The Carry Trade and Uncovered Interest Parity when Markets are Incomplete*

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Abstract

We propose a new model to explain the failure of UIP and the profitability of the carry trade and to link these two phenomena to the Balassa-Samuelson effect and the Backus-Smith puzzle. The key features of our model are market incompleteness and partial risk sharing through tradable goods. In the model, carry trade profits are due to two independent channels. First, a purely nominal channel, which works even in complete markets, makes the carry trade risky due to (endogenously) counter-cyclical inflation. Second, a real channel, which, due to imperfect risk sharing, makes the carry trade risky exactly when risk sharing is needed most. The model is consistent with several empirical facts. In particular: (i) real and nominal currency appreciations are positively related to *domestic* output and consumption growth, (ii) carry trade profits are positively related to output and consumption growth, but negatively to inflation in the target (high interest rate) country, (iii) ex-ante, target countries are smaller and have higher expected inflation volatility, but there do not appear to be systematic differences between high and low interest rate countries in loadings on world output growth, in expected output growth, or in output volatility. Leading existing models give opposite predictions.

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1 Introduction

In a simple, risk neutral setting, uncovered interest parity (UIP) predicts that the currency of a country with a high interest rate is expected to depreciate so that the profit is the same as from investing in a low interest rate bond. Empirically, UIP fails and the expected profit from the carry trade (investing in a high interest rate bond abroad, and then converting back to the home currency) is positive. In other words, there is a positive correlation between interest rate differentials, and the currency appreciation.

In this paper, we link the failure of UIP and the profitability of the carry trade to two widely studied (though rarely in the in context of UIP) phenomena in international economics. The first is the Balassa-Samuelson effect, i.e., the empirical observation that countries which experience higher output growth also experience currency appreciation.[1] The second is the Backus-Smith puzzle, i.e., the empirical observation that countries which experience higher consumption growth also experience currency appreciation. This observation is referred to as a puzzle because most models with complete financial markets imply exactly the opposite relationship: countries with higher consumption growth should experience currency depreciations.[2,3]

We propose a model with incomplete financial markets and internationally tradable goods to explain the failure of UIP and the profitability of the carry trade, which at the same time, is consistent with the data regarding both the Balassa-Samuelson effect and the Backus-Smith puzzle. To the best of our knowledge, ours is the first model to study UIP and the carry trade in an incomplete market setting. As will be discussed below, our mechanism is quite different from previous explanations of the carry trade, and our model leads to additional empirical implications.

[1] A common way to rationalize this is through a model of tradable and non-tradable goods.

[2] Complete markets imply that $m_H - m_F = \Delta er$. Here m is the log of the stochastic discount factor and $e = \log(E)$ where E is the amount of foreign currency per unit of home. Thus, $\Delta e > 0$ implies an appreciation of home currency. This relation can be derived through no arbitrage, see Backus and Smith (1993). For example, with CRRA preferences, $m = -\gamma \Delta c$, thus $\Delta c_H - \Delta c_F = -\frac{1}{\gamma}\Delta e$. The intuition is that after a positive productivity shock at home, the domestic price falls, implying a a currency depreciation.

[3] The Backus-Smith puzzle is usually stated in terms of second moments, i.e., countries with a volatile real exchange rate do not tend to have volatile real consumption. However a more direct implication of complete markets is to look at the relationship between realized exchange rate growth and realized consumption growth. Complete markets usually imply a negative relationship between currency appreciation and consumption growth—in the data it is positive (see Section 2 below). Although we do not focus on second moments, our model's implications are consistent with the data. In particular: across the different specifications of our model that we consider in Section 4, there is a positive relationship between exchange rate volatility and consumption growth volatility; within any model, exchange rate volatility is higher than consumption growth volatility.

In our model, carry trade profits come from one of two distinct channels. The first, which may happen even in a model of perfect risk sharing, is a nominal channel that relies on counter-cyclical inflation. The carry trade is risky because inflation in the high interest rate country is highest when output is low, which erodes the real value of investing in nominal bonds.[4] The second, which may happen even in a purely real model, relies on imperfect risk sharing leading to pro-cyclical movements in a country's currency (the Balassa-Samuelson effect and the Backus-Smith puzzle). The carry trade is risky because the currency of the high interest rate country depreciates when risk sharing is needed most and when output is low. Both channels have empirical implications that are broadly consistent with the data, but not with existing complete markets models.

Most existing explanations of the carry trade assume that financial markets are complete, and that there is a strong home bias in consumption (often resulting in zero trade in goods markets); they attribute positive carry trade profits to high risk (covariance with consumption growth). However, as mentioned above, the assumption of complete markets typically implies that there is a negative relationship between a country's output and consumption growth relative to other countries, and the appreciation of its currency relative to other countries.[5] In this setting, the carry trade is most profitable after a negative shock to the high interest rate country; this is counterfactual.

The positive relationship between currency appreciation and relatively strong local economic growth is not only true empirically, but widely consistent with practitioner intuition and the financial press. For example *The New York Times* writes "Across trading desks in New York, London and elsewhere, analysts are rushing to raise their dollar forecasts based on the resurgence in the American economy. The recent rally in the dollar... underscores expectations that the United States economy will continue to grow at a faster clip than that of Europe, Japan and even large emerging markets."[6] Furthermore, empirically, the carry trade is most profitable when output growth is high and inflation is low in the high interest rate country. Our model reproduces these patterns.

We argue that the carry trade is risky because the foreign currency will depreciate exactly when the foreign country suffers negative shocks. These negative shocks will also affect domestic consumption negatively due to trade. To the best of our knowledge, this channel is missing from all the

[4]Hollifield and Yaron (2001) argue that carry trade profits are mostly due to real sources.

[5]One exception is Colacito and Croce (2013), which we discuss below.

[6]Landon Thomas Jr. "Buoyant Dollar Recovers Its Luster, Underlining Rebound in U.S. Economy," Sept 25, 2014, *The New York Times*.

previous models explaining the carry trade. Furthermore, unlike some of the previous explanations of the carry trade and the failure of UIP, our model does not require cross-country differences in risk aversion (Verdelhan (2010), Heyerdahl-Larsen (2014)), volatility (Colacito (2009), Bansal and Shaliastovich (2012), Tran (2013)), expected growth (Colacito and Croce (2013)), loading on aggregate productivity risk (Colacito, Croce, Gavazzoni, and Ready (2014)), exporter versus importer status (Ready, Roussanov, and Ward (2014)), or exotic preferences.

Given the abundance of models, ultimately, the fundamental difference between countries, which is responsible for interest rate differentials and carry trade profits is an empirical question. Our model can accommodate the above cross-country differences, but given the empirical evidence in Section 2, in our preferred version of the model, high interest rate countries simply have higher sovereign default risk. Furthermore, unlike most previous models, all versions of our model produce the aforementioned positive correlation between currency appreciation and positive economic news - this is a feature of the imperfect risk sharing within our model, which is absent in earlier explanations of the carry trade.

Our model can also reproduce two key empirical observations about the carry trade, which motivated previous models. As in the data (Lustig and Verdelhan (2007)), both our model and complete markets models suggest that the carry trade is risky because it is positively correlated with world output and consumption growth. Additionally, as in the data (Lustig, Roussanov, and Verdelhan (2011)), in our model the carry trade is more profitable when world volatility is high. What is different from previous work is the mechanism, as well as additional empirical implications. Let ΔE be the appreciation of Home currency and R_{CT} the realized carry trade profit of borrowing in Home and investing in Foreign. Consistent with the data, our model implies that $corr(\Delta Y_F - \Delta Y_H, \Delta E) < 0$ and that $corr(\Delta Y_F - \Delta Y_H, R_{CT}) > 0$; most complete markets model imply the opposite. Thus, by integrating trade with frictions in financial markets, we show that both are important to explain prices and quantities jointly.

An example may be in order. Consider the popular carry trade of borrowing Japanese yen and investing in Australian dollars. The carry trade is profitable because it carries consumption risk, that is, the Australian dollar depreciates against the Japanese yen exactly when Japanese consumption is low (Lustig and Verdelhan (2007)). However, the mechanism and empirical implications in our model are very different from complete markets. In complete markets, this trade is risky because the Australian dollar will depreciate when Japan experiences worse shocks than Australia; this makes Japanese output relatively low, and due to home bias, will in turn make Japanese consumption low. In our model, this trade is risky because the Australian dollar will depreciate when Australia experiences worse shocks than Japan; this makes Australian output relatively low and, through trade, will also affect Japanese consumption.

We now summarize our main findings. We identify two sources of carry trade profits, one nominal and one real. The nominal can occur even when markets are complete and risk sharing is perfect, while the real one relies on imperfect risk sharing. Our first set of results concerns cases when risk sharing is perfect:

- 1A) Only two nominal bonds are oftentimes enough to allow for perfect risk sharing despite incomplete markets. This is because inflation causes nominal bond payoffs to become equity-like.
- 1B) When risk sharing is perfect, the carry trade can be profitable only due to counter-cyclical inflation. This result is different from existing complete markets models because we allow for trade.[7]
- 1C) Carry trade target countries tend to have higher inflation volatility; moreover, the realized carry trade profit is positively correlated with realized output growth (in the carry target country) and negatively with realized inflation (in the carry target country). This result is consistent with the data, but not consistent with past explanations of the carry trade, where carry trade profits are highest following negative shocks in the carry country.
- 1D) Carry trade target countries have a larger loading on aggregate risk. Within the context of the model, they are larger, more volatile, and grow faster. This result is analogous to existing complete market models, but is not consistent with the data.

Our second set of results concerns cases when risk sharing is not perfect:

- 2A) Adding symmetric non-tradable shocks to a model with otherwise perfect risk sharing can improve the model's performance on consumption and risk sharing, but does not affect the carry trade or UIP (see footnote 7).

[7] Asymmetric non-tradable shocks would affect the model's implications for the carry trade and UIP. This is the channel in Colacito (2009), Bansal and Shaliastovich (2012), Tran (2013).

- 2B) Goods markets frictions, which make risk sharing imperfect, do not help with explaining the carry trade or UIP.
- 2C) Financial market frictions, which make markets incomplete, usually do help with explaining the carry trade and UIP. This explanation can work together with counter-cyclical inflation, but does not rely on counter-cyclical inflation.
- 2D) Just as 1C), carry trade target countries tend to have higher inflation volatility; moreover, the realized carry trade profit is positively correlated with realized output growth and negatively with realized inflation. This result is consistent with the data, but not consistent with past explanations of the carry trade where carry trade profits are highest following negative shocks in the carry country. Furthermore, the above relationship between output and carry trade profit holds even when inflation is exactly zero.
- 2E) Unlike 1D), carry trade target countries do not need to be larger, more volatile, or to grow faster.

Our model is not the first to study incomplete markets in an international setting. Most work has focused only on the Backus-Smith puzzle (Backus and Smith (1993)), that is explaining a positive relationship between currency appreciation and consumption growth. Our model's setup is closest to Benigno and Thoenissen (2008), as they employ the same two ingredients: two nominal bonds and tradable and non-tradable goods. Corsetti, Dedola, and Leduc (2008) is a related model but with only one bond. Recently, Rabitsch (2014) explores the implications of incomplete markets for UIP. In her setup, borrowing constraints lead to imperfect risk sharing and to pro-cyclical movements in a country's currency. However, as there is only one bond, the carry trade is not possible in her economy. Hassan and Mano (2014) argue that the failure of UIP and the carry trade may be distinct phenomena. In our model they are closely related.

Although most complete markets models (counterfactually) imply a negative relationship between consumption growth and currency appreciation, one exception is the complete markets economy of Colacito and Croce (2013). In their model, like in other complete markets models, a positive (long run) productivity shock leads to a currency depreciation. However, the positive productivity shock also leads to lower (relative) consumption growth as the country saves more for precautionary reasons. As a result, like in our model and in the data, their model achieves a positive correlation between consumption growth differentials and currency appreciation. At the same time, due to cross-sectional differences in volatility and expected growth, the UIP does not hold in their model, as in the data. Although our model targets related phenomena, our mechanism is very different because in our model markets are incomplete and risk sharing is imperfect. Our framework differs in two important aspects from existing studies that rely on complete financial markets. First, in our model there is a *positive* correlation between currency appreciation (of the carry trade target country) and carry trade profit, productivity shocks, output growth, and consumption growth. Second, to explain violations of UIP and profitability of the carry trade we do not need to rely on asymmetries in expected growth rates, or expected volatility across countries.

The rest of the paper is laid out as follows. Section 2 reviews existing empirical evidence regarding the carry trade, presents new empirical results, and relate these results to past models of the carry trade. Section 3 describes our model. Section 4 explains why this model can explain these empirical results. Section 5 concludes.

2 Empirical evidence

The goal of this project theoretical: we aim to build a relatively flexible model which can explain the profitability of the carry trade and the failure of UIP, and for the model's implications to be consistent with exchange rate behavior in the real world. In order to know which specifications of the model are most relevant empirically, and to help interpret the model's results, in this section we carry out some elementary empirical analysis. Some of the empirical results below confirm the findings of past studies, while others are new. Additionally, in Appendix A we explore the mechanism behind two of the leading models used for understanding the carry trade: Verdelhan (2010) and Bansal and Shaliastovich (2012). Below, we will compare the empirical implications from these two models to our model and to the data.

Note that in this paper the notation is always such that E is the nominal amount of foreign currency per unit of home currency. Therefore, $E_{t+1} - E_t > 0$ means that the Home currency (in this case the U.S. dollar) appreciates against the Foreign currency.

The data is described in detail in Appendix C and comes from DataStream. The data covers 40 countries between 1982 and 2013. Not all countries have data for all years, resulting in an unbalanced panel. To study empirical relationships between variables of interest, we employ one of three empirical approaches. First, we run univariate pooled regressions across all countries and years; this results in 736 data points. The results of this approach are on the left of Table 1, here we present the correlation between the variables, the slope, and the t-statistic. Second, we run univariate regressions in each country. On the right of Table 1 we present the average correlation, the average slope, the fraction of positive slopes (negative in Panel D), and the fraction of slopes with a t-statistic above 2. Third, we sort all countries in a particular year into quintiles based on the variable of interest and compare the averages in each quintile. For brevity we do not present the sorted results, however, they are always consistent with the first two approaches.

In panel A of Table 1 we reproduce the well known failure of UIP and profitability of the carry trade. In the panel we report the results (pooled and by country) of the following regression

$$\mathcal{Y}_{t+1} = a + b\left(\log(R_t) - \log(R_t^{US})\right) + \epsilon_{t+1} \tag{1}$$

where R_t and R_t^{US} denote, respectively the interest rate in the foreign and in the domestic (U.S.) country. In the row labeled “UIP”, $\mathcal{Y}_{t+1} = -\Delta E = -\log(\frac{E_{t+1}}{E_t})$, i.e., the foreign currency appreciation against the U.S. dollar, and in the row labeled “CARRY”, $\mathcal{Y}_{t+1} = \log(R^C_{t+1}) = -\log(\frac{E_{t+1}}{E_t}) + \log(R_t) - \log(R_t^{US})$, i.e., the carry trade profit when the U.S. is the low interest rate country.

Note that if UIP holds, then $\log(\frac{E_{t+1}}{E_t}) = \log(R_t) - \log(R_t^{US})$ implying that the slope coefficient in the regression (1) with $\mathcal{Y}_{t+1} = -\log(\frac{E_{t+1}}{E_t})$ is equal to -1. We find the slope to be insignificant from zero; point estimates are slightly negative in the pooled, but positive in the country by country approach. The failure of UIP suggests positive carry trade profits; indeed, we find carry trade profits to be positive in both empirical approaches. These results are consistent with prior literature.[8]

In Panel B of Table 1 we present the relationship between output growth differentials and currency appreciation. Specifically, we estimate the following regression (pooled and by country)

$$\mathcal{Y}_{t+1} = a + b\left(\Delta\log(GDP_t) - \Delta\log(GDP_t^{US})\right) + \epsilon_{t+1}. \tag{2}$$

The dependent variable is real output growth. The independent variable $\mathcal{Y}_{t+1}$ is either the nominal appreciation of foreign currency, $-(\log(E_{t+1}) - \log(E_t))$ or the real appreciation of foreign currency,

[8]See Hodrick (2014) for a recent survey of the empirical literature on foreign exchange markets.

$-(\log(ER_{t+1}) - \log(ER_t))$. The real exchange rate ER_t is defined as $ER_t = E_t \frac{P_t^{\mathrm{US}}}{P_t}$, where P_t^{US} and P_t are, respectively, the U.S. and foreign price indices; note that we define a large ER to mean a strong U.S. dollar. Countries with higher realized real output growth tend to contemporaneously experience both real and nominal currency appreciations. This is the Balassa-Samuelson effect (Balassa (1964), Samuelson (1964)) and is consistent with prior empirical findings.[9]

Corsetti, Dedola, and Leduc (2008) also find a similar positive relationship between realized consumption growth and real currency appreciations for the OECD. Similarly, Devereux and Hnatkovska (2014) find a positive relationship between realized consumption growth and real currency appreciations in international data, but a negative relationship in intra-national; this is suggests the importance of market incompleteness for international settings.[10]

As mentioned in the introduction, a positive relationship between currency appreciation and either output or consumption growth is inconsistent with the implications of most perfect risk sharing models (this is part of the Backus-Smith puzzle, Backus and Smith (1993)), although it is consistent with the model we develop in Section 3. If risk sharing is perfect then differences in output growth could be smoothed out through trade and would not necessarily translate into differences in consumption growth. In the data, the correlation between output growth and consumption growth is 0.74, suggesting risk sharing is far from perfect.[11] The Backus-Smith puzzle is evident in Panel C, which is identical to Panel B but uses consumption growth instead of output growth. Countries with higher realized consumption growth tend to contemporaneously experience both real and nominal currency appreciations.

Panels D and E of Table 1 document the negative relationship between real output growth and realized inflation. Supply shocks should lead to a negative relationship, while demand shocks to a positive relationship. As will be discussed in Section 4, our model implies that if risk sharing is perfect, then this relationship must be negative for the carry trade to be profitable (if risk sharing is imperfect the carry trade may be profitable regardless of how inflation behaves). Panel D uses the same cross-country data as panels A and B. The relationship between inflation and GDP growth is negative but insignificant, however if instead we regress inflation relative to U.S. inflation on output growth relative to U.S. output growth then the relationship is negative and significant. For our

[9]See Tica and Druzic (2006) for a survey of empirical evidence on the Balassa-Samuelson effect.

[10]Devereux and Hnatkovska (2009) explain this in a model of incomplete risk sharing and both within and across country shocks.

[11]It is also 0.74 is we instead use consumption and output growth relative to the U.S.

model, this second, relative relationship is more relevant. In Panel E, we test the same relationship but for a longer time series available for the U.S. only. We find this relationship to be negative but insignificant for GDP, however, negative and significant when we replace GDP by TFP.[12] This finding is consistent with the interpretation of TFP as a measure of supply shocks. Since GDP is a combination of demand and supply shocks, it is not surprising that we see a stronger relationship in TFP than in GDP.

In Table 2 we explore how country characteristics relate to the carry trade. Carry trade target countries are, by definition, countries with relatively higher interest rates. In panel A we regress several country characteristics at t and $t+1$ on the interest rate differential at t. Carry trade target (high interest rate) countries tend to be smaller, this was pointed out by Hassan (2013). However, these are not countries which have a higher loading on world output growth, either backward or forward looking. Perhaps surprisingly, these are also not countries that tend experience high output growth or high output volatility going forward;[13] the coefficients for both are actually negative, though insignificant. However, high interest rate differential countries do tend to experience high inflation and high inflation volatility going forward; perhaps higher inflation is not surprising as the high nominal interest rate, in part, reflects inflation expectations.[14]

As shown in Appendix A, existing models rely on certain ex-ante differentials between countries to explain the carry trade. In particular, in habit models, such as Verdelhan (2010), the target country needs to be bigger;[15] in the data target countries are smaller. On the other hand, the driving force in Bansal and Shaliastovich (2012) is heteroscedasticity. This model requires the target country to have lower expected volatility of output growth, consumption growth, or inflation; in the data target countries do not appear to have lower expected output volatility, and actually have higher expected inflation volatility.[16]

[12]TFP data comes from John Fernald's website.

[13]We define volatility as the realized annual standard deviation in years $t+1$ through $t+4$.

[14]The results in this table are consistent with Gourio, Simer, and Verdelhan (2013), who also find little relationship between high interest rate countries and output volatility. However, they do find that high interest rate countries have higher consumption growth volatility.

[15]This is because for this channel to work, the target country must have a lower risk aversion. A lower risk aversion is usually associated with bigger countries—positive shocks lead to larger size and to lower risk aversion as consumption is further from habit. An exception is Heyerdahl-Larsen (2014), where deep habit preferences allow the target country to be smaller.

[16]The key driving force in Bansal and Shaliastovich (2012) is a higher volatility of shocks in the low interest rate country; because there is perfect home bias, non-tradable output is equal to total output. The carry trade in Tran (2013) is profitable for exactly the same reason as in Bansal and Shaliastovich (2012), but the model allows for both tradables and non-tradables. The non-tradable sector needs to be either more volatile or more important for low interest rate countries. He provides some evidence that this is indeed true.

One existing model that does not rely on the above differences is Ready, Roussanov, and Ward (2014), where carry trade target countries are commodity exporters. These countries endogenously have a higher interest rate but are also safer because they are insulated from aggregate shocks resulting in positive carry trade profits.

As will be shown below, our model does not rely on ex-ante differences in size, output volatility, inflation volatility, growth, or loading on world output to explain the carry trade. As in the data, in most versions of our model, carry trade target countries do have higher expected inflation volatility. Additionally, we solve versions of our model where the target country is smaller, as in the data.

In panel B of Table 2 we link *realized* carry trade profits to the same characteristics. In particular, we regress the realized profit from the carry trade at $t+1$ on the interest rate differential at t and on one of the aforementioned characteristics. The interest rate coefficient has similar magnitude and significance as in the univariate regression of carry trade profits on interest rate differentials in panel A of Table 1.

Realized carry trade profits do not seem related to country size or to past loading on world output growth. There is some evidence that countries with a low realized loading on world output have higher realized carry trade profits.

Realized carry trade profits are higher when the target country experiences high output growth (insignificant for output shocks but significant for output shocks relative to the U.S.). This result is contrary to what is implied by most complete markets models, where currency depreciates after positive shocks. This is perhaps the sharpest difference between existing models and our model. Consistent with the data, our model produces a positive relationship, due to a combination of currency appreciation (through the Balassa-Samuelson effect) and fewer realized defaults.

Realized carry trade profits are higher when the target country experiences low realized inflation. This result is also consistent with our model, however, note that both in our model and in the data, inflation and output are negatively correlated thus the two results above are linked.[17] This result is also consistent with Bansal and Shaliastovich (2012), where counter-cyclical inflation is one of two channels that can lead to a carry trade. Unlike our model, in Bansal and Shaliastovich (2012), the process for inflation is specified exogenously.

[17] When we include both output growth and inflation in the same regression, output remains significant but inflation is no longer so.

The relationships between realized carry trade profits, realized output shocks, and realized inflation are consistent with recent empirical work by Kim (2014) who investigates two specific carry trade strategies: borrowing in USD to invest in the relatively higher interest rate Australian Dollar (AUD), and borrowing in Japanese Yen (JPY) to invest in the relatively higher interest rate USD. For AUD, carry trade returns are highest when inflation and unemployment in the high interest rate country (Australia) are unexpectedly low. For JPY, carry trade returns are highest when machine orders in the low interest rate country (Japan) are unexpectedly low, and retail sales growth in the U.S. is unexpectedly high.

Note that the above results are about realized output growth in the high interest rate (carry trade target) country, or relative output growth between the high and low interest rate countries. Most studies have focused on the relationship between carry trade profits and world quantities. Gourio, Simer, and Verdelhan (2013) show that realized carry trade profits are higher when realized world output growth is high, that is the carry trade loads on aggregate risk. This is also true in our model, however, due to imperfect risk sharing, our model's more direct predictions are about country specific output. Lustig, Roussanov, and Verdelhan (2011) show that carry trade profits are highest when world volatility is high, this is also true in our model.

3 Model

We consider a two-period general equilibrium model of a world economy consisting of two countries: Home (H) and Foreign (F). Each country produces two types of goods: Tradable (T) and Non Tradable (N). We assume that, within each country financial markets are complete and therefore allocations are determined in each economy by the optimal consumption and saving decisions of a representative household. The agents in each country receive a country-specific endowment of both tradable and non-tradable goods and can only trade in two nominal riskless bonds denominated in the domestic and foreign currency, respectively. This setup is standard, except for the market incompleteness (which resembles Corsetti, Dedola, and Leduc (2008) and Benigno and Thoenissen (2008)); later we also allow for default, which is non-standard. In what follows we describe the household optimization problem and derive the equilibrium conditions.

We denote by $C_{\mathrm{H,t}}$, $t = 0, 1$, the composite consumption good in H given by

$$C_{\mathrm{H,t}} = \left((1-\theta)(C^{\mathrm{T}}_{\mathrm{H,t}})^{\alpha} + \theta(C^{\mathrm{N}}_{\mathrm{H,t}})^{\alpha}\right)^{\frac{1}{\alpha}}, \quad 0 < \theta < 1, \tag{3}$$

where $C^{\mathrm{T}}_{\mathrm{H,t}}$ and $C^{\mathrm{N}}_{\mathrm{H,t}}$ denote, respectively, consumption of the T and N goods at time $t = 0, 1$. The parameter θ in (3) is the share of non-tradable good in the consumption basket and α is the intratemporal elasticity of substitution between T and N goods.[18]

The agent receives an exogenous endowment of both tradable and non-tradable good at each date. Specifically, we denote by $Y^{\mathrm{T}}_{\mathrm{H},0}$ and $Y^{\mathrm{N}}_{\mathrm{H},0}$ the time-zero endowments of T and N goods and by $Z_{\mathrm{H}}Y^{\mathrm{T}}_{\mathrm{H},1}$ and $Z_{\mathrm{H}}Y^{\mathrm{N}}_{\mathrm{H},1}$ the time-1 endowments, with Z_{H} representing a random productivity shock. The ratio $Y^{\mathrm{T}}_{\mathrm{H},1}/Y^{\mathrm{T}}_{\mathrm{H},0}$ and $Y^{\mathrm{N}}_{\mathrm{H},1}/Y^{\mathrm{N}}_{\mathrm{H},0}$ are the expected growth in the T and N endowment, respectively. If $Z_{\mathrm{H}}Y^{\mathrm{T}}_{\mathrm{H,t}} > C^{\mathrm{T}}_{\mathrm{H,t}}$ country H exports to country F at time $t = 0, 1$.

The household in the H country solves the following problem

$$\max_{\{C^{\mathrm{T}}_{\mathrm{H},0}, C^{\mathrm{N}}_{\mathrm{H},0}, B^{\mathrm{H}}_{\mathrm{H}}, B^{\mathrm{F}}_{\mathrm{H}}, C^{\mathrm{T}}_{\mathrm{H},1}, C^{\mathrm{N}}_{\mathrm{H},1}\}} \left(C^{1-\rho}_{\mathrm{H},0} + \beta\mathbb{E}\left[C^{1-\gamma}_{\mathrm{H},1}\right]^{\frac{1-\rho}{1-\gamma}}\right)^{\frac{1}{1-\rho}}, \quad 0 < \beta < 1, \tag{4}$$

with $\mathbb{E}$ denoting expectation conditional on information at time 0, subject to the following budget constraints at time 0 and 1

$$P^{\mathrm{T}}_{\mathrm{H},0}C^{\mathrm{T}}_{\mathrm{H},0} + P^{\mathrm{N}}_{\mathrm{H},0}C^{\mathrm{N}}_{\mathrm{H},0} + q_{\mathrm{H}}B^{\mathrm{H}}_{\mathrm{H}} + \frac{q_{\mathrm{F}}}{E_0}B^{\mathrm{F}}_{\mathrm{H}} = P^{\mathrm{T}}_{\mathrm{H},0}Y^{\mathrm{T}}_{\mathrm{H},0} + P^{\mathrm{N}}_{\mathrm{H},0}Y^{\mathrm{N}}_{\mathrm{H},0} \tag{5}$$

$$P^{\mathrm{T}}_{\mathrm{H},1}C^{\mathrm{T}}_{\mathrm{H},1} + P^{\mathrm{N}}_{\mathrm{H},1}C^{\mathrm{N}}_{\mathrm{H},1} = P^{\mathrm{T}}_{\mathrm{H},1}Z_{\mathrm{H}}Y^{\mathrm{T}}_{\mathrm{H},1} + P^{\mathrm{N}}_{\mathrm{H},1}Y^{\mathrm{N}}_{\mathrm{H},1} + B^{\mathrm{H}}_{\mathrm{H}} + \frac{1}{E_1}B^{\mathrm{F}}_{\mathrm{H}} \tag{6}$$

where $P^{\mathrm{T}}_{\mathrm{H,t}}$ and $P^{\mathrm{N}}_{\mathrm{H,t}}$, $t = 0, 1$, denote the price in H currency of T and N goods; $B^{\mathrm{H}}_{\mathrm{H}}$ and $B^{\mathrm{F}}_{\mathrm{H}}$ denote the quantities of domestic and foreign bonds held by the household; q_{H} and q_{F} denote the nominal prices of the domestic and foreign bond; and E_t, $t = 0, 1$, denotes the exchange rate, expressed as the relative price of the foreign composite consumption good to the domestic composite consumption good. Note that a large E implies a strong home currency.

In one version of the model, we add a cost of holding financing securities. In this case, in the utility function (equation (4)), $C_{\mathrm{H},0}$ is replaced by $C_{\mathrm{H},0} - \kappa\left(\frac{q_{\mathrm{H}}B^{\mathrm{H}}_{\mathrm{H}}}{P_{\mathrm{H},0}}\right)^2 - \kappa\left(\frac{q_{\mathrm{F}}B^{\mathrm{F}}_{\mathrm{H}}}{E_0 P_{\mathrm{H},0}}\right)^2$ where the

[18] Cobb-Douglas consumption aggregation is a special case of the aggregation (3), with $\alpha = 0$. More generally, $\alpha = -\infty$ implies $C_{\mathrm{H,t}} = \min(C^{\mathrm{T}}_{\mathrm{H,t}}, C^{\mathrm{N}}_{\mathrm{H,t}})$ or perfect complements; $\alpha = 0$ is Cobb-Douglas; $\alpha = 1$ implies $C = (1-\theta)C^{\mathrm{T}}_{\mathrm{H,t}} + \theta C^{\mathrm{N}}_{\mathrm{H,t}}$, or perfect substitutes; $\alpha \to \infty$ implies $C^{\mathrm{T}}_{\mathrm{H,t}} = \max(C^{\mathrm{T}}_{\mathrm{H,t}}, C^{\mathrm{N}}_{\mathrm{H,t}})$.

second and third terms are reduced-form utility costs of positions in domestic and foreign bonds, respectively. For example these can be monitoring costs. In most versions of the model $\kappa = 0$ and these costs are absent.

Consumption is determined by the the household's intra-temporal first order conditions combined with aggregate budget constraint:

$$C^{\mathrm{T}}_{\mathrm{H},0} = Y^{\mathrm{T}}_{\mathrm{H},0} - \frac{1}{P^{\mathrm{T}}_{\mathrm{H},0}} \left(q_{\mathrm{H}} B^{\mathrm{H}}_{\mathrm{H}} + \frac{q_{\mathrm{F}}}{E_0} B^{\mathrm{F}}_{\mathrm{H}} \right) \tag{7}$$

$$C^{\mathrm{T}}_{\mathrm{H},1} = Z_{\mathrm{H}} Y^{\mathrm{T}}_{\mathrm{H},1} + \frac{1}{P^{\mathrm{T}}_{\mathrm{H},1}} \left(B^{\mathrm{H}}_{\mathrm{H}} + \frac{1}{E_1} B^{\mathrm{F}}_{\mathrm{H}} \right) \tag{8}$$

$$\frac{P^{\mathrm{N}}_{\mathrm{H},0}}{P^{\mathrm{T}}_{\mathrm{H},0}} = \frac{\theta}{1-\theta} \left(\frac{C^{\mathrm{T}}_{\mathrm{H},0}}{C^{\mathrm{N}}_{\mathrm{H},0}} \right)^{1-\alpha} \tag{9}$$

$$\frac{P^{\mathrm{N}}_{\mathrm{H},1}}{P^{\mathrm{T}}_{\mathrm{H},1}} = \frac{\theta}{1-\theta} \left(\frac{C^{\mathrm{T}}_{\mathrm{H},1}}{C^{\mathrm{N}}_{\mathrm{H},1}} \right)^{1-\alpha} \tag{10}$$

Market clearing condition for the N good implies

$$C^{\mathrm{N}}_{\mathrm{H},0} = Y^{\mathrm{N}}_{\mathrm{H},0} \quad \text{and} \quad C^{\mathrm{N}}_{\mathrm{H},1} = Y^{\mathrm{N}}_{\mathrm{H},1}. \tag{11}$$

Bond holdings are determined by the household's inter-temporal first order condition (Euler equations)

$$q_{\mathrm{H}} = \mathbb{E}\left[\mathbb{M}_1 \frac{P_{\mathrm{H},0}}{P_{\mathrm{H},1}} \right] \quad \text{and} \quad q_{\mathrm{F}} = \mathbb{E}\left[\mathbb{M}_1 \frac{P_{\mathrm{H},0} E_0}{P_{\mathrm{H},1} E_1} \right], \tag{12}$$

where

$$\mathbb{M}_1 = \beta \left(\frac{C_{\mathrm{H},0}}{\mathbb{E}\left[C^{1-\gamma}_{\mathrm{H},1} \right]^{\frac{1}{1-\gamma}}} \right)^{\rho-\gamma} \left(\frac{C_{\mathrm{H},1}}{C_{\mathrm{H},0}} \right)^{-\gamma} \tag{13}$$

denotes the real stochastic discount factor. An equivalent set of intra- and inter-temporal first order condition holds for the household in the F country.

We define the aggregate H price level at time t as

$$P_{\mathrm{H,t}} = \frac{P^{\mathrm{T}}_{\mathrm{H,t}} C^{\mathrm{T}}_{\mathrm{H,t}} + P^{\mathrm{N}}_{\mathrm{H,t}} C^{\mathrm{N}}_{\mathrm{H,t}}}{C_{\mathrm{H,t}}}. \tag{14}$$

Using the intra-temporal first order conditions the above definition implies

$$P_{\mathrm{H,t}} = \left((1-\theta)^{\frac{1}{1-\alpha}}(P^{\mathrm{T}}_{\mathrm{H,t}})^{\frac{\alpha}{\alpha-1}} + \theta^{\frac{1}{1-\alpha}}(P^{\mathrm{N}}_{\mathrm{H,t}})^{\frac{\alpha}{\alpha-1}}\right)^{\frac{\alpha-1}{\alpha}}. \tag{15}$$

We model trade frictions between countries by assuming the existence of "iceberg" shipping costs $\tau > 0$. This implies that for every unit of home (foreign) good shipped abroad, only a fraction $1-\tau$ arrives at destination. If markets are competitive, no arbitrage implies that (see, e.g., Obstfeld and Rogoff (2001))

$$P^{\mathrm{T}}_{\mathrm{H,t}} = \begin{cases} (1-\tau)P^{\mathrm{T}}_{\mathrm{F,t}}\frac{1}{E_t}, & \text{if } Y^{\mathrm{T}}_{\mathrm{H,t}} > C^{\mathrm{T}}_{\mathrm{H,t}} \\ \frac{P^{\mathrm{T}}_{\mathrm{F,t}}}{1-\tau}\frac{1}{E_t} & \text{if } Y^{\mathrm{T}}_{\mathrm{H,t}} \leq C^{\mathrm{T}}_{\mathrm{H,t}} \end{cases}, \quad \text{for} \quad t = 0, 1. \tag{16}$$

The bond market clearing condition implies

$$B^{\mathrm{H}}_{\mathrm{F}} = -B^{\mathrm{H}}_{\mathrm{H}} \quad \text{and} \quad B^{\mathrm{F}}_{\mathrm{F}} = -B^{\mathrm{F}}_{\mathrm{H}}. \tag{17}$$

From the above equation, all quantities of interest are a function of 8 unknowns: q_{H}, q_{F}, $B^{\mathrm{H}}_{\mathrm{H}}$, $B^{\mathrm{F}}_{\mathrm{H}}$, $P^{\mathrm{T}}_{\mathrm{H,0}}$, $P^{\mathrm{T}}_{\mathrm{F,0}}$, $P^{\mathrm{T}}_{\mathrm{H,1}}$, $P^{\mathrm{T}}_{\mathrm{F,1}}$. These unknowns will be determined by household's 4 Euler equations, and by monetary policy. We define monetary policy as the Central Bank's choice of money supply $M_{\mathrm{H,0}}$ and $M_{\mathrm{H,1}}$, where $M_{\mathrm{H,1}}$ is potentially a function of the realized shocks $(Z_{\mathrm{H}}, Z_{\mathrm{F}})$. Specifically, to close the model and determine the equilibrium price level, we assume that the amount of money $M_{\mathrm{H,0}}$, $M_{\mathrm{H,1}}$, $M_{\mathrm{F,0}}$ and $M_{\mathrm{F,1}}$ in the H and F country satisfy

$$M_{\mathrm{H,0}} = P^{\mathrm{T}}_{\mathrm{H,0}}Y^{\mathrm{T}}_{\mathrm{H,0}} + P^{\mathrm{N}}_{\mathrm{H,0}}Y^{\mathrm{N}}_{\mathrm{H,0}} \tag{18}$$

$$M_{\mathrm{H,1}} = P^{\mathrm{T}}_{\mathrm{H,1}}Z_{\mathrm{H}}Y^{\mathrm{T}}_{\mathrm{H,1}} + P^{\mathrm{N}}_{\mathrm{H,1}}Y^{\mathrm{N}}_{\mathrm{H,1}} \tag{19}$$

Rearranging the above equation it is possible to write $P^{\mathrm{T}}_{\mathrm{H,t}}$ as a function of $M_{\mathrm{H,t}}$, $C^{\mathrm{T}}_{\mathrm{H,t}}$ (which itself is a function of $P^{\mathrm{T}}_{\mathrm{H,t}}$), and the 4 unknowns: q_{H}, q_{F}, $B^{\mathrm{H}}_{\mathrm{H}}$, $B^{\mathrm{F}}_{\mathrm{H}}$. Given an arbitrary monetary policy, the equilibrium is determined by the solution of 4 equations (Euler equations) and 4 unknowns.

We set $P^{\mathrm{T}}_{\mathrm{H,0}} = P^{\mathrm{T}}_{\mathrm{F,0}} = 1$, without loss of generality and investigate three possible monetary policies:

1. Constant tradable prices: $P^{\mathrm{T}}_{\mathrm{H,t}} = P^{\mathrm{T}}_{\mathrm{F,t}} = 1$, which implies a fixed exchange rate regime.

2. Zero expected inflation: $\mathbb{E}\left[P_{\mathrm{H},1}\right] = P_{\mathrm{H},0}$. The $t = 1$ money supply is a function of $t = 0$ variables only, i.e. it is a constant. We believe this case to be relevant empirically because if the central bank cannot observe economic shocks in real time, but if low inflation is desired (for example due to menu costs), then the central bank can only target zero expected inflation; of course, empirically, inflation volatility is not zero. This implies that the money supply at $t = 1$ is a function of $t = 0$ variables only.

3. Zero realized inflation: $P_{\mathrm{H},1} = P_{\mathrm{H},0}$. The $t = 1$ money supply is potentially a function of $t = 1$ variables.

In the next section we analyze the implications of each of these three monetary policy regimes on the the uncovered interest parity, the profitability of the carry trade and on the correlation between real exchange and output growth (Balassa-Samuelson effect).

4 Analysis

4.1 Calibration

We now describe the parameter choices under which we solve the model. We will begin the analysis with with logarithmic utility ($\rho = \gamma = 1$) and a Cobb-Douglas aggregator ($\alpha = 0$) over tradable and non-tradable consumption, this simple utility allows us to build qualitative intuition. In particular, households maximize:

$$U = \log(C_0) + \beta\mathbb{E}\left[\log(C_1)\right] \tag{20}$$

$$C_t = (C_t^T)^{1-\theta}(C_t^N)^{\theta} \tag{21}$$

We set $\theta = 0.5$, which implies that the fraction of non-tradables in consumption is 50%.[19]

In the general case of Epstein-Zin-Weil preferences (Epstein and Zin (1989),Weil (1989)) with a CES consumption aggregator, we set the intertemporal elasticity of substitution, $\frac{1}{\rho}$ to 1.5 and the risk aversion, γ to 7.5. This is a popular calibration of the long run risk model, used in Bansal and Yaron (2004). In Appendix B we also present results for $\alpha = 0.5$, which implies a higher

[19] Corsetti, Dedola, and Leduc (2008) estimate that 53% of consumption in the U.S. is non-traded; Stockman and Tesar (1995) find a value of 50% for OECD countries.

degree of substitutability between T and N goods, and $\alpha = -1.0$ which implies a lower degree of substitutability.[20] The bond issuing cost κ and the iceberg shipping cost τ are initially set to zero but we solve versions of the model in which we allow for the existence of shipping costs. We always keep the above parameters symmetric across both countries.

In order to speak to the carry trade, there needs to be some asymmetry across the two countries, otherwise there could be no ex-ante differentials in interest rates or forward rates.[21] In Sections 4.2.3 and 4.3.1 we will explore four sources of asymmetry. The first source of asymmetry is higher expected growth in the F tradable sector (section 4.2.3), specifically, $Y^{\mathrm{N}}_{\mathrm{H},0} = Y^{\mathrm{N}}_{\mathrm{H},1} = Y^{\mathrm{N}}_{\mathrm{F},0} = Y^{\mathrm{N}}_{\mathrm{F},1} = Y^{\mathrm{T}}_{\mathrm{H},0} = Y^{\mathrm{T}}_{\mathrm{H},1} = Y^{\mathrm{T}}_{\mathrm{F},0} = 1$ and $Y^{\mathrm{T}}_{\mathrm{F},1} = 1.1$, under the distributional assumption for the shocks discussed next, this implies that expected GDP growth in F is approximately 5% higher than in H. The second source of asymmetry is higher volatility of shocks in F (section 4.2.3). When there is no asymmetry in volatility, both countries face equal sized, independent productivity shocks in the tradable sector: $Z_{\mathrm{H}} = Z_{\mathrm{F}} = (0.93, 1.0, 1.07)$ with equal probability of each of nine states of the word, implying real GDP volatility of around 3%. When there is asymmetry in volatility the shocks are $Z_{\mathrm{F}} = (0.9067, 1.0, 1.0933)$ and $Z_{\mathrm{H}} = (0.9534, 1.0, 1.9466)$, implying real GDP volatility of around 2% in H and 4% in F (these numbers are similar across all of our models). The third sources of asymmetry is smaller size of F, specifically, $Y^{\mathrm{T}}_{\mathrm{H},0} = Y^{\mathrm{T}}_{\mathrm{H},1} = Y^{\mathrm{N}}_{\mathrm{H},0} = Y^{\mathrm{N}}_{\mathrm{H},1} = 1.0$ and $Y^{\mathrm{T}}_{\mathrm{F},0} = Y^{\mathrm{T}}_{\mathrm{F},1} = Y^{\mathrm{N}}_{\mathrm{F},0} = Y^{\mathrm{N}}_{\mathrm{F},1} = 0.9$ (section 4.2.3). The fourth source of asymmetry is the possibility of sovereign default in the bonds of F. The specifics of default will be discussed in Section 4.3.1. Usually we will study each asymmetry one at a time, although at times we present results of some combinations.

Our preferred model is presented in section 4.3.1. However, we find it useful to build up intuition with a set of intermediate results.

4.2 Perfect risk sharing

In this subsection, we analyze models with perfect risk sharing. Complete markets imply perfect risk sharing, but the converse is not true. Many of the models we study in this section have incomplete financial markets which are enough for perfect risk sharing nevertheless. We find that

[20] Stockman and Tesar (1995) estimate $\alpha = -1.27$ for a sample of developed and developing countries; Mendoza (1991) estimates $\alpha = -0.35$ for industrialized countries; Obstfeld and Rogoff (2001) set $\alpha = 5/6$.

[21] In a dynamic setting, one can model all countries being unconditionally symmetric but for there to be transitory differences at any point in time, for example Verdelhan (2010) follows this approach. However, since there are only two periods in our model, we need to allow for unconditional asymmetry.

when risk sharing is perfect, then the only source of carry trade profits is inflation risk.[22] In the next subsection, we will show that with imperfect risk sharing, the carry trade may be profitable for reasons other than inflation risk.

At this stage, it may also be useful to briefly discuss cases we do not solve. With one exception, our model does not appear to have a solution for zero realized inflation (or equivalently, for the case in which the two traded assets are real bonds). Appendix D.2 provides a proof that no solution exists for the case where both countries are ex-ante identical but face different ex-post shocks. Numerically, we are unable to find solutions for the ex-ante asymmetric cases as well. The one exception is when we allow for sovereign default, in this case the model with zero realized inflation (or equivalently with real rather than nominal bonds) does have a solution, and we present those results below.

We also do not present cases where *expected* inflation is something other than zero, or where there are non-zero differences in *expected* inflation across the two countries. As shown in the empirical section, differences in expected inflation are one reason for differences in nominal exchange rates. However, we have solved versions of all of the models to be discussed below, but in which expected inflation is higher in the Foreign country. Although this has an effect on the nominal interest rate differential, it has no effect on carry trade profits because in our two-period setting there is no risk associated to changes in expected inflation and so any higher expected inflation is fully offset by a depreciating nominal currency, leaving the real exchange rate unchanged. In other words, in our model, UIP holds in regards to differences in *expected* inflation. As will be seen below, in our model, only shocks to inflation matter for the carry trade and violation of UIP.

4.2.1 Complete markets and symmetry

We begin by analyzing a case with symmetry and complete markets. Utility is logarithmic and $\alpha = 0$. This case is identical to the model described earlier, however, instead of only two nominal bonds, there are Arrow-Debreu securities which pay one tradable good in state i at $t = 1$, and zero in all other states; their nominal price at $t = 0$ is q_i. We describe the solution to this problem in the appendix. Because markets are complete, risk sharing is perfect, resulting in the

[22]This result is unique to our environment. For example, with perfect Home bias—that is, no trade—it is possible to for the carry trade to be profitable even in a complete market, for example Verdelhan (2010).

consumption growth of the two countries being perfectly correlated. The Home country always consumes a constant fraction ξ of tradables.[23] The holdings of Arrow-Debreu securities by Home has a correlation of 1.0 with $Z_{\mathrm{F}} - Z_{\mathrm{H}}$, thus Home invests in the Foreign tradable sector and sells the Home tradable sector to Foreign investors. In other words, perfect risk sharing implies large cross-holdings of Foreign equity; this is very intuitive.

Table 3 presents results from several models. Each model is a column in the table, and the rows are separated into four sections: model description (Parameters), risk sharing, Balassa-Saumelson effect, and Carry Trade. If country F has a higher interest rate, then we define the carry trade return as $R_{CT} = R_F \frac{E_0}{E_1} - R_H$, and the reverse carry trade return as $\hat{R}_{CT} = R_H \frac{E_1}{E_0} - R_F$; if H has a higher interest rate, then we define the carry trade return as $R_{CT} = R_H \frac{E_1}{E_0} - R_F$ and the reverse carry trade return as $\hat{R}_{CT} = R_F \frac{E_0}{E_1} - R_H$. In summary,

$$R_{CT} = \begin{cases} R_F \frac{E_0}{E_1} - R_H, & \text{if } R_F > R_H \\ R_H \frac{E_1}{E_0} - R_F & \text{if } R_H > R_F \end{cases}, \quad \text{and} \quad \hat{R}_{CT} = \begin{cases} R_H \frac{E_1}{E_0} - R_F, & \text{if } R_F > R_H \\ R_F \frac{E_0}{E_1} - R_H & \text{if } R_H > R_F \end{cases}. \tag{22}$$

Due to Jensen's inequality, the expected currency appreciation may be positive in both directions and the carry trade may be profitable in both directions. For this reason we report the expected difference in currency appreciation $\mathbb{E}\left[\frac{E_0}{E_1} - \frac{E_1}{E_0}\right]$ and the expected difference in carry trade return $\mathbb{E}\left[R_{CT} - \hat{R}_{CT}\right]$. However, this issue matters only for low risk aversion cases (log utility in Table 3 in particular) where expected return differentials are very small and Jensen's inequality dominates. Although we report all statistics in the same way for consistency, the later results, where the calibration is more realistic, typically have only one country expecting a currency appreciation and only one direction of the carry trade being profitable.

The first two columns in Table 3 present the complete market model under the two alternative monetary policy assumptions, these are models 1 and 2 (FE for fixed exchange rate, ZEI for zero expected inflation). The real side of both models is identical. Since markets are complete, risk sharing is perfect, as is the correlation of consumption growth; the two countries also have identical

[23] In Appendix D.1, equation (D34) we show that $\xi = \frac{Y_{\mathrm{H},0} + (1/P^{\mathrm{T}}_{\mathrm{H},0}) \sum_{i=1}^{K} q_i Z^T_{\mathrm{H},i} Y^{\mathrm{T}}_{\mathrm{H},1}}{Y^{\mathrm{T}}_{\mathrm{H},0} + Y^{\mathrm{T}}_{\mathrm{F},0} + (1/P^{\mathrm{T}}_{\mathrm{H},0}) \sum_{i=1}^{K} q_i (Z^T_{\mathrm{H},i} Y^{\mathrm{T}}_{\mathrm{H},1} + Z^T_{\mathrm{F},i} Y^{\mathrm{T}}_{\mathrm{F},1})}$ for the case of log utility, where $i = 1, \ldots, K$, denotes future states.

consumption volatility. Furthermore, the real exchange rate (ER) is constant in these models, thus, the Balassa-Samuelson effect is absent.[24]

Although these two models are identical on the real side, due to different monetary policies, their nominal sides are quite different. The Balassa-Saumelson effect is about real quantities, but we can explore its nominal analog—this is very relevant for UIP and the carry trade. In the model with a fixed exchange rate, by construction, the nominal exchange rate is constant, the two nominal bonds have identical returns, and the carry trade profit is exactly zero. Not so in the model of zero expected inflation. In this model nominal rates have a volatility of 4.0% and, most interestingly, consistent with the data, currency appreciation is positively related to output growth. The realized carry trade return is no longer exactly zero, but has a volatility of 4.0%, however, since this model is perfectly symmetric, the expected return on the carry trade is the same in either direction. As will be shown below, the positive relationship between currency appreciation and output growth is crucial for understanding why UIP may fail and why the carry trade is profitable in this model.

The reason a country's currency appreciates after a positive shock in this model is that the money supply is chosen by the government before the shock is realized. Since, as in the quantity theory of money, the money supply must be equal to total amount of goods sold, an unexpected positive shock to local output implies a fall in local prices when money supply is unchanged. This can also be seen in the following equation, which can be derived from equations (9), (11), and (19):

$$P^{\mathrm{T}}_{\mathrm{H},1} = \frac{(1-\theta)M_{\mathrm{H},1}}{(1-\theta)Y^{\mathrm{T}}_{\mathrm{H},1}Z_{\mathrm{H}} + \xi\theta(Y^{\mathrm{T}}_{\mathrm{H},1}Z_{\mathrm{H}} + Y^{\mathrm{T}}_{\mathrm{F},1}Z_{\mathrm{F}})} \tag{23}$$

Combining this with equation (16), it is clear that relatively more positive shocks to Home tradable output ($Z_{\mathrm{H}} > Z_{\mathrm{F}}$) lead to lower Home prices ($P^{\mathrm{T}}_{\mathrm{H},1} < P^{\mathrm{T}}_{\mathrm{F},1}$), and a strengthening of the Home currency ($E_1 > E_0$). Thus, in our model, under a zero expected inflation monetary policy, inflation is *counter-cyclical*. As discussed earlier in the empirical section, this is consistent with the data. As will be explained below, this alone could lead to positive carry trade profits once we introduce asymmetry.

[24]It is interesting to contrast models 1 and 2 with models which assume complete markets but no trade (for example Verdelhan (2010) or Colacito and Croce (2011)). As discussed earlier, for log utility, complete markets imply $\Delta log(C_H) - \Delta log(C_F) = -\Delta log(ER)$. This equation holds in models 1 and 2, but the relationship is trivial—both sides are exactly zero. To break the triviality of this example, a mechanism like Home bias, asymmetry in preferences, or non-tradable shocks is needed.

4.2.2 Incomplete markets and symmetry

We will now shut down complete markets and explore the implications of market incompleteness for currency and carry trade profits. The financial market consists of two nominal bonds, each denominated in a country's own currency. There is also a spot market in which currency can be exchanged. Each country's government controls the money supply, thus the monetary policy will affect the real return on each of the nominal bonds. Clearly, the choice of monetary policy affects the risk associated with each bond, and the risk premium; it also affects risk sharing and welfare.[25]

As in complete markets, we begin by analyzing the case of a fixed exchange rate, this is Model 4 in Table 3. Under this policy, there is effectively one nominal bond because any interest rate differential between the two bonds would lead to an arbitrage. With effectively one nominal bond, neither country buys any foreign securities. This is because with a fixed exchange rate and equal nominal returns (due to no arbitrage), the return on the bond can not be state contingent. Thus, there is no risk sharing and the correlation between Home and Foreign consumption growth is zero. If the Foreign country had higher expected growth (not presented for brevity), the high growth country (F) would borrow at $t = 0$ to smooth consumption intertemporally. However even intertemporal risk sharing would be imperfect, in expectation: $\Delta y_H < \Delta c_H < \Delta c_F < \Delta y_F$. The Foreign country's expected consumption growth is lower than its expected output growth but is higher than Home's expected consumption growth because risk limits the amount the Foreign country is willing to borrow. Since the exchange rate is constant, and both bonds have identical nominal returns, UIP holds and the carry trade earns exactly zero profit; this would be the case in this economy even if we introduced asymmetry. Interestingly, the Balassa-Samuelson effect is present.

Now consider both central banks setting inflation to zero in expectation; this is Model 3 in Table 3. The results are drastically different from the fixed exchange rate case. Although financial markets are incomplete, the two nominal bonds are enough for perfect risk sharing; the intuition is explained below. As a result, this economy behaves exactly like a complete market economy with zero expected inflation (Model 1). Although our focus is not welfare, these results suggest that when financial markets are incomplete, common currency areas such as the E.U. are worse off from

[25] An interesting direction would be to study optimal monetary policy and the strategic interactions between governments in this setting.

a welfare perspective then floating exchange rate regimes. To our knowledge, this point has not been made before.

The intuition for why this monetary policy, combined with nominal bonds, achieves perfect risk sharing is as follows. After a positive output shock to F, prices in F fall because monetary policy is not state dependent and goods are relatively more abundant. Holders of nominal F-bonds can now buy more goods. Thus, H investors in F-bonds can purchase more tradable goods and ship them back to Home; an alternative way of saying this is that the currency of F appreciates ($E = \frac{P_F^T}{P_H^T}$ falls) making H-investors better off. Thus, F-bonds behave like F-equity and are a perfect risk sharing security (we posit that F-equity, if available, would be an even better risk sharing security as it is not monetary policy dependent).

Model 11 in Table 4 is the equivalent of Model 3 solved under a more realistic utility calibration, where $\gamma = 7.5$ and $\frac{1}{\rho} = 1.5$. Because these parameters are identical to those used by Bansal and Yaron (2004), we refer to this calibration as the "BY calibration". These results from the recursive utility case of Model 11 are similar to the logarithmic utility case of Model 3. In particular, risk sharing is perfect, the real exchange rate is constant, and the strength of the nominal currency is pro-cyclical.

We have found that even with asymmetry, the two safe nominal bonds in a zero expected inflation environment are always enough to achieve perfect risk sharing as long as the consumption aggregator is Cobb-Douglas ($\alpha = 0$) and we do not add frictions ($\kappa = 0$ and $\tau = 0$), or extra shocks (ie to the non-tradable sector), or default. For this reason, we will not present any more complete markets results as they are identical to the incomplete market results we present. Further below we will explore frictions, alternative consumption aggregation, and sovereign default—each of these will lead to interesting implications for UIP and the carry trade.

4.2.3 Incomplete markets and asymmetry

In this section we will maintain the assumptions of Cobb-Douglas consumption aggregation ($\alpha = 0$) but we allow for either logarithmic ($\gamma = \rho = 1$) or more general ($\gamma \neq 1$, $\rho \neq 1$) preferences. We also allow for asymmetry across the two countries (differences in expected growth, volatility, or size), and allow for frictions ($\kappa > 0$ or $\tau > 0$).

When risk sharing is perfect, we show in Appendix A that countries with a higher loading on the aggregate shock—that is, high growth, more volatile, and larger countries[26]—have higher interest rates. The carry trade is profitable by borrowing in the low interest rate (low growth, low volatility, small size) country and investing in the high interest rate country.[27] The intuition is that the zero expected inflation monetary policy causes inflation to be counter-cyclical;[28] this leads to carry trade target countries having higher inflation volatility, and carry trade profits being risky because they are positively correlated with output and negatively with inflation in the target country.

Indeed, as we have shown in Panels D and E of Table 1 and Panels A and B of Table 2, in the data inflation is counter-cyclical, carry trade target countries have higher inflation volatility, and the carry trade profit is positively related to output growth but negatively to inflation in the target country. These results are opposite to what is implied by complete markets models without a tradable sector because positive consumption (and therefore output) growth are associated with currency depreciations.[29] This implies that the carry trade profit is highest when consumption growth in the target country is lowest.

Although the mechanism in our model is somewhat different (counter-cyclical inflation versus perfect home bias), the finding that countries with a higher loading on aggregate risk have higher interest rates and are carry targets is analogous to some of the previous explanations of the carry trade. For example, in Verdelhan (2010) risk sharing is perfect and larger countries are carry trade targets.

One problem for perfect risk sharing models is that, as shown in Panel A of Table 2, high interest rate countries do not appear to be larger, more volatile, or higher growth. Note that these results are specific to a perfect risk sharing environment. Once we move to a richer setting with

[26]Carry trade profits have been linked to aggregate consumption risk (Lustig and Verdelhan (2007)) and one can think of a world where different countries have different loadings on some aggregate world shock in addition to idiosyncratic risk. Our simple two country example cannot properly model such a world CAPM, since all risk is aggregate. However, loosely speaking, growth, volatility, and size are related to a country's loading on world output in our economy.

[27]Due to Jensen's inequality, when risk aversion is sufficiently low, there is an exception to this statement for the case of asymmetry in volatility (see Appendix A).

[28]Counter-cyclical inflation usually arises from supply shocks as higher output leads to lower prices. On the other hand, demand shocks combined with some form of rigidity could lead to pro-cyclical inflation as higher demand induces both higher output and higher prices. An unexpected shock to the money supply and sticky prices may also lead to higher prices and higher output. In the real world demand, supply, and monetary shocks could all be important, however, in Table2 we document that empirically, inflation is indeed counter-cyclical.

[29]An exception is Colacito and Croce (2013) where positive (long run) output growth is associated with negative (relative) consumption growth and currency depreciations.

imperfect risk sharing, we will show that it is not difficult to reverse these results; for example, to have the smaller country both have a higher interest rate, and expected currency appreciation, or to have no differences in output capabilities and still have carry trade profits due to differences in sovereign default risk.

We will now explain the mechanism in more detail. Recall that two nominal bonds are enough for perfect risk sharing, this implies that under symmetric utility, the real interest rates are identical in both countries. Therefore, only differences in inflation behavior can cause differences in nominal rates, and carry trade profits. A positive (negative) shock to the tradable sector of either country leads to an abundance (scarcity) of tradable goods. This results in relatively low (high) prices in relatively good (bad) states of the world. In other words, this results in counter-cyclical inflation.

This also implies pro-cyclical currency returns because $E_1 = \frac{P^{\mathrm{T}}_{\mathrm{F},1}}{P^{\mathrm{T}}_{\mathrm{H},1}}$ e.g., a positive shock to F leads to an appreciation of F currency and hence a drop in E_1. Since the bonds have no default risk, only unexpected shocks to exchange rates can lead to unexpected carry trade profits. As explained above, positive (negative) output shocks lead to lower (higher) inflation, stronger (weaker) currency, and positive (negative) carry trade profits. The output shocks of a country with a higher loading on aggregate risk are more important for aggregate output, and because of perfect risk sharing, for consumption as well. Therefore, investing in the nominal bonds of that country is riskier.

The carry trade is the sum of currency appreciation and interest rate differentials. It is also useful to think about the effects on currency and interest rate differentials individually. The country with a higher loading on aggregate output has a currency that is more strongly correlated with aggregate consumption, therefore its currency appreciates on average.

The country with a higher loading on aggregate output also (with one exception discussed below) has a higher nominal interest rate. Note that the nominal bond price is:

$$q = \mathbb{E}\left[\mathbb{M}_{t+1}\frac{P_t}{P_{t+1}}\right] = corr\left[\mathbb{M}_{t+1}, \frac{P_t}{P_{t+1}}\right]\sigma\left[\frac{P_t}{P_{t+1}}\right]\sigma\left[\mathbb{M}_{t+1}\right] + \mathbb{E}[\mathbb{M}_{t+1}]\mathbb{E}\left[\frac{P_t}{P_{t+1}}\right]. \tag{24}$$

Since the real stochastic discount factor $\mathbb{M}_{t+1}$ is identical for both countries due to perfect risk sharing, the nominal interest rate can be high (q is low) if $corr\left[\mathbb{M}_{t+1}, \frac{P_t}{P_{t+1}}\right]\sigma\left[\frac{P_t}{P_{t+1}}\right]$ is large in magnitude (more negative)[30], or $\mathbb{E}[\frac{P_t}{P_{t+1}}]$ is small. The country with a higher loading on aggregate

[30]Note that $corr[M_{t+1}, \frac{P_t}{P_{t+1}}] < 0$. This is because $\mathbb{M}_{t+1}$ is counter-cyclical, as is inflation ($\frac{P_{t+1}}{P_t}$).

risk has both a more negative $corr\left[M_{t+1}, \frac{P_t}{P_{t+1}}\right]$, and a more volatile inflation (high $\sigma\left[\frac{P_{t+1}}{P_t}\right]$ and high $\sigma\left[\frac{P_t}{P_{t+1}}\right]$). This pushes interest rates up. At the same time, due to Jensen's inequality, the country with a higher loading on aggregate risk has a higher $\mathbb{E}[\frac{P_t}{P_{t+1}}]$, pushing interest rates down.[31] In most cases, this Jensen's inequality effect is small and the high aggregate risk country has a higher nominal interest rate. The only exception is the case when the volatility of shocks in F is much higher than in H, and the risk aversion is sufficiently low. In this case, $\sigma[M_{t+1}]$ is very small and the term $\mathbb{E}[\frac{P_t}{P_{t+1}}]$ dominates, resulting in a lower nominal interest rate in the high aggregate risk country.

We solved our model for cases where F has identical output to H at $t = 0$ but higher expected growth in the tradable sector, F has higher volatility of output shocks than H, and the output of F in all states is proportionally smaller than in H. These are respectively Models 5, 9, and 10 in Table 3 for logarithmic utility, and Models 12, 13, 14 in Table 4 for Epstein-Zin-Weil utility. For comparison, we also present perfectly symmetric cases (Models 3 and 11).

Risk sharing is still perfect and the real exchange rate is still constant in all cases. As explained above, the strength of the nominal currency is pro-cyclical, which is consistent with the data. In all cases, the country with a higher loading on aggregate risk—higher growth, higher volatility, bigger size—expects its currency to appreciate, and in all cases (with the exception of Model 9, as explained above), the same country has a higher nominal interest rate resulting in positive expected carry trade profits.

For the cases of Epstein-Zin-Weil utility, due to higher risk aversion,[32] the magnitude of the carry trade return is economically significant, though still smaller than the data. For example, for the case of higher volatility (model 13), the expected return is 0.45% with a volatility of 4.53%.[33]

4.2.4 Additional shocks and frictions

As discussed above, under a zero expected inflation regime, two nominal bonds are enough for perfect risk sharing. We are interested in economies with imperfect risk sharing; a simple starting

[31]Because of convexity, Jensen's inequality implies $\mathbb{E}\left[\frac{P_t}{P_{t+1}}\right] > \frac{\mathbb{E}[P_t]}{\mathbb{E}[P_{t+1}]} = 1$.

[32]The elasticity of intertemporal substitution, $1/\rho$, does not play an important role here. We have also solved this case for CRRA utility where $\gamma = \frac{1}{\rho}$ and results are similar.

[33]Due to Jensen's Inequality, the reverse carry trade has an expected return of -0.26% with a volatility of 4.52%.

point is a cost on issuing bonds. We introduce a quadratic cost whose strength is parameterized by $\kappa = 0.001$, this case is in Model 7 of Table 3.[34] For example, this cost may represent information, monitoring, or creating collateralizable assets. With this cost, the face value of the issued bonds is roughly half of the frictionless case.

Prior to discussing this case, it is useful to review the intuition behind the Balassa-Saumelson effect, that is the observation that real currency appreciation is positively correlated with output shocks. The basic intuition can be seen in a static, one period model, however, this easily extends to a dynamic model in which financial markets (or rather lack of them) do not allow agents to trade intertemporally.[35] Since there is only one tradable good, there can be no mutual gains from trade, therefore each country just consumes its endowment. Consider a monetary policy keeping the price of tradables equal to 1,[36] a shock to tradable output must therefore increase the price of non-tradables, as they are relatively less abundant. This leads to a rise in the overall price level, and a real currency appreciation.

When $\kappa > 0$, the model gets closer to this simple example of the Balassa-Saumelson effect. Due to costs, there are fewer financial claims traded—in this parametrization, the total value of financial securities traded is 56% of the perfect risk sharing case (not reported in tables). This leads to less intertemporal trade, and to each country's consumption more closely resembling its output. Risk sharing is no longer perfect and consumption growth is no longer perfectly correlated across countries. Furthermore, the consumption of the high growth country (F) is now more volatile. A positive shock to tradable output in H leads to a real appreciation of H's currency. This also leads to higher carry trade profits for two reasons. First, because the high growth country's consumption growth is now closer to its output growth, its real interest rate is higher which leads to a larger nominal interest rate differential. Second, because the real exchange rate is now volatile and procyclical, the carry trade is even more risky as the Foreign currency will depreciate by even more when the Foreign country suffers negative shocks. Although the cost κ is somewhat reduced form and ad hoc, this example suggests that financial market incompleteness is a promising route for explaining

[34] Benigno and Thoenissen (2008) explore a similar cost, which helps to break risk sharing and explain the Backus-Smith puzzle. Alternately, Rabitsch (2014) assumes a fixed borrowing constraint, which, like a cost, limits financial trade and therefore risk sharing.

[35] In such a model, there is no trade because there is no way for one country to promise its future output in return for another country's excess output today.

[36] This assumption is innocuous. Since consumption is pre-determined by endowment, monetary policy cannot have an effect on real quantities, including the real exchange rate.

the failure of UIP. We will see a similar pattern later, when financial market incompleteness alone will lead to imperfect risk sharing.

As an alternative to restricting the financial market, we can restrict the trade in goods. In particular, we allow for an iceberg cost $\tau = 0.01$ on transporting tradable goods abroad, this case is in Model 8 of Table 3. As with the financial cost, this reduces risk sharing, with the high growth country (F) now having a higher volatility of consumption growth and imperfect consumption correlation; higher expected consumption growth leads to a rise in the interest rate differential. However, the carry trade profit is weakened because currency depreciation more than offsets the rise in interest rates. This can be seen in equation (16), and is oftentimes referred to as the terms-of-trade effect. When a country is importing (exporting), as F is doing at $t = 0$ ($t = 1$), its currency appreciates (depreciates) relative to the zero cost case, to compensate exporters for the shipping cost. Thus, the currency of the high growth country is expected to depreciate resulting in a drag on carry trade profit. Additionally, the real currency return is counter-cyclical—opposite from the Balassa-Samuelson effect. For the calibration in model 8, the carry trade profit is still positive, but lower than the no cost case; for higher τ, the carry trade profit can be negative. Thus, in the context of this model, unlike financial frictions, real goods frictions do not help explain the carry trade.

We have also solved cases identical to the ones above but with shocks to the non-tradable sector in each country; these shocks are of the same size as the tradable shocks and uncorrelated with other shocks. For brevity, we report only results for the case of higher growth in F; the model without non-tradable shocks is model 5, the one with tradable shocks is model 6 in Table 3. Non-tradable shocks drastically reduce the correlation in consumption growth to a number that is similar to the data. This also increases the volatility of the real exchange rate, which was previously zero. Neither of these results is particularly surprising. However, the model still does not produce any correlation between the real exchange rate and shocks (the Balassa-Samuelson effect), and the results on UIP and the carry trade are virtually identical to the model without non-tradable shocks. Non-tradable shocks matter for risk sharing, but symmetric non-tradable shocks appear orthogonal to the risk associated with the carry trade.[37] We found this to be the case any time we extended a model with perfect risk sharing to allow for symmetric non-tradable shocks. For this reason, we choose to

[37] Asymmetry in non-tradable shocks (Bansal and Shaliastovich (2012), Tran (2013)) or asymmetry in preferences (Verdelhan (2010), Heyerdahl-Larsen (2014)) would affect carry trade profit.

focus only on results where there are no non-tradable shocks. Still, it is useful to know that adding non-tradable shocks to a model can improve its quantitative performance along some dimensions (decrease consumption correlation, increase real exchange rate volatility) without affecting UIP.

4.3 Financial Market Frictions and Imperfect risk sharing

As discussed above, two nominal bonds in a zero expected inflation environment are enough to achieve perfect risk sharing. There are several ways to break perfect risk sharing in this model. Earlier we saw that frictions in the goods market, and non-tradable shocks can break perfect risk sharing but do not help explain the carry trade or the failure of UIP. On the other hand, frictions in the financial market do help. An alternative way to break perfect risk sharing is to allow for an alternate consumption aggregator ($\alpha \neq 0$). We will continue setting $\alpha = 0$ and explore how financial market frictions, in particular the possibility of default, affect risk sharing, the real exchange rate, and UIP; these results are in section 4.3.1. In Appendix B, we show that for reasonable alternative values of α the results remain quite similar. For the sake of brevity, we will focus on the BY calibration only.

4.3.1 Sovereign default

The profitability of the carry trade suggests that there is something fundamentally different between countries with high and low interest rates. These differences could be due to differences in production capabilities (growth, volatility, or size). Indeed, as seen in Table 2 smaller countries do tend to have higher interest rates, however, empirically, it is difficult to find systematic differences in output growth or volatility between these countries. One reason for interest rate differentials is differences in expected inflation, however in our model, differences in expected inflation are fully offset by expected currency depreciation and therefore cannot affect the carry trade. Another potential reason for interest rate differentials is default risk. We explore this type of asymmetry in this section.[38]

We assume that the bonds of F may suffer a nominal default in some states of the world. This may be because the financial market in F is not sophisticated enough to commit to always pay

[38]Several studies have tied carry trade profits to downside risk and the peso problem, these include Burnside (2011), Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011), and Lettau, Maggiori, and Weber (2014). Default, either actual or through inflation, would be one way in which this downside risk could arise.

back its promises. In particular, F only repays 75% of the nominal value of its debt in any default state. For simplicity, we assume default happens in only one state of the world and the nominal bond is fully repaid in all others.[39] As an alternative, we have also solved cases where the nominal value is repaid in full, however the money supply is increased in one state of the world, leading to high inflation and a smaller payout in real terms. For the sake of brevity, we do not present these results, though they are similar to the cases of nominal default.

The above assumption about default implies an interest rate differential of roughly 3% between F and H in our model.[40] In the data, when countries are sorted on interest rate differential versus the U.S., the lowest quintile is around -2% while the highest is above 7%, suggesting that our default assumption may be quite conservative.

We consider two kinds of default that differ depending on the state in which default occurs. We define a *systemic default* as an event that occurs when both countries suffer the most negative realization of the shocks. Alternatively, we define a *strategic default* as an event that occurs when H suffers the most negative realization of the shock, while F has the most positive realization. When there is no default, this is the state of the world with the most goods being shipped from F to H, because H redeems its highly valuable (in real terms) F-bond. We call this strategic default because this is the state of the world in which F is most incentivized to default. We believe systematic default is most relevant empirically and examine these cases in the main text. In Appendix B we present results for the case of strategic default.

Recall that the reason that two nominal bonds were enough for perfect risk sharing is that their real returns were perfectly inversely correlated with the price level, making them behave like equity. Now that the nominal payoff of one of the bonds is no longer constant, these two bonds are no longer enough for perfect risk sharing and, as a result, consumption growth correlation is below one. In Model 15 of Table 4 we present the case of perfectly symmetric countries, except the nominal bonds issued by F potentially undergo systematic default, while the bonds issued by H do not. Risk sharing is no longer perfect because the nominal payoffs of the two bonds are no longer constant, thus, the intuition given earlier for why nominal bond payoffs look like real equity in this monetary regime no longer holds. These cases are analogous to $\kappa > 0$ in that far fewer bonds are

[39] In our baseline model, there are three possible states for each country's tradable output: low, medium, and high. In total, there are nine equally likely states and default occurs in one.

[40] The probability of default is 1/9 and the loss given default is 25% implying an approximate spread of $1/9 \times 0.25$.

issued, however instead of an issuance cost, here the reason for less financial trade is that these bonds are an imperfect risk sharing tool.[41]

The consumption growth correlation between the two countries is now only 0.36—similar to the data. The volatility of Foreign consumption is lower than Home. This is because when Foreign defaults and Home does not, the Foreign country is, in effect, getting a transfer from Home. Since default happens in the worst aggregate state (low output in both), F has relatively high consumption in the worst state while H has relatively low consumption, resulting in a relatively smoother consumption profile for F. To compensate Home investors for low consumption in the default state, F-bonds pay a higher nominal rate and H investors have higher consumption in non-default states. Note that since there are no asymmetries in production, neither country would be a net borrower if risk sharing was perfect. This is not quite the case anymore, however the net positions as of $t = 0$ are still close to zero.

Real domestic currency appreciation is now quite volatile and positively related to domestic output shocks. This is the Balassa-Samuelson effect and it is present in all models with default risk. As mentioned above, far fewer bonds are issued and far less risk sharing occurs. As a result, a positive domestic output shock leads to an abundance of tradables not only in production but also in consumption. Although the price level falls after a positive shock, the price of tradables falls by relatively more because, due to the relative scarcity of non-tradables, the price of non-tradables relative to tradables must rise. The definition of the real exchange rate ($ER = E_1 \frac{P_{\mathrm{H},1}}{P_{\mathrm{F},1}} = \frac{P^{\mathrm{T}}_{\mathrm{F},1}}{P^{\mathrm{T}}_{\mathrm{H},1}} \frac{P_{\mathrm{H},1}}{P_{\mathrm{F},1}}$) makes it clear that the real currency must appreciate. This intuition is identical to the case of $\kappa > 0$.

Real domestic currency appreciation is also positively related to the difference between domestic and foreign consumption growth (not in tables). This is because risk sharing is no longer perfect and there is a positive relationship between output growth differentials and consumption growth differentials; this would not be the case if risk sharing was perfect. As mentioned earlier, the Backus-Smith puzzle is that under perfect risks sharing, real domestic currency appreciation is negatively related to the difference between domestic and foreign consumption growth despite a positive relationship in the data. Thus, our model resolves the Backus-Smith puzzle, however, we

[41] The nominal value of outstanding bonds relative to the perfect risk sharing case is roughly 29% (not reported).

do not view this as our main contribution because Benigno and Thoenissen (2008) and Corsetti, Dedola, and Leduc (2008) have shown that this is possible once risk sharing is imperfect.

The nominal exchange rate is even more volatile than the real and the the Foreign currency appreciates on average.[42] The carry trade is profitable in expectation because it is risky from the perspective of Home investors. The expected return is 0.79% with a volatility of 9.90%; the expected return and Sharpe ratio rise with a larger loss due to default, or a larger volatility of aggregate shocks. Although in this model, the Foreign currency appreciates in the defaulting state, it is not nearly enough to offset the loss due to default in this very risky aggregate state.[43] As with the case of perfect risk sharing, and consistent with the empirical results in Table 2, carry trade profits are highest when output is high and inflation is low. Note that the high interest country (F) has exactly the same size, expected output growth, and expected output volatility as the low interest country (H).

We also interact default with other asymmetries. The results for F having sovereign default risk and higher output volatility are presented in Model 16 of Table 4. Recall that when output was symmetric (Model 15), default resulted in lower consumption volatility for Foreign compared to Home. This need not be the case if the output volatility in Foreign is higher. Because of imperfect risk sharing, each country's consumption will look more similar to its output, as a result, for this calibration of default, the ratio of Foreign to Home consumption volatility is 1.08. The carry trade profit is now quantitatively bigger: 1.35% with a volatility of 10.71%.

The results for F having sovereign default risk and smaller size are presented in model 17 of Table 4. Recall that when risk sharing was perfect (Model 10), the bigger country always had the higher interest rate and was the carry trade target; this was inconsistent with the data where size was negatively related to interest rate differentials. Now that we have allowed for default risk, the smaller country can have a higher interest rate and carry trade profits, as in the data. Furthermore, as in the data, realized carry trade profits are still positively related to output growth and negatively to inflation.

[42] In non-defaulting states, the nominal currency of F depreciates slightly on average. However, in the defaulting state it spikes up because of the overabundance of tradables. As a result, it appreciates on average.

[43] This does not happen in the version of the model in which default happens through inflation rather than insolvency. We also believe this would not happen if default triggers additional negative consequences for the economy, as in Xu (1989).

Model 17 is especially good at replicating the patterns observed in the data and described in Tables 1 and 2. Risk sharing is imperfect and consumption correlation is far below one. Both the real and nominal currency returns are volatile and positively correlated with output shocks. Both countries have the same expected output growth and output volatility. The smaller country (F) has a high nominal rate, a higher volatility of inflation, and its currency is expected to appreciate. The carry trade of borrowing in the larger country (H) to invest in the smaller country (F) is profitable on average; it is risky because it is most profitable in states of the world where realized output growth is high and realized inflation is low. Additionally, this model is consistent with empirical facts documented in Lustig and Verdelhan (2007) and Lustig, Roussanov, and Verdelhan (2011): realized carry trade profit is positively correlated with world output growth, and expected returns and Sharpe ratios are higher when world volatility is higher.[44]

Although the carry trade profit in this model is still far short of the data (0.74% with a volatility of 9.88%), the profit and Sharpe ratio would be higher with either a higher volatility of shocks, a higher loss to default, or a higher risk aversion. For example, doubling the volatility of output shocks, or loss to default, or risk aversion each more than doubles the expected carry trade return while having a much smaller effect on carry trade return volatility. We also believe that a dynamic model with long-run risk style shocks would deliver quantitatively stronger results than our two period model.

4.3.2 Real bonds or zero inflation

Earlier we showed that if the asset market allows for perfect risk sharing, then carry trade profits must come from inflation risk only and inflation must be counter-cyclical. It would have been interesting to compare those results to an alternative inflation policy, specifically a case of zero realized inflation. Note that if realized inflation is exactly zero, then nominal bonds are equivalent to real bonds and we would have been solving the equivalent of a real economy. Unfortunately, it appears that no solution exists for the models we solved earlier under a zero inflation policy. Fortunately, once we allow for defaultable bonds and incomplete risk sharing (as in the previous subsection), a solution does exist.

[44]Since the model only has 2 periods, it cannot have heteroscedasticity. Instead, we solve the model with both countries having either low, or high output volatility.

In Models 18–20 of Table 4 we present the same sovereign default cases of the previous section (models 15-17) but under a zero inflation policy, or equivalently with the asset market allowing for two real bonds. As can be seen, the key results of the previous section hold and the results are quite similar quantitatively. Perhaps this is not surprising because given our calibration, the real effects on UIP and the carry trade from incomplete risk sharing were quantitatively much stronger than the nominal effects from counter-cyclical inflation.

5 Conclusion

We propose a fairly general model which can produce a failure of UIP and a profitability of the carry trade in multiple environments. When risk sharing is perfect, the carry trade is profitable only due to higher inflation risk in the carry country. On the other hand, when risk sharing is not perfect, the carry trade is also profitable because negative shocks in the target country lead to a real currency depreciation, thus lowering carry trade profits. We show that financial market frictions and shocks in the tradable sector are crucial for understanding the carry trade; on the other hand frictions in the goods market or shocks to the non-tradable sector do not appear to play a large role.

We argue that complete markets models have trouble matching many of the empirical observations relating to UIP and the carry trade. On the other hand, our incomplete market model is able to qualitatively, and sometimes quantitatively, match the data. Still, our model is a simple two period model whose quantitative implications must be taken with a grain of salt. The quantitative importance of the channels we identify must be tested in a seriously dynamic model. We leave this to future work.

A Relation to other models

Consider a model where markets are complete and there is no trade due to perfect home bias ($\theta = 1$), thus output is equal to consumption. Utility is CRRA but risk aversion can be time varying. The logarithm of the real stochastic discount factor is $m_{t+1} = \gamma_t \Delta c_{t+1}$. The logarithm of the nominal stochastic discount factor is $mn_{t+1} = m_{t+1} - i^H_{t+1} = -\gamma_t \Delta c_{t+1} - i^H_{t+1}$. Let e_t be the log of foreign currency per unit of home, thus a high e_t indicates a strong home currency.

The currency appreciation is:

$$\Delta e_{t+1} = m^H_{t+1} - m^F_{t+1} - (i^H_{t+1} - i^F_{t+1}) = -\gamma^H_t \Delta c^H_{t+1} + \gamma^F_t \Delta c^F_{t+1} - (i^H_{t+1} - i^F_{t+1}) \tag{A1}$$

The nominal interest rate in either country is given by $r_{t+1} = -log(E_t[e^{m_{t+1} - i_{t+1}}])$. Assuming log normal shocks:

$$\begin{aligned} r_{t+1} &= -E_t[m_{t+1}] + E_t[i_{t+1}] - 0.5\sigma^2_t[m_{t+1}] - 0.5\sigma^2_t[i_{t+1}] + cov_t[m_{t+1}, i_{t+1}] && \text{(A2)} \\ &= \gamma_t E_t[\Delta c_{t+1}] + E_t[i_{t+1}] - 0.5\gamma^2_t \sigma^2_t[\Delta c_{t+1}] - 0.5\sigma^2_t[i_{t+1}] - \gamma_t cov_t[\Delta c_{t+1}, i_{t+1}] && \text{(A3)} \end{aligned}$$

The interest rate differential between the two countries is:

$$\begin{aligned} r^F_{t+1} - r^H_{t+1} = \; & \gamma^F_t E_t[\Delta c^F_{t+1}] - \gamma^H_t E_t[\Delta c^H_{t+1}] \\ & -0.5\left((\gamma^F_t)^2 \sigma^2_t[\Delta c^F_{t+1}] - (\gamma^H_t)^2 \sigma^2_t[\Delta c^H_{t+1}]\right) \\ & +E_t[i^F_{t+1}] - E_t[i^H_{t+1}] \\ & -0.5\left(\sigma^2_t[i^F_{t+1}] - \sigma^2_t[i^H_{t+1}]\right) \\ & -\gamma^F_t cov_t[\Delta c^F_{t+1}, i^F_{t+1}] + \gamma^H_t cov_t[\Delta c^H_{t+1}, i^H_{t+1}] \end{aligned} \tag{A4}$$

Define the carry trade r^{CT}_{t+1} as borrowing in H and investing in F, regardless of which country has a higher interest rate. The expected carry trade return is:

$$\begin{aligned} E_t[r^{CT}_{t+1}] &= r^F_{t+1} - r^H_{t+1} - E_t[\Delta e_{t+1}] \\ &= -0.5\left((\gamma^F_t)^2 \sigma^2_t[\Delta c^F_{t+1}] - (\gamma^H_t)^2 \sigma^2_t[\Delta c^H_{t+1}]\right) \\ &\quad -0.5\left(\sigma^2_t[i^F_{t+1}] - \sigma^2_t[i^H_{t+1}]\right) \\ &\quad -\gamma^F_t cov_t[\Delta c^F_{t+1}, i^F_{t+1}] + \gamma^H_t cov_t[\Delta c^H_{t+1}, i^H_{t+1}] \end{aligned} \tag{A5}$$

Note that as in our model, expected differences in inflation play no role in carry trade profits because any differences in nominal interest rates are offset by expected currency movements. The unexpected carry trade profit depends only on the shocks to the exchange rate, since the interest rate differential is not a function of $t+1$ shocks:

$$\begin{aligned} r^{CT}_{t+1} - E_t[r^{CT}_{t+1}] &= -(\Delta e_{t+1} - E_t[\Delta e_{t+1}]) \\ &= \gamma^H_t \left(\Delta c^H_{t+1} - E_t[\Delta c^H_{t+1}]\right) \\ &\quad -\gamma^F_t \left(\Delta c^F_{t+1} - E_t[\Delta c^F_{t+1}]\right) \\ &\quad + \left(i^H_{t+1} - E_t[i^H_{t+1}\right) \\ &\quad - \left(i^F_{t+1} - E_t[i^F_{t+1}\right) \end{aligned} \tag{A6}$$

Now consider two of the leading explanations for carry trade profits, Verdelhan (2010) and Bansal and Shaliastovich (2012). Both models assume complete financial markets but perfect home bias (no trade). The former relies on time varying risk aversion, while the later on time varying consumption and inflation volatility. The former uses habit preferences to achieve differences in risk aversion, though we believe that it is differences in risk aversion, rather than habit preferences that matter. The later uses Epstein-Zin-Weil preferences, though we believe that it is differences in volatility, rather than preferences that matter.

Starting with Verdelhan (2010), where $i_t = 0$. The simplest way to gain intuition is to assume that $\sigma_t[\Delta c^H_{t+1}] = \sigma_t[\Delta c^H_{t+1}]$ and $E_t[\Delta c^F_{t+1}] = E_t[\Delta c^H_{t+1}]$. Country H has higher risk aversion than country F: $\gamma^H_t > \gamma^F_t$. In this case the above equations reduce to:

$$\begin{aligned} r^F_{t+1} - r^H_{t+1} &= \left(\gamma^F_t - \gamma^H_t\right) E_t[\Delta c_{t+1}] \\ &\quad 0.5\left((\gamma^H_t)^2 - (\gamma^F_t)^2\right)\sigma^2_t[\Delta c_{t+1}] \end{aligned} \tag{A7}$$

$$E_t[r^{CT}_{t+1}] = 0.5\left((\gamma^H_t)^2 - (\gamma^F_t)^2\right)\sigma^2_t[\Delta c_{t+1}] \tag{A8}$$

$$r^{CT}_{t+1} - E_t[r^{CT}_{t+1}] = \gamma^H_t \Delta c^H_{t+1} - \gamma^F_t \Delta c^F_{t+1} \tag{A9}$$

If $\gamma^H_t > \gamma^F_t$ then this trade will always be profitable on average, from equation (A8). However, for it to be defined as the carry trade, the Foreign interest rate needs to be higher. This will happen if the second term in equation (A7) dominates; this is the precautionary saving channel and it will dominate if the risk aversion difference needs to be sufficiently high. This model implies that the realized carry trade profits are highest when consumption growth in the carry target country (F) is low. Additionally, in a multi-period, dynamic model with habit, in order for H to have a higher risk aversion, H must have received recent negative shocks. This implies that the carry trade target country (F) is bigger.

Continuing with Bansal and Shaliastovich (2012), here inflation is no longer zero and, like in our model, the carry trade profit can be attributed to one of two channels, one real and one nominal. Although Bansal and Shaliastovich (2012) employ Epstein-Zin-Weil preferences, the intuition is easiest to see with CRRA utility with a constant risk aversion. Additionally, for simplicity assume that $E_t[\Delta c^F_{t+1}] = E_t[\Delta c^H_{t+1}]$ but $\sigma_t[\Delta c^H_{t+1}] > \sigma_t[\Delta c^F_{t+1}]$. In this case the above equations reduce to:

$$\begin{aligned} r^F_{t+1} - r^H_{t+1} &= 0.5\gamma^2\left(\sigma^2_t[\Delta c^H_{t+1}] - \sigma^2_t[\Delta c^F_{t+1}]\right) \\ &\quad + E_t[i^F_{t+1}] - E_t[i^H_{t+1}] \\ &\quad 0.5\left(\sigma^2_t[i^H_{t+1}] - \sigma^2_t[i^F_{t+1}]\right) \\ &\quad \gamma\left(cov_t[\Delta c^H_{t+1}, i^H_{t+1}] - cov_t[\Delta c^F_{t+1}, i^F_{t+1}]\right) \end{aligned} \tag{A10}$$

$$\begin{aligned} E_t[r^{CT}_{t+1}] &= 0.5\gamma^2\left(\sigma^2_t[\Delta c^H_{t+1}] - \sigma^2_t[\Delta c^F_{t+1}]\right) \\ &\quad 0.5\left(\sigma^2_t[i^H_{t+1}] - \sigma^2_t[i^F_{t+1}]\right) \\ &\quad \gamma\left(cov_t[\Delta c^H_{t+1}, i^H_{t+1}] - cov_t[\Delta c^F_{t+1}, i^F_{t+1}]\right) \end{aligned} \tag{A11}$$

$$\begin{aligned} r^{CT}_{t+1} - E_t[r^{CT}_{t+1}] &= \gamma\left(\Delta c^H_{t+1} - \Delta c^F_{t+1}\right) \\ &\quad + \left(i^H_{t+1} - i^F_{t+1}\right) \end{aligned} \tag{A12}$$

First consider the real side. If $\sigma_t[\Delta c^H_{t+1}] > \sigma_t[\Delta c^F_{t+1}]$ then country F has a higher interest rate and the carry trade of borrowing in H and investing in F is profitable. Thus, carry trade target countries should have a lower expected consumption growth volatility. Additionally, as in Verdelhan (2010), this model implies that the realized carry trade profits are highest when consumption growth in the carry target country (F) is low.

Next, consider the nominal side. All else equal, if $\sigma_t[i^H_{t+1}] > \sigma_t[i^F_{t+1}]$ then country F has a higher interest rate and the carry trade of borrowing in H and investing in F is profitable. Thus, carry trade target countries should have a lower expected inflation volatility. Additionally, as in our model, this model implies that the realized carry trade profits are highest when realized inflation in the target country (F) is low. In this model, carry trade profits can also come through lower (more negative) expected covariance of inflation with consumption growth in the target country. Finally, all else equal, the country with a higher expected inflation has a higher interest rate, however this has no effect on carry trade profits.

It is straight forward to map the intuition above into our two-period framework. In particular, we set $\theta = 1$ so that there is no trade and we assume CRRA preferences ($\gamma = \frac{1}{\rho}$); these results are presented in Table A3. Models 36 and 37 present models with real bonds and, asymmetries in, respectively, risk aversion or output volatility. Country F has a higher interest rate, and the carry trade from H to F yields a positive expected profit. However, the strength of a country's currency is negatively correlated with its output shocks, implying that the carry trade is most profitable when the target country experiences negative shocks. Recall that in the data, as in our models with trade, a country's currency is positively correlated with its output shocks, and the carry trade is most profitable when the target country experiences positive shocks.

In Bansal and Shaliastovich (2012), the carry trade is profitable due to one or two channels. The real channel relies on asymmetry in volatility, as in model 37. This is exactly the same channel as in Tran (2013). The nominal channel relies on counter-cyclical inflation, which is related to the nominal channel in our model. However, unlike our model, Bansal and Shaliastovich (2012) assume an exogenous process for inflation. In model 38, we solve a model with asymmetry in volatility but endogenous inflation due to a zero expected inflation monetary policy. As in all of our models, inflation is negatively correlated with output shocks.

Unlike our baseline model with trade, where endogenous inflation can strengthen the carry trade profit, here it weakens the carry trade profit. This is because in this model, carry trade profits are a result of counter-cyclical real currency returns—the carry trade is most profitable in bad times for the target country. The nominal exchange rate is less volatile than the real because high inflation in bad times strengthens the exchange rate; it therefore weakens the carry trade profit.

B Additional results

B.1 CES Consumption Aggregation

In the main text we have solved the model with Cobb-Douglas aggregation of tradable and non-tradable consumption ($\alpha = 0$). This led to perfect risk sharing because the change in prices caused nominal bonds to behave like real equity. The intuition is related to equation (9):

$$\frac{C^{\mathrm{T}}_{\mathrm{H},1}}{C^{\mathrm{N}}_{\mathrm{H},1}} = \left(\frac{1-\theta}{\theta}\right)^{\frac{1}{1-\alpha}} \left(\frac{P^{\mathrm{T}}_{\mathrm{H},1}}{P^{\mathrm{N}}_{\mathrm{H},1}}\right)^{\frac{1}{1-\alpha}}$$

When $\alpha = 0$, the price level of tradables is directly inversely related to the consumption of tradables. When the money supply is not a function of the realized shock at $t = 1$ (for example, under a zero expected inflation monetary policy), this equation can be combined with the money supply equation (19) to show that the payoff of the nominal bond is linearly related to tradable output. When $\alpha \neq 0$, this is no longer the case and two nominal bonds are no longer enough for perfect risk sharing.

Results with $\alpha = 0.5$ and $\alpha = -1.0$ are presented in Table A1. $0 < \alpha < 1$ implies that the two goods are more substitutible than the Cobb-Douglas case, which is consistent with Obstfeld and Rogoff (2001); $\alpha < 0$ implies they are more complimentary and are consistent with Mendoza (1991) and Stockman and Tesar (1995). The models are identical to those in Table 4, except for the difference in α. For brevity we present only the cases with higher volatility of F tradable shocks (21 and 25); symmetry in production but systematic default in F (Models 22 and 26); higher volatility of F tradable shocks and systematic default in F (Models 23 and 26); smaller size in F and systematic default in F (Models 24 and 27).

For the cases with no default, despite imperfect risk sharing, the results are quite similar to the perfect risk sharing case of $\alpha = 0$. Risk sharing is still very close to perfect with correlations of consumption growth nearly one. The ratios in the volatility of consumption are not exactly one, but also relatively close to one. Interestingly, the real exchange rate is no longer constant, however its volatility is still quite low. The magnitudes of the carry trade profits are also quite similar to those with $\alpha = 0$.

Setting $\alpha > 0$ seems to somewhat increase carry trade profits relative to $\alpha = 0$ case, while $\alpha < 0$ somewhat decreases carry trade profits. However, the results are quite similar to the $\alpha = 0$ case.

B.2 Strategic default

In Table A2 we present results for Strategic default; Models 29–34 are analogous to the systematic default cases 15–20 in Table 4. The carry trade profits are smaller quantitatively. This is because this default is not as costly from a risk sharing perspective.

The outstanding nominal value of bonds is lower in the systematic case because default occurs in the state of the world when insurance is most valuable (low output in both countries), thus, the nominal bond which defaults in the systematic state is less useful from a risk sharing perspective. Contrast that with strategic default where default occurs in a neutral aggregate state; in this state Foreign output is high and Foreign tradable prices are low. Although Home investors are unhappy about the bond defaulting, at least they get 75% of a relatively valuable (in real terms) promised payment; furthermore they get the full payout (plus the higher interest to compensate for default) in the state where both countries experience the worst shock.

The currency return in the case of strategic default is even more volatile and pro-cyclical than in the case of systematic default. This is because in addition to the channel described in the main text (fewer bonds traded leads to pro-cyclicality), there is a second channel present. In the default state, tradable goods are even more abundant in Foreign consumption because Home investors suffered a loss due to default; this causes an even bigger appreciation of the Foreign real currency (fall in ER). Since strategic default happens in the bad state for H but good for F, this causes the real currency return to be even more correlated with local output shocks.

C Data

Our currency data set consists of daily observations of US-dollar based spot and forward exchange rates for 1 month, 3 months and 1 year maturities, for 39 countries. These countries are as follows: Australia, Austria, Belgium, Canada, Czech Republic, Denmark, France, Germany, Hong Kong, Hungary, Iceland, Indonesia, India, Ireland, Italy, Japan, Korea, Kuwait, Malaysia, Mexico, Netherlands, New Zealand, Norway, Philippines, Poland, Saudi Arabia,Singapore, Slovak Republic, South Africa, Spain, Sweden, Switzerland, Taiwan, Thailand, Turkey, UAE, UK. We extract the forward rates and spot rates from DataStream. Our data gathering procedure follows the one used by Hassan and Mano (2014). We gather spot rates and forward rates from four sources of data DataStream: World Markets PLC/Reuters (WM/R), thomson/Reuters (T/R), HSBC, and Barclays Bank PLC (BB). The range of each of these data sources differ, however if for an observation we have data from different sources we follow a priority rule. If data is available from WM.R, we use that, otherwise we select from Thomson/Reuters, HSBC and Barclays Bank data sets in order. Following Hassan and Mano (2014), we also exclude observations for which covered interest parity seems to be violated.[45]

Our data set for macroeconomic indicators such as GDP and inflation comes from WorldBank. This includes all countries in the DataStream dataset except for Taiwan, and adds Finland, Greece, Portugal, and the U.S.

D Solution of the model

The H household problem is

$$\max_{\{C^{\mathrm{T}}_{\mathrm{H},0},C^{\mathrm{N}}_{\mathrm{H},0},B^{\mathrm{H}}_{\mathrm{H}},B^{\mathrm{F}}_{\mathrm{H}},C^{\mathrm{T}}_{\mathrm{H},1,i},C^{\mathrm{N}}_{\mathrm{H},1,i}\}_{i=1}^{K}} \left(C_{\mathrm{H},0}^{1-\rho} + \beta \mathbb{E}\left[\widetilde{C}_{\mathrm{H},1}^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}}, \quad 0 < \beta < 1, \tag{D13}$$

where

$$\widetilde{C}_{\mathrm{H},1} = (C_{\mathrm{H},1,1}, \ldots, C_{\mathrm{H},1,K}) \tag{D14}$$

$$C_{\mathrm{H},0} = ((1-\theta)(C^{\mathrm{T}}_{\mathrm{H},0})^{\alpha} + \theta (C^{\mathrm{N}}_{\mathrm{H},0})^{\alpha})^{\frac{1}{\alpha}} \tag{D15}$$

$$C_{\mathrm{H},1,i} = \left((1-\theta)(C^{\mathrm{T}}_{\mathrm{H},1,i})^{\alpha} + \theta (C^{\mathrm{N}}_{\mathrm{H},1,i})^{\alpha}\right)^{\frac{1}{\alpha}}, \quad 0 < \theta < 1, \quad i = 1, \ldots, K, \tag{D16}$$

subject to the budget constraints

$$P^{\mathrm{T}}_{\mathrm{H},0} C^{\mathrm{T}}_{\mathrm{H},0} + P^{\mathrm{N}}_{\mathrm{H},0} C^{\mathrm{N}}_{\mathrm{H},0} + q_{\mathrm{H}} B^{\mathrm{H}}_{\mathrm{H}} + \frac{q_{\mathrm{F}}}{E_0} B^{\mathrm{F}}_{\mathrm{H}} = P^{\mathrm{T}}_{\mathrm{H},0} Y^{\mathrm{T}}_{\mathrm{H},0} + P^{\mathrm{N}}_{\mathrm{H},0} Y^{\mathrm{T}}_{\mathrm{H},0} \tag{D17}$$

$$P^{\mathrm{T}}_{\mathrm{H},1,i} C^{\mathrm{T}}_{\mathrm{H},1,i} + P^{\mathrm{N}}_{\mathrm{H},1,i} C^{\mathrm{N}}_{\mathrm{H},1,i} = P^{\mathrm{T}}_{\mathrm{H},1,i} Z^{T}_{\mathrm{H},i} Y^{\mathrm{T}}_{\mathrm{H},1} + P^{\mathrm{N}}_{\mathrm{H},1,i} Z^{N}_{\mathrm{H},i} Y^{\mathrm{N}}_{\mathrm{H},1} + B^{\mathrm{H}}_{\mathrm{H}} + \frac{1}{E_{1,i}} B^{\mathrm{F}}_{\mathrm{H}}, \tag{D18}$$

[45] These observations are excluded from the sample and are as follow: Italy 1/1985 and 2/1985; Switzerland 2/1985; Germany 2/1985; United Kingdom 3/1985; Belgium 7/1990; Indonesia 12/1997, 3/1998, 5/1998-7/1998, 2/2001-11/2002, Austria 1/1990-2/1990; Spain 9/1987, 5/1988; Ireland 11/1986, 11/1987, 1/1989, 1/1991, 9/1992-11/1992, 1/1993; Belgium 2/1985; and Norway 2/1985. Also South Africa 8/1985 and Turkey before 11/2001 has been excluded following Lustig et al. (2011) due to large covered interest parity departures. Malaysia 8/1998-6/2005 and Indonesia 1/2003-5/2007 have also been removed because forward rates are zero.

and the resource constraints

$$C^{\mathrm{N}}_{\mathrm{H},1} = Y^{\mathrm{N}}_{\mathrm{H},0} \tag{D19}$$

$$C^{\mathrm{N}}_{\mathrm{H},1,i} = Z^{N}_{\mathrm{H},i} Y^{\mathrm{N}}_{\mathrm{H},1}, \quad i = 1, \ldots, K. \tag{D20}$$

The expectation $\mathbb{E}^{\pi}[x] = \sum_{i=1}^{k} \pi_i x_i$.

Solution details

$(D17)$ and $(D19) \Rightarrow \quad C^{\mathrm{T}}_{\mathrm{H},0} = Y^{\mathrm{T}}_{\mathrm{H},0} - \frac{1}{P^{\mathrm{T}}_{\mathrm{H},0}}(q_{\mathrm{H}} B^{\mathrm{H}}_{\mathrm{H}} + \frac{q_{\mathrm{F}}}{E_0} B^{\mathrm{F}}_{\mathrm{H}})$

$(D18)$ and $(D20) \Rightarrow \quad C^{\mathrm{T}}_{\mathrm{H},1,i} = Z^{T}_{\mathrm{H},i} Y^{\mathrm{T}}_{\mathrm{H},1} + \frac{1}{P^{\mathrm{T}}_{\mathrm{H},1,i}}(B^{\mathrm{H}}_{\mathrm{H}} + \frac{1}{E_1} B^{\mathrm{F}}_{\mathrm{H}})$

$C^{\mathrm{T}}_{\mathrm{H},0}, C^{\mathrm{T}}_{\mathrm{H},1,i}, B^{\mathrm{H}}_{\mathrm{H}}, B^{\mathrm{F}}_{\mathrm{H}}$ are determined from the Lagrangian:

$$\mathcal{L} = \underbrace{(C^{1-\rho}_{\mathrm{H},0} + \beta \mathbb{E}[\widetilde{C}^{1-\gamma}_{\mathrm{H},1}]^{\frac{1-\rho}{1-\gamma}})^{\frac{1}{1-\rho}}}_{u} + \lambda_0 (P^{\mathrm{T}}_{\mathrm{H},0} Y^{\mathrm{T}}_{\mathrm{H},0} + P^{\mathrm{N}}_{\mathrm{H},0} Y^{\mathrm{N}}_{\mathrm{H},0} - P^{\mathrm{T}}_{\mathrm{H},0} C^{\mathrm{T}}_{\mathrm{H},0} - P^{\mathrm{N}}_{\mathrm{H},0} C^{\mathrm{N}}_{\mathrm{H},0} - q_{\mathrm{H}} B^{\mathrm{H}}_{\mathrm{H}} - \frac{q_{\mathrm{F}}}{E_0} B^{\mathrm{F}}_{\mathrm{H}})$$
$$+ \sum_{i=1}^{k} \lambda_{1,i} (P^{\mathrm{T}}_{\mathrm{H},1,i} Z^{T}_{\mathrm{H},i} Y^{\mathrm{T}}_{\mathrm{H},1} + P^{\mathrm{N}}_{\mathrm{H},1,i} Z^{N}_{\mathrm{H},i} Y^{\mathrm{N}}_{\mathrm{H},1} + B^{\mathrm{H}}_{\mathrm{H}} + \frac{1}{E_{1,i}} B^{\mathrm{F}}_{\mathrm{H}} - P^{\mathrm{T}}_{\mathrm{H},1,i} C^{\mathrm{T}}_{\mathrm{H},1,i} - P^{\mathrm{N}}_{\mathrm{H},1,i} C^{\mathrm{N}}_{\mathrm{H},1,i})$$

FOC for consumption

a) $C^{\mathrm{T}}_{\mathrm{H},0} : \frac{1}{1-\rho} u^{\rho} (1-\rho) C^{-\rho}_{\mathrm{H},0} \frac{\partial C_{\mathrm{H},0}}{\partial C^{\mathrm{T}}_{\mathrm{H},0}} = \lambda_0 P^{\mathrm{T}}_{\mathrm{H},0}$

b) $C^{\mathrm{N}}_{\mathrm{H},0} : \frac{1}{1-\rho} u^{\rho} . (1-\rho) C^{-\rho}_{\mathrm{H},0} \frac{\partial C_{\mathrm{H},0}}{\partial C^{\mathrm{N}}_{\mathrm{H},0}} = \lambda_0 P^{\mathrm{N}}_{\mathrm{H},0}$

From a) and b): $\Rightarrow \quad \frac{P^{\mathrm{N}}_{\mathrm{H},0}}{P^{\mathrm{T}}_{\mathrm{H},0}} = \frac{\partial C_{\mathrm{H},0} / \partial C^{\mathrm{N}}_{\mathrm{H},0}}{\partial C_{\mathrm{H},0} / \partial C^{\mathrm{T}}_{\mathrm{H},0}} = \frac{\frac{\alpha}{\alpha} C^{(1-\alpha)}_{\mathrm{H},0} \theta (C^{\mathrm{N}}_{\mathrm{H},0})^{\alpha-1}}{\frac{\alpha}{\alpha} C^{(1-\alpha)}_{\mathrm{H},0} (1-\theta)(C^{\mathrm{T}}_{\mathrm{H},0})^{\alpha-1}} = \frac{\theta}{1-\theta} (\frac{C^{\mathrm{T}}_{\mathrm{H},0}}{C^{\mathrm{N}}_{\mathrm{H},0}})^{1-\alpha}$
$\stackrel{(D19)}{=} \frac{\theta}{1-\theta} (\frac{C^{\mathrm{T}}_{\mathrm{H},0}}{Y^{\mathrm{N}}_{\mathrm{H},0}})^{1-\alpha} \Rightarrow \quad \frac{P^{\mathrm{N}}_{\mathrm{H},0}}{P^{\mathrm{T}}_{\mathrm{H},0}} = \frac{\theta}{1-\theta} (\frac{C^{\mathrm{T}}_{\mathrm{H},0}}{Y^{\mathrm{N}}_{\mathrm{H},0}})^{1-\alpha}$

c) $C^{\mathrm{T}}_{\mathrm{H},1,i} : \frac{1}{1-\rho} u^{\rho} \beta \frac{1-\rho}{1-\gamma} \mathbb{E}^{\pi} [C^{1-\gamma}_{\mathrm{H},1}]^{\frac{\gamma-\rho}{1-\gamma}} (1-\gamma) \pi_i C^{-\gamma}_{\mathrm{H},1,i} \frac{\partial C_{\mathrm{H},1,i}}{\partial C^{\mathrm{T}}_{\mathrm{H},1,i}} = \lambda_{1,i} P^{\mathrm{T}}_{\mathrm{H},1,i}$

d) $C^{\mathrm{N}}_{\mathrm{H},1,i} : \frac{1}{1-\rho} u^{\rho} \beta \frac{1-\rho}{1-\gamma} \mathbb{E}^{\pi} [C^{1-\gamma}_{\mathrm{H},1}]^{\frac{\gamma-\rho}{1-\gamma}} (1-\gamma) \pi_i C^{-\gamma}_{\mathrm{H},1,i} \frac{\partial C_{\mathrm{H},1,i}}{\partial C^{\mathrm{N}}_{\mathrm{H},1,i}} = \lambda_{1,i} P^{\mathrm{N}}_{\mathrm{H},1,i}$

From c) and d): $\Rightarrow \quad \frac{P^{\mathrm{N}}_{\mathrm{H},1,i}}{P^{\mathrm{T}}_{\mathrm{H},1,i}} = \frac{\partial C_{\mathrm{H},1,i} / \partial C^{\mathrm{N}}_{\mathrm{H},1,i}}{\partial C_{\mathrm{H},1,i} / \partial C^{\mathrm{T}}_{\mathrm{H},1,i}} = \frac{\frac{\alpha}{\alpha} C^{(1-\alpha)}_{\mathrm{H},1,i} \theta (C^{\mathrm{N}}_{\mathrm{H},1,i})^{\alpha-1}}{\frac{\alpha}{\alpha} C^{(1-\alpha)}_{\mathrm{H},1,i} (1-\theta)(C^{\mathrm{T}}_{\mathrm{H},1,i})^{\alpha-1}} = \frac{\theta}{1-\theta} (\frac{C^{\mathrm{T}}_{\mathrm{H},1,i}}{C^{\mathrm{N}}_{\mathrm{H},1,i}})^{1-\alpha} \stackrel{(D20)}{=}$
$\frac{\theta}{1-\theta} (\frac{C^{\mathrm{T}}_{\mathrm{H},1,i}}{Z^{N}_{\mathrm{H},i} Y^{\mathrm{N}}_{\mathrm{H},1}})^{1-\alpha} \Rightarrow \frac{P^{\mathrm{N}}_{\mathrm{H},1,i}}{P^{\mathrm{T}}_{\mathrm{H},1,i}} = \frac{\theta}{1-\theta} (\frac{C^{\mathrm{T}}_{\mathrm{H},1,i}}{Z^{N}_{\mathrm{H},i} Y^{\mathrm{N}}_{\mathrm{H},1}})^{1-\alpha}.$

FOC for assets

e) $B_{\mathrm{H}}^{\mathrm{H}}: -\lambda_0 q_{\mathrm{H}} + \sum_{i=1}^{k} \lambda_{1,i} = 0 \Rightarrow q_{\mathrm{H}} = \frac{1}{\lambda_0}\sum_{i=1}^{k}\lambda_{1,i}$

from FOC($C_{\mathrm{H},0}^{\mathrm{T}}$), $\lambda_0 = \frac{1}{P_{\mathrm{H},0}^{\mathrm{T}}} u^{\rho} C_{\mathrm{H},0}^{-\rho} C_{\mathrm{H},0}^{(1-\alpha)}(1-\theta)(C_{\mathrm{H},0}^{\mathrm{T}})^{\alpha-1} = \frac{1}{P_{\mathrm{H},0}^{\mathrm{N}}} u^{\rho} C_{\mathrm{H},0}^{-\rho} C_{\mathrm{H},0}^{(1-\alpha)}(\theta)(C_{\mathrm{H},0}^{\mathrm{N}})^{\alpha-1}$

from FOC($C_{\mathrm{H},1,i}^{\mathrm{T}}$), $\lambda_{1,i} = \frac{1}{P_{\mathrm{H},1,i}^{\mathrm{T}}} u^{\rho}\beta\mathbb{E}[C_{\mathrm{H},1}^{1-\gamma}]^{\frac{\gamma-\rho}{1-\gamma}} \pi_i C_{\mathrm{H},1,i}^{-\gamma} C_{\mathrm{H},1,i}^{(1-\alpha)}(1-\theta)(C_{\mathrm{H},1,i}^{\mathrm{T}})^{\alpha-1} =$
$\frac{1}{P_{\mathrm{H},1,i}^{\mathrm{N}}} u^{\rho}\beta\mathbb{E}[C_{\mathrm{H},1}^{1-\gamma}]^{\frac{\gamma-\rho}{1-\gamma}} \pi_i C_{\mathrm{H},1,i}^{-\gamma} C_{\mathrm{H},1,i}^{(1-\alpha)}\theta(C_{\mathrm{H},1,i}^{\mathrm{N}})^{\alpha-1}$

$$q_{\mathrm{H}} = \frac{P_{\mathrm{H},0}^{\mathrm{T}}}{u^{\rho} C_{\mathrm{H},0}^{(1-\alpha)-\rho}(1-\theta)(C_{\mathrm{H},0}^{\mathrm{T}})^{(\alpha-1)}} u^{\rho}\beta\mathbb{E}[\widetilde{C}_{\mathrm{H},1}^{1-\gamma}]^{\frac{\gamma-\rho}{1-\gamma}}(1-\theta)\sum_{i=1}^{K}\pi_i \frac{1}{P_{\mathrm{H},1,i}^{\mathrm{T}}} C_{\mathrm{H},1,i}^{(1-\alpha)-\gamma}(C_{\mathrm{H},1,i}^{\mathrm{T}})^{\alpha-1}$$
$$= \beta\left(\frac{C_{\mathrm{H},0}}{\mathbb{E}[\widetilde{C}_{\mathrm{H},1}^{1-\gamma}]^{\frac{1}{1-\gamma}}}\right)^{\rho-\gamma} . \sum_{i=1}^{K}\pi_i \frac{P_{\mathrm{H},0}^{\mathrm{T}}}{C_{\mathrm{H},0}^{1-\alpha-\gamma}(C_{\mathrm{H},0}^{\mathrm{T}})^{\alpha-1}} \frac{C_{\mathrm{H},1,i}^{1-\alpha-\gamma}(C_{\mathrm{H},1,i}^{\mathrm{T}})^{\alpha-1}}{P_{\mathrm{H},1,i}^{\mathrm{T}}}$$
$$= \beta\left(\frac{C_{\mathrm{H},0}}{\mathbb{E}[\widetilde{C}_{\mathrm{H},1}^{1-\gamma}]^{\frac{1}{1-\gamma}}}\right)^{\rho-\gamma} . \mathbb{E}\Big[\left(\frac{\widetilde{C}_{\mathrm{H},1}}{C_{\mathrm{H},0}}\right)^{-\gamma} \underbrace{\frac{\widetilde{C}_{\mathrm{H},1}^{(1-\alpha)}(\widetilde{C}_{\mathrm{H},1}^{T})^{\alpha-1}}{C_{\mathrm{H},0}^{(1-\alpha)}(C_{\mathrm{H},0}^{\mathrm{T}})^{\alpha-1}} \frac{P_{\mathrm{H},0}^{\mathrm{T}}}{\widetilde{P}_{\mathrm{H},1}}}_{\equiv \frac{P_{\mathrm{H},0}}{P_{\mathrm{H},1}} \text{see Claim 1}}\Big]$$

Claim 1. $P_{J,0} = ((1-\theta)^{\frac{1}{1-\alpha}}(P_{J,0}^{T})^{\frac{\alpha}{\alpha-1}} + \theta^{\frac{1}{1-\alpha}}(P_{J,0}^{N})^{\frac{\alpha}{\alpha-1}})^{\frac{\alpha-1}{\alpha}}, \quad J = H, F$

$P_{J,1,i} = ((1-\theta)^{\frac{1}{1-\alpha}}(P_{J,1,i}^{T})^{\frac{\alpha}{\alpha-1}} + \theta^{\frac{1}{1-\alpha}}(P_{J,1,i}^{N})^{\frac{\alpha}{\alpha-1}})^{\frac{\alpha-1}{\alpha}}, \quad J = H, F, \quad i = 1, ..., K$

Proof: Let P denote either $P_{\mathrm{J},0}$ or $P_{\mathrm{J},1,i}$, $i = 1, ..., K$.
Let C^T denote either $C_{\mathrm{J},0}^{\mathrm{T}}$ or $C_{\mathrm{J},1,i}^{\mathrm{T}}$, $i = 1, ..., K$
Let C^N denote eiither $C_{\mathrm{J},0}^{\mathrm{N}}$ or $C_{\mathrm{J},1,i}^{\mathrm{N}}$, $i = 1, ..., K$
Price index: $C = ((1-\theta)(C^T)^{\alpha} + \theta(C^N)^{\alpha})^{\frac{1}{\alpha}} \qquad P = \frac{P^T C^T + P^N C^N}{C}$
$= \frac{P^T + P^N(\frac{C^N}{C^T})}{\frac{C}{C^T}}$
$= \frac{P^T + P^N(\frac{C^N}{C^T})}{((1-\theta)+\theta(\frac{C^N}{C^T})^{\alpha})^{\frac{1}{\alpha}}}$

From FOC for $t = 0$ and $t = 1$: $\frac{C^N}{C^T} = (\frac{\theta}{1-\theta})^{\frac{1}{1-\alpha}}(\frac{P^T}{P^N})^{\frac{1}{1-\alpha}}$

$$\Rightarrow P = \frac{P^T + P^N(\frac{\theta}{1-\theta})^{\frac{1}{1-\alpha}}(\frac{P^T}{P^N})^{\frac{1}{1-\alpha}}}{((1-\theta)+\theta(\frac{\theta}{1-\theta})^{\frac{\alpha}{1-\alpha}}(\frac{P^T}{P^N})^{\frac{\alpha}{1-\alpha}})^{\frac{1}{\alpha}}} = \frac{P^T + (\frac{\theta}{1-\theta})^{\frac{1}{1-\alpha}}(P^N)^{\frac{\alpha}{\alpha-1}}(P^T)^{\frac{1}{1-\alpha}}}{((1-\theta)+\theta^{\frac{1}{1-\alpha}}(\frac{1}{1-\theta})^{\frac{\alpha}{1-\alpha}}(\frac{P^N}{P^T})^{\frac{\alpha}{\alpha-1}})^{\frac{1}{\alpha}}} =$$
$$\frac{(P^T)^{1/(1-\alpha)}((P^T)^{\alpha/(\alpha-1)} + \frac{\theta^{1/(1-\alpha)}}{(1-\theta)^{1/(1-\alpha)}}(P^N)^{\alpha/(\alpha-1)})}{\frac{(P^T)^{1/(1-\alpha)}}{(1-\theta)^{1/(1-\alpha)}}((1-\theta)^{1/(1-\alpha)}(P^T)^{\alpha/(\alpha-1)} + \theta^{1/(1-\alpha)}(P^N)^{\alpha/(\alpha-1)})^{1/\alpha}} =$$

$$\frac{\frac{(P^T)^{1/(1-\alpha)}}{(1-\theta)^{1/(1-\alpha)}}((P^T)^{\alpha/(\alpha-1)}(1-\theta)^{1/(1-\alpha)}+\theta^{1/(1-\alpha)}(P^N)^{\alpha/(\alpha-1)})}{\frac{(P^T)^{1/(1-\alpha)}}{(1-\theta)^{1/(1-\alpha)}}((1-\theta)^{1/(1-\alpha)}(P^T)^{\alpha/(\alpha-1)}+\theta^{1/(1-\alpha)}(P^N)^{\alpha/(\alpha-1)})^{1/\alpha}} =$$

$$((1-\theta)^{1/(1-\alpha)}(P^T)^{\alpha/(\alpha-1)} + \theta^{1/(1-\alpha)}(P^N)^{\alpha/(\alpha-1)})^{(\alpha-1)/\alpha} \qquad \blacksquare$$

Claim 2. $C_{J,0} = (1-\theta)^{-1/(1-\alpha)}(\frac{P^T_{J,0}}{P_{J,0}})^{\frac{1}{1-\alpha}} C^T_{J,0} \quad J = H, F$

$C_{J,1,i} = (1-\theta)^{-1/(1-\alpha)}(\frac{P^T_{J,1,i}}{P_{J,1,i}})^{\frac{1}{1-\alpha}} C^T_{J,1,i} \quad J = H, F \;\; ; \;\; i = 1, ..., K$

Proof: Let P denote either $P_{\text{J},0}$ or $P_{\text{J},1,i}, i = 1, ..., K$ and C^N and C^T as we claim in claim 1. From the budget constrain: $PC = P^T C^T + P^N C^N$ we have $\Rightarrow C = \frac{P^T}{P}C^T + \frac{P^N}{P}C^N$. If we substitute C^T from the FOC: $\Rightarrow C^N = (\frac{\theta}{1-\theta})^{\frac{1}{1-\alpha}}(\frac{P^T}{P^N})^{\frac{1}{1-\alpha}}C^T$ we get:

$$C = \frac{P^T}{P}C^T + \frac{P^N}{P}(\frac{\theta}{1-\theta})^{\frac{1}{1-\alpha}}(\frac{P^T}{P^N})^{\frac{1}{1-\alpha}}C^T = (\frac{P^T}{P})^{\frac{1}{1-\alpha}}[(\frac{P^T}{P})^{\frac{-\alpha}{1-\alpha}} + (\frac{\theta}{1-\theta})^{\frac{1}{1-\alpha}}(\frac{P^N}{P})^{\frac{-\alpha}{1-\alpha}}]C^T$$

$$= (\frac{P^T}{P})^{\frac{1}{1-\alpha}}C^T \frac{\overbrace{((1-\theta)^{\frac{1}{1-\alpha}}(P^T)^{\frac{\alpha}{\alpha-1}} + \theta^{\frac{1}{1-\alpha}}(P^N)^{\frac{\alpha}{\alpha-1}})}^{=P^{\frac{\alpha}{\alpha-1}} \text{ by Claim 1}}}{(1-\theta)^{1/(1-\alpha)}P^{\alpha/\alpha-1}}$$

$$= (\frac{P^T}{P})^{1/(1-\alpha)}C^T\frac{P^{\frac{\alpha}{\alpha-1}}}{(1-\theta)^{1/1-\alpha}P^{\alpha/(\alpha-1)}} = (1-\theta)^{-1/(1-\alpha)}(\frac{P^T}{P})^{\frac{1}{1-\alpha}}C^T. \qquad \blacksquare$$

Hence, using Claims 1 and 2 in the Euler equation for q_{H} we have:

$$q_{\text{H}} = \beta(\frac{C_{\text{H},0}}{[\widetilde{C}^{1-\gamma}_{\text{H},1}]^{\frac{1}{1-\gamma}}})^{\rho-\gamma}\mathbb{E}[(\frac{\widetilde{C}_{\text{H},1}}{C_{\text{H},0}})^{-\gamma}\frac{\widetilde{C}^{(1-\alpha)}_{\text{H},1}(\widetilde{C}^{\text{T}}_{\text{H},1})^{\alpha-1}}{C^{(1-\alpha)}_{\text{H},0}(\widetilde{C}^{\text{T}}_{\text{H},1})^{\alpha-1}}\frac{P^{\text{T}}_{\text{H},0}}{P^{\text{T}}_{\text{H},1}}]$$

$$= \beta(\frac{C_{\text{H},0}}{[\widetilde{C}^{1-\gamma}_{\text{H},1}]^{\frac{1}{1-\gamma}}})^{\rho-\gamma}\mathbb{E}[(\frac{\widetilde{C}_{\text{H},1}}{C_{\text{H},0}})^{-\gamma}\frac{\overbrace{(1-\theta)^{-1}\frac{(\widetilde{P}^{\text{T}}_{\text{H},1})}{(P_{\text{H},1})}(\widetilde{C}^{\text{T}}_{\text{H},1})^{(1-\alpha)}(\widetilde{C}^{\text{T}}_{\text{H},1})^{\alpha-1}}^{\text{Claim 2}}}{(1-\theta)^{-1}\frac{(P^{\text{T}}_{\text{H},0})}{P_{\text{H},0}}(C^{\text{T}}_{\text{H},0})^{1-\alpha}(C^{\text{T}}_{\text{H},0})^{\alpha-1}}\frac{P^{\text{T}}_{\text{H},0}}{\widetilde{P}^{\text{T}}_{\text{H},1}}]$$

Then we get $q_{\text{H}} = \beta(\frac{C_{\text{H},0}}{[\widetilde{C}^{1-\gamma}_{\text{H},1}]^{\frac{1}{1-\gamma}}})^{\rho-\gamma}\mathbb{E}[(\frac{\widetilde{C}_{\text{H},1}}{C_{\text{H},0}})^{-\gamma}\frac{P_{\text{H},0}}{P_{\text{H},1}}]$

$$\text{Nominal SDF} : \tilde{M}^{\$} = \beta\underbrace{(\frac{\widetilde{C}_{\text{H},1}}{C_{\text{H},0}})^{-\gamma}}_{\text{CRRA}} \underbrace{(\frac{C_{\text{H},0}}{\mathbb{E}[\widetilde{C}^{1-\gamma}_{\text{H},1}]^{\frac{1}{1-\gamma}}})^{\rho-\gamma}}_{\text{Epstein-Zin } =0 \text{ if } \rho=\gamma} \frac{P_{\text{H},0}}{P_{\text{H},1}}$$

Finally, q_{F} is given from FOC $(B^{\text{F}}_{\text{H}})$

$$(B^{\text{F}}_{\text{H}}) : -\lambda_0\frac{q_{\text{F}}}{E_0} + \sum_{i=1}^{K}\lambda_{1,i}\frac{1}{E_{1,i}} = 0$$

Following the same steps as we did before we get:

$$q_{\text{F}} = \beta(\frac{C_{\text{F},0}}{[\widetilde{C}^{1-\gamma}_{\text{F},1}]^{\frac{1}{1-\gamma}}})^{\rho-\gamma}\mathbb{E}[(\frac{\widetilde{C}_{\text{F},1}}{C_{\text{F},0}})^{-\gamma}\frac{P^{\text{T}}_{\text{F},0}E_0}{P^{\text{T}}_{\text{F},1}\widetilde{E}_1}]$$

Unknowns: $q_{\text{H}}, q_{\text{F}}, B^{\text{H}}_{\text{H}}, B^{\text{F}}_{\text{H}}, P^{\text{T}}_{\text{H},0}, P^{\text{N}}_{\text{H},0}, P^{\text{T}}_{\text{H},1}, P^{\text{N}}_{\text{H},1}, P^{\text{T}}_{\text{F},0}, P^{\text{N}}_{\text{F},0}, P^{\text{T}}_{\text{F},1}, P^{\text{N}}_{\text{F},1}$.

Set $P^{\text{T}}_{\text{H},0} = P^{\text{T}}_{\text{F},0} = 1$ w.l.o.g. $\Rightarrow$ 10 Unknowns

8 constraints: a) 4 Euler equations and (b) 4 Money supply equations: $M_{\text{H},0}, M_{\text{F},0}$, $M_{\text{H},1}$, and $M_{\text{F},1}$. The money supply at time zero pins down $P^{\text{N}}_{\text{H},0}$ and $P^{\text{N}}_{\text{F},0}$ leaving only 8 unknowns.

From the money supply constraint we can pin down $P_{\text{H},1}$ and $P_{\text{F},1}$. Specifically,

$$M_{\text{H},1} = P^{\text{T}}_{\text{H},1} Z^{T}_{\text{H}} Y^{\text{T}}_{\text{H},1} + P^{\text{N}}_{\text{H},1} Z^{N}_{\text{H}} Y^{\text{N}}_{\text{H},1}.$$

Using the FOC of the household and the fact that $C^{\text{N}}_{\text{H},1} = Z^{N}_{\text{H}} Y^{\text{N}}_{\text{H},1}$ we obtain

$$P^{\text{T}}_{\text{H},1} = \frac{(1-\theta)M_{\text{H},1} - \theta\left(B^{\text{H}}_{\text{H}} + \frac{1}{E_1}B^{\text{F}}_{\text{H}}\right)\left(\frac{Z^{N}_{\text{H}} Y^{\text{N}}_{\text{H},1}}{C^{\text{T}}_{\text{H},1}}\right)^{\alpha}}{Z^{T}_{\text{H}} Y^{\text{T}}_{\text{H},1}\left(1-\theta+\theta\left(\frac{Z^{N}_{\text{H}} Y^{\text{N}}_{\text{H},1}}{C^{\text{T}}_{\text{H},1}}\right)^{\alpha}\right)}.$$

Similar derivation hold for the case of the F price, $P^{\text{T}}_{\text{F},1}$.

D.1 Complete markets

Let us suppose that there exists a complete set of K Arrow-Debreu (AD) securities that pay one unit of tradable good in state $i = 1, \ldots, K$ and zero otherwise. We assume that household have logarithmic preferences and that the composite consumption good is obtained via a Cobb-Douglas aggregator ($\alpha = 0$) over tradable and non-tradable consumption. In what follows, we describe the problem of the H household. The problem of the F household is similar. The households in the H country choose consumption and holdings of AD securities s_i, $i = 1, \ldots, K$ in order to maximize:

$$U = \log(C_0) + \beta \mathbb{E}\left[\log(C_1)\right] \tag{D21}$$

$$C_t = (C^{\text{T}}_{\text{H,t}})^{1-\theta}(C^{\text{N}}_{\text{H,t}})^{\theta}, \quad t = 0, 1. \tag{D22}$$

subject to the budget constraints

$$P^{\mathrm{T}}_{\mathrm{H},0}C^{\mathrm{T}}_{\mathrm{H},0} + P^{\mathrm{N}}_{\mathrm{H},0}C^{\mathrm{N}}_{\mathrm{H},0} + \sum_{i=1}^{K} q_i s_i = P^{\mathrm{T}}_{\mathrm{H},0}Y^{\mathrm{T}}_{\mathrm{H},0} + P^{\mathrm{N}}_{\mathrm{H},0}Y^{\mathrm{N}}_{\mathrm{H},0} \tag{D23}$$

$$P^{\mathrm{T}}_{\mathrm{H},1}C^{\mathrm{T}}_{\mathrm{H},1} + P^{\mathrm{N}}_{\mathrm{H},1}C^{\mathrm{N}}_{\mathrm{H},1} = P^{\mathrm{T}}_{\mathrm{H},1}Z^{T}_{\mathrm{H},i}Y^{\mathrm{T}}_{\mathrm{H},1} + P^{\mathrm{N}}_{\mathrm{H},1}Y^{\mathrm{N}}_{\mathrm{H},1} + \sum_{i=1}^{K} P^{\mathrm{T}}_{\mathrm{H},1,i}s_i \tag{D24}$$

where q_i denotes the price of the AD security i. FOC with respect to $C^{\mathrm{T}}_{\mathrm{H,t}}$ and the budget constraints imply

$$\frac{C^{\mathrm{T}}_{\mathrm{H,t}}}{C^{\mathrm{N}}_{\mathrm{H,t}}} = \frac{1-\theta}{\theta}\frac{P^{\mathrm{N}}_{\mathrm{H,t}}}{P^{\mathrm{T}}_{\mathrm{H,t}}}, \quad t = 0,1 \tag{D25}$$

from which we obtain

$$C^{\mathrm{N}}_{\mathrm{H},0} = \theta\left(P^{\mathrm{T}}_{\mathrm{H},0}Z^{T}_{\mathrm{H},i}Y^{\mathrm{T}}_{\mathrm{H},0} + P^{\mathrm{N}}_{\mathrm{H},0}Y^{\mathrm{N}}_{\mathrm{H},0} - \sum_{i=1}^{K} q_i s_i\right)\frac{1}{P^{\mathrm{N}}_{\mathrm{H},0}} \tag{D26}$$

$$C^{\mathrm{T}}_{\mathrm{H},0} = (1-\theta)\left(P^{\mathrm{T}}_{\mathrm{H},0}Y^{\mathrm{T}}_{\mathrm{H},0} + P^{\mathrm{N}}_{\mathrm{H},0}Y^{\mathrm{N}}_{\mathrm{H},0} - \sum_{i=1}^{K} q_i s_i\right)\frac{1}{P^{\mathrm{T}}_{\mathrm{H},0}} \tag{D27}$$

and

$$C^{\mathrm{N}}_{\mathrm{H},1} = \theta\left(P^{\mathrm{T}}_{\mathrm{H},1}Z^{T}_{\mathrm{H},i}Y^{\mathrm{T}}_{\mathrm{H},1} + P^{\mathrm{N}}_{\mathrm{H},1}Y^{\mathrm{N}}_{\mathrm{H},1} - \sum_{i=1}^{K} P^{\mathrm{T}}_{\mathrm{H},1,i}s_i\right)\frac{1}{P^{\mathrm{N}}_{\mathrm{H},1}} \tag{D28}$$

$$C^{\mathrm{T}}_{\mathrm{H},1} = (1-\theta)\left(P^{\mathrm{T}}_{\mathrm{H},1}Z^{T}_{\mathrm{H},i}Y^{\mathrm{T}}_{\mathrm{H},1} + P^{\mathrm{N}}_{\mathrm{H},1}Y^{\mathrm{N}}_{\mathrm{H},1} + \sum_{i=1}^{K} P^{\mathrm{T}}_{\mathrm{H},1,i}s_i\right)\frac{1}{P^{\mathrm{T}}_{\mathrm{H},1}}. \tag{D29}$$

Combining the above with the FOC with respect to s_i and assuming that the money supply is such that $P^{\mathrm{T}}_{\mathrm{H},0} = P^{\mathrm{T}}_{\mathrm{F},0}$, we obtain that the AD price is given by

$$q_i = \pi_i \frac{C^{\mathrm{T}}_{\mathrm{H},0}}{C^{\mathrm{T}}_{\mathrm{H},1,i}} = \pi_i \frac{C^{\mathrm{T}}_{\mathrm{F},0}}{C^{\mathrm{T}}_{\mathrm{F},1,i}}, \tag{D30}$$

where π_i is the probability of state i. Combining the budget constraints we obtain

$$P^{\mathrm{T}}_{\mathrm{H},0}C^{\mathrm{T}}_{\mathrm{H},0}+P^{\mathrm{N}}_{\mathrm{H},0}C^{\mathrm{N}}_{\mathrm{H},0}+\sum_{i=1}^{K} q_i\left(C^{\mathrm{T}}_{\mathrm{H},1,i} + \frac{P^{\mathrm{N}}_{\mathrm{H},1,i}}{P^{\mathrm{T}}_{\mathrm{H},1,i}}C^{\mathrm{N}}_{\mathrm{H},1,i}\right) = P^{\mathrm{T}}_{\mathrm{H},0}Y^{\mathrm{T}}_{\mathrm{H},0}+P^{\mathrm{N}}_{\mathrm{H},0}Y^{\mathrm{N}}_{\mathrm{H},0}+\sum_{i=1}^{K} q_i\left(Z^{T}_{\mathrm{H},i}Y^{\mathrm{T}}_{\mathrm{H},1} + \frac{P^{\mathrm{N}}_{\mathrm{H},1,i}}{P^{\mathrm{T}}_{\mathrm{H},1,i}}Y^{\mathrm{N}}_{\mathrm{H},1,i}\right), \tag{D31}$$

from which, after imposing the resource constraint $C^{\mathrm{N}}_{\mathrm{H,t}} = Y^{\mathrm{N}}_{\mathrm{H,t}}$, $t = 0, 1$, we obtain

$$C^{\mathrm{T}}_{\mathrm{H},0} + \frac{\sum_{i=1}^{K} q_i C^{\mathrm{T}}_{\mathrm{H},1,i}}{P^{\mathrm{T}}_{\mathrm{H},1,i}} = Y^{\mathrm{T}}_{\mathrm{H},0} + \frac{\sum_{i=1}^{K} q_i Z^{T}_{\mathrm{H},i} Y^{\mathrm{T}}_{\mathrm{H},1}}{P^{\mathrm{T}}_{\mathrm{H},1,i}}. \tag{D32}$$

No arbitrage implies that $P^{\mathrm{T}}_{\mathrm{H},1,i} = \frac{P^{\mathrm{T}}_{\mathrm{F},1,i}}{E_{1,i}}$. From (D25) we have

$$\frac{C^{\mathrm{T}}_{\mathrm{H},0}}{C^{\mathrm{T}}_{\mathrm{F},0}} = \frac{C^{\mathrm{T}}_{\mathrm{H},1,i}}{C^{\mathrm{T}}_{\mathrm{F},1,i}}, \quad \forall\ i = 1, \ldots, K. \tag{D33}$$

The above relationship, in turn implies that $C^{\mathrm{T}}_{\mathrm{H,t}} = \xi Y^{\mathrm{T}}_{\mathrm{H,t}}$ and $C^{\mathrm{T}}_{\mathrm{F,t}} = \xi Y^{\mathrm{T}}_{\mathrm{F,t}}$, $t = 0, 1$, where

$$\xi = \frac{Y_{\mathrm{H},0} + \sum_i q_i Z^{T}_{\mathrm{H},i} Y^{\mathrm{T}}_{\mathrm{H},1} \frac{1}{P^{\mathrm{T}}_{\mathrm{H},0}}}{Y^{\mathrm{T}}_{\mathrm{H},0} + Y^{\mathrm{T}}_{\mathrm{F},0} + \sum_i q_i (Z^{T}_{\mathrm{H},i} Y^{\mathrm{T}}_{\mathrm{H},1} + Z^{T}_{\mathrm{F},i} Y^{\mathrm{T}}_{\mathrm{F},1}) \frac{1}{P^{\mathrm{T}}_{\mathrm{H},0}}}. \tag{D34}$$

From (D30) we then have

$$q_i = \pi_i \frac{Y^{\mathrm{T}}_{\mathrm{H},0} + Y^{\mathrm{T}}_{\mathrm{F},0}}{Z^{T}_{\mathrm{H},i} Y^{\mathrm{T}}_{\mathrm{H},1} + Z^{T}_{\mathrm{F},i} Y^{\mathrm{T}}_{\mathrm{F},1}}, \quad i = 1, \ldots, K. \tag{D35}$$

The prices q_i and the multiplier ξ fully characterize optimal consumption and the relative prices of T and N goods. Monetary policy can be used to determine the level of prices $P^{\mathrm{T}}_{\mathrm{H},1}$ and $P^{\mathrm{T}}_{\mathrm{F},1}$.

D.2 No solution for the case of zero realized inflation

We consider the case of a log utility households where the H and F countries are perfectly symmetric ex-ante and the composite consumption good is represented by a Cobb-Douglas aggregator, i.e., $\alpha \to 0$ in (3) For $\alpha \to 0$, the price index (15) becomes

$$P_{\mathrm{H,t}} = \kappa \left(P^{\mathrm{T}}_{\mathrm{H,t}}\right)^{1-\theta} \left(P^{\mathrm{N}}_{\mathrm{H,t}}\right)^{\theta}, \quad \kappa \equiv \left(\frac{1}{1-\theta}\right)^{1-\theta} \left(\frac{1}{\theta}\right)^{\theta}. \tag{D36}$$

Without loss of generality we assume that $P_{\mathrm{H},0} = P_{\mathrm{F},0}$. Zero realized inflation implies that $P_{\mathrm{H},1} = P_{\mathrm{F},1} = 1$. From the first order conditions with respect to consumption, and using the resource constraint $C^{\mathrm{N}}_{\mathrm{H},1} = Y^{\mathrm{N}}_{\mathrm{H},1}$, we have

$$\frac{P^{\mathrm{N}}_{\mathrm{H},1}}{P^{\mathrm{T}}_{\mathrm{H},1}} = \frac{\theta}{1-\theta} \frac{C^{\mathrm{T}}_{\mathrm{H},1}}{Y^{\mathrm{N}}_{\mathrm{H},1}} \tag{D37}$$

Using the time-1 budget constraints of the H and F household, impose the assumption of zero realized inflation $P_{\mathrm{H},1} = P_{\mathrm{F},1} = 1$ and the optimality condition (D37) we can express the time-1 consumption of T good in H and F good as follows

$$C^{\mathrm{T}}_{\mathrm{H},1} = Z_{\mathrm{H}} Y^{\mathrm{T}}_{\mathrm{H},1} + \frac{1}{P^{\mathrm{T}}_{\mathrm{H},1}} B^{\mathrm{H}}_{\mathrm{H}} + \frac{1}{P^{\mathrm{T}}_{\mathrm{F},1}} B^{\mathrm{F}}_{\mathrm{H}} = \left(P^{\mathrm{T}}_{\mathrm{H},1}\right)^{1/\theta} Y^{\mathrm{N}}_{\mathrm{H},1} (1-\theta)^{1/\theta} \tag{D38}$$

$$C^{\mathrm{T}}_{\mathrm{F},1} = Z_{\mathrm{F}} Y^{\mathrm{T}}_{\mathrm{F},1} + \frac{1}{P^{\mathrm{T}}_{\mathrm{H},1}} B^{\mathrm{H}}_{\mathrm{F}} + \frac{1}{P^{\mathrm{T}}_{\mathrm{F},1}} B^{\mathrm{F}}_{\mathrm{F}} = \left(P^{\mathrm{T}}_{\mathrm{F},1}\right)^{1/\theta} Y^{\mathrm{N}}_{\mathrm{F},1} (1-\theta)^{1/\theta} \tag{D39}$$

where we used the fact that the nominal exchange rate $E_1 = P^{\mathrm{T}}_{\mathrm{F},1}/P^{\mathrm{T}}_{\mathrm{H},1}$. By symmetry, $B^{\mathrm{H}}_{\mathrm{H}} = -B^{\mathrm{F}}_{\mathrm{H}}$ and by market clearing $B^{\mathrm{H}}_{\mathrm{H}} = -B^{\mathrm{H}}_{\mathrm{F}}$. Hence we define $B \equiv B^{\mathrm{H}}_{\mathrm{H}}$ from which it follows that $B^{\mathrm{F}}_{\mathrm{H}} = -B$, $B^{\mathrm{H}}_{\mathrm{F}} = -B$, and $B^{\mathrm{F}}_{\mathrm{F}} = B$. Imposing $q_{\mathrm{H}} = q_{\mathrm{F}}$ and $E_1 = 1$ in the Euler equations we obtain

$$\mathbb{E}\left[\frac{(P^{\mathrm{T}}_{\mathrm{H},1})^{\frac{1-\theta}{\theta}}}{Y^{\mathrm{N}}_{\mathrm{H},1}}\right] = \mathbb{E}\left[\frac{(P^{\mathrm{T}}_{\mathrm{H},1})^{\frac{1}{\theta}}(P^{\mathrm{T}}_{\mathrm{F},1})^{-1}}{Y^{\mathrm{N}}_{\mathrm{H},1}}\right] \tag{D40}$$

$$\mathbb{E}\left[\frac{(P^{\mathrm{T}}_{\mathrm{F},1})^{\frac{1-\theta}{\theta}}}{Y^{\mathrm{N}}_{\mathrm{F},1}}\right] = \mathbb{E}\left[\frac{(P^{\mathrm{T}}_{\mathrm{F},1})^{\frac{1}{\theta}}(P^{\mathrm{T}}_{\mathrm{H},1})^{-1}}{Y^{\mathrm{N}}_{\mathrm{F},1}}\right] \tag{D41}$$

By using symmetry, imposing, wlog, that $Y^{\mathrm{N}}_{\mathrm{H},1} = Y^{\mathrm{N}}_{\mathrm{F},1} = 1$, and adding up (D38) and (D39), we can characterize the equilibrium as the solution of the following system of two non-linear equation in the two unknown B and $P^{\mathrm{T}}_{\mathrm{H},1}$

$$\left(P^{\mathrm{T}}_{\mathrm{H},1}\right)^{-1/\theta}(1-\theta)^{1/\theta} = Z_{\mathrm{H}} + B\left(\frac{1}{P^{\mathrm{T}}_{\mathrm{H},1}} - \frac{1}{P^{\mathrm{T}}_{\mathrm{F},1}}\right) \tag{D42}$$

$$\mathbb{E}\left[(P^{\mathrm{T}}_{\mathrm{H},1})^{\frac{1-\theta}{\theta}}\right] = \mathbb{E}\left[(P^{\mathrm{T}}_{\mathrm{H},1})^{\frac{1}{\theta}}(P^{\mathrm{T}}_{\mathrm{F},1})^{-1}\right] \tag{D43}$$

where

$$P^{\mathrm{T}}_{\mathrm{F},1} = \Big((1-\theta)^{-1/\theta}(Z_{\mathrm{H}} + Z_{\mathrm{F}}) - (P^{\mathrm{T}}_{\mathrm{H},1})^{-1/\theta}\Big)^{-\theta}. \tag{D44}$$

We claim that equation (D43) can be satisfied only in states for which $Z_{\mathrm{H}} = Z_{\mathrm{F}}$. To see this, note that for states in which $Z_{\mathrm{H}} = Z_{\mathrm{F}}$, by symmetry, $P^{\mathrm{T}}_{\mathrm{H},1} = P^{\mathrm{T}}_{\mathrm{F},1}$. Suppose instead that there are two states on which Z_{H} and Z_{F} take different values, e.g., $a = (Z_{\mathrm{H}} = 1+\epsilon, Z_{\mathrm{F}} = 1-\epsilon)$ or $b = (Z_{\mathrm{H}} = 1-\epsilon, Z_{\mathrm{F}} = 1+\epsilon)$, with equal probability. Let $x \equiv P^{\mathrm{T}}_{\mathrm{H},1}$ and $y \equiv P^{\mathrm{T}}_{\mathrm{F},1}$ in state a, where y

is obtained from (D44). The system (D42)-(D43) can then be written as

$$x^{-1/\theta}(1-\theta)^{1/\theta} = 1+\epsilon+B\left(\frac{1}{x}-\left(2(1-\theta)^{-1/\theta}-x^{-1/\theta}\right)^{\theta}\right) \tag{D45}$$

$$x^{\frac{1-\theta}{\theta}}+y^{\frac{1-\theta}{\theta}} = x^{1/\theta}y^{-1}+y^{1/\theta}x^{-1}, \tag{D46}$$

where $y=\left(2(1-\theta)^{-1/\theta}-x^{-1/\theta}\right)^{-\theta}$. Let $z\equiv\frac{x}{1-\theta}$, then the above system can be written as

$$z^{-1/\theta} = 1+\epsilon+B\left(\frac{1}{z}-\left(2-z^{-1/\theta}\right)^{\theta}\right)\frac{1}{1-\theta} \tag{D47}$$

$$z^{\frac{1-\theta}{\theta}}+\left(2-z^{-1/\theta}\right)^{\theta-1} = z^{1/\theta}+(2-z)^{-1}+z^{-1}(2-z)^{1/\theta}. \tag{D48}$$

Note that $z=1$ is the only solution for equation (D48). To see this, let $q\equiv z^{-1/\theta}(2-z^{-1/\theta})^{-1}$ in (D48), which can then be rewritten as

$$1+q^{1-\theta}=q^{-\theta}+q. \tag{D49}$$

Let us define the function $f(q)=1-q+q^{1-\theta}-q^{-\theta}$. Direct inspection shows that $f(1)=0$ and that a global maximum is achieved at $q=1$. Therefore, $q=1$ is the only solution to (D48). From $q\equiv z^{-1/\theta}(2-z^{-1/\theta})^{-1}$, $q=1$ implies $z=1$. But $z=1$ can be a solution for (D47) only if $\epsilon=0$. Therefore we showed that an equilibrium with zero realized inflation is impossible unless both countries are perfectly identical ex-post, i.e., $\epsilon=0$.

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Table 1: Empirical Results

In panel A we report the relationship between interest rate differentials and either currency appreciation (UIP), or the carry trade profit (CARRY). The right hand side variable is always $log(R_t) - log(R_t^{US})$; the left hand side variable is either foreign currency appreciation against the U.S. dollar $-\Delta E_{t+1} = -log(E_{t+1} - E_t)$ or carry trade profit $R_{t+1}^C = -log(E_{t+1} - E_t) + log(R_t) - log(R_t^{US})$. In panel B we report the relationship between output growth and currency appreciation. The right hand side variable is always $log(GDP_t) - log(GDP_t^{US})$; the left hand side variable is foreign currency appreciation against the U.S. dollar $-log(E_{t+1} - E_t)$. Panel C is identical to Panel B but output growth is replaced by consumption growth. In Panel D we report the relationship between GDP growth and inflation. In the first line, the right and left hand side variables are GDP growth and inflation; in the second line they are GDP growth relative to the U.S., and inflation relative to the U.S. Panels A, B, C, and D report pooled regressions for an unbalanced panel of all countries (left), or summaries from separate regressions for each country (left). In country by country results, we report the average correlation, the average slope, the fraction of positive slopes (negative in panel D), and the fraction of significant slopes. Data for panels A, B, C, and D is annual 1982-2013 for 40 countries; it comes from Datastream and is described in detail in the appendix. In panel E we report the relationship between GDP growth and inflation or TFP growth and inflation for the U.S. only; this data is quarterly 1947-2013 and comes from BEA.

	Pooled			Country by Country			
	Corr	Slope	t-stat	E[Corr]	E[Slope]	%Slope	%t-stat
	Panel A: UIP and the Carry Trade $\mathcal{Y}_{t+1} = a + b\left(\log(R_t) - \log(R_t^{US})\right) + \epsilon_{t+1}$						
UIP: $\mathcal{Y} = -\Delta E$	-0.0636	-0.1792	-1.5209	0.0583	0.2625	55.26	7.89
CARRY: $\mathcal{Y} = R^C$	0.2801	0.8208	6.9675	0.3131	1.2625	92.11	31.58
	Panel B: Balassa-Saumelson Effect $\mathcal{Y}_{t+1} = a + b\left(\Delta\log(GDP_t) - \Delta\log(GDP_t^{US})\right) + \epsilon_{t+1}$						
Nominal Exchange: $\mathcal{Y} = -\Delta E$	0.0771	0.3154	2.2103	0.1455	1.0429	74.29	28.57
Real Exchange: $\mathcal{Y} = -\Delta ER$	0.0826	0.3437	2.1803	0.1255	0.8507	65.71	20.00
	Panel C: Backus-Smith Puzzle $\mathcal{Y}_{t+1} = a + b\left(\Delta\log(C_t) - \Delta\log(C_t^{US})\right) + \epsilon_{t+1}$						
Nominal Exchange: $\mathcal{Y} = -\Delta E$	0.0841	0.3347	1.9410	0.1609	0.9188	77.14	17.14
Real Exchange: $\mathcal{Y} = -\Delta ER$	0.0997	0.4047	2.2465	0.1182	0.7916	71.43	17.14
	Panel D: GDP and Inflation						
$i_{t+1} = a + b\Delta GDP_{t+1} + \epsilon_{t+1}$	-0.0439	-0.1102	-1.1981	-0.0676	-0.2566	46.15	17.95
$i_{t+1} - i_{t+1}^{US} = a + b(\Delta GDP_{t+1} - \Delta GDP_{t+1}^{US}) + \epsilon_{t+1}$	-0.0912	-0.2190	-2.3447	-0.1864	-0.4097	78.95	28.95
	Panel E: GDP and Inflation (USA)						
$i_{t+1} = a + b\Delta GDP_{t+1} + \epsilon_{t+1}$	-0.0670	-0.0440	-0.7609				
$i_{t+1} = a + b\Delta TFP_{t+1} + \epsilon_{t+1}$	-0.2265	-0.0409	-2.9728				

Table 2: Carry Trade Characteristics

The top panel shows coefficients from a univariate regression of a country's interest differential versus the U.S. at t, on one of several characteristics at t or $t+1$. The frequency is annual. The bottom panel shows coefficients from a bivariate regression of realized carry trade profit at $t+1$ one the interest rate differential at t and on one of several characteristics at t or $t+1$. All regressions are for an annual unbalanced panel of 40 countries from 1982 to 2013. The data is from DataStream and is described in the appendix. The characteristics are size of a country's GDP relative to U.S. GDP at t, the beta of a country's GDP growth on world GDP growth from $t-3$ to t, the same beta from $t+1$ to $t+4$, GDP growth realized $t+1$, GDP growth relative to U.S. GDP growth realized at $t+1$, the realized volatility of GDP growth from $t+1$ to $t+4$, inflation realized at $t+1$, inflation relative to U.S. inflation realized at $t+1$, and the realized volatility of inflation from $t+1$ to $t+4$.

x	$Size_t$	$\beta^{\Delta y}_{t-3,t}$	$\beta^{\Delta y}_{t+1,t+4}$	Δy_{t+1}	$\Delta y_{t+1} - \Delta y^{US}_{t+1}$	$\sigma^{\Delta y}_{t+1,t+4}$	i_{t+1}	$i_{t+1} - i^{US}_{t+1}$	$\sigma^{i}_{t+1,t+4}$
	Panel A: $x = \gamma_0 + \gamma_r(r_t - r^{US}_t)$								
γ_r	-5.0184	-0.0134	0.0077	-0.0597	-0.0384	-0.0049	0.5489	0.5662	0.0957
t-stat	-2.3643	-0.5630	0.5837	-1.3983	-0.9450	-0.3999	17.3574	17.7806	3.8687
	Panel B: $r^{CT}_{t,t+1} = \gamma_0 + \gamma_r(r_t - r^{US}_t) + \gamma_x x$								
γ_r	0.8148	0.7753	0.8103	0.8315	0.8391	0.8067	1.0891	0.9492	0.9320
t-stat	6.7682	6.0600	6.6484	7.0057	7.2516	6.4965	7.0297	6.0385	6.6201
γ_x	-0.0025	-0.0000	-0.6895	0.0672	0.3022	-0.8464	-0.4767	-0.2150	-1.2659
t-stat	-0.8366	-0.7820	-2.1414	0.5328	2.1168	-2.0812	-2.3528	-1.0972	-1.8427

Table 3: Results with log utility

This table presents results from models with log utility and Cobb-Douglas consumption aggregation. In the assets row, AD indicates a full set of Arrow-Debreu securities and NB indicates only nominal bonds in each country. In the money row, FE (fixed exchange) indicates monetary policy which sets the prices of tradables to always be one, this is one way (but not the only way) of maintaining a fixed exchange rate regime; ZEI indicates monetary policy which targets zero expected inflation. ΔX indicates $\frac{X_{t+1}}{X_t} - 1$. The nominal exchange rate E indicates the strength of Home currency (Foreign currency per unit of Home), the real exchange rate is defined as the nominal exchange rate multiplied by the price level: $ER = E\frac{P_H}{P_F}$. When the interest rate is higher in F, we define the carry trade return as $R_{CT} = \mathbb{E}\left[R_F \frac{E_0}{E_1} - R_H\right]$ and the reverse carry trade return as $\hat{R}_{CT} = \mathbb{E}\left[R_H \frac{E_1}{E_0} - R_F\right]$; when the interest rate is higher in H, we define the carry trade return as $R_{CT} = \mathbb{E}\left[R_H \frac{E_1}{E_0} - R_F\right]$ and the reverse carry trade return as $\hat{R}_{CT} = \mathbb{E}\left[R_F \frac{E_0}{E_1} - R_H\right]$. All expected returns and volatility are given as percent, ie 0.1 indicates 0.1%.

Model	1	2	3	4	5	6	7	8	9	10
Parameters										
Assets	AD	AD	NB	NB	NB	NB	NB	NB	NB	NB
Money	ZEI	FE	ZEI	FE	ZEI	ZEI	ZEI	ZEI	ZEI	ZEI
$\frac{Y^{\mathrm{T}}_{\mathrm{F},0}}{Y^{\mathrm{T}}_{\mathrm{H},0}}$	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.9
$\frac{Y^{\mathrm{T}}_{\mathrm{F},1}}{Y^{\mathrm{T}}_{\mathrm{H},1}}$	1.0	1.0	1.0	1.0	1.1	1.1	1.1	1.1	1.0	0.9
$\frac{\sigma(Z_{\mathrm{F}})}{\sigma(Z_{\mathrm{H}})}$	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	2.0	1.0
$\frac{\sigma(Z^N)}{\sigma(Z^T)}$	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0
κ	0.0	0.0	0.0	0.0	0.0	0.0	0.001	0.0	0.0	0.0
τ	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.01	0.0	0.0
Risk sharing										
$c(\Delta C_H, \Delta C_F)$	1.0000	1.0000	1.0000	0.0000	1.0000	0.3336	0.3011	0.9730	1.0000	1.0000
$\frac{\sigma(\Delta C_F)}{\sigma(\Delta C_H)}$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0737	1.0156	1.0000	1.0000
Balassa-Saumelson effect										
$\sigma(\Delta ER)$	0.0000	0.0000	0.0000	4.0524	0.0000	4.0524	4.2016	0.4714	0.0000	0.0000
$c(Z_H - Z_F, \Delta ER)$	0.0000	0.0000	0.0000	0.9995	0.0000	0.0000	0.2636	-0.8165	0.0000	0.0000
$\sigma(\Delta E)$	4.0487	0.0000	4.0487	0.0000	4.0443	4.0443	5.1568	3.6530	4.2705	4.0490
$c(Z_H - Z_F, \Delta E)$	0.9950	0.0000	0.9950	0.0000	0.9996	0.9996	0.9994	0.9960	0.9997	0.9995
Carry trade and UIP										
$R_F - R_H$	0.0000	0.0000	0.0000	0.0000	0.0020	0.0020	0.0022	0.6875	-0.0549	-0.0043
$\mathbb{E}\left[\frac{1}{\Delta E} - \Delta E\right]$	0.0000	0.0000	0.0000	0.0000	0.0020	0.0020	0.0052	-1.3390	0.1096	0.0000
$\mathbb{E}\left[R_{CT} - \hat{R}_{CT}\right]$	0.0000	0.0000	0.0000	0.0000	0.0059	0.0059	0.0097	0.0048	0.0002	0.0086
$\sigma(R_{CT})$	4.0445	0.0000	4.0445	0.0000	4.1349	4.1349	5.2720	3.7517	4.2644	4.0448

Table 4: Results with Epstein-Zin-Weil utility

This table presents results from models with Epstein-Zin-Weil utility and Cobb-Douglas consumption aggregation. In the money row, ZEI indicates monetary policy which targets zero expected inflation (M_1 is set at $t = 0$), while ZI indicates monetary policy which targets zero realized inflation (M_1 is set at $t = 1$). For all models in this table, the only assets available are nominal bonds (sometimes with sovereign default risk). If default occurs, it occurs in just one state: the state where both countries experience the worst negative shock (Systematic default). ΔX indicates $\frac{X_{t+1}}{X_t} - 1$. The nominal exchange rate E indicates the strength of Home currency (Foreign currency per unit of Home), the real exchange rate is defined as the nominal exchange rate multiplied by the price level: $ER = E\frac{P_H}{P_F}$. When the interest rate is higher in F, we define the carry trade return as $R_{CT} = \mathbb{E}\left[R_F \frac{E_0}{E_1} - R_H\right]$ and the reverse carry trade return as $\hat{R}_{CT} = \mathbb{E}\left[R_H \frac{E_1}{E_0} - R_F\right]$; when the interest rate is higher in H, we define the carry trade return as $R_{CT} = \mathbb{E}\left[R_H \frac{E_1}{E_0} - R_F\right]$ and the reverse carry trade return as $\hat{R}_{CT} = \mathbb{E}\left[R_F \frac{E_0}{E_1} - R_H\right]$. All expected returns and volatility are given as percent, ie 0.1 indicates 0.1%.

Model	11	12	13	14	15	16	17	18	19	20
					Parameters					
Default	No	No	No	No	Syst	Syst	Syst	Syst	Syst	Syst
Money	ZEI	ZEI	ZEI	ZEI	ZEI	ZEI	ZEI	ZI	ZI	ZI
$\frac{Y^{\mathrm{T}}_{\mathrm{F},0}}{Y^{\mathrm{T}}_{\mathrm{H},0}}$	1.0	1.0	1.0	0.9	1.0	1.0	0.9	1.0	1.0	0.9
$\frac{Y^{\mathrm{T}}_{\mathrm{F},1}}{Y^{\mathrm{T}}_{\mathrm{H},1}}$	1.0	1.1	1.0	0.9	1.0	1.0	0.9	1.0	1.0	0.9
$\frac{\sigma(Z_{\mathrm{F}})}{\sigma(Z_{\mathrm{H}})}$	1.0	1.0	2.0	1.0	1.0	2.0	1.0	1.0	2.0	1.0
					Risk sharing					
$c(\Delta C_H, \Delta C_F)$	1.0000	1.0000	1.0000	1.0000	0.3622	0.4621	0.3516	0.1113	0.2199	0.1010
$\frac{\sigma(\Delta C_F)}{\sigma(\Delta C_H)}$	1.0000	1.0000	1.0000	1.0000	0.6874	1.0818	0.6766	0.8006	1.3338	0.7953
					Balassa-Saumelson effect					
$\sigma(\Delta ER)$	0.0000	0.0000	0.0000	0.0000	2.9098	2.6424	2.9061	3.6437	3.4647	3.6431
$c(Z_H - Z_F, \Delta ER)$	0.0000	0.0000	0.0000	0.0000	0.7721	0.6904	0.7688	0.9578	0.8602	0.9267
$\sigma(\Delta E)$	4.0487	4.0443	4.2705	4.0490	6.5408	6.3597	6.5334	3.6437	3.4647	3.6431
$c(Z_H - Z_F, \Delta E)$	0.9996	0.9996	0.9997	0.9995	0.9613	0.9588	0.9607	0.9578	0.8602	0.9267
					Carry trade and UIP					
$R_F - R_H$	0.0000	0.0274	0.2995	-0.0032	3.2971	3.6017	3.2484	3.4996	3.3633	3.4728
$\mathbb{E}\left[\frac{1}{\Delta E} - \Delta E\right]$	0.0000	0.0002	0.1098	0.0000	0.5842	0.9083	0.5772	0.2841	1.0738	0.2818
$\mathbb{E}\left[R_{CT} - \hat{R}_{CT}\right]$	0.0000	0.0569	0.7089	0.0641	1.3063	2.3284	1.2027	1.4594	1.9939	1.4038
$\sigma(R_{CT})$	4.0469	4.1094	4.2748	4.0465	9.3340	10.1024	9.3161	7.8283	7.9441	7.8167

Table A1: Results with Epstein-Zin-Weil utility and CES consumption

This table presents results from models with Epstein-Zin-Weil utility and CES consumption aggregation, with $\alpha = 0.5$ or $\alpha = -1.0$. For all models in this table, the only assets available are nominal bonds (sometimes with sovereign default risk) and the monetary policy is to set expected inflation to zero. If default occurs, it occurs in just one state: the state where both countries experience the worst negative shock (Systematic default). ΔX indicates $\frac{X_{t+1}}{X_t} - 1$. The nominal exchange rate E indicates the strength of Home currency (Foreign currency per unit of Home), the real exchange rate is defined as the nominal exchange rate multiplied by the price level: $ER = E\frac{P_H}{P_F}$. When the interest rate is higher in F, we define the carry trade return as $R_{CT} = \mathbb{E}\left[R_F\frac{E_0}{E_1} - R_H\right]$ and the reverse carry trade return as $\hat{R}_{CT} = \mathbb{E}\left[R_H\frac{E_1}{E_0} - R_F\right]$; when the interest rate is higher in H, we define the carry trade return as $R_{CT} = \mathbb{E}\left[R_H\frac{E_1}{E_0} - R_F\right]$ and the reverse carry trade return as $\hat{R}_{CT} = \mathbb{E}\left[R_F\frac{E_0}{E_1} - R_H\right]$. All expected returns and volatility are given as percent, ie 0.1 indicates 0.1%.

Model	21	22	23	24	25	26	27	28
				Parameters				
α	0.5	0.5	0.5	0.5	-1.0	-1.0	-1.0	-1.0
Default	No	Syst	Syst	Syst	No	Syst	Syst	Syst
$\frac{Y^{\mathrm{T}}_{\mathrm{F},0}}{Y^{\mathrm{T}}_{\mathrm{H},0}}$	1.0	1.0	1.0	0.9	1.0	1.0	1.0	0.9
$\frac{Y^{\mathrm{T}}_{\mathrm{F},1}}{Y^{\mathrm{T}}_{\mathrm{H},1}}$	1.0	1.0	1.0	0.9	1.0	1.0	1.0	0.9
$\frac{\sigma(Z_{\mathrm{F}})}{\sigma(Z_{\mathrm{H}})}$	2.0	1.0	2.0	1.0	2.0	1.0	2.0	1.0
				Risk sharing				
$c(\Delta C_H, \Delta C_F)$	1.0000	0.2393	0.3548	0.2276	1.0000	0.5618	0.6190	0.5559
$\frac{\sigma(\Delta C_F)}{\sigma(\Delta C_H)}$	0.9995	0.7292	1.1772	0.7203	1.0026	0.6538	0.9748	0.6425
				Balassa-Saumelson effect				
$\sigma(\Delta ER)$	0.00002	1.6401	1.5145	1.6386	0.0120	4.6117	4.1242	4.6099
$c(Z_H - Z_F, \Delta ER)$	0.7252	0.8415	0.7577	0.8394	-0.8705	0.6594	0.5892	0.6551
$\sigma(\Delta E)$	4.2700	5.4945	5.4978	5.4907	4.2640	7.8452	7.4118	7.8361
$c(Z_H - Z_F, \Delta E)$	0.9998	0.9878	0.9859	0.9876	0.9995	0.9007	0.9038	0.8993
				Carry trade and UIP				
$R_F - R_H$	0.2991	3.3865	3.6823	3.3435	0.2997	3.2576	3.5579	3.2048
$\mathbb{E}\left[\frac{1}{\Delta E} - \Delta E\right]$	0.1095	0.4336	0.6535	0.4309	0.1092	0.5619	1.0433	0.5477
$\mathbb{E}\left[R_{CT} - \hat{R}_{CT}\right]$	0.7080	1.4073	2.2910	1.3195	0.7088	1.0771	2.2786	0.9576
$\sigma(R_{CT})$	4.2746	9.3347	10.0429	9.3256	4.2677	9.0896	10.0161	9.0611

Table A2: Alternative assumption about default

This table presents results from models with Epstein-Zin-Weil utility and Cogg-Douglas consumption aggregation. In the money row, ZEI indicates monetary policy which targets zero expected inflation (M_1 is set at $t = 0$), while ZI indicates monetary policy which targets zero realized inflation (M_1 is set at $t = 1$). If default occurs, it occurs in just one state: it is the state in which Home experiences the worst negative shock, Foreign experiences the best positive shock (Strategic). ΔX indicates $\frac{X_{t+1}}{X_t} - 1$. The nominal exchange rate E indicates the strength of Home currency (Foreign currency per unit of Home), the real exchange rate is defined as the nominal exchange rate multiplied by the price level: $ER = E\frac{P_H}{P_F}$. When the interest rate is higher in F, we define the carry trade return as $R_{CT} = \mathbb{E}\left[R_F\frac{E_0}{E_1} - R_H\right]$ and the reverse carry trade return as $\hat{R}_{CT} = \mathbb{E}\left[R_H\frac{E_1}{E_0} - R_F\right]$; when the interest rate is higher in H, we define the carry trade return as $R_{CT} = \mathbb{E}\left[R_H\frac{E_1}{E_0} - R_F\right]$ and the reverse carry trade return as $\hat{R}_{CT} = \mathbb{E}\left[R_F\frac{E_0}{E_1} - R_H\right]$. All expected returns and volatility are given as percent, ie 0.1 indicates 0.1%.

Model	29	30	31	32	33	34
			Parameters			
Default	Strat	Strat	Strat	Strat	Strat	Strat
Money	ZEI	ZEI	ZEI	ZI	ZI	ZI
$\frac{Y^{\mathrm{T}}_{\mathrm{F},0}}{Y^{\mathrm{T}}_{\mathrm{H},0}}$	1.0	1.0	0.0	1.0	1.0	0.0
$\frac{Y^{\mathrm{T}}_{\mathrm{F},1}}{Y^{\mathrm{T}}_{\mathrm{H},1}}$	1.0	1.0	0.9	1.0	1.0	0.9
$\frac{\sigma(Z_{\mathrm{F}})}{\sigma(Z_{\mathrm{H}})}$	1.0	2.0	1.0	1.0	2.0	1.0
			Risk sharing			
$c(\Delta C_H, \Delta C_F)$	0.0803	0.1172	0.0806	0.0126	0.0051	0.0122
$\frac{\sigma(\Delta C_F)}{\sigma(\Delta C_H)}$	0.9908	1.5817	0.9904	1.0033	2.0420	1.0033
			Balassa-Saumelson effect			
$\sigma(\Delta ER)$	3.8570	3.9601	3.8537	3.9940	4.2766	3.9942
$c(Z_H - Z_F, \Delta ER)$	0.9206	0.8751	0.9203	0.9907	0.9986	0.9906
$\sigma(\Delta E)$	7.8960	8.1389	7.8924	3.9940	4.2766	3.9942
$c(Z_H - Z_F, \Delta E)$	0.9741	0.9638	0.9740	0.9907	0.9986	0.9906
			Carry trade and UIP			
$R_F - R_H$	2.5214	3.3865	3.6823	2.5168	2.0400	2.5247
$\mathbb{E}\left[\frac{1}{\Delta E} - \Delta E\right]$	1.2886	1.8383	1.2821	0.8333	2.1723	0.8341
$\mathbb{E}\left[R_{CT} - \hat{R}_{CT}\right]$	0.1514	0.8754	0.1156	0.0409	0.4114	0.0571
$\sigma(R_{CT})$	6.0561	6.0948	6.0535	7.3510	7.2284	7.3563

Table A3: Comparison to existing models

This table presents results from models no trade ($\theta = 1$), and CRRA preferences. Asymmetries are either due to differences in risk aversion, or in output volatility. For all models in this table, the local stochastic discount factor prices the local bond, while the difference between the two countries' stochastic discount factors determines the exchange rate. In the money row, ZEI indicates monetary policy which targets zero expected inflation (M_1 is set at $t = 0$), while ZI indicates monetary policy which targets zero realized inflation (M_1 is set at $t = 1$). The nominal exchange rate E indicates the strength of Home currency (Foreign currency per unit of Home), the real exchange rate is defined as the nominal exchange rate multiplied by the price lesvel: $ER = E\frac{P_H}{P_F}$. When the interest rate is higher in F, we define the carry trade return as $R_{CT} = \mathbb{E}\left[R_F\frac{E_0}{E_1} - R_H\right]$ and the reverse carry trade return as $\hat{R}_{CT} = \mathbb{E}\left[R_H\frac{E_1}{E_0} - R_F\right]$; when the interest rate is higher in H, we define the carry trade return as $R_{CT} = \mathbb{E}\left[R_H\frac{E_1}{E_0} - R_F\right]$ and the reverse carry trade return as $\hat{R}_{CT} = \mathbb{E}\left[R_F\frac{E_0}{E_1} - R_H\right]$. All expected returns and volatility are given as percent, ie 0.1 indicates 0.1%.

Model	35	36	37	38
		Parameters		
Money	ZI	ZI	ZI	ZEI
$\frac{\gamma_H}{\gamma_F}$	1.0	2.5	1.0	1.0
$\frac{\sigma(Z^N_{\mathrm{F}})}{\sigma(Z^N_{\mathrm{H}})}$	1.0	1.0	2.0	2.0
		Risk sharing		
$c(\Delta C_H, \Delta C_F)$	0.0000	0.0000	0.0000	0.0000
$\frac{\sigma(\Delta C_F)}{\sigma(\Delta C_H)}$	1.0000	1.0000	0.5000	0.5000
		Balassa-Saumelson effect		
$\sigma(\Delta ER)$	17.6039	13.2673	18.4790	18.4790
$c(Z_H - Z_F, \Delta ER)$	-0.9953	-0.9182	-0.9964	-0.9964
$\sigma(\Delta E)$	17.6039	13.2673	18.4790	14.7019
$c(Z_H - Z_F, \Delta E)$	-0.9953	-0.9182	-0.9964	-0.9977
		Carry trade and UIP		
$R_F - R_H$	0.0000	0.7157	1.1873	0.8739
$\mathbb{E}\left[\frac{1}{\Delta E} - \Delta E\right]$	0.0000	-0.1827	-0.4105	-0.4874
$\mathbb{E}\left[R_{CT} - \hat{R}_{CT}\right]$	0.0000	1.2560	1.9881	1.2734
$\sigma(R_{CT})$	17.4462	13.2434	18.4052	14.6589