When transparency improves, must prices reflect fundamentals better?*

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Abstract

No. Regulation often mandates increased transparency to improve how well prices reflect fundamentals. We show that such policy can be counterproductive. We study the optimal decision of an investor who can choose to acquire costly information not only about asset fundamentals but also about the behavior of liquidity traders. We characterize how changing the cost of information acquisition affects the extent to which prices reflect fundamentals. When liquidity trading is price-dependent (e.g., due to forced deleveraging), surprising results emerge: higher transparency, even if exclusively targeting fundamentals, can decrease price informativeness, and cheaper access to non-fundamental information can improve efficiency.

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The sharp decline in ABS prices during the financial crisis precipitated a period of extraordinary uncertainty. As Figure 1 shows, the AAA ABX issued at par in the second half of 2006 (i.e., ABX AAA 06-2) was trading near 30 cents on the dollar by mid-2009, while the index tracking a basket of AA securities (i.e., ABX AA 06-2) was trading below 10 cents. This collapse led to a “cascading vicious circle of falling asset prices, margin calls, fire sales, deleveraging, and further asset price deflation” (Acharya, Philippon, Richardson, and Roubini (2009)). Investors struggled to reconcile these prices with the credit rating and security data they had been provided, and were unsure whether these price drops were driven by asset fundamentals or the insolvency and forced liquidation of other investors.\(^1\) Moreover, as Figure 1 suggests, the uncertainty lasted for many months and was not apparently resolved by the evolution of prices.\(^2\)

The above episode highlights that while investors, regulators and academics often turn to security prices to infer the “market’s expectation” of fundamentals, frictions can drive a wedge between the two and amplify investor uncertainty, especially during crises. A natural regulatory response during such episodes is to improve transparency. For instance, the events of the subprime crisis triggered the introduction of higher requirements for disclosure of loan-level data.

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\(^1\)In fact, Stanton and Wallace (2011) point out that market prices for AAA ABX indices in June 2009 were “inconsistent with any reasonable assumptions for future default rates;” even assuming a recovery rate below any ever observed before in the United States implied default rates in excess of 100%. This suggests that these price changes were not completely driven by fundamental shocks.

\(^2\)While the AAA and AA 06-2 indices fell in lockstep between 2007 and mid-2009, the AAA tranches had recovered to almost 60 cents on the dollar by the end of 2012, but the AA tranches stayed around 10 cents. At the same time, the spread between the 06-1 indices has remained relatively stable since mid-2009.
as part of the Dodd-Frank Act (2010), disclosure of bank stress test results (see Goldstein and Sapra (2012)) and the provision of forward guidance on monetary policy (see Bernanke (2013)).

In such situations, a question naturally arises: when investors face uncertainty along multiple dimensions and choose what information to acquire, do these policies necessarily increase the extent to which prices reflect fundamentals?

Our analysis suggests that the answer to this question is no. In the presence of frictions, investors often choose to learn not only about asset fundamentals (e.g., cash-flows, systematic risk), but also about other traders. Equilibrium prices reflect both types of information, and as a result, need not reflect fundamentals more accurately when learning becomes easier. Surprisingly, we show that even when the increase in transparency targets fundamental information, price informativeness about fundamentals can decrease. Similarly, making it easier to learn about other traders exclusively can lead to an increase in price efficiency.

We consider a three-date (two-period) model. At date 3, the risky asset pays a terminal dividend, which reflects the asset’s fundamental value. We study the information acquisition and trading decisions of a risk-neutral investor, who maximizes terminal wealth, faces quadratic transaction costs, and anticipates trading the risky security with liquidity (noise) traders at dates 1 and 2. She is also subject to a liquidity shock — with a positive probability, she is forced to liquidate her position before trading at date 2 and exit the market. As a result, her optimal demand for the asset at date 1 depends not only on her beliefs about fundamentals, but her beliefs about the price at date 2 — this implies that she has an incentive to learn about both fundamentals and the actions of other traders.

The key feature that distinguishes our model from standard, noisy rational expectations (RE) models, is that the liquidity trading can be price-dependent: a component of the aggregate noise trader demand is generated by feedback trading. The investor is uncertain about the

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3Prior to the financial crisis, issuers of MBS were only required to provide aggregate data, such as the weighted-average coupon, or distributional data, including the number of borrowers with a FICO score in a given range. In response, Congress passed Title IX of the Dodd-Frank Act (2010), which requires issuers of asset-backed securities to disclose mortgage-level data. Similarly, the Sarbanes-Oxley Act (2002), which encourages greater disclosure within the financial statements of publicly-traded firms, was passed partly in response to the lack of transparency brought to light by the accounting scandals of the early 2000’s (e.g., Enron, WorldCom and Tyco).

4She is replaced by another, identical risk-neutral investor, who lives for the remaining period. As we discuss in Section 2.4, the liquidation shock provides a transparent motivation for the investor to learn about the intermediate price. Other incentives to do the same (e.g., short-term, performance-based compensation for money managers) should lead to similar results.

5While the assumption of price-independent noise is often made for modeling convenience, price-dependent noise trading is likely to be quite relevant. As we discuss in Section 2.4, such trading behavior may arise not only due to behavioral biases (e.g., extrapolative expectations), but also as a consequence of institutional and market frictions such as forced deleveraging during financial crises and performance-flow sensitivity in delegated asset management. A number of recent papers consider alternative specifications in which noise trading depends on the contemporaneous price (e.g., Goldstein, Ozdenoren, and Yuan (2013b), Goldstein and Yang (2014a)).
fundamental value of the asset and the demand from liquidity traders, but can choose the precision of her information about either dimension, subject to a cost of information acquisition. The investor derives value from trading the risky asset in the presence of feedback trading, and the extent to which she can take advantage of such speculative opportunities depends on what she learns about fundamentals and liquidity demand.

To highlight the intuition for how higher transparency can reduce price informativeness, consider the following example. Suppose the asset’s fundamental value is uniformly distributed around $100, feedback trading is equally likely to be positive or negative (and is independent of the fundamental value) and the price-independent component of noise trading is zero. Without any additional information, the initial price of the risky asset is its unconditional expected value, $100.

First, consider the case when the investor learns the asset’s fundamental value is $50 before trading at date 1, but learns nothing about liquidity demand. Then, the date 1 price is $50 — the price is efficient and it accurately reflects the investor’s expectation of the asset’s fundamentals. Next, note that the anticipation of liquidity demand in the future can distort current prices. To see this, suppose the investor also learns that feedback trading is more likely to be positive: for instance, there could be investors or intermediaries who, faced with financing constraints, are forced to delever as prices fall. Given the negative signal about fundamentals, she anticipates liquidity traders will sell the asset, depressing tomorrow’s price. Consequently, the current price is below her conditional expectation of $50 — learning about feedback traders decreases price accuracy.

Importantly, liquidity demand responds endogenously to price changes in our model: the feedback demand tomorrow depends upon the price today, which in turn, depends on the investor’s expectation of feedback demand tomorrow. Crucially, this price-dependence implies that learning about fundamentals and liquidity trading is complementary. More precise information about fundamentals leads to a bigger change in today’s price, which leads to more

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6 As we discuss in Section 2.4, such trading may be increasingly relevant given recent developments in financial markets and is especially important in times of financial market stress.

7 Note that trading frictions are crucial to generating valuable speculative opportunities for the rational investor. In the absence of transaction costs, a risk-neutral investor is able to absorb all the demand from noise traders, and so force the price equal to her conditional expectation of the asset’s value; hence, there will be no price distortion.

8 Note that while the investor recognizes the impact of current prices on noise trader demand when forming her beliefs, she does not manipulate today’s price strategically — today’s price is simply a weighted-average of her expectation of the price tomorrow and the fundamental value of the asset. Moreover, trading frictions are crucial to generating valuable speculative opportunities for the rational investor. In the absence of transaction costs, a risk-neutral investor is able to absorb all the demand from noise traders, and so force the price equal to her conditional expectation of the asset’s value; hence, there will be no price distortion.
feedback trading tomorrow — this increases the value of learning more about liquidity trading. This complementarity gives rise to predictions that differ from those in linear, noisy RE models (e.g., Grossman and Stiglitz (1980), Kyle (1985)), where noise trading is price-independent.\(^9\) In such models, the price is a linear function of agents’ information about fundamentals and noise, and so improving information about fundamentals usually leads to higher price efficiency. In contrast, since the noise in our model is price-dependent, the price depends non-linearly on the investor’s information. Surprisingly, this implies that increasing information about fundamentals, when accompanied by sufficient learning about liquidity traders, can make the price less informative about fundamentals.

We consider two measures of price informativeness. The first, which we denote as accuracy, captures how closely, on average, the level of prices reflects fundamentals. The second, which we call efficiency, measures the error in the conditional expectation of fundamentals, given the information in the price.\(^10\) In the absence of feedback trading, the two measures are closely related — more learning about fundamentals (liquidity trading) leads to an increase (decrease) in both accuracy and efficiency. However, in the presence of price-dependent liquidity trading, we show that the two measures can move in opposite directions.

To see the relation between accuracy and efficiency, first note that in our example above, when the investor learns more about feedback trading, the price moves away from fundamentals (i.e., $50) — this is a decrease in accuracy. Now, consider an outside observer who forms an expectation about fundamentals, conditional on the date 1 price alone. When the investor does not learn about liquidity trader demand, the date 1 price reflects only her information about fundamentals. However, when she also learns about liquidity traders, the price is a noisy signal about fundamentals since it also reflects beliefs about liquidity demand. For instance, suppose the date 1 price is $50 (see Figure 2). This could correspond to a fundamental value of $50 and no price-dependent liquidity demand, to a higher fundamental value with some positive feedback demand (solid line), or to a lower fundamental value with some negative feedback (dotted line). Conditional on the price, the outside observer is more uncertain about fundamentals, i.e., efficiency is lower, when the investor learns about liquidity demand.

However, while more learning about liquidity traders always decreases accuracy, it can

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\(^9\) As we discuss in Section 1, learning about fundamentals may be complementary across investors in noisy RE models. However, this is distinct from the result we highlight — learning about fundamentals and noise trading is complementary for the single investor. In standard, noisy RE models with heterogeneous information, the equilibrium price serves the additional role of imperfectly “aggregating” private information about fundamentals. We consider a setup with a single, informed investor, and so the price “reflects” her information perfectly. However, the price may still be inefficient in that it does not reflect information about fundamentals accurately.

\(^10\) More precisely, given fundamentals $\phi$ and price $P$, accuracy captures $-E[(\phi - P)^2]$ and efficiency captures $-E[(\phi - E[\phi|P])^2]$.  

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The figure plots the price as a function of the date \( t = 0, 1, 2, 3 \) for the following cases. For the solid line, the fundamental value $70 and there is positive feedback trading. For the other three lines, the fundamental value is $30. The dotted line corresponds to the price path when feedback trading is negative and anticipated by the investor, the dashed line is when the feedback trading is positive and anticipated, and the dot-dashed line is when it is positive but unanticipated by the investor.

sometimes *increase* efficiency. For instance, an extremely low price is much more likely when fundamentals are low and feedback trading is positive, than when fundamentals are high. Furthermore, conditional on extreme price realizations, the difference in the relative likelihoods of the underlying fundamentals increases as the investor learns more about feedback trading. This implies that efficiency can *increase* with more learning about liquidity trading — when the investor learns more about feedback traders, small differences in beliefs about fundamentals can become amplified, making prices more informative about fundamentals.\(^{11}\) Figure 2 plots an instance — for the same fundamentals and feedback intensity, prices are lower when the investor learns about liquidity demand (dashed line) than when she does not (dot-dashed line).

We provide an analytical characterization of how transparency affects accuracy and efficiency, and then explore its implications using a series of examples. The complementarity between learning about fundamentals and feedback trading implies that an *increase* in overall transparency can *decrease* both efficiency and accuracy. Moreover, the complementarity introduces an amplification mechanism: small increases in transparency can lead to disproportionately large decreases in efficiency and accuracy. Finally, as the above discussion suggests, we find that efficiency and accuracy do not always respond in the same direction with changes

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\(^{11}\)This suggests one explanation for why the ABX AA 06-1 and ABX AA 06-2 have such different outcomes despite moving in tandem through mid-2009 — while the 06-2 index remained relatively stable and never rose above 20 cents between 7/09 and the 7/12, the 06-1 index was more volatile and oscillated between 20 and 60 cents over the same period. Investors may have interpreted the more extreme price decline in 06-2 before 7/09 as a more informative signal about fundamentals, while being more uncertain about interpreting the price changes in the 06-1 index.
in transparency. In particular, while more learning about feedback trading always decreases accuracy, it can actually increase efficiency. Similarly, while learning more about fundamentals tends to increase efficiency, it can also lead to a decrease in accuracy.

The complementarity between learning along different dimensions also generates counterintuitive predictions when transparency is targeted along a specific dimension. When the increased transparency targets fundamental information, the investor learns more about fundamentals. But this makes learning about feedback trading more valuable, and the resulting increase in learning about noise trading can actually decrease accuracy and efficiency. As a result, policy that targets higher transparency about fundamentals (e.g., forward guidance by central banks, disclosure of stress tests outcomes) may be counterproductive, especially during crises accompanied by deleveraging cycles. Similarly, a decrease in transparency about noise trading leads to less learning about feedback trading, but this reduces the value of learning about fundamentals. In some instances, this can lead to a decrease in price efficiency and accuracy. This cautions against policies that limit the availability or timeliness of information about other participants, as have been recently proposed to mitigate the adverse effects of high-frequency traders (e.g., Harris (2013)).

Instead of dampening our results, we show that allowing feedback trading to respond to an increase in transparency can amplify the effects on price uninformativeness. When learning becomes easier for the investor, a natural response for liquidity traders is to reduce the intensity of their feedback trading. But such a response decreases the investor’s opportunities to speculate, and therefore reduces her incentive to acquire information. As a result, when the response by liquidity traders is large enough, we show that an increase in transparency can actually lead to less learning about fundamentals, which can lead to lower efficiency and accuracy.

Since accuracy and efficiency can respond to policy changes differently, our results highlight the importance of clarifying which measure of price informativeness is being targeted for empirical and policy analysis. Moreover, it is not immediate which measure is the right one, especially when investors, academics and regulators are uncertain about the structure of the economy. While efficiency is the appropriate theoretical measure for agents within the model, it is difficult to measure in practice since it requires knowledge of the joint distribution of fundamentals, liquidity demand and prices. In contrast, accuracy is easier to estimate empirically, may be more robust to mis-specification and appears to match “real-world” measures commonly used by empirical studies and regulators.

Our analysis recommends against the construction of naive price signals to infer changes

\footnote{Filardo and Hofmann (2014) empirically evaluate the impact of forward guidance by the Federal Reserve, the Bank of Japan, the ECB, and the Bank of England on the level and volatility of interest rate expectations, and discuss its role in potentially encouraging excessive risk-taking by investors.}
in fundamentals, since prices combine investor beliefs about fundamentals and other traders non-linearly. We show that an increase in transparency is more likely to decrease price informativeness when investors are already sufficiently informed about fundamentals. Given recent technological and regulatory changes that improve access to information, the mechanism we describe is increasingly important and may help explain the increased importance of sophisticated institutional investors that focus on learning about the behavior of other investors (e.g., statistical arbitrageurs, high frequency traders). 

The paper proceeds as follows. The next section discusses the relevant related literature. Section 2 describes the model, characterizes the equilibrium, and solves for the investor’s optimal information acquisition. Section 3 presents the main analysis of the paper. It describes how price accuracy and efficiency change with general and targeted transparency, and how our results are affected when feedback trading can respond to changes in transparency. Section 4 discusses some implications of our analysis and concludes. Proofs for the main results can be found in the Appendix.

1 Related Literature

A number of papers, including DeLong et al. (1990), Cutler, Poterba, and Summers (1991), Hong and Stein (1999), and Barberis, Greenwood, Jin, and Shleifer (2014), explore the impact of feedback trading on asset prices. In our paper, we extend the analysis of DeLong et al. (1990) by endogenizing the information received by the rational investor. Empirically, Greenwood and Shleifer (2014) use survey data to document the existence of investors with extrapolative expectations, which can induce feedback demand. We show that the anticipation of this demand (not simply the demand shock itself) can reduce the extent to which prices reflect fundamental information.

Our paper is related to the large literature on endogenous information acquisition in financial markets. Counter to the standard intuition of Grossman and Stiglitz (1980), a number of papers have identified different channels through which learning about fundamentals can be complementary across investors. Most closely related are papers in which this complemen-

\textsuperscript{13} Along these lines, in their Concept Release on Equity Market Structure (Federal Register Volume 75, Issue 13, 2010), the Securities and Exchange Commission (SEC) specifically requested comment on “two types of directional strategies that may present serious problems in today’s market structure – order anticipation and momentum ignition,” and their role in price discovery. Order anticipation strategies seek to “ascertain the existence of one or more large buyers (sellers) in the market and to buy (sell) ahead of the large orders with the goal of capturing a price movement in the direction of the large trading interest (a price rise for buyers and a price decline for sellers).” A momentum ignition strategy involves initiating “a series of orders and trades... in an attempt to ignite a rapid price move either up or down.”

tarity results from the fact that learning more about fundamentals makes the price a more informative signal about noise trading (as in Avdis (2012) and Cespa and Vives (2014)) — hence, acquiring fundamental information can become more valuable as the number of informed investors increases. Our analysis focuses on a single investor and highlights complementarity across different payoff components — learning about fundamentals makes learning about other investors more valuable. As such, it is closely related to that of Goldstein and Yang (2014b) who show that acquiring information about different components of fundamentals can be complementary.\textsuperscript{15} Moreover, we consider a setting in which an investor can obtain a direct signal about aggregate noise trader demand, similar to Ganguli and Yang (2009). The feature that distinguishes our model from this class of linear RE models, however, is that the noise trading in our setting is price-dependent: more precise learning about fundamentals leads to larger price changes today, which increases future demand from feedback traders, thereby increasing the value of information about others. The endogenous nature of noise trading generates predictions that do not naturally arise in linear RE models — for instance, an increase in transparency about fundamentals can lead to a decrease in price informativeness, even when the investor learns more about fundamentals.

Finally, our paper relates to the broad literature that studies the costs and benefits of higher transparency and disclosure. Our model is stylized to highlight a novel tradeoff associated with increased transparency, and as such, abstracts from other tradeoffs already analyzed in the literature (see Goldstein and Sapra (2012) and Bond, Edmans, and Goldstein (2012) for recent surveys). On the one hand, improved transparency and disclosure can decrease adverse selection across market participants, reveal valuable information to real decision makers (and hence induce better allocative efficiency), and provide better market and supervisory discipline for firms. On the other hand, such changes can also reduce risk-sharing (i.e., the Hirshleifer (1971) effect), lead to over-investment in disclosure (e.g., Fishman and Hagerty (1989)), induce risk-shifting and short-termism in managerial decisions (e.g., Sapra (2002)), generate inefficient coordination on public information (e.g., Morris and Shin (2002)), crowd out the ability of managers or regulators to learn from the market (e.g., Bond and Goldstein (2013), Goldstein and Yang (2014c)) and reduce expected returns for investors (e.g., Kurlat and Veldkamp (2013)).

\textsuperscript{15}In their noisy RE model, learning about the first component of fundamentals reduces uncertainty about trading on, and encourages learning about, the second component.
2 The Model

This section presents the benchmark model. We consider the information acquisition decision of the risk-neutral investor (i.e., the optimal choice of precision of the signals), subject to a general cost function. The first subsection describes the setup of the model. Section 2.2 characterizes the financial market equilibrium (prices and quantities) and discusses some salient features of our setting. Section 2.3 formally characterizes the investor’s optimal information acquisition problem, and discusses the incentives for the investor to learn along various dimensions. Section 2.4 provides a discussion of our assumptions and results.

2.1 Model Setup

Assets and payoffs. There are three dates (i.e., \( t \in \{1, 2, 3\} \)) and two assets. The risk-free security is in perfectly elastic supply, and the net risk-free rate is normalized to zero. The risky asset is in zero net supply, is traded at dates \( t \in \{1, 2\} \) at price \( P_t \), and pays a liquidating dividend \( \phi + \theta \) at date 3, where \( \phi \) and \( \theta \) are independent and have finite first and second moments.\(^{16}\) The predictable component of the asset’s payoff is captured by \( \phi \) — the realization of \( \phi \) is revealed at date 2, and the rational investor can acquire costly information about \( \phi \) prior to trading at date 1. In contrast, \( \theta \) reflects the residual / unpredictable component of the asset’s payoff since no information about \( \theta \) is revealed before the payoff is realized at \( t = 3 \). Without loss of generality, we set \( \mathbb{E}_0[\theta] = 0 \) and \( \mathbb{E}_0[\phi] = 0 \). We make additional distributional assumptions about \( \phi \) when we solve for the optimal information acquisition decisions in Section 2.3.

Investor preferences, information and beliefs. There are two types of traders in the model who exist in equal measure: noise traders and a risk-neutral investor. We assume that the noise traders’ aggregate demand for the risky asset, denoted by \( Z_t \), is given by:

\[
Z_t = Z_{t-1} + \beta(P_{t-1} - P_{t-2}) + u_t, \tag{1}
\]

where \( u_t \) and \( \beta \) are independent of each other and of \( \phi \) and \( \theta \), and for completeness, we set \( P_0 = P_{-1} = \mathbb{E}_0[\phi + \theta] = 0 \).\(^{17}\) There are two components to noise trading in our model. The first component, \( u_t \), captures price-independent liquidity demand, and corresponds to the standard

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\(^{16}\) The assumption of zero net supply is without loss of generality. An alternative formulation, which generates identical results, specifies that the risk-neutral investor holds the entire supply of the risky asset \( Q \neq 0 \) at date zero.

\(^{17}\) If we allowed for trading at date 0, before the investor updates her beliefs based on her acquired signals, the equilibrium price would be \( \mathbb{E}_0[\phi + \theta] \). For expositional clarity, we do not show the equilibrium derivation here.
specification of aggregate liquidity shocks in most rational expectations models. The second component, $\beta (P_{t-1} - P_{t-2})$, captures price-dependent feedback trading. As we show below, the rational investor will generally have an incentive to learn about both components of noise. How much is learned about each component, and the impact this learning has on the extent to which prices reflect information, will differ in important ways. In order to maintain tractability, we do not explicitly model the preferences of noise traders. Our reduced form specification is in line with the standard approach in the literature (e.g., DeLong et al. (1990), Barberis et al. (2014)), and is consistent with empirical and survey evidence, as we discuss in Section 2.4.

We assume that there is a representative, risk-neutral investor who maximizes terminal wealth, subject to quadratic transaction costs. With probability $\rho \equiv \Pr (\xi = 1)$ she is subject to forced liquidation before trading at date 2, where $\xi \in \{0, 1\}$ is an indicator variable for the liquidity shock which is independent of both fundamental and noise shocks. As such, when $\rho \neq 0$, she has incentive to learn about factors which affect the short-term value of the asset. Denote her optimal demand at date $t$ by $x_t$. Conditional on being able to trade at date 2 (i.e., if $\xi = 0$), the investor chooses a limit order $x_2$ to maximize

$$V_2 = \max_x \mathbb{E}_2 \left[ x (\phi + \theta - P_2) - \frac{\xi}{2} (x - x_1)^2 + x_1 (P_2 - P_1) - \frac{\xi}{2} (x_1)^2 \right].$$

Since she is forced to liquidate before trading at date 2 with probability $\rho$, at date 1 she chooses a limit order $x_1$ to maximize

$$V_1 = \max_x \mathbb{E}_1 \left[ (1 - \rho) V_2 (x, \phi, P_2, P_1) + \rho \left\{ x (P_2 - P_1) - \frac{\xi}{2} x^2 \right\} \right].$$

If forced to liquidate (i.e., $\xi = 1$), she is replaced by an investor who lives for one period and submits a limit order $x^\rho$ to maximize

$$V^\rho_2 = \max_x \mathbb{E}_2^\rho \left[ x (\phi + \theta - P_2) - \frac{\xi}{2} x^2 \right].$$

The price of the risky asset is set by market clearing. At date 2, the market clearing condition is given by

$$\xi x_2^\rho + (1 - \xi) x_2 + Z_2 = 0,$$

and at date 1, the market clearing condition is given by

$$x_1 + Z_1 = 0.$$
2.2 Financial Market Equilibrium

We solve for the equilibrium by working backwards. At date 3, all uncertainty is resolved, and investors are paid the realized dividend, $\phi + \theta$.

**Date 2:** Before trade occurs, $\phi$ is publicly revealed. If the risk-neutral investor is not forced to liquidate before trading at date 2, her optimal demand maximizes the objective in (2) and is given by

$$x_2 = x_1 + \frac{1}{c} (\phi - P_2),$$

and her value function simplifies to

$$V_2 = \frac{1}{2c} (\phi - P_2)^2 + x_1 (\phi - P_1) - \frac{c}{2} x_1^2.$$  

If she is forced to liquidate, the optimal demand of the investor who replaces her solves (4), and is given by

$$x_2^\rho = \frac{1}{c} (\phi - P_2).$$

The market clearing condition at date 2 implies that, in either case, the date 2 price is given by

$$P_2 = \phi + c (Z_2 + (1 - \xi) x_1).$$

**Date 1:** Before trading, the investor observes her private information about fundamentals and noise trading. She maximizes the objective function in (3), which after substituting in (8), simplifies to

$$V_1 = \max_x \mathbb{E}_1 \left[ (1 - \rho) \frac{1}{2c} (\phi - P_2)^2 + x ((1 - \rho) \phi + \rho P_2 - P_1) - \frac{c}{2} x^2 \right].$$

This implies that the optimal date 1 demand is given by

$$x_1 = \frac{1}{c} \left( (1 - \rho) \mathbb{E}_1 [\phi] + \rho \mathbb{E}_1 [P_2] - P_1 \right),$$

and market clearing at date 1 implies that the price is given by

$$P_1 = (1 - \rho) \mathbb{E}_1 [\phi] + \rho \mathbb{E}_1 [P_2] + c Z_1.$$  

Ignoring noise trader demand, the date 1 price is a weighted-average of the investor’s expectation of the asset’s short- and long-term value, a result of the investor’s partial myopia. Finally, note that since $P_0 = P_{-1} = 0$, $Z_1 = u_1$ and $Z_2 = \beta P_1 + u_2 + u_1$.\(^{19}\) Combining this and the expression

\(^{19}\)The assumption that the price-independent component of noise trader demand follows a random walk is not necessary for our results to hold. Qualitatively similar results arise, for instance, if this component follows
for the date 2 price in (10) implies that the date 1 price is

\[ P_1 = \frac{\mathbb{E}_1[\phi] + \rho c \mathbb{E}_1[u_2] + c(1 + \rho^2)u_1}{1 - \rho c \mathbb{E}_1[\beta]} . \]  

(14)

It is important to note that for this to be an equilibrium price, one needs \( \mathbb{E}_1(\beta) \leq 1/c \) — otherwise, the equilibrium does not exist (see DeLong et al. (1990)). By conditioning on the price at date 1, the investor can infer \( Z_1 = u_1 \) perfectly. We can now simplify the objective function in (3):

\[ V_1 = (1 - \rho) \frac{\phi}{2} \mathbb{E}_1[(\beta P_1 + u_2 + \xi u_1)^2] + \frac{c}{2} u_1^2 . \]  

(15)

The following proposition summarizes these results and characterizes the equilibrium prices.

**Proposition 1.** Suppose \( \beta < 1/c \). Then equilibrium prices are given by:

\[ P_1 = \frac{\mathbb{E}_1[\phi] + \rho c \mathbb{E}_1[u_2] + c(1 + \rho^2)u_1}{1 - \rho c \mathbb{E}_1[\beta]}, \quad \text{and} \quad P_2 = \phi + c(\beta P_1 + u_2 + \xi u_1) . \]  

(16)

### 2.3 Optimal Information Acquisition

In order to completely characterize the optimal information acquisition decision, we specify distributional assumptions. Suppose that \( \phi \sim N(0, \sigma^2_f) \), \( \theta \sim N(0, \sigma^2_\theta) \), \( u_t \sim N(0, \sigma^2_{u,t}) \) and \( \beta \sim \{-b, b\} \) with equal probability and \( 0 < b < c \).\(^{20}\) Furthermore, suppose that the information acquisition technology allows the investor to acquire the following signals:\(^{21}\)

\[ S_f = \phi + e_f, \quad \text{where} \quad e_f \sim N(0, \sigma^2_{f, e}) \]  

(17)

\[ S_u = u_2 + e_u, \quad \text{where} \quad e_u \sim N(0, \sigma^2_{u, e}) \]  

(18)

\[ S_b = \begin{cases} 
\beta & \text{with probability } q_b \\
-\beta & \text{with probability } 1 - q_b 
\end{cases} \]  

(19)

For convenience, we define \( \kappa_f \equiv \frac{\sigma^2_f}{\sigma^2_f + \sigma^2_{f, e}} \), \( \kappa_u \equiv \frac{\sigma^2_{u,2}}{\sigma^2_{u,2} + \sigma^2_{u, e}} \) and \( \kappa_b \equiv (2q_b - 1) \), and note that

\[ \mathbb{E}[\phi|S_f] = \kappa_f S_f, \quad \mathbb{E}[u_t|S_{ut}] = \kappa_u S_u, \quad \mathbb{E}[\beta|S_b] = \kappa_b S_b . \]  

(20)

Note that for \( i \in \{f, u, b\} \), \( \kappa_i \in [0, 1] \) is a normalized measure of the precision of signal \( S_i \). When \( \kappa_i = 1 \), the signal \( S_i \) is perfectly informative; when \( \kappa_i = 0 \), it is perfectly uninformative.

\(^{20}\)Note that \( \beta \) must be bounded above; we chose the binomial distribution for tractability in updating.

\(^{21}\)Given the assumption that \( \phi \) is normally-distributed, Mackowiak and Wiederholt (2009) prove that the optimal signal structure is also Gaussian. We conjecture an analogous result holds for the Bernoulli distribution.
Given these distributional assumptions, we can now rewrite the date 1 price in terms of the investor’s signals. First, denote

\[ y \equiv \kappa_f S_f + \rho\kappa_u S_u + c \left( 1 + \rho^2 \right) u_1. \]  

(21)

In the absence of feedback trading (i.e., \( \beta = 0 \)), this linear combination of \( S_f, S_u \) and \( u_1 \) is the date 1 price (i.e., \( P_1 = y \)). More generally, the date 1 price can be expressed as:

\[ P_1 = \frac{\kappa_f S_f + \rho\kappa_u S_u + c(1+\rho^2)u_1}{1 - \rho c \kappa_b S_b} = \frac{y}{1 - \rho c \kappa_b S_b}. \]  

(22)

As the investor learns more about both fundamentals and noise trading, the variance of the price increases, which in turn affects both the investor’s expected utility (29) as well as an uninformed observer’s ability to extract information. We denote

\[ \alpha \equiv \frac{\rho^2 c^2 \sigma_{u,2}^2}{\sigma_f^2} \quad \text{and} \quad \omega \equiv \frac{c^2(1+\rho^2)^2 \sigma_{u,1}^2}{\sigma_f^2}, \]

(23)

so that the variance of \( y \) is given by:

\[ \sigma_y^2 = \kappa_f \sigma_f^2 + \kappa_u \rho^2 c^2 \sigma_{u,2}^2 + c^2 \left( 1 + \rho^2 \right)^2 \sigma_{u,1}^2 = \sigma_f^2 (\kappa_f + \alpha \kappa_u + \omega). \]  

(24)

Given this parameterization, \( \alpha \) represents the relative variation in the date 1 price generated by uncertainty about \( u_2 \) versus prior uncertainty about fundamentals (i.e., \( \phi \)). Similarly, \( \omega \) represents the relative variation in the price due to uncertainty about noise trading at date 1 versus prior fundamental uncertainty.

For a given choice of precisions \( \{\kappa_f, \kappa_u, \kappa_b\} \), the investor pays a cost \( C(\kappa_f, \kappa_u, \kappa_b, h) \), where \( h \) parameterizes transparency. In particular, we assume

\[ C_h \equiv \frac{\partial C}{\partial h} < 0, \quad C_{hf} \equiv \frac{\partial^2 C}{\partial h \partial \kappa_f} \leq 0, \quad C_{hu} \equiv \frac{\partial^2 C}{\partial h \partial \kappa_u} \leq 0, \quad C_{hb} \equiv \frac{\partial^2 C}{\partial h \partial \kappa_b} \leq 0. \]  

(25)

As such, increasing transparency, \( h \), decreases the (marginal) cost of learning about \( \phi, u_2 \) and \( \beta \). We also assume that the cost function is increasing and convex in the precisions i.e.,

\[ C_i \equiv \frac{\partial C}{\partial \kappa_i} > 0, \quad C_{ii} \equiv \frac{\partial^2 C}{\partial \kappa_i^2} > 0, \]

(26)

and is separable along the three dimensions i.e., for \( i \in \{f, u, b\} \) and \( j \in \{f, u, b\} \neq i \), \( C_{ij} = 0 \).

Before trading at date 1, the investor optimally chooses \( \{\kappa_f, \kappa_u, \kappa_b\} \) to maximize her ex-
pected utility subject to the cost function $C(\kappa_f, \kappa_u, \kappa_b, h)$. Given the characterization of the financial market equilibrium, the investor’s optimal choice of signals can be represented as:

$$\{\kappa_f, \kappa_u, \kappa_b\} = \arg \max_{\kappa_f, \kappa_u, \kappa_b} E_0 [V_1] - C(\kappa_f, \kappa_u, \kappa_b, h)$$

(27)

$$= \arg \max_{\kappa_f, \kappa_u, \kappa_b} E_0 [(1 - \rho) \frac{c}{2} E_1 [(\beta P_1 + u_2 + \xi u_1)^2]] - C(\kappa_f, \kappa_u, \kappa_b, h)$$

(28)

$$= \arg \max_{\kappa_f, \kappa_u, \kappa_b} (1 - \rho) \frac{c}{2} \left[ \frac{1+x^2}{(1-x^2)^2} b^2 \sigma_f^2 + \frac{2 x^2}{1-x^2} (\kappa_u \sigma_u^2, (1+\rho^2) \sigma_{u,1}^2) \right] - C(\kappa_f, \kappa_u, \kappa_b, h)$$

(29)

$$\equiv \arg \max_{\kappa_f, \kappa_u, \kappa_b} V_0 - C(\kappa_f, \kappa_u, \kappa_b, h),$$

(30)

where $x \equiv \rho cb \kappa_b$. The second equality follows from the fact that only the distribution of the date 1 price $P_1$ depends on the investor’s choice of precisions $\{\kappa_f, \kappa_u, \kappa_b\}$, and the third equality follows from the distributional assumptions. The characterization of the value function highlights the important role of feedback traders — in the absence of price-dependent noise trading (i.e., when $b = 0$), the representative investor has no incentive to acquire information.

Consider the marginal value of learning along each dimension:

$$V_f \equiv \frac{\partial V_0}{\partial \kappa_f} = (1 - \rho) \frac{c}{2} \left[ \frac{1+x^2}{(1-x^2)^2} b^2 \sigma_f^2 \right]$$

(31)

$$V_u \equiv \frac{\partial V_0}{\partial \kappa_u} = (1 - \rho) \frac{c}{2} \left( \frac{1+x^2}{(1-x^2)^2} b^2 \rho^2 \sigma_f^2 + \frac{2 x^2}{1-x^2} \right) \sigma_u^2, \quad \sigma_{u,1}^2$$

(32)

$$V_b \equiv \frac{\partial V_0}{\partial \kappa_b} = \frac{(1-\rho) \rho c^2 b x (b^2 (3+x^2) \sigma_f^2 (\kappa_f + \alpha \kappa_u + \omega) + 2 (1-x^2) (\kappa_u \sigma_u^2) + (1+\rho^2) \sigma_{u,1}^2)}{(1-x^2)^3}$$

(33)

These expressions provide some intuition for the investor’s optimal choice of information. First, in the absence of noise trading at date 1 (i.e., when $\sigma_{u,1}^2 = 0$), the investor will only learn about $\beta$ if she is also learning about $\phi$ or $u_2$. If the investor does not learn about either $\phi$ or $u_2$ (i.e., $\kappa_f = \kappa_u = 0$), and if $\sigma_{u,1}^2 = 0$, the date 1 price must be equal to the date 0 price. As a result, irrespective of the level of $\beta$, there would be no feedback demand from noise traders, and consequently, no trading opportunity for the investor, which leaves her date 0 utility unchanged. The opposite does not hold; if the investor is not learning about $\beta$, she will still have an incentive to learn about both $\phi$ and $u_2$. Second, feedback trading introduces complementarity in learning — the marginal value of learning about $\phi$ and $u_2$ is increasing in how much the investor knows about $\beta$ (i.e., increasing in $\kappa_b$, through $x$), and the marginal value of learning about $\beta$ is increasing in $\kappa_f$ and $\kappa_u$. Finally, the marginal value of learning along a particular dimension is increasing in the prior uncertainty that the investor faces along that dimension. For instance, learning about $\phi$ ($u_2$) is more valuable when $\sigma_f^2$ ($\sigma_{u,2}^2$, respectively) is higher. Similarly, the marginal value of learning about $\beta$ is increasing in $b$, higher values of
which imply higher uncertainty about the feedback effect.

If ease of learning corresponds to sophistication, the above characterization suggests that more sophisticated investors choose to learn more about the behavior of other traders. Although at first glance this may appear inconsistent with the standard notion of financial sophistication, it provides a natural interpretation for the behavior of extremely sophisticated institutional investors (e.g., statistical arbitrageurs and high-frequency traders) who focus much of their attention on learning about other market participants. Given the increase in transparency and technological sophistication over the last few decades, our results can help explain the recent increase in popularity of strategies that exploit predictability in order-flow, and why investors are willing to pay for such information.

Under sufficient regularity assumptions on the cost of acquiring information, the following proposition characterizes the investor’s optimal information acquisition decision.

**Proposition 2.** Suppose for every $h$, the cost function $C$ is separable, increasing, and convex in the choice of precisions $\{\kappa_f, \kappa_u, \kappa_b\}$. Then, the optimal choice of precisions $\kappa_f(h)$, $\kappa_u(h)$ and $\kappa_b(h)$ is characterized by $V - C \geq 0$ and the following first order conditions:

\[
V_f - C_f \leq 0, \quad V_u - C_u \leq 0, \quad V_b - C_b \leq 0,
\]

where the equalities are strict when the corresponding choice of precisions is strictly greater than zero (i.e., $V_i - C_i = 0$ when $\kappa_i(h) > 0$).

**2.4 Discussion**

Transactions costs play a critical role in generating the feedback bubble. The investor’s expected utility is increasing in the cost parameter $c$ — in fact, $V_0$ is zero, its lowest value, when there is no cost to trade. Intuitively, when the investor is unconstrained, the price of the risky asset is unaffected by the actions of the noise traders and simply reflects the fundamental value (i.e., $P_2 = \phi$ and $P_1 = E_1[\phi]$). In order for feedback traders to have an effect on the price $P_2$, and consequently for valuable speculative opportunities to exist, the investor must be constrained. While transactions costs provide a transparent way to capture such a constraint, alternative assumptions (e.g., risk-aversion) suffice as well. Similarly, the liquidity shock $\xi$ plays an important role: in the absence of a liquidity shock (i.e., if $\rho = 0$), the investor is long-lived and her demand at date 1 only depends on her beliefs about fundamentals. In turn, this implies she has no motive to learn about noise traders. Other motivations (e.g., short-term, performance-based compensation for institutional investors) that generate similar incentives for the investor to learn about intermediate prices would yield similar implications.
While we focus on the case of a single investor, one can show that, in a setting with \( N > 1 \), identically-informed, risk-neutral investors, price accuracy decreases with \( N \), but persists except in the limit when investors are perfectly competitive.\(^{23}\) While heterogeneity in information acquisition across investors is a natural extension to the current setup, a model with asymmetric information and learning from prices does not seem analytically tractable in our setting, since the price depends non-linearly on the investor’s conditional expectations. We hope to explore this in future work.

The economic relevance of the mechanism we describe relies, in part, on the presence of price-dependent noise (i.e., feedback traders). A number of papers, including Lakonishok, Shleifer, and Vishny (1992), Grinblatt, Titman, and Wermers (1995), Wermers (1999), and Grinblatt and Keloharju (2000), document trading behavior by both individuals and institutions which is consistent with feedback trading. Cohen and Shin (2003) find evidence of positive feedback trading in the U.S. Treasury market during a period of high market stress. A standard explanation for feedback trading is that investors exhibit extrapolative expectations, and a number of papers provide survey evidence consistent with such behavior (e.g., Frankel and Froot (1987); Malmendier and Nagel (2011); and Greenwood and Shleifer (2014)). While feedback trading can naturally arise through such behavioral channels (see Shleifer and Summers (1990) and Hirshleifer (2001) for comprehensive surveys), there may be other reasons for such predictability in trading. We focus our discussion on two alternatives: deleveraging episodes and fund-flows to delegated portfolio managers.

The 2007 subprime crisis has highlighted the impact of leverage constraints, especially on financial institutions like hedge funds. As security prices fell, lenders demanded higher margins and more collateral, which forced hedge funds to delever by selling the underlying assets, further lowering prices (see Acharya et al. (2009) for a narrative of the financial crisis). Exacerbating the issue, hedge funds were also hit with large increases in redemptions over this period; in fact, their redemptions exceeded those suffered by mutual funds. The impact of their delevering was economically significant. For instance, Ben-David, Franzoni, and Moussawi (2012) show that hedge funds reduced their U.S. equity holdings by about 6% in each of the third and fourth quarters of 2007 and by about 15% in each of the third and fourth quarters of 2008, on average. Furthermore, about 80% of this decrease can be explained by redemptions and reduced leverage. Finally, Ang, Gorovyy, and Van Inwegen (2011) and Aragon and Strahan (2012) document that this deleveraging is predictable, and has price impact on the underlying assets. Taken together, this evidence suggests that deleveraging episodes can generate feedback trading by financing-constrained, sophisticated investors.

\(^{23}\)This result is reminiscent of the literature on informed trading that extends the Kyle (1985) model to multiple informed traders (e.g., Holden and Subrahmanyam (1992), Foster and Viswanathan (1993)).
Delegated asset management, and the nature of investor behavior therein, provides another natural channel through which predictable feedback trading may arise, even in the absence of leverage crises. A number of papers, including Chevalier and Ellison (1997) and Sirri and Tufano (1998), document a strong relation between past performance and mutual fund flows. Coval and Stafford (2007) and Lou (2012) show that such flows generate predictable price pressure in the underlying stocks.24 This predictability in trading behavior by mutual funds corresponds closely to the feedback trading we consider.25 Moreover, there is evidence to suggest that, as in our model, some investors learn about, and could profitably trade on, this predictability in mutual fund demand. For instance, Chen, Hanson, Hong, and Stein (2008) document that, for an individual stock, short interest tends to increase prior to its sale by mutual funds which experience large outflows. Similarly, Dyakov and Verbeek (2013) present evidence that a trading strategy which uses public information to predict price pressure (and trades accordingly) can generate excess returns.

3 The extent to which prices reflect fundamentals

This section presents the main analysis of the paper. We define two measures — accuracy and efficiency — to capture the extent to which asset prices reflect fundamentals. Accuracy measures the unconditional mean squared error between the date 1 price and the fundamentals $\phi$. Efficiency measures the mean squared error between fundamentals $\phi$ and the date 1 expectation of fundamentals, conditional on the price at date 1. We describe conditions in our setting under which an increase in one measure may be accompanied by a decrease in the other. In Section 3.2, we then characterize conditions under which, given the endogenous choice of precisions described above, an increase in transparency (i.e., an increase in $h$) can lead to decreases in accuracy and efficiency. Intuitively, when the investor chooses to learn about noise trading sufficiently faster than she learns about fundamentals, price accuracy and efficiency fall. We consider specific examples of cost functions to illustrate our results in Section 3.3. In Section 3.4 we consider the impact of targeted transparency on price informativeness. Finally, in Section 3.5 we allow feedback trading to respond endogenously to the information environment.

24For instance, Lou (2012) documents that redemptions lead to a one-for-one reduction in positions, while for a dollar increase in assets under management, mutual funds tend to scale up existing positions by approximately sixty cents.

25The mechanism described above would tend, on average, to lead to an effect which is consistent with a positive feedback effect, but it does not capture the possibility of manager heterogeneity. Cici (2012) finds that a sizable minority (between 22% and 55%) of mutual fund managers tend to sell winners more than losers, that this behavior is persistent over time, and that they tend to buy more of their losers, consistent with a negative feedback effect. This behavior is exacerbated by inflows and outflows.
3.1 Price Accuracy and Price Efficiency

We define two measures of the extent to which prices reflect fundamentals. The first measure captures how close the date 1 price is to fundamentals in expectation.\textsuperscript{26}

\textbf{Definition 1.} Price \textit{accuracy} of the date 1 price is given by \( A \equiv -\mathbb{E} \left[ (\phi - P_1)^2 \right] \).

The second measure captures how close the conditional expectation of fundamentals, given the date 1 price, is to fundamentals in expectation.

\textbf{Definition 2.} Price \textit{efficiency} of the date 1 price is given by \( E \equiv -\mathbb{E} \left[ (\phi - \mathbb{E}[\phi|P_1])^2 \right] \).

Note that estimating efficiency empirically not only requires observations of \( \phi \) and \( P_1 \), but also requires that the observer know the structure of the economy, and in particular, the prior distribution of fundamentals and noise trading. In contrast, price accuracy can be interpreted as a more robust measure since it can be estimated using observations of \( \phi \) and \( P_1 \) alone. Furthermore, note that in the absence of transaction costs (i.e., \( c = 0 \)) and with perfect transparency, the price at date 1 would be equal to the predictable component of the payoff, \( \phi \). Hence, our notion of accuracy captures how informative the date 1 price is about the predictable component of fundamentals, relative to its \textit{frictionless} benchmark.\textsuperscript{27} Finally, while efficiency captures the notion of price informativeness for agents within the model, accuracy seems to more closely match the concept of price informativeness used in the empirical literature and by market participants and regulators in practice.

In the absence of feedback trading (i.e., when \( \beta = 0 \)), the two measures are intimately related.\textsuperscript{28} In this case, since \( P_1 = y \) (the linear component of the price), accuracy is given by

\[
A = -\mathbb{E} \left[ (\phi - y)^2 \right] = \sigma_f^2 \left( \kappa_f - \alpha \kappa_u - \omega - 1 \right),
\]

and efficiency is given by

\[
E = -\mathbb{E} \left[ \text{var} \left( \phi \mid y \right) \right] = \sigma_f^2 \left( \kappa_f + \alpha \kappa_u + \omega - 1 \right).
\]

\textsuperscript{26}While the focus of our analysis is on price informativeness at date 1, one could also consider how well the date 2 price reflects fundamentals. In our model, this is not very interesting: the investor’s value function increases as price accuracy at date 2 falls, and so as we make it easier to learn, this measure must fall.

\textsuperscript{27}The difference between \( P_1 \) and \( \phi \) is dependent upon the realization of the signals selected, which is random. We take the date 0 expectation over the possible signal realizations to simplify our characterization of the results. This also ensures our measure of efficiency can be empirically implemented — an econometrician who is able to observe prices and fundamentals (i.e., \( \phi \)), but not the information of investors, would still be able to construct our measure. Finally, our measure is common in the theoretical literature (e.g., Gao (2008)).

\textsuperscript{28}This is also true in standard noisy RE models, where the price is a linear function of normally distributed shocks.
Note that in this case both accuracy and efficiency increase in the precision of fundamental information (i.e., $\kappa_f$) and decrease in the quality of information about noise trading (i.e., $\kappa_u$). More generally, however, since the price depends non-linearly on the investor’s beliefs about feedback traders, the two measures are generically different. This is summarized in the following result.

**Theorem 1.** Efficiency and accuracy can be expressed as

\[
E = \sigma_f^2 \left( \frac{\kappa_f^2}{\kappa_f + \alpha \kappa_u + \omega} (1 - G(x)) - 1 \right), \quad \text{and} \quad A = \sigma_f^2 \left( \frac{1 - 2x^2}{(1-x^2)^2} \kappa_f - \frac{1 + x^2}{(1-x^2)^2} (\alpha \kappa_u + \omega) - 1 \right)
\]

where $x \equiv \rho c b \kappa_b$ and $G(x)$ is given by

\[
G(x) = \mathbb{E} \left[ 2x^2 (1 - x^2) s^2 \left( \frac{2x^2}{e^{(1-x)^2} s^2 (1+x(1-x)^2)^2} + \frac{2x^2}{e^{(1+x)^2} s^2 (1+x(1-x)^2)^2} \right) \right],
\]

where the expectation is evaluated over the standard normal random variable $s$.

The above expressions for $E$ and $A$ immediately imply the following results.

**Proposition 3.** Efficiency always increases in $\kappa_f$, always decreases in $\kappa_u$, and decreases in $\kappa_b$ when $\frac{d}{d \kappa_b} G > 0$. Accuracy always decreases in $\kappa_u$, always decreases in $\kappa_b$, and decreases in $\kappa_f$ when $3 (\rho c b \kappa_b)^2 > 1$.

Recall that under our distributional assumptions, the price at date 1 is given by

\[
P_1 = \frac{\kappa_f S_f + \rho c \kappa_u S_u + c(1 + \rho^2) u_1}{1 - \rho c \kappa_b S_b} = \frac{\kappa_f}{1 - \rho c \kappa_b S_b} \phi + \frac{\kappa_f e_f + \rho c \kappa_u S_u + c(1 + \rho^2) u_1}{1 - \rho c \kappa_b S_b}.
\]

The result that efficiency and accuracy fall when the investor learns more about $u_2$ (i.e., $\kappa_u$ increases) is intuitive — in this case, the price becomes a noisier signal about fundamentals. Similar results obtain with increases in $\sigma_{u,1}^2$ and $\sigma_{u,2}^2$ — increasing the variance of $u_1$ and $u_2$ also make the price a noisier signal of $\phi$ on average.

When the investor learns more about fundamentals (i.e., $\kappa_f$ increases), the price becomes a more informative signal about $\phi$, which leads to greater efficiency. However, the multiplicative term in the price, $\frac{1}{1 - \rho c \kappa_b S_b}$, changes the extent to which the level of $P_1$ responds to better information about fundamentals. Note that on average, the multiplier term is increasing in $b \kappa_b$ since

\[
\mathbb{E} \left[ \frac{1}{1 - \rho c \kappa_b S_b} \right] = \frac{1}{1 - (\rho c b \kappa_b)^2}.
\]
Intuitively, when $b\kappa_b$ is low, either because the feedback effect (i.e., $b$) is small or the investor knows little about it (i.e., $\kappa_b$ is low), the multiplier term is close to one, on average, and so learning more about fundamentals (i.e., higher $\kappa_f$) pushes the level of the price closer to $\phi$. However, when the feedback effect is large and the investor learns about it precisely (i.e., $b\kappa_b$ is large enough), the multiplier term is large and learning more about $\phi$ pushes the price away from the fundamental value.

This multiplier effect also implies that accuracy always decreases as the investor learns more about $\beta$ — the multiplier is equal to one when the investor chooses to learn nothing about $\beta$ (i.e., when $\kappa_b = 0$). However, the effect of an increase in $\kappa_b$ on efficiency is more nuanced. To see this, note that conditional on a specific value of $S_b$, the price is a linear signal about $\phi$, since

$$P_1 = \begin{cases} \frac{\kappa_f}{1 - \rho c b \kappa_b} \phi \frac{\kappa_f}{1 - \rho c b \kappa_b} + \frac{\kappa_f e + \rho c \kappa_b u + \rho (1 + \rho^2) u_1}{1 - \rho c b \kappa_b} & \text{when } S_b = b \\ \frac{\kappa_f}{1 + \rho c b \kappa_b} \phi \frac{\kappa_f}{1 + \rho c b \kappa_b} + \frac{\kappa_f e + \rho c \kappa_b u + \rho (1 + \rho^2) u_1}{1 + \rho c b \kappa_b} & \text{when } S_b = -b \end{cases}$$

(42)

However, for an investor who does not observe the realization of $S_b$, but instead must form an expectation of $\phi$ conditional on $P_1$ only, the uncertainty about $S_b$ generates additional uncertainty about fundamentals, as it is not clear which of the two signals in (42) is being observed. Specifically, note that the law of total variance implies that

$$\text{var} (\phi|P_1) = \mathbb{E} [\text{var} (\phi|P_1, S_b)|P_1] + \mathbb{E} [\text{var} (\phi|P_1, S_b)|P_1].$$

(43)

The second term in this decomposition, $\mathbb{E} [\text{var} (\phi|P_1, S_b)|P_1]$, captures the additional uncertainty generated by the fact that $S_b$ is uncertain. As we show in the proof of Proposition 1, this term depends on the observer’s conditional variance of $S_b$, upon observing $P_1$. Moreover, we show that the unconditional expectation of this term is non-monotonic in $\kappa_b$ on average and is captured by $G(x)$. At $k_b = 0$, there is no additional uncertainty since there is no effect of $S_b$ on the price. Increasing $\kappa_b$ when it is low increases the additional uncertainty, since the price becomes more sensitive to the realization of $S_b$. However, when $\kappa_b$ is sufficiently large, the price is sufficiently sensitive to the realization of $S_b$, so that it becomes increasingly easy to distinguish whether $S_b = b$ or $S_b = -b$ from the price itself. In this region, increasing $\kappa_b$ further decreases the variance about $S_b$, conditional on the price, which consequently decreases the additional variance in the inference of fundamentals. To summarize, an increase in $\kappa_b$ leads to a decrease in efficiency when it increases the additional variance due to $S_b$ being unknown i.e., when it increases $\mathbb{E} [\text{var} (\phi|P_1, S_b)|P_1]$, or equivalently, when $\frac{d}{d\kappa_b} G > 0$.

The function $G(x)$ cannot be computed in closed form. However, since $x \equiv \rho c b \kappa_b \in [0, 1]$, $G(x)$ can be completely characterized by its plot over the region $x \in [0, 1]$. As is apparent from
Figure 3, $G(x)$ is always non-negative and single peaked at $x^* \approx 0.7$. This implies that we can ensure that price efficiency always decreases in $\kappa_b$ by imposing $p_{cb} \leq x^*$. We impose this restriction in our numerical illustrations below.

### 3.2 When does greater transparency decrease accuracy / efficiency?

We use the above results to characterize conditions for which accuracy and efficiency can decrease with an increase in transparency. Recall that the investor optimally chooses precisions $\kappa_f$, $\kappa_u$ and $\kappa_b$, subject to a cost function $C(\kappa_f, \kappa_u, \kappa_b, h)$, which is separable, increasing, and convex in the precisions (i.e., equation (26) holds), and where $h$ parameterizes transparency (i.e., equation (25) holds). In this section, we consider a general decrease in transparency, which implies that an increase in $h$ leads to a decrease in the marginal cost of acquiring information across all dimensions. One might expect that increasing transparency should lead to an increase in the extent to which prices reflect fundamentals. The following result characterizes conditions under which this is not the case.

**Theorem 2.** Let $x = p\kappa_b$, and suppose the investor’s optimal choice of precisions, as characterized in Proposition 2, is given by $\kappa_f(h)$, $\kappa_u(h)$ and $\kappa_b(h)$. Then,

(i) Efficiency $E$ decreases in transparency (i.e., $h$) if and only if

$$
\frac{2}{\kappa_f} \frac{d\kappa_f}{dh} - \frac{\sigma_f^2}{\sigma_b^2} \left( \frac{d\kappa_f}{dh} + \alpha \frac{d\kappa_u}{dh} \right) - \frac{p_{cb}}{1-G(x)} \frac{dG(x)}{dx} \frac{d\kappa_b}{dh} < 0.
$$

(ii) Accuracy $A$ decreases in transparency (i.e., $h$) if and only if

$$
\frac{d\kappa_f}{dh} - \left[ \alpha \frac{d\kappa_u}{dh} + x^2 \left( \frac{dG(x)}{dx} + \alpha \frac{d\kappa_u}{dh} \right) \right] - \frac{2x((1+3x^2)\kappa_f + (3+x^2)(\alpha \kappa_u + \omega))}{(1-x^2)} \frac{p_{cb} d\kappa_b}{dh} < 0
$$

The result follows from simplifying the conditions for which $\frac{dA}{dh} < 0$ and $\frac{dE}{dh} < 0$, given the expressions in Theorem 1. Standard intuition suggests that both accuracy and efficiency would
fall with an increase in transparency when the resulting increase in precision about fundamentals (i.e., $\kappa_f$) is sufficiently less than the increase in precision about noise trading (i.e., $\kappa_u$ and $\kappa_b$). Interestingly, however, this intuition does not always hold. Specifically, note that while the coefficient on $\frac{d\kappa_u}{dh}$ is always negative in both inequalities, the coefficient on $\frac{d\kappa_b}{dh}$ can be positive in (44), and the coefficient on $\frac{d\kappa_f}{dh}$ can be negative in (45). As such, an increase in $\kappa_u$ due to higher transparency always increases the likelihood that accuracy and efficiency fall. However, while accuracy is always more likely to fall with an increase in $\kappa_b$, it may be that learning about feedback trading makes it more likely that efficiency increases (because when $\frac{dG(x)}{dx} < 0$, it becomes easier to extract information about $\phi$ from the price, as discussed above). Similarly, while learning more about fundamentals always makes it more likely that efficiency increases, it may lead to lower accuracy when $\kappa_b$ (and hence $x = \rho c\kappa_b$) is sufficiently high, due to the multiplier effect.

To gain a better understanding of the underlying mechanism, first consider condition (44). The first term — the semi-elasticity of $\kappa_f$ with respect to transparency — captures the increase in price efficiency as a result of more learning about fundamentals. Note that this effect decreases as $\kappa_f$ approaches 1 — the marginal increase in price informativeness decreases as $\kappa_f$ increases. The second term — the sensitivity of $\sigma^2_y$ with respect to transparency — captures the increase in the variance of $y$ as a result of more learning about fundamentals (captured by the $\frac{d\kappa_f}{dh}$ term) and about $u_2$ (captured by the $\frac{d\kappa_u}{dh}$ term). The increased variance of $y$ makes it more difficult to infer $\phi$ from the price. When there is relatively more prior uncertainty about $u_2$ (i.e., when $\alpha$ is higher), learning more about $u_2$ makes it more likely that efficiency will fall. The third term measures the effect of a change in $\kappa_b$. In the region when $\frac{dG(x)}{dx}$ is negative, learning more about feedback trading makes the price more informative about fundamentals. On the other hand, when $\frac{dG(x)}{dx}$ is positive, efficiency is more likely to fall with an increase in transparency, and the effect of learning about feedback trading can actually increase as $\kappa_b$ gets larger. To summarize, an increase in transparency is most likely to decrease efficiency when the investor already chooses to learn a lot about fundamentals and sufficient information about feedback trading.\textsuperscript{29} Paradoxically, making it easier to learn is worse for efficiency when the investor already possesses precise information about fundamentals.

Turning to condition (45), we obtain a similar decomposition. The first term captures the increase in accuracy due to more learning about fundamentals — the price reflects $\phi$ more accurately as the investor learns more about it. The second term captures the decrease in accuracy resulting from more learning about $u_2$ (the $\frac{d\kappa_u}{dh}$ term) and, because of the multiplier effect, from more learning about fundamentals (the $x^2 \frac{d\kappa_f}{dh}$ term). In particular, note that when

\textsuperscript{29}These are not necessary conditions. As we show below, efficiency can fall when the investor is not learning anything about feedback trading.
there is no feedback effect (i.e., \( b = x = 0 \)), the coefficient on \( \frac{d\kappa_f}{dh} \) in the second term is zero — in the absence of the feedback effect, accuracy always increases with the quality of information about fundamentals. Finally, the third term captures the decrease in accuracy due to more learning about feedback trading — learning more about feedback trading always decreases accuracy, even though at times, it may increase price efficiency.

### 3.3 Illustrative Examples

Theorem 2 describes the necessary conditions under which accuracy and efficiency fall with an increase in transparency. Our characterization of these conditions relies upon not just the assumed parameters of the model, but crucially upon the endogenous choice of optimal precisions made by the investor. Below, we demonstrate that under reasonable specifications of the cost function, these conditions can arise naturally.

#### 3.3.1 Log-linear cost of precision

To ensure that the optimal choice of precision is interior (i.e., \( \kappa \in (0, 1) \)), the marginal cost of \( \kappa \) should be zero at \( \kappa = 0 \) and infinite at \( \kappa = 1 \). A parsimonious functional form which features both properties assumes a marginal cost that is proportional to \( \frac{\kappa}{1-\kappa} \), which can be generated by the following cost function:

\[
C(\kappa_f, \kappa_u, \kappa_b) = e^{-h} \left[ \sum_{i \in \{f, u, b\}} e^{-h_i} (-\kappa_{i} - \log(1 - \kappa_{i})) \right],
\]

where the parameters \( \{h, h_f, h_u, h_b\} \) capture different dimensions of transparency. We refer to the parameter \( h \) as general transparency, since it uniformly affects the marginal cost of acquiring information along any of the three dimensions. In contrast, the targeted transparency parameter \( h_i \) (for \( i \in \{f, u, b\} \)) affects the marginal cost of increasing \( \kappa_i \), but not \( \kappa_j \) for \( j \neq i \). This allows us to capture the possibility that learning certain types of information may be more costly than others. Moreover, in Section 3.4, we study the effects of targeted changes in transparency by characterizing the effects of changing \( h_i \), specifically.\(^{*}\)

For tractability, we first consider the case when there is no noise trading at date 1 (i.e., \( \sigma_{u,1}^2 = 0 \)). Under this assumption, we show that the investor does not initially learn about

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\(^{*}\)This parameterization implies that a shift in transparency affects both the total and marginal cost of information. Given (25) and (26), this affects only whether \( \kappa \) is zero or non-zero (i.e., the initial decision to learn). In the log-linear setting considered here, and in the presence of date 1 noise, as is shown below, \( \kappa \) is always non-zero, and so our assumption is without loss of generality. In contrast, for the entropy-based cost function we consider in Section 3.3.2, the marginal cost of information is not zero when \( \kappa = 0 \), and consequently, the investor chooses not to become informed at all if transparency is too low.
feedback trading for sufficiently low transparency. This leads us to consider two cases: when
the investor optimally chooses to only learn about \( \phi \) and \( u_2 \) (i.e., the optimal choice of \( \kappa_b = 0 \)),
and when the investor begins to learn along all three dimensions. Finally, we numerically
illustrate that our conclusions are robust to \( \sigma_{u,1}^2 \neq 0 \).

**Corollary 1.** Let \( \sigma_{u,1}^2 = 0 \). There exists \( \bar{h} \) such that for all values of \( h < \bar{h} \), the investor chooses
only to learn about \( \phi \) and \( u_2 \). Otherwise, the investor learns along along all three dimensions.

The above result immediately follows from the expressions for the marginal value of learning
along each dimension that are given in (31), (32) and (33). In particular, while \( V_f \) and \( V_u \) are
non-zero when \( \kappa_f = \kappa_u = \kappa_b = 0 \), \( V_b = 0 \) in this case.

**Case 1** (\( \kappa_f, \kappa_u \neq 0, \kappa_b = 0 \)): Using equations (31), (32), and Proposition 2, it can be shown
that the optimal precisions are given by

\[
\kappa_f = \frac{(1-\rho)\frac{\bar{b}}{b} \frac{\bar{c}}{c} \sigma_e^{k+h_f} + \rho \sigma_e^{h_f}}{1+(1-\rho)\frac{\bar{b}}{b} \frac{\bar{c}}{c} \sigma_e^k}, \quad \text{and} \quad \frac{d\kappa_f}{dh} = \frac{\kappa_f}{1+(1-\rho)\frac{\bar{b}}{b} \frac{\bar{c}}{c} \sigma_e^k},
\]  \( \kappa_u = \frac{\alpha(1-\rho)\frac{\bar{b}}{b} \frac{\bar{c}}{c} \sigma_e^{h+hu}}{1+\alpha(1-\rho)\frac{\bar{b}}{b} \frac{\bar{c}}{c} \sigma_e^h}, \quad \text{and} \quad \frac{d\kappa_u}{dh} = \frac{\kappa_u}{1+\alpha(1-\rho)\frac{\bar{b}}{b} \frac{\bar{c}}{c} \sigma_e^h}. \) \tag{47, 48}

Taking the ratio of \( \frac{d\kappa_u}{dh} \) and \( \frac{d\kappa_f}{dh} \), we see that the relative rate at which the investor learns about
\( \phi \) and \( u_2 \) as transparency improves depends upon (i) how much information the investor is
currently choosing to learn, (ii) the relative prior uncertainty (i.e., \( \alpha \)), and (iii) the relative
ease of learning (i.e., \( h_f \) versus \( h_u \)). Theorem 2 and the equilibrium solution above imply that

\[
\kappa_f < \kappa_u \left( \frac{\alpha(1-\rho)\frac{\bar{b}}{b} \frac{\bar{c}}{c} \sigma_e^{h+hu}}{1+\alpha(1-\rho)\frac{\bar{b}}{b} \frac{\bar{c}}{c} \sigma_e^h} - 2\alpha \right) \quad \text{and} \quad \kappa_f < \kappa_u \left( \frac{\alpha(1-\rho)\frac{\bar{b}}{b} \frac{\bar{c}}{c} \sigma_e^{h+hu}}{1+\alpha(1-\rho)\frac{\bar{b}}{b} \frac{\bar{c}}{c} \sigma_e^h} \right), \tag{49}
\]

respectively. In the limit, as general transparency increases (i.e., \( h \) tends to infinity), efficiency
falls if \( 2\alpha < e^{(h_f-h_u)} - 1 \), while accuracy falls when \( 0 < e^{(h_f-h_u)} - 1 \). Thus, if it is relatively
easier to learn about fundamentals (\( h_f > h_u \)), and general transparency is already sufficiently
high, both price efficiency and accuracy will fall as general transparency increases. It is also
clear that, if efficiency falls when transparency improves, accuracy must as well; however, the
opposite need not be true. As discussed above, efficiency is most likely to fall when \( \kappa_f \) is high,
while \( \kappa_u \) is low. If \( \alpha \) is too large, the investor will learn about both relatively equally (or will
learn more about \( u_2 \)), so that this condition no longer holds.

**Case 2** (\( \kappa_f, \kappa_u, \kappa_b \neq 0 \)): At \( \bar{h} \), the investor begins to learn about the feedback effect, decreasing
both efficiency and accuracy. Complementarity, however, implies that this increases the
marginal value of learning about both \( u_2 \) and \( \phi \), the latter of which could counteract any fall
in efficiency due to learning about \( \beta \). The following result shows that, even with this comple-

\[
\]
Figure 4: Log-linear cost of precision with no date 1 noise

This figure plots (a) optimal choice of precisions, (b) efficiency, and (c) accuracy, as a function of general transparency $h$, when the cost of precision is given by (46). The other parameters are set to the following values: $\rho = 0.6$, $c = 1$, $b = 1$, $\sigma^2_f = 1$, $\sigma^2_{u,2} = 0.4$, $h_f = 4$, $h_u = 0$, $h_b = -4$. The choice of precisions in panel (a) are given by: $\kappa_f$ (solid), $\kappa_u$ (dashed) $\kappa_b$ (dot-dashed). The dotted lines in panels (b) and (c) mark the level of maximum efficiency and accuracy, respectively, for the parameter range plotted.

Proposition 4. Let $\sigma^2_{u,1} = 0$. For sufficiently low $h_b$, and sufficiently high $h_f$, increasing transparency leads to a fall in efficiency at $h = \bar{h}$. Furthermore, $\mathcal{E}(\bar{h}) > \lim_{h \to \infty} \mathcal{E}(h)$. Accuracy falls for all $h > \hat{h}$, where $\kappa_b(\hat{h}) = \frac{1}{\sqrt{3} \rho cb}$.

Figure 4 illustrates both cases. The intuition is as follows. As transparency increases, initially the investor chooses to learn only about $\phi$ and $u_2$ (Case 1). Since it is sufficiently more costly to learn about noise trading than fundamentals (i.e., $h_f > h_u$), the investor chooses to learn faster about $\phi$ initially — in this region, efficiency and accuracy increase with transparency. However, given decreasing returns to scale, once the investor’s choice of $\kappa_f$ is large enough, she chooses to learn about noise trading (i.e., $u_2$) at a faster rate — in this region, efficiency and accuracy begin to decrease. Finally, when general transparency is high enough, the investor chooses to learn along all three dimensions (Case 2). She continues to learn about $u_2$ at a faster rate, which in combination with the information she obtains about feedback trading, increases the rate at which efficiency and accuracy fall. Moreover, at this point, the complementarity in learning kicks in. This makes learning about $\phi$ and $u_2$ more valuable, which triggers even faster learning along both dimensions. However, as she has little remaining information to learn about $\phi$, the effect of learning about noise dominates, which leads to efficiency and accuracy falling more steeply when the investor begins to learn about the feedback effect.

Figure 5 illustrates the effect of introducing date 1 noise (i.e., $\sigma^2_{u,1} \neq 0$), holding the other parameters fixed. The plots suggest that our general conclusions are robust to its inclusion.
This figure plots (a) optimal choice of precisions, (b) efficiency, and (c) accuracy, as a function of general transparency $h$, when the cost of precision is given by (46). The other parameters are set to the following values: $\rho = 0.6$, $c = 1$, $b = 1$, $\sigma_f^2 = 1$, $\sigma_{u,2}^2 = \sigma_{u,1}^2 = 0.4$, $h_f = 4$, $h_u = 0$, $h_b = -4$. The choice of precisions in panel (a) are given by: $\kappa_f$ (solid), $\kappa_u$ (dashed) $\kappa_b$ (dot-dashed). The dotted lines in panels (b) and (c) mark the level of maximum efficiency and accuracy, respectively, for the parameter range plotted.

The main effect of introducing date 1 noise is to increase the value of learning about feedback trading. As a result, the investor chooses to start learning about $\beta$ for a lower level of $h$ (compared to the case when $\sigma_{u,1}^2 = 0$). However, as before, when learning about fundamentals is already sufficiently easier than learning about noise trading, increasing transparency can decrease efficiency and accuracy.

Note that for accuracy to fall with an increase in transparency, it need not be the case that learning about fundamentals is easier than learning about noise trading: in fact, it may be more likely when the opposite is true. If learning about feedback trading is sufficiently easy, and the investor obtains a precise enough signal about $\beta$, further learning along any dimension (caused by the increase in transparency) will cause accuracy to fall.

### 3.3.2 Entropy-based cost of precision

To facilitate a comparison to the information acquisition literature, and to highlight the robustness of our results, we consider a cost function where the cost of learning along a particular dimension is proportional to the corresponding reduction in entropy.$^{31}$ Recall that for a normal random variable $X$ with variance $\sigma^2$, the entropy is given by $H(X) = \frac{1}{2} \log_e (2\pi e \sigma^2)$, and for a random variable $Y$ drawn from a binomial distribution with probabilities $\{p, 1-p\}$, the

$^{31}$A growing literature characterizes the information acquisition decisions of investors who face an entropy-based, information processing constraint (e.g., Sims (2003), Peng and Xiong (2006), Van Nieuwerburgh and Veldkamp (2010)).
entropy is given by \( H(Y) = -p \log_2(p) - (1 - p) \log_2(1 - p) \). This implies that

\[
H(\phi) - H(\phi | S_f) = -\frac{1}{2} \log_2(1 - \kappa_f),
H(u_2) - H(u_2 | S_u) = -\frac{1}{2} \log_2(1 - \kappa_u)
\] (50)

and

\[
H(\beta) - H(\beta | S_b) = \frac{1}{2} \left((1 + \kappa_b) \log_2(1 + \kappa_b) + (1 - \kappa_b) \log_2(1 - \kappa_b)\right).
\] (51)

As before, in order to allow for heterogeneity in the cost of information across dimensions, we specify the cost function as follows:

\[
C(\kappa_f, \kappa_u, \kappa_b) = e^{-h} \left[ e^{-h_f} (H(\phi) - H(\phi | S_f)) + e^{-h_u} (H(u_2) - H(u_2 | S_u)) + e^{-h_b} (H(\beta) - H(\beta | S_b)) \right],
\] (52)

so that \( h \) parametrizes general transparency and \{\( h_f, h_u, h_b \)\} reflect targeted transparency. Since \( \phi, u_2, \) and \( \beta \) are independent, the special case when \( h_f = h_u = h_b = 0 \) corresponds to the case when the cost of acquiring information is proportional to the reduction in total entropy, since

\[
C(\kappa_f, \kappa_u, \kappa_b) = e^{-h} \left[ (H(\phi) - H(\phi | S_f)) + (H(u_2) - H(u_2 | S_u)) + (H(\beta) - H(\beta | S_b)) \right] (53)
= e^{-h} (H(\phi, u_2, \beta) - H(\phi, u_2, \beta | S_f, S_u, S_b)).
\] (54)

The cost specification in (52) differs from the log-linear specification in (46) along two important dimensions. First, the marginal cost at \( \kappa_i = 0 \) is no longer zero. This implies that the marginal benefit of learning along any dimension must be sufficiently high before the investor chooses to learn along this dimension. More specifically, when transparency is low enough (i.e., \( h \) is low enough), the investor may optimally choose to obtain no information. Second, while the marginal cost for \( \phi \) and \( u_2 \) is infinite at \( \kappa_f = 1 \) and \( \kappa_u = 1 \), respectively, the marginal cost at \( \kappa_b = 1 \) is finite. This implies that, unlike the log-linear specification, the investor can choose to learn perfectly about feedback trading (when doing so is sufficiently valuable).

Despite these differences, our main conclusions remain true. As before, if learning about fundamentals is sufficiently easier than learning about noise trading, increasing general transparency can decrease efficiency and accuracy. Figure 6 provides an illustration. Since learning about fundamentals is cheaper than learning about noise trading (i.e., \( h_f > h_u, h_b \)), the investor initially learns about \( \phi \). In contrast to the log-linear cost specification (illustrated in Figures 4 and 5), however, the investor does not always learn about \( u_2 \). This is because, unlike the previous case, the marginal cost of learning about \( u_2 \) is non-zero, even when \( \kappa_u = 0 \). As a result, the investor only begins to learn about \( u_2 \) when general transparency is sufficiently high. At this point, since the investor has already learned a lot about fundamentals (i.e., \( \kappa_f \) is high),
an increase in transparency leads to a bigger change in $\kappa_u$ than in $\kappa_f$. As a result, efficiency and accuracy fall. Similarly, when transparency is high enough for the investor to also start learning about feedback trading, an increase in transparency leads to a much smaller increase in $\kappa_f$ relative to the increase in $\kappa_u$ and $\kappa_b$, and as a result, efficiency and accuracy fall more steeply. Notably, since the investor can learn $\beta$ perfectly with finite cost (see expression (52)), the complementarity in learning about feedback and the other dimensions can lead to a sharp jump in learning about feedback trading (i.e., $\kappa_b$) in response to a small change in transparency. This suggests that not only can an increase in transparency lead to lower efficiency and accuracy, but small changes in transparency may have disproportionately large effects on the extent to which prices reflect fundamentals.

### 3.4 Targeted Transparency

The last subsection highlights that when general transparency increases, both efficiency and accuracy can fall. It is reasonable to consider, however, the effect of actions which directly target the transparency of certain types of information. Many prominent examples of financial market regulation share a stated objective, held by regulators, market participants, and political figures alike, to make relevant financial information more readily available. These examples include the Securities Act of 1933 (requires registration of traded securities), Regulation FD (standardizes information made available to the public), and Sarbanes-Oxley (targets increased accuracy of financial statements). By making such information more readily available, the hope is that prices will more fully reflect this information, leading to increased price informativeness.
This figure plots (a) optimal choice of precisions, (b) efficiency, and (c) accuracy, as a function of targeted transparency $h_f$, when the cost of precision is given by (46). The other parameters are set to the following values: $\rho = 0.6$, $c = 1$, $b = 1$, $\sigma^2_f = 1$, $\sigma^2_{u,2} = \sigma^2_{u,1} = 0.4$, $h = 4$, $h_u = 1$, $h_b = -3$. The choice of precisions in panel (a) are given by: $\kappa_f$ (solid), $\kappa_u$ (dashed) $\kappa_b$ (dot-dashed). The dotted lines in panels (b) and (c) mark the level of maximum efficiency and accuracy, respectively, for the parameter range plotted.

However, when investors have incentives to learn about fundamentals and other traders, a natural question arises: what do investors do with the resources they had previously devoted to learning about fundamentals? Do they simply obtain better information? Or do they choose to divert some resources to other, non-fundamental factors?

As transparency does not enter either the efficiency or accuracy measures directly, but only through the optimal precisions chosen by the investor, we can apply Theorem 2 for this analysis. Instead of considering the change in precisions as a result of changes in general transparency (i.e., $\frac{d\kappa}{dh}$), we apply the conditions to changes in targeted transparency along dimension $i$ (i.e, $\frac{d\kappa}{dh_i}$). It is clear that the direct effect of increasing transparency along a particular dimension is to induce the investor to learn more precisely along this dimension. When there is no learning about feedback traders, the implications of this direct effect are clear: accuracy and efficiency always fall when we make learning about noise traders (specifically, $u_2$) easier, whereas accuracy and efficiency always rise when transparency about fundamental information increases.

In our setting, however, this is not the end of the story. Because learning about feedback trading introduces complementarities across signals, learning more along the “cheaper” dimension makes learning along the other dimensions more valuable, creating an indirect effect of targeted transparency. The relevant metric, as Theorem 2 makes clear, is how much these precisions change relative to each other, or how strong the direct effect is relative to the indirect effect. As a result, increasing fundamental transparency can have counterintuitive implications. We illustrate these using the log-linear and entropy-based cost specifications from the previous subsection.

First, we consider an example using the log-linear cost specification of (46) illustrated in
This figure plots (a) optimal choice of precisions, (b) efficiency, and (c) accuracy, as a function of targeted transparency $h_f$, when the cost of precision is given by (52). The other parameters are set to the following values: $\rho = 0.6, c = 1, b = 1, \sigma_f^2 = 2, \sigma_u^2 = 0.4, \sigma_u^1 = 0, h = 0, h_u = 4, h_b = 0$. The choice of precisions in panel (a) are given by: $\kappa_f$ (solid), $\kappa_u$ (dashed) $\kappa_b$ (dot-dashed). The dotted lines in panels (b) and (c) mark the level of maximum efficiency and accuracy, respectively, for the parameter range plotted.

As fundamental transparency (i.e., $h_f$) increases, efficiency increases but accuracy decreases. This is due to the multiplier term (41) in the price that is generated due to learning about feedback trading. Specifically, recall from Proposition 3 that accuracy can decrease in $\kappa_f$ when the feedback effect is sufficiently large (i.e., $3(\rho cb\kappa_b)^2 > 1$). As a result, when learning about feedback trading is sufficiently easy, either because overall transparency is high, or information about other traders is easy to acquire, an increase in fundamental transparency can lead to a decrease in price accuracy.

However, the complementarity in learning can generate more surprising results. Note that since an increase in fundamental transparency leads to an increase in $\kappa_f$, it can also lead to an increase in $\kappa_b$ (and consequently, $\kappa_u$), since learning more about fundamentals makes learning about noise trading more valuable. If the increase in learning about noise trading is sufficiently larger than the increase in learning about fundamentals, both efficiency and accuracy can decrease with an increase in fundamental transparency! Figure 8 provides an illustration of this effect for the entropy-based cost specification of (52). As the plots suggest, this counterintuitive result is more likely to occur when the investor learns a lot about fundamentals from the beginning.

The complementarity can move in the opposite direction as well: if making it easier to learn about feedback trading causes the investor to increase $\kappa_b$, then it can also cause her to learn more about $\phi$. Standard intuition suggests that when learning about feedback trading becomes cheaper, investors choose to learn more along this dimension, which should decrease efficiency and accuracy. However, in our model, learning about feedback trading makes it more valuable to learn about fundamentals. Consequently, as Figure 9 illustrates, an increase in
transparency about feedback trading can result in an increase in efficiency and accuracy when the precision of the investor’s fundamental information is sufficiently sensitive. With further increases in targeted transparency, accuracy begins to fall since the investor learns sufficiently about feedback trading (i.e., $\kappa_b$ is large enough), which leads the multiplier effect to dominate. However, efficiency can still be increasing in this region.

From a policy perspective, these effects change the calculus on the value of improving access to financial data. If investors simply divert their resources to learning about non-fundamental factors, price efficiency can fall. This is more likely to be true in economies in which (i) investors are relatively sophisticated and (ii) fundamental information is currently widely available, if not perfectly transparent. This is especially true in light of the estimated cost of these regulations (see, for instance, Iliev (2010)). Similarly, decreasing the availability or timeliness of information about other participants, which is currently a popular proposal to mitigate the negative impact of high frequency trading (e.g., see Harris (2013)), can lead to a decrease in price informativeness by decreasing the incentives of sophisticated investors to acquire fundamental information.

### 3.5 When feedback trading can respond to transparency

Our analysis so far has focused on the case when feedback trading is unaffected by changes to the information environment. While endogenizing the behavior of the noise traders is beyond the scope of this paper, in this section, we consider the effect of allowing the intensity of feedback trading to respond to transparency. In particular, the following result characterizes conditions under which efficiency and accuracy fall in transparency, when the parameter $b$
responds (sufficiently smoothly) to changes in the investor’s access to information.

**Theorem 3.** Suppose $b$ is a continuous and differentiable function of $h$. Let $x = \rho c \kappa b$, and suppose the investor’s optimal choice of precisions, as characterized in Proposition 2, is given by $\kappa_f(h)$, $\kappa_u(h)$ and $\kappa_b(h)$, where $\kappa_i(h) \equiv \kappa_i(h; b(h))$ so that $\frac{d\kappa_i}{dh} = \frac{\partial \kappa_i}{\partial h} + \frac{\partial \kappa_i}{\partial b} \frac{db}{dh}$. Then,

(i) Efficiency $E$ decreases in transparency (i.e., $h$) if and only if

$$\frac{2}{\kappa_f} \frac{d\kappa_f}{dh} - \frac{\sigma_f^2}{\sigma_b^2} \left( \frac{d\kappa_f}{dh} + \alpha \frac{d\kappa_u}{dh} \right) - \frac{\rho c}{1 - G(x)} \frac{dG(x)}{dx} \left( b \frac{db}{dh} + \kappa_b \frac{db}{dh} \right) < 0. \quad (55)$$

(ii) Accuracy $A$ decreases in transparency (i.e., $h$) if and only if

$$\frac{d\kappa_f}{dh} - \left[ \alpha \frac{d\kappa_u}{dh} + x^2 \left( \frac{d\kappa_f}{dh} + \alpha \frac{d\kappa_u}{dh} \right) \right] - \frac{2x \left( (1 + 3x^2) \kappa_f + (3 + x^2)(\alpha \kappa_u + \omega) \right)}{(1 - x^2)} \frac{\rho c}{1 - G(x)} \frac{dG(x)}{dx} \left( b \frac{db}{dh} + \kappa_b \frac{db}{dh} \right) < 0. \quad (56)$$

Since higher transparency allows the investor to learn about, and trade against, feedback traders more cheaply, a natural specification would be to allow feedback traders to cut back on their trading as transparency increases. If we expect the intensity of feedback trading to decrease in response to increased transparency, then $\frac{db}{dh} \leq 0$. The above expressions are analogous to the conditions in Theorem 2, but account for the fact that a change in $b$ in response to changes in transparency has an additional effect on the optimal choice of precisions — the total change in precision $\frac{d\kappa_i}{dh}$ depends not only on $\frac{\partial \kappa_i}{\partial h}$ but also on $\frac{\partial \kappa_i}{\partial b} \frac{db}{dh}$.

Since learning about noise trading is less valuable when the intensity of feedback trading is lower, efficiency and accuracy should decrease more slowly (or even increase) with transparency. Figure 10 presents an instance of this. We assume that the intensity of feedback trading $b$ is given by $b = b_0 e^{-rh}$. Comparing the plots to those in Figure 4 (since both assume a log-linear cost of precision and the same parameter values), note that since the intensity of feedback trading decreases with transparency, the investor has a lower incentive to learn about noise trading. In fact, the investor no longer learns about $\beta$ in the plotted parameter range. Moreover, while efficiency and accuracy still decrease when transparency is sufficiently high, this only happens at a much higher level of transparency.

However, the dependence of learning about fundamentals on the intensity of feedback trading gives rise to a more counterintuitive result. If feedback trading decreases quickly enough with an increase in transparency, this can lead the investor to learn less about fundamentals, even though learning becomes cheaper. Figure 11 provides an illustration of this effect. Compared to the previous example, the only parameter change is the rate at which feedback trading decreases with $h$ (i.e., $r = 0.75$ instead of $r = 0.1$). The plots suggest that when the decrease in feedback trading is fast enough, learning along all three dimensions — and, in particular, about fundamentals — can decrease as transparency increases.
This figure plots (a) feedback intensity $b$, (b) optimal choice of precisions, (c) efficiency, and (d) accuracy, as a function of general transparency $h$, when the cost of precision is given by (46) and feedback intensity is given by $b = b_0 e^{-rh}$. The other parameters are set to the following values: $\rho = 0.6$, $c = 1$, $b_0 = 1$, $\sigma^2_f = 1$, $\sigma^2_u = 0.4$, $h_f = 4$, $h_u = 0$, $h_b = -4$ and $r = 0.1$. The choice of precisions in panel (b) are given by: $\kappa_f$ (solid), $\kappa_u$ (dashed) $\kappa_b$ (dot-dashed). The dotted lines in panels (c) and (d) mark the level of maximum efficiency and accuracy, respectively, for the parameter range plotted.

This result arises because the investor only chooses to acquire information in order to speculate against the feedback traders. In more general settings, if investors choose to learn about fundamentals for other motives, the result is not likely to be as dramatic. However, the example does highlight the importance of understanding why investors choose to acquire information in the first place. If investors are motivated to learn about fundamentals in order to speculate against other traders, increasing transparency can have unintended (and even counterproductive) consequences through its effect on the behavior of these other traders.

4 Discussion and Concluding Remarks

We consider an economy in which a risk-neutral investor chooses to learn about asset fundamentals and the trading behavior of other investors. We characterize the optimal information acquisition decision, and show how this affects the extent to which prices reflect fundamentals. Importantly, we show that improving transparency, even if targeted to fundamental information,
This figure plots (a) feedback intensity $b$, (b) optimal choice of precisions, (c) efficiency, and (d) accuracy, as a function of general transparency $h$, when the cost of precision is given by (46) and feedback intensity is given by $b = b_0 e^{-rh}$. The other parameters are set to the following values: $\rho = 0.6$, $c = 1$, $b_0 = 1$, $\sigma_f^2 = 1$, $\sigma_u^2 = 0.4$, $h_f = 4$, $h_u = 0$, $h_b = -4$ and $r = 0.75$. The choice of precisions in panel (b) are given by: $\kappa_f$ (solid), $\kappa_u$ (dashed) $\kappa_b$ (dot-dashed). The dotted lines in panels (c) and (d) mark the level of maximum efficiency and accuracy, respectively, for the parameter range plotted.

Information-processing constraint. The signals considered in our model are costly, directly reducing the investor’s utility. A commonly-used alternative is to impose an information-processing constraint on the investor’s learning problem. For instance, one could study the information acquisition decision subject to a capacity constraint which limits the reduction in total entropy for the investor. Such a budget constraint introduces a new channel through which some of our effects may be amplified. Consider the effect of increasing fundamental transparency. By making fundamental information cheaper, the investor learns more about $\phi$ — this is analogous to the substitution effect in a portfolio allocation problem. However, since her budget is fixed, this also frees up capacity to learn about noise traders — analogous to a wealth effect. Given the complementarity in learning about fundamentals and feedback, this additional channel can serve to amplify the decrease in efficiency and accuracy as a result of increased fundamental transparency.

Figure 11: Log-linear cost of precision with quickly decreasing $b$
Direct provision of information. In addition to changing the cost of acquiring information, policymakers and insiders can often directly provide public information about fundamentals to market participants (e.g., forward guidance by central banks, earnings guidance by firms). In the context of our model, this can be analyzed as a reduction in the prior uncertainty, $\sigma_f^2$, about fundamentals. The direct effect of such a change is to increase efficiency and accuracy. However, our model highlights an indirect effect: the reduction in prior uncertainty reduces the incentives of the investor to acquire information about fundamentals, which might lead to a decrease in efficiency and accuracy. While this crowding out effect is not unique to our setting (see Section 1), it suggests that the benefit of providing information to market participants may be much less than anticipated. This is an especially important consideration when information generation or provision of such public information is costly.

Empirical Implications. Testing our model directly presents a number of empirical challenges, including identifying proxies for both the informativeness of prices and the costs of information processing, as well as isolating the impact of price-dependent noise on prices. However, our model’s predictions are broadly consistent with the empirical evidence documented by Bai, Philippon, and Savov (2014). For the full sample of CRSP stocks, they find that price informativeness has steadily declined since the 1990’s, even though availability and ease of access to firm-specific information has likely increased. Our predictions are also consistent with evidence documented by Jorion, Liu, and Shi (2005), Duarte, Han, Harford, and Young (2008) and others, which suggests the introduction of Reg FD may have decreased the information content of stock prices, at least for a subset of firms.

The importance of our mechanism for explaining variation in returns (and price efficiency) is likely to differ across assets and over time. For instance, our results are more applicable to securities that are susceptible to price pressure and for which investors are able to obtain fundamental information that is not reflected in the price, provided the likelihood of positive feedback trading is high enough. This suggests that our mechanism should have the biggest impact for small-cap, illiquid stocks with low analyst coverage.\(^\text{32}\) Our mechanism may be more relevant when (ex-ante) fundamental uncertainty is high and market liquidity is low. Such conditions commonly arise during periods of market stress or financial crises, which is often when new regulatory policy is introduced to improve price efficiency.\(^\text{33}\)

Heterogeneity, Asymmetric Information, and Dynamics. We have kept the model as parsimonious as possible for tractability and to highlight clearly the intuition for our results.

\(^{32}\)Our analysis might partially explain the surprising results of Bai et al. (2014) with respect to small-cap stocks. Specifically, they note that their model would predict that price informativeness of small-cap stocks should have risen over the past two decades when in fact, it falls.

\(^{33}\)If such periods also coincide with higher intensity of feedback trading (as suggested by Cohen and Shin (2003)), the mechanism we describe may be amplified even further.
Introducing asymmetric information by allowing for heterogeneity across investors is a natural extension, as is generalizing the analysis to multiple periods. A general analysis of either extension requires characterizing how investors update their beliefs using the information in prices, however, and since the equilibrium price depends non-linearly on the information about fundamentals and noise trading, this class of model does not immediately appear to be analytically tractable. We hope to explore the feasibility of such extensions in future work.
References


Appendix

Proof of Proposition 2. The optimization problem is a standard constrained optimization problem. The conditions given in equation (34) are the standard Kuhn tucker conditions. If the objective function is concave, the Kuhn tucker conditions are both necessary and sufficient to guarantee a global maximum. The hessian of the objective function is given by

\[ H = \begin{bmatrix} -C_{ff} & 0 & V_{fb} \\ 0 & -C_{uu} & V_{ub} \\ V_{bf} & V_{bu} & V_{bb} - C_{bb} \end{bmatrix} \]

If the hessian matrix is negative definite, the objective function is concave and will have a unique global maximum. The determinant of hessian matrix \( H \) is negative if \( V_{bb} - C_{bb} < 0 \) i.e., if the cost function is sufficiently convex. Note that this is a sufficient condition but not a necessary condition.

Proof of Theorem 1. Let \( x = \rho cb \kappa_b \) and \( \sigma^2_y = \kappa_f \sigma^2_f + \rho^2 c^2 \kappa_b \sigma^2_{u,2} + c^2 (1 + \rho^2) \sigma^2_{u,1} \). Efficiency is

\[ \mathcal{E} = -E \left[ (\phi - E_1 [\phi | P_1])^2 \right] = -E [\text{var} (\phi | P_1)] \]

\[ = -E \left[ E [\text{var} (\phi | P_1, S_b)] - \text{var} [E (\phi | P_1, S_b)] P_1] \right]. \tag{58} \]

Conditional on \( P_1 \) and \( S_b \), the price is a linear signal about fundamentals, and so

\[ E (\phi | P_1, S_b) = \frac{\kappa_f \sigma^2_f}{\pi^2} \left( 1 - \rho c \kappa_b S_b \right) P_1, \quad \text{var} (\phi | P_1, S_b) = \sigma^2_f \left( 1 - \frac{\kappa_f \sigma^2_f}{\pi^2} \right). \tag{59} \]

Moreover, conditional on \( P_1 \), the probability that \( S_b = b \) is given by

\[ \pi = \frac{\frac{1}{\sigma^2_y} \phi \left( \frac{P_1}{\sigma^2_y} \right)}{\frac{1}{\sigma^2_y} \phi \left( \frac{P_1}{\sigma^2_y} \right) + \frac{1}{\sigma^2_y} \phi \left( \frac{P_2}{\sigma^2_y} \right)} = \frac{\frac{2 \rho c \kappa_b \mu^2}{\pi^2} P_1}{1 + \rho c \kappa_b (1 - \rho c \kappa_b) \mu^2} \tag{60} \]

where \( \phi (\cdot) \) is the pdf of the standard normal. Given these probabilities, we have

\[ \mathcal{E} = -\sigma^2_f \left( 1 - \frac{\kappa_f \sigma^2_f}{\pi^2} \kappa_f \right) - 4 \left( \frac{\kappa_f \sigma^2_f}{\pi^2} \right)^2 \rho^2 c^2 \kappa_b^2 b^2 \mathbb{E} \left[ \pi (1 - \pi) P_1^2 \right] \]

\[ = -\sigma^2_f + \frac{\kappa_f^2 (\sigma^2_f)}{\sigma^2_y} \left( 1 - \frac{4}{\sigma^2_y} \rho^2 c^2 \kappa_b^2 b^2 \mathbb{E} \left[ \pi (1 - \pi) P_1^2 \right] \right) \tag{61} \]

Let \( y = \kappa_f S_f + \rho c \kappa_u S_u + c (1 + \rho^2) u_1 \) and note that \( s \equiv \frac{y}{\sqrt{\sigma^2_y}} \sim N(0, 1) \). Then, we can express

\[ \mathbb{E} \left[ \pi (1 - \pi) P_1^2 \right] = \mathbb{E} \left[ \frac{\pi (1 - \pi)}{(1 - \rho c \kappa_b S_b)^2} y^2 \right] = \sigma^2_y \mathbb{E} \left[ \frac{\pi (1 - \pi)}{(1 - \rho c \kappa_b S_b)^2} s^2 \right]. \tag{63} \]

Moreover, note that

\[ \pi = \begin{cases} \frac{(1 - z) e^{z_2}}{1 + x + (1 - x) e^{z_2}}, & \text{when } S_b = b \\ \frac{(1 - z) e^{z_2}}{1 + x + (1 - x) e^{z_2}}, & \text{when } S_b = -b \end{cases} \tag{64} \]

Plugging this back into the expression for efficiency gives the expression in the result.
Similarly, note that accuracy can be expressed as
\[
A = -E \left[ \left( \phi - P_{1} \right)^{2} \right] = -E \left[ \left( \phi - E_{1} [\phi] \right)^{2} + (E_{1} [\phi] - P_{1})^{2} \right]
\]
(65)
\[
= -E \left[ \varphi + \left( \frac{\rho c E_{1} \beta}{1 - \rho c E_{1} \beta} \right)^{2} \left( E_{1} [\phi] \right)^{2} \right] = -E \left[ \left( \frac{\rho c E_{1} \beta}{1 - \rho c E_{1} \beta} \right)^{2} - 1 \right] \varphi (E_{1} [\phi]) - E \left[ \left( \frac{\rho c E_{1} \beta}{1 - \rho c E_{1} \beta} \right)^{2} \right] \left( \mu^{2} \sigma_{2}^{2} + \left( 1 + \mu^{2} \right)^{2} \sigma_{2}^{2} \right),
\]
(66)
(67)
(68)
which gives us the result.

**Proof of Corollary 1.** The Kuhn-Tucker conditions for the optimization problem are given by:
\[
\frac{\partial c}{\partial \kappa} = \frac{\partial V}{\partial \kappa} - e^{-h} h \kappa f \leq 0; \kappa f \geq 0; \kappa f \frac{\partial c}{\partial \kappa} = 0
\]
\[
\frac{\partial c}{\partial \kappa} = \frac{\partial V}{\partial \kappa} - e^{-h} \kappa u \leq 0; \kappa u \geq 0; \kappa u \frac{\partial c}{\partial \kappa} = 0
\]
\[
\frac{\partial c}{\partial \kappa} = \frac{\partial V}{\partial \kappa} - e^{-h} \kappa b \leq 0; \kappa b \geq 0; \kappa b \frac{\partial c}{\partial \kappa} = 0
\]
(69)
The first Kuhn-Tucker condition implies \( \kappa f \) is either \( = 0 \) or \( > 0 \). Suppose \( \kappa f = 0 \). Then the first Kuhn-Tucker equation cannot be satisfied since \( \frac{\partial c}{\partial \kappa} > 0 \), which is a contradiction. Similarly, we can rule out the case that \( \kappa u = 0 \). The third Kuhn Tucker condition implies \( \kappa b \) is either \( = 0 \) or \( > 0 \). So, the only possible choices are \( \{ \kappa f, \kappa u, \kappa b \} > 0 \) or \( \{ \kappa f, \kappa u \} > 0 \) and \( \kappa b = 0 \). For low values of transparency, \( h \), investors only learns about \( \{ \kappa f, \kappa u \} \) and for high values of \( h \), agent learns about all 3 dimensions. Let the transparency at which investor start increasing \( \kappa b \) be denoted \( \bar{h} \). At this transparency, all three Kuhn Tucker conditions hold with equality and \( \kappa b = 0 \). For \( h < \bar{h} \), the optimal precisions are solved using the first two Kuhn Tucker conditions and \( \kappa b = 0 \). They are given by
\[
\kappa f (\bar{h}) = \frac{(1-\rho) \frac{2}{1+\rho} b^2 \sigma_f^2 \bar{h} + \beta}{1+\rho \frac{2}{1+\rho} b^2 \sigma_f^2 \bar{h}}, \quad \kappa u (\bar{h}) = \frac{1+\rho \frac{2}{1+\rho} b^2 \sigma_f^2 \bar{h} + \beta}{1+\rho \frac{2}{1+\rho} b^2 \sigma_f^2 \bar{h}},
\]
As transparency \( (\bar{h}) \) increases, optimal \( \kappa f \) and \( \kappa u \) increases and this increases the marginal value of learning about feedback traders(\( \beta \)). The cutoff \( \bar{h} \) is the point at which investor starts learning about feedback traders and at this transparency, (69) holds for \( \kappa b \rightarrow 0 \). This implies
\[
e^{-\bar{h}+h_b} = \lim_{(\kappa_b) \rightarrow 0} \frac{\partial V}{\partial \kappa_b} = (1-\rho) \frac{2}{1+\rho} b^2 \sigma_f^2 \left( 6b^2 \left( \kappa f (\bar{h}) \sigma_f^2 + \kappa u (\bar{h}) \rho^2 \sigma_u^2 \right) + 4 \kappa u (\bar{h}) \sigma_u^2 \right)
\]
(70)
The cutoff \( \bar{h} \) solves above equation. As \( \bar{h} \) increases, left side of the equation decreases and right side increases, which implies a unique solution. For \( h > \bar{h} \), the optimal signal precisions solve the Kuhn Tucker conditions with equality.

**Proof of Proposition 4.** Denote \( z = (1-\rho) \frac{2}{1+\rho} b^2 \sigma_f^2 \). Then the optimal signal precisions at \( h = \bar{h} \) can be rewritten as
\[
\kappa f (\bar{h}) = \frac{z \kappa f + \sigma_f^2}{1+ze^{\bar{h}+h_b}}, \quad \kappa u (\bar{h}) = \frac{z \kappa u + \rho^2 \sigma_u^2}{1+ze^{\bar{h}+h_b}}, \quad \kappa b (\bar{h}) = 0
\]
and \( \bar{h} \) solves the equation
\[
e^{-\bar{h}+h_b} = (1-\rho) \frac{2}{1+\rho} b^2 \sigma_f^2 \left( 6b^2 \sigma_f^2 \left( \frac{z \kappa f + \sigma_f^2}{1+ze^{\bar{h}+h_b}} + \frac{z \kappa u + \rho^2 \sigma_u^2}{1+ze^{\bar{h}+h_b}} \right) + 4 \frac{z \kappa u + \rho^2 \sigma_u^2}{1+ze^{\bar{h}+h_b}} \right)
\]
(71)
Since \( \kappa_b \) increases smoothly at \( h = \bar{h} \), we can show that the optimal \( \kappa f (h) \) and \( \kappa u (h) \) are continuous functions of \( h \). This implies
\[
\frac{d}{dh} E \approx \frac{\kappa_f^2 (\sigma_f^2)^2}{\sigma_b^2} \left( -G' (\rho c \kappa_b) \rho c \kappa_b b + \frac{\sigma_f^2 \kappa f \kappa_f^2 (\sigma_f^2)^2 - \kappa_f^2 (\sigma_f^2)^2}{\sigma_b^2} \kappa_f^2 \sigma_f^2 + \rho^2 c^2 \kappa_b \sigma_b^2 \right) \left( 1 - G (\rho c \kappa_b) \right)
\]
(72)
Using the facts that $\kappa'_b > 0$ and $G'(\cdot) > 0$, we can write

$$\frac{d}{dh} E < \frac{\sigma_f^2 \kappa_f'(\sigma_f^2)^2 - \kappa_f'(\sigma_f^2)^2 \left[ \kappa_f'(\sigma_f^2 + \rho^2 \sigma_u^2) \kappa_f' \right]}{\sigma_y^2} \cdot (1 - G(\rho \kappa_f b))$$

So, the sufficient condition for price efficiency to decrease with transparency is $(\kappa_f + 2 \alpha \kappa_u) \kappa'_f < \alpha \kappa_f' \kappa'_u$. Also,

$$\kappa'_f(h) = \frac{ze^{h+\kappa_f}}{(1+ze^{h+\kappa_f})^2}, \quad \kappa'_u(h) = \frac{\alpha ze^{h+\kappa_u}}{(1+\alpha ze^{h+\kappa_u})^2}$$

At $h = \bar{h}$, price efficiency decreases in transparency if

$$\left( \frac{e^{\bar{h}+\kappa_f}}{1+ze^{\bar{h}+\kappa_f}} + 2 \frac{\alpha ze^{\bar{h}+\kappa_u}}{1+\alpha ze^{\bar{h}+\kappa_u}} \right) \frac{1}{1+ze^{\bar{h}+\kappa_f}} < \frac{\alpha ze^{\bar{h}+\kappa_u}}{(1+\alpha ze^{\bar{h}+\kappa_u})^2}$$

From equation (71), observe that as $h_b$ decreases, the cutoff $\bar{h}$ increases and $h_b + \bar{h}$ tends to a constant. As cost of learning about feedback increases, investors have less incentive to learn about it. Given this, the price efficiency decreases in transparency if $(1 + 2\alpha) < e^{h_f - h_u}$. This condition is true for sufficiently high $h_f$ or sufficiently low $h_u$. Hence there exists a cutoff for $h_b$ below which price efficiency decreases with transparency at $h = \bar{h}$.

Price efficiency at $h = \bar{h}$ and $h = \infty$ are given by

$$E_{\bar{h}} = -\sigma_f^2 \left( 1 - \frac{\kappa_f(\bar{h})^2}{\kappa_f(h) + \alpha \kappa_u(h)} \right), \quad E_{\infty} = -\sigma_f^2 \left[ \frac{\alpha + G(\rho \kappa_f b)}{\alpha + 1} \right]$$

We want the conditions under which $E_{\bar{h}} > E_{\infty}$. This can be rewritten as

$$\left( \frac{e^{\bar{h}+\kappa_f}}{1+ze^{\bar{h}+\kappa_f}} + \frac{\alpha ze^{\bar{h}+\kappa_u}}{1+\alpha ze^{\bar{h}+\kappa_u}} \right)^2 > \frac{1 - G(\rho \kappa_f b)}{1 + \alpha}$$

From equation (71), as $h_b$ decreases, the cutoff $\bar{h}$ increases and $h_b + \bar{h}$ tends to a constant. The above condition reduces to $\frac{1}{1+\alpha} > \frac{1 - G(\rho \kappa_f b)}{1+\alpha}$ which is obviously true. This implies that there exists a cutoff for $h_b$ below which price efficiency decreases with transparency.