Wolf Pack Activism *

Alon Brav† Amil Dasgupta‡ Richmond Mathews§

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Abstract

It is alleged that activist hedge funds congregate around a common target, with one acting as the “lead” activist and others as peripheral activists, or “wolf pack” members. We model this phenomenon as a coordination game, and show that the concentration of capital and skill matters: Holding constant total activist ownership, the presence of a lead activist increases the probability of successful activism due to improved coordination among activists. We model the dynamics of share acquisition by wolf pack members and the lead activist: Block acquisition by the lead activist spurs significant entry by wolf pack members, while the lead activist acquires only if the expected wolf pack is large enough. Finally, we provide predictions concerning which wolf pack activists will buy ahead of the lead activist, and which will wait to acquire until after the lead activist’s stake is announced.

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‡Duke and NBER

§London School of Economics, CEPR, and ECGI

§University of Maryland, College Park
1 Introduction

Activist hedge funds have been the most prominent and successful proponents of institutional shareholder activism in recent decades (Gillan and Starks 2007). They have delivered significant shareholder value both in the short-term (Brav, Jiang, Partnoy, and Thomas, 2008) and in the long-term (Bebchuk, Brav, and Jiang, 2013). Yet, the transformative effect of these activist hedge funds is typically achieved via relatively small holdings: According to Brav, Jiang, and Kim (2010) the median stake of activist hedge funds at the beginnings of an activist campaign is only 6.3%. How do relatively small blockholders have such transformational influence? One possibility is that hedge funds (and other activist shareholders) act in groups: Multiple funds implicitly coordinate together to achieve change in a target firm, thus radically enhancing the power of the lead activist who files the 13D.

Such group activism has already been alleged by market observers, who note that multiple hedge funds or other activist investors sometimes congregate around a common target, with one acting as the “lead” activist and others as peripheral activists, or “wolf pack” members (see, e.g., Briggs (2006)). Since U.S. disclosure rules (Regulation 13D) require investors to file together as a group when their activities are formally coordinated, much of this activity is ostensibly uncoordinated. For example, Briggs (2006) quotes one target manager as saying that “This form of parallel action, driven by numerous independent decisions by like-minded investors, as opposed to explicit cooperation agreements among participants, has allowed hedge funds to avoid being treated as a ‘group’ for purposes of Regulation 13D.” In such situations, peripheral funds may trade before an activism campaign has been announced, by predicting which firms will be targeted, or they may wait until after an initial 13D announcement. In either case, these funds often join together with the lead activist to form a larger voting block and ultimately place greater pressure on target management. Companies have responded to such tactics by, for example, changing the provisions of their shareholder
rights plans, or “poison pills," or by pursuing legal action when they believe investors are, in effect acting as a group and thereby violating disclosure rules. Nathan (2009) writes:

This could be a particularly important issue for public companies because the wolf pack tactic...has been used virtually exclusively by the activist investor community in campaigns against corporations, often culminating in successful proxy contests or other change-of-control events as documented in the CSX case. The market’s knowledge of the formation of a wolf pack (either through word of mouth or public announcement of a destabilization campaign by the lead wolf pack member) often leads to additional activist funds entering the fray against the target corporation, resulting in a rapid (and often outcome determinative) change in composition of the target’s shareholder base seemingly overnight.

Similarly, Smilan, Becker and Holbrook (2006) note that “even large and well-known public companies are vulnerable to attacks by these wolf packs.” They suggest that companies aggressively pursue prosecution of groups of activists that may have violated the group reporting provisions.

In this paper we provide a model of wolf pack activism. We model activism in a target firm by many activist investors—one large activist and many small ones—and the process by which such activist investors build up their equity stake. The stake-building process anticipates the activism stage to follow. Our (static) model of coordinated activism focuses on the interaction between the large activist and the small ones, and highlights the catalytic effect of a lead activist on the strategies of small ones. Our (dynamic) model of block building anticipates the coordinated activism process, and traces how small and large activists anticipate each others’ actions in making their acquisition decisions.
We start our analysis with the activism stage, when the activists’ stakes have already been set. At this stage, each activist must decide whether to “engage” the target, i.e., exert influence (through talking with management, making public statements, proposing new actions, voting, etc.) to try to improve the firm’s decisions, and hence its value. Engaging the target costs each activist something, but also provides a potential private benefit if the activism is successful (in addition to the value appreciation in their shares). Activism is ultimately successful if the number of activists who choose to engage is sufficient to overcome the level of insider entrenchment, which is ex ante random. Thus, the activists play a coordination game in which each player’s potential payoff is increasing in the number of other activists who engage. Such coordination games generally admit multiple equilibria, so, using insights from the global games literature (Carlsson and van Damme 1993, Morris and Shin 1998), we assume each activist receives a noisy signal of the firm’s level of entrenchment prior to choosing whether to engage. Allowing the noise to become small results in a unique equilibrium outcome, in which activism is successful for all levels of entrenchment below some critical level, and is unsuccessful otherwise.

Our analysis of the coordination game builds on the methodology introduced by Corsetti, Dasgupta, Morris, and Shin (2004) and focuses on how the presence of the large activist, who has potentially better information, and also potentially higher costs of engagement and higher private benefits to success, affects the coordination outcome. The main result is that, holding the aggregate size of all activists holdings constant, the presence of a large activist increases the level of coordination and leads to value-increasing activism more often. We call this the “Coordination Effect” of the large activist. An implication of this result is that, even when a significant number of shares are held by potential activists, the arrival of a “lead” activist who holds a larger block may be a necessary catalyst for a successful campaign, which is consistent with the activist strategies that are well documented in the empirical literature.
Modeling activism as a coordination game also sheds important light on the importance of the “wolf pack” of small activists whose actions ultimately support the lead activist. In particular, our analysis of the earlier stake acquisition process reveals an important effect of the availability of wolf pack members on the lead activist’s willingness to buy a stake. In particular, the larger is the wolf pack the activist can expect to exist at the time of the campaign, the more likely it is that buying a stake will be profitable given the activist’s opportunity cost of tying up capital. Note that this is true even though, in our model, there are no trading profits associated with activist purchases (i.e., we assume that the passive shareholders know that any given activist is buying, and can accurately assess the probability of an ex post successful campaign). This result provides a foundation for the findings of Brav et al (2008) that high institutional ownership is a cross-sectional predictor of 13D filings.

The dynamics of our share acquisition game also provide direct theoretical foundations for the phenomena noted by Nathan (2009): In our model, the acquisition of a position by the large activist (in effect, a 13D filing) precipitates the immediate entry of a significant additional number of small shareholder activists. While these activists know about the potential for activism at the firm before the lead activist buys in, other attractive uses of funds keep them from committing capital to the firm before they are sure that a lead activist will emerge. Others with lower opportunity costs may be willing to buy in earlier, as the real (but smaller) chance of successful engagement in the absence of a lead activist provides sufficient potential returns. Thus, our model predicts that late entrants to activism will be those who have relatively higher opportunity costs of tying up capital. One potential way to interpret this is that more concentrated, smaller, and more “specialized” vehicles (such as other activist funds) may be more inclined to acquire a stake only after the filing of a 13D by a lead activist. This, again, is in keeping with Nathan’s description.

Our analysis is related to the past theoretical literature on the influence of block-
holders in corporate governance. Papers in this literature tend to focus either on blockholders who, like here, exercise “voice” by directly intervening in the firm’s activities (Shleifer and Vishny, 1986; Kyle and Vila, 1991; Burkart, Gromb, and Panunzi, 1997; Bolton and von Thadden, 1998; Maug, 1998; Kahn and Winton, 1998; Faure-Grimaud and Gromb, 2004), or those who use informed trading, also called “exit,” to improve stock price efficiency and encourage correct actions by managers (Admati and Pfleiderer, 2009; Edmans, 2009). Dasgupta and Piacentino (2014) show that the ability to use exit as a governance mechanism is hindered when the blockholder is a flow-motivated fund manager. Some other papers suggest that blockholders improve decisions by directly providing information to decision makers (see Cohn and Rajan, 2012; Edmans, 2011). Our paper is distinct from all of these in its focus on implicit coordination between different block investors in their value creating activity.

Several existing papers discuss the implications of having multiple blockholders, but from very different perspectives. Zwiebel (1995) models the sharing of private control benefits as part of a coalitional bargaining game, and derives the equilibrium number and size of blockholders who try to optimally capture these benefits. Noe (2002) studies a model in which strategic traders may choose to monitor management, which improves value. In the model, monitoring activities by different investors are perfect substitutes (i.e., if any one investor monitors, the full improvement in value is achieved), and the strategic investors play mixed strategies, where they generally mix between monitoring and buying vs not monitoring and selling. Instead of studying coordination among these monitors, therefore, the paper’s focus is on showing that there can be multiple monitors despite the substitutability because of the financial market trading opportunities. Attari, Banerjee, and Noe (2006) show that institutional investors may strategically “dump” shares to induce activists to buy and then intervene directly in the firm’s management. There the different blockholders play very distinct roles, as only the activist’s direct intervention matters for the governance outcome. Edmans
and Manso (2011) model a group of equal-size block holders and ask whether their impact on corporate governance through both exit and voice is larger or smaller than if the same block were held by a single entity. Their main result is that while having a disaggregated stake makes voice less productive due to free rider problems, it helps make the exit channel more effective since the blockholders trade more aggressively when competing for trading profits. We take a very different perspective, asking how the activities of blockholders of different size affect their ability to coordinate around a target, and how it affects their initial decision to buy a block.

2 The Model

Consider a publicly traded firm which is a potential target for shareholder activists, i.e., if activist investors are present and engage with management—and if such an engagement is successful—there will be an increase in firm value. The firm currently does not have a measurable presence of activist investors. Of the total outstanding equity, which we normalize to measure 1, a measure $1 - \bar{A} \in (0, 1)$ is owned by entrenched shareholders. These shareholders do not trade their shares and are opposed to change in the firm. The degree to which they are entrenched is measured by $\eta$ where $\eta \sim N(\mu_\eta, \sigma^2_\eta)$. Denote by $\alpha_\eta = \frac{1}{\sigma^2_\eta}$ the precision of $\eta$. For example, $\eta$ could be determined jointly by what percentage of these shares are owned by management and by other corporate charter provisions limiting the efficacy of any activist engagement. The remainder, measure $\bar{A}$ of shares, is initially held by passive non-entrenched shareholders. These shareholders are not willing to engage in activism directly but are willing to sell their shares to potential activists shareholders at fair value. Thus, the maximum measure of potential activists who may hold shares in this firm is $\bar{A} < 1$.

There is a large activist fund manager, $L$, whom we shall also refer to as the large activist. All quantities relating to $L$ are subscripted $L$. If $L$ is available for activism (which occurs with probability $p_L$) then he enters the model at a date that we label
$t = 1$ and considers whether to acquire a stake in the firm. $L$ faces a capital constraint $A_L << \bar{A}$. Conditional on being available for activism, $L$ has an opportunity cost of capital $k_L$. If $L$ is not available for activism, nothing happens at $t = 1$. The events at $t = 1$ are publicly observed.

There is a continuum of small activists of measure 1. All quantities relating to small activists are subscripted $s$. These activists are aware that there is a date $t = 1$ when $L$ may enter and establish a position in the firm. These activists may, in turn, purchase shares in the firm, either before they know whether $L$ will be available for activism, at a date that we label $t = 0$, or after they know whether $L$ is available for activism and whether she has established a position in the firm, i.e., at some date $t = 2$. Each activist may only acquire shares once, but those small activists who do not acquire shares at $t = 0$ have the option of acquiring shares at $t = 2$. Each small activist has an opportunity cost of acquiring a share in the firm: Small activist $i$ has opportunity cost $k^i_s$, where $k^i_s$ is distributed $U[0, \bar{k}]$ for some $\bar{k} > 0$ in the population of small activists.

At some later date $t = 3$, each activist, whether small or large, has the option of engaging ($a_s = E$ or $a_L = E$) or not engaging ($a_s = N$ or $a_L = N$) firm management in order to induce value enhancing changes in the firm. The engagement can be successful or not. If the engagement is successful, the price of the firm’s shares rises to $P_1$. If the engagement fails, the price of the firm’s shares falls to $P_0$. Not engaging is a costless action for both large and small activists.

A small activist who does not engage receives a payoff of $P_1$ if any engagement by others is successful, capturing a free rider benefit, and a payoff of $P_0$ otherwise. Engagement entails a private cost of $c_s$, which we interpret to be effort cost required for engagement. This may represent the effort of formulating and articulating arguments for changes in target strategy, or—in the case of a campaign led by a large activist—the effort of conducting research to support the effort of the lead activist and of credibly communicating support for the campaign to target management. A small activist who
does engage receives a payoff of $\beta_s + P_1 - c_s$ if the engagement is successful, where $\beta_s > c_s$ measures the excludable benefits earned by participants in a successful activist engagement. For example, if an activist campaign succeeds in appointing new board members, these board members are more likely to be friendly to those activists who installed them. Further, successful engagement may also endow small activists with soft information and also earn them the kinship of other activist investors, including the large activist if present. If the campaign fails, the payoff to a small activist who engages is $P_0 - c_s$. We assume that $\beta_s - c_s \leq \overline{k}$, that is, there exist some small activists for whom the returns to activism are dominated by their opportunity costs.

If the large activist does not engage she receives a payoff of $A_L P_1$ if any engagement by others is successful, and a payoff of $A_L P_0$ otherwise. Engagement entails a private effort cost of $c_L$. This may represent effort spent on pressuring management via discussion, visible publicity campaigns, and proxy proposal formulation and sponsorship. If the large activist engages she receives a payoff of $\beta_L + A_L P_1 - c_L$ if the engagement is successful, where $\beta_L > c_L$ again represents the excludable benefits earned from successful engagement. In addition to incorporating the interpretations offered above for $\beta_s$, $\beta_L$ can be interpreted to include the reputational benefits that accrue to a large activist hedge fund manager from leading a successful activist campaign. Such reputational benefits may enable successful activist fund managers to attract investor capital in the future. If the campaign fails, the payoff to the large activist who engages is $A_L P_0 - c_L$.

Our model requires no restriction on the relative values of $\beta_L$ and $\beta_s$ and of $c_L$ and $c_s$. However, we believe that a natural interpretation is that $\beta_L$ and $c_L$ are larger than $\beta_s$ and $c_s$ respectively. This is because leading an activist campaign is likely to be both more costly and more rewarding than simply participating in one.

The success or failure of any engagement is determined by whether the collective support for engagement is sufficient to overcome the opposition of entrenched shareholders. Let $A_s^0$ denote the mass of small activists who acquire shares in the firm at
$t = 0$, $A_s^2$ the mass of small activists who acquire shares in the firm at $t = 2$, and thus $A_s = A_s^0 + A_s^2$ the total mass of small activists present. Thus, overall the total mass of activists present at $t = 3$ is

$$A = \begin{cases} A_s & \text{if } L \text{ does not acquire a block} \\ A_L + A_s & \text{otherwise} \end{cases}.$$  \hfill (1)

Let $e$ denote the total mass of activist shares that engage and $e_s$ the proportion of small activist who engage. Thus:

$$e = \begin{cases} A_s e_s & \text{if } L \text{ does not engage} \\ A_L + A_s e_s & \text{otherwise} \end{cases}.$$  \hfill (2)

An engagement succeeds if $e \geq \eta$ and fails otherwise.

While our success condition, $e \geq \eta$, is naturally interpreted as engagement vs entrenchment, it accommodates other, broader, interpretations. Any mechanism where a higher level of engagement makes value enhancement more likely, given a set of firm fundamentals, can be accommodated in the model. For example, imagine that value enhancement is achieved via a restructuring of the target firm, and that some firms are more complicated to restructure than others. Imagine that activists bring a plethora of restructuring skills to the table. The greater the measure of activists who engage, the broader the set of skills that are brought to the restructuring process. Thus, $\eta$ could be reinterpreted as the complexity of restructuring: Engagement is successful if sufficiently many activists, and thus a group with a sufficiently rich set of skills, engage with the target firm given its fundamental level of complexity.

If $\eta$ is common knowledge, then for each $\eta \in (1_L, A_L, A)$, where $1_L$ is an indicator function equaling one if the lead activist has bought a stake and zero otherwise, there exist multiple pareto ranked equilibria with full engagement or no engagement. If $\eta < 0$ it is dominant to engage. If $\eta > A$ it is dominant not to engage. We shall sometimes refer to these regions of $\eta$ where one action choice or another is dominant as dominance regions.
Activists who have acquired a position in the firm observe $\eta$ with small amounts of idiosyncratic noise at the beginning of $t = 3$. The noise in observing entrenchment can be thought to be the result of (potentially imperfect) due diligence (research) carried out by each activist into the target firm. Each small activist $i$ receives a private signal $x_{s,i} = \eta + \sigma_s \epsilon_i$ where $\epsilon_i$ is standard normal, independent of $\eta$ and iid across small activists. Denote $\alpha_s = 1/\sigma_s^2$, the precision of each small activist’s signal. The large activist receives a private signal $x_{L,i} = \eta + \sigma_L \epsilon$ where $\epsilon$ is standard normal, independent of $\eta$ and of the $\epsilon_i$’s. Denote $\alpha_L = 1/\sigma_L^2$, the precision of the large activist’s signal.

We now solve the game by backward induction. We first take as given the activist presence in the firm, and solve for the activism game at $t = 3$. Subsequently, we solve for the endogenous stake purchase decisions of each type of activist.

3 Activism

Given the bipartite characterization of $A$ in (1) our backward induction solution requires that we solve two versions of the activism game, without and with a large activist.

3.1 Only small activists: $A = A_s$

We look for equilibria in threshold strategies: Each small activist $i$ engages if and only if his private signal $x_{s,i}$ is weakly below some threshold $x_s^*$.

Since $x_{s,j}|\eta \sim N(\eta, \sigma_s^2)$, if activists follow such strategies, then, for each $\eta$, the measure of engagement is given by $A_s \Pr(x_{s,j} \leq x_s^*|\eta) = A_s \Phi\left(\frac{x_s^* - \eta}{\sqrt{\alpha_s}}\right)$. Thus, engagement is successful if and only if

$$A_s \Phi\left(\frac{x_s^* - \eta}{\sqrt{\alpha_s}}\right) \geq \eta.$$ 

The LHS is decreasing in $\eta$, the RHS is increasing in $\eta$, and there exists $\eta_s^*$ such that
engagement is successful if and only if \( \eta \leq \eta^*_s \), where \( \eta^*_s \) is defined by

\[
A_s \Phi \left( \sqrt{\alpha_s} (x^*_s - \eta^*_s) \right) = \eta^*_s. 
\] (3)

Given this, the expected payoff of any activist \( j \) from engaging is given by

\[
\Pr (\eta \leq \eta^*_s | x_{s,j}) (\beta_s + P_1) + (1 - \Pr (\eta \leq \eta^*_s | x_{s,j})) P_0 - c_s, 
\]

whereas the expected payoff from not engaging is given by

\[
\Pr (\eta \leq \eta^*_s | x_{s,j}) P_1 + (1 - \Pr (\eta \leq \eta^*_s | x_{s,j})) P_0. 
\]

Thus, the net expected payoff from engagement is given by

\[
\Pr (\eta \leq \eta^*_s | x_{s,j}) \beta_s - c_s 
\]

which is clearly decreasing in \( x_{s,j} \). The existence of the dominance regions and continuity jointly imply that there exists \( x^*_s \in \mathbb{R} \) such that

\[
\Pr (\eta \leq \eta^*_s | x^*_s) \beta_s - c_s = 0. 
\]

Further, since \( \eta | x_{s,j} \sim N \left( \frac{\alpha_x \mu + \alpha_s x_{s,j}}{\alpha_y + \alpha_s}, \frac{1}{\alpha_y + \alpha_s} \right) \), we have the following condition:

\[
\Phi \left( \sqrt{\alpha_x} + \alpha_s \left( \eta^*_s - \frac{\alpha_x \mu + \alpha_s x^*_s}{\alpha_y + \alpha_s} \right) \right) = \frac{c_s}{\beta_s}. 
\] (4)

Solving (3) for \( x^*_s \) gives

\[
x^*_s = \eta^*_s + \frac{1}{\sqrt{\alpha_s}} \Phi^{-1} \left( \eta^*_s \frac{A_s}{A_s} \right). 
\]

Substituting into (4) gives:

\[
\Phi \left( \sqrt{\alpha_x} + \alpha_s \left( \eta^*_s - \frac{\alpha_x \mu + \alpha_s \left( \eta^*_s + \frac{1}{\sqrt{\alpha_s}} \Phi^{-1} \left( \eta^*_s \frac{A_s}{A_s} \right) \right)}{\alpha_y + \alpha_s} \right) \right) = \frac{c_s}{\beta_s}, 
\]

i.e.,

\[
\Phi \left( \eta^*_s \frac{\alpha_x}{\sqrt{\alpha_x} + \alpha_s} - \frac{\alpha_x \mu}{\sqrt{\alpha_x} + \alpha_s} - \frac{\sqrt{\alpha_s}}{\sqrt{\alpha_x} + \alpha_s} \Phi^{-1} \left( \eta^*_s \frac{A_s}{A_s} \right) \right) = \frac{c_s}{\beta_s}. 
\]
Taking the limit as $\alpha_s \to \infty$ (i.e., noise in observing $\eta$ vanishes, we have

$$\Phi \left( -\Phi^{-1} \left( \frac{\eta^*_s}{A_s} \right) \right) = \frac{c_s}{\beta_s},$$

so that

$$\eta^*_s = A_s \left( 1 - \frac{c_s}{\beta_s} \right).$$

We have just proved:

**Proposition 1** In the limit as $\alpha_s \to \infty$, the unique monotone equilibrium of the activism game when only small activists are present is given by:

$$x^*_s = \eta^*_s = A_s \left( 1 - \frac{c_s}{\beta_s} \right).$$

The uniqueness result above stands in sharp contrast to the multiplicity in the game with common knowledge. It may seem surprising since the information held by activists about $\eta$ in the limit as $\alpha_s \to \infty$ is identical to that of the game in which $\eta$ is commonly known. To appreciate the difference between the two cases, it is important to recognize that the payoffs of any given activist are determined jointly by the $\eta$ (an exogenous variable) and $e$ (the endogenous measure of other activists who engage). Thus both uncertainty about $\eta$, i.e., uncertainty about firm fundamentals, and uncertainty about the actions of other activists, i.e., *strategic uncertainty*, is relevant to each activist. When $\eta$ is common knowledge, it is clear that there is neither uncertainty about firm fundamentals nor strategic uncertainty. In $\alpha_s \to \infty$ limit, there is again, no uncertainty about firm fundamentals. However, interestingly, strategic uncertainty does *not* vanish in the $\alpha_s \to \infty$ limit. As $\alpha_s \to \infty$, each activist remains highly uncertain about his *relative* ranking in the population of activists. In particular, each activist has *uniform* beliefs over the *proportion* of activists who have received signals about $\eta$ which are lower than his own. A discussion of the theoretical foundation for this result can be found in Morris and Shin (2002).
Using this characterization of strategic uncertainty delivers an alternative method for computing the threshold $\eta^*_s$, as follows. The activist with signal $x^*_s$ must be indifferent between engaging and not engaging. Further, all activists with signals lower than his will wish to engage. Thus, the proportion of agents with signals lower than his is simply $e_s$. In the limit as $\alpha_s \rightarrow \infty$, the activist with signal $x^*_s$ believes that $e_s \sim U(0, 1)$. Then, this activist’s evaluation of the probability of successful engagement, $\Pr (e_s A_s \geq \eta^*_s)$, can be rewritten as $1 - \frac{\eta^*_s}{A_s}$, giving this to the indifference condition:

$$\beta_s \left(1 - \frac{\eta^*_s}{A_s}\right) = c_s,$$

which immediately implies that $\eta^*_s = A_s \left(1 - \frac{c_s}{\beta_s}\right)$, as above.

### 3.2 Large and small activists: $A = A_L + A_s$

The equilibrium is now characterized by four threshold parameters. Define $x^*_s$ as before as the signal threshold of the small activists. Define $x^*_L$ as the signal threshold of the large activist. The threshold level of entrenchment for engagement success is clearly dependent on whether the large activist engages or not, a binary variable. Thus, there are two threshold engagement levels: If the large activist does engage we denote the threshold level by $\eta^*_L$ whereas if he does not we denote the threshold level by $\eta^*_L$.

We now can write down the four conditions that jointly define these thresholds. The first three are immediate, by analogy to the case of small activists only:

$$A_s \Phi \left(\sqrt{\alpha_s} \left(x^*_s - \eta^*_L\right)\right) = \eta^*_L$$

$$A_L + A_s \Phi \left(\sqrt{\alpha_s} \left(x^*_s - \eta^*_L\right)\right) = \eta^*_L$$

$$\Phi \left(\sqrt{\alpha_s + \alpha_L} \left(\eta^*_L - \frac{\alpha_s \mu_s + \alpha_s x^*_L}{\alpha_s + \alpha_L}\right)\right) = \frac{c_L}{\beta_L}. $$

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In this equilibrium, engagement succeeds with or without the large activist if \( \eta \leq \eta_L \) and if \( \eta > \eta_L \), it succeeds only if the large activist engages. Thus, engagement succeeds in that \( \left( \eta \leq \eta_L \right) \cup \left( \eta > \eta_L \cap x_L \leq x_L^* \right) \). Thus, since the marginal small activist who receives signal \( x_s^* \) must be indifferent between engaging or not, we have:

\[
\Pr \left( \eta \leq \eta_L \mid x_s^* \right) + \Pr \left( \eta \in \left( \eta_L, \eta_L^* \right), x_L \leq x_L^* \right) = \frac{c_s}{\beta_s},
\]
i.e.,

\[
\Phi \left( \sqrt{\alpha_{\eta} + \alpha_s} \left( \eta_L - \frac{\alpha_{\eta} \mu_L + \alpha_s x_s^*}{\alpha_{\eta} + \alpha_s} \right) \right) + \int_{\eta_L}^{\eta_L^*} \Phi \left( \sqrt{\alpha_L (x_L^* - \eta)} \right) \sqrt{\frac{\alpha_L}{2\pi} e^{-\frac{(\eta - \frac{\alpha_{\eta} \mu_L + \alpha_s x_s^*}{\alpha_{\eta} + \alpha_s})^2}{2}}} d\eta = \frac{c_s}{\beta_s}. \tag{8}
\]
Solving (7) gives

\[
\eta_L^* = \frac{\alpha_{\eta} \mu_L + \alpha_L x_L^*}{\alpha_{\eta} + \alpha_L} + \frac{1}{\sqrt{\alpha_{\eta} + \alpha_L}} \Phi^{-1} \left( \frac{c_L}{\beta_L} \right)
\]
\[
= \frac{\alpha_{\eta} \mu_L + \alpha_L x_L^*}{\alpha_{\eta} + \alpha_L} + \frac{1}{\sqrt{\alpha_{\eta} + \alpha_L}} \Phi^{-1} \left( \frac{c_L}{\beta_L} \right).
\]

Now, taking \( \alpha_L \to \infty \) (i.e., letting the large activist’s signal noise vanish) we have \( x_L^* = \eta_L^* \), which then means that for \( \eta < \eta_L^* \), \( \Phi \left( \sqrt{\alpha_L (x_L^* - \eta)} \right) \to 1 \), and thus (8) reduces to

\[
\Pr \left( \eta \leq \eta_L^* \mid x_s^* \right) = \frac{c_s}{\beta_s},
\]
i.e.,

\[
\Phi \left( \sqrt{\alpha_{\eta} + \alpha_s} \left( \eta_L^* - \frac{\alpha_{\eta} \mu_L + \alpha_s x_s^*}{\alpha_{\eta} + \alpha_s} \right) \right) = \frac{c_s}{\beta_s}. \tag{9}
\]

Intuitively, when the large activist is very well informed, she always engages whenever \( \eta \leq \eta_L^* \), because engagement will succeed given her participation for such entrenchment levels. Solving \( x_s^* \) from (6) gives

\[
x_s^* = \eta_L^* + \frac{1}{\sqrt{\alpha_s}} \Phi^{-1} \left( \frac{\eta_L^* - A_L}{A_s} \right).
\]
Substituting into (9) gives
\[
\Phi \left( \sqrt{\frac{\alpha_\eta + \alpha_s}{\alpha_\eta + \alpha_s}} \left( \eta^*_L - \frac{\alpha_\eta \mu_\eta + \alpha_s \left( \eta^*_L + \frac{1}{\sqrt{\alpha_\eta}} \Phi^{-1} \left( \frac{\eta^*_L - A_L}{A_s} \right) \right)}{\alpha_\eta + \alpha_s} \right) \right) = \frac{c_s}{\beta_s},
\]
i.e.,
\[
\Phi \left( \eta^*_L \frac{\alpha_\eta}{\sqrt{\alpha_\eta + \alpha_s}} - \frac{\alpha_\eta \mu_\eta \sqrt{\alpha_\eta + \alpha_s} - \alpha_s \sqrt{\alpha_\eta + \alpha_s}}{\alpha_\eta + \alpha_s} \Phi^{-1} \left( \frac{\eta^*_L - A_L}{A_s} \right) \right) = \frac{c_s}{\beta_s}.
\]
Taking the limit as \( \alpha_s \to \infty \) (i.e., noise in observing \( \eta \) vanishes), we have
\[
\Phi \left( -\Phi^{-1} \left( \frac{\eta^*_L - A_L}{A_s} \right) \right) = \frac{c_s}{\beta_s},
\]
so that
\[
\eta^*_L = A_L + A_s \left( 1 - \frac{c_s}{\beta_s} \right).
\]

To summarize:

**Proposition 2** In the ordered limit as \( \alpha_L \to \infty \) and \( \alpha_s \to \infty \), the unique monotone equilibrium of the activism game when large and small activists are both present is given by:
\[
x^*_s = x^*_L = \eta^*_L = A_L + A_s \left( 1 - \frac{c_s}{\beta_s} \right).
\]

We have taken the ordered limit as \( \alpha_L \to \infty \) and \( \alpha_s \to \infty \), which can be reinterpreted as the limit as \( \alpha_s \to \infty, \alpha_L \to \infty, \) and \( \frac{\alpha_L}{\alpha_s} \to \infty \), i.e., this is the case where noise vanishes but the large activist is much better informed than the small activists. This is the natural case to consider. Note that since it is clear that \( \eta^*_L > \eta_L \) and the large activist engages whenever \( \eta \leq \eta^*_L \), the large activist also engages whenever \( \eta \leq \eta_L \), making \( \eta_L \) an irrelevant part of equilibrium. Below we work only with \( \eta^*_L \).

### 3.3 Isolating the impact of the large activist: The Coordinating Effect

Does the presence of a large activist have a tangible effect on the probability of successful engagement over and above the impact arising from the presence of dispersed
activists? In order to isolate the potential effect cleanly we must control for total activist holdings. In other words, we must consider the change in the efficacy of activism when, for a given total activist holding, we replace the large activist by an equal measure of dispersed activists.

In our dynamic model, the share acquisition decisions of small activists at \( t = 0 \) anticipate the potential arrival of the large activist which—if it occurs—may potentially spur further share acquisitions by other dispersed activists. Thus, fixing an initial set of parameters, it is never the case in equilibrium that the total size of the activist base is identical with and without the presence of the large activist. Nevertheless, our model provides the basis for carrying out a comparative statics exercise which pinpoints the impact of the large activist: We compare the efficacy of activism under two potential ownership structures. Under the first ownership structure there are only small activists in a total measure \( A^T \) (i.e., \( A_s = A^T \)). Under the second ownership structure a measure \( A_L \) of the small activists are replaced by the single large activist \( L \), so that \( A_s + A_L = A^T \). By using Propositions 1 and 2, we can compare the entrenchment levels below which activism succeeds under the two ownership structures:

**Corollary 3** There exists a range of entrenchment levels of measure \( A_L \frac{\alpha}{\beta} \) for which engagement is successful in a target firm if and only if a large activist is present.

The result follows from comparing \( \eta^*_s \) (for \( A_s = A^T \)) and \( \eta^*_L \) (for \( A_s = A^T - A_L \)):

\[
\eta^*_L - \eta^*_s = A_L + (A^T - A_L) \left( 1 - \frac{c_s}{\beta_s} \right) - A^T \left( 1 - \frac{c_s}{\beta_s} \right) = A_L \frac{c_s}{\beta_s} > 0.
\]

In words, fixing the size of the activist base, if a measure of dispersed activists is replaced by a single large activist, activism becomes more effective. To appreciate the forces behind this result, let us compare the engagement threshold of the small activists. Under the ownership structure with only small activists, this engagement threshold is \( A^T \left( 1 - \frac{c_s}{\beta_s} \right) \), i.e., small activists will engage only when they (correctly) believe \( \eta < A^T \left( 1 - \frac{c_s}{\beta_s} \right) \). Under the alternative ownership structure where a measure
of small activists are replaced by a single large activist, the engagement threshold rises to \( A_L + (A^T - A_L) \left( 1 - \frac{\alpha}{\beta} \right) \). In other words, the presence of a well-informed large activist in their midst makes small activists more aggressive in their engagement strategy: The presence of a large activist has a coordinating effect on small activists.

4 Late Entry

Small activists who did not acquire a position in the firm at \( t = 0 \) have the option of doing so at \( t = 2 \). The strategy of small activists at \( t = 2 \) are conditioned on the actions of the large activist, who chooses at \( t = 1 \), and on their private opportunity cost of capital, \( k^i_s \). Since the incentive to acquire is decreasing in \( k^i_s \), we focus on strategies in which small activists acquire if and only if their opportunity cost \( k^i_s \) is below some threshold value, i.e., monotone strategies (as in the activism game). Accordingly, we characterize two thresholds: \( k^*_2 (A_L) \) and \( k^*_2 (0) \), representing the cases where the large activist holds a position in the firm and where she does not, respectively.

What about the small activists who acquired shares at \( t = 0 \), before knowing whether \( L \) would enter or not? Using the same reasoning as above, denote the threshold for purchase at \( t = 0 \) by \( k^*_0 \). We guess (and later verify) that \( k^*_0 \leq \min \{ k^*_2 (A_L), k^*_2 (0) \} \), i.e., it is only activists with strictly lower opportunity costs who shall choose to acquire positions before they know whether \( L \) enters or not. Further, we assume that if any activist is indifferent between entry at \( t = 0 \) and \( t = 2 \), they enter at \( t = 0 \). For example, this could be because there are small trading profits available if these activists trade prior to the 13D announcement because they are better able than passive holders to predict the availability of the lead activist. For parsimony, we do not model this asymmetric information trading game, but we believe it would not significantly alter the model’s qualitative results.

By definition, activists who acquire a position in the firm at any date \( t \), purchase their shares from passive shareholders. Since these passive shareholders are rational,
share the same information at the point of acquisition as the small activist (recall that the activists’ private signals are only received at the beginning of \(t = 3\)), and are only willing to trade at fair value, the sole source of gains for activists arises from their net private rents \((\beta_s - c_s)\) from successful activism. In turn, since the activism game at \(t = 3\) is played with vanishing noise, small activists engage only when engagement is successful. Thus, they receive \(\beta_s - c_s\) in the event that engagement is successful and nothing otherwise. Engagement succeeds whenever the level of entrenchment is below the relevant threshold, which in turn depends on the size of the activist base.

In case \(L\) is present, under our maintained hypothesis that \(k_0^* \leq \min \{k^* (A_L), k^* (0)\}\), the mass of activists is given by \(A_L + \frac{k_2^*(A_L)}{k}\), where \(A_s = \frac{k_2^*(A_L)}{k} = \Pr (k_s^* \leq k^* (A_L))\). Given this mass of activists, Proposition 2 implies that the entrenchment threshold in the activism game is \(A_L + \frac{k_2^*(A_L)}{k} \left(1 - \frac{c_s}{\beta_s}\right)\), so that the expected payoff from share acquisition for any given small activist is:

\[
\Pr \left( \eta \leq A_L + \frac{k_2^*(A_L)}{k} \left(1 - \frac{c_s}{\beta_s}\right) \right) (\beta_s - c_s)
\]

while his opportunity cost is \(k_s^i\). For consistency with the monotone strategy with threshold \(k_2^* (A_L)\), the small activist with opportunity cost \(k_2^* (A_L)\) must be exactly indifferent, i.e., \(k_2^* (A_L)\) is implicitly determined by

\[
\Pr \left( \eta \leq A_L + \frac{k_2^*(A_L)}{k} \left(1 - \frac{c_s}{\beta_s}\right) \right) (\beta_s - c_s) = k_2^* (A_L).
\]  

It is easy to see that as long as there is sufficient volatility in entrenchment levels, there exists a unique such threshold \(k_2^* (A_L)\):

**Lemma 4** There exists a \(\sigma_{\eta} \in \mathbb{R}_{++}\) such that if \(\sigma_{\eta} \geq \sigma_{\eta}\) there is a unique solution to (10).

The proof is in the appendix. The intuition for uniqueness is as follows: Both sides of the equation implicitly defining \(k_2^* (A_L)\) are increasing in \(k_2^* (A_L)\). Under these circumstances, a sufficient condition for uniqueness is that rates of change with respect
to $k_2^* (A_L)$ are strictly ranked. The left hand side is a scaled probability in $\eta$. As long as the density function of $\eta$ is sufficiently spread out, the left hand side will always increase slower than the right hand side (the 45 degree line), giving rise to uniqueness.

In case $L$ is absent, as long as $k_0^* \leq \min \{k_2^* (A_L), k_2^* (0)\}$, the mass of activists is given by $\frac{k_2^* (0)}{k}$. Given this mass of activists, Proposition 1 implies that the entrenchment threshold in the activism game is $\frac{k_2^* (0)}{k} \left(1 - \frac{c_s}{\beta_s}\right)$, so that $k_2^* (0)$ is implicitly defined by:

$$\Pr \left(\eta \leq \frac{k_2^* (0)}{k} \left(1 - \frac{c_s}{\beta_s}\right) (\beta_s - c_s) = k_2^* (0)\right).$$

(11)

The sufficient condition for the uniqueness of $k_2^* (0)$ is identical to that for $k_2^* (A_L)$. Thus, we state without proof:

**Lemma 5** If $\sigma_\eta \geq \sigma_\eta$, there is a unique solution to (11).

Given Lemmas 4 and 5, we can now compare the thresholds $k_2^* (A_L)$ and $k_2^* (0)$ to determine the effect of the entry of the large activist on subsequent entry by small activists. We show:

**Proposition 6** $k_2^* (A_L) > k_2^* (0)$.

The proof is in the appendix. The intuition for this result can be understood as follows. The reason small activists may acquire shares in the firm even though they trade with rational traders who charge the full expected continuation value is due to their expected future net private benefits from successful coordinated engagement. Such benefits must be offset against their opportunity costs, $k_2^*$, giving rise to a threshold level of opportunity costs below which share acquisition occurs and above which it does not. Anything that increases expected private benefits, increases incentives to acquire blocks and moves the opportunity cost threshold upwards.

Consider the small activist with opportunity cost $k_2^* (0)$. This activist is exactly indifferent between acquiring a share and not acquiring a share if the large activist
does not participate, in which case—by monotonicity—exactly \( \frac{k_s^*(0)}{k} \) small activists will participate, giving rise to a expected net benefit from share acquisition of

\[
\Pr \left( \eta \leq \frac{k_s^*(0)}{k} \left( 1 - \frac{c_s}{\beta_s} \right) \right) (\beta_s - c_s).
\]

However, imagine now that the large activist does participate. Even if small activists did not change their behavior, the probability of successful engagement would rise to \( \Pr \left( \eta \leq A_L + \frac{k_s^*(0)}{k} \left( 1 - \frac{c_s}{\beta_s} \right) \right) \), and thus the activist with opportunity cost \( k_s^*(0) \) would no longer be exactly indifferent between acquiring a share or not: He would strictly prefer to acquire shares. By continuity, this means that some small activists with strictly higher opportunity costs would strictly prefer to participate. In other words, the threshold level of opportunity cost would increase.

The implication of this result is that the entry of a large activist spurs additional entry by small activists: A wolf pack forms, given the presence of a leader.

5 The Lead Activist

When the large activist enters (with probability \( p_L \)), there is an existing base of small activists of size \( \frac{k_0^*}{k} \). Given our earlier analysis, we know that if \( L \) enters, the size of the activist base will increase to \( A_L + \frac{k_s^*(A_L)}{k} \), giving rise to an expected payoff for entry of:

\[
A_L \Pr \left( \eta \leq A_L + \frac{k_s^*(A_L)}{k} \left( 1 - \frac{c_s}{\beta_s} \right) \right) (\beta_L - c_L)
\]

which will be compared to \( L \)'s opportunity cost \( k_L \). Accordingly, for a given \( (A_L, k_L) \), the large activist will enter only if the anticipated activist ownership \( \frac{k_s^*(A_L)}{k} \) is large enough, which we summarize in the next proposition:

**Proposition 7** For a given \( (A_L, k_L, \beta_L, c_L, \beta_s, c_s) \) the large activist enters only if \( \frac{k_s^*(A_L)}{k} \) is large enough.
A simple proxy for the presence of small activists is the measure of institutional ownership. Interpreted in that light, this results provides a justification for the finding of Brav et al (2008) that the targets of hedge fund activism tend to be firms with high institutional ownership.

6 Early Entry

At \( t = 0 \) small activists have the option of buying into the firm before they know whether \( L \) will enter, or to wait until uncertainty is resolved. Note that since there is a \( 1 - p_L \) probability that \( L \) is unavailable for activism, there is always ex ante uncertainty with regard to \( L \)'s presence. The behavior of small activists is characterized by a threshold: Activists with opportunity costs below \( k^*_0 \) will enter early (by our tie-breaking assumption) and those with higher opportunity costs will wait until \( t = 2 \). Note that, since it is costless to wait and verify whether \( L \) is present (because the transaction price for share acquisition is always fair and the private benefits are received after \( t = 3 \)) a small activist can only wish to buy a share at \( t = 0 \) if his opportunity cost of ownership is low enough that he would prefer to own regardless of whether \( L \) enters or not. In other words, \( k^*_0 \) is defined by:

\[
\Pr \left( \eta \leq \frac{k^*_0}{k} \left( 1 - \frac{c_s}{\beta_s} \right) \right) (\beta_s - c_s) = k^*_0,
\]

which has a unique solution if \( \sigma_\eta \geq \sigma_\eta \). But notice that this condition is identical to (11) and thus \( k^*_0 = k^*_2(0) \), and in turn since \( k^*_2(0) < k^*_2(A_L) \), we have \( k^*_0 \leq \min \{ k^*_2(A_L), k^*_2(0) \} \) as conjectured above.

7 Wolf-Pack Formation

In this section, we summarize the empirical implications of our model for the dynamics of wolf pack formation. Consider the case in which there is sufficient cross-sectional
variation in entrenchment (i.e. $\sigma_y \geq \sigma_x$) and in which activists are well informed by the end of the game of the level of entrenchment of their targets, while the large activist (if present) is even better informed than the small activists (i.e., in the ordered limit as $\alpha_L \to \infty$ and $\alpha_s \to \infty$). Then, our game has a unique equilibrium, and this equilibrium delivers the following dynamic implications:

1. Some small activists (those with opportunity costs smaller than $k_2^*(0) = k_0^*$) acquire positions in the target firm at $t = 0$ in potential anticipation of the large activist’s arrival.

2. If the large activist is available for activism at $t = 1$, she acquires a stake in the firm if and only if she correctly predicts that there will be a sufficiently large activist base given her opportunity cost of acquiring a stake (i.e., if she believes that the total mass of small activists at $t = 0$, $k_2^*(0)$ and the incremental set that will acquire a position at $t = 2$ if the large activist enters, $k_2^*(A_L) - k_2^*(0)$ is large enough).

3. Conditional on the large activist’s entry at $t = 1$ there will be additional entry by small activists (a measure $k_2^*(A_L) - k_2^*(0)$).

Imagine that the entry of the large activist is synonymous with the filing of a 13D. Then, combining these dynamic implications delivers two empirical implications:

**Implication 1**: Firms in which 13Ds are filed will have substantially higher activist presence ($A_L + \frac{k_2^*(A_L)}{k}$) than firms in which they are not ($\frac{k_2^*(0)}{k}$).

The empirical content of this depends on our definition of an activist. If we define an activist by “institutional trader,” then this result captures the Brav et al (2008) finding.
**Implication 2:** There will be significant additional accumulation of activist shares following a 13D filing (a measure \( \frac{k_2^L(A_L) - k_2^L(0)}{\kappa} \) of additional small activists will enter conditional on the large activist’s entry).

This seems to be what is captured by the Nathan quote in the introduction.

**Implication 3:** Late entrants to wolf packs have higher opportunity costs of locking up capital than early entrants.

This also seems to be consistent with the Nathan quote in the introduction.

It may be worth seeing a numerical example to get a sense of the quantitative implications of this highly stylized model. In constructing the example, we set \( c_s = 0.7 \), \( \beta_s = 1 \) (i.e., we allow only for a small private benefit to small activists), and set \( \kappa = 0.35 \) (note that this satisfies the assumption that \( \beta_s - c_s \leq \kappa \)). We set \( A_L = 10\% \). Finally, we set \( \mu_\eta = 0.3, \sigma_\eta = 0.4 \). While we do not compute whether \( \sigma_\eta \) satisfies the sufficient condition for uniqueness, our computations demonstrate the existence of a unique equilibrium in each case.

Simple computations show that \( k_2^L(0) \simeq 0.085 \) while \( k_2^L(A_L) \simeq 0.12 \). Thus, the initial activist base is approximately 24% and conditional on entry of the large activist, the activist base rises to approximately 34%, i.e., the proportionate increase in the activist base (not accounting for the actual 13D filer) as a result of the 13D filing is approximately \( \frac{34\%-24\%}{24\%} = 42\% \). Thus, even in the simplest formulation, our model can deliver a significant pack formation, conditional on a 13D filing.

**8 Appendix**

**Proof of Lemma 4:** Existence follows immediately, because for \( k_2^L(A_L) = 0 \) the left hand side is bigger than the right hand side, whereas, since \( \beta_s - c_s < \kappa \), for \( k_2^L(A_L) = \kappa \)
the left hand side is smaller than the right hand side. Since \( \eta \sim N (\mu_{\eta}, \sigma_{\eta}^2) \), taking the derivative with respect to \( k_2^*(A_L) \) of the left hand side gives:

\[
\frac{1}{\overline{k}} \frac{(\beta_s - c_s)^2}{\beta_s} \phi_{\mu_{\eta}, \sigma_{\eta}^2} \left( A_L + \frac{k_2^*(A_L)}{\overline{k}} \left( 1 - \frac{c_s}{\beta_s} \right) \right) > 0.
\]

Since \( \phi_{\mu_{\eta}, \sigma_{\eta}^2} (\cdot) < \frac{1}{\sqrt{2\pi}\sigma_{\eta}} \), for any given \( \overline{k}, \beta_s, \) and \( c_s \), there exists a \( \sigma_{\eta} \in \mathbb{R}_{++} \) such that if \( \sigma_{\eta} \geq \sigma_{\eta} \) the rate of increase of the left hand side is strictly smaller than 1, the rate of increase of the right hand side. Then, the intersection point is unique. ■

**Proof of Proposition 6:** When \( \sigma_{\eta} \geq \sigma_{\eta} \), \( k_2^*(0) \) is uniquely defined by (11) while \( k_2^*(A_L) \) is uniquely defined by (10). Note first that for \( A_L = 0 \), (11) coincides with (10), so that"

\[ k_2^*(A_L) |_{A_L = 0} = k_2^*(0). \]

Further note that the left hand side and right hand side of (10) are both increasing in \( k_2^*(A_L) \) but only the left hand side is increasing in \( A_L \). This implies that \( \frac{dk_2^*(A_L)}{dA_L} > 0 \), so that \( k_2^*(A_L) > k_2^*(0) \). ■
References


