Aggregate Effects of Collateral Constraints*

Sylvain Catherine† Thomas Chaney‡ Zongbo Huang§
David Sraer¶ David Thesmar‖

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Abstract

We structurally estimate a dynamic model with heterogeneous firms and collateral constraints. Embedding this model in a general equilibrium framework allows us to quantify the impact of financing frictions on aggregate output and welfare. The structural estimation is based on the well identified causal effect of collateral shocks on firm level corporate investment in the United States. The estimates imply that lifting financing frictions would increase welfare by 9.4% and aggregate output by 11%. Half of this aggregate output gain are due to an increase in the aggregate stock of capital, one quarter is due to a lower aggregate labor supply, while the remaining quarter is due to a higher aggregate productivity from a better allocation of inputs across heterogeneous firms.

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†HEC Paris
‡Sciences Po and CEPR
§Princeton University
¶UC Berkeley, CEPR and NBER
‖MIT and CEPR
There is an accumulating body of evidence showing the causal effect of financing frictions on firms’ investment decisions at the micro-level. While this literature safely rejects the null hypothesis that firms are unconstrained financially, it does not measure if these constraints matter quantitatively. In this paper, we use a quantitative model that matches these findings to investigate the aggregate effects of financing frictions. We focus on a pervasive source of financing friction – collateral constraints. Our approach expands on the existing literature by (i) estimating our structural model using well-identified firm-level evidence that collateral constraints causally affect investment and (ii) nesting this model in a general equilibrium framework with heterogenous firms to study aggregate effect of collateral constraints. Our estimated model shows that even in a developed country like the U.S., collateral constraints can have a large effect on welfare. Compared to a counterfactual economy without collateral constraints, welfare in our constrained economy is lower by 9.4%, and output by 11%. Of this output loss, about a quarter can be attributed to lower aggregate TFP due to input misallocation. The remaining output loss is due to a lower aggregate capital stock when firms are constrained. Thus, collateral constraints induce significant misallocations, but their impact on the aggregate capital stock is larger.

We estimate our structural model by targeting the sensitivity of investment to exogeneous shocks to firms’ real estate value. Starting with Gan (2007) and Chaney et al. (2012), a large literature documents how corporate investment responds to real estate shocks and argues that such sensitivity is evidence of financing constraints, insofar that real estate shocks are shocks to debt capacity that are uncorrelated with investment opportunity. Relying on this insight, we use this sensitivity to identify the parameter governing financing constraints in our model. The existing literature that estimates similar models (see for instance Hennessy and Whited (2007)) typically targets capital structure decisions such as the average debt to capital ratio. We argue however that this moment is driven by many other forces (trade credit, inventory, unsecured debt

1See, among many others, Lamont (1997), Rauh (2006), Chaney et al. (2012), Blanchard et al. (1994) for the effect of financial frictions on investment and Benmelech et al. (2010) or Chodorow-Reich (2013) for the effect of financial frictions on employment

2The costs of input misallocation is the focus of Hsieh and Klenow (2009), Moll (2014), Midrigan and Xu (2014).
capacity) that are not captured by the model. If one targets leverage, these forces are absorbed by the final estimate of financing constraints. In contrast to leverage, causal estimates coming from the reduced form literature are purely attributable to financing constraints. This should lead to more reliable estimates of financing constraints parameters. We show that, in our data, targeting the leverage ratio leads to underestimating the effect of financing constraints. The intuition is that the sensitivity of investment to real estate value is relatively low in the data, indicating a relatively low pledgeability of capital. Leverage is, on the other side, relatively large in the data, so that an estimation procedure that seeks to match leverage will assume that capital is easily collateralized. This makes financing constraints less binding. At the aggregate level, when targeting leverage, the estimated aggregate output loss is only half as large as when targeting the sensitivity of investment to real estate shocks.

We start by documenting how, on a panel of U.S. firms, corporate investment and leverage responds to shocks to real estate value. Repeating earlier analysis (Chaney et al., 2012) with slightly different specifications, we find that a $1 increase in real estate value leads to a $0.04 increase in investment and a $0.04 increase in financial debt. While these estimates allow to comfortably reject the null that firms are not financially constrained, they do not tell us whether these constraints matter quantitatively.

To assess whether these micro-level elasticities have significant aggregate implications, we proceed in two steps. First, we set-up a structural model of firms dynamics. The model builds on the standard neo-classical model of investment with adjustment costs (Jorgenson, 1963; Lucas, 1967; Hayashi, 1982). To this standard model, we add only one key ingredient. We assume that firms face a collateral constraint: The amount they can borrow every period is limited by how much tangible assets—including real estate—they own. Each period, the value of real estate assets fluctuates randomly, creating variations in the collateral constraint, thus mimicking our reduced-form empirical design.\(^3\) We estimate this model through a Simulated Method of Moments. In addition

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\(^3\)While we do not explicitly micro-found the collateral constraint, it emanates naturally from limited enforcement models (Hart and Moore, 1994).
to the standard moments used in the structural corporate finance literature, our estimation procedure explicitly targets the sensitivity of investment to variations in local real estate prices. We show that the model manages to fit the targeted moments and some non-targeted ones precisely. It also has well-behaved comparative statics properties, which ensures a precise parameter estimation. We also show a simple ratio of sales to capital is a good measure of financing constraints, as argued in the development literature (Hsieh and Klenow, 2009).

In a second step, the estimated model is nested in a simple general equilibrium where firms compete for customers and for capital goods. We simulate two economies: One in which firms face the estimated collateral constraints, and a counterfactual economy where firms face no borrowing constraint. We compute output and welfare loses from financing constraints by comparing the two economies. We find aggregate welfare loss from financing constraints of 9.4% and output loss of 11%. Such losses arise in part from the misallocation of inputs across heterogeneous producers (Hsieh and Klenow, 2009; Moll, 2014; Midrigan and Xu, 2014) and in part from a sub-optimal aggregate capital stock. While both channels matter, we find aggregate capital matters more than misallocations.

**Related Literature.** Our focus on collateral constraints is rooted in a large array of empirical evidence on the importance of collateral constraints. It is well documented that collateral plays a key role in financial contracting. More redeployable assets receive larger loans and loans with lower interest rates (Benmelech et al., 2005). The value of collateral affects the relative ex post bargaining power of borrowers and lenders (Benmelech and Bergman, 2008). Beyond these effects on financial contracting, collateral values also affect real outcomes at the micro-economic level: Firms with more valuable collateral invest more (Gan, 2007; Chaney et al., 2012); individuals with more valuable collateral are more likely to start up new businesses (Schmalz et al., Forthcoming). In addition, many empirical evidence point to the prevalence of real estate collateral in loan contracts (Davydenko and Franks, 2008; Calomiris et al., 2015). Our paper adds to the literature by bridging the gap between microeconomic evidence on the role of collateral constraints and the
macroeconomic effect of financial frictions.

Our paper also contributes to the long-standing literature in corporate finance investigating the real effects of financing frictions. This literature has traditionally explored the effect of financing frictions on corporate investment. A key challenge is to find exogenous variations in financing capacity that are not correlated with investment opportunities. For instance, Lamont (1997) overcomes this challenge by showing that non-oil divisions of oil conglomerates increase their investment when oil prices increase. Rauh (2006) shows that firms with underfunded defined benefit plans need to make financial contributions to their pension fund, depriving them of available cash-flows and leading to reduced investment.\(^4\)

Several important papers have developed a structural quantitative approach to estimate the effect of financing frictions. This literature is reviewed in Strebulaev and Whited (2012). In a seminal contribution, Hennessy and Whited (2007) apply SMM to a dynamic model to infer the magnitude of financing costs. They find that for small firms, the estimated marginal equity flotation costs is about 10.7% of capital and bankruptcy costs 15.1%. Hennessy and Whited (2005) develop a dynamic trade-off model, which they structurally estimate to explain several empirical findings inconsistent with the static trade-off theory. Lin et al. (2011) examines the impact of the divergence between corporate insiders’ control rights and cash-flow rights on firms’ external finance constraints from a generalized method of moments estimation of an investment Euler equation and show that the agency problems associated with the control-ownership divergence can have a real impact on corporate financial and investment outcomes. Nikolov and Whited (2014) estimate a dynamic model of finance and investment with different sources of agency conflicts between managers and shareholders to analyze the role of agency conflicts in corporate policies and investment. Our contribution to this literature is twofold. First, we include coefficient estimates from a reduced-form regression identifying the effect of collateral constraints on investment and

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\(^4\)See Bakke and Whited (2012) for a discussion of this identification strategy.

debt as targeted moments. We show that these moments are crucial in identifying the strength of financial frictions in our data. Second, we nest our investment model into a general equilibrium model, which allows us to account for general equilibrium effects in our counterfactuals. In contrast, the literature typically only considers partial equilibrium counterfactuals. In that sense, our model is close to Gourio and Miao (2010) who focus on taxation. Compared to their paper, we focus on model estimation and the effect of financing constraints.

Finally, our paper contributes to the important macroeconomic literature on the aggregate effects of financial frictions. Restuccia and Rogerson (2008), Hsieh and Klenow (2009) and Bartelsman et al. (2013) emphasize the effect of misallocation of resources across heterogeneous firms on aggregate TFP and welfare. Midrigan and Xu (2014) focus on financing frictions as a source of misallocation. They calibrate a model of establishment dynamics with financing constraints and find that financing frictions cannot explain large aggregate TFP losses from misallocation. In contrast, Moll (2014) shows that for a TFP persistence parameter in the empirically relevant range, financial frictions can matter in both the short and the long run. Buera et al. (2011) develop a quantitative framework to explain the relationship between aggregate/sector-level TFP and financial development across countries and show that financial frictions account for a substantial part of the observed cross-country differences in output per worker, aggregate TFP, sector-level relative productivity, and capital-to-output ratios. Beyond misallocation, a large literature has investigated the effects of financing friction on aggregate TFP growth and welfare. Jeong and Townsend (2007) develop a method of growth accounting based on an integrated use of transitional growth models and micro data and find that in Thailand, between 1976 and 1996, 73 percent of TFP growth is explained by occupational shifts and financial deepening. Amaral and Quintin (2010) present calibrated simulations of a model of economic development with limited enforcement and find that the average scale of production rise with the quality of enforcement. Riddick and Whited (2009) study the costly reallocation of capital across heterogeneous firms. They infer the cost of reallocation from a calibrated model and show that reallocation costs need to be strongly countercyclical to be consistent with the observed dispersion of productivity. Our contribution to this
literature is that we base our quantification exercise on an estimation procedure that targets moments from a reduced-form analysis exploiting exogenous shocks to financing capacity. Second, our paper combines adjustment costs with financing frictions. Asker et al. (2014) consider the effect of adjustment costs on static misallocation measures, but their economy does not feature a financing friction. In contrast, our approach delivers interesting implications on the interaction between adjustment costs and credit frictions.

We present reduced form evidence of the effect of collateral values on both investment and employment in Section 1. We present our formal model of firm dynamics with collateral constraints in Section 2. We structurally estimate the model using US firm level data in Section 3. Section 4 describes and implements the general equilibrium analysis. Section 5 discusses robustness and implements a policy experiment.

1 Reduced form evidence

We estimate the investment and borrowing sensitivity to real estate value as in Chaney et al. (2012). The construction of the data is detailed in this paper. The dataset is a panel of publicly listed firms from 1993 to 2006 extracted from COMPUSTAT. We require that these firms supply information about the accounting value and cumulative depreciation of land and buildings (items ppenb, ppenli, dpacb, dpaci) in 1993. We then combine this information with office prices in the city where headquarters are located, in order to obtain a measure of the market value of each firm’s real estate holdings. We call this measure REValue\(_{it}\) for firm \(i\) at date \(t\). We require that this variable is available for all firms, so we end up with a panel of 20,074 observations corresponding to 2,218 firms which are followed from 1993 until 2006 unless they drop out of the panel before (only 676 firms are still present in 2006).

We then run the following regression:

\[
\frac{Y_{it}}{k_{it-1}} = \beta \frac{\text{REValue}_{it}}{k_{it-1}} + \frac{1}{k_{it-1}} + \text{Offprice}_{it} + a_i + \epsilon_{it}
\]
where \( k_{it-1} \) is the lagged stock of productive capital (item ppent). \( \text{Offprice}_it \) is the level office prices, which is available from Global Realanalytics for 64 MSAs. We further add a firm fixed effect \( (a_i) \) and cluster error terms \( \epsilon_{it} \) at the firm level. We are interested in \( \beta \), the sensitivity of \( Y_{it} \) to real estate value. We report descriptive statistics for these variables in Table 1.

We look at two different left hand-side variables \( Y_{it} \): Capital expenditures (item capx) and net debt increase (sum of changes in long term debt – item dltt – and short term debt – item dlc). The sensitivity of investment to real estate value is equal to 0.04 with a t-stat of 6.1. This can be interpreted as a $0.04 investment response per $1 increase in real estate value. This number is close to main estimate of Chaney et al. (2012), the difference coming from the set of controls used. We opt here for a simpler specification with fewer controls, in order to restrict ourselves to variables available in the model simulations. The sensitivity of net borrowing to real estate value is also found to be equal to 0.04, with a t-stat of 4.5. In the following Section, we will estimate a model that matches the first coefficient (the investment sensitivity), and will look at the second coefficient (the borrowing sensitivity) as a non-targeted moment.

2 The model

In this Section, we explain our model. The economy is populated with heterogenous, financially constrained, firms who use capital and labor. A representative consumer consumes the final good and supplies the capital.

2.1 Production technology and demand

The firm-level model is close to Hennessy and Whited (2007) in the sense that it includes a tax shield for debt and a large cost of equity issuance (in our case, infinite) and Midrigan and Xu (2014) in the sense that firms face a collateral constraint. The firm’s shareholder is assumed risk-neutral and has a time discount rate of \( r \). Firm \( i \) produces output \( q_{it} \) combining capital \( k_{it} \) and
efficiency units of labor $l_t$ into a Cobb-Douglas production function with capital share $\alpha$

$$q_{it} = F(e^{z_{it}}, k_{it}, l_{it}) = e^{z_{it}} (k_{it}^{\alpha} l_{it}^{1-\alpha})$$

with $z_{it}$ the firm’s log total factor productivity which is assumed to follow an AR(1) process:

$$z_{it} = \rho z_{it-1} + \epsilon_{it}$$

where we denote $\sigma^2$ the variance of the innovation $\epsilon_{it}$. The firm faces a downward sloping demand curve with constant elasticity $\phi$,

$$q_{it} = Qp_{it}^{-\phi}$$

where $Q$ is aggregate spending and will be determined in equilibrium (see Section 4).

Labor is fully flexible, and $w$ is the wage – also determined in equilibrium. As labor is a static input, the total revenue of the firm net of labor input is

$$R(z_{it}; k_{it}) \equiv \max_{l_{it}} p_{it} q_{it} - wl_{it} = bQ^{1-\theta} w^{-(1-\alpha)\theta} e^{((\theta/\alpha)z_{it})} k_{it}^{\theta},$$

with $b$ a scaling constant and $\theta \equiv \frac{\alpha(\phi-1)}{1+\alpha(\phi-1)}$.

### 2.2 Input dynamics

Capital accumulation is subject to depreciation, time to build, and adjustment costs. At date $t$, gross investment $i_{it}$ is given by:

$$k_{it+1} = k_{it} + i_{it} - \delta k_t$$

where $\delta$ is the depreciation rate. In period $t$, investing $i_{it}$ entails a convex cost of $\frac{e^{\frac{i_{it}^2}{2k_{it}}}}{2}$. The firm pays in period $t$ for capital that will only be used in production in period $t + 1$: This one period time to build for capital is conventional in the macro literature (Hall, 2004; Bloom, 2009) and acts as an additional adjustment cost. We do not, however, include fixed adjustment costs to our model.
as in Gourio and Kashyap (2007): This choice is motivated by the fact that we seek to match firm-level data, for which investment is not very lumpy. In our data (described in Section 1), only 4% of the observations have an investment rate smaller than 2% of capital.\footnote{To compute the investment rate, we divide item capx by lagged item ppent} Thus, investment in firm-level data is significantly less lumpy than in plant-level data.

We believe that it is important to add adjustment costs to the model since they generate in the data patterns that may be similar to financing constraints. For instance, adjustment costs make capital vary less than firm output: This generates a natural dispersion in capital productivities, exactly like financing constraints do (Asker et al., 2014). It is therefore useful to have a model with both adjustment costs and financing constraints, in particular since we will see that our reduced form moment helps identifying financing constraints separately from adjustment costs.

### 2.3 Financing frictions and capital structure

The firm finances investment out of retained earnings and debt issuance to outside investors. $d_t$ is net debt, so that $d_t < 0$ means that the firm holds cash. As is standard in the structural corporate finance literature (Hennessy and Whited, 2005), we assume that debt has a maturity of 1 period. We set up the model so that debt is risk-free and thus pays interest rate $r$ – see Section 4. Thus, $d_t$ is the amount of debt issued at date $t$, and the firm commits to repay $(1 + r)d_{t+1}$ at date $t + 1$. Finally, we also assume that the interest rate the firm receives on cash is lower than the interest rate it has to pay on its debt. We model this by assuming that, if the firm has negative net debt, it faces a negative cash outflow of $(1 + (1 - m)r)d_{t+1}$ (where $m > 0$).

Consistently with the corporate finance literature, we also assume that debt is tax free. This creates an incentive for firms to increase their debt. Other papers make alternative assumptions that make debt attractive to firms, either by making debt holders intrinsically more patient than shareholders, or by assuming the shareholders seek to smooth consumption (for instance through log utility as in Midrigan and Xu (2014)). We assume that the firm’s profits net of interest payments are taxed at rate $\tau$. We also make the assumption that capital depreciation $\delta k_{it}$ is deductible
from taxable income. Hence, taxable income is given by revenue, minus interest payments, minus depreciation. Tax proceeds are rebated to the representative consumer – see Section 4.

The financing frictions come from the combination of two constraints. First, we assume, to simplify exposition, that firms cannot issue equity. This is an approximation but this assumption is not material to our results. Second, we assume that the firm faces a collateral constraint which reflects limited debt enforcement (Hart and Moore, 1994). The idea is that, in case of default between date $t$ and $t + 1$, the financier will seize and liquidate the firm’s collateral, only realizing a fraction $s$ of its value. $s$ captures the quality of debt enforcement, but also the extent capital can be redeployed and sold. This collateral is made of depreciated productive capital $(1 - \delta)k_{it}$ and real estate $p_t h$. $p_t$ is the time varying price of real estate, while $h$ is the firm’s holding of real estate, exogenous in our model. We assume log $p_t$ to be a discretized AR(1) process. Since the lender can only realize a fraction $s$ of the value of the assets, the borrowing constraint is given by:

$$(1 + r)d_{it+1} \leq s ((1 - \delta)k_{it} + p_t h)$$

This constraint makes debt risk-free, as in Gourio and Miao (2010). It is not material to our qualitative and quantitative results. Its sole purpose is to clarify the exposition.

Besides, we abstract from issues related to real estate ownership heterogeneity. This is why we assume the amount of real estate $h$ is the same for all firms, and constant throughout the life of each firm. It would be interesting to see how the heterogeneity in $h$ interacts with productivity dispersion to affect the aggregate impact of financing constraints. Such an analysis would, however, require an explicit theory of real estate ownership. We expect the decision to buy real estate to interact with investment decisions in subtle ways. For instance, firms may choose to purchase real estate in anticipations of events of high productivity and low debt capacity. Such analysis is beyond the scope of this paper, which focuses on measuring and aggregating financial frictions, so we defer this question to future research.

Consistent with the classical tradeoff theory, the capital structure decision reflects the tradeoff
between two forces. On the one hand, the tax shield of debt encourages firms to borrow more. On the other, too much debt leads to debt overhang, as the firm may be unable to invest enough when productivity increases.

2.4 The optimization problem

The firm is infinitely lived, but may disappear every period with probability \( d \). Every period, capital and debt are chosen optimally to maximize a discounted sum of per period cash flows, subject to a financing constraints. The firm takes as given its productivity, local real estate prices, and forms expectation for future productivities and real estate prices according to their actual stochastic processes.

Define as 
\[
V(S_{it}; X_{it})\]
the value of the discounted sum of cash flows given the exogenous state variables \( X_{it} = \{z_{it}, p_t\} \) and the past endogenous state variables \( S_{it} = \{k_{it}, d_{it}\} \). Shareholders are assumed to be perfectly diversified so their discount rate is the same as risk-free debt \( r \).

This value function \( V \) is the solution to the following Bellman equation,

\[
\begin{align*}
V(S_{it}; X_{it}) &= \max_{S_{it+1}} \left\{ e(S_{it}, S_{it+1}; X_{it}) + \frac{d}{1+r}E[V(S_{it+1}; X_{it+1})|X_{it}] + \frac{d}{1+r} (k_{it+1} - (1 + \tilde{r}_{it})d_{it+1}) \right\} \\
\text{s.t.} & \quad (1 + r)d_{it+1} \leq s ((1 - \delta)k_{it} + p_t h) \\
& \quad e(S_{it}, S_{it+1}; X_{it}) \geq 0 \\
\text{with} & \quad e(S_{it}, S_{it+1}; X_{it}) = bQ^{1-\theta} u_t^{-(1-\frac{\alpha}{\theta})} e^{(\theta/\alpha)z_{it}} k_{it}^\theta - i_{it} - \frac{\sigma^2}{2k_{it}} + d_{it+1} - (1 + \tilde{r}_{it})d_{it} \\
& \quad i_{it} = k_{it+1} - (1 - \delta) k_{it} \\
& \quad \tilde{r}_{it} = r \text{ if } d_{it} < 0 \text{ and } (1 - m) r \text{ if } d_{it} \geq 0
\end{align*}
\]

(6)

where the second term in the maximand \( \frac{d}{1+r} (k_{it+1} - (1 + \tilde{r}_{it})d_{it+1}) \) corresponds to the shareholder’s payoff in case of firm death. This term is important in order to avoid biasing the firm towards borrowing. Without this term, since bankers can recover capital in case of death, shareholders have an incentive to borrow more in order to transfer value from the states of nature where
they cannot consume to states of nature where the firm survives. We do not want this effect to drive capital structure decisions in our model.

Aggregate demand $Q$ and the real wage $w$ are equilibrium variables that the firms take as given (we determine them in Section 4). Given the absence of aggregate uncertainty and the steady state assumption, they are fixed over time. Due to downward sloping demand, firms have an optimal scale of production. A firm initially below this level accumulates capital. Once the target scale is reached, firms replace depleted capital. When faced by a productivity shock, the firm adjusts towards its new desired steady state. The firm refrains from adjusting capital instantaneously because of convex adjustment costs. Finally, spending on adjusting capital is bound by the collateral constraint. When the value of a firm’s real estate assets increases, the collateral constraint is relaxed, and the firm finances more of the cost of adjusting towards its desired scale. This will generate the sensitivity of investment to real estate value that we have documented in Section 1.

3 Structural Estimation

3.1 Estimation procedure

We estimate the key parameters of the model via a Simulated Method of Moments. The entire procedure is described in detail in Appendix A. We look for the set of parameters $\hat{\Omega}$ such that model-generated moments $m(\hat{\Omega})$ on simulated data fit a pre-determined set of data moments $m$. If we could solve the model analytically, we could just invert the system of equations given by model-based moments. Because our model does not have an analytic solution, we need to use indirect inference to perform the estimation. Such inference is done in two steps:

1. For a given set of parameters, we need to solve the model numerically, which means solving the Bellman problem (6) and obtain the policy function $S_{it+1} = (d_{it+1}, k_{it+1})$ as a function of $S_{it} = (d_{it}, k_{it})$ and exogenous variables $X_{it} = (z_{it}, p_t)$. To numerically solve the model,
we need to discretize the state space \((S, X)\) into a grid that is as fine as possible. We need a fine grid in order to obtain minimize numerical errors in the presence of hard financing constraints. We believe this is critical as a 1-2% numerically generated error is already too big given that our goal is to quantify aggregate effects of this order of magnitude. The challenge however is that we have two endogenous state variables and 2 exogenous ones. As we discuss in the Appendix, this would make the resolution time on a conventional CPU last several hours. This would be acceptable if we just wanted to calibrate the model, but not if we want to estimate it, i.e. solve it many times in order to find the parameters that fit the data moments. This is why we are using a GPU which lowers the resolution time to a few minutes. The model resolution step is described in Appendix A.1.

2. Our parameter estimates \(\hat{\Omega}\) minimize the distance from simulated to data moments \(m\):

\[
\hat{\Omega} = \arg \min_{\Omega} (m - \hat{m}(\Omega))^\prime W (m - \hat{m}(\Omega))
\]

where the weighting matrix \(W\) is the inverse of the variance-covariance matrix of data moments. As is typical in this type of estimation, we face two key technical issues at this stage. First, we need to minimize the numerically induced error in estimating the model-generated moments \(\hat{m}(\Omega)\). This error arises from the fact that we compute these moments on simulated data of finite size. Second, given the large number of parameters (5 in our preferred specification), we need to make sure the algorithm does not leads to a local optimum. We describe this second step in detail in Appendix A.2.

3.2 Predefined and Estimated Parameters

The model has 14 parameters. We calibrate 9 of them using estimates from the literature or the data, and estimate the 5 remaining ones.

*Predefined parameters.*— Our 9 calibrated parameters are as follows. We set the capital share \(\alpha = 1/3\) from Bartelsman et al. (2013) and the demand elasticity \(\sigma = 5\) from Broda and Weinstein
(2006) (which would lead to mark-ups of 25% in the absence of adjustment costs). Real estate prices $\log p_t$ follow a discretized AR(1) process. We estimate this AR(1) process on de-trended logged real estate prices and find a persistence 0.62 and innovation volatility 0.06. Both AR(1) processes for $\log z_t$ and $\log p_t$ are discretized using Tauchen’s method. The rate of obsolescence of capital is set at $\delta = 6\%$ as in Midrigan and Xu (2014). The risk-free borrowing rate $r$ is fixed at 3%, while the lending rate is set to $(1 - m)r = 2\%$. We fix the death rate $d$ to 8% which corresponds to the turnover rate of firms in our data. Finally, we set $w = 0.03$ (\$ 30,000) and $Q = 1$ for the estimation. They will, however, be endogenously determined in general equilibrium in our counterfactual analyses.

Estimated parameters.— We estimate 5 deep parameters but focus the discussion on 4 of them. Our efforts focus on 4 main parameters: the persistence $\rho$ and innovation volatility $\sigma$ of log productivity, the collateral parameter $s$ and the adjustment cost $c$. To these 4 parameters, we add a fifth one, the amount of real estate collateral available $h$, which is here to match the average ratio of real estate to capital $h/k_t$ in the data. This last moment conditions is essentially a normalization of the model.

3.3 Data Moments

We compute the moments on the COMPUSTAT sample described in Section 1. We describe them here and have a short heuristic discussion about their “identifying” power. In the next section, we will discuss identification more systematically by showing how model-generated moments vary with parameters.

First, in the spirit of Midrigan and Xu (2014), we use the short- and long-term volatility of output to estimate the persistence and volatility of the productivity process. We thus compute the volatility of change in log sale (COMPUSTAT item: sale) which is in our sample equal to 0.327 ($\log \text{sales}_{it} - \log \text{sales}_{it-1}$). We also compute the volatility of 5-year change in log sales ($\log \text{sales}_{it} - \log \text{sales}_{it-5}$), which is in our sample equal to 0.911. The fact that 5-year growth is less than 5 times more volatile than 1-year growth indicates mean-reversion and hence contribute
to the identification of the persistence parameter of productivity. We chose these two moments, instead of seeking to directly match the persistence coefficient of log sales. This is to make the estimation less sensitive to our assumption that productivity is an AR1 process. By targeting long run volatility, the estimated persistence parameter of productivity will take into account the fact that the real process may have longer memory than an AR1.

Second, we use the autocorrelation of investment to identify adjustment costs (as for instance in Bloom (2009)). For each firm in our panel we compute the ratio $\frac{i_{it}}{k_{it}}$ of capital expenditures (COMPUESTAT item: capx) to lagged capital stock (COMPUESTAT item: ppent). We then compute the correlation between $\frac{i_{it}}{k_{it}}$ and $\frac{i_{it-1}}{k_{it-1}}$, and find 0.43 in our data. This large correlation needs adjustment costs to be matched by the model: Adjustment costs compel the firm to smooth its investment policy over time when responding to a productivity shock. This intuition holds in the absence of financing constraints. We will see whether it still holds when we add them to the model, since these frictions typically also have the effect of limiting firm adjustment (Asker et al., 2014).

Third, we use two alternative moments to estimate the collateral constraint parameter $s$. The first moment is net book leverage, a moment typically used in the literature (Hennessy and Whited, 2007; Midrigan and Xu, 2014). Book leverage is computed as net financial debt (COMPUESTAT items: dlc + dltt) minus cash holdings (COMPUESTAT item: che), normalized by total assets (COMPUESTAT item: at). This definition reflects the notion that cash is equivalent to negative debt, as it is the case in our model. We obtain an average of 0.313 in our data. Leverage directly identifies the collateral parameter $s$ as higher collateral value unambiguously leads to more borrowing in the model. Yet, as we discuss more extensively below, the leverage moment is not ideal to identify financing constraints for measurement issues, but also because of measurement issues and model identification problem. For instance, accounting depreciation may underestimate the quantity of capital in the firm, and lead to an overstatement of the average $d_{t+1}/k_{t}$ which is generated by the model. This would lead to an overestimation of $s$. Similarly, financial debt typically includes unsecured debt, which is not part of our model (see Section 5.3 for such an
extension). As a result, net leverage overstates the extent to which collateral can be pledged. Finally, model identification issues arise because the firm may not be financially constrained yet choose to lever up for tax purposes. This would lead to misattribute corporate leverage to collateral constraints. Because of all these limitations of the leverage moment, we use instead the sensitivity of investment to real estate value, computed in Section 1. This measure is a more direct measure of the extent of financing constraints. Because it is also an informative and natural moment, we also look at the sensitivity of debt issuance to real estate value. We never target this second moment in our estimation, but it turns out our main model matches it very well (more on this below).

Finally, we also estimate the parameter $h$, the quantity of real estate held by the average firm, by asking the model to match the fraction of real estate in asset holding. This part of our estimation is conceptually less important, the goal is mostly to make sure that, given the firm size implied by the other parameters, firms in the models hold the same amount of real estate as in the data. We compute this number in COMPUSTAT by taking the ratio of real estate holdings (land + buildings) in 1993 normalized by total assets, and obtain 0.14. By adjusting $h$, our estimation procedure matches this moment perfectly; we view this part of the estimation as a normalization more than anything else. As a result, we omit discussion of this parameter from this point on.

### 3.4 Parameter Identification

This Section discusses identification of the parameters of the model. In Appendix Figures C.1-C.4, we reproduce how moments vary as a function of model parameters. We also show, in Table 2, the elasticities of each moments with respect to estimated parameters – a simple transformation of the Jacobian matrix. All this analysis is about local identification, in the sense that we operate around around our main SMM estimate for $(s, c, \rho, \sigma)$ – which we discuss in detail in the next Section.

Let us first discuss the graphical evidence. In Figures C.1-C.4, we first offer visual evidence that different parameters are not identified by the same moments. To construct each one of these 4 figures, we vary one of the four parameters $(s, c, \rho, \sigma)$ while leaving the others equal to the SMM
estimate. We vary these parameters over a wide range (for instance, \( \rho \) goes from 0.3 to 0.95), but the discussion remains on local identification in the sense that the other parameters are fixed at the SMM estimate. All figures are reported using the same scale for each moment. This facilitates the visual observation of which parameter affects each moment. Last, we do not “smooth out” these comparative statics. They are reported exactly as the computer produces them: This explains why they sometimes appear choppy. They are however, relatively smooth: This comes from our relatively dense grid for capital (about 300 points), debt (29 points) and productivity (51 points, see Appendix A for details).

In Figure C.1, we see that the collateral parameter \( s \) has the biggest impact on mean leverage and the investment and debt sensitivities to real estate prices. This is intuitive. Quite obviously, a higher \( s \) unambiguously leads to bigger leverage: The firm takes on more debt if it is allowed to. The sensitivity moments are non-monotonic. The intuition here is that, when \( s \) is low, the firm is “closer to its constraint”: An increase in the pledgeability of capital leads to an increase in debt capacity, which raises debt and investment. But when \( s \) is high, any increase in \( s \) reduces financial constraints so much that investment and capital structure decision are more determined by value maximization, and less by the fluctuations of real estate prices. Around the SMM estimate (represented by a vertical line), both moments are very smooth and increasing functions of \( s \), suggesting that the optimization algorithm behaves well locally. Looking at log production volatility, we also see that \( s \) unambiguously increases the long-term volatility of production: When the firm is less constrained, it has a bigger ability to adapt its capital stock to productivity shocks. This increases the volatility of sales. In our model, financially constrained firms adapt their capital stock less, and are thus less volatile.

In Figure C.2, we essentially see that the autocorrelation of investment is the moment that “identifies” the adjustment cost parameter \( c \): As \( c \) goes up, firms smooth their investments, which increases the autocorrelation of investment. From the top-left figure, we also see that adjustments costs tend to reduce short-term volatility. This is where adjustment costs have an impact similar to financing constraints: They prevent firms from fully adjusting their capital stock to productivity
shocks and therefore reduces sales volatility. Figures C.4 and C.3 look at the effect of the two parameters governing the log productivity process on the key moments. Clearly, conditional volatility $\sigma$ has a nearly linear impact on short-term volatility – though a much smaller one on long-term volatility. Simultaneously, persistence $\rho$ has a very large impact on long-term volatility, and almost no first-order impact on short-term volatility. Combined together, these two observations are consistent with the idea that the ratio of 1-year to 5-year volatilities allows to identify persistence. One last interesting observation is that persistence has a sizable positive impact on the autocorrelation of investment, which is intuitive: Firms can afford to “take their time” to adapt to productivity shocks, since they are more permanent.

Overall, the comparative statics shown in Figures C.1-C.4 confirm that our model behaves well around the SMM estimate. The moments vary smoothly, and, most of the time, monotonously, with parameters. Such smoothness arises from the granularity of our discretization of the problem (300 grid points for capital, 29 for debt, 51 for productivity) and the fact that we simulate the model on a very large dataset: 1,000,000 firms over 10 years (we discard the first 90 years of data to remove transition dynamics, as explained in the Appendix). While there remains a bit of numerical noise in the sensitivity moments, that noise is relatively minor thanks to this strategy.

Which moments identify which parameter the most? In Table 2, we estimate the matrix of elasticities of moments to parameters, around the SMM (Hennessy and Whited (2007), for instance, implement the same exercise). These elasticities are computed using the simulations exploited in Figures C.1-C.4. For each moment $m_n$, and each parameter $\omega_k$, we compute the following elasticity:

$$
\epsilon_{n,k} = \frac{m_n^+ - m_n^-}{\omega_k^+ - \omega_k^-} \times \frac{\hat{\omega}_k}{\hat{m}_n} \approx \frac{\partial \log(\hat{m}_n)}{\partial \log(\hat{\omega}_k)}
$$

where $\hat{\omega}_k$ is the parameter value at the SMM estimate and $\hat{m}_n$ the corresponding value for moment $n$. $\hat{\omega}_k^+$ is the parameter value located right above on the grid used to plot Figures C.1-C.4. $m_n^+$ is the corresponding moment obtained with this parameter, keeping the other parameters $\hat{\omega}_{k'}$ at the SMM estimate. Similarly, $m_n^-$ is the moment value corresponding to first parameter value $\hat{\omega}_k^-$. 

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below the SMM estimate. We thus obtained matrix of partial elasticities of moments to parameters around the SMM estimate.

Table 2 shows that both average leverage and investment sensitivities have a large elasticity with respect to the collateral parameter $s$. Technically, both moments thus “identify” the collateral parameter well, in the sense that they contribute to reducing the standard error on $s$ the most. The downside of the leverage moment is that it does not generically distinguishes data generated by unconstrained firms from data generated by constrained firms; The sensitivity moment does. The intuition is that, for unconstrained firms, investment is fully insensitive to real estate value – it just depends on investment opportunities – while unconstrained firms may hold positive debt for tax shield purposes. As a result, a model with unconstrained firms can fit the leverage moment, but not the investment sensitivity coefficient.

The other elements of Table 2 confirm our prior discussion. The adjustment costs $c$ is mostly identified by the autocorrelation of investment, in the direction that we expect. It is also identified by the leverage ratio: This is just the outcome of that fact that debt is used to pay for these costs. Hence, a lower leverage will “force” the estimation procedure to set a lower value for adjustment costs. Given the number of “real world” determinants of leverage, this is not a desirable property. Second, productivity volatility $\sigma_z$ is mostly identified by the short- and long-term volatilities of sales, but the financial moments also play role. This is the corporate propensity to save: When risk is higher, firms prefer to hold some debt capacity for risk management purposes (Riddick and Whited, 2009). This reduces leverage, but also the sensitivities of debt and investment to real estate shocks. Last, productivity persistence $\rho$ affects long-term volatility but not short-term one. The persistence coefficient also strongly affects the other moments.

3.5 Estimation results

We report the results of the SMM estimation in Table 3. One key contribution of the paper is to use the sensitivity of investment to real estate value as an identifying moment. To highlight the contribution of this moment, we thus report two sets of results: one set of result where the SMM
targets the mean leverage to identify financing constraints – as the existing literature does – and one set of results where the SMM instead targets the sensitivity moment. Each column correspond to a model specification (with or without adjustment cost) and a set of targeted moments (leverage or investment sensitivity to real estate shocks). The last column corresponds to the data.

We first study the stripped down version of the model without adjustment cost ($c = 0$). There are thus 3 parameters to estimate: the persistence ($\rho$) and variance ($\sigma$) of log productivity, as well as the collateral coefficient $s$. In column 1 of Table 3, the SMM targets “traditional moments”, ie the short- and long-term volatilities of log sales, and the average leverage. The estimation procedure matches all the targeted moments up to the second decimal, but it does poorly on the non targeted ones. The sensitivity of investment and debt to real estate value is high (three times their value from the data: 0.12 instead of 0.04 in both cases). The autocorrelation of investment has the wrong sign, due to the absence of adjustment costs. In column 2, we target investment sensitivity to real estate instead of leverage. To match this moment, the SMM sets the value of $s$ to a much lower level (0.133 instead of 0.495). This is because in this range of parameters, the sensitivity moment is an increasing function of $s$: the less pledgeable real estate is, the less investment reacts to its fluctuations (see Figure C.1). A lower $s$ means a lower debt capacity, so average leverage in this model is very small (almost zero actually), very far off its data value. The autocorrelation of investment turns slightly positive, but still too small.

We introduce adjustment costs in columns 3 and 4. They allow to match the autocorrelation of investment exactly whether we target mean leverage (column 3) or the investment sensitivity coefficient (column 4). The sensitivity moment however leads – like before – to a much smaller collateral coefficient $s$ (0.189 vs 0.422), for the same reason as before. Comparing estimations with (column 4) and without adjustment cost (column 2), we also see that the introduction of adjustment costs increases the estimate of $s$. This happens because adjustment costs reduce the reactivity of investment to real estate shocks, so that $s$ has to increase more to match the data moment. The sensitivity moments (of investment and debt) are both matched perfectly – recall that the debt sensitivity coefficient is not targeted. The leverage ratio is higher than in
column 2 – the firm now has to pay for adjustment costs – but not quite what it is in the data. This is not a big source of concern because in the real world, leverage is affected by many other factors (working capital management, moral hazard etc) that are not in the model. We thus make SMM4 our preferred specification. We do, however, propose a variant of our model designed to simultaneously match the sensitivity moment and the average leverage – see Section 5.3.

The calculation of standard errors is detailed in Appendix A. It is done by bootstrapping. We draw 100 data samples and compute the set of targeted moments for each sample. We then run a SMM procedure for each one of these samples, and compute standard errors as the empirical s.d. of these parameters. To save on computing time, we estimate these 100 SMMs in parallel. Each time we solve the model with a new set of parameters, we check whether these parameters improve the matching each one of the 100 moments. All parameters are estimated with a t-stat between 15 and 100. Such precision is not rare in SMM estimation. The collateral coefficient $s$ is however, much less precisely estimated (with a t-stat slightly above 3). This precision mostly comes from the fact that the data moments are very precisely estimated.

### 3.6 Determinants of financing constraints

In this Section, we briefly discuss how firm characteristics covary with the extent financing constraints. We use our preferred specification of Table 3, Panel A, column 4. We define a firm to be financially constrained when its capital stock is less than 80% of its frictionless capital stock. To compute the frictionless capital stock, we solve the model with the same parameters, except that we remove the no equity issuance constraint. This allows us to compute the capital for each state $(S, X)$. We then simulate a large panel dataset of 1,000,000 firms taken over 10 years (we remove the first 90 years to ensure firms are in steady state). We then take various firm characteristics $x$, sort firms into 20 equal-sized bins of $x$ and compute the fraction of constrained firms in each of these bins. This allows to see how, in the cross-section of firms, financing constraint covary with firm characteristics.

We report the results of this investigation in Figure 1. Panel A shows that more productive
firms are more constrained. This happens because most productive firms are on average former unproductive firms and as a result inherit from their history a small capital stock: This prevents them from growing as much as they would in the absence of collateral constraint (notice that adjustment costs are taken into account in our computation of the unconstrained level of capital). This pattern explains the results of Panels B-E which investigate the relationship between constraints and characteristics that are typically observable in firm data. Panel B shows the effect of size in our model: larger firms tend to be slightly more constrained, but this is because they are typically more productive. This relation is significant but not very sharp because larger firms also have more collateral and as a result are less constrained. As shown in Panel C, the relation is stronger with sales growth: Growing firms are the ones that experienced positive productivity shocks. As a result, they are more likely to start with low collateral but many investment opportunities. Panels D and E show a very strong relation with the ratio of profit or sales to capital. This illustrates that this ratio, which captures the marginal productivity of capital, is a good measure of financing constraints as it captures the effective wedge firms are facing with respect to actual capital cost. Panel F illustrates the non-monotonic relation between constraints and market to book. Low MB firms have few investment opportunities and are thus less constrained. High MB firms have lots of capital but a low productivity and are thus less constrained.

Overall, these comparative static properties are well behaved. The main takeaway of this analysis is that the ratio of sales to capital is a good measure of financing constraints, as put forward in the macro literature (Hsieh and Klenow, 2009).

4 General Equilibrium Analysis

We now have a fully estimated model of firm behavior under financial constraints. To estimate the effect of this model on aggregate production and TFP, we need to plunge it into a simple macro-model that accounts for general equilibrium feedbacks. In this Section, we describe this model and implement the quantitative analysis.
4.1 General equilibrium model

By clearing the goods and labor market equilibria, the model endogenizes aggregate demand $Q$ and the real wage $w$. The model consists of the following simple assumptions.

4.1.1 Firms

A large number $N$ of firms indexed by $i$ produce intermediate inputs, in quantity $q_{it}$, at price $p_{it}$. All intermediate inputs are combined into a CES-composite final good through the following CRS production function:

$$Q_t = \left( \sum_{i=1}^{N} q_{it}^{\frac{\phi-1}{\phi}} \right)^{\frac{\phi}{\phi-1}}$$

(7)

The final good is produced competitively. The demand for input $i$ is thus given by $q_{it} = Q_t \left( \frac{p_{it}}{P_t} \right)^{-\phi}$, with $P_t = \left( \sum_i p_{it}^{1-\phi} \right)^{\frac{1}{1-\phi}}$. We normalize $P_t$ to 1 and derive the demand function in equation (2).

4.1.2 Consumption and consumer behavior

The final good serves as (i) consumption good, (ii) investment good and (iii) to pay for adjustment costs. The final good market equilibrium thus writes:

$$Q_t = C_t + \text{Adj. Cost}_t + I_t$$

(8)

with $C_t$ being aggregate consumption, Adj. Cost$_t = \sum_i \frac{i_{it}^2}{k_{it}}$ is the sum of all adjustment costs, and $I_t = \sum_i i_{it}$ is aggregate investment.

Consumption goes to a representative consumer that maximizes inter-temporal utility over consumption and labor supply:

$$U_s = \sum_{t \geq s} \beta^{t-s} u_t \text{ with } u_t = C_t - \bar{L}^{-\frac{1}{\epsilon}} \frac{L_t^{1+\frac{1}{\epsilon}}}{1 + \frac{1}{\epsilon}}$$

(9)

where $L_t$ are aggregate hours worked, $\bar{L}$ is a simple scaling constant, and $\epsilon$ is the Frisch elasticity.
of labor supply. With quasi-linear preferences, the Hicksian, Marshallian and Frisch elasticities of labor supply are all equal to $\epsilon$. Labor supply is a static decision given by

$$L_t^s = L w_t^\epsilon.$$  \hspace{1cm} (10)

The consumption Euler equation ties the equilibrium interest rate $r_t$ to the discount rate $\beta$, and so we take interest rate $r_t = 1/\beta - 1$ as fixed throughout all counterfactuals.

4.1.3 Steady state assumption and equilibrium definition

We assume that the economy is in steady state. Intermediate good producers produce according to the technology described and estimated in the previous section. The log productivity shocks $z_{it}$ that they face have no aggregate component. Given our assumption that the number of firms is large, aggregate output $Q$ and wage $w$ are constant over time. We are thus exactly in the case studied in the previous Sections.

The equilibrium $(Q, w)$ of this economy is defined by two equations: the labor market equilibrium and the final good aggregator:

$$\bar{L}w^\epsilon = \sum_{i=1}^{N} l^d (w, Q; z_{it}, k_{it} (w, Q))$$  \hspace{1cm} (11)

$$Q = \sum_{i=1}^{N} p_{it} q (w, Q; z_{it}, k_{it} (w, Q))$$  \hspace{1cm} (12)

where $l^d()$ is the numerically obtained labor demand function which is a function of each firm state variable and aggregate equilibrium $(Q, w)$. Similarly $pq()$ is the supply function which, for each firm, associates state variables and macroeconomic conditions to its dollar sales. The equilibrium $(Q, w)$ is the solution of these two conditions. We solve this problem by iteration, using a variant of the Newton-Raphson algorithm. Our approach broadly consists of postulating a given equilibrium $(Q_n, w_n)$, then check if aggregate labor and product supply given these values is above or below
$(Q_n, w_n)$. We then adjust $(Q_{n+1}, w_{n+1})$ accordingly. This approach assumes that there is a unique fixed point and that the contraction mapping theorem applies in our setting. We describe our methodology in detail in Appendix B.

In our quantitative exercise, we focus on the following aggregate quantities. Aggregate output $Q$ and real wage $w$ are direct outcomes of the algorithm. Aggregate employment is given by the supply curve equation: $L = L w^\epsilon$. Aggregate log TFP is classically given by $\log Q - \alpha \log K - (1 - \alpha) \log L$, where $K$, the aggregate capital stock in the steady state, is computed as the sum of capital stocks over all firms. Finally, welfare is a function of $(Q, w)$, the aggregate capital stock $K$ and aggregate adjustment cost

$$U = \frac{1}{1 - \beta} \left( Q - \delta K - \text{Adj. Cost} \frac{L w^{1+\epsilon}}{1 + \frac{1}{\epsilon}} \right).$$

### 4.2 The aggregate effect of financing constraints

We are now in a position to evaluate the aggregate effect of financing constraints. Compared to the firm-level model, the macroeconomic model has a few additional free parameters. Following Chetty (2012), we set the labor elasticity $\epsilon = 0.50$. We adjust $L$ and the number of firms $N$ so that the equilibrium parameter chosen for the estimation process ($Q = 1$ and $w = 0.03$) are actual equilibrium parameters when firm parameters are at the SMM estimate.

To measure the aggregate impact of financing constraints, we present all aggregates (output $Q$, wage $w$, TFP and welfare) in log deviations from the “unconstrained” benchmark. The unconstrained benchmark corresponds to a model where firms have the same parameters, except that we remove the no equity issuance constraint. We label this variant the “unconstrained model”. In this model, firms are still constrained as to how much debt they can raise (they still face the collateral constraint), but they are not constrained in terms of investment. The collateral constraint always binds: As in the constrained model, debt gives firms a tax advantage, but, unlike the constrained model, storing debt capacity in anticipation for future investment needs is not necessary, since equity is freely available and fairly priced at rate $r$. 

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We first ask how the estimation method affects the aggregate effect of financing constraints. We implement this exercise in Table 4. First, we see that estimations targeting the sensitivity moment (columns 2 and 4) generate a TFP loss which is twice as large as estimations targeting the leverage ratio (columns 1 and 3). In our preferred specification (column 4), we find a TFP loss of 2.7%, compared to 1.5% in column . The explanation is simple: When the estimation targets leverage, it fixes the collateral parameter \( s \) so as to explain the entirety of leverage. The procedure attributes all the leverage present in the data to collateralized debt. As a result, targeting leverage leads to a large estimate of \( s (0.42) \). This estimate is larger than actual net leverage (0.31 in our data), because in the model firms seek to keep spare debt capacity and therefore hold less debt than they actually can. On the contrary, the rather low sensitivity of investment to collateral value leads to a small level of pledgeability of capital and \( s = 0.189 \) in column 4. Because the collateral constraint is relatively tighter when targeting the sensitivity moment, loss from financing constraints are bigger. This first result suggests that, at least in our set-up, targeting the “real moments” allows to get a bigger estimate of financing constraints.

Second, we see in column 4 that the output loss with respect to the financially unconstrained equilibrium is as big as 11%. More than half of this loss is accounted for by smaller aggregate capital: \( 0.192 \times 0.3 = 6.5\% \). About a quarter comes from misallocation (TFP is lower by 3%). These two effects combined reduce the productivity of labor which in turn depresses labor supply: This last bit accounts for a small quarter of the overall effect. Hence, even though misallocation is non-negligible, the output loss from financing constraints mostly comes from aggregate underinvestment: Firms are constrained and therefore the consumer undersaves and supplies too little labor. Because all of these effects combined, removing these constraints has a big effect on welfare (9.4%). The welfare effect is two times smaller (5.1%) when one uses the estimate obtained by targeting the leverage moment. Adjustment costs tend to attenuate the cost of financing constraints a little bit (with the two sets of moments). This happens because with adjustment costs, firms already underadjust to productivity shocks, even in the absence of financing constraints. But overall, omitting adjustment costs in our estimation of the effect of
financing constraint does not materially change the outcome of the analysis.

In Figure 2, we plot the GE effects of progressively increasing \( s \) from 0 to 1. Again, we do not smooth the graphs, so that the effect of numerical errors appears very clearly. For the other parameters than \( s \), we use the estimates coming from our preferred specification, obtained by targeting the investment sensitivity moment, and including adjustment costs (as in Table 3, Panel A, column 4). For each value of \( s \), we compute the deviation of each equilibrium quantity w.r.t. the “unconstrained benchmark” which as before correspond to the general equilibrium of the economy where firms follow the same model except that firms have no equity issuance constraint. Finally, to highlight the precision of our aggregate estimates, we represent, via a light blue bar, the parameter region within 2 standard errors of our main estimate of \( s \). Since it is relatively precise (a standard error of .008 for a point estimate of 0.189), this region is relatively narrow which means that our aggregate effects are precisely estimated: The margin is 0.5 percentage point for TFP, about 2 ppt for output, and up to 5 ppt for capital. In all cases, 0 is very far from the range of estimated effects.

In Figure 2, the loss from financing constraints of each aggregate, for \( s = 0.189 \), is by definition the same as the one reported in Table 4. Clearly, when \( s = 1 \) (100% of the capital stock can be pledged to lenders), there is no significant difference between the constrained and the unconstrained equilibrium: When capital is highly pledgeable, firms have no problem investing optimally. Consistently with the previous discussion, we also see that for a given increase in \( s \), log capital is the variable that reacts the most. Employment reacts much less due to the small elasticity of labor supply that we use. Finally, we also report the cross-sectional dispersion of the log MRPK \((\log p q / k_i)\), which is a measure of distortions in Hsieh and Klenow (2009). This object does not have to be zero even in the absence of financial frictions due to the presence of adjustment costs and time-to-build in investment Asker et al. (2014). To account for this, like for the other aggregates, we report the s.d. of log MRPK in excess to what it would be in the absence of financing constraint. From the chart, it is clear that, as financing constraints become less binding \((s \to 1)\), the dispersion in capital productivities decreases.
4.3 Productivity persistence and misallocation

In this Section, we analyze the effect of productivity persistence on misallocation of inputs across firms. Some recent papers on financing frictions emphasize the idea that the persistence of productivity should reduce financing frictions (Moll (2014), Buera et al. (2011)). These paper start from the intuition that, is productivity shocks are very persistent, firms have enough time to “grow out” of financing constraints: currently productive firms have been so for long enough to have the cash to finance their investment. We check whether this intuition holds in our model. To do this, we vary $\rho$, and compute misallocation as the standard deviation of log MRPK ($\log \frac{p_k}{k}$) at equilibrium, in the spirit of Hsieh and Klenow (2009) and Midrigan and Xu (2014). When this dispersion is large, marginal productivities of capital are very different from one another, indicating significant misallocation (since they should be equal in the absence of financial and adjustment frictions).

In Figure 3, we show that the s.d. of log MRPK is actually increasing with productivity persistence, indicating more misallocation, not less. The intuition is that, in our model with adjustment frictions, a higher $\rho$ leads to a bigger need to adjust capital: When productivity increases, the increase is expected to last longer, which justifies a larger adjustment. But in this case, the collateral constraint binds more, which pushes firms away from having similar capital productivities. This makes misallocation increase with productivity persistence in our model.

5 Discussion

5.1 Policy Experiments

In this Section, we use our model to investigate the effect of an investment tax credit. More specifically, we assume that each firm in our sample receives a subsidy equal to $x i_{it}$, where $i_{it}$ is the firm’s investment and $x$ is a fraction equal to 5, 10 and 15%. Implicitly, we assume that this subsidy is a linear function of investment, i.e. that it becomes a tax as soon as investment is negative. This feature avoids the emergence of short capital cycles where firms buy capital to
enjoy the subsidy, and sells it the next year. Finally, this subsidy is financed via a lump-sum tax raised on household income. We make this assumption in order to focus on the effect of the ITC.

We report the results of this policy experiment in Table 7. We find that the effect of the investment tax credit to be very large. With a tax credit of 5%, we obtain an increase in the capital stock of 11%, an increase in aggregate employment of about 1.4% and as a result rise in output of 4.3%. TFP is unaffected – this is true for all levels of subsidy. This happens because the investment tax credit is untargeted: it creates as much overinvestment (many firms in our sample are unconstrained) as it reduces underinvestment. Even though misallocation is not improved due to the policy, welfare increases. In our model, this happens because corporate profits are taxed at a high rate (30%) which depresses investment significantly. The ITC partially undoes the depressing effect of the corporate profit tax, and hence enhances welfare.

5.2 Model Identification

This section shows that targeting the investment sensitivity moment dominates the leverage moment when we try to “identify” financial constraints in the data. We do this by simulating data without financing constraints, and show that the sensitivity moment allows to reject a model with financing constraints, while the leverage moment does not.

More precisely, we implement the following experiment. We start from the model with parameters of our preferred specification (Table 3, Panel A, column 4). We remove the no equity issuance constraint and simulate a dataset of firms. These firms are by definition unconstrained. Assume now we compute the following moments on this dataset: long- and short-term volatility of log sales, autocorrelation of investment and investment sensitivity to real estate value. These moments are reported in column 3 of Table 5. The sensitivity moment is unsurprisingly equal to 0 (-0.001 in the Table): the investment policy is optimal, and is thus unaffected by real estate shocks since they are uncorrelated with productivity shocks. As a result, an econometrician that would seek to fit a model with no equity issuance constraint would find it impossible to match this moment unless $s = 1$, i.e. firms have so much debt capacity that they are unconstrained.
An econometrician targeting this moment would thus rightly conclude that the firm is, in fact, unconstrained. The model would be wrong (in fact there is no issuance constraint but $s < 1$) but the aggregate effect of financing constraints would be correctly inferred to be tiny (as shown in Figure 2, there is no loss from financing constraints when $s = 1$).

Let us now take the perspective of an econometrician who estimates a model with financing constraints, but now targets mean leverage instead of the sensitivity moment. The result of this exercise is shown in Table 5, column 2. The outcome of the estimation is a relatively high level of collateral $s = 0.435$ compared to our baseline preferred estimate (shown in column 1) but also a more persistent productivity shocks ($\rho = .943$). This leads to big aggregate effects of collateral constraints: a TFP loss of 2.8% and an output loss of 12.9%. Thus, an estimation of the constrained model targeting the leverage moments entirely misses the fact that firms are not constrained in these data. The estimation procedure attributes the fact that firms take on debt to financing needs, while in fact this is purely done for tax optimization purposes. Obviously, the estimation fails to match the investment sensitivity moment which is equal to .320 while, in the (simulated) data the moment is equal to 0.

### 5.3 Robustness: Accounting for residual leverage

This section discusses the robustness of our conclusion to a setting where firms have spare debt capacity in addition to the collateralized debt that is the focus of our study. A potential concern with our baseline specification is that it fails to match the mean leverage ratio (see Table 3, Panel A, column 4). The tension is between the investment sensitivity moment and the debt ratio. If $s$ is such that the leverage ratio (0.313 in our data) is matched, $s$ has to be large, which leads to a counterfactually large investment to real estate sensitivity. If however one matches the investment moment (equal to 0.04 in our data), $s$ has to be small (0.189) and leverage has to be even smaller. So far, our argument for matching the investment sensitivity moment is that it is more specific to the presence of financing constraint (see Section 5.2). Leverage is determined by a host of firm characteristics (unsecured debt capacity, trade credit, inventories etc) that are not in the model.
But in this case the danger is that, when we account for these other sources of external funding, firms may have enough debt capacity to avoid financing constraints.

In this Section we show that this is not the case. We modify the baseline model by adding some debt capacity \( s_0 \) to the borrowing constraint. This coefficient \( s_0 \) captures debt capacity that is not collateralized and therefore left out of the model.

\[
(1 + r)d_{it+1} \leq \bar{d} + s ((1 - \delta)k_{it} + p_t h)
\]  

We estimate this new model and report the results in Table 6. We target both leverage and investment sensitivity moments, in addition to the other moments (short- and long-term volatilities and autocorrelation of investment). We find that we are now able to target both leverage and investment sensitivity. \( \bar{d} \) is very high (0.45) and \( s \) is very close to our previous estimate (0.254 instead of 0.189). The productivity shock process remains pretty similar. Interestingly, however, the aggregate impact of financing constraints is not smaller than in the model which does not fit leverage (i.e. where \( \bar{d} = 0 \)). Firms have a higher debt capacity, but this higher debt capacity is fixed, it is like free cash (i.e. cash that is not penalized in terms of returns). As a result, firms lever up more (this is why we can now match the leverage ratio) in order to minimize taxes. The overall borrowing constraint does not bind less because the extra debt capacity is used for tax optimization. Overall, a version of our model that matches both leverage and the investment sensitivity to real estate value does feature very similar aggregate effect of financing constraints. Extra, uncollateralized, debt does not make firms less financially constrained while allowing us to match the empirical leverage ratio of the data.

**Conclusion**

This paper provides a quantification of the aggregate effects of a specific source of financing frictions, collateral constraints. We build a simple dynamic general equilibrium model with heterogeneous firms and collateral constraints. To estimate this model structurally, we match not
only key features of firm level dynamics, but also well identified reduced form evidence that an increase in the value of a firm’s collateral leads to an increase in investment. The estimated model is then used to simulate a counterfactual economy where financing frictions are lifted. Welfare increases by 9.4% and aggregate output by 11%. Quantitatively, only one quarter of these gains can be attributed to a more efficient allocation of inputs across heterogeneous firms – more productive firms are able to obtain more financing and expand – while half of these gains are due to a higher aggregate stock of capital, and the remaining quarter to a lower aggregate labor supply. So financing frictions cause substantially less misallocation than underinvestment.

One limitation of this analysis is that the shocks to collateral value that we use to identify the effect of collateral constraints at the firm-level are exogenous in the model. Yet in equilibrium, increased investment and hiring at the local level will clearly feed back into local real estate prices. In addition, since households are not fully mobile across regions, variations in real estate prices will induce variations in wages faced by firms, which will affect their local input choices. Endogenizing the housing market and incorporating it into our quantitative analysis is an important step that we plan to tackle in future research.
References


Figures

Figure 1: Financing constraints as a function of firm characteristics

Note: This Figure shows how the extent of financing constraints covaries with firm characteristics, in the cross-section of simulated firms. We simulate a dataset of 1,000,000 firms over 10 years using parameters from our preferred specification (Table 3, Panel A, column 4). We remove the first 90 years to make sure firms are in steady state. For each characteristic $x$, we then sort firms into 20 quantiles of $x$, and for each quantile compute the average fraction of constrained firms in our simulated data. We label a firm constrained if its capital stock is less than 80% of its unconstrained capital stock. Unconstrained capital stock is computed after solving the same model, with the same parameters but without the no equity issuance constraint. The conditioning variable $x$ is given by $z$ (Panel A), $\log k$ (Panel B), $\log p_{qt} - \log p_{qt-1}$ (Panel C), $\frac{pl_{-wt}}{k}$ (Panel D), $\frac{pl}{k}$ (Panel E), and $\frac{V}{k}$ (Panel F).
Figure 2: General equilibrium effect of pledgeability $s$

Note: This figure reports the general equilibrium effects of changing the collateral parameter $s$ from 0 (full financial constraints) to 1 (100% of the capital stock can be pledged to lenders). We use the model with adjustment costs and estimated targeting the investment sensitivity moment (thus using the parameters reported in Table 3, Panel A, column 4). All aggregates are represented in deviation with respect to the unconstrained benchmark: For each value of $s$, we compute the general equilibrium of the economy populated with constrained firms, and also the GE of the economy populated by firms with the same parameters, but without the no equity issuance constraint. We then compute the log difference of output, welfare, employment, capital stock, TFP and the difference in the s.d. of log sales to capital ratio (MRPK). We then try all values of $s$ from 0 to 1, spaced by .1. The vertical redline correspond to the SMM estimate of $s$ (.189). Reading: When $s$ increases from .1 to .6, the loss of log capital stock w.r.t. the unconstrained benchmark goes from -.2 to -.1.
Figure 3: Productivity persistence $\rho$ and the dispersion of capital productivity

Note: This figure reports the effect on capital misallocation of changing the log productivity persistence $\rho$ from 0.35 (low persistence) to .95 (high persistence). We use the model with adjustment costs and estimated targeting the investment sensitivity moment (thus using the parameters reported in Table 3, Panel A, column 4). In the spirit of Hsieh and Klenow (2009), we measure misallocation as the s.d. of log sales to capital ratio (MRPK).
### Tables

Table 1: Summary statistics: COMPUSTAT Extract

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>s.d.</th>
<th>Obs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment$<em>{it}/k</em>{it-1}$</td>
<td>.37</td>
<td>.42</td>
<td>20,074</td>
</tr>
<tr>
<td>Net borrowing / $k_{it-1}$</td>
<td>.05</td>
<td>.48</td>
<td>19,998</td>
</tr>
<tr>
<td>Real estate value$_{it}$</td>
<td>.77</td>
<td>1.27</td>
<td>20,074</td>
</tr>
<tr>
<td>$\frac{1}{k_{it}}$</td>
<td>.42</td>
<td>.65</td>
<td>20,074</td>
</tr>
<tr>
<td>Office price</td>
<td>.67</td>
<td>.21</td>
<td>20,074</td>
</tr>
</tbody>
</table>

Source: COMPUSTAT for accounting items and Global RealAnalytics for office prices. The construction of this data is described in detail in Chaney et al. (2012). The dataset is an extract of COMPUSTAT. It contains all firms present in 1993 who report accounting value and cumulative depreciation of land and buildings. These firms are then followed until they exit the sample or until 2006. We also require that office price data are available in the city where these firms have their headquarter in 1993. The variables shown are used in the two regression presented in Section 1.
Table 2: Elasticity of Moments with respect to Parameters

<table>
<thead>
<tr>
<th></th>
<th>s.d. $\Delta \log q$</th>
<th>s.d. $\Delta_5 \log q$</th>
<th>$\frac{d_i}{k_i}$</th>
<th>$\beta(\text{Inv, RE})$</th>
<th>$\text{corr}(\frac{i_t}{k_{t-1}}, \frac{i_{t+1}}{k_t})$</th>
<th>$\beta(\text{Debt, RE})$</th>
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</thead>
<tbody>
<tr>
<td>Pledgeability $s$</td>
<td>.077</td>
<td>.16</td>
<td>1.3</td>
<td>-1.3</td>
<td>-0.44</td>
<td>-.99</td>
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<tr>
<td>Adjustment cost $c$</td>
<td>-.041</td>
<td>-.063</td>
<td>.34</td>
<td>.014</td>
<td>.42</td>
<td>.011</td>
</tr>
<tr>
<td>Volatility $\sigma$</td>
<td>.97</td>
<td>.92</td>
<td>-1.4</td>
<td>-.48</td>
<td>-.15</td>
<td>-.76</td>
</tr>
<tr>
<td>Persistence $\rho$</td>
<td>.081</td>
<td>1</td>
<td>-2.2</td>
<td>-.99</td>
<td>2</td>
<td>-2</td>
</tr>
</tbody>
</table>

Note: This table reports the elasticity of various moments with respect to the structural parameters that we estimate. First, we start with the SMM estimate $\hat{\Omega}$ of the parameters $\Omega$. For each $k = 1, \ldots, 4$, we set $\omega_l = \hat{\omega}_l$ for all $l \neq k$, and vary the parameter $\omega_k$ around the estimated $\hat{\omega}_k$ in order to compute the elasticity of moments to parameters in the vicinity of the SMM estimate. For each moment $m_n$, we compute:

$$
\epsilon_{n,k} = \frac{m_n^+ - m_n^-}{\omega_k^+ - \omega_k^-} \times \frac{\hat{\omega}_k}{m_n}
$$

where $\hat{m}_n$ is the $n^{th}$ data moment. $m_n^+$ is the moment based on data simulated with parameter $\omega_k^+$. Likewise, $m_n^-$ is the average of moments based on data simulated with parameters $\omega_k^-$. $\omega_k^+$ and $\omega_k^-$ are parameter values right above and right below the SMM estimate $\hat{\omega}_k$, when the interval of definition of $\omega$ is graded on a scale going from 0 to 10 as in Figures C.1-C.4. Reading: Around the SMM estimate, a 1% increase in $s$ is associated with a 1% increase in the sensitivity of investment to real estate and a 1.5% increase in leverage.
Table 3: **Main results**

<table>
<thead>
<tr>
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<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
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<td>No adj cost,</td>
<td>Adj cost,</td>
<td>Adj cost,</td>
<td>Data</td>
</tr>
<tr>
<td></td>
<td>Lev. target</td>
<td>Inv. target</td>
<td>Lev. target</td>
<td>Inv. target</td>
<td></td>
</tr>
<tr>
<td>(\rho)</td>
<td>0.917</td>
<td>0.919</td>
<td>0.865</td>
<td>0.895</td>
<td></td>
</tr>
<tr>
<td>(\sigma)</td>
<td>0.623</td>
<td>0.725</td>
<td>0.818</td>
<td>0.820</td>
<td></td>
</tr>
<tr>
<td>(s)</td>
<td>0.495</td>
<td>0.133</td>
<td>0.422</td>
<td>0.189</td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td>0.050</td>
<td>0.045</td>
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**Panel A: Estimated Parameters**

<table>
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<tr>
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<th>(4)</th>
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<td>Adj cost,</td>
<td>Adj cost,</td>
<td>Data</td>
</tr>
<tr>
<td></td>
<td>Lev. target</td>
<td>Inv. target</td>
<td>Lev. target</td>
<td>Inv. target</td>
<td></td>
</tr>
<tr>
<td>(\rho)</td>
<td>0.917</td>
<td>0.919</td>
<td>0.865</td>
<td>0.895</td>
<td></td>
</tr>
<tr>
<td>(\sigma)</td>
<td>0.623</td>
<td>0.725</td>
<td>0.818</td>
<td>0.820</td>
<td></td>
</tr>
<tr>
<td>(s)</td>
<td>0.495</td>
<td>0.133</td>
<td>0.422</td>
<td>0.189</td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td>0.050</td>
<td>0.045</td>
<td></td>
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</table>

**Panel B: Moments (targeted in **bold**)**

<table>
<thead>
<tr>
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<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
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<td>No adj cost,</td>
<td>Adj cost,</td>
<td>Adj cost,</td>
<td>Data</td>
</tr>
<tr>
<td></td>
<td>Lev. target</td>
<td>Inv. target</td>
<td>Lev. target</td>
<td>Inv. target</td>
<td></td>
</tr>
<tr>
<td>Std of 1-year sales growth</td>
<td>0.327</td>
<td>0.327</td>
<td>0.327</td>
<td>0.327</td>
<td>0.327</td>
</tr>
<tr>
<td>Std of 5-year sales growth</td>
<td>0.909</td>
<td>0.910</td>
<td>0.910</td>
<td>0.911</td>
<td>0.911</td>
</tr>
<tr>
<td>Real-Estate to assets</td>
<td>0.140</td>
<td>0.140</td>
<td>0.140</td>
<td>0.140</td>
<td>0.140</td>
</tr>
<tr>
<td>Net debt to assets</td>
<td>0.300</td>
<td>0.013</td>
<td>0.315</td>
<td>0.095</td>
<td>0.313</td>
</tr>
<tr>
<td>(\beta(Inv, RE))</td>
<td>0.126</td>
<td>0.038</td>
<td>0.082</td>
<td>0.040</td>
<td>0.040</td>
</tr>
<tr>
<td>Autocorrelation of investment</td>
<td>-0.057</td>
<td>0.064</td>
<td>0.436</td>
<td>0.436</td>
<td>0.436</td>
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<tr>
<td>(\beta(Debt, RE))</td>
<td>0.124</td>
<td>0.037</td>
<td>0.084</td>
<td>0.038</td>
<td>0.039</td>
</tr>
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</table>

This table reports the results of our SMM estimations. The estimation procedure is described in the text and in Appendix A. Columns (1)-(4) correspond to SMMs using different sets of parameters and targeting different sets of moments. Columns 1 and 2 assume not adjustment cost \((c = 0)\), while columns (3) and (4) allow for them. Estimations reported in columns (1) and (3) target “classic” moments: short- and long-term volatilities of log sales, mean leverage, and the autocorrelation of investment in column (3). By contrast, columns (2) and (4) target the sensitivity of investment to real estate value instead of mean leverage. For each one of these estimations, panel A shows the estimated parameters, along with standard errors (obtained via bootstrapping 100 times) in brackets. Panel B shows the value of a set of moments, measured on simulated data (with 1,000,000 observations). Moments in bold are the ones that are targeted by the estimation. The other moments are not targeted. The last column (labeled “data”) features the moments from the data.
Table 4: **Aggregate effects of collateral constraints as a function of targeted moments**

<table>
<thead>
<tr>
<th></th>
<th>(1) No adj cost, Lev. target</th>
<th>(2) No adj cost, Inv. target</th>
<th>(3) Adj cost, Lev. target</th>
<th>(4) Adj cost, Inv. target</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Targeted moments</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std of 1-year sales growth</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Std of 5-year sales growth</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Real-Estate to assets</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Net debt to assets</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>$\beta(Inv, RE)$</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>Autocorrelation of investment</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td><strong>Panel B: Loss from financial constraint in general equilibrium</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log(TFP)</td>
<td>0.015</td>
<td>0.034</td>
<td>0.015</td>
<td>0.027</td>
</tr>
<tr>
<td>log(output)</td>
<td>0.081</td>
<td>0.160</td>
<td>0.061</td>
<td>0.110</td>
</tr>
<tr>
<td>log(wage)</td>
<td>0.054</td>
<td>0.106</td>
<td>0.040</td>
<td>0.073</td>
</tr>
<tr>
<td>log(L)</td>
<td>0.027</td>
<td>0.053</td>
<td>0.020</td>
<td>0.036</td>
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<tr>
<td>log(K)</td>
<td>0.157</td>
<td>0.296</td>
<td>0.107</td>
<td>0.192</td>
</tr>
<tr>
<td>log(welfare)</td>
<td>0.063</td>
<td>0.131</td>
<td>0.051</td>
<td>0.094</td>
</tr>
</tbody>
</table>

This table reports the results of the general equilibrium analysis for different SMM estimates shown in Table 3. The general equilibrium analysis is detailed in Appendix B and in the main text. Columns (1)-(4) correspond to SMMs using different sets of parameters and targeting different sets of moments. Columns 1 and 2 assume not adjustment cost ($c = 0$), while columns (3) and (4) allow for them. Estimations reported in columns (1) and (3) target “classic” moments: short- and long-term volatilities of log sales, mean leverage, and the autocorrelation of investment in column (3). By contrast, columns (2) and (4) target the sensitivity of investment to real estate value instead of mean leverage. For each one of these estimations, panel A simply recalls the targeted moments. Panel B reports the result of the GE analysis. All results are shown as log deviations with respect to the unconstrained benchmark. The unconstrained benchmark correspond to an equilibrium where firms face the same set of parameters as in the SMM estimate – as reported in the same column, Table 3, panel A – but no constraint on equity issuance. Thus, even though in this benchmark economy firms are subject to the borrowing constraint, they are financially unconstrained in that their investment can reach the first best. Hence, in the first SMM (targeted leverage, no adjustment cost), the aggregate TFP loss compared to a case without financing constraint is $e^{0.015} \approx 1.5\%$. 

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Table 5: Estimating a constrained model on unconstrained data

<table>
<thead>
<tr>
<th></th>
<th>Baseline Result</th>
<th>Estimation on unconstrained model data</th>
<th>Unc. model data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Estimated Parameters</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.895</td>
<td>0.943</td>
<td></td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0.820</td>
<td>0.810</td>
<td></td>
</tr>
<tr>
<td>( s )</td>
<td>0.189</td>
<td>0.435</td>
<td></td>
</tr>
<tr>
<td>( c )</td>
<td>0.045</td>
<td>0.023</td>
<td></td>
</tr>
</tbody>
</table>

**Panel B: Moments (matched in bold fonts)**

<table>
<thead>
<tr>
<th></th>
<th>Baseline Result</th>
<th>Estimation on unconstrained model data</th>
<th>Unc. model data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std of 1-year sales growth</td>
<td>0.327</td>
<td>0.360</td>
<td>0.367</td>
</tr>
<tr>
<td>Std of 5-year sales growth</td>
<td>0.911</td>
<td>1.116</td>
<td>1.171</td>
</tr>
<tr>
<td>Real-Estate to assets</td>
<td>0.140</td>
<td>0.152</td>
<td>0.152</td>
</tr>
<tr>
<td>Net debt to assets</td>
<td>0.095</td>
<td>0.168</td>
<td>0.168</td>
</tr>
<tr>
<td>( \beta(\text{Inv, RE}) )</td>
<td>0.040</td>
<td>0.320</td>
<td>-0.001</td>
</tr>
<tr>
<td>Autocorrelation of investment</td>
<td>0.436</td>
<td>0.307</td>
<td>0.426</td>
</tr>
<tr>
<td>( \beta(\text{Debt, RE}) )</td>
<td>0.038</td>
<td>0.307</td>
<td>0.069</td>
</tr>
</tbody>
</table>

**Panel C: Loss from financial constraint in general equilibrium**

<table>
<thead>
<tr>
<th></th>
<th>Baseline Result</th>
<th>Estimation on unconstrained model data</th>
<th>Unc. model data</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \log(\text{TFP}) )</td>
<td>0.027</td>
<td>0.028</td>
<td></td>
</tr>
<tr>
<td>( \log(\text{output}) )</td>
<td>0.110</td>
<td>0.129</td>
<td></td>
</tr>
<tr>
<td>( \log(L) )</td>
<td>0.073</td>
<td>0.043</td>
<td></td>
</tr>
<tr>
<td>( \log(\text{welfare}) )</td>
<td>0.094</td>
<td>0.106</td>
<td></td>
</tr>
</tbody>
</table>

This table reports the result of our SMM estimation on a dataset simulated by a model without financing friction. We start with the model of Table 3, column 4. We remove the no equity constraint, and simulated a panel dataset of firms. We then compute various moments and report them in column 3. We then estimate a model with no equity issuance constraint that matches four targeted moments: mean leverage, long- and short ter log sales volatility, and the autocorrelation of investment (along with the ratio of real estate value to assets). We report the results of this investigation in Column 2. Panel A reports the estimated parameters, Panel B the moments (targeted and non-targeted, to be compared with the simulated data moments in column 3), and Panel C computes the implied GE losses from financial constraints by comparing output, TFP, labor and welfare with a model with the same parameters but no equity issuance constraints. Column 1 recalls the baseline results (Tables 3-4, column 4) for comparison.
Table 6: Robustness to additional unmodelled debt capacity

<table>
<thead>
<tr>
<th></th>
<th>(1) Baseline</th>
<th>(2) Uncollateralized Debt Capacity</th>
<th>(3) Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Estimated Parameters</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.895</td>
<td>0.877</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.041)</td>
<td></td>
<td></td>
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<tr>
<td>$\sigma$</td>
<td>0.820</td>
<td>0.821</td>
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</tr>
<tr>
<td></td>
<td>(.116)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$s$</td>
<td>0.189</td>
<td>0.254</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.059)</td>
<td></td>
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<td>$\bar{d}$</td>
<td>-</td>
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<td></td>
</tr>
<tr>
<td>$c$</td>
<td>0.045</td>
<td>0.052</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.004)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Panel B: Targeted moments</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std of 1-year sales growth</td>
<td>0.327</td>
<td>0.327</td>
<td>0.327</td>
</tr>
<tr>
<td>Std of 5-year sales growth</td>
<td>0.911</td>
<td>0.906</td>
<td>0.911</td>
</tr>
<tr>
<td>Real-Estate to assets</td>
<td>0.140</td>
<td>0.140</td>
<td>0.140</td>
</tr>
<tr>
<td>Net debt to assets</td>
<td>0.095</td>
<td>0.300</td>
<td>0.313</td>
</tr>
<tr>
<td>$\beta(Inv,RE)$</td>
<td>0.040</td>
<td>0.040</td>
<td>0.040</td>
</tr>
<tr>
<td>Autocorrelation of investment</td>
<td>0.436</td>
<td>0.439</td>
<td>0.436</td>
</tr>
<tr>
<td>$\beta(Debt,RE)$</td>
<td>0.038</td>
<td>0.039</td>
<td>0.039</td>
</tr>
<tr>
<td>Panel C: Loss from financial constraint in general equilibrium</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log(TFP)</td>
<td>0.027</td>
<td>0.027</td>
<td></td>
</tr>
<tr>
<td>log(output)</td>
<td>0.110</td>
<td>0.122</td>
<td></td>
</tr>
<tr>
<td>log(wage)</td>
<td>0.073</td>
<td>0.081</td>
<td></td>
</tr>
<tr>
<td>log(welfare)</td>
<td>0.094</td>
<td>0.100</td>
<td></td>
</tr>
</tbody>
</table>

This table reports the estimation result of a model identical to our main specification except that the borrowing constraint contains a fixed parameter:

$$(1 + r)d_{t+1} \leq \bar{d} + s ((1 - \delta)k_t + p_t h).$$

The estimation procedure is described in the text and in Appendix A. The general equilibrium analysis is detailed in Appendix B and in the main text. We target the following moments: short- and long-term volatilities of log sales, the autocorrelation of investment and both mean leverage and investment sensitivity to real estate value. Panel A shows the estimated parameters, along with standard errors in brackets (obtained via bootstrapping 100 times). Panel B shows the value of a set of moments, measured on simulated data (with 1,000,000 observations). Moments in bold are the ones that are targeted by the estimation. Panel C reports the result of the GE analysis. All results are shown as % losses with respect to the unconstrained benchmark. The unconstrained benchmark correspond to an equilibrium where firms face the same set of parameters as in the SMM estimate – as reported in Panel A – but no constraint on equity issuance. Column 1 just reports the results of our baseline preferred estimation for comparison. Column 2 reports the estimate of the model with fixed debt capacity targeting the 5 moments. Column 3 reports the data moments.
Table 7: Macro effect of an investment subsidy

<table>
<thead>
<tr>
<th>Subsidy (share of investment)</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate subsidy (% of output)</td>
<td>.007</td>
<td>.015</td>
<td>.024</td>
</tr>
<tr>
<td>$\Delta \log$ Output</td>
<td>.043</td>
<td>.089</td>
<td>.14</td>
</tr>
<tr>
<td>$\Delta \log$ Capital</td>
<td>.11</td>
<td>.23</td>
<td>.36</td>
</tr>
<tr>
<td>$\Delta \log$ Labor</td>
<td>.014</td>
<td>.03</td>
<td>.046</td>
</tr>
<tr>
<td>$\Delta \log$ TFP</td>
<td>0</td>
<td>0</td>
<td>-.001</td>
</tr>
<tr>
<td>$\Delta \log$ Welfare</td>
<td>.029</td>
<td>.059</td>
<td>.089</td>
</tr>
</tbody>
</table>

This Table reports the aggregate equilibrium impact of tax subsidies. We start with the model and parameter estimates of Table 3, column 4. To cash flows of firm $i$ at date $t$, we add a tax free subsidy equal to $x I_{it}$ where $I_{it}$ is the investment of firm $i$ at date $t$ and $x$ is a fraction equal to 5, 10 and 15%. Note that this subsidy becomes a tax when the firm’s investment becomes negative. This subsidy is assumed to be levied from household in a non-distorsionary manner. For each one of these three policies, we compute the equilibrium and report the change in log aggregates compared to the case without subsidy. For instance, we find that giving firms a subsidy equal to 5% of their investment leads to an increase in aggregate output of 4.3%.
APPENDIX

This Appendix contains: the method used to solve and estimate the model (Section A), the method we use to compute the general equilibrium of our model (Section B) and the additional comparative static results in partial equilibrium designed to show that the model is well behaved around the estimate (Section C).
A Solving the model and Estimation

This Appendix details the algorithms used to solve the model and estimate it. To estimate the model, one needs to find the set of parameters such that model-generated moments fit a predetermined set of data moments. Because our model does not have an analytic solution, we need to use indirect inference to perform the estimation. Such inference is done in two steps:

- For a given set of parameters, we need to solve the model numerically, which means solving the Bellman problem (6) and obtain the policy function $S_{t+1} = (D_{t+1}, K_{t+1})$ as a function of $S_t = (D_t, K_t)$ and exogenous variables $X_t = (z_t, p_t)$.

- We then use this resolution technique to estimate the parameters that match best a set of moments chosen from the data. We explain the methodology (Simulated Method of Moments) and the numerical algorithm that we use to implement it.

A.1 Solving the model numerically

In this section, we describe how we numerically solve the firm’s problem with given parameters.

A.1.1 Grid definition

In order to solve the model numerically, we need to discretize the state space $(S; X)$. Let us start with the two exogenous variables. The log productivity process $z$ is discretized using the standard Tauchen method on 51 grid points. Log real estate prices are also an AR1, discretized using the Tauchen method on grid 11 points. For both variables, we set the bounds of the grid at -2.5 and 2.5 standard deviations.

Capital choice is discretized over a range going from $K_{min}$ $K_{max}$. $K_{min}$ is the smallest level of capital chosen by a firm without adjustment costs and financing constraint. For this particular case, we can solve the capital decision analytically. In most cases, this number should serve as a lower bound because adjustment costs would prevent firms from adjusting all the way down to this level; and financial constraints would push them to keep more capital as precautionary savings. Since we did not, however, establish this result analytically, we check that $K_{min}$ is always “far enough” from the lowest simulated value of capital. Similarly, $K_{max}$ is the capital stock chosen by a non-constrained firm, without adjustment cost, facing the highest productivity level in the grid. Again, we expect this level to be above the upper bound of capital for a constrained firm with adjustment costs. We check that this is the case in our estimates. We then form an equally spaced grid for log capital between log $K_{min}$ and log $K_{max}$, with increment of log$(1 + \delta/2)$. Thus, the capital grid is geometrically spaced using $(1 + \delta/2)$ as the multiplying coefficient, i.e. the $k^{th}$ point is equal to $K_{min} \times (1 + \delta/2)^k$ until $K_{max}$. Given that $K_{min}$ and $K_{max}$ are functions of productivity, the grid thus depends on the persistence $\rho$ and volatility $\sigma$ of log productivity. Bigger persistence or volatility leads to bigger grid. In our preferred specification, grid size has 270 points. We will take this as a reference when we later discuss grid size, bearing in mind that, in fact, the capital grid is a function of parameter values.

Finally, the grid of debt $D_t$ is defined as a function of the amount of capital $K_t$. This adaptive feature of the debt grid comes from the fact that the amount of debt is bounded above by a
function of capital: Larger firms can borrow more. We restrict future period debt $D'$ to the $[-4\bar{D}; \bar{D}]$ interval, where $\bar{D} = s((1 - \delta)K' + E(\rho'|p_{\text{max}})H)$ and $p_{\text{max}}$ is the maximum house price level. The grid interval is thus a function of the model parameters $s$ but also $\rho$ and $\sigma$ via the grid of $K'$. The upper bound is a natural consequence of the collateral constraint: The model imposes that it cannot be exceeded. The lower bound is somewhat arbitrary as their is in theory no upper bound as to how much cash the firm may decide to hold. We check that there is no accumulation of cash at this bound during the estimation process. Within this interval, the grid is geometrically spaced so that it is denser closer to the constraint, i.e. right below $\bar{D}$. We implement this by setting the $k^{\text{th}}$ grid point at $\bar{D}(1 - 0.001 \times e^{3k})$ until it reaches $-4\bar{D}$. Thus, the grid size of debt does not depend on parameters (in constrast to the capital grid size) and always has 29 points.

### A.1.2 Bellman resolution algorithm

We solve the firm’s problem using policy iteration. This algorithm is based on the fact that the value function is the solution of a fixed point problem generated by a contraction mapping.

Before starting to iterate, we compute profit flows $e(S, S'; X)$ using the production and cost functions, for all possible values of $S$ and $X$ on the grid. We set $e$ to “missing” when $(S, S'; X)$ are such that $e < 0$ – the no equity issuance constraint is violated, or when the borrowing constraint is violated. So profits are only defined when both financing constraints are satisfied.

Then, in order to initiate the process, we start with the value function $V_0(S; X) = 1$. We then look for the policy function $(K'_0, D'_0) = P_0(S; X)$ which solves:

$$
P_0(S; X) = \text{argmax}_{S'}\{e(S, S'; X) + \frac{1}{1 + r}\}
$$

for each state of $(S; X)$. Then, we iterate the following loop (where $n \geq 1$ is the number of the loop):

1. Start from $(K'_{n-1}, D'_{n-1}) = P_{n-1}(S; X)$, the policy function obtained from the previous round; and $V_{n-1}(S; X)$, the value function obtained from the previous round. For every point $(S; X)$ on the grid, we compute the value function $V_n$ that satisfies:

$$
V_n(S; X) = e(S, P_{n-1}(S; X); X) + \frac{1 - d}{1 + r} \mathbb{E}_{X'}[V_{n-1}(P_{n-1}(S; X'); X')] |X| + \frac{d}{1 + r} (K'_{n-1} - (1 + \bar{\gamma}_t)D'_{n-1})
$$

2. We then use the new value function $V_n$ maximize the continuation value for each state $(S; X)$, and obtain the new policy function $P_{n-1}(S; X)$.

$$
P_n(S; X) = \text{argmax}_{S'}\{e(S, S'; X) + \frac{1 - d}{1 + r} \mathbb{E}_{X'}[V_n(S'; X') |X] + \frac{d}{1 + r} (K' - (1 + \bar{\gamma}_t)D')\}
$$

3. We stop when $P_n = P_{n-1}$.

The contraction mapping theorem ensures that we are guaranteed to find a good approximation of the value function $V(S; X)$ and the policy function $S' = P(S; X)$ defined over the grid. The computationally costly step is the maximization in step 2. with respect to $S'$. This consists of $29 \times 270 \times 51 \times 11 = 4,392,630$ optimizations of vectors with $29 \times 270 = 7,830$ points. This is
where parallelization achieved through a GPU accelerates the process. For the range of parameters we explore, we typically solve the model in about 2 minutes with a GPU (Nvidia K80), compared to several hours with a CPU. What prevents us from having a finer grid is the RAM of the GPU, since the computer needs to create the maximand in step 2., a $29 \times 270 \times 29 \times 270 \times 51 \times 11 \approx 34$ bn numbers array.

The above algorithm is the standard policy function iteration algorithm. We make two adjustments to adapt it to our setting. First, in order to reduce computing time, we first solve the model with a coarser grid, and then solve it again on the grid describe above. To define this coarser grid, we divide the resolution of the control ($K$ and $D$) grids by two. This divides computing time by four in the first step but only gives us the value and policy functions on the coarser grid. We then re-run the algorithm on the finer grid with the “coarser” policy and value functions as starting point. Convergence occurs much more quickly.

The other adjustment that we make to the method is related to the treatment of missing values, which in our set-up occur when any of the financing constraints are violated. Without modification, the policy iteration algorithm does not behave well in the presence of missing values. This is because, for some given value functions $V_{n-1}$, there may exist some $(S, X)$ for which there is no acceptable policy $S'$. In this case, the optimal policy function $P_n(S, X)$ is not defined everywhere on the grid (note $(S_0, X_0)$ such states for which the policy is not defined). When this happens, the next iteration value $V_n(S; X)$ is non-defined for all $(S, X)$ which lead with non zero probability to states $(S_0, X_0)$. As we iterate, missing value progressively spread to the entire grid and the algorithm is blocked. To solve this problem, we modify step 1. of the algorithm by requiring that $V_n(S; X)$ replaces $V_{n-1}(S; X)$ if and only if $V_n(S; X)$ is non missing. This prevents missing values from spreading to the entire grid of states $(S; X)$.

A.2 Estimation

We now proceed to estimate the parameters $(s, c, \rho, \sigma)$ for which the model best matches a pre-defined set of moment (we experiment with different set of moments and models in the main text).

A.2.1 Estimation method: SMM

We estimate the key parameters of the model by simulated method of moments (SMM), which minimizes the distance between moments from real data and simulated data. Let us call $m$ the vector of moments computed from the actual data, and let us call $\Omega$ the moments generated by the model with parameters $\Omega$. The SMM procedure searches the set of parameters that minimizes the weighted deviations between the actual and simulated moments,

$$ (m - \hat{m}(\Omega))' W (m - \hat{m}(\Omega)) $$

We detail the various components of our implementation in the following sections.
A.2.2 Empirical moments \( m \) and Weight matrix \( W \)

The empirical moments are computed in a simple way, and the definitions are given in the main text, in Section 3.3.

The weight matrix \( W \) adjusts for the fact that some moments are more precisely estimated than others. It is computed as the inverse of the variance-covariance matrix of actual moments estimated by bootstrap with replacement on the actual data. To compute the elements of this matrix, we repeat 100 times the following procedure. Using our dataset, we draw, with replacement, \( N \) firms with their entire history where \( N \) is the number of firms in the sample (we use the `bsample` command in Stata, clustered at the firm level, this ensures that the bootstrapped sample has the same firm histories as the data source). We then compute the moments, and store them. Once we have performed this procedure 100 times, we compute the empirical variance-covariance matrix of the moments, and invert it.

A.2.3 Model-generated moments \( m \)

Once, for a given set of parameter \( \Omega \), we have solved the model (Appendix A.1), we need to simulate data in order to compute the moments. To maximize the variability, we simulate a balanced panel of 1,000,000 firms over 100 years, and only keep the last 10 years to make sure we are in steady state for each of them. For each firm, we simulate a path of log productivities \( z_{it} \) and a path of log real estate prices \( p_{it} \). This makes the variability of real estate prices larger than in the data, where prices only vary at the city (MSA) level. Recall however that our objective in this simulation is not to replicate the variability of the data, but ideally to estimate model-generated moments. If we had closed forms for the model, we could measure these moments without infinite precision. The problem here comes from the fact that we cannot directly compute these moments but have to “estimate” them. Ideally, we would want to generate an infinitely large simulated dataset in order to compute the model-generated moments exactly, but computational constraints make it infeasible. 100,000 firms over 100 years already generate arrays with 10m entries. Allowing real estate prices to vary at the firm-level is a way to make sure the sensitivity to prices model-generated moments are estimated as well as possible.

A.2.4 Optimization algorithm

We now know how to compute the objective function (14). In this Section, we explain how to minimize it. Since in our most preferred specification we have 5 parameters, we need to make sure that we are indeed reaching a global minimum. We do this by implementing the following two-step procedure, inspired from Guvenen et al. (2014):

- We generate 1,000 quasi-random vectors of parameters \( \Omega \) taken from a Halton sequence. The Halton sequence is a deterministic sequence of numbers that has the property of covering the parameter space evenly. For each of these parameters, we solve the model to obtain the policy function, simulate a dataset, compute the moments and therefore the distance to data moments (14).

- We then use the lowest points (in terms of objective function) as starting points for minimization. We iterate on the following loop. We begin with parameter estimate \( \hat{\Omega}_1 \) for which
the objective function is the lowest. We then use the Nelder-Mead method (command `fminsearch` in Matlab) to perform a local optimization starting from this point. We then compute the objective function $O_1$. We then move to the second lowest parameter estimate ($\hat{\Omega}_2$) and compute the objective function $O_2$. We iterate on this, and stop as soon as $O_n = 0$. Among the lowest parameters, a large fraction typically leads to the same parameters for which the objective function is equal to 0. This gives an indication that our objective function is wel-behaved.

There is no general theoretical results arguing that this technique dominates other popular algorithms adapted for large dimension optimization. In our setting however, we found that the genetic algorithm and simulated annealing were much slower at converging. Also, this approach allows to “control” the smoothness of the objective function. For instance, within the lowest 20 parameters isolated after step 1., it would be worrisome if minimizations starting from each of these parameters gave inconsistent parameters. On the contrary, they tend to be very consistent. The only cases where convergence goes to alternative choice of parameters than the one we present are cases where the objective function is much bigger than zero (i.e. other local optima). Finally, the best argument in favor of our selected estimates is the well-behaved comparative statics we present in Appendix C.

A.2.5 Standard errors

We estimate our standard errors using a block-bootstrap procedure. As for the computation of the variance-covariance matrix, we start by generating $B = 100$ datasets of $N$ firms drawn without replacement from the data, and then compute the vector of targeted moments for each dataset. To preserve the panel structure we make sure to draw firms and not observations (hence the “block” in block-bootstrapping). The result is a set of 100 vectors $m_b$, for each of whom we seek the vector of model parameters $\Omega_b$ that minimizes

$$f_b (\Omega_b) = (m_b - \hat{m}(\Omega_b))' W (m_b - \hat{m}(\Omega_b))$$

(15)

To reduce computing time we estimate the 100 parameters $\Omega_b$ in parallel. We implement this idea with the following algorithm. We define a new objective function as the sum of all 100 objective functions, that is

$$F (\Omega_1, \ldots, \Omega_B) = \sum_{b=1}^{B} f_b (\Omega_b)$$

(16)

We then construct $\hat{M}$ as the list of $B$ parameters $\Omega_b$, each corresponding to a sample $b$. We then iterate on this list, updating some of these parameters at each round. The algorithm works as follows.

1. $\hat{M}$ is initialized. We set each parameter $\Omega_b$ of the list to the SMM estimate $\hat{\Omega}$ (so they are all identical). Let $b^*$ be the sample for which $f_b(\hat{\Omega})$ is that highest. This corresponds to the bootstrapped sample for which the main SMM estimate fits the moments the worst.
2. We use the Nelder-Mead simplex algorithm to improve the estimate $\Omega_{b^*}$ of the least well matched sample $b^*$. Specifically, we use Matlab fminsearch function with the following options:

- The initial simplex $\Delta_{b^*}$ is computed using the current estimate of $\Omega_{b^*}$ as an “initial guess”
- If fminsearch reaches a maximum of 50 iterations, $\Delta_{b^*}$ is reinitialized using the best available estimate of $\Omega_{b^*}$ as an “initial guess”.

We then use the outcome of this procedure to update the parameter estimate of sample $b^*$ in the list $\hat{M}$.

3. For each vector $m_b$, we find in $\hat{M}$ the vector $\Omega_b$ that minimizes $f_b(\Omega_b)$. We then find the sample $b^*$ for which the objective function $f_{b^*}$ has the highest value.

4. If the standard deviations of $\Omega_b$ have moved by less than 1% over the last 500 evaluations, and if the value of $F$ is less than one tenth of its initial value, then the procedure stops. Otherwise, it goes back to step 2.

Standard errors of $\Omega$ are estimated using the standard deviation of the $\Omega_b$. The fact the value of $F$ is divided by at least ten indicates that the dispersion of $\Omega_b$ is sufficient to explain 90% of the (weighted) dispersion of $\Omega_b$. To reach that point, our procedure typically takes the equivalent of 2-3 SMMs to converge, and is thus about 30 times faster than running all 100 SMMs sequentially.
B General Equilibrium Computation

In this Section, we describe how we compute the general equilibrium of an economy populated by firms whose behavior is described by the model estimated and solved in Appendix A. First, recall that this model is estimated assuming aggregate demand $Y = 1$ and aggregate wage $w = .03$.

The economy is described in detail in Section 4 in the main text. There is a large number of firms (a continuum in the model), each of them facing an idiosyncratic path of productivity and of real estate prices. The behavior of each of these firms is described by the dynamic model with adjustment costs, time-to-build capital and the collateral constraint. All firms are monopolist intermediate input producers which are combined with elasticity of substitution $\theta$. As a result $\theta$ measures the intensity of competition between intermediate producers ($\theta = 1$ means perfect competition). The final good is then consumed by a representative producer with linear utility, Frisch elasticity of labor supply $\epsilon$ and subjective discount rate $r$. Consumption equals production minus adjustment costs and investment. The price of the final good is normalized to 1 without loss of generality. This economy has no aggregate uncertainty and the equilibrium is uniquely described by aggregate production $Y$ and real wage $w$ which are fixed over time.

Start from a set of SMM estimates $\hat{\Omega}$. Our goal is to investigate the GE consequences of a change in parameter $\omega$ from its estimated value $\hat{\omega}$ to another $\omega'$. This change affects firm’s behavior, hence aggregate labor demand and aggregate production. This in turn affects the wage and aggregate demand which in turn changes firm behavior. The following algorithm finds the fixed point of this problem such that: (1) aggregate production of all firms equal aggregate demand $Y$ in firms’ problems and (2) the labor market clears such that aggregate labor demand equals labor supply at prevailing wage.

We proceed in three main steps:

1. Find number of firms $N$ and the labor supply $L_0$ at wage $w_0 = .03$, so that the estimated model is at equilibrium with salary $w_0 = .03$ and aggregate production $Y = 1$. This will become part of the structure of the economy.
   (a) Simulate the data with 100,000 firms, $w = .03$, $Y = 1$ and parameters $\hat{\Omega}$.
   (b) Compute mean labor demand $l$ and mean revenue $py$.
   (c) Set $N = \frac{1}{py}$ and $L_0 = \frac{L}{py}$. With such parameters, the economy with $N$ firms and labor supply parameter $L_0$ is at equilibrium with $w_0 = .03$ and $Y = 1$.

2. Change one of the parameters to its new value $\omega'$. Given this, we loop to find the new equilibrium $w$ and $Y$.
   (a) Set $w_0 = .03$ and $Y_0 = 1$.
   (b) Initiate round number $n = 1$. Then,
      i. Solve the model with $w_{n-1}$ and $Y_{n-1}$ and simulate 100,000 firms.
      ii. Compute average revenue $py_n$ and average labor demand $l_n$ and multiply both by $N$ to obtain aggregate production $Y_n^*$ and aggregate labor demand $L_n$.
      iii. Compute labor market clearing wage $w_n^* = w_0(L_n/L_0)^{1/\epsilon}$
      iv. Take $w_n = (w_{n-1})^{\lambda} (w_n^*)^{1-\lambda}$ and $Y_n = (Y_{n-1})^{\lambda} (Y_n^*)^{1-\lambda}$
v. go back to step (iii), until convergence in $Y$ and $w$.

(c) compute aggregates:

- $Y, K = \sum_i k_i, L = \sum_i l_i$
- $\log \text{TFP} = \log Y - \alpha \log K - (1 - \alpha) \log L$
- $W = Y - \delta K - \frac{w_0}{1 + 1/e} \frac{L^{1+1/s}}{L_0^{1/s}}$
C Additional tables

Figure C.1: Sensitivity of moments to pledgeability $s$

Note: In this figure, we set all estimated parameters ($s, c, \rho, \sigma$ and $H$) at their SMM estimate in our preferred specification – as per column 4, Table 3. We fix $w$ and $Y$ at their reference levels: $w = .03$ and $Y = 1$. We then vary $s$ from 0 to 1. For each value of $s$ that we choose, we solve the model, simulate the data, and compute four target moments, plus the average leverage ratio and the sensitivity of debt issuance to real estate value. Each panel corresponds to one moment. The red vertical line corresponds to the SMM estimate of $s$. 

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Figure C.2: Sensitivity of moments to adjustment costs $c$

Note: In this figure, we set all estimated parameters ($s, c, \rho, \sigma$ and $H$) at their SMM estimate in our preferred specification – as per column 4, Table 3. We fix $w$ and $Y$ at their reference levels: $w = .03$ and $Y = 1$. We then vary $c$ from 0 to .1. For each value of $c$ that we choose, we solve the model, simulate the data, and compute four target moments, plus the average leverage ratio and the sensitivity of debt issuance to real estate value. Each panel corresponds to one moment. The red vertical line corresponds to the SMM estimate of $c$. 
Figure C.3: Sensitivity of moments to productivity volatility $\sigma$

Note: In this figure, we set all estimated parameters ($s, c, \rho, \sigma$ and $H$) at their SMM estimate in our preferred specification – as per column 4, Table 3. We fix $w$ and $Y$ at their reference levels: $w = .03$ and $Y = 1$. We then vary $\sigma$ from 0 to 1. For each value of $\sigma$ that we choose, we solve the model, simulate the data, and compute four target moments, plus the average leverage ratio and the sensitivity of debt issuance to real estate value. Each panel corresponds to one moment. The red vertical line corresponds to the SMM estimate of $\sigma$. 

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Figure C.4: Sensitivity of moments to productivity persistence $\rho$

Note: In this figure, we set all estimated parameters ($s, c, \rho, \sigma$ and $H$) at their SMM estimate in our preferred specification – as per column 4, Table 3. We fix $w$ and $Y$ at their reference levels: $w = .03$ and $Y = 1$. We then vary $\rho$ from 0 to 1. For each value of $\rho$ that we choose, we solve the model, simulate the data, and compute four target moments, plus the average leverage ratio and the sensitivity of debt issuance to real estate value. Each panel corresponds to one moment. The red vertical line corresponds to the SMM estimate of $\rho$. 