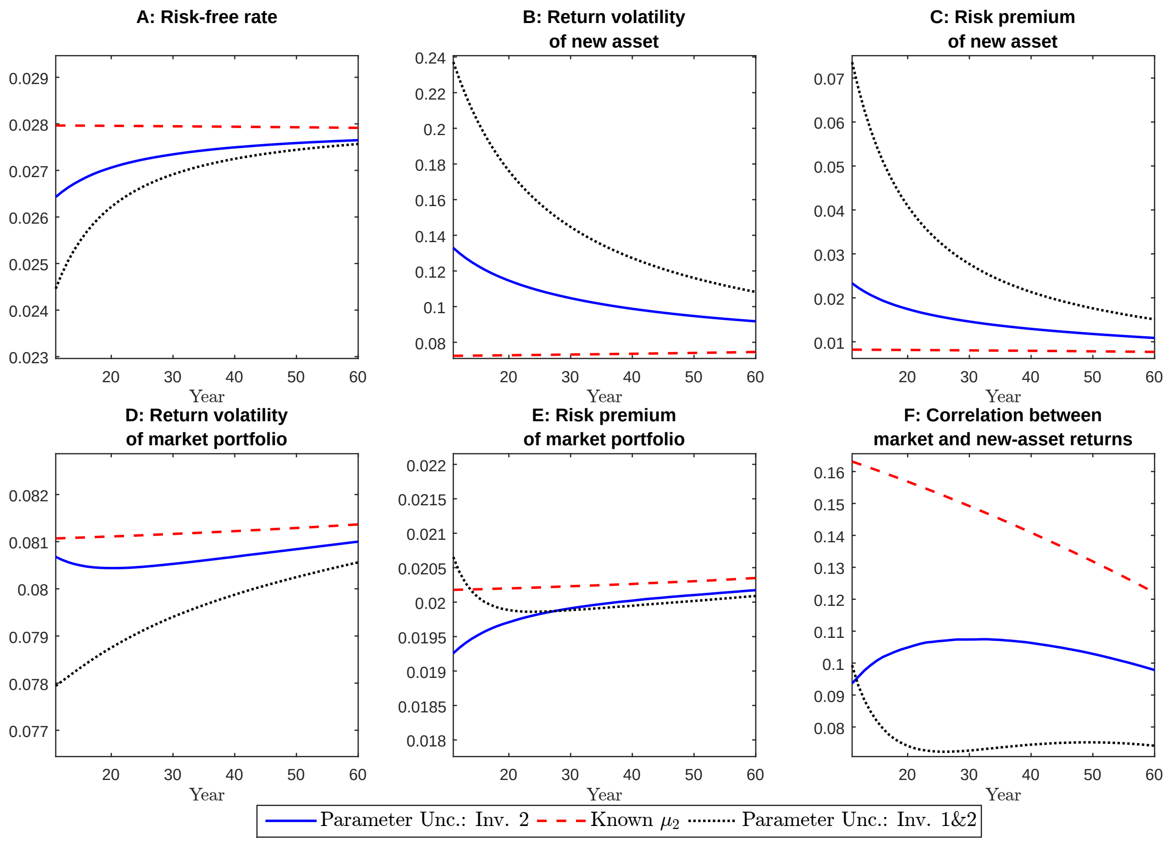
Note, the volatility of the SDF for both investors is lower with heterogeneous beliefs. Specifically, while with homogenous beliefs shocks to perceived dividend growth affect both investors symmetrically, with heterogeneous beliefs these shocks are partially absorbed. This is because, on the one hand, experienced investors provide a risk-sharing opportunity for the inexperienced investors, which reduces the sensitivity of the inexperienced investors' continuation utility to shocks to the perceived growth rate. On the other hand, for the experienced investors, who now know the true growth rate, the SDF is no longer affected directly by learning, but only indirectly because of changes in the beliefs of the inexperienced investors.

4.5 Moments of Asset Returns

The implications of parameter uncertainty along with rational learning for the return moments of the three financial assets are illustrated in Figure 7. Note that in the absence of parameter uncertainty, only the dividend share of the market portfolio fluctuates over time, while the consumption share and beliefs are constant. Because the mean of the

Figure 7: Moments of Asset Returns

This figure shows the dynamics of the return moments of the financial assets over time, based on the parameter values described in Section 4.1. Panel A shows the average risk-free rate. Panels B and C depict the average return volatility and the average risk premium for the new asset. Panels D and E show the same quantities for the market portfolio, and Panel F shows the average return correlation between the market portfolio and the new asset.



dividend share of the market portfolio is essentially constant over time, the average return moments in this setting are also very stable. In contrast, if some investors need to learn about the expected dividend growth of the new asset, then we observe strong time-variation in the return moments of the assets.

Particularly, in the two economies with parameter uncertainty—the economy in which both groups of investors need to learn and the economy where only one group of investors needs to learn about the expected dividend growth of the new asset, the risk-free rate, $r_{0,t}$, shown in Panel A of Figure 7, is lower than in the economy without parameter uncertainty and increases over time. The explanation for this effect is the precautionary-savings motive that leads to an excess demand from the inexperienced investors for the bond, thereby pushing up its price, or equivalently, driving down the risk-free rate. Over time, as the precautionary-savings motive weakens because of the increase in the posterior precision, so does the demand for the bond, leading to a gradual increase in the risk-free rate. Note that in the economy in which both agents face parameter uncertainty, the effect is substantially stronger, as both investors exhibit a strong demand for the bond—even though in equilibrium they hold none of it because of market clearing.

Before turning to the two risky assets' return moments, recall that we focus on levered claims on the individual trees, with a leverage factor $\lambda = 2.5$. We denote the levered returns by $r^L_{n,t}$ in contrast to the simple returns, which are denoted by $r_{n,t}$. In the presence of parameter uncertainty, our findings for the return moments of the asset that has an uncertain mean are similar to those in Collin-Dufresne, Johannes, and Lochstoer (2016a) for the single-risky-asset case. In particular, as shown in Panels B and C of Figure 7, the risk premium of the new asset class, $E_t[r^L_{2,t+1}] - r_{0,t}$, and its volatility, $\mathrm{Vol}_t(r^L_{2,t+1})$, are considerably higher than without parameter uncertainty, and they are declining over time.

There is considerable empirical support for declining risk premia and return volatility over time for various alternative assets in the literature: for example, Ineichen Research and Management (2013, their Figure 4) provides this evidence for hedge-fund returns, and Harris, Jenkinson, and Kaplan (2014) show this for private-equity returns. Similarly, evidence for the decline in volatility for hedge-fund returns is provided by Adrian (2007) and for emerging-market returns by Dimson, Marsh, and Staunton (2014, their Table 4).

To understand the reason for the higher return volatility, recall that positive (negative) dividend shocks for the new asset class are associated with upward (downward) revisions in investors' perceived dividend growth. If the substitution effect dominates the wealth effect, that is, with EIS > 1, these upward (downward) revisions lead to a higher (lower) price-dividend ratio for the new asset class. Accordingly, positive dividend news coincides with a higher price-dividend ratio, and vice versa for negative news, thereby amplifying the variations in its dividend and creating "excess" volatility. Over time, the decline in posterior variance reduces this amplification mechanism, explaining the decline in volatility.

To see the impact of learning on the new asset's risk premium, note that the risk premium is given by the covariance between an asset's return and the SDF:

$$\begin{aligned} E_t[r^L_{n,t}] - r_{0,t} &= -\lambda \operatorname{Cov}_t\big(M_{k,t+1}, r^L_{n,t+1}\big) \\ &= -\lambda \operatorname{Corr}_t\big(M_{k,t+1}, r_{n,t+1}\big) \operatorname{Vol}_t\big(M_{k,t+1}\big) \operatorname{Vol}_t\big(r_{n,t+1}\big). \end{aligned} \tag{8}$$

In the presence of rational learning, the correlation between the new asset's return and the SDF decreases (that is, becomes *more negative*). This is because an upward revision in the perceived growth rate leads to a *simultaneous* increase in the new asset's price and a decline in the SDF (due to an increase in the continuation utility) and vice versa for downward revisions. The increases in the volatility of the new asset's returns and the volatility of the SDF, which were explained above, along with this change in correlation, all contribute positively to the observed increase in the risk premium. Over time, as the impact of parameter uncertainty declines, so does the risk premium.

Observe from Figure 7, that there are substantial differences in the magnitudes of the effects between the two economies with and without heterogeneous beliefs. These differences can be directly attributed to the risk-sharing that takes place in the setting where investors have heterogeneous beliefs.[32] Particularly, as described above, the risk-sharing reduces the volatility of the SDF and, at the same time, also limits the amplification mechanism that generates excess volatility, so that both—return volatility and the risk premium—are considerably lower with heterogeneous beliefs.

Turning to the market portfolio, Panel D of Figure 7 shows that the return volatility of the market is affected much less by parameter uncertainty and, therefore, is quite stable

[32] These results are comparable to the findings in Chen, Joslin, and Tran (2012), who show that the presence of just a small proportion of optimistic investors reduces substantially the risk premium associated with disasters, due to the strong risk-sharing motives. However, note that in our economy all investors are fully rational.

over time. To understand why, note that positive news about the new asset reduces the demand for the market, and hence, its price, keeping all else constant. This implies that if positive news about the market coincides with positive news about the new asset, then the reduced demand for the market attenuates the increase in its price. On the other hand, if positive news about the market coincides with negative news about the new asset, then the increased demand for the market leads to a larger increase in its price. Given that the cash flows of the market and new asset are uncorrelated, there is equal probability of these two cases, and hence, the two effects mostly cancel out, leaving volatility of the market largely unchanged.

Most intriguing, however, is that the risk premium of the market portfolio, shown in Panel E of Figure 7, changes over time in opposite directions for the two economies exhibiting parameter uncertainty. That is, in the homogenous-beliefs case, the risk premium is high initially and decreases over time, while in the setting with heterogeneous beliefs, the premium is low initially and increases over time.[33] To understand the difference, recall from equation (8) that an asset's risk premium is driven by its return volatility, the volatility of the SDF, and the correlation of its return with the SDF. Parameter uncertainty in general leads to an increase in the SDF volatility, thus pushing up the risk premium. However, heterogeneity across the two groups of investors more than compensates for this effect by increasing the correlation (rendering it *less negative*) between the SDF and the market return, so that the risk premium is initially lower than in the case with homogenous investors.

Similarly, the correlation between the returns of the two risky assets also behaves differently in the economies with homogeneous and heterogeneous beliefs.[34] In the

[33]Note that in the absence of parameter uncertainty, the risk premium of the market portfolio is actually higher than the risk premium for the new asset class. This is driven exclusively by our choice of *symmetric* dividend growth rates and dividend growth volatilities for the two assets. As described in Cochrane, Longstaff, and Santa Clara (2008), in such a setting, the asset with the larger dividend share—in our case the market portfolio—earns a higher risk premium because it constitutes a larger fraction of aggregate consumption and, thus, covaries more with it.

[34]Note that even though dividends are uncorrelated, returns are correlated in the case in which the dividend growth rate is known. This is due to the common variation in the two asset's expected returns, which is described in detail in Cochrane, Longstaff, and Santa Clara (2008).

homogenous-beliefs case, a positive shock to the new asset's cash flow, which implies an increase in the perceived growth rate, makes both investors feel wealthier, so that they increase their demand for both stocks, driving up both prices at the same time, and vice versa for a negative shock, thus initially *increasing* return correlation. In contrast, with heterogeneous beliefs, a positive shock to the cash flow of the new asset leads to a wealth transfer away from inexperienced investors, because they are under-invested in the new asset class. As a reaction to this wealth shock, the inexperienced investors disproportionally sell the market portfolio in which they were over-invested, driving down its price. Thus, a positive shock to the new asset's dividends coincides with a decrease in the price of the market portfolio and vice versa for negative news. Accordingly, we initially observe a lower return correlation in the heterogeneous-beliefs economy, which increases over time as the effect of parameter uncertainty diminishes.[35]

5 Effects of Illiquidity

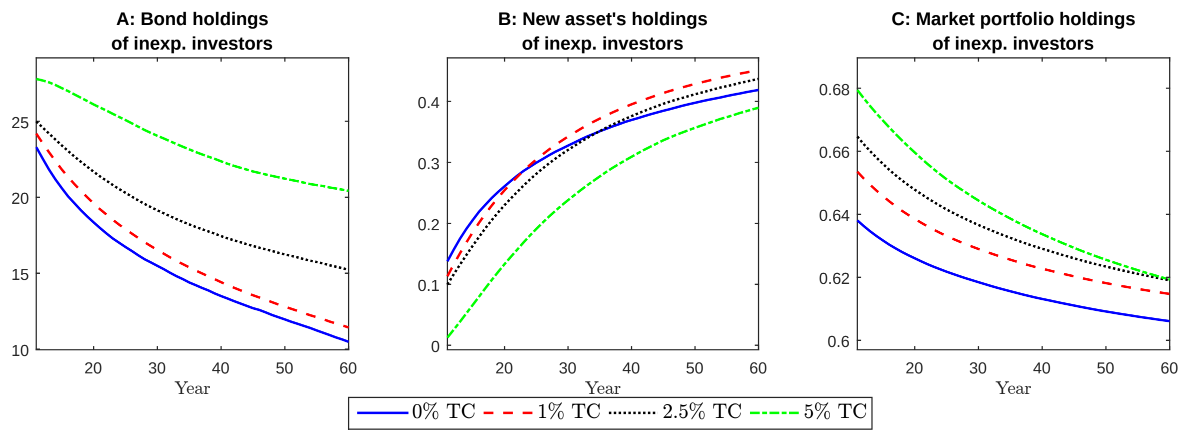
We now study the impact of illiquidity on the "transitional dynamics" of the new asset class. Specifically, we introduce transaction costs for the new asset class into the economy with heterogeneous beliefs, that is, where only a single group of investors faces parameter uncertainty. In the other two economies with investors who have homogenous beliefs—that is, either both investors know the mean dividend growth or both investors have to learn about it—there is no trade in equilibrium, so illiquidity plays no role. For the numerical analysis we rely on the same set of parameter values as in the previous section, which are presented in Table 1. We consider transaction costs of up to 5%, which seems realistic given the empirical estimates for some of the alternative asset classes.[36]

[35]Empirical evidence of this increase can be found, for example, in the correlation between emerging- and developed-market returns, as shown by Christoffersen, Errunza, Jacobs, and Langlois (2012) and Dimson, Marsh, and Staunton (2014, their Figure 5).

[36]See footnote 2.

Figure 8: Asset Allocation with Transaction Costs

The figure shows the evolution of the inexperienced investors' portfolio positions over time, based on the parameter values described in Section 4.1. Panels A to C show the average number of shares of the bond, the new asset and the market portfolio held by the inexperienced investors, respectively.

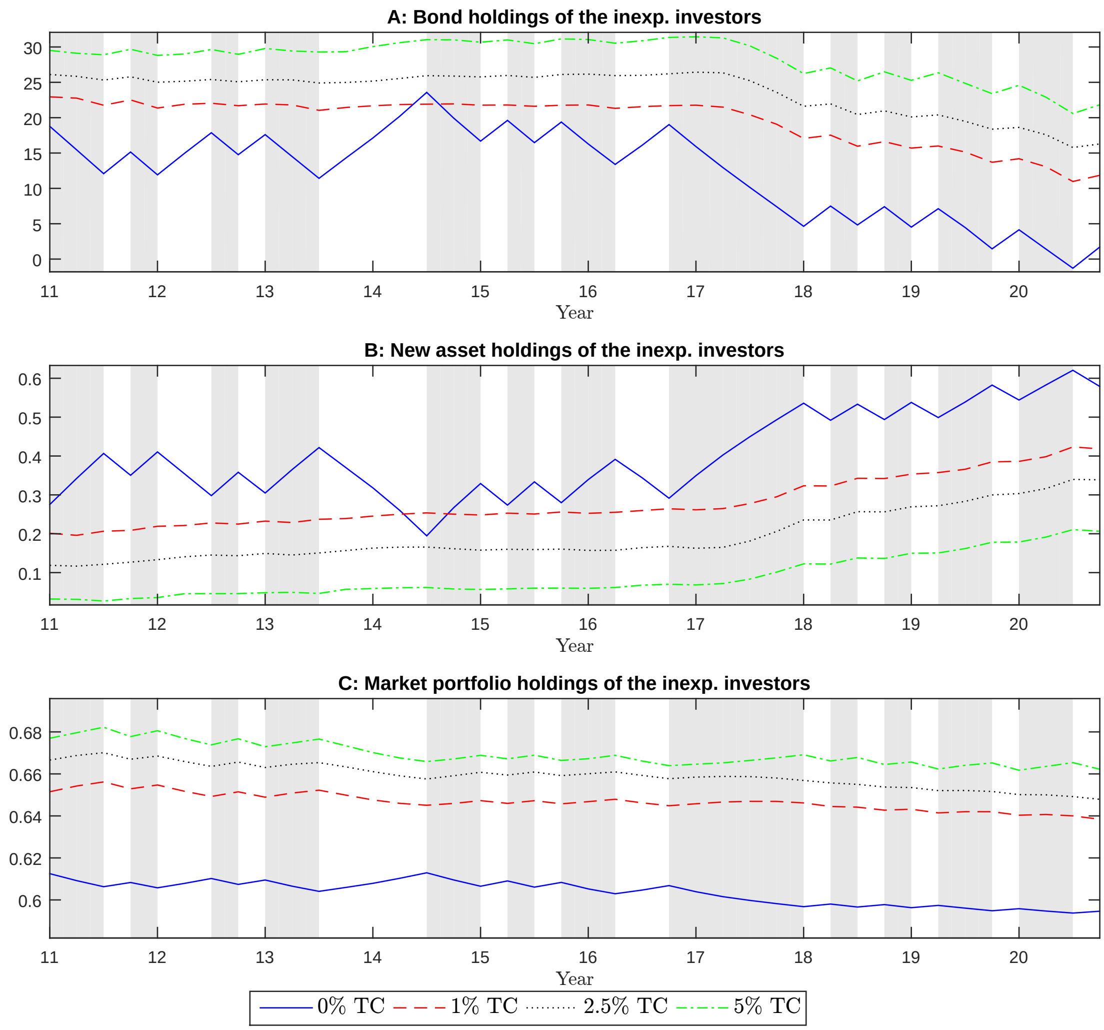


5.1 Asset Allocation

Figure 8 shows the dynamics of the average portfolio positions for the inexperienced investors over time. Panel B shows that the introduction of transaction costs leads, for the early years, to a reduction in the portfolio position of inexperienced investors in the new asset. In particular, because of the presence of a transaction cost, the inexperienced investors build up their position in the new asset more gradually. For the later years, however, the holdings in the new asset are similar across different levels of transaction costs. This occurs because the inexperienced investors know that, because of the deterministic reduction in posterior variance, they will increase their holdings in the new asset over time, and so in order to reduce unnecessary round-trip costs, they are actually not willing to sell the new asset even after negative cash-flow news.

The initially smaller positions of inexperienced investors in the new asset are offset by larger positions in the bond (Panel A) and the market portfolio (Panel C). To understand the effect for the market portfolio, recall that transaction costs prevent these investors from increasing their holdings in the new asset class as quickly as they desire. Given that

Figure 9: Asset Allocation with Transaction Costs over a Simulated Path
This figure shows the dynamics of the asset allocation decisions of the inexperienced investors in the presence of transaction costs for a particular simulated path of the economy, which is illustrated in Figure 2. Panel A shows the holdings in the bond, Panel B the holdings in the new asset, and Panel C the holdings in the market portfolio. The shaded gray areas indicate periods of positive dividend realizations for the new asset, leading to an upward revision in the perceived growth rate.



the returns of the two risky assets are correlated, inexperienced investors use the market portfolio as a substitute for their desired holdings in the new asset class.[37]

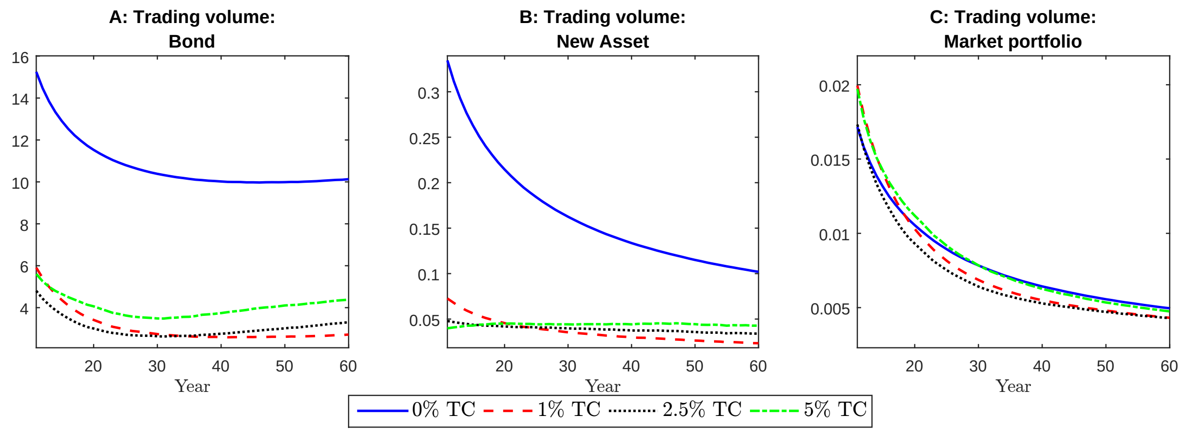
[37] To interpret the magnitudes, keep in mind that the price of the market portfolio is considerably higher than the price of the new asset class, so small changes in the market portfolio are sufficient to offset large changes in the new asset.

The results presented above capture the inexperienced investors' trade-off between their desire to increase investment in the new asset class over time, as their *posterior variance* declines, and the fact that trading is costly. In contrast, the portfolio holdings on the sample path, shown in Figure 9, illustrate the trade-off between the investors' desire to change their holdings in reaction to updates in their *posterior mean* and the transaction cost. In the frictionless economy, upward (downward) revisions in the beliefs always lead to an increase (decrease) in their holdings of the new asset (Panel B) and an opposite change in the bond position (Panel A). With transaction costs, the investors now face a trade-off: while the current update of the perceived growth rate may suggest an increase in holdings, they understand that next period's shock to the perceived growth rate might imply a reduction in holdings, thus leading to unnecessary round-trip costs. Accordingly, in the presence of transaction costs, investors react less to shocks to their beliefs—often not at all until they have substantially revised their beliefs—which leads to "portfolio inertia." Empirical evidence of this infrequent trading for various alternative assets is provided by Ang, Papanikolaou, and Westerfield (2014, their Table 1). Over time, the increase in the posterior precision leads the inexperienced investors to favor purchases of the new asset and an increase in the portfolio weight allocated to it.

The portfolio positions in the bond along the sample path, depicted in Panel A of Figure 9, essentially mirror this pattern. As the inexperienced investors reduce their trading in the new asset, they also trade less in the bond because the bond is used to finance trading in the new asset. In contrast, the changes in the holdings of the market portfolio in the presence of transaction costs, shown in Panel C of Figure 9, are much more intriguing: the market portfolio is used as a *substitute* for trading in the new asset. That is, the decision of the inexperienced investors to increase their holdings of the new asset less than in the frictionless case is associated with an increase in their holdings of the market portfolio. For some periods, the holdings in the market portfolio actually perfectly track the shocks to the perceived growth rate of the new asset; that is, dividend shocks in the new asset *spill over* to the traditional asset.

Figure 10: Turnover with Transaction Costs

The figure shows the evolution of the assets' turnover over time, based on the parameter values described in Section 4.1. Panels A to C show the total number of shares of the bond, the new asset and the market portfolio traded, on average, within a year, respectively.

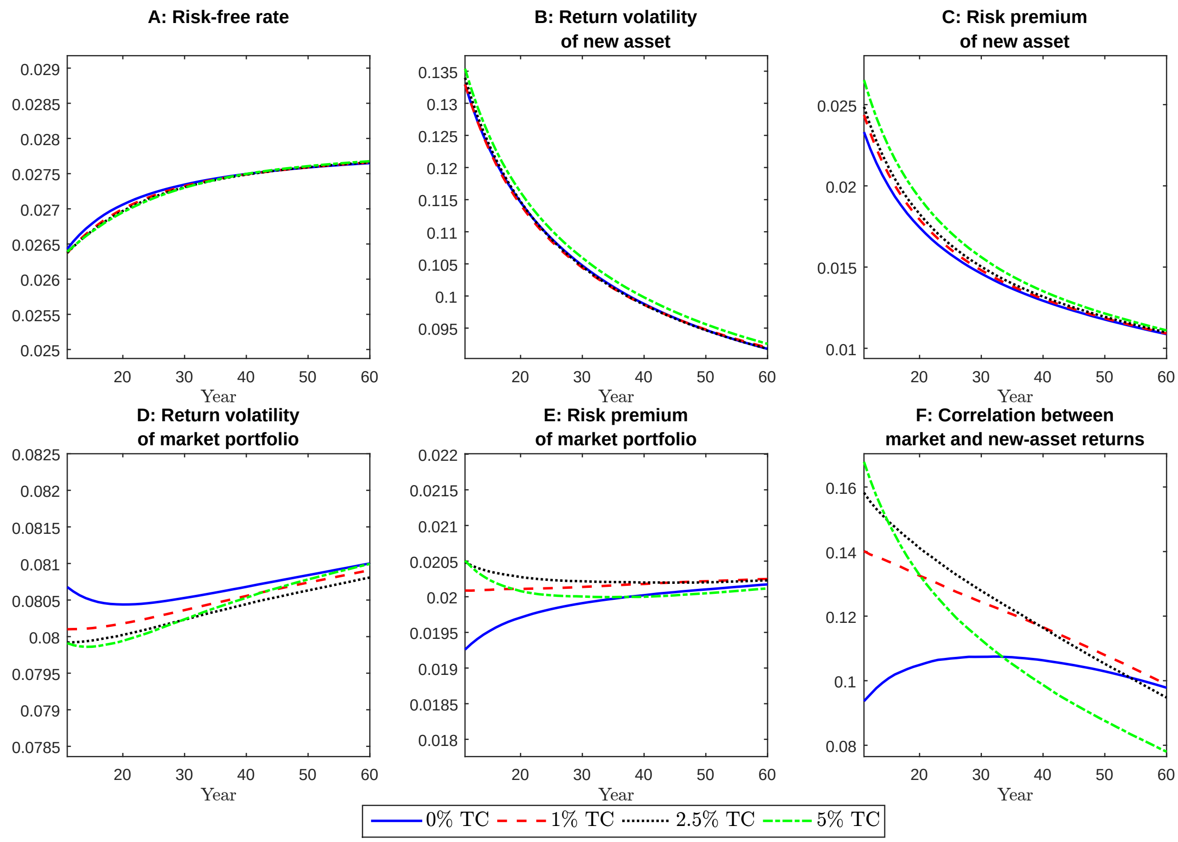


The impact of illiquidity on the inexperienced investors' trading in reaction to changes in their posterior mean, which, in general, constitutes the majority of the trading, can be directly related to the impact of transaction costs on the assets' turnover, shown in Figure 10. That is, transaction costs reduce the trading volume in the new asset class and, with it, the turnover in the bond. In contrast, the trading volume in the market portfolio initially increases because of its role as a substitute for trading the new asset.

5.2 Consumption Policies

Unreported results show that the patterns observed for the investors' holdings are directly reflected in their optimal consumption policies. Specifically, as the inexperienced investors' over-investment in the bond increases with transaction costs, their portfolio return declines, and, thus, their expected consumption growth rate drops relative to the frictionless case. An opposite effect can be observed for the experienced investors so that the gap in expected consumption growth widens. This, in turn, leads to an acceleration in the decline in the inexperienced investors' consumption share. In contrast, consumption

Figure 11: Moments of Asset Returns with Transaction Costs
This figure shows the dynamics of the return moments of the financial assets over time for different levels of transaction costs, based on the parameter values described in Section 4.1. Panel A shows the average risk-free rate. Panels B and C depict the average return volatility and the average risk premium for the new asset. Panels D and E show the same quantities for the market portfolio, and Panel F shows the average return correlation between the market portfolio and the new asset.



growth volatilities are only marginally affected and quickly converge to their frictionless counterparts.

5.3 Moments of Asset Returns

The dynamics of returns of financial assets in the presence of transaction costs are depicted in Figure 11. While changes in the risk-free rate (Panel A) and in the new asset's return volatility (Panel C) are small, the risk premium for the new asset increases monotonically (Panel B). The small increase in volatility is driven by a wealth effect. Specifically, recall that with transaction costs inexperienced investors are even more under-invested in the new asset class, so that positive dividend news for this asset leads to an even stronger wealth transfer in favor of the experienced investor. As the portfolio of experienced investors is tilted toward the new asset, this creates additional price pressure, further amplifying the effect of dividend shocks compared to the frictionless economy, thus increasing volatility. The risk premium of the new asset class increases monotonically with transaction costs, leading to a *liquidity* premium that can be in the range of 0.2–0.3% p.a., which is quite substantial given that the overall risk premium is in the range of 2.5–3.0% p.a.[38] This liquidity premium is driven by an increase in the volatility of the SDF. Over time, as the posterior variance decreases and with it the volatility of the SDF, we observe a gradual decline in the liquidity premium.

The volatility of the market portfolio declines slightly with transaction costs. This effect is driven by the price implication of "substitute trading," which absorbs some of the shocks to the dividends of the new asset. In particular, observe that the inexperienced investors would like to increase their holdings in the new asset in reaction to positive perceived growth news about the new asset. However, in the presence of transaction costs, they limit their purchases of the new asset and instead invest in the market portfolio, which is less costly to trade, thus pushing up its price. Accordingly, positive perceived growth news for the new asset, which implies a decrease in the SDF (due to an increase in continuation utility), also leads to an increase in the price of the market portfolio. This leads to a decrease in the correlation with the SDF (rendering it *more negative*) and explains the increase in the risk premium. Moreover, this simultaneous increase in the price of the new asset and the market portfolio also explains the increase in the correlation between the two assets, as shown in Panel F of Figure 11.

[38]Empirical evidence of liquidity premia for private equity is provided by Franzoni, Nowak, and Phalippou (2012) and Harris, Jenkinson, and Kaplan (2014).

6 Robustness

To study the robustness of our results, we now evaluate the impact of parameter learning and illiquidity for several variations of the benchmark economy.

While in our benchmark model the investors' portfolios are unconstrained, in practice it is difficult to short a new asset class. Therefore, we also solve our economy with *short-sale constraints* imposed on the new asset class.[39] While short-sale constraints lead, on average, to a higher initial investment into the new asset class by the inexperienced investors, and thus, a smaller position in the bond, the dynamics over time are comparable to those in the benchmark case. Moreover, short-sale constraints lead to a reduction in the risk premium and volatility of the new asset; however, both are declining over time, as in our benchmark analysis.

As noted in Section 4.2, our benchmark model is non-stationary because the distribution of the dividend share becomes degenerate in the long-run. Therefore, we also consider an economy with a *mean-reverting dividend share* which we achieve by means of a state-dependent expected dividend growth rate for the market portfolio. That is, if the dividend share of the market portfolio is below its long-term mean, its expected dividend growth rate increases and vice versa. Qualitatively, this change has no effect on our results.

We also study an economy with *correlated dividends*, keeping aggregate consumption growth unchanged. In such a setting, the dividend shocks of the market also provide information for the expected dividend growth rate of the new asset, thus affecting revisions of the perceived growth rate. Accordingly, changes in the perceived dividend growth rate are more volatile, so that inexperienced investors further reduce their holdings in the new asset class during the initial years and instead invest more into the bond and the (more correlated) market portfolio. As a result, the risk premium for the new asset class is higher in the early years, but over time still declines as in the benchmark economy.

[39] There is no need to impose short-sale constraints on the market, as investors naturally abstain from shorting it.

Next, we consider a variety of economies, changing one parameter at a time. First, we reduce the *trading and learning frequency*, Δ_t, to $1/2$, which implies that investors update their beliefs only every six months instead of every three months. Thus, the decline in the posterior variance is slower, making the effects of parameter uncertainty more long-lasting; for example, the excess volatility and the risk premium stemming from parameter learning persist for longer. In contrast, setting the *elasticity of intertemporal substitution*, ψ, to 1.5, instead of 2.0 in the base case, or the *initial share of wealth* of the inexperienced investors to $1/2$ instead of $2/3$ in the base case, slightly weakens the impact of parameter uncertainty. This is because the effect of parameter uncertainty on the SDF weakens and the demand effects stemming from the inexperienced investors' portfolio choices diminish, respectively. However, the dynamics over time are comparable to the ones studied in the benchmark model.

Reducing the *initial dividend share* of the market portfolio from 0.8 in the benchmark case to 0.7 or increasing the *number of periods*, T, from 600 to $1,200$, leads only to small quantitative shifts in the level of portfolio holdings and asset returns, and leaves the transitional dynamics over time unchanged. Similarly, widening the *truncation bounds* to $[-0.53\%, 1.43\%]$ has practically no impact, implying that our results are not driven by extreme beliefs. Finally, in an economy in which the *cash-flow volatility* of the new asset is twice as high as the one for the market, instead of being the same as in the benchmark case, the return volatility and risk premium for the new asset are substantially higher than for the market portfolio, but their dynamics over time are unchanged.

7 Conclusion

New asset classes, such as private equity, hedge funds, natural resources, and real assets, should play a crucial role in the portfolios of all investors because of the substantial diversification benefits they offer. However, because many investors are uncertain about the cash-flow dynamics of these "alternative assets" and these assets are relatively illiquid, a majority of investors hold only a small percentage of their wealth in alternative assets.

To understand how the movement of capital into new asset classes affects the returns of new and traditional assets, we develop a general equilibrium model with the following key characteristics: (1) three assets—a risk-free bond, a traditional risky asset, and a new "alternative" asset; (2) parameter uncertainty about the expected growth rate of the cash flows of the new asset, along with rational learning and recursive preferences; (3) heterogeneity in beliefs of investors about the cash flows of the new asset; and (4) transaction costs for trading the new asset class.

This model helps one to understand the large differences in the asset-allocation decisions of experienced and less-experienced investors and how this difference evolves over time. That is, in the presence of parameter uncertainty, inexperienced investors tilt their portfolios away from the new asset, due to a negative *intertemporal hedging demand*. In contrast, experienced investors over-weight the new asset, thereby capturing the risk premium associated with the new asset, which leads to higher expected portfolio returns. Parameter learning also has substantial effects on asset returns. The *rational learning* of inexperienced investors' *amplifies* price movements in the new asset class because positive (negative) cash-flow news leads to a higher (lower) price-dividend ratio. This higher return volatility, together with a higher volatility of the stochastic discount factor, lead to an additional risk premium for the new asset. Over time, as the inexperienced investors' beliefs become more precise, their holdings in the new asset increase and the return volatility and additional risk premium decline.

Finally, in the presence of transaction costs, a *liquidity premium* for the new asset arises. Moreover, inexperienced investors rebalance their holding of the new asset by a smaller amount in response to updates in their beliefs. Instead, they use the liquid market portfolio as a substitute for costly trading in the new asset class, which gives rise to a risk premium on the market portfolio and also creates, in the early years, *excess correlation* with the return of the new asset.

Our analysis shows that, while inexperienced investors may be reluctant to invest in new asset classes, which they perceive to be excessively risky, the gains from investing in these assets can be substantial, particularly in the early years when the new asset classes become available for trading. This suggests that there are considerable benefits from undertaking the necessary research in alternative assets that enables one to invest in these assets.

A Optimality Conditions and Equilibrium

A.1 Investors' Optimality Conditions

The objective of each investor k is to maximize her expected lifetime utility given in equation (2), by choosing consumption, $C_{k,t}$, and the holdings—in terms of number of shares—in the three financial assets, $\theta_{k,n,t}$, $n \in \{0,1,2\}$:[40]

$$V_{k,t}\big(\{\theta_{k,n,t-1}\}\big) = \max_{C_{k,t},\{\theta_{k,n,t}\}} \left[(1-\beta)\, C_{k,t}^{1-\frac{1}{\psi}} + \beta E_t^k \left[V_{k,t+1}\big(\{\theta_{k,n,t}\}\big)^{1-\gamma}\right]^{\frac{1}{\phi}}\right]^{\frac{\phi}{1-\gamma}},$$

subject to the budget equation

$$\begin{aligned} C_{k,t} + \theta_{k,0,t}\, S_{0,t} + \sum_{n=1}^{2}\big(\theta_{k,n,t} - \theta_{k,n,t-1}\big)\, S_{n,t} + \kappa\,\big|\theta_{k,2,t} - \theta_{k,2,t-1}\big|\, S_{2,t} \\ \leq \theta_{k,0,t-1} + \sum_{n=1}^{2}\theta_{k,n,t-1}\, D_{n,t} + \xi_{k,t}. \end{aligned} \tag{A1}$$

To make the problem more suitable for dynamic programming, we decompose the holdings in the new asset, $\theta_{k,2,t}$ by investor k at time t into three components: the incoming holdings, $\theta_{k,2,t-1}$; the number of shares bought, $\check{\theta}_{k,2,t} \geq 0$; and, the number of shares sold, $\hat{\theta}_{k,2,t} \geq 0$:

$$\theta_{k,2,t} = \theta_{k,2,t-1} + \check{\theta}_{k,2,t} - \hat{\theta}_{k,2,t}. \tag{A2}$$

Accordingly, we can simplify the budget equation (A1) to

$$\begin{aligned} C_{k,t} + \theta_{k,0,t}\, S_{0,t} + \big(\theta_{k,1,t} - \theta_{k,1,t-1}\big)\, S_{1,t} + \check{\theta}_{k,2,t} S_{2,t}(1+\kappa) - \hat{\theta}_{k,2,t} S_{2,t}(1-\kappa) \\ \leq \theta_{k,0,t-1} + \sum_{n=1}^{2}\theta_{k,n,t-1}\, D_{n,t} + \xi_{k,t}. \end{aligned} \tag{A3}$$

[40]For ease of exposition, in the following derivations we do not explicitly write the dependence of $V_{k,t}$ on the incoming (i.e., date $t-1$) asset holdings, $\{\theta_{k,n,t-1}\}$.

The Lagrangian of the investor's optimization problem can then be written as:

$$\mathcal{L}_{k,t} = \sup_{C_{k,t},\theta_{k,0,t},\theta_{k,1,t},\check{\theta}_{k,2,t},\hat{\theta}_{k,2,t}} \inf_{\eta_{k,t},\mu_{k,1,t},\mu_{k,2,t}} \left[(1-\beta)\, C_{k,t}^{1-\frac{1}{\psi}} + \beta E_t^k \left[V_{k,t+1}^{1-\gamma}\right]^{\frac{1}{\phi}}\right]^{\frac{\phi}{1-\gamma}}$$

$$+ \eta_{k,t} \left(\theta_{k,0,t-1} + \sum_{n=1}^{2} \theta_{k,n,t-1}\, D_{n,t} + \xi_{k,t} - C_{k,t} - \theta_{k,0,t} S_{0,t} - \left(\theta_{k,1,t} - \theta_{k,1,t-1}\right) S_{1,t}\right.$$

$$\left. - \check{\theta}_{k,2,t} S_{2,t}(1+\kappa) + \hat{\theta}_{k,2,t} S_{2,t}(1-\kappa)\right) + \mu_{k,1,t}\check{\theta}_{k,2,t} + \mu_{k,2,t}\hat{\theta}_{k,2,t},$$

where $\eta_{k,t}$ denotes the Lagrange multiplier associated with budget equation (A3), and $\mu_{k,1,t}$ and $\mu_{k,2,t}$ denote the Lagrange multipliers associated with the inequality conditions for the number of shares bought and sold, respectively.

The corresponding Karush-Kuhn-Tucker first-order conditions are given by:

$$\frac{\partial \mathcal{L}_{k,t}}{\partial C_{k,t}} = \frac{1}{1-\frac{1}{\psi}} \left\{(1-\beta)\, C_{k,t}^{1-\frac{1}{\psi}} + \beta E_t^k \left[V_{k,t+1}^{1-\gamma}\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right\}^{\frac{1}{1-\frac{1}{\psi}}-1} (1-\beta) \left(1 - \frac{1}{\psi}\right) C_{k,t}^{-\frac{1}{\psi}} - \eta_{k,t}$$

$$= (1-\beta)\; C_{k,t}^{-\frac{1}{\psi}}\, V_{k,t}^{\frac{1}{\psi}} - \eta_{k,t} \equiv 0; \tag{A4}$$

$$\frac{\partial \mathcal{L}_{k,t}}{\partial \eta_{k,t}} = \theta_{k,0,t-1} + \sum_{n=1}^{2} \theta_{k,n,t-1}\, D_{n,t} + \xi_{k,t} - C_{k,t} - \theta_{k,0,t} S_{0,t} - \left(\theta_{k,1,t} - \theta_{k,1,t-1}\right) S_{1,t}$$

$$- \check{\theta}_{k,2,t} S_{2,t}(1+\kappa) + \hat{\theta}_{k,2,t} S_{2,t}(1-\kappa) \equiv 0; \tag{A5}$$

$$\frac{\partial \mathcal{L}_{k,t}}{\partial \theta_{k,n,t}} = \frac{1}{1-\frac{1}{\psi}}\, V_{k,t}^{\frac{1}{\psi}}\, \beta\, \frac{1-\frac{1}{\psi}}{1-\gamma}\, E_t^k \left[V_{k,t+1}^{1-\gamma}\right]^{\frac{\gamma-\frac{1}{\psi}}{1-\gamma}} (1-\gamma)\; E_t^k \left[V_{k,t+1}^{-\gamma} \frac{\partial V_{k,t+1}}{\partial \theta_{k,n,t}}\right] - \eta_{k,t} S_{n,t}$$

$$= \beta\, V_{k,t}^{\frac{1}{\psi}}\, E_t^k \left[V_{k,t+1}^{1-\gamma}\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}-1}\; E_t^k \left[V_{k,t+1}^{-\gamma} \frac{\partial V_{k,t+1}}{\partial \theta_{k,n,t}}\right] -\; \eta_{k,t}\, S_{n,t} \equiv 0, \quad \text{for } n \in \{0, 1\}; \tag{A6}$$

$$\frac{\partial \mathcal{L}_{k,t}}{\partial \check{\theta}_{k,2,t}} = \beta\, V_{k,t}^{\frac{1}{\psi}}\, E_t^k \left[V_{k,t+1}^{1-\gamma}\right]^{\frac{\gamma-\frac{1}{\psi}}{1-\gamma}}\; E_t^k \left[V_{k,t+1}^{-\gamma} \frac{\partial V_{k,t+1}}{\partial \theta_{k,2,t}} \underbrace{\frac{\partial \theta_{k,2,t}}{\partial \check{\theta}_{k,2,t}}}_{=1}\right] -\; \eta_{k,t}\, S_{2,t}\,(1+\kappa) + \mu_{k,1,t} \equiv 0; \tag{A7}$$

$$\frac{\partial \mathcal{L}_{k,t}}{\partial \hat{\theta}_{k,2,t}} = \beta\, V_{k,t}^{\frac{1}{\psi}}\, E_t^k \left[V_{k,t+1}^{1-\gamma}\right]^{\frac{\gamma-\frac{1}{\psi}}{1-\gamma}}\; E_t^k\left[V_{k,t+1}^{-\gamma}\frac{\partial V_{k,t+1}}{\partial \theta_{k,2,t}}\underbrace{\frac{\partial \theta_{k,2,t}}{\partial \hat{\theta}_{k,2,t}}}_{=-1}\right] + \eta_{k,t}\, S_{2,t}\,(1-\kappa) + \mu_{k,2,t} \equiv 0; \tag{A8}$$

$$\mu_{k,1,t}\,\check{\theta}_{k,2,t} \equiv 0; \quad \mu_{k,2,t}\,\hat{\theta}_{k,2,t} \equiv 0; \quad \check{\theta}_{k,2,t} \geq 0; \quad \hat{\theta}_{k,2,t} \geq 0; \quad \mu_{k,1,t} \geq 0; \quad \mu_{k,2,t} \geq 0. \tag{A9}$$

Equating equations (A7) and (A8), one can define a new variable $R_{k,2,t}$, which captures the shadow cost of trading and subsumes the two Lagrange multipliers, $\mu_{k,1,t}$ and $\mu_{k,2,t}$:

$$\eta_{k,t} R_{k,2,t} S_{2,t} \hat{=} \eta_{k,t}\, S_{2,t}\,(1+\kappa) - \mu_{1,t} = \eta_{k,t}\, S_{2,t}\,(1-\kappa) + \mu_{2,t}, \tag{A10}$$

which, in turn, allows us to eliminate one of the two equations, (A7) or (A8), and rewrite the other one as:

$$\beta\, V_{k,t}^{\frac{1}{\psi}}\, E_t^k \left[V_{k,t+1}^{1-\gamma}\right]^{\frac{\gamma-\frac{1}{\psi}}{1-\gamma}}\; E_t^k\left[V_{k,t+1}^{-\gamma}\frac{\partial V_{k,t+1}}{\partial \theta_{k,2,t}}\right] - \eta_{k,t} R_{k,2,t} S_{2,t} \equiv 0. \tag{A11}$$

Moreover, we can now rewrite the complementary-slackness conditions in (A9) as:

$$\big((1+\kappa) - R_{k,2,t}\big)\,\check{\theta}_{k,2,t} \equiv 0; \qquad \big(R_{k,2,t} - (1-\kappa)\big)\,\hat{\theta}_{k,2,t} \equiv 0. \tag{A12}$$

Note, equation (A10) also implies that

$$R_{k,2,t} = (1+\kappa) - \frac{\mu_{k,1,t}}{\eta_{k,t}\, S_{2,t}} = (1-\kappa) + \frac{\mu_{k,2,t}}{\eta_{k,t}\, S_{2,t}},$$

so that $R_{k,2,t}$ is bounded between $(1-\kappa)$ and $(1+\kappa)$:

$$1 - \kappa \leq R_{k,2,t} \leq 1 + \kappa. \tag{A13}$$

Finally, using the Envelope Theorem we can compute the derivatives of the value function $V_{k,t}$ with respect to $\theta_{k,n,t-1}$, $n \in \{0,1,2\}$:

$$\frac{\partial V_{k,t}}{\partial \theta_{k,0,t-1}} = \frac{\partial \mathcal{L}_{k,t}}{\partial \theta_{k,0,t-1}} = \eta_{k,t}; \tag{A14}$$

$$\frac{\partial V_{k,t}}{\partial \theta_{k,1,t-1}} = \frac{\partial \mathcal{L}_{k,t}}{\partial \theta_{k,1,t-1}} = \eta_{k,t}\,(D_{1,t} + S_{1,t}); \tag{A15}$$

$$\begin{aligned}\frac{\partial V_{k,t}}{\partial \theta_{k,2,t-1}} = \frac{\partial \mathcal{L}_{k,t}}{\partial \theta_{k,2,t-1}} &= \eta_{k,t}\, D_{2,t} + \beta\, V_{k,t}^{\frac{1}{\psi}}\, E_t^k\left[V_{k,t+1}^{1-\gamma}\right]^{\frac{\gamma-\frac{1}{\psi}}{1-\gamma}}\; E_t^k\left[V_{k,t+1}^{-\gamma}\frac{\partial V_{k,t+1}}{\partial \theta_{k,2,t}}\,\underbrace{\frac{\partial \theta_{k,2,t}}{\partial \theta_{k,2,t-1}}}_{=1}\right] \\ &= \eta_{k,t}\, D_{2,t} + \eta_{k,t} R_{k,2,t} S_{2,t} = \eta_{k,t}\,(D_{2,t} + R_{k,2,t} S_{2,t}),\end{aligned} \tag{A16}$$

where we used equation (A11) to derive equation (A16).

In summary, the optimality conditions for each investor k are given by the following set of equations. First, the budget equation arising from (A5):

$$\begin{aligned} C_{k,t} + \theta_{k,0,t}\, S_{0,t} + \left(\theta_{k,1,t} - \theta_{k,1,t-1}\right) S_{1,t} + \check{\theta}_{k,2,t} S_{2,t}(1+\kappa) - \hat{\theta}_{k,2,t} S_{2,t}(1-\kappa) \\ \leq \theta_{k,0,t-1} + \sum_{n=1}^{2} \theta_{k,n,t-1}\, D_{n,t} + \xi_{k,t}, \end{aligned} \tag{A17}$$

which equates the uses and sources of funds. Second, the pricing equations, arising from equations (A6) and (A11), in combination with equations (A14) to (A16), which equate the price of an asset to the expected payoff from holding it:

$$\begin{aligned} S_{0,t} &= E_t^k\left[M_{k,t+1}\right], \\ S_{1,t} &= E_t^k\left[M_{k,t+1}\left(S_{1,t+1} + D_{1,t+1}\right)\right], \\ S_{2,t} &= \frac{1}{R_{k,2,t}}\, E_t^k\left[M_{k,t+1}\left(R_{k,2,t+1}\, S_{2,t+1} + D_{2,t+1}\right)\right], \end{aligned}$$

where the stochastic discount factor $M_{k,t+1}$, given in (3), subsumes the Lagrange multiplier $\eta_{k,t}$ from equation (A4). Note, for the new asset ($n = 2$), if one were to trade the asset the investment / proceeds need to be net of transaction costs, which is captured by the time t and $t+1$ shadow costs of trading $R_{k,2,t}$ and $R_{k,2,t+1}$, respectively.

Third, the complementary slackness conditions (A12) associated with the non-negative purchasing and selling decision variables, and the corresponding inequality conditions from (A9) and (A13):

$$\big((1+\kappa) - R_{k,2,t}\big)\,\check{\theta}_{k,2,t} = 0, \qquad \big(R_{k,2,t} - (1-\kappa)\big)\,\hat{\theta}_{k,2,t} = 0, \tag{A18}$$

$$\check{\theta}_{k,2,t} \geq 0, \qquad \hat{\theta}_{k,2,t} \geq 0, \qquad 1-\kappa \leq R_{k,2,t} \leq 1+\kappa. \tag{A19}$$

A.2 Characterization of Equilibrium

Equilibrium in the economy can then be characterized by the following set of equations: the budget equation (A17), complementary slackness equations (A18), and the inequality conditions (A19) for both groups of investors, $k \in \{1,2\}$. Moreover, the "kernel conditions" that equate the prices of the assets across investors:

$$E_t^1\left[M_{1,t+1}\right] = E_t^2\left[M_{2,t+1}\right], \tag{A20}$$

$$E_t^1\left[M_{1,t+1}\left(S_{1,t+1} + D_{1,t+1}\right)\right] = E_t^2\left[M_{2,t+1}\left(S_{1,t+1} + D_{1,t+1}\right)\right], \tag{A21}$$

$$\frac{1}{R_{1,2,t}}\,E_t^1\left[M_{1,t+1}\left(R_{1,2,t+1}\,S_{2,t+1} + D_{2,t+1}\right)\right] = \frac{1}{R_{2,2,t}}\,E_t^2\left[M_{2,t+1}\left(R_{2,2,t+1}\,S_{2,t+1} + D_{2,t+1}\right)\right], \tag{A22}$$

and the market-clearing conditions:[41]

$$\sum_{k=1}^{2}\theta_{k,0,t} = 0; \qquad \sum_{k=1}^{2}\theta_{k,1,t} = 1; \qquad \sum_{k=1}^{2}\left(\theta_{k,2,t-1} + \check{\theta}_{k,2,t} - \hat{\theta}_{k,2,t}\right) = 1. \tag{A23}$$

B Numerical Algorithm

As described in Section 3.3, we use the time-shift proposed by Dumas and Lyasoff (2012) to obtain a recursive system. That is, at date t, the "shifted" system of equations consists of the date-t kernel conditions (A20)–(A22), the market-clearing conditions (A23), the date $t+1$ budget constraints (A17), the portfolio-flow equations (A2),[42] complementary-slackness conditions (A18), and the inequality conditions (A19).

[41] By Walras' law, market clearing in the asset markets guarantees market clearing for the consumption good.

[42] Next period's portfolio-flow equations get re-introduced into the system because of the time-shift.

In particular, assuming that there are J future states (nodes), denoted by $j = 1, \ldots, J$, the shifted system of equations can be written as:

$$C_{k,t+1,j} + \theta_{k,0,t+1,j} S_{0,t+1,j} + \big(\theta_{k,1,t+1,j} - \theta_{k,1,t}\big)\, S_{1,t+1,j} + \check{\theta}_{k,2,t+1,j} S_{2,t+1,j}(1+\kappa)$$
$$-\hat{\theta}_{k,2,t+1,j} S_{2,t+1,j}(1-\kappa) \leq \theta_{k,0,t} + \sum_{n=1}^{2} \theta_{k,n,t}\, D_{n,t+1,j} + \xi_{k,t+1,j}, \quad \forall k, j;$$
$$E_t^1\left[M_{1,t+1}\right] = E_t^2\left[M_{2,t+1}\right];$$
$$E_t^1\Big[M_{1,t+1}\Big(S_{1,t+1} + D_{1,t+1}\Big)\Big] = E_t^2\Big[M_{2,t+1}\Big(S_{1,t+1} + D_{1,t+1}\Big)\Big];$$
$$\frac{1}{R_{1,t}}\, E_t^1\Big[M_{1,t+1}\Big(R_{1,t+1}\, S_{2,t+1} + D_{2,t+1}\Big)\Big] = \frac{1}{R_{2,t}}\, E_t^2\Big[M_{2,t+1}\Big(R_{2,t+1}\, S_{2,t+1} + D_{2,t+1}\Big)\Big];$$
$$\theta_{k,2,t+1,j} = \theta_{k,2,t} + \check{\theta}_{k,2,t+1,j} - \hat{\theta}_{k,2,t+1,j}, \quad \forall k, j;$$

$$\sum_{k=1}^{2} \theta_{k,0,t} = 0; \qquad \sum_{k=1}^{2} \theta_{k,1,t} = 1; \qquad \sum_{k=1}^{2} \big(\theta_{k,2,t-1} + \check{\theta}_{k,2,t} - \hat{\theta}_{k,2,t}\big) = 1;$$
$$\big((1+\kappa) - R_{k,t+1,j}\big)\, \check{\theta}_{k,2,t+1,j} = 0; \qquad \big(R_{k,t+1,j} - (1-\kappa)\big)\, \hat{\theta}_{k,2,t+1,j} = 0;$$
$$\check{\theta}_{k,2,t+1,j} \geq 0; \qquad \hat{\theta}_{k,2,t+1,j} \geq 0; \qquad 1-\kappa \leq R_{k,t+1,j} \leq 1+\kappa, \quad \forall k, j.$$

Not counting the inequality conditions, there are $2K+3+2K+3+4K$ equations. The unknowns are next period's consumption, $C_{k,t+1,j}$, for both investors over the J states, next period's buying as well as selling decision variables, $\check{\theta}_{k,2,t+1,j}$ and $\hat{\theta}_{k,2,t+1,j}$, for both investors and J states, next period's shadow prices, $R_{k,t+1,j}$, for both investors and J states, and today's holdings in the assets, $\theta_{k,n,t}$, for both investors; in total $2K + 4K + 2K + 6$ unknowns—matching the number of equations.

This system of equations is solved recursively, starting from date $T-1$ to date 0. That is, at each date t, we solve the equation system outlined above over the *grid of the state variables*. In the next recursive step, when solving the equation system for date $t-1$, we interpolate over the grid the date-t variables required by the equation system; that is, the optimal date-t portfolio positions, $\theta_{k,n,t}$ and corresponding security prices, $S_{n,t}$, using the terminal conditions $\theta_{k,n,T} = 0$ and $S_{n,T} = 0$, $\forall n, k$.

In our economy, there are five state variables: (i) the dividend share of the first risky security $\delta_{1,t} \equiv D_{1,t}/\big(D_{1,t} + D_{2,t}\big) \in (0,1)$, the dynamics of which follow from the joint dividend dynamics in (1); (ii) the inexperienced investors' beliefs regarding the expected dividend growth of the new asset class, $\mu_{2,t}$, which we truncate at $[\underline{\mu}_2, \bar{\mu}_2]$, with the dynamics specified in (4); (iii) the (deterministic) posterior variance of the inexperienced investors' beliefs, $A_t\,\sigma_2^2$; (iv) the consumption share of the experienced investors, $\omega_{1,t} \equiv C_{1,t}/\big(C_{1,t} + C_{2,t}\big) \in (0, 1)$; and (v) the ratio of the two investors' shadow prices of trading $R_{2,2,t}/R_{1,2,t} \in [(1-\kappa)/(1+\kappa), (1+\kappa)/(1-\kappa)]$—the dynamics of last two state variables being determined endogenously by the investors' optimal choices.

After solving the system for all dates $t \in \{0, ..., T-1\}$, one has solved all equations from the global system—except the budget equations, the portfolio-flow equations, and the complementary-slackness conditions for the initial date ($t = 0$), which have not been used so far because of the time shift. So, at the initial date, we need to solve a system consisting of these equations based on interpolating functions for the initial-date prices, $S_{n,0}$, and holdings, $\theta_{k,n,0}$. The endowed holdings $\theta_{k,n,-1}$ for the three assets are exogenous to the system and reflect the initial period's incoming (endowed) wealth of the two investors.

We approximate the joint dynamics of the dividends in (1) using a four-node tree with growth realizations $\{(u_1, u_2), (d_1, u_2), (u_1, d_2), (d_1, d_2)\}$ that have equal probabilities, where $u_n \equiv \mu_n + \sigma_n$ and $d_n \equiv \mu_n - \sigma_n$ are chosen to match the expected dividend growth and volatility of asset n. Under the inexperienced investors' probability measure, the probabilities are set to $p_{2,t}/2$ for the first two nodes and to $\big(1 - p_{2,t}\big)/2$ for the last two nodes, with $p_{2,t}$ chosen to match the inexperienced investors' perceived dividend growth, $\mu_{2,t}$.

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