Good Disclosure, Bad Disclosure

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Abstract

We study the implications of public information in a model where market prices convey information to relevant decision makers and the fluctuation of market prices is driven by multiple factors. Disclosure has a positive direct effect of providing new information and an indirect effect of changing the price informativeness. If disclosure is about a variable of which real decision makers are well informed, then the indirect effect is also positive, so that the direct effect is amplified, leading to a positive overall effect on real efficiency. If disclosure is about a variable that real decision markers care to learn much, then the indirect effect is negative and the direct effect is attenuated. Moreover, in markets which aggregate private information effectively, the negative indirect effect can dominate, so that disclosure can harm real efficiency.

Keywords: Disclosure, Price Informativeness, Feedback Effects, Real Efficiency.

JEL Classifications: D61, G14, G30, M41

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1 Introduction

There is abundant public information in reality. Government agencies, such as the Bureau of Economic Analysis, frequently publish economic statistics about the economic conditions. Corporate firms, either mandatorily or voluntarily, make various announcements about their operations to the public. In effect, regulators and academics often view disclosure as one of the most powerful policy tools,[1] and many recent government policies, such as Sarbanes-Oxley, RegFD, and Dodd-Frank, are involved with increased disclosure requirements.[2] One underlying premise for promoting public disclosure is that it can improve real efficiency by providing new information to relevant decision makers. For example, the FASB states: "The benefits of financial reporting information include better investment, credit, and similar resource allocation decisions, which in turn result in more efficient functioning of the capital markets and lower costs of capital for the economy as a whole."[3] In this paper, we propose a framework to examine whether and when this premise holds.

In our model economy, speculators trade one risky asset in the financial market based on their private and public information. The cash flow of the risky asset is in turn determined by the relevant decision makers who have access to a production technology. Real decision makers use the pubic information and more importantly, the asset price formed in the financial market, to guide their investments.[4] It is their actions that establish the effect that public information has on the real economy. Our mechanism works through the interactions between the exogenous public information and the endogenous price information, both of which affect the forecast quality of real decision makers.

The production technology determining the asset cash flow is involved with two indepen-

[1] Greenstone et al. (2006, p. 399) state: "Since the passage of the Securities Act of 1933 and the Securities Exchange Act of 1934, the federal government has actively regulated U. S. equity markets. The centerpiece of these efforts is the mandated disclosure of financial information."

[2] For instance, Sarbanes-Oxley Act was passed as an "act to protect investors by improving the accuracy and reliability of corporate disclosures made pursuant to the securities laws, and for other purposes."

[3] FASB Financial Accounting Series, NO.1260-001 July 6 2006, "Conceptual Framework for Financial Reporting: Objective of Financial Reporting and Qualitative Characteristics of Decision-Useful Financial Reporting Information," Section QC53, p. 35.

[4] Recent empirical studies document that asset prices indeed contain valuable information relevant to real decisions and that decision makers also appear to learn information from prices (e.g., Luo, 2005; Chen, Goldstein, and Jiang, 2007; Edmans, Goldstein, and Jiang, 2012; Foucault and Frésard, 2014). See Bond, Edmans, and Goldstein (2012) for a recent survey.

dent productivity factors – factor $\tilde{a}$ and factor $\tilde{f}$ – such as a macro factor and a firm-specific factor as in Veldkamp and Wolfers (2007), or a permanent factor and a transitory factor as in Liu, Wang, and Zha (2013). The dichotomy between factors $\tilde{a}$ and $\tilde{f}$ is defined based on information asymmetry between real decision makers and speculators: Real decision makers know better about one factor ($\tilde{a}$) than the other ($\tilde{f}$), and hence they are more keen to learn about the factor ($\tilde{f}$) of which they are relatively uninformed.[5] Speculators have noisy private information about both factors $\tilde{a}$ and $\tilde{f}$. The equilibrium asset price will therefore convey information about $\tilde{a}$ and $\tilde{f}$ through their trading, which is useful for real decision makers to make investments.

There are many empirical settings that naturally fit our model. One example is a financially-constrained firm that needs fund from capital providers (such as banks) to make investments. Real decision makers in this example are capital providers and speculators are hedge funds or mutual funds who trade the firm's shares. The asset is the firm's stock, whose cash flow depends on the aggregate industry productivity (factor $\tilde{a}$) and firm-specific productivity (factor $\tilde{f}$). It seems reasonable that capital providers might have better information about the industry than the specific firm. Public disclosure can either come from the government or from the firm. If it is from the government, then it is more likely to convey information about the aggregate economy (factor $\tilde{a}$), and if it is from the firm, then it is more likely firm specific (factor $\tilde{f}$).

Another example is an index on a particular industry, whose cash flow depends on the technology of the composing companies (factor $\tilde{a}$) and on the demand for the companies' products (factor $\tilde{f}$). Real decision makers are the managers of those companies included in the index and speculators are still mutual and/or hedge funds. It is arguable that company managers know better about their production technology than the market demand. More generally, it is natural that in the modern economy cash flows depend on multiple sources of uncertainty – such as developments of different countries for a multinational firm that operates in several countries, or success of different lines of business for a conglomerate that operates across different business lines – and that due to limited information processing

[5] So, the notation "$\tilde{a}$" refers to information *already* known by real decision makers, while the notation "$\tilde{f}$" means information that they need to *forecast*.

capacity, relevant decision makers have comparative advantages in processing different types of information.

The key insight emerging from our analysis is that whether public disclosure is "good" in terms of promoting real efficiency depends on whether it makes speculators trade on the right private information such that the overall forecast quality of real decision makers gets improved.[6] In general, public disclosure has two effects on real decision makers' forecast quality. The direct effect is to provide new information, and it is always positive, because real decision makers always become better informed after observing some information, however noisy. The second effect is an indirect effect: Public disclosure affects speculators' trading, which in turn affects the price informativeness about factor $\tilde{f}$ that real decision makers care to learn. This indirect effect can be positive or negative. When it is positive, the direct effect is amplified, leading to a strong overall effect on real efficiency. When the indirect effect is negative, the direct effect is attenuated or even overturned, making the overall effect modest or even negative.

When public disclosure is primarily a signal about factor $\tilde{a}$ of which real decision makers have already had a good knowledge, the indirect effect is positive. To see this, note that the speculators' trading is determined by the public signal about $\tilde{a}$ and their private signals about $\tilde{a}$ and $\tilde{f}$, because the future asset cash flows are affected by both factors $\tilde{a}$ and $\tilde{f}$ and these signals convey information about these factors. When the public signal mainly provides information about $\tilde{a}$ and when it becomes more accurate, speculators will put a higher weight on this public information and less on their private information in forecasting factor $\tilde{a}$. As a result, their private information based trading will reflect more of their private information about factor $\tilde{f}$ that real decision makers care to learn. So, the indirect effect of disclosing information about factor $\tilde{a}$ is positive, which will amplify the direct effect and generate a strong overall effect on real efficiency.

When public disclosure is mainly a signal about factor $\tilde{f}$ that real decision makers wish to learn, the indirect effect becomes negative. This is because a more accurate public signal

[6] Guttman, Kremer, and Skrzypacz (2014) construct a dynamic voluntary disclosure model and argue that the equilibrium reaction to voluntarily disclosed information depends not only on what is disclosed but also when. Their conclusion complements our message that real efficiency implications of disclosure depends on what type of information is disclosed.

about $\tilde{f}$ will render speculators to put a lower weight on their own private information about $\tilde{f}$, which reduces the informational content about $\tilde{f}$ in the price. This negative indirect effect of disclosure will attenuate the positive direct effect, thereby making the overall effect modest. In addition, the indirect effect can be so strong that it even dominates the direct effect, leading to a negative overall effect of disclosure on real efficiency. This is true when the market aggregates speculators' private information very effectively. So, although it appears attractive to disclose information about some variable that relevant decision makers care to learn the most, the overall impact of such disclosure can be counter-productive. Paradoxically, this concern is particularly severe in those markets which function well in terms of aggregating private information.

We also show that disclosing different types of information often has contrasting implications for speculators' welfare, which is defined as their ex-ante trading profits from the financial market. Specifically, disclosing information about $\tilde{a}$ either monotonically decreases speculators' welfare or it first increases and then decreases speculators' welfare. By contrast, disclosing information about $\tilde{f}$ has exactly the opposite effects. That is, it either monotonically increases speculators' welfare or it first decreases and then increases speculators' welfare. In addition, we argue that the two factor structure, which is defined in terms of information asymmetry, is crucial in delivering our results on real efficiency. Finally, we show that our results are qualitatively robust to many variations and extensions of our baseline model, including economies with exogenous cash flows, economies with two separate public signals, economies with real decision makers receiving noisy signals about both factors, and economies with speculators observing prices.

1.1 Related Literature

Our paper is broadly related to two literatures: the accounting literature on the real effects of disclosing accounting information, and the finance literature on the real effects of financial markets (which is labeled "feedback effects" in this literature). Kanodia (2007) and Bond, Edmans, and Goldstein (2012) provide a review of these two literatures, respectively. Our paper introduces the interactions among public disclosure, trading by informed speculators, and learning by real decision makers, and provides new real efficiency implications of disclosure based on these interactions. Below, we discuss in detail several studies that also explore adverse efficiency implications of public information through various mechanisms and are thus most closely related to our paper.

Amador and Weill (2010) construct a monetary model and show that releasing public information about monetary and/or productivity shocks can reduce welfare through reducing the informational efficiency of the good price system, which relates to the indirect effect of disclosure in our financial model. However, our analysis highlights the different implications of different types of disclosure – i.e., disclosing information about factor $\tilde{a}$ increases the price informativeness to real decision makers and therefore amplifies the positive direct effect of disclosure, while disclosing information about $\tilde{f}$ does the opposite. In contrast, in Amador and Weill (2010), the indirect effect of disclosing information is always negative. Moreover, the two dimensional uncertainty ($\tilde{a}$ and $\tilde{f}$) is crucial in driving the results in our model, because as we show in Section 5.2.1, the indirect effect of disclosure vanishes in an economy with one-dimensional uncertainty. This is not the case in Amador and Weill (2010).

Bond and Goldstein (2014) analyze a trading model where a government learns from asset prices and intervenes in the asset's cash flows. Their analysis also suggests that disclosure can either reduce or raise price informativeness, depending on the type of information being disclosed. However, both their mechanism and results are different from ours. First, their mechanism works through a risk-return trade off faced by risk-averse traders. On the one hand, disclosure affects the importance of traders' own information in predicting the government's private information that affects asset cash flows. On the other hand, it affects the uncertainty faced by traders through influencing their information sets. In our model, all agents are risk neutral and so our results are not driven by any kind of risk-return trade-off. Second, in Bond and Goldstein (2014), disclosing information about a variable that the government wishes to learn – i.e., the counterpart of public information about $\tilde{f}$ in our model – makes the price completely useless and its overall effect is to harm the government's intervention decision. By contrast, in our model, although such public disclosure can be detrimental to price informativeness, the positive direct effect of disclosure providing new information may still dominate. So, the overall effect of disclosing information that real decision makers wish to learn can be positive or negative, depending on the effectiveness of information aggregation in financial markets.

A few other recent papers also present models in which disclosure harms price efficiency or investment efficiency, albeit through different channels. In Edmans, Heinle, and Huang (2013), only hard information (such as earnings) can be disclosed, and disclosing hard information distorts manager's investment incentives by changing the relative weight between hard and soft information. In Han, Tang, and Yang (2014), disclosure attracts noise trading which reduces price informativeness and harms managers' learning quality. In Gao and Liang (2013), disclosure crowds out private information production, reduces price informativeness, and so harms managers' learning and investments. In Banerjee, Davis, and Gondhi (2014), public information can lower price efficiency because facing more fundamental information traders choose to acquire non-fundamental information exclusively. Our results highlight the importance of disclosing different types of fundamental information, and they are not driven by anything related to manager incentives, noise trading attraction, or private information production.

Some early papers, such as Hirshleifer (1971) and Hakannson, Kunkel, and Ohlson (1982), have pointed out that public information destroys risk sharing opportunities and thereby impairs the social welfare. In our paper, all agents are risk neutral, and our focus is on real investment efficiency instead of risk sharing. Gao (2010) and Cheynel (2013) have recently considered the welfare implications of corporate disclosure, but both papers do not allow real decision makers to learn from the asset price, which is the key in driving our results. A recent line of research rely on payoff externality and hence coordination motives across economic agents to show that public information release may harm welfare, for example, Morris and Shin (2002), Angeletos and Pavan (2004), Angeletos and Pavan (2007), Goldstein, Ozdenoren, and Yuan (2011), Vives (2013), and Colombo, Femminis, and Pavan (2014). In contrast with this line of work, our results are not driven by any kind of payoff externality – speculators do not care about what other speculators do in our setting. Instead, our results are generated from the indirect effect of disclosure on real decision makers' forecast quality through changing the price informativeness.

2 The Model

2.1 Environment

We consider a variation of the models studied by Goldstein, Ozdenoren, and Yuan (2013) and Goldstein and Yang (2013). There are three dates, $t = 0, 1, 2$. At date 0, a continuum $[0, 1]$ of "speculators" trade one risky asset based on their diverse private signals and a common public signal about factors related to the asset's future cash flows. The equilibrium asset price aggregates speculators' private information through their trading. Assuming a continuum of speculators endowed with diverse signals captures the idea that the financial market aggregates value-relevant information that is inherently dispersed among market participants. At date 1, a continuum $[0, 1]$ of "real decision makers," who see the public signal and the equilibrium asset price, make inference from the price to guide their actions, which in turn determine the cash flow of the risky asset that was traded in the previous period. As will become clear later, in our baseline model all real decision makers are identical, and so we can simply replace the continuum with a single real decision maker without affecting our analysis. We keep a continuum of real decision makers to admit a macro interpretation of our model, as we explain shortly. At date 2, the cash flow is realized, and all agents get paid and consume.

Our model admits two interpretations – a micro interpretation and a macro interpretation. At a micro level, the risky asset can be interpreted as a stock of a financially-constrained firm which needs capital from outside capital providers to make investments. Real decision makers in this case are capital providers, such as banks, equity investors, and venture capital firms. At a macro level, the risky asset can be interpreted as an index on a particular industry or on the aggregate stock market, and real decision makers are the managers of those companies included in the index. Under both interpretations, speculators can be thought of as mutual and/or hedge funds who have private information about the future value of the asset. We are agnostic to the two interpretations, but for simplicity, we have adopted the first one and call real decision makers as capital providers in presenting the model.

2.2 Investment

The firm in our economy has access to the following production technology:

$$q(k_j) = \tilde{A}\tilde{F}k_j,$$

where k_j is the amount of capital that the firm raises from capital provider j at date 1, $q(k_j)$ is the date-2 output that is generated by the investment k_j, and $\tilde{A} \geq 0$ and $\tilde{F} \geq 0$ are two productivity factors. Let $\tilde{a}$ and $\tilde{f}$ denote the natural logs of $\tilde{A}$ and $\tilde{F}$, i.e., $\tilde{a} \equiv \log \tilde{A}$ and $\tilde{f} \equiv \log \tilde{F}$. We assume that $\tilde{a}$ and $\tilde{f}$ are normally distributed as follows:[7]

$$\tilde{a} \sim N\left(0, \tau_a^{-1}\right) \text{ and } \tilde{f} \sim N\left(0, \tau_f^{-1}\right),$$

where $\tilde{a}$ and $\tilde{f}$ are mutually independent, and $\tau_a > 0$ and $\tau_f > 0$, respectively, are their precision (inverse of variance).

Factors $\tilde{A}$ and $\tilde{F}$ represent two dimensions of uncertainty that affect the cash flow of the traded firm. For example, one dimension can be a factor related to aggregate economy, and the other one can be firm-specific (e.g., Greenwood, MacDonald, and Zhang, 1996, p. 97; Veldkamp and Wolfers, 2007). Also, $\tilde{a}$ can be thought of as the permanent component in the total productivity and the factor $\tilde{f}$ is the transitory component, as in Liu, Wang, and Zha (2013). More generally, cash flows depend on the demand for firms' products and the technology they develop, and on the success of firms' operations in traditional lines of business and in new speculative lines of business, and thus the feature of multiple dimensions of uncertainty follows directly. Several papers in the finance literature have also specified that the value of the traded security is affected by more than one fundamental; e.g., Froot, Scharfstein, and Stein (1992), Goldman (2005), Kondor (2012), and Goldstein and Yang (2014), among others. In Section 5.2.1, we show that this feature of two-dimensional uncertainty is essential in generating our results.

The two-dimensional uncertainty serves to capture the idea that real decision makers might be more informed about some particular aspect of the firm, which is a natural feature of the modern economy to the extent that decision makers often have comparative advantages in processing different types of information. So, in our setting, it is the nature of information

[7] The assumption that $\tilde{a}$ and $\tilde{f}$ have a mean of 0 is without loss of generality. Assuming non-zero means is equivalent to renormalizing the cost parameter c introduced shortly.

asymmetry that characterizes the dichotomy between factors $\tilde{a}$ and $\tilde{f}$. Specifically, we assume that capital providers have better information about factor $\tilde{a}$ than factor $\tilde{f}$. In the baseline model analyzed in this section, we consider an extreme version of this asymmetric knowledge by assuming that capital providers know perfectly factor $\tilde{a}$ but nothing about factor $\tilde{f}$ beyond the prior distribution. Capital providers are essentially identical in the baseline model, because they have access to the same investment technology and information set. In Section 6.2, we extend our model to equip each capital provider with differential noisy signals about the two factors, and show that our results go through as long as the signal quality about one factor is sufficiently different from the signal quality about the other factor.

At date $t = 1$, each capital provider j chooses the level of capital (and investment) k_j. As in in Goldstein, Ozdenoren, and Yuan (2013), providing capital incurs a private cost according to the following functional form:

$$c\left(k_j\right) = \frac{1}{2} c k_j^2,$$

where the constant $c > 0$ controls the size of the cost relative to the output $q\left(k_j\right)$. The cost can be the monetary cost of raising the capital or the private effort incurred in monitoring the investment.

We also follow Goldstein, Ozdenoren, and Yuan (2013) and assume that each capital provider j captures proportion $\beta \in (0,1)$ of the full output $q\left(k_j\right)$ by providing k_j, and thus his payoff from the investment is $\beta q\left(k_j\right)$. Capital provider j chooses k_j to maximize the payoff $\beta q\left(k_j\right)$ he captures from the firm minus his cost $c\left(k_j\right)$ of raising capital, conditional on his information set. All capital providers have the same information set, denoted by $\mathcal{I}_R$ (with the subscript "R" indicating "*real* decision makers"), which consists of factor $\tilde{a}$, a public signal $\tilde{\omega}$, and the asset price $\tilde{P}$ (we will elaborate on $\tilde{\omega}$ and $\tilde{P}$ in the subsequent subsections shortly). Therefore, capital provider j chooses k_j to solve

$$\max_{k_j} E\left[\beta \tilde{A} \tilde{F} k_j - \frac{1}{2} c k_j^2 \middle| \mathcal{I}_R\right].$$

The solution to this maximization problem is:

$$k_j^* = \frac{\beta \tilde{A} E\left(\tilde{F} \middle| \mathcal{I}_R\right)}{c}. \tag{1}$$

2.3 Private and Public Information

Each speculator i observes two private noisy signals about $\tilde{a}$ and $\tilde{f}$, respectively:

$$\tilde{x}_i = \tilde{a} + \tilde{\varepsilon}_{x,i} \text{ and } \tilde{y}_i = \tilde{f} + \tilde{\varepsilon}_{y,i},$$

where $\tilde{\varepsilon}_{x,i} \sim N(0, \tau_x^{-1})$ (with $\tau_x > 0$), $\tilde{\varepsilon}_{y,i} \sim N\left(0, \tau_y^{-1}\right)$ (with $\tau_y > 0$), and they are mutually independent and independent of $\left\{\tilde{a}, \tilde{f}\right\}$. The market price $\tilde{P}$ will aggregate speculators' private signals $\{\tilde{x}_i, \tilde{y}_i\}$ through their trading in the financial market, and hence $\tilde{P}$ will contain information about $\tilde{a}$ and $\tilde{f}$, which is useful for capital providers to make real investment decisions.

All agents, including speculators and capital providers, observe a public signal $\tilde{\omega}$, which communicates a linear combination of the two productivity factors with some error as follows:

$$\tilde{\omega} = \mu_a \tilde{a} + \mu_f \tilde{f} + \tilde{\varepsilon}_\omega, \tag{2}$$

where μ_a and μ_f are two constants, and $\tilde{\varepsilon}_\omega \sim N(0, \tau_\omega^{-1})$ (with $\tau_\omega \geq 0$) is independent of $\left\{\tilde{a}, \tilde{f}\right\}$. The constants μ_a and μ_f determine what information the signal $\tilde{\omega}$ conveys. If $\frac{Var(\mu_a \tilde{a})}{Var\left(\mu_f \tilde{f}\right)}$ is large, then the variations in $\mu_a \tilde{a} + \mu_f \tilde{f}$ are largely driven by factor $\tilde{a}$, and so $\tilde{\omega}$ is mainly a signal about $\tilde{a}$. If $\frac{Var(\mu_a \tilde{a})}{Var\left(\mu_f \tilde{f}\right)}$ is small, then by the same reason, $\tilde{\omega}$ is primarily a signal about factor $\tilde{f}$.

Parameter τ_ω controls the precision of the public signal $\tilde{\omega}$. The public signal can represent public announcements made by a firm, and the firm can influence the disclosure precision by, for example, affecting the degree of access they may provide to outside analysts. Alternatively, the public signal can be some economics statistics published by government agencies. For example, a current policy debate is on how much information governments should release about the outcomes of bank stress tests, and this amount of public information corresponds to the size of τ_ω in our model. Similarly, public information $\tilde{\omega}$ can represent forward guidance about the expected path of future interest rates that is provided by many central banks. In our analysis, we will follow the literature (e.g., Morris and Shin, 2002; Amador and Weill, 2010) and conduct comparative statics exercises with respect to parameter τ_ω to examine the real efficiency effect of public information.

Note that in equation (2) we have specified that public information generally conveys information about both factors, as long as both μ_a and μ_f are not zero. This specification captures the idea that due to technical reasons the providers of public information may have difficulty in separating the two factors when collecting information. Alternatively, equation (2) can be viewed as a parsimonious modeling device to capture two types of disclosure. That is, if we set $\mu_a = 0$ and $\mu_f \neq 0$, then $\tilde{\omega}$ is a public signal about factor $\tilde{f}$, and similarly, if we set $\mu_f = 0$ and $\mu_a \neq 0$, then $\tilde{\omega}$ is a public signal about factor $\tilde{a}$. So by deriving equilibrium outcomes under the more general specification of (2), our analysis can naturally cover both degenerate cases. Finally, in Section 6.1, we also analyze a variation of our model by specifying two separate public signals – each of which conveys information about one factor, respectively – and show that our results hold.

2.4 Trading and Price Formation

At date $t = 0$, speculators submit market orders as in Kyle (1985) to trade the risky asset in the financial market. They can buy or sell up to a unit of the risky asset, and thus speculator i's demand for the asset is $d(i) \in [-1, 1]$. This position limit can be justified by borrowing/short-sales constraints faced by speculators. As argued by Goldstein, Ozdenoren, and Yuan (2013), the specific size of this position limit is not crucial, and what is crucial is that speculators cannot take unlimited positions. Speculators are risk neutral, and therefore they choose their positions to maximize the expected trading profits conditional on their information sets $\mathcal{I}_i = \{\tilde{x}_i, \tilde{y}_i, \tilde{\omega}\}$.

The traded asset is a claim on the portion of the aggregate output that remains after removing capital providers' share.[8] Specifically, the aggregate output is

$$\tilde{Q} \equiv \int_0^1 q\left(k_j^*\right) dj = \tilde{A}\tilde{F} \int_0^1 k_j^* dj = \tilde{A}\tilde{F}K^*,$$

where $K^* \equiv \int_0^1 k_j^* dj$ is the aggregate investment in equilibrium. So, after removing the β

[8] As explained in Goldstein, Ozdenoren, and Yuan (2013), for technical reasons, we do not assume that the asset is a claim on the net return from the investment. Specifically, under the current assumption, the expected cash flow of the security for a speculator is expressed as one exponential term (given our lognormal distributions), which is crucial for our ability to find a linear solution. If the cash flow from the traded security was proportional to $\int_0^1 \left[\tilde{A}\tilde{F}k_j - c(k_j)\right] dj$, we would have two exponential terms, which would render the steps for finding a linear solution impossible. See Goldstein, Ozdenoren, and Yuan (2013) for more discussions on the nature of the traded asset. We believe that our results do not rely on the assumption about the asset's cash flow. See Section 5.2.2 for more discussions.

fraction of $\tilde{Q}$, the remaining $(1-\beta)$ fraction constitutes the cash flow on the risky asset:

$$\tilde{V} \equiv (1-\beta)\,\tilde{Q} = (1-\beta)\,\tilde{A}\tilde{F}K^{*}.$$

A speculator's profit from buying one unit of the asset is given by $\tilde{V}-\tilde{P}$, and similarly, his profit from shorting one unit is $\tilde{P}-\tilde{V}$. So, speculator i chooses demand $d(i)$ to solve:

$$\max_{d(i)\in[-1,1]} d(i)\,E\left(\tilde{V}-\tilde{P}\middle|\mathcal{I}_i\right). \tag{3}$$

Since each speculator is atomistic and is risk neutral, he will optimally choose to either buy up to the one-unit position limit, or short up to the one-unit position limit. We denote the aggregate demand from speculators as $D \equiv \int_0^1 d(i)\,di$, which is the fraction of speculators who buy the asset minus the fraction of those who short the asset.

As in Goldstein, Ozdenoren, and Yuan (2013), to prevent a price that fully reveals the factor $\tilde{f}$ to capital providers, we assume the following noisy supply curve provided by (unmodeled) liquidity traders:

$$L\left(\tilde{\xi},\tilde{P}\right) \equiv 1-2\Phi\left(\tilde{\xi}-\lambda\log\tilde{P}\right), \tag{4}$$

where $\tilde{\xi} \sim N\left(0,\tau_{\xi}^{-1}\right)$ (with $\tau_{\xi}>0$) is an exogenous demand shock independent of other shocks in the economy. Function $\Phi(\cdot)$ denotes the cumulative standard normal distribution function, which is increasing. Thus, the supply curve $L\left(\tilde{\xi},\tilde{P}\right)$ is strictly increasing in the price $\tilde{P}$ and decreasing in the demand shock $\tilde{\xi}$. The parameter $\lambda>0$ captures the elasticity of the supply curve with respect to the price,[9] and it can be interpreted as the liquidity of the market in the sense of price impact: When λ is high, the supply is very elastic with respect to the price and thus, the demand from informed speculators can be easily absorbed by noise trading without moving the price very much. In our baseline model, we need to assume $\lambda>0$ to determine the price, and in Section 6.3 we will relax this assumption by allowing speculators to observe prices and show that our main results are robust.

The basic features assumed in (4) are that the supply is increasing in price $\tilde{P}$ and also has a noisy component $\tilde{\xi}$, both of which are standard in the literature. It is also common in the literature to assume particular functional forms to obtain tractability. The specific functional form assumed here is close to that in Angeletos and Werning (2006), Hellwig, Mukherji, and

[9] See Banerjee, Davis, and Gondhi (2014) for a discussion on the economic relevance of price-dependent noise trading.

Tsyvinski (2006), Dasgupta (2007), and Albagli, Hellwig, and Tsyvinski (2012, 2014). As usual, the noisy supply component $\tilde{\xi}$ represents trading coming from (unmodeled) agents who trade for liquidity or hedging needs (e.g., Dow and Rahi, 2003). We do not endogenize the actions of these traders in our setting, because doing so here breaks the loglinear structure of the model, which makes impossible an analytical characterization.

The market clears by equating the aggregate demand D from speculators with the noisy supply $L\left(\tilde{\xi}, \tilde{P}\right)$:

$$D = L\left(\tilde{\xi}, \tilde{P}\right). \tag{5}$$

This market clearing condition will determine the equilibrium price $\tilde{P}$.

After completing the description of our model, we see that our setup differs from the one analyzed by Goldstein, Ozdenoren, and Yuan (2013) in two important ways. First, the investment productivity has two sources of uncertainty ($\tilde{a}$ and $\tilde{f}$) in our model, while it has only one dimensional uncertainty ($\tilde{f}$) in Goldstein, Ozdenoren, and Yuan's (2013) economy. As we mentioned before, this two factor structure forms the basis of our idea of different types of disclosure. Second, the information structure is quite different. Goldstein, Ozdenoren, and Yuan (2013) specify that the noise in speculators' private information contains both an idiosyncratic term and a common term, and by doing so, they can study how traders coordinate on trading on rumors represented by the common noise term. In contrast, we shut down the common noise term in speculators' information and introduce a public signal about productivity factors to explore the real efficiency implications of different types of information disclosure.

2.5 Equilibrium Definition

The exogenous parameters in our model are: τ_a, the prior precision of factor $\tilde{a}$; τ_f, the prior precision of factor $\tilde{f}$; τ_ω, the precision of public information; τ_x, the precision of speculators' private signals about factor $\tilde{a}$; τ_y, the precision of speculators' private signals about factor $\tilde{f}$; τ_ξ, the precision of noise trading; λ, the elasticity of noisy supply; μ_a, the loading on factor $\tilde{a}$ in the public signal; μ_f, the loading on factor $\tilde{f}$ in the public signal; β, the fraction of the output captured by capital providers; and c, the parameter controlling the relative size of investment costs. So, the tuple

$$\mathcal{E} = \left\{\tau_a, \tau_f, \tau_\omega, \tau_x, \tau_y, \tau_\xi, \lambda, \mu_a, \mu_f, \beta, c\right\}$$

defines an economy. For a given economy, we consider an equilibrium which involves the optimal decisions of each player (capital providers and speculators) and the statistical behavior of aggregate variables (K, D, and $\tilde{P}$).

Each player's optimal decisions will be a function of their information sets. For capital providers, their optimal investments k_j^* given by (1) will be a function of their information set $\mathcal{I}_R = \left\{\tilde{a}, \tilde{P}, \tilde{\omega}\right\}$. That is, $k_j^* = k\left(\tilde{a}, \tilde{P}, \tilde{\omega}\right)$. Since they are identical, the aggregate investment function $K\left(\tilde{a}, \tilde{P}, \tilde{\omega}\right)$ will be the same as the individual investment function: $K^* = K\left(\tilde{a}, \tilde{P}, \tilde{\omega}\right) = k\left(\tilde{a}, \tilde{P}, \tilde{\omega}\right)$. Speculators' optimal trading strategies d_i^* will be a function of their information set $\mathcal{I}_i = \{\tilde{x}_i, \tilde{y}_i, \tilde{\omega}\}$. That is, $d_i^* = d\left(\tilde{x}_i, \tilde{y}_i, \tilde{\omega}\right)$. In aggregate, the noise terms $\tilde{\varepsilon}_{x,i}$ and $\tilde{\varepsilon}_{y,i}$ in their signals $\tilde{x}_i$ and $\tilde{y}_i$ will wash out, and so the aggregate demand D for the risky asset is a function of $\tilde{a}$, $\tilde{f}$ and $\tilde{\omega}$:

$$D = D\left(\tilde{a}, \tilde{f}, \tilde{\omega}\right) = \int_0^1 d\left(\tilde{x}_i, \tilde{y}_i, \tilde{\omega}\right) di = E\left[d\left(\tilde{x}_i, \tilde{y}_i, \tilde{\omega}\right) \middle| \tilde{a}, \tilde{f}, \tilde{\omega}\right], \tag{6}$$

where the expectation is taken over $(\tilde{\varepsilon}_x, \tilde{\varepsilon}_y)$ in (6).

The market clearing condition (5) will therefore determine the price $\tilde{P}$ as a function of productivity factors $\left\{\tilde{a}, \tilde{f}\right\}$, the public signal $\tilde{\omega}$, and the noise trading shock $\tilde{\xi}$: $\tilde{P} = P\left(\tilde{a}, \tilde{f}, \tilde{\omega}, \tilde{\xi}\right)$. An equilibrium is defined formally as follows.

Definition 1 *An equilibrium consists of a price function,* $P\left(\tilde{a}, \tilde{f}, \tilde{\omega}, \tilde{\xi}\right) : \mathbb{R}^4 \to \mathbb{R}$*, an investment policy for capital providers,* $k\left(\tilde{a}, \tilde{P}, \tilde{\omega}\right) : \mathbb{R}^3 \to \mathbb{R}$*, a trading strategy of speculators,* $d\left(\tilde{x}_i, \tilde{y}_i, \tilde{\omega}\right) : \mathbb{R}^3 \to [-1, 1]$*, and the corresponding aggregate demand function for the asset* $D\left(\tilde{a}, \tilde{f}, \tilde{\omega}\right)$*, such that:*

(a) for capital provider j*,* $k\left(\tilde{a}, \tilde{P}, \tilde{\omega}\right) = \frac{\beta \tilde{A} E\left(\tilde{F} \middle| \tilde{a}, \tilde{P}, \tilde{\omega}\right)}{c}$*;*

(b) for speculator i*,* $d\left(\tilde{x}_i, \tilde{y}_i, \tilde{\omega}\right)$ *solves (3);*

(c) the market clearing condition (5) is satisfied; and

(d) the aggregate asset demand is given by (6).

3 Equilibrium Characterization

In this section, we illustrate the steps for constructing an equilibrium. It turns out that the equilibrium characterization boils down to a fixed point problem of solving the weight that speculators put on the signal $\tilde{y}_i$ about factor $\tilde{f}$ when they trade the risky asset. Specifically, we first conjecture a trading strategy of speculators and use the market clearing condition to determine the asset price and hence the information that capital providers can learn from the price. We then update capital providers' beliefs and characterize their investment rule, which in turn determines the cash flow of the traded asset. Finally, given the implied price and cash flow in the first two steps, we solve for speculators' optimal trading strategy, which compares with the initial conjectured trading strategy to complete the fixed point loop.

3.1 The Information that Capital Providers Learn from the Price

We conjecture that speculators buy the asset if and only if a linear combination of their (private and public) signals is above a cutoff g, and sell it otherwise. That is, speculators buy the asset whenever $\tilde{x}_i + \phi_y \tilde{y}_i + \phi_\omega \tilde{\omega} > g$, where ϕ_y, ϕ_ω, and g are endogenous parameters that will be determined in equilibrium. Note that $\tilde{x}_i + \phi_y \tilde{y}_i + \phi_\omega \tilde{\omega} > g$ is equivalent to $\frac{\tilde{\varepsilon}_{x,i}+\phi_y\tilde{\varepsilon}_{y,i}}{\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}} > \frac{g-\left(\tilde{a}+\phi_y\tilde{f}\right)-\phi_\omega\tilde{\omega}}{\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}$, and hence speculators' aggregate purchase can be characterized by $1-\Phi\left(\frac{g-\left(\tilde{a}+\phi_y\tilde{f}\right)-\phi_\omega\tilde{\omega}}{\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}\right)$. Similarly, their aggregate selling is $\Phi\left(\frac{g-\left(\tilde{a}+\phi_y\tilde{f}\right)-\phi_\omega\tilde{\omega}}{\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}\right)$. Thus, the net holding from speculators is:

$$D\left(\tilde{a},\tilde{f},\tilde{\omega}\right) = 1-2\Phi\left(\frac{g-\left(\tilde{a}+\phi_y\tilde{f}\right)-\phi_\omega\tilde{\omega}}{\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}\right). \tag{7}$$

The market clearing condition (5) together with equations (4) and (7) indicates that

$$1-2\Phi\left(\frac{g-\left(\tilde{a}+\phi_y\tilde{f}\right)-\phi_\omega\tilde{\omega}}{\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}\right) = 1-2\Phi\left(\tilde{\xi}-\lambda\log\tilde{P}\right),$$

which implies that the equilibrium price is given by:

$$\tilde{P} = \exp\left(\frac{\tilde{a}+\phi_y\tilde{f}+\phi_\omega\tilde{\omega}-g}{\lambda\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}+\frac{\tilde{\xi}}{\lambda}\right). \tag{8}$$

Recall that capital providers have the information set $\left\{\tilde{a},\tilde{P},\tilde{\omega}\right\}$, and so they know the realizations of $\tilde{a}$ and $\tilde{\omega}$. As a result, the price $\tilde{P}$ is equivalent to the following signal in predicting factor $\tilde{f}$:

$$\tilde{s}_p \equiv \frac{\lambda\sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}} \log \tilde{P} - \tilde{a} - \phi_\omega \tilde{\omega} + g}{\phi_y} = \tilde{f} + \tilde{\varepsilon}_p, \tag{9}$$

where

$$\tilde{\varepsilon}_p \equiv \frac{\sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}}{\phi_y} \tilde{\xi}, \tag{10}$$

which has a precision of

$$\tau_p \equiv \frac{1}{Var\left(\tilde{\varepsilon}_p\right)} = \frac{\phi_y^2 \tau_x \tau_y \tau_\xi}{\tau_y + \phi_y^2 \tau_x}. \tag{11}$$

The endogenous precision τ_p captures how much information capital providers can learn from the price. It is crucially related to real efficiency through guiding capital providers' investment decisions. The public information precision τ_ω affects τ_p only through its effect on ϕ_y. Specifically, if speculators trade more aggressively on their information about $\tilde{f}$ (i.e., when ϕ_y becomes higher), the price is more informative about factor $\tilde{f}$, all other things being equal. As a result, capital providers can glean more information from the price.

3.2 The Optimal Investment Policy of Capital Providers

Capital providers have information set $\mathcal{I}_R = \left\{\tilde{a}, \tilde{P}, \tilde{\omega}\right\}$. We have already characterized how they use the price $\tilde{P}$ to form a signal $\tilde{s}_p$ in predicting the factor $\tilde{f}$. Regarding the public signal $\tilde{\omega}$ in (2), they can use their knowledge of $\tilde{a}$ to transform $\tilde{\omega}$ into the following signal in predicting $\tilde{f}$:

$$\tilde{s}_\omega \equiv \frac{\tilde{\omega} - \mu_a \tilde{a}}{\mu_f} = \tilde{f} + \mu_f^{-1} \tilde{\varepsilon}_\omega,$$

which has a precision of $\mu_f^2 \tau_\omega$. That is, capital providers' information set is equivalent to:

$$\mathcal{I}_R = \left\{\tilde{a}, \tilde{s}_p, \tilde{s}_\omega\right\},$$

where the two signals $\tilde{s}_p$ and $\tilde{s}_\omega$ are useful for predicting $\tilde{f}$.

By Bayes' rule and equation (1), we compute capital providers' optimal investment as follows:

$$k_j^* = \exp\left[\left(\log\frac{\beta}{c} + \frac{1}{2}\frac{1}{\tau_f + \mu_f^2\tau_\omega + \tau_p}\right) + \tilde{a} + \frac{\mu_f^2\tau_\omega}{\tau_f + \mu_f^2\tau_\omega + \tau_p}\tilde{s}_\omega + \frac{\tau_p}{\tau_f + \mu_f^2\tau_\omega + \tau_p}\tilde{s}_p\right]. \tag{12}$$

3.3 The Optimal Trading Strategy of Speculators

Using the expression of $\tilde{P}$ in (8), the cash flow expression $\tilde{V} = (1-\beta)\tilde{A}\tilde{F}K^*$, and the investment rule in (12), we can compute the expected price and cash flow conditional on speculator i's information set $\{\tilde{x}_i, \tilde{y}_i, \tilde{\omega}\}$ as follows:

$$E\left(\tilde{P}\middle|\tilde{x}_i, \tilde{y}_i, \tilde{\omega}\right) = \exp\left(b_0^p + b_x^p\tilde{x}_i + b_y^p\tilde{y}_i + b_\omega^p\tilde{\omega}\right), \tag{13}$$

$$E\left(\tilde{V}\middle|\tilde{x}_i, \tilde{y}_i, \tilde{\omega}\right) = \exp\left(b_0^v + b_x^v\tilde{x}_i + b_y^v\tilde{y}_i + b_\omega^v\tilde{\omega}\right), \tag{14}$$

where the coefficients b's are given in the appendix.

Speculator i will choose to buy the asset if and only if $E\left(\tilde{V}\middle|\tilde{x}_i, \tilde{y}_i, \tilde{\omega}\right) > E\left(\tilde{P}\middle|\tilde{x}_i, \tilde{y}_i, \tilde{\omega}\right)$. Thus, we have

$$\begin{aligned} E\left(\tilde{V}\middle|\tilde{x}_i, \tilde{y}_i, \tilde{\omega}\right) &> E\left(\tilde{P}\middle|\tilde{x}_i, \tilde{y}_i, \tilde{\omega}\right) \Leftrightarrow \\ \left(b_x^v - b_x^p\right)\tilde{x}_i + \left(b_y^v - b_y^p\right)\tilde{y}_i + \left(b_\omega^v - b_\omega^p\right)\tilde{\omega} &> \left(b_0^p - b_0^v\right). \end{aligned}$$

Recall that we conjecture speculators' trading strategy as buying the asset whenever $\tilde{x}_i + \phi_y\tilde{y}_i + \phi_\omega\tilde{\omega} > g$. So, to be consistent with our initial conjecture, we require that in equilibrium,

$$\phi_y = \frac{b_y^v - b_y^p}{b_x^v - b_x^p}, \tag{15}$$

$$\phi_\omega = \frac{b_\omega^v - b_\omega^p}{b_x^v - b_x^p}, \tag{16}$$

provided that $\left(b_x^v - b_x^p\right) > 0$. The right-hand-side $\frac{b_y^v - b_y^p}{b_x^v - b_x^p}$ of (15) depends only on ϕ_y (through the term of ϕ_y in b_y^p and b_x^p and the term of τ_p in b_y^v and b_x^v). Therefore, we use (15) to compute ϕ_y, and then plug this solved ϕ_y into equation (16) to compute ϕ_ω.

To summarize, we have the following characterization proposition.

Proposition 1 *The equilibrium is characterized by the weight ϕ_y that speculators put on the private signal $\tilde{y}_i$ about factor $\tilde{f}$, and ϕ_y is determined by condition (15), as long as $b_x^v > b_x^p$.*

4 Good Disclosure, Bad Disclosure

4.1 Measurement of Efficiency

In this section we study the normative implications of disclosure. Our analysis will focus on real efficiency – we say that disclosure is "good" if it improves real efficiency in equilibrium, and that disclosure is "bad" if it harms real efficiency. Ideally, we should conduct a full welfare analysis by examining how public disclosure affects the expected utility levels of all agents in the economy. However, as we mentioned before, in order to solve the model in closed form, we have assumed that noise traders trade the risky asset according to equation (4) to meet their unmodeled liquidity/hedging needs, and it is challenging to endogenize noise trading fully. This precludes a welfare analysis on these (unmodeled) noise traders. So, we focus our analysis on real efficiency implications and relegate to Section 5.1 a discussion on the welfare implications for speculators.

Nonetheless, we believe that our analysis on real efficiency is of its own interest for two reasons. First, as in standard economics textbook analysis, any welfare maximizing allocation must also be production efficient, and so our real efficiency analysis can be viewed as a first step toward a full welfare analysis. Second, to the extent that noise traders' unmodeled liquidity/hedging needs are not affected by public disclosure, real efficiency is a reasonable measure for the aggregate welfare.[10] Specifically, there are three categories of agents in the economy – speculators, noise traders, and capital providers. The welfare of speculators can be measured by their ex-ante expected trading profit evaluated in equilibrium. As for noise traders, we can follow the microstructure literature and use expected trading revenue as a proxy for their welfare to capture the idea that they are better off if they can realize their hedging or liquidity needs at a lower expected opportunity cost (e.g., Chowdhry and Nanda, 1991; Subrahmanyam, 1991; Leland, 1992). Under this interpretation, what speculators gain in trading is exactly equal to what noise traders lose, and so the total welfare of these two groups of traders is not affected by public information. The welfare of capital providers is $E\left[\beta q\left(k_j^*\right) - c\left(k_j^*\right)\right]$, and we can show that in equilibrium, it is equal to real efficiency scaled by a constant $\frac{\beta}{2-\beta}$. As a result, the total welfare across all agents in the economy is

[10] We thank Alessandro Pavan for suggesting this interpretation.

proportional to real efficiency.

We follow Goldstein, Ozdenoren, and Yuan (2013) and measure real efficiency by the ex-ante expected net benefit of investment evaluated in equilibrium. We can compute

$$
\begin{aligned}
RE &\equiv E\left[\tilde{A}\tilde{F}K^{*} - \int c\left(k_j^{*}\right) dj\right] \\
&= \frac{\beta}{c}\left(1 - \frac{\beta}{2}\right)\exp\left[\frac{2}{\tau_a} + \frac{2}{\tau_f} - Var\left(\tilde{f}\middle|\, \tilde{s}_\omega, \tilde{s}_p\right)\right], \qquad (17)
\end{aligned}
$$

where

$$
Var\left(\tilde{f}\middle|\, \tilde{s}_\omega, \tilde{s}_p\right) = \frac{1}{\tau_f + \mu_f^2 \tau_\omega + \tau_p}. \qquad (18)
$$

In our model, disclosure affects real efficiency through changing capital providers' information set. The more information that capital providers have, the better are their investment decisions, and so is real efficiency in equilibrium. This fact is clearly captured by expression (17): Recall that real decision makers know factor $\tilde{a}$ and so they only need to forecast the other factor $\tilde{f}$, and so the term $Var\left(\tilde{f}\middle|\, \tilde{s}_\omega, \tilde{s}_p\right)$ captures the efficiency loss relative to a full information allocation.

Equation (18) demonstrates that public information has two effects on capital providers' information set (and hence real efficiency). The first is a direct effect of providing new information, which is related to the term $\mu_f^2 \tau_\omega$ in (18). The second effect is an endogenous indirect effect: Public information affects the trading of speculators (more specifically, the loading ϕ_y on private information about $\tilde{f}$), and hence the price informativeness about factor $\tilde{f}$, which in turn affects the amount of information that capital providers can learn from the price (i.e., the term τ_p in (18)). Formally, by (17) and (18), we have

$$
\underbrace{\frac{\partial RE}{\partial \tau_\omega}}_{\text{total effect}} \propto \frac{\partial\left(\tau_f + \mu_f^2 \tau_\omega + \tau_p\right)}{\partial \tau_\omega} = \underbrace{\mu_f^2}_{\text{direct effect}} + \underbrace{\frac{\partial \tau_p}{\partial \tau_\omega}}_{\text{indirect effect}}, \qquad (19)
$$

where,

$$
\frac{\partial \tau_p}{\partial \tau_\omega} = \frac{2\tau_p \tau_y}{\phi_y\left(\tau_y + \phi_y^2 \tau_x\right)} \frac{\partial \phi_y}{\partial \tau_\omega}, \qquad (20)
$$

which follows from applying the chain rule to equation (11).

Clearly, in equation (19), the direct effect of disclosure is always positive, as long as $\mu_f > 0$ (i.e., as long as public disclosure has some information about factor $\tilde{f}$). However, the indirect effect $\frac{\partial \tau_p}{\partial \tau_\omega}$ can be positive or negative. If it is positive, then the direct effect of disclosure is amplified, leading to a very strong overall effect. By contrast, if the indirect effect is negative, then it attenuates or even overturns the direct effect, making the overall effect of disclosure modest or even counterproductive. In the following two subsections, we show that the sign of the indirect effect depends on the type of information being disclosed. Specifically, releasing public information about factor $\tilde{a}$ generates a positive indirect effect, while releasing public information about factor $\tilde{f}$ generates a negative indirect effect, which can dominate the positive direct effect so that disclosing information about $\tilde{f}$ can harm real efficiency, provided that the market aggregates speculators' private information effectively.

4.2 The Effect of Disclosure about Factor $\tilde{a}$

As we discussed before, when $\frac{Var(\mu_a \tilde{a})}{Var\left(\mu_f \tilde{f}\right)}$ is very large, the public signal $\tilde{\omega}$ in (2) is primarily a signal about factor $\tilde{a}$. For simplicity, we assume $\mu_f = 0$ and normalize μ_a as 1, so that $\tilde{\omega}$ degenerates to

$$\tilde{\omega} = \tilde{a} + \tilde{\varepsilon}_\omega.$$

In this case, since capital providers know factor $\tilde{a}$ perfectly and the public signal $\tilde{\omega}$ does not provide information about the other factor $\tilde{f}$, the direct effect of public disclosure vanishes (i.e., $\frac{\partial \mu_f^2 \tau_\omega}{\partial \tau_\omega} = \mu_f^2 = 0$ in (19)). Therefore, the only channel for public disclosure to affect real efficiency is through its indirect effect on the endogenous precision of the information that capital providers can learn from the asset price (i.e., $\frac{\partial RE}{\partial \tau_\omega} \propto \frac{\partial \tau_p}{\partial \tau_\omega}$ in (19)).

We can compute that the terms b's in equation (15) are as follows:

$$\begin{aligned} b_x^p &= \frac{\frac{\tau_x}{\tau_a+\tau_x+\tau_\omega}}{\lambda\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}, b_y^p = \frac{\frac{\phi_y\tau_y}{\tau_f+\tau_y}}{\lambda\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}, \\ b_x^v &= \frac{2\tau_x}{\tau_a+\tau_x+\tau_\omega}, \text{ and } b_y^v = \left(1+\frac{\tau_p}{\tau_f+\tau_p}\right)\frac{\tau_y}{\tau_f+\tau_y}. \end{aligned}$$

Thus, as the supply elasticity λ becomes large, b_x^p and b_y^p approach to 0, and thus the condition of (15) determining ϕ_y degenerates to:

$$\phi_y \approx \frac{b_y^v}{b_x^v} = \frac{\left(1+\frac{\tau_p}{\tau_f+\tau_p}\right)\frac{\tau_y}{\tau_f+\tau_y}}{\frac{2\tau_x}{\tau_a+\tau_x+\tau_\omega}}. \tag{21}$$

Note that as $\lambda \to \infty$, we always have $b_x^v > b_x^p$, so that the condition in the Proposition 1 is always satisfied.

Equation (21) makes sense. Recall that speculators buy the asset whenever $\tilde{x}_i + \phi_y \tilde{y}_i + \phi_\omega \tilde{\omega} > g$, and so the sensitivity ϕ_y captures how aggressively they trade on their private information $\tilde{y}_i$ relative to their private information $\tilde{x}_i$. When λ is large, the price is highly elastic, so that the price is not much affected by speculators' trade (see equation (8)). As a consequence, speculators will trade mainly based on their expectations $E\left(\tilde{V}\middle|\tilde{x}_i, \tilde{y}_i, \tilde{\omega}\right)$ in (14) about the asset's future cash flow. It is therefore reasonable that ϕ_y is mainly determined by the ratio $\frac{b_y^v}{b_x^v}$ of the two coefficients that speculators put on signals $\tilde{y}_i$ and $\tilde{x}_i$, respectively, in forecasting the future value of the asset.

Using the expression of τ_p in (11) and applying the implicit function theorem to (21), we can show

$$\frac{\partial \phi_y}{\partial \tau_\omega} = \frac{\frac{\phi_y}{\tau_a + \tau_x + \tau_\omega}}{1 - \frac{2\tau_p \tau_y \tau_f}{\left(\tau_f + \tau_p\right)\left(\tau_f + 2\tau_p\right)\left(\tau_y + \phi_y^2 \tau_x\right)}} > 0. \tag{22}$$

That is, public disclosure about factor $\tilde{a}$ causes speculators to trade more aggressively on their private information about the other factor $\tilde{f}$.

The intuition for this result lies in equation (21). Note that in the expression of $E\left(\tilde{V}\middle|\tilde{x}_i, \tilde{y}_i, \tilde{\omega}\right)$ in (14), the private signal $\tilde{x}_i$ and the public signal $\tilde{\omega}$ are useful for predicting $\tilde{a}$, while the private signal $\tilde{y}_i$ is useful for predicting $\tilde{f}$. When τ_ω increases, so that the public signal becomes a more informative signal about $\tilde{a}$, speculators put a higher weight b_ω^v on the signal $\tilde{\omega}$ and a lower weight b_x^v on the signal $\tilde{x}_i$ in predicting $\tilde{a}$. Other things being equal, this increases ϕ_y since $\phi_y = \frac{b_y^v}{b_x^v}$ in (21). However, this is not the end of the story, because there is a further "multiplier effect" (as captured by the denominator in (22)): The increased ϕ_y improves τ_p in (11), and so capital providers glean more information on $\tilde{f}$ from the price, making the asset cash flow $\tilde{V}$ more responsive to $\tilde{f}$ through capital providers' investments, which, in turn, causes speculators to rely more on their private private $\tilde{y}_i$ – which is a signal about $\tilde{f}$ – in making their forecasts, which increases b_y^v in (14). So, ϕ_y continues to increase through (21). This amplification chain continues on and on till it converges to a much higher level of ϕ_y.

Since $\frac{\partial \phi_y}{\partial \tau_\omega} > 0$ by (22), we have $\frac{\partial \tau_p}{\partial \tau_\omega} > 0$ as well in (20). That is, capital providers learn more information about factor $\tilde{f}$ from the price. Thus, real efficiency improves with disclosure, since when the public signal communicates information only about factor $\tilde{a}$, it affects real efficiency only through its effect on the price informativeness about factor $\tilde{f}$.

To summarize, we have the following proposition.

Proposition 2 *Suppose that the supply elasticity λ is high and that public information $\tilde{\omega}$ is a signal about factor $\tilde{a}$ (i.e., $\mu_a = 1$ and $\mu_f = 0$).*
(a) There exists a unique equilibrium that is characterized by the relative weight $\phi_y > 0$ on private signals $\tilde{y}_i$ about the other factor $\tilde{f}$ in speculators' trading strategy, which is determined by equation (21).
(b) Increasing the precision τ_ω of public disclosure
(i) increases the relative weight $\phi_y > 0$ on private signals $\tilde{y}_i$ (i.e., $\frac{\partial \phi_y}{\partial \tau_\omega} > 0$);
(ii) increases the precision τ_p that capital providers learn from the price regarding factor $\tilde{f}$ (i.e., $\frac{\partial \tau_p}{\partial \tau_\omega} > 0$), and so the indirect effect is positive; and
(iii) increases real efficiency RE (i.e., $\frac{\partial RE}{\partial \tau_\omega} > 0$).

Panels (a1) and (a2) of Figure 1 graphically illustrate Proposition 2. Here, we simply set the precision of all random variables to be 1; that is, $\tau_a = \tau_f = \tau_x = \tau_y = \tau_\xi = 1$. The patterns are quite robust with respect to changes in these precision parameter values. We set the supply elasticity λ at 1. We choose $\mu_a = 0.8$ and $\mu_f = 0.2$, so that public disclosure $\tilde{\omega}$ is mainly a signal about factor $\tilde{a}$. Note that under this parameter configuration, $\tilde{\omega}$ also provides information about factor $\tilde{f}$, and the direct effect of disclosure is positive (i.e., $\mu_f^2 > 0$ in equation (19)).[11] In Panel (a1), we plot the weight ϕ_y that speculators put on the private signal $\tilde{y}_i$ against the precision τ_ω of the public signal. In Panel (a2), we plot three variables against τ_ω: (i) $\mu_f^2 \tau_\omega$, the direct effect of public disclosure on capital providers' forecast precision by providing new information about $\tilde{f}$; (ii) τ_p, the indirect effect of public disclosure on capital providers' forecast precision by affecting the informational content in the price; and (iii) $\mu_f^2 \tau_\omega + \tau_p$, which is a proxy for real efficiency, since by (17) and (18), real efficiency RE is a monotonic transformation of $\mu_f^2 \tau_\omega + \tau_p$.

[Insert Figure 1 Here]

[11] In Section 6.2, we extend the model to allow capital providers to see noisy signals about both factors, and in that extension, even a public signal of the form $\tilde{a} + \tilde{\varepsilon}_\omega$, which corresponds to $\mu_f = 0$ and $\mu_a = 1$ in (2), has a positive direct effect on real efficiency.

We see that in Panel (a1), as Proposition 2 predicts, increasing the precision τ_ω of the public signal increases the weight ϕ_y that speculators put on the private signal $\tilde{y}_i$ about factor $\tilde{f}$. This in turn means that in Panel (a2), the precision τ_p of information that capital providers learn from the price increases, because τ_p increases with ϕ_y by (11). Clearly, the direct effect $\mu_f^2\tau_\omega$ of disclosure increases with τ_ω as well in Panel (a2). As a result, the indirect effect of disclosure amplifies the direct effect, and the overall effect of disclosure is to increase real efficiency $\mu_f^2\tau_\omega + \tau_p$.

4.3 The Effect of Disclosure about Factor $\tilde{f}$

When $\frac{Var(\mu_a\tilde{a})}{Var(\mu_f\tilde{f})}$ is very small, public disclosure $\tilde{\omega}$ in (2) is primarily a signal about $\tilde{f}$. For simplicity, we assume $\mu_a = 0$ and normalize μ_f as 1, so that $\tilde{\omega}$ degenerates to

$$\tilde{\omega} = \tilde{f} + \tilde{\varepsilon}_\omega.$$

In this case, both effects of public disclosure are active in equation (19). First, the public signal $\tilde{\omega}$ directly benefits capital providers by providing information that they wish to learn. Second, it affects the trading behavior of speculators and hence the informational content in the price, thereby indirectly affecting capital providers' forecast.

We can compute that the terms b's in (15) are as follows:

$$\begin{aligned} b_x^p &= \frac{\frac{\tau_x}{\tau_a+\tau_x}}{\lambda\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}, b_y^p = \frac{\frac{\phi_y\tau_y}{\tau_f+\tau_y+\tau_\omega}}{\lambda\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}, \\ b_x^v &= \frac{2\tau_x}{\tau_a+\tau_x}, \text{ and } b_y^v = \left(1+\frac{\tau_p}{\tau_f+\tau_\omega+\tau_p}\right)\frac{\tau_y}{\tau_f+\tau_y+\tau_\omega}. \end{aligned}$$

Thus, as the supply elasticity λ becomes large, b_x^p and b_y^p approach to 0, and therefore the condition of (15) determining ϕ_y degenerates to

$$\phi_y \approx \frac{b_y^v}{b_x^v} = \frac{\frac{\tau_y(\tau_f+2\tau_p+\tau_\omega)}{(\tau_f+\tau_p+\tau_\omega)(\tau_f+\tau_y+\tau_\omega)}}{\frac{2\tau_x}{\tau_a+\tau_x}}. \tag{23}$$

Applying the implicit function theorem to the above equation, we can show:

$$\frac{\partial\phi_y}{\partial\tau_\omega} = -\frac{\phi_y\left(\frac{\tau_p}{(\tau_f+2\tau_p+\tau_\omega)(\tau_f+\tau_p+\tau_\omega)}+\frac{1}{\tau_f+\tau_y+\tau_\omega}\right)}{1-\frac{2\tau_p\tau_y(\tau_f+\tau_\omega)}{(\tau_f+\tau_p+\tau_\omega)(\tau_f+2\tau_p+\tau_\omega)(\tau_y+\phi_y^2\tau_x)}} < 0. \tag{24}$$

That is, public disclosure about factor $\tilde{f}$ causes speculators to trade less aggressively on their own private information about $\tilde{f}$.

Similar to the result for disclosure about $\tilde{a}$, the intuition here also lies in equation (23) that determines the equilibrium value of ϕ_y. By (23), ϕ_y is determined by the ratio $\frac{b_y^v}{b_x^v}$. When the public information $\tilde{\omega}$ is mainly a signal about $\tilde{f}$, increasing its precision τ_ω will decrease the weight b_y^v on speculators' own private signal $\tilde{y}_i$ in predicting $\tilde{f}$. This tends to decrease ϕ_y through $\phi_y = \frac{b_y^v}{b_x^v}$ in (23). In addition, there is a multiplier effect, as captured by the denominator in (24): The decreased ϕ_y reduces τ_p, which causes capital providers to glean less information about $\tilde{f}$, making the asset value $\tilde{V}$ less sensitive to $\tilde{f}$. So, in anticipation of this outcome, speculators trade more aggressively on their private information $\tilde{x}_i$ about the other factor $\tilde{a}$ and less aggressively on information $\tilde{y}_i$ about $\tilde{f}$ (that is, b_x^v becomes higher and b_y^v becomes lower), which further reduces ϕ_y through (23), until the equilibrium value ϕ_y reaches a much lower level.

The effect on τ_p of disclosure is still given by equation (20). Since $\frac{\partial \phi_y}{\partial \tau_\omega} < 0$ by (24), we have $\frac{\partial \tau_p}{\partial \tau_\omega} < 0$ as well. That is, capital providers learn less information from the price as a result of more disclosure about factor $\tilde{f}$, so that the indirect effect of disclosing information about factor $\tilde{f}$ is negative in (19). This negative indirect effect attenuates the positive direct effect, causing the overall effect of disclosure on real efficiency to be modest or ambiguous. This result is somehow paradoxical: Recall that factor $\tilde{f}$ is the variable that capital providers care to learn, and it is also disclosing information about this same variable that gives rise to a counterproductive indirect effect through affecting the price informativeness. In other words, the strength of the real efficiency effect of disclosing information about a variable that capital providers care to learn is inherently limited.

Finally, it is possible that the overall real efficiency effect of disclosing information about factor $\tilde{f}$ is negative, depending on whether the negative indirect effect dominates the positive direct effect. We can show that this possibility occurs at low levels of disclosure when the precision τ_ξ of noise trading is large. The intuition is as follows. First, when the disclosure level is sufficiently high, the positive direct effect always dominates. For instance, if $\tau_\omega \to \infty$, capital providers would know factor $\tilde{f}$, and the allocation would be the first best, which achieves the maximum real efficiency. So, only when the disclosure level τ_ω is low, is it possible for the negative indirect effect to dominate. Second, suppose τ_ω is low and the precision τ_ξ of noise trading is large. When there is little noise trading, the market aggregates speculators' private information effectively. Note that the indirect effect operates through price informativeness, and so it is particularly strong. By contrast, when τ_ξ is small, the market has a lot of noise trading, and its information aggregation role is limited, thereby weakening disclosure's indirect effect via price informativeness.

Again, this result is kind of paradoxical in the sense that market efficiency can be misaligned with real efficiency. Regulators and academics often view promoting market efficiency as one desirable goal. For instance, O'Hara (1997, p. 270) states: "How well and how quickly a market aggregates and impounds information into the price must surely be a fundamental goal of market design." However, our analysis suggests that the negative indirect effect of disclosure on real efficiency is particularly strong exactly in those markets which effectively aggregate traders' private information.[12]

To summarize, we have the following proposition.

Proposition 3 *Suppose that the supply elasticity λ is high and that public information $\tilde{\omega}$ is a signal about factor $\tilde{f}$ (i.e., $\mu_a = 0$ and $\mu_f = 1$).*

(a) There exists a unique equilibrium that is characterized by the relative weight $\phi_y > 0$ on private signals $\tilde{y}_i$ about factor $\tilde{f}$ in speculators' trading strategy, which is determined by equation (23).

(b) Increasing the precision τ_ω of public disclosure

(i) decreases the relative weight ϕ_y on private signals $\tilde{y}_i$ (i.e., $\frac{\partial \phi_y}{\partial \tau_\omega} < 0$);

(ii) decreases the precision τ_p that capital providers learn from the price regarding the factor $\tilde{f}$ (i.e., $\frac{\partial \tau_p}{\partial \tau_\omega} < 0$), and so the indirect effect is negative;

(iii) increases real efficiency RE at high levels of disclosure (i.e., $\frac{\partial RE}{\partial \tau_\omega} > 0$ for large τ_ω); and

(iv) decreases (increases) real efficiency RE at low levels of disclosure if the precision τ_ξ of noise trading is large (small) (i.e., for small τ_ω, $\frac{\partial RE}{\partial \tau_\omega} < 0$ if τ_ξ is large, and $\frac{\partial RE}{\partial \tau_\omega} > 0$ if τ_ξ is small).

Panels (b1)-(c2) of Figure 1 graphically illustrate Proposition 3. As in Panels (a1)-(a2), we set $\tau_a = \tau_f = \tau_x = \tau_y = \lambda = 1$ in all these four panels. But here we set $\mu_a = 0.2$ and

[12] See Goldstein and Yang (2013) for a comprehensive study on the connect and disconnect between market efficiency and real efficiency.

$\mu_f = 0.8$, so that public information $\tilde{\omega}$ is primarily a signal about factor $\tilde{f}$. In Panels (b1)-(b2), we choose $\tau_\xi = 0.5$, so that the level $\frac{1}{\tau_\xi}$ of noise trading is relatively high and the market does not aggregate private information that much. In Panels (c1)-(c2), we choose $\tau_\xi = 10$, and thus the level of noise trading is low and the market aggregates private information effectively.

In Panels (b1) and (c1), we see that, consistent with Proposition 3, the relative weight ϕ_y that speculators put on private information $\tilde{y}_i$ decreases with the precision τ_ω of public disclosure. This translates to a decreasing τ_p as a function of τ_ω in Panels (b2) and (c2), which corresponds to the negative indirect effect of disclosure. As a result, the direct effect of increasing τ_ω, as manifested by the increasing $\mu_f^2\tau_\omega$, is attenuated by the negative indirect effect in both panels.

In addition, in Panel (b2) where τ_ξ is relatively small, the direct effect dominates and real efficiency $(\mu_f^2\tau_\omega+\tau_p)$ increases with τ_ω. By contrast, in Panel (c2) where τ_ξ is relatively large, the indirect effect dominates for low levels of disclosure while the direct effect dominates for high levels of disclosure, so that there exists a U-shape between real efficiency and disclosure. This U-shape pattern shares some similarity to the main result of Morris and Shin (2002, p. 1529), and so it has similar implications for optimal disclosure. That is, there may be technical constraints in achieving precision beyond some upper bound, so that a social planner may be restricted to choosing a disclosure level τ_ω from some given interval $[0, \bar{\tau}_\omega]$. Then, we will see a "bang-bang" solution to the choice of optimal τ_ω^* in which the socially optimal real efficiency entails either providing no public information at all (i.e., setting $\tau_\omega^* = 0$), or providing the maximum feasible amount of public information (i.e., setting $\tau_\omega^* = \bar{\tau}_\omega$).

5 Discussions

In this section, we first show that disclosing different types of information often has opposite implications for speculators' welfare, and then we discuss the roles of the assumptions in driving our results.

5.1 Implications for Speculators' Welfare

In Section 4.1, we have argued that real efficiency is a good proxy for the aggregate welfare. Still, we may be interested in welfare distributions, and therefore we examine the implications for speculators' welfare in this subsection. Note that the welfare of noise traders would be the negative of speculators' welfare plus fixed unmodeled hedging benefits, and thus combining the analysis with our previous analysis on real efficiency (which is proportional to capital providers' welfare) will give us a complete picture of welfare distributions.

We can measure speculators' welfare as their ex-ante expected trading gains in equilibrium:

$$\begin{aligned} W_S &\equiv E\left[d^*(i)\, E\left(\tilde{V} - \tilde{P}\middle|\, \tilde{x}_i, \tilde{y}_i, \tilde{\omega}\right)\right] \\ &= E\left[1_{\left\{E\left(\tilde{V}\middle|\tilde{x}_i,\tilde{y}_i,\tilde{\omega}\right) > E\left(\tilde{P}\middle|\tilde{x}_i,\tilde{y}_i,\tilde{\omega}\right)\right\}} E\left(\tilde{V} - \tilde{P}\middle|\, \tilde{x}_i, \tilde{y}_i, \tilde{\omega}\right)\right] \\ &\quad + E\left[1_{\left\{E\left(\tilde{P}\middle|\tilde{x}_i,\tilde{y}_i,\tilde{\omega}\right) \geq E\left(\tilde{V}\middle|\tilde{x}_i,\tilde{y}_i,\tilde{\omega}\right)\right\}} E\left(\tilde{P} - \tilde{V}\middle|\, \tilde{x}_i, \tilde{y}_i, \tilde{\omega}\right)\right], \end{aligned} \tag{25}$$

where $1_{\{\cdot\}}$ is the indicator function. The complexity of expression (25) precludes an analytical analysis. But we can numerically compute W_S easily, since by equations (13) and (14), the two terms in (25) are simply expectations on a bivariate normal distribution.

Figure 2 reports the results. We still set $\tau_a = \tau_f = \tau_x = \tau_y = \lambda = 1$. In Panels (a) and (b), we choose $\mu_a = 0.8$ and $\mu_f = 0.2$, so that public information is mainly a signal about factor $\tilde{a}$, while in Panels (c) and (d), we choose $\mu_a = 0.2$ and $\mu_f = 0.8$, making public information mainly a signal about factor $\tilde{f}$. We find that disclosing different types of information also has contrasting implications for speculators' welfare W_S. Specifically, disclosing information about $\tilde{a}$ either monotonically decreases W_S (Panel (a)) or it first increases and then decreases W_S (Panel (b)). By contrast, disclosing information about $\tilde{f}$ has exactly the opposite effects: It either monotonically increases W_S (Panel (d)) or it first decreases and then increases W_S (Panel (c)).

[Insert Figure 2 Here]

The intuitions for these results are as follows. In our model, there are two sources of uncertainty and speculators make profits from noise traders based on their private information about both factors. Releasing public information about one factor tends to have opposite effects on trading profits that are driven by different types of private information. Take as an example disclosing information about factor $\tilde{a}$ in Panels (a) and (b). First, like traditional asymmetric information models with exogenous cash flows, releasing public information about $\tilde{a}$ weakens speculators' information advantage about factor $\tilde{a}$ and reduces their profits made from trading on this factor. Second, as Proposition 2 shows, releasing information about $\tilde{a}$ makes the price more informative about factor $\tilde{f}$, which means that capital providers now can learn more information about $\tilde{f}$, making cash flows more responsive to factor $\tilde{f}$ and hence strengthening speculators' information advantage about $\tilde{f}$. The patterns in Panels (a) and (b) are determined by the relative strength of these two opposite effects. Also note that speculators make more profits from trading on $\tilde{a}$ than on $\tilde{f}$, because capital providers know more about $\tilde{a}$ and so variations in cash flows are driven more by $\tilde{a}$ as well. This implies that when the level $\frac{1}{\tau_\xi}$ of noise trading is large, speculators are making a lot of money from trading on factor $\tilde{a}$, and so the negative effect of disclosing information about $\tilde{a}$ is particularly strong and it dominates the positive effect for all levels of disclosure, which explains the monotonic pattern in Panel (a). A symmetric argument explains the inverted U-shape in Panel (b).

5.2 The Role of Assumptions

Relative to traditional asymmetric information models, our setup has two important features: two dimensional uncertainty, and feedback effects which endogenously determine the cash flow of the traded asset. In this subsection, we show that the feature of two dimensional uncertainty is more essential in delivering our results. Specifically, in Section 5.2.1, we show that shutting down two dimensional uncertainty implies that disclosure does not affect price informativeness and therefore has no indirect effect at all. By contrast, in Section 5.2.2, we analyze a model with exogenous cash flows and show that all our results still hold in this alternative setting.

5.2.1 The Role of Two Dimensional Uncertainty

Suppose we shut down the uncertainty related to factor $\tilde{a}$ by letting τ_a approach infinity, so that $\tilde{a}$ becomes common knowledge. Then, speculators will no longer rely on their signals $\tilde{x}_i$ in forming their trading strategies. We thus conjecture that speculators buy the asset whenever $\tilde{y}_i + \phi_\omega^{1\,\dim}\tilde{\omega} > g^{1\,\dim}$, where $\phi_\omega^{1\,\dim}$ and $g^{1\,\dim}$ are endogenous parameters. We can follow similar steps as in Section 3 and show that speculators' aggregate demand for the risky asset is $D^{1\,\dim}\left(\tilde{f}, \tilde{\omega}\right) = 1 - 2\Phi\left(\frac{g^{1\,\dim} - \tilde{f} - \phi_\omega^{1\,\dim}\tilde{\omega}}{\sqrt{\tau_y^{-1}}}\right)$. So, using market clearing condition (5), we can find that the equilibrium price would change to:

$$\tilde{P}^{1\,\dim} = \exp\left(\frac{\tilde{f}}{\lambda\sqrt{\tau_y^{-1}}} + \frac{\tilde{\xi}}{\lambda} - \frac{g^{1\,\dim}}{\lambda\sqrt{\tau_y^{-1}}} + \frac{\phi_\omega^{1\,\dim}\tilde{\omega}}{\lambda\sqrt{\tau_y^{-1}}}\right).$$

Given that capital providers know public information $\tilde{\omega}$, the price $\tilde{P}^{1\,\dim}$ is equivalent to the following signal in predicting $\tilde{f}$:

$$\tilde{s}_p^{1\,\dim} = \tilde{f} + \sqrt{\tau_y^{-1}}\tilde{\xi},$$

which has a precision of

$$\tau_p^{1\,\dim} \equiv \frac{1}{Var\left(\sqrt{\tau_y^{-1}}\tilde{\xi}\right)} = \tau_y\tau_\xi.$$

Clearly, the amount $\tau_p^{1\,\dim}$ of information that capital providers learn from the price is not affected by the public information precision τ_ω, which therefore shuts down the mechanism emphasized in our analysis. So, all our main results, such as the amplification or attenuation of the direct effect of disclosure and the negative overall effect on real efficiency, vanish in this alternative economy with unidimensional uncertainty.

Proposition 4 *In the economy with unidimensional uncertainty, disclosure does not affect the amount of information that capital providers learn from prices and so the indirect effect of disclosure is inactive.*

5.2.2 Exogenous Cash Flow of the Risky Asset

We now shut down the feature that real investments affect the cash flow of the traded asset. Specifically, we assume that the asset's cash flow is fixed at $\tilde{V} \equiv \tilde{A}\tilde{F}K_0$, where $K_0 > 0$ is a constant. There are still real decision makers who see the price (as well as factor $\tilde{a}$

and public disclosure $\tilde{\omega}$) and make investments according to equation (1). In this setting, the informational content of the price still affects real efficiency as given by (17), but real investments do not affect the cash flow of the traded asset. This alternative setting is similar to those adopted by Subrahmanyam and Titman (1999) and Foucault and Gehrig (2008), who interpret the traded asset as the asset in place of a firm and the real investments as non-tradable growth options of the same firm. Alternatively, we can interpret the traded asset as the stock on a public firm, whose stock price conveys information to other private firms that operate in the same industry and make real investments such as R&D.

The derivations of equilibrium in this economy are almost identical to those for the baseline model, except that in the trading stage speculators now use their information set $\{\tilde{x}_i, \tilde{y}_i, \tilde{\omega}\}$ to simply forecast an exogenous cash flow $\tilde{A}\tilde{F}K_0$. We can show that our results in Propositions 2 and 3 continue to hold in this economy with exogenous cash flows. So, the endogenous feature of cash flow of the traded asset is not crucial in driving our results (which also echoes our footnote 8). Formally, we have the following proposition.

Proposition 5 *Suppose that the supply elasticity λ is high in the economy where the cash flow of the traded asset is exogenous.*

(a) If public information $\tilde{\omega}$ is a signal about factor $\tilde{a}$ (i.e., $\mu_a = 1$ and $\mu_f = 0$), then: (i) there is a unique equilibrium characterized by the relative weight on private signals about factor $\tilde{f}$ in speculators' trading strategy, $\phi_y = \frac{\tau_y(\tau_a+\tau_x+\tau_\omega)}{\tau_x(\tau_f+\tau_y)}$; (ii) increasing the precision τ_ω of public disclosure increases the relative weight ϕ_y, the precision τ_p that capital providers learn from the price, and real efficiency RE.

(b) If public information $\tilde{\omega}$ is a signal about factor $\tilde{f}$ (i.e., $\mu_a = 0$ and $\mu_f = 1$), then: (i) there is a unique equilibrium characterized by the relative weight on private signals about factor $\tilde{f}$ in speculators' trading strategy, $\phi_y = \frac{\tau_y(\tau_a+\tau_x)}{\tau_x(\tau_f+\tau_y+\tau_\omega)}$; (ii) increasing the precision τ_ω of public disclosure decreases the relative weight ϕ_y and the precision τ_p that capital providers learn from the price, and it increases real efficiency RE if and only if τ_ω is large or the precision τ_ξ of noise trading is small.

Nonetheless, by comparing our baseline model with this alternative setting with exogenous cash flow, we find that endogenizing cash flow makes speculators trade less aggressively on their private information about factor $\tilde{f}$, so that capital providers learn less information and real efficiency is lower, which is summarized in Corollary 1. This is because the cash flow is more sensitive to factor $\tilde{a}$ in the endogenous case than in the exogenous case. Formally, in the alternative economy in this section, the cash flow of the traded asset is exogenously set at $\tilde{V} \equiv \tilde{A}\tilde{F}K_0$, which is equally sensitive to both factors, while in the economy in our baseline model, the cash flow is endogenously given by $\tilde{V} = (1-\beta)\,\tilde{A}\tilde{F}K^* = \frac{\beta(1-\beta)}{c}\tilde{A}^2\tilde{F}E\left(\tilde{F}\middle|\mathcal{I}_R\right)$, which is more sensitive to $\tilde{a}$ than $\tilde{f}$ (for instance, if capital providers do not learn any information so that $E\left(\tilde{F}\middle|\mathcal{I}_R\right) = E\left(\tilde{F}\right)$, then the endogenous cash flow is twice as sensitive to factor $\tilde{a}$ as to factor $\tilde{f}$). This result suggests that designing asset with cash flows more sensitive to factor $\tilde{f}$ can improve real efficiency.

Corollary 1 *Suppose that the supply elasticity λ is high and that public information $\tilde{\omega}$ is a signal about one factor. Then, speculators put a higher weight on private signals about factor $\tilde{f}$ and real efficiency is higher in economies with exogenous cash flow than in those with endogenous cash flow.*

6 Variations and Extensions

In this section, we show the robustness of our results to alternative settings by analyzing variations and extensions of the baseline model.

6.1 Two Public Signals

In our baseline model, in (2) we specify public disclosure as a signal about a linear combination of both factors. Although this specification is reasonable – for instance, the providers of public disclosure have technical constrains in separating the two factors – it is also interesting to ensure that our results hold when there are two separate public signals, each of which conveys information about one factor only. Specifically, in this subsection, we assume two pieces of public information as follows:

$$\tilde{\omega}_a = \tilde{a} + \tilde{\varepsilon}_{\omega a} \text{ and } \tilde{\omega}_f = \tilde{f} + \tilde{\varepsilon}_{\omega f},$$

where $\tilde{\varepsilon}_{\omega a} \sim N(0, \tau_{\omega a}^{-1})$ (with $\tau_{\omega a} \geq 0$) and $\tilde{\varepsilon}_{\omega f} \sim N\left(0, \tau_{\omega f}^{-1}\right)$ (with $\tau_{\omega f} \geq 0$) are mutually independent and independent of other random variables. Parameters $\tau_{\omega a}$ and $\tau_{\omega f}$ control the precision of the two public signals, respectively. All the other features of the model are the same as before.

We conjecture that speculators buy the asset whenever $\tilde{x}_i + \phi_y \tilde{y}_i + \phi_{\omega a}\tilde{\omega}_a + \phi_{\omega f}\tilde{\omega}_f > g$, where ϕ's and g are endogenous parameters. Following similar steps as in the baseline model, we can show that the price information to capital providers is still given by a signal $\tilde{s}_p$, as specified by equations (9)-(11). Now capital providers have the information set $\{\tilde{a}, \tilde{\omega}_a, \tilde{\omega}_f, \tilde{p}\} = \{\tilde{a}, \tilde{\omega}_a, \tilde{\omega}_f, \tilde{s}_p\}$. By noting that $\tilde{\omega}_a$ is redundant given $\tilde{a}$, capital provider j's decision problem at $t = 1$ is:

$$\begin{aligned} k_j^* &= \arg\max_{k_j} E\left(\beta \tilde{A}\tilde{F}k_j - \frac{c}{2}k_j^2 \middle| \tilde{a}, \tilde{\omega}_f, \tilde{s}_p\right) = \frac{\beta \tilde{A} E\left(\tilde{F}\middle| \tilde{\omega}_f, \tilde{s}_p\right)}{c} \\ &= \exp\left[\left(\log\frac{\beta}{c} + \frac{1}{2}\frac{1}{\tau_f + \tau_{\omega f} + \tau_p}\right) + \tilde{a} + \frac{\tau_{\omega f}}{\tau_f + \tau_{\omega f} + \tau_p}\tilde{\omega}_f + \frac{\tau_p}{\tau_f + \tau_{\omega f} + \tau_p}\tilde{s}_p\right]. \end{aligned}$$

We can also compute real efficiency as

$$RE = \frac{\beta}{c}\left(1 - \frac{\beta}{2}\right)\exp\left(\frac{2}{\tau_a} + \frac{2}{\tau_f} - \frac{1}{\tau_f + \tau_{\omega f} + \tau_p}\right).$$

Back to date 0, speculators forecast the cash flow and the asset price as follows:

$$\begin{aligned} E\left(\tilde{V}\middle| \tilde{x}_i, \tilde{y}_i, \tilde{\omega}\right) &= \exp\left(b_0^v + b_x^v \tilde{x}_i + b_y^v \tilde{y}_i + b_{\omega a}^v \tilde{\omega}_a + b_{\omega f}^v \tilde{\omega}_f\right), \\ E\left(\tilde{P}\middle| \tilde{x}_i, \tilde{y}_i, \tilde{\omega}\right) &= \exp\left(b_0^p + b_x^p \tilde{x}_i + b_y^p \tilde{y}_i + b_{\omega a}^p \tilde{\omega}_a + b_{\omega f}^p \tilde{\omega}_f\right),, \end{aligned}$$

where the coefficients b's are given in the appendix. They will purchase one unit of the asset if and only if $E\left(\tilde{V}\middle| \tilde{x}_i, \tilde{y}_i, \tilde{\omega}\right) > E\left(\tilde{P}\middle| \tilde{x}_i, \tilde{y}_i, \tilde{\omega}\right)$. So, comparing with the initially conjectured trading strategy, we have the following system determining the equilibrium:

$$\phi_y = \frac{b_y^v - b_y^p}{b_x^v - b_x^p}, \phi_{\omega a} = \frac{b_{\omega a}^v - b_{\omega a}^p}{b_x^v - b_x^p}, \text{ and } \phi_{\omega f} = \frac{b_{\omega f}^v - b_{\omega f}^p}{b_x^v - b_x^p},$$

where the first equation has only one unknown ϕ_y.

We can show that our results in Propositions 2 and 3 continue to hold in this economy with two independent public signals. Formally, we have the following proposition.

Proposition 6 *Suppose that the supply elasticity λ is high in the economy with two independent public signals about the two factors.*

(a) There exists a unique equilibrium characterized by the relative weight $\phi_y > 0$ on private

signals about factor $\tilde{f}$ in speculators' trading strategy.

(b) Increasing the precision $\tau_{\omega a}$ of public disclosure about factor $\tilde{a}$ increases the relative weight ϕ_y in speculators' trading strategy, the precision τ_p that capital providers learn from the price, and real efficiency RE.

(c) Increasing the precision $\tau_{\omega f}$ of public disclosure about factor $\tilde{f}$ decreases the relative weight ϕ_y and the precision τ_p that capital providers learn from the price, and it increases real efficiency RE if and only if τ_ω is large or the precision τ_ξ of noise trading is small.

6.2 Capital Providers Receive Noisy Signals about Factors

In our baseline model, we have assumed that capital providers know factor $\tilde{a}$ perfectly and know nothing about factor $\tilde{f}$, so that they care only about the price's informational content about $\tilde{f}$. In this subsection, we extend our model by assuming that capital providers receive noisy signals about both factors, and show that all our results go through as long as capital providers wish to learn one productivity factor more than the other.

Specifically, we now endow each capital provider j with two private signals

$$\tilde{z}_j = \tilde{a} + \tilde{\varepsilon}_{z,j} \text{ and } \tilde{s}_j = \tilde{f} + \tilde{\varepsilon}_{s,j},$$

where $\tilde{\varepsilon}_{z,j} \sim N\left(0, \tau_z^{-1}\right)$ (with $\tau_z > 0$) and $\tilde{\varepsilon}_{s,j} \sim N\left(0, \tau_s^{-1}\right)$ (with $\tau_s > 0$) are mutually independent and they are independent of all other random variables. We keep intact all the other features of the model. Our baseline model corresponds to the case of $\tau_z = \infty$ and $\tau_s = 0$. If $\frac{\tau_z}{\tau_a}$ and $\frac{\tau_s}{\tau_f}$ are sufficiently different, then capital providers are more keen to learn one factor than the other.

We still consider trading strategies that speculators buy the asset if and only $\tilde{x}_i + \phi_y \tilde{y}_i + \phi_\omega \tilde{\omega} > g$, where ϕ_y, ϕ_ω, and g are endogenous parameters determined in equilibrium. So, their aggregate demand $D\left(\tilde{a}, \tilde{f}, \tilde{\omega}\right)$ is still given by equation (7), and the market clearing condition still implies a price function $P\left(\tilde{a}, \tilde{f}, \tilde{\omega}, \tilde{\xi}\right)$ in equation (8). However, because now capital providers do not observe $\tilde{a}$ perfectly, the price is no longer a signal about $\tilde{f}$ given by (9); but instead, it is a signal about both $\tilde{a}$ and $\tilde{f}$ as follows:

$$\tilde{s}_p^{ext} = \frac{\tilde{a}}{\phi_y} + \tilde{f} + \tilde{\varepsilon}_p, \tag{26}$$

where $\tilde{\varepsilon}_p$ is still defined by $\tilde{\varepsilon}_p \equiv \frac{\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}{\phi_y}\tilde{\xi}$.

Each capital provider j's optimal investment decision is:

$$k_j^* = \arg\max_{k_j} E\left(\beta\tilde{A}\tilde{F}k_j - \frac{c}{2}k_j^2\middle|\, \tilde{z}_j, \tilde{s}_j, \tilde{\omega}, \tilde{s}_p^{ext}\right) = \frac{\beta E\left(e^{\tilde{a}+\tilde{f}}\middle|\, \tilde{z}_j, \tilde{s}_j, \tilde{\omega}, \tilde{s}_p^{ext}\right)}{c}. \tag{27}$$

We can still show that capital providers follow a loglinear investment rule:

$$k_j^* = \exp\left(h_0 + h_z\tilde{z}_j + h_s\tilde{s}_j + h_\omega\tilde{\omega} + h_p\tilde{s}_p^{ext}\right),$$

where h's are endogenous constants that depend on $\left(\tau_a, \tau_f, \tau_z, \tau_s, \mu_a, \mu_f, \phi_y, \tau_p\right)$. We then follow steps similar to the baseline model and show that the characterization of the equilibrium boils down to one equation in terms of the loading ϕ_y that speculators put on their private signals $\tilde{y}_i$. The complexity of the inference problem induced by the price signal $\tilde{s}_p^{ext}$ in (26) precludes a full analytical characterization of the equilibrium, and therefore we rely on numerical analysis.

[Insert Figure 3 Here]

In the right panels of Figure 3, we plot real efficiency, $RE = E\left[\tilde{A}\tilde{F}K^* - \int c\left(k_j^*\right)dj\right]$, against the precision τ_ω of public information. In the left panels, we also plot the direct and indirect effects of public information on the inference problem of capital providers. Specifically, in the first order condition (27) of capital providers' decision problem, they wish to forecast the total productivity $\tilde{a}+\tilde{f}$ using the information set $\left\{\tilde{z}_j, \tilde{s}_j, \tilde{\omega}, \tilde{s}_p^{ext}\right\}$. Note that capital providers have private information $\{\tilde{z}_j, \tilde{s}_j\}$, and the forecast precision given their own information is $\frac{1}{Var\left(\tilde{a}+\tilde{f}\middle|\tilde{z}_j,\tilde{s}_j\right)}$. After adding the public disclosure $\tilde{\omega}$, their forecast precision increases to $\frac{1}{Var\left(\tilde{a}+\tilde{f}\middle|\tilde{z}_j,\tilde{s}_j,\tilde{\omega}\right)}$, and so the difference of $\frac{1}{Var\left(\tilde{a}+\tilde{f}\middle|\tilde{z}_j,\tilde{s}_j,\tilde{\omega}\right)} - \frac{1}{Var\left(\tilde{a}+\tilde{f}\middle|\tilde{z}_j,\tilde{s}_j\right)}$ captures how much extra precision that capital providers obtain by directly observing the public signal $\tilde{\omega}$ (This is consistent with that we use $\mu_f^2\tau_\omega = \frac{1}{Var\left(\tilde{a}+\tilde{f}\middle|\tilde{a},\tilde{s}_j,\tilde{\omega}\right)} - \frac{1}{Var\left(\tilde{a}+\tilde{f}\middle|\tilde{a},\tilde{s}_j\right)}$ to measure the direct learning from disclosure in the baseline model). So, we measure the direct effect of disclosure as follows:

$$\text{Direct Effect} \equiv \frac{\partial}{\partial\tau_\omega}\left[\frac{1}{Var\left(\tilde{a}+\tilde{f}\middle|\, \tilde{z}_j, \tilde{s}_j, \tilde{\omega}\right)} - \frac{1}{Var\left(\tilde{a}+\tilde{f}\middle|\, \tilde{z}_j, \tilde{s}_j\right)}\right],$$

which is always positive. Similarly, we will use the difference $\frac{1}{Var\left(\tilde{a}+\tilde{f}\middle|\tilde{z}_j,\tilde{s}_j,\tilde{\omega},\tilde{s}_p^{ext}\right)} - \frac{1}{Var\left(\tilde{a}+\tilde{f}\middle|\tilde{z}_j,\tilde{s}_j,\tilde{\omega}\right)}$ to capture how much extra information that capital providers can learn from the price, and therefore we define the indirect effect of disclosure as follows:

$$\text{Indirect Effect} \equiv \frac{\partial}{\partial \tau_\omega}\left[\frac{1}{Var\left(\tilde{a}+\tilde{f}\,\middle|\,\tilde{z}_j,\tilde{s}_j,\tilde{\omega},\tilde{s}_p^{ext}\right)}-\frac{1}{Var\left(\tilde{a}+\tilde{f}\,\middle|\,\tilde{z}_j,\tilde{s}_j,\tilde{\omega}\right)}\right].$$

The indirect effect can be positive or negative.

In all panels of Figure 3, we set $\tau_a = \tau_f = \tau_x = \tau_y = \lambda = 1$, $\beta = \frac{1}{2}$, and $c = 1$. We also choose $\tau_z = 5$ and $\tau_s = 1$, so that capital providers know more about factor $\tilde{a}$ than factor $\tilde{f}$. In Panels (a1) and (a2), we set $\mu_a = 0.8$ and $\mu_f = 0.2$, making public information $\tilde{\omega}$ primarily a signal about factor $\tilde{a}$. We also arbitrarily choose $\tau_\xi = 1$ in these two panels. In contrast, in the remaining four panels (Panels (b1)-(c2)), we set $\mu_a = 0.2$ and $\mu_f = 0.8$, making public information $\tilde{\omega}$ primarily a signal about factor $\tilde{f}$. Also, in Panels (b1) and (b2), we choose $\tau_\xi = 0.5$, so that the market doe not aggregate private information that much, and in Panels (c1) and (c2), we choose $\tau_\xi = 10$ to make the market aggregate private information effectively.

We see that Figure 3 delivers the same message as Figure 1. In Panels (a1) and (a2), when pubic information is mainly a signal about $\tilde{a}$, increasing the precision of public disclosure both directly and indirectly benefit capital providers' learning. That is, the indirect effect amplifies the direct effect of disclosure, leading to a positive total effect on real efficiency. In Panels (b1)-(c2), when public information is mainly a signal about $\tilde{f}$, public disclosure directly improves but indirectly harms capital providers' learning. That is, the indirect effect attenuates the direct effect, and the overall effect of disclosure is ambiguous. In addition, when the market does not aggregate speculators' private information effectively, the positive direct effect of public disclosure always dominates and the overall effect of disclosure is to improve real efficiency (Panels (b1) and (b2)). In contrast, when the market aggregates speculators' private information effectively, the indirect effect dominates for small levels of disclosure, so that the overall effect is that real efficiency can decrease with disclosure precision (Panels (c1) and (c2)).

6.3 Speculators Submit Demand Schedules and Observe Prices

In the baseline model, we have assumed that speculators submit market orders and that noise trading depends on prices to clear the market. Now we consider a variation in which speculators submit price-contingent demand schedules, so that they can effectively condition their trades on prices. That is, as in Albagli, Hellwig, and Tsyvinski (2012, 2014), each speculator decides how many shares to trade at the prevailing price $\tilde{P}$, in exchange for cash. This variation serves two purposes. First, we can check the robustness of our main results. Second, in this alternative setting, we can set $\lambda = 0$ to make noise trading independent of prices and still we can clear the market through speculators' trading.

Now we conjecture that speculators buy the asset whenever $\tilde{x}_i + \phi_y \tilde{y}_i + \phi_\omega \tilde{\omega} - \phi_p \tilde{p} > g$, where $\tilde{p} = \log \tilde{P}$ and ϕ's and g are endogenous parameters. Then, we follow similar steps as in the baseline model and show that the price is

$$\tilde{P} = \exp\left(\frac{-g + \tilde{a} + \phi_y \tilde{f} + \phi_\omega \tilde{\omega} + \sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}\tilde{\xi}}{\lambda\sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}} + \phi_p}\right),$$

which, to capital providers, is still a signal $\tilde{s}_p$ in predicting $\tilde{f}$, as specified by equations (9)-(11). So, capital providers' decision problem does not change and their investment policy is still given by (12). Accordingly, the expression of real efficiency is still given by (17).

However, when we go back to date 0, speculators' forecast problem changes. They now can condition on the information in prices to make forecast about future cash flow as follows:

$$E\left(\tilde{V}|\tilde{x}_i, \tilde{y}_i, \tilde{\omega}, \tilde{p}\right) = b_0^v + b_x^v \tilde{x}_i + b_y^v \tilde{y}_i + b_\omega^v \tilde{\omega} + b_p^v \tilde{p},$$

where b's are given in the appendix. Since they know the price $\tilde{p}$, they do not need to forecast it. As a result, speculator i will buy the asset if and only if

$$b_0^v + b_x^v \tilde{x}_i + b_y^v \tilde{y}_i + b_\omega^v \tilde{\omega} + b_p^v \tilde{p} > \tilde{p} \Leftrightarrow b_0^v + b_x^v \tilde{x}_i + b_y^v \tilde{y}_i + b_\omega^v \tilde{\omega} - \left(1 - b_p^v\right)\tilde{p} > 0,$$

which compares with the conjectured trading strategy, yielding the following equations that determine the equilibrium:

$$\phi_y = \frac{b_y^v}{b_x^v}, \phi_\omega = \frac{b_\omega^v}{b_x^v}, \text{ and } \phi_p = \frac{1 - b_p^v}{b_x^v}.$$

We can show that the supply elasticity λ in noise trading does not affect the value of ϕ_y. As a result, it does not affect the information precision τ_p that capital providers can learn from the price and hence real efficiency in equilibrium. Its only role is to change the value of ϕ_p. We summarize this result in the following proposition.

Proposition 7 *In the economy where speculators submit price-contingent demand schedules,* *the supply elasticity λ in noise trading has no effect on real efficiency.*

In Figure 4, we numerically examine the implications of disclosure in this economy. Similar to Figure 1, we here have set $\tau_a = \tau_f = \tau_x = \tau_y = \tau_\xi = \lambda = 1$. In Panels (a1) and (a2), we choose $\mu_a = 0.8$ and $\mu_f = 0.2$ to make public disclosure mainly a signal about factor $\tilde{a}$, while in Panels (b1)-(c2) we choose $\mu_a = 0.2$ and $\mu_f = 0.8$ to make public disclosure mainly a signal about factor $\tilde{f}$. We find that our main results continue to hold qualitatively. First, consistent with Proposition 2, disclosing information about $\tilde{a}$ makes speculators trade more aggressively on their private information about $\tilde{f}$ in Panel (a1), which in turn improves capital providers' learning from the price in Panel (a2). This means that the indirect effect of disclosing information about $\tilde{a}$ is positive and it can amplify the direct effect, thereby making the overall effect positive. Second, consistent with Proposition 3, disclosing information about $\tilde{f}$ causes speculators to trade less aggressively on private information about $\tilde{f}$ in Panels (b1) and (b2), thereby harming capital providers' learning from the price in Panels (c1) and (c2). That is, the indirect effect of disclosing information about $\tilde{f}$ is negative and it attenuates the positive direct effect on real efficiency.

[Insert Figure 4 Here]

However, we notice that Panel (c2) of Figure 4 is different from our previous Panel (c2) of Figure 1. Here, we find that even when the variance of noise trading is relatively small, the indirect effect does not dominate the direct effect. What accounts for this difference is the following. In this alternative economy, speculators can also observe prices, and so part of effect of disclosure on the weights ϕ_y that speculators put on their private signals about $\tilde{f}$ is absorbed by the price information when they make predictions about the asset's cash flow. This weakens the indirect effect which operates through the responsiveness of ϕ_y to disclosure. Despite the difference between Panel (c2) of Figure 4 and Panel (c2) of Figure 1, we want to emphasize that our main message continues to be valid. That is, when disclosure is about factor $\tilde{a}$, the indirect effect is negative (i.e., τ_p is decreasing in both Panels (b2) and (c2) of Figure 4), and this negative indirect effect becomes stronger as the noise trading level in the market becomes smaller (i.e., $\mu_f^2 \tau_\omega + \tau_p$ is increasing in Panel (b2), while it is almost flat in Panel (c2)).

7 Conclusion

Public disclosure has been an important component of financial regulation. Various agencies, corporate companies and governments, for example, frequently make public announcements to the financial market. The financial market is not just a sideshow, because the market price conveys valuable information relevant for real decision makers, such as managers and capital providers, to make decisions. Moreover, the fluctuations of asset price are driven by multiple factors, and real decision makers care to learn some factors more than others. In this paper, we have proposed a framework to study what kind of public information is "good" in the sense that it strongly improves the forecast quality of real decision makers and thus real efficiency.

We find that public information release generally has two effects on real efficiency. First, it benefits real decision makers' forecast problem by directly providing new information. Second, it can benefit or harm real decision makers' learning indirectly by affecting the informational content of asset price. When this indirect effect is positive (negative), the direct effect is amplified (attenuated), leading to a strong (weak) overall effect on real efficiency. We show that the indirect effect is positive when public disclosure is mainly a signal about something of which real decision makers are already well informed. In this case, public information always improves real efficiency. In contrast, when public disclosure mainly conveys information that real decision makers care to learn much, the indirect effect is generally negative, which tends to attenuate the positive direct effect. In addition, when the financial market is a very effective device of aggregating traders' private information, the negative indirect effect can even dominate the positive direct effect for small levels of public disclosure, so that public disclosure can actually harm real efficiency.

Appendix

The Expressions of the Coefficients b's in Equations (13) and (14)

Define $\Delta^p \equiv Var\left(\tilde{a}+\phi_y\tilde{f}\middle|\tilde{x}_i,\tilde{y}_i,\tilde{\omega}\right)$, and let δ_x^p, δ_y^p, and δ_ω^p be the loadings of $\tilde{x}_i$, $\tilde{y}_i$ and $\tilde{\omega}$ in the expression of $E\left(\tilde{a}+\phi_y\tilde{f}\middle|\tilde{x}_i,\tilde{y}_i,\tilde{\omega}\right)$, respectively. Then, we have:

$$\begin{aligned} b_0^p &= \frac{-g}{\lambda\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}+\frac{1}{2\lambda^2\tau_\xi}+\frac{\Delta^p}{2\lambda^2\left(\tau_x^{-1}+\phi_y^2\tau_y^{-1}\right)}, \\ b_x^p &= \frac{\delta_x^p}{\lambda\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}, b_y^p=\frac{\delta_y^p}{\lambda\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}, \text{ and } b_\omega^p=\frac{\delta_\omega^p+\phi_\omega}{\lambda\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}. \end{aligned}$$

Similarly, define $\Delta^v \equiv Var\left(\tilde{a}+\frac{1+\frac{\tau_p}{\tau_f+\mu_f^2\tau_\omega+\tau_p}}{2-\frac{\mu_f^2\tau_\omega}{\tau_f+\mu_f^2\tau_\omega+\tau_p}\frac{\mu_a}{\mu_f}}\tilde{f}\middle|\tilde{x}_i,\tilde{y}_i,\tilde{\omega}\right)$, and let δ_x^v, δ_y^v, and δ_ω^v be the loadings of $\tilde{x}_i$, $\tilde{y}_i$ and $\tilde{\omega}$ in the expression of $E\left(\tilde{a}+\frac{1+\frac{\tau_p}{\tau_f+\mu_f^2\tau_\omega+\tau_p}}{2-\frac{\mu_f^2\tau_\omega}{\tau_f+\mu_f^2\tau_\omega+\tau_p}\frac{\mu_a}{\mu_f}}\tilde{f}\middle|\tilde{x}_i,\tilde{y}_i,\tilde{\omega}\right)$, respectively. Then, we have:

$$\begin{aligned} b_0^v &= \left[\log\frac{\beta(1-\beta)}{c}\right]+\frac{\tau_p\left(\tau_f+\mu_f^2\tau_\omega+\tau_p\right)+1}{2\tau_p\left(\tau_f+\mu_f^2\tau_\omega+\tau_p\right)^2}+\frac{\Delta^v}{2}\left(2-\frac{\mu_f^2\tau_\omega}{\tau_f+\mu_f^2\tau_\omega+\tau_p}\frac{\mu_a}{\mu_f}\right)^2, \\ b_x^v &= \left(2-\frac{\mu_f^2\tau_\omega}{\tau_f+\mu_f^2\tau_\omega+\tau_p}\frac{\mu_a}{\mu_f}\right)\delta_x^v, b_y^v=\left(2-\frac{\mu_f^2\tau_\omega}{\tau_f+\mu_f^2\tau_\omega+\tau_p}\frac{\mu_a}{\mu_f}\right)\delta_y^v, \text{ and} \\ b_\omega^v &= \frac{\mu_f^2\tau_\omega}{\tau_f+\mu_f^2\tau_\omega+\tau_p}\frac{1}{\mu_f}+\left(2-\frac{\mu_f^2\tau_\omega}{\tau_f+\mu_f^2\tau_\omega+\tau_p}\frac{\mu_a}{\mu_f}\right)\delta_\omega^v. \end{aligned}$$

Proof of Proposition 2

Given that Part (b) has been proved in the text, we here prove Part (a). By the expression of τ_p in (11), we have $\lim_{\phi_y\to 0}\frac{b_y^v}{b_x^v}>0$ and $\lim_{\phi_y\to\infty}\frac{b_y^v}{b_x^v}<\infty$ in equation (21). So, by the intermediate value theorem, we know that there exists $\phi_y>0$ satisfying equation (21).

We next prove the uniqueness. If we can prove that at the equilibrium level of ϕ_y, the right-hand-side $\frac{b_y^v}{b_x^v}$ in equation (21) always crosses the 45 degree line from above, then the equilibrium is unique. That is, we need to show $\frac{\partial}{\partial\phi_y}\frac{b_y^v}{b_x^v}<1$ for those values of ϕ_y satisfying equation (21). Direct computation shows:

$$\frac{\partial}{\partial\phi_y}\frac{b_y^v}{b_x^v}=\frac{\frac{\tau_y}{\tau_f+\tau_y}}{\frac{2\tau_x}{\tau_a+\tau_x+\tau_\omega}}\frac{\tau_f}{(\tau_f+\tau_p)^2}\frac{\partial\tau_p}{\partial\phi_y}. \tag{A1}$$

By the expression of τ_p in (11), we can compute

$$\frac{\partial \tau_p}{\partial \phi_y} = \tau_y \tau_\xi \frac{2\phi_y \tau_x \tau_y}{\left(\tau_y + \phi_y^2 \tau_x\right)^2}, \tag{A2}$$

which is plugged in (A1), yielding

$$\frac{\partial}{\partial \phi_y} \frac{b_y^v}{b_x^v} = \frac{\frac{\tau_y}{\tau_f + \tau_y}}{\frac{2\tau_x}{\tau_a + \tau_x + \tau_\omega}} \frac{\tau_f}{\left(\tau_f + \tau_p\right)^2} \tau_y \tau_\xi \frac{2\phi_y \tau_x \tau_y}{\left(\tau_y + \phi_y^2 \tau_x\right)^2}. \tag{A3}$$

By (21), we have

$$\frac{\frac{\tau_y}{\tau_f + \tau_y}}{\frac{2\tau_x}{\tau_a + \tau_x + \tau_\omega}} = \frac{\phi_y}{1 + \frac{\tau_p}{\tau_f + \tau_p}}$$

which is plugged into (A3), yielding

$$\frac{\partial}{\partial \phi_y} \frac{b_y^v}{b_x^v} = \frac{1}{1 + \frac{\tau_p}{\tau_f + \tau_p}} \frac{\tau_f}{\left(\tau_f + \tau_p\right)^2} \tau_y \tau_\xi \frac{2\phi_y^2 \tau_x \tau_y}{\left(\tau_y + \phi_y^2 \tau_x\right)^2}. \tag{A4}$$

By the expression of τ_p in (11), we have

$$\phi_y^2 \tau_x = \frac{\tau_p \tau_y}{\tau_y \tau_\xi - \tau_p}, \tag{A5}$$

which is plugged into (A4),

$$\frac{\partial}{\partial \phi_y} \frac{b_y^v}{b_x^v} = \frac{2\tau_p \tau_f}{\left(\tau_f + 2\tau_p\right)\left(\tau_f + \tau_p\right)} \frac{\tau_y \tau_\xi - \tau_p}{\tau_y \tau_\xi} < 1,$$

since $2\tau_p \tau_f < \left(\tau_f + 2\tau_p\right)\left(\tau_f + \tau_p\right)$ and $0 < \tau_y \tau_\xi - \tau_p < \tau_y \tau_\xi$. QED.

Proof of Proposition 3

Part (a). We follow the same methodology of proving Part (a) of Proposition 2. First, in (23), we have $\lim_{\phi_y \to 0} \frac{b_y^v}{b_x^v} > 0$ and $\lim_{\phi_y \to \infty} \frac{b_y^v}{b_x^v} < \infty$, and so by the intermediate value theorem, we have the existence of the equilibrium.

Second, for the uniqueness, we will show that in (23), the right-hand-side always crosses the 45 degree line from above at equilibrium. That is, we establish $\frac{\partial}{\partial \phi_y} \frac{b_y^v}{b_x^v} < 1$. Direct computation shows

$$\frac{\partial}{\partial \phi_y} \frac{b_y^v}{b_x^v} = \frac{\frac{\tau_y}{\tau_f + \tau_y + \tau_\omega}}{\frac{2\tau_x}{\tau_a + \tau_x}} \frac{\tau_f + \tau_\omega}{\left(\tau_f + \tau_p + \tau_\omega\right)^2} \tau_y \tau_\xi \frac{2\phi_y \tau_x \tau_y}{\left(\tau_y + \phi_y^2 \tau_x\right)^2}. \tag{A6}$$

By (23), we have

$$\frac{\frac{\tau_y}{\tau_f + \tau_y + \tau_\omega}}{\frac{2\tau_x}{\tau_a + \tau_x}} = \frac{\phi_y \left(\tau_f + \tau_p + \tau_\omega\right)}{\tau_f + 2\tau_p + \tau_\omega},$$

which is plugged into (A6), yielding

$$\frac{\partial}{\partial \phi_y}\frac{b_y^v}{b_x^v}=\frac{1}{\tau_f+2\tau_p+\tau_\omega}\frac{\tau_f+\tau_\omega}{\tau_f+\tau_p+\tau_\omega}\tau_y\tau_\xi\frac{2\phi_y^2\tau_x\tau_y}{\left(\tau_y+\phi_y^2\tau_x\right)^2}. \tag{A7}$$

Then, inserting (A5) into (A7), we have:

$$\frac{\partial}{\partial \phi_y}\frac{b_y^v}{b_x^v}=\frac{2\tau_p\left(\tau_f+\tau_\omega\right)}{\left(\tau_f+2\tau_p+\tau_\omega\right)\left(\tau_f+\tau_p+\tau_\omega\right)}\frac{\tau_y\tau_\xi-\tau_p}{\tau_y\tau_\xi}<1.$$

Part (b). Since the other results have been proved in the text, we only need to examine the real efficiency implications. By (24) and (A5), we can compute the indirect effect of disclosure as

$$\frac{\partial \tau_p}{\partial \tau_\omega}=-2\tau_p\frac{\tau_y\tau_\xi-\tau_p}{\tau_y\tau_\xi}\frac{\frac{\tau_p}{\left(\tau_f+2\tau_p+\tau_\omega\right)\left(\tau_f+\tau_p+\tau_\omega\right)}+\frac{1}{\tau_f+\tau_y+\tau_\omega}}{1-\frac{2\tau_p\left(\tau_f+\tau_\omega\right)}{\left(\tau_f+\tau_p+\tau_\omega\right)\left(\tau_f+2\tau_p+\tau_\omega\right)\frac{\tau_y\tau_\xi}{\tau_y\tau_\xi-\tau_p}}}.$$

Clearly, as $\tau_\omega\to\infty$, we have $\frac{\partial\tau_p}{\partial\tau_\omega}\to 0$, and so disclosure only has the positive direct effect. This establishes Part (b.iii).

To show Part (b.iii), we examine the behavior of the indirect effect $\frac{\partial\tau_p}{\partial\tau_\omega}$ at $\tau_\omega=0$. Consider the process of $\tau_\xi\to\infty$ or $\tau_\xi\to 0$. If $\lim_{\tau_\xi}\frac{f_1(\tau_\xi)}{f_2(\tau_\xi)}=0$, then we denote $f_1=o\left(f_2\right)$, meaning that f_1 converges at a faster rate than f_2. If $\lim_{\tau_\xi}\frac{f_1(\tau_\xi)}{f_2(\tau_\xi)}$ is bounded (but different from 0), then we denote $f_1=O\left(f_2\right)$, meaning that f_1 and f_2 converge at the same rate. By (23) and (11), we have $\phi_y=O\left(1\right)$ and $\tau_p=O\left(\tau_\xi\right)$. By (24) and the orders of ϕ_y and τ_p, we have

$$\left.\frac{\partial\phi_y}{\partial\tau_\omega}\right|_{\tau_\omega=0}=-\frac{\phi_y}{\tau_f+\tau_y}+o\left(1\right).$$

So, by (20), we have

$$\left.\frac{\partial\tau_p}{\partial\tau_\omega}\right|_{\tau_\omega=0}=-\frac{2\phi_y^2\tau_x\tau_y^2}{\left(\tau_y+\phi_y^2\tau_x\right)^2\left(\tau_f+\tau_y\right)}\tau_\xi+o\left(\tau_\xi\right).$$

Thus, by (19), we have

$$\left.\frac{\partial RE}{\partial\tau_\omega}\right|_{\tau_\omega=0}\propto 1+\left.\frac{\partial\tau_p}{\partial\tau_\omega}\right|_{\tau_\omega=0}=1-\frac{2\phi_y^2\tau_x\tau_y^2}{\left(\tau_y+\phi_y^2\tau_x\right)^2\left(\tau_f+\tau_y\right)}\tau_\xi+o\left(\tau_\xi\right),$$

when $\mu_f=1$ and $\mu_a=0$. As a result, $\left.\frac{\partial RE}{\partial\tau_\omega}\right|_{\tau_\omega=0}<0$ for sufficiently large τ_ξ, and $\left.\frac{\partial RE}{\partial\tau_\omega}\right|_{\tau_\omega=0}>0$ for sufficiently small τ_ξ. QED.

Proof of Proposition 5

By Bayes' rule, we can directly compute

$$\begin{aligned} b_x^v &= \delta_x = \tau_x \frac{\tau_f + \tau_y + \mu_f^2 \tau_\omega - \mu_a \mu_f \tau_\omega}{\tau_a \tau_f + \tau_a \tau_y + \tau_f \tau_x + \tau_x \tau_y + \tau_a \mu_f^2 \tau_\omega + \mu_a^2 \tau_f \tau_\omega + \mu_a^2 \tau_y \tau_\omega + \mu_f^2 \tau_x \tau_\omega}, \\ b_y^v &= \delta_y = \tau_y \frac{\tau_a + \tau_x + \mu_a^2 \tau_\omega - \mu_a \mu_f \tau_\omega}{\tau_a \tau_f + \tau_a \tau_y + \tau_f \tau_x + \tau_x \tau_y + \tau_a \mu_f^2 \tau_\omega + \mu_a^2 \tau_f \tau_\omega + \mu_a^2 \tau_y \tau_\omega + \mu_f^2 \tau_x \tau_\omega}. \end{aligned}$$

So, as $\lambda \to \infty$, we have

$$\phi_y \approx \frac{b_y^v}{b_x^v} = \frac{\delta_y}{\delta_x} = \frac{\tau_y \left(\tau_a + \tau_x + \mu_a^2 \tau_\omega - \mu_a \mu_f \tau_\omega \right)}{\tau_x \left(\tau_f + \tau_y + \mu_f^2 \tau_\omega - \mu_a \mu_f \tau_\omega \right)}. \tag{A8}$$

Part (a). Now suppose $\mu_a = 1$ and $\mu_f = 0$. By (A8), we can compute

$$\phi_y = \frac{\tau_y (\tau_a + \tau_x + \tau_\omega)}{\tau_x (\tau_f + \tau_y)}. \tag{A9}$$

The other results follow directly.

Part (b). Now suppose $\mu_a = 0$ and $\mu_f = 1$. By (A8), we can compute

$$\phi_y = \frac{\tau_y (\tau_a + \tau_x)}{\tau_x (\tau_f + \tau_y + \tau_\omega)}. \tag{A10}$$

So, direct computation shows

$$\frac{\partial \phi_y}{\partial \tau_\omega} = -\frac{\tau_y (\tau_a + \tau_x)}{\tau_x (\tau_f + \tau_y + \tau_\omega)^2} < 0. \tag{A11}$$

The implications for τ_p follow from (A11) and (20). Finally, combining equations (A10)-(A11) with equations (11), (19), and (20), we have

$$\begin{aligned} \frac{\partial RE}{\partial \tau_\omega} &\propto \frac{\partial (\tau_f + \tau_\omega + \tau_p)}{\partial \tau_\omega} \\ &= 1 - \frac{2\tau_y^2 (\tau_a + \tau_x)^2 \tau_x (\tau_f + \tau_y + \tau_\omega)}{\begin{pmatrix} \tau_x \tau_\omega^2 + 2\tau_f \tau_x \tau_\omega + 2\tau_x \tau_y \tau_\omega \\ +\tau_a^2 \tau_y + \tau_f^2 \tau_x + \tau_x \tau_y^2 + \tau_x^2 \tau_y + 2\tau_a \tau_x \tau_y + 2\tau_f \tau_x \tau_y \end{pmatrix}^2} \tau_\xi. \end{aligned}$$

So, $\frac{\partial RE}{\partial \tau_\omega} > 0$ if $\tau_\omega \to \infty$ or $\tau_\xi \to \infty$, and the real efficiency implications follow directly. QED.

Proof of Corollary 1

The result on ϕ_y follows from comparison between equations (21) and (A9) and comparison between equations (23) and (A10). The result on real efficiency follows directly from the fact that RE is increasing in ϕ_y by equations (11), (17), and (18). QED.

Proof of Proposition 6

The coefficients b's in speculators forecast about the cash flow and the price are

$$\begin{aligned}
b_0^p &= -\frac{g}{\lambda\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}+\frac{1}{2\lambda^2\tau_\xi}+\frac{1}{2\lambda^2\left(\tau_x^{-1}+\phi_y^2\tau_y^{-1}\right)}\left(\frac{1}{\tau_a+\tau_x+\tau_{\omega a}}+\frac{\phi_y^2}{\tau_f+\tau_y+\tau_{\omega f}}\right),\\
b_x^p &= \frac{1}{\lambda\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}\frac{\tau_x}{\tau_a+\tau_x+\tau_{\omega a}}, b_y^p=\frac{\phi_y}{\lambda\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}\frac{\tau_y}{\tau_f+\tau_y+\tau_{\omega f}},\\
b_{\omega a}^p &= \frac{1}{\lambda\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}\left(\phi_{\omega a}+\frac{\tau_{\omega a}}{\tau_a+\tau_x+\tau_{\omega a}}\right), b_{\omega f}^p=\frac{1}{\lambda\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}\left(\phi_{\omega f}+\frac{\phi_y\tau_{\omega f}}{\tau_f+\tau_y+\tau_{\omega f}}\right),
\end{aligned}$$

$$b_0^v=\log\left[\frac{\beta(1-\beta)}{c}\right]+\frac{\tau_f+\tau_{\omega f}+2\tau_p}{2\left(\tau_f+\tau_{\omega f}+\tau_p\right)^2}+\frac{2}{\tau_a+\tau_x+\tau_{\omega a}}+\frac{1}{2}\left(1+\frac{\tau_p}{\tau_f+\tau_{\omega f}+\tau_p}\right)^2\frac{1}{\tau_f+\tau_y+\tau_{\omega f}},$$

$$\begin{aligned}
b_x^v &= \frac{2\tau_x}{\tau_a+\tau_x+\tau_{\omega a}}, b_y^v=\left(1+\frac{\tau_p}{\tau_f+\tau_{\omega f}+\tau_p}\right)\frac{\tau_y}{\tau_f+\tau_y+\tau_{\omega f}}, \text{ and}\\
b_{\omega a}^v &= \frac{2\tau_{\omega a}}{\tau_a+\tau_x+\tau_{\omega a}}, b_{\omega f}^v=\frac{\tau_{\omega f}}{\tau_f+\tau_{\omega f}+\tau_p}+\left(1+\frac{\tau_p}{\tau_f+\tau_{\omega f}+\tau_p}\right)\frac{\tau_{\omega f}}{\tau_f+\tau_y+\tau_{\omega f}}.
\end{aligned}$$

Part (a). As $\lambda\to\infty$, we have $b_x^p\to 0$ and $b_y^p\to 0$, and so ϕ_y is determined by

$$\phi_y=\frac{b_y^v}{b_x^v}=\frac{\left(1+\frac{\tau_p}{\tau_f+\tau_{\omega f}+\tau_p}\right)\frac{\tau_y}{\tau_f+\tau_y+\tau_{\omega f}}}{\frac{2\tau_x}{\tau_a+\tau_x+\tau_{\omega a}}}. \tag{A12}$$

When $\phi_y=0$, we have $\left.\frac{b_y^v}{b_x^v}\right|_{\phi_y=0}>0$ in (A12). When $\phi_y\to\infty$, we have $\left.\frac{b_y^v}{b_x^v}\right|_{\phi_y\to\infty}<\infty$. So, by the intermediate value theorem, there exists a $\phi_y>0$ satisfying equation (A12), which establishes the existence of the equilibrium.

We prove the uniqueness by showing that the RHS of (A12) crosses 45 degree line from above, that is, at equilibrium, we have $\frac{\partial}{\partial\phi_y}\frac{b_y^v}{b_x^v}<1$. Specifically, direct computation shows

$$\frac{\partial}{\partial\phi_y}\frac{b_y^v}{b_x^v}=\frac{\frac{\tau_y}{\tau_f+\tau_y+\tau_{\omega f}}}{\frac{2\tau_x}{\tau_a+\tau_x+\tau_{\omega a}}}\left[\frac{\tau_f+\tau_{\omega f}}{\left(\tau_f+\tau_{\omega f}+\tau_p\right)^2}\right]\tau_y\tau_\xi\frac{2\phi_y\tau_x\tau_y}{\left(\tau_y+\phi_y^2\tau_x\right)^2}.$$

Then, by (A12), we have $\frac{\frac{\tau_y}{\tau_f+\tau_y+\tau_{\omega f}}}{\frac{2\tau_x}{\tau_a+\tau_x+\tau_{\omega a}}}=\frac{\phi_y}{\left(1+\frac{\tau_p}{\tau_f+\tau_{\omega f}+\tau_p}\right)}$, which is plugged into the above equation, together with (A5), yielding

$$\frac{\partial}{\partial\phi_y}\frac{b_y^v}{b_x^v}=\frac{2\tau_p\left(\tau_f+\tau_{f\omega}\right)}{\left(\tau_f+2\tau_p+\tau_{f\omega}\right)\left(\tau_f+\tau_p+\tau_{f\omega}\right)}\frac{\tau_y\tau_\xi-\tau_p}{\tau_y\tau_\xi}<1.$$

Part (b). Applying the implicit function theorem to equations (11) and (A12), we can show

$$\frac{\partial\phi_y}{\partial\tau_{\omega a}} = \frac{\frac{\phi_y}{\tau_a+\tau_x+\tau_{\omega a}}}{1-\frac{2\tau_p\tau_y\left(\tau_f+\tau_{\omega f}\right)}{\left(\tau_f+\tau_{\omega f}+\tau_p\right)\left(\tau_f+\tau_{\omega f}+2\tau_p\right)\left(\tau_y+\phi_y^2\tau_x\right)}} > 0.$$

The other results follow directly from $\frac{\partial\tau_p}{\partial\tau_{\omega a}} = \frac{2\tau_p\tau_y}{\phi_y\left(\tau_y+\phi_y^2\tau_x\right)}\frac{\partial\phi_y}{\partial\tau_{\omega a}}$ and $\frac{\partial RE}{\partial\tau_{\omega a}} \propto \frac{\partial\left(\tau_f+\tau_{\omega f}+\tau_p\right)}{\partial\tau_{\omega a}} = \frac{\partial\tau_p}{\partial\tau_{\omega a}}$.

Part (c). Applying the implicit function theorem to equations (11) and (A12), we can show

$$\frac{\partial\phi_y}{\partial\tau_{\omega f}} = -\frac{\phi_y\left[\frac{\tau_p}{\left(\tau_f+\tau_{\omega f}+2\tau_p\right)\left(\tau_f+\tau_{\omega f}+\tau_p\right)}+\frac{1}{\tau_f+\tau_y+\tau_{\omega f}}\right]}{1-\frac{2\tau_p\tau_y\left(\tau_f+\tau_{\omega f}\right)}{\left(\tau_f+\tau_{\omega f}+\tau_p\right)\left(\tau_f+\tau_{\omega f}+2\tau_p\right)\left(\tau_y+\phi_y^2\tau_x\right)}} < 0. \tag{A13}$$

By (11), direct computation shows

$$\frac{\partial\tau_p}{\partial\tau_{\omega f}} = -\frac{\left[\frac{\tau_p}{\left(\tau_f+\tau_{\omega f}+2\tau_p\right)\left(\tau_f+\tau_{\omega f}+\tau_p\right)}+\frac{1}{\tau_f+\tau_y+\tau_{\omega f}}\right]}{1-\frac{2\tau_p\tau_y\left(\tau_f+\tau_{\omega f}\right)}{\left(\tau_f+\tau_{\omega f}+\tau_p\right)\left(\tau_f+\tau_{\omega f}+2\tau_p\right)\left(\tau_y+\phi_y^2\tau_x\right)}}\frac{2\tau_p\tau_y}{\left(\tau_y+\phi_y^2\tau_x\right)} < 0. \tag{A14}$$

Plugging (A5) into the above equation yields

$$\frac{\partial\tau_p}{\partial\tau_{\omega f}} = -\frac{\frac{\tau_p}{\left(\tau_f+\tau_{\omega f}+2\tau_p\right)\left(\tau_f+\tau_{\omega f}+\tau_p\right)}+\frac{1}{\tau_f+\tau_y+\tau_{\omega f}}}{1-\frac{2\tau_p\tau_y\left(\tau_f+\tau_{\omega f}\right)}{\left(\tau_f+\tau_{\omega f}+\tau_p\right)\left(\tau_f+\tau_{\omega f}+2\tau_p\right)\left(\tau_y+\frac{\tau_p\tau_y}{\tau_y\tau_\xi-\tau_p}\right)}}\frac{2\tau_p\tau_y}{\left(\tau_y+\frac{\tau_p\tau_y}{\tau_y\tau_\xi-\tau_p}\right)},$$

and thus, as $\tau_{\omega f}\to\infty$, we have $\frac{\partial\tau_p}{\partial\tau_{\omega f}}\to 0$ by noting that τ_p is bounded. So, we will have $\frac{\partial RE}{\partial\tau_{\omega a}}\to 1$ as $\tau_{\omega f}\to\infty$.

Now suppose $\tau_{\omega f}=0$ and we consider the process of $\tau_\xi\to 0$ or $\tau_\xi\to\infty$. By equations (11) and (A12), we know that $\phi_y = O(1)$ and $\tau_p = O(\tau_\xi)$. So, combining this order information with equation (A13), we have

$$\left.\frac{\partial\phi_y}{\partial\tau_{\omega f}}\right|_{\tau_{\omega f}=0} = -\frac{\phi_y}{\tau_f+\tau_y}+O(1).$$

Then, by (11), we have

$$\left.\frac{\partial\tau_p}{\partial\tau_{\omega f}}\right|_{\tau_{\omega f}=0} = -\frac{2\phi_y^2\tau_x\tau_y^2}{\left(\tau_y+\phi_y^2\tau_x\right)^2\left(\tau_f+\tau_y\right)}\tau_\xi+o(\tau_\xi),$$

which implies that $\left.\frac{\partial RE}{\partial\tau_{\omega a}}\right|_{\tau_{\omega f}=0} \propto \frac{\partial\left(\tau_f+\tau_{\omega f}+\tau_p\right)}{\partial\tau_{\omega f}} = 1+\left.\frac{\partial\tau_p}{\partial\tau_{\omega f}}\right|_{\tau_{\omega f}=0} > 0$ if and only if τ_ξ is sufficiently small. QED.

Proof of Proposition 7

Speculator i's information set is $\{\tilde{x}_i, \tilde{y}_i, \tilde{\omega}, \tilde{p}\}$. To speculators, the price is equivalent to the following signal

$$\tilde{t}_p \equiv \left(\lambda\sqrt{\tau_x^{-1} + \phi_y^2\tau_y^{-1}} + \phi_p\right)\tilde{p} + g - \phi_\omega\tilde{\omega} = \tilde{a} + \phi_y\tilde{f} + \sqrt{\tau_x^{-1} + \phi_y^2\tau_y^{-1}}\tilde{\xi}.$$

Define $\theta \equiv \left(2 - \frac{\mu_f^2\tau_\omega}{\tau_f+\mu_f^2\tau_\omega+\tau_p}\frac{\mu_a}{\mu_f} - \frac{\tau_p}{\tau_f+\mu_f^2\tau_\omega+\tau_p}\frac{1}{\phi_y}\right)$. Let $\Delta^v \equiv Var\left(\theta\tilde{a} + \tilde{f}\middle| \tilde{x}_i, \tilde{y}_i, \tilde{\omega}, \tilde{t}_p\right)$ and let δ_x, δ_y, $\delta_{\omega 0}$, and δ_{p0} be the loadings of $\tilde{x}_i, \tilde{y}_i, \tilde{\omega}$, and $\tilde{t}_p$ in the expectations $E\left(\theta\tilde{a} + \tilde{f}\middle| \tilde{x}_i, \tilde{y}_i, \tilde{\omega}, \tilde{t}_p\right)$, respectively. Then, we can compute

$$\begin{aligned}
b_0^v &= \left(\log\frac{\beta(1-\beta)}{c}\right) + \frac{1}{2}\frac{1}{\tau_f + \mu_f^2\tau_\omega + \tau_p} + \frac{\Delta^v}{2} + \left(\delta_{p0} + \frac{\tau_p}{\tau_f + \mu_f^2\tau_\omega + \tau_p}\frac{1}{\phi_y}\right)g, \\
b_x^v &= \delta_x, b_y^v = \delta_y, b_\omega^v = \left(\delta_{\omega 0} + \frac{\mu_f^2\tau_\omega}{\tau_f + \mu_f^2\tau_\omega + \tau_p}\frac{1}{\mu_f}\right) - \left(\delta_{p0} + \frac{\tau_p}{\tau_f + \mu_f^2\tau_\omega + \tau_p}\frac{1}{\phi_y}\right)\phi_\omega, \\
\text{and } b_p^v &= \left(\delta_{p0} + \frac{\tau_p}{\tau_f + \mu_f^2\tau_\omega + \tau_p}\frac{1}{\phi_y}\right)\left(\lambda\sqrt{\tau_x^{-1} + \phi_y^2\tau_y^{-1}} + \phi_p\right).
\end{aligned}$$

Note that $\phi_y = \frac{b_y^v}{b_x^v} = \frac{\delta_y^v}{\delta_x^v}$, and that δ_x and δ_y are not affected by parameter λ. Therefore, the equilibrium value of ϕ_y is not affected by λ. Real efficiency implications follow directly from equations (11) and (17). QED.

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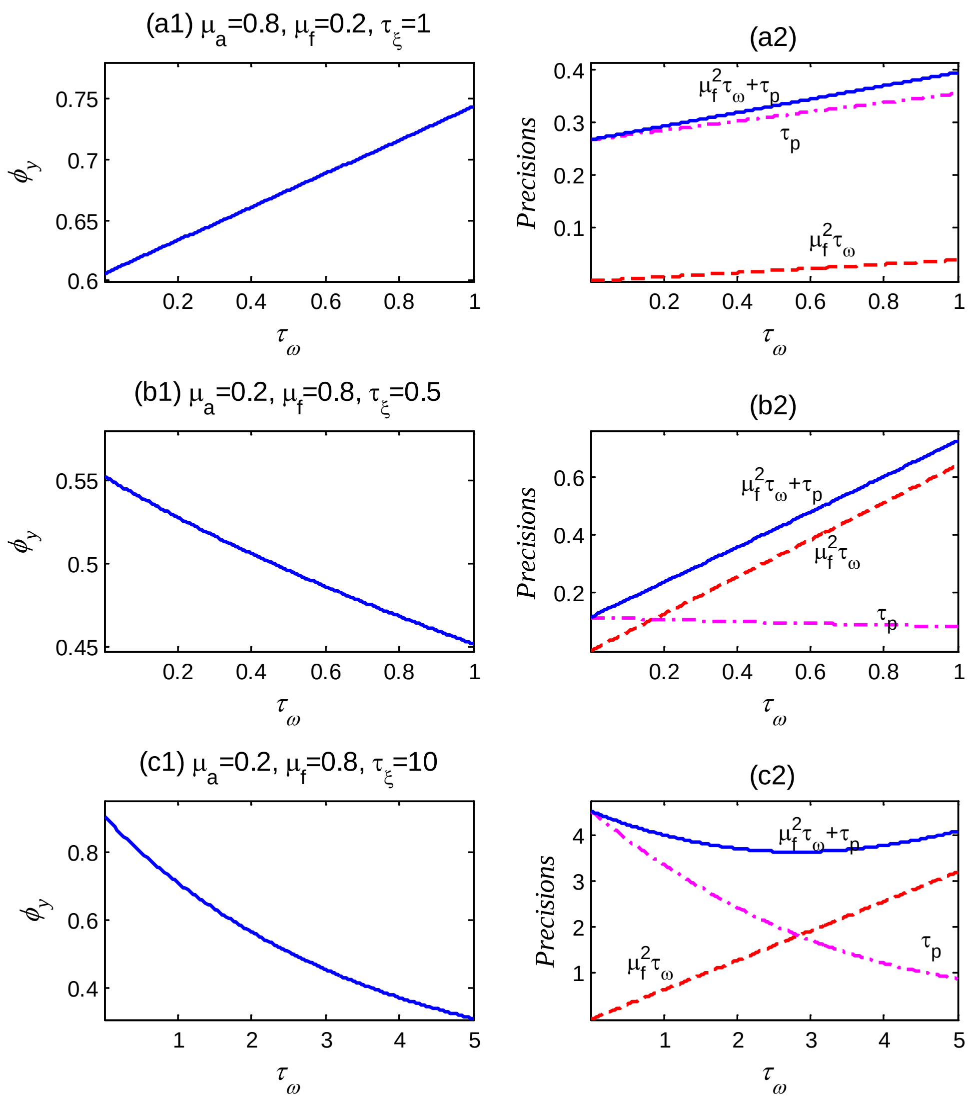
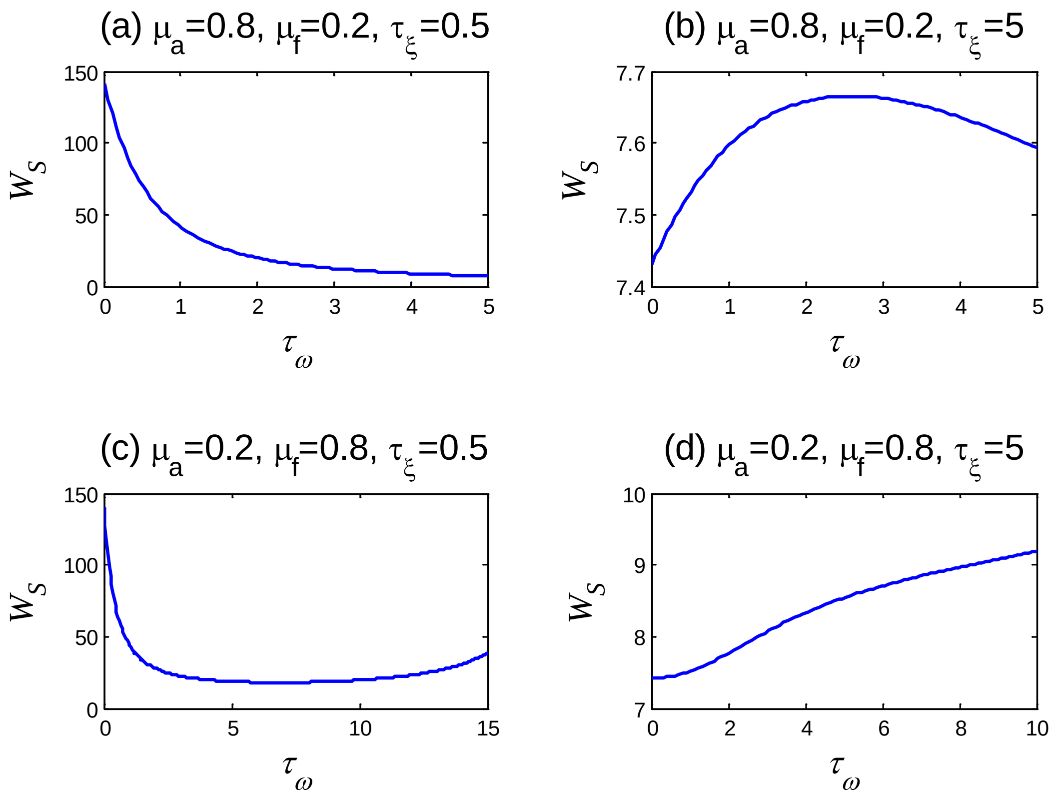
Figure 1: Implications of Disclosure for Trading and Real Efficiency



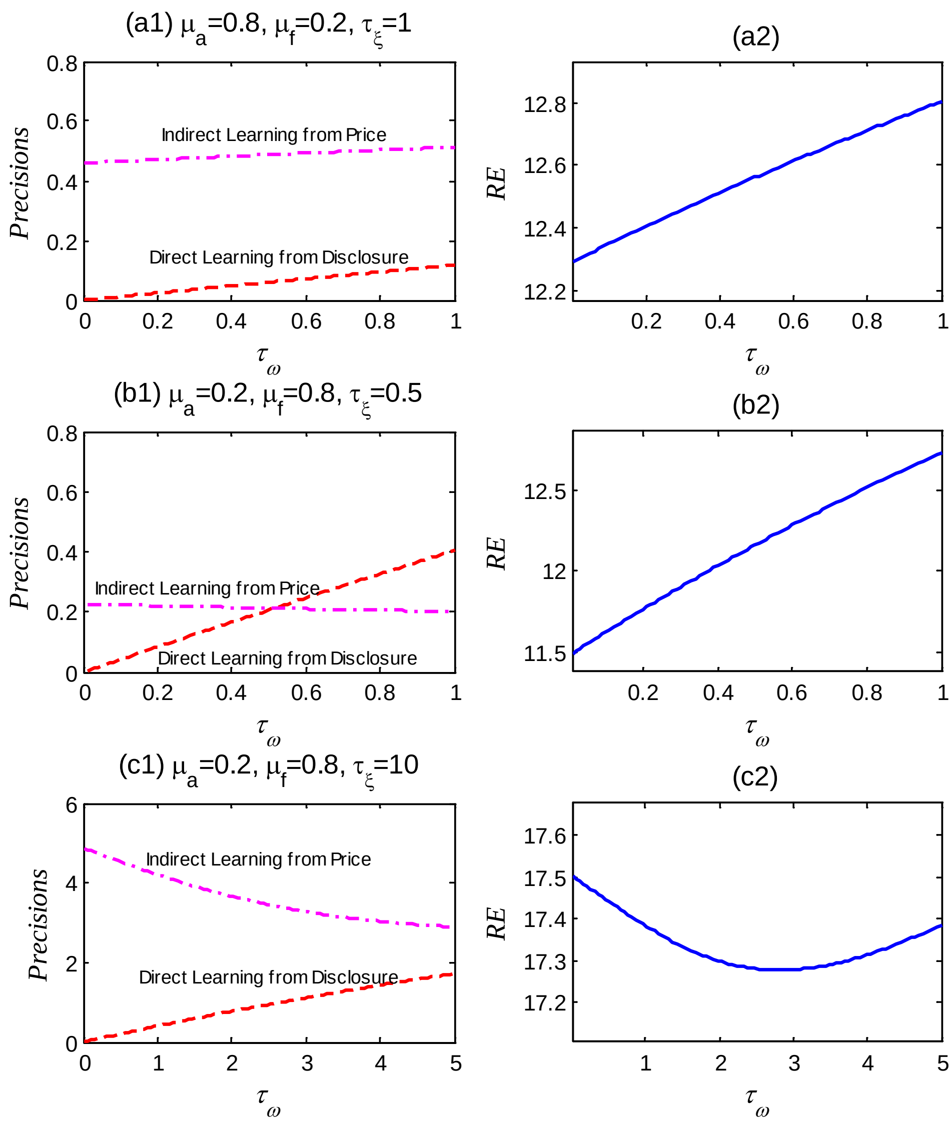
This figure plots the trading and real efficiency implications of public information release in the baseline model. Parameter τ_ω controls the precision of public information. Parameter ϕ_y measures speculators' trading aggressiveness on their private information about factor $\tilde{f}$ that capital providers care to learn. Parameter τ_p is the endogenous precision of the information that capital providers can learn from the price. In all panels, we have set $\tau_a = \tau_f = \tau_y = \tau_s = \tau_\xi = \lambda = 1$. In Panels (a1) and (a2), $\mu_a = 0.8$, $\mu_f = 0.2$, and $\tau_\xi = 1$. In Panels (b1) and (b2), $\mu_a = 0.2$, $\mu_f = 0.8$, and $\tau_\xi = 0.5$. In Panels (c1) and (c2), $\mu_a = 0.2$, $\mu_f = 0.8$, and $\tau_\xi = 10$.

Figure 2: Implications of Disclosure for Speculators' Welfare



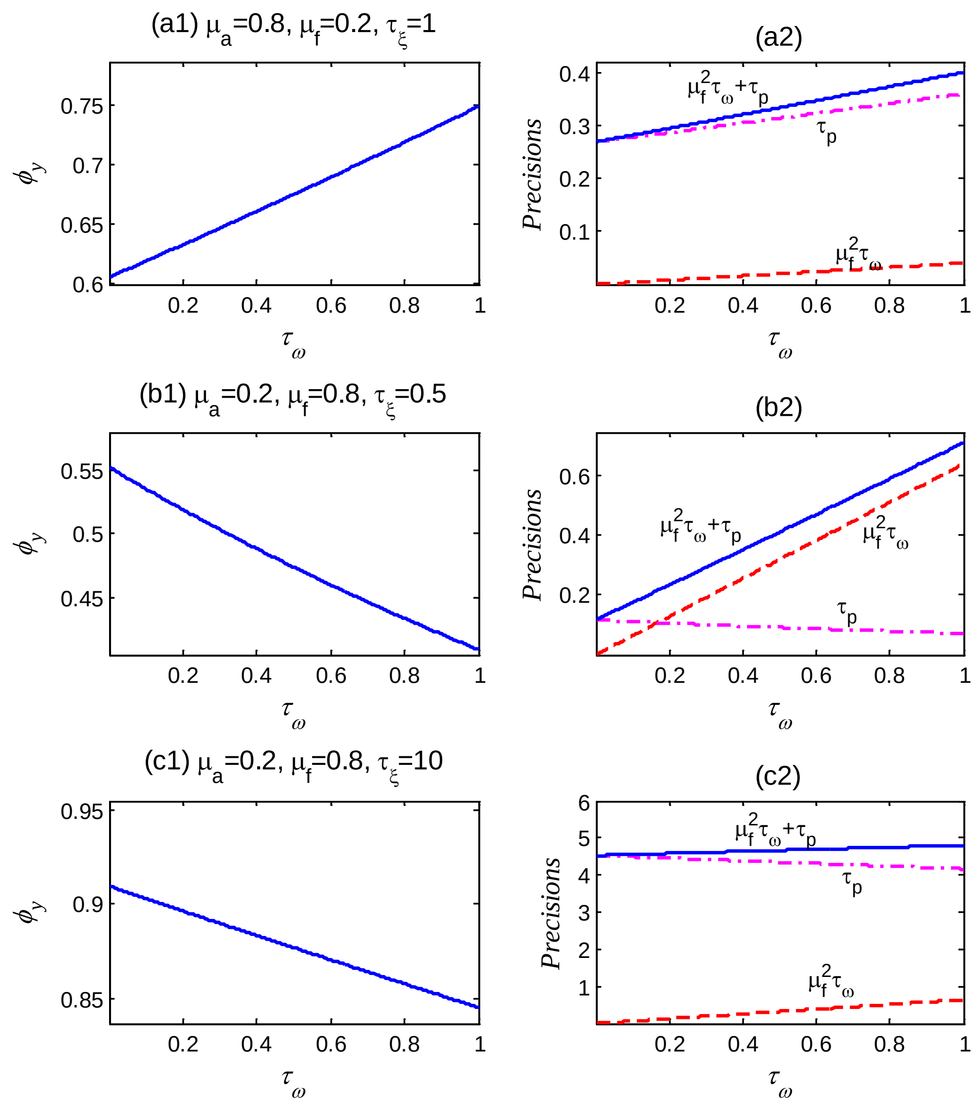
This figure plots implications of public information release for speculators' welfare in the baseline model. Parameter τ_ω controls the precision of public information. The other parameter values are $\tau_a = \tau_f = \tau_x = \tau_y = \lambda = 1$.

Figure 3: Real Efficiency Effect of Disclosure in Economies Where Capital Providers Receive Noisy Signals about Both Factors



This figure plots the real efficiency implications of public information in the extended economies in which capital provider receive noisy signals about factors $\tilde{a}$ and $\tilde{f}$. The left panels plot the direct and indirect effects of disclosure on capital providers' forecast probelm. The right panels plot real efficiency against the precision of public disclosure. In all panels, we have set $\tau_a = \tau_f = \tau_y = \tau_s = \tau_\xi = \lambda = 1$, $\tau_z = 5$, β=1/2, and $c = 1$. In Panels (a1) and (a2), $\mu_a = 0.8$, $\mu_f = 0.2$, and $\tau_\xi = 1$. In Panels (b1) and (b2), $\mu_a = 0.2$, $\mu_f = 0.8$, and $\tau_\xi = 0.5$. In Panels (c1) and (c2), $\mu_a = 0.2$, $\mu_f = 0.8$, and $\tau_\xi = 10$.

Figure 4: Implications of Disclosure for Trading and Real Efficiency in Economies Where Speculators Observe Prices



This figure plots the trading and real efficiency implications of public information release in economies where speculators submit demand schedules. Parameter τ_ω controls the precision of public information. Parameter ϕ_y measures speculators' trading aggressiveness on their private information about factor $\tilde{f}$. Parameter τ_p is the endogenous precision of the information that capital providers can learn from the price. In all panels, we have set $\tau_a = \tau_f = \tau_y = \tau_s = \tau_\xi = \lambda = 1$. In Panels (a1) and (a2), $\mu_a = 0.8$, $\mu_f = 0.2$, and $\tau_\xi = 1$. In Panels (b1) and (b2), $\mu_a = 0.2$, $\mu_f = 0.8$, and $\tau_\xi = 0.5$. In Panels (c1) and (c2), $\mu_a = 0.2$, $\mu_f = 0.8$, and $\tau_\xi = 10$.