Stock Market Coverage

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Abstract

Coverage by security analysts and media journalists are found to increase stock prices. We provide a theory using an assignment approach. Coverage is generated by a labor market of agents who produce public signals about firm profits of heterogeneous precision. Firms seeking to maximize share price compete for accurate coverage as it improves price efficiency and lowers the risk premium required by investors. Positive-assortative-matching, when large and volatile firms pay more for accurate coverage, often arises. Small or less-volatile firms might prefer to be neglected since they can only hire less-accurate agents. This sorting effect yields lower expected returns for larger and more-volatile stocks, reconciling the small-firm and idiosyncratic-volatility puzzles.
1. Introduction

A large empirical literature points to the importance of coverage, be it by security analysts or media journalists, for the stock market. To start, coverage has been shown to be associated with greater price efficiency, both in the cross-section of stocks and using exogenous shocks to analyst and media coverage. By generating public signals about earnings for investors, coverage leads to more informative prices about fundamental value.\(^1\) Moreover, coverage is also associated with higher mean market valuations. For instance, stocks with little coverage, or neglected stocks, have been shown to significantly out-perform other stocks, consistent with investors in neglected stocks requiring a higher risk premium. The economic effects are large: small stocks with no media coverage out-perform stocks with coverage by 10% per annum. A similar set of quasi-experiments as for the price efficiency effects of coverage show substantial improvements in mean share price.\(^2\) Importantly, coverage is correlated with firm characteristics but is most heavily skewed with large firms: most large firms have coverage while most small firms have zero coverage.\(^3\) This latter fact points to the need for a theory that can address potentially important cross-sectional sorting effects.

The main framework used to think about coverage and asset pricing is Merton (1987)'s investor recognition model in which incomplete information gives rise to a segmented version of the Capital Asset Pricing Model (Sharpe (1964)). Firms with greater number of investors, or more recognition, have higher prices due to a lower risk premium. The literature since has loosely connected coverage with investor recognition. But the model does not explicitly

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\(^1\) Analyst coverage in the cross-section is correlated with more informative prices and deeper markets (see, e.g., Brennan and Subrahmanyam (1995)). Exogenous shocks to analyst coverage generated by brokerage house mergers (Hong and Kacperczyk (2010)) or closures (? ) show that the effect of coverage on price efficiency is causal. Media coverage variation due to differential investor access to local newspapers or newspaper strikes lead to similar causal conclusions for the benefits of coverage for stock market pricing (Engelberg and Parsons (2011) and Peress (2014)).

\(^2\) The earliest study on the neglected firm effect by Arbel et al. (1983) finds that stocks with zero or low analyst coverage out-perform stocks with high analyst coverage. Similarly, Tetlock (2007) and Fang and Peress (2009) find that stocks with more media coverage have higher prices or lower expected returns. Subsequent studies such as Foerster and Karolyi (1999) used quasi-experiments to establish the importance of coverage or investor recognition in explaining the neglect effect.

\(^3\) In addition to firm size, coverage increases with institutional ownership and stock price volatility (see, e.g., Bhushan (1989)). Similar sorting patterns hold for business press coverage Solomon and Soltes (2011).
model coverage, i.e. does not endogenize recognition. At the same time, theories used to think about cross-sectional variation in coverage (see, e.g, Brennan and Hughes (1991)) focus on the institutional incentives of brokerage houses, be it to increase trading revenues or investment banking business. But these theories do not speak to the concomitant asset pricing patterns. Yet the evidence, especially recent causal studies pointing to a significant effect of coverage on asset markets, requires a richer theory.

To this end, we provide a different take on stock market coverage in which coverage is endogenously determined in a labor market between heterogenous firms with different levels of capital investments (indexed by $k$) or different levels of fundamental pay-off volatility (indexed by $\sigma$) seeking to maximize their share price and agents who can generate coverage, defined as a public signal about the firm’s fundamental pay-off. The firm might be issuing equity. Alternatively, as we consider in the extension, the firm might be financially constrained and worried that they might have to issue some equity with some probability (Froot et al. (1993)), in which case the firm maximizes a mean-variance objective function around the stock price.

The agents have heterogeneous precision (indexed by $h$), which is known to the market, and the outcome of the labor market matching between firms and agents determines the accuracy or precision of coverage across firms. Heterogeneity of ability for coverage agents and the observability of ability are realistic assumptions. For instance, Stickel (1992) find that there is substantial heterogeneity in earnings forecast ability, which is persistent and correlated with appearance in year-end professional rankings and compensation. Similarly, Dougal et al. (2012) find that journalist fixed effects as opposed to simply the newspaper influences stock prices, pointing to the importance of the ultimate journalist talent in affecting prices.

Our stock market coverage theory based on matching in labor markets can simultaneously rationalize all the aforementioned asset pricing and coverage patterns. At the same time,  

\footnote{Or short-termism due to managerial career concerns might lead the firm to care about the short-term stock price (Stein (1989)).}
it generates new testable implications, calls attention to the importance of the accuracy of coverage and not just simply the raw number covering and can potentially explain a number of outstanding asset pricing anomalies. In practice, a great deal of coverage is indeed initiated when firms want to sell shares.\(^5\) They pay brokerage houses or media outlets for coverage indirectly, through investment banking business or advertising (Gentzkow et al. (2006)), or pay investor relations firms directly to garner coverage from analysts or the media (Bushee and Miller (2012), ?).

But more talented analysts, journalists or investor relations professionals ultimately get paid significantly more and are assigned to cover larger firms.\(^6\) For instance, Hong and Kubik (2003) find security analysts with superior earnings forecasting track records are more likely to work for top-tier brokerage houses with investment banking business and assigned to cover large stocks with equity issuance needs. Following the classical assignment models, we for simplicity and tractability abstract away from the intermediate layer, and simply consider the matching between agents of different talent or accuracy (which we assume is known) and firms with varying levels of capital investments or fundamental pay-off volatility.\(^7\)

There are three dates. At \(t = 0\), a firm issuing equity and seeking to maximize its share price decides which agent it will hire, i.e. the assignment function \(\mu(y)\) mapping agent \(h\) to a firm of characteristic \(y\), where \(y\) can denote either the level of capital or firm fundamental volatility. We consider as our baseline model heterogeneity in either \(k\) or \(\sigma\). We then allow for both dimensions to vary simultaneously in an extension. Firms pay a competitive wage to hire an agent. Each agent works for and covers one firm, producing the information for that firm at a fixed cost.\(^8\) At \(t = 1\), asset markets are open for each firm. The asset market set-up

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\(^5\)For instance, Krigman et al. (2001) find that coverage is the main reason for firms changing investment bankers when they go to public markets.

\(^6\)Indeed, Karolyi and Liao (2015) find that investor relations activity are also associated with a lower cost of capital using a comprehensive sample of international firms.

\(^7\)A typical starting point for thinking about assignment problems with heterogeneous agents is the model of Becker (1973) and Rosen (1974). Building on this, there are extensive works that study labor market sorting. In our context, we can think of brokerage houses and media companies act as a screening mechanism for human capital, thereby making the types of labor known.

\(^8\)In practice, an analyst might cover multiple firms though they are all typically in the same industry (e.g. technology). So that the best analysts might cover the largest technology firms. Traditional assignment
follows a traditional noisy rational expectations set-up. Investors submit price-contingent demand based on their own private signals and whatever public signal the firm purchases in the labor market for coverage.\textsuperscript{9} There are also noise traders in the market. At $t = 2$, the pay-off of the firm is realized.

We solve for an equilibrium consisting of: (1) optimality of firm coverage decisions; (2) optimality of investor’s decisions; (3) market clearing in the labor market, i.e. an assignment function $\mu(y)$ and wage function $\omega(h)$ based on the optimal choices of the firms for coverage and the talent distribution of agents; and (4) market-clearing in the asset market.

In equilibrium, we show that, all else equal, coverage improves the estimation of fundamentals by investors, i.e. the market is more efficient. Since investors are risk-averse, this also means a higher stock price for the company at $t = 1$. In other words, coverage improves both price efficiency and the mean price, consistent with empirical evidence on the importance of coverage for the pricing of the stock market.

But more importantly, our model also generates a host of additional testable predictions. First, we show that there is positive assortative matching in the labor market in that firms with a higher level of $k$ hire analysts with a higher precision $h$. This result follows from a closed-form characterization of the matching surplus function between the firms and the agents. It holds for any distribution of precision among agents $G^A(h)$ and distribution of firm characteristics $G^F(k, \sigma)$. Holding fixed $\sigma$, firms with a higher $k$ have more risk that the investors have to bear. Hence, higher $k$ firms benefit the most from paying for the most precise agents. Holding fixed $k$, firms with a higher $\sigma$ have more risk that investors have to bear and hence pay more for accurate coverage.

There is evidence in the literature consistent with positive assortative matching. In ad-\textsuperscript{9}models can only handle matching of one firm per worker, though recent models can be extended to allow for multiple workers per firm. We leave this for future research. As a first step, we can think of our model as focusing on the extensive margin of whether a firm purchases coverage or not. This extensive margin as we mentioned in the beginning is a prominent aspect of the data on coverage.

\textsuperscript{9}We consider an asset market along the line of Grossman and Stiglitz (1980); Diamond and Verrecchia (1981). Specifically, we apply the analytical techniques recently developed in Albagli et al. (2011) to solve for equilibrium price.
dition to those cited earlier (see, e.g., Hong and Kubik (2003) on assignments of talented analysts to cover bigger firms), Frankel et al. (2006) find that analyst research is more informative for larger and more volatile firms. Moreover, the compensation of the top analysts are highly skewed. During the Internet Bubble Period of 1997-2000, security analysts’ pay were especially skewed as the prices of dot-com stocks were noisy and underlying dot-com pay-offs were also highly uncertain, consistent with our model’s prediction.

When there are fixed costs, firms below a certain cut-off $k^*$ do not pay for coverage since they do not benefit as much as high $k$ firms, consistent the cross-sectional coverage evidence cited above. Small firms are only able to compete for and hire low precision agents since the high precision ones work for large firms and get paid more. So the zero coverage effect in our model is magnified by the labor market matching effect. We show that, absent this labor market matching effect, the cut-off firm size $k^*$ below which there would would be no coverage is larger.

Beyond the findings about coverage, our model can potentially rationalize the small firm effect, whereby small firms have lower prices and higher expected returns than large firms (see, e.g., Banz (1981)). Absent coverage, it should be large firms that have lower prices and higher expected returns since investors have to bear more risk. But we show that the positive assortative matching effect can over-turn this risk effect since large firms get more accurate coverage in equilibrium and have lower perceived risk from the point of view of investors.

We then fix $k$ and consider heterogeneity of firms in another dimension, namely the volatility of pay-offs. By the same logic as positive assorting matching on $k$, we get positive assortative matching for $\sigma$. Moreover, we also get an additional implication, which is that our model can now explain the idiosyncratic volatility effect whereby high idiosyncratic volatility firms have lower expected returns than high idiosyncratic volatility firms (see, e.g., Ang et al. (2006)). The logic is similar as that for the small firm effect. More volatile firms pay for more accurate coverage and this positive sorting effect can dominate the underlying fundamental
volatility effect.

We show that our competitive-sorting effect uniquely leads to the small firm and idiosyncratic volatility effects. To show this, we consider a related but alternative set-up where accuracy is the same across analysts but firms can hire more than one analyst. We show that larger and more volatile firms hire more analysts. But since the wage is the same across agents, small and less volatile firms still hire enough analysts so that the risk premium still rises with firm size and volatility.

The empirical literature thus far has focused on the number of analysts covering a firm, but our model points to the importance of the accuracy of this coverage for determining stock prices, especially in explaining the small firm effect and the idiosyncratic volatility effect. In other words, the small firm and idiosyncratic volatility effects are strongly related to the accuracy of the coverage for these firms.

Our model generates additional testable predictions, which we elaborate on below, regarding when the small firm effect and idiosyncratic volatility effect are likely to be the strongest. The risk premium of a firm depends on the strength of the competitive-sorting effect, which is strongest when there are scarce talented agents (i.e. a supply of precision effect) and the market is noisier (i.e. a demand effect whereby firms have a greater incentive to hire coverage).

One metric for the strength of this competitive-sorting effect is the wage distribution. The wage of an agent is rising in his talent. The stronger is our competitive-sorting effect, the steeper the wage profile is with talent. Hence, these asset pricing puzzles are larger when the wage profile for agents is steeper.\footnote{10} Wage distribution results are also emphasized in the CEO matching literature (see, e.g., Terviö (2008); Gabaix and Landier (2006)). However, rather than complementarities embedded in the firm production function, as in these CEO matching models, we derive complementarity endogenously from the firm’s stock pricing and trading environment.

\footnote{10}Wages also rise with firm size, firm pay-off volatility, and the ratio of noise trading to the precision of private signals in the market.
We show that we can also construct an equilibrium in which we simultaneously allow for sorting in the two dimensions of \( k \) and \( \sigma \). The sorting then happens on an index weighting \( k \) and \( \sigma \). As long as \( k \) and \( \sigma \) are not negatively correlated in the cross-section, we get similar conclusions as in the equilibrium when sorting on just \( k \) or just \( \sigma \). There is positive sorting in both dimensions of \( k \) and \( \sigma \).

We then extend the firm’s objective function to consider not just the maximization of mean share price. Financial constraints and agency frictions can lead the firm to act as if it were risk-averse (see, e.g., Froot et al. (1993)). If the firm acts risk averse, they care not only about the mean share price at \( t = 1 \) but also the variance of the share price at which they can sell. We show that our key results remain as long as the firm is not too risk averse. The reason is that paying for coverage, i.e. a public signal, can actually increase the volatility of the stock price at \( t = 1 \) from the perspective of the firm. This variance effect pushes the firm away from purchasing coverage. The matching surplus function in this case ends up being non-monotonic in the precision of coverage \( h \), i.e. all else equal, firms might not want to purchase the most precise signal. Interestingly, in this middle region, small firms do not purchase coverage even if there are no fixed costs.

Our model shares similarities with models of voluntary firm disclosure (see, e.g., Diamond and Verrecchia (1991)), in which firms pre-commit to disclosing a public signal of their earnings to improve the price efficiency and risk discount for their shares. Results depend on the nature of the cost of disclosure, which is exogenously given and requires convexity in the cost structure. In the context of our coverage model, the cost of coverage is the wage paid to the agents, which is endogenously determined. The convexity of wages we derived can be thought of as creating a form of convex costs from the point of view of the firm for choosing the precision of coverage. Of course, our model generates a host of additional implications absent in these models.

Finally, we have not modeled the potential bias in coverage as a result of conflicts of interest or incentive issues (see, e.g., Michaely and Womack (1999), Hong and Kubik (2003),
Dyck and Zingales (2003)). One could introduce bias into our setting by assuming the noise traders are influenced by the bias. But assuming that there is talent in spinning the news to get stock prices higher, we would end up with the same outcome.

Our paper proceeds as follows. We describe our model in Section 2. We derive a number of testable implications in Section 3. We consider a number of extensions of our model in Section 4. We conclude in Section 5.

2. Model

2.1. Setup

The model lasts for three periods. There is a continuum of heterogeneous firms who issue equity through stock markets with measure one. On the other side, there is a distribution of coverage agents (analysts, journalists, investor relations professionals) with measure one, denoted by $G^A(h)$ with support $[h_L, h_U] \equiv H$, who differ in terms of skill (i.e., the precision of information they can produce). The quality of information for each firm’s performance thus depends on which agent a firm hires. At date 0, the allocation of agents across firms and agents’ fees are pinned down in a competitive assignment equilibrium. At date 1, agents produce information, and trade takes place in the stock markets. Finally, at date 2, the cash flow is realized, and all agents consume their realized gains.

**Firm:** Firms seek to maximize mean share price.\footnote{In Section 5, we extend our model to include firms that seek to maximize a mean-variance objective function.} A firm’s capital stock is denoted by $k$ and each firm owns a risky project with volatility $\sigma$. The end-of-period cash flows for firm with capital $k$ and with project volatility $\sigma$, is given by

$$\pi = k\theta,$$

where $\theta$ is a firm-specific payoff drawn from a Normal distribution with mean $\bar{\theta}$ and variance

In Section 5, we extend our model to include firms that seek to maximize a mean-variance objective function.
\( \sigma^2 \). We assume that the fundamental payoffs are uncorrelated across firms. Let \( y = (k, \sigma) \in Y \subseteq \mathbb{R}^2_+ \) represent firm types. Denote the joint c.d.f. of \((k, \sigma)\) by \( G^F(k, \sigma) \). All firms originally own \((1 + \psi)\) measure of shares and want to raise capital by issuing one measure of their equity to investors.

**Agent:** When the agent \( h \in H \) works for firm \( y \), he can produce a report \( z \) at cost \( C \) at date 1. Coverage, or the report, is a noisy, unbiased signal regarding the payoff:

\[
z = \theta + \sigma_h \eta, \tag{2}\]

where \( \eta \sim N(0, 1) \), and the variance of the report is parameterized as

\[
\sigma^2_h = \frac{1}{h}. \tag{3}\]

Higher values of \( h \) denote more precise agents.

**Labor Market for Agents:** At date 0, each firm can at most hire one agent to cover the firm assuming the firm hires any at all. The fee paid to the agent, denoted by \( \omega(h) \), is paid at date 0. That is, the fee is independent of the signal and realized payoff.\(^{12}\) The fee \( \omega(h) \) will be determined in equilibrium. The payoff of agent \( h \) is then given by \( \omega(h) - C \).

It is assumed the firm is be able to pre-commit to the hiring decision. At date 0, given the fee required to hire agent \( \omega(h) \), a firm of type \( y = (k, \sigma) \), rationally anticipating how different agents affect the stock price at date 1, chooses the optimal agent to maximize the firm’s expected payoff,

\[
U^*(k, \sigma) = \max_h E \left[ \left( \frac{\psi}{1 + \psi} \right) k\theta + \tilde{p}_{hy} \right] - \omega(h), \tag{4}\]

where \( \tilde{p}_{hy} \) denotes the realized share price at date 1 for firm \( y \) if it hires agent \( h \). That is, a firm

\(^{12}\)Since the fee is not contingent on the report \( z \), we thus assume away the agent’s incentive for biased report. In practice, analyst and media incentives might be biased due to conflicts of interest as pointed out in the Introduction.
effectively maximizes its expected share price. As standard in a noisy rational expectations stock market equilibrium, the realized price will be a function of the fundamental $\theta$, public signal $z$, and the demand of noisy traders $\tilde{u}$. That is, $\tilde{p}_{hy} = P(\theta, z, u|h, y)$.

**Financial Markets:** In the market for stock $(k, \sigma)$, there is a unit measure of a continuum of risk-averse investors. They have CARA expected utility with coefficient of risk aversion $\gamma_I$. Investors are imperfectly and heterogeneously informed. Specifically, each investor receives a private signal

$$x_i = \theta + \sigma_I \epsilon_i,$$

where $\epsilon_i \sim N(0, 1)$. For each stock, all investors observe the public signal $z$ produced by the agent and know its precision. These claims are also traded by noise traders. To solve the model in closed form, we assume that noise traders purchase a random quantity $\Phi(\tilde{u})$ of stock, where $\tilde{u} \sim N(0, \sigma_u)$ and $\Phi$ is the standard normal CDF. This specific functional form assumed here is close to that in Hellwig et al. (2006).

For simplicity, one can imagine that there is a segmented market for each stock and each investor only has access to one market. Nevertheless, this assumption can be further relaxed since all the fundamental payoffs are uncorrelated. In the Appendix, we show that our result remains unchanged when each investor has access to $N$ markets using our set-up below.

More specifically, each investor can purchase at most one share for each stock or none at all based on their information set. That is, they submit a price-contingent demand schedule, which specifies their demand $d_i(p) \in \{0, 1\}$ conditional on price $p$ to solve:

$$\max_{d \in \{0, 1\}} \left\{ dE[(k\theta - p + \psi)|x_i, z, p] - \frac{\gamma_I}{2} \text{Var}(k\theta - p|x_i, z, p) \right\}. $$

(6)

These bid functions determine the aggregate demand by informed investors. We maintain the restriction on demand for tractability. Alternatively, one can allow investors to submit a bidding schedule $d_i(p) \in \mathbb{R}$, which will not change the key economic results.\textsuperscript{13}

\textsuperscript{13}In the alternative setup, one would need to use a different assumption on the noise traders’ demand. Specifically, the noise traders’ demand is given by $\tilde{u}$ instead of $\Phi(\tilde{u})$. As standard, there exists a price which
the demand from the noisy traders, the auctioneer selects a price $P$ to clear the market.

**Equilibrium:** An equilibrium consists of an assignment $\mu(y) : Y \rightarrow H$, competitive fee for agents $\omega(h) : H \rightarrow \mathbb{R}^+$, demand function for each investor in the market $(h,y)$, $D(x_i, z, p|h, y)$, and a price function $P(\theta, z, u|h, y)$ such that the following three conditions are met.

First, in the labor market for agents, the optimality conditions for both firms and agents are satisfied, which means that, given the wage $\omega(h)$, $\mu(y)$ is the type of agent that firm $y$ optimally chooses to hire. That is, $\mu(y)$ maximizes (4). Second, in each market $(h,y)$, investors choose their demand schedules to maximize (6). Third and lastly, the market-clearing condition holds for both the agent and asset markets.

### 2.2. Characterization

We first analyze the surplus generated by agent $h$ for firm $y$, taking into account how the produced information affects the price movement in the stock market at $t = 0$. With this surplus function, we then analyze the matching of agents and firms in the labor market.

#### 2.2.1. Financial Markets

When firm $y$ hires agent $h$, investors thus obtain a public signal with precision $h = \frac{1}{\sigma_h}$. Aggregating the demand decisions of all investors in market $(h,y)$, market clearing then implies

$$
\int D(x_i, z, p|h, y)dF(x_i|\theta) + \Phi(\tilde{u}) = 1.
$$

(7)

From the investor’s optimization problem (6),

$$
D(x_i, z, p|h, y) \in \arg\max_{d \in \{0, 1\}} \left\{ dE[(\frac{k\theta}{1+\psi} - p)|x_i, z, p] - \frac{\gamma I d^2 Var(\frac{k\theta}{1+\psi} - p|x_i, z, p)}{2} \right\}
$$

$$
= \arg\max_{d \in \{0, 1\}} \left\{ dE[(\frac{k\theta}{1+\psi} - p)|x_i, z, p] - \frac{\gamma I}{2} \frac{k^2 d^2}{(1+\psi)^2} Var(\theta|x_i, z, p) \right\}.
$$

is linear in $(\theta, z, \tilde{u})$.  

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Since $\text{Var}(\theta|x_i, z, p)$ is constant over the realization of $(x_i, z, p)$, the demand $D(x_i, z, p|h, y) \in \{0, 1\}$ can be characterized by a cutoff, such that $D(x_i, z, p|h, y) = 1$ if and only if $x_i > \hat{x}(z, p)$.

Recall that each investor receives a private signal $x_i = \theta + \sigma_I \epsilon_i$, where $\epsilon_i \sim N(0, 1)$. This cut-off equilibrium then implies that only investors with good signals will buy, i.e. those investors with

$$\epsilon_i > \frac{\hat{x}(z, p) - \theta}{\sigma_I}. \quad (8)$$

With our specifications, the market-clearing condition can then be conveniently rewritten as

$$1 - \Phi\left(\frac{\hat{x}(z, p) - \theta}{\sigma_I}\right) + \Phi(\tilde{u}) = 1. \quad (9)$$

For the market to clear,

$$\hat{x} = \theta + \sigma_I \tilde{u}. \quad (10)$$

Hence, observing price in our model is informationally equivalent to a public signal, i.e. this cut-off value $\hat{x}$.\footnote{In general, as shown in Albagli et al. (2011), there exists a random variable that is only a function of $\theta$ and $\tilde{u}$, and contains the same information as the price.}

An investor’s information set can be summarized by $I_i = (x_i, z, \hat{x})$. Thus, the conditional expectation of the fundamental is given by

$$E[\theta|I_i] = \frac{\Sigma}{\sigma_x^2} \hat{x} + \frac{\Sigma}{\sigma_I} x_i + \frac{\Sigma}{\sigma_h^2} z + \frac{\Sigma}{\sigma_I^2 \sigma_u^2} \hat{x}, \quad (11)$$

where $\Sigma = \left(\frac{1}{\sigma_x^2} + \frac{1}{\sigma_I^2} + \frac{1}{\sigma_h^2} + \frac{1}{\sigma_I^2 \sigma_u^2}\right)^{-1}$. For the cut-off investor $\hat{x}$, the price must be equalized to the payoff of holding one share. Hence,

$$P(\theta, z, u|h, y) = \left(\frac{k}{1 + \psi}\right) E[\theta|x_i = \hat{x}, z, \hat{x}] - \frac{\gamma_I}{(1 + \psi)^2} \left(\frac{k^2}{\sigma_x^2}\right) \text{Var}(\theta|x, z, \hat{x})$$

$$= \frac{k}{1 + \psi} \left(\frac{h_\theta \hat{\theta} + h_z \hat{z} + h_m (\theta + \sigma_I \tilde{u})}{h_\theta + h + h_m}\right) - \frac{\gamma_I}{(1 + \psi)^2} \left(\frac{k^2}{\sigma_x^2}\right) \text{Var}(\theta|x, z, \hat{x}), \quad (12)$$
where
\[ h_m \equiv \frac{1}{\sigma_I^2} + \frac{1}{\sigma_I^2 \sigma_u^2} \]  \hspace{2cm} (13)
depends on market noise and
\[ h_\theta \equiv \frac{1}{\sigma^2} \]  \hspace{2cm} (14)
depends on firm volatility.

It is worth noting that the risk-aversion of investors does not affect any variance function relating to price but only changes the price level. The second term thus represents the risk premium for firm \( y \) that hires analyst \( h \), which has the following expression

\[ \tilde{R}(k, \sigma, h) \equiv \frac{\gamma_I k^2}{2 (1 + \psi)^2} \text{Var}(\theta|x, z, p) = \frac{\gamma_I k^2}{2(1 + \psi)^2(h_m + h + h_\theta)}. \]  \hspace{2cm} (15)

Let
\[ \varphi(h, \sigma) \equiv \frac{\text{Var}(\theta|x, z, p)}{2(1 + \psi)^2} \]  \hspace{2cm} (16)
and observe that it depends only on the underlying information structure, and not on the firm size. Hence, the risk premium can be expressed as a multiplicative function of firm size and the information precision. Lemma 1 summarizes the properties of the risk premium.

**Lemma 1.** The risk premium for firm \( (k, \sigma) \) that hires analyst \( h \) is given by \( \gamma_I k^2 \varphi(h, \sigma) \), which strictly decreases with the precision of the agent (i.e., \( \frac{\partial \varphi(h, \sigma)}{\partial h} < 0 \ \forall h \)) and strictly increases with firm size \( k \), as well as with firm volatility \( \sigma_\theta^2 \).

That is, a higher level of precision decreases the risk premium charged by investors, since it improves investor’s estimation of the fundamental pay-off. Clearly, all things being equal, the risk premium is higher for larger firms and for a more volatile firms, since the investors have to bear more risk for those firms. But of course, not all things will be equal in equilibrium, as firms with different characteristics have varying incentives or purchase varying degrees of accurate coverage, or even none at all.
2.2.2. The Labor Market for Coverage Agents

As is well-known in a matching model, the allocation of agents across firms depends on the property of the matching surplus of firm and agent. In our model, this surplus is driven by the coverage effect in financial markets. Given Equation (12), which takes into account the resulting risk premium when hiring an agent $h$, the firm’s expected utility can then be conveniently rewritten as the expected payoff of the project minus the risk premium and the agent fee:

$$U(y, h) = k \bar{\theta} - \gamma_1 k^2 \varphi(h, \sigma) - \omega(h).$$  \hspace{1cm} (17)

That is, intuitively, the benefit of hiring a more precise agent leads to a lower risk premium and thus a lower cost of capital. On the other hand, the cost is simply the agent fee. And, one would expect that a more precise agent is also more costly in equilibrium as a result of competition.

The surplus $\Omega(y, h)$ generated between firm $y$ and agent $h$, which is defined as the sum of the agents’ payoff minus their outside option, $\Omega(y, h) \equiv \{U(y, h) - U(y, \emptyset)\} + \{\omega(h) - C\}$, where $\emptyset$ denotes the case in which a firm chooses to hire no agent. The first term thus represents the gain of firm $y$ when he hires agent $h$ relatively to no hiring. The second term represents the payoff of a worker, which is the fee minus the production cost. And the worker’s unemployed value is normalized to zero. Hence, the surplus is simply the value of coverage, which decreases the risk premium, minus the production cost:

$$\Omega(y, h) = \gamma_1 k^2 \{\varphi(\emptyset, \sigma) - \varphi(h, \sigma)\} - C$$ \hspace{1cm} (18)

To understand how different firm size and pay-off volatility determine the allocation of agents, we first analyze each heterogeneity separately and use these analyses to establish our
main results. That is, in our basic model, we consider an environment in which firms only differ either in their size \( k \) or volatility \( \sigma \). In the extension section, we then consider joint, two-dimensional heterogeneity. Lemma 2 establishes that, assuming fixed firm volatility, there must be positive assortative matching between more precise agents and larger firms. Similarly, all else being equal, a more volatile firm must hire a better agent.

**Lemma 2.** There is positive assortative matching (PAM) (1) by size: a larger firm hires a more precise agent, \( \frac{\partial \mu(k,\sigma)}{\partial k} \geq 0 \); and (2) by volatility: a more volatile firm hires a more precise agent, \( \frac{\partial \mu(k,\sigma)}{\partial \sigma} \geq 0 \).

It is well-known that the sorting between heterogeneous agents and firms in a competitive equilibrium is determined by the *complementarity* of the surplus function. Hence, the results in Lemma 2 can be derived from Equation (18). Clearly, the marginal value of more precise information strictly increases with size:

\[
\frac{\partial^2 \Omega(y,h)}{\partial k \partial h} = -2\gamma_1 k \frac{\partial \varphi(h,\sigma)}{\partial h} > 0.
\]

(19)

Since firms with a higher \( k \) have more risk that the investors have to bear, a better estimation reduces the risk premium more substantially. One can also show that there is complementarity between volatility and precision; that is,

\[
\frac{\partial^2 \Omega(y,h)}{\partial \sigma \partial h} \propto -\frac{\partial^2 \varphi(h,\sigma)}{\partial h \partial \sigma} > 0.
\]

(20)

The intuition is also simple: when returns are more volatile, an increase in the precision of public information substantially improves investor’s estimation by more than if firm returns are less volatile.

Proposition 1 characterizes the equilibrium outcome when firms differ only in either size \( k \) or volatility \( \sigma \). According to Lemma 2, there is positive sorting for both cases. Hence, the equilibrium outcome exhibits similar patterns. With a slight abuse of notation, we keep \( y \)
as the index of firm types; however, in the case in which firms are different only in terms of their size, a higher index \( y \) simply represents a firm with a larger size \( k \). Similarly, in the case in which firms are different only in terms of their volatility, a higher \( y \) is then interpreted as a firm with higher volatility \( \sigma \).

**Proposition 1.** The equilibrium is characterized by the marginal agent \( h^* \) and firm \( y^* \) that jointly solve \( \Omega(y^*, h^*) = 0 \) and \( G^A(h^*) = G^F(y^*) \). Given these,

1. Firms with type \( y < y^* \) do not hire agents, \( \mu(y) = \emptyset \forall y < y^* \). And, the equilibrium assignment function for firms with type \( y \geq y^* \) solves the differential equation

\[
\mu'(y) = \frac{dG^F(y)}{dG^A(\mu(y))}
\]

with the initial condition \( \mu(y^*) = h^* \).

2. The fee for agents \( \omega(h) \) is given by

\[
\omega(h) = \omega(h^*) + \int_{h^*}^{h_U} \Omega_h(\mu^{-1}(\tilde{h}), \tilde{h})d\tilde{h},
\]

where \( \Omega_h(y, h) \) denotes the partial derivative respect to \( h \), and \( \mu^{-1} \) denotes the inverse of \( \mu \), representing the type of firm assigned to agent \( h \). And \( \omega(h^*) = C \).

3. The market price for firm \( y \) is then characterized by Equation (12), setting \( (h, y) = (\mu(y), y) \), and the demand function for firm \( y \) is given by \( D(x_i, z, p | \mu(y), y) \).

The equilibrium outcome can be understood as follows. All agents with higher precision than the cutoff type \( h^* \) will be hired. Given the market-clearing condition in the labor market, the total measures of agents that are hired must equal the total measures of active firms; hence there exists a cutoff type of firm \( y^* \) such that

\[
\int_{y^*}^{y_U} dG^F(y) = \int_{h^*}^{h_U} dG^A(h),
\]

which explains the condition \( G^A(h^*) = G^F(y^*) \).
Furthermore, conditional on being hired, the least-precise agent must work for the lowest-type firm, \( \Omega(y^*, h^*) = 0 \), thus guaranteeing that the surplus between the least-precise agent and the lowest type of firm is exactly zero, taking into account the fixed cost \( C \). Given this, one can then easily see that all the matching surpluses generated by inactive firms and agents must be negative, which explains why no agent \( h \leq h^* \) is hired.

As a result of positive sorting, the assignment function \( \mu(y) \), which represents the type of agent that a firm will hire, must then solve

\[
\int_{\mu(y)}^{h_U} dG^A(h) = \int_{y_U}^{y_U} dG^F(\tilde{y}),
\]

which explains the result in point (1) of the Proposition 1. The agent fee in Equation (22) is pinned down in a standard way (as in Terviö (2008)), which shows that the marginal increase in the fee of agent \( h \) is his contribution to the surplus \( \Omega(h^{-1}(h), h) \) within the match, given his optimal assignment \( \mu^{-1}(h) \). Lastly, given the equilibrium sorting between firm \( y \) and agent \( h \), the characterization of the market clearing price and investors’ demand function in each asset market is then provided in Section 2.2.1.

3. Implications

We now establish empirical implications for cross-sectional variation in coverage across different stocks, asset returns and agent fees or wages. To highlight the unique force of sorting, it is useful to establish a counterfactual environment in which sorting is not allowed. Specifically, we refer the environment in which all firms are allowed to hire an average agent,

\[
h \equiv \int h dG^A(h),
\]

as the no-sorting benchmark.\(^{15}\)

\(^{15}\)One can also interpret this environment as one in which the skills of agents are unobserved.
3.1. Coverage and Neglected Stocks

As established in Proposition 1, larger firms or more volatile firms will have more precise coverage. Moreover, which firms and how many of them will have zero coverage (i.e., being neglected) can be seen from the cutoff type $y^*$. Formally, define

$$N^s \equiv \eta (\{y : \mu(y) = \emptyset, \forall y \in Y\}) \tag{26}$$

as the measure of firms who receive zero coverage. In other words, $1 - N^s$ then represents the extensive margin of aggregate market coverage.

It is worth noting that, due to the fix cost of information production or coverage, one would expect that some firms might not hire agents. Hence, to emphasize our neglect effect as a result of the competition (i.e. sorting), we compare our results to the environment in which all firms can choose to hire the average quality of agents $h = \int h dG^A(h)$. and let $\mu^{ns}(y) \in \{\emptyset, \bar{h}\}$ denote whether a firm $y$ has agent coverage in that case. Hence, the measure of firms who receive zero coverage without sorting is then given by

$$N^{ns} \equiv \eta (\{y : \mu^{ns}(y) = \emptyset, \forall y \in Y\}) \tag{27}$$

Proposition 2. (Results on the Measure of Neglected Firms)

1. There exists $\bar{c} > 0$ and $\underline{c} > 0$ such that, sorting strictly increases the effect of neglect $N^s > N^{ns}$ for $C \in (\underline{c}, \bar{c})$.

2. Consider two talent distributions, where $G^A_1(h)$ first-order stochastically dominates $G^A_2(h)$. The measure of neglected stocks must be weakly higher under the distribution $G^A_2(h)$.

3. The measure of neglected stocks $N^s$ increases with information cost $C$, and decreases with market noise ($\sigma_I$ and $\sigma_u$), and investor’s risk aversion ($\gamma_I$).

4. With heterogeneity in size, $N^s$ decreases with volatility; and with heterogeneity in volatility, $N^s$ decreases with firm size.
The first result establishes how the competition leads to more firms being neglected. Consider the case when \( \Omega(h, y_L) \geq 0 \), the value of information is then positive for all firms without sorting, taking into consideration the production cost. However, in a competitive market of agents, smaller (or less volatile) firms would only hire agents with relatively low precision. The value of such a relatively low precision no longer compensates for the information cost \( C \) so that \( \Omega(h_L, y_L) < 0 \). As a result, these firms do not hire agents in equilibrium. The condition on the production cost simply guarantees that the cost is not too high so that it is valuable to hire an average analyst but it is also not too low so that the worst agent does not generate enough surplus to compensate the production cost.

The rest of the results on the other hand highlight how the marginal value of information depends on the underlying parameters. Intuitively, the marginal value is high when the market is relatively noisy, or when firms are larger or have more volatile fundamentals. Hence, the surplus increases in those cases, which leads to more aggregate market coverage and thus less neglected stocks.

One way to test Proposition 2 is to regress the fraction of neglected firms in the cross-section on various proxies for fundamental uncertainty or degree of noise trading. Such proxies are widely available in the literature. Most existing empirical studies simply recognize the neglected firm effect but our model offer some guidance on empirical work identifying the underlying market forces driving it.

Lastly, Proposition 3 establishes the informativeness of price across firms, highlighting the effect of cross-sectional variation in coverage on price efficiency. Observe that for firm \( y = (k, \sigma) \) with an agent \( h \), the conditional distribution of the return \( k\theta \) given \( p \), is

\[
N(k\bar{\theta} + \frac{cov(p, \theta)}{var(p)} (p - \bar{p}_{hk}), k^2 Var(\theta|p_{hy})),
\]

where \( \bar{p}_{hy} \) is the average price, \( \bar{p}_{hy} = k\bar{\theta} - \gamma_1 k^2 \varphi(h, \sigma) \). All else equal, the informativeness of price, which is defined as \( \{Var(\theta|p_{hy})\}^{-1} \), clearly increases with the precision of agents.
This immediately predicts that the informativeness of price increases in firm sizes due to the coverage effect.

On the other hand, with heterogeneity in volatility only, if the coverage is homogenous across firms, one would then expect that the informativeness of price decreases with firm volatility. Our model shows that, the informativeness of price may in fact increase with firm volatility due to the coverage effect. Moreover, such coverage effect is stronger whenever a more precise information can have a greater impact to the market. This happens when investors are less informed ex-ante and when all firms have relatively risky projects.

**Proposition 3.** (1) With heterogeneity in size only, the informativeness of price increases in firm size. (2) With heterogeneity in volatility only, the informativeness of price increases in volatility when investors are less informed (large enough $\sigma_I$) and when the level of volatility of all firms are relatively high.

These predictions have not to our knowledge been carefully tested but measures of informativeness of prices for fundamental value can be constructed by forecasting regressions of fundamentals on price and calculating the extent to which price is an accurate proxy. One can then take these measures and regress them on the predicted firm characteristics both in the cross-section and over time.

### 3.2. Asset Pricing Puzzles

In this section, we examine how the cross-sectional variation in coverage affects the expected return across firms. Given that the firm hires an agent with precision $\mu(y)$, the (unconditional) asset return for firm $y$ is then given by

$$R(y) \equiv E[E[k\theta - p|p]] = \gamma_I k^2 \varphi(\mu(y), \sigma),$$

(29)

which is effectively the risk premium. Note that, the key difference from Lemma (1) is that we now take into account the equilibrium assignment $\mu(y)$ to calculate the risk premium for
3.2.1. Neglected Firm Effect

Recall that the neglected firm effect (Arbel et al. (1983), Foerster and Karolyi (1999)) finds that zero or low analyst coverage out-perform stocks with high analyst coverage. This effect has been typically interpreted as investor recognition along the lines of Merton (1987), in which neglected stocks have investor recognition, a lower investor base and less risk-sharing. Hence they require a large risk premium than covered stocks. The issue of this explanation is that coverage is loosely linked to investor recognition.

In contrast, our model delivers a more direct link between coverage and expected returns via Lemma 1. Lemma 1 shows that stocks with coverage or more precise coverage have more efficient prices and a lower risk premium, all else equal.

3.2.2. Small Firm Effect

Our model can potentially rationalize the small firm effect, whereby small firms have lower prices and higher expected returns than large firms (see, e.g., Banz (1981)). Notice again from Lemma 1 that in our model absent coverage, it should be large firms with lower prices and higher expected returns since investors have to bear more risk. But we show that the positive assortative matching effect can over-turn this risk effect.

To see why, we consider an environment in which firms have the same volatility but different capital stock $k$. Hence, setting $y = k$, the type of agent that is hired by firm $k$ is then given by $\mu(k)$ in Proposition (1). The effect of size on expected return is then given by

$$R'(k) = \gamma_I \{2k \varphi(\mu(k), \sigma) + k^2 \varphi_h(\mu(k), \sigma) \mu'(k)\}$$

$$= \frac{\gamma_k k}{2(1 + \psi)^2} \left( \frac{2(h_m + \sigma^{-2} + \mu(k)) - k \mu'(k)}{h_m + \sigma^{-2} + \mu(k)} \right),$$

(30)

where $\mu'(k) = \frac{g'(k)}{g'(\mu(k))}$ is determined by the distribution of firms and agents for $\mu(k) \neq \emptyset$. It
is worth noting that, when all firms hire an average analyst (i.e., no sorting), $\mu'(k) = 0$ and, clearly, the risk premium increases with firm size. This highlights the unique force coming from sorting, which has an opposite effect on expected returns. By hiring a better agent in equilibrium, a larger firm provides more precise information to investors, leading to a lower risk premium. When this coverage force is stronger, the model then predicts lower returns for larger firms.

These two competing forces therefore suggest that, in general, firm size can have ambiguous effects on returns. As shown in Equation (30), the coverage effect is stronger with a steeper assignment function, which highlights the affect of underlying talent distribution. To illustrate the impact of talent distribution, we consider a uniform distribution of both firms and agents. That is, $G_F(k) = \frac{k - k_L}{k^U - k_L}$ and $G_A(h) = \frac{h - h_L}{h^U - h_L}$. Hence, according to Equation (21),

$$\mu'(k) = \frac{g_F(k)}{g_A(\mu(k))} = \frac{h^U - h_L}{k^U - k_L}$$

is a constant ratio of two densities function. A decrease in $h_L$ thus represents a decrease in the average quality of agents, and thus talent agents become more scarce. Hence, for small enough $h_L$, that is, when there is relatively scarce agent talent, the coverage effect dominates in the sense that $R'(k) < 0$.

Motivated by the existing empirical evidence on the small firms effect, we provide testable prediction on the comparison of the risk premium for larger firms vs. smaller firms. To do so, one divide firms into two groups: the small-firm group contain firms with size that is smaller than the $X_1$th percentile and the large-firm group includes firms with size that is larger than the $X_2$th percentile, where $X_1 \leq X_2$. The average risk premium for small-firm group and for large-firm group is then given by, respectively, $R_S(X_1) \equiv E[R(k) | G_F(k) \leq X_1]$ and $R_L(X_2) \equiv E[R(k) | G_F(k) \geq X_2]$.

Below, we analyze how market noise and talent distribution affect these two forces and establish the condition under which the coverage effect dominates.

**Proposition 4.** (Coverage Effect and Firm Size) (1) Given any $X_1 \leq X_2$, there exists $\bar{c} \geq 0$ and $\bar{h}_m > 0$, such that the average risk premium for small-firm group is higher than that of larger-firm group for any $C \leq \bar{c}$ and $h_m < \bar{h}_m$. That is, the coverage effect dominates,
\( \bar{R}_S(X_1) > \bar{R}_L(X_2) \), when there is enough market noise (i.e., large \( \sigma_u \) and \( \sigma_I \)) and when the production cost is small enough.

(2) Given any \( X_1 \leq X_2 \), the coverage effect dominates when there is relatively scarce agent talent and a small enough production cost. That is, \( \bar{R}_S(X_1) > \bar{R}_L(X_2) \) for small enough \( h^L \) and \( C \).

Intuitively, coverage effect plays a role only if firms do hire analysts in equilibrium, which explains why a low production cost is a necessary condition in Proposition 4. Furthermore, the coverage effect dominates when the sorting force is strong. This happens when there are less talents available, so that the variation across firms increases (i.e., a higher \( \mu'(k) \)). Or, when the market is relatively noisy, so that an increase in the precision has a greater impact.

To see why, consider an extreme case when the market itself is very informative (i.e., a large enough \( h_m \)), in this case, according to (30), the coverage effect is relatively small so that \( R'(k) > 0 \).

3.2.3. Idiosyncratic Volatility Effect

Our model can also explain the idiosyncratic volatility effect whereby high idiosyncratic volatility firms have lower expected returns than high idiosyncratic volatility firms (see, e.g., Ang et al. (2006)). The logic is similar as that for the small firm effect. More volatile firms pay for more accurate coverage and this positive sorting effect can dominate the underlying firm volatility effect.

We now turn to study the environment in which firms differ in their volatility \( \sigma \) but have the same size \( k \).

\[
R'(\sigma) = \gamma_I k^2 \{ \varphi_\sigma(\mu(\sigma), \sigma) + \varphi_h(\mu(\sigma), \sigma) \mu'(\sigma) \}
= \frac{\gamma_I k^2}{2(1 + \psi)^2} \left( \frac{2\sigma^{-3} - \mu'(\sigma)}{(h_m + \frac{1}{\sigma^2} + \mu(\sigma))^2} \right),
\]

(31)
where \( \mu'(\sigma) = \frac{g^F(\sigma)}{g^A(\mu(\sigma))} \). Again, Equation (31) shows the unique force coming from the sorting.
Without sorting (i.e. $\mu'(\sigma) = 0$), the expected return should increase with firm volatility.

Furthermore, one can see that the coverage effect dominates with the level of firm volatility is high. That is, $R'_(\sigma) < 0$ for large enough $\sigma$. In that case, an agent with a higher precision is relatively valuable. To give a concrete example, suppose that the volatility of all firms is given by $\sigma' = \sigma + \Delta$. Hence, a higher $\Delta > 0$ represents a higher level of volatility for all firms without changing the density ratio.

Same as before, we derive our testable predictions on volatility by dividing firms into two groups: the stable-firm group contain firms with volatility that is smaller than the $X_1$th percentile and the volatile-firm group includes firms with volatility that is larger than the $X_2$th percentile, where $X_1 \leq X_2$. The average risk premium for stable-firm group and for volatile-firm group is then given by, respectively, $\bar{R}_S(X_1) \equiv E[R(\sigma)|G_F(\sigma) \leq X_1]$ and $\bar{R}_L(X_2) \equiv E[R(\sigma)|G_F(\sigma) \geq X_2]$.

**Proposition 5.** (Coverage Effect and Firm Volatility)

(1) Consider a shift in the volatility distribution, where $\sigma' = \sigma + \Delta \forall \sigma$ where $\Delta > 0$. Given any $X_1 \leq X_2$, there exists $\bar{c} \geq 0$ and $\bar{\Delta} > 0$, such that the average risk premium for stable-firm group is higher than that of larger-firm group for any $C \leq \bar{c}$ and for $\Delta > \bar{\Delta}$. That is, the coverage effect dominates, $\bar{R}_S(X_1) > \bar{R}_L(X_2)$, when the level of volatility is higher enough and when the production cost is small enough.

(2) Given any $X_1 \leq X_2$, the coverage effect dominates when there is relatively scarce agent talent and a small enough production cost. That is, $\bar{R}_S(X_1) > \bar{R}_L(X_2)$ for small enough $h^L$ and $C$.

This proposition also offers testable implications on when the idiosyncratic volatility effect is likely to be stronger or weaker for future empirical work. Similar as before, the coverage effect is stronger when there is less talents available and when the projects are riskier so that the precision of agent is relatively valuable.
3.3. Compensation for Coverage

In addition to the coverage and pricing results, we can also derive testable predictions about the cross-sectional distribution of wages. To see the prediction on the wage distribution, it is convenient to look at the wage profile $\omega[i]$, where $\omega[i]$ denotes the $i$th percentile wage. As shown in Terviö (2008), the wage profile as a function of $i$th percentile can then be rewritten

$$\omega'[i] = \Omega_h(y[i], h[i])h'[i]$$  

(32)

where $y[i] = y$ s.t. $G^F(y) = i$ and $h[i] = h$ s.t. $G^A(h) = i$. To see this, one can think of the agents or the firms are ordered by their type. Hence, $h[i]$ is the precision of an $i$ quantile individual and $y[i]$ is the size (or volatility) of an $i$ quartile firm. This expression highlights the fact that, as a result of PAM (in 2) and labor market clearing condition, an $i$ quantile agent then must match with an $i$ quantile firms.  

A higher slope of the wage profile then predicts a more dispersed wage distribution and a stronger superstar effect.

Proposition 6. (1) The slope of the wage profile increases with the market noise ($\sigma_I$ and $\sigma_u$). That is, $\frac{\partial}{\partial \sigma_I} \omega'[i] > 0$ and $\frac{\partial}{\partial \sigma_u} \omega'[i] > 0$. (2) If the type of all firms is multiplied by $\alpha$, $w'[i]$ increases with $\alpha$. (3) With uniform distribution of agents, $G^A(h) = \frac{h-h_I}{h^U-h_L}$, $\omega'[i]$ increases when there is relatively scarce agent talent (i.e., lower $h_L$).

In other words, the wages of agents are convex in the talent distribution of agents. They rise with firm size, firm pay-off volatility, and the ratio of noise trading to the precision of private signals in the market. These results are similar to CEO matching literature (see, e.g., Terviö (2008); Gabaix and Landier (2006)). Rather than complementarities embedded

\[^{16}\text{From the market clearing condition (23), } \mu^{-1}(h[i]) = y[i]. \text{ Furthermore, according to Equation (32), } \omega'(h) = \Omega_h(\mu^{-1}(h), h). \text{ Hence, Equation (32) can be derived from } \omega'[i] = \omega'(h)h'[i].\]
in the firm production function, we derive complementary endogenously from the firm’s stock pricing and trading environment.

There is evidence in the literature consistent with the result (2) on wage convexity. During the Internet Bubble Period of 1997-2000, security analysts’ pay were highly skewed as the prices of dot-com stocks were noisy and underlying dot-com pay-offs were also highly uncertain. Top analysts such as Henry Blodgett and Mary Meeker had compensation in the millions of dollars per year similar to top investment bankers. After the dot-com bubble, it is generally thought that compensation was significantly scaled back because of regulatory reforms such as Reg-FD which curtailed conflicts of interests in the industry.

Our model predicts that, beyond regulatory pressures, this compensation is also fundamentally tied to the uncertainty in the stock market and the scarcity of talent. Indeed, during the Second Internet Boom of the mid 2000s, compensation for analysts have returned with again very skewed pay-offs for the top analysts.17

The compensation predictions of our model can, like the coverage and asset pricing results, also be readily tested. Specifically, combining Proposition (6) and the predictions from Proposition (4) and (5), it shows that the coverage effect is positively correlated with the wage skewness. In other words, we provide a testable prediction: a higher $\omega'[^i]$ tends to coexists with the small firms effect as well as the idiosyncratic volatility effect.

3.4. Alternative Set-up: Homogeneous Precision and Hiring Multiple Agents

Our theory rationalizes the small firm effect and the idiosyncratic volatility effect by establishing that the coverage effect can over-turn the original risk effect for larger firms or for more volatile firms, and the talent distribution plays an unique role. Such a force would not appear in a setting where firms are competing for extensive margin (i.e., quantities of information).

17Susan Craig, "Star Analysts Are Back (No Autographs, Please), August 20, 2011, NYTIMES DealBook
To see this clearly, it is useful to consider an alternative setting, where all agents have the same precisions $h_0$ while firms are allowed to hire multiple agents instead. For the sake of illustration, we assume that the precision of the public signal is simply given by $nh_0$, where $n$ is the number of agents that a firm hires (i.e., the intensive margin). Firm’s optimization problem can then be expressed as,

$$\max_n \left\{ -\gamma I k^2 \varphi(n, \sigma) - nw \right\}, \quad (33)$$

where $w$ represents the competitive wages and $\varphi(n, \sigma) \equiv \frac{1}{2(1+\psi)^2(h_m+nh_0+\frac{1}{\sigma^2})}$. Let $n^*(y)$ denote the solution to (33). One can easily show that, for the same intuition, the larger firms or the more volatile firms would have a higher incentive to hire more agents, which effectively maps to a more precise public signal. That is, $n^*(y)$ is strictly increasing in $k$ or $\sigma$. In this case, the risk premium is then given by $R(y) = \frac{\gamma I k^2}{2(1+\psi)^2(h_m+n^*(y)+\frac{1}{\sigma^2})}$. The counterpart of Equation (30) is then given by

$$R'(k) = \gamma I \left\{ 2k\varphi(n^*(k), \sigma) + k^2\varphi_n(n^*(k), \sigma)\frac{dn^*(k)}{dk} \right\}.$$

In a competitive equilibrium, the marginal return of adding one more agent must be equalized. That is,

$$\gamma I k^2 \varphi_n(n^*(y), \sigma) = w \forall y. \quad (34)$$

Hence, the expression of $\frac{dn^*(k)}{dk}$ is then given by

$$\frac{dn^*(k)}{dk} = -\frac{2\varphi_n(n^*, \sigma)}{k\varphi_{nn}(n^*, \sigma)},$$

and therefore one can show that the coverage effect would never over-turn the original size effect since $R'(k) > 0$. \(^{18}\)

---

\(^{18}\)This can be seen from: $R'(k) = \frac{\gamma I k}{2(1+\psi)^2} \left\{ \frac{2}{h_m+nh_0+\frac{1}{\sigma^2}} - \frac{2h^2}{(h_m+nh_0+\frac{1}{\sigma^2})^2} \right\} \left(\frac{h_m+nh_0+\frac{1}{\sigma^2}}{2}\right)^3 > 0.$
Similarly, when firms are different in volatility, the fact that the marginal value is equalized suggests that the total precision \( h_m + (n(\sigma)h_0 + \frac{1}{\sigma^2}) \equiv H(\sigma) \) must be the same for all firms. As a result, \( R'(\sigma) = 0 \). Again, this shows that the coverage effect is not enough to explain the idiosyncratic volatility effect without the sorting force.

4. Extension

4.1. Two-dimensional Heterogeneity

We now consider an environment in which firms differ in two dimensions, size \((k)\) and volatility \((\sigma)\). Recall that the marginal of precision for firm \(y = (k, \sigma)\) is given by

\[
\frac{\partial \Omega(y, h)}{\partial h} = \frac{\gamma I}{2(1 + \psi)^2} \left( \frac{k}{\frac{1}{\sigma^2} + h_m + h} \right)^2.
\] (35)

Intuitively, firms with higher marginal value (either because of being larger size or more volatile) hire a more skilled agent. It is therefore convenient to rank firms by their marginal value of precision, denoted by

\[
q(y, h) = \frac{k}{\frac{1}{\sigma^2} + h_m + h}.
\] (36)

That is, given any precision \(h\), firms with a higher index \(q\) would be willing to pay more for the coverage.

As shown in Lemma 3, the positive sorting can then be effectively understood between the weighted index \(q\) and agent \(h\). The characterization will then be similar to Proposition 1, where firms with higher index \(q\) matches with better agents.\(^{19}\)

**Lemma 3.** (1) If \(h = \mu(y) = \mu(y')\), then \(q(y', h) = q(y, h)\); (2) If \(q(y', h) > q(y, h)\), then \(\mu(y') > \mu(y)\).

Since \(q(k, \sigma, h)\) increases with firm size and with volatility, our basic mechanism continuous to hold and the cross-sectional variation in coverage remains intact. On the other hand,\(^{19}\) See appendix for the detailed characterization.
the variation across firms would decrease when size and volatility are negatively correlated. For example, consider the special case when firm size is perfectly negatively correlated with volatility, where the volatility of the firm with size $k$ is given by $\sigma^2 = \frac{1}{k}$. While the positive sorting still holds for larger firms, all firms have more similar marginal values, implying a lower competition.

The additional force in this two dimensional heterogeneity, on the other hand, is that firms have to compete for both dimensions. To see this clearly, consider firms who have the same marginal value of hiring a more skilled agent, the marginal rates of substitution between size and volatility is given by

$$\frac{dk}{d\sigma}|_{q(k,\sigma,h)=\bar{q}} = -\left\{ \frac{2k}{\sigma^2(h_m+h)+\sigma} \right\}. \tag{37}$$

A higher substitution rate thus implies a relatively more volatile and small firm can outbid a relatively large but less volatile firm. Proposition below formally shows that the volatility factor become more important when the market is exposed to more market noise. Hence, if two firms hire the same agent under the market precision $h_m$, the more volatile firm $y'$ must hire a more skilled agent when the levels of noise trading increases $h'_m < h_m$. This can be seen immediately from (37): a lower market precision implies a higher substitution rate.

Proposition 7. Consider two firms $(y',y)$, where $k' < k$ and $\sigma' > \sigma$, if $\mu(\sigma',k';h_m) = \mu(\sigma,k;h_m) = h$, then $\mu(\sigma',k';h'_m) > \mu(\sigma,k;h'_m)$ for $h'_m < h_m$.

4.2. Risk-Averse Firm

Finally, we consider how our results change when we consider a firm that acts risk-averse. Risk averse behavior might follow from financial constraints or agency frictions as we alluded to in the Introduction. Assume that firms have CARA expected utility with coefficient of risk aversion $\gamma_f$ and, for simplicity, assume that firms are liquidating all shares and thus $\psi = 0$. The expected utility of firm $k$ when hiring analyst $h$ is then given by $U(y,h) =$
\[ \mathbb{E}[W_{hy}] - \frac{\gamma_f}{2} \text{Var}[W_{hy}], \] where \( W_{hy} = k \bar{\theta} - \gamma f k^2 \varphi(h, \sigma) - \omega(h) \). The firm’s expected payoff becomes

\[ U(y, h) = k \bar{\theta} - \frac{\gamma f}{2} k^2 \text{Var}(\theta | x, z, p) - \frac{\gamma f}{2} \text{Var}(p_{hy}) - \omega(h), \] (38)

where

\[ \text{Var}(p_{hy}) = k^2 \frac{(h_\theta + h_m) h_\theta^{-1} + h + h_m (1 + \sigma^2_u)}{(h_\theta + h + h_m)^2}. \] (39)

One can show that price volatility can be non-monotonic in the precision of coverage:

\[ \frac{\partial \text{Var}(p_{hy})}{\partial h} = k^2 \left\{ \frac{(h_\theta + h) + h_m (1 - 2 \sigma^2_u)}{(h_\theta + h + h_m)^3} \right\}. \]

Observe that when the market price is already relatively precise (i.e., few noise trading and thus small \( \sigma^2_u \)), the variance of wealth strictly increases with the precision \( h \). Hence, hiring a analyst can help decrease variance only when the market is exposed to large levels of noise trading (with large \( \sigma^2_u \)). Such an effect can be understood from investors’ coordination motives through market price. More precise public information, by reducing investor’s reliance on private information, can then amplify the price sensitivity to the noise in public information.\textsuperscript{20} This explains why a more precise public information may increase the price volatility. This channel disappears when investors don’t learn from the price (i.e., in the limit case when price is not informative, \( \sigma^2_u \rightarrow \infty \)) and, therefore, precision decreases volatility when a high enough level of market noise.

The value of precision is thus determined by two forces: the variance effect as well as the risk premium effect in our basic model. Below, we first establish the condition under which our positive sorting result remains intact. Specifically, as long as investors are less risk averse than firm, there is complementarity between precision and larger firms, or more volatile firms. In this case, the equilibrium can then be characterized by Proposition 1 with

\textsuperscript{20}The bias toward the public signal has been pointed out in Morris and Shin (2002) and Allen et al. (2006). Although there is no explicit coordination motive in the rational expectations equilibrium, when individual’s willingness to pay is related to their expectation of aggregate demand, asset prices tend to overweight public information and hence the noise.
a modified surplus function

\[ \Omega(y,h) = \gamma_I k^2 \{ \varphi(0,\sigma) - \varphi(h,\sigma) \} + \frac{\gamma_I}{2} \{ \text{Var}(p_{0y}) - \text{Var}(p_{hy}) \} - C. \] (40)

**Lemma 4.** PAM holds as long as \( \gamma_I \geq \gamma_f \). That is, a larger or a more volatile firm hires a more precise agent.

Next, we turn to the case when firms are relatively risk averse \( \gamma_I < \gamma_f \). In this case, the variance effect dominates and firm’s value no longer monotonically increases in the precision. As shown in Proposition below, only when the market is exposed to a large noisy trading, firms have incentives to hire analysts and no agent is hired whenever the market noisy is small enough.

In the case that the firms do hire agents, when firms differ in size, the larger firms hires the one who produces the highest value (but not necessarily the one with highest precision). On the other hand, when firms differ in volatility, PAM between volatility and precision continuous to hold for larger enough noisy trading.

**Proposition 8.** (1) When \( \gamma_I < \gamma_f \) and the market is relatively precise (small enough \( \sigma_u \)), no agent is hired and thus \( \mu(y) = \emptyset \).

(2) The heterogeneity in Size: When \( \gamma_I < \gamma_f \) and for larger enough noisy trading \( \sigma_u > \hat{\sigma}_u \): there exists a cutoff \( h^* \geq 0 \) and \( k^* \geq 0 \) such that all agents with lower precision (i.e., \( h \leq h^* \)) are hired. Only larger firms hire analysts but the assignment \( \mu(k) \) is non-monotonic in \( k \), where the middle size firm \( k_m \in [k^*,k^U] \) hires the most precise agent.

(3) The heterogeneity in Volatility: When \( \gamma_I < \gamma_f \), PAM holds only if there is larger enough noisy trading.

Result (3) thus implies that all our pricing predictions on volatility effect due to PAM remain intact as long as there is enough noisy trading. On the other hand, when no agent is hired, all pricing results collapse to the defaulted environment. That is, the risk premium is higher for larger firms and for a more volatile firms. The non-monotonicity in result (2), on
the other hand, suggests that the effect of size on expected return is also monotonic. Same as before, the smaller firms will not have coverage and hence, a relatively high premium. However, conditional on being coverage, the middle size firm $k_m$ hires the most precise agent, thus have a lowest risk premium. All larger firms, on the other hand, must have a higher premium than this middle size firm, $R(k) > R(k_m) \forall k \geq k_m$.

5. Conclusion

Motivated by recent empirical studies pointing to the importance of coverage for the functioning of asset markets, we provide a theory of stock market coverage emphasizing the sorting or assignment of firms of heterogeneous characteristics and in a labor market of agents who generate a public signal about the firm’s fundamental value, i.e. coverage, with heterogeneous precision. In a noisy rational-expectations stock market equilibrium, we show that there is positive assortative matching—large firms with more investments or more volatile firms benefit more and pay more for accurate coverage as it leads to greater price efficiency and less risk discount. It turns out that this assortative matching effect has wide-reaching implications for thinking about a host of issues in stock markets, including coverage patterns, wage distributions and even well known asset pricing anomalies including the neglected firm effect, small firm effect, and idiosyncratic volatility effect.

The model generates a number of new testable implications for future empirical work. The distinguishing feature of our model is that coverage, compensation and stock pricing are all endogenously determined across firms. As such, there are a large number of sharp predictions that have not yet been tested as we explained. On the theoretical front, it would be interesting to extend our model to account for the following. First, we follow traditional assignment models in allowing for only one agent to be hired per firm. But large firms and more volatile firms typically have more than one analysts following them. Extending the assignment framework in this direction, with both heterogeneity in talent
and multiple hires, might be fruitful. Second, we might also change the noisy rational expectations framework and allow coverage to potential influence naive or noise traders. While the crux of our implications are likely to remain the same, these extensions would generate additional testable implications and further enhance our understanding of how coverage affects stock markets.
References


A. Appendix

A.1. Omitted Proofs

A.1.1. Proof for Lemma 1

Proof.

\[ \text{Var}(\theta|x, z, p) = \sigma^2 - \begin{bmatrix} \sigma^2 & \sigma^2 \\ k \frac{(h+h_m)\sigma^2}{1+\psi(h+h_m+h_\theta)} & k \frac{(h+h_m)\sigma^2}{1+\psi(h+h_m+h_\theta)} \end{bmatrix}^T \Sigma^{-1}_{dd} \begin{bmatrix} \sigma^2 \\ k \frac{(h+h_m)\sigma^2}{1+\psi(h+h_m+h_\theta)} \end{bmatrix} = \frac{1}{(h_m + h + \frac{1}{\sigma^2})}, \]

where \( \Sigma_{dd} \equiv \begin{bmatrix} \sigma^2 + \sigma_I^2 & \sigma^2 \\ \sigma^2 & \sigma^2 + \sigma_h^2 \end{bmatrix} \). Hence, the expression for the risk premium is given by

\[ \gamma_1 k^2 \varphi(h, \sigma) = \frac{\gamma_1 k^2}{2(1 + \psi)^2} \text{Var}(\theta|x, z, p) = \frac{\gamma_1 k^2}{2(1 + \psi)^2(h_m + h + \frac{1}{\sigma^2})}. \]

Since \( \frac{\partial \varphi(h, \sigma)}{\partial h} = \frac{1}{2(1 + \psi)^2(h_m + h + \frac{1}{\sigma^2})} < 0 \) and \( \frac{\partial \varphi(h, \sigma)}{\partial \sigma} = \frac{\sigma^{-3}}{(1 + \psi)^2(h_m + h + \frac{1}{\sigma^2})^2} > 0 \), the risk premium strictly decreases with precision \( h \) and increases with size \( k \) and volatility \( \sigma \).

\[ \square \]

A.1.2. Proof for Lemma 2

Proof. from Equation (18), \( \frac{\partial^2 \Omega(y, h)}{\partial k \partial h} = -2\gamma_1 k \frac{\partial \varphi(h, \sigma)}{\partial h} > 0 \) and

\[ \frac{\partial^2 \Omega(\sigma, h)}{\partial \sigma \partial h} = -\gamma_1 k \frac{\partial^2 \varphi(h, \sigma)}{\partial h \partial \sigma} = \frac{\gamma_1 k^2 \sigma^{-3}}{(1 + \psi)^2(h_m + h + \frac{1}{\sigma^2})^3} > 0. \]

\[ \square \]
A.1.3. Proof for Proposition 1

Proof. Given Lemma 2, the characterization for the labor market is standard in the assignment model. Specifically, the competitive fee is obtained by applying the analytical solution developed in Terviö (2008). By construction, given that \( \Omega(y^*, h^*) = 0 \), all the matching surpluses generated by inactive firms \( y < y^* \) and agents \( h < h^* \) must be negative as \( \Omega_y < 0 \) and \( \Omega_h < 0 \). Given the assignment, the market clearing price and investors’ demand function in each asset market is then provided in Section 2.2.1 by setting the precision of agent to be \( h = \mu(y) \). That is

\[
P(\theta, z, u|\mu(y), y) = \left(\frac{k}{1 + \psi}\right) E[\theta|x_i = \hat{x}, z, \hat{x}] - \frac{k^2}{2(1 + \psi)^2}\left(h_m + \mu(y) + \frac{1}{\sigma^2}\right).\]

And, one can verify that the demand \( D(x_i, z, p|\mu(y), y) \in \{0, 1\} \) indeed satisfies the cutoff rule: \( D(x_i, z, p|\mu(y), y) = 1 \) if and only if \( x_i > \hat{x}(z, p) \).

\[\square\]

A.1.4. Proof for Proposition 2

Proof. For part (1), let \( C \) solve \( \Omega(h_L, y_L) = 0 \); hence, \( \Omega(h_L, y_L) < 0 \) for \( C > C \). Furthermore, since \( h^* \) increases with \( C \), there exists \( \bar{c} \) such that \( h^* < h \) for \( C < \bar{c} \). Hence, for \( C \in (C, \bar{c}) \), there exists \( h^* > h_L \) and \( y^* > y_L \) such that \( \Omega(h^*, y^*) = 0 \) and \( \Omega(h, y^*) > 0 \). Hence, there exists \( \hat{y} = y^* - \epsilon \) and \( \epsilon > 0 \) such that \( \Omega(h, \hat{y}) = 0 \) and \( \mu_{ns}(y) = h \forall y \geq \hat{y} \). Hence, \( N_{ns} > N^s \). For part (2), given \( G^A_1(h) \leq G^A_2(h) \), consider the marginal type \( (h_1^*, y_1^*) \) under distribution \( G^A_1(h) \). Hence, by market clearing, \( 1 - G^F(y_1^* ) = 1 - G^A_1(h_1^*) \geq 1 - G^A_2(h_1^*) \), this implies that the analyst that is hired by firm \( y_1^* \) must be weakly lower that \( h_1^* \). Hence, by construction, \( \Omega(h_1^*, y_1^*) \leq 0 \) and therefore \( y_2^* \geq y_1^* \). The comparative statics in (3) and (4) can be obtained by looking at the complementarity between information and the variable \( x \). If \( \Omega_{hx} > 0 \), an increase in \( x \) thus increases the aggregate coverage and decrease the neglect effect.

\[\square\]
A.1.5. Proof for Proposition 3

Proof. One can show that \( \{ \text{Var}(\theta|\text{phy}) \}^{-1} = \frac{(h+h_m)^2}{h+h_m(1+\sigma_u^2)} + \frac{1}{\sigma^2} \). Hence, \( \text{Var}(\theta|\text{phy}) \) is only a function of \((h, \sigma^{-2}, h_m)\) but is independent of firm sizes. Given the assignment \( \mu(y) \), the informativeness of firm \( y \) is then \( \tau(y) = \frac{(\mu(\sigma)+h_m)^2}{\mu(\sigma)+h_m(1+\sigma_u^2)} + \frac{1}{\sigma^2} \) and the informativeness clearly increases with the agent’s precision. Hence, when firms differ in size, conditional on having coverage, the price for larger firms is more informative as a result of positive sorting established in Lemma 2. When firms differ in volatility, there are two effects on the price informativeness,\( \tau'(\sigma) = -2 + \frac{(\mu(\sigma)+h_m)(\mu(\sigma)+h_m(1+2\sigma_u^2))}{(\mu(\sigma)+h_m(1+\sigma_u^2))^2} \mu'(\sigma), \)

where \( \mu'(\sigma) = \frac{g^F(\sigma)}{g^A(\mu(\sigma))} \). Hence, when the level of volatility of all firms is multiplied by \( \alpha > 1, \tau'(\sigma) > 0 \) for large \( \alpha \). Lastly, since \( \frac{\partial \tau'(\sigma)}{\partial h_m} = \frac{-2h\sigma_u^2 h_m \mu'(\sigma)}{(h+h_m(1+\sigma_u^2))^2} < 0 \) and \( h_m = \frac{1+\sigma_u^2}{\sigma_I \sigma_u^2} \), \( \tau'(\sigma) > 0 \) for large enough \( \sigma_I \).

\( \square \)

A.1.6. Proof for Proposition 4

Proof.

Lemma 5. For \( C = 0 \), \( R'(k) < 0 \) \( \forall k \) for (1) large enough \( \sigma_u \) and \( \sigma_I \) and for (2) small enough \( h^L \) with uniform distribution.

Proof. When \( C = 0 \), all firms hire. Result (1) can be seen from

\( R'(k) = \frac{\gamma_1 k}{2(1+\psi)^2} \left( \frac{2(h_m+h_\theta+\mu(k)-k\mu'(k))}{(h_m+h_\theta+\mu(k))^2} \right) < 0. \)

(2) With uniform distribution, \( \mu'(k) = \frac{k^U-h^L}{k^U-k}\). For smaller \( h^L \) (i.e., less talent analysts), all firms \( k < k^U \) must hire an agent with a lower precision. That is, conditional on hiring, \( \mu(k; h^U) < \mu(k; h^L) \) for \( h^L < h^U \) and for \( k < k^U \). Hence, for small enough \( h^L \), \( R'(k) < 0. \)
Let $R(k; C)$ denote the risk premium for firm $k$ with production cost $C$. Clearly, given Lemma (5), $R'(k; 0) < 0 \forall k$, it is clear that $R_S(X_1; 0) > R_L(X_2; 0)$ holds. For any $C > 0$, this implies that some smaller firms do not hire agents, which means that $R(k; C) > R(k; 0)$ if $\mu(k; C) = \emptyset$. For any given $X_1$, there exists $\bar{c}$ such that all firms above $X_1$ quantile hire an agent. In this case, for any $C < \bar{c}$, $R_S(X_1; C) > R_S(X_1; 0) > R_L(X_1; 0) = R_L(X_1; C)$. The first inequality follows from the fact that the average risk premium in small-firm group must increase in the fixed cost (as some firms do not hire agents in this case), and the last equality follows from the fact that all firms above $X_1$ quantile hires an agent and therefore the risk premium are not affected by the fixed cost in this case.

A.1.7. Proof for Proposition 5

Proof. Recall that $R'(\sigma + \Delta) = \frac{\gamma \sigma^2}{2(1+\psi)^2} \left( \frac{2(\sigma + \Delta)^3 - \mu'(\sigma)}{(h_m + \frac{\mu}{2} + \mu(\sigma))^2} \right)$. Hence, $R'(\sigma) < 0$ for a larger enough $\Delta$ and for a low enough $h^L$, which leads to a steeper density function $\mu'(\sigma) = \frac{h^U - h^L}{g(\sigma)}$. Similar as in Proof for Proposition 4, $R'(\sigma) > 0$ is a sufficient condition to guarantee that $R_S(X_1) > R_L(X_2)$ for a small enough $C \leq \bar{c}$.

A.1.8. Proof for Proposition 6

Proof. (1) Changes of market noise $\sigma_I$ and $\sigma_u$ only affects $\Omega_h(y, h)$. Since $\frac{\partial^2 \Omega(y, h)}{\partial h \partial h_m} < 0$ and $\frac{\partial h_m}{\partial x} < 0$ for $x \in \{ \sigma_I, \sigma_m \}$, $\frac{\partial^2 \Omega(y, h)}{\partial h \partial h_m} \frac{\partial h_m}{\partial x} > 0$. Therefore, the slope increases with the market noise.

(2) When all firm types are multiplied by $\alpha$, since $\frac{\partial^2 \Omega(y, h)}{\partial h \partial y} > 0$ for $y \in \{ k, \sigma \}$, a higher $\alpha$ leads to a higher marginal value for any match $\Omega_h(\alpha y, h)$ and hence a higher $\omega'[i]$.

(3) With uniform distribution, $h[i] = (h^U - h^L)i + h^L$ and $h'[i] = \frac{1}{g'(h[i])} = (h^U - h^L)$. 40
Hence,
\[ \omega'[i] = \frac{k[i]^2}{h_m + \sigma[i]^2 + h[i]}(h^U - h^L) \]
and \( \frac{\partial \omega'[i]}{\partial h^L} > 0. \)

\[ \square \]

A.1.9. Proof for Lemma 3 and Characterization

Proof. Given that \( \mu(y) = \arg \max_h \{k\tilde{\theta} - \gamma k^2 \phi(h, \sigma) - \omega(h)\} \), if \( h = \mu(y) = \mu(y') \), then,
\[ \frac{\gamma}{2(1+\psi)^2} \left( \frac{k}{\frac{1}{\sigma} + h_m + h} \right)^2 = \omega'(h). \]
Hence, \( q(y, h) = q(y', h) \). Furthermore, when \( q(y', h) > q(y, h) \), Suppose that \( \mu(y') < \mu(y) \),
\[ q(y', \mu(y)) - \omega(\mu(y)) > q(y, \mu(y)) - \omega(\mu(y)) = 0 = q(y', \mu(y')) - \omega(\mu(y')) \]
Then, \( q(y', \mu(y)) - q(y', \mu(y')) > \omega(\mu(y)) - \omega(\mu(y')) > 0 \), which is a contradiction, since \( q(y, h) \) decreases with \( h \).
\[ \square \]

The sorting can then be understood between the weighted index \( q \) and agent \( h \). The characterization will then be similar to Proposition 1 with a modified distribution. Given the initial distribution \( G(k, \sigma) \) and given any value of \( h \), let \( F(q, h) \) denote the measure of firms \( (k, \sigma) \) whose weight index \( q(y, h) \) is weakly below \( q \). The market clearing condition can then be rewritten as:
\[ \int_{\mu(q)}^{h^U} dG^A(\bar{h}) = \int_q^{q^H} dF(\tilde{q}, \mu(\tilde{q})), \]
where \( q^H \equiv \sup\{q(y, h^U) : y \subset Y\} \). The initial condition is then given by \( \mu(q^H) = h^U \). The assignment \( \mu(\tilde{q}) \) thus specific the type agent that is hired by the firm with index \( q(y, \mu(q)) \). In other words, \( \mu(\tilde{q}) \) agent will be hired by the set of firms\{\( y : q(y, \mu(\tilde{q})) = \tilde{q} \}\}. And firms only hire if and only if \( \Omega(y, \mu(q)) \geq 0 \).
A.1.10. Proof for Proposition (7)

Proof. This can be seen immediately from (37): a lower market precision implies a higher substitution rate. □

A.1.11. Proof for Proposition 8 and Lemma 4

Proof. Given \( y = (k, \sigma) \), and \( \gamma_f > 0 \),

\[
\frac{\partial \Omega(k, h)}{\partial h} = \frac{k^2}{2} \left( \frac{(\gamma_I - \gamma_f)(h_\theta + h + h_m) + h_m \gamma_f 2 \sigma_u^2}{(h_m + h_\theta + h)^3} \right)
\]

(41)

\[
\frac{\partial^2 \Omega(\sigma, h)}{\partial h \partial \sigma^2} = \frac{k^2}{2} \left( \frac{-2(\gamma_I - \gamma_f)}{(h_m + h_\theta + h)^3} - 3 \frac{h_m \gamma_f 2 \sigma_u^2}{(h_m + h_\theta + h)^4} \right) \frac{\partial h_\theta}{\partial \sigma^2}
\]

(42)

Hence, when \( \gamma_I \geq \gamma_f \), \( \frac{\partial^2 v(y, h)}{\partial y \partial h} \geq 0 \), \( \forall h \geq 0, y \in \{k, \sigma\} \). This establishes Lemma 4 and thus the characterization can be obtained directly from Proposition 1. We now turn to the characterization when \( (\gamma_I - \gamma_f) < 0 \).

The heterogeneity in size When \( (\gamma_I - \gamma_f) < 0 \), the value of precision can be non-monotonic. It is convenient to rank analysts by their value \( v(h) \) to firm, where \( v(h) \equiv \frac{\gamma_f}{2} \left( \frac{1}{h_m + h_\theta + h} \right) + \frac{\gamma_f}{2} \left( h_\theta + h_m \right)^{-1} \left( h_\theta + h + h_m \right)^{1+\sigma_u^2} \) and the surplus function can then be rewritten as \( \Omega(k, h) = k^2 v(h) - C \). In other words, indices \( v \) are ordered such that they increase the utility of firm. As a result, there must be positive sorting between larger firms and the agents that generate higher value (not necessarily the one with more precisions). Hence, an equilibrium can be characterized by a cutoff type \( v^* \) such that \( \Omega(k^*, v^*) = 0 \) and analysts are hired if and only if \( v \geq v^* \).

Furthermore, from equation (41), set \( \hat{h} \) that solves \( \Lambda(h) \equiv (\gamma_I - \gamma_f)(h_\theta + h) + \frac{\sigma_u^2 + 1}{\sigma_f^2 \sigma_u^2} (\gamma_I - \gamma_f) + \gamma_f 2 \sigma_u^2 = 0 \). That is, when \( (\gamma_I - \gamma_f) < 0 \), \( \hat{h} \) is the local maximum of \( v(h) \). If \( \hat{h} = \left( \frac{\sigma_u^2 + 1}{\sigma_f^2 \sigma_u^2} (\gamma_I - \gamma_f) + \gamma_f 2 \sigma_u^2 \right) \gamma_I - \gamma_f \) < 0, \( v(h) \leq 0 \), \( \forall h \). In other words, none of analyst generates positive surplus. On the other hand, if \( \hat{h} > 0 \), then there exists \( \bar{h} > 0 \) such that \( \Omega(k, \bar{h}) = 0 \). Since \( \frac{\partial \hat{h}}{\partial \sigma_u^2} > 0 \), there exists \( \sigma_u \) such that for \( \hat{h} > 0 \) if and only \( \sigma_u > \sigma_u \). When \( \hat{h} > 0 \),
the equilibrium can be characterized in Proposition 1 by replacing types \( h \) with \( v \) instead, where the density of type \( v \) can be obtained from \( dG^A(h) \). Specifically, let \( v_1^{-1} \) be the partial inverse of \( v \) with the domain \( h \in [h^L, \hat{h}] \) and \( v_2^{-1} \) be the partial inverse of \( v \) with the domain \( h \in [\hat{h}, h^U] \). Hence, \( g^v(x) = dG^A(v_1^{-1}(x)) + dG^A(v_2^{-1}(x)) \). The fact that PAM holds regarding \((h, v)\) and \( v \) is monotonic in \( h \) thus establishes the Proposition.

**The heterogeneity in volatility** From equation (42),

\[
\frac{\partial^2 \Omega(\sigma, h)}{\partial h \partial \sigma^2} \propto 2 \left\{ (\gamma_I - \gamma_f) \left( \frac{1}{\sigma^2} + h \right) + \frac{3 \sigma_\sigma^2 + 1}{2 \sigma^2 \sigma_u^2} (2(\gamma_I - \gamma_f) + \gamma_f \sigma_u^2) \right\}.
\]

When \((\gamma_I - \gamma_f) < 0\), if \( \sigma_u^2 \) is large enough so that \( \frac{\partial \Omega(\sigma, h)}{\partial h} \geq 0 \) (and thus \( \frac{\partial^2 \Omega(\sigma, h)}{\partial h \partial \sigma^2} \geq 0 \)) then, \( \frac{\partial \Omega(\sigma, h)}{\partial h} > 0 \) and (thus \( \frac{\partial^2 \Omega(\sigma, h)}{\partial h \partial \sigma^2} \geq 0 \)), \( \forall \sigma, h \). Hence, PAM holds. Similarly, when \( \sigma_u^2 \) is small enough so that \( \frac{\partial \Omega(\sigma, h)}{\partial h} < 0 \) then \( \Omega(\sigma, h) < 0 \) \( \forall \sigma, h \). And no agent is hired.

\[ \Box \]

**A.2. Setting with Multiple assets**

Our results can be easily extended to the environment where the same (deep-pocket) investor \( i \) can invest multiple assets in different markets and submit the demand function \( d^j_i(p^j) \in \{0, 1\} \) for firm \( j \). Specifically, assume that each investor has access to \( N \) market and his private signal for each market (i.e., firm) is given by \( x^j_i = \theta^j + \sigma_I \epsilon_i \), And \( I_i = (x^j_i, z^j, p^j)_{j=1,\ldots,N} \) that contain the information for each market \( j \). The investor’s wealth is the given by \( W_i = \sum_{j \in N} d^j_i \left( \frac{k \theta^j}{1 + \psi} - p^j \right) \). Hence, an investor then solves the following maximization problem:

\[
U^i = \max_{d^j_i} E_{I_i} \left[ \sum_{j \in N} d^j_i (\theta^j - p^j) \right] - \frac{\gamma_I}{2} \text{Var} \left( \sum_{j \in N} (d^j_i)^2 \left( \frac{k \theta^j}{1 + \psi} - p^j \right) \middle| I_i \right).
\]

\[
= \sum_{j \in N} \left\{ \max_{d^j_i} E_{I_i} \left[ d^j_i (\theta^j - p^j) \right] - \frac{\gamma_I k^2(d^j_i)^2}{2(1 + \psi)^2} \text{Var}(\theta^j \middle| I_i) \right\}
\]
Given that all assets and signals are uncorrelated, one can see that our result remains unchanged since the optimization for each asset can be solved separately.