Learning across Peer Firms and Innovation Waves

Merih Sevilir
Indiana University-Bloomington

Abstract

This paper presents a model where firms learn from each other’s innovation, and innovation by one firm spurs subsequent innovations by peer firms, leading to the emergence of innovation waves. In our model, innovation of a firm reaches peer firms competing in similar markets. Peer firms receiving the innovation can either expropriate it, or learn from it to generate a subsequent innovation. As an innovation reaches a greater number of peer firms, each peer firm’s expropriation incentives decline while its incentives to generate a subsequent innovation increase. In addition, as the intensity of the innovation transmission across peer firms gets larger, expropriation incentives decline even further. Hence, perhaps surprisingly, as the innovation travels to a greater number of peer firms at a greater speed, and becomes easier to be expropriated, it becomes less desirable to expropriate it. Our results are consistent with the empirical evidence presented in Matray (2014) that innovation by one firm spurs innovation by neighboring peer firms, and firms innovate by building on previous technologies produced by peer firms. Our model predicts that a concentrated mass of interconnected firms that both compete and learn from each other as well as a speedy flow of ideas from one firm to another are necessary conditions for innovation waves to emerge. Finally, our analysis has implications for the innovation output of a firm and industry after a merger wave. A wave of consolidating mergers in an industry will reduce the number of firms, and increase each firm’s incentive to expropriate a given innovation, with a negative effect on incentives to invest in innovation in the first place.
1 Introduction

This paper presents a model where firms learn from the innovation of their peers and advance it into a subsequent innovation, leading to the emergence of innovation waves. It is often argued that firms in similar markets and close geographies learn from each other, resulting in the diffusion of innovation across peer firms. In fact, recent work by Matray (2014) finds evidence that innovation by one firm spurs future innovations by neighboring firms, and the degree of innovation spillovers gets stronger in the degree of technological proximity and the intensity of information flow among peer firms. It is not obvious, however, if innovation spillovers across peer firms would arise in equilibrium as it might be attractive for firms to expropriate the innovation of peer firms given that expropriation is easier and less costly than learning from a current innovation and advancing into a subsequent innovation. This paper develops a model and identifies the conditions under which innovation by one firm leads to subsequent innovations by peers firms, despite the fact that expropriation of an innovation is less costly than learning from and advancing the innovation.

In our model, an innovation generated by a firm travels to peer firms competing in product markets involving similar technologies. Firms receiving an innovation of a peer firm have the ability to expropriate it at no cost, or invest in a subsequent innovation at a cost by learning from the current innovation. When the current innovation reaches a greater number of peer firms, each peer firm’s incentives to expropriate become weaker since a larger number of firms expropriating the same innovation results in a smaller payoff for each firm given that firms compete over similar technologies. Hence, a given firm receiving the current innovation finds it more desirable to invest to generate a subsequent innovation, as the number of other peer firms with the ability to expropriate the current innovation increases. This leads to a second factor which dilutes expropriation incentives even further. Greater incentives to innovate by learning from a current innovation implies a greater probability that the current innovation will be displaced by a subsequent innovation in a short period of time. This enhances the incentives of peer firms further to move away from expropriating the current innovation to work towards a subsequent innovation, given that the current innovation is likely to be short-lived.
Interestingly, consistent with the empirical results in Matray (2014), firms’ willingness to learn from peer firms’ innovations and to advance them into future innovations becomes stronger as the intensity of information flow across peer firms increases. This is because with a greater intensity of information flow, innovations reach a greater number of peer firms at a greater speed, reducing the desire to expropriate them.

Our model leads to emergence of innovation waves where an innovation by one firm leads to future innovations by peer firms, as documented empirically. There are two conditions for an innovation wave to emerge. The first is a critical mass of interconnected peer firms that both compete with, and learn from each other. The second is an intense flow of innovation among the peer firms. Under these conditions, peer firms prefer costly investment to learn from each other’s innovation and generate their own innovations, to expropriating each other’s innovations at no cost.

Our paper’s predictions are also consistent with the findings in Uribe (2014) documenting that start-up firms receiving financing from the same venture capitalist (VC) are more likely to cite each other’s patents. To the extent that having a common VC investor facilitates knowledge flow among the VC’s portfolio companies, a VC with a greater ability to facilitate knowledge flow would reduce expropriation incentives and enhance innovation incentives of each start-up in his portfolio. Consistent with this interpretation, Uribe (2014) finds that the cross-citation intensity of start-ups in the VC’s portfolio is greater for more experienced VCs, that is, for VCs with a greater efficiency to facilitate information flow among startups in their portfolio.

Our model has implications for the innovation output of a firm and industry after a merger wave. A wave of consolidating mergers in an industry will reduce the number of firms, and increase each firm’s incentive to expropriate a given innovation, with a negative effect on incentives to invest in innovation in the first place. Similarly, our results are also consistent with the clustering patterns observed in IPOs. In a survey of CFOs, Braun and Fawcett (2006) find that one of the important concerns for private firms in deciding whether to go public is the potential loss of proprietary information to competing firms. Our paper shows that as long as there is a sufficient
number of peer firms with the ability to learn about a firm’s proprietary information, incentives to expropriate the firm’s critical information will be weaker, while incentives to learn from that information to produce new information will be stronger. Hence, the potential negative effect of being a public firm on innovation incentives will be smaller, and the incentives to go public will be greater as the number of peer firms going public increases in an industry.

In our model, it is not possible to protect an innovation through property rights or patents. Hence, a firm with an innovation faces the risk of expropriation if its innovation travels to peer firms competing over similar technologies. However, as the innovation travels to a greater number of peer firms, and with greater probability, expropriation risk becomes smaller. Hence, in our model, although the presence of competing firms creates the risk of expropriation, a more intense competition and a more intense information flow among competitors may help resolve the risk of expropriation. The intuition that existence of competition may prevent innovation expropriation is also in the model in Anton and Yao (1994) which analyzes a problem faced by an inventor when selling a valuable idea which can be easily expropriated due to the absence of property rights. They show that the inventor can capture the full value of its invention by revealing the invention to a competitor of the buyer. The threat of competition prevents the buyer of the invention from expropriating or stealing the invention due to the inventor’s ability to sell the invention to a competing buyer and reducing the original buyer’s profits. In our set-up, different from Anton and Yao (1994), competing peer firms can choose between expropriation and innovation through learning from the innovating firm. As the innovation of an inventor reaches a greater number of firms, each competing firm’s incentives to expropriate decline and their incentives to learn from the innovation and to advance it become stronger. Hence, in our model it is not the threat of revealing an innovation to a competitor which prevents expropriation, but rather the actual flow of the innovation to competitors makes it less likely for the innovation to be expropriated and more likely to be advanced. The existence of a sufficient number of competitors not only prevents expropriation of current innovations, but also leads to the emergence of future innovations where competing firms learn from each other, and advance each other’s innovation into subsequent ones.
There is no scope for current innovations to spur future innovations and to lead to innovation waves in Anton and Yao (1994) because competitors can only steal an existing innovation, with no ability to learn from it.

Biais and Perotti (2008) focuses on the complementarity between different dimensions of an innovative idea to mitigate the risk of the idea being stolen. In their model, an entrepreneur needs experts to evaluate her innovative idea along different dimensions. Sharing the idea creates the risk of the idea being stolen by experts. Forming a partnership with the experts can prevent the idea from being stolen since it gives each experts access to the other’s expertise. Interestingly, very valuable ideas cannot be shared since it is too attractive for experts to steal them. In our model, as an innovative idea reaches a greater number of competing firms, the idea is more likely to spur future valuable ideas. This leads to a novel insight that incentives to create innovative ideas depend positively on whether innovative ideas flow freely and are shared among competing firms.

In our model, an innovation of a firm travels to other firms which can learn from it to generate a subsequent innovation. Stein (2008) studies a similar situation where firms exchange ideas with their competitors since doing so increases the probability of generating a new idea due to complementarity in the information structure. In our paper, subsequent innovations occur only if the current innovation reaches a sufficient number of peer firms where each firm finds expropriation undesirable. Otherwise, if the innovation travels to only a small number of peer firms, it is expropriated, and as an anticipation of this, in equilibrium, there is no investment in the creation of the innovation in the first place. Hence, complementarity among current and future innovations arises only if a current innovation travels with a greater probability to a sufficiently large number of firms that compete with each other. This allows our paper to generate new implications on the intensity of information flow among peer firms and the emergence of innovation waves through learning across peer firms, consistent with the recently documented empirical evidence on the innovation spillovers among peer firms in Matray (2014).

This paper is organized as follows. Section 2 presents the model. Section 3 analyzes the
model. Section 4 discusses the implications and predictions of the model. Section 5 concludes. All proofs are in the Appendix.

2 The Model

In our model, we have two different settings with risk-neutral firms and no discounting. The first setting has two peer firms, firm 1 and firm 2, and the second setting has three peer firms--firm 1, firm 2 and firm 3. In each setting, firm 1 decides at $t = 0$ whether to invest $K$, with $K > 0$, to generate an innovation. If it invests $K$, it is successful in generating an innovation with probability $p$, with $0 < p \leq 1$ and obtains payoff $y_1$ from its innovation at $t = 1$. After firm 1 implements its innovation and realizes the first period payoff $y_1$, competing firms learn about firm 1’s innovation with probability $q$, with $0 < q \leq 1$ where $q$ measures the efficiency of innovation transmission from one firm to another.

In the two-firm setting, if firm 1 generates an innovation and if its innovation travels to firm 2, firm 2 has two strategies to choose from. It either expropriates the innovation at no cost, or innovates through costly investment by learning from the current innovation of firm 1.\footnote{Bessein and Maskin (2009), Scotchmer (1991,1996), among others, make a a similar assumption on the sequential nature of innovation in that future innovations build on current innovations.} If it chooses expropriation, firm 1 and firm 2 compete in the product market where each firm obtains payoff $y_2$, with $y_2 < y_1$ at $t = 2$. The assumption that firm 2 obtains the same payoff as firm 1 from expropriating firm 1’s innovation is only for analytical simplicity, and our results hold under an alternative assumption that firm 2 obtains a lower payoff than firm 1 when it expropriates firm 1’s innovation. If firm 2 chooses innovation, it invests $k$, with $k < K$ and obtains a subsequent innovation to firm 1’s current innovation with probability $p$ at $t = 2$, which displaces the current innovation of firm 1. Conditional on firm 2 being successful, it obtains payoff $Y_1$ with $Y_1 > y_1$ from its innovation at $t = 2$, and since firm 1’s innovation is displaced, firm 1 obtains zero payoff. The assumption $Y_1 > y_1$ implies that firm 2’s innovation builds on the innovation of firm 1 and represents an improvement to firm 1’s innovation. The assumption $k < K$ captures the notion...
that learning from firm 1’s innovation reduces the cost of generating a follow-up innovation. Finally, the assumption that the probability of a successful innovation $p$ is the same for both firm 1 and firm 2 is not important for our results, and is made only for keeping the model more tractable.

In the three-firm setting, if firm 1 is successful in generating an innovation, after implementing the innovation and obtaining the first period payoff $y_1$ at $t = 1$, its innovation travels to firm 2 and firm 3, each with probability $q$. Firm 2 and firm 3 choose between expropriation and innovation conditional on firm 1’s innovation reaching them. If firm 1’s innovation travels to only one of firm 2 or firm 3, everything remains the same as in the two-firm setting described above. If it travels to both firm 2 and firm 3, there are three possibilities. Both firms choose expropriation, one firm chooses expropriation and the other chooses innovation, and both firms choose innovation.

If both firm 2 and firm 3 choose expropriation, the three firms compete in the product market and each obtains payoff $y_3$ at $t = 2$, with $y_3 < y_2$.

If one firm chooses innovation, say firm 2, and the other, firm 3, chooses expropriation, firm 2 invests $k$ at $t = 1$ and obtains an innovation with probability $p$ at $t = 2$. Its innovation displaces firm 1’s innovation and firm 2 obtains payoff $Y_1$ from its innovation. Firm 1 and firm 3 obtain zero payoffs given that firm 1’s innovation is displaced. If firm 2 fails in generating a subsequent innovation, it obtains zero, and firm 1 and firm 3 obtain payoff $y_2$ from firm 1’s innovation. Note that although expropriation is less costly than innovation, firm 2 investing in innovation cannot switch to expropriation if it fails to generate an innovation.

If both firm 2 and firm 3 choose innovation, and if both firms succeed, we assume that they compete in the product market where each firm obtains payoff $Y_2$ at $t = 2$, with $Y_2 < Y_1$, while firm 1 obtains zero payoff given that its innovation is displaced. If one of the firm 2 and firm 3 is successful, and the other fails, the successful firm obtains payoff $Y_1$ from its innovation, and firm 1 and the failed firm obtain zero payoffs. If both firm 2 and firm 3 fail in generating an innovation and displacing firm 1’s innovation, firm 1 obtains the second period payoff $y_1$ from its
innovation at $t = 2$.

We characterize the product market competition and specify the payoffs $y_1, y_2, y_3, Y_1$ and $Y_2$ in the next section under a model of Cournot competition.

We assume initially that subsequent innovations obtained by firm 2 and firm 3 do not travel back to firm 1, or to each other. In Section 3.3, we relax this assumption and modify the model where innovation transfers are possible between any of the three firms, and establish that the main results of the model are robust under that more general setting as well.

### 2.1 Product market and the payoffs

An innovation refers to a new product with a marginal cost of production $c$, with $0 < c < 1$, and an inverse demand function $P = a - QT$ where $P$ is the price of the good, $QT$ is the total quantity of the product produced. For analytical simplicity $a$ is normalized to 1.

If firm 1 generates an innovation, and if it is the only firm commercializing its innovation, we assume that based on the Cournot model of product market, it sells quantity $Q^*_1$ per period to maximize its profit $(P - c - Q^*_1)Q^*_1$. It sets the optimal quantity produced $Q^*_1 = \frac{1-c}{2}$, and obtains profits $\frac{(1-c)^2}{4}$. Hence, $y_1 = \frac{(1-c)^2}{4}$.

If firm 1 generates an innovation, and if its innovation travels to another firm, say firm 2, and is expropriated by firm 2, then the two firms compete in the product market under Cournot competition where each produces the product at the same cost $c$. Let $Q^*_1$ and $Q^*_2$ denote the quantities produced by firm 1 and firm 2 respectively. Firm 1 chooses $Q^*_1$ to maximize its profit given by $(P - c - Q^*_1 - Q^*_2)Q^*_1$, and firm 2 chooses $Q^*_2$ to maximize $(P - c - Q^*_1 - Q^*_2)Q^*_2$, resulting in optimal quantities $Q^*_1 = Q^*_2 = \frac{(1-c)}{3}$ and profits $\frac{(1-c)^2}{9}$ for each firm. Hence, $y_2 = \frac{(1-c)^2}{9}$.

If firm 1 generates an innovation, and its innovation travels to both firm 2 and firm 3, is expropriated by both firms, under Cournot competition, following the profit maximization under Cournot competition with three firms, it is immediate to show that each of the three firms produces optimal quantity $Q^*_1 = Q^*_2 = Q^*_3 = \frac{(1-c)}{4}$, and obtains profit $\frac{(1-c)^2}{16}$. Hence, $y_3 = \frac{(1-c)^2}{16}$. 7
As mentioned before, the assumption that firms expropriating firm 1’s innovation has the same cost $c$ of production as firm 1 and hence, all firms set same quantities and obtain same profits is only for analytical tractability. All our results go through if firm 2 and firm 3 produce at a higher cost of production than firm 1 when expropriating firm 1’s innovation.

We specify now the payoffs if firm 2 and/or firm 3 do not expropriate firm 1’s innovation, but innovate by learning from it. First, consider the case where firm 1’s innovation travels to only one firm, say firm 2, and is advanced into a subsequent innovation. We assume that a subsequent innovation to firm 1’s innovation represents an improvement over firm 1’s innovation. Specifically, it reduces the cost of production from $c$ to $\beta c$ with $0 \leq \beta < 1$. Hence, if firm 2 obtains a subsequent innovation, it sells quantity $Q^1_{2,i}$ where superscript $i$ refers to innovation, to maximize its profit $(P - \beta c - Q^2_{2,i}) Q^2_{2,i}$. We assume that $\beta \leq \frac{2c-1}{c}$ which implies that firm 1 produces zero, while firm 2 produces $Q^2_{2,2} = \frac{1-\beta c}{2}$, obtains profit $\frac{(1-\beta c)^2}{4}$. Hence, $Y_1 = \frac{(1-\beta c)^2}{4}$.

The assumption $\beta \leq \frac{2c-1}{c}$ implies that firm 1’s innovation is displaced and it is no longer optimal for firm 1 to compete in the product market with firm 2 given that firm 2’s innovation is superior to firm 1’s innovation.

If firm 1’s innovation travels to both firm 2 and firm 3, and one firm chooses innovation, say firm 2, and the other, firm 3, chooses expropriation, if firm 2 is successful, it again sells optimal quantity $Q^1_{2,2} = \frac{1-\beta c}{2}$, obtains profit $Y_1 = \frac{(1-\beta c)^2}{4}$ while firm 1 and firm 3 produce zero and obtain zero profits, given $\beta \leq \frac{2c-1}{c}$.

If one firm succeeds, and the other firm fails, the successful firm produces quantity $\frac{1-\beta c}{2}$ and obtains profit $Y_1 = \frac{(1-\beta c)^2}{4}$. If both firms fail, they produce zero quantity and obtain zero profits. In this case, since firm 1’s innovation is not displaced, it continues to obtain profits $y_1$ from its innovation.
In analyzing the model, for analytical tractability, we set $\gamma = \frac{Y_1}{y_1} = \frac{Y_2}{y_2} = \frac{(1-\beta c)^2}{(1-c)^2}$ where $\gamma$ refers to the value of a subsequent innovation(s) by firm 2 and/or firm 3 relative to firm 1’s innovation.

3 Equilibrium Analysis

3.1 Two-firm setting

We proceed with the analysis of the model in the two-firm setting. Firm 1 has two strategies available at $t = 0$, either to "invest in innovation", denoted by $I$, or "not to invest in innovation", denoted by $NI$. If firm 1 chooses $I$, is successful and its innovation travels to firm 2, firm 2 has two strategies available: "expropriate the innovation", denoted by $e$, or "innovate by learning from firm 1’s innovation", denoted by $i$.

Suppose firm 1 chooses $I$ by investing $K$, it is successful and its innovation travels to firm 2. Using backward induction, firm 2 compares its expected profit from expropriation and that from innovation to determine its optimal strategy. If it expropriates, it obtains $y_2$, given that it competes with firm 1 in the product market. If it invests $k$ to innovate, it obtains a subsequent innovation with probability $p$ with profit $Y_1$, yielding an expected payoff $-k + pY_1$. We assume $k < pY_1$, which implies that investment to advance firm 1’s innovation into a subsequent innovation has a positive NPV. Comparing the two payoffs, it is immediate to see that firm 2 chooses expropriation for $k > pY_1 - y_2$, and innovation for $k \leq pY_1 - y_2$. Noting that $y_2 = \frac{(1-c)^2}{y}$ and $Y_1 = \frac{(1-\beta c)^2}{4} = \frac{2(1-c)^2}{4}$, it follows that firm 2 chooses expropriation for $k > k_1 \equiv \frac{9p\gamma - 4}{36}(1-c)^2$.

Let $k > k_1$ so that conditional on receiving firm 1’s innovation, firm 2 chooses expropriation. Let $\pi_{F1}^2(I; e)$ denote in the two-firm setting firm 1’s ex ante expected profit from choosing innovation, $I$ at $t = 0$, and firm 2 choosing expropriation, $e$ at $t = 1$, conditional on firm 1’s innovation traveling to firm 2. It follows that

$$\pi_{F1}^2(I; e) = -K + p(y_1 + qy_2 + (1-q)y_1);$$

$$= -K + p(2y_1 - q(y_1 - y_2)).$$
After investing $K$, with probability $p$, firm 1 is successful in generating an innovation, and captures the first period payoff $y_1$. Its innovation travels to firm 2 with probability $q$, and is expropriated, yielding payoff $y_2$ for each firm. With probability $1 - q$, firm 1’s innovation does not reach firm 2, and firm 1 obtains the second period payoff $y_1$ from its innovation.

Firm 1 does not invest in innovation if its ex ante expected profit from doing so is negative, that is, if $\pi_{F1}(I; e) < 0$, or equivalently $K > p(2y_1 - q(y_1 - y_2))$. Plugging $y_1 = \frac{(1-c)^2}{4}$, $y_2 = \frac{(1-c)^2}{9}$, firm 1 does not invest in innovation for $K > K_1 \equiv p(\frac{18-5q}{90})(1 - c)^2$. Hence, there is the classical problem of underinvestment in innovation due to the expropriation behavior of firm 2, as first discussed in Arrow (1962).

The following Lemma presents the conditions under which firm 1 does not invest in innovation, anticipating the expropriation behavior of firm 2.

**Lemma 1** There exist values of $k$ and $K$ such that for $k > k_1$ and $K > K_1$, firm 1 does not invest in innovation at $t = 0$.

Note that our earlier assumption that $k < pY_1$ implies that the NPV created from firm 1 investing in innovation, and firm 2 learning from firm 1’s innovation and advancing it into a subsequent innovation denoted by $NPV(I, i)$ is positive. That is, $NPV(I, i) = -K + p(y_1 + q(pY_1 - k) + (1 - q)y_1) > 0$. Hence, for $K_1 < K < pY_1$, there is inefficiency in that firm 1 does not invest in innovation, and there is no possibility for a potential innovation by firm 1 to be advanced into a subsequent innovation by firm 2. We show in the next section that the presence of another firm, that is, firm 3, and firm 1’s innovation traveling to both firm 2 and firm 3 may eliminate this inefficiency and restore firm 1’s incentives to invest in innovation.

### 3.2 Three-firm setting

In this section, in addition to firm 1 and firm 2, we have firm 3, which receives firm 1’s innovation with the same probability $q$ as firm 2. Hence, conditional on firm 1 investing in innovation and being successful, with probability $q^2$, its innovation travels to both firm 2 and firm 3. With probability $q(1 - q)$, its innovation travels to only to firm 2 but not to firm 3. With the same
probability \((1-q)q\), it does not travel to firm 2, but travels to only firm 3. Finally, with probability \((1-q)^2\), its innovation travels to neither firm 2 nor firm 3.

First consider the states where firm 1’s innovation travels to only firm 2 or firm 3, which happens with probability \(2q(1-q)\). From the discussion in the previous setting with two firms, we already know that the firm receiving the innovation finds it optimal to expropriate it for \(k > k_1\). Let \(k > k_1\) for the remainder of this section so that there is always expropriation if firm 1’s innovation travels to only firm 2 or firm 3.

In the state where firm 1’s innovation travels to both firm 2 and firm 3, it is not obvious any longer that firm 2 and firm 3 would necessarily choose expropriation. Expropriating firm 1’s innovation, relative to innovation by learning from it, is less desirable for two reasons. The first is that there would be three firms competing over the same innovation. In situations where only firm 2 or firm 3 receives the innovation, it chooses expropriation and obtains expropriation payoff \(y_2\). Now with three firms, although the cost \(k\) of advancing the innovation into a subsequent innovation remains the same, expropriation strategy is less desirable since it yields each of the two expropriating firms payoff \(y_3\), with \(y_3 < y_2\). This may induce one of the firms, say firm 2, to switch to innovation since doing so may result in a subsequent innovation where firm 2 captures the full payoff from it. However, this will not restore firm 1’s incentives to invest in innovation as long as firm 3 still chooses to expropriate. Importantly, once firm 2 switches to innovation, there is an additional economic reason which may induce firm 3 to switch to innovation as well. To see this, suppose that due to the dilution in expropriation payoff from \(y_2\) to \(y_3\), firm 2 deviates to innovation. Although it may seem at first that firm 3 would enjoy expropriating firm 1’s innovation given that it is the only expropriating firm with payoff \(y_2\), this argument is incomplete. The reason is that with probability \(p\), firm 2 will generate a subsequent innovation, which will displace firm 1’s innovation and drive the expropriation payoff from firm 1’s innovation to 0. This implies that firm 3 would be able to capture the expropriation payoff \(y_2\) only if firm 2 fails to generate an innovation, which happens with probability \(1-p\). Hence, firm 2 switching to innovation reduces firm 3’s expected expropriation payoff and under certain conditions, firm
3 chooses to innovate as well. Interestingly, while with only firm 2 there might be no way to prevent expropriation of firm 1’s innovation, with both firm 2 and firm 3, each firm receiving firm 1’s innovation might unilaterally find it optimal to innovate and advance firm 1’s innovation rather than to expropriate it. Restoration of incentives to innovate, in turn, may increase the ex ante expected profit of firm 1 from investing in innovation. Hence, it is possible that in the two-firm setting, there is never innovation creation, and hence, transmission of innovation from one firm to another does not take place, in the three-firm setting, not only firm 1 invests in innovation, but also its innovation leads to the creation of subsequent innovations by firm 2 and firm 3. The analysis below formalizes this intuition and shows that the higher the level of \( q \), that is, the greater the probability that firm 1’s innovation reaches both firm 2 and firm 3, the lower the expropriation risk is. Hence, expropriation would be a smaller concern when the speed or the probability of transmission of innovation among competing firms is greater, and when an innovation reaches a greater number of competing firms with the ability to expropriate it.

To see formally the intuition behind the two reasons deterring expropriation and encouraging innovation in the three-firm setting, suppose that firm 1’s innovation travels to both firm 2 and firm 3, and both firms choose expropriation. Since now there are three firms competing over the same innovation, each firm’s payoff is diluted to \( y_3 \). Hence, firm 2 and firm 3’s expropriation payoffs, denoted by \( v_{F2}^{e,e} \) and \( v_{F3}^{e,e} \) respectively are given by \( v_{F2}^{e,e} = v_{F3}^{e,e} = y_3 \). For the expropriation strategy by both firm 2 and firm 3 to be an equilibrium outcome, neither firm should find deviation to innovation desirable, given that the other firm chooses expropriation. Suppose that firm 2 deviates to innovation. It spends \( k \), and is successful in generating a subsequent innovation with probability \( p \) and obtains payoff \( Y_1 \). Hence, firm 2’s expected payoff from innovation, given that firm 3 chooses expropriation, denoted by \( v_{F2}^{i,e} \) is given by \( v_{F2}^{i,e} = -k + pY_1 \). Comparing firm 2’s innovation payoff to its expropriation payoff, we find that firm 2 finds it optimal to deviate to innovation if \( v_{F2}^{i,e} = -k + pY_1 \geq v_{F2}^{e,e} = y_3 \), or equivalently if \( k \leq pY_1 - y_3 \). Plugging \( y_3 = \frac{(1-c)^2}{16} \) and \( Y_1 = \frac{(1-\beta c)^2}{4} = \frac{\gamma(1-c)^2}{4} \), firm 2 chooses to innovate for \( k \leq k_2 = \frac{4p(1-c)}{16} \), and chooses to expropriate for \( k > k_2 \). Recall that in the two-firm setting, firm 2 expropriates for \( k > k_1 \).
Hence, given that we have \( k_1 < k_2 \), for \( k \) such that \( k_1 < k \leq k_2 \), firm 2 chooses innovation in the three-firm setting, conditional on firm 1’s innovation reaching both firm 2 and firm 3, while it chooses expropriation in the two-firm setting. However, firm 2 choosing innovation and firm 3 choosing expropriation is not necessarily an equilibrium outcome. To establish the equilibrium, we need to analyze firm 3’s optimal strategy given that firm 2 switches to innovation. Let \( v_{F_3}^{i,i} \) denote firm 3’s expected payoff from deviating to innovation, given that firm 2 chooses innovation. With probability \( p_2 \), both firms are successful in generating a subsequent innovation to firm 1’s innovation, and with probability \( p(1 - p) \), only firm 3 is successful. If both firms are successful, they compete by producing a similar product to each other under Cournot competition where each firm obtains payoff \( Y_2 \). With probability \( p(1 - p) \), firm 3 is the only successful firm, and obtains profit \( Y_1 \). Hence, firm 3’s expected payoff from deviating to innovation, given that firm 2 also chooses innovation, is \( v_{F_3}^{i,i} = -k + p^2 \times Y_2 + p(1 - p) \times Y_1 = -k + pY_1 - p^2(Y_1 - Y_2) \). Comparing \( v_{F_3}^{i,i} \) with \( v_{F_3}^{i,e} \), it is immediate to see that firm 3 finds it optimal to deviate to innovation if \( v_{F_3}^{i,i} = -k + pY_1 - p^2(Y_1 - Y_2) \leq v_{F_3}^{i,e} = (1 - p)y_2 \), or if
\[
 k \leq pY_1 - p^2(Y_1 - Y_2) - (1 - p)y_2. \tag{2}
\]
Plugging \( y_2 = \frac{(1-c)^2}{9} \), \( Y_1 = \frac{\gamma(1-c)^2}{4} \) and \( Y_2 = \frac{\gamma(1-c)^2}{9} \) into (2), firm 3 deviates to innovation for
\[
 k \leq k_3 \equiv \frac{p\gamma (9 - 5p) - 4(1 - p)}{36} (1 - c)^2. \tag{3}
\]
Hence, conditional on firm 1’s innovation reaching both firm 2 and firm 3, for \( k \leq k_2 \) and \( k \leq k_3 \), the unique equilibrium of the subgame at \( t = 1 \) is such that both firm 2 and firm 3 choose innovation where they learn from firm 1’s innovation. Since we have \( k_3 < k_2 \) for all parameter values, both firm 2 and firm 3 choose innovation for \( k \) such that \( k \leq k_3 \).

The following lemma presents this result formally.\(^2\)

---

\(^2\)This assumption that firm 2 and firm 3 compete even though they are successful in generating an innovation reduces their incentives to switch to innovation from expropriation, and hence, makes it harder for us to prove that innovation incentives are stronger in the three-firm setting than in the two-firm setting. Put differently, our results will go through more easily under a more realistic assumption that firm 2 and firm 3’s innovations are not perfectly substitutes, but are differentiated from each other to a certain degree.
Lemma 2 Let \( \frac{4(1-p)}{p(9-5p)} < \gamma \). There exist values of \( k \) such that for \( k \leq k_3 \), conditional on firm 1’s innovation traveling to both firm 2 and firm 3, the unique equilibrium of the subgame played by firm 2 and firm 3 is such that both firms choose innovation over expropriation.

It is worth discussing the result in Lemma 2 in the context of Anton and Yao (1994). Anton and Yao (1994) does not consider competing firms to learn from each other, and shows that with an increase in the number of competing firms, expropriation of an innovation becomes less desirable, due to dilution in expropriation payoff. In our paper, competing firms choose between expropriation and innovation, and although dilution in expropriation payoff is necessary for firms not to expropriate, it is not sufficient alone to induce each competing firm to deviate to innovation. As we discuss above, even though firm 2 deviates to innovation, as long as firm 3 still expropriates, firm 1 would not invest in innovation ex ante, and there would not be any innovation created in the first place. In addition, in their model, the innovation of an inventor is revealed to only one buyer and the presence of a second buyer (i.e., competitor) acts only as a threat to prevent the first buyer from expropriating the innovation. In fact, the second buyer never learns about the innovation. In our paper, under certain conditions, the innovation of firm 1 reaches all competitors who choose to learn from it. This, in turn, makes the emergence of a wave of subsequent innovations possible. Hence, a novel feature of our model is that current innovations spur future innovations by competing firms and leads to the emergence of innovation waves. Competing firms play a more active role in shaping innovation incentives than acting purely as a threat.

As mentioned before, until now we rule out the possibility that subsequent innovations generated by firm 2 and firm 3 may travel back to firm 1. We relax this assumption in Section 3.3 and show that our main results are robust to allowing subsequent innovations of firm 2 and firm 3 to travel back to firm 1, as well as to each other.

In the two-firm setting, from Lemma 1, we have that for \( k > k_1 \) firm 2 always expropriates firm 1’s innovation, conditional on firm 1’s innovation traveling to firm 2. In the three-firm setting, for values of \( k \) such that \( k_1 < k \leq k_3 \), firm 1’s innovation is expropriated if it reaches only firm 2 or
firm 3 and it is not expropriated if it reaches both firms, implying that the probability of firm 1’s innovation being expropriated in the three-firm setting may be smaller than that in the two-firm setting. Hence, perhaps surprisingly, an increase in the number of firms with the potential to expropriate the same innovation leads to less expropriation. The intuition for this result is due to two reasons mentioned previously: the first is that as a given firm’s innovation reaches a greater number of competing firms, expropriation results in a smaller payoff for each firm considering it. This leads some of the firms to deviate to innovation, and makes expropriation even less desirable by increasing the probability of a subsequent innovation, which displaces the current innovation and eliminates the payoff from its expropriation.

The following proposition summarizes this result formally.

**Proposition 1** Let \( \frac{4(1-p)}{p(9-5p)} \leq \gamma < \frac{4}{5p} \). There exist values of \( k \) such that for \( k_1 < k \leq k_3 \), in the two-firm setting, conditional on firm 1 investing in innovation, being successful and its innovation reaching firm 2, there is always expropriation by firm 2. In the three-firm setting, however, firm 1’s innovation is expropriated only if it reaches firm 2 or firm 3 alone. If it reaches both firms, it is not expropriated, but advanced into subsequent innovations.

An important question that follows is how the equilibrium behavior of firm 2 and firm 3 if they both receive firm 1’s innovation restores incentives of firm 1 to invest in innovation at \( t = 0 \), given that subsequent innovations displace firm 1’s innovation and drive its second period profit to 0. To understand this, we next analyze firm 1’s innovation decision, anticipating the equilibrium behavior of firm 2 and firm 3. Let \( k \leq k_3 \) so that both firm 2 and firm 3 find it optimal to innovate rather than to expropriate, conditional on both receiving firm 1’s innovation. Suppose that firm 1 invests in innovation at \( t = 0 \), succeeds with probability \( p \), and obtains the first period payoff \( y_1 \). Conditional on its innovation traveling to both firm 1 and firm 2, and given the equilibrium behavior of firm 2 and firm 3, firm 1’s second period expected payoff at \( t = 2 \), denoted by \( v_{F1}^{i,i} \), is given by

\[
v_{F1}^{i,i} = (p^2 + 2p(1-p)) \times 0 + (1-p)^2 y_1.
\]
To see the computation of (4), in the second period, firm 1 obtains payoff $y_1$ with probability $(1 - p)^2$; that is, only if both firm 2 and firm 3 fail in generating a subsequent innovation. With probability $p^2$, both firm 2 and firm 3 are successful in generating a subsequent innovation and with probability $2p(1-p)$, only one of them is successful in generating a subsequent innovation. In both cases, subsequent innovation(s) displace firm 1’s innovation and drive down firm 1’s second period payoff to 0.3

Turning our attention now to firm 1’s ex ante expected profit at $t = 0$, let $\pi_1^3(I; i, i)$ denote firm 1’s ex ante expected profit from investing in innovation, conditional on firm 2 and firm 3 choosing innovation rather than expropriation if firm 1’s innovation travels to both of them at $t = 1$. It follows that

$$\pi_1^3(I; i, i) = -K + p(y_1 + q^2v_{F1}^{i,i} + 2q(1 - q)y_2 + (1 - q)^2y_1);$$

$$= -K + p((1 + q^2(1 - p)^2 + (1 - q)^2)y_1 + 2q(1 - q)y_2).$$

To see the computation of (5), after investing $K$, firm 1 is successful with probability $p$ in generating an innovation. At $t = 1$, it obtains the first period payoff from its innovation. Its second period payoff depends on whether its innovation travels to both firm 2 and firm 3, and whether it is expropriated or not. With probability $q^2$, its innovation travels to both firm 2 and firm 3, and as we show in (4), firm 1 obtains $v_{F1}^{i,i}$. With probability $2q(1 - q)$, its innovation travels to only firm 2 or firm 3, and is expropriated, given that there is only one firm to expropriate it, as we determine in the two-firm setting, yielding firm 1 payoff $y_2$. Finally, with probability $(1 - q)^2$, firm 1’s innovation travels to neither firm, yielding firm 1 payoff $y_1$.

Plugging $y_1 = \frac{(1-c)^2}{4}$, and $y_2 = \frac{(1-c)^2}{9}$ into (5) gives the ex ante expected profit of firm 1 as

$$\pi_1^3(I; i, i) = -K + p(18 - 9p(2 - p) q^2 - 10q(1 - q)) \frac{36}{(1 - c)^2}.$$
\( \pi_1^3(I; i, i) \) is non-negative for \( K \leq K_2 \), and hence, for \( K \leq K_2 \), firm 1 finds it optimal to invest in innovation at \( t = 0 \). Since we have from Lemma 1 that firm 1 does not invest in innovation in the two-firm setting for \( K > K_1 \), for values of \( K \) such that \( K_1 < K \leq K_2 \), there will be investment in innovation in the three-firm setting, but not in the two-firm setting.

The following proposition identifies conditions under which there is no investment in innovation by firm 1 in the two-firm setting, while there is investment in innovation in the three-firm setting.

**Proposition 2** Let \( q \geq \frac{5}{10 - 9p(2 - p)} \), and \( \frac{4(1 - p)}{p(9 - 9p)} \leq \gamma < \frac{4}{5p} \). There exist \( K \) and \( k \) such that for \( K_1 < K \leq K_2 \) and \( k_1 < k \leq k_3 \), there is no investment in innovation by firm 1 in the two-firm setting. In the three-firm setting, firm 1 invests in innovation, and conditional on firm 1’s innovation reaching both firm 2 and firm 3, there is subsequent investment in innovation by firm 2 and firm 3. In addition, for \( q \geq \frac{5}{10 - 9p(2 - p)} \) firm 1’s ex ante expected profit is increasing in \( q \) in the three-firm setting.

Although subsequent innovations by firm 2 and 3 displace firm 1’s innovation, firm 1 can still be better off in the three-firm setting than in the two-firm setting in terms of generating greater profits from investing in innovation. The reason is that with probability \( q^2 \), firm 2 and firm 3 do not expropriate firm 1’s innovation but rather invest in generating a subsequent innovation. With probability \((1 - p)^2\), both fail in generating an innovation and displacing firm 1’s innovation. Hence, as long as the value of \( p \) is not too high, there is a greater likelihood that firm 1 captures the full second period payoff from its innovation in the three-firm setting.

Proposition 2 shows that for high values of \( q \) and intermediate values of \( \gamma \), innovation incentives in the three-firm setting are stronger than those in the two-firm setting. The reason for why a high level of \( q \) implies stronger innovation incentives in the three-firm setting is the following. Conditional on firm 1 being successful in generating an innovation, the probability that firm 1’s innovation is expropriated is \( q \) in the two-firm setting while it is \( 2q(1 - q) \) in the three-firm setting. Since \( 2q(1 - q) < q \) for \( q > \frac{1}{2} \), an increase in the probability of innovation...
transmission from firm 1 to other firms leads to a lower probability of expropriation for high values of $q$. Similarly, an increase in $q$ leads to a greater probability $q^2$ that firm 1’s innovation travels to both firm 2 and firm 3, and is advanced into subsequent innovations rather than being expropriated. Hence, for sufficiently large values of $q$, in the three-firm setting an increase in $q$ also leads to greater profits from investing in innovation for firm 1, especially when $p$, that is, the probability of firm 2 and/or firm 3 displacing firm 1’s innovation is not too high. Note that the condition $q \geq \frac{5}{10-9p(2-p)}$ is more likely to be satisfied for smaller values of $p$, consistent with the intuition that firm 1’s innovation is less likely to be displaced for smaller values of $p$, leading to greater ex ante profits and incentives for firm 1 from investing in innovation.

The intuition for why innovation incentives are stronger in the three-firm setting relative to two-firm setting for intermediate values of $\gamma$ is the following. For large values of $\gamma$, the value of subsequent innovations is sufficiently high that expropriation incentives are minimum even in the two-firm setting. For very small values of $\gamma$, innovation incentives are not strong enough in the three-firm setting given that if both firm 2 and firm 3 are successful in generating a subsequent innovation, they end up competing in the product market, each firm capturing a smaller payoff. Hence, for innovation incentives to be stronger in the three-firm setting than two-firm setting, $\gamma$ cannot be too large or too small.

Until now, we assume that subsequent innovations by firm 2 and firm 3 are internalized only by the firms generating them. In other words, we rule out the possibility that a subsequent innovation by, for example, firm 2 can travel to firm 1 and/or firm 3 and is either expropriated or advanced into a subsequent innovation. The next section relaxes this assumption and investigates the robustness of the results in a more general setting.

### 3.3 Bidirectional Innovation Transmission

In the previous section, firm 1’s innovation generates profits for two periods, unless it is displaced by firm 2 and/or firm 3. After the first period-production, it travels to firm 2 and firm 3, and can either be expropriated or be advanced into a subsequent innovation. We assume that subsequent
innovations generated by firm 2 and firm 3 last only one period, and do not travel to firm 1 or to each other. In this section, we extend our model such that innovation transmissions are possible between any of the two firms. In other words, subsequent innovations generated by firm 2 and firm 3 travel to firm 1 as well as between them. Extending the model in this way is important for two reasons. The first is that the possibility of future innovations traveling back to firm 1 affects firm 1’s ex ante incentives to invest in innovation. The second is that allowing innovation transfers, say from firm 2 to firm 3, may reduce firm 3’s incentives to invest in innovation since by choosing the expropriation strategy, firm 3 will have the ability to expropriate firm 1’s innovation, and even if firm 1’s innovation gets displaced by firm 2, firm 3 can still expropriate firm 2’s innovation as long as there is some chance that firm 2’s innovation travels to firm 3. This same line of argument applies to firm 2 as well. Put differently, firm 2 may choose the expropriation strategy anticipating that it will have an ability to expropriate firm 1’s innovation as well as firm 3’s innovation in case firm 3 displaces firm 1’s innovation. This might lead to a free-rider problem and an inefficient equilibrium where both firm 2 and firm 3 choose expropriation over innovation, and in anticipation, firm 1 may choose not to invest in innovation even in the three-firm setting. Hence, allowing innovation transfers between any two firms is important for establishing the robustness of our results established in the previous section.

We modify the model such that if firm 1’s innovation travels to firm 2 and firm 3, and if they choose innovation, and are successful, their innovation generates profits for two periods at \( t = 2 \) and \( t = 3 \), instead of just for one period at \( t = 2 \). This implies that, for example, if firm 2 generates a subsequent innovation to firm 1’s innovation, after it implements the innovation and obtains the payoff from the innovation at \( t = 2 \), its innovation travels to firm 1 and firm 3, each with probability \( q \). In an infinite time horizon model, as long as an innovation flows from the innovating firm to two other firms at the same time, the basic intuition of the model from the earlier analysis will continue to hold and the two firms will not expropriate the innovation but will learn from it. In our simpler model with a finite horizon, since \( t = 3 \) is the final date of the model, we assume that firm 1 and firm 3 receiving firm 2’s innovation always find it optimal
to expropriate it. Our objective with this modification is to show that our main result that innovation incentives are greater in the three-firm setting continues to hold even when we allow for the possibility that firm 2 and firm 3’s subsequent innovations are expropriated by each other as well as by firm 1, the same way as firm 1’s innovation can be expropriated.

### 3.3.1 Two-firm setting

As in the previous section, firm 1 chooses between $I$ and $NI$ at $t = 0$, and, if it chooses $I$, it is successful in obtaining an innovation with probability $p$ at $t = 1$. After it obtains the first period payoff from its innovation, the innovation travels to firm 2 with probability $q$ at which point firm 2 makes a decision about whether to expropriate the innovation, or to innovate by learning from it. If firm 2 chooses expropriation, it obtains profit $y_2$ at $t = 2$ and the game ends. If it chooses innovation, it invests $k$, and is successful in generating an innovation with probability $p$, and obtains the first period payoff $Y_1$ from its innovation at $t = 2$. Different from the previous section, this subsequent innovation travels back to firm 1 with probability $q$, and firm 1 makes a decision between expropriation and innovation. Since $t = 3$ is the last date in the model, we assume that firm 1 always finds it optimal to expropriate firm 2’s innovation, yielding each firm payoff $Y_2$, at $t = 3$.

Given that firm 1 expropriates the subsequent innovation of firm 2 with probability $q$, firm 2’s investment in innovation results in an expected payoff $-k + p(Y_1 + qY_2 + (1 - q)Y_1)$. Firm 2 compares this payoff to that from expropriating firm 1’s innovation, given by $y_2$. It chooses to expropriate firm 1’s innovation for $k > k_4 \equiv p(2Y_1 - q(Y_1 - Y_2)) - y_2$. Plugging the values of $y_2, Y_1$ and $Y_2$, firm 2 expropriates firm 1’s innovation for $k > k_4 = \frac{p(18 - 5q) - 4}{36}(1 - c)^2$. Suppose $k > k_4$ holds. Firm 1’s ex ante expected profit from investing in innovation at $t = 0$ is then given by $-K + p(y_1 + qy_2 + (1 - q)y_1)$, the same as in the previous section, given that for $k > k_4$, it anticipates the expropriation behavior of firm 2. The only difference from the earlier analysis is that the threshold level of $k$ above which firm 2 expropriates firm 1’s innovation is given by $k_4$, with $k_4 > k_1$. This implies that firm 2’s expropriation incentives are weaker now due to the
simple reason that its innovation lasts two periods and generates a greater payoff than before. However, for \( k > k_4 \), there is no change in the expropriation behavior of firm 2, and, hence, in the ex ante innovation incentives of firm 1. The following Lemma presents this result.

**Lemma 3** There exist values of \( k, K \) such that for \( k > k_4 \) and \( K > K_1 \) there is no investment in innovation in the two-firm setting.

### 3.3.2 Three-firm setting

We now proceed with the three-firm setting where each subsequent innovation generated by firm 2 and firm 3 travels to firm 1 with probability \( q \) as well as to each other.

Suppose firm 1 chooses \( I \) at \( t = 0 \), and is successful with probability \( p \). Let \( v^S \) denote firm 1’s expected payoff conditional on success. Its ex ante expected profit from investing in innovation at \( t = 0 \) denoted by \( \pi^3_{F1}(I) \) is given by

\[
\pi^3_{F1}(I) = -K + pv^S. \tag{7}
\]

As before, firm 1 obtains the first period payoff \( y_1 \) from its innovation, and in the second period with probability \( q^2 \), firm 1’s innovation travels to both firm 2 and firm 3. Let \( v^2_{F1} \) denote firm 1’s second period expected payoff in this state where subscript 2 refers to the number of firms receiving firm 1’s innovation. With probability \( 2q(1 - q) \), its innovation travels only to firm 2 or firm 3. As we show in the previous section, it is expropriated for \( k > k_4 \), yielding firm 1 second period payoff \( y_2 \). Suppose \( k > k_4 \) holds and let \( v^1_{F1} \equiv y_2 \) denote firm 1 second period payoff conditional on its innovation traveling to only one of firm 2 or firm 3. With probability \( (1 - q)^2 \), it does not travel to either firm 2 or firm 3. Let \( v^0_{F1} \) is defined as firm 1’s second period payoff conditional on its innovation not traveling to either firm, with \( v^0_{F1} \equiv y_1 \). Hence, we can write \( v^S \) in (7) as

\[
v^S = y_1 + q^2 v^2_{F1} + 2q(1 - q)v^1_{F1} + (1 - q)^2 v^0_{F1}; \tag{8}
\]

Plugging \( v^1_{F1} = y_2 \) and \( v^0_{F1} = y_1 \) into (8), we obtain

\[
v^S = y_1 + q^2 v^2_{F1} + 2q(1 - q)y_2 + (1 - q)^2 y_1. \tag{9}
\]
To determine $v_{F1}^2$, we need to understand the equilibrium behavior of firm 2 and firm 3 when firm 1’s innovation travels to both firms, which happens with probability $q^2$. First suppose that both firm choose expropriation, each obtaining $v_{F2}^{e,e} = v_{F3}^{e,e} = y_3$. Now, consider that firm 2 deviates to innovation while firm 3 chooses expropriation. Firm 2’s expected payoff from innovation, given firm 3 chooses expropriation denoted by $v_{F2}^{i,e}$ is

$$v_{F2}^{i,e} = -k + p(Y_1 + q^2 Y_3 + 2q(1 - q)Y_2 + (1 - q)^2 Y_1).$$  \hspace{1cm} (10)$$

To see the derivation of (10), if firm 2 is successful in generating a subsequent innovation, after it obtains the first period payoff $Y_1$, with probability $q^2$, its innovation travels to both firm 1 and firm 3, and is expropriated, yielding each of the three firms payoff $Y_3$. Under Cournot competition with three-firms, it is straightforward to show that $Y_3 = \frac{(1 - \beta)^2}{\beta} = \frac{\gamma(1 - c)^2}{16}$.

The innovation travels to either firm 1 or firm 3 with probability $2q(1 - q)$, and is expropriated, yielding firm 2 payoff $Y_2$. Finally, with probability $(1 - q)^2$, the innovation does not travel to either firm 1 or firm 3, and firm 2 obtains second period payoff $Y_1$. Firm 2 deviates to innovation from expropriation if and only if $v_{F2}^{i,e} \geq v_{F2}^{e,e}$, or equivalently if $k \leq k_5$ where $k_5$ is defined as

$$k_5 \equiv p(Y_1 + q^2 Y_3 + 2q(1 - q)Y_2 + (1 - q)^2 Y_1) - y_3.$$  \hspace{1cm} (11)$$

Plugging $y_3, Y_1, Y_2$, and $Y_3$ into (11), we find that firm 2 deviates to innovation for

$$k \leq k_5 = \frac{\gamma pq (13q - 40) + 9(8\gamma p - 1)}{144} (1 - c)^2.$$$ \hspace{1cm} (12)$$

Consider now firm 3’s decision between expropriation and innovation, given that firm 2 chooses innovation. Firm 3’s expected payoff from expropriation given that firm 2 chooses innovation denoted by $v_{F3}^{i,e}$ is

$$v_{F3}^{i,e} = (1 - p)y_2 + p(q^2 Y_3 + q(1 - q)Y_2).$$  \hspace{1cm} (13)$$

As before, given that firm 2 chooses innovation, firm 3 expropriates firm 1’s innovation in the first period as long as firm 2 fails to displace firm 1’s innovation, which happens with probability $(1 - p)$. Different from the previous analysis where innovation transfers between firm 2 and firm 3 are not possible, now if firm 2 is successful in generating an innovation, its innovation travels to
firm 1 and firm 3 with probability $q^2$, where the three firms compete and each firm obtains payoff $Y_3$. With probability $q(1 - q)$, firm 2’s innovation travels to only firm 3, and is expropriated by firm 3, yielding firm 3 payoff $Y_2$. Firm 3 compares its expected payoff from expropriation given in (13) to its expected payoff from innovation in deciding whether to deviate to innovation. Hence, we proceed to obtain the expected payoff for firm 3 from deviating to innovation given that firm 2 chooses innovation. Let $v_{F3}^{ij}$ denote expected payoff of firm 3 given that both firm 2 and firm 3 choose to innovate by learning from firm 1’s innovation. We have

$$v_{F3}^{ij} = -k + p^2(Y_2 + q^2Y_3 + 2q(1 - q)Y_3 + (1 - q)^2Y_2)$$

(14)

$$+ p(1 - p)(Y_1 + q^2Y_3 + 2q(1 - q)Y_2)$$

(15)

$$+(1 - p)p(q^2Y_3 + q(1 - q)Y_2).$$

(16)

The first line in (14) presents, net of cost $k$ of investment in innovation, the expected payoff of firm 3 from investing in innovation, conditional on both firm 2 and firm 3 succeed in generating a subsequent innovation, and end up competing in the product market, each obtaining payoff $Y_2$ in the first period.\(^4\) In the second period, with probability $q^2$, both firms’ innovations travel to firm 1 with probability $q^2$, and only one of the firms’ innovation travels to firm 1 with probability $2q(1 - q)$. In each case, given that there is only one date $t = 3$ left in the game, firm 1 expropriates the innovation(s) it receives, yielding each of the three firms payoff $Y_3$. With probability $(1 - q)^2$, neither firms’ innovation travels to firm 1, and hence, firm 2 and firm 3 obtain payoff $Y_2$ from their innovations.

The second line in (14) corresponds to the case where only firm 3 succeeds in generating a subsequent innovation. After it obtains the first period payoff $Y_1$ from its innovation, the

\(^4\)Note that when both firm 2 and firm 3 are successful in generating an innovation, each firm’s innovation travels to the other firm with probability $q$. Since we assume that these subsequent innovations are substitutes, and the two firms compete in the product market despite being successful in innovation, learning about each other’s innovation does not matter. Also note that the assumption that the two firms end up with perfectly substitute innovations if both of them are successful makes it harder for firm 2 and firm 3 to favor innovation over expropriation. Hence, our result that innovation incentives are stronger when an innovation reaches a greater number of firms would hold more easily in a set-up where subsequent innovations of firm 2 and firm 3 are not perfect substitutes.
innovation travels to both firm 1 and firm 2, and is expropriated, yielding firm 3 payoff $Y_3$. With probability $2q(1-q)$, it travels to only firm 1 or firm 3, and is expropriated, yielding firm 3 payoff $Y_2$. Finally, line 3 in (14) gives the expected payoff for firm 3 conditional on firm 3 failing and firm 2 succeeding in generating a subsequent innovation. In this case, with probability $q^2$, firm 2’s innovation travels to both firm 1 and firm 3, and is expropriated, yielding firm 3 payoff $Y_3$. With probability $q(1-q)$, it travels to only firm 3, and is expropriated, yielding firm 3 payoff $Y_2$.

Firm 3 compares its expected payoff from innovation given in (14) to that from expropriation given in (13), and chooses to innovate if $v_i^{i;e}_{F3} \geq v_i^{i;i}_{F3}$, or equivalently for $k \leq k_6$ where $k_6$ is defined as

\[
k_6 = p^2(Y_2 + q^2Y_3 + 2q(1-q)Y_3 + (1-q)^2Y_2) + p(1-p)(Y_1 + q^2Y_3 + 2q(1-q)Y_2) + (1-p)p(q^2Y_3 + q(1-q)Y_2) - (1-p)y_2 - p(q^2Y_3 + q(1-q)Y_2).
\]

Plugging $y_2$, $Y_1$, $Y_2$, and $Y_3$ into (17), we find that firm 3 deviates to innovation for

\[
k \leq k_6 = \frac{p\gamma((37p - 23)q^2 - (62p - 32)q - 4(p - 9)) - 16(1-p)}{144} (1-c)^2.
\]

For values of $k$ such that $0 < k \leq \min\{k_5, k_6\}$, both firm 2 and firm 3 unilaterally choose to deviate to innovation, and as before, in the state where firm 1’s innovation reaches both firm 2 and firm 3, it is not expropriated, but advanced into subsequent innovations by firm 2 and firm 3. Different from the earlier analysis, the subsequent innovations of firm 2 and firm 3 travel to firm 1 as well as to each other, and are expropriated in the last period. It is possible to show that we have $k_6 < k_5$ for all parameter values. Hence, it follows that firm 2 and firm 3 choose innovation for $k \leq k_6$ in the three-firm setting. In addition, for values of $k_4 < k \leq k_6$, there is always expropriation in the two-firm setting while in the three-firm setting, there is expropriation only if firm 1’s innovation reaches one of firm 2 or firm 3. Otherwise, if it reaches both firms, there is
no expropriation. The following proposition presents the conditions under which expropriation incentives are weaker in the three-firm setting than in the two-firm setting.

**Proposition 3** Let $q = 1$ and $\frac{7}{23} < p \leq \frac{9}{29}$. There exist values $\gamma$ and $k$ such that for $\frac{16(1-p)}{p(45-29p)} < \gamma < \frac{16}{7+29p}$ and $k_4 < k \leq k_6$, firm 1’s innovation is always expropriated in the two-firm setting, while it is never expropriated in the three-firm setting.

For analytical tractability, we set $q = 1$ in Proposition 3. This implies that firm 1’s innovation travels to other firms with probability 1. Hence, it is always expropriated by firm 2 in the two-firm setting while it is never expropriated in the three-firm setting. Importantly, firm 2 and firm 3 choose not to expropriate even though in the last period of the model, their innovations end up being expropriated for sure given that $q = 1$. As mentioned before, this result is important for establishing the robustness of the main intuition of the model.

Having established the conditions under which firm 1’s innovation is not expropriated, conditional on reaching both firm 2 and firm 3, we now turn our attention to firm 1’s ex ante incentives to invest in innovation, anticipating the subsequent equilibrium behavior of firm 2 and firm 3. Recall from (5) firm 1’s ex ante expected profit from investing in innovation at $t = 0$, is

$$\pi_{F_1}^3(I) = -K + pv^S,$$

(19)

where $v^S$ is given by (9),

$$v^S = y_1 + q^2v_{F_1}^2 + 2q(1 - q)y_2 + (1 - q)^2y_1.$$  

(20)

As explained before, if firm 1 is successful in generating an innovation, it obtains the first period payoff $y_1$. In the second period, its innovation reaches one of firm 2 or firm 3 with probability $2p(1 - p)$, and is expropriated, yielding firm 1 payoff $y_2$. With probability $(1 - q)^2$, it does not reach any of the firms, yielding firm 1 payoff $y_1$. With probability $q^2$, it reaches both firm 2 and firm 3. Suppose that the conditions in Proposition 3 hold, implying that both firm 2 and firm 3 choose innovation over expropriation. Firm 1’s expected payoff $v_{F_1}^2$ is then given by

$$v_{F_1}^2 = p^2(q^2Y_3 + 2q(1 - q)Y_3) + 2p(1 - p)(q^2Y_3 + q(1 - q)Y_2) + (1 - p)^2y_1.$$  

(21)
With probability $p^2$, both firm 2 and firm 3 succeed in generating a subsequent innovation. Both innovations travel to firm 1 with probability $q^2$, only one of them travels to firm 1 with probability $2q(1 - q)$. In both cases, given the last period of the model, firm 1 chooses to expropriate the innovation(s) and the three firm end up competing, each obtaining payoff $Y_3$. With probability $2p(1 - p)$, only one of firm 2 or firm 3, say firm 2, succeeds in generating an innovation. The innovation travels to both firm 1 and firm 3 with probability $q^2$, and is expropriated, yielding each of the three firms payoff $Y_3$. With probability $q(1 - q)$, it reaches only firm 1, and is expropriated, yielding firm 1 payoff $Y_2$. Finally, with probability $(1 - p)^2$, both firm 2 and firm 3 fail to generate an innovation and displace firm 1’s innovation, in which case, firm 1 obtains second period payoff $y_1$ from its own innovation.

Plugging $y_1, Y_2, Y_3$ into (21), and plugging $v^2_{F_1}$ back into (20), and $v^S$ back into (19), we obtain firm 1’s ex ante expected profit from investing in innovation as

$$
\pi^3_{F_1}(I) = -K + \frac{p(\gamma pq^3(5pq + 32 - 14(p + q)) + 4q(10(q - 1) + 9pq(p - 2)) + 72)}{144}(1 - c)^2. \quad (22)
$$

Firm 1 invests in innovation for $K \leq K_3$ where $K_3$ is defined as

$$
K_3 = \frac{p(\gamma pq^3(5pq + 32 - 14(p + q)) + 4q(10(q - 1) + 9pq(p - 2)) + 72)}{144}(1 - c)^2. \quad (23)
$$

For values of $K$ such that $K_1 < K \leq K_3$, there will not be investment in innovation in the two-firm setting while firm 1 will invest in innovation in three-firm setting. The following Proposition shows that under the conditions identified in Proposition 3, such values of $K$ always exist, implying that investment in innovation is a negative NPV project in the two-firm setting and a positive NPV project in the three-firm setting.

**Proposition 4** Let $q = 1$ and $\frac{7}{23} < p \leq \frac{9}{29}$. There exist values $\gamma$, $k$ and $K$ such that for $\frac{16(1-p)}{p(45 - 29p)} < \gamma < \frac{16}{17 + 29p}$, $k_4 < k \leq k_6$, and $K_1 < K \leq K_3$, there is no investment in innovation in the two-firm setting while there is investment in innovation in the three-firm setting.

Compared to the earlier analysis where subsequent innovations of firm 2 and firm 3 do not travel to firm 1, and hence, firm 1 has no ability to expropriate them, it is obvious that firm 1
obtains a greater expected profit from investing in innovation, and hence, has greater incentives to invest in innovation at $t = 0$. However, what is critical is that firm 1’s expected profit is conditional on firm 2 and firm 3 choosing innovation over expropriation, despite the fact that their innovations will flow to other firms, and end up being expropriated.

4 Implications

Our model shows that expropriation incentives decline and innovation incentives improve as the number of firms which compete with each other as well as learn from each other increases. This result suggests that a concentrated mass of interconnected firms that both compete and learn from each other will lead to an emergence of innovation waves where firms build on current innovations of their competitors to generate follow-up innovations. This implies a shorter life cycle for innovations where a current innovation spurs future innovations, but at the same time ends up getting displaced by future innovations. This implication of our model is consistent with serial innovations observed in high-tech industries where firms continuously learn from each other and advance each other’s innovations.

Our model is also consistent with the clustering patterns observed in IPOs. In a survey of CFOs, Braun and Fawcett (2006) find that one of the important concerns for private firms in deciding whether to go public is the potential loss of proprietary information to competing firms. Our paper shows that as long as there is a sufficient number of competing firms which learn about a firm’s proprietary information, incentives to expropriate the firm’s critical information will be weaker, while incentives to learn from that information to produce new information will be stronger. This result is consistent with the clustering patterns of IPO activity over time and across industries in that the cost of doing an IPO in terms of loss of proprietary information of a private firm will be smaller when the firm’s proprietary information flows freely to a greater number of firms, that is, when a greater number of firms go public around the same time, or when there is already sufficient number of public firms in the private firm’s industry.

In our model, a high value of $q$ implies a greater probability of an innovation reaching a
greater number of competing firms, and a lower probability for the innovation to be expropriated. This result suggests that a free circulation of ideas among competing firms enhances innovation incentives, and result in a series of innovations by competing firms. For sufficiently high values of $q$, an increase in the number of competing firms results in a greater likelihood of each firm innovating and obtaining greater profits as well as a more intense competition in the product market. This result suggests that a series of mergers or a merger wave in an industry would reduce innovation incentives given that post-merger firms would have reduced incentives to invest in innovative ideas, anticipating that their ideas are more likely to be expropriated due to the fact that they would circulate among a smaller number of firms.

5 Conclusions

This paper studies a model of learning across peer firms where an innovation by a firm leads to subsequent innovations by peer firms. As an innovation of a firm travels to a greater number of peer firms, it becomes less likely to be expropriated, but to be advanced into a subsequent innovation, due to the intuition that expropriation payoff declines in the number of firms which expropriate the same innovation. As a result, some peer firms find it optimal to learn from the current innovation to generate its own innovation. Interestingly, this creates an additional mechanism which reduces the desirability of expropriation even further, since subsequent innovations make current innovations obsolete and drive their payoff to zero.

Our model’s results are consistent with the empirical evidence in Matray (2014) that innovation by one firm fosters innovation by neighboring firms through a channel of cross learning. In addition, our model shows that the larger the intensity of information flow among peer firms is, the less likely an innovation gets expropriated, consistent with the finding in Matray (2014). This result is based on the intuition that a more efficient circulation of an innovation implies that the innovation reaches with a greater probability to a greater number of peer firms, reducing for each firm’s desire to expropriate the innovation.

Our paper provides an explanation for why clusters of interconnected firms that both compete
and learn from each other is a necessary condition for innovation incentives to take-off. This is consistent with observed free flow of ideas across competing firms in Silicon Valley where companies learn from each other while they intensely compete in similar product, labor and capital markets.
References


Matray, A, (2014), The Local Innovation Spillovers of Listed Firms, working paper.


Appendix

Proof of Lemma 1: From the discussion preceding the proposition, we have that for $k > pY_1-y_2$, if firm 2 receives firm 1’s innovation, it always expropriates it. From (1) firm 1’s ex ante expected profit conditional on its innovation being expropriated is negative for $K > p(2y_1-q(y_1-y_2))$. Plugging the values of $y_1$, $y_2$ and $Y_1$, we obtain that firm 1 does not invest in innovation for $k > k_1 = (\frac{9p-4}{36})(1-c)^2$ and $K > K_1 = p(\frac{18-5q}{36})(1-c)^2$. Note that the values of $k > 0$ and $K > 0$, with $k < K$ such that $k > k_1$ and $K > K_1$ always exist.
Proof of Lemma 2: Starting from the case where both firm 2 and firm 3 choose expropriation, for \( k \leq k_2 \), firm 2 finds it optimal to switch to innovation. Hence, expropriation by each firm cannot be an equilibrium for \( k \leq k_2 \). For \( k \leq k_3 \), firm 3 finds it optimal to switch to innovation as well, given that firm 2 chooses innovation. Hence, for \( k \leq k_2 \) and \( k \leq k_3 \), innovation by firm 2 and expropriation by firm 3 cannot be an equilibrium either, and the unique equilibrium is where both firm 2 and firm 3 choose innovation. Since we have \( k_3 < k_2 \), for \( k \leq k_3 \) the unique equilibrium of the innovation game at \( t = 1 \) is that both firm 2 and firm 3 invest in innovation, conditional on firm 1’s innovation reaching both of them.

Proof of Proposition 1: From Lemma 2, we have that in the three-firm setting, firm 2 and firm 3 choose innovation in equilibrium for \( k \leq k_3 \). From the discussion in the text and Lemma 1, we have that firm 2 chooses expropriation for \( k > k_1 \). Hence, for values of \( k \) such that \( k_1 < k \leq k_3 \), there is always expropriation in the two-firm setting provided firm 1’s innovation travels to firm 2, while in the three-firm setting, there is expropriation if firm 1’s innovation travels to only firm 2 or firm 3. If it travels to both firms, firm 2 and firm 3 choose innovation. Note that for \( \frac{4(1-p)}{p(9-5p)} \leq \gamma < \frac{4}{5p} \), we have \( k_1 < k_3 \) and \( k_3 > 0 \) so that the values of \( k > 0 \) such that \( k_1 < k \leq k_3 \) always exist.

Proof of Proposition 2: For \( k_1 < k \leq k_3 \), firm 1’s ex ante expected profit from investing in innovation in the two-firm setting is given by (1) and in the three-firm setting is given by (5). Firm 1 invests in innovation in the three-firm setting for \( K \leq K_2 = p \left( \frac{18 - p(2-p)q^2 - 10q(1-q)}{36} \right) (1 - c)^2 \) and does not invest in innovation in the two-firm setting for \( K > K_1 = p \left( \frac{18 - 5q}{36} \right) (1 - c)^2 \). This implies that for values \( k \) and \( K \) such that \( k_1 < k \leq k_3 \) and \( K_1 < K \leq K_2 \), there is investment in innovation in the three-firm setting, and no investment in innovation in the two-firm setting. For \( \frac{4(1-p)}{p(9-5p)} \leq \gamma < \frac{4}{5p} \), we have \( k_1 < k_3 \) and \( k_3 > 0 \) so that the values of \( k > 0 \) such that \( k_1 < k \leq k_3 \) always exist. It follows immediately from the definitions of \( K_1 \) and \( K_2 \) that for \( q \geq \frac{5}{10-9p(2-p)} \), we have \( K_1 < K_2 \) so that the values of \( K \) such that \( K_1 < K \leq K_2 \) always exist. Taking the partial derivative of (5) with respect to \( q \) yields \( \frac{\partial \pi_3(I;i,i)}{\partial q} = p \left( \frac{q(20-2p(2-p))}{36} \right) (1 - c)^2 \). It is immediate to see that \( \frac{\partial \pi_3(I;i,i)}{\partial q} \geq 0 \) for \( q \geq \frac{5}{10-9p(2-p)} \).

Proof of Lemma 3: Conditional on firm 1’s innovation traveling to firm 2, firm 2 expropriates firm 1’s innovation for \( k > k_4 \). Firm 1’s expected profit from investing in innovation is given by \( p \left( \frac{18 - 5q}{36} \right) (1 - c)^2 - K \).
, conditional on its innovation traveling to firm 2 and is expropriated by firm 2. Hence, for \( k > k_4 \) and \( K > K_1 = p(\frac{18 - 5q}{36})(1 - c)^2 \), firm 1 chooses not to invest in innovation for \( k \), given that doing so leads to negative NPV. Note that from the definitions of \( k_4 \) and \( K_1 \), it is immediate to see that the values of \( k > 0 \) and \( K > 0 \), with \( k < K \) and \( k > k_4 \) and \( K > K_1 \) always exist.

Proof of Proposition 3: Setting \( q = 1 \), and using the definitions of \( k_5 \) and \( k_6 \), it is possible to show that \( k_6 < k_5 \), implying that \( k_6 = \min(k_5, k_6) \). From the definitions of \( k_4 \) and \( k_6 \), it follows that for \( \frac{7}{23} < p \leq \frac{9}{29} \), and \( \frac{16(1-p)}{p(45 - 29p)} < \gamma < \frac{16}{7 + 29p} \), we have \( \max(0, k_4) < k_5 \), implying that there always exist values of \( k \) such that \( \max(0, k_4) < k_6 < k_5 \). For such values of \( k \), if firm 1’s innovation travels to firm 2 in the two-firm setting, it is always expropriated. In the three-firm setting, if it travels to only firm 2 or firm 3, it is expropriated while if it travels to both firms, firm 2 and firm 3 choose innovation over expropriation.

Proof of Proposition 4: Setting \( q = 1 \) and \( \frac{7}{23} < p \leq \frac{9}{29} \), and from the definitions of \( K_1 \) and \( K_3 \), it follows that \( 0 < K_1 < K_3 \) for \( \gamma < \frac{36p(2-p)(2-p) + 16}{52p - 9p^2 (2-p)} \). Since \( \frac{16}{7 + 29p} < \frac{36p(2-p)(2-p) + 16}{52p - 9p^2 (2-p)} \), for \( \gamma \) such that \( \frac{16(1-p)}{p(45 - 29p)} < \gamma < \frac{16}{7 + 29p} \), values of \( K \) such that \( K_1 < K < K_3 \) always exist, and for such values of \( K \), investing in innovation has a negative NPV in the two-firm setting while it has a positive NPV in the three-firm setting.