The Incentive Channel of Capital Market Interventions*

[PRELIMINARY AND INCOMPLETE]

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March 15, 2017

Abstract

We study how policy interventions designed to jump-start liquidity in frozen collateralized lending markets affect the private incentives to maintain or produce high-quality assets. Interventions may reinforce or destroy private incentives depending on whether expected future liquidity increases the relative value of owning a high-quality asset. When adverse selection stems primarily from uncertainty about returns on investment opportunities, intervention boosts private incentives and leads to faster recovery. In contrast, if adverse selection stems from the collateral value of assets, markets become subject to "intervention traps" – expectations concerning future interventions eliminate private incentives to improve the quality of collateral, which stunts recovery and warrants continued market intervention. The adverse effects of interventions may therefore be particularly pronounced in settings where dispersed collateral values are at the root of market breakdowns to begin with.

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1 Introduction

Monetary authorities increasingly rely on asset purchase programs to stimulate economic activity and to restore liquidity in financial markets during times of stress. These programs, which initially focused on purchases of government debt, have more recently expanded to include a broader set of private sector financial assets such as corporate bonds and mortgage-backed securities. For example, the European Central Bank began trading investment grade corporate bonds for government debt with higher collateral capacity using the Eurosystem's Corporate Sector Purchase Programme starting June 8, 2016. Such interventions raise the concern that the opportunity to pass of relatively low-quality assets to monetary authorities may affect the incentives of private agents to produce high-quality assets in the first place, thereby hampering the recovery of impaired markets.

We explore this incentive channel of capital market interventions in a parsimonious dynamic model of collateralized lending under asymmetric information. Market liquidity hinges on investors' beliefs about the value of collateral. As a result, large aggregate shocks to asset quality may result in the breakdown of lending markets impaired by severe adverse selection. This provides the scope for the government to improve welfare through capital market intervention that can restore market functions essential to the financing of productive investment opportunities in the economy.

While intervention leads to immediate gains by supporting lending markets, long-term implications critically depend on how it interacts with financial markets' own devices to endogenously self-repair and improve liquidity. To analyze this, we introduce two channels through which markets endogenously recover from episodes of illiquidity. First, market participants can exert costly effort to maintain or improve the quality of their collateral. Second, market participants may voluntarily exit capital markets to pursue an outside option, which may be particularly attractive to owners of low-quality assets who reap lower benefits from future market liquidity than the owners of good assets. We show that in the absence of interventions, both channels may boost average asset quality and lead to an endogenous recovery of market liquidity.

Whether such market discipline alone is sufficient to restore liquidity depends on the size of the initial asset quality shock and private effort incentives. Naturally, private incentives are determined by the difference in value of good and bad assets (*value differential*). In our setting, good assets and bad assets may differ both in their collateral capacity (*collateral differential*) and their expected returns conditional on investment (*return differential*). Agents can only consume the collateral value of the asset when markets are illiquid, but can invest and earn the expected return when markets are liquid. As a result, the value differential is equal to the collateral differential when markets are illiquid. When markets are liquid, adverse selection implies that the fraction of collateral that must be pledged conditional on market liquidity is higher when average asset quality is lower. Hence the value differential is given by the sum of the return differential and the collateral differential adjusted by an adverse selection discount. Effort incentives are thus determined by expectations of future market liquidity.

Specifically, future market liquidity increases effort incentives today if the return differential is higher than the collateral differential. Since market liquidity itself depends on average asset quality, the model features an effort complementarity: each agent is more willing to exert effort if others do, too. When current asset quality is not too low, this complementarity is sufficient for an endogenous recovery of market liquidity. If instead the collateral differential is larger than the return differential, future market liquidity may hamper current effort incentives. The reason is that the value differential is now approximately equal to the (small) return differential, and may thus not be large enough to sustain effort. While this effect weakens the recovery of average asset quality, the competitive equilibrium nevertheless features a strictly positive probability of market recovery.

In this setting, we study the dynamic effects of market interventions designed to alleviate market breakdowns. We endow the regulator with a simple objective function: to intervene whenever markets inefficiently break down within a given period. Period-by-period, the efficiency gain from such interventions stems from the notion in Holmstöm and Tirole (1998) that the government can use its taxation power to render cash flows pledgeable that could otherwise not be pledged. We then study the dynamic implications of such statically optimally policies. In line with the intuition outlined above, our basic result is that interventions may hurt or help the recovery of asset quality depending on whether or not future liquidity boosts effort incentives today. If liquidity and effort incentives are complementary, then interventions lead to a faster recovery; if they are substitutes, then interventions lead to a slower recovery. In the latter case, adverse dynamic effects of interventions can be severe. In particular, we find that markets may enter an "intervention trap," in which agents exert no effort given that they expect regulators to intervene, owners of bad assets do not exit, asset quality does not improve, and regulators are forced to intervene time and again.

This observation is fairly general, in that it applies to all policies that achieve their intended goal of alleviating market breakdowns. Indeed, it also applies to conventional monetary policy that improves market liquidity by lowering the risk-free rate. Our findings thus suggest that the incentive effects of market interventions may be particularly harmful when dispersion in collateral values is high. Through the lens of our model, we thus caution against the expansion of asset purchase programs to risky and opaque asset classes. Interestingly, these are perhaps the asset classes where adverse selection and market breakdowns are likely to be of greater concern ex-ante.

In a second step, we ask how the design of interventions affect the evolution of asset quality. We first show that internally financed interventions may drive effort incentives down below those of the competitive equilibrium even when externally financed interventions would boost incentives above the competitive equilibrium. The reason is that ex-post taxation of market participants leads to an endogenous risk-shifting motive by shifting tax incidence to the owners of high-quality assets. Second, we argue that policies that are *not* conditional on future asset quality – due to information frictions or announcement lags – may further reduce effort incentives by removing the effort complementarity that underpins market discipline. These incentive costs of market intervention must be traded off against the benefits of improving market liquidity within each period.

Related Literature. Our paper is most closely related to Tirole (2012) and Philippon and Skreta (2012), who study how public interventions can jump-start markets frozen due to adverse selection, and how to best design such interventions. Their focus lies on studying how participation constraints for the government program depend on the endogenous response of the competitive allocation. Fuchs and Skrzpacz (2015) consider a similar setting in which private trading occurs dynamically, and show an initial subsidy and subsequent tax on trade can improve allocations, fixing the distribution of asset quality. Our approach is to analyze how the expectation of future interventions affects private incentives to produce high-quality collateral, and to study conditions under which interventions may either lead to a sustained lack of high-quality collateral, or complement private effort incentives.

A key feature of our model is that all lending must be collateralized as in Gorton and Ordoñez (2014) or Fostel and Geanakoplos (2015). This structure implies a divergence between pledgable cash flows and those that are inalienable from the asset holder. When markets are liquid, collateral is pledged at a pooling price, thereby diluting the private value of holding a good collateral asset and reducing effort incentives. Hence it is precisely when there is dispersion in collateral values that expected market liquidity hampers effort incentives.

Choi, Santos, and Yorulmazer (2016) also study a model of collateralized lending and ask which types of collateral central banks should lend against to boost asset market liquidity. They show that when the central bank can distinguish between good and bad collateral, policies demanding good collateral in exchange for liquidity provisions impose negative externalities on private markets. In our setting, we assume that the central bank is as as uninformed as financial market participants when assessing collateral quality, and we study the endogenous dynamics of asset quality under interventions.

Layout. Section 2 lays out the model environment and discusses preliminary results that will guide our analysis. Section 3 use a baseline two-period model to discuss the key channels affecting the evolution of asset quality. Section 4 discusses the incentive channel of public interventions, while Section 5 characterizes the evolution of asset quality in multi-period dynamic settings. Section 6 concludes. All proofs are in Appendix A.

2 Setting

2.1 Environment

Time is discrete and finite. Periods are indexed by $t = 1, 2, \dots, T$. There is a measure one of *farmers*, a measure one of *investors*, and a *regulator*. All agents are risk-neutral with discount factor $\beta \leq 1$. Farmers are long-lived and are each endowed with a tree of quality θ, which can be either *good* or *bad*, $\theta \in \{g, b\}$. Trees represent investment opportunities. Trees require 1 unit of water (capital) to produce fruit for consumption. Investors live for one period only and are the only agents endowed with water (capital). Farmers can thus invest only if they successfully borrow from investors. Investors have access to a risk-free investment opportunity with return r_f. This risk-free return thus determines investors' outside option when deciding whether to lend to farmers. If farmers obtain capital from investors, the returns on investment in a tree are as follows.

Assumption 1 (Payoffs)**.** *A tree of type θ produces a return of*

$$\tilde{R}_\theta = \begin{cases} R_\theta & w.p. \quad p_\theta \\ 0 & w.p. \quad 1 - p_\theta \end{cases}$$

where $p_g \geq p_b$, $E[\tilde{R}_g] \geq E[\tilde{R}_b]$, and $E[R_g] > 1$.

That is, good farmers are weakly more likely to be successful than bad farmers, earn a weakly higher expected return than bad farmers, and the NPV of investing in a good tree is strictly positive. We are agnostic with respect to the NPV of investing in a bad tree.

Each tree requires inalienable farmer human capital. As a result, trees are not traded. Trees can, however, be chopped down for consumption as wood at the end of every period. A tree of type θ yields L_θ units of wood. When trees are chopped down, the corresponding farmer exits. Wood is transferable to investors. As we describe below, wood will therefore serve as collateral, and we refer to L_θ as a tree's collateral value. We assume that only good trees have enough collateral value to sustain borrowing of the required 1 unit of capital, and that only farmers know the type of their tree.

Assumption 2 (Collateral Quality)**.** $L_g > 1 + r_f \geq 1 > L_b$.

Assumption 3 (Asymmetric Information)**.** *Only farmers know the type of their tree.*

These two assumptions combine to produce an adverse selection problem, such that lending markets will be liquid only if the average quality of assets is not too low.

2.2 Interpretation of the Production Technology

We now briefly discuss and interpret the production technology. If investment takes place, we can write the high-state payoff as $R_\theta + L_\theta$, while the low-state payoff is L_θ. Hence we can interpret L_θ as a risk-free salvage value of the production technology, and R_θ as a risky growth option. Two aspects of this assumption are of note. First, only the salvage value can be pledged as collateral. This is consistent with the results in Fostel and Geanakoplos (2015) that only the worst-case return is pledgeable in the optimal contract in binomial economues. Second the salvage value of the production technology can be consumed even when there is no investment. This means that it should be interpreted as the value of assets in place similar to the legacy value of assets in Tirole (2012) and Philippon and Skreta (2012).

We can thus summarize the difference in expected returns across θ-types by the *productivity differential* $\Delta Y = p_g R_G + L_g - p_b R_b - L_g$. This productivity differential can then be decomposed into two parts. The first is the *return differential*

$$\Delta \tilde{R} = p_g R_g - p_b R_b$$

summarizing the difference in the expected payoffs of each type's growth option. The second is the *collateral differential*

$$\Delta L = L_g - L_b,$$

which summarizes the difference in collateral values. The relative size of the collateral differential and the return differential will turn out the be the key determinant of monitoring incentives with and without government interventions.

2.3 Capital Market

Farmers must obtain financing from investors, who competitively offer debt financing. As outlined above, we assume that the returns from investment are not directly pledgeable. Instead, farmers must pledge the wood that can be obtained from chopping down trees as collateral. Investors offer one-period debt contracts $\{\chi_t, B_t\}$ with liquidation rights, where χ_t is the fraction of the wood owed to investors conditional on forced liquidation, and B_t the face value of debt. Investors' option to liquidate is triggered when a farmer fails to pay B_t.

The aggregate realization of λ_t is assumed to be observed by all agents prior to markets opening, but the individual type is not. Lending contracts can therefore not be contingent on the farmer's type. Since the returns on investment are not pledgeable, moreover, contracts must be incentive compatible given the farmer's option to default and surrender his collateral. Upon success, a farmer weakly prefers paying B_t to defaulting if B_t is lower than the *maximum sustainable debt level*, $\bar{B}_{\theta,t}(\chi_t)$, i.e.

$$B_t \leq \bar{B}_{\theta,t}(\chi_t, e_{\theta,t}) \equiv \tilde{V}_{\theta,t}(e_{\theta,t}) - (1-\chi_t)L_\theta.$$

Accordingly, the effective repayment received by investors when the farmer's investment was successful is:

$$\tilde{B}_{\theta,t}(\chi_t, e_{\theta,t}) = \begin{cases} B_t & \text{if} \quad B_t \leq \bar{B}_{\theta,t}(\chi_t, e_{\theta,t}) \\ \chi_t L_\theta & \text{if} \quad B_t > \bar{B}_{\theta,t}(\chi_t, e_{\theta,t}). \end{cases}$$

Given a required return of r_t, investors' break even condition conditional on λ_t is:

$$\lambda_t \left[p_g \tilde{B}_{g,t}(\chi_t, e_{\theta,t}) + (1-p_g)\chi_t L_g \right] + (1-\lambda_t) \left[p_b \tilde{B}_{b,t}(\chi_t, e_{\theta,t}) + (1-p_b)\chi_t L_b \right] = 1 + r_t. \quad (1)$$

The funding capacity of farmers thus depends on (i) the fraction of good assets, (ii) farmers' effort choices, and (iii) the continuation value of trees. All three will be determined in equilibrium.

2.4 The Evolution of Asset Quality

Due to asymmetric information about tree quality, lending markets will be liquid only if a sufficiently large number of trees are of good quality. Hence, the key state variable in our setting is the fraction of good trees at the beginning of every period period, which we denote by λ_t. Accordingly, this section describes the evolution of this average asset quality.

The first channel through which asset quality evolves is that farmers can exert effort to improve (or maintain) the quality of their trees. Specifically, each farmer can choose to privately exert effort $e \in \{0, 1\}$ at non-pecuniary cost c at the end of every period. The quality of a farmer's tree in period $t+1$ depends on the farmer's effort decision and the current quality of the tree. Specifically, we assume that

$$Prob\left[\theta_{t+1} = g|\theta_t, e\right] = \begin{cases} 1 & \text{if } \theta_t = g \text{ and } e = 1 \\ 1-\pi_g & \text{if } \theta_t = g \text{ and } e = 0 \\ \pi_b & \text{if } \theta_t = b \text{ and } e = 1 \\ 0 & \text{if } \theta_t = b \text{ and } e = 0 \end{cases} \tag{2}$$

The parameters π_g and π_b thus denote the incremental benefit of effort for maintaining or producing a good tree. We assume the outcome of each farmer's effort is independently distributed, so that there is no randonmess in the aggregate law of motion for λ_t conditional on effort decisions. We denote the equilibrium *value of a tree of type* θ at the beginning of period t by $V_{\theta,t}$, and define the value difference to be $\Delta V_t \equiv V_{g,t} - V_{b,t}$. Given that the quality of each individual tree tomorrow is a random variable whose distribution depends on the current quality, we denote the *continuation value* of holding a tree of type θ once production has occurred (i.e., at the end of Stage 3 in Figure 1 below) by, respectively,

$$\tilde{V}_{g,t}(e_{g,t}) = \max\left\{\beta\left(V_{g,t+1} - (1-e_{g,t})\pi_g \Delta V_{t+1}\right) - e_{g,t}c, L_g\right\}$$

and

$$\tilde{V}_{b,t}(e_{b,t}) = \max\left\{\beta\left(V_{b,t+1} + e_{b,t}\pi_b \Delta V_{t+1}\right) - e_{b,t}c, L_b\right\},$$

where $e_{\theta,t}$ denotes the period t effort choice of type θ. The max operator ensures that $\tilde{V}_{g,t}(e_{g,t}) \geq L_\theta$ because the tree can be always be chopped down for consumption at the end of a period. Note that $\tilde{V}_{\theta,T}(e_{\theta,T}) = L_\theta$ for any $e_{\theta,T}$ because there is no production after period T. The continuation value is thus equal to the future value of the tree to its owner. Because investors can always liquidate the tree once collateral has been posted, it also represents farmers' potential losses from liquidation, and thus the degree to which fruit can be made contractible by the threat of liquidation conditional on no ex-post renegotiation. If instead contracts must be renegotiation-proof, then farmers could never credibly pledge more than the posted collateral. We discuss how our results depend on the choice of contracting convention as we go along.

The second channel affecting the evolution of asset quality is the liquidition of trees. Liquidation occurs either because farmers find it in their private interest to chop down their trees and exit (voluntary liquidation), or because investors force liquidation of trees posted as collateral by farmers after failure (involuntary liquidation).[1] Denote the fraction of θ-farmers who borrow and invest by $\phi^I_{\theta,t}$, and the fraction of θ-farmers who voluntarily liquidate conditional on not being involuntary liquidated by $\phi^{VL}_{\theta,t}$. Then the fraction of θ-farmers who remain once liquidation decisions have been made is

$$\phi^R_{\theta,t}\left(\phi^{VL}_{\theta,t},\phi^I_{\theta,t}\right) \equiv \left(1-\phi^{VL}_{\theta,t}\right)\left[1-\phi^I_{\theta,t}(1-p_\theta)\right]. \tag{3}$$

Farmers who exit through liquidation are replaced by an equal mass of new entrants so as to keep the total mass of farmers constant at one. We assume that entry is random process, with a fraction δ of new farmers entering with a good tree. Here, δ is a continuous random variable distributed according to p.d.f. f with support $[\underline{\delta},\bar{\delta}]$ and associated c.d.f. F. This assumption is shorthand for a richer model of entry and exit where farmers must pay a fixed cost of entry prior to knowing the productivity of their assets.

The realization of δ is publicly observed, and the remaining non-liquidated farmers make their effort choice conditional on this observation.[2] We denote the fraction of remaining θ-farmers who exert effort by $\phi^E_{\theta,t}$. As a result, the fraction of good farmers at the beginning of period $t+1$ conditional on δ follows the law of motion

$$\lambda_{t+1} = \left[\phi^R_{g,t}(1-\pi_g+\phi^E_{g,t}\pi_g)+(1-\phi^R_{g,t})\delta\right]\lambda_t + \left[\phi^R_{b,t}\phi^E_{b,t}\pi_b+(1-\phi^R_{b,t})\delta\right](1-\lambda_t) \tag{4}$$

The evolution of λ is thus determined by farmers' effort choices and the replacement of liquidated trees by new entrants of random quality.

[1] Note that this implies that involuntary liquidation can only arise if markets were liquid at the beginning of the period.

[2] Assuming that effort takes place conditional on the realization of δ simplifies the analysis of the effort choice but does not affect the economic mechanism.

2.5 Timing

The timing within each period $t < T$ is summarized in Figure 1. Period T ends in stage 5 with all farmers liquidating their trees. Liquidations and δ are pre-determined at the time of the effort choice. Agents thus know that there will be at least

$$\underline{\lambda}_t \equiv \left[\phi^R_{g,t}(1-\pi_g) + (1-\phi^R_{g,t})\delta\right]\lambda_t + (1-\phi^R_{b,t})\delta(1-\lambda_t) \tag{5}$$

good trees in period $t+1$. We will therefore sometimes work with the reduced-form law of motion

$$\lambda_{t+1} = \underline{\lambda}_t + \phi^E_{g,t}\pi_g\phi^R_{g,t}\lambda_t + \phi^E_{b,t}\pi_b\phi^R_{b,t}(1-\lambda_t) \tag{6}$$

in order to analyze the effort choice.

1. Quality of trees determined. Aggregate shock γ realized.
2. Investors receive endowment.
3. Borrowing and lending using collateralized debt contracts. Investment.
4. Output of fruit realized.
5. Voluntary and involuntary liquidation of trees. Accounts settled and consumption.
6. Liquidated trees replaced by new farmers. Average quality δ realized.
7. Effort choice of non-liquidated farmers..

Figure 1: Timing of Events

2.6 Preliminary Analysis

In this section, we state three preliminary results that will guide the analysis going forward.

First, we describe the conditions under which farmers are willing to exert effort. Since the benefit of effort is that it allows farmers to upgrade or maintain the quality of their tree in the next period, the value of effort is pinned down by the difference in the value of good trees and bad trees tomorrow. The increment in the probability of having a good tree after exerting effort is π_θ. A farmer of type θ thus exerts effort in period t if

$$c \leq \bar{c}_{\theta,t+1} \equiv \pi_\theta \Delta V_{t+1}. \tag{7}$$

Second, we turn to the liquidity of capital markets under the assumption that farmers offer a pooling contract (B_t, χ_t). We later verify that there is indeed no separating equilibrium. Given the pooling contract, each farmer's pledgeable income is strictly increasing in χ_t, since both the maximum sustainable debt level $\tilde{B}_{b,t}(\chi_t, e_{\theta,t})$ and the pledged collateral is strictly increasing in χ_t. Farmers' *aggregate* pledgeable income in turn is increasing in λ_t. Given an effort decision, farmers are able to obtain funding in period t only if

$$\lambda_t \geq \bar{\lambda}_t \equiv \frac{1 + r_f - \left[p_b \tilde{B}_{b,t}(1, e_{\theta,t}) + (1 - p_b) L_b\right]}{\left[p_g \tilde{B}_{g,t}(1, e_{\theta,t}) + (1 - p_g) L_g\right] - \left[p_b \tilde{B}_{b,t}(1, e_{\theta,t}) + (1 - p_b) L_b\right]} \tag{8}$$

The numerator depicts the degree to which the expected repayments from bad farmers fall short of the required interest rate given $\chi_t = 1$. The denominator depicts the difference in expected repayments between good and bad farmers $\chi_t = 1$. Markets can thus not sustain borrowing if the fraction of low-quality borrowers is too high.

Third, we turn to the farmer's decision of whether to invest in period t if they are able to obtain funding. The key downside to investing is that the risk of failure exposes farmers to the risk of involuntary liquidation. As a result, farmers invest only if $p_\theta \left[R_\theta - B_t + \tilde{V}_{\theta,t}\right] + (1 - p_\theta)(1 - \chi_t) L_\theta \geq \tilde{V}_{\theta,t}$. This condition can be rearranged to give the participation constraint

$$(1 - p_\theta)\tilde{V}_{\theta,t} \leq p_\theta(R_\theta - B_t) + (1 - p_\theta)(1 - \chi_t) L_\theta. \tag{9}$$

The condition states that farmers do not want to participate if the expected loss from liquidation (as measured by the continuation value of the tree) is larger than the flow payoff from investing in period t.

3 Baseline Two-period Model

In this section, we use a baseline two-period model ($T = 2$) to transparently lay out the main mechanisms that determine market liquidity and output. In later sections, we highlight how these mechanisms carry over to longer time horizons.

Prior to analyzing the model with asymmetric information, we characterize the perfect information benchmark. If investors observe the type of each farmer, then contracts can be fully contingent on the farmer's type. Even though the effort decision is unobservable, type-contingent contracts are sufficient to attain the first-best outcome.

Proposition 1. *When the tree types are observable to investors, the first best outcome is attainable.*

The intuition is contracts will sufficiently type-dependent to deter excessive investment by bad types while allowing farmers to retain enough skin in the game to make efficient effort decisions.

3.1 Asymmetric Information without Government Intervention

The perfect information case shows that even though the effort choice is unobservable and contracts are not fully enforceable, type-contingent contracting is sufficient to obtain the efficient outcome. We now introduce asymmetric information concerning the type of farmers' trees. A key implication is that investors are unable to distinguish between good and bad farmers, giving rise to adverse selection.

To begin, we characterize the $t = 2$ continuation value conditional on market liquidity, $V_{\theta,2}(\lambda_2)$. When markets are illiquid at time $t = 2$ (that is, $\lambda_2 < \bar{\lambda}_2$) then farmers' payoffs are solely determined by the liquidation value of their trees: $V_{\theta,2} = L_\theta$. When markets are liquid (that is, $\lambda_2 \geq \bar{\lambda}_2$), farmers' borrowing capacity is given by $\tilde{B}_{\theta,2} = \chi_2 E[L_\theta]$, where χ_2

is determined such that investors earn a required return r_f:[3]

$$\chi_2^*(\lambda_2 L_g + (1-\lambda_2)L_b) = 1 + r_f$$

A θ-farmer's expected payoff conditional on markets being liquid is

$$V_{\theta,2} = p_\theta R_\theta + (1-\chi_2^*)L_\theta. \tag{10}$$

It follows that the utility difference between good and bad farmers in the second period is

$$\Delta V_2(\lambda) = \begin{cases} \Delta L & \text{if} \quad \lambda_2 < \bar{\lambda}_2 \\ \Delta\tilde{R} + \left(1 - \frac{1+r_f}{L_b+\lambda_2(L_g-L_b)}\right)\Delta L & \text{if} \quad \lambda_2 \geq \bar{\lambda}_2. \end{cases}$$

Since farmers exert effort if $c \leq \bar{c}_{\theta,2} \equiv \beta\pi_\theta \Delta V_2(\lambda)$, effort incentives are partly determined by the average asset quality λ_2. A closer look at the effort condition reveals an important insight: conditional on markets being liquid, farmers' effort choices are *complementary*. The proof follows from noting that χ_2 is strictly decreasing in λ_2 in the interval $[\bar{\lambda}_2, 1]$.

Lemma 1. *Suppose $\lambda_2 > \bar{\lambda}_2$. Then $\Delta V_2(\lambda)$ strictly increases in λ.*

Complementaries between farmers' effort arise due to the joint determination of effort and market liquidity. Market liquidity enhance the value of having a high quality assets, as the marginal value of a good tree is greater conditional on investments being feasible and lower adverse selection corresponds to a lower subsidization from high to low type farmers. As more farmers exert effort, the average quality of assets rises, which in turn improves liquidity.

Going forward, we focus on the interesting case in which the cost of effort is sufficiently high such that farmers' do not necessarily exert effort and equilibrium effort decisions are partially pinned down by the expected aggregate evolution of asset quality.

[3] In the final period, borrowing capacities are constrained by the collateral value, since no future periods of investment remain. Note also that the face-value of the contract is indeterminate, since all farmers will default if the required interest payment is too large. However, the de-facto repayment is pinned down by the farmer's optimal default decisions.

Assumption 4. $c > \min\{\Delta L, \Delta\tilde{R}\}$

Under Assumption 4, two main cases arise. If $\Delta L > \Delta\tilde{R}$, then effort incentives are primarily determined by the collateral differential, and farmers are willing to exert effort only if markets are illiquid tomorrow with sufficiently large probability or if χ_2 is sufficiently low. If instead $\Delta\tilde{R} > \Delta L$, then market liquidity tomorrow boosts effort incentives today, while future illiquidity tends to hamper effort incentives. That is, future liquidity has different effects on current effort incentives depending on whether good trees differ from bad trees primarily due to their collateral differential or their return differential. As we discuss below, the implications for effort of market interventions that boost future liquidity will naturally inherit the same properties.

Yet even in the case where the value differential is primarily driven by the return differential ($\Delta\tilde{R} > \Delta L$) it is not sufficient for markets to be merely liquid tomorrow in order for farmers to exert effort. In particular, since $c > \Delta\tilde{R}$, average asset quality tomorrow must attain at least the threshold value $\hat{\lambda}_t$ defined by

$$\beta\pi_\theta\Delta V_2(\hat{\lambda}_t) = c \tag{11}$$

with $\hat{\lambda}_t \geq \bar{\lambda}_t$. Hence effort is incentive compatible and markets benefit from effort complementarity only if the average asset quality is expected to reach a fixed threshold tomorrow. Naturally, this complementarity operates through the share of trees that must be pledged tomorrow, χ_2, which is decreasing in λ_2.[4]

To ensure that coordinated effort is a viable means of improving average asset quality, we assume that there exists some $\lambda_t' \leq \bar{\lambda}_t$ such that when all farmers coordinate effort, it is in fact incentive compatible.

Assumption 5. *π_θ is sufficiently large such that $\bar{\lambda}_t + (1-\bar{\lambda}_t)\pi_\theta > \hat{\lambda}_t$*

For simplicity, we also assume for now that both types of farmers face the same incentive constraint.

Assumption 6 (Identical payoff to effort)**.** $\pi_g = \pi_b = \pi$.

[4]Note also that since the average quality tomorrow is partly determined by δ, the effort decision is probabilistic at the time of voluntary liquidation, and deterministic after observing δ.

This simplification allows us to focus on symmetric effort decisions, and thereby highlight how aggregate coordination motives shape the evolution of asset quality. In order to highlight when miscoordination may occur, we focus on selecting equilibria with coordination whenever possible. We call an equilibrium in which all farmers find it incentive compatible to exert effort as a *coordinated equilibrium*:

Definition 1. *A* coordinated equilibrium *is an equilibrium in which all farmers exert effort.*

Against this background, we begin our analysis by studying how markets can endogenously recovery from episodes of illiquidity. To this end, we assume that the initial assset quality is such that markets must be illiquid in period 1.

Recovering from Illiquidity. Suppose that markets are illiquid today, i.e. $\lambda_t < \bar{\lambda}_t$. Though markets are illiquid today, if $\lambda_t \geq \lambda_t'$, coordinated effort occurs in equilibrium, which dramatically improves tomorrow's asset quality and restores market liquidity. Specifically, given that all farmers remain in the market and exert effort, asset quality tomorrow is

$$\lambda_{t+1} = \pi + (1-\pi)\lambda_t.$$

Since for any $\lambda_t > \lambda_t'$, $\pi + (1-\pi)\lambda_t \geq \bar{\lambda}_t$, asset quality is sufficiently high such that markets recover: [5]

Proposition 2. *Suppose that $\lambda_t > \lambda_t'$. Then, a coordinated equilibrium exists with liquid markets at time $t = t+1$.*

In other words, a coordinated equilibrium exists as long as the initial asset quality is not too poor, and farmers find it incentive-compatible to collectively improve asset quality through effort, thereby restoring market liquidity in the next period.

Next, we consider cases in which asset quality is severely impaired initially (i.e. $\lambda_t \leq \lambda_t'$). An important distinction arises between two cases: (i) $\Delta L < c < \Delta\tilde{R}$ and (ii) $\Delta L > c > \Delta\tilde{R}$. Consider the first case in which $\Delta L < \Delta\tilde{R}$. This condition implies that farmers *only* find it optimal to exert effort when markets are liquid and when adverse selection is expected to

[5] If contracts are renegotiation proof, then $\bar{\lambda}_t$ is time-invariant.

be sufficiently low tomorrow. As a result, when $\lambda_t \leq \lambda_t'$, illiquidity is so severe that effort alone is not sufficient to recover. Fortunately, we can show that severe illiquidity can induce farmers to voluntary exit, which allows for asset quality to improve through the entry of new farmers.

To illustrate this mechanism, assume and verify the existence of a candidate equilibrium in which all good farmers choose to remain in the market (i.e. $\phi_{g,1}^{VL} = 0$), a positive fraction of bad farmers choose to exit at $t = 1$ (i.e. $\phi_{b,1}^{VL} > 0$).[6] This directly corresponds to $\phi_{g,t}^{R} = 1$ and $\phi_{b,t}^{R} = \left(1 - \phi_{b,t}^{VL}\right)$, and $\underline{\lambda}_t = (1 - \pi_g)\lambda_t + \delta\phi_{b,t}^{VL}(1 - \lambda_t)$. Given that all remaining farmers exert effort, tomorrow's asset quality is given by:

$$\lambda_{t+1} = \pi + (1 - \pi)\lambda_t + (\delta - \pi)\phi_{b,t}^{VL}(1 - \lambda_t) \tag{12}$$

To show that voluntary liquidations can improve market liquidity, assume that $\bar{\delta}$ be such that

$$\pi + (1 - \pi)\lambda_1 + (\bar{\delta} - \pi)(1 - \lambda_1) > \lambda_2', \tag{13}$$

and define the cutoff $\delta^*(\phi_{b,1}^{VL})$ such that

$$\pi + (1 - \pi)\lambda_1 + (\delta^*(\phi_{b,1}^{VL}) - \pi)\phi_{b,1}^{VL}(1 - \lambda_1) = \bar{\lambda}_2. \tag{14}$$

Conditional on $\phi_{b,1}^{VL}$, farmers then exert effort if and only if $\delta \geq \delta^*(\phi_{b,1}^{VL})$, and markets are liquid when they do so. Condition (13) ensures that there exists $\delta \in [\underline{\delta}, \bar{\delta}]$ such that this condition holds for at least one $\phi_{b,1}^{VL} \in [0, 1]$.

Now, consider the liquidation decision of bad farmers. A bad farmer's payoff to exiting in period 1 is $V_{b,1}^{VL} = L_b$. The payoff to staying in the market conditional on $\phi_{b,1}^{VL}$ is

$$V_{b,2}(\phi_{b,1}^{VL}) = \beta \int_{\underline{\delta}}^{\delta^*(\phi_{b,1}^{VL})} L_b dF(\delta) + \beta \int_{\delta^*(\phi_{b,1}^{VL})}^{\bar{\delta}} \left[\tilde{p}\tilde{R} + \left(1 - \frac{1 + r_f}{L_b + \lambda_2^*(L_g - L_b)}\right)\tilde{L} - \frac{c}{\beta}\right] dF(\delta) \tag{15}$$

where λ_2^* is the equilibrium asset quality in period two conditional on δ, $\phi_{b,1}^{VL}$, and the

[6] This candidate equilibrium is in the same spirit as a coordinated equilibrium, since only through the voluntary exit of bad farmers does average asset quality improve.

optimal effort decision, and ˜ denotes the random project quality of the individual farmers' tree.Hence, $\phi_{b,1}^{VL}$ fraction of bad farmers choose voluntary liquidation such that $V_{b,2}(\phi_{b,1}^{VL}) = L_b$. We show that with a combination of voluntary exit and effort, market liquidity is restored in expectation:

Proposition 3 (Self-cleansing through exit)**.** *Suppose that markets are illiquid at $t = 1$. Then, a fraction $\phi_{b,1}^{VL}$ of low-quality farmers voluntarily exit and remaining farmers exert effort such that markets are liquid $t = 2$ with probability $1 - F(\delta^*(\phi_{b,1}^{VL}))$.*

Proof. If no bad farmer voluntarily exits, then $\delta^*(0) > \bar{\delta}$ and markets are illiquid tomorrow with probability one. But then $V_{b,2} = \beta L_b > L_b$, and all bad farmers prefer to exit. Hence $\phi_{b,1}^{VL}$ must be such that $V_{b,2}(\phi_{b,1}^{VL}) = V_{b,1}^{VL}$. This requires $\phi_{b,1}^{VL} > 0$ and that markets open with positive probability. □

Since ΔV_2 is positive and increasing in λ_2^*, it follows that good farmers must strictly prefer to stay rather than liquidate. If condition (13) is satisfied, then there exists an equilibrium in which bad farmers voluntarily liquidate, and markets become liquid with positive probability in the next period. If instead the condition is not satisfied (which is the case if $\lambda_1 = 0$ and $\bar{\delta} - \pi < \bar{\lambda}_2$, for example), then exit and effort are not sufficient to restore markets in period 2. As we highlight below, market interventions may interfere with voluntary liquidations by raising the continuation value of all farmers.

Next, we characterize equilibria for the case where $\Delta\tilde{R} < \Delta L$ and $\lambda_t < \lambda_t'$. Here, effort coordination is non-monotonic with the level of adverse selection. When the value differential is primarily determined by the collateral differential, then a coordinated equilibrium arises, as $c < \beta\pi\Delta L$. However, for barely liquid market conditions, effort incentives are destroyed, as farmers' payoffs are predominately determined by asset payoff $\tilde{R}$, which are not as different. To highlight this difference, we assume that $L_b < \beta(L_b + \pi\delta L)$, which ensures that no voluntary liquidation occurs.

Proposition 4. *Define $\lambda_t'' + \pi(1 - \lambda_t'') = \bar{\lambda}_t$ and suppose that $\Delta\tilde{R} < \Delta L$.*

- *for $\lambda_t < \lambda_t''$, all farmers exert effort, and markets are illiquid at time $t = t + 1$;*
- *for $\lambda_t \in (\lambda_t'', \lambda_t')$, a fraction $\alpha(\lambda_t, \delta_t)$ fraction of framers exert effort, and markets are liquid at time $t = t + 1$ with probability ρ.*

An important difference from the previous section is that the value of improving the liquidation value is sufficiently valuable such that even if the market is illiquid, farmers find it incentive compatible to exert effort. Because $\Delta\tilde{R} < c$, farmers find it optimal to *shirk* conditional on expectations that markets are to be liquid in the next period. This brings rise to a partial effort equilibrium with probabilistic recovery from illiquidity.

3.2 Discussion

In the model, we showed how markets coped with severe adverse selection absent market intervention. If the initial asset quality is not too poor, then farmers could collectively improve market quality and restore market functions in the next period.

If asset quality is severely impaired to begin with, then market liquidity could be restored through a combination of voluntary exit and effort. This was achieved primarily through two channels. First, when discounted future expected payoff is lower than the liquidation value of a bad tree, bad farmers opt to voluntarily exit, allowing for average asset quality to stochastically improve with new entrants. Second, when effort is incentive compatible even if markets are expected to be illiquid, partial effort exerted by farmers endogenously improve the quality of existing trees. Together, market participants endogenously improve market integrity. Moreover, effort incentives were shaped by whether the value differential was primarily determined by the collateral differential or by the return differential, as this distinction determined whether future expected liquidity harmed or helped current effort incentives. This feature will be crucial in understanding the effects of public interventions precisely because their goal is to alter future market liquidity.

4 Restoring Markets through Intervention

In the previous section, we showed how markets could recuperate from large adverse shocks by endogenously improving market quality, either through liquidation of bad assets or improving existing stock of assets. The analysis of our setting absent market intervention provides us with a useful benchmark to assess the marginal impact in the presence of market intervention. In this section, we analyze interventionist policies and evaluate the potential for market intervention to restore liquidity and their interactions with markets' self regulatory functions.

4.1 Market Stimulus

To evaluate the impact of government intervention, we extend the model by introducing a regulator that can implement policies to improve market liquidity.

The key inefficiency is the impairment of funding markets caused by adverse selection concerning collateral quality of farmers' trees. Given that a shortage of collateral is at the crux of market breakdown, a natural policy to consider is one in which the regulator injects safe collateral into the economy, which can relax the borrowing constraint faced by farmers and revitalize funding required for investments. This form of policy resembles central bank policies aimed at improving the collateral quality in markets in order to stimulate lending and stability in financial markets:

- Fed's Term Securities Lending Facility (stopped in 2010)
- ECB's Securities Lending Programme (continued).

When the binding constraint on investment is the lack of confidence in the safe return of investments, a central bank can reinforce funding markets by insuring market participants from downside resulting from adverse selection.

To study the impact of such policy, we first lay out a collateral-swap program that could loosen investors' participation condition, and correspondingly a policy rule that governs how the regulator implements the liquidity provision policy over time.

Specifically, suppose that the regulator can issue one period zero-coupon bonds with face value 1. The regulator can use these bonds to operate an repo program, which allows investors to ex-ante exchange the underlying collateral on their contracts with a risk-free bond at an exchange rate of q bonds per tree. In other words, the program offers contract to investors that offers payment in the form of government bonds in the case that farmers fail to pay out. As such, if a farmer's project succeeds, investors are paid back, and the government returns the tree. If the project fails, investors receive the risk-free bond, and the government chops down the tree.

The policy at hand is one which is aimed at encouraging investors to finance farmers, who may have positive NPV projects, but not sufficiently high quality collateral. By loosening investors' participation condition, this policy can be effective even at the zero lower bound (i.e. when $r_f = 0$), which allows us to infer how unconventional policies impact market functions. In Section 4.2, we specifically show how insights relay to policies aimed at affecting the effective interest rate, primarily through the risk-free rate.

Market intervention acts as a subsidy to investors to revitalize market funding conditions. Market intervention can improve welfare through two channels. First, intervention relaxes farmers' borrowing constraints that can prevent otherwise socially efficient investment. As such, relieving market illiquidity in the short-term directly improves welfare. Second, by restoring markets, market quality improves through forced liquidation, which bad farmers face with greater likelihood.

We consider the following policy implementation of market intervention:

Definition 2 (Contingent Liquidity)**.** *A policy of* contingent liquidity *is one in which the government intervenes if and only if at the beginning of the period, markets are illiquid* and *expected payoff from projects is positive.*

In other words, contingent liquidity policy is one in which the government intervenes if and only if the asset quality is too low to sustain markets.

We analyze the augmented two-period model by solving backwards, starting with $t = 2$. To simplify the analysis, we assume that renegotiation proof contracts are used, which limits the set of contracts to fully collateralized loans. Consider the first case in which $\lambda > \bar{\lambda}$. Since markets are liquid, trivially, government intervention does not occur. As such, the farmers' expected payoffs are equivalent to that in Equation 10:

$$V_{\theta,2} = p_\theta R_\theta + (1 - \chi_2^*) L_\theta$$

Consider the second case in which $\lambda < \bar{\lambda}$. Since markets are illiquid, market intervention occurs under Definition 2. In general, two conditions determine q. Given a required rate of return r_f, the government must offer q that is sufficiently large such that investors' incentive condition is satisfied, i.e. $q \geq \frac{1+r_f}{\chi_t^{INT}}$. In turn, investors must offer a contract that satisfies

$\chi_t^{INT} \leq 1$. Farmers' payoff is given by:

$$V_{\theta,2}^{INT} = p_\theta R_\theta + (1 - \chi_2^{INT}) L_\theta$$

The government minimizes costs by setting $q = 1 + r_f$, which also requires that investors offer $\chi_t^{INT} = 1$, implying:

$$V_{\theta,2}^{INT} = p_\theta R_\theta.$$

Combining the two cases, under intervention we have:

$$\Delta V_2(\lambda) = \begin{cases} p_g R_g - p_b R_b & \text{if} \quad \lambda_2 < \bar{\lambda}_2 \\ p_g R_g - p_b R_b + \left(1 - \frac{1+r_f}{L_b + \lambda_2 (L_g - L_b)}\right)(L_g - L_b) & \text{if} \quad \lambda_2 \geq \bar{\lambda}_2. \end{cases} \tag{16}$$

Given farmers' payoffs conditional on λ_2, we can analyze farmers' effort decisions in the presence of government intervention. Consider a farmer's incentive to exert effort. As before, a complementary exists when equilibrium $\lambda_2 \geq \bar{\lambda}_2$. Consequently, the same threshold λ_t', where coordinate effort occurs for $\lambda_t > \lambda_t'$ exists.

For $\lambda_t < \lambda_t'$, the introduction of market intervention results in two important changes. First, note that:

$$\beta p_\theta R_\theta > L_\theta \text{ for } \theta = g, b.$$

This implies that the continuation value may increases for all farmers. As a result, under a contingent liquidity policy, farmers find it more attractive to hold their assets.

Lemma 2. *Under market intervention, no farmers voluntarily exit.*

While continuation values have improved, intervention has a more complicated impact on farmers' incentives to exert effort. When $\lambda_t < \lambda_t'$, the impact of market intervention hinges on the nature of the economy. To analyze the overall interaction between market intervention and self-regulatory mechanisms, we consider the two main cases (i)$\Delta L < \Delta \tilde{R}$ and (ii) $\Delta L > \Delta \tilde{R}$.

Consider the first case when $\Delta L < \Delta\tilde{R}$. Under this condition, continuation values monotonically increase for all farmers. In addition, farmers' outcomes improve even when $\lambda < \bar{\lambda}_2$, effort becomes desirable conditional on every state of the world. Relative to the case without intervention, contingent liquidity policy improves farmers' collective effort to improve market quality, as exerting effort becomes a dominant strategy:

Lemma 3. *Suppose that $\Delta L < c < \Delta\tilde{R}$. Under market intervention, effort is undertaken by all farmers.*

Given Lemma 2 and 3, we can characterize the evolution of λ_t under market intervention. Given any $\lambda_t < \bar{\lambda}_t$, λ_{t+1} is:

$$\lambda_{t+1} = p_g\lambda_t + p_b\pi(1-\lambda_t) + \delta_t((1-p_g)\lambda_t + (1-p_b)(1-\lambda_t)).$$

Using this equation, we can back out the likelihood of market liquidity at $t = t+1$ conditional on λ_t:

$$1 - F\left(\frac{\bar{\lambda}_{t+1} - \left[p_g\lambda_t + p_b\pi(1-\lambda_t)\right]}{(1-p_g)\lambda_t + (1-p_b)(1-\lambda_t)}\right).$$

Given this characterization, we can compare how aggregate asset quality improves with and without market intervention. Recall, absent intervention, markets required private incentives to voluntarily exit to enable probabilistic recovery. As β increases, the efficacy of voluntary exit diminishes, resulting in the following:

Proposition 5. *Suppose that $\Delta L < c < \Delta\tilde{R}$. Then, the probability of self-sufficient market liquidity is (weakly) greater under market intervention for sufficiently large β.*

Proof. It suffices to show that there exists sufficiently large β such that $\delta^*(\phi^V L_{b,1}(\lambda_t)) > \frac{\bar{\lambda}_{t+1} - \left[p_g\lambda_t + p_b\pi(1-\lambda_t)\right]}{(1-p_g)\lambda_t + (1-p_b)(1-\lambda_t)}$. We show by using a limit argument. Note that as $\beta \to 1$, $\phi^V L_{b,1}(\lambda_t)$ approaches 0 since $L_b < p_b R_b$. This implies that $\delta^*(\phi^V L_{b,1}(\lambda_t)) \to \bar{\delta}$. Hence, $\delta^*(\phi^V L_{b,1}(\lambda_t)) > \frac{\bar{\lambda}_{t+1} - \left[p_g\lambda_t + p_b\pi(1-\lambda_t)\right]}{(1-p_g)\lambda_t + (1-p_b)(1-\lambda_t)}$ for sufficiently large β. □

In addition to the relative strength between voluntary and involuntary liquidation as a channel through which market quality improves, intervention restores private incentives to exert effort even when $\lambda_2 < \bar{\lambda}_2$. Let $\lambda_{t,t+1}$ denote the quality of assets after voluntary/involuntary exit and entry. Since effort is not exerted absent market intervention, this directly implies:

Proposition 6. *Suppose that $\Delta L < c < \Delta\tilde{R}$. For any $\lambda_{1,2} < \lambda_2'$, recovery is faster under market intervention, i.e. $E[\lambda_{2,3}^{INT}|\lambda_{1,2}] < E[\lambda_{2,3}|\lambda_{1,2}]$.*

When $\Delta\tilde{R} > c > \Delta L$, effort is incentive compatible only under market liquidity. Market intervention, which enables investment to occur, improves effort. As a result, when voluntary liquidation becomes less effective, intervention works as a catalyst to improve market integrity by aligning farmers' incentives to exert effort.

Consider the second case, in which $\Delta L < \Delta\tilde{R}$. As before, continuation values monotonically increase for all farmers. However, while farmers' outcomes improve even when $\lambda < \bar{\lambda}_2$, effort becomes *less* desirable conditional on every state of the world. Relative to the case without intervention, contingent liquidity policy destroys farmers' incentives to improve market quality.

Lemma 4. *Suppose that $\Delta\tilde{R} < c < \Delta L$. Under market intervention, farmers exert effort only if $\lambda_1 > \lambda_t'$.*

As before, given farmers' equilibrium effort decisions, we can characterize dynamics for λ_t. For any $\lambda_t < \bar{\lambda}_t$, λ_{t+1} is:

$$\lambda_{t+1} = \begin{cases} p_g(1-\pi)\lambda_t + \delta_t((1-p_g)\lambda_t + (1-p_b)(1-\lambda_t)) & \text{if } \lambda_1 < \lambda_t' \\ p_g\lambda_t + p_b\pi(1-\lambda_t) + \delta_t((1-p_g)\lambda_t + (1-p_b)(1-\lambda_t)) & \text{if } \lambda_1 \geq \lambda_t' \end{cases}$$

The likelihood of market liquidity at $t = t+1$ conditional on λ_t:

$$\begin{cases} 1 - F\left(\frac{\bar{\lambda}_{t+1} - p_g(1-\pi)\lambda_t}{(1-p_g)\lambda_t + (1-p_b)(1-\lambda_t)}\right) & \text{if } \lambda_1 < \lambda_t' \\ 1 - F\left(\frac{\bar{\lambda}_{t+1} - \left[p_g\lambda_t + p_b\pi(1-\lambda_t)\right]}{(1-p_g)\lambda_t + (1-p_b)(1-\lambda_t)}\right) & \text{if } \lambda_1 \geq \lambda_t' \end{cases}$$

Given this characterization, we can analyze the recovery speed between $\Delta\tilde{R} < c < \Delta L$. As opposed to the earlier case, market intervention stifles private incentives to exert effort when involuntary liquidation does not shock the asset quality above a threshold λ'. Since absent intervention, effort is (partially) exerted by farmers in order to improve collateral value, when the incentive channel is sufficiently important, self sufficient liquidity is more likely under no intervention:

Proposition 7. *Suppose that $\Delta\tilde{R} < c < \Delta L$. Then, the probability of self-sufficient market liquidity is lower under market intervention when for sufficiently large π.*

Proof. Under market intervention, investment occurs at $t = 1$. Through involuntary liquidation and the entry of new farmers, aggregate asset quality becomes:

$$\lambda_{1,2} = \lambda_1 p_g + \delta_1(\lambda_1(1 - p_g) + (1 - \lambda_1(1 - p_b))$$

Self-sufficient market liquidity requires that:

$$\lambda_1 p_g + (1 - \lambda_1)p_b\pi + \delta_1(\lambda_1(1 - p_g) + (1 - \lambda_1)(1 - p_b)) \geq \hat{\lambda}_2.$$

This implies that δ_1 must satisfy:

$$\delta_1 \geq \frac{\hat{\lambda_2} - \lambda_1 p_g - (1 - \lambda_1)p_b\pi}{\lambda_1(1 - p_g) + (1 - \lambda_1)(1 - p_b)}$$

Hence, the probability of self-sufficient markets is:

$$1 - F\left(\frac{\hat{\lambda_2} - \lambda_1 p_g - (1 - \lambda_1)p_b\pi}{\lambda_1(1 - p_g) + (1 - \lambda_1)(1 - p_b)}\right)$$

Recall, the probability of liquidity at time $t = 2$ absent market intervention is:

$$\begin{cases} 1 & \text{if } \lambda_1 > \lambda_1' \\ \frac{\Delta L - c}{\Delta L - \Delta\tilde{R}} & \text{if } \lambda_1 \in (\lambda_1'', \lambda_1') \\ 0 & \text{if } \lambda_1 < \lambda_1'' \end{cases}$$

Since the probability of recovery under market intervention is in $(0, 1)$, the probability of recovery is greater, relative to no intervention, for $\lambda_1 \leq \lambda_1''$ and less for $\lambda_2 \geq \lambda_1'$. For the intermediate region $(\lambda_1'', \lambda_1')$, the probability of recovery is greater absent recovery when:

$$\begin{aligned}
\frac{\Delta L - c}{\Delta L - \Delta \tilde{R}} &> 1 - F\left(\frac{\hat{\lambda_2} - \lambda_1' p_g - (1-\lambda_1')p_b\pi}{\lambda_1'(1-p_g) + (1-\lambda_1')(1-p_b)}\right) \\
&= 1 - F\left(\frac{\lambda_1'(1-p_g) + (1-\lambda_1')(1-p_b)\pi}{\lambda_1'(1-p_g) + (1-\lambda_1')(1-p_b)}\right)
\end{aligned}$$

Since $\frac{\Delta L - c}{\Delta L - \Delta \tilde{R}} < 1$ and $F \to 0$ as $\pi \to 1$, there exists some threshold π' for which recovery is faster absent recovery when $\pi > \pi'$. □

As recovery is tied to the aggregate asset quality, we can also show that intervention may slow down recovery relative to the case without intervention when π is sufficiently large:

Proposition 8. *Suppose that $\Delta \tilde{R} < c < \Delta L$ and $E[\delta] \leq \frac{\bar{\lambda}}{1-p_b}$. Then, $E[\lambda_2^{INT} | \lambda_1] < \lambda_2$ for sufficiently high π.*

Proof. First, consider the evolution of λ_t under contingent liquidity policy. Under market intervention, $\lambda_1 p_g + (1-\lambda_1)p_b$ measure of farmers' projects fail and are subject to liquidation. Conditional on realized δ_1, effort decisions are made by farmers. Let $\bar{\delta}_1$ is such that:

$$\lambda_1 p_g + (1-\lambda_1)p_b\pi + \bar{\delta}_1(\lambda_1(1-p_g) + (1-\lambda_1)(1-p_b)) = \hat{\lambda}_2.$$

For any $\delta_1 \geq \bar{\delta}_1$, a coordinated equilibrium exists. This implies that with probability $1 - F(\bar{\delta}_1)$ liquidity is restored. In this case, λ_t at the beginning and end of period 2 is given by:

$$\begin{aligned}
\lambda_2^{INT}(\delta_1 > \bar{\delta}_1) &= \lambda_1 p_g + (1-\lambda_1)p_b\pi + \bar{\delta}_1(\lambda_1(1-p_g) + (1-\lambda_1)(1-p_b)) \\
\lambda_{2,3}^{INT} &= \lambda_2 p_g + \delta_2(\lambda_1(1-p_g) + (1-\lambda_1)(1-p_b)) \geq \hat{\lambda}_2.
\end{aligned}$$

For any $\delta_1 < \bar{\delta}_1$, effort is not exerted by farmers as under a contingent liquidity policy, as

$c > \Delta V = \tilde{R}$. In this case, λ_2^{INT} at the beginning and end of period 2 is given by:

$$\lambda_2^{INT}(\delta < \bar{\delta}_1) = \lambda_1 p_g(1-\pi) + \delta_1(\lambda_1(1-p_g) + (1-\lambda_1)(1-p_b))$$
$$\lambda_{2,3}^{INT} = \lambda_2 p_g + \delta_2(\lambda_2(1-p_g) + (1-\lambda_2)(1-p_b))$$

Expected market quality at $t = 2$ is given by:

$$\begin{aligned} E_1[\lambda_2^{INT}|\lambda_1] &= \int_{\underline{\delta}}^{\bar{\delta}_1} \lambda_1 p_g(1-\pi) + \delta_1(\lambda_1(1-p_g) + (1-\lambda_1)(1-p_b))dF(\delta) \\ &+ \int_{\bar{\delta}_1}^{\bar{\delta}} \lambda_1 p_g + (1-\lambda_1)p_b\pi + \delta_1(\lambda_1(1-p_g) + (1-\lambda_1)(1-p_b))dF(\delta) \\ &= \lambda_1 p_g(1-\pi) + (\lambda_1(1-p_g) + (1-\lambda_1)(1-p_b))E[\delta] + \pi(\lambda_1 p_g + (1-\lambda_1)p_b)(1-F(\bar{\delta}_1)), \end{aligned}$$

where $\bar{\delta}_1 = \frac{\hat{\lambda}_2 - \lambda_1 p_g}{\lambda_1(1-p_g)+(1-\lambda_1)(1-p_b)}$. Similarly, market quality at the end of $t = 2$ is given by:

$$\begin{aligned} E[\lambda_{2,3}^{INT}|\lambda_1] &= (1-F(\bar{\delta}_1))[\lambda_2^{INT}(\delta_1 > \bar{\delta}_1)p_g + E[\delta](\lambda_2^{INT}(\delta_1 > \bar{\delta}_1)(1-p_g) + (1-\lambda_2^{INT}(\delta_1 > \bar{\delta}_1))(1-p_b))] \\ &+ F(\bar{\delta}_1)[\lambda_2^{INT}(\delta_1 < \bar{\delta}_1)p_g + E[\delta](\lambda_2^{INT}(\delta_1 < \bar{\delta}_1)(1-p_g) + (1-\lambda_2^{INT}(\delta_1 < \bar{\delta}_1))(1-p_b))] \end{aligned}$$

Now, consider the evolution of λ_t absent market intervention. Without intervention, market are illiquid for any $\lambda_1 < \bar{\lambda}_1$. For any $\lambda_1 \in (\lambda_1', \bar{\lambda}_1)$, a coordinated equilibrium exists, in which case liquidity is restored with probability 1, and $\lambda_2 = \lambda_1 + (1-\lambda_1)\pi$. At the end of period 2, λ_t is given by: $\lambda_{2,3} = \lambda_2 p_g + \delta_2(\lambda_2(1-p_g) + (1-\lambda_2)(1-p_b))$. For any $\lambda_1 \in (\lambda_2'', \lambda_1')$, a mixed equilibrium arises, in which case liquidity is restored with probability ρ, and $\lambda_2 \approx \bar{\lambda}_2$. When investment occurs (with probability ρ), $\lambda_{2,3} = \bar{\lambda}_2 p_g + \delta_2(\bar{\lambda}_2(1-p_g) + (1-\bar{\lambda}_2)(1-p_b))$.

Market quality at $t = 2$ without intervention is:

$$E_1[\lambda_2|\lambda_1] = \begin{cases} \lambda_1 + (1-\lambda_1)\pi & \text{if } \lambda_1 > \lambda_1' \\ \bar{\lambda}_2 & \text{if } \lambda_1 \in (\lambda_1'', \lambda_2'') \\ \lambda_1 + (1-\lambda_1)\pi & \text{if } \lambda_1 < \lambda_2'' \end{cases}$$

Market quality at the end of $t = 2$ without intervention is:

$$E_1[\lambda_{2,3}|\lambda_1] = \begin{cases} \lambda_2 p_g + E[\delta](\lambda_2(1-p_g) + (1-\lambda_2)(1-p_b)) & \text{if } \lambda_1 > \lambda_1' \\ \rho(\bar{\lambda}_2 p_g + \delta_2(\bar{\lambda}_2(1-p_g) + (1-\bar{\lambda}_2)(1-p_b))) + (1-\rho)\bar{\lambda}_2 & \text{if } \lambda_1 \in (\lambda_1'', \lambda_2'') \\ \lambda_1 + (1-\lambda_1)\pi & \text{if } \lambda_1 < \lambda_2'' \end{cases}$$

It remains to show that when $E[\delta] < \bar{\lambda}/(1-p_b)$, $\lambda_2 > \lambda_2^{INT}$ and $\lambda_{2,3} > \lambda_{2,3}^{INT}$.

Claim: there exists some cutoff for which π is sufficiently high such that $E[\lambda_2] > E[\lambda_2^{INT}]$. Consider the limit case as $\pi \to 1$. Note that $F \to 1$ as $\pi \to$. Then for $\lambda_1 > \lambda_1'$ or $\lambda_1 < \lambda_1''$:

$$E[\lambda_2 - \lambda_2^{INT}|\lambda_1] \to 1 - E[\delta](\lambda_1(1-p_g) + (1-\lambda_1)(1-p_b)) > 0.$$

Next, consider when $\lambda_1 \in (\lambda_1'', \lambda_1')$. When $\bar{\lambda} - E[\delta](1-p_b) > 0$, $E[\lambda_2 - \lambda_2^{INT}|\lambda_1] > 0$. Finally, consider $E[\lambda_{2,3} - \lambda_{2,3}^{INT}|\lambda_1]$. Since $E[\lambda_2|\lambda_1] \geq E[\lambda_2^{INT}|\lambda_1]$ for sufficiently large π, it follows directly that for any λ_1, $E[\lambda_{2,3} - \lambda_{2,3}^{INT}|\lambda_1]$. □

Together, this shows that when private incentives are tied to the collateral differential, improving liquidity through intervention may hinder the incentives to improve quality. This adaptation may result in slower recovery of λ_t to levels at which markets operate without liquidity. As expectations about contingent market intervention destroy incentives to exert effort, an intervention trap arises. The existence of policy aimed at improving liquidity drags recovery, which justifies the need for continued intervention.

4.2 Interest Rate Policy

Importantly, these insights on the interaction between government intervention and market self-regulation apply to a broader set of policies. This includes conventional monetary stimulus, such as interest rate policy. In this section, we show that our result outlined in Proposition 8 extends to interest rate policy.

Consider an alternative contingent liquidity policy through which the regulator chooses r_f at the beginning of each period after observing λ_t. A contingent liquidity policy entails setting r_f such that investors break even given λ_t:

Lemma 5. *Under contingent liquidity policy, the regulator sets $r_f = \cdot E[L_\theta]$ when $\lambda_t < \lambda_t'$.*

In other words, the regulator lowers the riskfree rate low enough such that investors' incentives to finance farmers are restored at $\chi_t^* = 1$. Note that when $\chi_t^* = 1$, effort is not exerted when $c > \Delta\tilde{R}$ since $\Delta V_2 = \Delta\tilde{R}$. As a result, we find a result analogous to Lemma 4:

Lemma 6. *Suppose that the regulator uses an interest rate policy of contingent liquidity. Then, when $\Delta L > c > \Delta\tilde{R}$, effort is exerted only if $\lambda_t > \lambda_t'$.*

When market intervention results in market liquidity contingent on farmers pledging full collateral, i.e. $\chi = 1$, then farmers' incentives become identical to that under a collateral-based policy of intervention. Together, this implies that interest rate policy qualitatively exhibits the same drawbacks as collateral-based policies outlined in Proposition 8:

Proposition 9. *Suppose that $\Delta\tilde{R} < c < \Delta L$ and $E[\delta] \leq \frac{\bar{\lambda}}{1-p_b}$. Then, recovery is slower under market intervention relative to no intervention, i.e. $E[\lambda_2^{INT}|\lambda_1] < \lambda_2$ for sufficiently high π.*

4.3 Discussion

In this section, we outlined the interaction between market intervention and market self-regulation. Market intervention strictly improves welfare by restoring liquidity to a frozen market, thereby reinvigorating investment into positive NPV projects held by agents. Whether this liquidity provision is facilitated through collateral lending, subsidization (i.e. insurance), or by affecting the risk-free rate, the regulator is uniquely positioned to immediately revitalize markets by resolving adverse selection problems that prohibit investors from lending to farmers. While the immediate welfare benefits are clear, we show that intervention may interfere or improve market recovery. This required us to study a setting in which absent regulatory intervention, markets could endogenously repair itself and eventually restore liquidity.

Our analysis in a three period setting highlights how the nature of the interaction between intervention and market self-regulation depends on whether private incentives to improve asset quality is concentrated in the collateral value or investment value. Specifically, when the return differential, i.e. $\Delta\tilde{R}$ is high, gains from improving asset quality are realized conditional on investment. As a consequence, market intervention, which artificially improve liquidity and enables investment, also improves private incentives to jointly improve asset quality and collateral value. Endogenous increases in the aggregate collateral stock enable markets to reach a state of self-sufficient liquidity.

When the collateral differential, i.e. ΔL is high, gains from improving asset quality are realized when farmers retain a sufficiently high fraction of collateral. Importantly, because market intervention primarily targets the restoration of market liquidity, it necessarily offer some form of collateral-independent subsidization to farmers holding low quality assets. In this case, intervention interferes with private incentives of farmers to improve asset quality, which are essential to lowering adverse selection. Market participants adapt to expectations that intervention will resolve future illiquidity, in turn destroying endogenous channels through which asset quality improves. As a result, contrary to the previous case, intervention drags recovery.

5 Dynamic Implications

The previous section lays out the mechanisms through which market intervention affects endogenous channels through which markets improve. This meant that depending on the underlying structure of assets, intervention could improve or worsen recovery from severe adverse selection.

The analysis can be flexibly extended to understand the impact of market intervention in a dynamic setting. To illustrate this, consider an extension of the setting to $T > 2$, and consider the dynamics of λ_t, which governs aggregate asset quality. Extrapolating the dynamics outlined in Propositions 6 and 8 offers insights into the dynamic implications of the interaction between market intervention and market self-regulation. Let us measure the speed of recovery, measured the minimum time period, τ at which $\lambda_{\tau,\tau+1} \geq \lambda'_\tau$. As a coordinated equilibrium arises whenever $\lambda_{t,t+1} \geq \lambda'_t$, recovery entails average asset quality improving to the point at which self-sufficient liquidity is realized. When the incentive channel plays an important role, i.e. π large, endogenous improvements in asset quality critically hinge on whether market intervention improves or harms incentives. By Lemmas 3 and 4, market intervention improves and harms incentives when $\Delta L < \Delta \tilde{R}$ and $\Delta L > \Delta \tilde{R}$, respectively.

[To be completed.]

6 Conclusion

In this paper, we study how government interventions that provide liquidity to frozen collateralized lending markets affect private incentives of agents to maintain or produce high-quality assets. We develop a model in which agents may endogenously improve asset quality which lowers adverse selection faced by investors that finance investments. Market intervention directly restores market liquidity but complements or subdues effort incentives depending on whether future expected liquidity increases the relative value of owning a high-quality asset. When adverse selection stems primarily from uncertainty about the collateral value of assets, the model gives rise to "intervention traps" –the expectation of future interventions eliminate private effort incentives to improve quality, in turn necessitating continued intervention by the policy maker. Instead, when adverse selection stems primarily from uncertainty about investment opportunities, interventions improves private incentives and speeds recovery. We show that these insights extend not only to collateral-based interventions but also to interest-rate policies aimed at increasing investors' tolerance toward adverse selection concerning asset quality. This suggests that depending on the underlying source of adverse selection, policies geared toward stimulating economic activity, by improving or destroying private incentives, may speed or slow down long-term market recovery.

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A Proofs

Proof of Proposition 1. We denote equilibrium outcomes under perfect information by superscript $\star$. Given Assumption 2, bad farmers cannot obtain credit in the second period. As a result, the value of having a bad tree in the second period under full information is $V^{\star}_{b,2} = L_b$. Good farmers instead can offer a contract $(\chi_{g,2}, B_{g,2})$. Without loss of generality, we assume $B_{g,2} \leq R_g$. Taking the required return r_2 as given, the investor's break even condition is

$$p_g B_{g,2} + (1 - p_g)\chi_{g,2} L_g = 1 + r_2.$$

The continuation value in period 2 is $\tilde{V}_{g,2} = L_g$. This implies that the maximum feasible debt level is $\bar{B}_{g,t}(\chi_{g,2}) = \chi_{g,2} L_g$. Without loss of generality, we can restrict attention to contracts with $B_{g,2} \geq \chi_{g,2} L_g$. Then $B_{g,2} = \chi_{g,2} L_g$, so that the investor break-even condition is $\chi_{g,2} L_g = B_{g,2} = 1 + r_2$. Under Assumption 2 good farmers have enough collateral to borrow when investors receive their outside option. Accordingly, good farmers obtain funding by issuing a bond of face value $B_{g,2} = 1 + r_2$ and offering a fraction $\chi^*_{g,2} = \frac{1+r}{L_g}$ of their tree as collateral. In the final period ($t = 2$), all farmers liquidate their trees. We can characterize the value of a good tree at the beginning of time $t = 2$:

$$
\begin{aligned}
V^\star_{g,2} &= p_g \left(R_g - B_{g,2} + L_g\right) + (1 - p_g)\left(-\chi_{g,2} L_g + L_g\right) \\
&= p_g R_g + (1 - \chi_{g,2}) L_g \\
&= p_g R_g + L_g - (1 + r_2),
\end{aligned}
$$

and the utility difference under perfect information is

$$
\Delta V^\star_2 \equiv V^\star_{g,2} - V^\star_{b,2} = p_g R_g + \left(L_g - L_b\right) - (1 + r_2).
$$

Now, consider the first period. By Condition (7), a farmer of type θ exerts effort if the cost of effort is sufficiently low:

$$
c \leq \pi_\theta \Delta V^\star_2 \tag{IC}
$$

Taking the optimal effort decision $e^\star_{\theta,1}$ as given, the continuation values are

$$
\tilde{V}^\star_{g,1} = V^\star_{g,2} - (1 - e^\star_{g,1}) \pi_g \Delta V^\star_2 \qquad \text{and} \qquad \tilde{V}_{b,t} = V^\star_{b,2} + e^\star_{b,t} \pi_b \Delta V^\star_2.
$$

Assuming without loss of generality that farmers do not default when their tree yields fruit, the type-contingent break-even conditions for investors are

$$
p_\theta B_{1,\theta} + (1 - p_\theta) \chi_{1,\theta} L_\theta = 1 + r.
$$

This implies that a θ-farmer can obtain financing at $t = 1$ if

$$L_\theta \geq 1 + r - p_\theta \left[\tilde{V}^\star_{\theta,1} - L_\theta\right].$$

Note, this condition is always satisfied for good farmers. Since bad farmers only have access to a negative NPV project, they never attempt to acquire funding in the first period. $\square$

Proof of Proposition 4. Since $c < \beta\pi\Delta L$, all farmers choose to exert effort when markets are expected to be illiquid. This implies that when $\lambda_1 \leq \lambda''_t$, a coordinated equilibrium arises. Next, consider when $\lambda \in (\lambda''_t, \lambda'_t)$. An important difference from the previous section is that the value of improving the liquidation value is sufficiently valuable such that even if the market is illiquid, farmers find it incentive compatible to exert effort. Because $\Delta\tilde{R} < c$, farmers find it optimal to *shirk* conditional on expectations that markets are to be liquid in the next period. This brings rise to a partial effort equilibrium with probabilistic recovery from illiquidity.

Finally, consider when $\lambda_1 \in (\lambda''_2, \lambda'_2)$. To identify a solution, we now introduce a small degree of correlation in the effort realization of bad agents' effort. As before, let bad agents who exert effort turn into good agents w.p. π, and let the realization of outcomes be *i.i.d.* only for a fraction $1-\alpha$ of bad agents. This implies that precisely a fraction π of these agents turns into good agents. For the remaining fraction $1-\alpha$, assume that the effort realization the outcome is correlated. In particular, we assume that a fraction $\tilde{\pi}$ of the fraction $1-\alpha$ turns into good agents, where $\tilde{\pi}$ is a continuous random variable distributed according c.d.f G with $E[\tilde{\pi}] = \pi$. For simplicity, let the effort technology for good agents be the same as before, with the effort realization continuing to be *i.i.d.*. Note that this structure leaves the individual decision problem unchanged, in that the probability of becoming a good agent is fixed at π. Given this structure, the law of motion for λ is

$$\begin{aligned}\lambda'(\mu, \tilde{\pi}|\lambda) &= \lambda\mu + \lambda(1-\mu)(1-\pi) + (1-\alpha)(1-\lambda)\mu\pi + \alpha(1-\lambda)\mu\tilde{\pi} \\ &= \lambda\mu + \lambda(1-\mu)(1-\pi) + (1-\lambda)\mu\pi + \alpha(1-\lambda)\mu\left(\tilde{\pi} - \pi\right)\end{aligned}$$

where μ denotes the fraction of agents of either type who exert effort.[7] Let

$$\begin{aligned}\lambda^*(\mu|\lambda) &= E_{\tilde{\pi}}\lambda(\mu,\tilde{\pi}|\lambda) = \lambda\mu + \lambda(1-\mu)(1-\pi) + (1-\lambda)\mu\pi \\ &= \lambda(1-\pi) + \pi\mu\end{aligned}$$

denote *expected asset quality tomorrow*. Then we can write the law of motion simply as

$$\lambda'(\mu,\tilde{\pi}|\lambda = \lambda^*(\mu|\lambda) + \alpha(1-\lambda)\mu\,(\tilde{\pi}-\pi)\,,$$

i.e. expectation plus some aggregate noise.

Equilibrium mixing requires $c' = E_{\lambda'}\Delta V(\lambda')$, where we know from Figure **??** that ΔV is discontinuous in λ'. In the region where mixing is necessary, we know that $\lambda' > \bar{\lambda}$ if $\mu = 1$, but $\lambda' < \bar{\lambda}$ if $\mu = 0$. Given μ, define $\bar{\pi}(\mu)$ to be the realization of $\tilde{\pi}$ such that markets switch from being illiquid to liquid, i.e.

$$\bar{\pi}(\mu) = \pi + \frac{\bar{\lambda} - \lambda^*(\mu)}{\alpha(1-\lambda)\mu} \qquad \text{where} \qquad \bar{\lambda} = \frac{1 + r_f - L_b}{\Delta L}$$

Since $\lambda^*(\mu)$ is strictly increasing in μ and $\lambda^*(1) \geq \bar{\lambda}$, there exists a μ such that $\bar{\pi}(\mu) \in [\pi^0, \pi^1]$. Indeed, equilibrium mixing *requires* μ to be such that $\bar{\pi}(\mu) \in [\pi^0, \pi^1]$. If it were not, then market liquidity would be deterministic, in which case we already know that there does not exist an equilibrium. Note also that $\bar{\pi}(\mu)$ is strictly decreasing in μ. Denote tomorrow's value of χ conditional on λ by $\chi(\mu, \tilde{\pi})$. Then we can write the indifference condition as

$$c' = G(\bar{\pi})\Delta L + \int_{\bar{\pi}}^{\pi^1} \left[\Delta R + (1 - \chi(\mu,\tilde{\pi}))\Delta L\right] g(\tilde{\pi}) d\tilde{\pi}$$

We now argue that the mixing probability μ^* that satisfies this indifference condition is well-defined, determined by fundamentals, and independent of the particular distribution for $\tilde{\pi}$ in the limit as aggregate noise approaches to zero. Guess and verify that $\Delta V \approx \Delta R$

[7]Given that $\pi_g = \pi_b$ we can restrict attention to symmetric strategies. However, we should check if there is any social efficiency gain in having a larger fraction of good types exert effort.

conditional on liquidity. Given this approximation, the indifference condition for effort is

$$c' = G(\bar{\pi})\Delta L + (1 - G(\bar{\pi}))\Delta R = \Delta R + G(\bar{\pi})\,(\Delta L - \Delta R)$$

To satisfy this condition, we must have that $G(\bar{\pi}) = \frac{c' - \Delta R}{\Delta L - \Delta R}$, and, equivalently, $\bar{\pi}(\mu^*) = G^{-1}(\frac{c' - \Delta R}{\Delta L - \Delta R})$. Recall that $\bar{\pi}(\mu) = \pi + \frac{\bar{\lambda} - \lambda^*(\mu)}{\alpha(1-\lambda)\mu}$. Solving for μ then yields

$$\mu^*(\alpha) = \frac{\bar{\lambda} - \lambda(1-\pi)}{\pi + \alpha(1-\lambda)(G^{-1}(\frac{c' - \Delta R}{\Delta L - \Delta R}) - \pi)},$$

Given this solution, we now consider the limit economy where aggregate noise goes to zero, i.e. $\alpha \to 0$. First, note that $\lim_{\alpha \to 0} \mu^*(\alpha) = \frac{\bar{\lambda} - \lambda(1-\pi)}{\pi}$. In the region we are focusing on, we must have that $\lambda^*(1) \geq \bar{\lambda}$. Hence $\lambda \geq \underline{\lambda} \equiv \frac{\bar{\lambda} - \pi}{1-\pi}$, which implies that $\frac{\bar{\lambda} - \lambda(1-\pi)}{\pi} \leq 1$, so that the mixing probability in the limit economy is well-defined. Indeed, $\mu = 1$ if $\lambda = \underline{\lambda}$, and $\mu^* < 1$ if $\lambda > \underline{\lambda}$. Next, note that $\lim_{\alpha \to 0} \lambda^* = \bar{\lambda}$. This implies that $\lim_{\alpha \to 0} \Delta V = \Delta R$, i.e. the approximation $\Delta V \approx \Delta R$ conditional on market liquidity becomes arbitrarily accurate as $\alpha \to 0$. Hence the limit economy with essentially no aggregate noise in effort outcomes allows for the existence of equilibrium with well-defined mixing probabilities without affecting individual indifference conditions. □