Aggregate implications of corporate debt choices∗

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Abstract

The composition of corporate borrowing between bank loans and publicly traded debt varies substantially, both across firms and over the business cycle. This paper develops a new model of firm dynamics, where firms choose both the scale and composition of their borrowing, in order to understand the aggregate implications of corporate debt choices. In choosing debt composition, firms trade-off the flexibility that banks provide in case of financial distress, with the lower intermediation costs offered by markets in normal times. The steady-state of the model captures key cross-sectional facts about debt composition: firms sort between a mixed-debt and a market-debt regime, and shift from the former to the latter as their credit risk declines. Aggregate shocks to banks’ intermediation costs generate a large recession, accompanied by a sharp drop in the total share of intermediation conducted by banks. In this model, the aggregate decline in borrowing and investment reflects not only lower bank loan issuance by mixed-debt firms (the traditional “lending channel”), but also a reduction in total borrowing by a subset firms that retire bank loans and issue publicly traded debt in their place (a new “substitution channel”). When replacing loans by public debt, these firms indeed lose the flexibility provided by banks in bad times; they attempt to offset this increase in financial fragility by deleveraging. I show that this mechanism can be quantitatively large, accounting for as much as a third of the decline in investment among rated firms in the Great Recession.

Keywords : banks, bonds, financial structure, financial frictions, firm dynamics, output, investment, productivity risk.

JEL Classification Numbers: E22, E23, G21, G33.

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1 Introduction

How do financial frictions affect aggregate investment? In the aftermath to the 2007-2009 recession, which was marked by a dramatic 18.2% collapse in fixed capital formation, the question has attracted renewed attention among policymakers and economists.\(^1\) Because debt is the main source of external financing for investment projects, much of the theoretical literature on the topic has focused on understanding the effects of frictions to total debt issuance. In order to represent these frictions, this literature has assumed that all corporate debt issuance occurs via a single financial intermediary, typically thought of as a bank. In these models, firms effectively face a single borrowing constraint, and use a single type of debt.

While this approach may be useful to capture the constraints faced by small firms, it ignores the fact that larger firms in fact have access to and make use of various types of debt instruments. In fact, on the eve of the Great Recession, bank loans only accounted for 31.3% of total debt of the non-financial corporate sector in the US, with bonds making up the lion’s share of the remainder (63.0%).\(^2\) Moreover, there is substantial evidence that the debt mix used by firms with access to public debt markets varies, both in the cross-section and over time. In the cross-section, Rauh and Sufi (2010) study a sample of rated US firms, and show that the majority of them borrow simultaneously from banks and bond markets, with only the very largest specializing in bond borrowing. Over the business cycle, Adrian et al. (2012) and Becker and Ivashina (2014) find evidence of substitution between bank loans and corporate bonds among rated firms as credit conditions tighten. Yet, while debt composition is shaped by idiosyncratic and aggregate factors, its implications for aggregate investment elude standard models, where total borrowing is the only margin of choice. This is not an omission of minor importance: in 2007, firms with access to public debt markets – those for which debt composition was a relevant choice – accounted for 82.5% of the aggregate stock of debt, and 65.0% of aggregate investment.\(^3\)

The goal of this paper is to provide a theoretical framework to study the implications of debt composition for aggregate investment. I address three specific questions. First, what drives cross-sectional differences in debt structure, and how do they relate to firm-level investment? Second, how do aggregate shocks, and in particular shocks to bank credit supply, affect aggregate investment, when firms can endogenously adjust debt composition? Third, can policy affect the way firms choose their financing mix, say by encouraging bond issuance in lieu of bank borrowing, and if so, are such policies necessarily beneficial to aggregate investment?

In order to answer these questions, I develop a macroeconomic model with heterogeneous firms in which both the scale and composition of debt are endogenous. Section 2 describes this model. It is the first model of

\(^1\) The source is the BEA private nonresidential fixed investment series (A008RC1). Appendix A contains all references to
\(^2\) These ratios are constructed from table L.102 of Flow of Funds, the balance sheet of the non-financial corporate sector in the US. Appendix A contains more details on the use of the Flow of Funds data.
\(^3\) These ratios compare total debt and total investment of non-financial firms in Compustat, to total borrowing from the the Flow of Funds, and to private nonresidential fixed investment from the BEA. Appendix A contains more details on the use of the Flow of Funds data.
firm dynamics with financial frictions to endogenize jointly borrowing composition, investment choices, and firm growth. In this economy, firms can finance investment either internally (through the accumulation of retained earnings), or by issuing two types of debt: bank loans and market debt. Credit is constrained by the fact that firms have limited liability, and default entails deadweight losses of output. The central assumption of the model is that banks and market lenders differ in their ability to deal with financial distress. Specifically, I assume that bank loans can be restructured when firms’ revenues are low, whereas market liabilities cannot be reorganized. In this sense, banks offer more flexibility than market lenders when a firm is in financial distress. On the other hand, outside of financial distress, I assume that bank lending is more restrictive than market lending. In the model, this difference is captured through differences in intermediation costs between bank and market lenders. The higher intermediation costs of banks are reflected in the equilibrium terms of lending contracts, which firms must honor outside of financial distress. In choosing the scale and composition of borrowing, firms therefore trade-off the higher flexibility of bank debt in financial distress, with the lower costs associated with market financing in normal times.4

Section 3 analyzes firms’ financial policies in steady-state, in a calibration of the model to US data. The model endogenously generates a distribution of firms across levels of internal finance, and along this distribution, firms differ in their choices of debt composition and investment. The model has two key predictions. First, firms sort into two financial regimes: a “mixed-finance” regime, in which they borrow simultaneously from banks and bond markets, and a “market-finance” regime, in which they borrow only from public debt markets. Second, the transition between these two regimes occurs as a firm’s internal finance increases and its credit risk falls. The joint determination of debt structure and investment is crucial to understanding these results. First, the endogeneity of investment creates a complementarity between bank and market finance: as market debt issuance increases, firms are able to invest more and operate at larger scales; this improves their liquidation value, which may in turn relax their bank borrowing constraint. This complementarity is what drives some firms to use a combination of bank and market debt. Second, because of decreasing returns, firms have an ex-ante optimal scale of operation, irrespective of internal resources. As firms grow and accumulate internal resources, they require less leverage in order to approach that scale. Lower leverage implies a smaller probability of financial distress, and therefore smaller gains associated with bank debt flexibility. When internal finance is large enough, relative to the optimal scale of investment, these gains become negligible and firms only use market debt.

These cross-sectional predictions line up closely with recent evidence on the debt structure of rated firms. Rauh and Sufi (2010) study the cross-sectional variation in debt structure of rated US firms, and

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4 In section 2, I discuss at length the empirical support for the view that banks are more flexible creditors than markets in financial distress. I also provide different interpretations of the assumption that banks provide a more constraining form of debt in normal times than markets.
emphasize two key findings: first, the majority of firms in their sample (68.3%) use a combination of bank loans and publicly traded debt, with the remainder using only publicly traded debt; second, firms move from the former to the latter group as their credit quality increases.\footnote{In related work, Colla et al. (2013) document similar patterns of debt choices among rated firms. Section 3 discusses this evidence in more detail.} The model’s prediction that firms sort into mixed-finance and market-finance regimes matches the first finding. It does so both qualitatively, and quantitatively: the baseline calibration of the model, which matches aggregate evidence on the ratio of bank to total debt of medium and large US firms, implies that 67.2% of firms in the model use a mixed debt structure, strikingly close to the 68.3% found by Rauh and Sufi (2010). The model is thus capable of capturing both the intensive and extensive margins aspects of debt choices. The finding that the majority of firms operate using a combination of debt instruments is particularly important; as emphasized by Rauh and Sufi (2010), many existing models of debt composition instead tend to predict that firms issue exclusively one type of debt security. Additionally, in the model, the key driver of the decision of firms to switch from the mixed-finance to the market-finance regime is the decline of their ex-ante probability of financial distress as they grow and accumulate internal finance. As a result, the model predicts a tight connection between measures of risk, such as the ex-ante liquidation probability, or the volatility of profits, and debt composition, as is the case in the data.

Given its ability to match key cross-sectional facts about debt structure, the remainder of the paper uses this model to study the aggregate implications of debt heterogeneity. Section 4 focuses on the transmission of financial shocks. I study the perfect foresight response of the model to a bank credit supply shock, which I capture as a permanent increase in the intermediation costs of banks. The magnitude of this shock is chosen so that the model matches the large and persistent drop in the share of bank debt to total debt of medium and large US firms during the Great Recession, which fell from 24.0% in 2007Q3 to 19.8% in 2011Q3.\footnote{I document this drop using data from the Quarterly Financial Report of manufacturing firms. It mirrors the drop in the aggregate bank share, which also reflects a reduction in bank borrowing by smaller firms. The aggregate bank share from 31.3% in 2007Q3 to 24.3% in 2011Q3, and has not recovered since. I choose to focus on data from the QFR because of the lack of publicly available time-series evidence on the total composition of debt of rated firms. This issue is discussed in more detail in section 4.} The model a qualitatively and quantitatively similar drop in the bank share in response to a bank credit supply shock that increases bank intermediation costs, relative to market intermediation costs, by roughly a third (or 44bps) relative to steady-state.

In addition to its effects on the bank share, the shock also generates a large recession, with investment contracting by 25.2% over the first three years. The shock affects investment in two ways. On one hand, firms react to the permanently higher bank intermediation costs by reducing their borrowing from banks. This traditional “lending channel” effect accounts for about two thirds of the response of investment, and is limited to mixed-finance firms, who were heavily relying on bank loans before the shock. The remaining third of the response of output is accounted for by a novel propagation mechanism. It only affects firms which,
prior to the shock, had accumulated sufficient internal finance to be close to the market-finance regime, but were still relying on bank debt. The bank credit supply shock causes these firms to make the switch between financial regimes: in effect, their response is to retire all outstanding bank debt and issue bonds in their place. However, the model predicts that the substitution is less than one for one. As a result, the investment of these switching firms drops, relative to their pre-shock investment levels. The negative effect of this “debt substititon channel” can be interpreted as a precautionary response of firms. Moving from the mixed- to the market-finance regime strips firms from the flexibility that bank debt traditionally gave them. As a result, for a comparable leverage, switching firms are more prone to liquidation risk. To put it otherwise, switching between financial regime increases the financial fragility of these firms. In order to offset this increase in financial fragility, they choose to deleverage relative to their pre-shock state. Numerical decompositions of the impact of these two channels on investment suggest that the “substitution channel” can be quantitatively important: in the baseline calibration of the model, it accounts for one third of the total decline in investment of firms.

Section 5 concludes by studying the real effects of policies aimed at incentivizing firms to rely more heavily on market credit. I analyze two specific examples: German efforts to develop a bond market specifically targeted for medium-sized firms, and an Italian fiscal reform extending tax deductibility of interest payments to bond issues by private firms. As for the propagation of financial shocks, the effects of these policies are best understood in terms of a “lending channel” and a “substitution channel” effect. On the one hand, they boost aggregate investment by lowering the cost of market debt issuance (the “credit channel” effect). On the other, they induce medium-sized firms that were previously partially bank-financed to switch entirely to market finance. As a result of their increased fragility, these firms borrow less, and their output and investment falls (the “substitution channel” effect). The net effect of the policy on aggregate investment and output in steady-state is in general positive, but this comes at the expense of a precautionary reduction in leverage and activity of medium-sized firms.

Related literature  This paper builds on the extensive literature on corporate debt structure, following the seminal contributions of Diamond (1991), Rajan (1992), Besanko and Kanatas (1993) and Bolton and Scharfstein (1996). The assumption that bank and market lending differ in their degree of flexibility in times of financial distress builds on the insight of Bolton and Scharfstein (1996) that the dispersion of market creditors reduces individual incentives to renegotiate debt payments, and may create holdout problems that impede efficient restructuring. In terms of its description of banks as flexible creditors, the framework studied

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7The term “precautionary” is used here somewhat loosely, as firms value the stream of future dividends in a risk-neutral manner. However, the combination of decreasing returns and borrowing constraints implies that firms’ valuation of internal finance flows is concave, and particularly so for small levels of internal finance. This is a common feature of firm dynamics models with financial frictions; see, for example, Cooley and Quadrini (2001).
in this paper is closest to the continuous-time framework of Hackbarth, Hennessy, and Leland (2007). This paper, along with most of theoretical literature on the topic, focuses on the structure of financing, given a fixed scale of investment; by contrast, in the model I propose, the scale of investment is also determined endogenously, which allows me to draw cross-sectional and aggregate implications of debt structure for output and investment.

This paper is also related to the literature on firm growth and financial frictions. My model’s key friction is limited liability, as in Cooley and Quadrini (2001), Clementi and Hopenhayn (2006) or Hennessy and Whited (2007). In particular, the connection between the firms’ optimal financial policies and their steady-state growth dynamics follows closely Cooley and Quadrini (2001). I contribute to this literature by introducing an endogenous debt structure choice, and illustrating its implications for firm growth and the distribution of firms across levels of internal finance in steady-state.

The macroeconomic implications of debt heterogeneity have been addressed by relatively few papers. Bolton and Freixas (2006), in the context of a static model, show that, by affecting the spread of bank loans over corporate bonds, monetary policy can lower banks’ equity-capital base, in turn leading to a contraction in corporate credit. This is in contrast to the traditional view that the “bank lending channel” operates through reductions in bank reserves. My model does not distinguish between causes of contractions bank lending; its focus is purely and squarely on their consequences for firm-level and aggregate investment. De Fiore and Uhlig (2011) and De Fiore and Uhlig (2014) study an asymmetric information model of bond and bank borrowing, and show that it accounts well for long-run differences between the Euro Area and the US to the extent that information availability on corporate risk is lower in Europe. They also provide a model-based assessment of the changes in corporate debt composition in the US during the Great Recession, relying on a combination of different shocks, among which an increase in firm-level uncertainty and in the intermediation costs of banks. Aside from the differences in the role that banks play in our respective setups, the cross-sectional predictions of the models differ: firms in their model indeed specialize in either bank or market borrowing. As a result, amplification effects of financial shocks on borrowing and investment arising from debt substitution within firms are weaker in their framework.

2 A dynamic model of debt composition

This section describes a frictional model of firm investment, in which heterogeneous firms with limited internal finance must raise outside funding from financial intermediaries. The novelty of the model is that outside funding can take two forms: bank debt \(b_t\) or market debt \(m_t\). A firm’s stock of internal finance is denoted by \(e_t\); it represents total retained earning accumulated up to time \(t\). Time is discrete. Firms are infinitely-lived, but all debt contracts mature within one period.
In this economy, firm investment is constrained for two reasons. First, firms have limited liability. A firm can choose to default on its debt obligations, and this default may entail the liquidation of the firm and the transfer of its assets to creditors. Liquidation is inefficient: it involves deadweight losses. Second, firms cannot issue equity. Absent either friction, the firm would be able to finance investment to its optimal scale. With frictions to debt and equity issuance, the firm will attempt to accumulate retained earnings in order to fund investment internally, and limit its dependence on debt.\(^8\)

In order to clarify the exposition, sections 2.1-2.8 focus on the description of the model, while sections 2.9 is devoted to the discussion of key assumptions on bank flexibility, bank seniority, and intermediation costs, and their relationship to evidence on bank and market lending.

### 2.1 Overview of firms’ problem

Figure 1 summarizes the timing of each individual firms’ problem. There are three stages within each period: first, the choice of debt structure \((b_t, m_t)\); second, the settlement of debt contracts; third, the issuance of dividends.

At the beginning of the period, each firm is characterized by its internal finance \(e_t\). The present discounted value of future dividends for a firm with internal finance \(e_t\) is given by \(V(e_t)\); managers are risk-neutral and all discount dividends using the same discount factor \(\beta\). The firm operates a decreasing returns to scale technology that takes capital \(k_t\) as sole input. The firm’s total resources after production are given by:

\[
\pi_t = \pi(\phi_t, k_t) = \phi_t k_t^{\zeta} + (1 - \delta)k_t,
\]

where \(0 < \zeta < 1\) denotes the degree of returns to scale, \(0 < \delta < 1\) denotes the rate of depreciation of capital, and \(\phi_t \geq 0\) denotes the productivity level of the firm. \(\phi_t\) is drawn from a distribution \(F(\cdot)\) with mean \(E(\phi)\) and standard deviation \(\sigma(\phi)\).\(^9\) Total investment in capital is given by \(k_t = e_t + b_t + m_t\). Both the scale

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\(^8\)The assumption that firms cannot issue equity \textit{at all} is not necessary for the main results of the paper to hold. It is however necessary to assume that equity issuance is costly. This can be done by introducing a fixed marginal issuance cost of equity, which needs to be strictly larger than the marginal cost of lending of banks. However, the assumption of infinite equity issuance costs, effectively maintained here, allows the paper to focus on the relationship between debt composition and investment. Results on a version of the model with equity issuance are available from the author upon request.

\(^9\)In this section, productivity is \textit{i.i.d.} across firms and over time. Section discusses 4 deals with time variation in \(E(\phi)\) and \(\sigma(\phi)\). Additionally, to ensure uniqueness of lending contracts, \(F(\cdot)\) must have a strictly increasing hazard rate; see Crouzet.
of borrowing \( (b_t + m_t) \) and its composition \( \left( \frac{b_t}{b_t + m_t}, \frac{m_t}{b_t + m_t} \right) \) are chosen by the firm before the realization of productivity. Thus, the terms of the debt contracts, denoted by \( R_{b,t} \) (for bank debt) and \( R_{m,t} \) (for market debt), cannot be indexed by \( \phi_t \).\(^{10}\) The terms of the debt contracts represent the gross promised repayments to the intermediaries (principal plus interest), due after productivity has been realized.

Debt contracts are settled in the interim stage. The value of a firm entering this stage with resources \( \pi_t \) and liabilities \( (R_{b,t}, R_{m,t}) \) is given by \( V_s(\pi_t; R_{b,t}, R_{m,t}) \). Given the realization of its current productivity \( \phi_t \), the firm can choose to either fulfill its promise to repay \( (R_{b,t}, R_{m,t}) \) to its creditors, or attempt to renegotiate down these liabilities. If the realization of productivity is sufficiently bad, the firm may also be forced into liquidation. The details of the settlement process are spelled out below.

Firms that are not liquidated during debt settlement then allocate remaining funds \( n_t \) between dividends and retained earnings, which constitute next period’s internal finance \( e_{t+1} \). After this dividend issuance choice, only a fraction \( (1 - \eta) \) of firms survive; the rest are destroyed, along with their internal finance stock \( e_{t+1} \).\(^{11}\) The value of a firm with remaining funds \( n_t \) after settlement is denoted by \( V^c(n_t) \).

A recursive formulation of the firm’s problem can be obtained by solving the model backwards within each period, starting with the dividend issuance stage.

2.2 Dividend issuance

Given the value function \( V(\cdot) \), the dividend issuance problem of a firm after debt settlement is given by:

\[
V^c(n_t) = \max_{\text{div}_t, e_{t+1}} \text{div}_t + (1 - \eta)\beta V(e_{t+1})
\]

\[
\text{s.t. } \text{div}_t + e_{t+1} \leq n_t, \quad \text{div}_t \geq 0
\]

(1)

Here, \( \text{div}_t \) denotes dividends issued, and \( \eta \) denotes the probability of exogenous exit. The dividend policy of the firm is given by the following lemma.\(^{12}\)

**Lemma 1 (Dividend policy)** If \( V \) is continuous on \( \mathbb{R}_+ \) and satisfies \( V(0) \geq 0 \), then \( V^c \) is continuous and strictly increasing on \( \mathbb{R}_+ \), and satisfies \( V^c(0) \geq 0 \). If, moreover, (1) \( V \) is left and right-hand differentiable for all \( e > 0 \), and (2) there exists \( \bar{e} > 0 \) such that:

\[
\forall e \in [0, \bar{e}], \quad (1 - \eta)\beta V'_-(e) > 1 \quad \text{and} \quad \forall e \in [\bar{e}, +\infty[, \quad (1 - \eta)\beta V'_+(e) \leq 1
\]

(2)

\(^{10}\)They also depend on the amounts \( b_t \) and \( m_t \) borrowed by the firm, as well as its internal funds \( e_t \) and potentially its value function \( V(\cdot) \). In order to alleviate notation, I omit this in the exposition of the model.

\(^{11}\)The assumption of exogenous exit is not necessary to guarantee the existence of a solution to each firm’s individual problem, but it guarantees the existence of a stationary distribution of firms across levels of \( e_t \). See the proof of proposition 3 for details.

\(^{12}\)This lemma generalizes the results of Cooley and Quadrini (2001), by relaxing the assumption of concavity of the value function. The resulting dividend policy is however the same. The properties of the value function that guarantee the optimality of the dividend issuance policy discussed above will hold as a result of the existence of decreasing returns at the firm level. This will be formally established as part of the proof of proposition 3.
then, the firm’s optimal dividend policy is given by:

$$
div(n_t) = \begin{cases} 
0 & \text{if } 0 \leq n_t < \bar{e} \\
n_t - \bar{e} & \text{if } n_t \geq \bar{e}
\end{cases}
$$

If the value function $V$ were additionally assumed to be differentiable, $\bar{e}$ would satisfy: $(1 - \eta)\beta V'(\bar{e}) = 1$.

Intuitively, a firm with earnings $n_t$ below $\bar{e}$ finds that the marginal benefit of reinvested earnings is always above their shadow marginal cost. This shadow marginal cost is the marginal value of dividends, which is equal to 1. The firm therefore chooses the corner solution $e_{t+1} = n_t$, reinvests all earnings, and issues no dividends. On the other hand, a firm with earnings above $\bar{e}$ is able to equate the marginal value of reinvested earnings with their marginal cost, by setting $e_{t+1} = \bar{e}$. As a result, it issues $n_t - \bar{e}$ dividends.

The relevant state-space of the firm’s problem is therefore $[0, \bar{e}]$, since surviving firms will never enter the period with more than $\bar{e}$ of internal finance.\(^\dagger\)

### 2.3 Debt settlement

At the debt settlement stage, the firm has three options: liquidation; debt restructuring; or full payment of its liabilities. Let $V^L_t, V^R_t$ and $V^P_t$ denote the respective values of the firm under each option. The firm faces the discrete choice problem:

$$
V^s(\pi_t, R_{b,t}, R_{m,t}) = \max_{L,R,P} (V^L_t, V^R_t, V^P_t)
$$

(3)

I next describe in more detail each of these three options. I delay the discussion of key assumptions embodied in this description to the next paragraph.

**Liquidation** ($V^L_t$) In liquidation, the firm is shut down and its resources $\pi_t$ are passed onto creditors. I make the following assumption about liquidation:

**Assumption 1 (Liquidation losses)** The liquidation value of the firm given by:

$$
\tilde{\pi}_t = \chi \pi_t \quad , \quad 0 \leq \chi \leq 1.
$$

\(^\dagger\)The key constraint in this problem is that firms are not allowed to issue equity: $d\pi_t \geq 0$. This is a sufficient assumption to guarantee a non-degenerate solution to the problem. In general, following Cooley and Quadrini (2001), it is necessary that equity issuance be costly, that is, that the marginal cost of equity issuance be strictly larger than 1. Indeed, imagine that the firm were allowed to issue negative dividends at a marginal cost of 1. In that case, the firm can always achieve the interior optimum, by using the dividend policy $d\pi_t = n_t - \bar{e}$, even when $\bar{e} > n_t$. All firms would then operate under identical debt structures and scales, and would only differ in the interim period, after idiosyncratic productivity has been realized.
When $\chi < 1$, the transfer of the firm’s resources to creditors involves a deadweight loss. In that case, lenders charge the firm a liquidation premium. Absent this loss (when $\chi = 1$), if all participants are risk-neutral, lenders charge the risk-free rate, and firms can reach their ex-ante optimal size.

With multiple creditors, one must take a stance on how liquidation resources are allocated among stakeholders. I assume that a claim by a stakeholder to liquidation resources can only be repaid if all stakeholders placed higher in the priority structure have been fully repaid.\footnote{This is similar to the Absolute Priority Rule (APR) which governs chapter 7 corporate bankruptcies in the US.} In this model, there are three stakeholders: bank lenders; market lenders; and the firm itself. The firm is the last residual claimant. I furthermore assume that bank lenders are senior to market lenders in the priority structure.\footnote{See section \ref{sec:assumptions} for a discussion of this assumption, and results under alternative priority structures.} Payoffs to stakeholders are then given by:

$$
\begin{align*}
\tilde{R}_{b,t} & = \min (R_{b,t}, \chi \pi_t) \quad \text{(bank lenders)} \\
\tilde{R}_{m,t} & = \min (\max(0, \chi \pi_t - R_{b,t}), R_{m,t}) \quad \text{(market lenders)} \\
V_I^L & = \max (0, \chi \pi_t - R_{b,t} - R_{m,t}) \quad \text{(firm)}
\end{align*}
$$

Restructuring ($V_I^R$) In restructuring, firms attempt to renegotiate their liabilities, conditional on the realization of their productivity. The crucial distinction between banks and market lenders lies with their ability to carry out this restructuring.

**Assumption 2 (Debt flexibility)** Only bank debt can be restructured; market debt is not flexible.

I model the restructuring process as a two-stage game between the firm and the bank. This game is summarized in figure 2. The firm moves first, and offers to repay the bank an amount $l_t$, instead of the promised amount $R_{b,t}$. The bank can choose to accept or reject the offer. In case the offer is rejected, liquidation ensues, and all parties receive the liquidation payoffs described by (4). In this process, market lenders only have an indirect role, and their liabilities remain untouched in any successful restructuring agreement.
The optimal action of the bank is to accept the firm’s offer, if and only if it exceeds its reservation value, that is, if and only if \( l_t \geq \min(R_{b,t}, \chi \pi_t) \). The value of an offer \( l_t \) to the firm is therefore:

\[
\tilde{V}_R^R (l_t; \pi_t, R_{b,t}, R_{m,t}) = \begin{cases} 
V^C (\pi_t - l_t - R_{m,t}) & \text{if } l_t \geq \min(R_{b,t}, \chi \pi_t) \\
V^L & \text{if } l_t < \min(R_{b,t}, \chi \pi_t)
\end{cases}
\]

The firm chooses its restructuring offer, \( l_t \), in order to maximize this value, subject to the constraint that its net resources, after the restructuring offer, must be positive:

\[
V_t^R = \max_{l_t} \tilde{V}_R^R (l_t; \pi_t, R_{b,t}, R_{m,t}) \quad \text{s.t. } \pi_t - l_t - R_{m,t} \geq 0 \tag{5}
\]

**Payment (\( V_t^P \))** A firm can only fully pay its creditors if its resources exceed its total liabilities; otherwise, it is liquidated. Therefore,

\[
V_t^P = \begin{cases} 
V^C (\pi_t - R_{m,t} - R_{b,t}) & \text{if } \pi_t \geq R_{b,t} + R_{m,t} \\
V^L & \text{if } \pi_t < R_{b,t} + R_{m,t}
\end{cases} \tag{6}
\]

**Debt settlement outcomes** Given values of \( \pi_t, R_{m,t} \) and \( R_{b,t} \), a solution to problem (3), subject to (4), (5) and (6), is called a debt settlement outcome. The following proposition describes the equilibrium debt settlement outcomes.

**Proposition 1 (Debt settlement outcomes)**

- **When** \( \frac{R_{b,t}}{\chi} \geq \frac{R_{m,t}}{1-\chi} \), the firm chooses to repay its creditors in full if and only if \( \pi_t \geq \frac{R_{b,t}}{1-\chi} \). It successfully restructures its debt if and only if \( \frac{R_{m,t}}{1-\chi} \leq \pi_t < \frac{R_{b,t}}{1-\chi} \), and it is liquidated when \( \pi_t < \frac{R_{m,t}}{1-\chi} \).

- **When** \( \frac{R_{b,t}}{\chi} < \frac{R_{m,t}}{1-\chi} \), the firm repays its creditors in full if and only if \( \pi_t \geq R_{b,t} + R_{m,t} \), and it is liquidated otherwise.

Moreover, in all debt settlement outcomes resulting in restructuring, the bank obtains its reservation value \( \chi \pi_t \), and in all debt settlement outcomes resulting in liquidations, \( V_t^L = 0 \).

Figure 3 offers a graphical representation of the two situations. Two points are worth noting. First, the firm does not necessarily prefer restructuring to repayment. The surplus that the firm can extract from the bank in restructuring is the difference between its bank liabilities and the banks’ reservation value, \( R_{b,t} - \chi \pi_t \). The firm will only attempt a restructuring if this surplus is positive. Although the firm has bargaining power, since it moves first, the bank’s ability to force liquidation in restructuring renegotiations effectively limits the firm’s incentives to “abuse” restructuring.
Second, restructuring will not always save the firm from liquidation. This is because of the indirect role of market lenders in the restructuring process. When market liabilities are relatively large \( \frac{R_{b,t}}{1-\chi} < \frac{R_{m,t}}{1-\chi} \), even though the firm can extract a positive surplus from the bank in restructuring, that surplus is never sufficient to avoid liquidation. On the other hand, when market liabilities are relatively small \( \frac{R_{b,t}}{1-\chi} < \frac{R_{m,t}}{1-\chi} \), restructuring can be the best option for the firm. In some cases \( \frac{R_{m,t}}{1-\chi} \leq \pi_t \leq \frac{R_{b,t}}{1-\chi} \), it is because restructuring allows the firm to avoid liquidation. In others \( R_{b,t} + R_{m,t} \leq \pi_t \leq \frac{R_{b,t}}{1-\chi} \), the firm restructures for opportunistic reasons: it could pay in full both creditors, but instead decides to exert its bargaining power over the bank and reduce its bank liabilities.

Additionally, proposition 1 indicates that liquidation never involves a strictly positive payment to the firm \( V^L_t = 0 \). This result is intuitive. Imagine indeed that \( V^L_t > 0 \). This is possible only if both bank and market lenders have been repaid, so that necessarily \( \chi \pi_t - R_{b,t} - R_{m,t} > 0 \). But this implies \( \pi_t - R_{b,t} - R_{m,t} \geq \chi \pi_t - R_{b,t} - R_{m,t} > 0 \), so that the firm would be better off by simply paying its creditors.\(^{16}\)

### 2.4 Debt pricing and feasible debt structures

Banks and market lenders are perfectly competitive financial intermediaries, and have constant marginal lending costs \( r_b \) (for banks) and \( r_m \) (for markets); I come back below to the equilibrium determination of these lending costs. Perfect competition implies that lenders make zero expected profits on each loan.\(^{17}\)

Therefore, equilibrium lending terms \( R_{b,t} \) and \( R_{m,t} \) must satisfy:

\[
\begin{align*}
\int_{\phi_t \geq 0} \hat{R}_b(\phi_t, e_t + b_t + m_t, R_{b,t}, R_{m,t})dF(\phi_t) &= (1 + r_b)b_t \\
\int_{\phi_t \geq 0} \hat{R}_m(\phi_t, e_t + b_t + m_t, R_{b,t}, R_{m,t})dF(\phi_t) &= (1 + r_m)m_t
\end{align*}
\]

Here, \( \hat{R}_i(\phi_t, k_t, R_{b,t}, R_{m,t}), i = b, m \), is short-hand notation for the gross return on lending for each type

---

\(^{16}\)Note that the decision to liquidate, repay or restructure does not depend on the continuation value function \( V^c(\cdot) \). Instead, it takes the form of a simple threshold rule for \( \pi_t \). This is because the continuation value function is increasing. Because \( V^c(0) \geq 0 \), the liquidation decision also takes the form of a simple threshold rule.

\(^{17}\)In particular, perfect competition precludes financial intermediaries from imposing tougher lending terms on certain firms in order to subsidize lending to others.
of financial intermediary. This return depends on the realization of the idiosyncratic productivity shock $\phi_t$, as well as the firm’s debt structure, since both determine the firms’ resources $\pi_t$ and the associated debt settlement outcomes.\footnote{For example, when $\frac{R_{m,t}}{\chi} \leq \frac{R_{b,t}}{1-\chi}$, the gross lending return function for market lenders is given by:

$$\tilde{R}_m(\phi_t, k_t, R_{b,t}, R_{m,t}) = \begin{cases} 0 & \text{if } \pi(\phi_t, k_t) \leq \frac{R_{m,t}}{\chi} \\ R_{m,t} & \text{if } \pi(\phi_t, k_t) > \frac{R_{m,t}}{\chi} \end{cases}$$

The first case corresponds to realization of the productivity shock sufficiently low that the firm is forced into liquidation. The second case corresponds to realizations of the productivity shock such that the firm will either choose to pay its creditors in full, or will be able to successfully restructure debt payments with the bank.}

The lending menu $S(e_t)$ is defined as the set of all debt structures $(b_t, m_t) \in \mathbb{R}^2_+$ for which there exists a unique solution to (7). It is the set of feasible debt structures for a firm with internal finance $e_t$.

**Proposition 2** The lending menu $S(e_t)$ is non-empty and compact for all $e_t \geq 0$. Moreover, $S(e_t)$ can be partitioned into two non-empty and compact subsets $S_K(e_t)$ and $S_R(e_t)$, such that:

- The lending terms $(R_{b,t}, R_{m,t})$ satisfy $\frac{R_{b,t}}{1-\chi} \geq \frac{R_{m,t}}{\chi}$ if and only if $(b_t, m_t) \in S_R(e_t)$;
- The lending terms $(R_{b,t}, R_{m,t})$ satisfy $\frac{R_{b,t}}{1-\chi} < \frac{R_{m,t}}{\chi}$ if and only if $(b_t, m_t) \in S_K(e_t)$.

Proposition 2 provides a partition of the feasible set of debt structures according to the nature of the debt settlement outcomes that they lead to. In the first subset, $S_R(e_t)$, debt structures $(b_t, m_t)$ imply liabilities $(R_{b,t}, R_{m,t})$ such that restructuring is sometimes successful during debt settlement. On the other hand, in the second subset, $S_K(e_t)$, debt structures are such that restructuring never occurs during debt settlement.

The subsets $S_K(e_t)$ and $S_R(e_t)$ are depicted in figure 4. Their relative location conveys the same intuition that underpins proposition 1: ex-post, debt structures tend to lead to successful restructuring when they are tilted towards bank loans (i.e., towards the lower left part of the positive orthant in figure 4). On the other
hand, when the debt structure is tilted towards market debt (i.e., towards the upper part of the positive orthant in figure 4), firms never successfully restructure debt ex-post.\textsuperscript{19}

2.5 The firm’s dynamic debt structure problem

The firm’s problem can now be written recursively as:

\[
V(e_t) = \max_{(b_t,m_t) \in S(e_t)} \int_{\max(n^F_t,n^R_t) \geq 0} V^s(\pi(\phi_t,k_t),R_{b,t},R_{m,t}) dF(\phi_t) \tag{8}
\]

s.t.

\[
\begin{align*}
\pi(\phi_t,k_t) &= \phi_t k_t^2 + (1 - \delta)k_t & \text{(Capital structure)} \\
(1 + r_b)b_t &= \int_{\phi_t}^{\phi_t} R_b(\phi_t,k_t, R_{b,t}, R_{m,t}) dF(\phi_t) & \text{(Debt pricing, bank)} \\
(1 + r_m)m_t &= \int_{\phi_t}^{\phi_t} R_m(\phi_t,k_t, R_{b,t}, R_{m,t}) dF(\phi_t) & \text{(Debt pricing, market)} \\
V^s(\pi(\phi_t,k_t),R_{b,t},R_{m,t}) &= V^c(\max(n^F_t,n^R_t)) & \text{(Debt settlement)} \\
n^F_t &= \pi(\phi_t,k_t) - R_{b,t} - R_{m,t} & \text{(Repayment)} \\
n^R_t &= \pi(\phi_t,k_t) - \chi\pi(\phi_t,k_t) - R_{m,t} & \text{(Restructuring)} \\
V^c(n_t) &= \max_{0 \leq \eta \leq n_t} n_t - \epsilon_{t+1} + (1 - \eta)\beta V(e_{t+1}) & \text{(Dividend issuance)}
\end{align*}
\]

This formulation incorporates results from the three stages of the firm’s problem discussed in the previous paragraphs. First, the debt structure chosen by the firm at the beginning of the period must be feasible: \((b_t,m_t) \in S(e_t)\). Second, the expression for the value of the firm at the debt settlement stage, \(V^s(\pi(\phi_t,k_t),R_{b,t},R_{m,t})\), uses the results of proposition 1.\textsuperscript{20} Finally, the value of the firm at the dividend issuance stage is the sum of the current value of dividends issued and the discounted value of future dividends. A solution to this problem is characterized by a threshold for internal finance \(\bar{\epsilon}\) as well as a value function \(V\) and policy functions \(b\) and \(m\), all defined on \([0,\bar{\epsilon}]\).

2.6 Entry and exit

The are two sources of firm exit in this economy. First, some firms are endogenously liquidated at the debt settlement stage. Given that \(\phi_t\) is independent of \(e_t\), the fraction of existing firms with internal finance \(e_t\) that are liquidated is given by \(F\left(\phi\left(e_t, b(e_t), m(e_t)\right)\right)\). Here, \(\phi(e_t, b_t, m_t)\) denotes the threshold such that firms with a productivity realization \(\phi_t \leq \phi(e_t, b_t, m_t)\) are liquidated.\textsuperscript{21} Second, a fraction \(\eta\) of firms exogenously exit after the dividend issuance stage.

\textsuperscript{19}Specifically, debt structures in \(S_{b}(e_t)\) have a ratio of bank to total debt strictly above a threshold \(s^{R,max}\), where \(\frac{1 - s^{R,max}}{s^{R,min}}\) is the slope of the upper straight line in figure 4. On the other hand, debt structures in \(S_{K}(e_t)\) have a ratio of bank to total debt strictly below a threshold \(s^{K,max}\), where \(\frac{1 - s^{K,max}}{s^{K,min}}\) is the slope of the lower straight line in figure 4.

\textsuperscript{20}In particular, the firm is liquidated if and only if its resources under both restructuring and payment are strictly negative, so that the integrand in the objective function of the firm is truncated at 0.

\textsuperscript{21}A complete characterization of the restructuring and liquidation thresholds is given in appendix B.
Let $\mu_t$ denote the distribution of firms over $[0, \bar{e}]$ at the beginning of period $t$. The total mass of firms exiting during period $t$ is given by:

$$\delta^e(\mu_t) = \int_{e \in [0, \bar{e}]} \left( F\left( \phi\left( e_t, \hat{b}(e_t), \hat{m}(e_t) \right) \right) + \left( 1 - F\left( \phi\left( e_t, \hat{b}(e_t), \hat{m}(e_t) \right) \right) \right) \eta \right) d\mu_t(e_t).$$

The fraction $\delta^e(\mu_t)$ of exiting firms is replaced by an identical number of entering firms at the beginning of the following period. Entry involves two costs: the internal funds $e^e_t$ of entering firms; and a fixed entry cost $\kappa$. The surplus associated with entering with internal finance $e^e_t$ is given by $\beta V(e^e_t) - (\kappa + e^e_t)$. There is free entry, so that the surplus associated with entering is 0 and $e^e_t$ must solve:

$$\beta V(e^e_t) = \kappa + e^e_t.$$  \hfill (9)

Given the entry scale, the mass of exiting firms, and the firm's optimal policy functions along with their dividend issuance policies, the law of motion for the distribution of firms across levels of internal finance can be expressed as: $\mu_{t+1} = M(\mu_t)$ where $M : \mathcal{M}(\bar{e}) \to \mathcal{M}(\bar{e})$ is a transition mapping over firm measures, and $\mathcal{M}(\bar{e})$ denotes the set of measures on $[0, \bar{e}]$ that are absolutely continuous with respect to the Lebesgue measure.\textsuperscript{22}

### 2.7 Financial intermediation

Intermediaries raise funds in order to extend credit to firms. I assume that they face an identical opportunity cost of funds, but different intermediation costs.

**Assumption 3 (Financial intermediation costs)** Banks and market lenders have a common opportunity cost of funds, given by $r = \frac{1}{\beta} - 1$. Their intermediation costs per unit of credit extended are given $\gamma_b$ and $\gamma_m$. The wedge between bank and market-specific intermediation costs is strictly positive: $\gamma_b - \gamma_m > 0$.

Equilibrium financial intermediation costs for banks and markets are thus given by:

$$r_m = r + \gamma_m, \quad r_b = r + \gamma_b, \quad r = \frac{1}{\beta} - 1.$$  \hfill (10)

The restriction that $r = \frac{1}{\beta} - 1$ can be thought of as a general equilibrium outcome. Indeed, it would hold in a model in which intermediaries raise deposits from a representative, risk-neutral household. In such a model, perfect competition in the market for deposits would impose that $\beta(1 + r) = 1.\textsuperscript{23}$ Alternatively,

\textsuperscript{22}The detailed expression of the mapping $M$ is reported in appendix B.

\textsuperscript{23}The online appendix to the paper spells out a general equilibrium version of this model with risk-neutral households. This model is notationally more burdensome, but leads to identical equilibrium outcomes.
the restriction $\beta (1 + r) = 1$ would hold if both financial intermediaries and firms had access to a risk-free technology offering a rate of return $r$. I discuss assumption 3 in more detail in section 2.9.

2.8 Equilibrium

Definition 1 (Recursive competitive equilibrium) A recursive competitive equilibrium of this economy is given by value functions $V$, $V^s$ and $V^c$, an upper bound on internal finance $\bar{e}$, policy functions $\hat{b}$, $\hat{m}$, equilibrium lending costs $r_b$ and $r_m$, equilibrium lending terms $R_b$ and $R_m$, an entry size $e^e$, a distribution of firm size $\mu$ and a transition mapping $M$, such that:

- given $\bar{e}$, the value functions solve problem (8), and $\hat{b}$, $\hat{m}$, are the associated policies;
- given the value function $V$, the upper bound $\bar{e}$ satisfies condition (2);
- equilibrium lending costs satisfy (10);
- equilibrium lending terms satisfy the zero profit conditions of intermediaries (7);
- the entry scale $e^e$ satisfies the free-entry condition (9);
- the transition mapping $M$ is consistent with firms’ policies and with the entry scale of firms $e^e$;
- the distribution $\mu$ is a fixed point of $M$.

Proposition 3 (Existence of a recursive competitive equilibrium) There exists a recursive competitive equilibrium of this economy.

The proof of proposition 3 is given in appendix B. It uses two key insights mentioned in the exposition of the model: (1) the structure of the feasible set of debt contracts for a firm only depends on its internal finance $e_t$; (2) the feasible set can be partitioned into two subsets, $S_R(e_t)$ and $S_K(e_t)$, associated with the two types of debt settlement outcomes. Both insights follow from proposition 2.

The first step of the proof is to establish the existence of a unique solution to problem (8). In general, this is a triple fixed point problem, where the value function $V(\cdot)$, the upper bound $\bar{e}$, and the constraint correspondence $S(\cdot) : [0, \bar{e}] \to \mathbb{R}_+^2$ must be simultaneously determined. However, given the first insight, the problem reduces to a double fixed point problem in $\bar{e}$ and $V(\cdot)$, analogous to Cooley and Quadrini (2001). However, unlike that framework, the resulting value function $V$ need not be globally concave, so that standard approaches do not apply. This is because, as suggested by the second insight, the value function $V$ is the upper envelope of two functions: $V(e_t) = \max_{K,R} (V_K(e_t), V_R(e_t))$, where $V_K(e_t)$ denotes the continuation value of a firm restricted to use debt structures that are in $S_K(e_t)$, and $V_R(e_t)$ is analogously defined. $V$ does not inherit the concavity of $V_K$ and $V_L$, and instead has a downward kink where they intersect. However, using the generalized envelope theorems of Milgrom and Segal (2002), one can establish the right and left-hand differentiability of $V$ required by lemma 1, and thus the existence of the solution to (8).
The second step of the proof of proposition 3 is to derive the expression of the transition mapping $M$, and show that it has a fixed point. This part of the proof also uses the partition of the feasible set of debt structures between $S_R(\epsilon_t)$ and $S_K(\epsilon_t)$, because transition probabilities between levels of internal finance depend on the type of debt settlement outcomes chosen by the firm. Given the expression constructed for $M$, standard approaches can be used to prove that $M$ has a fixed point.\footnote{Only existence is established. The sufficient uniqueness conditions of monotonicity of the transition kernel developed in Stokey, Lucas, and Prescott (1989), or the weaker conditions of Hopenhayn and Prescott (1987), do not hold for every point of the firms’ state-space, because of the non-monotonocities in firms’ investment policies as they transition across financial regimes, as described below. However, the issue of uniqueness never arises numerically.}

2.9 Discussion

I now come back to the discussing key assumptions about liquidation costs, seniority, the flexibility of banks relative to market lenders, and intermediation costs.

**Liquidation and seniority** The first key assumption about liquidation (assumption 1) is that it involves losses of output: $\chi < 1$. This assumption is common to many models in which the underlying financial friction is limited liability. It embodies the notion that bankruptcy and liquidation are costly processes, and is supported by evidence on changes in asset values of firms that go through bankruptcy proceedings (see, for example, Bris, Welch, and Zhu (2006)).

The second key assumption is the seniority of bank lenders in liquidation. This assumption is motivated by two considerations. First, empirically, bank loans tend to be either senior or secured by liens on assets. Giving banks the first claim in liquidation in the model captures this empirical regularity. Second, in the model, given the choice, firms may prefer to put banks at the top of the priority structure. There are two opposing forces that affect how desirable firms find bank seniority in this model, both having to do with how much firms value the flexibility offered by banks. On the one hand, seniority increases the reservation value of the bank during restructuring negotiations. This means that it is more difficult for firms to obtain large concessions from banks, which implies that bank debt is effectively less flexible. This tends to make seniority less attractive for firms. On the other hand, if liquidation actually occurs, banks’ payoff is larger if they are senior. The debt pricing condition (7) then implies that the “liquidation risk premium” charged by banks is lower. It is therefore cheaper and easier for firms to issue bank debt when it is senior. This makes seniority more attractive to firms. In a static version of this model, one can establish formally that the former effect (seniority reduces the concessions that can be obtained from banks in restructuring) is dominated by the latter effect (bank seniority reduces the “liquidation risk premium” charged by banks), provided that output losses in liquidation are not too large.\footnote{See Crouzet (2013) for the analysis of optimal seniority in a static version of this model.} While such a result cannot be established for the dynamic version of the model studied in this paper, it suggests that bank seniority can improve firms’ overall ability to issue...
Bank debt and use its flexibility.\footnote{In a closely related setup, Hackbart, Hennessy, and Leland (2007) establish that bank debt seniority is the optimal priority structure. The optimality of bank seniority is also a feature of other models of debt structure, in which banks’ role is to provide ex-ante monitoring of projects, such as for example Besanko and Kanatas (1993), Park (2000), or DeMarzo and Fishman (2007). The rationale for bank seniority, in these models, is that it increases banks’ return on monitoring, by allowing them to seize more output in liquidation. This is distinct from, but related to the model I consider here, where seniority allows firms to operate at a larger scale early on.}

**Bank flexibility** The assumption that banks are more flexible in distress than markets (assumption 2) receives considerable support in the data. Gilson, Kose, and Lang (1990) show that, in a sample of 169 financially distressed firms, the single best predictor of restructuring success is the existence of bank loans in the firm’s debt structure. Denis and Mihov (2003), in a sample of 1560 new debt financings by 1480 public companies, show that bank debt issuances have more flexibility in the timing of borrowing and payment, and that firms with higher revenue volatility tend to issue more bank debt. Bolton and Scharfstein (1996) provide a theoretical rationale for bank flexibility, by noting that ownership of market debt tends to be more dispersed than ownership of bank debt. This creates a free-rider problem, as market creditors have little individual incentive to participate in debt renegotiations. To the extent that they have an informational advantage over other creditors, banks may also be more flexible in restructuring negotiations because they are more precisely aware of the going concern value of the firm, as in Rajan (1992). The precise cause of banks’ greater flexibility in distress is not explicitly modelled in this paper; rather, the focus is on the implications of this difference in flexibility for firms’ investment choices.

**Intermediation costs** The motivations underlying assumption 3 are the following. First, the fact that financial intermediation is costly ($\gamma_b, \gamma_m > 0$) is not controversial: Philippon (2012) provides recent and comprehensive evidence that overall intermediation costs in the US financial sector have averaged approximately 2% between 1870 and 2012. The assumption specific to this model is that these intermediation costs are larger for banks than for markets. This assumption captures three key differences between the costs associated with bank and market lending:

1. Bank lenders place more stringent requirements on borrowers outside of financial distress than markets, in particular, tighter loan covenants, as documented by Demiroglu and James (2010) and Rauh and Sufi (2010). The positive lending wedge is a reduced-form way of capturing tighter bank lending requirements: indeed, in the model, the wedge will be reflected in higher equilibrium lending terms for banks loans ($R_{b,t}$) outside of financial distress.

2. Banks specialize in costly activities related to lending and which markets typically shun. In particular, banks engage in screening and monitoring of borrowers, as documented in, for example, Berger and
Udell (1995), Houston and James (1996) or Mester, Nakamura, and Renault (2007). The positive lending wedge then captures costs associated with these bank-specific activities.

3. Banks face specific regulatory environments that have an impact on their lending costs. In particular, capital requirements force banks to issue additional equity in order to expand their deposit and lending base. However, banks typically finds it costly to adjust their equity base (see Adrian and Shin (2011) for evidence on this topic). This mechanism contributes to making marginal loan issuance more costly for banks.

3 Financial policies in steady-state

I now turn to a description of the equilibrium financial policies of firms. The model has no closed form solution. Instead, this section focuses on the properties of a numerical solution, using a baseline calibration of the model to the US before the Great Recession. This calibration is which summarized in table 1.27

3.1 Baseline calibration

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<th>Description</th>
<th>Value</th>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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<td>ζ</td>
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<td>E(φ)</td>
<td>average productivity</td>
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<td>δ</td>
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<td>γ_m</td>
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<td>bank intermediation costs</td>
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</table>

Table 1: Baseline calibration of the model.

The frequency of the model is annual. Standard model parameters (the discount rate, the rate of depreciation of capital and the degree of returns to scale) are calibrated using existing estimates from the literature.

Idiosyncratic productivity shocks follow a Weibull distribution.28 I pick the location and scale parameters to match two targets: (1) \( \bar{k} = \left( \frac{E(\phi) \zeta}{\delta + \gamma + \gamma_m} \right) \) = 100 and (2) \( \sigma(\text{log}(\phi)) = 0.62 \). The former is a normalization that ensures that firms’ total size is smaller than or equal to \( \bar{k} \). The latter value corresponds to estimates of the cross-sectional standard deviation of output-based measures of firm-level (log) productivity reported by Bartelsman, Haltiwanger, and Scarpetta (2009) for the US manufacturing sector.29 The choice of the fixed entry cost \( \kappa \) implies that the size of entrants is 5% of the maximum size of firms in the economy.

27The numerical solution procedure is standard and is described in the online appendix to the paper.
28The Weibull distribution has a strictly increasing hazard rate, which is sufficient to ensure the uniqueness of the lending terms that solve the zero-profit conditions (7).
29See their online data appendix at http://econweb.umd.edu/~haltiwan/BHS_jobflows_productivity/. I use their estimates for the US manufacturing sector for consistency with the data on debt composition used below. This value is line with other estimates of output-based productivity dispersion for the US, such as for example Hsieh and Klenow (2009).
In order to calibrate the magnitude of deadweight losses in liquidation, I use evidence reported by Bris, Welch, and Zhu (2006). They analyze a sample of 61 chapter 7 liquidations in Arizona and New York between 1995 and 2001. Their median estimate of the change in asset values pre- to post- chapter 7 liquidation is 38%. I therefore use $\chi = 0.38$.

As a proxy for market-specific lending costs $\gamma_m$, I use existing estimates of underwriting fees for corporate bond issuances. Fang (2005) studies a sample of bond issuances in the US, and finds an average underwriting fee of 0.95%, while Altinkilic and Hansen (2000), in a sample including lower-quality issuances, find an average underwriting fee of 1.09%. Given this evidence, I set market-specific intermediation costs to $\gamma_m = 0.0100$.

Finally, given all the other parameters, I choose the intermediation costs of banks, $\gamma_b$, in order to match the aggregate bank share of medium and large US manufacturing firms in 2007Q3, as reported in the Quarterly Financial Report of manufacturing firms (QFR). Given the model’s focus on firms with access to public debt markets, I only use data on firms with more than $250m in assets. At that date, outstanding bank debt of these firms accounted for 24.0% of their total outstanding debt. Given other parameters, the model generates a bank share of 24.0% for bank-specific intermediation costs of $\gamma_b = 0.02655$, or 266 basis point above the risk-free rate in the model, which is set to 4%. This implies a bank lending rate of 6.63% for the very safest firms, which is in the same order of magnitude as the real monthly Bank Prime Loan Rate during 2007Q3 (6.19%).

Using the QFR data to calibrate the model has two advantages. First, the QFR provides an accurate measure of the bank share: it is one of the few balance sheet datasets in which firms are asked to report separately bank debt from other debt outstanding. Second, the QFR has a large coverage over time and in the cross-section: it covers the universe of all medium and large manufacturing firms at a quarterly frequency. This will be useful in section 4, when I compare the business-cycle implications of the model with the data. However, two limitations of the QFR could introduce bias in the measurement of the bank share: the restriction to manufacturing firms; and the inability to explicitly separate firms with access to public debt markets from others. In Appendix A, however, I show that the QFR bank share is similar to what can be obtained using two alternative sources of evidence: the work of Rauh and Sufi (2010) and Colla et al. (2013). The small differences seems to be mainly driven by composition: QFR firms are smaller, on
3.2 Optimal debt structure

The key properties of the firm’s optimal debt structure are reported in figures 5(a) and 5(b), and summarized in the following result. This result has no analytical proof, but holds for all numerical calibrations under which I have solved the model, provided that $\theta = \gamma_b - \gamma_m > 0$.

**Numerical Result 1 (The firm’s optimal debt structure)**  Let $\hat{b}(e_t)$ and $\hat{m}(e_t)$ denote the policy func-
tions associated with the solution to problem (8). There exists a unique value of internal finance $e^* \in [0, \bar{e}]$ such that:

- For $e_t \in [0, e^*]$, $\hat{b}(e_t) > 0$, $\hat{m}(e_t) > 0$, and $\left(\hat{b}(e_t), \hat{m}(e_t)\right) \in S_R(e_t)$;
- For $e_t \in [e^*, \bar{e}]$, $\hat{b}(e_t) = 0$, $\hat{m}(e_t) \geq 0$, and $\left(\hat{b}(e_t), \hat{m}(e_t)\right) \in S_K(e_t)$.

This result, illustrated in figure 5(a), indicates that firms’ debt structures fall under two broad categories. Firms with internal funds below $e^*$ will choose a “mixed” debt structure, involving a combination of bank and market debt. On the other hand, large firms – those firms with internal funds strictly above $e^*$ – choose a “market-only” debt structure. As firms grow by accumulating internal funds from retained earnings, they will therefore switch from a mixed debt structure to a market-only debt structure.

The intuition for this result is as follows. Because of decreasing returns, firms with small $e_t$ have a larger financing gap, and tend to be more leveraged than firms with large $e_t$. This also implies that they have a higher probability of financial distress. Since, all other things equal, borrowing more from banks reduces the expected losses associated with financial distress, firms with small $e_t$, seeking high leverage, have a strong incentive to use bank debt. One should therefore expect the composition of their debt to be more tilted towards bank loans. More generally, the trade-off between bank flexibility in times of financial distress, and the costs associated with using bank loans in normal times, changes with the level of the firm’s internal resources $e_t$, and ultimately affects the firms’ choice of debt structure.

This logic does not account for the fact that firms switch in a discrete manner to market finance when $e_t \geq e^*$. To understand this, it is useful to think back to the results of proposition 1. This proposition indicates that, if a firm’s market liabilities are excessively large, it never manages to restructure bank debt in bad times. In that case, the flexibility associated with bank debt is irrelevant to the firm since, in equilibrium, the firm will never be able to actually use that flexibility to avoid liquidation. But recall that, because the lending wedge $\theta = \gamma_b - \gamma_m$ is strictly positive, borrowing from banks results in large liabilities $R_{b,t}$ outside of financial distress; that is, outside of financial distress, bank lending is a costly source of debt. For firms which are unable to use the flexibility of bank debt to avoid liquidation, the net benefit of substituting a unit of bank debt for a unit of market debt is therefore always strictly positive. As a result, these firms choose the corner solution $\hat{b}(e_t) = 0$, $\hat{m}(e_t) > 0$.

Apart from the decline of the bank share with internal finance, the second key aspect of firms’ optimal debt choices is that they imply non-monotonicities in the borrowing and investment policies of firms. This is illustrated in figure 5(b), which reports bank borrowing $\hat{b}(e_t)$ (left panel), market borrowing $\hat{m}(e_t)$ (middle panel), and total assets $\hat{k}(e_t) = e_t + \hat{b}(e_t) + \hat{m}(e_t)$ (right panel), as a function of internal funds $e_t$. In each
of the two regions $e_t \leq e^*$ and $e_t > e^*$, the amounts borrowed from banks and markets are increasing (or equal to 0 for bank borrowing when $e_t > e^*$). However, when a firm crosses the threshold $e^*$, total assets and total borrowing fall: bank debt is not replaced one for one by market debt.

This effect can be interpreted as a precautionary response of firms that migrate from a mixed to a market debt structure. At the point $e_t = e^*$, firms are exactly indifferent between mixed and market debt structures. Imagine that firms were to choose the same overall leverage under the market debt structure than under the mixed debt structure. Since, under the market debt structure, firms have no option to restructure debt in bad times, their ex-ante likelihood of liquidation would be much higher than under the mixed debt structure. Switching firms offset this higher “financial fragility” by operating under a lower leverage. This feature of the optimal debt structure is one example of the real implications of imperfect substituability between types of debt.

### 3.3 Debt structure implications and cross-sectional evidence

Figure 6 reports, in the left panel, the steady-state distribution of firms across levels of internal finance. The key property of this distribution is that it is strongly skewed to the left. This occurs both because the entry size of firms is relatively small, and because small firms are much more frequently liquidated than large firms. Because of this left-skewness, the share of bank loans as a fraction of total borrowing is high for most firms, and only declines for the largest firms. This is reported in the right panel of figure 6. Each point in this graph corresponds to the median bank share of firms within a particular decile of the distribution of internal finance. As a result of the left-skewness of the distribution, in the baseline US calibration, 67.2% of firms use a mixed debt structure; only the top 32.8% of firms are purely market-financed.

Are these predictions about variation in debt structure consistent with the data? Rauh and Sufi (2010) provide evidence on the composition of debt in a sample of publicly traded and rated US firms between
1996 and 2006, combining data from firms’ 10-K filings with data on issue origination. They have two key findings. First, a majority of firms simultaneously have outstanding issues of different types. 68.3% of firm-year observations use at least two types of debt; among firms using bank debt (52.6% of the total), 70.5% also use straight or convertible bond debt. Second, the degree to which the debt structure of firms is “spread out” across types is strongly related to firms’ credit ratings. Investment-grade firms (with ratings of BBB and above) mostly use senior unsecured debt (bond and program debt), while speculative grade firms (with ratings of BB and below) use a combination of secured debt, senior unsecured debt, and subordinated bonds. Moreover, their results indicate that “the increase in secured debt as credit quality deteriorates is driven almost exclusively by an increase in secured bank debt, and the increase in subordinated debt is driven almost exclusively by an increase in subordinated bonds and convertibles”.

The cross-sectional variation of debt structure in the model is strongly consistent with these two facts. First, in the model, a majority of firms use a mixed debt structure, combining bank with market debt – the first of the two key findings of Rauh and Sufi (2010). In fact, the fraction of firms using a mixed debt structure in the model is surprisingly close to the data (68.3% vs. 67.2%), despite the fact that this moment is not a target of the calibration. This suggests that the mechanisms at work in the model can capture both the intensive and extensive margin components of debt choices. The intuition behind the fact that most firms, in the model, use both bank and market debt, instead of specializing, is that by increasing market debt issuance, firms increase the scale of total investment, and therefore liquidation value. This in turn loosens firms’ borrowing constraint with respect to banks, who are senior in the priority structure, and allows firms to increase bank debt issuance. In this manner, the endogenous investment scale decision creates a complementarity between types of debt, which accounts for the existence of an interior solution, consistent with the data.

Second, the model is qualitatively consistent with the empirical finding that as credit quality improves, firms move away from mixed debt structures, and use mostly bond and program debt. Indeed, in the model, the key driver of the shift from mixed to market debt structure is that, as firms’ internal finance increases and their leverage falls, the probability that they will face financial distress ex-post declines, making the use of bank debt less attractive. Firms with market debt structures, in the model, have generically lower ex-ante liquidation probabilities than firms with mixed debt structures. Thus, to the extent that credit ratings proxy

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38 Table 2, p. 11.
39 Program debt encompasses debt issuances which do not require an SEC filing, such as commercial paper, medium-term notes and shelf-registered debt.
40 As emphasized by Rauh and Sufi (2010), many existing models of the choice between bank and non-bank debt, in which investment scale is not endogenous, instead have the prediction that firms issue a single type of debt; this is the case, for example, in Diamond (1991), Chemmanur and Fulghieri (1994), Cantillo and Wright (2000) or Bolton and Freixas (2000). Among models addressing simultaneously the choice of priority and type of debt in an optimal contracting framework, Besanko and Kanatas (1993) and Park (2000) both find interior solutions. However, in these models, the generic intuition for the result is different from the model studied here: combining a senior, monitoring creditor (a bank) and junior creditors reduces the bank’s stake in the firm, making any liquidation threats by the bank more credible. By allowing other creditors in the debt structure, banks are therefore better able to deter sub-optimal effort or risk-shifting behavior by the firm.
for a firm’s credit or liquidation risk, the model captures well the empirical correlation between credit ratings and the concentration of debt structure.

Colla, Ippolito, and Li (2013) provide additional evidence on the debt structure of both rated and unrated US firms, using information on debt structure gathered from 10-K filings. Their findings support the evidence of Rauh and Sufi (2010) for rated firms. Specifically, they find that 36.6% of firm-year observations of rated firms concentrate 90% or more of their debt within a single type (mostly senior bonds and notes); the remaining 63.4% are diversified across debt types.\footnote{See their Table IV.} They also find that firms that move from investment to speculative grade tend to increase their reliance on term loans and junior bonds and notes, instead of using exclusively on senior bonds and notes: for example, drawn credit lines and term loans together account for over 30% of total debt for firms with a credit rating of BB and below, whereas they account for less than 6% of total debt for firms with a credit rating of AA and above.\footnote{Colla, Ippolito, and Li (2013) also document the fact that unrated firms seem to be more likely to specialize in particular forms of debt issuances. They show 51.3% of unrated firms specialize in a single type of debt; of these specialized firms, 27.2% of them use only credit lines or term loans. Debt specialization does arise in the model, but only for the very largest firms. Potential drivers of debt specialization among unrated firms are outside of the scope of this model; see section III of Colla et al. (2013) for a detailed discussion.}

Thus, the model’s cross-sectional predictions about debt choices of firms with access to public debt markets align very closely with evidence on US firms with credit ratings.

### 3.4 Other aspect of firms’ policies

Does the model’s ability to account for cross-sectional variation in debt structure come at the expense of other predictions? Figure 7 addresses this question. With respect to financial policies, the model predicts that (1) firms with low internal resources are generally more leveraged (bottom row, right panel), as previously mentioned; and that (2) firms with low internal resources have a higher rate of profit, but distribute less dividends (top row, middle panel). These features of firms’ financial policies are broadly similar to the results of Cooley and Quadrini (2001), and consistent with empirical facts on financial behavior of firms documented in, e.g., Fazzari, Hubbard, and Petersen (1988) and Gilchrist and Himmelberg (1998). With respect to firm dynamics, the key predictions of the model can be summarized as follows: (1) smaller firms experience a higher rate of growth, either measured in terms of internal funds, output, or total assets (top row, right graph; bottom row, left and middle graphs); (2) smaller firms have a higher volatility of growth (bottom row, right graph).\footnote{The online appendix to the paper gives the exact definition of these variables in terms of the optimal policy functions of firms.} These facts also align well with empirical evidence on the relationship between size and growth dynamics.\footnote{See for example Evans (1987), or more recently Davis, Haltiwanger, Jarmin, and Miranda (2006).} Note that, as was the case for financial policies, the switch between financial regimes is associated with changes in the growth dynamics of firms. Namely, firms immediately to the left of
the switching threshold experience negative expected growth rates, as they anticipate the fact that reaching the switching threshold will imply a reduction in the scale of their operations. The volatility of firms that switch to market-only debt also increases, in line with the intuition that the debt structure adopted by these firms increases their exposure to liquidation risk.

This section has established that the model’s predictions are consistent with firm-level evidence on patterns of debt choices and growth dynamics. I now turn to its implications for aggregate activity.

4 Debt choices and the transmission of aggregate shocks

This section focuses on the business cycle implications of debt heterogeneity. The discussion centers mostly on shocks to the lending wedge $\theta = \gamma_b - \gamma_m$. From the standpoint of the model, such shocks offer the best account of observed changes in debt structure in the US during the Great Recession.

The central finding of this section is that debt choices introduce a new channel of propagation of aggregate shocks, which operates through substitution between bank and market finance, and affects both the scale and composition of aggregate borrowing. In response to an increase in the relative cost of intermediation
of banks, this channel is quantitatively important: it accounts for approximately one third of the decline in investment in the numerical example analyzed in this section.

4.1 Debt structure during the Great Recession

In the US, the 2007-2009 recession was accompanied by large and persistent changes in debt composition. Figure 8 illustrates these changes. First, the aggregate bank share fell substantially. For the universe of non-financial firms, the bank share fell from 31.1% in 2007Q3 to 24.3% in 2011Q3 (left panel, right axis). A similar pattern can be documented for firms with more than $250m in assets in the QFR: their bank share fell from 24.0% in 2007Q3 to 19.8% in 2011Q3 (left panel, left axis). Second, in the cross-section of firms, changes in debt composition differed substantially: debt substitution was salient only for the largest firms. The middle and right panels of figure 8 show the changes, relative to 2008Q3, in bank and non-bank liabilities of firms with assets between $250m and $1bn, and firms with assets above $1bn dollars, respectively. In the former group, both bank and non-bank liabilities fell, with bank debt showing the largest drop relative to peak: debt substitution was limited. In the latter group, on the other hand, the decline in bank liabilities was accompanied by an increase in non-bank liabilities. Thus, the decline in the aggregate bank share reflected substantially different patterns of debt choices across firms.

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45 This graph uses data from the Quarterly Financial Report of manufacturing firms (QFR). Appendix A discusses this data and the construction of the time series of figure 8 in more detail.

46 These patterns echo other evidence on the 2007-2009 recession, most notably, Adrian, Colla, and Shin (2012). They are also consistent with evidence from other periods or other countries on the effects of shocks to the supply of bank credit on debt composition, such as Becker and Ivashina (2014) and Baumann, Hoggarth, and Pain (2003).
4.2 The effects of a shock to the lending wedge

In the model, the decline in the aggregate bank share along with the patterns of debt choices across firms emerge naturally in response to a shock to the lending wedge $\theta = \gamma_b - \gamma_m$. To illustrate this, I compute the perfect foresight response of the model to an exogenous increase in the lending wedge, driven by an increase in bank lending costs $\gamma_{b,t}$. The economy starts from the steady-state described in section 2. I choose the path of $\gamma_{b,t}$ to match, quantitatively, the fall in the bank share of firms with more than $250m$ in assets documented in figure 8. The path of the bank share, and the implied path of the lending wedge, are reported on the left column of figure 9. The model requires a long-run increase in $\gamma_b$ of 44bps in order to match the observed decline in the bank share.

Figure 9 indicates that the increase in $\gamma_b$ is associated with a 3.5% fall in output and a 25.2% fall in investment over the first three years, in line with the peak to trough drop in output and private investment observed during the 2007-2009 recession. The response of output displays endogenous persistence, since output continues declining even after $\theta$ has reached its new long-run value. At that point, investment
instead starts recovering, having overshot relative to its long-run value.\footnote{Aggregate investment is defined as: }\footnote{Aggregate investment is defined as: }I_t = K_t - (1 - \delta)K_{t-1}, with: K_t = \int_{e \in [0,\bar{e}_t]} (e_t + \hat{b}(e_t) + \hat{m}(e_t)) d\mu(e_t).

The effect of the shock on borrowing in the cross-section is also consistent with the evidence on the response of debt composition of medium and large firms documented in the middle and right panels of figure 8. In order to compare the model’s predictions with this evidence, I compute the change in bank and market liabilities of firms of two size groups in the model, using a threshold for internal finance to define the two groups.\footnote{Specifically, I construct the size groups as follows. I first determine a cutoff for internal finance, \( e_{S/L} \), such that, in year 0, firms with \( e_0 > e_{S/L} \) account for a fraction \( S/L \) of the total stock of internal finance in the economy. For \( t \geq 1 \), I define borrowing by small and large firms, in bank and market debt, as:}

\begin{align*}
BS_t &= \int_{0 \leq e_t \leq e_{S/L}} \hat{b}(e_t) d\mu_t(e_t) , \\
MS_t &= \int_{0 \leq e_t \leq e_{S/L}} \hat{m}(e_t) d\mu_t(e_t) , \\
BL_t &= \int_{e_{S/L} \leq e_t \leq \bar{e}_t} \hat{b}(e_t) d\mu_t(e_t) , \\
ML_t &= \int_{e_{S/L} \leq e_t \leq \bar{e}_t} \hat{m}(e_t) d\mu_t(e_t) ,
\end{align*}

where, along the perfect foresight path, \( \hat{b}(\cdot), \hat{m}(\cdot) \) and \( \bar{e}_t \) characterize firms’ policies, and \( \mu_t \) is the distribution of firms. This definition is analogous to that of the data because it uses fixed cut-offs on successive cross-sections to define firm groups. Qualitative results on the borrowing of these groups are similar if one uses a cutoff for assets instead of internal finance. I choose \( S/L = 55.7\% \). This is the ratio of internal finance of manufacturing firms with more than 1 billion dollars in assets, to internal finance of all firms with more than $250m in assets in the QFR in 2007Q3. Internal finance computed as the difference between non-financial assets and total debt, as in the model.

4.3 Propagation mechanisms

In order to understand the results, it is useful to first analyze the short-run response of firms’ policy functions for total assets. Figure 10 illustrates the two key mechanisms through which the shock affects these policies in the short run.

The first mechanism is illustrated in the top panel of figure 10: the increase in the lending wedge results in a higher cost of bank borrowing outside of financial distress, and makes bank lending less attractive for mixed-finance firms. Those firms issue less bank debt and reduce their scale of operation. This is the traditional “bank lending” channel, present in models with a single borrowing constraint. The model however makes two additional predictions about this channel. First, for mixed-debt firms, the shock also leads to a fall in market borrowing. This is a manifestation of the fact that, when firms are in the mixed-finance regime, market and bank debt display a degree of complementarity. Intuitively, this complementarity arises because issuing market liabilities can partially relax a firm’s bank borrowing constraint by increasing its scale of operation and therefore its value in liquidation. The second additional prediction of the model is that, although market-financed firms are not directly affected by the increase in the lending wedge, they still somewhat reduce their borrowing from markets. These firms anticipate the fact that, given a sufficiently
Figure 10: “Credit” and “substitution” channels in the response of firms to the lending wedge shock.
bad sequence of shocks \( \phi_t \), they will have to revert to mixed debt structures and face the now tighter lending terms of banks. They seek to avoid this by reducing their leverage.

The second mechanism by which the shock affects output and investment is that it changes the threshold at which firms switch from a mixed debt structure to a market-financed debt structure. This is illustrated in the bottom panel of figure 10. This threshold shifts to the left, and all firms between the pre- and post-shock threshold switch to a market-financed debt structure. A central prediction of the model is that this switch has real effects: it affects the scale at which switching firms choose to operate. Firms that switch lose the flexibility associated with bank finance, and become more exposed to liquidation risk. Put differently, these firms would face excessively high liquidation premia, under a market-financed structure, in order to continue operating at the same scale. As a result, their total borrowing, and therefore their output and investment, drop below what they would be under a mixed debt structure. The additional fall in borrowing due to the substitution channel corresponds to the shaded area in the bottom panel of figure 10.

Quantitatively, the contribution of the substitution channel to the total decline in borrowing can be large. One way to gauge this contribution is to compute what total investment would have been in period 2 (once \( \gamma_{b,t} \)), had switching firms been constrained to keep using a mixed debt structure. In that counterfactual case, the drop in aggregate investment between year 1 and year 2 would have been 10.4%, instead of 15.1%. The substitution channel thus accounts for about a third of the decline in aggregate investment in the model.

The previous discussion helps understand the short-run response of the economy to the shock. In the long-run, the shock has protracted effects on firms’ ability to accumulate internal finance, both because lending terms of banks worsen, and because the shock induces firms to leverage less. As a result, the steady-state distribution of firms across levels of internal finance shifts to the left: in the long run, the median stock of internal finance of firms in the economy falls. The slow adjustment of firms’ internal funds in the new environment with tougher lending bank lending standards is what drives the endogenous persistence in the response output and investment, since, after the lending wedge has stabilized to a new, higher level, the threshold \( e^* \) does not change.

4.4 Can other aggregate shocks account for the behavior of the bank share?

The previous discussion focused on the effects of an increase in the lending wedge \( \gamma_{b,t} - \gamma_{m,t} \): as discussed, such a shock generates a deep recession accompanied by a fall in the aggregate bank share. Shocks to \( \mathbb{E}(\phi) \), and shocks that increase \( \gamma_b \) and \( \gamma_m \) jointly (and thus do not affect the lending wedge), also generate large recessions, but have limited effects on debt composition at the aggregate and firm level.\(^{49}\) A recession driven by an increase in idiosyncratic volatility, on the other hand, is accompanied by an increase in the aggregate

\(^{49}\)Detailed results for the perfect foresight response of the economy to shocks to aggregate productivity, productivity dispersion are available from the author upon request.
bank share. This reflects the greater demand for debt flexibility when firm-level uncertainty is high. The response of large firms also involves debt substitution, but in the opposite direction, away from market debt and into bank debt. Thus, in this model, dispersion-driven recessions are associated with changes in corporate debt structure that are at odds with the patterns documented in figure 8.

Summarizing, this section has shown that lending wedge shocks can generate large recessions accompanied by substitution toward market borrowing, both at the firm and aggregate levels. However, debt substitution also forces some firms to reduce leverage, and therefore investment. Deleveraging is optimal from the standpoint of switching firms: relying only on market debt deprives them from the flexibility offered by banks in bad times. But it amplifies the response of investment and aggregate activity to the shock. The following section turns to the policy implications of the model, and investigates how the substitution channel may play out in the context of programs aimed at encouraging firms to use market-based intermediation.

5 Corporate finance policies and their real effects

As a result of the contraction in bank lending that followed the Great Recession, some countries have considered encouraging market-based intermediation as a remedy to lagging investment. The Bank of England, for example, included corporate bonds in the Asset Purchase Facility that was set up in January 2009 in order to provide liquidity to credit markets, with the explicit goal of stimulating primary market issuance. In other countries, in particular in the Euro area, developments unrelated to monetary policy also encouraged bond issuance by firms facing a tightening in the supply of bank credit. This section discusses two examples: a new exchange for bond issuances by German firms of the “mittelstand”, and an Italian tax reform introducing tax deductibility of interest payments on bonds issued by private firms. I what follows, I use the model to gauge the potential effects of these two policies on debt and investment choices of firms.

5.1 The Bondm market

The Bondm exchange was launched by Boerse Stuttgart in 2010 with the goal of providing a more favorable environment for bond issuance for mid-size companies than existing markets. Bondm provides firms with a primary market for new issuances, and also operates a secondary exchange in which private and retail investors trade existing issues. Participation is restricted to firms with less than €250m in assets, and

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50 In a February 2010 speech, the Markets Executive Director of the Bank of England described the objectives of the program as follows: “It was clear there was a substantial liquidity premium in the market. [...] There was a risk that secondary market illiquidity might have restricted the strength of the primary market, so constraining the availability of credit.”.

51 The Euro Area Bank Lending Survey indicates that between 2008 and 2009, half of Euro area banks tightened their margins on “normal” loans; two thirds tightened their margins on “risky” loans. An additional tightening of bank lending standards also took place in 2010, and coincided with a pick-up in the issuance of new corporate bonds.
to issuances between €50m and €150m. Crucially, it aims at making bond issuances attractive to mid-size businesses by reducing intermediation costs. Bondm issuances do not need to be individually rated, and do not need to be underwritten by a bank. Underwriting requirements are particularly costly, as few European investment banks specialize in underwriting small issuers.\footnote{For more details on the German corporate bond market during the recession, and the Bondm exchange, see the case study of the German mid-cap D"urr in Hillion et al. (2012).}

In the context of the model described in section 2, a simple way to capture the advantages offered by Bondm to bond issuance for mid-size firms is to let market intermediation costs depend on internal funds $e_t$:

$$
\gamma_m(e_t) = \begin{cases} 
\tilde{\gamma}_m & \text{if } e_t \leq e_{sm} \\
\gamma_m & \text{if } e_t > e_{sm}, \\
\tilde{\gamma}_m < \gamma_m.
\end{cases}
$$

The introduction of the Bondm exchange corresponds to an economy in which $e_{sm} > 0$, as opposed to the baseline case where $e_{sm} = 0$. $\tilde{\gamma}_m \leq \gamma_m$ denotes intermediation costs for mid-size firms ($e_t \leq e_{sm}$) that use Bondm for their issuances.

Figure 11 compares a baseline economy (black line), to an economy with low market intermediation costs for mid-size firms (grey line). The baseline economy differs from the calibration of section 1 with respect to the magnitude of market intermediation costs. I assume that they are given by $\gamma_m = 0.016$, consistent with the evidence provided by Santos and Tsatsaronis (2003) on the differences in issuance costs between the US and the Euro area. In the alternative economy, the intermediation costs for Bondm-type issuances is set to $\tilde{\gamma}_m = 0.010$.\footnote{The threshold $e_{sm}$ is set to $e_{sm} = e^*$, where $e^*$ is the switching threshold in the baseline economy. The qualitative effects of the policy would be identical if $e_{sm} < e$, but this choice makes the graphical discussion that follows clearer.}

The left panel of figure 11 indicates that the policy has limited effects on the size distribution. Its effects on total output and investment are best understood by looking at changes in firms’ total assets.

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Figure 11: The effect of lower intermediation costs on firms’ choice of scale.
\[ \hat{k}(e_t) = e_t + \hat{b}(e_t) + \hat{m}(e_t), \]
which are reported in the right panel of figure 11.

This response depends on internal funds \( e_t \). There are three cases. First, the lower intermediation costs create a new threshold \( \hat{e}^* \). Firms with internal funds \( e_t \leq \hat{e}^* \) keep relying on bank loans even when there are lower market intermediation costs. However, these firms increase total borrowing: lower intermediation costs indeed encourage them to increase market debt issuance, which additionally relaxes their bank borrowing constraint, because of the complementarity between forms of borrowing in the mixed-finance regime.

The second case is that of firms with internal funds \( e_t > e^* \), where \( e^* \) is the threshold above which firms cannot take direct advantage of low market intermediation costs (i.e., they cannot issue on the Bondm market because they are too large). These firms nevertheless increase bond issuance in response to the lower intermediation costs. This is because they anticipate that, now that market borrowing is cheaper at small scales, they will be able to operate more profitably if a string of bad shocks pushes them back to a mixed debt structure.

The third case relates to intermediate firms, those with \( \hat{e}^* < e_t \leq e^* \). These firms are large enough that the policy will induce them to switch to an entirely market-financed debt structure, but small enough that they will still benefit from the Bondm intermediation costs for their bond issuances. The switch makes the debt structure of these firms more fragile, in the sense that they lose the option to restructure debt in bad times. As a result, some of these firms deleverage: their total borrowing is smaller than in the world with higher intermediation costs, and they operate at a smaller scale. Surprisingly, the lower intermediation costs therefore result in a reduction in total debt issuance and total investment by these firms.

How do these two effects – the “lending channel” effect that stimulates borrowing and investment by firms with \( e_t \leq \hat{e}^* \) and \( e_t > e^* \), and the “substitution channel” effect that depresses borrowing and investment by firms with \( \hat{e}^* < e_t \leq e^* \) – measure up against each other? Overall, the effect of the policy \( e_{sm} > 0 \) on total output and investment is positive: they increase, respectively, by 5.6% and 7.1% under the policy of lower issuance costs for mid-size firms. However, the negative effect on investment by medium-sized firms is sizeable, because the mass of firms that are in this region is large (see the left panel of figure 11). Specifically, these firms account for a -3.1% decline in aggregate investment, and a -2.0% decline in aggregate output, relative to the initial steady-state.

5.2 An Italian tax reform

To the extent that they discriminate between bank and market liabilities, taxes are an alternative tool that can be used as an instrument to influence firms’ choice of debt structure. A 2012 Italian tax reform meant to improve access to bond markets for private companies includes the tax treatment of interest payments

\[ \text{Visually, this is indicated by the fact that the light grey line is below the black line on the right panel of figure 11.} \]
to corporate bonds as a part of a larger array of policy tools. Specifically, the reform allows private firms to deduct interest paid on bonds in the same way as interest paid on other debt, in line with the tax rules imposed on large firms.

The model of section 2 does not explicitly incorporate tax deductibility of debt. However, in appendix C, I show that a simple extension can accommodate this. Let cash on hand of the firm in repayment be given by:

$$n_t^P = (1 - \tau)\pi_t - (1 - \tau_b)R_{b,t} - (1 - \tau_m)R_{m,t},$$

where $\tau$ denotes the marginal corporate tax rate. Then, $\tau_b \leq \tau$ and $\tau_m \leq \tau$ reflect preferential tax treatments of different types of debt instruments.

The two tax regimes that existed in Italy prior to the reform were, respectively, $\tau_b = \tau$ and $\tau_m = 0$ (only bank debt is interest-deductible); and which $\tau_b = \tau_m = \tau$ (both types of debt are treated identically for tax purposes). The policy experiment then consists in comparing an economy where only sufficiently large firms enjoy preferential tax treatment for market debt ($\tau_b = \tau_m = \tau$ for $e_t > e^*$; $\tau_b = \tau$ and $\tau_m = 0$ for $e_t \leq e^*$), to an economy in which all firms enjoy the same tax shields for market and bank debt ($\tau_m = \tau_b = \tau$ for all $e_t$).

The impact of this change in tax policy on borrowing and investment are qualitatively similar to the previous experiment. Intuitively, a differential treatment of bank and market debt ($\tau_m < \tau_b$) directly affects the relative cost of market and bank debt outside of liquidation. Introducing a tax shield on market debt issuance for mid-size firms will therefore have similar effects on borrowing as lowering market intermediation costs. Namely, the tax shield boosts total borrowing by the smallest and the largest firms, because of the “credit channel” effect discussed above. However, it reduces total borrowing by the mass of intermediate firms that switch entirely to market-financed debt structure in response to the introduction of the tax shield.

The results of both policy exercises in this section show that attempts to do encourage bond issuance by midsize firms, while they may stimulate investment in aggregate, may also result in inadvertent effects for certain firms, by inducing them firms to adopt more fragile debt structures. In turn, this increase in financial fragility has real implications, as these firms may end up reducing borrowing and investment. These results emphasize the importance of cross-sectional variation in debt structure in understanding the transmission of policies to investment and borrowing when firms have access to public debt markets.

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55 The reform also relaxes the requirements to find a sponsor to guarantee the issuance of the bond, and eliminates existing limits on indebtedness as a fraction of net worth for private firms. See http://www.paulhastings.com/Resources/Upload/Publications/2351.pdf for more details on the contents of the reform.

56 Appendix C reports these results.
6 Conclusion

The composition of corporate borrowing between bank loans and market debt exhibits substantial variation, across firms and over time. This paper starts from the view that banks specialize in providing a flexible form of borrowing to firms, albeit at the cost of tougher lending standards in normal times. Embedding this trade-off between flexibility and cost into a simple model of firm dynamics leads to cross-sectional predictions that align well with firm-level evidence on the composition of debt. In response to an aggregate shock to bank intermediation costs, the model generates a drop in output and investment, accompanied by changes in debt composition which, both at the aggregate and the firm level, resemble those experienced in the US during the 2007-2009 recession.

A key finding of the paper is that, aside from the traditional “lending channel” effects of financial shocks or policies, debt heterogeneity introduces a new “substitution channel”: in response to aggregate shocks affecting the relative cost of bank debt, some firms substitute towards bond issuances, but do so less than one-for-one, and as result, experience a deleveraging episode. Firms respond in this fashion because switching from a mixed debt structure to a purely bond-financed debt structure exposes them to more liquidation risk, precisely because bond and bank financing do not offer the same degree of flexibility in distress. The deleveraging that accompanies debt substitution by certain firms in the model can thus be interpreted as a precautionary response to their increases financial fragility, when more of external financing is obtained via public debt markets.

These results speak to the idea that bond markets could act as an alternative to traditional, bank-based intermediation in times when the latter is impaired. Providing support to corporate bond markets has indeed been one the explicitly stated goals of the recent quantitative easing policies, for example in the UK and the US. The policy experiments of section 5 suggest that these interventions could inadvertently have adverse effects on the investment of firms of intermediate size and credit risk, by inducing them to adopt a fragile debt structure. A precise analysis of this question, as well as the related issue of long-run effects of changes in bank regulation on borrowing by corporates, would require a more precise modelling of the balance sheet dynamics of banking and market intermediaries. I leave this to future research.

57 For a description of the quantitative easing policies of the Bank of England and their effects on corporate bond issuance, see Joyce, Tong, and Woods (2011). For the effects of Fed’s Large Scale Asset Purchase programs on corporate bond yields, see Gilchrist and Zakrajšek (2013).
References


A Data appendix

Main measures of the bank share and debt composition The bank share of all non-financial corporate businesses mentioned in the introduction, and reported in figure 8, is constructed using table L.102 of the Flow of Funds, the balance sheet of the nonfinancial corporate sector. It is defined as the ratio of the sum of depository institution loans (line 27, series fl103168005) and other loans and advances (line 28, series fl103169005), to total credit market instruments outstanding (line 23, series fl104104005).

The bank share used in the calibration of the model is constructed using data from the Quarterly Financial Report (QFR) for manufacturing firms. This dataset contains information on firms’ balance sheets and income statements, and is reported in semi-aggregated form (by asset size categories). In order to focus on firms with potential access to public debt markets, I use only the two top asset size groups: firms with between $250m and $1bn dollars in assets, and firms with more than $1bn in assets (asset size groups 9 and 10 in the QFR data). I also focus only on manufacturing firms (industry group TMFG in the QFR data), since the absence of size brackets for wholesale and retail firms do not allow me to narrow down the sample to firms in those industries with more than $250m in assets.

For each size group $i$, bank debt $b_{i,t}$ is defined as the sum of short-term bank debt ($stbank$ in the QFR data), currently due long-term bank debt ($instbank$), and bank debt due in more than one year ($ltbnkdebt$). Total debt $d_{i,t}$ is defined as the sum of bank debt and commercial paper ($compaper$), total bond debt (the sum of $instbonds$ and $ltbnddebt$), and total other debt (the sum of $stdebtoth$, $instother$ and $ltothdebt$). The bank share reported in the left panel of figure 8 is then given by $BS_t = \sum_i b_{i,t} \sum_i d_{i,t}$.

The middle and right panels of figure 8 report changes in the levels of bank and non-bank debt for each of the two groups of firms (firms with $250m - $1bn in assets, and firms with more than $1bn in assets). I define market debt for each group as: $m_{i,t} = d_{i,t} - b_{i,t}$. The series reported are normalized relative to 2008Q3, and are weighted so that the sum of market and bank debt adds up to the total change in debt. Specifically, the change in bank debt for group $i$ is defined as: $\gamma_{b,i,t} = \frac{b_{i,t} - b_{i,t_0}}{b_{i,t_0}} \left( \frac{b_{i,t}}{b_{i,t_0}} - 1 \right)$, where $t_0$ is 2008Q3. The series for the change in market debt is defined analogously. All series in the middle and right panels of figure 8 are smoothed by a 2 by 4 MA smoother.

Alternative measures of the bank share While the QFR data can be used to construct a time series for the bank share in a transparent and consistent manner, it has two main limitations: (1) it is restricted to manufacturing firms and (2) it may include data on firms that do not have access to public debt markets.

An alternative data source is the firm-level dataset created by Rauh and Sufi (2010). It combines data from Compustat on capital structure, from SDC and Dealscan data on debt issuances, and from companies’ 10K data, in order to provide a breakdown of firms’ debt by instrument. The data made available by the authors is composed of a random sample of approximately 220 firms per year, between 1996 and 2006. The sample is limited to rated firms (as measured by the existence of an S&P credit rating), thus narrowing the focus to firms with access to public debt markets. Industries other than manufacturing are represented: while manufacturing firms make up 44.3% of the sample, utilities, services, and wholesale and retail account for 18.0%, 17.2% and 11.4% of the sample.

I look at the subset of firm-year observations with debt \((d)\) above $1 m. I denote the total debt of a firm-year observation by \(d_{i,t}\). The data breaks down debt instruments into seven categories. In order to compute the bank share, I define bank debt \(b_{i,t}\) as the sum of three instruments: outstanding bank debt (bankout), privately placed debt (pp), and mortgage or equipment debt (mgeq). Following the definitions of debt categories provided by Rauh and Sufi (2010), these are the debt instruments most likely to be held by a bank or by a concentrated group of creditors. Additionally, I keep only observations for which the discrepancy between total debt \(d_{i,t}\) and the sum of all seven debt instrument groups are within 1% (or $1 m) of each other. These criteria narrow down the sample to 2322 firm-year observations (out of the initial 2453). I then define the bank share for each year as: \(BS_t = \frac{\sum_i b_{i,t}}{\sum_i d_{i,t}}\), in order to maintain comparability with the QFR data.

With this measure of the bank share, the average annual bank share in the sample is 21.2\% (the median is 20.9\%), close to the 24.0\% documented in the QFR. The somewhat lower bank share may be driven the fact that firms are, on average, larger in this sample than in the QFR (the assets of the median firm are $1.4bn). The average bank share when restricting the sample to firms with assets between $250m and $1bn is 42.6\% (that share is 51.3\% in the QFR in 2007Q3), but these firms only account for 4.6\% of total debt in the dataset (whereas they account for 8.6\% of total debt in the QFR in 2007Q3).

An alternative data source for the measurement of debt composition is the work of Colla et al. (2013), who study debt composition by instrument in a sample of 16115 observations of non-financial US firms between 2002 and 2009. Of these 16115 firm-year observations, 9968 (or approximately 60\%) correspond to rated firms (again measured by the existence of an S&P rating). They report a systematic breakdown of debt in seven instrument classes, two of which (drawn credit lines and term loans) can be used to measure outstanding bank debt. Among rated firms, the average share of bank debt to total debt, as measured using these two instruments, is 19.3\%. This number is close to the aggregate bank share computed above in the \(BS_t = \frac{\sum_i b_{i,t}}{\sum_i d_{i,t}}\), in order to maintain comparability with the QFR data. Since this number is computed by averaging unweighted means of different rating classes, the discrepancy may be driven by variation in the number of firms in different rating groups.\(^{59}\)

Overall, both the data compiled by Rauh and Sufi (2010) and the evidence reported by Colla et al. (2013) indicate that the bank share of firms with access to public debt markets is close to the figure of 24.0\% obtained using the two largest asset size classes from the 2007Q3 QFR. Thus, biases in the measurement of the bank share introduced by the fact that the QFR is limited to manufacturing, and the fact that firms without access to public debt markets may be included in the QFR totals, are likely to be small.

Data on firm-level debt composition I additionally use firm-level data to illustrate cross-sectional variation in debt structure. Firm-level data is drawn from the dataset created by Rauh and Sufi (2010). This dataset draws from Compustat (for balance-sheet data) and Dealscan (for bond issuances), and has direct measures of total debt \(d_{j,t,US}\). Internal finance \(e_{j,t,US}\) is defined as the difference between the “debt plus equity” and the “debt” variables. Finally, bank debt \(b_{j,t,US}\) is defined as outstanding bank loans. All other variable definitions are identical.

\(^{59}\)A direct computation of the aggregate bank share, as measured in the QFR and in the Rauh and Sufi (2010) data, is not feasible, since the firm-level data used by Colla et al. (2013) is not available publicly. Instead, the figure of 19.3\% is computed using the information from their table VI, p. 2130.
B Proofs

Proof of lemma 1.

Since \( V \) is continuous, the existence and continuity of \( V^c \) follows from the theorem of the maximum. Let \((n^1_t, n^2_t) \in \mathbb{R}^2_+ \) such that \( n^1_t > n^2_t \), and let \( e^2_{t+1} \) be a value for next period net worth that solves problem (1), when \( n_t = n^2_t \). We have \( e^2_{t+1} \leq n^2_t < n^1_t \), so \( e^2_{t+1} \) is also feasible when \( n_t = n^1_t \). Therefore, \( V^c(n^1_t) \geq n^1_t - e^2_{t+1} + (1 - \eta)\beta V(e^2_{t+1}) > n^2_t - e^2_{t+1} + (1 - \eta)\beta V(e^2_{t+1}) = V^c(n^2_t) \). This proves that \( V^c \) is strictly increasing. Finally, when \( n_t = 0 \) the feasible set contains only \( \text{div}_t = 0, e_{t+1} = 0 \). So \( V^c(0) = (1 - \eta)\beta V(0) \). Therefore when \( V(0) \geq 0 \), \( V^c(0) \geq 0 \).

Next, define the function \( F(e) \equiv (1 - \eta)\beta V(e) - e \) on \( \mathbb{R}_+ \). \( F \) is continuous whenever \( V \) is continuous. Since \( V \) is also assumed to be left and right-hand differentiable everywhere, \( F \) has directional derivatives, given by \( F'_+(e) = (1 - \eta)\beta V'_+(e) - 1 \) and \( F'_-(e) = (1 - \eta)\beta V'_-(e) - 1 \). Assumption (2) in lemma 1 implies that, \( \forall e \in [0, \bar{e}], F'_-(e) > 0 \). As a continuous function with at least one strictly positive Dini differential, \( F \) is thus strictly increasing on \([0, \bar{e}] \) (condition (a)). Likewise, because of assumption (2), \( F \) is weakly decreasing on \([\bar{e}, +\infty) \). As a result, \( \forall e > \bar{e}, F(e) \geq F(e) \) (condition (b)). It is straightforward to check that conditions (a) and (b) are equivalent to the optimality of the dividend policy described in lemma 1.

Proof of proposition 1. A useful result for the proof is that \( V^c(n_t) \geq n_t \) when \( V(0) \geq 0 \). This is established by noting that the dividend policy \( e_{t+1} = 0 \) is always feasible at the dividend issuance stage, and that the value of this policy is \( n_t + (1 - \eta)\beta V(0) \geq n_t \).

Assume, first, that \( \frac{R_{m,t}}{1 - \chi} \leq \frac{R_{b,t}}{1 - \chi} \). Then, \( \frac{R_{m,t}}{1 - \chi} \leq R_{b,t} + R_{m,t} \leq \frac{R_{b,t}}{1 - \chi} \). The proof proceeds by comparing \( V^L_t \), \( V^R_t \) and \( V^P_t \), the values of the firm under liquidation, restructuring or repayment, for each realization of \( \pi_t \). There are five possible cases:

- when \( \pi_t \geq \frac{R_{b,t} + R_{m,t}}{1 - \chi} \), we have \( V^L_t = \chi \pi_t - R_{b,t} - R_{m,t} < \pi_t - R_{b,t} - R_{m,t} \leq V^c(\pi_t - R_{b,t} - R_{m,t}) = V^P_t \). Moreover, since \( \pi_t \geq \frac{R_{b,t} + R_{m,t}}{1 - \chi} \), the reservation value of the bank is \( R_{b,t} \), so the best restructuring offer for the firm is \( l_t = R_{b,t} \). Therefore \( V^P_t = V^R_t \); I will assume the firm chooses repayment.

- when \( \frac{R_{b,t} + R_{m,t}}{1 - \chi} < \pi_t \geq \frac{R_{b,t}}{1 - \chi} \), we have \( V^L_t = 0 \leq V^c(\pi_t - R_{b,t} - R_{m,t}) = V^P_t \), since \( \pi_t \geq \frac{R_{b,t}}{1 - \chi} \geq R_{b,t} + R_{m,t} \). \( V^c(0) \geq 0 \) and \( V^c \) is strictly increasing. (\( V^L_t < V^R_t \) so long as \( \pi_t > R_{b,t} + R_{m,t} \)). Moreover, \( V^R_t = V^P_t \) for the same reason as above. Again, the firm chooses repayment.

- when \( \frac{R_{b,t}}{1 - \chi} \geq \pi_t \geq R_{b,t} + R_{m,t} \), the reservation value of the bank is \( \chi \pi_t \). The restructuring offer at which the participation constraint of the bank binds, \( \hat{l} = \chi \pi_t \), is feasible because \( \pi_t - l_t - R_{m,t} = (1 - \chi)\pi_t - R_{m,t} \geq 0 \).

So \( V^R_t \geq V^c((1 - \chi)\pi_t - R_{m,t}) \). This implies \( V^R_t > V^c(\pi_t - R_{b,t} - R_{m,t}) = V^P_t \), since \( V^c \) is strictly increasing and \( (1 - \chi)\pi_t - R_{m,t} > \pi_t - R_{b,t} - R_{m,t} \). For the same reasons as above, \( V^R_t > V^P_t \). So the firm chooses to restructure. Because \( V^c \) is increasing, the optimal restructuring offer makes the participation constraint of the bank bind: \( \hat{l} = \chi \pi_t \).

- when \( R_{b,t} + R_{m,t} \geq \pi_t \geq \frac{R_{m,t}}{1 - \chi} \), we have \( V^L_t = 0 < V^c((1 - \chi)\pi_t - R_{m,t}) = V^R_t \), where again the properties of \( V^c \) were used. Moreover, \( V^P_t = V^L_t \), since the firm does not have enough funds to repay both its creditors. So
the firm chooses to restructure, again with $\hat{t} = \chi \pi_t$.

- when $\pi_t < \frac{R_{m,t}}{1-x}$, the firm is liquidated because any restructuring offer consistent with the participation constraint of the bank will leave the firm unable to repay market creditors. Since in that case, $0 > \chi \pi_t - R_{b,t} - R_{m,t}$, the liquidation value for the firm is $V^{L}_t = 0$.

This shows that when $\frac{R_{m,t}}{1-x} \leq \frac{R_{b,t}}{x}$, the firm repays when $\pi_t \geq \frac{R_{b,t}}{x}$, restructures when $\frac{R_{b,t} + R_{m,t}}{x} \geq \pi_t \geq \frac{R_{m,t}}{1-x}$, and is liquidated otherwise. Moreover, this also establishes the two additional claims of the proposition, in the case $\frac{R_{m,t}}{1-x} \leq \frac{R_{b,t}}{x}$: $V^{L}_t = 0$ whenever liquidation is chosen, and the restructuring offer always makes the participation constraint of the bank bind: $\hat{t} = \chi \pi_t$. The claims of the proposition when $\frac{R_{m,t}}{1-x} > \frac{R_{b,t}}{x}$ can similarly be established, by focusing on the three sub-cases $\pi_t \geq \frac{R_{b,t} + R_{m,t}}{x}$, $\frac{R_{b,t} + R_{m,t}}{x} > \pi_t \geq \frac{R_{b,t}}{x}$ and $\frac{R_{b,t}}{x} > \pi_t$. ■

Proof of proposition 2. Given the results of proposition 1, the debt settlement outcomes yield the same conditional return functions for banks and market lenders, $\tilde{R}_{b,t}(\pi_t, R_{b,t}, R_{m,t})$ and $\tilde{R}_{m,t}(\pi_t, R_{b,t}, R_{m,t})$, as those reported in the appendix to Crouzet (2013). Proposition 2 is a subset of the results of proposition 2 in that paper. ■

Proof of proposition 3.

The proof of the existence of a recursive competitive equilibrium of the economy of section 2 proceeds by first establishing the existence and unicity of a solution to the individual firm problem, then by establishing the existence of a steady-state distribution.

Firm problem Throughout, the firm’s problem is restated in terms of the variables $d_t = b_t + m_t$ and $s_t = \frac{b_t}{s_t + m_t}$. $d_t$ denotes total borrowing by a firm, and $s_t$ denotes the share of borrowing that is bank debt. Note that $(d_t, s_t) \in \mathbb{R}_+ \times [0,1]$. With some abuse of notation, the set of feasible debt structures $(d_t, s_t)$ is still denoted by $S(e_t)$, and its partition established in proposition 2 as $(S_K(e_t), S_L(e_t))$. Additionally, the functions $G: \mathbb{R}_+ \rightarrow \mathbb{R}_+$, $I(., e_t + d_t): \mathbb{R}_+ \rightarrow \mathbb{R}_+$ and $M(., e_t + d_t): \mathbb{R}_+ \rightarrow \mathbb{R}_+$ by:

$$G(x) = x(1 - F(x)) + \int_0^x \phi d F(\phi)$$

$$I(x; e_t + d_t) = x(1 - F(x)) - F(x)(1 - \delta)(e_t + d_t)^{1-\zeta}$$

$$M(x; e_t + d_t) = (1 - \chi)I(x; e_t + d_t) + \chi G(x)$$

Following lemmas 2 to 4 in the appendix to Crouzet (2013), $G$ is strictly increasing on $\mathbb{R}_+$, while $I$ and $M$ have unique maxima. I denote these inverse mappings by $G^{-1}(\cdot)$, $I^{-1}(\cdot; e_t + d_t)$ and $M^{-1}(\cdot; e_t + d_t)$. They are defined, respectively, on $[0, \mathbb{E}(\phi)]$, $[0, I(e_t + d_t)]$ and $[0, M(e_t + d_t)]$, where $I(e_t + d_t)$ is the global maximum of $I$ and similarly $M(e_t + d_t)$ is the global maximum of $M$.

These functions are useful because their inverse mappings determine the terms of debt contracts $(R_{b,t}, R_{m,t})$ for given $(e_t, d_t, s_t)$. Following the results of proposition 1, the thresholds for restructuring and liquidation, in terms

\[ R_{b,t} = R_b(d_t, s_t, e_t) = \begin{cases} 
(1 + \tau_b)d_t s_t & \text{if } 0 \leq (1 + \tau_b)d_t s_t < (1 - \delta)(e_t + d_t)^{1-\zeta} \\
\chi (1 - \delta)(e_t + d_t) + \chi (e_t + d_t)^{1-\zeta} G^{-1}(\frac{1 + \tau_b}{\chi (e_t + d_t)^{1-\zeta}}) & \text{if } (1 - \delta)(e_t + d_t)^{1-\zeta} \leq (1 + \tau_b)d_t s_t \leq \mathbb{E}(\phi) + (1 - \delta)(e_t + d_t)^{1-\zeta} 
\end{cases} \]

Expressions for $R_b(d_t, s_t, e_t)$ and $R_m(d_t, s_t, e_t)$ that use these inverse mappings, for other cases, are reported in the appendix.
of the productivity shock $\phi_t$, are given by:

$$
\phi_R(e_t, d_t, s_t) = \frac{R_m(d_t, s_t, e_t) - (1-\chi)(1-\delta)(e_t + d_t)}{(1-\chi)(e_t + d_t)^\zeta} \\
\overline{\phi}_R(e_t, d_t, s_t) = \frac{R_b(d_t, s_t, e_t) - (1-\delta)(e_t + d_t)}{\chi(e_t + d_t)} \\
\underline{\phi}_R(e_t, d_t, s_t) = \frac{R_b(d_t, s_t, e_t) + R_m(d_t, s_t, e_t) - (1-\delta)(e_t + d_t)}{(e_t + d_t)^\zeta}
$$

(liquidation threshold when $(d_t, s_t) \in S_R(e_t)$)

(restructuring threshold when $(d_t, s_t) \in S_R(e_t)$)

(liquidation threshold when $(d_t, s_t) \in S_R(e_t)$)

The proof combines these expressions with the inverse mappings defined above to express liquidation thresholds directly as functions of $e_t, d_t$ and $s_t$. There are three steps:

**Step 1:** Reformulate the problem as the combination of a discrete choice and continuous choice problem.

**Step 2:** Show that the functional mapping $T$ associated with this new formulation maps the space $C(E)$ of real-valued, continuous functions on $[0, E]$, with the sup norm $\| \cdot \|_s$, onto itself, where $E > 0$ is an arbitrarily large upper bound for equity. Additionally, show that $T(C_0(E)) \subseteq C_0(E)$, where $C_0(E) = \{ V \in C(E) \text{ s.t. } V(0) \geq 0 \}$. Since $(C(E), \| \cdot \|_s)$ is a complete metric space and $C_0(E)$ is a closed subset of $C(E)$ under $\| \cdot \|_s$ which is additionally stable through $T$, if $T$ is a contraction mapping, then its fixed point must be in $C_0(E)$.

**Step 3:** Check that $T$ satisfies Blackwell’s sufficiency conditions, so that it has a unique fixed point $V$.

**Step 4:** Check that $V$ is left- and right-differentiable everywhere, and that its differentials are bounded from below (close to 0) and from above (for large $e$), so that the conditions of lemma 1 are verified.

Note that step 2 is crucial because lemma 1 requires the continuity of $V$ and the fact that $V(0) \geq 0$ for $V^c$ to be continuous, strictly increasing and satisfy $V^c(0) \geq 0$. In turn, these three conditions are necessary for characterizing the set of feasible debt structures, that is, for proposition 2 to hold.

**Step 1:** Define the mapping $T$ on $C(E)$ as:

$$
\forall e_t \in [0, E], \quad TV(e_t) = \max_{R, K} \left( T_R V(e_t), T_K V(e_t) \right) \tag{A1}
$$

where the mappings $T_R$ and $T_K$, also defined on $C(E)$, are given by:

$$
n_R(e_t, d_t, s_t) = \max_{0 \leq e_{t+1} \leq n_t} \left( n_t - e_{t+1} + (1-\eta)\delta V(e_{t+1}) \right)
$$

$$
\phi_R(e_t, d_t, s_t) = \begin{cases} 
\phi_t(e_t + d_t)^\zeta + (1-\delta)(e_t + d_t) - (1-r_m)d_t s_t - (1+r_h)d_t(1-s_t) & \text{if } \phi_R(e_t, d_t, s_t) = 0 \\
(1-\chi)(\phi_t - \phi_R(e_t, d_t, s_t))(e_t + d_t)^\zeta & \text{if } 0 < \phi_R(e_t, d_t, s_t) \leq \phi_t \leq \overline{\phi}_R(e_t, d_t, s_t) \\
(\phi_t - \chi \phi_R(e_t, d_t, s_t) - (1-\chi)\chi e_t d_t) (e_t + d_t)^\zeta & \text{if } \overline{\phi}_R(e_t, d_t, s_t) \leq \phi_t \leq \phi_R(e_t, d_t, s_t)
\end{cases}
$$

$$
\overline{\phi}_R(e_t, d_t, s_t) = \begin{cases} 
I^{-1} \left( \frac{(1+r_m)d_t(1-s_t) - (1-\chi)(1-\delta)(e_t + d_t)}{(1-\chi)(e_t + d_t)^\zeta} ; e_t + d_t \right) & \text{if } (1-\chi)(1-\delta)(e_t + d_t) \leq (1+r_m)d_t(1-s_t) \\
\chi(1-\delta)(e_t + d_t) & \text{if } \chi(1-\delta)(e_t + d_t) \leq (1+r_h)d_t s_t \\
\chi(1-\delta)(e_t + d_t) + (1-\chi)(e_t + d_t)^\zeta \hat{I}(e_t + d_t) & \text{if } \chi(1-\delta)(e_t + d_t) > (1+r_h)d_t s_t
\end{cases}
$$

1 of Crouzet (2013).
∀ argument similar to the proof of lemma 1, \( \text{maps } R \rightarrow \mathbb{R} \)
\[ O \text{ is increasing,lemma 1. Moreover } \]
\[ \text{TV}\]
\[ \text{Here, } \bar{V} \]
\[ \text{Consider a solution } V \]
\[ \text{and: } \]
\[ \forall t \in [0, E], \quad T_K V (e_t) = \max_{(d_t, s_t) \in S_K (e_t)} \int_{d_t \geq \underline{\phi}_K (e_t, d_t, s_t)} V^c (n_K (\phi_t; e_t, d_t, s_t)) dF (\phi_t) \quad \text{(A1-K)} \]

\[ n_K (\phi_t, e_t, d_t, s_t) = \left\{ \begin{array}{ll}
\phi_t (e_t + d_t) + (1 - \delta) (e_t + d_t) - (1 - r_m) d_t s_t - (1 + r_b) d_t (1 - s_t) & \text{if } \phi_t (e_t, d_t, s_t) = 0 \\
(\phi_t - \underline{\phi}_K (e_t, d_t, s_t)) (e_t + d_t) & \text{if } 0 < \phi_t (e_t, d_t, s_t) \leq \phi_t
\end{array} \right\} \]

\[ \hat{\phi}_K (e_t, d_t, s_t) = \left\{ \begin{array}{ll}
0 & \text{if } 0 \leq (1 + r_m (1 - s_t) + r_b s_t) d_t < (1 - \delta) (e_t + d_t)

M^{-1} \left( (1 + r_m (1 - s_t) + r_b s_t) d_t - (1 - \delta) (e_t + d_t) \right) ; e_t + d_t & \text{if } (1 - \delta) (e_t + d_t) \leq (1 + r_m (1 - s_t) + r_b s_t) d_t

\leq (1 - \delta) (e_t + d_t) + (e_t + d_t) \delta \bar{M} (e_t + d_t)
\end{array} \right. \]

Step 2: Let \( V \in C (E) \). By lemma 1, the associated continuation value \( V^c \) is continuous on \( \mathbb{R}^+ \). Moreover, since \( I^{-1} \) and \( G^{-1} \) are continuous functions of \( e_t, d_t \) and \( s_t \), the functions \( n_R, \underline{\phi}_R \) and \( \bar{\phi}_R \) are continuous in their \((e_t, d_t, s_t)\) arguments. Define the mapping \( O_R : [0, E] \times [0, \bar{d} (E)] \times [0, 1] \rightarrow \mathbb{R}^+ \) by

\[ O_R (e_t, d_t, s_t) = \int_{d_t \geq \underline{\phi}_R (e_t, d_t, s_t)} V^c (n_R (\phi_t; e_t, d_t, s_t)) dF (\phi_t). \]

Here, \( \bar{d} (E) \) denotes the upper bound on borrowing for the maximum level of equity.\(^61\) By continuity of \( V^c, n_R, \underline{\phi}_R \) and \( \bar{\phi}_R \), the integrand in \( O_R \) is continuous on the compact set \([0, E] \times [0, \bar{d} (E)] \times [0, 1]\), and therefore uniformly continuous. Hence, \( O_R \) is continuous on \([0, E] \times [0, \bar{d} (E)] \times [0, 1]\). The constraint correspondence \( \Gamma_R : e_t \rightarrow S_R (e_t) \) maps \([0, E]\) into \([0, \bar{d} (E)] \times [0, 1]\). The characterization of the set \( S_R (e_t) \) in proposition 4 of Crouzet (2013) moreover shows that the graph of the correspondence \( \Gamma_R \) is closed and convex. Theorems 3.4 and 3.5 in Stokey, Lucas, and Prescott (1989) then indicate that \( \Gamma_R \) is continuous. Given that \( O_R \) is continuous and \( \Gamma_R \) compact-valued and continuous, the theorem of the maximum applies, and guarantees that \( T_K V \in C (E) \).

By analogously defining a mapping \( O_K (e_t, d_t, s_t) \) for the problem A1-K, one can also prove that \( T_K V \in C (E) \). Therefore, \( TV = \max (T_K V) \in C (E) \). Moreover, let \( V \in C_0 (E) \). Then \( V^c (0) \geq 0 \) and \( V^c \) is increasing, by lemma 1. Moreover \( S_R (0) \neq \emptyset \), so one can evaluate \( O_R \) at some \((d_{t,0}, s_{t,0}) \in S_R (0)\). Since \( n_R \geq 0, V^c (0) \geq 0 \) and \( V^c \) is increasing, \( O_R (0, d_{t,0}, s_{t,0}) \geq 0 \). Therefore \( T_K V (0) \geq 0 \), so \( TV (0) \geq 0 \) and \( TV \in C_0 (E) \).

Step 3: First, let \((V, W) \in C (E)\) such that \( \forall e_t \in C (E), V (e_t) \geq W (e_t) \). Pick a particular \( e_t \in [0, E] \). By an argument similar to the proof of lemma 1, \( \forall n_t \geq 0, V^c (n_t) \geq W^c (n_t) \), where \( W^c \) denotes the solution to the dividend issuance problem when the continuation value is \( W \) (and analogously for \( V \)). Since the functions \( \phi_R, \bar{\phi}_R \) and \( n_R \) are independent of \( V \), this inequality implies \( O_R^W (e_t, d_t, s_t) \geq O_R^W (e_t, d_t, s_t) \) for any \((d_t, s_t) \in S_R (e_t)\), where the notation \( O_R^W \) designates the objective function in problem (A1-R) when the continuation value function is \( W \) (and analogously

\(^{61}\)See the results of proposition 4 of Crouzet (2013) for a proof that such an upper bound always exist.
for \( V \). Thus \( T_R V(e_t) \geq T_R W(e_t) \). Similarly, one can show that \( T_K V(e_t) \geq T_K W(e_t) \). Therefore, \( TV(e_t) \geq TW(e_t) \), and \( T \) has the monotonicity property.

To establish the discounting property, it is sufficient to note that \((V + a)'(nt) = V'(nt) + \beta a\), so that for any \( e_t \in [0, E] \) and \((dt, st) \in S_R(e_t)\), \( O^V_R + a(e_t, dt, st) = O^V_R(e_t, dt, st) + \left(1 - F \left( \left( \frac{\phi_R(e_t, dt, st)}{T_R(e_t, dt, st)} \right) \right) \right) \beta a \leq O^V_R(e_t, dt, st) + \beta a\). This shows that \( T_R(V + a)(e_t) \leq T_R V(e_t) + \beta a \). A similar claim can be made for \( T_K \). Therefore, the operator \( T \) has the discounting property. The Blackwell sufficiency conditions hold, so that \( T \) is a contraction mapping. As a contracting mapping on a complete metric space, it has a unique fixed point, which I denote \( V \) in what follows.

**Step 4:** First, I establish the fact that \( T_K V \) is differentiable at any \( e_t \in [0, E] \). For any \((e_t, dt, st)\) such that \( \phi_K(e_t, dt, st) > 0 \), using the change of variable \( nt = nK(\phi_t, e_t, dt, st) \) we can rewrite:

\[
O_K(e_t, dt, st) = \int_{nt \geq 0} V'(nt) dG_K(nt; e_t, dt, st),
\]

\[
G_K(\phi_t; e_t, dt, st) \equiv F \left( \frac{\phi_K(e_t, dt, st)}{e_t + dt} \right).
\]

Given the continuous differentiability of \( \phi_K \), this expression establishes that \( O_K(e_t, dt, st) \) is continuously differentiable in \((dt, st)\). This implies that \( T_K V(e_t) \) is differentiable. Similarly, one can establish that \( T_R V(e_t) \) is differentiable. Theorem 3 of Milgrom and Segal (2002) then applies to the family of functions \((T_K V(e_t), T_R V(e_t))\) (this family is equidifferentiable because it is finite, and each of the two functions is continuously differentiable), so that \( TV \) has left and right-hand derivatives for all \( e > 0 \). Moreover, except at points where \( TV_R(e_t) = TV_K(e_t) \), the left and right hand side derivative of \( TV \) are both given by \( V'_L(e_t) = V'_R(e_t) = TV'(e_t) \), where \( i = R \) if \( T_R V(e_t) > T_K V(e_t) \), and vice-versa. Since \( TV = V \), this also holds for \( V \).

When \( e_t \) is very large, firms have no default risk and can operate at the unconstrained optimal scale:

\[
k^* = \frac{E(\phi) \zeta}{\delta + r_m}
\]

In this case, their flow profits are given by \((1 + r_m)e + E(\phi)(k^*)^\delta - \delta k^*\), so that the marginal value of internal finance is \( T_K V'(e_t) = T_R V'(e_t) = (1 + r_m) \). Thus, provided that \((1 + r_m)(1 - \eta)\beta < 1, (1 - \eta)\beta TV'(e_t) = (1 - \eta)\beta V'(e_t) < 1\) for sufficiently large \( e_t \). When \( e_t \) is small, the lower bound \( V'(nt) \geq nt \), along with expression 11, implies that:

\[
O_K(e_t, dt, st) \geq (e_t + dt)^\delta \left( E(\phi) - \phi_K(e_t, dt, st) \right) \geq (e_t + dt)^\delta \left( E(\phi) - \phi_K(0, dt, st) \right),
\]

for small \((e_t, dt)\), where the second inequality follows from the fact that \( \phi_K(e_t, dt, st) \) is decreasing in \( e_t \). The derivative of the right-hand side with respect to \( e_t \) is proportional to \( \zeta(e_t + dt)^{\zeta - 1} \), which becomes infinitely large for \((e_t, dt) \to 0\) when \( \zeta < 1 \). Thus in turn implies that \( T_K V'(e_t) \) becomes infinitely large for small \( e_t \). A similar reasoning holds for \( T_R V'(e_t) \), and therefore, for \( V'(e_t) \). Thus, for small enough \( e_t, (1 - \eta)\beta V'(e_t) > 1 \). This establishes the existence of the threshold \( \bar{e} \) described in lemma 1.

**Existence of an invariant measure** 1 next prove that, given a solution to problem (8), an invariant measure of firms across levels of \( e_t \) exists. I start by introducing some preliminary notation. \( \tilde{E} = [0, \bar{e}] \) denotes the state-space of the firm problem (8). \((\tilde{E}, \tilde{E})\) is the measurable space composed of \( \tilde{E} \) and the family of Borel subsets of \( \tilde{E} \). For any value \( e_t \in \tilde{E} \), \( \tilde{d}(e_t) \) and \( \tilde{s}(e_t) \) denote the policy functions of the firms. The fact that these policy functions are such
that \((\hat{d}(e_t)), \hat{s}(e_t)) \in S_R(e_t)\) will be denoted by \(e_t \in \hat{E}_R\), and \(e_t \in \hat{E}_K\) for the other case. \(\hat{\phi}_R(e_t)\) and \(\hat{\phi}_K(e_t)\) denote the liquidation threshold implied by the firm’s policy functions when \(e_t \in \hat{E}_R\), while \(\hat{\phi}_K(e_t)\) denotes the liquidation threshold when \(e_t \in \hat{E}_K\). I use the notation: \(\hat{r}(e_t) = r_m(1 - \hat{s}(e_t)) + r_b \hat{s}(e_t)\).

**Transition function** Let \(N(e_t, e_{t+1})\) denote the probability that a firm will a level of internal finance of at least \(e_{t+1}\), given that its current internal finance level is \(e_t\). Three elements contribute to this probability: the exogenous exit probability; conditional on survival, the firm’s borrowing and dividend policy; and the replacement of exiting firms by new ones, entering at scale \(e^\epsilon\). The exact expression of transition probabilities depend both on whether \(e_t \in \hat{E}_K\), or \(e_t \in \hat{E}_R\); and on the amount borrowed by firms in each case; it is straightforward but tedious to construct, and is reported in detail in the online appendix.

This expression indicates that \(N(e_t, \cdot)\) is weakly increasing, has limits 0 and 1 at \(-\infty\) and \(+\infty\) and is everywhere continuous from above. Following theorem 12.7 of St rocky, Lucas, and Prescott (1989), there is therefore a unique probability measure \(\hat{Q}(e_t, \cdot)\) on \((\mathbb{R}, B(\mathbb{R}))\) such that \(\hat{Q}(e_t, [\cdot, e_t]) = N(e_t, e)\ \forall e \in \mathbb{R}\). This measure is 0 for \(e < 0\) and 1 for \(e > \epsilon\), and so the restriction of \(\hat{Q}\) to \(\hat{E}\) is also a probability measure on \((\hat{E}, \hat{\mathcal{E}})\), which can be denoted \(\hat{Q}(e_t, \cdot)\). Moreover, fixing \(e \in \hat{E}\), the function \(\hat{Q}(\cdot, [\cdot, e_t]) : \hat{E} \to [0, 1]\) is measurable with respect to \(\hat{\mathcal{E}}\).

Indeed, given \(e \in \hat{E}\), the definition of \(N(e_t, e)\) indicates that the set \(H(z, e) = \left\{ e_t \in \hat{E} \text{ s.t. } Q(e_t, [0, e]) \leq z \right\} = \left\{ e_t \in \hat{E} \text{ s.t. } \hat{Q}(e_t, [0, e]) \leq z \right\} = \left\{ e_t \in \hat{E} \text{ s.t. } N(e_t, e) \leq z \right\}\) is an element of \(\hat{\mathcal{E}}\). This in turn implies that the function \(\hat{Q}(\cdot, \mathbf{E})\) is \(\hat{\mathcal{E}}\)-measurable for any \(\mathbf{E} \in \hat{\mathcal{E}}\). The function \(Q : \hat{E} \times \hat{\mathcal{E}} \to [0, 1]\) is such that \(Q(e_t, \cdot)\) is therefore a probability measure for any \(e_t \in \hat{E}\), and \(Q(\cdot, \mathbf{E})\) is \(\hat{\mathcal{E}}\)-measurable \(\forall \mathbf{E} \in \hat{\mathcal{E}}\); hence, \(Q\) is a transition function.

**Feller property** To establish that the transition function \(Q\) has the Feller property, one must show that \(\forall e_t \in \hat{E}\) and \(\forall (e_{n,t})_n \in \hat{E}^N\) such that \(e_{n,t} \to e_t\), \(Q(e_{n,t}, \cdot) \Rightarrow Q(e_t, \cdot)\), where \(\Rightarrow\) denotes weak convergence. To establish this, given the definition of \(Q\) it is sufficient to show that \(N(e_{n,t}, e_{t+1}) \to N(e_t, e_{t+1})\) pointwise, at all values of \(e_{t+1}\) and \(e_t\) where \(N(\cdot, e_{t+1})\) and \(N(\cdot, e_t)\) are continuous. This excludes, in particular, the cases \(e_{t+1} = \epsilon\) or \(e_{t} = \epsilon\). I give the proof for the case \(e_t \in \hat{E}_K\); the proof for the other case \((e_t \in \hat{E}_R)\) is similar.

First consider a simple case, when \(e_t\) satisfies:

\[
(1 + \hat{r}(e_t))\hat{d}(e_t) > (1 - \delta)(e_t + \hat{d}(e_t))
\]

Because \(e_t \mapsto \eta + (1 - \eta)F\left(\underline{\hat{\phi}}_K(e_t) + \frac{e_{t+1}}{(e_t + \hat{d}(e_t))}\right)\) is continuous on \(\hat{E}_K, e \mapsto N(e, e_{t+1})\) is continuous in a neighborhood of \(e_t\). Additionally, since \(\hat{r}\) and \(\hat{d}\) is continuous, the inequality (12) holds for \(e_{n,t}\), when \(n\) is sufficiently large. Combining these two observations implies that \(N(e_{n,t}, e_{t+1}) \to N(e_t, e_{t+1})\). The case when the inequality above

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62For example, if \(z \leq \eta\) and \(0 < \epsilon < \eta\), the intersection of the set \(H(z, e)\) with \(\hat{E}_K\) is given by:

\[
H(z, e) \cap \hat{E}_K = \left\{ e_t \in \hat{E}_K \text{ s.t. } (1 - \delta)(e_t + \hat{d}(e_t)) \geq (1 + \hat{r}(e_t))\hat{d}(e_t) + \hat{e} \right\} \cup \left\{ e_t \text{ s.t. } (1 + \hat{r}(e_t))\hat{d}(e_t) + \hat{e} > (1 - \delta)(e_t + \hat{d}(e_t)) \geq (1 + \hat{r}(e_t))\hat{d}(e_t) \text{ and } c(e_t) > \epsilon \right\}
\]

This set is the inverse image of \([0, +\infty[\) by the function \(g : \hat{E}_K \to B, x \mapsto (1 - \delta)(x + \hat{d}(x)) - (1 + \hat{r}(x))\hat{d}(x) - x\). Since the policy functions are continuous, \(g\) is continuous. Since the inverse image of an open set by a continuous function is an open set, \(H(z, e) \cap \hat{E}_K\) is an open set, and hence a Borel set. The intersection \(H(z, e) \cap \hat{E}_K\) has a more complicated expression, but also boils down to a finite union of sets that are open because of the continuity of policy functions. Given that \(\hat{E}_K\) and \(\hat{E}_R\) are intervals that form a partition of \(\hat{E}\), this implies that \(H(z, e) = (H(z, e) \cap \hat{E}_K) \cup (H(z, e) \cup \hat{E}_K)\) is a finite union of Borel sets, and therefore a Borel set. This line of reasoning applies for all \(0 \leq z \leq 1\) and \(0 < e \leq \eta\).
holds in the reverse direction is handled similarly.

Now consider a knife-edge case:

\[(1 + \hat{r}(e_t))\hat{d}(e_t) = (1 - \delta)(e_t + \hat{d}(e_t))\]  

(13)

The problem is that the sequence \((e_{n,t})_n\) can have elements that satisfy either \((1 + \hat{r}(e_{n,t}))\hat{d}(e_{n,t}) \geq (1 - \delta)(e_{n,t} + \hat{d}(e_{n,t}))\), which correspond to different expressions for \(N(e_{n,t}, e_{t+1})\); one must check therefore check that \(N(. , e_{t+1})\) is continuous at \(e_t\) such that (13) holds. At such a point, \(e(\hat{e}) = 0\), so that:

\[
\lim_{\varepsilon \searrow \hat{e}} N(e_t, e_{t+1}) = \lim_{\varepsilon \searrow \hat{e}} \eta + (1 - \eta)F\left(\hat{\phi}_K(e_t) + \frac{e_{t+1}}{(e_t + \hat{d}(e_t))^\xi}\right) = \eta + (1 - \eta)F\left(\frac{e_{t+1}}{(e_t + \hat{d}(e_t))^\xi}\right).
\]

Moreover, \(\hat{\phi}_K(e_t) = 0\), so that:

\[
\lim_{\varepsilon \uparrow \hat{e}} N(e_t, e_{t+1}) = \eta + (1 - \eta)F\left(\hat{\phi}_K(e_t) + \frac{e_{t+1}}{(e_t + \hat{d}(e_t))^\xi}\right) = \eta + (1 - \eta)F\left(\frac{e_{t+1}}{(e_t + \hat{d}(e_t))^\xi}\right).
\]

This establishes the continuity of \(N(., e_{t+1})\) at points \(e_t\) such that equation (13) holds; thus, \(\forall e_{t+1} \in \bar{E}, e_{t+1} \neq \hat{e}, \quad N(., e_{t+1})\) is continuous on \(\bar{E}_K\).

Finally, given that the transition function \(Q\) has the Feller property, theorem 12.10 of Stokey, Lucas, and Prescott (1989) indicates that there exists a probability measure on \(\bar{E}\) that is invariant under \(Q\).

C The Italian tax reform

In this appendix, I first briefly describe the introduction of tax shields into the model. I then illustrate the effects of the Italian reform described in section 5.2 on the borrowing and investment choices of firms.

Introducing tax shields As mentioned in section 5.2, when gross income and debt payments are subject to differential taxation, the cash on hand of firm that repays its creditors can generally be written as:

\[n^R_t = (1 - \tau)\pi_t - (1 - \tau_b)R_{b,t} - (1 - \tau_m)R_{m,t}.\]

One must specify how the firm and its creditor’s income are taxed under payment, restructuring and liquidation. I make two key assumptions in this regard:

Assumption 4 (Tax treatment of restructuring and liquidation)

- Income tax liabilities are senior to bank and market debt payments in liquidation;
- There are no tax shields for debt payments that have been restructured.

The first assumption is innocuous, and simply guarantees that firms will not find it beneficial to default in order to avoid the payment of tax liabilities. The second assumption guarantees that, when tax shields are identical \((\tau_b = \tau_m)\),

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the restructuring choices of the firm are similar to the baseline model; it therefore helps to focus the discussion on the effects of asymmetric tax treatment of debt.

With these assumptions, the payoffs to stakeholders in liquidation are given by:

\[
\tilde{R}_{b,t} = \min \left( R_{b,t}, (1 - \tau) \chi \pi_t \right) \quad \text{(bank lenders)}
\]

\[
\tilde{R}_{m,t} = \min \left( \max(0, (1 - \tau) \chi \pi_t - R_{b,t}), R_{m,t} \right) \quad \text{(market lenders)}
\]

\[
n_L^t = \max \left( 0, (1 - \tau) \chi \pi_t - R_{b,t} - R_{m,t} \right) \quad \text{(firm)}
\]

Moreover, in restructuring the firm will drive the bank down to its reservation value, \((1 - \tau) \chi \pi_t\). In that case, given the second assumption, the cash on hand of the firm after restructuring will be given by:

\[
n_L^t = (1 - \tau) \pi_t - (1 - \tau) \chi \pi_t - (1 - \tau_m) R_{m,t}.
\]

Given this, the following lemma is straightforward to establish.

**Lemma 2 (Debt settlement outcomes)** Assume that \(V^c(.)\) is increasing, and \(V^c(0) \geq 0\). Then, there are two types of debt settlement outcomes:

- **When** \(\frac{(1 - \tau_b) R_{b,t}}{\chi} \geq \frac{(1 - \tau_m) R_{m,t}}{1 - \chi}\), the firm chooses to repay its creditors in full, if and only if, \(\pi_t \geq \frac{(1 - \tau_m) R_{b,t}}{\chi}\).
  
  It successfully restructures its debt, if and only if, \(\frac{(1 - \tau_m) R_{m,t}}{1 - \chi} \leq \pi_t < \frac{(1 - \tau_b) R_{b,t}}{\chi}\), and it is liquidated when \(\pi_t < \frac{(1 - \tau_m) R_{m,t}}{1 - \chi}\).

- **When** \(\frac{(1 - \tau_b) R_{b,t}}{\chi} < \frac{(1 - \tau_m) R_{m,t}}{1 - \chi}\), the firm repays its creditors in full if and only if \(\pi_t \geq \frac{(1 - \tau_b) R_{b,t} + (1 - \tau_m) R_{m,t}}{(1 - \tau)}\), and it is liquidated otherwise.

Moreover, in any successful restructuring offer, the bank obtains its reservation value \((1 - \tau) \chi \pi_t\), and in all debt settlement outcomes resulting in liquidations, \(n_L^t = 0\).

Thus, under the two assumptions above, the structure of the debt settlement outcomes is similar to the baseline model: when the firm’s bank liabilities are large enough, it will sometimes restructure debt contracts conditional on its productivity realizations; otherwise, it never uses the restructuring option. The difference is that, when \(\tau_b \neq \tau_m\), the firm’s decision to restructure debt contracts also depends on the relative values of the tax shield on bank and market debt, and not simply on liquidation losses (associated with the parameter \(\chi\)).

**Effects of the reform** Lemma 2 be used to fully characterize the set of feasible debt contracts, and therefore the general formulation of firms’ optimal debt structure problem when there are differential tax treatments of debt. These derivations are available upon request.

I next turn to the effects of the policy experiment described in section 5.2. Namely, I compare firm-level borrowing and investment in an economy without and with the tax reform. In the baseline economy, all firms enjoy tax shields for bank debt, but only large firms enjoy a tax shield for market debt issuance:

\[
\tau_b(e_t) = \tau \quad \forall e_t,
\]

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Figure 12: The effect of the tax reform on borrowing and investment.

\[
\tau_m(e_t) = \begin{cases} 
0 & \text{if } e_t \leq e_{sm} \\
\tau & \text{if } e_t > e_{sm}
\end{cases}
\]

In the reformed economy, however, the tax shield applies to all debt issuances of all firms:

\[
\tau_b(e_t) = \tau_m(e_t) = \tau \quad \forall e_t.
\]

As in the case of the experiment of section 5.1, so as to clarify the exposition of the effects of the subsidy, I set the threshold for the reform at \(e_{sm} = e^*\). Moreover, I focus on the borrowing policies of firms in the static version of the model. Indeed, the results of section 5.1 suggest that the bulk of the effects of this type of policy is mediated by firms’ borrowing decisions, rather than by long-run changes in the firm size distribution.

Figure 12 reports the results of the experiment, when the tax rate is \(\tau = 2.5\%\). The reform has little incidence on the debt composition of firms, but induces a measure of firms to switch to pure market finance (left panel). In doing so, these firms operate at a lower scale than they otherwise would have (right panel). The investment of all other firms, however, gains from the introduction of the tax shield. These results are qualitatively analogous to those obtained in the experiment of section 5.1.