Information Aggregation and Asset Prices in Large Markets with Institutional Investors

Matthijs Breugem and Adrian Buss*

March 15, 2016

Abstract

We study the joint determination of endogenous information acquisition and equilibrium asset prices in a rational expectation equilibrium model with a continuum of asset managers who care about their performance relative to a benchmark and have CRRA preferences. In the presence of benchmarking, managers are less willing to deviate from the benchmark and, thus, to speculate based on private information, such that less of a manager’s private information gets incorporated into prices. As benchmarking also reduces the fraction of managers that endogenously decide to acquire private information, prices are substantially less informative in the presence of institutional investors. The benchmark asset is therefore perceived to be more risky, leading to a decline in price, which can dominate the positive price effect stemming from the managers’ excess demand due to index-hedging, and a substantial increase in return volatility.

*Matthijs Breugem is at Frankfurt School of Finance and Management gGmbH, Sonnemannstrasse 9-11, 60314 Frankfurt am Main, Germany. e-mail: m.breugem@fs.de. Adrian Buss is at INSEAD, Boulevard de Constance, 77305 Fontainebleau Cedex, France. For useful comments or suggestions to this paper and its previous versions we thank Joel Peress, Zacharias Sautner, Larissa Schaefer, Gunther Strobl, Laura Veldkamp, Jing Zeng and seminar participants at Frankfurt School of Finance and Management.
1 Introduction

Nowadays, a substantial part of financial wealth is managed by professional asset managers. For example, French (2008) finds that while direct holdings of individuals declined from 47.9% of the market in 1980 to only 21.5% in 2007, the fraction managed by mutual funds increased from 4.6% to 32.4% over the same horizon, with the remaining fraction managed by other institutional investors, such as endowment or pension funds, who also directly or indirectly employ asset managers. One of the most prevalent features of the asset management industry is that a manager’s performance is measured against a “benchmark,” directly determining his compensation, reputation and career outlook, but also implicitly affecting future inflows into the manager’s fund.

Given the size and importance of the asset management industry, the fact that an asset manager’s performance is measured relative to a benchmark has important implications for financial markets. In our analysis, we focus on the implications of “benchmarking” for one of the most fundamental roles of financial markets: the aggregation and dissemination of information. Specifically, our goal is to demonstrate how, in the presence of benchmarking, asset managers’ valuation and willingness to acquire private information is affected, how this influences price informativeness, and through this channel prices itself as well as the return distribution, all determined endogenously in equilibrium.

We consider a noisy rational expectations equilibrium (REE) model with a continuum of investors that care not only about their own wealth but also about their performance relative to a benchmark. That is, investors have an incentive to deliver a high return when the return on the benchmark is high which we integrate into a constant relative risk-aversion utility function. Investors can trade two assets: a riskless bond as well as the risky benchmark. In addition, they can acquire costly private information regarding the payoff of the benchmark, endogenously giving rise to two groups of investors. First, ‘informed’ investors who are willing to pay the cost – each of them receiving a separate piece of private information. Second, ‘uninformed’ investors who do not possess any private information. Both groups can learn
from the equilibrium price which is, due to a noisy and unobservable supply, not perfectly revealing.

We find that in the presence of benchmarking, an individual investor who takes prices as given and is provided with private information for free incorporates a smaller fraction of the information into his demand schedule. For example, the difference between the investor’s demand following good cash flow news and his demand following bad cash flow news decreases substantially in the degree of benchmarking. That is, information is of less value for an investor whose performance is measured relative to a benchmark. This result is quite intuitive as, in the presence of benchmarking, investors hedge against changes in the benchmark so that they can post a high return when the benchmark is high. Accordingly, they are unwilling to deviate from the benchmark and, thus, speculate less, so that less of their private information gets incorporated into their demand.

In equilibrium and with endogenous information acquisition, we show that this behavior of individual investors implies that the fraction of investors who are willing to acquire costly private information declines with the degree of benchmarking. Taken together, the fact that a smaller fraction of investors acquires information, and those that acquire information incorporate less of it into asset prices, substantially reduces the informativeness of asset prices. Accordingly, the presence of investors whose performance is measured relative to a benchmark has an adverse effect on financial markets’ effectiveness of aggregating and providing information. Because prices are less informative, the ‘information gap’ between investors that acquire private information and those that only infer information from equilibrium prices widens.

Our analysis also captures, as an extreme case, exchange-traded funds (ETFs) which basically just correspond to ‘fully benchmarked’ investors. As intuition suggests, we show that for managers of ETFs private information has no value as they cannot deviate from the index. Accordingly, in this setup asset prices do not incorporate any private information. This suggests that the recent surge of capital flows into ETFs might lead to a deterioration
of price informativeness for a large set of financial markets which, however, could lead to profitable investment opportunities for the subset of managers that is informed.

In equilibrium, the investors’ information acquisition choices and their use of private information has also a direct impact on the price of the benchmark asset. That is, due to the decline in price informativeness, investors can infer less information from an asset’s price, so that the asset is perceived more risky. Therefore, benchmarking leads to a decline in price through an ‘information channel’ which, in turn, increases the expected return of the benchmark asset. Due to decline in price informativeness, we also find a substantial increase in the return volatility of the benchmark asset. As the increase in volatility is quantitatively stronger than the increase in expected return, the Sharpe ratio declines with the degree of benchmarking in our model.

The decline in the price of the benchmark asset through our ‘information channel’ is contrary to the ‘excess demand effect’, discussed in Basak and Pavlova (2013) and Buffa, Vayanos, and Woolley (2015), that drives up the price of the benchmark asset due to the excess demand for hedging purposes. In our model both effects are present, i.e., we have excess demand for the benchmark asset due to hedging, but at the same time the price of the benchmark asset is less informative. We find that in our model either effect can dominate, with the information channel typically dominating for low degrees of benchmarking, and the excess demand effect being the dominating force for high degrees of benchmarking.

On the technical side, our assumption of a constant relative risk-aversion (CRRA) utility function, in contrast to the constant absolute risk-aversion (CARA) utility typically used in rational expectations models, is realistic and key for our results. Specifically, while in a CARA setup with relative performance evaluation investors would still have an excess demand for the benchmark asset due to hedging, benchmarking will not affect their information acquisition decisions and the value they assign to private valuation. That, however, would imply that a manager of an exchange-traded fund would actually be willing to invest the same amount of capital to acquire information as a manager that has only mild benchmarking concerns,
which seems rather implausible. In contrast, due to the relative wealth concerns of CRRA utility, the information acquisition choices of these managers differ substantially from each other in our model.

Our analysis is related to two strands of the literature. First, the growing literature on the implications of institutional investors on equilibrium asset prices. Brennan (1993) studies a static, CARA utility setup in which investors’ utility depends on their performance relative to an index. He shows that in this setup a two-factor model arises, with one of the factors being the market and the other one being the benchmark. Basak and Pavlova (2013) study a dynamic setup with multiple risky assets as well as CRRA preferences over terminal wealth and a benchmark. The excess demand created by investors’ hedging decisions raises the price of the assets included in the index, and increases their volatility as well as their correlation with each other. Similar to our paper, both papers incorporate benchmarking directly into investors’ preferences. However, in contrast to our model they assume that information is symmetric and exogenous, therefore, leaving aside information aggregation and dissemination.

Cuoco and Kaniel (2011) as well as Buffa, Vayanos, and Woolley (2015) model portfolio delegation as well as the contracts between asset managers and individual investors explicitly. Cuoco and Kaniel (2011) consider a dynamic setup with CRRA preferences and two risky stocks in which asset managers receive a fee, driven by their own performance and their performance relative to a benchmark, for managing the risky part of their investors’ portfolios. While benchmarking raises prices and lowers expected returns, they find ambiguous results for return volatilities. Buffa, Vayanos, and Woolley (2015) study the joint determination of managers’ contracts and equilibrium prices in a dynamic setup with multiple risky assets and CARA preferences. Agency frictions lead to contracts that depend on managers’ performance relative to a benchmark, which, in turn, leads to an increase in asset prices. In contrast to our paper, both papers model contracts explicitly, but similar to the papers discussed above abstract from private information and information acquisition.
García and Vanden (2009) and Qiu (2012) bridge the gap to the second, closely related strand of literature that focuses on information aggregation in financial markets. Both papers study asset management in a static setting in which managers with CARA-utility receive private signals. While García and Vanden (2009) show that competition between fund managers makes price more informative, Qiu (2012) finds ambiguous results for price informativeness if managers’ performance is evaluated against their peers. Relative to these papers, our assumption of CRRA preferences is key, as this gives rise to relative wealth concerns which lower price informativeness.

The rational expectations literature, focusing on information aggregation and price informativeness, typically works under the CARA-normal framework as this allows for tractable solutions. Among others, Grossman (1976), Grossman and Stiglitz (1980), and Diamond and Verrecchia (1981), who study a setup with two (informed and uninformed) groups of agents, Hellwig (1980), who considers a continuum of asymmetrically informed agents, Admati (1985) who considers multiple assets, and Wang (1993) who studies a dynamic economy, document how information is incorporated into prices and how much of it is reflected by asset prices. In contrast to these papers, we allow for CRRA preferences that feature performance evaluation relative to a benchmark.

Thus, we also contribute to the growing but still nascent literature that studies information aggregation outside the CARA-normal framework. That is, a set of papers relaxes the assumption of normally distributed payoffs but maintain CARA utility. For example, Breon-Drish (2015) solves a one-asset economy for distributions that are members of the “exponential family”, and Chabakauri, Yuan, and Zachariadis (2015) extend this to a multi-asset economy. Another stream of papers focuses on relaxing the assumption of CARA-preferences. For instance, Barlevy and Veronesi (2000) and Albagli, Hellwig, and Tsyvinski (2014) study risk neutral investors who face portfolio constraints, and Peress (2003) studies general preferences using a (small risk) log-linearization. In our paper, we are going to allow for both, general preferences and non-normally distributed payoffs, but, therefore, have to rely on a
numerical algorithm to solve for the equilibrium. Accordingly, our paper is also related to Bernardo and Judd (2000) who solve a two agent economy similar to Grossman and Stiglitz (1980) with general preferences and payoff distributions. In contrast, our methodology allows for a continuum of asymmetrically informed investors, similar to Hellwig (1980), and we incorporate relative performance concerns into our utility function.

The remainder of the paper is organized as follows. Section 2 describes the details of the model. Section 3 discusses the implications of benchmarking for investors’ asset demand as well as how they incorporate information into their demand. Sections 4 and 5 describe how benchmarking affects price informativeness and asset prices. Finally, Section 6 concludes. The Appendix provides detailed information on our numerical solution approach.

2 Model

Our setup integrates the institutional investors à la Cuoco and Kaniel (2011) and Basak and Pavlova (2013) in a rational expectations economy with information acquisition. More specifically, our economy is similar to Hellwig (1980) yields the following key differences: First, we consider institutional investors who have utility over (i) their own performance and (ii) their performance relative to an index or benchmarking portfolio. The focus of this paper is to analyze how the second component in their utility function—typical to institutional investors—affect the financial market equilibrium. Since wealth effects are critical to our setup, we employ CRRA utility rather than the CARA preferences typically considered in the bulk of the literature on financial markets with asymmetric information. In order to solve such an economy, we develop a new numerical method that can tackle rational expectation economies with general preferences and payoff distributions. Appendix B discusses our method in detail.1

1In the remainder of this paper we maintain a notation that distinguishes actual realizations of random variables (denoted with bar; e.g., \(\bar{x}\)), the actual random variable (denoted with tilda; e.g., \(\tilde{x}\)), and scalars are denoted without bars, hats or tildas (e.g., \(c\)).
2.1 Agents and assets

We consider a two-period economy with time \( t \in \{0, T\} \). The economy is populated with a unit mass of ex-ante identical agents \( i \in [0, 1] \) who maximize their utility \( U \) over their final period consumption \( c_{i,T} \). Specifics of the utility function will be discussed shortly. Trading takes place at \( t = 0 \), and consumption only takes place at \( t = T \).

There are two investment opportunities (assets) available to each agent. The first asset is in perfectly elastic supply and yields a unit payoff with certainty. The second asset is risky, has inelastic supply \( z \), and produces a risky payoff \( \tilde{v} \) distributed with density \( f_v \). Importantly, \( \tilde{z} \) is unobservable and distributed according to \( f_z \).\(^2\) Initially, agents hold zero riskless assets and an identical share of the risky asset. Portfolio holdings for the riskless and risky asset are denoted by \( \theta_{i,t}^B \) and \( \theta_{i,t}^S \) respectively.

In the bulk of our analysis we assume signals and payoffs are binomially distributed; i.e., \( v, y \in \{v_L, v_H\} = \{\mu_v - \sigma_v, \mu_v + \sigma_v\} \). While our methodology is capable of handling more complex distributions, the binomial assumption provides tremendous computational simplicity. Our results remain valid under more general payoff distributions, such as (log-) normally distributed fundamentals. The exact values of parameters in our analysis is shown in Table 1.

2.2 Institutional investors and incentive schemes

Agents in our economy represent institutional investors that are subject to an incentive scheme. Agents have no prior wealth and have CRRA utility over (i) their terminal wealth and (ii) their performance above an index (or benchmarking portfolio) \( I \):

\[
U(w_i, I) = \begin{cases} 
\frac{(A+w_i-B \times I)^{1-\gamma}}{1-\gamma} & \gamma \neq 1 \\
\log (A + w_i - B \times I) & \gamma = 1 
\end{cases}
\]  

(1)

The parameter \( B \) is the degree to which agents are incentivized towards outperforming

\(^2\)The random supply of the risky asset is a source of noise that prevents asset prices from being perfectly revealing. The assumption is identical to the employment of noise traders with random demand \(-z\).
the reference portfolio. The parameter $A$ is the fixed compensation component similar to the incentive scheme employed in Cuoco and Kaniel (2011). Since we have only one stock in our setup, we set the index level equal to the payoff of the risky security $\tilde{v}$.

There is no broad consensus what the preference structure of an institutional trader should be modeled. However, Basak and Pavlova (2013) point out for any candidate solution, it is vital that (i) the marginal utility is increasing with the index and (ii) the utility itself is decreasing in the index. Since in Basak and Pavlova (2013) only marginal utilities are used in their system of equations, they resort to the tractable specification $U(w_i, I) = \left(1 + \hat{B} \times I\right) \times \log(w_i)$, where $\hat{B}$ denotes the degree of benchmarking. However, in order to valuate information, our equation system consists of the utility function itself and therefore it is important that our objective function satisfies criterion (ii) as well. Our simple preference structure, a special case of the incentive scheme in Cuoco and Kaniel (2011) and mentioned in Basak and Pavlova (2013) as a valid alternative to theirs, satisfies both criteria.\footnote{Our preference structure is a case of Cuoco and Kaniel (2011) when in their paper $\alpha = A$, $\beta + \gamma = 1$, $\gamma = B$. Importantly, we arrive to similar conclusions when utility is specified over $A + w_i (1 + B) - B \times I$ in which case the dependency of $\beta$ and $\gamma$ is broken and $\alpha = A$, $\beta = 1$, $\gamma = B$.} \footnote{The utility mentioned in the footnote of Basak and Pavlova (2013) is for $B = 1$, since we are interested in the effect of benchmarking, we introduce the parameter $B$ to study the implications of incentive schemes on a continuum scale.} \footnote{The signal is uninformative about the risky asset’s payoff only when $y_i \perp v$, that is, when $f_{v,y_i} = f_v \times f_{y_i}$. In contrast, the signal is perfectly informative when $f_{v,y_i} = 1_{y_i=v}$.}

Note that in our setup gains and losses are equally rewarded or penalized. These types of incentive schemes are known as *fulcrum performance fees*. As Cuoco and Kaniel (2011) point out, the 1970 Amendment of the Investment Advisers Act of 1940 imposes mutual funds’ performance fees to be of this type, although hedge funds’ performance fees are less restrictive.

### 2.3 Acquisition of private information

At time $t = 0$, before trading, agents can decide to acquire a private signal $\tilde{y}_i$ at cost $\kappa$. The observation of $\tilde{y}_i$ is useful as it is informative about $\tilde{v}$. The joint distribution of the two random variables is denoted by $f_{v,y_i}$.\footnote{The utility mentioned in the footnote of Basak and Pavlova (2013) is for $B = 1$, since we are interested in the effect of benchmarking, we introduce the parameter $B$ to study the implications of incentive schemes on a continuum scale.} Now, despite that agents have homogeneous
endowments and preferences, trading occurs due to endogenous differences in information allocation.

In our binomial setup, the density $f_{v,y_i}$ is given by:

$$f_{v,y_i} = \begin{cases} 
\frac{\rho}{2} & y_i = v \\
\frac{1-\rho}{2} & y_i \neq v 
\end{cases}$$

where $\rho \in \left[\frac{1}{2}, 1\right)$ denotes the exogenous quality of information, with $\rho = 0.5$ being uninformative and $\rho = 1$ being totally informative.\(^6\)

Similar to Hellwig (1980), each agent $i$ that is informed receives a separate draw from the distribution of $\tilde{y}_i$. Since there is a continuum of informed agents (for $\lambda > 0$), by Borel’s law of large numbers the density of allocated signals is exclusively dictated by the realization of $\tilde{v}$.

We next label the two groups of agents that can arise endogenously: “Informed” agents have paid $\kappa$ to be (privately) informed. “Uninformed” agents have not incurred this cost but do not possess private information. Both groups can learn from equilibrium price $p$. In equilibrium, the fraction of informed agents $\lambda$ is such that the expected utility of the two groups coincides.\(^7\) Without loss of generality we label agents $i \in [0, \lambda)$ to be informed and agents $i \in [\lambda, 1]$ to be uninformed.

### 2.4 Demand schedules

After the information acquisition stage, each agent solves the following portfolio optimization problem:

$$V_i(\lambda) = \max_{\theta_i^{S,T}} \left\{ \mathbb{E}_i \left[ U \left( w_i,T, I \right) \right] \mid w_i,T = \theta_i^{S,T} \times v + \theta_i^{B,T} \times p + \kappa \times \mathbb{1}_{i<\lambda} = \theta_i^{S,0} \times p \right\}$$  \(2)$$

Recall that informed agents have a superior information set $\mathcal{F}_i$ that is captured in the

\(^6\)The variable $\rho$ can be mapped to explained variance ($R^2$) as follows: $R^2(\rho) = 1 - \frac{\text{Var}[\tilde{v} | \tilde{y} = y]}{\text{Var}[\tilde{v}]} = 1 - \frac{\mathbb{E}[\tilde{v}^2 | \tilde{y} = y] - \mathbb{E}[\tilde{v} | \tilde{y} = y]^2}{\mathbb{E}[\tilde{v}^2] - \mathbb{E}[\tilde{v}]^2} = 1 - \frac{4\rho \sigma_v^2 (1-\rho)}{\sigma_v^2} = (1-2\rho)^2$

\(^7\)If information is too cheap, then all agents will be informed. Likewise, if information is too expensive, no agent will be informed.
expectation symbol $E_i$. However, the cost reflected to becoming privately informed is taken into account in the budget constraint. We highlight that $F_i$ does not only contain private information but also public information that is reflected in equilibrium prices. The learning from prices channel is described in the next section.

We next substitute the budget equations to obtain the following Lagrangian:

$$L_i = E_i \left[ U \left( \theta_{i,T} \times v + \left( \theta_{i,0}^S - \theta_{i,T}^S \right) \times p - \kappa \times 1_{i \leq \lambda} - B \times I \right) \right]$$

Taking the derivative with respect to $\theta_{i,T}$ yields the first order conditions that pins down the risky asset portfolio holdings:

$$(v - p) \times E_i \left[ U' \left( \theta_{i,T}^* \times v + \left( \theta_{i,0}^S - \theta_{i,T}^S^* \right) \times p - \kappa \times 1_{i \leq \lambda} - B \times I \right) \right] = 0 \quad (3)$$

Riskless asset holdings can be obtained by substituting the optimal $\theta_{i,T}^*$ into the budget constraint at $t = 0$. Note that equilibrium portfolio holdings are functions of the triple $\{v, y_i, z\}$ for $i \leq \lambda$ (informed agents) and a function of $\{v, z\}$ for $i > \lambda$ (uninformed agents).

### 2.5 Equilibrium information allocation

For a given cost of information $\kappa$, the equilibrium number of informed agents $\lambda^*$ is such that agents are indifferent between acquiring information or not. However, if the cost of information is too high or too low, there might be not an interior $\lambda^*$ available. In this case, all agents could be either informed or uninformed. Therefore, the fraction of informed agents is determined as follows in equilibrium:

$$\lambda^* = \begin{cases} 
0 & \text{if } V_{i=0}(0) < V_{i>0}(0) \\
1 & \text{if } V_{i<1}(1) > V_{i=1}(1) \\
\lambda & \text{ otherwise}
\end{cases} \quad (4)$$
2.6 Market clearing

Equilibrium prices $p$ for risky assets are pinned down by the market clearing condition:

$$\bar{\theta}_T^S \equiv \int_0^1 \theta_{i,T}^S di = \bar{z}$$  \hspace{1cm} (5)

Although the demand of informed traders is a function of $\tilde{y}_i$, the total demand of informed traders is independent on the individual realizations of private signals: The benefit of working with a continuum of agents is that the distribution of signals realizations across agents is a function of $\tilde{v}$ only. Consequently, aggregate demand $\bar{\theta}_T^S$ and therefore equilibrium prices $p$ are a function of $\{\tilde{v}, \tilde{z}\}$.

2.7 Information content of equilibrium prices

In our model, agents are fully rational and know the model setup of our economy. Specifically, they are aware that observed realizations of asset prices must be consistent with the market clearing equation (5). Specifically, can compute the likelihood for each potential pair $\{v, z\}$ that is consistent with the observed equilibrium asset prices. Moreover, by submitting demand schedules, they can condition down their trades upon $p$. Since there is no hierarchical information structure, $p$ provides valuable information content that is both valuable to informed and uninformed agents.

In this subsection, we discuss learning from prices for the binomial model, which is graphically illustrated in Figure (1). The methodology for general distributions and its numerical implementation are more precisely described in Appendix (B).

It is important to point out that all agents in the model can compute $\bar{\theta}_T^S$ for all realizations of observable and unobservable random variables. Using the market clearing equation, agents internalize that $\{v, z\} \in \{\{v_L, \bar{\theta}_{T,xL}^S\}, \{v_H, \bar{\theta}_{T,xH}^S\}\}$, where aggregate demand depends on equilibrium prices. Updating beliefs now consist of computing the relative likelihood of the two possible realization pairs.
2.7.1 Learning for uninformed agents

For privately uninformed agents, who can only use the equilibrium price to update their beliefs, the likelihood that $v = \tilde{v}$ is now given by:

$$L(\tilde{v} = v_x|p) = \Pr[\tilde{v} = v_x | y_i] f_z(\bar{\theta}_{T,vx}^S) \quad \forall x \in \{H, L\}$$

The posterior probability distribution is now:

$$\Pr[\tilde{v} = v_x | p] = \frac{L(\tilde{v} = v_x|p)}{L(\tilde{v} = v_L|p) + L(\tilde{v} = v_H|p)}$$

$$= \frac{\Pr[\tilde{v} = v_x] f_z(\bar{\theta}_{T,vx}^S)}{\Pr[\tilde{v} = v_L] f_z(\bar{\theta}_{T,vL}^S) + \Pr[\tilde{v} = v_H] f_z(\bar{\theta}_{T,vH}^S)} \quad \forall x \in \{H, L\} \quad (6)$$

2.7.2 Learning for informed agents

Privately informed agents can use their own signal as well as the equilibrium price to learn about the risky asset’s fundamental. This leads to the following likelihood:

$$L(\tilde{v} = v_x|p, y_i) = \Pr[\tilde{v} = v_x|y_i] f_z(\bar{\theta}_{T,vx}^S) \quad \forall x \in \{H, L\}$$

The posterior probability distribution is now:
\[
\Pr [\bar{v} = v_x | p, y_i] = \frac{L (\bar{v} = v_x | p, y_i)}{L (\bar{v} = v_L | p, y_i) + L (\bar{v} = v_H | p, y_i)} \\
= \frac{\Pr [\bar{v} = v_x | y_i] f_z (\tilde{\theta}_T, v_x)}{\Pr [\bar{v} = v_L | y_i] f_z (\tilde{\theta}_T, v_L) + \Pr [\bar{v} = v_H | y_i] f_z (\tilde{\theta}_T, v_H)} \quad \forall x \in \{H, L\}
\]

It follows immediately from the above equation that \(\Pr [\bar{v} = v_x | p, y_i] = \Pr [\bar{v} = v_x | p]\) when \(\rho = \frac{1}{2}\), since in this case \(L (\bar{v} = v_x | p, y_i) = L (\bar{v} = v_x | p)\). On the contrary, when \(\rho = 1\) (i.e., the private signal is perfectly informative), \(\Pr [\bar{v} = v_x | y_i] = 1_{v_x = y_i}\) and therefore \(\Pr [\bar{v} = v_x | p, y_i] = 1_{v_x = y_i}\).

2.8 Description of an equilibrium

The goal is to find functions \(\theta^*_i, T\) and \(p\) and scalar \(\lambda\) such that (i) markets clear according to (5), (ii) the first order conditions (3) are satisfied under the updated information set of investors (6) and (7), and (iii) private signals are allocated according to (4). Details on the numerical method to solve the economy are presented in Appendix B.

3 Demand for Assets and Information

In this section we now study the impact of benchmarking on investors’ demand for the benchmark asset as well as the value they assign to private information – in a setup where prices as well as private information and its quality are exogenously given. This ‘partial equilibrium view’ of the model will help us understand the equilibrium results presented in the following sections. All illustrations presented below are based on the parameter values outlined in Table 1 in the Appendix.
3.1 Asset Demand

In a first step, we are going to discuss how the degree of benchmarking affects an investor’s demand for the benchmark asset, taken its price as given and assuming that the investor has access to private information. Recall that in our binomial setup, an investor will, with equal probability, receive a signal indicating a high future cash flow for the benchmark asset (‘good’ cash flow news) or, alternatively, a signal indicating a low future cash flow for the benchmark asset (‘bad’ cash flow news). Conditional on the signal, the investor will form his demand for the benchmark asset.

Figure 2 Panel (a) presents the unconditional expected demand for the benchmark asset for various degrees of benchmarking. Benchmarking increases the investor’s demand due to a hedging component. Intuitively, in the presence of benchmarking the investor has an incentive to report a high return when the benchmark return is high, so that his hedging portfolio correlates positively with the return on the benchmark. This hedging portfolio can therefore
be easily implemented by acquiring additional shares of the benchmark itself, explaining the positive relation between the degree of benchmarking and the investor’s demand for the benchmark asset displayed in Panel (a).

Relative performance considerations not only affect the expected demand for the benchmark asset, but also substantially affect the investor’s demand for the benchmark asset conditional on the signal realization. Panel (b) of Figure 2 shows that, in the absence of benchmarking, the investor’s demand for the benchmark asset is given by about 0.375 as well as about 1.55 shares in case he receives bad or good cash flow news, respectively. As the degree of benchmarking increases the difference between his demand conditional on good and bad cash flow news shrinks substantially. That is, in the presence of benchmarking, the investor is unwilling to deviate considerably from the benchmark, even if the private information that he receives indicates a low or high future cash flow. In the limit he poses the almost exactly the same demand whether he receives good or bad news, i.e., essentially ignores the private information that he has. Again, this result is quite intuitive as deviations from the index performance at the terminal date become more costly (in terms of utility losses) as the degree of benchmarking increases, limiting the investor’s willingness to deviate from the benchmark in his investment decisions today, and his willingness to speculate based on the private information that he has received.

3.2 Value of Information

The fact that, in the presence of relative performance concerns, an investor is less willing to speculate based on his private information and, instead, his portfolio closely mimics the benchmark, has direct implications for the value that an investor associates with private information. Specifically, to measure the value of information for an individual investor, we compute the number of risk-free assets that he is willing to pay to become informed, keeping the quality of information exogenous.

Figure 3 shows that the investor’s willingness to pay for information, i.e., his private
valuation of information, is decreasing linearly in the degree of benchmarking. Intuitively, this is a direct consequence of the result that, in the presence of benchmarking, the investor is less willing to deviate from the benchmark and, accordingly, profits less from private information available to him. In the limit, private information is of no value to the investor, as the incentives posed by relative performance evaluation are so strong that his portfolio perfectly mimics the benchmark, and, as we have learned above, his asset demand is independent of the signal he receives.

4 Equilibrium Price Informativeness

In this section we are now going to study the ability of financial markets to aggregate and disseminate information in the presence of relative performance evaluation. For this, we focus on our equilibrium model in which asset prices, but also the investors’ information acquisition is determined endogenously. In a first step, we will describe the endogenous information acquisition choice of investors in the presence of benchmarking. Then, we discuss how the
investors’ valuation of private information and their decision to acquire costly information affect price informativeness.

4.1 Information Acquisition

In the previous section we had already shown that, in absence of any equilibrium effects, i.e., with the price of the asset and private information given exogenously, benchmarking increases an investor’s unconditional demand for the benchmark asset, but substantially reduces his willingness to deviate from the benchmark, i.e., to speculate based on the private information that he has received. Therefore, private information is of less value to an investor whose performance is measured relative to a benchmark.

We are now going one step further and study the full equilibrium in which investors can endogenously choose to acquire costly private information – with only the quality of the information being specified exogenously. This information acquisition choice gives endogenously
rise to two groups of investors. First, ‘informed’ investors who are willing to pay the cost and, accordingly, are going to receive private information. Second, ‘uninformed’ investors who choose not acquire information and, therefore, possess no private information. Both groups can learn about the signals that the fraction of informed investors has received from the equilibrium price which is, however, due to a noisy and unobservable supply, not going to reflect all private information.

Figure 4 depicts the fraction of informed investors as a function of the degree of benchmarking for a low as well as a high level of information acquisition costs. Focusing on the setup with the low cost, we can see that for low degrees of benchmarking all investors are willing to acquire private information. However, as the degree of benchmarking further increases the fraction of investors that acquire information and, thus, are informed shrinks. This effect is a direct consequence of the partial equilibrium results presented in the previous section. Specifically, because investors are less willing to deviate from the benchmark as the degree of relative performance evaluation increases, they are unwilling to speculate based on private information, so that the cost of acquiring information dominates the benefits of being better informed. Specifically, for a certain degree of benchmarking, i.e., with \( B \geq 0.6 \), all investors decide not to acquire private information, but, instead just to mimic the benchmark. Interestingly, this implies that in these cases the whole asset management industry has endogenously decided to invest passively, driven exclusively by relative performance evaluation.

In the setup with high information acquisition costs, the fraction of investors that is willing to acquire costly information is below 1 even in the setup without benchmarking. As the degree of benchmarking increases, the fraction declines and, at some point approaches zero, comparable to the setup with low costs for acquiring information.

### 4.2 Price Informativeness

As benchmarking reduces both, the value of a given piece of private information to an individual investor as well as the fraction of investors who are willing to acquire private informa-
Figure 5: Price informativeness as a function of benchmarking. Left panel (a). Equilibrium with exogenously imposed information: Steeper incentive schemes decrease information content of prices. The effect is shown for various levels of exogenously imposed $\lambda$. Right panel (b): Equilibrium view with endogenous information. Both the posterior information of the informed trader and the uninformed trader decreases as benchmarking increases. Note that the difference between the information sets of the two investors increases as $B$ goes up.

In our model, one would expect that benchmarking has a substantial impact on the informativeness of prices in equilibrium. In the following we are therefore going to study how benchmarking affects the informational content in asset prices, measured by the part of the variance of the fundamental that is explained by the price of the underlying asset – in terms of $R^2$.

To disentangle the two effects, i.e., that (1) a given investor incorporates less of his private information into the asset’s price, and (2) less investors acquire private information, we are first going to take a step back, and analyze price informativeness in a setup where a fraction $\lambda$ of investors is exogenously assumed to be informed with the remaining fraction $1 - \lambda$ being uninformed. This basically removes the information choice, so that we can exclusively study the impact of the investors’ willingness to incorporate their private information into the price.

Figure 5 Panel (a) shows the results of this exercise for three different levels of $\lambda$, i.e., fractions of informed investors. Obviously, if none of the investors are informed, price infor-
mativeness is zero, as none of the variance can be explained by the asset’s price. In contrast, in the absence of benchmarking and with 50% of the investors being informed, about 20% (in terms of $R^2$) of the variance of the fundamental is explained by the price of the asset. As the degree of benchmarking increases and with it the willingness of the investors to deviate from the benchmark declines, investors speculate less based on their private information, so that less of it is incorporated into prices. A similar effect can be observed in case all investors are provided with private information, only that the fraction of the variance explained is, as expected, higher in this case. As benchmarking approaches its limit, investors only mimic the benchmark and abstain from speculating, so that one approaches the case in which prices cannot explain any part of the variance of the fundamental.

In the second step, we are now going to study the full equilibrium economy again, i.e., with endogenous information acquisition. In this setup, we should see an even stronger decline in price informativeness as, on top of the effect discussed above, the fraction of informed investors declines with the degree of benchmarking. Specifically, Figure 5 Panel (b) illustrates the fraction of the variance of the fundamental, again measure in terms of $R^2$, that is explained by an investor’s private information only, by the asset prices only, as well as jointly by private information and the price, for the case of high information acquisition costs.

First, note that the fraction of the variance explained by an investor’s private information \( \{y_i\} \), conditional on his decision to acquire information, is always constant, with a value slightly below 5%, as the quality of the signal is exogenous in our model. Second, in the absence of benchmarking about 48% of the variance of the fundamental is explained by the price. This is slightly below the fraction of the variance explained by the price reported in Panel (a) for \( \lambda = 100\% \), as the fraction of investors that endogenously decide to acquire information is only about 80% for the high information costs (cf. Figure 4). Third, as expected, the fraction explained by the price and private information jointly is slightly higher than the fraction for the price alone, as not all information of an investor’s private
information is revealed in the price.

Focusing on the effects of benchmarking, we can see that the fraction of the variance of the fundamental that is explained by the price or jointly by the price and an investor’s private information declines considerably faster than in Panel (a), as now also the effect of the smaller fraction of informed investors kicks in, and amplifies the effect of each investor incorporating less of his private information. Already for $B = 0.5$, the price is totally uninformative as no investor acquires information.

Interestingly, the difference between the fraction of the variance of the fundamental that is explained jointly by the price and private information and the fraction that is explained by the price only increases with the degree of benchmarking (not shown). That is, the higher the degree of benchmarking the more profitable is the private information for the (small) subgroup of informed investors.

5 Asset Prices and Returns

Finally, we now want to study how the decline in price informativeness in the presence of relative performance evaluation affects the price of the benchmark asset as well as its return.

5.1 Asset Prices

For this analysis, we again first go back to the economy in which a certain fraction of investors is exogenously provided with private information. Similar to the discussion above, this allows us to disentangle the effect of lower price informativeness because each investor incorporates less information into the price from the effect of a declining fraction of informed investors.

Figure 6 Panel (a) depicts how the price of the benchmark asset changes as we increase the degree of benchmarking – again for three different fractions of informed investors. First, note that in the absence of private information ($\lambda = 0$), i.e., in a setup with symmetric information, we get an increase in the price of the benchmark asset due to ‘excess demand’, stemming from the investors’ hedging component, in line with the demand pattern illustrated
in Figure 2 and similar to the results in Basak and Pavlova (2013) as well as Buffa, Vayanos, and Woolley (2015).

However, for the setup in which all investors are exogenously provided with private information (\(\lambda = 100\%\)) the price of the benchmark asset declines for low degrees of benchmarking. Intuitively, due to the decline in price informativeness discussed above, individual investors perceive the benchmark asset to be more risky and, thus, require a price concession. For low degrees of benchmarking this effect actually dominates the excess demand effect, so that the price of the benchmark asset declines. For high degrees of benchmarking the excess demand effect starts to dominate, explaining the U-shaped pattern in the price of the benchmark asset. We can also observe this, though in smaller magnitude, for the setup in which half of the
investors are informed ($\lambda = 50\%$).

Re-introducing the effects from the investors’ endogenous information acquisition choice strengthens the decline in price informativeness in the presence of benchmarking, as was discussed above. This, in turn, makes investments into the benchmark asset being perceived even more risky, so that investors require a more substantial price concession. This is illustrated in Figure 6 Panel (b). The price of the benchmark asset declines much faster than in the setup with exogenous information, dominating for high and low levels of information costs, the excess demand effect for a wide range of the degree of benchmarking.

5.2 Asset Returns

As the last part of our analysis, we also want to study the implications of benchmarking for the return distribution of the benchmark asset. Specifically, Figure 7 shows the expected return, its volatility and the Sharpe ratio of the benchmark asset plotted against the degree of benchmarking in the full equilibrium economy, i.e., with endogenous information acquisition.

As a direct analogue to Figure 6 Panel (b) that depicts the price of the benchmark asset, Panel (a) of Figure 7 shows that the expected return initially increases in the degree of benchmarking due to the price concessions that investors require as a reaction to the lower price informativeness. In contrast, for high degrees of benchmarking the excess demand effect dominates, lowering the expected return of the asset. The higher risk that can be attributed to the lower level of price informativeness can also be directly seen from Panel (b), showing a substantial increase in volatility, specifically so in the region where the reduction in price informativeness is the driving force. Finally, Panel (c) shows that the increase in volatility actually dominates the initial increase of the expected return in the degree of benchmarking, so that the Sharpe ratio, i.e., the compensation per unit of risk, declines in the degree of benchmarking.
Figure 7: *Equilibrium implication of benchmarking on returns.* Endogenous information: Panel (a): expected return of the benchmark asset, Panel (b): return volatility of the benchmark asset, and Panel (c) Sharpe ratio of the benchmark asset. The shaded area marks the entire collection of curves ranging from $\kappa = 0$ (and therefore $\lambda = 1$, top part of area) to $\kappa = \infty$ (and therefore $\lambda = 0$, bottom part of area).
6 Conclusion

The asset management industry plays an important role in determining the prices of assets traded in financial markets. However, asset managers are also crucial for information acquisition and, thus, the ability of financial markets to aggregate and disseminate information.

In this paper, we study the joint determination of endogenous information acquisition and equilibrium asset prices in a noisy rational expectation model. Key to our analysis is the assumption that asset managers care not only about their own performance, but also about their performance relative to a benchmark. We incorporate this incentive into CRRA preferences which gives rise to relative wealth concerns. This is realistic and crucial for our result as CARA preferences would suggest that each investor independent of his degree of benchmarking, and therefore his willingness to deviate from the benchmark, would acquire the same amount of information which seems quite implausible.

We show that even though asset managers increase their demand for the benchmark asset in the presence of relative performance evaluation due to index-hedging, they incorporate less of their private information into asset prices. This effect is quite intuitive as benchmarking reduces an investor’s willingness to deviate from the benchmark and, thus, to speculate based on private information. As benchmarking at the same time also reduces the fraction of investors that are willing to acquire costly information and, thus, not only less of the information of each informed manager is incorporated into prices, but also the group of informed managers shrinks, prices are considerably less informative in the presence of institutional investors.

The reduction in price informativeness has direct implications for the asset’s price which is determined simultaneously with the information choice of the asset managers in equilibrium. Specifically, as the price is less informative, it is perceived more risky by the asset managers, so that they require a higher expected return, leading to a decline in price. This ‘information effect’ on asset prices is complementary to the excess demand effect, which implies a higher asset prices and is due to managers’ hedging concerns, studied in the previously literature.
In equilibrium, the information effect typically dominates for low to medium degrees of benchmarking and the excess demand effect dominates for high degrees of benchmarking, leading to a U-shaped price function. Due to the decline in price informativeness in the presence of institutional investors, returns also become considerably more volatile.

Our analysis has important implications for the efficiency of financial markets to incorporate and provide information. Specifically, our results suggest that the recent surge of capital flows into exchange-traded funds might lead to a decline in price informativeness with adverse effects on asset prices and return volatility. On the other hand, this might give rise to profitable investment opportunities for those investors that are willing to acquire information and are willing to deviate from benchmarks.

Our model also gives endogenously rise to two groups of asset managers: one group that acquires costly information and, accordingly, actively invests, as well as a second group that abstains from acquiring information and passively mimics the benchmark. The size of each group of investors is exclusively driven by the degree of benchmarking in the asset management industry. Our paper can therefore also be related to the optimal size of active and passive portfolio management within the asset management industry.
References


## A Parameters

<table>
<thead>
<tr>
<th>group</th>
<th>description</th>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>preferences</td>
<td>risk aversion</td>
<td>$\gamma$</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>degree of benchmarking</td>
<td>$B$</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>fixed compensation</td>
<td>$A$</td>
<td>0.04</td>
</tr>
<tr>
<td>asset payoffs</td>
<td>distribution</td>
<td>$f_v$</td>
<td>binomial</td>
</tr>
<tr>
<td></td>
<td>mean</td>
<td>$\mu_v$</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>deviation from mean</td>
<td>$\sigma_v$</td>
<td>0.35</td>
</tr>
<tr>
<td>random supply</td>
<td>distribution</td>
<td>$f_z$</td>
<td>log-normal</td>
</tr>
<tr>
<td></td>
<td>mean</td>
<td>$\mu_z$</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>standard deviation</td>
<td>$\sigma_z$</td>
<td>0.125</td>
</tr>
<tr>
<td>initial portfolio</td>
<td>riskless asset</td>
<td>$\theta_{i,0}^B$</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>risky asset</td>
<td>$\theta_{i,0}^S$</td>
<td>1</td>
</tr>
<tr>
<td>information</td>
<td>reference cost</td>
<td>$\kappa_{\text{high}}$</td>
<td>0.0100</td>
</tr>
<tr>
<td></td>
<td>lower cost</td>
<td>$\kappa_{\text{low}}$</td>
<td>0.0085</td>
</tr>
<tr>
<td></td>
<td>fraction of informed agents</td>
<td>$\lambda$</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>quality (conditional probability)</td>
<td>$\rho$</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>quality (explained variance)</td>
<td>$R^2$</td>
<td>0.04</td>
</tr>
<tr>
<td>discretization</td>
<td># gridpoints $f_{v,y}$</td>
<td>$M \times M$</td>
<td>$2 \times 2$</td>
</tr>
<tr>
<td></td>
<td># gridpoints $f_z$</td>
<td>$N$</td>
<td>51</td>
</tr>
</tbody>
</table>

Table 1: Reference parameter values used in figures unless otherwise stated.
B Information Aggregation in Large Markets with General Preferences and Payoff Distributions

This section introduces a generalized model of Hellwig (1980). Specifically, we allow for general preferences and payoff distributions. This allows us to take a look at the quantities such as wealth effects, incentive schemes, behavioral biases, and even trading frictions such as transaction costs. Our setup describes an environment with dispersed information in a large market. In the spirit of Grossman and Stiglitz (1980), we endogenize the fraction of informed traders.

B.1 Summary of the economy

Our setup regarding agents and assets is a general version of the one described in 2.1: The economy runs for two periods $t \in \{0, T\}$, is populated by a unit mass of ex-ante identical agents $i \in [0, 1]$ who maximize their utility $U$ over their final period consumption $c_{i,T}$. Riskless assets are supplied perfectly elastically and yields a unit payoff with certainty. The second asset is risky, has an uncertain inelastic supply $z \sim f_z$, and produces a risky payoff $v \sim f_v$.

Agents can decide to acquire a private signal $y_i$ that is informative about $v$ at cost $\kappa$. The joint distribution of the two random variables is denoted by $f_{v,y}$. Agents use their private signal as well as information aggregated into equilibrium prices to compute expected asset payoffs.

B.2 Discretization of Random Variables

We approximate the distributions of random variables on a grid. We propose to discretize $f_{v,y}$ with $M \times M$ grid points and $f_z$ with $N$ grid points. It is important to take $N$ large enough as to ensure that the desired distribution is approximated accurately. In this paper we use binomial asset payoffs and therefore $M = 2$. When continuous distributions are used,

8Portfolio holdings for the riskless and risky asset are denoted by $\theta_{i,t}^B$ and $\theta_{i,t}^S$ respectively.
a (much) larger number of grid points should be used to ensure a solution of appropriate quality. We denote the approximated grids by \( \tilde{f}_{v,y} \) and \( \tilde{f}_z \).

**B.3 Learning**

The key contribution of our method is to model learning from prices outside the CARA/normal framework. While departure from normality has received lots of attention in the recent literature (e.g., Breon-Drish (2015)), no closed-form is available to deal with general preferences. Close to our method is Bernardo and Judd (2000), we approximate a Grossman and Stiglitz (1980) type of economy. Our method is capable of handling disperse information across investors, which is more consistent with the notion of large capital markets (as mentioned by Hellwig (1980)). In addition, we believe that our method is faster to implement for higher precision and more convenient by requiring no guessing about orthogonal functions in their projection step.\(^9\)

By submitting demand schedules, agents can condition down their trades upon the observable equilibrium price \( p \). Since demand schedules are aggregated through the market clearing condition, equilibrium prices convey useful information about the signal realizations of other agents, which could be used to learn about the risky assets’ fundamental.

Any agent can compute the aggregate demand \( \tilde{\theta}_T^S = \tilde{\theta}_T^S + \tilde{\theta}_T^U \) = \( \int_0^\lambda \theta_{i,y,T}^S di + (1 - \lambda) \theta_{i,\emptyset,T}^S \) for all realizations of observable and unobservable random variables. Informed and uninformed demand are separately presented to highlight their heterogeneous dependence on private signals. Using the market clearing equation, agents internalize the values of \( \{ \tilde{v}, \tilde{z} \} \) that are consistent with observed prices. Updating beliefs now consist of computing the relative likelihood of all the possible realization pairs.

\(^9\)In the projection step of Bernardo and Judd (2000), one needs to show orthogonality to all (!) functions. Since this is not possible, the authors show orthogonality to a well-chosen set of polynomials instead.
B.3.1 Learning for uninformed agents

For privately uninformed agents, who can only use the equilibrium price to update their beliefs, the likelihood that \( \hat{v} = v \) is now given by:\(^{10}\)

\[
L (\hat{v} = \hat{v}|p(v, z)) = f_v (\hat{v}) f_z (\tilde{\theta}_{I,T} (p(v, z), \tilde{y}(\hat{v})) + \tilde{\theta}_{U,T} (p(v, z)))
\]

Therefore, the posterior density is given by:

\[
f_{v|p}(\hat{v} = \hat{v}|p(v, z)) = \frac{L (\hat{v} = \hat{v}|p(v, z))}{\int_{-\infty}^{\infty} L (\hat{v} = \hat{v}|p(v, z)) \, dv} = \frac{f_v (\hat{v}) f_z (\tilde{\theta}_{I,T} (p(v, z), \tilde{y}(\hat{v})) + \tilde{\theta}_{U,T} (p(v, z)))}{\int_{-\infty}^{\infty} f_v (\hat{v}) f_z (\tilde{\theta}_{I,T} (p(v, z), \tilde{y}(\hat{v})) + \tilde{\theta}_{U,T} (p(v, z))) \, d\hat{v}}
\]

Since we work on a discretized grid, the above equation is approximated by:\(^{11}\)

\[
\hat{f}_{v|p} (\hat{v} = \hat{v}_m|p(v_m, z_n)) = \frac{L (\hat{v} = \hat{v}_m|p(v_m, z_n))}{\sum_{m=1}^{M} L (\hat{v} = \hat{v}_m|p(v_m, z_n))} = \frac{\hat{f}_v (\hat{v}_m) \hat{f}_z (\tilde{\theta}_{I,T} (p(v_m, z_n), \tilde{y}(\hat{v}_m)) + \tilde{\theta}_{U,T} (p(v_m, z_n)))}{\sum_{n=1}^{N} \hat{f}_v (\hat{v}_m) \hat{f}_z (\tilde{\theta}_{I,T} (p(v_m, z_n), \tilde{y}(\hat{v}_m)) + \tilde{\theta}_{U,T} (p(v_m, z_n)))}
\]

B.3.2 Learning for informed agents

Privately informed agents can use their signal as an additional to the publicly observable equilibrium price. Conditional on their (bivariate) information set, the likelihood that \( v = \hat{v} \) is now given by:

\[
L (\hat{v}|p(v, z), y_i) = f_{v|y_i}(\hat{v}) f_z (\tilde{\theta}_{I,T} (p(v, z), \tilde{y}(\hat{v})) + \tilde{\theta}_{U,T} (p(v, z)))
\]

Therefore, the posterior density is given by:

\(^{10}\)To be precise on notation, the potential (possibly off-equilibrium) realizations of random variables are denoted with a hat, where actual realizations are denoted without hat.

\(^{11}\)\( \hat{f}_{v|p} \) denotes the approximation of \( f_{v|p} \)
\[ f_{v|y_i}(\tilde{v} = \hat{v}|p(v, z), y_i) = \frac{L(\tilde{v} = \hat{v}|p(v, z), y_i)}{\int_{-\infty}^{+\infty} L(\tilde{v} = \hat{v}|p(v, z), y_i) \, dv} = \frac{\int_{-\infty}^{+\infty} f_{v|y_i}(\tilde{v}) f_z(\hat{\theta}_{I,T}(p(v, z), \tilde{y}^{(0)})) + \hat{\theta}_{U,T}(p(v, z)))}{\int_{-\infty}^{+\infty} f_{v|y_i}(\tilde{v}) f_z(\hat{\theta}_{I,T}(p(v, z), \tilde{y}^{(0)})) + \hat{\theta}_{U,T}(p(v, z))} \, d\tilde{v} \]

Since we work on a discretized grid, the above equation is approximated by:

\[ \tilde{f}_{v|p,y_i}(\tilde{v} = \hat{v}_m|p(v, z), y_i) = \frac{L(\tilde{v} = \hat{v}_m|p(v, z), y_i)}{\sum_{m=1}^{M} L(\tilde{v} = \hat{v}_m|p(v_m, z_n), y_i)} \]

\[ = \frac{\sum_{m=1}^{M} f_{v|y_i}(\tilde{v}_m) f_z(\hat{\theta}_{I,T}(p(v_m, z_n), \tilde{y}^{(0)})) + \hat{\theta}_{U,T}(p(v_m, z_n))}{\sum_{m=1}^{M} f_{v|y_i}(\tilde{v}_m) f_z(\hat{\theta}_{I,T}(p(v_m, z_n), \tilde{y}^{(0)})) + \hat{\theta}_{U,T}(p(v_m, z_n))} \] (9)

Regarding special cases, firstly note that we can immediately see that \( \tilde{f}_{v|p,y_i}(\tilde{v}_m) = \tilde{f}_{v|p}(\tilde{v}_m) \) when private signals are uninformative, that is, when \( \tilde{f}_{v|y_i} = \tilde{f}_v \). Second, when the signal is perfectly informative, that is, when \( \tilde{f}_{v|y_i}(\tilde{v}_m) = 1 \) \( \hat{v}_m = y_i \), then \( \tilde{f}_{v|p,y_i}(\tilde{v}_m) = \tilde{f}_{v|y_i}(\tilde{v}_m) = 1 \) \( \hat{v}_m = y_i \). Importantly, note that for discrete distributions, as is done in our paper for asset payoffs and signals, the approximated posterior is identical to the true posterior.

“Learning” equations (8) and (9) add a total of \( M \times (M \times N) + M \times (M \times M \times N) = (M^2 \times N) \times (1 + M) \) equations to the system and pin down an equal number of unknowns contained in \( \tilde{f}_{v|p} \) and \( \tilde{f}_{v|p,y_i} \).

**B.4 First order conditions**

First order conditions (regarding risky asset demand) are derived in Section 2.4 and should be computed for every realization of \{\tilde{v}, \tilde{z}\} for the uninformed agent and every realization of \{\tilde{v}, \tilde{y}_i, \tilde{z}\} for the informed agent. In our discretized framework, this accounts for \( M \times N + M^2 \times N \) first order equations and an equal number of (portfolio holdings) unknowns.

\[ \tilde{f}_{v|p} \] denotes the approximation of \( f_{v|p} \)
B.5 Information Allocation

There is one information allocation condition, namely (4), that pins down the equilibrium fraction of informed agents $\lambda$. Since (4) is not a differentiable function, we try out all three scenarios and select the unique case under which the system is solved.

B.6 Market clearing

Finally, the market clearing equations are stated by (5). Note that these equations must hold for every realization of $\{\tilde{v}, \tilde{z}\}$. The market clearing conditions pin down equilibrium prices $p$. In our discretized setup, the market clearing conditions can be stated as:

$$\hat{\theta}_T^S = \lambda \times \left( \sum_{m=1}^{M} \tilde{f}_{y_m|v}(y_m) \times \theta_{i,y_m,T}^S \right) + (1 - \lambda) \times \theta_{i,0,T}^S = z_n$$

Where $\tilde{f}_{y_m|v} = \frac{f_{y_m|v}}{f_v}$ governs the distribution of signals as a function of the realization of the fundamental. Overall, there are $M \times N$ equations that pin down an identical number of equilibrium prices.

B.7 System overview

The combination of the learning equations, first order conditions, information allocation equation and market clearing equations yield $1 + (2 \times M + 2 \times M^2 + M^3) \times N$ equations and an identical number of unknowns. It is now clear that assuming a binomial distribution of payoffs and signals greatly simplifies the problem. In particular, setting $M = 2$ reduces the system to $1 + 20N$ equations. Further numerical convenience can be obtained by directly substituting the leaning equations in the first order conditions. The resulting fixed point problem is solved using a standard Newton algorithm.