Carlstrom and Fuerst meets Epstein and Zin:
The Asset Pricing Implications of Contracting Frictions

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Abstract

Models with financial frictions have been shown to create amplification and persistence effects in macroeconomic fluctuations. We test the ability that Costly State Verification (CSV) has to generate empirically plausible risk exposures in asset markets, when households have Epstein and Zin (1989) preferences and productivity shocks are in the style of Long Run Risks. Under the setup of Carlstrom and Fuerst (1997), alongside these mechanisms, we find that the CSV friction is negligible in augmenting the aggregate equity premium. Additionally we find that the separation between the elasticity of intertemporal substitution and risk aversion plays a key role in explaining financial market dynamics; in particular, we are only able to generate sizable equity premium when the elasticity is greater than one. Moreover, while the contracting friction provides volatility to the price of capital, its contribution is not significant. Instead physical adjustment costs of capital are much more meaningful in reaching realistic quantitative targets.

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1 Introduction

The success of recursive preferences in endowment-based economies (see for example, Bansal and Yaron (2004)) has led to an offspring in research that asks whether households with Epstein and Zin (1989) utility in a production-based, general equilibrium framework, can also generate plausible macroeconomic and financial dynamics. In a parallel line of work, financial frictions, such as those discussed in Bernanke, Gertler and Gilchrist (1999) via Costly State Verification (CSV), imply an amplification mechanism that increases the *quantity of risk* associated with total factor productivity shocks. To the extent that market dividends covary positively with aggregate fluctuations and have sufficiently positive market prices of risk, we ask whether the inclusion of Epstein-Zin preferences, in presence of such bankruptcy frictions, is able to amplify the risk exposure of financial assets.

We test these implications through the tractable framework of Carlstrom and Fuerst (1997). There are four agents in the model – (1) households who make consumption and savings decisions under Epstein-Zin preferences, (2) intermediaries who borrow household savings and lend them out, (3) risk-neutral entrepreneurs who take household savings and invest into a capital generating technology, and (4) final goods producers who rent capital and labor, set marginal prices, and close the economy. There is moral hazard between the intermediaries and entrepreneurs, regarding the capital-generating productivity, which leads to a static contract that allows for a role in monitoring costs; this eventually has a dynamic role in general equilibrium.

As households own capital in this economy we are able to calculate rates of return on capital from their perspective; that is to say the rate of return from purchasing a unit of capital today, collecting dividends, and reselling the undepreciated portion to the market tomorrow. Under a baseline calibration that has neglibile bankruptcy costs, where the monitoring costs in the aforementioned contract are close to zero, and implements quadratic costs of capital adjustments we are able to hit many key macro-financial moments including the correlation of consumption and investment growth, the relative volatility of consumption growth, the volatility of output growth, and the level of the risk free rate. Additionally we receive an equity risk premium of about three percent (3%) and a Sharpe ratio of .4, values that get close to empirical estimates.

Having fitted the data reasonably well with a model that only implements convex adjustment costs, we proceed to measure the marginal role of bankruptcy costs. What we find is stark in that the role of monitoring has very little effect on the aggregate market return. In particular, increasing monitoring costs from zero to twenty percent raises the levered household capital return by only thirty basis points, from the baseline result. While the contracting friction acts as an implicit adjustment cost, distorting the price of capital away from one and decreasing investment volatility, in reality, it is much weaker relative to a standard convex adjustment cost. Put in another way, the overall volatility of the cost of capital is less sensitive to a perturbation in the agency friction than it is to that of a change in the physical capital adjustment.

Another key result the model produces is regarding the role of Epstein-Zin preferences. We examine various values of the intertemporal elasticity of substitution while fixing risk aversion. What we find is that there is a clear, monotonically decreasing risk premium when we shift IES from two to below one. As our model setup is dependent on shocks to growth rates of productivity as opposed to levels of productivity the IES plays a strong role in influencing the persistence of investment flows. As a result, the procycality of dividend payments are diminished when IES decreases. This result is very similar to those in Croce(2014) and Favilukis and Lin (2013).

Previous Literature

Our paper contributes to many areas of work, but we start by describing its context in the macroeconomic literature involving financial frictions. In this area, financial frictions were first motivated as mechanisms to generate endogenous magnification in aggregate fluctuations. In particular, we can think of Kiyotaki and Moore (1997), Bernanke, Gertler and Gilchrist (1999), and Carlstrom and Fuerst (CF, 1997) as primary examples of models in which firms' financing constraints create distortions in capital markets. In Kiyotaki and Moore, firms are forced to pledge collateral for borrowing, which creates distortions across borrowers of different types. In CF, firms (entrepreneurs) can only borrow on the basis of their ex-ante net worth, which factors into a costly state verification agreement with intermediaries. As our model builds upon CF we also will use the net worth channel to create magnified cyclical variations. And as business cycle variations are magnified in quantities, we expect assets with positive exposures (betas) to be subject to higher risk premia.

If we interpret the net worth channel as an active financial constraint for entrepreneurs, this paper also connects to the literature that discusses the connection between firm financing constraints and asset pricing. Gomes, Yaron and Zhang (2006) quantitatively examines whether financial constraints are significantly priced in asset returns, through the Euler condition of a firm that dynamically chooses capital each period. Similarly, Whited and Wu (2006) construct a model-based financial constraints index that predicts returns significantly, relative to other predictive factors in the literature.

More relevantly, our work relates to the production-based asset pricing literature that attempts to jointly explain financial and macroeconomic dynamics. For example, one of the earlier contributors to this field, Jermann (1998), embeds habit preferences and adjustment costs to capital, in an otherwise standard real business cycle (RBC) framework to generate larger equity premia in dividend-paying assets. A central tenet of RBC models is the quick adjustment of investment to shocks in TFP. As a result, firms pay out dividends that tend to be countercyclical, resulting in asset returns that hedge the business cycle and generate low ex-ante risk premia as a result. To get around this large volatility of investment, adjustment costs to capital help slow down the rate of cyclical investment and increase its autocorrelation. This is also the spirit behind the "time-to-plan" assumption in Boldrin, Christiano and Fisher(2001). We will also be utilizing adjustment costs to capital in our model.

Starting from the Long Run Risks literature (Bansal and Yaron (2004)), the use of Epstein and Zin (1989) preferences has helped explain asset pricing dynamics, by separating the coefficient of risk aversion from the elasticity of intertemporal substitution. From a production economy standpoint, Tallarini (2000) and Kaltenbrunner and Lochstoer (2011) have both shown the benefits of utilizing recursive preferences to better match macro-financial data. In our model, the Epstein-Zin motive, namely the difference between risk aversion and the reciprocal of EIS, will play a key role in generating sizeable, positive equity premia.

Our article can best be thought of as a combination of Gomes, Yaron and Zhang (GYZ, 2003) and Croce (2014). While GYZ studies the asset pricing implications of the Carlstrom and Fuerst model, it does not allow for EZ preferences and also does not employ *shocks to growth rates* of productivity as in Croce (2014) and Favilukis and Lin(2013). We find that both of these mechanisms are crucial to better capture financial market dynamics and overturn many of the results in GYZ. For example, GYZ generates a very low excess return to capital over the risk free asset. We are able to get over this problem while still capturing salient macroeconomic properties.

The roadmap for our paper is as follows. In the next section we document the general equilibrium model that we will use to study the role of financial frictions in asset prices. In the following section we document the quantitative details of the model calibration, including parameter choice and the data used. In the fourth section, we document the main results regarding the role of contracting frictions. In the fifth and final section, we conclude.

2 Model

In this section we detail the model, which follows largely from Carlstorm and Fuerst (1997), additionally encorporating Epstein and Zin (1989) preferences and convex adjustment costs.

The model consists of four agents: (1) *Households* that exhibit recursive preferences (2) *Intermediaries* whose role wil largely be to propagate a costly state verification contract (3) *Entrepreneurs* who are capital-producing and make investment decisions under the previously stated friction (4) and *Final Goods Producers* that convert capital goods into consumption goods for the household and entrepreneur. Through the satisfaction of each one of these agent's problems and market clearing, equilibrium will sustain.

2.1 Households

Identical households are infinitely lived with Epstein-Zin (EZ) preferences over consumption and labor at wage w_t. They also own and rent capital, k_{t+1}, to the final goods producer at rate r_t. The decision problem can be written in the following manner:

$$U_t = \underset{\{c_t, l_t, k_{t+1}\}}{\text{Max}} \left((1-\beta)\left(V(c_t, l_t)\right)^{\frac{1-\gamma}{\theta}} + \beta \left(E_t \left[U_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right)^{\frac{\theta}{1-\gamma}}$$

$$\text{s.t.} \qquad c_t + q_t k_{t+1} + q_t \Phi_a \left(k_t, k_{t+1}\right) k_t = w_t l_t + r_t k_t + q_t(1-\delta)k_t$$

where $V(c_t, l_t)$ represents the *intra*temporal utility from consumption and labor choice. The price of capital is q_t and δ is its depreciation rate. The EZ parameters are standard with γ representing risk aversion, ψ representing IES, and $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$. Additionally, there are adjustment costs that directly enter the household's problem, as provided by $\Phi_a \times k$. Because the previous expression is in terms of capital units, it is multiplied by the price of capital to maintain consumption units.

In this model the household lends to an intermediary that provides an instantaneous, intraperiod return of one. Hence the household's problem does not need to take into account the "zero profit" intermediary. Similarly, the final goods producer is neoclassical and will provide zero profits to the households. Therefore the above budget constraint does not to take into account these two elements.

This is a fairly simple problem to solve which we can do so, analytically, using the first order and envelope conditions. Through the resulting algebra we receive an equation governing the trade-off of labor and consumption and the usual Euler equation:

$$
\begin{aligned}
& w_t V_c(c_t, l_t) = -V_l(c_t, l_t) \\
& E_t\left[M_{t+1} R^k_{t+1}\right] = 1
\end{aligned}
$$

$$
\begin{aligned}
\text{s.t.} \qquad & M_{t+1} = \beta \frac{U_{t+1}^{\frac{1}{\psi}-\gamma}}{\left(E_t\left[U_{t+1}^{1-\gamma}\right]\right)^{1-\frac{1}{\theta}}} \left(\frac{V_{t+1}}{V_t}\right)^{-\frac{1}{\psi}} \left(\frac{V_{c,t+1}}{V_{c,t}}\right) \\
& R^k_{t+1} = \frac{r_{t+1} + q_{t+1}(1-\delta) - q_{t+1}\left(\Phi_{a,t+1} + \frac{\partial \Phi_{a,t+1}}{\partial k_{t+1}} k_{t+1}\right)}{q_t\left(1 + \frac{\partial \Phi_{a,t}}{\partial k_{t+1}} k_t\right)}
\end{aligned}
$$

where $\Phi_{a,t}$ are adjustment costs at time t, evaluated at (k_t, k_{t+1}).

We will interpret the bottom quantity as the return on capital in the model. Put in another words, it is the return from purchasing a unit at time t, selling back the depreciated unit at time $t+1$ and collecting the appropriate dividend from the final goods producer. With adjustment costs, additional terms reflect the opportunity cost of investment. Note that when adjustment costs are set to zero, the expression boils down to:

$$
R^{k,noadj}_{t+1} = \frac{r_{t+1} + (1-\delta) q_{t+1}}{q_t}
$$

and the interpretation is straightforward.

2.2 Contract between Intermediary and Entrepreneurs

Intermediaries take household savings and lend to entrepreneurs at rate r^l_t. Because shocks are unobservable to the intermediaries, we solve a repeated one period contract to ensure banks are willing to participate. The entreprenurs face TFP shocks to their investment, represented by ω_t, which are IID random variables such that $\mathbb{E}(\omega_t) = 1$. Additionally, ω takes on a CDF $\Phi(\cdot)$ and PDF $\phi(\cdot)$ over non-negative support.

To finance his investment the entrepreneur needs to borrow (an external debt from intermediaries) over his current net worth, $i_t - n_t$. This will be at an endogenously determined interest rate, r_t^l. Note that ω is private to the entrepreneur and cannot be observed by outside investors. Essentially when the realization of ω is too low, the entrepreneur will not be able to repay the loan and will default. In this case he will give up all of his realized investment, $\omega_t i_t$. Hence, default only occurs when:

$$\omega_t < \bar{\omega}_t \equiv \frac{(1+r_t^l)(i_t - n_t)}{i_t}$$

Based on this cutoff value of ω, the bank's expected income of a loan of size $i_t - n_t$ will be given by:

$$\begin{aligned} q_t i_t g(\bar{w}_t) &\equiv q_t \left[\int_0^{\bar{\omega}_t} \omega_t i_t d(\Phi(\omega_t)) - \Phi(\bar{\omega}_t)\mu i_t + (1-\Phi(\bar{\omega}_t))(1+r_t^l)(i_t - n_t) \right] \\ &= q_t i_t \underbrace{\left[\int_0^{\bar{\omega}_t} \omega_t d(\Phi(\omega_t)) - \Phi(\bar{\omega}_t)\mu + (1-\Phi(\bar{\omega}_t))\bar{\omega}_t \right]}_{g(\bar{\omega}_t)} \end{aligned}$$

Similarly, the enterpreneur's expected income with the given loan size will be:

$$\begin{aligned} q_t i_t f(\bar{w}_t) &\equiv q_t \left[\int_{\bar{\omega}_t}^{\infty} (\omega_t i_t - (1+r_t^l)(i_t - n_t)) d(\Phi(\omega_t)) \right] \\ &= q_t i_t \underbrace{\left[\int_{\bar{\omega}_t}^{\infty} (\omega_t - \bar{\omega_t}) d(\Phi(\omega_t)) \right]}_{f(\bar{\omega}_t)} \end{aligned}$$

As we assume that each entrepreneur and intermediary will be risk neutral, this will allow us to formulate a contract in the form of Townsend (1979), where $\bar{\omega}_t$ is a function of the underlying state variables, before the shocks are realized. We maximize the expected income of the entrepreneur given that the lender is (at-least) returned his original loan amount. This results in the following contracting problem:

$$\begin{aligned} &\underset{\{\bar{\omega}_t, r_t^l, i_t\}}{\text{Max}} \; q_t i_t f(\bar{\omega}_t) \\ \text{s.t.} \qquad &q_t i_t g(\bar{\omega}_t) \geq i_t - n_t \end{aligned}$$

Notice that given i_t and n_t there is a one to one mapping from $\bar{\omega}_t$ to r_t^l – this comes from the definition of $\bar{\omega}_t$. Hence we will simply solve for i_t and $\bar{\omega_t}$. It can easily be shown that these values will satisfy the first order conditions:

$$
\begin{aligned}
& q_t f(\bar{\omega}_t) = \frac{f'(\bar{\omega}_t)}{g'(\bar{\omega}_t)}(q_t g(\bar{\omega}_t) - 1) \\
& i_t = \frac{n_t}{1 - q_t g(\bar{\omega}_t)} \\
\text{s.t.} \qquad & f'(\bar{\omega}_t) = -(1 - \Phi(\bar{\omega}_t)) \\
& g'(\bar{\omega}_t) = 1 - \Phi(\bar{\omega}_t) - \mu\phi(\bar{\omega}_t)
\end{aligned}
$$

To solve for the contract terms, notice first $\bar{\omega}_t = \bar{\omega}(q_t)$ to satisfy the first equation. For the given level of $\bar{\omega}_t$, q_t, and n_t, we can then solve for $i_t = i(q_t, n_t)$ from the second equation and also pin down r_t^l.

A few things of note. From this contract it is clear that as the participation constraint is binding from the intermediary's perspective, he will make zero profits. Hence the household does not receive anything from his lending to the intermediary, and resultingly savings does not enter the household's budget constraint. In terms of timing, the contract will determine what i_t is *before* the entrepreneur makes his consumption / saving decisions.

2.3 Entrepreneur

The entrepreneur is a risk neutral agent, maximizing expected lifetime discounted consumption. The entrepreneur's decision problem is the following:

$$
\begin{aligned}
& \underset{\{c_t^e, k_{t+1}^e\}}{\text{Max}} E \sum_{t=0}^{\infty} (\beta\gamma^e)^t c_t^e \\
\text{s.t.} \qquad & c_t^e + q_t k_{t+1}^e + q_t \Phi_a^e \left(k_t^e, k_{t+1}^e\right) k_t^e = q_t i_t f(\bar{\omega}_t) \\
& n_t = w_t^e + r_t k_t^e + q_t(1-\delta)k_t^e
\end{aligned}
$$

The first constraint represents the entrepreneur's budget constraint, where c_t^e is his consumption choice and k_{t+1}^e is his capital choice. For an investment of i_t (which is predetermined by the point in time of this decision) he receives $q_t i_t f(\bar{\omega}_t)$ as a return in the same period, where $f(\bar{\omega}_t)$ represents the share of surplus that he will be taking, as earlier decided in the contract between the entrepreneur and intermediary. The second constraint represents the components of net worth: wages w_t^e, rent on capital (at same rate as household) $r_t k_t^e$, and the current value of undepreciated capital, $q_t(1-\delta)k_t^e$. These are all observable at the time of the entrepreneur solving the problem. As before there are adjustment costs, $\Phi_a^e \times k^e$, adjusted by the capital price to place it in the consumption numeraire.

Using the definition of $f(\bar{\omega}_t)$ from above we can solve the previous problem. We can derive the following Euler equation:

$$E_t\left[\beta\gamma^e\left\{\frac{\left(\frac{q_{t+1}f(\bar{\omega}_{t+1})}{1-q_{t+1}g(\bar{\omega}_{t+1})}\right)(q_{t+1}(1-\delta)+r_{t+1})-q_{t+1}\left(\frac{\partial\Phi^e_{a,t+1}}{\partial k^e_{t+1}}k^e_{t+1}+\Phi^e_{a,t+1}\right)}{q_t\left(1+\frac{\partial\Phi^e_{a,t}}{\partial k^e_{t+1}}k^e_t\right)}\right\}\right]=1$$

where $\Phi^e_{a,t}$ is defined analogously to that as before, now evaluated at (k^e_t, k^e_{t+1}).

2.4 Final Goods Producer

The final goods firm exhibit constant returns to scale (CRS) with labor-augmenting shocks inside the production function. This will be given by:

$$Y_t = F(K_t, Z_tL_t, Z_tL^e_t)$$

where K_t denotes aggregate capital, L_t aggregate household labor, and L^e_t aggregate entrepreneur labor. Z_t represents a (symmetric) technology shock.

The share of entrepreneurs in the economy is η, while that of households is $1-\eta$, which suggest $K_t = (1-\eta)k_t + \eta k^e_t$. For labor market clearing, $L_t = (1-\eta)l_t$ and $L^e_t = \eta l^e_t = \eta$, where the last statement follows from labor not entering the entrepreneur's utility. Due to the CRS, zero-profit nature of the final goods producers we will have:

$$r_t = F_K(K_t, Z_tL_t, Z_tL^e_t)$$
$$w_t = F_L(K_t, Z_tL_t, Z_tL^e_t)$$
$$w^e_t = F_{L^e}(K_t, Z_tL_t, Z_tL^e_t)$$

at the equilibrium quantities.

2.5 Market Clearing

Beyond the labor markets clearing we will require that supply and demand match in the capital markets and goods markets. That is to say:

$$
\begin{aligned}
&\text{(Capital Markets)} \qquad K_{t+1} = (1-\delta)K_t + \underbrace{\eta i_t (1-\mu\Phi(\bar{\omega}_t))}_{\text{Investment across all entrepreneurs}} \\
&\text{(Goods Markets)} \qquad Y_t = F(K_t, Z_t L_t, Z_t L_t^e) = (1-\eta)c_t + \eta c_t^e + \eta i_t
\end{aligned}
$$

Notice the correction for the surplus loss in the capital markets clearing, as the capital decision is made at the end of the period while the investment decision is made earlier. Hence next period capital will reflect the intraperiod default of the fraction $\Phi(\bar{\omega}_t)$. Going forward we will denote aggregate consumption and investment as:

$$
\begin{aligned}
I_t &= \eta i_t \left(1 - \mu \Phi\left(\bar{\omega}_t\right)\right) \\
C_t &= \eta c_t^e + (1-\eta) c_t
\end{aligned}
$$

In measurement of aggregate quantities we compute growth rates of these variables.

2.6 Measurement of Quantities and Returns

We will be interested in examining log growth rates of macroeconomic variable X given by $\log\left(\frac{X_{t+1}}{X_t}\right)$ where $X_t \in \{Y_t, C_t, I_t\}$. The risk free rate we will be targeting will be given by:

$$
\exp\left(r_{t+1}^f\right) = \frac{1}{E_t\left[M_{t+1}\right]}
$$

We will focus on one capital return in the model – that of the household, denoted here by r^k:

$$
\begin{aligned}
\exp\left(r_{t+1}^k\right) &= \frac{r_{t+1} + q_{t+1}(1-\delta) - q_{t+1}\left(\Phi_{a,t+1} + \frac{\partial \Phi_{a,t+1}}{\partial k_{t+1}} k_{t+1}\right)}{q_t\left(1 + \frac{\partial \Phi_{a,t}}{\partial k_{t+1}} k_t\right)} \\
\exp\left(r_{t+1}^e\right) &= \frac{\left(\frac{q_{t+1} f(\bar{\omega}_{t+1})}{1 - q_{t+1} g(\bar{\omega}_{t+1})}\right)\left(q_{t+1}(1-\delta) + r_{t+1}\right) - q_{t+1}\left(\frac{\partial \Phi^e_{a,t+1}}{\partial k^e_{t+1}} k^e_{t+1} + \Phi^e_{a,t+1}\right)}{q_t\left(1 + \frac{\partial \Phi^e_{a,t}}{\partial k^e_{t+1}} k^e_t\right)}
\end{aligned}
$$

The last value specified, r^e, denotes a return-type object from the entrepreneur's perspective of owning and collecting on capital proceeds. For completeness, we also report this in the final tables but will not seek to utilize this number in our calibration procedure.

It is well founded in the data that excess returns in equities are levered and some volatility in the excess returns is due to idiosyncratic noise. To take our model's equity excess return to the data we finally define a levered, excess equity return given by:

$$R^{lev}_{ex,t+1} = 2 \times \left(r^k_{t+1} - r^f_{t+1} \right) + \sigma_{lev} \epsilon_{lev,t+1}$$

The leverage parameter here, two, comes from literature. It can be originally traced back to work by Rauh and Sufi (2012) as well as Garcia-Feijo and Jorgensen (2010).

3 Baseline Calibration and Data

In this section we specify the processes governing productivity shocks, household utility, and adjustment costs. We will also go over the baseline calibration which corresponds to a model with solely convex costs of adjustment to capital and roughly zero costs of bankruptcy (equivalently stated, a low value of μ). The goal in later parts of the paper will be to make adjustments to this baseline and observe marginal behavior of the model.

3.1 Long Run Productivity Shocks

As demonstrated in Croce (2014), the use of productivity shocks to *growth rates* of total factor productivity generates dynamics that are both empirically plausible as well as quantitatively advantageous for general equilibrium models. In particular, shocks to growth rates, as opposed to those to levels of productivity, modify the policy functions of next period capital, allowing for heteroskedastic volatility and autocorrelation of investment that are present in the data. Explained in Favilukis and Lin (2013), these modifications generate persistent investment flows that will result in a larger procyclicality of dividend flows and hence larger risk premia. In models where shocks are to the *level* of TFP, we are not able to generate such features.

We model log growth rates in productivity as having a small, persistent component:

$$\begin{aligned}\triangle z_{t+1} &= g_z + x_t + \varphi_z \epsilon_{z,t+1} \\ x_t &= \rho_x x_{t-1} + \varphi_x \epsilon_{x,t+1}\end{aligned}$$

The setup of TFP shocks here allows us to interpret movements in $\epsilon_{z,t+1}$ as "short run risks" and those in x_t as "long run risks." We follow from the literature and impose the ratio of $\frac{\varphi_x}{\varphi_z}$ to be 10% while the persistence of x_t will be $\rho_x = .96$ which is consistent with the estimate in Croce (2014). Similarly we will choose g_z to be roughly .5%, which allows us to match average growth rates ofthe US economy. All that is left to calibrate is φ_z which we do so to approximately match the volatility of US GDP.

3.2 Intratemporal Utility and Adjustment Costs

Given the generality of the model we will need to take a stand on the forms of household's intratemporal utility function, V_t. For the purposes of this exposition we will shut down the household's labor supply and make it inelastic. That is to say:

$$V(c_t, l_t) = c_t$$

and hence l_t is equal to one at all states. Additionally we will set the final goods production function to be Cobb-Douglas with labor-augmenting TFP shocks, which is to say:

$$\begin{aligned}Y_t &= K_t^{\alpha_k} \left(Z_t L_t\right)^{\alpha_l} \left(Z_t L_t^e\right)^{\alpha_e} \\ Z_t &= \exp\left(z_t\right)\end{aligned}$$

where $\alpha_k + \alpha_l + \alpha_e = 1$ and the transmission of z_t is as given in the previous subsection. Finally we set the adjustment costs to take a convex, quadratic form given by:

$$\begin{aligned}\Phi_t = \Phi\left(k_t, k_{t+1}\right) &= \frac{\phi_k}{2}\left(\frac{k_{t+1} - (1-\delta)k_t}{k_t} - \delta\right)^2 \\ \Phi_t^e = \Phi^e\left(k_t^e, k_{t+1}^e\right) &= \frac{\phi_k^e}{2}\left(\frac{k_{t+1}^e - (1-\delta)k_t^e}{k_t^e} - \delta\right)^2\end{aligned}$$

The use of this form of adjustment costs is certainly different than the irreversibility form used in Jermann (1998). As a result, investment by each individual is not limited to be be positive in theory. Furthermore, it might be the case that our results are not directly at a parallel of Kaltenbrunner and Lochstoer (2011) and Croce (2014). Nonetheless we find that the qualitative attributes of our model are similar relative to those of existing literature.

3.3 Baseline Calibration

In Table 1 we list the baseline calibration model where bankruptcy costs are set at very low levels. This is given through $\mu = .5\%$. Regarding other parameters of the model, we set the capital share of the final goods producer (α_k) to be .36 while setting the household labor share to be .6399 and the remainder to the entrepreneur share. This calibration follows directly from Carlstrom and Fuerst (1997) and Gomes, Yaron and Zhang (2003). Also similar to literature are the depreciation rate of capital ($\delta = .02$) and the share of entrepreneurs in the economy ($\eta = .10$). We also fix the household's risk aversion to a reasonable level ($\gamma = 15$) throughout our experiments. This value is in between those calibrated in Bansal and Yaron (2004) and Bansal and Shaliastovich (2013). We also fix the intertemporal elasticity of substitution at 2.5 in the baseline. This level of IES is necessary to merit reasonable consumption and dividend dynamics. We will show this more explicitly in our counterfactuals.

There are a few parameters that we have left to carefully calibrate; the first one being the subjective time discount factor of the household, β. Naturally we would expect β to influence the level of the risk free rate negatively. Different from the mechanism in Kaltenbrunner and Lochstoer (2011) however we do not get a sole identification of the risk free through the discount rate; rather the volatility of investment is also highly dependent on β (negatively). As β increases, the penchant to smooth consumption decreases which increases its volatility share. For a fixed output volatility and correlation of consumption and investment growth, this means that the volatility of investment will drop. Furthermore, this drop in the volatility of investment is mangified due to the persistence in investment from growth rate shocks (as opposed to level rate shocks). Altogether, this strong tradeoff makes it difficult to hit both moments simultaneously and hence we choose a β that gets the risk free rate at a reasonable expected level; in our calibration this amounts to 54 basis points.

As the baseline model is selected to operate with purely adjustment costs we modify the value of ϕ, which is the value of the multiplier on the household's quadratic adjustment costs.[1] What we find is that this parameter has a strong bearing on whether consumption and investment growth correlate in a positive manner. In particular, as ϕ increases so too does the value of $\rho(\triangle c, \triangle i)$. We set $\phi = 10$ to receive a correlation of about .67 which is very close to the annual data counterpart. Finally we have left one parameter to set which is the volatility parameter on the idiosyncratic noise in the levered excess equity return. We set this to 3.25% to match the annual standard deviation of about 6.5% as cited in Bansal and Yaron (2004). In many ways this helps us match the Sharpe ratio of the excess, levered equity return.

3.4 Data

The macroeconomic data we use as a calibration comparison come largely from the National Income Product Accounts (NIPA). Annual consumption from 1929 through 2008 is constructed as the sum of real, per capita, nondurables and services consumption. Similarly investment is taken to be the sum of real, per-capita, private residential and non-residential fixed investment.[2] To construct a comparable output series in the context of the model we sum the constructed consumption and investment series.

The annual return series for the realized excess equity returns is given through the excess market return on Ken French's website. We also take the nominal one month risk free rate on his website and subtract inflation constructed from the GDP deflator index to receive a measure for the real risk free rate.

4 Results

4.1 The Fit of the Baseline Model

Based on the parameters discussed in the previous section we examine the model's implied behavior through a long sample simulation of 50,000 quarters (the model is calibrated at a quarterly frequency). Following the simulation, we time aggregate the data by four quarters to receive annual data at a quarterly frequency. All reported statistics are hence in annual units. Table 2 provides the baseline fit of the model. Quantities in red indicate those that are used

[1]We fix the entrepreneur's adjustment cost parameter at zero as this plays a non-central role in our study of asset returns related to the household

[2]We do not subtract software processing equipment as the time series for this does not go back to 1929. Even if we are to extrapolate the series going backwards, investment statistics do not drastically change.

to calibrate the model (this is explained in detail in the previous section). We find that we are able to do a reasonable job in matching the relative volatility of consumption (.64 in the data versus .94 in the model), the volatility of output (3.36 in the data versus. 3.77 in the model), and the correlation of consumption and investment growth (.68 in the data versus .67 in the model). As discussed earlier we choose β to hit the expected level of the risk free rate and as a result the model warrants a reasonable risk free rate of 54 basis points.

While we reasonably succeed on our desired calibration metrics, we perform differentially with respect to other ones. In particular, the relative volatility of investment is too low in the model. This problem is also apparent in Favilukis and Lin (2013) and can trace its origins to the form of the growth rate shocks as described earlier. On the other hand we are able to capture the excess equity return reasonably well with an equity premium of about three percent (3.14 %) and a Sharpe ratio that is reasonable (.425). To better understand the mechanism through which we capture the equity premium, we provide a more granular image of the returns in Table 4. In the column titled $\mu = .5\%$ we outline the results of the baseline model. As the unlevered market return is only about 1.6% with a rather high Sharpe ratio of .90, leverage provides a much more reasonable final result. It is interesting to note that the "return-like" entrepreneur object, r^e, provides just under 7% of excess returns.

4.2 Adjustment versus Bankruptcy Costs

We start by examining the model results with respect to a perturbation of the household's adjustment costs, as given in Table 3. Broadly speaking, as we slow down the movement of capital in the model, this should lead to a lower volatility in investment, dividends that are more procyclical, and larger equity premia – phenomena that are present in Kaltenbrunner and Lochstoer(2011), among many other works. When adjustment costs are shut down (first column) , the relative share of investment volatility is much higher and levered returns collapse to .80% on an annual basis. The second column is our baseline model, and already represents a roughly four-fold increase in premia. In the final column we shift adjustment costs to a, perhaps, unreasonable level. All in all, the adjustment cost mechanism works similar to how we would expect it to.

Next, we explore the effects of bankruptcy costs. In Table 4 we incrementally shift the monitoring costs from the baseline (one half of a percentage point) upwards to ten and twenty percent. Largely, the trend is that there is a very marginal effect on this dimension, amounting to only about thirty basis points on the final levered market return. This is interesting to the extent that monitoring costs behave as an adjustment cost in this model, bringing the price of capital ("Average q" loosely speaking) away from unity. However the extent to which it behaves, as shown in the results, is very mild, further displayed by the very slight decrease in the investment volatility.

The minor effect of bankruptcy costs is also apparent in the impulse responses of the model, provided in Figure 1. We look at the change in the price of capital, output growth, investment growth, and the excess levered returns on capital, under different model configurations wtih respect to agency costs (given by $\mu = .05, .10, .20$). Following a positive, one standard deviation shock to the innovation on the long run component of TFP growth, ϵ_x, output increases about .5% while investment increases about 2% on an annual basis, under the baseline calibration. This is in conjunction with increases in the price of capital and excess levered returns on capital. When we increase agency costs to twenty percent, the impulse responses of output growth and investment growth are about the same. In the original results of CF and GYZ, the impulse responses from a pure agency cost model provided hump-shaped responses. However in our case this does not occur, as we already work in a parameter regime where there are strong physical costs to capital adjustment. It is important to make this distinction because we make much more progress in jointly satisfying macro-financial moments. In this sense, the original persistence and mangification effects of the CSV friction are washed out when we operate in a more realistic parameter space.

4.3 The Role of EZ

In Table 6 we examine the behavior of the model when shifting the intertemporal elasticity of substitution from $\psi = 2.5$ to values below 1. The first column represents the baseline version of the model. What we find is that the unlevered excess return on household capital decreases to -1.2%. A large amount of the shift downwards is due to the increase in the risk free rate. As ψ decreases agents are much more less willing to trade off consumption between periods. This will decrease the need for the hedging asset and the risk free rate increases in equilibrium as a result. Moreover, as ψ moves towards $\frac{1}{\gamma}$, the model returns to one with time-separable preferences, and wipes away any existing risk premia.

4.4 The Cyclicality of Returns

As documented in GYZ, one strong deficiency of the agency cost model is the inability to deliver counter-cyclicality of the default premium. In the context of our setup the default premium is considered as the difference between the capital returns of the entrepreneur and household, $r^e - r^k$. In Table 5, we report the cyclicality of returns in the model. We find that similar to GYZ, the model delivers slightly procyclical default premia (contemporaneous correlation with output growth of .07), which increases up to .42 as the agency friction increases. That is to say the inclusion of recursive preferences does not overturn this result.[3]

5 Conclusion

We revisit the quantitative role of financial frictions and discuss its impact on financial markets. In particular, the use of Epstein and Zin preferences and shocks to growth rates of productivity make a large impact on how we plausibly interpret the effects of Costly State Verification. After fitting a model jointly to business cycle and financial data, without monitoring costs associated with the entrepreneur's contract, we then activate the friction and examine its effects. Our setup sugests a very mild shift in total equity premia. Moreover, our paper supports earlier literature, regarding the minor role of financing frictions in aggregate asset pricing dynamics, but provides a baseline laboratory that is much closer to reality. In total, we are able to suggest that volatility of the price of capital due to physical adjustment costs, as opposed to contracting frictions, allows for more quantitatively satisfying results.

[3] One bright spot of the model is that it captures the slight positive correlation of (realized) excess market returns and output growth, which is about a quarter in the data. Further, while we don't document it explicitly, the model delivers highly countercyclical risk premia, which has been supported in the literature.

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Tables and Figures

Table 1: **Calibration Parameters for Baseline Model**

Parameter	Value	Selection Criteria
β	.991	Approximately match $E\left(r^f\right)$
ζ	1	Labor Supply Shut Down
δ	.02	
α_k	.3600	
α_l	.6399	
α_e	.0001	
η	.1	
σ_x	–	Set to receive steady bankruptcy rate, $\Phi(\bar{\omega}) = .974\%$
γ_e	–	Set to receive steady $R_d = 1.87\%$
μ	.5%	Key parameter to control amount of bankruptcy costs
γ	15	
ψ	2.5	
g_z	.45%	Approximately match $E\left(\triangle y\right)$
φ_z	2%	Approximately match $\sigma\left(\triangle y\right)$
ρ_x	.96	Croce(2014)
φ_x	$.1 \times \varphi_z$	Capture appropriate LRR portion of volatility
ϕ	10	
ϕ_e	0	
σ_{lev}	.0325	Bansal and Yaron (2004) and Croce (2014)

This table provides the baseline calibration for the model, at a quarterly frequency. All parameters with z and x subscripts refer to parameters that calibrate the growth rate process for TFP. The wedge in the discount rates of the entrepreneur and household is given through γ_e while that of the household is β. The level of monitoring costs for the intermediary is provided through μ.

Table 2: **Baseline Model Fit**

Statistic	Data (Annual, 1929 - 2008)	Baseline Model
Macro (Annual):		
$\sigma(\triangle c)/\sigma(\triangle y)$	.641	.937
$\sigma(\triangle i)/\sigma(\triangle y)$	4.46	1.68
$\sigma(\triangle y)(\%)$	3.36	3.77
$\rho(\triangle c, \triangle i)$	.678	.673
Financial (Annual):		
$E(r^f)(\%)$	.722	.54
$\sigma(r^f)(\%)$	3.78	.98
$E(R^{lev}_{ex,t+1})(\%)$	5.04	3.14
$\sigma(R^{lev}_{ex,t+1})(\%)$	20.31	7.39
$Sharpe(R^{lev}_{ex,t+1})$	.248	.425

This table provides the results of the baseline calibration. The data moments are constructed from annual data (1929 – 2008). Details for data construction are provided in the Appendix. The baseline calibration results are generated through a long sample simulation (population) of 50000 quarters. After a burn-in of 5000 quarters, we annualize all moments, in log terms and report figures above. All statistics refer to quantities that are defined in the main portion of the text.

Table 3: **Role of Household Adjustment Costs**

Statistic	$\phi = 0$	$\phi = 10$	$\phi = 20$
Macro (Annual):			
$\sigma(\triangle c)/\sigma(\triangle y)$	.808	.937	1.13
$\sigma(\triangle i)/\sigma(\triangle y)$	2.27	1.68	1.57
$\sigma(\triangle y)(\%)$	3.84	3.77	3.74
$\rho(\triangle c, \triangle i)$	.53	.67	.60
Financial (Annual):			
$E(r^f)(\%)$	.22	.54	.74
$\sigma(r^f)(\%)$	1.09	.98	.99
$E(r^k - r^f)(\%)$	.42	1.59	2.42
$\sigma(r^k - r^f)(\%)$	.92	1.77	2.33
$Sharpe(r^k - r^f)$	.45	.896	1.04
$E(r^e - r^f)(\%)$	5.69	6.86	6.18
$\sigma(r^e - r^f)(\%)$	2.43	3.21	2.56
$Sharpe(r^e - r^f)$	2.34	2.14	2.41
$E(R^{lev}_{ex,t+1})(\%)$	.80	3.14	4.81
$\sigma(R^{lev}_{ex,t+1})(\%)$	6.75	7.39	7.98
$Sharpe(R^{lev}_{ex,t+1})$	.118	.425	.602

This table provides the results of perturbing the level of adjustment costs, that enter the household's budget constraint. The results are generated through a long sample simulation (population) of 50000 quarters. After a burn-in of 5000 quarters, we annualize all moments, in log terms and report figures above. All statistics refer to quantities that are defined in the main portion of the text.

Table 4: **Role of Bankruptcy Costs**

Statistic	$\mu = .5\%$	$\mu = 10\%$	$\mu = 20\%$
Macro (Annual):			
$\sigma(\triangle c)/\sigma(\triangle y)$	.937	910	.927
$\sigma(\triangle i)/\sigma(\triangle y)$	1.68	1.55	1.54
$\sigma(\triangle y)(\%)$	3.77	3.76	3.76
$\rho(\triangle c, \triangle i)$	.673	.782	.78
Financial (Annual):			
$E(r^f)(\%)$	.54	.43	.40
$\sigma(r^f)(\%)$	.98	.99	1.00
$E(r^k - r^f)(\%)$	1.59	1.72	1.76
$\sigma(r^k - r^f)(\%)$	1.77	1.90	1.93
$Sharpe(r^k - r^f)$	.896	.9071	.909
$E(r^e - r^f)(\%)$	6.86	25.1	51.6
$\sigma(r^e - r^f)(\%)$	3.21	3.95	4.24
$Sharpe(r^e - r^f)$	2.14	6.36	12.17
$E(R^{lev}_{ex,t+1})(\%)$	3.14	3.41	3.48
$\sigma(R^{lev}_{ex,t+1})(\%)$	7.39	7.52	7.55
$Sharpe(R^{lev}_{ex,t+1})$	.425	.454	.461

This table provides the results of perturbing the level of monitoring costs, where we fix the adjustment cost parameter, $\phi = 10$. The results are generated through a long sample simulation (population) of 50000 quarters. After a burn-in of 5000 quarters, we annualize all moments, in log terms and report figures above. All statistics refer to quantities that are defined in the main portion of the text.

Table 5: **The Cyclicality of Returns – Current Model**

Statistic	Data (Annual, 1929 - 2008)	$\mu = .5\%$	$\mu = 10\%$	$\mu = 20\%$
$\rho(\triangle c, R_{ex}^{lev})$	.224	.298	.335	.080
$\rho(\triangle y, R_{ex}^{lev})$	.209	.363	.384	.162
$\rho(\triangle c, r^e - r^k)$	-.503	-.006	.247	.300
$\rho(\triangle y, r^e - r^k)$	-.539	.070	.388	.421
$\rho(\triangle y, m)$	NA	-.428	-.426	-.426

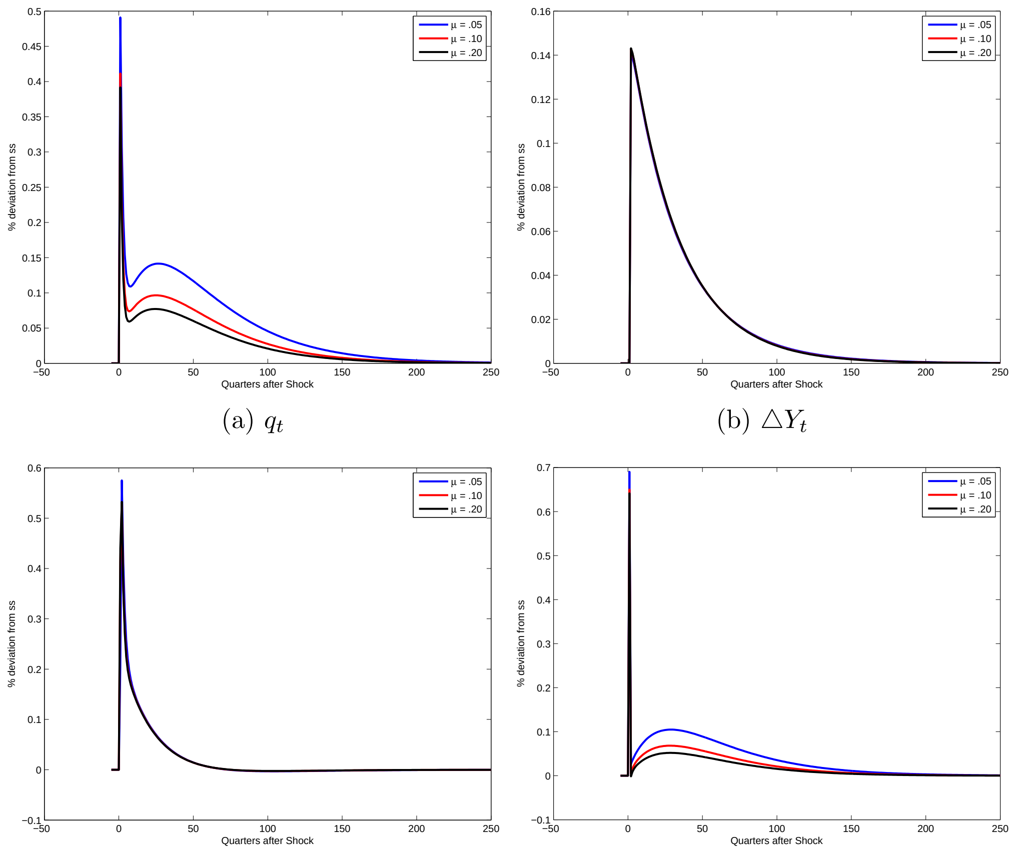
This table provides the results of perturbing the level of monitoring costs, where ρ represents contemporaneous correlation statistics. The results are generated through a long sample simulation (population) of 50000 quarters. After a burn-in of 5000 quarters, we annualize all moments, in log terms and report figures above. The last row is the correlation between time t output and the conditional expectation of excess returns (equity risk premia).

Table 6: **Role of IES**

Statistic	$\psi = 2.5$	$\psi = 1.5$	$\psi = .8$
Macro (Annual):			
$\sigma(\triangle c)/\sigma(\triangle y)$	.937	.905	.953
$\sigma(\triangle i)/\sigma(\triangle y)$	1.68	1.56	1.72
$\sigma(\triangle y)(\%)$	3.77	3.72	3.64
$\rho(\triangle c, \triangle i)$	.67	.84	.60
Financial (Annual):			
$E(r^f)(\%)$	.54	2.03	4.15
$\sigma(r^f)(\%)$	.98	1.48	2.46
$E(r_k - r_f)(\%)$	1.59	.73	-.59
$\sigma(r_k - r_f)(\%)$	1.77	1.65	2.14
$Sharpe(r_k - r_f)$	.896	.440	-.274
$E(r_e - r_f)(\%)$	6.86	5.87	4.81
$\sigma(r_e - r_f)(\%)$	3.21	3.14	4.62
$Sharpe(r_e - r_f)$	2.14	1.87	1.04
$E(R^{lev}_{ex})(\%)$	3.14	1.42	-1.21
$\sigma(R^{lev}_{ex})(\%)$	7.39	7.29	7.80
$Sharpe(R^{lev}_{ex})$	.425	.194	-.155

This table provides the results of perturbing the IES parameter that enters the household preferences, where we also fix the household adjustment costs parameter at $\phi = 10$. The calibration results are generated through a long sample simulation (population) of 50000 quarters. After a burn-in of 5000 quarters, we annualize all moments, in log terms and report figures above. All statistics refer to quantities that are defined in the main portion of the text.

Figure 1: Impulse Response Functions – Shifting μ



(a) q_t

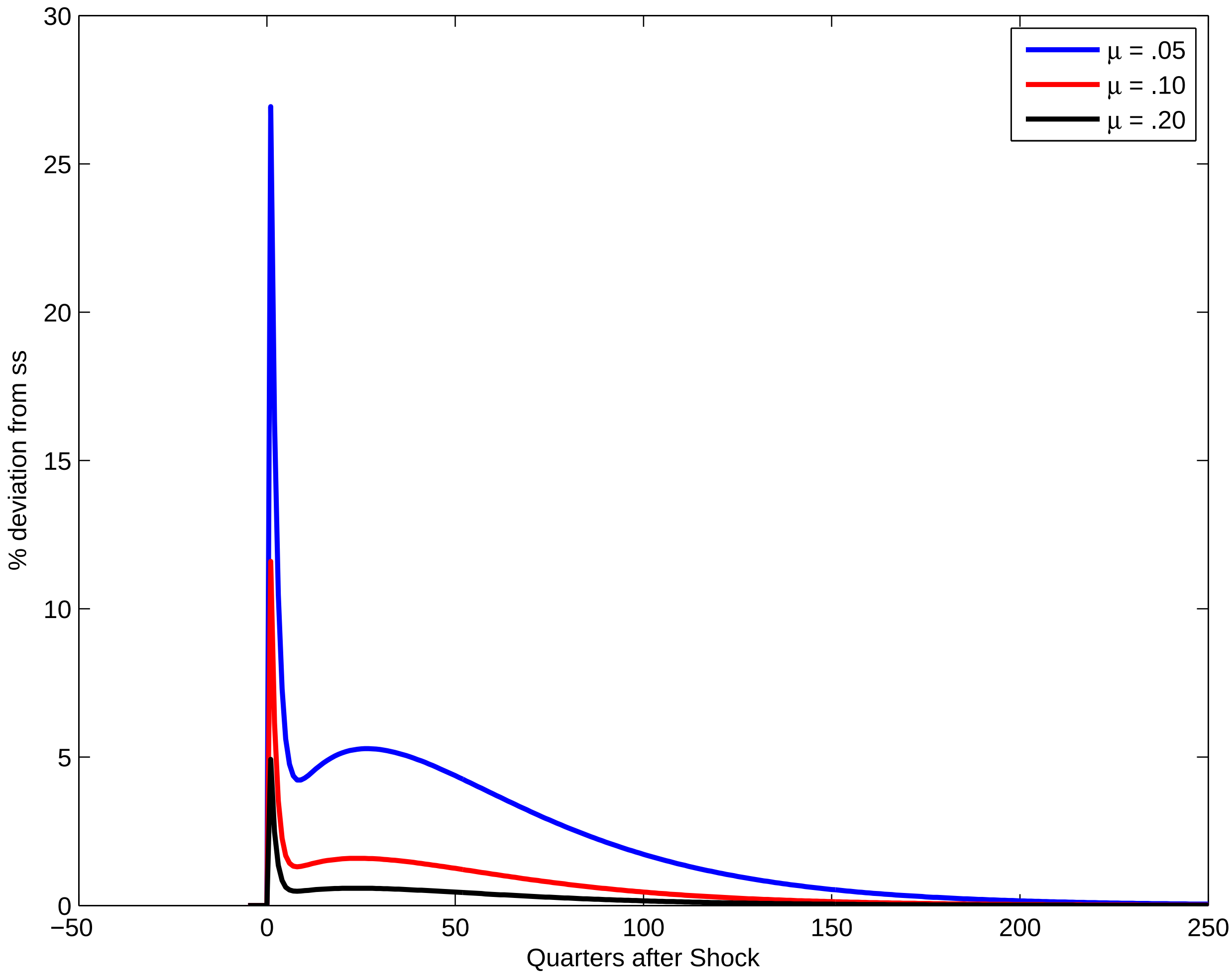
(b) $\triangle Y_t$

(c) $\triangle I_t$

(d) $\triangle R^{lev}_{ex,t+1}$

Impulse response functions are given here with respect to a one standard deviation shock to innovations of the long run component of TFP growth, ϵ_x. The blue, red, and black lines respectively represent different values of μ (.05, .10, .20). All units are given in quarterly values.

Figure 2: Impulse Response Functions – Effect on $\bar{\omega}$



Impulse response functions are given here with respect to a one standard deviation shock to innovations of the long run component of TFP growth, ϵ_x. The blue, red, and black lines respectively represent different values of μ (.05, .10, .20). All units are given in quarterly values.

Appendices

A Analytical Derivations

A.1 Household Problem

The decision problem of the Epstein-Zin household can be written as:

$$U_t = \underset{\{c_t, l_t, k_{t+1}, s_{t+1}\}}{\text{Max}} \left((1-\beta)\left(V(c_t, l_t)\right)^{\frac{1-\gamma}{\theta}} + \beta \left(E_t \left[U_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right)^{\frac{\theta}{1-\gamma}}$$

$$\text{s.t.} \qquad c_t + q_t k_{t+1} + q_t \Phi_a\left(k_t, k_{t+1}\right) k_t = w_t l_t + r_t k_t + q_t(1-\delta)k_t$$

where $V(c_t, l_t)$ represents the *intra*temporal utility from consumption and labor choice. The price of capital is q_t and δ is its depreciation rate. The EZ parameters are standard with γ representing risk aversion, ψ representing IES, and $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$. We proceed to solve the household problem with the following Lagrangian:

$$\mathcal{L} = U_t + \lambda_t \left(w_t l_t + r_t k_t + q_t(1-\delta)k_t - c_t - q_t k_{t+1} - q_t \Phi_a\left(k_t, k_{t+1}\right) k_t \right)$$

$$\frac{\partial \mathcal{L}}{\partial c_t}: \qquad \frac{\theta}{1-\gamma} \left(U_t^{\frac{1-\gamma}{\theta}} \right)^{\frac{\theta}{1-\gamma}-1} (1-\beta)(1-\frac{1}{\psi}) \left(V_t\right)^{-\frac{1}{\psi}} V_{c.t} - \lambda_t = 0$$

$$\frac{\partial \mathcal{L}}{\partial l_t}: \qquad \frac{\theta}{1-\gamma} \left(U_t^{\frac{1-\gamma}{\theta}} \right)^{\frac{\theta}{1-\gamma}-1} (1-\beta)(1-\frac{1}{\psi}) \left(V_t\right)^{-\frac{1}{\psi}} V_{l.t} + \lambda_t w_t = 0$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}}: \qquad \frac{\theta}{1-\gamma} \left(U_t^{\frac{1-\gamma}{\theta}} \right)^{\frac{\theta}{1-\gamma}-1} \beta \left(\frac{1}{\theta}\right) \left(E_t \left[U_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\theta}-1} E_t \left[(1-\gamma) U_{t+1}^{-\gamma} \frac{\partial U_{t+1}}{\partial k_{t+1}} \right] - \lambda_t \left(q_t \left[1 + \frac{\partial \Phi_{a,t}}{\partial k_{t+1}} k_t \right] \right) = 0$$

$$\text{Envelope}: \qquad \frac{\partial U_t}{\partial k_t} = \frac{\partial \mathcal{L}}{\partial k_t} = \lambda_t \left(r_t + q_t(1-\delta) - q_t \left[\Phi_{a,t} + \frac{\partial \Phi_{a,t}}{\partial k_t} k_t \right] \right)$$

If we bring λ terms to the right hand side of the first two equations, and divide the first equation by the second we will receive the following relationship governing consumption-labor tradeoffs:

$$w_t V_c(c_t, l_t) = -V_l(c_t, l_t)$$

To receive the Euler condition, we start by dividing the third FOC by the first one and receive:

$$q_t \left(1 + \frac{\partial \Phi_{a,t}}{\partial k_{t+1}} k_t \right) = E_t \left[\frac{\beta}{1-\beta} \frac{U_{t+1}^{-\gamma}}{\left(E_t \left[U_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\theta}-1}} \frac{V_t^{\frac{1}{\psi}}}{V_{ct}} \frac{\partial U_{t+1}}{\partial k_{t+1}} \right]$$

If we substiute the Envelope condition into $\frac{\partial U_{t+1}}{k_{t+1}}$ we receive:

$$
\begin{aligned}
& E_t\left[M_{t+1} R_{t+1}^k\right]=1 \\
\text{s.t.} \qquad & M_{t+1}=\beta \frac{U_{t+1}^{\frac{1}{\psi}-\gamma}}{\left(E_t\left[U_{t+1}^{1-\gamma}\right]\right)^{1-\frac{1}{\theta}}}\left(\frac{V_{t+1}}{V_t}\right)^{-\frac{1}{\psi}}\left(\frac{V_{c,t+1}}{V_{c,t}}\right) \\
& R_{t+1}^k=\frac{r_{t+1}+q_{t+1}(1-\delta)-q_{t+1}\left(\Phi_{a,t+1}+\frac{\partial \Phi_{a,t+1}}{\partial k_{t+1}} k_{t+1}\right)}{q_t\left(1+\frac{\partial \Phi_{a,t}}{\partial k_{t+1}} k_t\right)}
\end{aligned}
$$

which matches the result from the main text.

A.2 Intermediary Contract

The contract between intermediaries and entrpreneurs is a static, one period agreement, that maximizes the entrepreneur's portion of the surplus while ensuring that the participation constraint of the intermediary holds.

$$
\begin{aligned}
& \underset{\{\bar{\omega}_t, i_t\}}{\operatorname{Max}}\, q_t i_t f(\bar{\omega}_t) \\
\text{s.t.} \qquad & q_t i_t g(\bar{\omega}_t) \geq i_t-n_t
\end{aligned}
$$

where n_t and q_t are assumed to be known at the time of the contract.

As the lender is assumed to be risk neutral we know that his participation constraint will bind, which will result in:

$$
i_t=\frac{n_t}{1-q_t g(\bar{\omega}_t)}=i\left(\bar{\omega}_t\right)
$$

Notice that solving for i is interchangeable with ω_t as this is a one-to-one mapping (based on properties of f). Hence the maximization problem has ω_t as a sole control variable. The first order condition of the original maximand is:

$$
\begin{aligned}
\frac{\partial}{\partial \bar{\omega}_t}\left(q_t i_t f(\bar{\omega}_t)\right) & = q_t i_t f^{\prime}\left(\bar{\omega}_t\right)+q_t f\left(\bar{\omega}_t\right) i^{\prime}\left(\bar{\omega}_t\right)=0 \\
\Longleftrightarrow & = i_t f^{\prime}\left(\bar{\omega}_t\right)+f(\bar{\omega}_t) i_t \frac{q_t g^{\prime}(\bar{\omega}_t)}{1-q_t g(\bar{\omega}_t)}=0
\end{aligned}
$$

Canceling i_t and rearranging terms we receive the condition provided in the main text.

A.3 Entrepreneur Problem

The Lagrangian of the entrepreneur's problem can be written as:

$$
\mathcal{L}^e = \sum_{t=0}^{\infty} \sum_{s^t} \pi\left(s^t\right)\left(\beta \gamma^e\right)^t\left[c_t^e+\mu_t\left(q_t i_t f_t-c_t^e-q_t k_{t+1}^e-q_t \Phi_{a,t}^e k_t^e\right)\right]
$$

where the entrepreneur's choice variables are simply $\left\{c_t^e, k_{t+1}^e\right\}$ as investment, i_t is *predetermined* due to the ex-ante contract. μ_t denotes the Lagrange multiplier on the budget constraint. The first order conditions will be:

$$\frac{\partial \mathcal{L}^e}{\partial c_t^e}: \qquad \pi\left(s^t\right)\left(\beta\gamma^e\right)^t\left[1-\mu_t\right]=0$$

$$\iff \mu_t = 1$$

$$\frac{\partial \mathcal{L}^e}{\partial k_{t+1}^e}: \qquad \pi\left(s^t\right)\left(\beta\gamma^e\right)^t\left[\mu_t\left(-q_t\left(1+\frac{\partial \Phi_{a,t}^e}{\partial k_{t+1}^e}k_t^e\right)\right)\right]$$

$$+\sum_{s^{t+1}}\pi(s^{t+1})\left(\beta\gamma^e\right)^{t+1}\left[\mu_{t+1}\left(q_{t+1}f(\bar{\omega}_{t+1})\frac{\partial i_{t+1}}{\partial k_{t+1}^e}-q_{t+1}\left(\Phi_{a,t+1}^e+\frac{\partial \Phi_{a,t+1}^e}{\partial k_{t+1}^e k_{t+1}^e}\right)\right)\right]=0$$

$$\text{s.t.} \qquad \frac{\partial i_{t+1}}{\partial k_{t+1}}=\frac{\partial n_{t+1}/\partial k_{t+1}}{1-q_{t+1}g_{t+1}}=\frac{r_{t+1}+(1-\delta)q_{t+1}}{1-q_{t+1}g_{t+1}}$$

In the second condition, we treat $\bar{\omega}$ as a price which explains why there are no partials of $f(\bar{\omega})'$ or $g(\bar{\omega})'$ with respect to $k^{e'}$. Also we use the definition of net worth as earlier introduced in the text:

$$n_t = w_t^e + r_t k_t^e + q_t(1-\delta)k_t^e$$

If we rewrite the latter first order condition and plug in μ, we will receive the result from the text:

$$E_t\left[\beta\gamma^e\left\{\frac{\left(\frac{q_{t+1}f(\bar{\omega}_{t+1})}{1-q_{t+1}g(\bar{\omega}_{t+1})}\right)(q_{t+1}(1-\delta)+r_{t+1})-q_{t+1}\left(\frac{\partial \Phi_{a,t+1}^e}{\partial k_{t+1}^e}k_{t+1}^e+\Phi_{a,t+1}^e\right)}{q_t\left(1+\frac{\partial \Phi_{a,t}^e}{\partial k_{t+1}^e}k_t^e\right)}\right\}\right]=1$$

B Data Sources

Consumption

We construct real, per capita consumption (C_t) by summing real measures of nondurables and services consumption. Nominal consumption data for these items come from National Income and Product Accounts (NIPA) table 1.1.5. To get corresponding price deflators for real quantities, we use those in table 1.1.9. Per-capita measures utilize population data in NIPA table 2.1.

Investment

To measure investment (I_t) we use fixed investment, from NIPA table 1.1.5. We adjust this by its price deflator in table 1.1.9 and the population series from earlier.

Output

Output for model moments is provided by the sum of the constructed consumption and investment. As there is no government or trade in the model these are the only series of quantitative relevance for us.

Inflation and Financial Market Data

The annual, nominal data on risk free rates and market returns come from Ken French's website. We compute the market excess return as the annualized difference between the two. To calculate the real risk free rate, we adjust the risk free rate by the growth rate of the GDP deflator index from NIPA table 1.1.4.

To obtain statistics regarding credit spreads, we use Moody's data series on seasoned BBB and AAA corporate yields from the St. Louis FRED.