The Term Structure of Currency Carry Trade Risk Premia

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Abstract

High interest rate currencies yield high currency excess returns on short-term Treasury bill investments, but they tend to yield low local excess returns on long-term government bonds. At longer maturities, the low term premium offsets the high currency risk premium. Under no arbitrage conditions, this exact result obtains when global permanent innovations to the pricing kernels of different countries are the same and therefore do not to have permanent effects on exchange rates. In this case, the uncovered interest rate parity holds at long horizons. We derive parametric restrictions to match the downward sloping term structure of carry trade risk premia in a large class of affine term structure models.

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In this paper, we establish that the term structure of currency carry trade risk premia is downward-sloping: the returns to the currency carry trade are much smaller for long maturity bonds than for Treasury bills. Assuming that markets are complete, we derive a preference-free condition that links foreign and domestic long-term bond returns, expressed in a common currency, to the permanent components of the pricing kernels. The downward-sloping term structure of average carry trade returns is therefore informative about the temporal nature of risks that investors perceive in currency markets: carry trade returns are driven by asymmetric exposures to common shocks, but these shocks are perceived to be transitory in nature by bond market investors. Building on our preference-free condition, we derive analytical parameter restrictions in a large class of affine term structure models with common factors that ensure zero carry trade returns at long maturities. These parameter restrictions rule out the non-stationarity of the exchange rate in response to common shocks and enforce unconditional uncovered interest parity at very long horizons. Our work is the first to establish this connection between the stationarity of the exchange rate and the properties of foreign long-term bond returns and yields.

Carry trades at the short end of the maturity curve are akin to selling Treasury bills in funding currencies and buying Treasury bills in investment currencies. The exchange rate is here the only source of risk. The set of funding and investment currencies can be determined by the level of short-term interest rates or the slope of the yield curves, as noted by Ang and Chen (2010) and Berge, Jordà, and Taylor (2011). Likewise, carry trades at the long end of the maturity curve are akin to selling long-term bonds in funding currencies and buying long-term bonds in investment currencies. Each leg of the trade is subject to exchange rate and interest rate risk. The log return on a foreign bond position (expressed in U.S. dollars) in excess of the domestic (i.e., U.S.) risk-free rate is equal to the sum of the log excess bond return in local currency plus the return on a long position in foreign currency. Therefore, expected foreign bond excess returns converted in domestic currency are the sum of a local bond term premium and a currency risk premium. Each maturity component of the term structure of currency carry trade risk premia is defined by the average excess return obtained by selling and buying bonds of that maturity. The different maturities describe the whole term structure of currency risk.

We study empirically the term structure of currency carry trade risk premia through a cross-section of average portfolio excess returns. Our data pertain to either long time-series of G10 sovereign coupon bond returns over the 12/1950–12/2012 sample, or to a shorter sample (12/1971–12/2012) of G10 sovereign zero-coupon yield curves. Using zero-coupon bonds, Figure 1 offers a first glimpse at the term structure of currency risk. The figure, which is studied in details later in the paper, shows the dollar log excess returns as a function of the bond maturities, using the same set of funding and investment currencies. Investing in short-term bills of countries with flat yield curves (mostly high short-term interest rate) while borrowing at the same horizon in countries with steep yield curves (mostly low short-term interest rate countries) leads to positive excess returns on average. This is the classic carry trade, whose average excess return is represented here on the left hand side of the graph. Investing and borrowing in long-term bonds of the same countries, however, deliver negative excess returns on average. This is the carry trade at the long end of the term structure curve, represented here on the right hand side of the graph. As the maturity of the bonds increases, the average excess return decreases.

Using long time-series of G10 sovereign coupon bond returns delivers similar results. Between 12/1950 and 12/2012, the portfolio of flat-slope (mostly high short-term interest rate) currencies yields a one-month currency risk premium of 3.0% and a local term premium of −1.8% per annum (which sum to a bond premium in U.S. dollars of 1.2%). Over the same period, the portfolio of steep-slope (mostly low short-term interest rate) currencies yields a currency risk premium of 0.05% and a local term premium of 4.0% (which sum to a bond premium of 4.05%). The average spread in dollar risk premia between the low slope and high slope portfolios is thus 2.95% (3.0% – 0.05%) for Treasury bills, but it is −2.85% (1.2% – 4.05%) for the 10-year bond portfolios. Countries with a high currency risk premium tend to have a low bond term premium. The profitable bond strategy therefore involves shorting the usual carry trade investment currencies and going long in the funding currencies. We obtain similar results when sorting countries by the level of their short-term interest rates: the risk premia at the long end

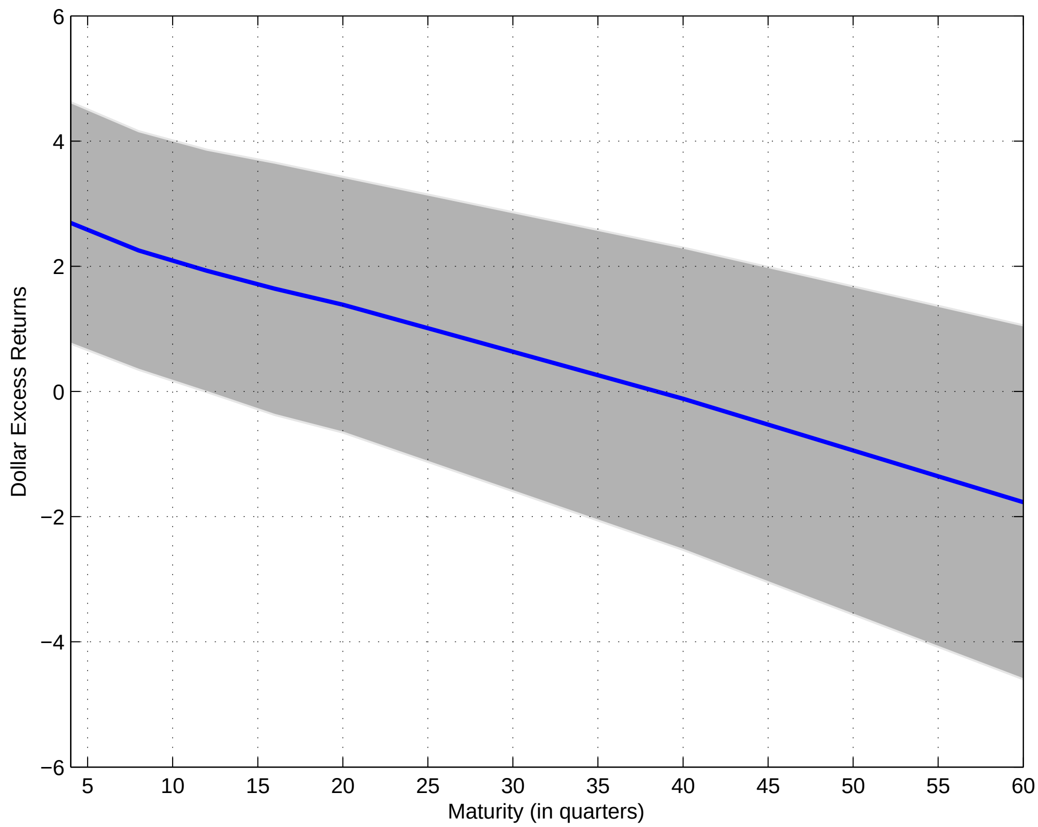

Figure 1: Term Structure of Dollar Bond Risk Premia — The figure shows the dollar log excess returns as a function of the bond maturities. Dollar excess returns correspond to the holding period returns expressed in U.S. dollars of investment strategies that go long and short foreign bonds of different countries. The unbalanced panel of countries consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. At each date t, the countries are sorted by the slope of their yield curves into three portfolios. The first portfolio contains countries with flat yield curves (mostly high interest rate) while the last portfolio contains countries with steep yield curves (mostly low interest rate countries). The first portfolio correspond to the investment currencies while the third one corresponds to the funding currencies. The slope of the yield curve is measured by the difference between the 10-year yield and the 3-month interest rate at date t. The holding period is one quarter. The returns are annualized. The shaded areas correspond to two standard deviations above and below each point estimates. Standard deviations are obtained by bootstrapping. Zero-coupon data are monthly, and the sample window is 4/1985–12/2012.

of the maturity curve are significantly smaller that those at the short end, as the difference in local currency bond term premia largely offsets the currency risk premium. As a result, the average returns on foreign long-term bonds, once converted into U.S. dollars, are small and rarely statistically different from the average return on U.S. long-term bonds.

Our empirical evidence on holding period bond returns complements the existing evidence on the uncovered interest rate parity condition in the long run. In the data, returns over long-term horizons, once converted into the same currency, appear similar across countries. In other words, differences in yields seem to correspond to cumulative changes in exchange rates over the life of the bonds (Meredith and Chinn, 2001). There are, however, few non-overlapping observations of long-term returns, and tests of the long-run U.I.P condition therefore lack power.

Assuming that markets are complete, we study how yield differences and one-period bond return differences across countries relate to the properties of the pricing kernels. We first show that cross-country differences in expected returns over long-term horizons, once converted into the same currency, correspond to cross-country differences in the entropies of the domestic and foreign stochastic discount factors (SDF). This result only relies on the market completeness and the absence of arbitrage. When SDFs are gaussian, the stationarity of exchange rates implies that U.I.P holds on average in the long-run. When SDFs are not gaussian, we find again, under mild regulatory restrictions on the SDFs, that long-run U.I.P holds on average when exchange rates are stationary.

We then show that the difference between domestic and foreign long-term bond risk premia, expressed in domestic currency, is pinned down by the difference in the entropies of the permanent components of the SDFs. The long-term bond risk premia, expressed in domestic currency, are higher on foreign bonds than on domestic bonds when there is less permanent risk in foreign countries' pricing kernels than at home. The temporary components of SDFs play no role because the currency exposure completely hedges the exposure of the long-short strategy in long-term bonds to the temporary pricing kernel shocks. This theoretical result speaks directly to our main empirical finding: while the usual carry trade delivers large and positive currency excess returns, the carry trade at the long end of the yield curve does not. Therefore, the permanent components of the SDF of investment currencies must be at least as risky as those of funding currencies, while the opposite is true for the overall SDFs. This result produces a novel restriction that all models in international finance need to satisfy in order to be consistent with the data. In the limit case when permanent shocks are the same across countries, bond returns, once converted in the same currency, should be the same, date by date.

We then use our preference-free theoretical results to derive parametric restrictions in several key affine term structure models, from the original Vasicek (1997) and Cox, Ingersoll, and Ross

(1985) to the most recent multi-factor models. The restrictions need to be satisfied to match our new facts about the cross-section of bond returns and exchange rates. Depending on whether the shocks are country-specific or global, we obtain country-specific or cross-country restrictions on the parameters of the models. Through the lenses of those term structure models, we propose a novel interpretation of carry trades at the short end of the yield curve. The insignificant carry trade risk premia at longer maturities rules out asymmetry in the loadings on the permanent global shocks; the asymmetry can only apply to transitory global shocks. The classic carry trade, at the short end of the yield curve, therefore compensates investors for exposure to transitory global shocks.

The rest of the paper is organized as follows. Section 1 rapidly reviews the literature. In Section 2, we examine the cross-section of bond excess returns in local currency and in U.S. dollars and we contrast it with the cross-section of currency excess returns. In Section 3, we derive the no-arbitrage, preference-free theoretical restrictions imposed on currency and term risk premia. To do so, we decompose pricing kernels into a permanent and a temporary component and link their properties to the downward term structure of currency risk. In Section 4, we apply this decomposition to affine term structure models and illustrate it quantitatively in a multi-country affine model. In Section 5, we present concluding remarks. The Appendix contains all proofs and an Online Appendix contains supplementary material not presented in the main body of the paper.

1 Related Literature

Our paper is related to four large strands of the literature: the carry trade returns, the empirical term premia across countries, the decomposition of SDFs, and term structure models.

Our paper builds on the vast literature on UIP condition and the currency carry trade [Engel (1996) and Lewis (2011) provide recent surveys]. We are the first to derive general conditions under which long-run unconditional UIP follows simply from market completeness: if all permanent shocks to the pricing kernel are common, then foreign and domestic yield spreads in dollars on long maturity bonds will be equalized, regardless of the properties of the pricing kernel.

Our focus is on the cross-sectional relation between the slope of the yield curve, interest rates and exchange rates. We study whether investors earn higher returns on foreign bonds from countries in which the slope of the yield curve is higher than the cross-country average. Prior work, from Campbell and Shiller (1991) to Bekaert and Hodrick (2001) and Bekaert, Wei, and Xing (2007), focus mostly on the time series, testing whether investors earn higher returns on foreign bonds from a country in which the slope of the yield curve is currently higher than average for that country. Chinn and Meredith (2004) document some time-series evidence that supports a conditional version of UIP at longer holding periods, while Boudoukh, Richardson, and Whitelaw (2013) show that past forward rate differences predict future changes in exchange rates. Some papers study the cross-section of bond returns: Koijen, Moskowitz, Pedersen, and Vrugt (2012) and Wu (2012) examine the currency-hedged returns on 'carry' portfolios of international bonds, sorted by a proxy for the carry on long-term bonds, but they do not examine the interaction between currency and term risk premia, the topic of our paper. Ang and Chen (2010) and Berge, Jordà, and Taylor (2011) have shown that yield curve variables can also be used to forecast currency excess returns. These authors do not examine the returns on foreign bond portfolios. Dahlquist and Hasseltoft (2013) study international bond risk premia in an affine asset pricing model and find evidence for local and global risk factors. Jotikasthira, Le, and Lundblad (2015) study the co-movement of foreign bond yields through the lenses of an affine term structure model. Our paper revisits the empirical evidence on bond returns without committing to a specific term structure model.

We interpret our empirical findings using a preference-free decomposition of the pricing kernel, building on recent work in the exchange rate and term structure literatures. On the one hand, at the short end of the maturity curve, currency risk premia are high when there is less overall risk in foreign countries' pricing kernels than at home (Bekaert, 1996; Bansal, 1997; and Backus, Foresi, and Telmer, 2001). High foreign interest rates and/or a flat slope of the yield curve mean less overall risk in the foreign pricing kernel. On the other hand, at the long end of the maturity curve, local bond term premia compensate investors for the risk associated with persistent innovations to the pricing kernel (Bansal and Lehmann, 1997; Hansen and Scheinkman, 2009; Alvarez and Jermann, 2005; Hansen, 2012; Hansen, Heaton, and Li, 2008; and Bakshi and Chabi-Yo, 2012). In this paper, we combine those two insights to derive preference-free theoretical results under the assumption of complete financial markets.

We apply the Alvarez and Jermann (2005) and Hansen and Scheinkman (2009) decomposition to a large set of term structure models, considering single- and multiple-factor models in the tradition of Vasicek (1977) and Cox, Ingersoll, and Ross (1985, denoted CIR). Models with heteroskedastic SDFs, following CIR, are naturally the most appealing, since currency risk premia, when shocks are Gaussian, are simply driven by the differences in conditional volatilities of the log SDFs. This extends earlier work by Backus, Foresi, and Telmer (2001), Hodrick and Vassalou (2002), Brennan and Xia (2006), Leippold and Wu (2007), Lustig, Roussanov, and Verdelhan (2011) and Sarno, Schneider, and Wagner (2012). Lustig, Roussanov, and Verdelhan (2011) focused on accounting for short-run uncovered interest rate parity condition (UIP) deviations and short-term carry trades respectively within this class of models. They show that asymmetric exposure to global innovations to the pricing kernel are key to understanding the global currency carry trade premium at short maturities.[1] This paper focuses on long-term carry trades.

2 The Cross-Section of Long-Term Bond Returns

This section documents the downward-sloping term structure of currency risk premia. We first define the bond and currency excess returns and then turn to the data and our results.

2.1 Definitions

Our empirical work focuses on three risk premia: the term premium in U.S. dollars, the currency risk premium, and the term premium in foreign currency. Let us first define them precisely.

[1]Taking this reasoning to the data, they identify innovations in the volatility of global equity markets as candidate shocks that explain the cross-section of short-term currency risk premia, while Menkhoff, Sarno, Schmeling, and Schrimpf (2012) propose the volatility in global currency markets instead.

Term Premium on Domestic Bonds $P_t^{(k)}$ denotes the price at date t of a zero-coupon bond of maturity k. The one-period return on the zero-coupon bond is $R_{t+1}^{(k)} = P_{t+1}^{(k-1)}/P_t^{(k)}$. The log excess returns, denoted $rx_{t+1}^{(k)}$, is equal to $\log R_{t+1}^{(k)}/R_t^f$, where the risk-free rate is $R_t^f = R_{t+1}^{(0)} = 1/P_t^{(1)}$. The expected log excess return on the zero-coupon bond with maturity k, or term premium, is:

$$E_t\left[rx_{t+1}^{(k)}\right] = E_t\left[\log R_{t+1}^{(k)}/R_t^f\right].$$

The yield spread is the log difference between the yield of the k-period bond and the risk-free rate: $y_t^{(k)} = -\log\left(R_t^f/(P_t^{(k)})^{1/k}\right)$.

Currency Risk Premium The nominal spot exchange rate in foreign currency per U.S. dollar is denoted S_t. When S increases, the U.S. dollar appreciates. Similarly, F_t denotes the one-period forward exchange rate, and f_t its log value. The log currency excess return corresponds to:

$$rx_{t+1}^{FX} = \log\left[\frac{S_t}{S_{t+1}}\frac{R_t^{f,*}}{R_t^f}\right] = (f_t - s_t) - \Delta s_{t+1},$$

when the investor borrows at the domestic risk-free rate, R_t^f, and invests at the foreign risk-free rate, $R_t^{f,*}$, and where the forward rate is defined through the covered interest rate parity condition: $F_t/S_t = R_t^{f,*}/R_t^f$. The currency risk premium is the expected value of the log currency excess return.

Term Premium on Foreign Bonds The log return on a foreign bond position (expressed in U.S. dollars) in excess of the domestic (i.e., U.S.) risk-free rate is denoted $rx_{t+1}^{(k),\$}$. It can be expressed as the sum of the log excess return in local currency plus the return on a long position in foreign currency:

$$rx_{t+1}^{(k),\$} = \log\left[\frac{R_{t+1}^{(k),*}}{R_t^f}\frac{S_t}{S_{t+1}}\right] = \log\left[\frac{R_{t+1}^{(k),*}}{R_t^{f,*}}\frac{R_t^{f,*}}{R_t^f}\frac{S_t}{S_{t+1}}\right] = rx_{t+1}^{(k),*} + rx_{t+1}^{FX}.$$

The first component of the foreign bond excess return is the excess return on a bond in foreign currency, while the second component represents the log excess return on a long position in foreign currency, given by the forward discount minus the rate of depreciation. Taking expectations, the total term premium in dollars thus consists of a foreign bond risk premium, $E_t[rx_{t+1}^{(k),*}]$, plus a currency risk premium, $(f_t - s_t) - E_t \Delta s_{t+1}$.

2.2 Data

We study the bond and currency risk premia in the data. Our benchmark sample consists of a small homogeneous panel of developed countries with liquid bond markets. This G-10 panel includes Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. The domestic country is the United States. It only includes one country from the eurozone, Germany. For those countries, we gather discount bonds and zero-coupon bonds.

In order to build the longest time-series possible, we obtain discount bond indices from Global Financial Data. The dataset includes a 10-year government bond total return index, in U.S. dollars and in local currency, for each of our target countries, as well as Treasury bill total return indices. The 10-year bond returns are a proxy for the bonds with the longest maturity.

While Global Financial Data offers, to the best of our knowledge, the longest time-series of government bond returns available, the series have three key limits. First, they pertain to discount bonds, while the theory developed later in this paper pertains to zero-coupon bonds. Second, they include default risk, while the theory focuses on default-free bonds. Third, they only offer 10-year bond returns, not the entire term structure of bond returns. To address these issues, we use zero-coupon bonds obtained from the estimation of term structure curves using government notes and bonds and interest rate swaps of different maturities; the time-series are shorter and dependent on the term structure estimations. In contrast, bond return indices, while spanning much longer time-periods, offer model-free estimates of bond returns. Our results turn out to be similar in both samples.

Our zero-coupon bond dataset covers the same benchmark sample of G10 countries from 12/1971 to 12/2012. To construct our sample, we use the entirety of the dataset in Wright (2011) and complement the sample, as needed, with sovereign zero-coupon curve data sourced from Bloomberg. The panel is unbalanced: for each currency, the sample starts with the beginning of the Wright (2011) dataset.[2] Yields are available at maturities from three months to 15 years, in three-month increments.

To focus on expected excess returns, we sort countries monthly into portfolios based on variables that can be used to predict bond and currency returns. Countries are sorted on the level of the short-term interest rates or the slope of their yield curves (measured by the spread between the 10-year bond yield and the one-month interest rate) and allocated to three portfolios. In all cases, portfolios formed at date t only use information available at that date. The log excess returns on currency (rx^{FX}), the log excess returns on the bond in local currency (e.g., $rx^{(10),*}$) and in U.S. dollars (e.g., $rx^{(10),\$}$) are first obtained at the country level. Returns are computed over three-month horizons. Then, the portfolio-level excess returns are obtained by averaging these log excess returns across all countries in a portfolio. We first describe results obtained with the 10-year bond indices and then turn to the zero-coupon bandits study the whole term structure.

2.3 Sorting Currencies by Interest Rates

Let us start with the classic portfolios of countries sorted by their short-term interest rates. Table 1 reports summary statistics on currency and bond excess returns. Clearly, the uncovered interest rate parity condition fails in the cross-section. As in the literature, average currency excess returns increase from low- to high-interest-rate portfolios, ranging from 0% to 3.1% per year over the last 60 years. The long-short currency carry trade implemented with short-term Treasury bills therefore delivers a 0.4 Sharpe ratio. Should investors trade long-term bonds instead of Treasury bills in the same countries? No. Local currency bond risk premia decrease from low- to high-interest-rate portfolios, from 2.2% to 0.4%. The decline in the local currency bond risk premia partly offsets the increase in currency risk premia. As a result, the average excess return on foreign bonds expressed in U.S. dollars measured in the high-interest-rate portfolio is only slightly higher than the average excess returns measured in the low-interest-

[2] The starting dates for each country are as follows: 2/1987 for Australia, 1/1986 for Canada, 1/1973 for Germany, 1/1985 for Japan, 1/1990 for New Zealand, 1/1998 for Norway, 12/1992 for Sweden, 1/1988 for Switzerland, 1/1979 for the U.K., and 12/1971 for the U.S. For New Zealand, the data for maturities above 10 years start in 12/1994.

Table 1: Interest Rate-Sorted Portfolios

		Panel A: 12/1950–12/2012				Panel B: 12/1971–12/2012			
Portfolio		1	2	3	$3-1$	1	2	3	$3-1$
$-\Delta s$	Mean	1.65	0.23	-0.28	-1.93	2.39	0.14	-0.49	-2.89
$f-s$	Mean	-1.64	0.81	3.39	5.03	-1.99	1.11	3.80	5.78
$y^{(10),*}-r^{*,f}$	Mean								
rx^{FX}	Mean	0.00	1.03	3.11	3.10	0.41	1.25	3.30	2.90
	s.e.	[1.02]	[0.91]	[1.20]	[1.03]	[1.56]	[1.37]	[1.75]	[1.52]
	Std	8.29	6.91	8.74	7.97	10.12	8.42	10.46	9.52
	SR	0.00	0.15	0.36	0.39	0.04	0.15	0.32	0.30
	s.e.	[0.13]	[0.13]	[0.14]	[0.15]	[0.16]	[0.16]	[0.17]	[0.17]
$rx^{(10),*}$	Mean	2.17	1.31	0.41	-1.76	2.60	1.66	0.56	-2.05
	s.e.	[0.50]	[0.53]	[0.64]	[0.61]	[0.72]	[0.77]	[0.92]	[0.87]
	Std	4.01	4.34	5.10	4.72	4.69	5.12	5.85	5.33
	SR	0.54	0.30	0.08	-0.37	0.55	0.32	0.10	-0.38
	s.e.	[0.14]	[0.13]	[0.13]	[0.12]	[0.17]	[0.16]	[0.16]	[0.15]
$rx^{(10),\$}$	Mean	2.17	2.34	3.52	1.35	3.01	2.90	3.86	0.85
	s.e.	[1.22]	[1.06]	[1.31]	[1.21]	[1.85]	[1.60]	[1.89]	[1.77]
	Std	10.00	8.22	9.94	9.32	12.09	9.91	11.68	10.95
	SR	0.22	0.28	0.35	0.14	0.25	0.29	0.33	0.08
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.16]	[0.16]	[0.17]	[0.16]
$rx^{(10),\$}-rx^{(10),US}$	Mean	0.65	0.82	2.00	1.35	0.48	0.37	1.33	0.85
	s.e.	[1.16]	[1.18]	[1.51]	[1.21]	[1.70]	[1.71]	[2.16]	[1.77]

Notes: The table reports the average change in exchange rates (Δs), the average interest rate difference ($f-s$), the average slope of the yield curve ($y^{(10),*}-r^{*,f}$), the average currency excess return (rx^{FX}), the average foreign bond excess return on 10-year government bond indices in foreign currency ($rx^{(10),*}$) and in U.S. dollars ($rx^{(10),\$}$), as well as the difference between the average foreign bond excess return in U.S. dollars and the average U.S. bond excess return ($rx^{(10),\$}-rx^{US}$). For the excess returns, the table also reports their annualized standard deviation (denoted Std) and their Sharpe ratios (denoted SR). The holding period is three months. The log returns are annualized. The balanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. The countries are sorted by the level of their short term interest rates into three portfolios. The standard errors (denoted s.e. and reported between brackets) were generated by bootstrapping 10,000 samples of non-overlapping returns.

rate portfolio. The long-short currency carry trade implemented with long-term government bonds does not deliver a significant average return. We obtain similar findings over a shorter, post-Bretton Woods sample. There is no evidence of statistically significant differences in dollar bond risk premia across the portfolios.

2.4 Sorting Currencies by the Slope of the Yield Curve

Table 2: Slope-Sorted Portfolios

		Panel A: 12/1950–12/2012				Panel B: 12/1971–12/2012			
Portfolio		1	2	3	3 − 1	1	2	3	3 − 1
$-\Delta s$	Mean	0.01	0.39	1.18	1.17	-0.08	0.61	1.51	1.60
$f-s$	Mean	2.96	0.42	-0.71	-3.68	3.31	0.54	-0.94	-4.25
$y^{(10),*}-r^{*,f}$	Mean	-0.20	0.33	0.77	0.97	-0.28	0.30	0.73	1.01
rx^{FX}	Mean	2.97	0.81	0.47	-2.50	3.23	1.15	0.58	-2.65
	s.e.	[1.08]	[1.03]	[0.95]	[0.87]	[1.65]	[1.55]	[1.44]	[1.26]
	Std	8.25	7.75	7.60	6.84	10.09	9.38	9.16	8.15
	SR	0.36	0.10	0.06	-0.37	0.32	0.12	0.06	-0.32
	s.e.	[0.14]	[0.13]	[0.13]	[0.15]	[0.17]	[0.16]	[0.16]	[0.18]
$rx^{(10),*}$	Mean	-0.86	1.33	3.33	4.19	-0.52	1.70	3.64	4.16
	s.e.	[0.58]	[0.51]	[0.58]	[0.60]	[0.85]	[0.75]	[0.81]	[0.85]
	Std	4.60	4.24	4.65	4.67	5.45	5.03	5.20	5.26
	SR	-0.19	0.31	0.72	0.90	-0.10	0.34	0.70	0.79
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.16]	[0.16]	[0.17]	[0.15]
$rx^{(10),\$}$	Mean	2.12	2.14	3.80	1.68	2.70	2.85	4.22	1.51
	s.e.	[1.19]	[1.17]	[1.18]	[1.08]	[1.81]	[1.74]	[1.73]	[1.56]
	Std	9.34	8.98	9.42	8.14	11.30	10.79	11.15	9.59
	SR	0.23	0.24	0.40	0.21	0.24	0.26	0.38	0.16
	s.e.	[0.13]	[0.13]	[0.13]	[0.12]	[0.16]	[0.16]	[0.16]	[0.15]
$rx^{(10),\$}-rx^{(10),US}$	Mean	0.60	0.62	2.28	1.68	0.17	0.32	1.69	1.51
	s.e.	[1.42]	[1.29]	[1.14]	[1.08]	[2.09]	[1.88]	[1.62]	[1.56]

Notes: The table reports the average change in exchange rates (Δs), the average interest rate difference ($f-s$), the average slope ($y^{(10),*}-r^{*,f}$), the average log currency excess return (rx^{FX}), the average log foreign bond excess return on 10-year government bond indices in foreign currency ($rx^{(10),*}$) and in U.S. dollars ($rx^{(10),\$}$), as well as the difference between the average foreign bond log excess return in U.S. dollars and the average U.S. bond log excess return ($rx^{(10),\$}-rx^{US}$). For the excess returns, the table also reports their annualized standard deviation (denoted Std) and their Sharpe ratios (denoted SR). The holding period of the returns is three months. Log returns are annualized. The balanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. The countries are sorted by the slope of their yield curves into three portfolios. The slope of the yield curve is measured by the difference between the 10-year yield and the one-month interest rate at date t. The standard errors (denoted s.e. and reported between brackets) were generated by bootstrapping 10,000 samples of non-overlapping returns.

A similar result appears with portfolios of countries sorted by the slope of their yield curve. There is substantial turnover in these portfolios, more so than in the usual interest rate-sorted portfolios, but the typical currencies in Portfolio 1 (flat yield curve currencies) are the Australian and New Zealand dollar and the British pound, whereas the typical currencies in Portfolio 3 (steep yield curve currencies) are the Japanese yen and the German mark. In other words, the flat slope currencies tend to be high interest rate currencies, while the steep slope currencies tend to be low interest rate currencies.

Consistent with this distribution of interest rates, average currency excess returns decrease across portfolios. Table 2 reports the annualized moments of log returns on the three slope-sorted portfolios. Average currency excess returns decline from 3.0% per annum on Portfolio 1 to 0.5% per annum on the Portfolio 3 over the last 60 years. Therefore, a long-short position of investing in steep-yield-curve currencies and shorting flat-yield-curve currencies delivers a currency excess return of -2.5% per annum and a Sharpe ratio of -0.4. Our findings confirm those of Ang and Chen (2010). The slope of the yield curve predicts currency excess returns very well. However, note that this result is not mechanical; the spread in the slopes (reported on the third line) is much smaller than the spread in excess returns.

Turning to the returns on local bonds, average bond excess returns increase across portfolios. Portfolio 1 produces negative bond excess returns of -0.9% per annum, compared to 3.3% per annum on Portfolio 3. Importantly, this strategy involves long positions in bonds issued by countries like Germany and Japan. These are countries with fairly liquid bond markets and low sovereign credit risk. As a result, credit and liquidity risk differences are unlikely candidate explanations for the return differences. Here again, the bond and currency excess returns move in opposite directions across portfolios.

Turing to the returns on foreign bonds in U.S. dollars, we do not obtain significant differences across portfolios. Average bond excess returns in U.S. dollars tend to increase from the first (flat-yield-curve) portfolio to the last (steep-yield-curve), but a long-short strategy does not deliver a significant excess return. Local bond and currency risk premier offset each other. We get similar findings when we restrict our analysis to the post-Bretton Woods sample.

2.5 The Term Structure of Currency Carry Trade Risk Premia

The previous results focus on the 10-year maturity and show that currency risk premia offset local currency term premia for coupon bond returns. We now turn to a full set of returns in the maturity spectrum, using the zero-coupon bond dataset.

The term structure of currency carry trade risk premia is clearly downward sloping: currency carry trade strategies that yield positive risk premia for short-maturity bonds yield lower (or even negative) risk premia for long-maturity bonds. As we move from the 4-quarter maturity to the 60-quarter maturity, the difference in the dollar term premium between Portfolio 1 (flat yield curve, mostly high short-term interest rate) currencies and Portfolio 3 (steep yield curve, mostly low short-term interest rate) currencies decreases from 2.69% to -1.77%. The former is significantly different from zero, whereas the latter is not. While investing in flat yield curve currencies and shorting steep yield curve currencies provides significant gains in the short end of the term structure, it yields negative returns in the long end.

To illustrate this finding, Figure 2 reports the local currency excess returns (in logs) in the top panel, and the dollar excess returns (in logs) in the bottom panel. The top panel in Figure 2 shows that countries with the steepest local yield curves (Portfolio 3, center) exhibit local bond excess returns that are higher, and increase faster with the maturity than the flat yield curve countries (Portfolio 1, on the left-hand side). Thus, ignoring the effect of exchange rates, investors should invest in the short-term and long-term bonds of steep yield curve currencies.

Considering the effect of currency fluctuations by focusing on dollar returns radically alters the results. Figure 2 shows that the dollar excess returns of Portfolio 1 are higher than those of Portfolio 3 at the short end of the yield curve, consistent with the carry trade results of Ang and Chen (2010). Yet, an investor who would attempt to replicate the short-maturity carry trade strategy at the long end of the maturity curve would incur losses on average: the long-maturity excess returns of flat yield curve currencies are lower than those of steep yield curve currencies, as currency risk premia more than offset term premia. This result is apparent in the lower panel on the right, which is the same as Figure 1 in the introduction. The term structure of currency carry trade risk premia slopes downwards.

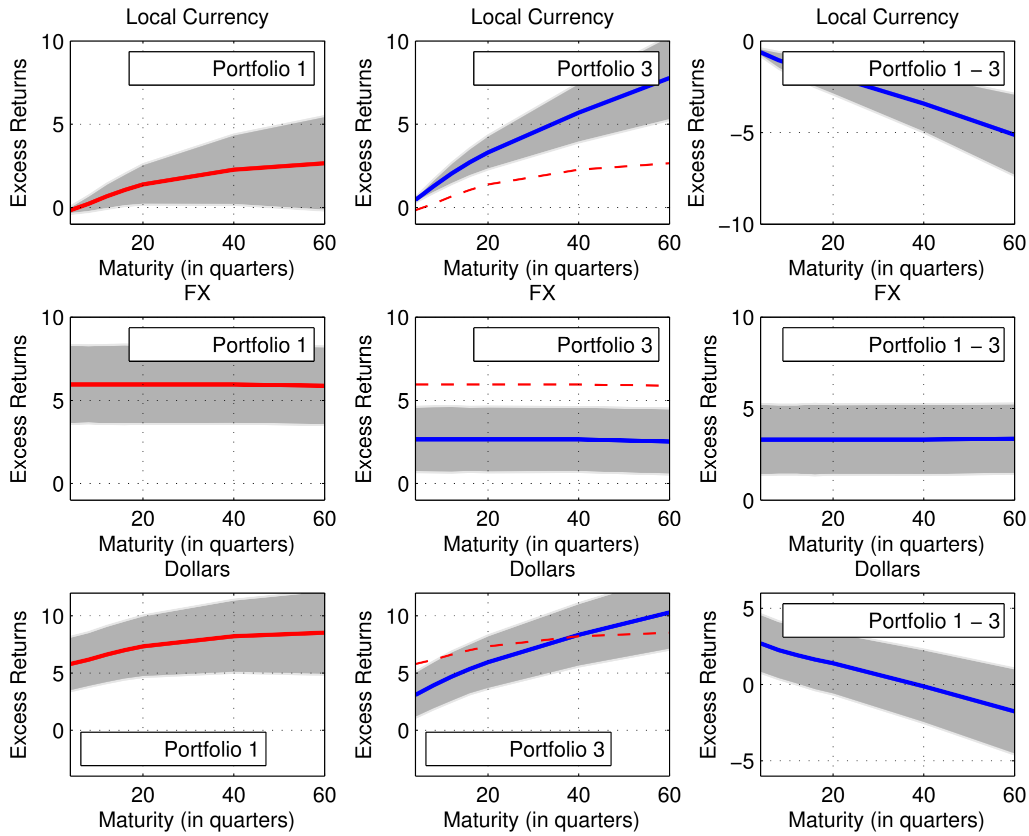

Figure 2: Term Structure of Dollar Bond Risk Premia — The figure shows the log excess returns on foreign bonds in local currency in the top panel, the currency excess return in the middle panel, and the log excess returns on foreign bonds in U.S. dollars in the bottom panel as a function of the bond maturities. The left panel focuses on Portfolio 1 (flat yield curve currencies) excess returns, while the middle panel reports Portfolio 3 (steep yield curve currencies) excess returns. The middle panels also report the Portfolio 1 excess returns in dashed lines for comparison. The right panel reports the difference. Data are monthly, from the zero-coupon dataset, and the sample window is 4/1985–12/2012. The unbalanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. The countries are sorted by the slope of their yield curves into three portfolios. The slope of the yield curve is measured by the difference between the 10-year yield and the 3-month interest rate at date t. The holding period is one quarter. The returns are annualized. The shaded areas correspond to two standard deviations above and below each point estimates. Standard deviations are obtained by bootstrapping.

2.6 Robustness Checks

We consider many robustness checks, studying (i) different time windows, (ii) different lengths of the bond holding period, and (iii) different samples of countries. All the results are reported in the Online Appendix. Here we simply describe the main findings.

The difference between currency excess returns and foreign bond excess returns appears robust across time windows. Figure 3 presents the cumulative three-month log returns on investments in foreign Treasury bills and foreign 10-year bonds, starting in 1950. Countries are sorted into portfolios based on the slope of their yield curves. The returns correspond to an investment strategy going long in Portfolio 1 (flat yield curves, mostly high short-term interest rates) and short in the Portfolio 3 (steep yield curves, mostly low short-term interest rates). Even when dividing the sample into two, three, or four sub periods, the main result remains: an investor buying short-term Treasury bills of countries with flat yield curves, mostly high short-term interest rates, while selling short-term Treasury bills of countries with steep yield curves, mostly low short-term interest rates, enjoys positive returns; investing in long-term bonds of the same countries results in negative returns. Again, currency and local term premia offset each other, and thus average carry trade returns are different at the short-end and the log end of the term structure.

Our results appear robust to the choice of the bond holding period. We consider investments of one, three, and twelve months. The patterns are similar. We sometimes obtain significant dollar term premia when investors only invest for one month, but such a strategy would entail large transaction costs, which would likely wipe out the returns. For longer holding periods, the dollar term premia are not significant.

Our findings appear also robust across samples of countries. With discount bonds, we consider two additional sets of countries: first, a larger sample of 20 developed countries (Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, and the U.K.), and second, a large sample of 30 developed and emerging countries (the same as above, plus India, Mexico, Malaysia, the Netherlands, Pakistan, the Philippines, Poland, South Africa, Singapore, Taiwan, and Thailand).

In the sample of developed countries, the steep-slope (low yielding) currencies are typically countries like Germany, the Netherlands, Japan, and Switzerland, while the flat-slope (high-yielding) currencies are typically Australia, New Zealand, Denmark and the U.K. At the one-month horizon, the 2.4% spread in currency excess returns obtained in this sample is more than offset by the 5.9% spread in local term premia. This produces a statistically significant 3.5%

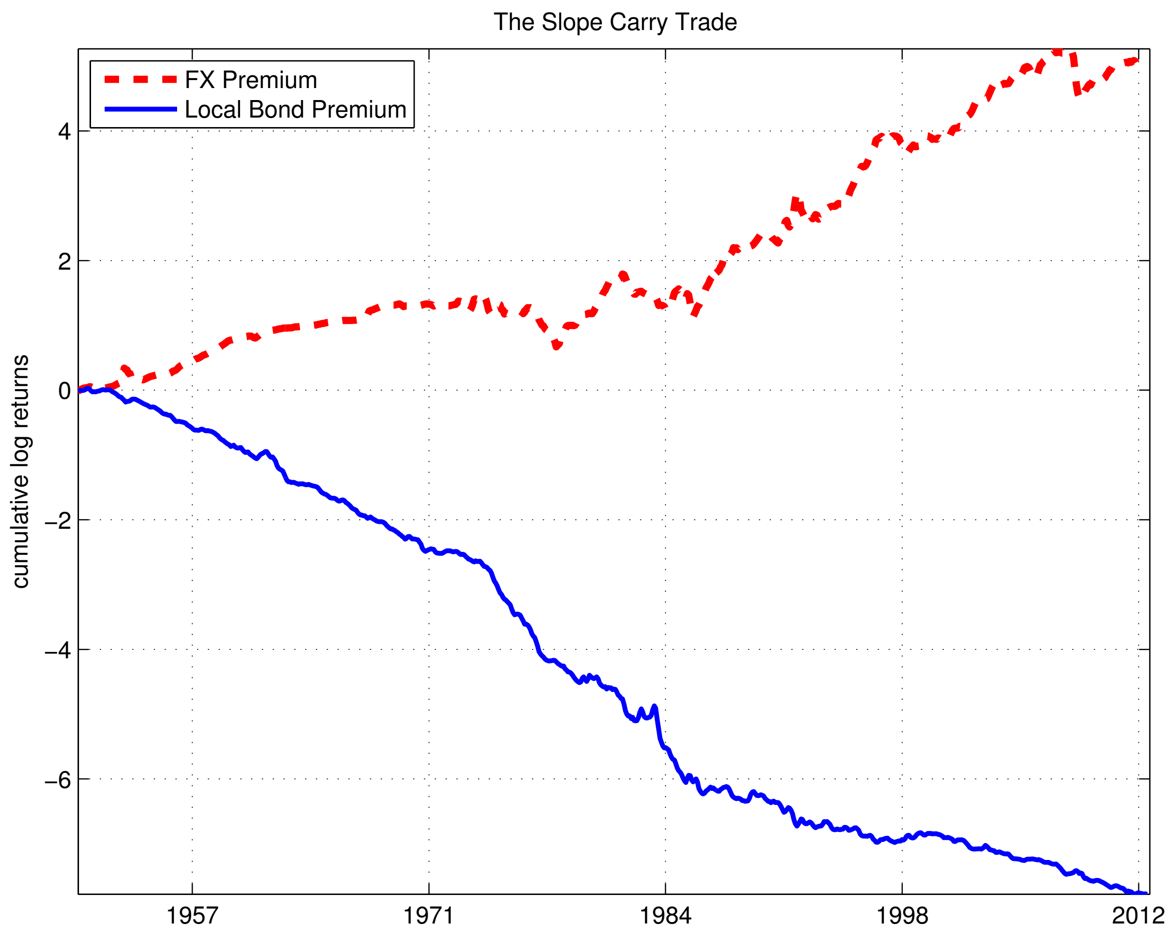

Figure 3: The Carry Trade and Term Premia: Conditional on the Slope of the Yield Curve – The figure presents the cumulative one-month log returns on investments in foreign Treasury bills and foreign 10-year bonds. The benchmark panel of countries includes Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. Countries are sorted every month by the slope of their yield curves into three portfolios. The slope of the yield curve is measured by the spread between the 10-year bond yield and the one-month interest rate. The returns correspond to an investment strategy going long in Portfolio 1 and short in the Portfolio 3. The sample period is 12/1950–12/2012.

return on a position that is long in the low yielding, high slope currencies and short in the high yielding, low slope currencies. These results are essentially unchanged in the post-Bretton-Woods sample. At longer horizons, the currency excess returns and the local risk premia almost fully offset each other.

In the entire sample of countries, including the emerging market countries, the difference in currency risk premia at the one-month horizon is 3.04% per annum, which is more than offset by a 8.37% difference in local term premia. As a result, investors earn 5.33% per annum on a long-short position in foreign bond portfolios of slope-sorted currencies. As before, this involves shorting the flat-yield-curve currencies, typically high interest rate currencies, and going long in the steep-slope currencies, typically the low interest rate ones. The annualized Sharpe ratio on this long-short strategy is 0.60.

We also construct an extended version of the zero-coupon dataset which, in addition to the countries of the benchmark sample, includes the following countries: Austria, Belgium, the Czech Republic, Denmark, Finland, France, Hungary, Indonesia, Ireland, Italy, Malaysia, Mexico, the Netherlands, Poland, Portugal, Singapore, South Africa, and Spain. The data for the aforementioned extra countries are sourced from Bloomberg.[3] Considering a large cross-section of countries does not change our main result. An investor who buys the one-year bonds of flat-yield curve currencies and shorts the one-year bonds of steep-yield-curve currencies realizes a dollar excess return of 4.1% per year on average. However, at the long end of the maturity structure this strategy generates negative and insignificant excess returns: the average annualized dollar excess return of an investor who pursues this strategy using 15-year bonds is -0.4%. The term structure of currency carry trade risk premia remains downward-sloping. We turn now to the interpretation of this empirical finding.

[3]The starting dates for the additional countries are as follows: 12/1994 for Austria, Belgium, Denmark, Finland, France, Ireland, Italy, the Netherlands, Portugal, Singapore, and Spain, 12/2000 for the Czech Republic, 3/2001 for Hungary, 5/2003 for Indonesia, 9/2001 for Malaysia, 8/2003 for Mexico, 12/2000 for Poland, and 1/1995 for South Africa.

3 The Foreign Term Premium and the Properties of SDFs

We begin by defining notation and then derive our main theoretical results, first on yields and then on bond returns.

3.1 Notation

In order to state our main results, we first need to introduce the domestic and foreign pricing kernels and stochastic discount factors.

Pricing Kernel and Stochastic Discount Factor The nominal pricing kernel is denoted $\Lambda_t(\varpi)$; it corresponds to the marginal value of a dollar delivered at time t in the state of the world ϖ. The nominal SDF is the growth rate of the pricing kernel: $M_{t+1} = \Lambda_{t+1}/\Lambda_t$. The price of a zero-coupon bond that matures k periods into the future is given by:

$$P_t^{(k)} = E_t\left(\frac{\Lambda_{t+k}}{\Lambda_t}\right).$$

Entropy Bond returns and SDFs are volatile, but not necessarily normally distributed. In order to measure the time-variation in their volatility, it is convenient to use entropy.[4] The dynamics of any random variable X_{t+1} are thus measured through the conditional entropy L_t, defined as:

$$L_t\left(X_{t+1}\right) = \log E_t\left(X_{t+1}\right) - E_t\left(\log X_{t+1}\right).$$

The conditional entropy of a random variable is determined by its conditional variance, as well as its higher moments; if $var_t\left(X_{t+1}\right) = 0$, then $L_t\left(X_{t+1}\right) = 0$, but the reverse is not generally true. If X_{t+1} is conditionally lognormal, then the entropy is simply the half variance of the log variable: $L_t\left(X_{t+1}\right) = (1/2)var_t\left(\log X_{t+1}\right)$. The relative entropy of the permanent and transitory components of the domestic and foreign SDFs turns out to be key to understanding the term structure of carry trade risk. Under regularity conditions, there is a higher-order expansion of

[4]Backus, Chernov, and Zin (2014) make a convincing case for the use of entropy in assessing macro-finance models.

$L_t(X_{t+1}) = \kappa_{2t}/2! + \kappa_{3t}/3! + \kappa_{4t}/4! + \ldots$ where κ_{it} are the cumulants of $\log X_t$. This follows directly from the cumulant-generating function of x_{t+1}.

Exchange Rates in Complete Markets When markets are complete, the change in the exchange rate corresponds to the ratio of the domestic to foreign SDFs:

$$\frac{S_{t+1}}{S_t} = \frac{\Lambda_{t+1}}{\Lambda_t}\frac{\Lambda_t^*}{\Lambda_{t+1}^*},$$

where * denotes a foreign variable. The no-arbitrage definition of the exchange rate comes directly from the Euler equations of the domestic and foreign investors, for any asset R^* expressed in foreign currency: $E_t[M_{t+1}R_{t+1}^* S_t/S_{t+1}] = 1$ and $E_t[M_{t+1}^* R_{t+1}^*] = 1$. When markets are complete, the SDF is unique, and thus the change in exchange rate is the ratio of the two SDFs. As Bekaert (1996) and Bansal (1997) show, in a lognormal model, the log currency risk premium equals the half difference between the conditional volatilities of the log domestic and foreign SDFs. Higher order moments are critical for understanding currency returns.[5] When higher moments matter and markets are complete, the currency risk premium is equal to the difference between the entropy of the domestic and foreign SDFs (Backus, Foresi, and Telmer, 2001):

$$E_t\left[rx_{t+1}^{FX}\right] = (f_t - s_t) - E_t(\Delta s_{t+1}) = L_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) - L_t\left(\frac{\Lambda_{t+1}^*}{\Lambda_t^*}\right).$$

Examining the conditional moments of one-period holding period returns on long maturity bonds, the focus of our paper, is not equivalent to studying the moments of long bond yields in tests of the long-horizon uncovered interest rate parity condition. We start with the latter, as it is the most common in the literature, and then move to the former.

[5]The literature on disaster risk in currency markets concurs. In earlier work, Brunnermeier, Nagel, and Pedersen (2009) show that risk reversals increase with interest rates. Jurek (2014) provides a comprehensive empirical investigation of hedged carry trade strategies. Gourio, Siemer, and Verdelhan (2013) study a real business cycle model with disaster risk. Farhi, Fraiberger, Gabaix, Ranciere, and Verdelhan (2013) estimate a no-arbitrage model with crash risk using a cross-section of currency options. Chernov, Graveline, and Zviadadze (2011) study jump risk at high frequencies. Gavazzoni, Sambalaibat, and Telmer (2012) show that lognormal models cannot account for the cross-country differences in carry returns and interest rate volatilities.

3.2 Theoretical Results on Yields

Long-Horizon Uncovered Interest Rate Parity The long-horizon uncovered interest rate parity condition states that the expected return over k periods on a foreign bond, once converted into domestic currency, is the same as the one on a domestic bond over the same investment horizon. The per period risk premium in logs on a long position in foreign currency over k periods consists of the yield spread minus the expected rate of depreciation over the holding period:

$$E_t[rx^{fx}_{t\to t+k}] = y^{k,*}_t - y^k_t - \frac{1}{k}E_t[\Delta s_{t\to t+k}].$$

The long-horizon uncovered interest rate parity condition assumes that this expected return is zero. It simply extends the usual uncovered interest rate parity condition to k periods. As is well-known, this expected excess return is the sum of a term premium and future currency risk premia. To see that, start from the definition of the currency risk premium: $E_t\Delta s_{t\to t+1} = (r^*_t - r_t) - E_t rx^{FX}_{t+1}$. Summing up over k periods leads to:

$$E_t\Delta s_{t\to t+k} \quad = \quad E_t\sum_{j=1}^{k}\left(r^*_{t+j-1} - r_{t+j-1}\right) - E_t\sum_{j=1}^{k} rx^{FX}_{t+j}.$$

The implied log currency risk premium over k periods is therefore equal to:

$$E_t[rx^{fx}_{t\to t+k}] \quad = \quad (y^{k,*}_t - y^k_t) + \frac{1}{k}E_t\sum_{j=1}^{k}\left(r_{t+j-1} - r^*_{t+j-1}\right) + \frac{1}{k}\sum_{j=1}^{k} E_t rx^{FX}_{t+j}.$$

The first two terms measure the deviations from the expectations hypothesis over the holding period k. The last term measures the deviations from short-run uncovered interest rate maturity over the holding period k. The next proposition shows that the expected excess return over k periods depends on the entropy of the cumulative SDFs.

Proposition 1. *The risk premium (per period) on foreign bonds of maturity k held over k periods equals the difference in the (per period) entropy of the pricing kernels over the holding* *period k:*

$$E_t[rx^{fx}_{t\to t+k}] = \frac{1}{k}\left[L_t\left(\frac{\Lambda_{t+k}}{\Lambda_t}\right) - L_t\left(\frac{\Lambda^*_{t+k}}{\Lambda^*_t}\right)\right].$$

Only differences in long-run per period entropy give rise to long run deviations from U.I.P. The risk premium on a long position in foreign currency is governed by how quickly entropy of the pricing kernel builds up at home and abroad over the holding period. We now consider the relation between foreign and domestic yields as we increase the holding period. To develop some intuition, we first consider the Gaussian case and then turn to the general result.

Gaussian Example If the pricing kernel is conditionally Gaussian over horizon k, the expression on the right hand side reduces to:

$$E_t[rx^{fx}_{t\to t+k}] = \frac{1}{2k}\left[var_t\left(\log\frac{\Lambda_{t+k}}{\Lambda_t}\right) - var_t\left(\log\frac{\Lambda^*_{t+k}}{\Lambda^*_t}\right)\right].$$

Let us assume that the variance of the one-period SDF is constant. The annualized variance of the increase in the log SDF can be expressed as follows:

$$\frac{var\left(\log\Lambda_{t+k}/\Lambda_t\right)}{kvar\left(\Lambda_{t+1}/\Lambda_t\right)} = 1 + 2\sum_{j=1}^{k-1}\left(1-\frac{j}{k}\right)\rho_j,$$

where ρ_j denotes the j-th autocorrelation (see Cochrane, 1988).[6] In the special case where the domestic and foreign countries share the same one-period volatility of the innovations, this expression for the long-run currency risk premium becomes:

$$E[rx^{fx}_{t\to t+k}] = var\left(\Delta\log\Lambda_{t+1}\right)\left[\sum_{j=1}^{k-1}\left(1-\frac{j}{k}\right)(\rho_j-\rho^*_j)\right].$$

This is the Bartlett kernel estimate with window k of the spread in the spectral density of the log SDF at zero, which measures the size of the permanent component of the SDF. More positive

[6]Cochrane (1988) uses these per period variances of the log changes in GDP to measure the size of the random walk component in GDP.

autocorrelation in the domestic than in the foreign pricing kernel tends to create long-term yields that are lower at home than abroad, once expressed in the same currency. The difference in yields, converted in the same units, is governed by a horse race between the speed of mean reversion in the pricing kernel at home and abroad.

In the long run, the foreign currency risk premium over many periods converges to the difference in the size of the random walk components:

$$
\begin{aligned}
\lim_{k\to\infty} E[rx^{fx}_{t\to t+k}] &= \frac{1}{2}var\left(\Delta\log\Lambda_{t+1}\right)\lim_{k\to\infty}\left[1+2\sum_{j=1}^{\infty}\rho_j\right]-\frac{1}{2}var\left(\Delta\log\Lambda_{t+1}\right)\lim_{k\to\infty}\left[1+2\sum_{j=1}^{\infty}\rho^*_j\right] \\
&= \frac{1}{2}\left[S_{\Delta\log\Lambda_{t+1}}-S_{\Delta\log\Lambda^*_{t+1}}\right],
\end{aligned}
$$

where S denotes the spectral density. The last step follows from the definition of the spectral density (see Cochrane, 1988). If the log of the exchange rate ($\log S_t$) is stationary, then the log of the foreign ($\log\Lambda^*_t$) and domestic pricing kernels ($\log\Lambda_t$) are cointegrated with co-integrating vector $(1,-1)$ and hence share the same stochastic trend component. This in turn implies that they have the same spectral density evaluated at zero. As a result, in the Gaussian case, exchange rate stationarity implies that the long-run currency risk premium goes to zero. In that case, exchange rate stationarity implies that U.I.P. holds in the long run.

In the non-Gaussian case, inter-temporal dependence in higher-order moments matters as well. In order to derive more general results, we use the Hansen and Scheinkman (2009) decomposition of the pricing kernel into a martingale and stationary component.

Martingale Component of the SDF We follow here the Hansen and Scheinkman (2009) approach. We consider a continuous-time, right continuous with left limits, strong Markov process X. The eigenpair (β,ϕ) solves the principal eigenvalue problem:

$$
\mathbb{M}_t\phi(x)=E\left[\Lambda_t\phi(X_t)|X_0=x\right]=e^{\beta t}\phi(x).
$$

If the expression above holds for a strictly positive ϕ, then we can decompose the pricing kernel as:

$$\Lambda_t = e^{\beta t} \frac{\phi(X_0)}{\phi(X_t)} \Lambda_t^{\mathbb{P}},$$

where $\Lambda^{\mathbb{P}}$ is guaranteed to be a martingale. The decomposition above implies that the one-period SDF is given by

$$M_{t+1} = \frac{\Lambda_{t+1}}{\Lambda_t} = e^{\beta} \frac{\phi(X_t)}{\phi(X_{t+1})} \frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}}$$

and satisfies

$$E\left[M_{t+1}\phi(X_{t+1})|X_t = x\right] = e^{\beta t}\phi(x).$$

Average Deviations from Long Run U.I.P. We can use this representation to further develop our understanding of the long run U.I.P. condition but we need to limit ourselves to *unconditional* risk premia. Unconditionally, under some additional assumptions, as the maturity of the bonds and the holding period increase, the risk of the persistent component dominates, and in the limit, only the differences in the entropy of the martingale components survive. The following corollary states these assumptions and this result precisely.

Corollary 1. *If the stochastic discount factors* $\frac{\Lambda_{t+1}}{\Lambda_t}$ *and* $\frac{\Lambda^*_{t+1}}{\Lambda^*_t}$ *are strictly stationary, and* $\lim_{k\to\infty}(1/k)L\left(E_t\frac{\Lambda_{t+k}}{\Lambda_t}\right) = 0$ *and* $\lim_{k\to\infty}(1/k)L\left(E_t\frac{\Lambda^*_{t+k}}{\Lambda^*_t}\right) = 0$*, then the per period long-run risk premium on foreign currency is given by:*

$$\lim_{k\to\infty} E[rx^{fx}_{t\to t+k}] = E[y_t^{\infty,*}] - E[y_t^{\infty}] - \lim_{k\to\infty}(1/k)E\Delta s_{t\to t+k} = E\left[L_t\left(\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right) - L_t\left(\frac{\Lambda^{\mathbb{P},*}_{t+1}}{\Lambda^{\mathbb{P},*}_t}\right)\right].$$

If there is no permanent component, or if the permanent component is common, which implies that exchange rates are stationary, the per period foreign currency risk premium converges to zero on average. In other words, a version of long-run U.I.P. obtains on average.

As shown in the literature, long run U.I.P is a potentially valid description of the data, but empirical tests lack power in finite sample. Intuitively, there are few non-overlapping observations of 10-year windows available so far. We thus turn now to the more powerful holding period returns and provide a theoretical framework to interpret our empirical results. Here again, the role of the martingale component of the SDF appears key. To the best of our knowledge, we are the first to demonstrate the connection between exchange rate stationarity, long bond return parity and long-run U.I.P in a no arbitrage framework.

3.3 Main Theoretical Results on Returns

In this section, we present our two key theoretical results on (i) the term structure of carry trade premia and (ii) the long-term bond return parity condition, and we contrast them with long-horizon versions of the uncovered interest rate parity condition.

The Term Structure of Carry Trade Risk Premia We fix the holding period, but instead we increase the maturity of the bonds. Thus, we characterize carry trade risk premia over short holding periods on longer maturity bonds. To do so, we appeal to a different decomposition that uses bond prices. These results will allow us to make contact with the returns on bond portfolios.

Alvarez and Jermann (2005) construct a pricing kernel representation using the price of the long term bond:

$$\Lambda_t = \Lambda_t^{\mathbb{P}} \Lambda_t^{\mathbb{T}}, \text{ where } \Lambda_t^{\mathbb{T}} = \lim_{k \to \infty} \frac{\delta^{t+k}}{P_t^{(k)}},$$

where the constant δ is chosen to satisfy the following regularity condition: $0 < \lim_{k \to \infty} \frac{P_t^{(k)}}{\delta^k} < \infty$ for all t. We also assume that, for each $t+1$, there exists a random variable x_{t+1} with finite expected value $E_t(x_{t+1})$ such that a.s. $\frac{\Lambda_{t+1}}{\delta^{t+1}} \frac{P_{t+1}^{(k)}}{\delta^k} \leq x_{t+1}$ for all k. Under those regularity conditions, the infinite maturity bond return is then:

$$R_{t+1}^{(\infty)} = \lim_{k \to \infty} R_{t+1}^{(k)} = \lim_{k \to \infty} P_{t+1}^{(k-1)} / P_t^{(k)} = \frac{\Lambda_t^{\mathbb{T}}}{\Lambda_{t+1}^{\mathbb{T}}}.$$

The permanent component, $\Lambda_t^{\mathbb{P}}$, is a martingale.[7] It is an important component of the pricing kernel. Alvarez and Jermann (2005) derive a lower bound on its volatility, and, given the size of

[7] Note that $\Lambda_t^{\mathbb{P}}$ is equal to:

$$\Lambda_t^{\mathbb{P}} = \lim_{k \to \infty} \frac{P_t^{(k)}}{\delta^{t+k}} \Lambda_t = \lim_{k \to \infty} \frac{E_t(\Lambda_{t+k})}{\delta^{t+k}}.$$

The second regularity condition ensures that the expression above is a martingale.

the equity premium relative to the term premium, conclude that the permanent component of the pricing kernel is large and accounts for most of the risk.[8] In other words, a lot of persistence in the pricing kernel is needed to deliver a low term premium and a high equity premium. In the absence of arbitrage, Alvarez and Jermann (2005) show that the local term premium in local currency is given by:

$$E_t\left[rx_{t+1}^{(\infty)}\right] = \lim_{k\to\infty} E_t\left[rx_{t+1}^{(k)}\right] = L_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) - L_t\left(\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}}\right).$$

The SDF representation defined here is subject to important limitations that need to be highlighted. Hansen and Scheinkman (2009) point out that this decomposition is not unique under the assumptions used in Alvarez and Jermann (2005). The temporary (or transient) and permanent components are potentially correlated, which may complicate their interpretation. Despite this limitation, this representation proves to be particularly useful when analyzing short-horizon returns on longer maturity bonds.

Proposition 2. *The foreign term premium on the long bond in dollars is equal to the domestic term premium plus the difference between the domestic and foreign entropies of the permanent components of the pricing kernels:*

$$E_t\left[rx_{t+1}^{(\infty),*}\right] + (f_t - s_t) - E_t[\Delta s_{t+1}] = E_t\left[rx_{t+1}^{(\infty)}\right] + L_t\left(\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}}\right) - L_t\left(\frac{\Lambda_{t+1}^{\mathbb{P},*}}{\Lambda_t^{\mathbb{P},*}}\right).$$

In case of an adverse temporary innovation to the foreign pricing kernel, the foreign currency appreciates, but this capital gain is exactly offset by the capital loss suffered on the longest maturity zero-coupon bond, as a result of the increase in foreign interest rates. Hence, interest rate exposure completely hedges the temporary component of the currency risk exposure, and the

[8] Alvarez and Jermann (2005) derive the following lower bound:

$$L_t\left(\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}}\right) \geq E_t\left(\log R_{t+1}\right) - E_t\left(\log R_{t+1}^{(\infty)}\right),$$

where R_{t+1} denotes any positive return and $R_{t+1}^{(\infty)}$ is the return on a zero-coupon bond of infinite maturity.

only source of priced currency risks in the foreign bond positions are the permanent innovations.

In order to produce a currency risk premium at longer maturities, entropy differences in the permanent component of the pricing kernel are required. If there are no such differences and domestic and foreign pricing kernels are identically distributed, then high local currency term premia coincide with low currency risk premia and vice-versa, and dollar term premia are identical across currencies.

Proposition 2 pertains to *conditional* holding period risk premier and is thus directly relevant to interpret the average excess returns obtained on portfolios of countries sorted by the relevant conditioning information, i.e., the level of the short-term interest rates and the slope of the yield curves.

Permanent Component of Exchange Rates Following the decomposition of the pricing kernel discussed above, exchange rate changes can be represented as the product of two components, defined below:

$$\frac{S_{t+1}}{S_t} = \left(\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}}\frac{\Lambda_t^{\mathbb{P},*}}{\Lambda_{t+1}^{\mathbb{P},*}}\right)\left(\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}}\frac{\Lambda_t^{\mathbb{T},*}}{\Lambda_{t+1}^{\mathbb{T},*}}\right) = \frac{S_{t+1}^{\mathbb{P}}}{S_t^{\mathbb{P}}}\frac{S_{t+1}^{\mathbb{T}}}{S_t^{\mathbb{T}}}.$$

Exchange rate changes capture the differences in both the transitory and the permanent component of the two countries' SDFs. Note that $S_{t+1}^{\mathbb{P}}$, the ratio of two martingales, is itself not a martingale in general, but in the class of affine term structure models that we consider in the next section, this exchange rate component is itself a martingale. If two countries share the same martingale component of the pricing kernel, then the resulting exchange rate is always stationary.

A New Long-Term Bond Return Parity Condition The exchange rate decomposition above implies an uncovered long-bond return parity condition when countries share permanent innovations to their SDFs.

Proposition 3. *If the domestic and foreign pricing kernels have common permanent innovations,* $\Lambda_{t+1}^{\mathbb{P}}/\Lambda_t^{\mathbb{P}} = \Lambda_{t+1}^{\mathbb{P},*}/\Lambda_t^{\mathbb{P},*}$ *for all states, then the one-period returns on the foreign longest* *maturity bonds in domestic currency are identical to the domestic ones:* $R_{t+1}^{(\infty),*}\frac{S_t}{S_{t+1}} = R_{t+1}^{(\infty)}$ *for all states.*

While Proposition 2 is about expected returns, Proposition 3 focuses on realized returns. In this polar case, even if most of the innovations to the pricing kernel are highly persistent, the shocks that drive exchange rates are not, because the persistent shocks are shared across countries. When bond parity holds, the exchange rate is a stationary process.

3.4 Finite- vs Infinite-Maturity Bond Returns

As we have shown, the foreign currency risk premia at 10 and 15 years are essentially the 10 and 15-year-window Bartlett kernel 'estimates' of the difference in the SDF's spectral density at zero. How good are these estimates? This matters because the theoretical restrictions that we derive pertain to the foreign currency risk premia on infinite maturity bonds. We provide some evidence that these estimates are likely to be rather good, at least when benchmarked against the dynamics of the U.S yield curve.

Using the state-of-the-art Joslin, Singleton, and Zhu (2011) term structure model, we show that the difference in 10- and 15-year log bond returns is a good proxy for the log returns on infinite-maturity bonds. We estimate a version of the Joslin, Singleton, and Zhu (2011) term structure model with three factors.[9] This Gaussian dynamic term structure model is estimated on constant maturity Treasury yields over the period from January 1990 to December 2007. The maturities considered are 6 months, and 1, 2, 3, 5, 7, and 10 years. The model-implied average one-month and thirty-year yields are 4.2% and 5.8%, as reported in Table 10 of Joslin, Singleton, and Zhu (2011).

Using the parameter estimates, we derive the implied bond returns for different maturities. Table 3 reports for different maturities (5, 10, 15, 20, 20, 100, 200 years and infinity), the average, standard deviation, variance, skewness, and kurtosis of simulated bond log excess returns. It also reports the correlation of the log excess returns of finite-maturity bonds with their infinite-maturity counterparts. This correlation is key for our purposes. For the 10-year bond, the

[9]We thank the authors for making their code available on their web pages.

correlation is 0.95; for the 15-year bond, this correlation is 0.98. Furthermore, the mean log excess return of either the 10-year bond or 15-year bond is not significantly different from that of the infinite-maturity bond at conventional levels of statistical significance. The simulation of the state-of-the-art model in the literature therefore suggests that our inference about infinite-maturity bonds from 10- or 15-year bonds is reasonable. Our empirical findings pertain to differences in log bond returns: long-term bonds, once converted in the same currency, offer similar holding period returns on average across countries. Since the long-term and infinite-maturity bond returns are highly correlated, cross-country differences in log returns are likely to be very similar for long-term and infinite maturity bonds.

Table 3: Simulated Bond Returns

maturity	mean	st. deviation	correlation	skewness	kurtosis
5	3.11	5.24	0.84	0.08	2.68
	[1.06]	[0.31]	[0.02]	[0.16]	[0.23]
10	4.31	9.08	0.95	-0.12	2.63
	[1.80]	[0.48]	[0.01]	[0.15]	[0.22]
15	5.03	12.41	0.98	-0.19	2.62
	[2.44]	[0.65]	[0.00]	[0.14]	[0.23]
20	5.51	15.36	0.99	-0.22	2.62
	[3.01]	[0.80]	[0.00]	[0.14]	[0.23]
30	6.03	20.21	1.00	-0.25	2.63
	[3.93]	[1.05]	[0.00]	[0.14]	[0.24]
100	5.91	32.90	1.00	-0.27	2.65
	[6.37]	[1.73]	[0.00]	[0.14]	[0.24]
200	5.72	34.73	1.00	-0.27	2.66
	[6.72]	[1.83]	[0.00]	[0.14]	[0.24]
∞	4.89	34.84	1.00	-0.27	2.66
	[6.74]	[1.84]	[0.00]	[0.14]	[0.24]

Notes: The table reports, for each maturity (in years), the annualized average (in percentage terms), annualized standard deviation (in percentage terms), skewness, and kurtosis of simulated bond log excess returns. The table also reports the correlation between each bond log excess return and the infinite-maturity bond log excess return. The simulated data come from the benchmark 3-factor model (denoted RPC) in Joslin, Singleton, and Zhu (2011) that sets the first 3 principal components of bond yields as the pricing factors. The standard errors (denoted s.e. and reported between brackets) were generated by block-bootstrapping 10,000 samples of 213 monthly observations.

Building on these results, we derive necessary conditions in a large class of affine term structure models that need to be satisfied in order to match the term structure of carry trade.

4 Implications for Affine Term Structure Models

In this section, we derive parametric restrictions for affine term structure models that imply zero carry trade risk premia at long maturities.[10] These sufficient conditions for zero carry trade risk premia at long horizons also rule out non-stationarity of the nominal exchange rate. For the sake of clarity, we start by analyzing the seminal Vasicek (1977) and Cox, Ingersoll, and Ross (1985) models and then turn to the most recent dynamic term structure models. For each of these models, we first analyze specifications with country-specific factors and then to turn to global factors that are common across countries. Carry trade risk premia arise from asymmetric exposures to global factors. If the entropy of the permanent component cannot differ across countries, then all countries' pricing kernels need the same loadings on the permanent component of the global factors.

All the proofs are in the Online Appendix; there we also show that the operator- and eigenfunction-based approach of Hansen and Scheinkman (2009) delivers the same decomposition as the Alvarez and Jermann (2005) approach.

For the reader's convenience, we repeat the three main equations that will be key to analyze the downward-sloping term structure of carry trade risk in the models:

$$
\begin{aligned}
E_t\left[rx_{t+1}^{FX}\right] &= (f_t - s_t) - E_t(\Delta s_{t+1}) = L_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) - L_t\left(\frac{\Lambda^*_{t+1}}{\Lambda^*_t}\right), \quad (1)\\
E_t\left[rx_{t+1}^{(\infty),*}\right] &= \lim_{k\to\infty} E_t\left[rx_{t+1}^{(k),*}\right] = L_t\left(\frac{\Lambda^*_{t+1}}{\Lambda^*_t}\right) - L_t\left(\frac{\Lambda^{*,\mathbb{P}}_{t+1}}{\Lambda^{*,\mathbb{P}}_t}\right), \quad (2)\\
E_t\left[rx_{t+1}^{(\infty),*}\right] + E_t\left[rx_{t+1}^{FX}\right] &= E_t\left[rx_{t+1}^{(\infty)}\right] + L_t\left(\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right) - L_t\left(\frac{\Lambda^{\mathbb{P},*}_{t+1}}{\Lambda^{\mathbb{P},*}_t}\right). \quad (3)
\end{aligned}
$$

As already noted, Equation (1) shows that the currency risk premium is equal to the difference between the entropy of the domestic and foreign SDFs (Backus, Foresi, and Telmer, 2001). Equation (2) shows that the term premium is equal to the difference between the total entropy of the SDF and the entropy of its permanent component (Alvarez and Jermann, 2005). Equation (3) shows that the foreign term premium in dollars is equal to the domestic term premium plus

[10] see Backus, Gregory, and Zin (1989) for one of the earliest analyses of long run bond yields and forward rates.

the difference in the entropy of the foreign and domestic permanent component of the SDFs.

4.1 Vasicek Model

In the single-factor Vasicek (1977) model, the log SDF is given by:

$$-\log M_{t+1} = y_{1,t} + \frac{1}{2}\lambda^2\sigma^2 + \lambda\varepsilon_{t+1},$$

where $y_{1,t}$, the short-term interest rate, is affine in a single factor:

$$\begin{aligned} x_{t+1} &= \rho x_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}\left(0, \sigma^2\right) \\ y_{1,t} &= \delta + x_t. \end{aligned}$$

In this model, x_t is the level factor and ε_{t+1} are shocks to the level of the term structure. Bond prices are exponentially affine: for any maturity n, bond prices are equal to $p_t^{(n)} = (-B_0^n - B_1^n x_t)$. The coefficients B_0^n and B_1^n satisfy first-order difference equations.[11] It follows directly that the temporary pricing component of the pricing kernel is:

$$\Lambda_t^{\mathbb{T}} = \lim_{n\to\infty} \frac{\beta^{t+n}}{P_t^n} = \lim_{n\to\infty} \beta^{t+n} e^{B_0^n + B_1^n x_t},$$

[11]The coefficients B_0^n and B_1^n satisfy the following recursions:

$$\begin{aligned} B_0^n &= \delta + B_0^{n-1} - \frac{1}{2}\sigma^2 (B_1^{n-1})^2 - \lambda B_1^{n-1}\sigma^2, \\ B_1^n &= 1 + B_1^{n-1}\rho. \end{aligned}$$

where the constant β is chosen in order to satisfy $0 < \lim_{n\to\infty} \frac{P_t^n}{\beta^n} < \infty$.[12] The temporary and the permanent component of the pricing kernel are thus equal to:

$$
\begin{aligned}
\frac{\Lambda^{\mathbb{T}}_{t+1}}{\Lambda^{\mathbb{T}}_t} &= \beta e^{B_1^\infty(x_{t+1}-x_t)} = \beta e^{-x_t + \frac{1}{1-\rho}\varepsilon_{t+1}}.\\
\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t} &= \beta^{-1} e^{-\delta - \frac{1}{2}\lambda^2\sigma^2 - (\frac{1}{1-\rho}+\lambda)\varepsilon_{t+1}}.
\end{aligned}
$$

When $\lambda = -B_1^\infty = -\frac{1}{1-\rho}$, the martingale component of the pricing kernel is constant and all the shocks that affect the pricing kernel are transitory. By using the expression for the bond risk premium in Equation (2), it is straightforward to verify that the expected log excess return of an infinite maturity bond is in this case:

$$
E_t[rx_{t+1}^{(\infty)}] = \frac{1}{2}\sigma^2\lambda^2.
$$

Model with Country-Specific Factor We start by examining the case in which each country has its own factor. We assume the foreign pricing kernel has the same structure, but it is driven by a different factor with different shocks:

$$
\begin{aligned}
-\log M^*_{t+1} &= y^*_{1,t} + \frac{1}{2}\lambda^{*2}\sigma^{*2} + \lambda^*\varepsilon^*_{t+1},\\
x^*_{t+1} &= \rho x^*_t + \varepsilon^*_{t+1}, \quad \varepsilon^*_{t+1} \sim \mathcal{N}\left(0, \sigma^{*2}\right)\\
y_{1,t} &= \delta^* + x^*_t.
\end{aligned}
$$

Equation (1) shows that the expected log currency excess return is constant: $E_t[rx^{FX}_{t+1}] = \frac{1}{2}Var_t(m_{t+1}) - \frac{1}{2}Var_t(m^*_{t+1}) = \frac{1}{2}\lambda^2\sigma^2 - \frac{1}{2}\lambda^{2*}\sigma^{*2}$.

Result 1. *In a Vasicek model with country-specific factors, the long bond uncovered return parity holds only if the model parameters satisfy the following restriction:* $\lambda = -\frac{1}{1-\rho}$.

Under these conditions, there is no martingale component in the pricing kernel and the foreign

[12] The limit of $B_0^n - B_0^{n-1}$ is finite: $\lim_{n\to\infty} B_0^n - B_0^{n-1} = \delta - \frac{1}{2}\sigma^2(B_1^\infty)^2 - \lambda B_1^\infty\sigma^2$, where B_1^∞ is $1/(1-\rho)$. As a result, B_0^n grows at a linear rate in the limit. We choose the constant β to offset the growth in B_0^n as n becomes very large: $\beta = e^{-\delta + \frac{1}{2}\sigma^2(B_1^\infty)^2 + \lambda B_1^\infty\sigma^2}$. This choice guarantees that the regularity conditions in Alvarez and Jermann (2005) are satisfied.

term premium on the long bond expressed in home currency is simply $E_t[rx_{t+1}^{(*,\infty)}] = \frac{1}{2}\lambda^2\sigma^2$. This expression equals the domestic term premium. The nominal exchange rate is stationary.

Symmetric Model with Global Factor Next, we examine the case in which the single state variable x_t is global. The foreign SDF is thus:

$$\begin{aligned} -\log M_{t+1}^* &= y_{1,t}^* + \frac{1}{2}\lambda^{*2}\sigma^2 + \lambda^*\varepsilon_{t+1}, \\ x_{t+1} &= \rho x_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}\left(0, \sigma^2\right) \\ y_{1,t} &= \delta^* + x_t. \end{aligned}$$

This case is key for our understanding of carry risk. Since carry trade returns are base-currency-invariant and obtained on portfolios of countries that average out country-specific shocks, heterogeneity in the exposure of the pricing kernel to *global* shocks is required to explain the carry trade premium (Lustig, Roussanov, and Verdelhan, 2011). Note that here $B_1^\infty = 1/(1-\rho)$ is the same for all countries, since it only depends on the persistence of the global state variable. Likewise, $\sigma = \sigma^{*2}$ in this case.

Result 2. *In a Vasicek model with a single global factor and permanent shocks, the long bond uncovered return parity condition holds only if the countries' SDFs share the same exposure (λ) to the global shocks.*

If countries SDFs share the same parameter λ, then the permanent components of their SDFs are perfectly correlated. In this case, the result is trivial, because the currency risk premium is zero, and the local term premia are identical across countries. we now turn to a model where the currency risk premium is potentially time-varying.

4.2 Cox, Ingersoll, and Ross (1985)

In the Cox, Ingersoll, and Ross (1985) model (denoted CIR), the stochastic discount factor evolves according to:

$$\begin{aligned} -\log M_{t+1} &= \alpha + \chi z_t + \sqrt{\gamma z_t} u_{t+1}, \quad u_{t+1} \sim \mathcal{N}(0,1) \\ z_{t+1} &= (1-\phi)\theta + \phi z_t - \sigma\sqrt{z_t} u_{t+1}. \end{aligned}$$

In the CIR model, log bond prices are also affine in the state variable z: $p_t^{(n)} = -B_0^n - B_1^n z_t$, where B_0^n and B_1^n are the solution to difference equations.[13] The temporary and martingale components of the SDF are:

$$\begin{aligned} \Lambda_t^{\mathbb{T}} &= \lim_{n\to\infty} \frac{\beta^{t+n}}{P_t^{(n)}} = \lim_{n\to\infty} \beta^{t+n} e^{B_0^n + B_1^n z_t}, \\ \frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} &= \frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}}\right)^{-1} = \beta^{-1} e^{-\alpha - \chi z_t - \sqrt{\gamma z_t} u_{t+1}} e^{-B_1^\infty[(\phi-1)(z_t-\theta) - \sigma\sqrt{z_t} u_{t+1}]}. \end{aligned}$$

where the constant $\beta = e^{-\alpha - B_1^\infty(1-\phi)\theta}$ is chosen to offset the growth in B_0^n as n becomes very large. The expected log excess return of an infinite maturity bond is then:

$$E_t\left[rx_{t+1}^{(\infty)}\right] = \left[B_1^\infty(1-\phi) - \chi + \gamma/2\right] z_t, \tag{4}$$

where B_1^∞ is defined implicitly in the following second-order equation: $B_1^\infty = \chi - \gamma/2 + B_1^\infty \phi - \left(B_1^\infty\right)^2 \sigma^2/2 + \sigma\sqrt{\gamma} B_1^\infty$.

[13] The bond price coefficients evolve according to the following second-order difference equations:

$$\begin{aligned} B_0^n &= \alpha + B_0^{n-1} + B_1^{n-1}(1-\phi)\theta, \\ B_1^n &= \chi - \frac{1}{2}\gamma + B_1^{n-1}\phi - \frac{1}{2}\left(B_1^{n-1}\right)^2 \sigma^2 + \sigma\sqrt{\gamma} B_1^{n-1}. \end{aligned}$$

Model with Country-specific Factors Suppose that the foreign pricing kernel is specified as above with the same parameters. The foreign country has its own factor z^*.

$$
\begin{aligned}
-\log M^*_{t+1} &= \alpha + \chi z^*_t + \sqrt{\gamma z^*_t}\, u^*_{t+1}, \\
z^*_{t+1} &= (1-\phi)\theta + \phi z^*_t - \sigma \sqrt{z^*_t}\, u^*_{t+1}.
\end{aligned}
$$

The foreign innovations u^*_{t+1} are not correlated with their domestic counterparts u_{t+1}. From Equation (1), it follows that the log currency risk premium is given by $E_t[rx^{FX}_{t+1}] = \frac{1}{2}\gamma(z_t - z^*_t)$.

Result 3. *In a symmetric CIR model (i.e., when countries share the same parameters) with country-specific factors, the long bond uncovered return parity condition holds only if the model parameters satisfy the following restriction:* $\chi/(1-\phi) = \sqrt{\gamma}/\sigma$.

In the CIR model, there are no permanent innovations to the pricing kernel provided that $B_1^\infty = \frac{\chi}{1-\phi}$. In this case, the second-order equation that defines B_1^∞ implies $B_1^\infty = \sqrt{\gamma}/\sigma$. This condition insures that the price of the long bond fully absorbs the cumulative impact of the innovations on the level of the pricing kernel. As a result, the permanent component of the pricing kernel is constant: $\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t} = \beta^{-1} e^{-\alpha - \chi\theta}$. The term premium on the infinite maturity bond in Equation (4) reduces to $(1/2)\gamma z_t$, the maximum log risk premium.

In the absence of a permanent component, the expected foreign log holding period return on a foreign long bond converted into U.S. dollars is equal to the U.S. term premium: $\frac{1}{2}\gamma z_t$. The nominal exchange rate has no permanent component $\left(S^{\mathbb{P}}_t / S^{\mathbb{P}}_{t+1} = 1\right)$, and hence is stationary.

Model with Global Factor Next, we consider a model in which $z_t = z^*_t$ is a single global state variable that drives the pricing kernel in all countries. The model is thus:

$$
\begin{aligned}
-\log M_{t+1} &= \alpha + \chi z_t + \sqrt{\gamma z_t}\, u_{t+1}, \\
-\log M^*_{t+1} &= \alpha^* + \chi^* z_t + \sqrt{\gamma^* z_t}\, u_{t+1}, \\
z_{t+1} &= (1-\phi)\theta + \phi z_t - \sigma \sqrt{z_t}\, u_{t+1}.
\end{aligned}
$$

Result 4. *In a CIR model with a global factor subject to permanent shocks, the long bond* *uncovered return parity condition holds only if the countries share the same parameters* γ *and* χ.

To understand this result, note that B_1^{∞} depends on χ and γ, as well as on the global parameters ϕ and σ. A necessary and sufficient condition is then the symmetry of the parameters that govern global exposure: $\gamma = \gamma^*$ and $\chi = \chi^*$. Under these conditions, the domestic and foreign pricing kernels react similarly to changes in the global "permanent " state variable and its innovations: the permanent component of the pricing kernel is common, the nominal exchange rate has no permanent component $\left(S_t^{\mathbb{P}}/S_{t+1}^{\mathbb{P}} = 1\right)$, and it is stationary.

4.3 Gaussian Dynamic Term Structure Models

We end this section with the $k-$factor heteroskedastic Gaussian Dynamic Term Structure Model (GDTSM) developed by Duffie and Kan (1996), which generalizes the Cox, Ingersoll, and Ross (1985) model. The log SDF is given by:

$$\begin{aligned}
-\log M_{t+1} &= y_{1,t} + \frac{1}{2}\Lambda' V(x_t)\Lambda + \Lambda' V(x_t)^{1/2}\varepsilon_{t+1}, \\
x_{t+1} &= \Gamma x_t + V(x_t)^{1/2}\varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}(0, I), \\
y_{1,t} &= \delta_0 + \delta_1' x_t,
\end{aligned}$$

where $V(x)$ is a diagonal matrix with entries $V_{ii}(x_t) = \alpha_i + \beta_i' x_t$. To be clear, x_t is a $k \times 1$ vector, and so are ε_{t+1}, Λ, δ_1, and β_i. But Γ and V are $k \times k$ matrices. The Cox, Ingersoll, and Ross (1985) model is a special case of this GDTSM model with $k = 1$, $\sigma = -\sqrt{\beta}$, $\Lambda = -\frac{1}{\sigma}\sqrt{\gamma}$, where β_i is a scalar, and $V_{ii}(x_t) = \alpha_i + \beta_i x_{it}$.[14]

For any maturity n, the zero coupon bond prices are exponentially affine, $P_t^{(n)} = \exp\left(-B_0^n - B_1^{n\prime} x_t\right)$, where the coefficients satisfy the following recursions:

$$\begin{aligned}
B_0^n &= \delta_0 + B_0^{n-1} - \frac{1}{2} B_1^{n-1\prime} V(0) B_1^{n-1} - \Lambda' V(0) B_1^{n-1}, \\
B_1^{n\prime} &= \delta_1' + B_1^{n-1\prime}\Gamma - \frac{1}{2} B_1^{n-1\prime} V_x B_1^{n-1} - \Lambda' V_x B_1^{n-1},
\end{aligned}$$

[14]The CIR model has no constant term in the square root component of the log SDF, but that does not imply $V(0) = 0$ here as the CIR model assumes that the state variable has a non-zero mean (while it is zero here).

where V_x denotes all the diagonal slope coefficients β_i of the V matrix.

We assume that all the shocks are global and that x_t contains global factors. Hence, the foreign pricing kernel is given by:

$$\begin{aligned} -\log M^*_{t+1} &= y^*_{1,t} + \frac{1}{2}\Lambda^{*\prime} V(x_t)\Lambda^* + \Lambda^{*\prime} V(x_t)^{1/2}\varepsilon_{t+1}, \\ x_{t+1} &= \Gamma x_t + V(x_t)^{1/2}\varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}(0, I), \\ y^*_{1,t} &= \delta^*_0 + \delta^{*\prime}_1 x_t, \end{aligned}$$

The only source of cross-sectional heterogeneity is the vector $\Lambda \neq \Lambda^*$.

Decomposition The temporary pricing component and the martingale component of the pricing kernel are then given by:

$$\frac{\Lambda^{\mathbb{T}}_{t+1}}{\Lambda^{\mathbb{T}}_t} = \beta e^{B_1^{\infty\prime}(x_{t+1}-x_t)} = \beta e^{B_1^{\infty\prime}(\Gamma-1)x_t + B_1^{\infty\prime} V(x_t)^{1/2}\varepsilon_{t+1}}, \tag{5}$$

$$\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t} = \beta^{-1} e^{-B_1^{\infty}(\Gamma-1)x_t - \delta_0 - \delta_1' x_t - \frac{1}{2}\Lambda' V(x_t)\Lambda - (\Lambda' + B_1^{\infty\prime})V(x_t)^{1/2}\varepsilon_{t+1}}. \tag{6}$$

We decompose the shocks into two groups: the first $h < k$ shocks affect both the temporary and the permanent SDF components and the last $k - h$ shocks are temporary. The temporary shocks are such that $\Lambda_{k-h} = -B^{\infty}_{1,k-h}$ (i.e., they do not affect the value of the permanent component of the SDF).

Special case Let us start with the special case of no permanent innovations: $h = 0$, the martingale component is constant. As can be shown in Equation (6), two conditions need to be satisfied for the martingale component to be constant: $\Lambda' = -B_1^{\infty\prime}$ and $B_1^{\infty}(\Gamma - 1) + \delta_1' + \frac{1}{2}\Lambda' V_x \Lambda = 0$. The second condition imposes that the cumulative impact on the pricing kernel of an innovation today given by $\left(\delta_1' + \frac{1}{2}\Lambda' V_x \Lambda\right)(1 - \Gamma)^{-1}$ equals the instantaneous impact of the innovation on the long bond price. The second condition is automatically satisfied if the first one holds, as can be verified from the implicit value of $B_1^{\infty\prime}$ implied by the law of motion of B_1. As a result, the martingale component is constant as soon as $\Lambda = -B_1^{\infty}$.

As implied by Equation (2), the term premium on an infinite-maturity zero coupon bond is:

$$E_t[rx_{t+1}^{(\infty)}] = -\delta_0 + \left((1-\Gamma)B_1^{\infty\prime} - \delta_1'\right)x_t. \tag{7}$$

In the absence of permanent shocks, when $\Lambda = -B_1^{\infty}$, this log bond risk premium equals half of the stochastic discount factor variance $E_t[rx_{t+1}^{(\infty)}] = \frac{1}{2}\Lambda' V(x_t)\Lambda$; it attains the upper bound on log risk premia. Consistent with the result in Equation (1), the expected log currency excess return is equal to:

$$E_t\left[rx_{t+1}^{FX}\right] = \frac{1}{2}\Lambda' V(x_t)\Lambda - \frac{1}{2}\Lambda^{*\prime} V(x_t)\Lambda^{*}. \tag{8}$$

Differences in the market prices of risk Λ imply non-zero currency risk premia. Adding the previous two expressions in Equations (7) and (8), we obtain the foreign bond risk premium in dollars. The foreign bond risk premium in dollars equals the domestic bond premium in the absence of permanent shocks: $E_t\left[rx_{t+1}^{(\infty),*}\right] + E_t\left[rx_{t+1}^{FX}\right] = \frac{1}{2}\Lambda' V(x_t)\Lambda$.

General case In general, there is a spread between dollar returns on domestic and foreign bonds. We describe a general condition for long-run uncovered return parity in the presence of permanent shocks.

Result 5. *In a GDTSM with global factors, the long bond uncovered return parity condition holds only if the countries' SDFs share the parameters $\Lambda_h = \Lambda_h^*$ and $\delta_{1h} = \delta_{1h}^*$, which govern exposure to the permanent global shocks.*

The log risk premia on the domestic and foreign infinite-maturity bonds (once expressed in the same currency) are identical provided that the entropies of the domestic and foreign permanent components are the same:

$$\begin{aligned}
(\Lambda_h' + B_{1h}^{\infty\prime})V(0)(\Lambda_h + B_{1h}^{\infty}) &= (\Lambda_h^{*\prime} + B_{1h}^{*\infty\prime})V(0)(\Lambda_h^{*} + B_{1h}^{\infty *}), \\
(\Lambda_h' + B_{1h}^{\infty\prime})V_x(\Lambda_h + B_{1h}^{\infty}) &= (\Lambda_h^{*\prime} + B_{1h}^{*\infty\prime})V_x(\Lambda_h^{*} + B_{1h}^{\infty *}).
\end{aligned}$$

These conditions are satisfied if that these countries share $\Lambda_h = \Lambda_h^*$ and $\delta_{1h} = \delta_{1h}^*$ which govern exposure to the global shocks. In this case, the expected log currency excess return is driven entirely by differences between the exposures to transitory shocks: Λ_{k-h} and Λ^*_{k-h}. If there are only permanent shocks ($h = k$), then the currency risk premium is zero.[15]

The long-term bond return parity condition therefore imposes clear restrictions on cross-country term structure models. The key take-away here is that the carry trade at the short end of the yield curve must compensate investors for bearing global, temporary risks. We have shown this result in different models. The intuition is simple. To generate carry trade risk premia, countries' pricing kernels need to have asymmetric exposures to global shocks (Lustig, Roussanov, and Verdelhan, 2011). However, these global shocks cannot have permanent effects. If they do, the models will generate counterfactual long-term carry premia as well.

4.4 Quantitative Analysis of a Calibrated Multi-Country Model

These restrictions matter. To show this, we end this paper with a specific, calibrated example of the N-country model developed by Lustig, Roussanov, and Verdelhan (2014) to match the cross-sectional evidence in the currency portfolios. The model does not satisfy the long term bond parity condition. The decomposition of the SDF shows why the model fails and why it cannot be fixed with only one source of heterogeneity.

Lustig, Roussanov, and Verdelhan (2014) Following Lustig, Roussanov, and Verdelhan (2014), we consider a world with N countries and currencies. Hodrick and Vassalou (2002) have argued that multi-country models can help to better explain interest rates, bond returns and exchange rates (also see recent work by Graveline and Joslin (2011) and Jotikasthira, Le, and Lundblad (2015) in the same spirit).

In the model, the risk prices associated with country-specific shocks depend only on the country-specific factors, but the risk prices of world shocks depend on world and country-specific

[15] To compare these conditions to the results obtained in the CIR model, recall that we have constrained the parameters in the CIR model such that: $\sigma^{CIR} = -\sqrt{\beta}$, and $\Lambda = -\frac{1}{\sigma^{CIR}}\sqrt{\gamma^{CIR}}$. Differences in Λ_h in the k-factor model are equivalent to differences in γ in the CIR model: in both cases, they correspond to different loadings of the log SDF on the "permanent" shocks. Differences in term premia can also come form differences in the sensitivity of the risk-free rate to the permanent state variable (i.e., different δ_1 parameters). These correspond to differences in χ in the CIR model.

factors. To describe these risk prices, the authors introduce a common state variable z_t^w shared by all countries and a country-specific state variable z_t^i. The country-specific and world state variables follow autoregressive square-root processes:

$$
\begin{aligned}
z_{t+1}^i &= (1-\phi)\theta + \phi z_t^i - \sigma\sqrt{z_t^i}u_{t+1}^i, \\
z_{t+1}^w &= (1-\phi^w)\theta^w + \phi^w z_t^w - \sigma^w\sqrt{z_t^w}u_{t+1}^w.
\end{aligned}
$$

Lustig, Roussanov, and Verdelhan (2014) assume that in each country i, the logarithm of the real SDF m^i follows a three-factor conditionally Gaussian process:

$$
-m_{t+1}^i = \alpha + \chi z_t^i + \sqrt{\gamma z_t^i}u_{t+1}^i + \tau z_t^w + \sqrt{\delta^i z_t^w}u_{t+1}^w + \sqrt{\kappa z_t^i}u_{t+1}^g,
$$

where u_{t+1}^i is a country-specific SDF shock while u_{t+1}^w and u_{t+1}^g are common to all countries SDFs. All of these three innovations are Gaussian, with zero mean and unit variance, independent of one another and over time. There are two types of common shocks. The first type u_{t+1}^w is priced identically in all countries that have the same exposure δ, and all differences in exposure are *permanent*. The second type of common shock, u_{t+1}^g, is not, as heterogeneity with respect to this innovation is *transitory*: all countries are equally exposed to this shock *on average*, but conditional exposures vary over time and depend on country-specific economic conditions

To be parsimonious, Lustig, Roussanov, and Verdelhan (2014) limit the heterogeneity in the SDF parameters to the different loadings δ^i on the world shock u_{t+1}^w; all the other parameters are identical for all countries. The model is therefore a restricted version of the GDTSM.

The log of the nominal bond yields are affine in the state variable z_t and z_t^w: $y_t^{i,n} = \frac{1}{n}\left(C_0^{i,n} + C_1^n z_t + C_2^{i,n} z_t^w\right)$, where the bond parameters are defined recursively.[16] In this model,

[16] Inflation depends on the same state variables, $\pi_{t+1}^i = \pi_0 + \eta^w z_t^w + \sigma_\pi \epsilon_{t+1}^i$, but inflation shocks are assumed orthogonal to the SDF shocks. The bond parameters evolve as:

$$
\begin{aligned}
C_0^{i,n} &= \alpha + \pi_0 - \frac{1}{2}\sigma_\pi^2 + C_0^{n-1} + C_1^{n-1}(1-\phi)\theta + C_2^{i,n-1}(1-\phi^w)\theta^w, \\
C_1^n &= \chi - \frac{1}{2}(\gamma+\kappa) + C_1^{n-1}\phi - \frac{1}{2}\left(C_1^{n-1}\right)^2\sigma^2 + \sigma\sqrt{\gamma}C_1^{n-1} \\
C_2^{i,n} &= \tau - \frac{1}{2}\delta^i + \eta^w + C_2^{i,n-1}\phi^w - \frac{1}{2}\left(C_2^{i,n-1}\right)^2(\sigma^w)^2 + \sigma^w\sqrt{\delta^i}C_2^{i,n-1}.
\end{aligned}
$$

the expected log excess return of an infinite maturity bond is then:

$$E_t[rx_{t+1}^{(i,\infty)}] = \left[C_1^{\infty}(1-\phi) - \chi + \frac{1}{2}(\gamma+\kappa)\right] z_t^i + \left[C_2^{i,\infty}(1-\phi^w) - \tau + \frac{1}{2}\delta^i - \eta^w\right] z_t^w.$$

The foreign currency risk premium is given by:

$$E_t[rx_{t+1}^{FX,i}] = -\frac{1}{2}(\gamma+\kappa)(z_t^i - z_t) + \frac{1}{2}(\delta - \delta^i)(z_t^w).$$

Investors obtain high foreign currency risk premia when investing in currencies whose exposure to the global shocks is smaller. That is the source of short-term carry trade risk premia. The foreign bond risk premium in dollars is simply given by the sum of the two expressions above:

$$\begin{aligned} E_t[rx_{t+1}^{(i,\infty)}] + E_t[rx_{t+1}^{FX,i}] &= \left[\frac{1}{2}(\gamma+\kappa)z_t + (C_1^{\infty}(1-\phi) - \chi)z_t^i\right] \\ &+ \left[\frac{1}{2}\delta + C_2^{i,\infty}(1-\phi^w) - \tau - \eta^w\right] z_t^w. \end{aligned}$$

We report in the Online Appendix the calibration and simulation of this model. It does not reproduce the long term bond parity condition. The simulation highlights a key tension: without the heterogeneity in δs, the model cannot produce short-term carry trade risk premia; the heterogeneity in δs, however, leads to counterfactual long-term carry risk premia. To understand this result, we implement again the decomposition of the pricing kernel into a transitory and permanent component and show that the calibration does not satisfy the restrictions highlighted above.

Next, we examine the conditions that are necessary for long-run uncovered bond return parity in this model.

Result 6. *The long-run uncovered bond return parity holds if* $C_1^{\infty}(1-\phi) = \chi$, $\kappa = 0$, *and* $C_2^{i,\infty}(1-\phi^w) = \tau + \eta^w$.

The first two restrictions rule out permanent effects of country-specific shocks. The last restriction rules out permanent effects of global shocks (u^w). When these restrictions are satisfied, the pricing kernel is not subject to permanent shocks. The U.S. term premium is simply $E_t[rx_{t+1}^{(\infty)}] = \frac{1}{2}(\gamma z_t + \delta z_t^w)$, which is equal to one-half of the variance of the log stochastic discount factor. As can easily be verified, the expected foreign log holding period return on a foreign long bond converted into U.S. dollars is equal to the U.S. term premium: $E_t[rx_{t+1}^{i,(\infty)}] + E_t[rx_{t+1}^{FX,i}] = \frac{1}{2}(\gamma z_t + \delta z_t^w)$. The higher foreign currency risk premium for investing in high δ countries is exactly offset by the lower bond risk premium.

The decomposition of the pricing kernel suggests a potential route to address this tension: two global state variables, one describing transitory shocks and one describing permanent shocks. As noted in the study of the term structure models, the heterogeneity in the SDFs' loadings on the *permanent* global shocks needs to be ruled out in order to match the long term bond parity. The heterogeneity in the SDFs' loadings on the *transitory* global shocks accounts for the carry trade excess returns at the short end of the yield curve. We sketch such a model in the Online Appendix, and show that the heterogeneity in the SDFs' loadings on pure *transitory* global shocks can only be obtained if countries differ in more than one dimension (δ, but also τ, ϕ^w, σ^w, or η^w, or a combination of those). A potential solution to the tension entails a very rich model that is beyond the scope of this paper. We leave the empirical estimation of a N-country model that replicates the term structure of currency trade risk premia as a challenge for the literature.

5 Conclusion

This paper presents some new empirical facts on currency and interest rate risk. In a cross-section of countries, we find that the term structure of currency risk premia is downward sloping. While carry trade strategies based on the three-month Treasury bills are highly profitable, carry trade strategies using long-maturity bonds are not.

This paper derives some novel preference-free results that rely only on market completeness and help understand the downward-sloping term structure of currency carry trade risk. We show that the difference between the domestic and foreign long-term bond risk premia, expressed in a common currency, reflects the difference in the entropy of the permanent components of the stochastic discount factor. These results entail novel parameter restrictions on global term structure models.

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Appendix

The Appendix starts with a review of Hansen and Scheinkman (2009) and its link to the Alvarez and Jermann (2005) decomposition used in the main text. The Appendix then gathers all the proofs of the theoretical results in the paper. To make sure that the paper is fully self-contained we reproduce here some proofs of intermediary results already in the literature, notably in Alvarez and Jermann (2005). The reader familiar with the literature can skip the intermediary steps. The Appendix ends with two examples: the case of no-permanent shocks and the case of power utility over consumption.

A Existence and Uniqueness of Multiplicative Decomposition of the SDF

Consider a continuous-time, right continuous with left limits, strong Markov process X and the filtration $\mathcal{F}$ generated by the past values of X, completed by the null sets. In the case of infinite-state spaces, X is restricted to be a semimartingale, so it can be represented as the sum of a continuous process X^c and a pure jump process X^j. The pricing kernel process Λ is a strictly positive process, adapted to $\mathcal{F}$, for which it holds that the time t price of any payoff Π_s realized at time s ($s \geq t$) is given by

$$P_t(\Pi_s) = E\left[\frac{\Lambda_s}{\Lambda_t}\Pi_s|\mathcal{F}_t\right].$$

The pricing kernel process also satisfies $\Lambda_0 = 1$. Hansen and Scheinkman (2009) show that Λ is a multiplicative functional and establish the connection between the multiplicative property of the pricing kernel process and the semigroup property of pricing operators $\mathbb{M}$.[17] In particular, consider the family of operators $\mathbb{M}$ described by

$$\mathbb{M}_t\psi(x) = E\left[\Lambda_t\psi(X_t)|X_0 = x\right]$$

where $\psi(X_t)$ is a random payoff at t that depends solely on the Markov state at t. The family of linear pricing operators $\mathbb{M}$ satisfies $\mathbb{M}_0 = \mathbb{I}$ and $\mathbb{M}_{t+u}\psi(x) = \mathbb{M}_t\psi(x)\mathbb{M}_u\psi(x)$ and, thus, defines a semigroup, called pricing semigroup.

Further, Hansen and Scheinkman (2009) show that Λ can be decomposed as

$$\Lambda_t = e^{\beta t}\frac{\phi(X_0)}{\phi(X_t)}\Lambda_t^{\mathbb{P}}$$

where $\Lambda^{\mathbb{P}}$ is a multiplicative functional and a local martingale, ϕ is a principal (i.e. strictly positive) eigenfunction of the extended generator of $\mathbb{M}$ and β is the corresponding eigenvalue (typically negative).[18] If, furthermore, $\Lambda^{\mathbb{P}}$ is martingale, then the eigenpair (β, ϕ) also solves the principal eigenvalue problem:[19]

$$\mathbb{M}_t\phi(x) = E\left[\Lambda_t\phi(X_t)|X_0 = x\right] = e^{\beta t}\phi(x).$$

Conversely, if the expression above holds for a strictly positive ϕ and $\mathbb{M}_t\phi$ is well-defined for $t \geq 0$, then $\Lambda^{\mathbb{P}}$ is a martingale. Thus, a strictly positive solution to the eigenvalue problem above implies a decomposition

$$\Lambda_t = e^{\beta t}\frac{\phi(X_0)}{\phi(X_t)}\Lambda_t^{\mathbb{P}}$$

[17] A functional Λ is multiplicative if it satisfies $\Lambda_0 = 1$ and $\Lambda_{t+u} = \Lambda_t\Lambda_u(\theta_t)$, where θ_t is a shift operator that moves the time subscript of the relevant Markov process forward by t periods. Products of multiplicative functionals are multiplicative functionals. The multiplicative property of the pricing kernel arises from the requirement for consistency of pricing across different time horizons.

[18] The extended generator of a multiplicative functional Λ is formally defined in Hansen and Scheinkman (2009) and, intuitively, assigns to a Borel function ψ a Borel function ξ such that $\Lambda_t\xi(X_t)$ is the expected time derivative of $\Lambda_t\psi(X_t)$.

[19] Since $\Lambda^{\mathbb{P}}$ is a local martingale bounded from below, it is a supermartingale. For $\Lambda^{\mathbb{P}}$ to be a martingale, additional conditions need to hold, as discussed in Appendix C of Hansen and Scheinkman (2009).

where $\Lambda^{\mathbb{P}}$ is guaranteed to be a martingale. The decomposition above implies that the one-period SDF is given by

$$M_{t+1} = \frac{\Lambda_{t+1}}{\Lambda_t} = e^{\beta} \frac{\phi(X_t)}{\phi(X_{t+1})} \frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}$$

and satisfies

$$E\left[M_{t+1}\phi(X_{t+1})|X_t = x\right] = e^{\beta t}\phi(x).$$

Hansen and Scheinkman (2009) provide sufficient conditions for the existence of a solution to the principal eigenfunction problem and, thus, for the existence of the aforementioned pricing kernel decomposition. Notably, multiple solutions may exist, so the pricing kernel decomposition above is generally not unique. However, if the state space is finite and the Markov chain is irreducible, then Perron-Frobenious theory implies that there is a unique principal eigenvector (up to scaling), and thus a unique pricing kernel decomposition. Although multiple solutions typically exist, Hansen and Scheinman (2009) show that the only (up to scaling) principal eigenfunction of interest for long-run pricing is the one associated with the smallest eigenvalue, as the multiplicity of solutions is eliminated by the requirement for stochastic stability of the Markov process X. In particular, only this solution ensures that the process X remains stationary and Harris recurrent under the probability measure implied by the martingale $\Lambda^{\mathbb{P}}$.

Finally, Hansen and Scheinkman (2009) show that the aforementioned pricing kernel decomposition can be useful in approximating the prices of long-maturity zero-coupon bonds. In particular, the time t price of a bond with maturity $t+k$ is given by

$$P_t^{(k)} = E\left[\frac{\Lambda_{t+k}}{\Lambda_t}|X_t = x\right] = e^{\beta k}E^{\mathbb{P}}\left[\frac{1}{\phi(X_{t+k})}|X_t = x\right]\phi(x) \approx e^{\beta k}E^{\mathbb{P}}\left[\frac{1}{\phi(X_{t+k})}\right]\phi(x)$$

where $E^{\mathbb{P}}$ is the expectation under the probability measure implied by the martingale $\Lambda^{\mathbb{P}}$ and the right-hand-side approximation becomes arbitrarily accurate as $k \to \infty$. Thus, in the limit of arbitrarily large maturity, the price of the zero-coupon bond depends on the current state solely through $\phi(x)$ and not through the expectation of the transitory component. Notably, this implies that the relevant ϕ is the one that ensures that X remains stationary under the probability measure implied by $\Lambda^{\mathbb{P}}$, i.e. the unique principal eigenfunction that implies stochastic stability for X, and β is the corresponding eigenvalue.

Indeed, Alvarez and Jermann (2005) *construct* a pricing kernel decomposition by considering a constant $\hat{\beta}$ that satisfies

$$0 < \lim_{k\to\infty} \frac{P_t^{(k)}}{\hat{\beta}^k} < \infty$$

and *defining* the transitory pricing kernel component as

$$\Lambda_t^{\mathbb{T}} = \lim_{k\to\infty} \frac{\hat{\beta}^{t+k}}{P_t^{(k)}} < \infty.$$

In contrast to Hansen and Scheinkman (2009), the decomposition of Alvarez and Jermann (2005) is constructive and not unique, as their Assumptions 1 and 2 do not preclude the existence of alternative pricing kernel decompositions to a martingale and a transitory component. Note that the Alvarez and Jermann (2005) decomposition implies that $\hat{\beta} = e^{\beta}$, where β is the smallest eigenvalue associated with a principal eigenfunction in the Hansen and Scheinkman (2009) eigenfunction problem.

B Proofs

- Proof of Proposition 1:

  Proof. In any no-arbitrage model, the yield spread equals the difference in the log of the expected rate of increase in the pricing kernels over the holding period k:

  $$y_t^{k,*} - y_t^k = -(1/k)\log E_t\left(\exp[\Delta \log \Lambda^*_{t\to t+k}]\right) + (1/k)\log E_t\left(\exp[\Delta \log \Lambda_{t\to t+k}]\right).$$

  Using the definition of entropy, we can restate this expression as:

  $$y_t^{k,*} - y_t^k = (1/k)E_t\Delta s_{t\to t+k} + (1/k)\left[L_t\left(\frac{\Lambda_{t+k}}{\Lambda_t}\right) - L_t\left(\frac{\Lambda^*_{t+k}}{\Lambda^*_t}\right)\right].$$

□

- Proof of Corollary 1:

Proof. Note that $L(x_{t+1}) = EL_t(x_{t+1}) + L_t(E(x_{t+1}))$. Given the stationary of the stochastic discount factor, $\lim_{k\to\infty}(1/k)L_t\left(E\frac{\Lambda_{t+k}}{\Lambda_t}\right) = 0$. Hence $\lim_{k\to\infty}(1/k)L\left(E\frac{\Lambda_{t+k}}{\Lambda_t}\right) = \lim_{k\to\infty}(1/k)EL_t\left(E\frac{\Lambda_{t+k}}{\Lambda_t}\right)$. Given our assumptions, it can be shown directly that:

$$\lim_{k\to\infty}(1/k)\left[L\left(\frac{\Lambda_{t+k}}{\Lambda_t}\right) - L\left(\frac{\Lambda^*_{t+k}}{\Lambda^*_t}\right)\right] = \left[L\left(\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right) - L\left(\frac{\Lambda^{\mathbb{P},*}_{t+1}}{\Lambda^{\mathbb{P},*}_t}\right)\right].$$

This result follows directly from the Alvarez-Jermann decomposition of the pricing kernel (see Alvarez and Jermann (2005)'s proposition 6).

We offer a different, shorter proof that directly exploits the Hansen-Scheinkman decomposition of the pricing kernel, but does not rely on the Alvarez-Jermann representation:

$$\frac{\Lambda_{t+k}}{\Lambda_t} = \frac{\Lambda^{\mathbb{P}}_{t+k}}{\Lambda^{\mathbb{P}}_t}\frac{\phi(X_t)}{\phi(X_{t+k})e^{\beta k}}.$$

Using a change of measure by exploiting the martingale property of the permanent component,

$$\lim_{k\to\infty}(1/k)\log E\frac{\Lambda^{\mathbb{P}}_{t+k}}{\Lambda^{\mathbb{P}}_t}\frac{\phi(X_t)}{\phi(X_{t+k})e^{\beta k}} = \lim_{k\to\infty}(1/k)\log\widehat{E}\frac{\phi(X_t)}{\phi(X_{t+k})e^{\beta k}} = 1,$$

where the last step follows because X is a Markov process. We also know that

$$\lim_{k\to\infty}(1/k)E\log\frac{\Lambda^{\mathbb{P}}_{t+k}}{\Lambda^{\mathbb{P}}_t}\frac{\phi(X_t)}{\phi(X_{t+k})e^{\beta k}} = E\log\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}.$$

As a result, we know that

$$\lim_{k\to\infty}(1/k)L\left(\frac{\Lambda_{t+k}}{\Lambda_t}\right) = 1 - E\log\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t} = L\left(\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right).$$

□

The same result can be derived in the framework of Alvarez and Jermann (2005), buildin on their Proposition 5. Alvarez and Jermann (2005) show that, when the limits of the k-period bond risk premium and the yield difference between the k-period discount bond and the one-period riskless bond (when the maturity k tends to infinity) are well defined and the unconditional expectations of holding returns are independent of calendar time, then the average term premium $E\left[\lim_{k\to\infty} rx^{(k),*}_{t+1}\right]$ equals the average yield spread $E[\lim_{k\to\infty} y^{(k),*}_t - y^{(1),*}_t]$. Substituting for the term premiums in Proposition 2 leads to:

$$E[y^{(\infty),*}_t] + E[\Delta s_{t+1}] = E[y^{(\infty),*}_t] + E\left[L_t\left(\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right) - L_t\left(\frac{\Lambda^{\mathbb{P},*}_{t+1}}{\Lambda^{\mathbb{P},*}_t}\right)\right].$$

Under regularity conditions, in a stationary environment, $E[\lim_{k\to\infty}\frac{1}{k}\sum_{j=1}^k \Delta s_{t+j}] = \lim_{k\to\infty}\frac{1}{k}\sum_{j=1}^k E[\Delta s_{t+j}]$ converges to $E[\Delta s_{t+1}]$. Using this result also produces the corollary.

- Proof of Proposition 2:

Proof. The proof builds on some results in Backus, Foresi, and Telmer (2001) and Alvarez and Jermann (2005). Specifically, Backus, Foresi, and Telmer (2001) show that the foreign currency risk premium is equal to the difference between domestic and foreign total SDF entropy:

$$(f_t - s_t) - E_t[\Delta s_{t+1}] = L_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) - L_t\left(\frac{\Lambda^*_{t+1}}{\Lambda^*_t}\right).$$

Furthermore, Alvarez and Jermann (2005) establish that total SDF entropy equals the sum of the entropy of the permanent SDF component and the expected log term premium:

$$L_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) = L_t\left(\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right) + E_t\left(\log\frac{R^{(\infty)}_{t+1}}{R^f_t}\right).$$

Applying the Alvarez and Jermann (2005) decomposition to the Backus, Foresi, and Telmer (2001) expression yields the desired result.

To derive the Backus, Foresi, and Telmer (2001) expression, consider a foreign investor who enters a forward position in the currency market with payoff $S_{t+1} - F_t$. The investor's Euler equation is:

$$E_t\left(\frac{\Lambda^*_{t+1}}{\Lambda^*_t}(S_{t+1} - F_t)\right) = 0.$$

In the presence of complete, arbitrage-free international financial markets, exchange rate changes equal the ratio of the domestic and foreign stochastic discount factors:

$$\frac{S_{t+1}}{S_t} = \frac{\Lambda_{t+1}}{\Lambda_t}\frac{\Lambda^*_t}{\Lambda^*_{t+1}},$$

Dividing the investor's Euler equation by S_t and applying the no arbitrage condition, the forward discount is:

$$f_t - s_t = \log E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) - \log E_t\left(\frac{\Lambda^*_{t+1}}{\Lambda^*_t}\right).$$

The second component of the currency risk premium is expected foreign appreciation; applying logs and conditional expectations to the no arbitrage condition above leads to:

$$E_t[\Delta s_{t+1}] = E_t\left(\log\frac{\Lambda_{t+1}}{\Lambda_t}\right) - E_t\left(\log\frac{\Lambda^*_{t+1}}{\Lambda^*_t}\right).$$

Combining the two terms of the currency risk premium leads to:

$$(f_t - s_t) - E_t[\Delta s_{t+1}] = \log E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) - E_t\left(\log\frac{\Lambda_{t+1}}{\Lambda_t}\right) - \log E_t\left(\frac{\Lambda^*_{t+1}}{\Lambda^*_t}\right) + E_t\left(\log\frac{\Lambda^*_{t+1}}{\Lambda^*_t}\right)$$

Applying the definition of conditional entropy in the equation above yields the Backus, Foresi, and Telmer (2001) expression.

To derive the Alvarez and Jermann (2005) result, first note that since the permanent component of the pricing kernel is a martingale, its conditional entropy can be expressed as follows:

$$L_t\left(\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right) = -E_t\left(\log\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right).$$

The definition of conditional entropy implies the following decomposition of total SDF entropy:

$$L_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) = \log E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) - E_t\left(\log\frac{\Lambda^{\mathbb{T}}_{t+1}}{\Lambda^{\mathbb{T}}_t}\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right)$$

or, using the above expression for the conditional entropy of the permanent SDF component:

$$L_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) = -\log R^f_t - E_t\left(\log\frac{\Lambda^{\mathbb{T}}_{t+1}}{\Lambda^{\mathbb{T}}_t}\right) + L_t\left(\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right).$$

The Alvarez and Jermann (2005) result hinges on:

$$\lim_{k\to\infty} R^{(k)}_{t+1} = \Lambda^{\mathbb{T}}_t/\Lambda^{\mathbb{T}}_{t+1}.$$

Under the assumption that $0 < \lim_{k\to\infty} \frac{P_t^{(k)}}{\delta^k} < \infty$ for all t, one can write:

$$\lim_{k\to\infty} R_{t+1}^{(k)} = \lim_{k\to\infty} \frac{E_{t+1}\left(\frac{\Lambda_{t+k}}{\Lambda_{t+1}}\right)}{E_t\left(\frac{\Lambda_{t+k}}{\Lambda_t}\right)} = \frac{\lim_{k\to\infty} \frac{E_{t+1}(\Lambda_{t+k}/\delta^{t+k})}{\Lambda_{t+1}}}{\lim_{k\to\infty} \frac{E_t(\Lambda_{t+k}/\delta^{t+k})}{\Lambda_t}} = \frac{\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_{t+1}}}{\frac{\Lambda_t^{\mathbb{P}}}{\Lambda_t}} = \Lambda_t^{\mathbb{T}}/\Lambda_{t+1}^{\mathbb{T}}.$$

Thus, the infinite-maturity bond is exposed only to transitory SDF risk. □

- Proof of Proposition 3:

Proof. As shown inAlvarez and Jermann (2005) (see the proof of Proposition 2), the return of the infinite maturity bond reflects the transitory SDF component:

$$\lim_{k\to\infty} R_{t+1}^{(k)} = \Lambda_t^{\mathbb{T}}/\Lambda_{t+1}^{\mathbb{T}}.$$

The result of Proposition 3 follows directly from the no-arbitrage expression for the spot exchange rate when markets are complete:

$$\frac{S_{t+1}}{S_t} = \frac{\Lambda_{t+1}}{\Lambda_t}\frac{\Lambda_t^*}{\Lambda_{t+1}^*}.$$

In this case,

$$\lim_{k\to\infty} \frac{S_t}{S_{t+1}}\frac{R_{t+1}^{(k),*}}{R_{t+1}^{(k)}} = \frac{S_t}{S_{t+1}}\frac{\lim_{k\to\infty} R_{t+1}^{(k),*}}{\lim_{k\to\infty} R_{t+1}^{(k)}} = \frac{S_t}{S_{t+1}}\frac{\Lambda_t^{\mathbb{T}}}{\Lambda_{t+1}^{\mathbb{T}}}\frac{\Lambda_{t+1}^{\mathbb{T},*}}{\Lambda_t^{\mathbb{T},*}} = \frac{\Lambda_{t+1}^{\mathbb{P},*}}{\Lambda_t^{\mathbb{P},*}}\frac{\Lambda_t^{\mathbb{P}}}{\Lambda_{t+1}^{\mathbb{P}}} = \frac{S_t^{\mathbb{P}}}{S_{t+1}^{\mathbb{P}}},$$

using the decomposition of exchange rate changes into a permanent and a transitory component:

$$\frac{S_{t+1}}{S_t} = \left(\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}}\frac{\Lambda_t^{\mathbb{P},*}}{\Lambda_{t+1}^{\mathbb{P},*}}\right)\left(\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}}\frac{\Lambda_t^{\mathbb{T},*}}{\Lambda_{t+1}^{\mathbb{T},*}}\right) = \frac{S_{t+1}^{\mathbb{P}}}{S_t^{\mathbb{P}}}\frac{S_{t+1}^{\mathbb{T}}}{S_t^{\mathbb{T}}}.$$

The exposure of the domestic and foreign infinite-maturity bonds to transitory risk fully offsets the transitory component of exchange rate changes, so only the exposure to the permanent part remains. □

C Examples

We now turn to two simple examples, one without permanent innovations and one with a homoskedastic SDF.

C.1 Special Case: No Permanent Innovations

Let us now consider the special case in which the pricing kernel is *not* subject to permanent innovations, i.e., $\lim_{k\to\infty} \frac{E_{t+1}[\Lambda_{t+k}]}{E_t[\Lambda_{t+k}]} = 1$. For example, the Markovian environment recently considered by Ross (2013) to derive his recovery theorem satisfies this condition. Building on this work, Martin and Ross (2013) derive closed-form expressions for bond returns in a similar environment. Alvarez and Jermann (2005) show that this case has clear implications for domestic returns: if the pricing kernel has no permanent innovations, then the term premium on an infinite maturity bond is the largest risk premium in the economy.[20]

The absence of permanent innovations also has a strong implication for the term structure of the carry trade risk premia. When the pricing kernels do not have permanent innovations, the foreign term premium in dollars equals the domestic term premium:

$$E_t\left[rx_{t+1}^{(\infty),*}\right] + (f_t - s_t) - E_t[\Delta s_{t+1}] = E_t\left[rx_{t+1}^{(\infty)}\right].$$

[20] If there are no permanent innovations to the pricing kernel, then the return on the bond with the longest maturity equals the inverse of the SDF: $\lim_{k\to\infty} R_{t+1}^{(k)} = \Lambda_t/\Lambda_{t+1}$. High marginal utility growth translates into higher yields on long maturity bonds and low long bond returns, and vice-versa.

The proof here is straightforward. In general, the foreign currency risk premium is equal to the difference in entropy. In the absence of permanent innovations, the term premium is equal to the entropy of the pricing kernel, so the result follows. More interestingly, a much stronger result holds in this case. Not only are the risk premia identical, but the returns on the foreign bond position are the same as those on the domestic bond position state-by-state, because the foreign bond position automatically hedges the currency risk exposure. As already noted, if the domestic and foreign pricing kernels have no permanent innovations, then the one-period returns on the longest maturity foreign bonds in domestic currency are identical to the domestic ones:

$$\lim_{k\to\infty} \frac{S_t}{S_{t+1}} \frac{R_{t+1}^{(k),*}}{R_{t+1}^{(k)}} = 1.$$

In this class of economies, the returns on long-term bonds expressed in domestic currency are equalized:

$$\lim_{k\to\infty} rx_{t+1}^{(k),*} + (f_t - s_t) - \Delta s_{t+1} = rx_{t+1}^{(k)}.$$

In countries that experience higher marginal utility growth, the domestic currency appreciates but is exactly offset by the capital loss on the bond. For example, in a representative agent economy, when the log of aggregate consumption drops more below trend at home than abroad, the domestic currency appreciates, but the real interest rate increases, because the representative agent is eager to smooth consumption. The foreign bond position automatically hedges the currency exposure.

We now turn a simple consumption-based example.

C.2 Homoskedastic SDF

Alvarez and Jermann (2005) propose the following example of an economy without permanent shocks: a representative agent economy with power utility investors in which the log of aggregate consumption is a trend-stationary process with normal innovations.

Example 1. *Consider the following pricing kernel (Alvarez and Jermann, 2005):*

$$\log \Lambda_t = \sum_{i=0}^{\infty} \alpha_i \epsilon_{t-i} + \beta \log t,$$

with $\epsilon \sim N(0, \sigma^2), \alpha_0 = 1$. *If* $\lim_{k\to\infty} \alpha_k^2 = 0$, *then the SDF has no permanent component. The foreign SDF is defined similarly.*

In this example, Alvarez and Jermann (2005) show that the term premium equals one half of the variance: $E_t\left[rx_{t+1}^{(\infty)}\right] = \sigma^2/2$, the highest possible risk premium in this economy, because the returns on the long bond are perfectly negatively correlated with the stochastic discount factor. When marginal utility is [temporarily] high, the representative agent would like to borrow, driving up interest rates and lowering the price of the long-term bond.

In this case, we find that the foreign term premium in dollars is identical to the domestic term premium:

$$E_t\left[rx_{t+1}^{(\infty),*}\right] + (f_t - s_t) - E_t[\Delta s_{t+1}] = \frac{1}{2}\sigma^2 = E_t\left[rx_{t+1}^{(\infty)}\right].$$

This result is straightforward to establish: recall that the currency risk premium is equal to the half difference in the domestic and foreign SDF volatilities. Currencies with a high local currency term premium (high σ^2) also have an offsetting negative currency risk premium, while those with a small term premium have a large currency risk premium. Hence, U.S. investors receive the same dollar premium on foreign as on domestic bonds. There is no point in chasing high term premia around the world, at least not in economies with only temporary innovations to the pricing kernel. Currencies with the highest local term premia also have the lowest (i.e., most negative) currency risk premia.

Building on the previous example, Alvarez and Jermann (2005) consider a log-normal model of the pricing kernel that features both permanent and transitory shocks.

Example 2. *Consider the following pricing kernel (Alvarez and Jermann, 2005):*

$$\log \Lambda_{t+1}^{\mathbb{P}} = -\frac{1}{2}\sigma_P^2 + \log \Lambda_t^{\mathbb{P}} + \varepsilon_{t+1}^P,$$
$$\log \Lambda_{t+1}^{\mathbb{T}} = \log \beta^{t+1} + \sum_{i=0}^{\infty} \alpha_i \varepsilon_{t+1-i}^T,$$

where α is a square summable sequence, and ε^P and ε^T are i.i.d. normal variables with mean zero and covariance σ_{TP}. A similar decomposition applies to the foreign SDF.

In this case, Alvarez and Jermann (2005) show that the term premium is given by the following expression: $E_t\left[rx_{t+1}^{(\infty)}\right] = \sigma_T^2/2 + \sigma_{TP}$. Only the transitory risk is priced in the market for long bonds. When marginal utility is temporarily high, interest rates increase because the representative agent wants to borrow, and long bonds suffer a capital loss. Permanent shocks to marginal utility do not have this effect. In this economy, the foreign term premium in dollars is:

$$E_t\left[rx_{t+1}^{(\infty),*}\right] + (f_t - s_t) - E_t[\Delta s_{t+1}] = \frac{1}{2}\left(\sigma^2 - \sigma_P^{2,*}\right).$$

Provided that $\sigma_P^{2,*} = \sigma_P^2$, the foreign term premium in dollars equals the domestic term premium:

$$E_t\left[rx_{t+1}^{(\infty),*}\right] + (f_t - s_t) - E_t[\Delta s_{t+1}] = \frac{1}{2}\sigma_T^2 + \sigma_{TP} = E_t\left[rx_{t+1}^{(\infty)}\right].$$

Online Appendix for "The Term Structure of Currency Carry Trade Risk Premia" —Not For Publication—

This Online Appendix describes additional empirical results on the cross-section of currency and term risk premia. Section A reports additional results on portfolios of countries sorted by the short-term interest rates. Section B reports similar results for portfolios of countries sorted by the slope of the yield curves. Section C reports additional results obtained with zero-coupon bonds to describe the whole term structure of currency carry trade risk premia. Section D reports additional theoretical results on dynamic term structure models, starting with the simple Vasicek (1977) and Cox, Ingersoll, and Ross (1985) one-factor models, before turning to their k-factor extensions and the model studied in Lustig, Roussanov, and Verdelhan (2014).

A Sorting Countries by Interest Rates

This section first focuses on our benchmark sample of G10 countries, and then turn to larger sets of countries. In each case, we consider three different bond holding periods (one, three, and twelve months), and two time windows (12/1950–12/2012 and 12/1971–12/2012).

A.1 Benchmark Sample

Figure 4 plots the composition of the three interest rate-sorted portfolios of the currencies of the benchmark sample, ranked from low to high interest rate currencies. Typically, Switzerland and Japan (after 1970) are funding currencies in Portfolio 1, while Australia and New Zealand are the carry trade investment currencies in Portfolio 3. The other currencies switch between portfolios quite often.

Table 4 reports the annualized moments of log returns on currency and bond markets. As expected [see Lustig and Verdelhan (2007) for a detailed analysis], the average excess returns increase from Portfolio 1 to Portfolio 3. For investment periods of one month, the average excess return on Portfolio 1 is -0.24% per annum, while the average excess return on Portfolio 3 is 3.26%. The spread between Portfolio 1 and Portfolio 3 is 3.51% per annum. The volatility of these returns increases only slightly from the first to the last portfolio. As a result, the Sharpe ratio (annualized) increases from -0.03 on Portfolio 1 to 0.40 on the Portfolio 3. The Sharpe ratio on a long position in Portfolio 3 and a short position in the Portfolio 1 is 0.49 per annum. The results for the post-Bretton-Woods sample are very similar. Hence, the currency carry trade is profitable at the short end of the maturity spectrum.

Recall that the absence of arbitrage implies a negative relationship between the equilibrium risk premium for investing in a currency and the SDF entropy of the corresponding country. Therefore, given the pattern in currency risk premia, high interest rate currencies have low entropy and low interest rate currencies have high entropy. As a result, sorting by interest rates (from low to high) seems equivalent to sorting by pricing kernel entropy (from high to low). In a log-normal world, entropy is just one half of the variance: high interest rate currencies have low pricing kernel variance, while low interest rate currencies have volatile pricing kernels.

Table 4 shows that there is a strong decreasing pattern in local currency bond risk premia. The average excess return on Portfolio 1 is 2.39% per annum and its Sharpe ratio is 0.68. The excess return decreases monotonically to -0.21% on Portfolio 3. Thus, there is a 2.60% spread per annum between Portfolio 1 and Portfolio 3.

If all of the shocks driving currency risk premia were permanent, then there would be no relation between currency risk premia and term premia. To the contrary, we find a very strong negative association between local currency bond risk premia and currency risk premia. Low interest rate currencies tend to produce high local currency bond risk premia, while high interest rate currencies tend to produce low local currency bond risk premia. The decreasing term premia are consistent with the decreasing entropy of the total SDF from low (Portfolio 1) to high interest rates (Portfolio 3) that we had inferred from the foreign currency risk premia. Furthermore, it appears that these are not offset by equivalent decreases in the entropy of the permanent component of the foreign pricing kernel.

The decline in the local currency bond risk premia partly offsets the increase in currency risk premia. As a result, the average excess return on the foreign bond expressed in U.S. dollars measured in Portfolio 3 is only

Table 4: Interest Rate-Sorted Portfolios: Benchmark Sample

Portfolio		1	2	3	3 − 1	1	2	3	3 − 1	1	2	3	3 − 1
Horizon		1-Month				3-Month				12-Month			
						Panel A: 12/1950–12/2012							
$-\Delta s$	Mean	1.43	0.35	-0.18	-1.62	1.65	0.23	-0.28	-1.93	1.78	0.22	-0.42	-2.20
$f-s$	Mean	-1.68	0.78	3.45	5.13	-1.64	0.81	3.39	5.03	-1.55	0.91	3.19	4.74
rx^{FX}	Mean	-0.24	1.13	3.26	3.51	0.00	1.03	3.11	3.10	0.23	1.12	2.77	2.54
	s.e.	[1.00]	[0.84]	[1.03]	[0.92]	[1.02]	[0.91]	[1.20]	[1.03]	[1.10]	[1.03]	[1.25]	[0.98]
	Std	7.92	6.64	8.10	7.22	8.29	6.91	8.74	7.97	9.20	7.32	9.59	8.03
	SR	-0.03	0.17	0.40	0.49	0.00	0.15	0.36	0.39	0.03	0.15	0.29	0.32
	s.e.	[0.13]	[0.13]	[0.14]	[0.14]	[0.13]	[0.13]	[0.14]	[0.15]	[0.13]	[0.13]	[0.14]	[0.15]
$rx^{(10),*}$	Mean	2.39	1.67	-0.21	-2.60	2.17	1.31	0.41	-1.76	1.98	1.07	0.85	-1.13
	s.e.	[0.45]	[0.47]	[0.57]	[0.57]	[0.50]	[0.53]	[0.64]	[0.61]	[0.58]	[0.65]	[0.72]	[0.65]
	Std	3.50	3.65	4.50	4.47	4.01	4.34	5.10	4.72	4.41	5.22	5.67	5.05
	SR	0.68	0.46	-0.05	-0.58	0.54	0.30	0.08	-0.37	0.45	0.20	0.15	-0.22
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]	[0.13]	[0.13]	[0.12]	[0.15]	[0.14]	[0.13]	[0.12]
$rx^{(10),\$}$	Mean	2.15	2.81	3.06	0.91	2.17	2.34	3.52	1.35	2.21	2.19	3.62	1.41
	s.e.	[1.18]	[0.98]	[1.16]	[1.09]	[1.22]	[1.06]	[1.31]	[1.21]	[1.26]	[1.14]	[1.42]	[1.23]
	Std	9.33	7.70	9.21	8.61	10.00	8.22	9.94	9.32	10.64	8.83	11.08	9.95
	SR	0.23	0.36	0.33	0.11	0.22	0.28	0.35	0.14	0.21	0.25	0.33	0.14
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]	[0.14]
$rx^{(10),\$} - rx^{(10),US}$	Mean	0.63	1.30	1.55	0.91	0.65	0.82	2.00	1.35	0.67	0.65	2.08	1.41
	s.e.	[1.22]	[1.09]	[1.32]	[1.09]	[1.16]	[1.18]	[1.51]	[1.21]	[1.33]	[1.22]	[1.61]	[1.23]
						Panel B: 12/1971–12/2012							
$-\Delta s$	Mean	2.16	0.30	-0.29	-2.45	2.39	0.14	-0.49	-2.89	2.61	0.05	-0.74	-3.35
$f-s$	Mean	-2.03	1.07	3.87	5.90	-1.99	1.11	3.80	5.78	-1.89	1.23	3.60	5.49
rx^{FX}	Mean	0.13	1.38	3.57	3.44	0.41	1.25	3.30	2.90	0.72	1.28	2.86	2.14
	s.e.	[1.52]	[1.26]	[1.50]	[1.35]	[1.56]	[1.37]	[1.75]	[1.52]	[1.67]	[1.55]	[1.82]	[1.42]
	Std	9.66	8.11	9.57	8.46	10.12	8.42	10.46	9.52	11.22	8.88	11.59	9.68
	SR	0.01	0.17	0.37	0.41	0.04	0.15	0.32	0.30	0.06	0.14	0.25	0.22
	s.e.	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.17]	[0.16]	[0.17]	[0.17]	[0.18]
$rx^{(10),*}$	Mean	2.82	2.12	-0.13	-2.95	2.60	1.66	0.56	-2.05	2.48	1.30	1.01	-1.47
	s.e.	[0.64]	[0.68]	[0.81]	[0.79]	[0.72]	[0.77]	[0.92]	[0.87]	[0.84]	[0.94]	[1.01]	[0.87]
	Std	4.12	4.33	5.17	5.09	4.69	5.12	5.85	5.33	5.17	6.06	6.31	5.45
	SR	0.68	0.49	-0.02	-0.58	0.55	0.32	0.10	-0.38	0.48	0.21	0.16	-0.27
	s.e.	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.16]	[0.16]	[0.15]	[0.19]	[0.18]	[0.16]	[0.15]
$rx^{(10),\$}$	Mean	2.95	3.49	3.45	0.50	3.01	2.90	3.86	0.85	3.20	2.58	3.87	0.67
	s.e.	[1.78]	[1.46]	[1.71]	[1.59]	[1.85]	[1.60]	[1.89]	[1.77]	[1.91]	[1.66]	[2.03]	[1.75]
	Std	11.33	9.34	10.82	10.06	12.09	9.91	11.68	10.95	12.85	10.44	13.01	11.70
	SR	0.26	0.37	0.32	0.05	0.25	0.29	0.33	0.08	0.25	0.25	0.30	0.06
	s.e.	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.16]	[0.16]	[0.17]	[0.18]	[0.16]
$rx^{(10),\$} - rx^{(10),US}$	Mean	0.44	0.99	0.94	0.50	0.48	0.37	1.33	0.85	0.64	0.01	1.31	0.67
	s.e.	[1.77]	[1.55]	[1.88]	[1.59]	[1.70]	[1.71]	[2.16]	[1.77]	[1.99]	[1.78]	[2.32]	[1.75]

Notes: The table reports the average change in exchange rates (Δs), the average interest rate difference ($f-s$), the average currency excess return (rx^{FX}), the average foreign bond excess return on 10-year government bond indices in foreign currency ($rx^{(10),*}$) and in U.S. dollars ($rx^{(10),\$}$), as well as the difference between the average foreign bond excess return in U.S. dollars and the average U.S. bond excess return ($rx^{(10),\$} - rx^{US}$). For the excess returns, the table also reports their annualized standard deviation (denoted Std) and their Sharpe ratios (denoted SR). The annualized monthly log returns are realized at date $t+k$, where the horizon k equals 1, 3, and 12 months. The balanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. The countries are sorted by the level of their short term interest rates into three portfolios. The standard errors (denoted s.e. and reported between brackets) were generated by bootstrapping 10,000 samples of non-overlapping returns.

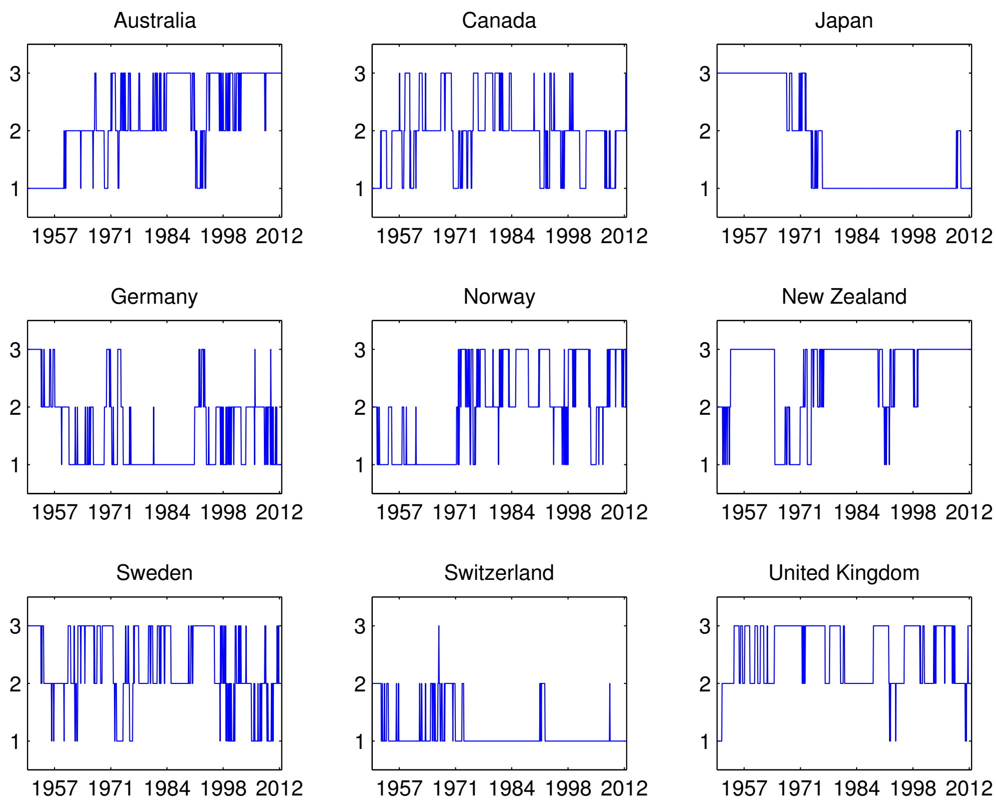

Figure 4: Composition of Interest Rate-Sorted Portfolios — The figure presents the composition of portfolios of 9 currencies sorted by their short-term interest rates. The portfolios are rebalanced monthly. Data are monthly, from 12/1950 to 12/2012.

0.91% per annum higher than the average excess returns measured in Portfolio 1. The Sharpe ratio on a long-short position in bonds of Portfolio 3 and Portfolio 1 is only 0.11. U.S. investors cannot simply combine the currency carry trade with a yield carry trade, because these risk premia roughly offset each other. Interest rates are great predictors of currency excess returns and local currency bond excess returns, but not of dollar excess returns. To receive long-term carry trade returns, investors need to load on differences in the quantity of permanent risk, as shown in Equation (3). The cross-sectional evidence presented here does not lend much support to these differences in permanent risk.

Table 4 shows that the results are essentially unchanged in the post-Bretton-Woods sample. The Sharpe ratio on the currency carry trade is 0.41, achieved by going long in Portfolio 3 and short in Portfolio 1. However, there is a strong decreasing pattern in local currency bond risk premia, from 2.82% per annum in Portfolio 1 to −0.13% in the Portfolio 3. As a result, there is essentially no discernible pattern in dollar bond risk premia.

Figure 5 presents the cumulative one-month log returns on investments in foreign Treasury bills and foreign 10-year bonds. Most of the losses are concentrated in the 1970s and 1980s, and the bond returns do recover in the 1990s. In fact, between 1991 and 2012, the difference is currency risk premia at the one-month horizon between Portfolio 1 and Portfolio 3 is 4.54%, while the difference in the local term premia is only 1.41% per annum. As a result, the un-hedged carry trade in 10-year bonds still earn about 3.13% per annum over this sample. However, this difference of 3.13% per annum has a standard error of 1.77% and, therefore, is not statistically significant.

As we increase holding period k from 1 to 3 and 12 months, the differences in local bond risk premia between portfolios shrink, but so do the differences in currency risk premia. Even at the 12-month horizon, there is no evidence of statistically significant differences in dollar bond risk premia across the currency portfolios.

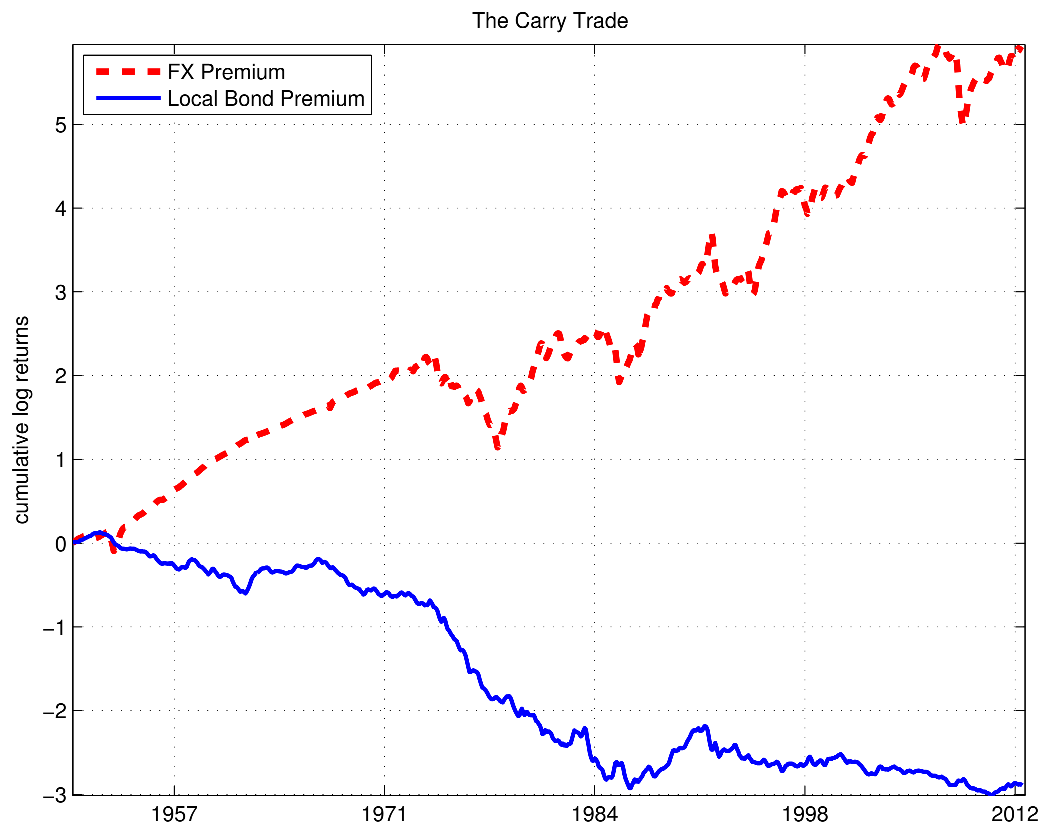

Figure 5: The Carry Trade and Term Premia – The figure presents the cumulative one-month log returns on investments in foreign Treasury bills and foreign 10-year bonds. The benchmark panel of countries includes Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. Countries are sorted every month by the level of their one-month interest rates into three portfolios. The returns correspond to a strategy going long in the Portfolio 3 and short in Portfolio 1. The sample period is 12/1950–12/2012.

A.2 Developed Countries

Very similar patterns of risk premia emerge using larger sets of countries. In the sample of developed countries, we sort currencies in four portfolios. Figure 6 plots the composition of the four interest rate-sorted currency portfolios. Switzerland and Japan (after 1970) are funding currencies in Portfolio 1, while Australia and New Zealand are carry trade investment currencies in Portfolio 4.

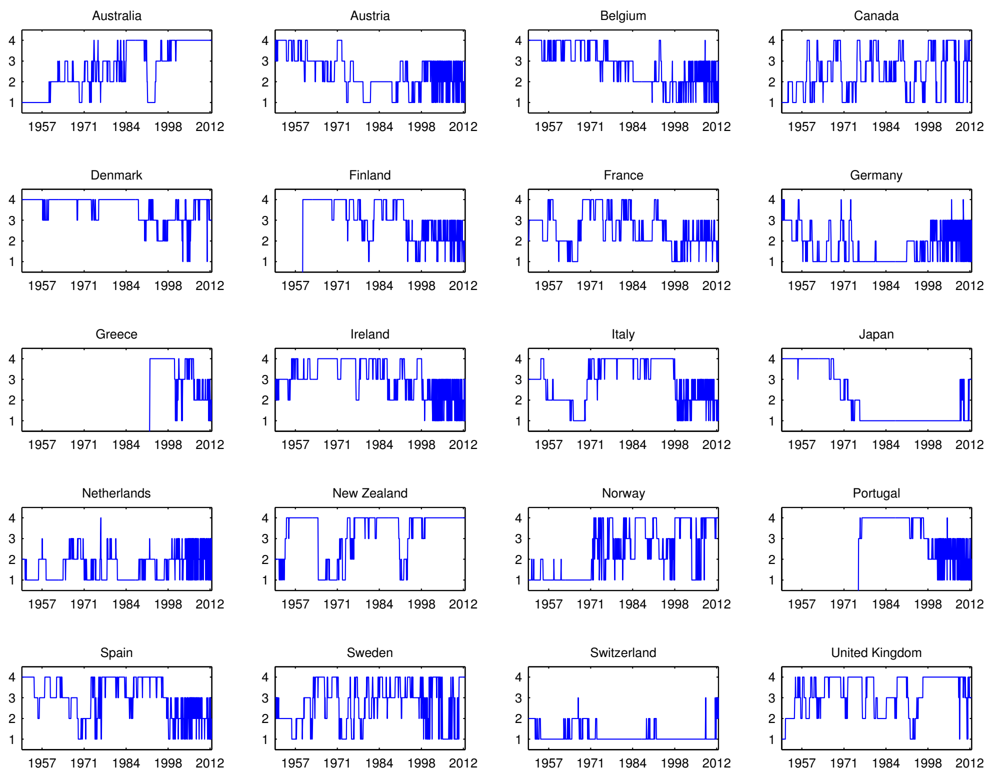

Figure 6: Composition of Interest Rate-Sorted Portfolios — The figure presents the composition of portfolios of 20 currencies sorted by their short-term interest rates. The portfolios are rebalanced monthly. Data are monthly, from 12/1950 to 12/2012.

Table 5 reports the results of sorting the developed country currencies into portfolios based on the level of their interest rate, ranked from low to high interest rate currencies. Essentially, the results are very similar to those obtained on the benchmark sample of developed countries. There is no economically or statistically significant carry trade premium at longer maturities. The 2.98% spread in the currency risk premia is offset by the negative 3.03% spread in local term premia at the one-month horizon against the carry trade currencies.

A.3 Whole Sample

Finally, Table 6 reports the results of sorting all the currencies in our sample, including those of emerging countries, into portfolios according to the level of their interest rate, ranked from low to high interest rate currencies. In the sample of developed and emerging countries, the pattern in returns is strikingly similar, but the differences are larger. At the one-month horizon, the 6.66% spread in the currency risk premia is offset by a 5.15% spread in local term premia. A long-short position in foreign bonds delivers an excess return of 1.51% per annum, which is not statistically significantly different from zero. At longer horizons, the differences in local bond risk premia are much smaller, but so are the carry trade returns.

Table 5: Interest Rate Sorted Portfolios: Developed sample

Portfolio		1	2	3	4	$4-1$	1	2	3	4	$4-1$	1	2	3	4	$4-1$
Horizon		1-month					3-month					12-month				
		Panel A: 1950-2012														
$-\Delta s$	Mean	1.30	0.58	0.06	-1.17	-2.47	1.40	0.37	0.19	-1.23	-2.63	1.54	0.38	-0.13	-1.13	-2.68
$f-s$	Mean	-1.41	0.39	1.51	4.03	5.44	-1.38	0.42	1.52	3.97	5.35	-1.26	0.53	1.56	3.79	5.05
r^{FX}	Mean	-0.11	0.97	1.56	2.86	2.98	0.02	0.79	1.71	2.74	2.72	0.28	0.91	1.43	2.65	2.37
	s.e.	[1.02]	[1.04]	[1.02]	[0.97]	[0.62]	[1.06]	[1.10]	[1.11]	[1.10]	[0.65]	[1.12]	[1.17]	[1.11]	[1.22]	[0.66]
	Std	8.02	8.26	7.96	7.67	4.87	8.36	8.43	8.28	8.18	5.25	9.27	8.76	8.79	9.24	5.36
	SR	-0.01	0.12	0.20	0.37	0.61	0.00	0.09	0.21	0.33	0.52	0.03	0.10	0.16	0.29	0.44
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]	[0.13]	[0.13]	[0.13]	[0.13]	[0.15]	[0.13]	[0.13]	[0.13]	[0.13]	[0.17]
$rx^{(10),*}$	Mean	3.00	1.90	1.05	-0.02	-3.03	2.46	1.59	1.08	0.48	-1.98	1.98	1.02	0.93	1.03	-0.95
	s.e.	[0.53]	[0.56]	[0.56]	[0.53]	[0.62]	[0.60]	[0.62]	[0.64]	[0.62]	[0.63]	[0.71]	[0.89]	[0.80]	[0.76]	[0.61]
	Std	4.15	4.40	4.41	4.14	4.81	4.53	4.84	5.06	4.95	4.91	5.00	5.81	6.24	5.88	4.52
	SR	0.72	0.43	0.24	-0.01	-0.63	0.54	0.33	0.21	0.10	-0.40	0.39	0.18	0.15	0.18	-0.21
	s.e.	[0.12]	[0.14]	[0.13]	[0.13]	[0.11]	[0.13]	[0.13]	[0.13]	[0.13]	[0.12]	[0.13]	[0.13]	[0.13]	[0.13]	[0.12]
$rx^{(10),\$}$	Mean	2.89	2.87	2.62	2.84	-0.05	2.48	2.38	2.79	3.22	0.74	2.26	1.93	2.36	3.68	1.42
	s.e.	[1.22]	[1.24]	[1.18]	[1.09]	[0.91]	[1.29]	[1.28]	[1.26]	[1.19]	[0.93]	[1.32]	[1.47]	[1.34]	[1.40]	[0.88]
	Std	9.59	9.86	9.26	8.62	7.13	10.24	10.05	9.74	9.22	7.45	10.74	10.66	10.91	10.60	7.60
	SR	0.30	0.29	0.28	0.33	-0.01	0.24	0.24	0.29	0.35	0.10	0.21	0.18	0.22	0.35	0.19
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]	[0.12]	[0.14]
$rx^{(10),\$} - rx^{(10),US}$	Mean	1.38	1.36	1.11	1.33	-0.05	0.96	0.86	1.27	1.70	0.74	0.71	0.38	0.81	2.14	1.42
	s.e.	[1.27]	[1.31]	[1.21]	[1.24]	[0.91]	[1.23]	[1.31]	[1.30]	[1.34]	[0.93]	[1.33]	[1.51]	[1.39]	[1.46]	[0.88]
		Panel B: 1971-2012														
$-\Delta s$	Mean	1.86	0.68	0.28	-1.41	-3.27	1.95	0.40	0.36	-1.49	-3.44	2.18	0.41	-0.17	-1.42	-3.60
$f-s$	Mean	-1.64	0.47	1.66	4.63	6.27	-1.59	0.51	1.68	4.56	6.15	-1.46	0.66	1.74	4.35	5.81
r^{FX}	Mean	0.22	1.16	1.94	3.22	3.00	0.36	0.91	2.04	3.07	2.71	0.71	1.07	1.57	2.93	2.22
	s.e.	[1.55]	[1.59]	[1.50]	[1.46]	[0.91]	[1.58]	[1.63]	[1.63]	[1.61]	[0.95]	[1.70]	[1.76]	[1.62]	[1.84]	[0.97]
	Std	9.80	10.13	9.54	9.30	5.83	10.23	10.32	9.98	9.89	6.26	11.33	10.69	10.65	11.16	6.45
	SR	0.02	0.11	0.20	0.35	0.51	0.03	0.09	0.20	0.31	0.43	0.06	0.10	0.15	0.26	0.34
	s.e.	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.16]	[0.17]	[0.16]	[0.17]	[0.19]
$rx^{(10),*}$	Mean	3.67	2.58	1.20	0.44	-3.23	2.95	2.16	1.25	1.07	-1.88	2.46	1.25	1.05	1.67	-0.80
	s.e.	[0.79]	[0.83]	[0.81]	[0.75]	[0.89]	[0.87]	[0.90]	[0.92]	[0.88]	[0.91]	[1.04]	[1.31]	[1.16]	[1.09]	[0.89]
	Std	4.98	5.29	5.17	4.77	5.62	5.40	5.75	5.95	5.76	5.76	5.92	6.82	7.39	6.79	5.28
	SR	0.74	0.49	0.23	0.09	-0.57	0.55	0.38	0.21	0.19	-0.33	0.42	0.18	0.14	0.25	-0.15
	s.e.	[0.15]	[0.17]	[0.16]	[0.16]	[0.14]	[0.16]	[0.16]	[0.16]	[0.15]	[0.15]	[0.16]	[0.16]	[0.16]	[0.17]	[0.15]
$rx^{(10),\$}$	Mean	3.89	3.73	3.14	3.66	-0.23	3.31	3.07	3.29	4.14	0.83	3.18	2.32	2.61	4.60	1.42
	s.e.	[1.85]	[1.87]	[1.74]	[1.62]	[1.33]	[1.92]	[1.90]	[1.83]	[1.73]	[1.35]	[1.96]	[2.17]	[1.92]	[2.03]	[1.31]
	Std	11.67	12.04	11.04	10.30	8.44	12.43	12.21	11.62	10.92	8.81	12.96	12.74	13.02	12.46	9.04
	SR	0.33	0.31	0.28	0.36	-0.03	0.27	0.25	0.28	0.38	0.09	0.25	0.18	0.20	0.37	0.16
	s.e.	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.17]	[0.16]	[0.17]
$rx^{(10),\$} - rx^{(10),US}$	Mean	1.38	1.23	0.63	1.15	-0.23	0.78	0.53	0.76	1.61	0.83	0.61	-0.24	0.05	2.03	1.42
	s.e.	[1.86]	[1.91]	[1.72]	[1.80]	[1.33]	[1.78]	[1.89]	[1.84]	[1.91]	[1.35]	[1.97]	[2.24]	[1.99]	[2.15]	[1.31]

Annualized monthly log returns realized at $t+k$ on 10-year Bond Index and T-bills for k from 1 month to 12 months. Portfolios of 21 currencies sorted every month by T-bill rate at t. The unbalanced panel consists of Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom.

As in the previous samples, the rate at which the high interest rate currencies depreciate (2.99% per annum) is not high enough to offset the interest rate difference of 6.55%. Similarly, the rate at which the low interest rate currencies appreciate (0.43% per annum) is not high enough to offset the low interest rates (3.52% lower than the U.S. interest rate). Uncovered interest rate parity fails in the cross-section. However, the bond return differences (in local currency) are closer to being offset by the rate of depreciation. The bond return spread is 4.63% per annum for the last portfolio, compared to an annual depreciation rate of 6.55%, while the spread on the first portfolio is −0.29%, compared to depreciation of −0.43%.

B Sorting Currencies by the Slope of the Yield Curve

This section presents additional evidence on slope-sorted portfolios, again considering first our benchmark sample of G10 countries before turning to larger sets of developed and emerging countries.

B.1 Benchmark Sample

Figure 7 presents the composition over time of portfolios of the 9 currencies of the benchmark sample sorted by the slope of the yield curve.

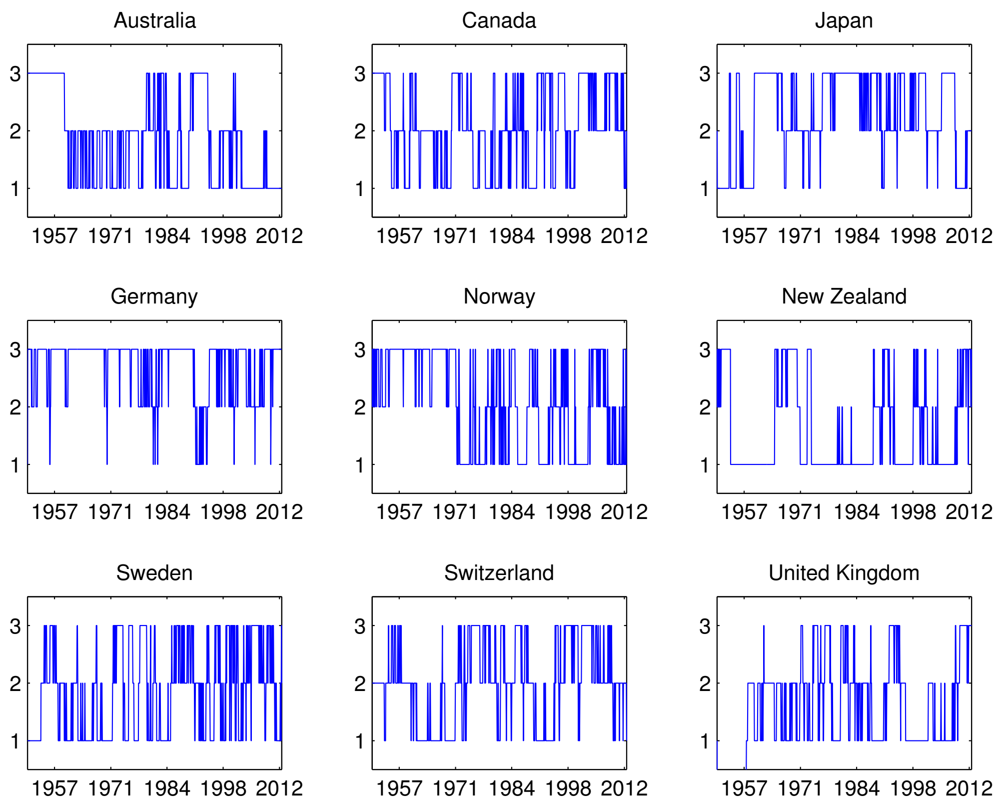

Figure 7: Composition of Slope-Sorted Portfolios — The figure presents the composition of portfolios of the currencies in the benchmark sample sorted by the slope of their yield curves. The portfolios are rebalanced monthly. The slope of the yield curve is measured as the 10-year interest rate minus the one-month Treasury bill rates. Data are monthly, from 12/1950 to 12/2012.

Table 7 reports the results of sorting on the yield curve slope on the benchmark G10 sample.

Table 6: Interest Sorted Portfolios: Whole sample

Portfolio		1	2	3	4	5	5 − 1	1	2	3	4	5	5 − 1	1	2	3	4	5	5 − 1
Horizon		1-month						3-month						12-month					
Panel A: 1950-2012																			
$-\Delta s$	Mean	0.43	-0.05	0.49	-0.63	-2.99	-3.41	0.64	0.05	0.29	-0.67	-3.09	-3.73	0.87	0.04	-0.06	-0.71	-2.95	-3.82
$f-s$	Mean	-1.81	-0.15	0.87	2.09	5.70	7.51	-1.72	0.45	0.89	2.11	5.59	7.31	-1.54	0.00	1.09	2.12	5.32	6.86
rx^{FX}	Mean	-1.38	-0.20	1.36	1.46	2.72	4.10	-1.08	0.50	1.17	1.44	2.50	3.58	-0.66	0.04	1.04	1.41	2.37	3.04
	s.e.	[0.82]	[0.94]	[0.94]	[0.91]	[0.84]	[0.63]	[0.84]	[1.03]	[0.96]	[0.97]	[0.95]	[0.68]	[0.87]	[1.08]	[1.04]	[1.03]	[1.05]	[0.68]
	Std	6.44	7.40	7.43	7.14	6.68	4.98	6.66	11.04	7.38	7.60	7.26	5.46	7.47	8.20	8.71	8.08	8.14	5.71
	SR	-0.22	-0.03	0.18	0.20	0.41	0.82	-0.16	0.05	0.16	0.19	0.35	0.66	-0.09	0.00	0.12	0.18	0.29	0.53
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]	[0.14]	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]	[0.16]	[0.13]	[0.13]	[0.13]	[0.14]	[0.14]	[0.19]
$rx^{(10),*}$	Mean	3.02	1.86	1.45	1.16	0.44	-2.58	2.56	1.05	1.26	1.14	0.99	-1.58	1.95	1.18	1.04	1.04	1.64	-0.31
	s.e.	[0.46]	[0.52]	[0.48]	[0.54]	[0.54]	[0.64]	[0.47]	[0.65]	[0.55]	[0.55]	[0.61]	[0.65]	[0.63]	[0.82]	[0.59]	[0.64]	[0.71]	[0.65]
	Std	3.59	4.05	3.79	4.17	4.29	5.09	3.96	9.22	4.28	4.57	5.00	5.35	4.33	5.24	6.62	5.25	5.52	5.19
	SR	0.84	0.46	0.38	0.28	0.10	-0.51	0.65	0.11	0.29	0.25	0.20	-0.30	0.45	0.22	0.16	0.20	0.30	-0.06
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.12]	[0.12]	[0.13]	[0.13]	[0.13]	[0.13]	[0.12]	[0.13]	[0.13]	[0.13]	[0.14]	[0.14]	[0.12]
$rx^{(10),\$}$	Mean	1.64	1.66	2.81	2.62	3.16	1.52	1.49	1.56	2.43	2.58	3.49	2.01	1.29	1.22	2.07	2.45	4.02	2.73
	s.e.	[0.99]	[1.12]	[1.08]	[1.09]	[1.05]	[0.97]	[1.00]	[1.26]	[1.11]	[1.08]	[1.16]	[1.01]	[1.03]	[1.35]	[1.24]	[1.21]	[1.25]	[0.95]
	Std	7.81	8.79	8.54	8.51	8.28	7.60	8.23	9.48	8.65	9.00	9.16	8.22	8.76	9.65	9.57	9.72	10.12	8.14
	SR	0.21	0.19	0.33	0.31	0.38	0.20	0.18	0.16	0.28	0.29	0.38	0.24	0.15	0.13	0.22	0.25	0.40	0.34
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]	[0.13]	[0.15]
$rx^{(10),\$} - rx^{(10),US}$	Mean	0.12	0.15	1.30	1.11	1.65	1.52	-0.04	0.04	0.91	1.06	1.97	2.01	-0.26	-0.33	0.53	0.91	2.47	2.73
	s.e.	[1.18]	[1.23]	[1.14]	[1.21]	[1.30]	[0.97]	[1.12]	[1.31]	[1.20]	[1.22]	[1.45]	[1.01]	[1.11]	[1.45]	[1.39]	[1.31]	[1.45]	[0.95]
Panel B: 1971-2012																			
$-\Delta s$	Mean	0.74	0.09	0.46	-0.24	-3.78	-4.52	0.99	0.05	0.41	-0.67	-3.78	-4.77	1.17	0.15	-0.03	-0.74	-3.61	-4.78
$f-s$	Mean	-2.13	-0.16	1.03	2.62	6.91	9.04	-2.02	-0.12	1.07	2.64	6.74	8.76	-1.86	0.02	1.16	2.64	6.40	8.26
rx^{FX}	Mean	-1.39	-0.08	1.50	2.37	3.13	4.52	-1.03	-0.07	1.48	1.97	2.96	3.99	-0.69	0.17	1.13	1.90	2.79	3.48
	s.e.	[1.17]	[1.37]	[1.28]	[1.26]	[1.20]	[0.88]	[1.19]	[1.46]	[1.39]	[1.42]	[1.38]	[0.95]	[1.27]	[1.52]	[1.37]	[1.44]	[1.54]	[1.07]
	Std	7.47	8.75	8.27	8.10	7.76	5.57	7.76	8.97	8.64	8.71	8.47	6.29	8.76	9.44	8.97	9.33	9.80	6.62
	SR	-0.19	-0.01	0.18	0.29	0.40	0.81	-0.13	-0.01	0.17	0.23	0.35	0.63	-0.08	0.02	0.13	0.20	0.28	0.53
	s.e.	[0.16]	[0.16]	[0.16]	[0.17]	[0.16]	[0.17]	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.19]	[0.16]	[0.16]	[0.16]	[0.17]	[0.18]	[0.27]
$rx^{(10),*}$	Mean	3.73	2.31	2.28	1.80	-0.04	-3.77	3.12	2.13	1.79	1.73	0.81	-2.31	2.52	1.60	1.56	1.19	1.73	-0.79
	s.e.	[0.67]	[0.74]	[0.68]	[0.70]	[0.71]	[0.85]	[0.73]	[0.88]	[0.78]	[0.82]	[0.79]	[0.89]	[0.85]	[1.15]	[0.82]	[0.98]	[1.00]	[0.82]
	Std	4.23	4.77	4.39	4.49	4.53	5.47	4.72	5.54	4.93	5.26	5.37	5.85	4.96	6.16	5.87	6.47	6.19	5.57
	SR	0.88	0.48	0.52	0.40	-0.01	-0.69	0.66	0.38	0.36	0.33	0.15	-0.40	0.51	0.26	0.27	0.18	0.28	-0.14
	s.e.	[0.15]	[0.17]	[0.16]	[0.16]	[0.16]	[0.15]	[0.15]	[0.17]	[0.17]	[0.17]	[0.16]	[0.15]	[0.17]	[0.16]	[0.17]	[0.17]	[0.18]	[0.16]
$rx^{(10),\$}$	Mean	2.34	2.24	3.77	4.17	3.09	0.75	2.09	2.06	3.27	3.70	3.77	1.68	1.83	1.77	2.69	3.08	4.52	2.69
	s.e.	[1.44]	[1.64]	[1.51]	[1.47]	[1.47]	[1.33]	[1.47]	[1.78]	[1.58]	[1.56]	[1.66]	[1.43]	[1.51]	[1.90]	[1.65]	[1.70]	[1.84]	[1.46]
	Std	9.13	10.43	9.74	9.37	9.45	8.48	9.71	10.96	10.01	9.97	10.49	9.43	10.29	11.13	10.84	11.14	12.07	9.49
	SR	0.26	0.21	0.39	0.45	0.33	0.09	0.22	0.19	0.33	0.37	0.36	0.18	0.18	0.16	0.25	0.28	0.37	0.28
	s.e.	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.15]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.17]	[0.18]	[0.18]	[0.20]
$rx^{(10),\$} - rx^{(10),US}$	Mean	-0.17	-0.27	1.27	1.67	0.59	0.75	-0.44	-0.47	0.74	1.17	1.24	1.68	-0.73	-0.79	0.13	0.52	1.96	2.69
	s.e.	[1.61]	[1.66]	[1.53]	[1.58]	[1.83]	[1.33]	[1.54]	[1.82]	[1.58]	[1.67]	[2.07]	[1.43]	[1.51]	[2.01]	[1.77]	[1.87]	[2.11]	[1.46]

Annualized monthly log returns realized at $t+k$ on 10-year Bond Index and T-bills for k from 1 month to 12 months. Portfolios of 30 currencies sorted every month by T-bill rate at t. The unbalanced panel consists of Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, India, Ireland, Italy, Japan Mexico, Malaysia, the Netherlands, New Zealand, Norway, Pakistan, the Philippines, Poland, Portugal, South Africa, Singapore, Spain, Sweden, Switzerland, Taiwan, Thailand, and the United Kingdom.

Table 7: Slope-Sorted Portfolios: Benchmark Sample

Portfolio		1	2	3	$3-1$	1	2	3	$3-1$	1	2	3	$3-1$
Horizon		1-Month				3-Month				12-Month			
						Panel A: 12/1950–12/2012							
$-\Delta s$	Mean	-0.01	0.77	0.83	0.84	0.01	0.39	1.18	1.17	-0.09	0.55	1.09	1.18
$f-s$	Mean	3.03	0.41	-0.77	-3.81	2.96	0.42	-0.71	-3.68	2.76	0.46	-0.55	-3.31
rx^{FX}	Mean	3.02	1.18	0.06	-2.97	2.97	0.81	0.47	-2.50	2.67	1.01	0.54	-2.13
	s.e.	[0.97]	[0.94]	[0.94]	[0.81]	[1.08]	[1.03]	[0.95]	[0.87]	[1.14]	[1.07]	[1.13]	[0.86]
	Std	7.59	7.37	7.36	6.30	8.25	7.75	7.60	6.84	9.05	8.31	8.39	6.65
	SR	0.40	0.16	0.01	-0.47	0.36	0.10	0.06	-0.37	0.30	0.12	0.06	-0.32
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]	[0.13]	[0.13]	[0.15]	[0.14]	[0.13]	[0.13]	[0.14]
$rx^{(10),*}$	Mean	-1.82	1.61	4.00	5.82	-0.86	1.33	3.33	4.19	-0.22	1.20	2.79	3.01
	s.e.	[0.50]	[0.46]	[0.51]	[0.54]	[0.58]	[0.51]	[0.58]	[0.60]	[0.62]	[0.69]	[0.65]	[0.58]
	Std	3.97	3.67	4.09	4.29	4.60	4.24	4.65	4.67	5.10	4.88	5.29	4.90
	SR	-0.46	0.44	0.98	1.35	-0.19	0.31	0.72	0.90	-0.04	0.25	0.53	0.61
	s.e.	[0.12]	[0.12]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.15]	[0.11]
$rx^{(10),\$}$	Mean	1.21	2.79	4.06	2.85	2.12	2.14	3.80	1.68	2.45	2.21	3.33	0.88
	s.e.	[1.09]	[1.07]	[1.12]	[0.99]	[1.19]	[1.17]	[1.18]	[1.08]	[1.28]	[1.18]	[1.31]	[1.12]
	Std	8.61	8.36	8.84	7.76	9.34	8.98	9.42	8.14	10.45	9.57	10.29	8.67
	SR	0.14	0.33	0.46	0.37	0.23	0.24	0.40	0.21	0.23	0.23	0.32	0.10
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.12]	[0.14]	[0.13]	[0.13]	[0.12]
$rx^{(10),\$} - rx^{(10),US}$	Mean	-0.30	1.28	2.55	2.85	0.60	0.62	2.28	1.68	0.91	0.66	1.79	0.88
	s.e.	[1.28]	[1.14]	[1.21]	[0.99]	[1.42]	[1.29]	[1.14]	[1.08]	[1.51]	[1.25]	[1.37]	[1.12]
						Panel B: 12/1971–12/2012							
$-\Delta s$	Mean	-0.08	1.21	1.03	1.11	-0.08	0.61	1.51	1.60	-0.30	0.76	1.47	1.77
$f-s$	Mean	3.40	0.54	-1.02	-4.42	3.31	0.54	-0.94	-4.25	3.08	0.57	-0.72	-3.80
rx^{FX}	Mean	3.32	1.75	0.01	-3.32	3.23	1.15	0.58	-2.65	2.78	1.33	0.75	-2.03
	s.e.	[1.45]	[1.41]	[1.37]	[1.16]	[1.65]	[1.55]	[1.44]	[1.26]	[1.71]	[1.62]	[1.66]	[1.22]
	Std	9.29	8.95	8.80	7.40	10.09	9.38	9.16	8.15	11.04	10.05	10.12	7.95
	SR	0.36	0.20	0.00	-0.45	0.32	0.12	0.06	-0.32	0.25	0.13	0.07	-0.26
	s.e.	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.16]	[0.16]	[0.18]	[0.17]	[0.16]	[0.16]	[0.17]
$rx^{(10),*}$	Mean	-1.68	1.94	4.56	6.24	-0.52	1.70	3.64	4.16	0.10	1.71	2.98	2.87
	s.e.	[0.74]	[0.69]	[0.72]	[0.76]	[0.85]	[0.75]	[0.81]	[0.85]	[0.88]	[1.02]	[0.91]	[0.83]
	Std. Dev.	4.67	4.37	4.63	4.84	5.45	5.03	5.20	5.26	5.95	5.73	5.86	5.54
	SR	-0.36	0.44	0.98	1.29	-0.10	0.34	0.70	0.79	0.02	0.30	0.51	0.52
	s.e.	[0.15]	[0.15]	[0.17]	[0.15]	[0.16]	[0.16]	[0.17]	[0.15]	[0.16]	[0.17]	[0.20]	[0.12]
$rx^{(10),\$}$	Mean	1.64	3.69	4.56	2.93	2.70	2.85	4.22	1.51	2.88	3.04	3.73	0.84
	s.e.	[1.63]	[1.59]	[1.62]	[1.41]	[1.81]	[1.74]	[1.73]	[1.56]	[1.89]	[1.77]	[1.91]	[1.63]
	Std	10.45	10.13	10.46	8.97	11.30	10.79	11.15	9.59	12.54	11.33	12.15	10.23
	SR	0.16	0.36	0.44	0.33	0.24	0.26	0.38	0.16	0.23	0.27	0.31	0.08
	s.e.	[0.16]	[0.16]	[0.16]	[0.15]	[0.16]	[0.16]	[0.16]	[0.15]	[0.17]	[0.17]	[0.17]	[0.15]
$rx^{(10),\$} - rx^{(10),US}$	Mean	-0.87	1.18	2.06	2.93	0.17	0.32	1.69	1.51	0.32	0.47	1.16	0.84
	s.e.	[1.85]	[1.63]	[1.69]	[1.41]	[2.09]	[1.88]	[1.62]	[1.56]	[2.23]	[1.87]	[1.98]	[1.63]

Notes: The table reports the average change in exchange rates (Δs), the average interest rate difference ($f-s$), the average log currency excess return (rx^{FX}), the average log foreign bond excess return on 10-year government bond indices in foreign currency ($rx^{(10),*}$) and in U.S. dollars ($rx^{(10),\$}$), as well as the difference between the average foreign bond log excess return in U.S. dollars and the average U.S. bond log excess return ($rx^{(10),\$} - rx^{US}$). For the excess returns, the table also reports their annualized standard deviation (denoted Std) and their Sharpe ratios (denoted SR). The annualized monthly log returns are realized at date $t+k$, where the horizon k equals 1, 3, and 12 months. The balanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. The countries are sorted by the slope of their yield curves into three portfolios. The slope of the yield curve is measured by the difference between the 10-year yield and the one-month interest rate at date t. The standard errors (denoted s.e. and reported between brackets) were generated by bootstrapping 10,000 samples of non-overlapping returns.

B.2 Developed Countries

Table 8 reports the results of sorting on the yield curve slope on the sample of developed countries. The results are commented in the main text.

B.3 Whole Sample

Table 9 reports the results obtained from using the entire cross-section of countries, including emerging countries. Here again, the results are commented in the main text.

C The Term Structure of Currency Carry Trade Risk Premia

This section reports additional results obtained with zero-coupon bonds to describe the whole term structure of currency carry trade risk premia. We start with our benchmark sample of G10 countries and then turn to a larger set of developed countries.

C.1 Benchmark Sample

Table 10 reports summary statistics on one-quarter holding period returns on zero-coupon bond positions with maturities from 4 (1 year) to 60 quarters (15 years).

At the short end of the maturity spectrum, it is profitable to invest in flat-yield-curve currencies and short the currencies of countries with steep yield curves: the annualized dollar excess return on that strategy using 1-year bonds is 4.10%. However, this excess return monotonically declines as the bond maturity increases: it is 2.33% using 5-year bonds and only 0.52% using 10-year bonds. At the long end of the maturity spectrum, this strategy delivers negative dollar excess returns: an investor who buys the 15-year bond of flat-yield-curve currencies and shorts the 15-year bond of steep-yield-curve currencies loses 0.42% per year on average. The term structure of currency carry trade risk premia is downward-sloping.

C.2 Developed Countries

Table 11 is the equivalent of Table 10 but for a larger set of developed countries. Results are very similar to those of our benchmark sample and are commented in the main text.

Table 8: Slope Sorted Portfolios: Developed sample

Portfolio		1	2	3	4	$4-1$	1	2	3	4	$4-1$	1	2	3	4	$4-1$
Horizon		1-month					3-month					12-month				
Panel A: 1950-2012																
$-\Delta s$	Mean	-0.78	0.14	0.05	0.67	1.45	-0.94	0.25	-0.06	0.68	1.62	-0.79	0.13	-0.01	0.48	1.28
$f-s$	Mean	3.69	1.64	0.82	-0.18	-3.87	3.60	1.63	0.85	-0.12	-3.71	3.33	1.62	0.90	0.06	-3.27
rx^{FX}	Mean	2.91	1.78	0.87	0.49	-2.42	2.66	1.88	0.79	0.56	-2.10	2.54	1.74	0.89	0.54	-1.99
	s.e.	[0.96]	[1.01]	[1.05]	[1.03]	[0.63]	[1.09]	[1.07]	[1.11]	[1.05]	[0.64]	[1.22]	[1.07]	[1.23]	[1.15]	[0.65]
	Std	7.62	7.92	8.16	8.08	5.03	8.33	8.08	8.59	8.09	4.95	9.32	8.68	9.44	8.67	4.88
	SR	0.38	0.22	0.11	0.06	-0.48	0.32	0.23	0.09	0.07	-0.42	0.27	0.20	0.09	0.06	-0.41
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.15]	[0.13]	[0.13]	[0.13]	[0.13]	[0.15]	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]
$rx^{(10),*}$	Mean	-1.96	0.27	2.27	3.95	5.90	-1.29	0.95	1.88	3.15	4.44	-0.33	1.08	1.60	2.20	2.52
	s.e.	[0.51]	[0.52]	[0.51]	[0.74]	[0.84]	[0.61]	[0.58]	[0.61]	[0.78]	[0.86]	[0.68]	[0.86]	[0.67]	[1.08]	[1.01]
	Std	4.05	4.08	4.02	5.84	6.60	4.89	4.72	4.67	6.43	7.01	5.86	5.82	5.68	6.87	6.97
	SR	-0.48	0.07	0.56	0.68	0.89	-0.26	0.20	0.40	0.49	0.63	-0.06	0.19	0.28	0.32	0.36
	s.e.	[0.13]	[0.13]	[0.13]	[0.14]	[0.16]	[0.13]	[0.13]	[0.13]	[0.14]	[0.15]	[0.13]	[0.14]	[0.13]	[0.13]	[0.18]
$rx^{(10),\$}$	Mean	0.95	2.05	3.14	4.44	3.48	1.37	2.83	2.67	3.71	2.34	2.21	2.82	2.49	2.74	0.53
	s.e.	[1.09]	[1.15]	[1.15]	[1.39]	[1.10]	[1.22]	[1.18]	[1.28]	[1.40]	[1.12]	[1.36]	[1.35]	[1.37]	[1.59]	[1.30]
	Std	8.59	9.06	8.97	10.91	8.72	9.51	9.23	9.72	11.27	9.09	10.86	10.37	11.18	11.36	9.45
	SR	0.11	0.23	0.35	0.41	0.40	0.14	0.31	0.27	0.33	0.26	0.20	0.27	0.22	0.24	0.06
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.12]	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]
$rx^{(10),\$}-rx^{(10),US}$	Mean	-0.56	0.53	1.63	2.93	3.48	-0.15	1.31	1.15	2.19	2.34	0.67	1.28	0.95	1.20	0.53
	s.e.	[1.25]	[1.23]	[1.19]	[1.46]	[1.10]	[1.40]	[1.24]	[1.26]	[1.42]	[1.12]	[1.47]	[1.35]	[1.43]	[1.63]	[1.30]
Panel B: 1971-2012																
$-\Delta s$	Mean	-0.96	0.18	0.29	0.84	1.80	-1.20	0.37	0.00	0.83	2.03	-1.18	0.27	0.05	0.54	1.73
$f-s$	Mean	4.29	1.89	1.03	-0.20	-4.49	4.18	1.87	1.06	-0.11	-4.29	3.87	1.86	1.12	0.13	-3.74
rx^{FX}	Mean	3.33	2.07	1.32	0.64	-2.69	2.98	2.24	1.06	0.72	-2.26	2.69	2.13	1.18	0.68	-2.01
	s.e.	[1.44]	[1.51]	[1.52]	[1.55]	[0.94]	[1.61]	[1.60]	[1.62]	[1.58]	[0.95]	[1.82]	[1.62]	[1.82]	[1.75]	[0.95]
	Std	9.22	9.71	9.75	9.90	5.99	10.13	9.87	10.35	9.90	5.94	11.35	10.55	11.43	10.61	5.90
	SR	0.36	0.21	0.14	0.07	-0.45	0.29	0.23	0.10	0.07	-0.38	0.24	0.20	0.10	0.06	-0.34
	s.e.	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.16]	[0.16]	[0.16]	[0.16]	[0.18]
$rx^{(10),*}$	Mean	-1.89	0.75	2.56	4.76	6.65	-1.10	1.72	2.32	3.50	4.60	0.07	1.81	2.03	2.17	2.10
	s.e.	[0.74]	[0.73]	[0.75]	[1.09]	[1.22]	[0.89]	[0.86]	[0.87]	[1.14]	[1.26]	[0.95]	[1.26]	[0.95]	[1.62]	[1.52]
	Std	4.77	4.75	4.77	6.92	7.79	5.77	5.58	5.50	7.61	8.33	6.85	6.77	6.69	8.12	8.33
	SR	-0.40	0.16	0.54	0.69	0.85	-0.19	0.31	0.42	0.46	0.55	0.01	0.27	0.30	0.27	0.25
	s.e.	[0.16]	[0.16]	[0.15]	[0.17]	[0.18]	[0.16]	[0.16]	[0.16]	[0.17]	[0.17]	[0.16]	[0.18]	[0.17]	[0.16]	[0.19]
$rx^{(10),\$}$	Mean	1.44	2.82	3.87	5.40	3.96	1.88	3.96	3.38	4.21	2.34	2.76	3.94	3.21	2.85	0.09
	s.e.	[1.60]	[1.70]	[1.66]	[2.09]	[1.63]	[1.77]	[1.75]	[1.85]	[2.07]	[1.66]	[1.97]	[1.97]	[1.97]	[2.40]	[1.95]
	Std	10.29	10.98	10.67	13.24	10.35	11.40	11.14	11.60	13.61	10.87	12.91	12.27	13.29	13.67	11.38
	SR	0.14	0.26	0.36	0.41	0.38	0.16	0.36	0.29	0.31	0.22	0.21	0.32	0.24	0.21	0.01
	s.e.	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.16]	[0.16]	[0.16]
$rx^{(10),\$}-rx^{(10),US}$	Mean	-1.06	0.31	1.37	2.90	3.96	-0.66	1.43	0.85	1.68	2.34	0.19	1.37	0.64	0.28	0.09
	s.e.	[1.76]	[1.74]	[1.66]	[2.14]	[1.63]	[2.01]	[1.79]	[1.80]	[2.08]	[1.66]	[2.15]	[1.96]	[2.06]	[2.43]	[1.95]

Annualized monthly log returns realized at $t+k$ on 10-year Bond Index and T-bills for k from 1 month to 12 months. Portfolios of 21 currencies sorted every month by the slope of the yield curve (10-year yield minus T-bill rate) at t. The unbalanced panel consists of Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom.

Table 9: Slope Sorted Portfolios: Whole sample

Portfolio		1	2	3	4	5	5 − 1	1	2	3	4	5	5 − 1	1	2	3	4	5	5 − 1
Horizon		1-month						3-month						12-month					
		Panel A: 1950-2012																	
$-\Delta s$	Mean	-2.15	-0.71	-0.18	0.27	-0.47	1.67	-2.47	-0.53	0.02	-0.11	-0.32	2.14	-2.19	-0.59	-0.15	0.02	-0.55	1.64
$f-s$	Mean	4.63	2.06	1.28	0.52	-0.08	-4.71	4.45	2.04	1.30	0.54	0.50	-3.95	4.12	1.99	1.30	0.74	0.27	-3.85
rx^{FX}	Mean	2.49	1.35	1.10	0.79	-0.55	-3.04	1.99	1.51	1.32	0.43	0.18	-1.81	1.93	1.40	1.15	0.76	-0.28	-2.21
	s.e.	[0.94]	[0.91]	[0.98]	[1.03]	[0.82]	[0.72]	[1.04]	[0.97]	[1.05]	[1.08]	[0.84]	[0.73]	[1.16]	[1.02]	[1.11]	[1.18]	[0.89]	[0.81]
	Std	7.48	7.10	7.70	8.00	6.37	5.65	8.11	7.66	7.92	8.14	8.91	8.56	8.90	8.16	8.76	9.70	6.87	6.06
	SR	0.33	0.19	0.14	0.10	-0.09	-0.54	0.24	0.20	0.17	0.05	0.02	-0.21	0.22	0.17	0.13	0.08	-0.04	-0.36
	s.e.	[0.14]	[0.13]	[0.13]	[0.13]	[0.13]	[0.16]	[0.14]	[0.13]	[0.13]	[0.13]	[0.13]	[0.16]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]
$rx^{(10),*}$	Mean	-3.32	-0.82	1.46	2.56	5.05	8.37	-2.56	-0.03	1.43	2.11	3.83	6.38	-1.32	0.32	1.50	1.55	3.07	4.40
	s.e.	[0.53]	[0.49]	[0.46]	[0.49]	[0.66]	[0.81]	[0.62]	[0.54]	[0.56]	[0.59]	[0.68]	[0.82]	[0.58]	[0.69]	[0.75]	[0.66]	[0.96]	[0.89]
	Std	4.14	3.88	3.65	3.91	5.20	6.31	4.87	4.42	4.40	4.62	8.46	9.18	5.05	5.42	5.57	6.80	6.06	6.36
	SR	-0.80	-0.21	0.40	0.66	0.97	1.33	-0.53	-0.01	0.32	0.46	0.45	0.70	-0.26	0.06	0.27	0.23	0.51	0.69
	s.e.	[0.11]	[0.12]	[0.13]	[0.14]	[0.15]	[0.15]	[0.11]	[0.13]	[0.13]	[0.13]	[0.15]	[0.16]	[0.14]	[0.14]	[0.14]	[0.14]	[0.12]	[0.17]
$rx^{(10),\$}$	Mean	-0.83	0.53	2.56	3.35	4.50	5.33	-0.57	1.48	2.74	2.54	4.00	4.57	0.61	1.72	2.65	2.31	2.80	2.19
	s.e.	[1.09]	[1.06]	[1.08]	[1.17]	[1.16]	[1.14]	[1.24]	[1.07]	[1.16]	[1.28]	[1.18]	[1.18]	[1.28]	[1.16]	[1.38]	[1.34]	[1.33]	[1.20]
	Std	8.80	8.23	8.53	9.17	9.08	8.95	9.84	8.71	9.04	9.73	9.35	9.63	10.70	9.59	10.73	10.83	9.29	9.32
	SR	-0.09	0.06	0.30	0.37	0.50	0.60	-0.06	0.17	0.30	0.26	0.43	0.47	0.06	0.18	0.25	0.21	0.30	0.23
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]	[0.12]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.12]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]
$rx^{(10),\$} - rx^{(10),US}$	Mean	-2.34	-0.99	1.04	1.84	2.99	5.33	-2.09	-0.04	1.22	1.02	2.48	4.57	-0.94	0.18	1.11	0.76	1.25	2.19
	s.e.	[1.32]	[1.19]	[1.19]	[1.19]	[1.33]	[1.14]	[1.50]	[1.21]	[1.22]	[1.27]	[1.33]	[1.18]	[1.53]	[1.21]	[1.51]	[1.30]	[1.45]	[1.20]
		Panel B: 1971-2012																	
$-\Delta s$	Mean	-2.91	-0.62	-0.10	0.13	-0.96	1.95	-3.30	-0.43	-0.07	-0.08	-0.85	2.45	-2.73	-0.65	0.03	-0.08	-1.05	1.68
$f-s$	Mean	5.54	2.37	1.53	0.66	-0.11	-5.65	5.30	2.35	1.55	0.68	0.05	-5.25	4.94	2.29	1.55	0.74	0.33	-4.61
rx^{FX}	Mean	2.63	1.75	1.44	0.79	-1.06	-3.70	2.01	1.91	1.49	0.60	-0.80	-2.81	2.20	1.65	1.58	0.66	-0.72	-2.93
	s.e.	[1.39]	[1.32]	[1.41]	[1.37]	[1.13]	[1.06]	[1.53]	[1.41]	[1.45]	[1.48]	[1.16]	[1.13]	[1.60]	[1.50]	[1.55]	[1.67]	[1.24]	[1.11]
	Std	8.95	8.39	9.02	8.74	7.27	6.80	9.72	9.07	9.38	9.11	7.25	7.19	10.49	9.75	10.37	10.24	7.80	7.02
	SR	0.29	0.21	0.16	0.09	-0.15	-0.54	0.21	0.21	0.16	0.07	-0.11	-0.39	0.21	0.17	0.15	0.06	-0.09	-0.42
	s.e.	[0.17]	[0.16]	[0.16]	[0.16]	[0.16]	[0.20]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.18]	[0.17]	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]
$rx^{(10),*}$	Mean	-3.73	-0.56	1.40	3.81	6.13	9.85	-2.73	0.47	1.54	2.93	5.15	7.87	-1.19	0.72	2.04	2.20	3.54	4.73
	s.e.	[0.77]	[0.72]	[0.66]	[0.70]	[0.90]	[1.11]	[0.89]	[0.78]	[0.84]	[0.80]	[0.93]	[1.15]	[0.82]	[1.00]	[1.12]	[0.89]	[1.32]	[1.23]
	Std	4.90	4.61	4.30	4.51	5.81	7.10	5.72	5.29	5.36	5.34	6.18	7.55	5.85	6.31	6.64	6.49	6.68	7.10
	SR	-0.76	-0.12	0.33	0.85	1.06	1.39	-0.48	0.09	0.29	0.55	0.83	1.04	-0.20	0.11	0.31	0.34	0.53	0.67
	s.e.	[0.14]	[0.15]	[0.16]	[0.16]	[0.18]	[0.18]	[0.14]	[0.16]	[0.16]	[0.16]	[0.18]	[0.18]	[0.16]	[0.18]	[0.19]	[0.18]	[0.16]	[0.20]
$rx^{(10),\$}$	Mean	-1.10	1.19	2.84	4.60	5.06	6.16	-0.72	2.39	3.03	3.53	4.35	5.07	1.02	2.37	3.62	2.86	2.82	1.80
	s.e.	[1.63]	[1.52]	[1.56]	[1.59]	[1.60]	[1.61]	[1.80]	[1.55]	[1.61]	[1.77]	[1.63]	[1.72]	[1.77]	[1.70]	[1.94]	[1.89]	[1.87]	[1.74]
	Std	10.49	9.74	10.03	10.17	10.31	10.25	11.69	10.33	10.66	11.09	10.49	11.16	12.56	11.25	12.53	12.62	10.38	10.66
	SR	-0.10	0.12	0.28	0.45	0.49	0.60	-0.06	0.23	0.28	0.32	0.41	0.45	0.08	0.21	0.29	0.23	0.27	0.17
	s.e.	[0.15]	[0.16]	[0.16]	[0.16]	[0.17]	[0.15]	[0.15]	[0.16]	[0.16]	[0.16]	[0.16]	[0.15]	[0.16]	[0.17]	[0.16]	[0.17]	[0.16]	[0.17]
$rx^{(10),\$} - rx^{(10),US}$	Mean	-3.60	-1.32	0.33	2.09	2.56	6.16	-3.25	-0.14	0.50	1.00	1.82	5.07	-1.55	-0.20	1.06	0.29	0.25	1.80
	s.e.	[1.89]	[1.63]	[1.65]	[1.56]	[1.78]	[1.61]	[2.17]	[1.68]	[1.64]	[1.74]	[1.77]	[1.72]	[2.17]	[1.72]	[2.12]	[1.78]	[2.00]	[1.74]

Annualized monthly log returns realized at $t+k$ on 10-year Bond Index and T-bills for k from 1 month to 12 months. Portfolios of 30 currencies sorted every month by the slope of the yield curve (10-year yield minus T-bill rate) at t. The unbalanced panel consists of Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, India, Ireland, Italy, Japan Mexico, Malaysia, the Netherlands, New Zealand, Norway, Pakistan, the Philippines, Poland, Portugal, South Africa, Singapore, Spain, Sweden, Switzerland, Taiwan, Thailand, and the United Kingdom.

Table 10: The Maturity Structure of Returns in Slope-Sorted Portfolios

Maturity		One Year				Five Years				Ten Years				Fifteen Years			
Portfolio		1	2	3	$3-1$	1	2	3	$3-1$	1	2	3	$3-1$	1	2	3	$3-1$
$-\Delta s$	Mean	2.80	2.51	2.96	0.16	2.80	2.51	2.96	0.16	2.80	2.51	2.96	0.16	2.75	2.55	2.87	0.13
$f-s$	Mean	3.15	0.79	-0.31	-3.46	3.15	0.79	-0.31	-3.46	3.15	0.79	-0.31	-3.46	3.12	0.73	-0.36	-3.48
rx^{FX}	Mean	5.95	3.30	2.64	-3.31	5.95	3.30	2.64	-3.31	5.95	3.30	2.64	-3.31	5.87	3.29	2.52	-3.35
	s.e.	[1.19]	[1.06]	[0.99]	[0.97]	[1.19]	[1.05]	[0.99]	[0.96]	[1.18]	[1.04]	[0.98]	[0.97]	[1.18]	[1.06]	[0.99]	[0.97]
	Std	10.98	9.61	9.00	8.88	10.98	9.61	9.00	8.88	10.98	9.61	9.00	8.88	11.00	9.68	9.09	8.94
	SR	0.54	0.34	0.29	-0.37	0.54	0.34	0.29	-0.37	0.54	0.34	0.29	-0.37	0.53	0.34	0.28	-0.38
	s.e.	[0.12]	[0.12]	[0.11]	[0.12]	[0.12]	[0.12]	[0.11]	[0.12]	[0.12]	[0.12]	[0.11]	[0.12]	[0.12]	[0.12]	[0.11]	[0.12]
$rx^{(k),*}$	Mean	-0.16	0.36	0.45	0.61	1.39	2.62	3.31	1.92	2.27	4.34	5.69	3.42	2.65	5.52	7.78	5.13
	s.e.	[0.11]	[0.09]	[0.08]	[0.11]	[0.61]	[0.51]	[0.51]	[0.50]	[1.06]	[0.88]	[0.90]	[0.79]	[1.43]	[1.22]	[1.28]	[1.13]
	Std	1.01	0.80	0.73	0.97	5.43	4.54	4.67	4.57	9.61	7.96	8.23	7.19	12.95	11.06	11.67	10.39
	SR	-0.16	0.45	0.62	0.63	0.26	0.58	0.71	0.42	0.24	0.55	0.69	0.48	0.20	0.50	0.67	0.49
	s.e.	[0.11]	[0.11]	[0.12]	[0.12]	[0.11]	[0.12]	[0.12]	[0.11]	[0.11]	[0.12]	[0.12]	[0.11]	[0.11]	[0.12]	[0.12]	[0.11]
$rx^{(k),\$}$	Mean	5.79	3.66	3.10	-2.69	7.34	5.92	5.95	-1.39	8.22	7.64	8.34	0.12	8.52	8.81	10.30	1.77
	s.e.	[1.19]	[1.05]	[1.01]	[0.96]	[1.35]	[1.10]	[1.18]	[1.02]	[1.60]	[1.26]	[1.39]	[1.21]	[1.84]	[1.50]	[1.63]	[1.42]
	Std	11.00	9.53	9.19	8.82	12.33	9.98	10.70	9.44	14.72	11.56	12.68	10.99	16.87	13.64	15.00	13.03
	SR	0.53	0.38	0.34	-0.31	0.60	0.59	0.56	-0.15	0.56	0.66	0.66	0.01	0.51	0.65	0.69	0.14
	s.e.	[0.12]	[0.12]	[0.11]	[0.12]	[0.11]	[0.11]	[0.11]	[0.11]	[0.11]	[0.11]	[0.11]	[0.11]	[0.11]	[0.11]	[0.12]	[0.11]
$rx^{(k),\$}-rx^{(k),US}$	Mean	8.77	6.64	6.08	-2.69	7.22	5.81	5.84	-1.39	5.71	5.13	5.83	0.12	4.43	4.72	6.20	1.77
	s.e.	[1.19]	[1.06]	[0.98]	[0.96]	[1.29]	[1.08]	[1.00]	[1.02]	[1.47]	[1.23]	[1.14]	[1.21]	[1.73]	[1.49]	[1.37]	[1.42]

Notes: The table reports summary statistics on annualized log returns realized on zero coupon bonds with maturity varying from $k=4$ to $k=60$ quarters. The holding period is one quarter. The table reports the average change in exchange rates ($-\Delta s$), the average interest rate difference ($f-s$), the average currency excess return (rx^{FX}), the average foreign bond excess return in foreign currency ($rx^{(k),*}$) and in U.S. dollars ($rx^{(k),\$}$), as well as the difference between the average foreign bond excess return in U.S. dollars and the average U.S. bond excess return ($rx^{(k),\$}-rx^{(k),US}$). For the excess returns, the table also reports their annualized standard deviation (denoted Std) and their Sharpe ratios (denoted SR). The balanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. The countries are sorted by the slope of their yield curves into three portfolios. The slope of the yield curve is measured by the difference between the 10-year yield and the 3-month interest rate at date t. The standard errors (denoted s.e. and reported between brackets) were generated by bootstrapping 10,000 samples of non-overlapping returns. Data are monthly, from the zero-coupon dataset, and the sample window is 4/1985–12/2012.

Table 11: The Maturity Structure of Returns in Slope-Sorted Portfolios: Extended Sample

Maturity		4-quarters						20-quarters						40-quarters						60-quarters					
Portfolio		1	2	3	4	5	5 − 1	1	2	3	4	5	5 − 1	1	2	3	4	5	5 − 1	1	2	3	4	5	5 − 1
$-\Delta s$	Mean	0.90	0.59	0.29	2.34	-0.30	-1.19	0.90	0.59	0.29	2.34	-0.30	-1.19	0.90	0.59	0.29	2.34	-0.30	-1.19	0.94	0.43	0.35	2.38	-0.39	-1.33
$f-s$	Mean	3.99	1.99	0.99	0.39	0.36	-3.63	3.99	1.99	0.99	0.39	0.36	-3.63	3.99	1.99	0.99	0.39	0.36	-3.63	3.93	2.00	0.95	0.32	0.32	-3.62
rx^{FX}	Mean	4.89	2.58	1.28	2.73	0.07	-4.82	4.89	2.58	1.28	2.73	0.07	-4.82	4.89	2.58	1.28	2.73	0.07	-4.82	4.87	2.43	1.30	2.70	-0.07	-4.94
	s.e.	[1.26]	[1.18]	[1.19]	[1.16]	[0.98]	[0.91]	[1.26]	[1.19]	[1.19]	[1.17]	[0.98]	[0.90]	[1.26]	[1.18]	[1.20]	[1.16]	[0.98]	[0.91]	[1.26]	[1.18]	[1.19]	[1.19]	[0.99]	[0.91]
	Std	11.00	10.36	10.44	10.24	8.56	7.95	11.00	10.36	10.44	10.24	8.56	7.95	11.00	10.36	10.44	10.24	8.56	7.95	11.01	10.34	10.44	10.44	8.65	7.99
	SR	0.44	0.25	0.12	0.27	0.01	-0.61	0.44	0.25	0.12	0.27	0.01	-0.61	0.44	0.25	0.12	0.27	0.01	-0.61	0.44	0.24	0.12	0.26	-0.01	-0.62
	s.e.	[0.13]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.11]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]
$rx^{(k),*}$	Mean	-0.09	0.11	0.32	0.33	0.63	0.72	1.11	2.12	2.56	2.73	3.60	2.50	1.59	2.69	4.16	4.32	5.90	4.31	2.42	2.84	6.36	4.78	7.78	5.36
	s.e.	[0.12]	[0.10]	[0.10]	[0.10]	[0.10]	[0.13]	[0.59]	[0.54]	[0.59]	[0.56]	[0.62]	[0.64]	[1.06]	[0.97]	[1.05]	[1.00]	[1.02]	[0.96]	[1.45]	[1.46]	[1.43]	[1.36]	[1.44]	[1.31]
	Std	1.04	0.88	0.88	0.84	0.88	1.15	5.13	4.70	5.22	4.96	5.42	5.53	9.34	8.56	9.19	8.78	9.01	8.44	12.67	12.82	12.55	11.92	12.70	11.76
	SR	-0.09	0.12	0.37	0.39	0.72	0.63	0.22	0.45	0.49	0.55	0.67	0.45	0.17	0.31	0.45	0.49	0.65	0.51	0.19	0.22	0.51	0.40	0.61	0.46
	s.e.	[0.11]	[0.12]	[0.11]	[0.12]	[0.11]	[0.11]	[0.12]	[0.12]	[0.12]	[0.13]	[0.12]	[0.11]	[0.12]	[0.12]	[0.12]	[0.13]	[0.13]	[0.11]	[0.12]	[0.12]	[0.12]	[0.13]	[0.12]	[0.11]
$rx^{(k),\$}$	Mean	4.80	2.69	1.60	3.06	0.70	-4.10	6.00	4.70	3.84	5.46	3.67	-2.33	6.48	5.27	5.45	7.05	5.97	-0.52	7.29	5.27	7.66	7.47	7.71	0.42
	s.e.	[1.26]	[1.17]	[1.17]	[1.16]	[1.00]	[0.91]	[1.35]	[1.26]	[1.23]	[1.30]	[1.22]	[1.05]	[1.59]	[1.48]	[1.50]	[1.55]	[1.48]	[1.26]	[1.87]	[1.84]	[1.78]	[1.82]	[1.80]	[1.54]
	Std	11.05	10.29	10.32	10.27	8.72	7.99	11.87	11.07	11.02	11.50	10.71	9.17	14.10	13.13	13.25	13.84	13.14	11.24	16.45	16.10	15.74	16.04	15.96	13.97
	SR	0.43	0.26	0.16	0.30	0.08	-0.51	0.51	0.42	0.35	0.47	0.34	-0.25	0.46	0.40	0.41	0.51	0.45	-0.05	0.44	0.33	0.49	0.47	0.48	0.03
	s.e.	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]
$rx^{(k),\$}-rx^{(k),US}$	Mean	7.66	5.56	4.47	5.92	3.57	-4.10	6.28	4.99	4.13	5.75	3.96	-2.33	4.89	3.68	3.86	5.46	4.38	-0.52	4.67	2.65	5.04	4.85	5.09	0.42
	s.e.	[1.26]	[1.16]	[1.16]	[1.14]	[0.97]	[0.91]	[1.34]	[1.23]	[1.13]	[1.20]	[1.13]	[1.05]	[1.54]	[1.41]	[1.33]	[1.39]	[1.34]	[1.26]	[1.77]	[1.76]	[1.57]	[1.63]	[1.66]	[1.54]

Notes: The table reports summary statistics on annualized log returns realized on zero coupon bonds with maturity varying from $k=4$ to $k=60$ quarters. The holding period is one quarter. The table reports the average change in exchange rates ($-\Delta s$), the average interest rate difference ($f-s$), the average currency excess return (rx^{FX}), the average foreign bond excess return in foreign currency ($rx^{(k),*}$) and in U.S. dollars ($rx^{(k),\$}$), as well as the difference between the average foreign bond excess return in U.S. dollars and the average U.S. bond excess return ($rx^{(k),\$}-rx^{(k),US}$). For the excess returns, the table also reports their annualized standard deviation (denoted Std) and their Sharpe ratios (denoted SR). The unbalanced panel consists of Australia, Austria, Belgium, Canada, the Czech Republic, Denmark, Finland, France, Germany, Hungary, Indonesia, Ireland, Italy, Japan, Malaysia, Mexico, the Netherlands, New Zealand, Norway, Poland, Portugal, Singapore, South Africa, Spain, Sweden, Switzerland, and the U.K. The countries are sorted by the slope of their yield curves into five portfolios. The slope of the yield curve is measured by the difference between the 10-year yield and the 3-month interest rate at date t. The standard errors (denoted s.e. and reported between brackets) were generated by bootstrapping 10,000 samples of non-overlapping returns. Data are quarterly and the sample window is 5/1987–12/2012.

Figure 8 shows the local currency log excess returns in the top panel, and the dollar log excess returns in the bottom panel as a function of the bond maturities for zero-coupon bonds of our extended sample of developed countries. The results are also commented in the main text.

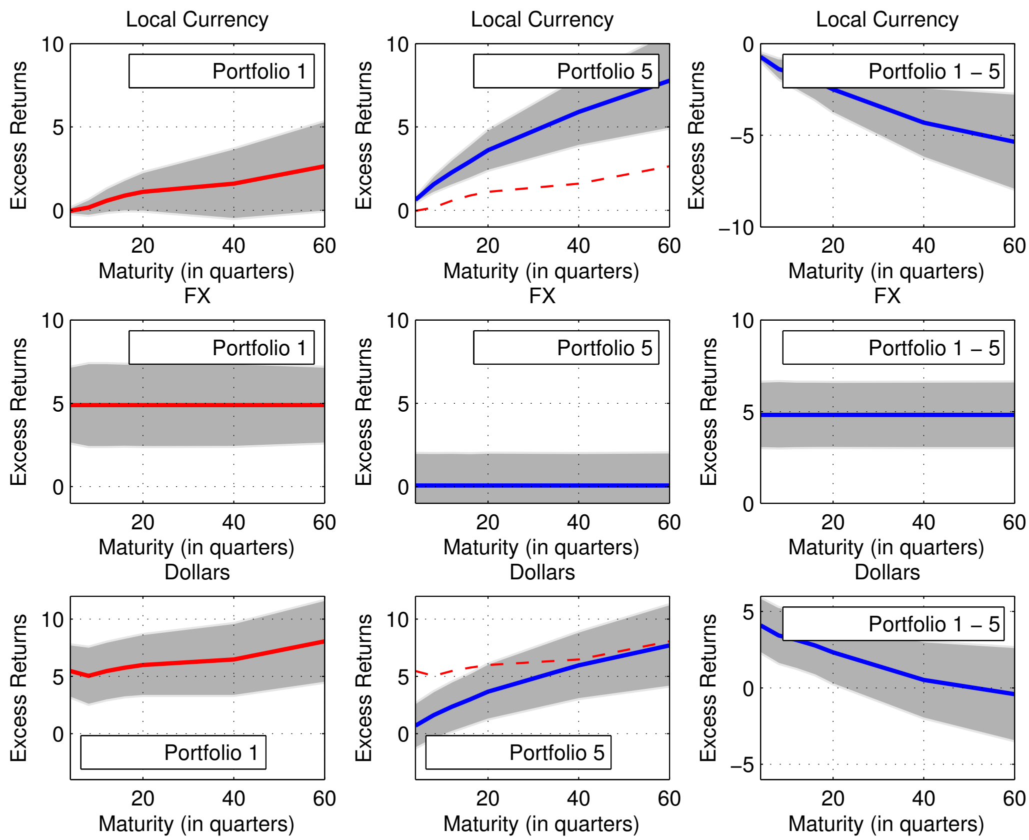

Figure 8: Term Structure of Dollar Bond Risk Premia: Extended Sample — The figure shows the local currency log excess returns in the top panel, and the dollar log excess returns in the bottom panel as a function of the bond maturities. The left panel focuses on Portfolio 1 (flat yield curve currencies) excess returns, while the middle panel reports Portfolio 5 (steep yield curve currencies) excess returns. The middle panels also report the Portfolio 1 excess returns in dashed lines for comparison. The right panel reports the difference. Data are monthly, from the zero-coupon dataset, and the sample window is 5/1987–12/2012. The unbalanced sample includes Australia, Austria, Belgium, Canada, the Czech Republic, Denmark, Finland, France, Germany, Hungary, Indonesia, Ireland, Italy, Japan, Malaysia, Mexico, the Netherlands, New Zealand, Norway, Poland, Portugal, Singapore, South Africa, Spain, Sweden, Switzerland, and the U.K. The countries are sorted by the slope of their yield curves into five portfolios. The slope of the yield curve is measured by the difference between the 10-year yield and the 3-month interest rate at date t. The holding period is one quarter. The returns are annualized.

D Dynamic Term Structure Models

This section reports additional results on dynamic term structure models, starting with the simple Vasicek one-factor model, before turning to essentially affine k-factor models and the model studied in Lustig, Roussanov, and Verdelhan (2011).

D.1 Vasicek (1977)

Model In the Vasicek model, the log SDF evolves as:

$$-m_{t+1} = y_{1,t} + \frac{1}{2}\lambda^2\sigma^2 + \lambda\varepsilon_{t+1},$$

where $y_{1,t}$ denotes the short-term interest rate. It is affine in a single factor:

$$\begin{aligned} x_{t+1} &= \rho x_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}\left(0, \sigma^2\right) \\ y_{1,t} &= \delta + x_t. \end{aligned}$$

In this model, x_t is the level factor and ε_{t+1} are shocks to the level of the term structure. The Jensen term is there to ensure that $E_t\left(M_{t+1}\right) = \exp\left(-y_{1,t}\right)$. Bond prices are exponentially affine. For any maturity n, bond prices are equal to $P_t^{(n)} = \exp\left(-B_0^n - B_1^n x_t\right)$. The price of the one-period risk-free note ($n = 1$) is naturally:

$$P_t^{(1)} = \exp\left(-y_{1,t}\right) = \exp\left(-B_0^1 - B_1^1 x_t\right),$$

with $B_0^1 = \delta, B_1^1 = 1$. Bond prices are defined recursively by the Euler equation: $P_t^{(n)} = E_t\left(M_{t+1}P_{t+1}^{(n-1)}\right)$, which implies:

$$-B_0^n - B_1^n x_t = -\delta - x_t - B_0^{n-1} - B_1^{n-1}\rho x_t + \frac{1}{2}\left(B_1^{n-1}\right)^2\sigma^2 x_t + \lambda B_1^{n-1}\sigma^2.$$

The coefficients B_0^n and B_1^n satisfy the following recursions:

$$\begin{aligned} B_0^n &= \delta + B_0^{n-1} - \frac{1}{2}\sigma^2(B_1^{n-1})^2 - \lambda B_1^{n-1}\sigma^2, \\ B_1^n &= 1 + B_1^{n-1}\rho. \end{aligned}$$

Decomposition (Alvarez and Jermann, 2005) We first implement the Alvarez and Jermann (2005) approach. The temporary pricing component of the pricing kernel is:

$$\Lambda_t^{\mathbb{T}} = \lim_{n\to\infty} \frac{\beta^{t+n}}{P_t^n} = \lim_{n\to\infty} \beta^{t+n} e^{B_0^n + B_1^n x_t},$$

where the constant β is chosen in order to satisfy Assumption 1 in Alvarez and Jermann (2005):

$$0 < \lim_{n\to\infty} \frac{P_t^n}{\beta^n} < \infty.$$

The limit of $B_0^n - B_0^{n-1}$ is finite: $\lim_{n\to\infty} B_0^n - B_0^{n-1} = \delta - \frac{1}{2}\sigma^2(B_1^\infty)^2 - \lambda B_1^\infty\sigma^2$, where B_1^∞ is $1/(1-\rho)$. As a result, B_0^n grows at a linear rate in the limit. We choose the constant β to offset the growth in B_0^n as n becomes very large. Setting $\beta = e^{-\delta + \frac{1}{2}\sigma^2(B_1^\infty)^2 + \lambda B_1^\infty\sigma^2}$ guarantees that Assumption 1 in Alvarez and Jermann (2005) is satisfied. The temporary pricing component of the pricing kernel is thus equal to:

$$\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}} = \beta e^{B_1^\infty(x_{t+1}-x_t)} = \beta e^{\frac{1}{1-\rho}(\rho-1)x_t + \frac{1}{1-\rho}\varepsilon_{t+1}} = \beta e^{-x_t + \frac{1}{1-\rho}\varepsilon_{t+1}}.$$

The martingale component of the pricing kernel is then:

$$\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} = \frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}}\right)^{-1} = \beta^{-1} e^{x_t - \frac{1}{1-\rho}\varepsilon_{t+1} - \delta - x_t - \frac{1}{2}\lambda^2\sigma^2 - \lambda\varepsilon_{t+1}} = \beta^{-1} e^{-\delta - \frac{1}{2}\lambda^2\sigma^2 - (\frac{1}{1-\rho}+\lambda)\varepsilon_{t+1}}.$$

In the case of $\lambda = -B_1^{\infty} = -\frac{1}{1-\rho}$, the martingale component of the pricing kernel is constant and all the shocks that affect the pricing kernel are transitory.

Decomposition (Hansen and Scheinkman, 2009) We now show that the Hansen and Scheinkman (2009) methodology leads to similar results. Guess an eigenfunction ϕ of the form

$$\phi(x) = e^{cx}$$

where c is a constant. Then, the (one-period) eigenfunction problem can be written as

$$E_t\left[\exp(-\delta - x_t - \frac{1}{2}\lambda^2\sigma^2 - \lambda\varepsilon_{t+1} + cx_{t+1})\right] = \exp(\beta + cx_t).$$

Expanding and matching coefficients, we solve for the constants c and β:

$$c = -\frac{1}{1-\rho}$$

$$\beta = -\delta + \frac{1}{2}\sigma^2\left(\frac{1}{1-\rho}\right)^2 + \lambda\sigma^2\left(\frac{1}{1-\rho}\right)$$

As shown above, the recursive definition of the bond price coefficients B_0^n and B_1^n implies that:

$$c = -B_1^{\infty}.$$

The transitory component of the pricing kernel is by definition:

$$\Lambda_t^{\mathbb{T}} = e^{\beta t - cx_t}$$

The transitory and permanent SDF component are thus:

$$\begin{aligned}
\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}} &= e^{\beta - c(x_{t+1}-x_t)} = e^{\beta - x_t + \frac{1}{1-\rho}\varepsilon_{t+1}} \\
\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} &= \frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}}\right)^{-1} = e^{-\delta - x_t - \frac{1}{2}\lambda^2\sigma^2 - \lambda\varepsilon_{t+1}} e^{-\beta + x_t - \frac{1}{1-\rho}\varepsilon_{t+1}} = e^{-\left[\frac{1}{2}\left(\frac{1}{1-\rho}\right)^2 + \lambda\frac{1}{1-\rho} + \frac{1}{2}\lambda^2\right]\sigma^2 - \left(\frac{1}{1-\rho}+\lambda\right)\varepsilon_{t+1}}
\end{aligned}$$

If $\lambda = -\frac{1}{1-\rho}$, then the martingale SDF component becomes

$$\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} = 1$$

so the entirety of the SDF is its transitory component.

Term and Risk Premium The expected log excess return of an infinite maturity bond is then:

$$E_t[rx_{t+1}^{(\infty)}] = -\frac{1}{2}\sigma^2(B_1^{\infty})^2 - \lambda B_1^{\infty}\sigma^2.$$

The first term is a Jensen term. The risk premium is constant and positive if λ is negative. The SDF is homoskedastic. The expected log currency excess return is therefore constant:

$$E_t[-\Delta s_{t+1}] + y_t^* - y_t = \frac{1}{2}Var_t(m_{t+1}) - \frac{1}{2}Var_t(m_{t+1}^*) = \frac{1}{2}\lambda\sigma^2 - \frac{1}{2}\lambda^*\sigma^{*2}.$$

D.2 Cox, Ingersoll, and Ross (1985) Model

Model The Cox, Ingersoll, and Ross (1985) model (denoted CIR) is defined by the following two equations:

$$\begin{aligned}
-\log M_{t+1} &= \alpha + \chi z_t + \sqrt{\gamma z_t}u_{t+1}, \qquad (9)\\
z_{t+1} &= (1-\phi)\theta + \phi z_t - \sigma\sqrt{z_t}u_{t+1},
\end{aligned}$$

where M denotes the stochastic discount factor. In this model, log bond prices are affine in the state variable z: $p_t^{(n)} = -B_0^n - B_1^n z_t$. The price of a one period-bond is: $P^{(1)} = E_t(M_{t+1}) = e^{-\alpha-(\chi-\frac{1}{2}\gamma)z_t}$. Bond prices are defined recursively by the Euler equation: $P_t^{(n)} = E_t(M_{t+1}P_{t+1}^{(n-1)})$. Thus the bond price coefficients evolve according to the following second-order difference equations:

$$\begin{aligned} B_0^n &= \alpha + B_0^{n-1} + B_1^{n-1}(1-\phi)\theta, \\ B_1^n &= \chi - \frac{1}{2}\gamma + B_1^{n-1}\phi - \frac{1}{2}\left(B_1^{n-1}\right)^2\sigma^2 + \sigma\sqrt{\gamma}B_1^{n-1}. \end{aligned} \tag{10}$$

Decomposition (Alvarez and Jermann, 2005) We first implement the Alvarez and Jermann (2005) approach. The temporary pricing component of the pricing kernel is:

$$\Lambda_t^{\mathbb{T}} = \lim_{n\to\infty} \frac{\beta^{t+n}}{P_t^{(n)}} = \lim_{n\to\infty} \beta^{t+n} e^{B_0^n + B_1^n z_t},$$

where the constant β is chosen in order to satisfy Assumption 1 in Alvarez and Jermann (2005):

$$0 < \lim_{n\to\infty} \frac{P_t^{(n)}}{\beta^n} < \infty.$$

The limit of $B_0^n - B_0^{n-1}$ is finite: $\lim_{n\to\infty} B_0^n - B_0^{n-1} = \alpha + B_1^\infty(1-\phi)\theta$, where B_1^∞ is defined implicitly in a second-order equation above. As a result, B_0^n grows at a linear rate in the limit. We choose the constant β to offset the growth in B_0^n as n becomes very large. Setting $\beta = e^{-\alpha - B_1^\infty(1-\phi)\theta}$ guarantees that Assumption 1 in Alvarez and Jermann (2005) is satisfied. The temporary pricing component of the SDF is thus equal to:

$$\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}} = \beta e^{B_1^\infty(z_{t+1}-z_t)} = \beta e^{B_1^\infty\left[(\phi-1)(z_t-\theta)-\sigma\sqrt{z_t}u_{t+1}\right]}.$$

As a result, the martingale component of the SDF is then:

$$\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} = \frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}}\right)^{-1} = \beta^{-1} e^{-\alpha-\chi z_t - \sqrt{\gamma z_t}u_{t+1}} e^{-B_1^\infty\left[(\phi-1)(z_t-\theta)-\sigma\sqrt{z_t}u_{t+1}\right]}. \tag{11}$$

Decomposition (Hansen and Scheinkman, 2009) We now show that the Hansen and Scheinkman (2009) methodology leads to similar results. We guess an eigenfunction ϕ of the form

$$\phi(x) = e^{cz}$$

where c is a constant. Then, the (one-period) eigenfunction problem can be written as

$$E_t\left[\exp(\alpha + \chi z_t + \sqrt{\gamma z_t}u_{t+1} + c z_{t+1})\right] = \exp(\beta + c z_t).$$

Expanding and matching coefficients, we get:

$$\beta = -\alpha + c(1-\phi)\theta$$

$$\left[\frac{1}{2}\sigma^2\right]c^2 + \left[\sigma\sqrt{\gamma} + \phi - 1\right]c + \left[\frac{1}{2}\gamma - \chi\right] = 0$$

so c solves a quadratic equation. The transitory component of the pricing kernel is by definition:

$$\Lambda_t^{\mathbb{T}} = e^{\beta t - c z_t}$$

The transitory and permanent SDF component are thus:

$$\begin{aligned} \frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}} &= e^{\beta - c(z_{t+1}-z_t)} = e^{\beta - c\left[(1-\phi)(\theta-z_t)-\sigma\sqrt{z_t}u_{t+1}\right]} = e^{-\alpha + c\left[(1-\phi)z_t + \sigma\sqrt{z_t}u_{t+1}\right]} \\ \frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} &= \frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}}\right)^{-1} = e^{-\alpha-\chi z_t-\sqrt{\gamma z_t}u_{t+1}} e^{\alpha - c\left[(1-\phi)z_t+\sigma\sqrt{z_t}u_{t+1}\right]} = e^{-[\chi + c(1-\phi)]z_t - \left[\sqrt{\gamma}+c\sigma\right]\sqrt{z_t}u_{t+1}} \end{aligned}$$

The law of motion of bond prices implies that $c = -B_1^\infty$. If $\chi = -c(1-\phi)$, then the quadratic equation for c becomes

$$\sigma^2 c^2 + 2\sigma\sqrt{\gamma}c + \gamma = 0$$

with unique solution $c = -\frac{\sqrt{\gamma}}{\sigma}$. Then, the martingale component of the SDF becomes

$$\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} = 1$$

so the entirety of the SDF is its transitory component.

Term Premium The expected log excess return is thus given by:

$$E_t[rx_{t+1}^{(n)}] = [-\frac{1}{2}\left(B_1^{n-1}\right)^2\sigma^2 + \sigma\sqrt{\gamma}B_1^{n-1}]z_t.$$

The expected log excess return of an infinite maturity bond is then:

$$\begin{aligned} E_t[rx_{t+1}^{(\infty)}] &= [-\frac{1}{2}\left(B_1^\infty\right)^2\sigma^2 + \sigma\sqrt{\gamma}B_1^\infty]z_t, \\ &= [B_1^\infty(1-\phi) - \chi + \frac{1}{2}\gamma]z_t. \end{aligned}$$

The $-\frac{1}{2}\left(B_1^\infty\right)^2\sigma^2$ is a Jensen term. The term premium is driven by $\sigma\sqrt{\gamma}B_1^\infty z_t$, where B_1^∞ is defined implicitly in the second order equation $B_1^\infty = \chi - \frac{1}{2}\gamma + B_1^\infty\phi - \frac{1}{2}\left(B_1^\infty\right)^2\sigma^2 + \sigma\sqrt{\gamma}B_1^\infty$.

Model with Country-specific Factors Consider the special case of $B_1^\infty(1-\phi) = \chi$. In this case, the expected term premium is simply $E_t[rx_{t+1}^{(\infty)}] = \frac{1}{2}\gamma z_t$, which is equal to one-half of the variance of the log stochastic discount factor.
Suppose that the foreign pricing kernel is specified as in Equation (9) with the same parameters. However, the foreign country has its own factor z^*. As a result, the difference between the domestic and foreign log term premia is equal to the log currency risk premium, which is given by $E_t[rx_{t+1}^{FX}] = \frac{1}{2}\gamma(z_t - z_t^*)$. In other words, the expected foreign log holding period return on a foreign long bond converted into U.S. dollars is equal to the U.S. term premium: $E_t[rx_{t+1}^{(\infty),*}] + E_t[rx_{t+1}^{FX}] = \frac{1}{2}\gamma z_t$.
This special case corresponds to the absence of permanent shocks to the SDF: when $B_1^\infty(1-\phi) = \chi$, the permanent component of the stochastic discount factor is constant. To see this result, let us go back to the implicit definition of B_1^∞ in Equation (11):

$$\begin{aligned} 0 &= \frac{1}{2}\left(B_1^\infty\right)^2\sigma^2 + (1-\phi-\sigma\sqrt{\gamma})B_1^\infty - \chi + \frac{1}{2}\gamma, \\ 0 &= \frac{1}{2}\left(B_1^\infty\right)^2\sigma^2 - \sigma\sqrt{\gamma}B_1^\infty + \frac{1}{2}\gamma, \\ 0 &= \left(\sigma B_1^\infty - \sqrt{\gamma}\right)^2. \end{aligned}$$

In this special case, $B_1^\infty = \sqrt{\gamma}/\sigma$. Using this result in Equation (11), the permanent component of the SDF reduces to:

$$\frac{M_{t+1}^{\mathbb{P}}}{M_t^{\mathbb{P}}} = \frac{M_{t+1}}{M_t}\left(\frac{M_{t+1}^{\mathbb{T}}}{M_t^{\mathbb{T}}}\right)^{-1} = \beta^{-1}e^{-\alpha-\chi z_t - \sqrt{\gamma z_t}u_{t+1}}e^{-B_1^\infty\left[(\phi-1)(z_t-\theta)-\sigma\sqrt{z_t}u_{t+1}\right]} = \beta^{-1}e^{-\alpha-\chi\theta},$$

which is a constant.

Model with Global Factors We assume that all the shocks are global and that z_t is a global state variable (and thus $\sigma = \sigma^*$, $\phi = \phi^*$, $\theta = \theta^*$). The state variable is referred as "permanent" if it has some impact on the permanent component of the SDF. The difference in term premia between the domestic and foreign bond (once expressed in the same currency) is pinned down by the difference in conditional variances of the permanent components of the SDFs. Therefore the two bonds have the same risk premia when:

$$\sqrt{\gamma} + B_1^\infty\sigma = \sqrt{\gamma^*} + B_1^{\infty *}\sigma$$

Note that B_1^{∞} depends on χ and γ, as well as on the global parameters ϕ and σ. The domestic and foreign infinite-maturity bonds have the same risk premia (once expressed in the same currency) when $\gamma = \gamma^*$ and $\chi = \chi^*$, i.e. when the domestic and foreign SDFs react similarly to changes in the global "permanent " state variable and its shocks

D.3 Multi-Factor Vasicek Models

Model Under some conditions, the previous results can be extended to a more k-factor model. The standard $k-$factor essentially affine model in discrete time generalizes the Vasicek (1977) model to multiple risk factors. The log SDF is given by:

$$-\log M_{t+1} = y_{1,t} + \frac{1}{2}\Lambda_t'\Sigma\Lambda_t + \Lambda_t'\varepsilon_{t+1}$$

To keep the model affine, the law of motion of the risk-free rate and of the market price of risk are:

$$\begin{aligned} y_{1,t} &= \delta_0 + \delta_1' x_t, \\ \Lambda_t &= \Lambda_0 + \Lambda_1 x_t, \end{aligned}$$

where the state vector ($x_t \in R^k$) is:

$$x_{t+1} = \Gamma x_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}(0, \Sigma).$$

x_t is a $k \times 1$ vector, and so are ε_{t+1}, δ_1, Λ_t, and Λ_0, while Γ, Λ_1, and Σ are $k \times k$ matrices.[21]

We assume that the market price of risk is constant ($\Lambda_1 = \mathbf{0}$), so that we can define orthogonal temporary shocks. We decompose the shocks into two groups: the first $h < k$ shocks affect both the temporary and the permanent SDF components and the last $k - h$ shocks are temporary.[22] The parameters of the temporary shocks satisfy $B_{1k-h}^{\infty\prime} = (I_{k-h} - \Gamma_{k-h})^{-1}\delta_{1k-h}' = -\Lambda_{0k-h}'$. This ensures that these shocks do not affect the permanent component of the SDF.

Symmetric Model with Global Factor Now we assume that x_t is a global state variable:

$$\begin{aligned} -\log M_{t+1}^* &= y_{1,t}^* + \frac{1}{2}\Lambda_t^{*\prime}\Sigma\Lambda_t^* + \Lambda_t^{*\prime}\varepsilon_{t+1}, \\ y_{1,t} &= \delta_0^* + \delta_1^{*\prime} x_t, \\ \Lambda_t^* &= \Lambda_0^*, \\ x_{t+1} &= \Gamma x_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}(0, \Sigma). \end{aligned}$$

In a multi-factor Vasicek model with global factors and constant risk prices, long bond uncovered return parity obtains only if countries share the same Λ_h and δ_{1h}, which govern exposure to the permanent, global shocks.

This condition eliminates any differences in permanent risk exposure across countries.[23] The nominal exchange rate has no permanent component $\left(\frac{S_t^{\mathbb{P}}}{S_{t+1}^{\mathbb{P}}} = 1\right)$. From equation (1) , the expected log currency excess return is equal to:

$$E_t[rx_{t+1}^{FX}] = \frac{1}{2}Var_t(m_{t+1}) - \frac{1}{2}Var_t(m_{t+1}^*) = \frac{1}{2}\Lambda_0'\Sigma\Lambda_0 - \frac{1}{2}\Lambda_0^{*\prime}\Sigma\Lambda_0^*.$$

Non-zero currency risk premia will be only due to variation in the exposure to transitory shocks (Λ_{0k-h}^*).

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[21] Note that if $k = 1$ and $\Lambda_1 = 0$, we are back to the Vasicek (1977) model with one factor and a constant market price of risk. The Vasicek (1977) model presented in the first section is a special case where $\Lambda_0 = \lambda$, $\delta_0' = \delta$, $\delta_0' = 1$ and $\Gamma = \rho$.

[22] A block-diagonal matrix whose blocks are invertible is invertible, and its inverse is a block diagonal matrix (with the inverse of each block on the diagonal). Therefore, if Γ is block-diagonal and $(I - \Gamma)$ is invertible, we can decompose the shocks as described

[23] The terms δ_1' and $\delta_{1h}^{*\prime}$ do not appear in the single-factor Vasicek (1977) model of the first section because that single-factor model assumes $\delta_1 = \delta_{1h}^* = 1$.

D.4 Gaussian Dynamic Term Structure Models

Model The $k-$factor heteroskedastic Gaussian Dynamic Term Structure Model (DTSM) generalizes the CIR model. When market prices of risk are constant, the log SDF is given by:

$$\begin{aligned} -m_{t+1} &= y_{1,t} + \frac{1}{2}\Lambda' V(x_t)\Lambda + \Lambda' V(x_t)^{1/2}\varepsilon_{t+1}, \\ x_{t+1} &= \Gamma x_t + V(x_t)^{1/2}\varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}(0, I), \\ y_{1,t} &= \delta_0 + \delta_1' x_t, \end{aligned}$$

where $V(x)$ is a diagonal matrix with entries $V_{ii}(x_t) = \alpha_i + \beta_i' x_t$. To be clear, x_t is a $k \times 1$ vector, and so are ε_{t+1}, Λ, δ_1, and β_i. But Γ and V are $k \times k$ matrices. A restricted version of the model would impose that β_i is a scalar and $V_{ii}(x_t) = \alpha_i + \beta_i x_{it}$— this is equivalent to assuming that the price of shock i only depends on the state variable i.

Bond Prices The price of a one period-bond is:

$$P_t^{(1)} = E_t(M_{t+1}) = e^{-\delta_0 - \delta_1' x_t}.$$

For any maturity n, bond prices are exponentially affine, $P_t^{(n)} = \exp\left(-B_0^n - B_1^{n\prime} x_t\right)$. Note that B_0^n is a scalar, while B_1^n is a $k \times 1$ vector. The one period-bond corresponds to $B_0^1 = \delta_0$, $B_1' = \delta_1'$. Bond prices are defined recursively by the Euler equation: $P_t^{(n)} = E_t(M_{t+1} P_{t+1}^{n-1})$, which implies:

$$\begin{aligned} P_t^{(n)} &= E_t\left(\exp\left(-y_{1,t} - \frac{1}{2}\Lambda' V(x_t)\Lambda - \Lambda' V(x_t)^{1/2}\varepsilon_{t+1} - B_0^{n-1} - B_1^{n-1\prime} x_{t+1}\right)\right) \\ &= \exp\left(-B_0^n - B_1^{n\prime} x_t\right). \end{aligned}$$

This delivers the following difference equations:

$$\begin{aligned} B_0^n &= \delta_0 + B_0^{n-1} - \frac{1}{2}B_1^{n-1\prime} V(0) B_1^{n-1} - \Lambda' V(0) B_1^{n-1}, \\ B_1^{n\prime} &= \delta_1' + B_1^{n-1\prime}\Gamma - \frac{1}{2}B_1^{n-1\prime} V_x B_1^{n-1} - \Lambda' V_x B_1^{n-1}, \end{aligned}$$

where V_x denotes all the diagonal slope coefficients β_i of the V matrix.
The CIR model studied in the previous pages is a special case of this model. It imposes that $k = 1$, $\sigma = -\sqrt{\beta}$, and $\Lambda = -\frac{1}{\sigma}\sqrt{\gamma}$. Note that the CIR model has no constant term in the square root component of the log SDF, but that does not imply $V(0) = 0$ here as the CIR model assumes that the state variable has a non-zero mean (while it is zero here).

Decomposition (Alvarez and Jermann, 2005) From there, we can define the Alvarez and Jermann (2005) pricing kernel components as for the Vasicek model. The limit of $B_0^n - B_0^{n-1}$ is finite: $\lim_{n\to\infty} B_0^n - B_0^{n-1} = \delta_0 - \frac{1}{2}B_1^{\infty\prime} V(0) B_1^{\infty} - \Lambda_0' V(0) B_1^{\infty}$, where $B_1^{\infty\prime}$ is the solution to the second-order equation above. As a result, B_0^n grows at a linear rate in the limit. We choose the constant β to offset the growth in B_0^n as n becomes very large. Setting $\beta = e^{-\delta_0 + \frac{1}{2}B_1^{\infty\prime} V(0) B_1^{\infty} + \Lambda' V(0) B_1^{\infty}}$ guarantees that Assumption 1 in Alvarez and Jermann (2005) is satisfied. The temporary pricing component of the pricing kernel is thus equal to:

$$\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}} = \beta e^{B_1^{\infty\prime}(x_{t+1} - x_t)} = \beta e^{B_1^{\infty\prime}(\Gamma - 1)x_t + B_1^{\infty\prime} V(x_t)^{1/2}\varepsilon_{t+1}}.$$

The martingale component of the pricing kernel is then:

$$\begin{aligned} \frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} = \frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}}\right)^{-1} &= \beta^{-1} e^{-B_1^{\infty}(\Gamma - 1)x_t - B_1^{\infty\prime} V(x_t)^{1/2}\varepsilon_{t+1} - y_{1,t} - \frac{1}{2}\Lambda' V(x_t)\Lambda_t - \Lambda' V(x_t)^{1/2}\varepsilon_{t+1}} \\ &= \beta^{-1} e^{-B_1^{\infty}(\Gamma - 1)x_t - \delta_0 - \delta_1' x_t - \frac{1}{2}\Lambda' V(x_t)\Lambda - (\Lambda' + B_1^{\infty\prime}) V(x_t)^{1/2}\varepsilon_{t+1}}. \end{aligned}$$

For the martingale component to be constant, we need that $\Lambda' = -B_1^{\infty\prime}$ and $B_1^{\infty}(\Gamma - 1) + \delta_1' + \frac{1}{2}\Lambda' V_x \Lambda = 0$. Note that the second condition is automatically satisfied if the first one holds: this result comes from the implicit value of $B_1^{\infty\prime}$ implied by the law of motion of B_1. As a result, the martingale component is constant as soon as $\Lambda = -B_1^{\infty}$.

Decomposition (Hansen and Scheinkman, 2009) We guess an eigenfunction ϕ of the form

$$\phi(x) = e^{c'x}$$

where c is a $k \times 1$ vector of constants. Then, the (one-period) eigenfunction problem can be written as

$$E_t\left[\exp(-\delta_0 - \delta_1' x_t - \frac{1}{2}\Lambda' V(x_t)\Lambda - \Lambda' V^{1/2}(x_t)\varepsilon_{t+1} + c x_{t+1})\right] = \exp(\beta + c x_t)$$

Expanding and matching coefficients, we get:

$$\begin{aligned}
\beta &= -\delta_0 - \frac{1}{2}\Lambda' V(0)\Lambda + \frac{1}{2}(c-\Lambda)' V(0)(c-\Lambda) \\
0 &= c'(\Gamma - I) - \delta_1' + \sum_{i=1}^{k} c_i(c_i - 2\Lambda_i)\mu_i'.
\end{aligned}$$

The transitory component of the pricing kernel is by definition:

$$\Lambda_t^{\mathbb{T}} = e^{\beta t - c' x_t}$$

The transitory and permanent SDF component are thus:

$$\begin{aligned}
\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}} &= e^{\beta - c'(x_{t+1} - x_t)} = e^{\beta - c'(\Gamma - I)x_t - c' V^{1/2}(x_t)\varepsilon_{t+1}} \\
\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} &= \frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}}\right)^{-1} \\
&= e^{-\delta_0 - \delta_1' x_t - \frac{1}{2}\Lambda' V(x_t)\Lambda - \Lambda' V^{1/2}(x_t)\varepsilon_{t+1}} e^{-\beta + c'(\Gamma - I)x_t + c' V^{1/2}(x_t)\varepsilon_{t+1}} \\
&= e^{-\delta_0 - \beta + \left[c'(\Gamma - I) - \delta_1'\right]x_t - \frac{1}{2}\Lambda' V(x_t)\Lambda + (c-\Lambda)' V^{1/2}(x_t)\varepsilon_{t+1}}
\end{aligned}$$

If $\Lambda = c$, then the equations for β and c become:

$$\begin{aligned}
\beta &= -\delta_0 - \frac{1}{2}c' V(0) c \\
0 &= c'(\Gamma - I) - \delta_1' - \sum_{i=1}^{k} c_i^2 \mu_i'
\end{aligned}$$

The martingale component of the SDF is then

$$\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} = e^{-\delta_0 - \beta + \left[c'(\Gamma - I) - \delta_1'\right]x_t - \frac{1}{2}c' V(x_t) c} = 1$$

The entirety of the SDF is described by its transitory component in this case.

Term Premium The expected log holding period excess return is:

$$E_t[rx_{t+1}^{(n)}] = -\delta_0 + \left(-B_1^{n-1\prime}\Gamma + B_1^{n\prime} - \delta_1'\right)x_t.$$

The term premium on an infinite-maturity bond is therefore:

$$E_t[rx_{t+1}^{(\infty)}] = -\delta_0 + \left((1-\Gamma)B_1^{\infty\prime} - \delta_1'\right)x_t.$$

The expected log currency excess return is equal to:

$$E_t[-\Delta s_{t+1}] + y_t^* - y_t = \frac{1}{2}Var_t(m_{t+1}) - \frac{1}{2}Var_t(m_{t+1}^*) = \frac{1}{2}\Lambda' V(x_t)\Lambda - \frac{1}{2}\Lambda^{*\prime} V(x_t^*)\Lambda^*.$$

We assume that all the shocks are global and that x_t is a global state variable ($\Gamma = \Gamma^*$ and $V = V^*$, no country-specific parameters in the V matrix— cross-country differences will appear in the vectors Λ). Let us decompose the shocks into two groups: the first $h < k$ shocks affect both the temporary and the permanent SDF components and the last $k - h$ shocks are temporary. Temporary shocks are such that $\Lambda_{k-h} = -B_{1,k-h}^{\infty}$ (i.e., they do not affect the value of the permanent component of the SDF).
The risk premia on the domestic and foreign infinite-maturity bonds (once expressed in the same currency) will be the same provided that the entropy of the domestic and foreign permanent components is the same:

$$\begin{aligned}
(\Lambda_h' + B_{1h}^{\infty\prime})V(0)(\Lambda_h + B_{1h}^{\infty}) &= (\Lambda_h^{*\prime} + B_{1h}^{*\infty\prime})V(0)(\Lambda_h^* + B_{1h}^{\infty*}), \\
(\Lambda_h' + B_{1h}^{\infty\prime})V_x(\Lambda_h + B_{1h}^{\infty}) &= (\Lambda_h^{*\prime} + B_{1h}^{*\infty\prime})V_x(\Lambda_h^* + B_{1h}^{\infty*}).
\end{aligned}$$

To compare these conditions to the results obtained in the one-factor CIR model, recall that $\sigma^{CIR} = -\sqrt{\beta}$, and $\Lambda = -\frac{1}{\sigma^{CIR}}\sqrt{\gamma^{CIR}}$. Differences in Λ_h in the k-factor model are equivalent to differences in γ in the CIR model: in both cases, they correspond to different loadings of the log SDF on the "permanent" shocks. As in the CIT model, differences in term premia can also come form differences in the sensitivity of infinite-maturity bond prices to the global "permanent" state variable ($B_{1h}^{\infty\prime}$), which can be traced back to differences in the sensitivity of the risk-free rate to the "permanent" state variable (i.e., different δ_1 parameters).

D.5 Lustig, Roussanov, and Verdelhan (2014)

Model In each country i, the logarithm of the SDF m^i follows

$$-m_{t+1}^i = \alpha + \chi z_t^i + \sqrt{\gamma z_t^i}u_{t+1}^i + \tau z_t^w + \sqrt{\delta^i z_t^w}u_{t+1}^w + \sqrt{\kappa z_t^i}u_{t+1}^g.$$

The country-specific and world volatility components are governed by autoregressive square root processes:

$$\begin{aligned}
z_{t+1}^i &= (1-\phi)\theta + \phi z_t^i + \sigma\sqrt{z_t^i}u_{t+1}^i, \\
z_{t+1}^w &= (1-\phi^w)\theta^w + \phi^w z_t^w + \sigma^w\sqrt{z_t^w}u_{t+1}^w.
\end{aligned}$$

The change in the real exchange rate Δq^i between the home country and country i is: $\Delta q_{t+1}^i = m_{t+1} - m_{t+1}^i$, where q^i is measured in country i goods per home country good. An increase in q^i means a real appreciation of the home currency. For the home country (the United States), we drop the superscript. The log of the nominal pricing kernel in country i is simply given by the real pricing kernel less the rate of inflation π^i: $m_{t+1}^{i,\$} = m_{t+1}^i - \pi_{t+1}^i$. Inflation is composed of a country-specific component and a global component. We simply assume that the same factors driving the real pricing kernel also drive expected inflation. In addition, inflation innovations in our model are not priced. Thus, country i's inflation process is given by $\pi_{t+1}^i = \pi_0 + \eta^w z_t^w + \sigma_\pi \epsilon_{t+1}^i$, where the inflation innovations ϵ_{t+1}^i are independent and identically distributed gaussian. It follows that the nominal risk-free interest rates (in logarithms) are given by

$$r_t^{i,\$} \quad = \quad \pi_0 + \alpha + \left(\chi - \frac{1}{2}(\gamma+\kappa)\right)z_t^i + \left(\tau + \eta^w - \frac{1}{2}\delta^i\right)z_t^w - \frac{1}{2}\sigma_\pi^2.$$

Decomposition The log nominal bond prices are affine in the state variable z and z^w: $p_t^{i,(n)} = -C_0^{i,n} - C_1^n z_t - C_2^{i,n} z_t^w$. The price of a one-period nominal bond is:

$$P^{i,(0)} = E_t(M_{t+1}^{i,\$}) = E_t\left(e^{-\alpha-\chi z_t - \tau z_t^w - \sqrt{\gamma z_t^i}u_{t+1}^i - \sqrt{\delta^i z_t^w}u_{t+1}^w - \sqrt{\kappa z_t^i}u_{t+1}^g - \pi_0 - \eta^w z_t^w - \sigma_\pi \epsilon_{t+1}^i}\right).$$

As a result, $C_0^1 = \alpha + \pi_0 - \frac{1}{2}\sigma_\pi^2$, $C_1^1 = \chi - \frac{1}{2}(\gamma+\kappa)$, and $C_2^{i,1} = \tau - \frac{1}{2}\delta^i + \eta^w$. Bond prices are defined recursively by the Euler equation: $P_t^{i,(n)} = E_t(M_{t+1}^{i,\$} P_{t+1}^{i,(n-1)})$. This leads to the following difference equations:

$$\begin{aligned}
-C_0^{i,n} - C_1^n z_t - C_2^{i,n} z_t^w &= -\alpha - \chi z_t - \tau z_t^w - C_0^{n-1} - C_1^{n-1}\left[(1-\phi)\theta + \phi z_t\right] - C_2^{i,n-1}\left[(1-\phi^w)\theta^w + \phi^w z_t^w\right] \\
&+ \frac{1}{2}(\gamma+\kappa) z_t + \frac{1}{2}\left(C_1^{n-1}\right)^2 \sigma^2 z_t - \sigma\sqrt{\gamma} C_1^{n-1} z_t \\
&+ \frac{1}{2}\delta^i z_t^w + \frac{1}{2}\left(C_2^{i,n-1}\right)^2 (\sigma^w)^2 z_t^w - \sigma^w \sqrt{\delta^i} C_2^{i,n-1} z_t^w \\
&- \pi_0 - \eta^w z_t^w + \frac{1}{2}\sigma_\pi^2
\end{aligned}$$

Thus bond parameters evolve as:

$$\begin{aligned}
C_0^{i,n} &= \alpha + \pi_0 - \frac{1}{2}\sigma_\pi^2 + C_0^{n-1} + C_1^{n-1}(1-\phi)\theta + C_2^{i,n-1}(1-\phi^w)\theta^w, \\
C_1^n &= \chi - \frac{1}{2}(\gamma+\kappa) + C_1^{n-1}\phi - \frac{1}{2}\left(C_1^{n-1}\right)^2 \sigma^2 + \sigma\sqrt{\gamma} C_1^{n-1} \\
C_2^{i,n} &= \tau - \frac{1}{2}\delta^i + \eta^w + C_2^{i,n-1}\phi^w - \frac{1}{2}\left(C_2^{i,n-1}\right)^2 (\sigma^w)^2 + \sigma^w\sqrt{\delta^i} C_2^{i,n-1}.
\end{aligned}$$

The temporary pricing component of the pricing kernel is:

$$\Lambda_t^{\mathbb{T}} = \lim_{n\to\infty} \frac{\beta^{t+n}}{P_t^n} = \lim_{n\to\infty} \beta^{t+n} e^{C_0^{i,n} + C_1^n z_t + C_2^{i,n} z_t^w},$$

where the constant β is chosen in order to satisfy Assumption 1 in Alvarez and Jermann (2000): $0 < \lim_{n\to\infty} \frac{P_t^n}{\beta^n} < \infty$. The temporary pricing component of the SDF is thus equal to:

$$\frac{\Lambda_{t+1}^T}{\Lambda_t^T} = \beta e^{C_1^\infty (z_{t+1} - z_t) + C_2^{i,\infty}(z_{t+1}^w - z_t^w)} = \beta e^{C_1^\infty \left[(\phi-1)(z_t^i - \theta) - \sigma\sqrt{z_t^i} u_{t+1}^i\right] + C_2^{i,\infty}\left[(\phi^w - 1)(z_t^w - \theta^w) - \sigma\sqrt{z_t^w} u_{t+1}^w\right]}.$$

The martingale component of the SDF is then:

$$\begin{aligned}
\frac{\Lambda_{t+1}^P}{\Lambda_t^P} &= \frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\Lambda_{t+1}^T}{\Lambda_t^T}\right)^{-1} = \beta^{-1} e^{-\alpha - \chi z_t^i - \sqrt{\gamma z_t^i} u_{t+1}^i - \tau z_t^w - \sqrt{\delta^i z_t^w} u_{t+1}^w - \sqrt{\kappa z_t^i} u_{t+1}^g} \\
&\quad e^{C_1^\infty \left[(\phi-1)(z_t^i - \theta) - \sigma\sqrt{z_t^i} u_{t+1}^i\right] + C_2^{i,\infty}\left[(\phi^w - 1)(z_t^w - \theta^w) - \sigma\sqrt{z_t^w} u_{t+1}^w\right]}.
\end{aligned}$$

As a result, we need $\chi = C_1^\infty(1-\phi)$ to make sure that the country-specific factor does not contribute a martingale component. This special case corresponds to the absence of permanent shocks to the SDF: when $C_1^\infty(1-\phi) = \chi$ and $\kappa = 0$, the permanent component of the stochastic discount factor is constant. To see this result, let us go back to the implicit definition of B_1^∞ in Equation (11):

$$\begin{aligned}
0 &= -\frac{1}{2}(\gamma+\kappa) - \frac{1}{2}\left(C_1^\infty\right)^2 \sigma^2 + \sigma\sqrt{\gamma} C_1^\infty \\
0 &= \left(\sigma C_1^\infty - \sqrt{\gamma}\right)^2,
\end{aligned}$$

where we have imposed $\kappa = 0$. In this special case, $C_1^\infty = \sqrt{\gamma}/\sigma$. Using this result in Equation (11), the permanent component of the SDF reduces to:

$$\frac{\Lambda_{t+1}^P}{\Lambda_t^P} = \frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\Lambda_{t+1}^T}{\Lambda_t^T}\right)^{-1} = \beta^{-1} e^{-\tau z_t^w - \sqrt{\delta^i z_t^w} u_{t+1}^w} e^{C_2^{i,\infty}\left[(\phi^w - 1)(z_t^w - \theta^w) - \sigma\sqrt{z_t^w} u_{t+1}^w\right]}.$$

Term Premium The expected log excess return on a zero coupon bond is thus given by:

$$E_t[rx_{t+1}^{(n)}] = [-\frac{1}{2}\left(C_1^{n-1}\right)^2 \sigma^2 + \sigma\sqrt{\gamma} C_1^{n-1}] z_t + [-\frac{1}{2}\left(C_2^{i,n-1}\right)^2 \sigma^2 + \sigma\sqrt{\delta}^i C_2^{i,n-1}] z_t^w.$$

The expected log excess return of an infinite maturity bond is then:

$$E_t[rx_{t+1}^{(\infty)}] = [-\frac{1}{2}\left(C_1^\infty\right)^2 \sigma^2 + \sigma\sqrt{\gamma} C_1^\infty] z_t + [-\frac{1}{2}\left(C_2^{i,\infty}\right)^2 \sigma^2 + \sigma\sqrt{\delta}^i C_2^{i,\infty}] z_t^w.$$

The $-\frac{1}{2}\left(C_1^{\infty}\right)^2\sigma^2$ is a Jensen term. The term premium is driven by $\sigma\sqrt{\gamma}C_1^{\infty}z_t$, where C_1^{∞} is defined implicitly in the second order equation $B_1^{\infty} = \chi - \frac{1}{2}(\gamma+\kappa) + C_1^{\infty}\phi - \frac{1}{2}\left(C_1^{\infty}\right)^2\sigma^2 + \sigma\sqrt{\gamma}C_1^{\infty}$. Consider the special case of $C_1^{\infty}(1-\phi) = \chi$ and $\kappa = 0$ and $C_2^{i,\infty}(1-\phi) = \tau$. In this case, the expected term premium is simply $E_t[rx_{t+1}^{(\infty)}] = \frac{1}{2}(\gamma z_t + \delta z_t^w)$, which is equal to one-half of the variance of the log stochastic discount factor.

Country-specific Factors Suppose that the foreign pricing kernel is specified as in Equation (9) with the same parameters. However, the foreign country has its own factor z^*. As a result, the difference between the domestic and foreign log term premia is equal to the log currency risk premium, which is given by $E_t[rx_{t+1}^{FX}] = \frac{1}{2}\gamma(z_t - z_t^*)$. In other words, the expected foreign log holding period return on a foreign long bond converted into U.S. dollars is equal to the U.S. term premium: $E_t[rx_{t+1}^{(\infty),*}] + E_t[rx_{t+1}^{FX}] = \frac{1}{2}\gamma z_t$.

Calibration The calibration is reported in Table 12.

Table 12: Parameter estimates.

This table reports the parameter values for the estimated version of the model. The model is defined by the following set of equations:

$$\begin{aligned} -m_{t+1}^i &= \alpha + \chi z_t^i + \sqrt{\gamma z_t^i}u_{t+1}^i + \tau z_t^w + \sqrt{\delta^i z_t^w}u_{t+1}^w + \sqrt{\kappa z_t^i}u_{t+1}^g, \\ z_{t+1}^i &= (1-\phi)\theta + \phi z_t^i - \sigma\sqrt{z_t^i}u_{t+1}^i, \\ z_{t+1}^w &= (1-\phi^w)\theta^w + \phi^w z_t^w - \sigma^w\sqrt{z_t^w}u_{t+1}^w, \\ \pi_{t+1}^i &= \pi_0 + \eta^w z_t^w + \sigma_\pi \epsilon_{t+1}^i. \end{aligned}$$

The 17 parameters were obtained to match the moments in Table 12 under the assumption that all countries share the same parameter values except for δ^i, which is distributed uniformly on $[\delta_h, \delta_l]$. The home country exhibits the average δ, which is equal to 0.36. The standard errors obtained by bootstrapping are reported between brackets.

Stochastic discount factor					
α (%)	χ	τ	γ	κ	δ
0.76	0.89	0.06	0.04	2.78	0.36
State variable dynamics					
ϕ	θ (%)	σ (%)	ϕ^w	θ^w (%)	σ^w (%)
0.91	0.77	0.68	0.99	2.09	0.28
Inflation dynamics				Heterogeneity	
η^w	π_0 (%)	σ^π (%)		δ_h	δ_l
0.25	-0.31	0.37		0.22	0.49
Implied SDF dynamics					
$E(Std_t(m))$	$Std(Std_t(m))$ (%)	$E(Corr(m_{t+1}, m_{t+1}^i))$		$Std(z)$ (%)	$Std(z^w)$ (%)
0.59	4.21	0.98		0.50	1.32

In Lustig, Roussanov, and Verdelhan (2014) calibration, the conditions for long run bond parity are not satisfied. First, global shocks have permanent effects in all countries, because $C_2^{i,\infty}(1-\phi^w) < \tau + \eta^w$ for all $i = 1, \ldots, 30$. Second, the global shocks are not symmetric, because δ varies across countries. Third, countryin-specific shocks have permanent effects as well. As a result, both country-specific and global shocks have permanent

Table 13: Simulated Excess Returns on Carry Strategies

	Low	2	3	4	5	High
	Panel A: Interest rates, Bond Returns and Exchange Rates					
Δs	0.57	-0.38	-0.14	-0.70	-0.84	-1.25
$f-s$	-2.33	-1.33	-0.71	-0.15	0.40	1.35
$rx^{*,15}$	1.03	0.52	0.32	0.23	0.17	-0.16
$rx^{*,30}$	1.28	0.72	0.51	0.41	0.35	0.01
	Panel B: Carry Returns with Short-Term Bills					
rx^{fx}	-2.91	-0.95	-0.57	0.54	1.24	2.61
	Panel C: Carry Returns with Long-Term Bonds					
$rx^{\$,15}$	-1.88	-0.43	-0.25	0.77	1.41	2.45
$rx^{\$,30}$	-1.62	-0.23	-0.06	0.95	1.59	2.62

Notes: The table reports summary statistics on simulated data from the Lustig, Roussanov, and Verdelhan (2014) model. Data are obtained from a simulated panel with $T = 33600$ monthly observations and $N = 30$ countries. Countries are sorted by interest rates into six portfolios. Panel A reports the average change in exchange rate (Δs), the average interest rate difference or forward discount ($f - s$), the average foreign bond excess returns for bonds of 15- and 30-year maturities in local currency ($rx^{*,15}$, $rx^{*,30}$). Panel B reports the average log currency excess returns (rx^{fx}). Panel C reports the average foreign bond excess returns for bonds of 15- and 30-year maturities in home currency ($rx^{\$,15}$, $rx^{\$,30}$). The moments are annualized (i.e., means are multiplied by 12).

effects, but only the global shocks matter for dollar-neutral carry trades. The short-term carry premium is only partly offset by the spread in local currency long-term bond risk premia. The variation in local currency bond risk premia is too small. The long-term carry trade is profitable as a result.

Simulation Results We simulate the Lustig, Roussanov, and Verdelhan (2014) model, obtaining a panel of $T = 33600$ monthly observations and $N = 30$ countries. Each month, these 30 countries are ranked by their interest rates into six portfolios. Low interest rate currencies on average have higher exposure δ to the world factor. As a result, these currencies appreciate in case of an adverse world shocks. Long positions in these currencies earn negative excess returns (rx^{fx}) of -2.91% per annum. On the other hand, high interest rate currencies typically have high δ. Long positions in these currencies earn positive excess returns (rx^{fx}) of 2.61% per annum. At the short end, the carry trade strategy, which goes long in the sixth portfolio and short in the first one, delivers an excess return of 5.51% and a Sharpe ratio of 0.47.

This spread is only partly offset by higher local currency bond risk premia in the low interest rate countries with higher δ. The log excess return on the 30-year zero coupo bond is 1.28% in the first portfolio compared to 0.01 % in the last portfolio. While the low interest rate currencies do have steeper yield curves and higher local bond risk premia, this effect is too weak to offset the effect of the currency risk premia. The heterogeneity in δ's across countries generates substantial dispersion in exposure to the permanent component. As a result, our long-run uncovered bond parity condition is violated. At the 30-year maturity, the high-minus-low carry trade strategy still delivers an excess return of 4.24% and a Sharpe ratio of 0.36.

D.6 Lustig, Roussanov, and Verdelhan (2014) with Temporary and Permanent Shocks

The Lustig, Roussanov, and Verdelhan (2014) calibration fails to satisfy the long term bond return parity condition. We turn to a model that explicitly features global permanent and transitory shocks. We show that the heterogeneity in the SDFs' loadings on the *permanent* global shocks needs to be ruled out in order to match the empirical evidence on the term structure of carry risk.

Model We assume that in each country i, the logarithm of the real SDF m^i follows a three-factor conditionally Gaussian process:

$$-m_{t+1}^i = \alpha + \chi z_t^i + \sqrt{\gamma z_t^i} u_{t+1}^i + \tau^i z_t^w + \sqrt{\delta^i z_t^w} u_{t+1}^w + \tau^{\mathbb{P},i} z_t^{\mathbb{P},w} + \sqrt{\delta^{\mathbb{P}} z_t^{\mathbb{P},w}} u_{t+1}^w + \sqrt{\kappa z_t^i} u_{t+1}^g.$$

The inflation process is the same as before. Note that the model now features two global state variables, z_t^w and $z_t^{\mathbb{P},w}$. The state variables follow similar square root processes as in the previous model:

$$\begin{aligned}
z_{t+1}^i &= (1-\phi)\theta + \phi z_t^i - \sigma\sqrt{z_t^i} u_{t+1}^i, \\
z_{t+1}^w &= (1-\phi^w)\theta^w + \phi^w z_t^w - \sigma^w \sqrt{z_t^w} u_{t+1}^w. \\
z_{t+1}^{\mathbb{P},w} &= (1-\phi^{\mathbb{P},w})\theta^{\mathbb{P},w} + \phi^{\mathbb{P},w} z_t^w - \sigma^{\mathbb{P},w}\sqrt{z_t^{\mathbb{P},w}} u_{t+1}^{\mathbb{P},w}.
\end{aligned}$$

But one of the common factors, z_t^w, is rendered transitory by imposing that $C_2^{i,\infty}(1-\phi^w) = \tau^i$. To make sure that the global shocks have no permanent effect for each value of δ^i, we need to introduce another source of heterogeneity across countries. Countries must differ in τ, ϕ^w, σ^w, or η^w (or a combination of those). Without this additional source of heterogeneity, there are at most two values of δ^i that are possible (for each set of parameters). [24] Here we simply choose to let the parameters τ differ across countries.

Bond Prices Our model only allows for heterogeneity in the exposure to the transitory common shocks (δ^i), but not in the exposure to the permanent common shock ($\delta^{\mathbb{P}}$). The nominal log zero-coupon n-month yield of maturity in local currency is given by the standard affine expression $y_t^{(n)} = \frac{1}{n}\left(C_0^n + C_1^n z_t + C_2^n z_t^w + C_3^n z_t^{\mathbb{P},w}\right)$, where the coefficients satisfy second-order difference equations. Given this restriction, the bond risk premium is equal to:

$$\begin{aligned}
E_t[rx_{t+1}^{(i,\infty)}] &= \left[C_1^\infty(1-\phi) - \chi + \frac{1}{2}(\gamma+\kappa)\right] z_t + \frac{1}{2}\delta^i z_t^w \\
&+ \left[C_3^\infty(1-\phi^{\mathbb{P},w}) - \tau^{\mathbb{P}} + \frac{1}{2}\delta^{\mathbb{P}} - \eta^w\right] z_t^{\mathbb{P},w}.
\end{aligned}$$

The log currency risk premium is equal to $E_t[rx_{t+1}^{FX,i}] = (\gamma+\kappa)(z_t - z_t^i)/2 + (\delta - \delta^i)z_t^w/2$. The permanent factor $z_t^{w,\mathbb{P}}$ drops out. This also implies that the expected foreign log holding period return on a foreign long bond converted into U.S. dollars is equal to:

$$\begin{aligned}
E_t[rx_{t+1}^{(i,\infty)}] + E_t[rx_{t+1}^{FX,i}] &= \left[(C_1^\infty(1-\phi) - \chi)z_t^i + \frac{1}{2}(\gamma+\kappa)z_t\right] + \frac{1}{2}\delta z_t^w \\
&+ \left[C_3^\infty(1-\phi^{\mathbb{P},w}) - \tau^{\mathbb{P}} + \frac{1}{2}\delta^{\mathbb{P}} - \eta^w\right] z_t^{\mathbb{P},w}.
\end{aligned}$$

Given the symmetry that we have imposed, the difference between the foreign term premium in dollars and the domestic term premium is then given by: $[C_1^\infty(1-\phi) - \chi](z_t^i - z_t)$. There is no difference in long bond returns that can be traced back to the common factor; only the idiosyncratic factor. The spread due to the common factor is the only part that matters for the long-term carry trade, which approximately produces zero returns here.

Term Structure of Carry Trade Risk Premia To match short-term carry trade returns, we need asymmetric exposure to the transitory shocks, governed by (δ), but not to permanent shocks, governed by ($\delta^{\mathbb{P}}$). If the foreign kernel is less exposed to the transitory shocks than the domestic kernel ($\delta > \delta^i$), there is a large positive foreign currency risk premium (equal here to $(\delta - \delta^i)z_t^w/2$), but that premium is exactly offset by a smaller foreign term premium and hence does not affect the foreign bond risk premium in dollars. The countries with higher exposure will also tend to have lower interest rates when the transitory volatility z_t increases, provided that $\left(\tau - \frac{1}{2}\delta\right) < 0$. Hence, in this model, the high δ^i funding currencies in the lowest interest rate portfolios will tend to earn negative currency risk premia, but positive term premia. The reverse would be true for the

[24] This result appears when plugging the no-permanent-component condition in the differential equation that governs the loading of the bond price on the global state variable.

low δ^i investment currencies in the high interest rate portfolios. This model thus illustrates our main theoretical findings: chasing high interest rates does not necessarily work at the long end of the maturity spectrum. If there is no heterogeneity in the loadings on the permanent global component of the SDF, then the foreign term premium on the longest bonds, once converted to U.S. dollars is identical to the U.S. term premium.