Sequential Credit Markets
(Preliminary and Incomplete)

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ABSTRACT

Entrepreneurs who seek financing for projects typically do so in decentralized markets where they need to approach investors sequentially. We study how well such sequential markets allocate resources when investors have valuable but dispersed information. The sequential nature of the market introduces endogenous adverse selection which prevents information from being fully aggregated and leads to substantial investment inefficiencies. Contrary to what is commonly believed, we show that when the application history of a borrower becomes observable, e.g., through a credit bureau, it leads to more adverse selection, quicker market break downs, and higher rents for investors. Nevertheless, a sequential search market with a credit bureau can be more efficient than a centralized exchange where excessive competition may impede information aggregation. We also show that investors who rely purely on hard information in their lending decisions can out-compete better informed investors with soft information, and an introduction of interest rate caps can increase the efficiency of the market.

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The main role of primary financial markets is to channel resources to firms with worthwhile projects, a process that requires information about investment opportunities. Investors with expertise in evaluating projects, such as venture capitalists, business angels, or commercial banks, therefore can serve an important role for the growth of the real economy. Since no single investor usually has all the information for deciding whether a project should be pursued or not there is a need for financial markets to aggregate information.

The previous literature have studied extensively the question of information aggregation in centralized market places such as organized exchanges. However, primary capital markets for small- and medium sized firms are almost exclusively decentralized markets in which firms approach potential investors sequentially (one-by-one). This is true whether firms are seeking capital from banks or from equity investors such as business angels and venture capitalists.

In this paper, we ask how well we can expect these decentralized markets to solve the resource allocation problem when investors have valuable but dispersed information. We develop a general but tractable framework that allows us to study this question in a number of settings that differ in the amount of transparency about the application history of a potential borrower and how verifiable the information is that is used. Our model has implications for the efficiency of resource allocation and the informational rent captured by investors when the size of the market changes. We also contrast the solution to one in which investors compete to finance projects in a centralized auction, a case we have studied in a companion paper (Axelson and Makarov (2016)).

We consider a setting in which an entrepreneur with a project idea searches for credit by approaching potential financiers sequentially, until she either finds an investor who is willing to accept her terms for financing the project or runs out of options and abandons the project. Unlike standard search models which focus on the friction introduced by the cost of finding a counterparty, we are interested in the consequences of sequential interactions. We therefore assume that the entrepreneur is infinitely patient and has no search cost, so that all our results are driven by informational frictions. We assume that there is uncertainty about whether the project is worthwhile or not. Each investor, if approached, can do due diligence which results in a private signal about the prospects of the project. Although the aggregate information held by all investors is valuable for distinguishing good projects from bad, a number of problems impede information aggregation.

First, whenever the entrepreneur comes to an agreement with an investor, information aggregation stops although there is potentially valuable information held by investors who have not yet been approached. Second, when the entrepreneur does not
come to an agreement with an investor and continues her search, not all of the information held by the investor is passed on to the next investor she meets. In particular, each new investor faces an adverse selection problem created by the fact that the entrepreneur failed to receive financing in previous interactions with informed investors, which could potentially lead to a market break down.

We first show that despite these problems, in the benchmark case where the entrepreneur can commit to maximizing social surplus rather than her own expected profits, the market outcome will be as efficient as the one achieved in a centralized first-price auction where an optimally chosen set of market participants can be invited to compete for the financing of the project, and where the investment decision can be based on all bids received. The way this is accomplished is by asking for financing at an interest rate which is increasing in the number of rounds the entrepreneur has been on the market, where the interest rate is set such that each investor accepts if his signal is above the same optimally chosen cut-off rate.

We then turn to the more realistic case where the entrepreneur maximizes her own expected profits. We show that the efficiency of the market and the way surplus is split depends critically on whether the sequence in which investors is approached is observable or not. Both cases are empirically relevant. In most developed countries, any bank that is approached for financing will submit a credit check to a credit bureau or credit registry, and will also learn from the credit bureau who else has performed a credit check on the borrower in the past. Hence, the investor will learn how many times the borrower has applied for financing previously. We refer to the case where the sequence is observable as the “credit bureau” case. In the second case, which we refer to as the “no credit bureau” case, a lender does not know how many other lenders an applicant has visited before. This is commonly the case in less developed countries, in informal lending markets, and in non-bank markets such as when an entrepreneur seeks angel- or venture capital financing.

We show that a credit bureau can have either positive or negative effect on social surplus and the profit of the entrepreneur. To understand the trade-off, consider the case with a credit bureau in place. Each time an entrepreneur is rejected, the rejection is recorded in the credit bureau so that remaining investors revise their beliefs about the quality of the project downwards. The impact of a rejection on the beliefs of remaining investors depends on the terms at which they believe the entrepreneur was rejected—if they believe the entrepreneur asked for financing at very favorable terms (a low interest rate), a rejection is not such bad news. Because these terms are not directly observable, the entrepreneur cannot affect the beliefs of investors and improve her prospects in future rounds by asking for more favorable terms in the current round.
In equilibrium, this biases her towards asking for less favorable terms.

When there is no credit bureau, an investor cannot verify how many times an applicant has been rejected previously. This is potentially bad for an entrepreneur who has not been rejected, since she might be pooled with rejected entrepreneurs with worse credit quality. A first-time applicant therefore has an incentive to signal her type, and we show that she will always be able to do so by asking for more favorable financing terms (a lower interest-rate loan). This is a credible signal, because a request for more favorable terms has a higher probability of rejection, and rejection is less costly for a first-time applicant who has many investors left to visit. This logic extends to all rounds, leading to a fully separating equilibrium where the entrepreneur asks for slightly less favorable terms with each rejection. Thus, the need for signalling creates a credible way for the entrepreneur to ask for favorable terms early on.

Asking for favorable financing terms early on has two consequences. First, it reduces the rents to investors. We show that as the number of potential investors grows large, investors’ rent is competed away in the case of no credit bureau. In constrast, in the case of a credit bureau, investors continue to earn significant rents even though the entrepreneur has zero search costs and all the bargaining power.

Second, asking for favorable financing terms leads to more financing rounds relative to the case with a credit bureau because credit quality deteriorates slower with each rejection. In the case of no credit bureau, the entrepreneur can visit all the available investors. In contrast, in the case of a credit bureau, the entrepreneur might get locked out of the market after a single rejection even when there is a large set of potential investors.

The benefits of having extended search depend on the informational content of the signal distribution. The way many financing rounds are sustained is by asking for offers that only the most optimistic investor would accept, while less optimistic information is never incorporated in the financing decision. As a result, extended search is desirable in situations where the informational content of the signal distribution is concentrated towards the top. We show that for these situations, as the number of potential investors grows large, the social surplus without a credit bureau approaches that attained in a large first-price auction, which is also the maximal possible one.

However, extended search can lead to less informative financing decisions in situations where the informational content of the signal distribution is not concentrated towards the top. For these situations, the market with a credit bureau and few financing rounds turns out to be more efficient and can dominate even a centralized auction market. Although we have established that a central auction market with an optimally chosen number of investors is always better than a sequential market, it may
not always be easy to commit to limit the number of participants in an auction. In the credit bureau market, there is no need for such a commitment—the market breaks down endogenously after a limited set of rounds. Hence, the market with a credit bureau can lead to higher social surplus than a large auction market because it restricts the competition among investors, allowing them to utilize their information more efficiently. Surprisingly, the increased surplus can more than compensate for the higher rent left to investors, so that the entrepreneur can also be better off than in an auction market.

We also show that the sequential market with a credit bureau can have multiple equilibria, due to the feedback effect of equilibrium beliefs. When investors believe that rejected borrowers have low credit quality, rejection is more costly for entrepreneurs. Therefore, entrepreneurs will be more likely to ask for unfavorable financing terms in early rounds to avoid rejection, which means that rejection is a signal of worse quality—a self-fulfilling prophesy. Hence, equilibria with few financing rounds and equilibria with more financing rounds can coexist. The equilibria with few financing rounds are often worse for entrepreneurs because of the unfavorable financing terms, but can be good for social surplus. This gives the surprising implication that social welfare can be improved if the government imposes an interest rate cap. An interest rate cap will eliminate “sub-prime” markets for rejected borrowers, and hence will eliminate the socially inefficient equilibria with many financing rounds.

Finally, in our main analysis, all investors have access to privately observed “soft information.” We also consider an extension where some investors do not have such information, or can commit not to use it and instead only rely on publicly available “hard information” in their lending decisions. Surprisingly, we show that such lenders are sometimes able to out compete soft information lenders when there is a credit bureau, even though they have strictly less information. The reason is that a hard information lender never makes any rents, which for high credit quality entrepreneurs can make them more attractive despite the lower surplus created.

**Related literature:** [To be completed]

1. **Setup**

We consider a penniless entrepreneur seeking outside financing for a new project from a set of $N < \infty$ investors indexed by $i \in \{1, ..., N\}$.\(^{1}\) All agents are risk neutral.

\(^{1}\)Although we assume the entrepreneur has zero wealth to invest in the project, this is not essential for our results. Our results generalize to situations where the entrepreneur has either wealth or other assets to pledge against the project.
The project requires one unit of investment, and can be of two types: good \((G)\) and bad \((B)\), where the unconditional probability of the project being good is \(\pi\). If the project is good it pays \(1 + X\). Otherwise, it pays 0. We denote the net present value, or NPV, of the project by \(V\), a random variable that takes value \(X\) if the project is good and value \(-1\) if the project is bad.

No one knows the type of the project but investors have access to a screening technology. When an investor makes an investigation, he gets a privately observed informative signal \(s \in [0, 1]\) drawn from a distribution \(F_G(s)\) with density \(f_G(s)\) in case the project is good and from a distribution \(F_B(s)\) with density \(f_B(s)\) in case the project is bad. We make the following assumption about the signal distribution:

**ASSUMPTION 1:** Signals satisfy the monotone likelihood ratio property (MLRP):

\[
\forall s > s', \quad \frac{f_G(s)}{f_B(s)} \geq \frac{f_G(s')}{f_B(s')},
\]

Both \(f_G(s)\) and \(f_B(s)\) are continuously differentiable at \(s = 1\), \(f_B(1) > 0\), and \(\lambda \equiv f_G(1)/f_B(1) > 1\).

Without loss of generality, we will also assume that \(f_G(s)\) and \(f_B(s)\) are left-continuous and have right limits everywhere. Assumption 1 ensures that higher signals are at least weakly better news than lower signals. Assuming that densities are continuously differentiable at the top of the signal distribution simplifies our proofs, but is not essential for our results.

We denote the likelihood ratio at the top of the distribution by \(\lambda\), a quantity that will be important in our asymptotic analysis. Assuming \(\lambda > 1\) ensures that MLRP is strict over a set of non-zero measure, which in turn implies that as \(N \to \infty\), an observer of all signals would learn the true type with probability one. Therefore, for large enough \(N\), the aggregate market information is valuable for making the right investment decision.

To exclude trivial cases, we assume that the signal of a single investor can be sufficiently optimistic for the expected value of the project to be positive:

**ASSUMPTION 2:** \(E(V|S_i = 1) > 0\).

Although the signal space is continuous with no probability mass points, it can be used to represent discrete signals by letting the likelihood ratio \(f_G(s)/f_B(s)\) follow a step-function which jumps up at a finite set of points. All signals within an interval over which the likelihood ratio is constant are informationally equivalent and represent the same underlying discrete signal. Following Pesendorfer and Swinkels (1997), we
call such intervals “equivalence intervals.” Representing discrete signals as equivalence intervals is a convenient way of making strategies pure when they are mixed in the discrete space: one can think of a continuous signal $s$ as a combination of a discrete signal and a random draw from the equivalence interval, where a different draw can result in a different strategy even when the underlying discrete signal is the same.

The entrepreneur contacts investors sequentially in a random order. When contacting investor $i$ the entrepreneur makes a take-it-or-leave-it offer, in which she asks for the loan size of one in an exchange for the repayment of $1 + r_i$ in case the project is successful. The terms of the offer is private information to the entrepreneur and investor $i$. Based on the signal, the investor decides whether to approve the application or not. We assume that the contract is exclusive and cannot be taken to other investors. If the offer is accepted the project is financed and production commences. If the offer is rejected the entrepreneur goes to the next investor in line.

We first consider the case of one investor, which will allow us to develop some intuition and introduce notation that will be useful in the general case of $N$ investors.

### 1.1. One investor

Suppose there is only one investor and that the entrepreneur asks for a loan of size one and promises to repay $1 + r$. Because the investor is risk-neutral he will approve the loan if and only if he can break-even in expectation. Given his signal $s$ the investor updates the probability that the project is good according to Bayes’ law:

$$
\pi(s) = \frac{\pi f_G(s)}{\pi f_G(s) + (1 - \pi) f_B(s)}.
$$

The investor breaks even if $\pi(s)(1 + r) \geq 1$, or equivalently,

$$
\frac{\pi}{1 - \pi} \frac{f_G(s)}{f_B(s)} r \geq 1.
$$

Because signals have the monotone likelihood ratio property the investor’s decision rule is described by a screening threshold $s^*$:

$$
s^* = \arg \min_{s} \{ s : \frac{\pi}{1 - \pi} \frac{f_G(s)}{f_B(s)} r \geq 1 \}. \tag{1}
$$

If the investor’s signal $s$ is above or equal to $s^*$ the investor approves the loan. Otherwise, he rejects the loan. Equation (1) shows that there is a one-to-one relation between the interest rate and the screening threshold. Therefore, equivalently, the entrepreneur and the investor can contract on the screening threshold $s^*$ rather than on the interest
rate. If the entrepreneur asks for the screening threshold $s^*$ then the corresponding interest rate $r(\pi, s^*)$ is given by

$$r(\pi, s^*) = \frac{(1 - \pi) f_B(s^*)}{\pi f_G(s^*)},$$

and the expected profit of the entrepreneur is

$$\pi(1 - F_G(s^*)) (X - r(\pi, s^*)).$$

Hence, to maximize her profit the entrepreneur should ask the investor for the screening threshold level $s^*$ that solves

$$s^* = \arg \max_s (1 - F_G(s)) (X - r(\pi, s)).$$

In what follows, we denote the likelihood ratio $\pi/(1 - \pi)$ as $z$. Because there is one-to-one map between $\pi$ and $z$ one can always write $r(\pi, s^*)$ as $r(z, s^*)$. The following lemma collects comparative statics results in equilibrium with one investor, which will be important when we consider the general case of $N$ investors.

**Lemma 1:** Let

$$V^*(z) = \max_s (1 - F_G(s)) (X - r(z, s)),$$

$$s^*(z) = \max \{s : s = \arg \max_s (1 - F_G(s)) (X - r(z, s))\}. \quad (3)$$

Then $V^*(z)$ is an increasing function of $z$ and $s^*(z)$ is a decreasing function of $z$.

**Proof:** The first statement is obvious because $r(z, s)$ is a decreasing function of $z$. To prove the second statement, observe that by the definition of $s^*(z)$, for any $s > s^*(z)$ it must be that

$$(1 - F_G(s)) (X - r(z, s)) \leq (1 - F_G(s^*)) (X - r(z, s^*)), \quad (4)$$

which implies that for any $z' > z$,

$$(1 - F_G(s)) (X - r(z', s)) \leq (1 - F_G(s^*)) (X - r(z', s^*)).$$

Therefore, it must be that $s^*(z') \leq s^*(z)$. Q.E.D.

Lemma 1 shows that if there is a single investor then in equilibrium the expected profit of the entrepreneur increases and the screening threshold decreases in the probability of the project being good. Note that it is in the interest of the entrepreneur to
commit to not recontract with the investor if the offer is rejected because otherwise the investor would always take advantage of the entrepreneur in the first screening round.

We conclude this section by showing that the outcome described in (3) and (4) can be realized in a directed search model in which investors compete in the terms of financing. Consider the following modification of the original setup. Suppose there are many investors that simultaneously post interest rates. The entrepreneur then chooses one investor based on the contracts posted. When the investor receives the application, the investor starts an evaluation procedure which yields the signal $s$. Based on the signal, the investor decides whether to approve the application or not.

Clearly, if there is an investor that posts $r(z, s^*)$ the entrepreneur will opt for this contract because it maximizes her utility. In equilibrium, at least one investor must offer such a contract. If none of the investors offers such a contract then some investor can offer a slightly better contract for the entrepreneur and therefore, secure the entrepreneur’s application.

2. Many Investors

When there are many potential investors, the entrepreneur can try to obtain financing from other investors after a rejection. We assume that investors’ signals and contracts are private and are not shared with other investors. Investors, however, can agree to create a credit bureau that can keep a record of the entrepreneur’s rejections. Access to the credit bureau allows investors to update their beliefs about quality of the project. We first consider the case where the bureau exits and then where it does not. In what follows we assume that the entrepreneur commits not to visit the same investor twice.$^2$

2.1. Equilibrium with a credit bureau

In this section we assume that there are total of $N$ investors and that a rejection by any investor is observed by all investors. We are going to use the following notation. We denote investor $i$’s belief that the project is good before the investor observes its signal by $\pi_i$, and the corresponding likelihood ratio $\pi_i/(1 - \pi_i)$ by $z_i$. We denote investor $i$’s screening threshold by $s^*_i$, and the corresponding interest rate by $r(z_i, s^*_i)$. Note that

\[ z_i = \frac{\pi_i}{1 - \pi_i}, \quad s^*_i = \text{screening threshold} \]

\[ r(z_i, s^*_i) = \text{interest rate} \]

\[ 2^2 \text{It is clearly in the interest of the entrepreneur to commit not to re-visit the same investor when there is only one investor available. It is an open question whether this result holds for any number of investors.} \]
in equilibrium it must be that
\[ z_{i+1} = z_i \frac{F_G(s_i^*)}{F_B(s_i^*)}, \] (5)
and
\[ r(z_i, s_i^*) = \frac{1}{z_i} \frac{f_B(s_i^*)}{f_G(s_i^*)}. \] (6)

We denote the expected surplus of the entrepreneur if she applies to investor \( i \) conditional on the project being good by \( V_i \). We show later that \( V_i \) depends only on \( z_i \), which summarizes all the publicly available information about the project quality.

To develop intuition we first consider the case of two investors and later generalize it to the case of any number of investors. If the entrepreneur is rejected by the first investor, she has only one investor left to visit. Therefore, her problem reduces to the case of one investor where the entrepreneur asks for the screening threshold \( s^*_2 \) that maximizes her expected surplus:

\[ s^*_2 = \arg \max_s (1 - F_G(s)) (X - r(z_2, s)). \]

The expected surplus \( V_2 \) then is

\[ V_2 = (1 - F_G(s^*_2)) (X - r(z_2, s^*_2)). \]

Because the second investor does not observe the terms of the entrepreneur’s offer to the first investor the entrepreneur cannot influence the beliefs of the second investor. Therefore, the entrepreneur takes \( V_2 \) as given when she approaches the first investor and asks for the screening threshold \( s^*_1 \). Thus, the screening threshold \( s^*_1 \) solves

\[ s^*_1 = \arg \max_s (1 - F_G(s)) (X - r(z_1, s)) + F_G(s_1)V_2. \] (7)

Notice that the solution to (7) coincides with the solution to

\[ s^*_1 = \arg \max_s (1 - F_G(s)) ((X - V_2 - r(z_1, s)) , \]

which is the same problem as the problem with one investor where \( X \) is replaced with \( X - V_2 \). In equilibrium, \( s^*_1 \) and \( V_2 \) should agree with each other because \( V_2 \) is a function of \( z_2 \), which depends on \( s^*_1 \) as in (5). Therefore, finding a solution to (7) amounts to finding a fixed point. To show that it exists note that according to Lemma 1, \( V_2 \) is an increasing function of \( z_2 \), and therefore by (5) and the MLRP, is an increasing function of \( s^*_1 \). Also, by Lemma 1, \( s^*_1 \) is an increasing function of \( V_2 \). Let \( s^*_1 \) be a solution to (7) with \( V_2(z_1) \), which is the largest possible value for \( V_2 \). Consider the following iterations
\(i = 1, 2, \ldots\). Let \(s^*_1^i\) be the maximal solution of (7) with \(V_2(z_1 \frac{F_G(s^*_1^i)}{F_B(s^*_1^i-1)})\). With each iteration, both \(s^*_1^i\) and \(V_2(z_1 \frac{F_G(s^*_1^i-1)}{F_B(s^*_1^i-1)})\) decrease. Let \(s^* = \lim_{i \to \infty} s^*_1^i\). Then \(s^*_1\) solves (7) and agrees with the value of \(V_2\). Thus, we have proved that there is always an equilibrium in pure strategies when there are two investors to contact.

Inspecting the proof we can see that we use the fact that \(V_2\) is an increasing function of \(z_2\). If the expected surplus with two investors

\[
V_1 = (1 - F_G(s^*_1)) (X - r(z_1, s^*_1)) + F_G(s^*_1) V_2
\]

were an increasing function of \(z_1\) then following similar reasoning one could have proven the existence of an equilibrium in the case of three investors. However, as we show in Section 5, perhaps somewhat counterintuitively, the entrepreneur’s surplus might not be an increasing function of project quality. As a result, an equilibrium in pure strategies may not exist (it always exists in mixed strategies). Below we provide sufficient conditions for the existence of a pure-strategy equilibrium.

**PROPOSITION 1:** Suppose that \(f_B(s)/f_G(s)\) is a continuous function and for any \(z\) the expected surplus with one investor (3) is a quasi-concave function of \(s\). Then there exists a pure-strategy equilibrium in the game with any number of investors \(N\) and publicly recorded rejections. The equilibrium solves

\[
V_i \equiv \max_{s_i}(1 - F_G(s_i)) (X - r(z_i, s_i)) + F_G(s_i) V_{i+1},
\]

\(s.t.\quad r(z_i, s_i) \leq X, \quad i = 1, 2, \ldots, N,
\]

where \(V_{N+1} = 0\), \(r(z_i, s_i)\) is given by (6), and \(s^*_1\) solves (8).

**Proof:** Because the entrepreneur cannot affect investors’ beliefs one can view an optimization problem of setting the optimal screening threshold level at each investor \(i\), \(i = 1, 2, \ldots, N\) as if it is done by a fictitious agent \(i\). Each fictitious agent \(i\) takes decisions of other agents as given and solves (8). By assumption the payoff of each agent \(i\) is quasi-concave in his own action and continuously depends on the actions of other agents. Therefore, by the theorem of 1.2 of Fudenberg and Tirole (1991) there exists a pure-strategy equilibrium. Q.E.D.

We show in Section 5 that there can be multiple equilibria. In the next section we show that the conditions stated Proposition 1 are also sufficient for the existence of a pure-strategy equilibrium in the case in which rejections are not observed by other investors.
2.2. Equilibrium without a credit bureau

In this section we study the case in which rejections are not observed by other investors. Thus, an investor faces a pool of entrepreneurs with projects of heterogeneous quality. This opens up the possibility for an entrepreneur to signal her type by asking investors for different financing terms. In general, as is typical in dynamic signaling games, there could exist many equilibria unless one imposes some restrictions on out-of-equilibrium beliefs. We first show that only separating equilibria survive the Cho and Kreps intuitive criterion.

**Proposition 2:** Suppose there are \( N \) investors and a rejection by any investor is not observed by other investors. Then any equilibrium that survives the Cho and Kreps intuitive criterion must be separating. In any separating equilibrium screening thresholds decrease with the number of rejections.

**Proof:** See the Appendix.

Intuitively, the entrepreneur who has been rejected by \( i \) investors (low type) has less investors to visit compared to the entrepreneur who has been rejected by \( i - 1 \) investors (high type). As a result, the probability of receiving financing for the project is lower for the low type entrepreneur than for the high type entrepreneur. This gives an opportunity for the high type entrepreneur to separate herself from the low type entrepreneur by asking for a higher screening threshold or, equivalently, for a lower interest rate. The next proposition shows that the conditions stated in Proposition 1 are also sufficient for the existence of a pure-strategy fully separating equilibrium.

**Proposition 3:** Suppose that \( f_B(s)/f_G(s) \) is a continuous function and for any \( z \) the expected surplus with one investor (3) is a quasi-concave function of \( s \). Then there exists a pure-strategy separating equilibrium in the game with any number of investors \( N \) that satisfies the Cho and Kreps intuitive criterion. In the equilibrium, the entrepreneur who has been rejected \( i - 1 \) times asks for the screening threshold \( s^*_i \), which solve the following system of equations:

\[
V_N \equiv \max_{s_N} (1 - F_G(s_N)) (X - r(z_N, s_N)), \\
V_i \equiv \max_{s_i} (1 - F_G(s_i)) (X - r(z_i, s_i)) + F_G(s_i) V_{i+1}, \tag{9}
\]

\[
s_i \geq s^*_{i+1},
\]

\[
(1 - F_G(s_i)) (X - r(z_i, s_i)) + F_G(s_i) V_{i+2} \leq V_{i+1}. \tag{10}
\]
where \( V_{N+1} = 0 \), \( z_{i+1} = z_i \frac{F_G(s_i^*)}{F_B(s_i^*)} \), \( r(z_i^*, s_i) \) is given by (6), and \( s_i^* \) solves (9).

**Proof:** See the Appendix.

The system of equations (9) is similar to the system of equations (8), which describes an equilibrium with a credit agency. The main difference is the presence of the incentive compatibility constraint (10). It states that the entrepreneur who has been rejected \( i \) times does not want to mimic the entrepreneurs who have been rejected \( i - 1 \) times. In principle, one also has to check that the entrepreneur who has been rejected \( i - 1 \) times does not want to mimic the entrepreneurs who have been rejected \( i \) times. We show in the Appendix that the latter constraint never binds in equilibrium.

In equilibrium \( s_i^* > s_{i+1}^* \) because the constraint (10) is always violated at \( s_i^* = s_{i+1}^* \). Since \( z_i > z_{i+1} \) it follows that \( r(z_i, s_i) < r(z_{i+1}, s_{i+1}) \). If \( r(z_1, s_1) < X \) then \( r(z_N, s_N) < X \). Otherwise, the applicant who is rejected \( N-1 \) times will find it profitable to deviate and ask for \( r(z_1, s_1) \). Thus, the entrepreneur always visits all the available investors in equilibrium. Having established the existence of an equilibrium in the case with and without a credit bureau we next study their comparative advantages.

### 3. Social surplus and profit

The results of the previous section imply that in both cases, with and without a credit bureau, the likelihood of financing the project depends solely on the screening thresholds. Therefore, before we study welfare in sequential credit markets we first describe what outcome a social planner who maximizes social surplus could achieve by dictating the screening thresholds applied by investors in each of \( N \) sequential rounds. This gives us an upper bound on equilibrium surplus, which will be useful both as a benchmark and for establishing some of our results.

#### 3.1. Maximal social surplus with sequential search

In this section we study the problem of the social planner who chooses \( N \) screening thresholds \( \{s_i^*\}_{i=1}^N \) to maximize social surplus. We show that the maximal social surplus is the same as the one achieved in a first-price auction with the appropriate number of bidders; hence, no sequential search equilibrium does better than an auction.

The problem of the social planner is to optimally trade off rejection of good projects versus acceptance of bad projects:

\[
\max_{\{s_i^*\}_{i=1}^N} \pi X \left( 1 - \prod_{i=1}^N F_G(s_i^*) \right) - (1 - \pi) \left( 1 - \prod_{i=1}^N F_B(s_i^*) \right).
\]  

(11)
In the case of a single screening, maximization of (11) entails
\[ \pi X f_G(s^*_1) = (1 - \pi) f_B(s^*_1) \iff X = r(z, s^*_1). \] (12)

Condition (12) is intuitive. It means that the surplus is maximized at the screening level that makes the investment break even. If the project can be screened more than once, it is never optimal to lower the screening threshold below \( s^*_1 \). However, it can be optimal to increase it. The trade off is that on the one hand, the marginal project of quality \( z \) gets screened again and therefore, potentially screening accuracy increases. On the other hand, because the signal is not observed by other investors, if the project is rejected it is pooled with other projects and becomes of \( z \) quality. To better understand the above trade off the next proposition studies screening thresholds in the game with \( N \) screenings. We have

PROPOSITION 4: The optimal screening policy is to use the same screening level \( s^*_n < 1 \) for \( n \leq N \) rounds and set the screening level at 1 for remaining rounds. The optimal screening level is an increasing function of \( n \) and is a unique solution of
\[ \frac{\pi X}{1 - \pi} \frac{f_G(s^*_n)}{F_G(s^*_n)} \frac{F_G^{-1}(s^*_n)}{F_B^{-1}(s^*_n)} = 1. \] (13)

If \( \frac{f_G(s)}{F_B(s)} \) is a strictly decreasing function of \( s \) then \( n = N \) and the expected surplus strictly increases with the number of screenings. If \( \frac{f_G(s)}{F_B(s)} \) is a strictly increasing function for \( s \in [s^*_n, 1] \) then the maximal expected surplus is achieved with no more than \( n \) screenings.

Proof: See the Appendix.

Inspecting (13) we see that with \( n \) screenings the project is financed if and only if the maximal of \( n \) signals is higher than \( s^*_n \). The screening threshold level \( s^*_n \) is set to make the project just break-even when \( \max\{s_1, s_2, \ldots, s_n\} = s^*_n \). In Axelson and Makarov (2016) we show that this is also the outcome realized in the first-price auction with \( n \) bidders. Thus, no sequential credit market can do better than the first-price auction if the number of investors is chosen optimally. Proposition 4 shows that if \( \frac{f_G(s)}{F_B(s)} \) is a decreasing function of \( s \) then it is optimal to invite all investors to participate in the auction.

In Axelson and Makarov (2016) we show that unless the likelihood ratio \( f_G(s)/f_B(s) \) goes to infinity at the top of the signal distribution information aggregation fails. As a result, investment mistakes are not eliminated even when the market becomes infinitely large. The next lemma provides an upper bound on the maximal expected surplus that can be achieved with a screening technology that satisfies \( f_G(1)/f_B(1) = \lambda \).
LEMMA 2: The maximal expected social surplus with a screening technology that satisfies \( f_G(1)/f_B(1) = \lambda \) is no larger than \( \max(\pi(X - (\lambda z)^{-1}), 0) \) with equality if and only if the likelihood ratio takes only two values: 0 and \( \lambda \).

Proof: See the Appendix.

An important implication of Lemma 2 is that it provides an upper bound on the expected surplus the entrepreneur can get in sequential credit markets, which in turn allows us to derive bounds on the behavior of the screening thresholds. We now turn to the analysis of sequential credit markets.

3.2. Surplus in large sequential credit markets

In this section we analyse social surplus and the entrepreneur’s expected profit in large sequential markets. We first consider the case without a credit bureau and then the case with a credit bureau. The next proposition shows that if the strict MLRP holds then as the number of investors increases the entrepreneur’s expected profit approaches that generated in the first-price auction.

PROPOSITION 5: Suppose the strict MLRP holds and there exists a separating equilibrium in the case of \( N \) investors and no credit bureau. Then as \( N \) goes to infinity the entrepreneur’s surplus converges to that generated in the first-price auction.

Proof: See the Appendix.

In the proof we show that because of the incentive compatibility constraints (10) all screening thresholds converge to one and the interest rate in the last screening round converges to \( X \). Using the expression for the interest rate we can write the latter as

\[
\frac{1 - \pi f_B(s_N^*) \prod_{i=1}^{N-1} F_B(s_i^*)}{\pi f_G(s_N^*) \prod_{i=1}^{N-1} F_G(s_i^*)} \rightarrow X.
\]

Comparing the above expression to (13) we can see that the probabilities of being rejected in the sequential search and the first-price auction must converge, which leads to the statement of the proposition.

In Section 3.1 we show that if \( \frac{f_G(s)}{f_B(s)} \) is a strictly decreasing function then the first-price auction generates the maximal possible surplus, and the surplus increases with the number of investors. Proposition 5 thus implies that under the same conditions, in the sequential credit market without a credit bureau the entrepreneur benefits from the large market and manages to receive the maximum possible surplus in the limit.
In Section 3.1 we also show that if \( F_G(s) F_B(s) \) is a strictly increasing function at some neighborhood of \( s = 1 \) then large markets lead to suboptimal social surplus, and it is better for the entrepreneur to commit to seek financing from a restricted set of investors. If the entrepreneur can commit to do so then the market can obviously never be larger than what is optimal for the entrepreneur. However, restricting the set of potential investors may be difficult in practice because it is ex post optimal for the entrepreneur to apply to other investors if he is rejected at the chosen set of investors.

One can hope that the presence of a credit bureau can alleviate the commitment problem because with a credit bureau in place it can happen that once the entrepreneur is rejected at one investor her credit score deteriorates so much that she will no longer be able to obtain a loan from any other investor. In other words, a credit bureau can limit the size of the market. Proposition 6 shows that the equilibrium number of screenings also depends on the behavior of the likelihood ratio. When \( F_G(s) F_B(s) \) is a strictly decreasing function, and therefore having as many investors as possible is socially optimal, the entrepreneur visits all available investors in equilibrium with a credit bureau. If \( F_G(s) F_B(s)^2 \) is a strictly increasing function at some neighborhood of \( s = 1 \), and therefore, smaller markets are preferred, the entrepreneur is able to apply to only a finite number of investors.

Proposition 6: If \( F_G(s) F_B(s) \) is a strictly decreasing function of \( s \) then the entrepreneur applies to all available investors. If \( F_G(s) F_B(s)^2 \) is a strictly increasing function at some neighborhood of \( s = 1 \) then the entrepreneur applies only to a finite number of investors.

Proof: See the Appendix.

Proposition 6 suggests that the sequential credit market with a credit bureau can do a better job than an auction if the number of potential investors is outside the entrepreneur’s control. In Axelson and Makarov (2016) we show that in the case of \( f_B(s) = 1 \) for all \( s \in [0, 1] \), and \( f_G(s) = 0 \) for \( s \in [0, 1/2] \) and \( f_G(s) = 2 \) for \( s > 1/2 \), the surplus in the first-price auction is maximized with a single investor, with \( s_1^* \) being set to \( 1/2 \). Notice that the same solution is obtained in the sequential market with a credit bureau. However, in the case of a credit bureau one does not need to constrain the number of investors because the entrepreneur, if rejected, can no longer obtain credit from other investors. Thus, the market depth is set endogenously.

Proposition 6 shows that when \( F_G(s) F_B(s) \) is a strictly decreasing function of \( s \) the interest rates are set in such a way that the entrepreneur is able to apply to all available investors. We showed that without a credit bureau as the number of investors increases the total investors’ profit converges to zero. One could expect that a similar result holds...
when there is a credit bureau. However, this is not the case as the next proposition shows.

**PROPOSITION 7:** Suppose the strict MLRP holds. Then there is an $\varepsilon > 0$ such that for any number of investors $N$ the total expected profit of all investors in equilibrium with $N$ investors and a credit bureau is greater than $\varepsilon$.

**Proof:** See the Appendix.

It is interesting to contrast the above result with the no rent result in the case when there is no credit bureau. Because the terms of the application are not recorded but only its outcome the entrepreneur’s actions are influenced by beliefs of the investors. In particular, the screening threshold close to one in the first financing round cannot be sustained as an equilibrium outcome. If investors believe that the screening threshold is low then the rejection of application is very costly for the entrepreneur, which makes her ask for a high interest rate. The investors have such beliefs because if they were to believe that the entrepreneur asked for a high screening threshold then the entrepreneur would have strong incentives to deviate and ask for a lower screening threshold. In the absence of the credit bureau asking for a high screening threshold becomes credible because it is a signal that the entrepreneur’s project is of high quality and therefore commands better financing terms.

The credit bureau helps investors keep track of project quality and therefore eliminates the need for signaling. It is beneficial when the signal structure is such that it favors small markets because it makes it impossible for bad projects to be confused with good projects. However, Proposition 7 shows that establishing a credit bureau may also create costs because it limits the entrepreneur’s choice of credible actions.

If the strict MLRP holds then the first contacted investor breaks even if he finances the project with the signal equal to the screening threshold and makes positive expected profit if the signal is above it. Because the screening threshold is separated from one the probability of making positive profit does not vanish with the number of investors.

So far we have assumed that all investors have access to privately observed “soft information.” Next we consider an extension where some investors do not have such information, or can commit not to use it and instead only rely on publicly available “hard information” in their lending decisions.

### 4. Hard vs. Soft Information

In this section we use the following interpretation of our main setup. We assume that “hard information” about the project quality is public and therefore, is summa-
rized by the likelihood ratio $z$. We assume that $z$ is sufficiently high so that the project can be financed without any additional due diligence process.

As before, there are $N$ investors who get “soft information” about the project quality. Unlike hard information, soft information is private and is modelled as an informative signal $s \in [0, 1]$, the realization of which is privately observed by an investor and is an independent draw from a distributions $F_G(s)$ and $F_B(s)$.

In addition, we assume that there are investors who either do not have access to soft information, or can commit not to use it. We assume that these investors are competitive. The break-even interest rate is $1/z$ so that the entrepreneur gets expected profit of $\pi(X - 1/z)$ if she applies to this set of investors. We show next that such lenders are sometimes able to out compete soft information lenders when there is a credit bureau, even though they have strictly less information.

Figure 1 shows the entrepreneur’s profit in two cases: if she obtains financing from investors that use only hard information (blue line) and if she obtains financing from investors that use both hard and soft information (red line). It is assumed that $X = 1$, $N = 100$, $f_B(s) \equiv 1$, and $f_G(s) = 2s$. We can see that if the ex ante quality of the project is good then the entrepreneur is better off if she does not apply to investors that use soft information. The intuition for the above result is that a hard information lender never makes any rents. With a credit bureau in place Proposition 7 shows that investors earn some rent in equilibrium. The example demonstrates that the rent can be so large so that it outweighs the benefits of using information, even though the market as an aggregate possesses perfect information.

Our results in Axelson and Makarov (2016) imply that if the entrepreneur gets financing via auctions then she is always better off obtaining financing from investors who use both hard and soft information. Since as we show in Proposition 5 a large sequential market without a credit bureau delivers the same surplus to the entrepreneur as the first-price auction, we see that hard information lenders cannot compete with soft information lenders if there is no credit bureau.

5. Multiple equilibria

So far we have focused on large markets. We now consider small markets. In this section we show that there can be multiple equilibria under quite natural assumptions. In particular, we provide an example in which two equilibria exist in the case of two investors and a credit bureau. Suppose that $X = 1$, $f_B(s) \equiv 1$ and $f_G(s)$ is given by
the following equation:

\[ f_G(s) = 0.25 + \frac{1}{\exp(-100\left(s - \frac{1}{3}\right)) + 1} + \frac{0.25}{\exp(-100\left(s - \frac{2}{3}\right)) + 1}. \]  

(14)

Panel A of Figure 2 draws densities \( f_B(s) \) and \( f_G(s) \). The so defined densities \( f_B(s) \) and \( f_G \) represent a smoothed version of the case when investors’ signals takes three values: low, medium and high as depicted in Figure 2 Panel B. If the project is bad then any of the values is equally likely. If the project is good then the respective probabilities of low, medium and high signals are 1/12, 5/12, 1/2.

Figure 3 plots the expected profit of the entrepreneur as a function of the screening threshold \( s \) if there is only one investor available. Panels A, B, and C correspond to the three initial values of the likelihood ratio: \( z = 0.9, z = 0.95, \) and \( z = 1 \). We can see that two flat areas of \( f_G(s) \) lead to two humps in the expected surplus. Confirming the results of Lemma 1, at high values of \( z \) the entrepreneur’s profit is maximized at low screening thresholds while at low values of \( z \) the profit is maximized at high screening thresholds. There is a value of \( z \) (Panel B, \( z = 0.95 \)) at which the same expected surplus is achieved at two different values of \( s \). Even though if \( z \neq 0.95 \) there is a unique equilibrium in case of a single investor two equilibria can realize in the case of two investors.

In the first equilibrium, the second investor believes that the entrepreneur asks for a low screening threshold from the first investor. This makes it optimal for the entrepreneur to ask for a low screening threshold because the rejection then is very costly for the entrepreneur: If she is rejected she can no longer obtain financing from the second investor even with the most optimistic signal.

In the second equilibrium, the second investor believes that the entrepreneur asks for a high screening threshold. In this case, the cost of rejection is not so high because even if rejected the entrepreneur has still a chance to obtain financing from the second investor. As a result, it is optimal for the entrepreneur to try for a low interest rate and high screening threshold from the first investor.

For the two equilibria to exist it must be that the entrepreneur’s choice of thresholds is consistent with investors’ beliefs. This happens if the likelihood ratio \( z \) is such that \( z > 0.95 \) and \( z(X - V_1) < 0.95 \), where \( V_1 \) is the expected profit of the entrepreneur in the second equilibrium after she is rejected by the first investor.

If \( z \) is just below 0.95 then only the second equilibrium with two screenings exists because even with a single investor the entrepreneur is better off with a high screening threshold. Therefore, no matter what the second investor believes, the entrepreneur will ask the first investor for a high screening threshold. If \( z \) is just above 1.03 then
only the first equilibrium with one screening exists because even if the second investor believes that the screening threshold at the first investor is high the entrepreneur will find it profitable to deviate and ask for a low screening threshold. As a result, the entrepreneur can no longer take advantage of two investors and therefore can no longer attain a high expected surplus.

Panel A of Figure 4 plots the entrepreneur’s expected profit in the two equilibria as a function of her initial likelihood ratio \( z \). Panel B plots social surplus. The blue line corresponds to the first equilibrium with one screening; the red line to the second equilibrium with two screenings. We can see that the entrepreneur is better off in the second equilibrium, in which she can be screened twice. Social surplus, however, is higher in the first equilibrium, in which the entrepreneur is screened only once.

This gives the surprising implication that social welfare can be improved if the government imposes an interest rate cap. Figure 5 shows interest rates in the two equilibria. The blue line shows an interest rate in the first equilibrium with one screening. The red and magenta lines show interest rates in the second equilibrium. Naturally, an interest rate increases if the entrepreneur is rejected by the first investor. If there is an interest rate cap so that the rejected entrepreneur can no longer obtain financing in the second round then the second equilibrium is no longer sustainable. Thus, an interest rate cap can eliminate sub-prime markets for rejected borrowers, and hence can eliminate the socially inefficient equilibria with many financing rounds.

Panel A also illustrates, perhaps surprisingly, that the entrepreneur’s profit can be non-monotone in the ex-ante project’s quality. This happens because of the entrepreneur’s inability to commit to ask for a high screening threshold from the first investor, or in other words, for a low interest rate. As a result, investors get higher rent and the entrepreneur gets worse off.

6. Conclusion

To be completed.
References


Appendix. Proofs

Proof of Proposition 2:

Part 1. We first show that the entrepreneur who has been rejected $i$ times would always like to separate herself from those who have been rejected more than $i$ times. In what follows, we denote the entrepreneur who has been rejected $i$ times by $E_i$, $i = 1, \ldots, N$. As before, we denote the screening threshold that $E_i$ asks for by $s_i^*$. With private rejections, contrary to the case with a credit bureau, the beliefs about the project quality depend on the screening threshold $s_i^*$ asked by the entrepreneur. We denote the investor $i$'s belief that the project is good before the investor observes its signal by $\pi_i(s_i^*)$, and the corresponding likelihood ratio $\pi_i/(1 - \pi_i)$ by $z_i(s_i^*)$. We denote the expected surplus of the entrepreneur conditional on the project being good if she applies to investor $i$ being rejected $i-1$ times by $V_i(s_i^*)$.

Suppose contrary to the statement of the proposition that there is some pooling in equilibrium. Let $i$ be the first instance such that $E_i$ pulls with entrepreneurs rejected more than $i$ times. If $E_i$ and $E_{i+1}$ pull together then $s^* \equiv s_i^* = s_{i+1}^*$, $z \equiv z_i = z_{i+1}$, and $r(s, z) \equiv r(s_i^*, z_i) = r(s_{i+1}^*, z_{i+1})$. The expected surplus of $E_i$ and $E_{i+1}$ entrepreneurs are given by

\begin{align*}
V_i(s^*) &= (1 - F_G(s^*)) (X - r(z, s^*)) + F_G(s^*) V_{i+1}(s^*), \\
V_{i+1}(s^*) &= (1 - F_G(s^*)) (X - r(z, s^*)) + F_G(s^*) V_{i+2}(s_{i+2}^*).
\end{align*}

Let $\hat{z}$ be the likelihood ratio of the project if the investors knew that the entrepreneur is of type $E_i$. Clearly, $\hat{z} > z$. Let $\hat{s}$ be the largest screening threshold such that

\begin{equation}
(1 - F_G(\hat{s})) (X - r(\hat{z}, \hat{s})) + F_G(\hat{s}) V_{i+2}(s_{i+2}) = V_{i+1}(s^*).
\end{equation}

Suppose that investors believe that the entrepreneur if of type $E_i$ if she asks for the screening threshold $\hat{s}$. Then the type $E_{i+1}$ entrepreneur is indifferent between asking for $s^*$ and $\hat{s}$. Notice that if $X > r(\hat{z}, \hat{s})$ then $V_{i+1}(s^*) > V_{i+2}(s_{i+2}^*)$ because the type $E_{i+1}$ entrepreneur can always follow the strategy of the type $E_{i+2}$ entrepreneur. Therefore, Equation (A1) implies that

\begin{align*}
V_i(s^*) &= (1 - F_G(s^*)) (X - r(z, s^*)) + F_G(s^*) V_{i+1}(s^*) < \\
&< (1 - F_G(\hat{s})) (X - r(\hat{z}, \hat{s})) + F_G(\hat{s}) V_{i+1}(s^*) = V_i(\hat{s}).
\end{align*}

Hence, the type $E_i$ entrepreneur is better off and the type $E_{i+1}$ entrepreneur is worse off by deviating and asking for the screening threshold just above $\hat{s}$.

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Part 2. For any separating equilibrium to exist two types of incentive compatibility conditions must hold. First, it must be that the type $E_{i+1}$ entrepreneur does not want to pool with the type $E_i$ entrepreneur:

$$V_{i+1}(s_{i+1}^*) = (1 - F_G(s_{i+1}^*)) (X - r(z_{i+1}, s_{i+1})) + F_G(s_{i+1}^*) V_{i+2}(s_{i+2}^*) \geq$$

$$\geq (1 - F_G(s_i^*)) (X - r(z_i, s_i)) + F_G(s_i^*) V_{i+1}(s_{i+1}^*).$$

(A2)

Second, it must be that a type $E_i$ entrepreneur does not want to pool with a type $E_{i+1}$ entrepreneur:

$$V_n(s_i^*) = (1 - F_G(s_i^*)) (X - r(z_i, s_i^*)) + F_G(s_i^*) V_{i+1}(s_{i+1}^*) \geq$$

$$\geq (1 - F_G(s_{i+1}^*)) (X - r(z_{i+1}, s_{i+1}^*)) + F_G(s_{i+1}^*) V_{i+1}(s_{i+1}^*).$$

(A3)

Let

$$\Delta_i(s_i^*) = (1 - F_G(s_{i+1}^*)) (X - r(z_{i+1}, s_{i+1}^*)) - (1 - F_G(s_i^*)) (X - r(z_i, s_i^*)).$$

We can write constraints (A2) and (A3) as

$$(F_G(s_i^*) - F_G(s_{i+1}^*)) V_{i+2}(s_{i+2}^*) \leq \Delta_i(s_i^*) \leq (F_G(s_i^*) - F_G(s_{i+1}^*)) V_{i+1}(s_{i+1}^*).$$

(A4)

Since $V_{i+2}(s_{i+2}^*) \leq V_{i+1}(s_{i+1}^*)$ (A4) can hold only if $s_i^* \geq s_{i+1}^*$. Thus, in any separating equilibrium the screening thresholds decrease with the number of rejections.

Q.E.D.

Proof of Proposition 3:

The proof is similar to that of Proposition 1. The added complication here is that for any separating equilibrium to exist two incentive compatibility conditions (A2) and (A3) must hold. We first prove that in equilibrium, the constraint (A3) is not binding. As a result, one needs to check only the constraint (A2), which is the constraint (10) in the proposition.

To see this let $\Theta_i$ be the set of $s \geq s_{i+1}^*$ that satisfy the constraint (A2):

$$\Theta_i = \{s \geq s_{i+1}^* : \Delta_i(s) \geq (F_G(s) - F_G(s_{i+1}^*)) V_{i+2}(s_{i+2}^*)\}.$$

(A5)
Then
\[ s^*_i = \arg \max_{s \in \Theta_i} (1 - F_G(s)) (X - r(z_i, s)) + F_G(s)V_{i+1}(s^*_{i+1}), \]  
(A6)
\[
\text{s.t. } r(z_i, s) \leq X, \quad i = 1, 2, \ldots, N.
\]

Notice that \( \Theta_i \) is not an empty set because it always contains \( s = 1 \). Let \( \hat{s}_i \) be the smallest element of \( \Theta_i \). Then
\[
(1 - F_G(s^*_i)) (X - r(z_i, s^*_i)) + F_G(s^*_i)V_{i+1}(s^*_{i+1}) \geq \\
(1 - F_G(\hat{s}_i)) (X - r(z_i, \hat{s}_i)) + F_G(\hat{s}_i)V_{i+1}(s^*_{i+1}) = \\
(1 - F_G(s^*_{i+1})) (X - r(z_{i+1}, s^*_{i+1})) + F_G(s^*_{i+1})V_{i+1}(s^*_{i+1}) + \\
(1 - F_G(\hat{s}_i) - F_G(s^*_{i+1}))(V_{i+1}(s^*_{i+1}) - V_{i+2}(s^*_{i+2})).
\]  
(A7)
where the last equality follows from the definition of \( \hat{s}_i \). Notice that because \( s \geq s^*_{i+1} \) and \( V_{i+1}(s^*_{i+1}) \geq V_{i+2}(s^*_{i+2}) \) the inequality (A7) implies the constraint (A2).

We are now ready to complete the proof. As in Proposition 1, because the entrepreneur cannot affect the investors’ beliefs one can view an optimization problem of setting the optimal screening threshold at each investor \( i, i = 1, 2, \ldots, N \) as if it is done by a fictitious agent \( i \). Each fictitious agent \( i \) takes decisions of other agents as given and solves
\[
V_i(z) \equiv \max_{s_i} (1 - F_G(s_i)) (X - r(z_i, s_i)) + F_G(s_i)V_{i+1}, \\
\text{s.t. } r(z_i, s_i) \leq X, \quad i = 1, 2, \ldots, N - 1, \\
s_i \geq s^*_{i+1}, \\
(1 - F_G(s_i)) (X - r(z^*_i, s_i)) + F_G(s_i)V_{i+2}(z^*_{i+2}) \leq V_{i+1}(z^*_{i+1}).
\]

By assumption the payoff of each agent \( i \) is quasi-concave in his own action and continuously depends on the actions of other agents. Also, quasi-concavity of the payoff ensures that the action space of every agent that satisfies the constraint is a concave set. Therefore, by the theorem of 1.2 of Fudenberg and Tirole (1991) there exists a pure-strategy equilibrium.

Q.E.D.

Proof of Proposition 4:
Consider the maximization problem (11). Let \( n \leq N \) be the largest \( n \) such that the expected surplus generated with \( n \) screenings is strictly higher than that of generated
with \( n - 1 \). Then for all \( i > n \), \( s_i = 1 \) and for all \( i \leq n \), \( s_i \) satisfy the F.O.C.:

\[
\frac{f_G(s_j)}{f_B(s_j)} \prod_{i \leq n, i \neq j} \frac{F_G(s_i)}{F_B(s_i)} = \frac{1}{zX} \quad j = 1, \ldots, n.
\] (A8)

Let \( s_* = \min(\{s_i\}_{i=1}^n) \) and \( s^* = \max(\{s_i\}_{i=1}^n) \). Suppose that \( s_* \neq s^* \). Consider a change \( \Delta \) in the expected surplus if one changes the threshold \( s^* \) (or any if there are multiple \( s^* \)) to \( s_* \):

\[
\Delta = \prod_{s_i \neq s^*} F_B(s_i) \left( \left[ F_B(s^*) - zX F_G(s^*) \prod_{s_i \neq s^*} \frac{F_G(s_i)}{F_B(s_i)} \right] - \left[ F_B(s_*) - zX F_G(s_*) \prod_{s_i \neq s_*} \frac{F_G(s_i)}{F_B(s_i)} \right] \right)
\]

\[
= \prod_{s_i \neq s^*} F_B(s_i) \left( \left[ F_B(s^*) - F_G(s^*) \frac{f_B(s^*)}{f_G(s^*)} \right] - \left[ F_B(s_*) - F_G(s_*) \frac{f_B(s_*)}{f_G(s_*)} \right] \right),
\]

where we use the F.O.C. (A8). Because of the MLRP the function

\[
F_B(s) - F_G(s) \frac{f_B(s^*)}{f_G(s^*)}
\]

is nondecreasing between \( s_* \) and \( s^* \). Thus, the maximum surplus is achieved when all \( s_i \) equal to \( s_* \). The F.O.C. (A8) therefore becomes the F.O.C. (13). Equation (13) has a unique solution because of the MLRP.

To prove that the expected surplus strictly increases with the number of screenings if \( \frac{F_G(s_j)}{F_B(s_j)} \) is a strictly decreasing function of \( s \) we need to show that for any \( N \) the solution to the maximization problem (11) is interior. Suppose on the contrary that at some \( N \) it is optimal to set \( s_N \) to 1. Let \( N \) be the lowest number of screenings when this happens. The optimal screening threshold level \( s \) is the same in all \( N - 1 \) screenings and solves the F.O.C.

\[
\frac{f_G(s)}{f_B(s)} = \frac{F_B^{N-2}(s)}{F_G^{N-2}(s)} \frac{1}{zX}.
\]

Taking the derivative of the surplus with respect to \( s_N \) at \( s_N = 1 \) we have

\[
f_B(1) F_B^{N-1}(s) - zX f_G(1) F_G^{N-1}(s) = f_B(1) F_B^{N-1}(s) \left( 1 - \lambda \frac{F_G(s)}{F_B(s)} \frac{f_B(s)}{f_G(s)} \right) < 0,
\]

where we have use the F.O.C and where the last inequality follows from the fact that \( \frac{F_G(s)}{F_B(s)} \frac{f_B(s)}{f_G(s)} \) is a decreasing function of \( s \) and therefore takes the lowest value \( \lambda^{-1} \) at \( s = 1 \). As a result, it is suboptimal to set \( s_N \) to 1 and the solution must be indeed interior.

Finally, we prove that the maximal expected surplus can be achieved with no more than \( n \) screenings if \( \frac{F_G(s_j)}{F_B(s_j)} \) is a strictly increasing function for \( s \in [s^*_n, 1] \). We prove
earlier that Equation (13) has a unique solution, \( s^n_n \), which is strictly increasing in \( n \). We now show that the maximand in (11) is higher for \( s_i = s^*_n \), \( i = 1, 2, \ldots, n \) and \( s_{n+1} = 1 \) than for \( s_i = s^*_{n+1} \), \( i = 1, 2, \ldots, n + 1 \). To see this, we start at a point \( s^*_{n+1} \), and move the first \( n \) screening thresholds \( s \) down while moving the \( n + 1 \)'s screening threshold \( s_{n+1} \) up to hold \( F_B(s)^n F_B(s_{n+1}) \) constant:

\[
\frac{ds}{ds_{n+1}} = -\frac{F_B(s) f_B(s_{n+1})}{n f_B(s) F_B(s_{n+1})}.
\]

This changes the maximand with an amount proportional to

\[-F_G(s)^n f_G(s_{n+1}) - n F_G(s)^{n-1} f_G(s) F_G(s_{n+1}) \frac{ds_n}{ds},\]

which has the same sign as

\[
\frac{F_G(s_{n+1}) f_B(s_{n+1})}{F_B(s_{n+1}) f_G(s_{n+1})} - \frac{F_G(s) f_B(s)}{F_B(s) f_G(s)}.
\]

By the assumption of the Proposition, this change is positive for \( s_{n+1} > s^*_n \), and hence the maximand is increased by setting \( s_{n+1} = 1 \). But at \( s_{n+1} = 1 \), it is optimal to set all first \( n \) screening thresholds to \( s^*_n \).

**Q.E.D.**

**Proof of Lemma 2:** We first observe that the maximal expected surplus respects the order induced by MLPR on the space of signal distributions. Consider two cases of informative signals. Suppose that in both cases if the project is bad the signal is drawn from the same distribution \( F_B(s) \). At the same time, if the project is good then in the first case, the signal is drawn from a distribution \( F_G_1 \) with density \( f_G_1 \), and in the second case, from a distribution \( F_G_2 \) with density \( f_G_2 \). Suppose that for all \( s > s' \)

\[
\frac{f_G_1(s)}{f_G_2(s)} \geq \frac{f_G_1(s')}{f_G_2(s')}.
\]

then the maximal surplus in the first case is no less than that in the second case. This follows from the fact that MLRP implies the monotone probability ratio (Milgrom (1981)).

Suppose for now that \( f_B(s) \equiv 1 \). Then given \( \lambda \), the maximal expected surplus is achieved with \( f_G(s) = 0 \) for \( s \in [0, 1 - \lambda^{-1}] \) and \( f_G(s) = \lambda \) for \( s \in [1 - \lambda^{-1}, 1] \). Setting a screening threshold level to \( 1 - \lambda^{-1} \) ensures that good projects are always financed and bad projects are financed with probability \( \lambda^{-1} \). Thus, with a single screening the expected surplus is \( \pi(X - (\lambda z)^{-1}) \). Direct computations show that \( \frac{F_G(s) f_B(s)}{F_B(s) f_G(s)} \) is an
increasing function for \( s \in [1 - \lambda^{-1}, 1] \). Thus, by Proposition 4, \( \pi(X - (\lambda z)^{-1}) \) is in fact the maximal expected surplus. Finally, notice that assumption of \( f_B(s) \equiv 1 \) is innocuous. For an arbitrary \( f_B(s) \) the maximal surplus is achieved with \( f_G(s) = 0 \) for \( s \in [0, \bar{s}] \) and \( f_G(s) = \lambda f_B(s) \) for \( s \in [\bar{s}, 1] \), where \( \bar{s} \) is determined by the condition that \( \int_{\bar{s}}^1 \lambda f_B(s) ds = 1 \). Hence, \( \int_0^\bar{s} f_B(s) ds = 1 - \lambda^{-1} \).

**Q.E.D.**

**Proof of Proposition 5:**

The proof is done in two steps. First, we show that if \( r(z_N, 1) \) goes to \( X \) as \( N \) goes to infinity then the entrepreneur’s surplus converges to the one generated in the first-price auction. Second, we show that \( r(z_N, 1) \) must go to \( X \) in equilibrium as \( N \) goes to infinity.

**Step 1.** Suppose that \( r(z_N, 1) \rightarrow X \) as \( N \rightarrow \infty \). From the expression for social surplus (11) it is clear that if \( \prod_{i=1}^N F_G(s_i) \rightarrow F^N_G(s^*) \) and \( \prod_{i=1}^N F_B(s_i) \rightarrow F^N_B(s^*) \), where \( s^* \) is a screening threshold in the first-price auction, then surplus generated in sequential financing rounds without a credit bureau and the first-price auction are asymptotically the same.

Using expressions (5) and (6) for \( z_N \) and \( r(z_N, s_N) \) we can see that \( r(z_N, 1) = \left( \lambda z \prod_{i=1}^{N-1} \frac{F_G(s_i)}{F_B(s_i)} \right)^{-1} \). Therefore, \( r(z_N, 1) \to X \) as \( N \to \infty \) is equivalent to

\[
\lambda z \prod_{i=1}^{N-1} \frac{F_G(s_i)}{F_B(s_i)} \to 1. \tag{A9}
\]

If the entrepreneur is rejected \( N - 1 \) times then in the last round she solves

\[
V_N = \max_{s_N} (1 - F_G(s_N)) (X - r(z_N, s_N)).
\]

Thus, if \( r(z_N, 1) \to X \) and the strict MLRP holds then \( s_N \to 1 \). For each \( i \), define \( \Delta s_i = 1 - s_i \). Since for all \( i \) \( s_i > s_N \), \( \Delta s_i < \Delta s_N \) so all \( s_i, i = 1, ..., N \) converge to one. Taking the Taylor’s series of (A9) we have

\[
\sum_{i=1}^{N-1} \Delta s_i = a_1 + O(\Delta s_N), \quad a_1 = \frac{\ln(\lambda z X)}{\lambda - 1}. \tag{A10}
\]
Therefore,
\[
\prod_{i=1}^{N} F_G(s_i) = e^{-\lambda a_1} + O(\Delta s_N),
\]
\[
\prod_{i=1}^{N} F_B(s_i) = e^{-a_1} + O(\Delta s_N).
\]

We show in Axelson and Makarov (2016) that \(F_G^{N}(s^*)\) and \(F_B^{N}(s^*)\) converge to the same corresponding limits.

**Step 2.** We now show \(r(z_N, 1)\) goes to \(X\) in equilibrium. Suppose on the contrary that \(r(z_N, 1) < X - \epsilon\) for some \(\epsilon > 0\). Note that only a bounded number of screening thresholds can stay away from one as \(N\) goes to infinity. Let \(M\) be the maximal index such that \(s_{N-M} \to 1\) and \(s_{N-M+1} < 1 - \delta\) for some \(\delta > 0\) and infinitely many values of \(N\). Consider the problem of the entrepreneur who has been rejected \(N - M - 1\) times. She solves

\[
V_{N-M} = \max_{s_{N-M}} (1 - F_G(s_{N-M})) (X - r(z_{N-M}, s_{N-M})) + F_G(s_{N-M}) V_{N-M+1}, \quad (A11)
\]

\[
\text{s.t.} \quad r(z_i, s_i) \leq X, \quad i = 1, 2, \ldots, N - 1,
\]
\[
s_{N-M} \geq s_{N-M+1},
\]
\[
(1 - F_G(s_{N-M})) (X - r(z_{N-M}, s_{N-M})) + F_G(s_{N-M}) V_{N-M+2} \leq V_{N-M+1}.
\]

(A12)

As in Proposition 7 one can show that unconstrained solution to (A11) entails \(s_{N-M}\) to be bounded away from one. Since by assumption \(s_{N-M}\) goes to one it must be that the constraint (A12) binds. However, with \(s_{N-M+1}\) away from one, \(s_{N-M}\) converging to one, and \(r(z_{N-M}, s_{N-M}) < X - \varepsilon\), the constraint (A12) cannot bind.

\[Q.E.D.\]

**Proof of Proposition 6:**

**Part 1.** Suppose \(\frac{F_G(s)}{F_B(s)} f_B(s) f_G(s)\) is a strictly decreasing function of \(s\) and suppose that the entrepreneur is unable to contact some investors because of her deteriorated credit rating. Let \(z^*\) be her likelihood ratio and \(s^*\) be her screening threshold at the last contacted investor. Then \(z^* > (\lambda X)^{-1}\) and \(z^* F_G(s^*) / F_B(s^*) \leq (\lambda X)^{-1}\). With only one investor to visit \(s^*\) solves

\[
\max_{s} (1 - F_G(s)) \left( X - \frac{1}{z^* f_B(s)} \right). \quad (A13)
\]
From (A13) it is clear that \( s^* < 1 \) and that \( \frac{f_G(s^*)}{f_B(s^*)} > (z^*X)^{-1} \). By assumption, \( \frac{f_G(s)}{f_B(s)} \) is a strictly decreasing function. Therefore,

\[
\frac{F_G(s^*)}{F_B(s^*)} > \frac{F_B(1)}{F_B(1)} = \frac{1}{\lambda},
\]

Therefore, \( z^*F_G(s^*)/F_B(s^*) > (\lambda X)^{-1} \). As a result, the entrepreneur has a chance to get financing if he applies to another investor. Thus, we arrive at a contradiction.

**Part 2.** Suppose \( \frac{f_G(s)f_B(s)^2}{f_B(s)f_G(s)^2} \) is a strictly increasing function of \( s \) in some neighbourhood of \( s = 1 \) and suppose that the entrepreneur contacts all available investors. For simplicity, we assume that \( f_B(s) \equiv 1 \). Notice that

\[
\left( \frac{F_G(s)}{F_B(s)} \right)'_{s=1} = \lambda - 1.
\]

Since

\[
\left( \frac{F_G(s)}{F_B(s)} \frac{1}{f_G(s)^2} \right)' = \left( \frac{F_G(s)}{F_B(s)} \right)' \frac{1}{f_G(s)^2} + F_G(s) \left( \frac{1}{f_G(s)^2} \right)'
\]

the fact that \( \frac{f_G(s)f_B(s)^2}{f_B(s)f_G(s)^2} \) is a strictly increasing function at \( s = 1 \) implies that

\[
0 \leq f_G(s)'_{s=1} < \frac{\lambda(\lambda - 1)}{2}.
\]

(A14)

In the proof of Proposition 7 we show that unless \( z_i\lambda X, i = 1, 2, \ldots \) goes to one, \( s^*_i \) stay bounded away from one. However, in this case there is an \( i \) such that \( z_i\lambda X < 1 \). Thus, it must be that \( \lim_{i \to \infty} z_i\lambda X = 1 \).

Consider the F.O.C. for the expected surplus over one period (A13):

\[
-(1 - F_G(s)) \left( \frac{1}{f_G(s)} \right)' = f_G(s)zX - 1.
\]

(A15)

Let \( \hat{f} = f_G'(s)_{s=1} \). Let \( \Delta s = 1 - s \). Taking the Taylor's series of (A15) at \( s = 1 \) we have

\[
\frac{\Delta s}{\lambda} = \lambda zX - 1 - \hat{f}zX\Delta s + o(\Delta s).
\]

Hence,

\[
\Delta s^* = \frac{\lambda(\lambda zX - 1)}{\hat{f}(1 + \lambda zX)}.
\]

(A16)
Given $\Delta s^*$ the update likelihood ratio

$$zF_G(\Delta s^*)/F_B(\Delta s^*) = z(1-(\lambda - 1)\Delta s^*) + o(\Delta s^*) = z \left( 1 - \frac{\lambda(\lambda - 1)(\lambda zX - 1)}{f(1 + \lambda zX)} \right) + o(\Delta s^*).$$

Inequality (A14) implies that there is $0 < \gamma < 1/2$ such that $\hat{f} = \gamma \lambda (\lambda - 1)$. Notice that

$$z \left( 1 - \frac{(\lambda zX - 1)}{\gamma(1 + \lambda zX)} \right) < \frac{1}{\lambda X} \iff \gamma(1 + \lambda zX) < \lambda z X \iff \gamma < \frac{\lambda z X}{(1 + \lambda z X)}.$$

Thus, after being rejected the entrepreneur has no chance of obtaining a loan at the remaining investors.

Q.E.D.

Proof of Proposition 7:

Proposition 1 shows that the screening threshold at the first investors, $s_1^*$, solves (8). According to Lemma 1, $s_1^*$ is an increasing function of $V_2$. By Lemma 2, $V_2 \leq X - (\lambda z)^{-1}$, with the equality if and only if $f_G/f_B$ takes only two values: 0 and $\lambda$. Thus, in all other cases there exists $\delta > 0$ such that for any $N$, $X - V_2 > (\lambda z)^{-1} + \delta$.

Let $s^*$ be a solution to

$$s^* = \arg \max_s (1 - F_G(s)) \left( \frac{1}{z\lambda} + \frac{\delta}{z f_B(s)} \right).$$

Then $s^* < 1$ and for all $N$, $s_1^* < s^*$. The first contacted investor is break even if it finances the project with the signal equal to $s_1^*$ and makes positive expected profit if the signal is above $s_1^*$.

Q.E.D.
Figure 1. **Hard vs. Soft information.** Figure 1 blue line shows the entrepreneur’s profit if she obtains financing from investors that use only hard information. The red line shows the entrepreneur’s profit if she obtains financing from investors that use both hard and soft information.

Figure 2. **Multiple equilibria. Signal densities.** Figure 2 Panel A plots densities $f_B(s)$ and $f_G(s)$. Function $f_G(s)$ is defined in equation (14), and is a smoothed version of the case shown in Panel B.
Figure 3. Entrepreneur’s profit. Panels A, B, and C display the entrepreneur’s profit with one investor when $z = 0.9, z = 0.95, z = 1$ respectively. Other parameters are as follows: $X = 1$, densities $f_B$ and $f_G$ are displayed in figure 2 Panel A.

Figure 4. Multiple equilibria. Entrepreneur’s profit and social surplus. Figure 4 Panel A plots the entrepreneur’s expected profit in the two equilibria described in Section 5 as a function of her initial likelihood ratio $z$. Panel B plots social surplus. The blue line corresponds to the first equilibrium with one screening; the red line - to the second equilibrium with two screenings.
**Figure 5. Multiple equilibria. Interest rate.** Figure 5 shows interest rate the entrepreneur asks from investors. The blue line shows the interest rate in the equilibrium with single screening. The red and magenta lines show the interest rates in the equilibrium with two screenings, with the highest rate being asked if rejected at the first investor.