Career Concerns and Corporate Finance

Preliminary and Incomplete

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Abstract

We incorporate managerial career concerns as in Holmstrom (1999) into a dynamic corporate finance framework with financial constraints. Career concerns induce the manager to exert time-varying stochastic effort as she attempts to influence the market's belief of her talent. However, in equilibrium, career concerns cause the self-interested manager to work hard especially when she is more productive creating substantial value for shareholders even beyond the first-best MM benchmark value (for a firm that is wholly owned by a risk-neutral manager without career concerns.) Additionally, financial constraints and managerial career concerns jointly have important conceptual and quantitative effects on corporate investment, financing, and valuation.

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1 Introduction

In an influential survey of chief financial officers Graham, Harvey and Rajgopal (2005) have revealed the extent to which publicly traded corporations focus on earnings performance as opposed to share value, and how far they are willing to sacrifice net present value to be able to smooth earnings or reach earnings targets expected by analysts. As Stein (1989) has emphasized, one reason why managers focus on earnings performance is that they seek to not only maximize shareholder value but also worry about their careers and what a poor earnings performance reveals about their type. He shows that in an otherwise efficient capital market pursuit of such career concerns by managers can give rise to inefficient myopic behavior by managers. While Stein's model was deliberately set up to prove a point, that myopic behavior is compatible with efficient markets, in reality firms are not only run by career-motivated managers but also face external financing constraints. An obvious question then is how career-concerns interact with external financing constraints? Do external financing costs reinforce or mitigate managerial myopia? More generally, how are corporate financial decisions affected when firms face both external financing costs and are run by career-motivated managers?

These are the questions we address in this paper, by generalizing and adapting the classic Holmstrom (1999) career-concerns model to a growing firm facing external financing costs and that is run by a manager motivated by career concerns. More specifically, our model combines the key building blocks of the dynamic corporate finance model with external financing costs of Bolton, Chen, and Wang (2011) (BCW) with a more general formulation of the managerial career-concern model of Holmstrom (1999). The five main building blocks of the model are: 1) a long-run constant-returns-to-scale production function with convex investment adjustment costs and a constant capital depreciation rate (as in Hayashi, 1982); 2) external equity financing and payout costs; 3) constant cash carry costs; 4) a career-concerns problem arising from the fact that the firm's productivity depends on the manager's talent and effort.

For the career concern part of the model, we generalize Holmstrom (1999) in two important ways. First, Holmstrom assumes that the firm's productivity follows an i.i.d. process with the mean given by the sum of the manager's type and effort, and with a time-invariant manager type. Similarly, we assume that the firm's productivity follows a diffusion process with the drift given by the sum of the manager's type and effort, but we let the manager's type follow a mean-reverting Ornstein-Uhlenbeck process. Second, Holmstrom assumes for simplicity that production goes on forever even if the manager's productivity is negative. In contrast, we allow for endogenous liquidation or abandonment if it is not worth continuing production. As a result managers' careers have stochastic endogenous duration; 5) Finally, the last building block of the model is the dynamic financial management through four financial instruments: cash, equity, credit line, and derivatives (futures). This model is admittedly somewhat stylized, but it captures the essential moral hazard and financial constraint elements that managerial firms face in practice.

The central idea in Holmstrom (1999) is that managers attempt to boost the market's perception about their innate productivity by working hard, but in a rational-expectations equilibrium this is a self-defeating pursuit: the market expects managers to work hard and therefore is not fooled by their efforts to boost their reputation. In addition, given that managers' innate types are assumed to be time invariant, the precision of the market's estimate about the manager's type increases with time as more observations are obtained on the manager's performance. Managers respond to the increasing precision by reducing effort, as reputation becomes harder to change once market beliefs are more firmly set. Thus, a core result emerging from Holmstrom's analysis is that managers are over-worked (over-supply effort) when young and overly idle when old.

In our model the manager's innate type is mean-reverting. Moreover, we allow for optimal stopping of production when the manager's productivity is too low. As a result manager careers have a stochastic endogenous finite duration. At the upper limit, when managerial talent is arbitrarily large, the manager's career approaches infinity. At the lower boundary, when the manager's productivity is thought to be so low that the firm is indifferent between continuation or liquidation, the manager's career is arbitrarily short-lived. In this context, not surprisingly, the manager's effort into building her reputation is inversely related to the expected duration of her career. In other words, the higher the expected productivity of the manager, the longer her expected career, and the higher the effort she supplies to boost her reputation. This is a key difference between our model and Holmstrom (1999). While in his model, managerial effort is decreasing with time, in our model effort is time-invariant but is decreasing in the manager's expected productivity.

In the absence of any external financing costs, one important effect of the career-concerns agency problem is to amplify the manager's productivity through her effort supply: A more productive manager is expected to work longer and puts in more effort, so that the firm's productive asset is more valuable and induces higher investment. The other important effect is through investment. The optimality condition for investment for a firm run by a career-motivated manager is no longer the classical Q-theory of investment condition. The managerial firm sets investment so as to equalize the marginal cost of investment with the marginal managerial-reputation value of investment. Accordingly, a manager with a low productivity and an expected short career will underinvest, while a manager with a high productivity and long career will overinvest.

A career-motivated manager of a financially constrained firm distorts her effort supply more the more financially constrained the firm is, for two reasons. First, by boosting the short-term earnings performance the manager helps relax the financial constraint of the firm. Second, should the firm still need to raise costly external financing it will get better terms the better the market's beliefs are about the manager's innate productivity. Therefore, our model predicts that expected earnings performance other things equal is higher the closer the firm is to raising new external financing. This prediction is consistent with the evidence on operating performance pre and post equity issuance in Degeorge and Zeckhauser (1993), Teoh, Welch and Wong (1998a, 1998b), Rangan (1998), Degeorge, Patel, and Zeckhauser (1999) among others.

Related Literature: Our model is most closely related to the short-termism model of Stein (1989). Just as in our model with no external financing costs, career-motivated managers in Stein (1989) distort investment so as to improve their reported earnings performance and thus boost their reputation. Stein (1989) builds on Holmstrom (1999) to make the point that short-termist distortions can arise even when capital markets are efficient and even though the investment distortions don't fool investors and are self-defeating.

Broadly speaking, our paper also relates to the fast growing dynamic corporate finance literature. One approach to dynamic corporate decision making is to take a contracting perspective. An alternative approach is to take the transaction costs as given. These two approaches are complementary. Our paper draws insights from both strands of the dynamic corporate finance literature. We focus on managerial agency concerns as in Holmstrom (1982, 1999) to highlight the important impact of managerial labor market considerations on corporate finance. [TO BE COMPLETED.]

2 Model

We first describe the firm's physical production and investment technology. We go on to introduce the manager's career concern problem and the learning problem about managerial productivity based on realized performance. We then introduce the firm's external financing costs and its opportunity cost of holding cash. We complete the model description by stating the managerial firm's optimization problem.

2.1 Production Technology

Capital accumulation. The firm employs physical capital and a manager for production. The price of capital is normalized to one and we denote by K and I respectively the level of the capital stock and gross investment. As in standard capital accumulation models, we assume that the firm's capital stock K evolves according to:

$$dK_t = (I_t - \delta K_t)\, dt, \quad t \geq 0, \tag{1}$$

where $\delta \geq 0$ is the rate of depreciation.

Capital adjustment costs. Following the neoclassical investment literature (Hayashi (1982)), we assume that the firm's adjustment cost is homogeneous of degree one in I and K. In other words, the adjustment cost takes the form $G(I, K) = g(i)K$, where i is the firm's investment capital ratio ($i = I/K$), and $g(i)$ is an increasing and convex function. Finally, we assume that the firm can liquidate its assets at any time. The liquidation value L_t is proportional to the firm's capital, $L_t = lK_t$, where $l > 0$ is constant.

Firm productivity and managerial talent. The firm's operating revenue at time t is proportional to its capital stock K_t, and is given by $K_t dA_t$, where dA_t is the firm's revenue or productivity shock over time increment dt.

As in Holmstrom (1999), we assume that the firm's revenue depends on the manager's talent and her effort. Specifically, we assume that the cumulative productivity shock A follows:

$$dA_t = (\mu_t + a_t)dt + \sigma dZ_t^A, \quad t \geq 0\,, \tag{2}$$

where Z^A is the standard Brownian motion under the risk-neutral measure, σ is the constant volatility parameter, a_t denotes the manager's effort choice at time t, and μ_t represents the exogenous time-varying (innate) managerial skill or talent at time t.

Note that the realized productivity shock up to time t, $\{A_s; 0 \leq s \leq t\}$ is observable. Importantly, however, the career-concerns literature assumes that neither the market nor the manager observes μ_t. That is, both the market and the manager learn about the manager's innate talent from past performance observations. Additionally, the market does not observe the manager's effort choice a. Therefore, the manager and the market use realized output/productivity shock as signals to learn about the manager's talent μ_t. We use $\mathcal{F}_t^A$ to denote the information filtration generated by the history of cumulative productivity shocks $\{A_s; 0 \leq s \leq t\}$. Because managerial effort is not observed by the market, the manager can attempt to influence the market's learning by exerting effort a.

We further assume that managerial stochastic talent $\{\mu_t\}$ follows a mean-reverting Ornstein-Uhlenbeck (OU) process given by:

$$d\mu_t = \eta(\overline{\mu} - \mu_t)dt + \epsilon \, dZ_t^\mu \,, \tag{3}$$

where Z^μ is the standard Brownian motion, $\overline{\mu}$ is the long-run mean, η is the rate of mean reversion, and ϵ is the volatility parameter for the managerial talent process μ. Both the market and the manager know the managerial talent process μ, which means that $\overline{\mu}$, η, and ϵ are publicly known. Note that (3) is equivalent to a first-order autoregressive (AR) process in a standard discrete-time formulation where the AR coefficient is equal to $e^{-\eta}$. For simplicity, we assume that the two Brownian motions Z^A and Z^μ have zero correlation.

2.2 Learning via Kalman Filter

As in Holmstrom (1982, 1999), we look for a rational-expectation equilibrium for managerial talent. The market and the manager share the same prior μ_0 about managerial talent. Specifically, we assume that this prior is normally distributed with mean x_0 and variance ϕ_0, in that $\mu_0|\mathcal{F}_0^A \sim \mathcal{N}(x_0, \phi_0)$.

We use a_t^* to denote the time-t managerial effort level inferred in equilibrium. Conditional on the time-t information filtration $\mathcal{F}_t^A$, generated by the history of cumulative productivity shocks $\{A_s : 0 \le s \le t\}$ and the equilibrium managerial effort levels $\{a_s^* : 0 \le s \le t\}$, we define the posterior estimates of the mean and variance of mangerial talent μ_t as follows:

$$x_t = \mathbb{E}^*\left(\mu_t \middle| \mathcal{F}_t^A\right) , \quad \text{and} \quad \phi_t = \mathbb{E}^*\left[(\mu_t - x_t)^2 \middle| \mathcal{F}_t^A\right] . \tag{4}$$

Next, we construct an "innovations" process from productivity shocks A, which we refer to as Z^*, as follows:

$$dZ_t^* = \frac{1}{\sigma}\left(dA_t - (x_t + a_t^*)dt\right) . \tag{5}$$

Over a small time interval $(t, t+\Delta t)$, the innovation $(Z_{t+\Delta t}^* - Z_t^*)$ is proportional to the *unexpected* changes in productivity $(A_{t+\Delta t} - A_t - (x_t + a_t^*)\Delta t)$, which is given by the difference between the realized incremental productivity $(A_{t+\Delta t} - A_t)$ and the expected productivity $(x_t + a_t^*)\Delta t$. By construction, the time-t conditional mean of $(dA_t - (x_t + a_t^*)dt)$ is zero. The volatility parameter σ normalizes the conditional variance of the "unexpected" changes so that Z^* constructed via (5) is a standard Brownian motion process under $\mathcal{F}_t^A$ and equilibrium managerial effort choices $\{a_s^* : 0 \le s \le t\}$. The innovations process Z^* plays the role of "shocks" to the optimal estimates x_t as shown below.

Using the Kalman filter technique, we may write the dynamics of the optimal estimate of productivity x via a recursive structure and express the belief updating process as follows:

$$dx_t = \eta(\overline{\mu} - x_t)dt + \frac{\phi_t}{\sigma}dZ_t^* , \tag{6}$$

where ϕ_t/σ corresponds to the conditional volatility of the innovation shock Z_t^* as defined in (5). Note that x_t has the same rate of mean reversion η and the same long-run mean $\overline{\mu}$

as the unobservable managerial talent μ_t. However, due to learning the volatility parameter is ϕ_t/σ, which differs from the volatility parameter σ for μ_t.

The conditional variance ϕ_t satisfies the following Riccati equation:

$$d\phi_t = \left[-2\eta\phi_t + \epsilon^2 - \left(\frac{\phi_t}{\sigma}\right)^2\right] dt\,. \tag{7}$$

The solution of (7) for the conditional variance ϕ_t is given by:

$$\phi_t = \frac{\epsilon^2 \left(1 - e^{-2t\zeta}\right)}{(\zeta + \eta)\left(1 - e^{-2t\zeta}\right) + \zeta e^{-2t\zeta}}\,,$$

where

$$\zeta = \sqrt{\eta^2 + \frac{\epsilon^2}{\sigma^2}}\,. \tag{8}$$

In the limit as $t \to \infty$, ϕ_t reaches the following constant steady-state level, denoted as ϕ_∞:

$$\phi_\infty = \sigma^2 (\zeta - \eta)\,. \tag{9}$$

Without loss of key economic insights and for analytical simplicity we start the ϕ process with $\phi_0 = \phi_\infty$. Hence, we may write the belief updating process simply as follows:[1]

$$dx_t = \eta(\overline{\mu} - x_t)dt + \frac{\phi_\infty}{\sigma} dZ_t^*\,. \tag{10}$$

Next, we turn to analyze the manager's effort choice a_t. The key insight is that given a market's belief x_t about μ_t, the manager may have incentives to exert effort a_t deviating from the market's expectation a_t^* as she hopes to influence the market's inference. However, markets rationally anticipate managerial incentives, which in equilibrium yield inefficient provision of managerial efforts $a_t = a_t^*$.

[1]See Wang (2004) for an application of this learning process with steady-state variance in an explicitly-solved precautionary saving models with partially observed income.

2.3 Managerial Career Concerns

Given the manager's effort choice $a \in \mathcal{A}$, we construct the following process Z^a:

$$dZ_t^a = \frac{1}{\sigma}\left(dA_t - (x_t + a_t)dt\right) . \qquad (11)$$

From the manager's perspective, Z^a is a standard Brownian motion. By substituting (5) and (11) into the dynamics of x given in (10), we may express x as follows:

$$dx_t \quad = \quad \left[\eta(\overline{\mu} - x_t) + \frac{\phi_\infty}{\sigma^2}(a_t - a_t^*)\right] dt + \frac{\phi_\infty}{\sigma} dZ_t^a . \qquad (12)$$

Therefore, given the market expectation of managerial effort a^*, the manager may change the drift of x by exerting effort a that may potentially differ from a^*.

Integrating (12) from time 0 to t, we obtain the following market belief:

$$x_t = \left[x_0 e^{-\eta t} + (1 - e^{-\eta t})\overline{\mu}\right] + \frac{\phi_\infty}{\sigma^2}\int_0^t e^{-\eta(t-s)}(a_s - a_s^*)ds + \frac{\phi_\infty}{\sigma}\int_0^t e^{-\eta(t-s)} dZ_s^a . \qquad (13)$$

The first term in (13) is the weighted average of time-0 expected value of x_t. The second term shows the impact of managerial incentives on x and the third term in (13) givens the impact of shocks Z^a defined in (11) on x_t from the manager's perspective.

Given the equilibrium effort strategy a_t^*, over time interval $(t, t + dt)$, the market wage for the manager then satisfies:

$$C_t^* dt = \pi \mathbb{E}^* \left[dA_t | \mathcal{F}_t^A\right] = \pi(x_t + a_t^*) K_t dt , \qquad (14)$$

where π is a constant. Dividing (14) by K_t gives the scaled wage payment rate $c_t^* = C_t^*/K_t$:

$$c_t^* = \pi(x_t + a_t^*) . \qquad (15)$$

As in Holmstrom (1999), the wage c^* has both a reputational component x_t and an effort component a_t^*.

In Holmstrom (1999) the manager is paid the entire expected productivity in each period and hence $\pi = 1$. We allow for a more general specification with $0 < \pi \leq 1$. That is, the manager and the firm split the expected output $(x_t + a_t^*)$. Presumably, the firm also contributes to the value-added process and hence captures a fraction $(1 - \pi)$ of the expected output.

The firm's incremental operating cash flow dY_t over time interval $[t, t + dt)$ is then given by:

$$dY_t = K_t dA_t - I_t dt - G(I_t, K_t)dt - C_t^* dt, \quad t \geq 0, \tag{16}$$

where $G(I, K)$ is the capital adjustment cost introduced earlier, and C_t^* is the market wage for the manager. Note that dY_t deducts both capital investment and managerial labor costs. From the manager's perspective, we may express the cash flow (16) as

$$dY_t = K_t \left[(x_t + a_t)dt + \sigma dZ_t^a\right] - I_t dt - G(I_t, K_t)dt - C_t^* dt, \quad t \geq 0\,. \tag{17}$$

2.4 Financing Costs

We follow Bolton, Chen, and Wang (2011) in modeling external financing costs: Whenever the firm chooses to issue external equity, it incurs a fixed cost $\gamma_0 K$ and a marginal cost γ_1 per unit of external financing. Together these costs imply that the firm will optimally tap equity markets only intermittently, and when doing so it raises funds in lumps, consistent with observed firm behavior.

We denote by B_t the firm's cumulative external financing up to time t and hence by dB_t the firm's incremental external financing over time interval $(t, t + dt)$. Similarly, we denote by U_t the firm's cumulative (non-decreasing) payout, and by dU_t the incremental payout over time interval dt. In addition, let N_t denote the cumulative costs of external financing up to time t, and dN_t the incremental costs of raising incremental external funds dB_t. The cumulative external equity issuance B or payout U and the associated cumulative costs N are stochastic controls chosen by the shareholders.

We begin by assuming that the only source of external financing is equity. We later extend the analysis by letting the firm obtain a credit line. When the firm runs out of cash ($W_t = 0$), it has to either raise external funds to continue operating, or it must liquidate its assets. If the firm chooses to raise external equity, it must pay the financing costs specified above. In some situations the firm may prefer liquidation. For example when the manager's expected productivity is too low and the cost of financing is too high. Let τ denote the firm's (stochastic) liquidation time (if $\tau = \infty$, then the firm never closes down).

The rate of return that the firm earns on its cash inventory is the risk-free rate r minus a cash carry cost $\lambda > 0$ per unit of time when the firm's liquid wealth is positive, i.e. $W_t > 0$. Combining cash flow from operations dY_t (netting out of capital investment/adjustment costs and wage compensation) given in (16), with the firm's financing policy given by the cumulative payout process U and the cumulative external financing process B, the firm's cash inventory W evolves according to the following cash-accumulation equation:

$$dW_t = dY_t + (r - \lambda)\, W_t dt + dB_t - dU_t \,, W \geq 0 \,,$$

where the first term is the incremental operating cash flow given in (16), the second term is the interest income netting out of the cash carry cost, the third term dB_t is the cash inflow from external financing, and the last term dU_t captures payout, so that $(dB_t - dU_t)$ is the net cash flow from financing. This equation is a general accounting identity including cash flow from operations dY_t, cash flow from investments $(r - \lambda)W_t dt$, and cash flow from financing. $(dB_t - dU_t)$. Combining (17) and (15), the dynamic of the firm's cash inventory W evolves as follows

$$dW_t = [(x_t + a_t) - \pi(x_t + a_t^*)] K_t dt - I_t dt - G(I_t, K_t)dt + (r - \lambda)W_t dt + \sigma K_t dZ_t^a + dB_t - dU_t\,.$$

Managerial optimality and security pricing. Unlike in Bolton, Chen, and Wang (2011), the manager in our model chooses to maximize her own utility rather than the firm's value. In addition to being paid wages as described above, we also assume that the manager owns a constant fraction, α, of the firm. As in Holmstrom (1999) the manager also incurs a flow disutility measured by $F(a_t, K_t)$ when exerting effort. For simplicity, we assume that the disutility cost is homogeneous in firm size K. That is, we assume that $F(a, K) = f(a)K$, where $f(\cdot)$ is increasing and convex in a.

We can now state the manager's optimization problem. The risk-neutral manager optimally chooses corporate investment I_t, payout policy U_t, external financing policy B_t, effort level a_t, and liquidation time τ to maximize her objective function defined as follows:

$$\begin{aligned} P(K, W, x) = \max_{I,U,H,\tau,a} \mathbb{E}^a \Bigg[& \int_t^\tau e^{-r(s-t)} \left(C(a_s^*, K_s) - F(a_s, K_s) \right) ds \\ & + \alpha \left(\int_t^\tau e^{-r(s-t)} \left(dU_s - dB_s - dN_s \right) + e^{-r(\tau - t)} \left(lK_\tau + W_\tau \right) \right) \Bigg| \mathcal{F}_t^A \Bigg]. \end{aligned} \tag{18}$$

The first term in (18) is the present discounted value of her wage compensation net of her disutility costs and the second term gives the manager's share of firm value.

In the Appendix, we show that the first term in (18) can be written as:

$$\begin{aligned} & \mathbb{E}^a \left[\int_t^\tau e^{-r(s-t)} \left(c(a_s^*) - f(a_s) \right) K_s ds \middle| \mathcal{F}_t^A \right] \\ = \; & \mathbb{E}^a \left[\int_t^\tau e^{-r(s-t)} \left[\pi(x_s + a_s^*) + (a_s - a_s^*)\beta_s - f(a_s) \right] K_s ds \middle| \mathcal{F}_t^A \right], \end{aligned} \tag{19}$$

where

$$\beta_s = \frac{\phi_\infty}{\sigma^2}\mathbb{E}^a_s\left[\int_s^\tau e^{-(r+\eta)(u-s)}du\right]. \tag{20}$$

Given the manager's chosen dynamic policies the firm's total market value, denoted by $V(K, W, x)$, is as follows:

$$V(K,W,x) = \mathbb{E}^*\left[\int_t^\tau e^{-r(s-t)}\left(dU_s - dB_s - dN_s\right) + e^{-r(\tau-t)}\left(lK_\tau + W_\tau\right)\middle|\mathcal{F}^A_t\right]. \tag{21}$$

Similarly, given the manager's chosen dynamic policies, we can calculate the expected PDV (present discounted value) of the manager's wage, which we also refer to as managerial human capital denoted, by $H(K, W, x)$, as follows:

$$H(K,W,x) = \mathbb{E}^*\left[\int_t^\tau e^{-r(s-t)}C(a^*_s, K_s)ds\middle|\mathcal{F}^A_t\right]. \tag{22}$$

Therefore, at each moment t, the manager's total value (before netting out of the disutility costs) is given by $H(K, W, x) + \alpha V(K, W, x)$.

3 Model Solution

Our model involves both managerial optimality and a market equilibrium analysis for managerial compensation. We begin by writing the manager's dynamic optimization problem in recursive form (see the Appendix 3 for all the steps involved in transforming the manager's problem in (18) into the recursive formulation below).

Recursive formulation for the manager's problem. Using standard dynamic programming methods in the interior region, we obtain that the firm's value $P(K, W, x)$ satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:

$$
\begin{aligned}
rP(K,W,x) = \max_{I,a}\, & [\pi(x+a^*+(a-a^*)\beta) - f(a)]K + (I-\delta K)\,P_K \\
& + \eta(\overline{\mu}-x)P_x + [((x+a)-\pi(x+a^*))\,K + (r-\lambda)W - I - G(I,K)]P_W \\
& + \frac{\phi_\infty^2 P_{xx}}{2\sigma^2} + \frac{\sigma^2 K^2}{2} P_{WW} + \phi_\infty K P_{xW}. \qquad (23)
\end{aligned}
$$

The FOC with respect to managerial effort a satisfies

$$
f'(a) = \pi\beta + P_W\,, \qquad (24)
$$

where β is given by (20).

The manager's marginal benefit has two components: the contemporaneous marginal value of effort, measure by $\pi\beta$, and the manager's marginal value of cash P_W. The left side of (24) is the marginal disutility of effort. The manager optimally chooses effort a to equate the two sides of (24).

The FOC for investment is then:

$$
1 + G_I(I,K) = \frac{P_K(K,W)}{P_W(K,W)}. \qquad (25)
$$

The left side of (25) is the marginal adjustment cost. The right side is the ratio between the manager's marginal reputation value with respect to K and the manager's marginal value of cash. Again, the manager chooses I to optimally equate the two sides of (25).

By substituting the manager's effort and investment decisions into (23) and imposing the condition that market expectations are rational in equilibrium, we obtain the following equation for $P(K, W, x)$:

$$\begin{aligned} rP(K,W,x) = {} & (\pi(x+a^*) - f(a^*))\,K + (I^* - \delta K)\,P_K + \eta(\overline{\mu} - x)P_x \\ & +[(1-\pi)(x+a^*)K + (r-\lambda)W - I^* \; - G(I^*,K)]P_W \\ & + \frac{\phi_\infty^2 P_{xx}}{2\sigma^2} + \frac{\sigma^2 K^2}{2} P_{WW} + \phi_\infty K P_{xW}, \end{aligned} \tag{26}$$

where I^* solves (25) and a^* solves

$$f'(a^*) = \pi\beta + P_W = \pi\,(\zeta - \eta)\,\mathbb{E}_t^a\left(\int_t^\tau e^{-(r+\eta)(s-t)} ds\right) + P_W\,. \tag{27}$$

Homogeneity property. A key simplification in our setup is that the firm's three-state optimization problem can be reduced to a two-state problem by exploiting homogeneity with respect to K. That is, we can write the manager's value in the multiplicative form

$$P\,(K, W, x) = K \cdot p\,(w, x)\,, \tag{28}$$

where $w = W/K$ is the firm's cash-capital ratio.

Instead of solving for the manager's value $P\,(K, W, x)$ in a three-dimensional problem, we are thus able to solve for the manager's value-capital ratio $p\,(w, x)$. Note that marginal q is $P_K\,(K, W, x) = p\,(w, x) - wp_w\,(w, x)$, the marginal value of cash is $P_W\,(K, W, x) = p_w\,(w, x)$, and $P_{WW} = p_{ww}\,(w, x)\,/K$. Substituting these terms into (26) we obtain the following PDE for $p\,(w, x)$:

$$\begin{aligned} rp(w,x) \;\; = \;\; & (\pi(x+a^*) - f(a^*)) + (i-\delta)\,(p - wp_w) + \eta(\overline{\mu} - x)p_x \\ & +[(1-\pi)(x+a^*) + (r-\lambda)w - i - g(i)]p_w + \frac{\phi_\infty^2 p_{xx}}{2\sigma^2} + \phi_\infty p_{xw} + \frac{\sigma^2}{2} p_{ww}\,, \end{aligned} \tag{29}$$

where the optimal investment-capital ratio $i(w,x)$ satisfies the following FOC:

$$g'(i(w,x)) = \frac{p(w,x)}{p_w(w,x)} - w - 1\,. \tag{30}$$

Boundary Conditions: We begin with the upper boundary condition. Let $\overline{w}(x)$ denote this endogenous payout boundary. Intuitively, if the firm starts with a large amount of cash $(w > \overline{w}(x))$, then it is optimal for the firm to distribute the excess cash as a lump-sum payment and bring the cash-capital ratio w down to $\overline{w}(x)$. Moreover, the manager's value must be continuous before and after cash distribution. Therefore, for $w > \overline{w}(x)$, we have the following equation for $p(w,x)$:

$$p(w,x) = p(\overline{w}(x),x) + \alpha\,(w - \overline{w}(x))\,, \quad w > \overline{w}(x)\,, \tag{31}$$

which implies that

$$p_w(\overline{w}(x),x) = \alpha. \tag{32}$$

Since the payout boundary $\overline{w}(x)$ is optimally chosen, we also have the following "super contact" condition (see, e.g. Dumas (1991)):

$$p_{ww}(\overline{w}(x),x) = 0. \tag{33}$$

We now turn to the lower boundary condition. When the manager's productivity is very low or the cost of financing is very high, the firm will prefer liquidation to refinancing. In that case the manager's value upon liquidation is $p(\underline{w}(x),\, x)K = \alpha l K$. Therefore, we have

$$p(\underline{w}(x),x) = \alpha l. \tag{34}$$

And the optimality of the liquidation decision implies

$$p_w(\underline{w}(x), x) \geq 0. \tag{35}$$

The firm will issue equity in lumps when the manager's productivity is sufficiently high and the financing cost is low. Under homogeneity with respect to K, we can show that the total amount raised through the equity issue is nK, where $n > 0$ is endogenously determined as follows. First, the manager's value function is continuous before and after equity issuance, which implies the following condition for $p(w, x)$ at the boundary $\underline{w}(x)$:

$$p(\underline{w}(x), x) = p(\underline{w}(x) + n, x) - \alpha(\gamma_0 + (1 + \gamma_1)\, n). \tag{36}$$

The smooth-pasting condition is

$$p_w(\underline{w}(x), x) \geq p_w(\underline{w}(x) + n, x). \tag{37}$$

Note finally that n is optimally chosen, and this gives the following smoothing pasting boundary condition at n:

$$p_w(\underline{w}(x) + n, x) = \alpha(1 + \gamma_1). \tag{38}$$

We summarize this characterization of the optimal solution in the following proposition.

Proposition 1 *In the region where $0 \leq \underline{w}(x) < w < \overline{w}(x)$, the manager's scaled value function solves the following PDE:*

$$\begin{aligned} rp(w, x) \;&=\; (\pi(x + a^*) - f(a^*)) + (i - \delta)\,(p - wp_w) + [(1 - \pi)(x + a^*) + (r - \lambda)w - i - g(i)]p_w \\ &\quad + \eta(\overline{\mu} - x)p_x + \frac{\phi_\infty^2 p_{xx}}{2\sigma^2} + \phi_\infty p_{xw} + \frac{\sigma^2}{2} p_{ww}, \end{aligned} \tag{39}$$

subject to the following boundary conditions under external financing:

$$
\begin{aligned}
p_w(\overline{w}(x), x) &= \alpha, \quad p_{ww}(\overline{w}(x), x) = 0\,, \\
p(\underline{w}(x), x) &= p(\underline{w}(x) + n, x) - \alpha(\gamma_0 + (1+\gamma_1)\, n), \\
p_w(\underline{w}(x), x) &\geq p_w(\underline{w}(x) + n, x)
\end{aligned}
$$

Moreover, the optimal effort choice satisfies

$$f'(a^*) = \pi\beta + p_w\,.$$

And when $\tau \to \infty$:

$$f'(a^*) = \pi\frac{\zeta - \eta}{r + \eta} + p_w\,.$$

And the optimal investment strategy is given by

$$g'(i(w, x)) = \frac{p(w, x)}{p_w(w, x)} - w - 1\,. \tag{40}$$

And the optimal external financing strategy solves:

$$p_w(\underline{w}(x) + n, x) = \alpha(1 + \gamma_1)\,.$$

Under liquidation the lower boundary condition, $\underline{w}(x) \geq 0$, satisfies the following equations

$$p(\underline{w}(x), x) = \alpha l\,, \quad p_w(\underline{w}(x), x) \geq 0. \tag{41}$$

3.1 Generalization with Credit Line

Our baseline financial constraint model can be extended to allow for a credit line as in Bolton, Chen, and Wang (2011). This is an important extension to consider, as, in practice, many firms are able to secure such lines, and, for these firms, access to a credit line is an important alternative source of liquidity.

We model the credit line as a source of funding the firm can draw on at any time it chooses up to a limit. We set the credit limit to a maximum fraction of the firm's capital stock, so that the firm can borrow up to ρK, where $0 < \rho \leq l$ is a constant. The logic behind this assumption is that the firm must be able to post collateral to secure a credit line and the highest quality collateral does not exceed the fraction ρ of the firms capital stock. We may thus interpret ρK to be the firm's short-term debt capacity. Following Bolton, Chen, and Wang (2011), we treat ρ as exogenous in this paper. It is straightforward to endogenize the credit line limit by assuming that banks charge a commitment fee on the unused part of the credit line. We assume that the firm pays a constant spread $|\lambda_-| > 0$ over the risk-free rate on the amount of credit it uses.

For notational simplicity, we define the cost of the liquidity as $\lambda_+ > 0$ in the cash region ($W > 0$), and $\lambda_- < 0$ in the credit region ($W \leq 0$) as follows:

$$\lambda \;=\; \begin{cases} \lambda_+ \,, & w > 0 \,, \\ \lambda_- \,, & w \leq 0 \,. \end{cases} \tag{42}$$

With this notation, we may express all the results in Proposition 1 by requiring that the following innocuous condition holds: $-\rho < \underline{w}(x) < \overline{w}(x)$.

4 Benchmarks under Incomplete Information

In this section, we summarize results for two incomplete-market benchmarks:

- the case with no career concerns and no external financing costs
- the case with managerial career concerns but no external financing costs.

4.1 No Career Concerns and No External Financing Costs

First, we consider the case where the manager chooses a to maximize her value function without worrying about market expectations about her managerial skills. The manager still learns about her own type x as the market does. Given the belief x, the manager chooses a and i to maximize her value function

$$P(K,W,x) = p(w,x)K\,, \tag{43}$$

where

$$p(w,x) = \alpha(m(x)+w)\,. \tag{44}$$

For simplicity, we also refer to this case as the managerial first-best setting. Note that this case differs from the first-best case from shareholder's perspective because objective functions are different here for the manager and for shareholders and managers have full control over firm's decisions.

With perfect capital markets, the manager's scaled value per unit of ownership, $m(x)$, solves the following ODE:

$$(r+\delta-i(x))m(x) = \max_{a,i} \quad \frac{\pi(x+a)-f(a)}{\alpha}+(1-\pi)(x+a)-i(x)-g(i(x))+\eta(\overline{\mu}-x)m'(x)+\frac{\phi_\infty^2}{2\sigma^2}m''(x), \tag{45}$$

with the following conditions for the endogenous liquidation boundary:

$$m(\underline{x}) = l, \quad \text{and} \quad m'(\underline{x}) = 0\,. \tag{46}$$

In (45), the manager's optimal investment policy, denoted by $i_{mf}(x)$, satisfies:

$$g'(i_{mf}(x)) = m(x)-1\,, \tag{47}$$

and the managerial effort a_{mf} solves

$$f'(a_{mf}) = \pi + \alpha(1-\pi). \tag{48}$$

Equation (47) is the manager's FOC with respect to investment. Note that the manager's rationally equates marginal cost of investing to manager's marginal q. Equation (48) is the FOC for managerial effort which states that the marginal disutility $f'(a)$ equals the marginal benefit, given by the sum of π and $\alpha(1-\pi)$.

Using the optimal values of $i_{mf}(x)$ and a_{mf}, we can derive the implied value of the firm's capital, Tobin's q, as follows:

$$(r+\delta-i(x))q(x) = (1-\pi)(x+a_{mf}) - i(x) - g(i(x)) + \eta(\overline{\mu}-x)q'(x) + \frac{\phi_{\infty}^2}{2\sigma^2}q''(x), \tag{49}$$

with the following boundary condition:

$$q(\underline{x}) = l\,. \tag{50}$$

Similarly, we can calculate the managerial human capital as follows:

$$(r+\delta-i(x))h(x) = \pi(x+a_{mf}) + \eta(\overline{\mu}-x)h'(x) + \frac{\phi_{\infty}^2}{2\sigma^2}h''(x)\,, \tag{51}$$

with the following boundary condition:

$$h(\underline{x}) = 0\,. \tag{52}$$

4.2 Career Concerns and No External Financing Costs

Now, we consider the case with career concerns model in our investment setting with perfect capital markets. Again, the manager's scaled value function is given by $p(w,x) = \alpha(w+$

$m(x)$). The manager's scaled value function $m(x)$ solves the following ODE:

$$(r+\delta-i(x))m(x) = \frac{\pi(x+a^*)-f(a^*)}{\alpha}+(1-\pi)(x+a^*)-i(x)-g(i(x))+\eta(\overline{\mu}-x)m'(x)+\frac{\phi_\infty^2}{2\sigma^2}m''(x), \tag{53}$$

subject to the following boundary conditions for the endogenous liquidation decision:

$$m(\underline{x}) = l, \quad \text{and} \quad m'(\underline{x}) = 0\,. \tag{54}$$

Both the manager's effort and investment policies in (53) depend on x. The FOC for investment is

$$g'(i(x)) = m(x) - 1. \tag{55}$$

The FOC for effort a^* in (53) satisfies:

$$f'(a(x)) = \pi\beta(x) + \alpha\,.$$

where $\beta(x)$ solves the following ODE:

$$(r+\eta)\beta(x) = (\zeta-\eta) + \eta(\overline{\mu}-x)\beta'(x) + \frac{\phi_\infty^2}{2\sigma^2}\beta''(x)\,, \tag{56}$$

with the following boundary conditions:

$$\beta(\underline{x}) = 0 \quad \text{and} \quad \lim_{x\to\infty}\beta(x) = \frac{\zeta-\eta}{r+\eta}\,. \tag{57}$$

Importantly, unlike in Holmstrom (1999) managerial effort a in equilibrium depends on x. Additionally, we note that the liquidation option effectively shortens the impact of career concerns. As a result,

$$\beta(x) < \frac{\zeta-\eta}{r+\eta}\,. \tag{58}$$

Intuitively speaking, the manager's incentive to exert effort is mitigated by a shorter expected horizon due to endogenous liquidation.

After obtaining the equilibrium a^* and the optimal investment rule $i(x)$, we may solve firm's Tobin's q by using (49)-(50), and managerial human capital $h(x)$ by using (51)-(52).

5 Analysis

Our model is tractable for any homogeneous adjustment cost function $g(i)$. For illustrational purposes, we choose the following widely-used quadratic adjustment cost function:

$$g\left(i\right)=\frac{\theta_i}{2}\,i^2, \tag{59}$$

where the parameter θ_i measures the degree of the adjustment cost. Similarly, we also choose the disutility cost to be the following quadratic form:

$$f(a)=\frac{\theta_f}{2}\,a^2, \tag{60}$$

where the parameter θ_f measures the degree of the disutility cost.

Parameter choices. We set the long-run mean of the risk-adjusted productivity shock to $\overline{\mu}=18\%$ and the volatility of μ to be $\sigma=9\%$, which are in line with the estimates of Eberly, Rebelo, and Vincent (2009) for large U.S. firms. The risk-free rate is $r=6\%$. The adjustment cost parameter is set at $\theta=4$. As suggested for the liquidation value in Hennessy and Whited (2007), we take $l=0.9$. The other parameter values are set as follows: $\delta=0.18, \lambda=0.01, \epsilon=0.15, \eta=0.3, \pi=0.2$, and $\alpha=0.6$.

5.1 Incomplete-Information Benchmark Results

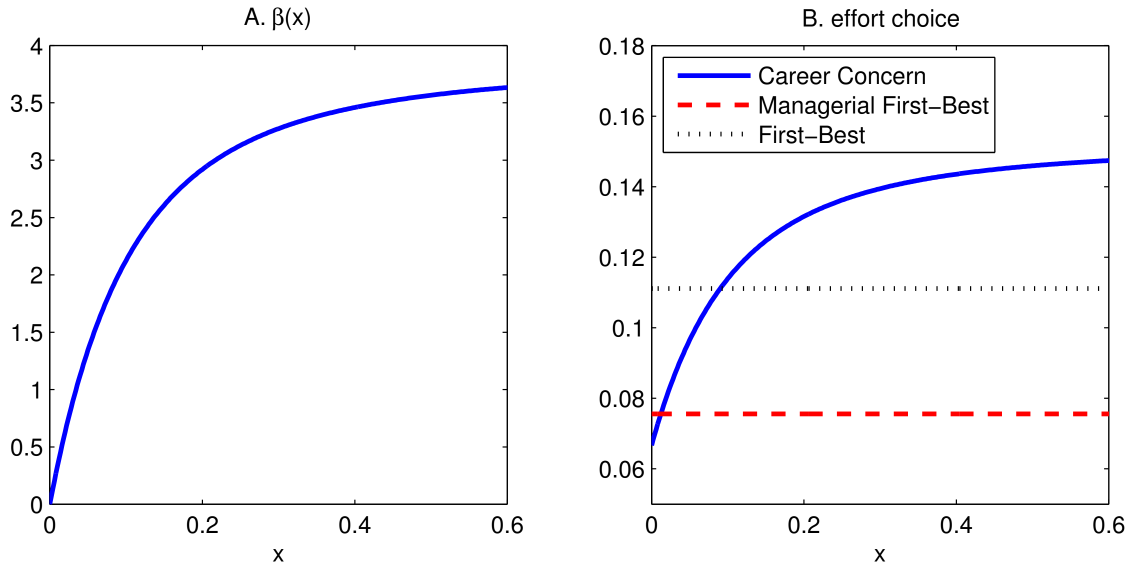

Figure 1: **Managerial duration $\beta(x)$ and effort choice $a(x)$.**

Managerial duration and effort choices. Panel A of Figure 1 plots the duration, $\beta(x)$, which measure the mean-reversion-adjusted horizon of career concerns in the model. Note that $\beta(x)$ increases with x. The firm will be liquidated when the estimated managerial talent x reaches the endogenous boundary $\underline{x}$ at which $\beta(x) = 0$. As $x \to \infty$, the firm's survival probability approaches unity, and the duration equals $(\zeta - \eta)/(r + \eta) = 3.87$.

Panel B of Figure 1 plots the optimal effort choice $a(x)$. The dotted line corresponds to the optimal effort choice under the first-best case where the manger owns 100% of the firm, (i.e., $\alpha = 1$) and has no career concerns. In our example, the first-best effort level is $a_{fb} = 0.111$. The dashed line gives the optimal effort choice under the managerial first-best case, where $a_{mf} = 0.076$. We see that the effort choice under the managerial first-best case is lower than that the first-best case. The more the manager owns the firm, the harder she works.

Importantly, for the case with career concerns, for sufficiently low values of x, the manager exerts low effort, as the firm is very close to being liquidated and $\beta(x) \to 0$. The manager

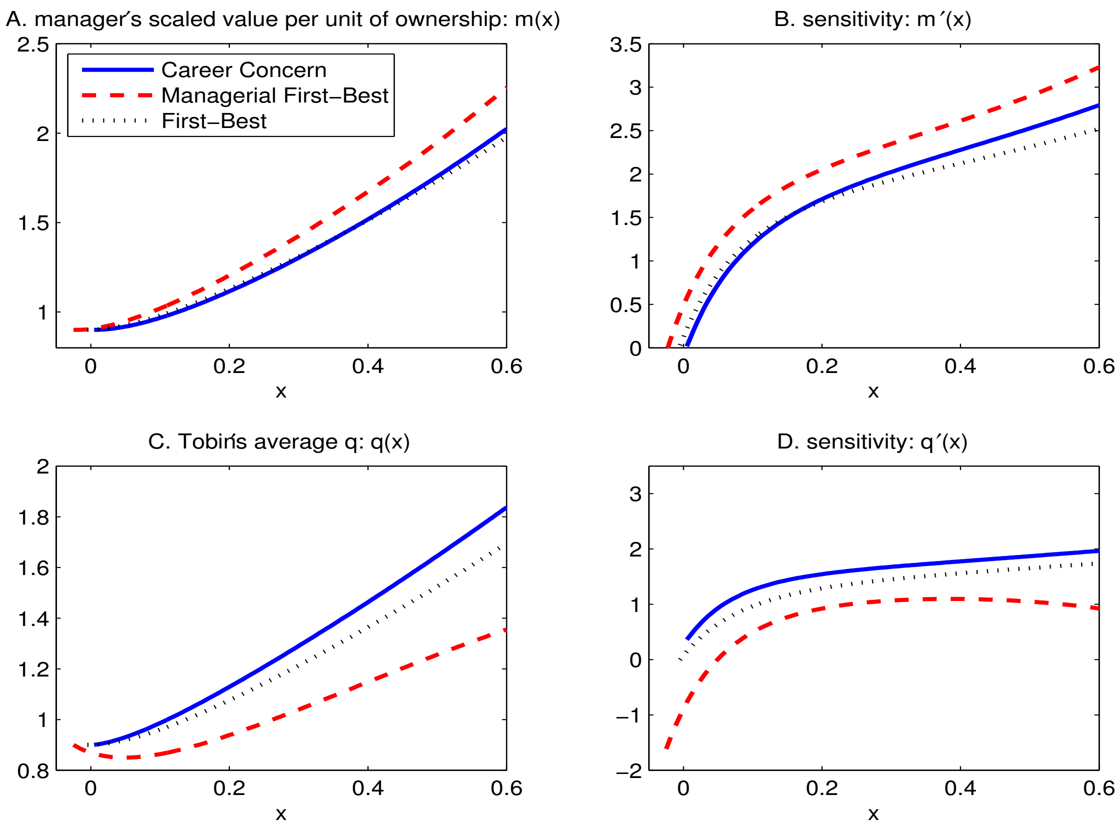

Figure 2: **The manager's q, $m(x)$, and the firm's Tobin's average q, $q(x)$, and their sensitivities $m'(x)$ and $q'(x)$.**

essentially has an abandonment option which discourages her from working hard. On the other hand, for sufficiently high values of x, the manager exert much more effort because the manager's duration $\beta(x)$ approaches the maximal value and hence the benefit of working hard is very high. For example, when $x = 0.6$, we have $\beta(0.6) = 3.5$ and $a(0.6) = 0.15$, which is much greater than the managerial first-best effort level $a_{mf} = 0.111$.

Manager's q and firm's Tobin's q. Panels A and C of Figure 2 plot both the manager's q, $m(x)$, and the firm's average q for investors, respectively. Additionally, we also plot their sensitivities $m'(x)$ and $q'(x)$ in Panels B and D in the same Figure. First, it shows that both the manager's and the firm's q increase in x, which is intuitive.

Second, managerial career concerns actually substantially increase firm value $q(x)$ compared even with the firm's first-best case. How can this be? This is because the manager rationally exerts excessive effort which increases the firm's productivity and hence firm's value of capital. Surprisingly, managerial career concerns have unintended positive consequences on shareholders' value.

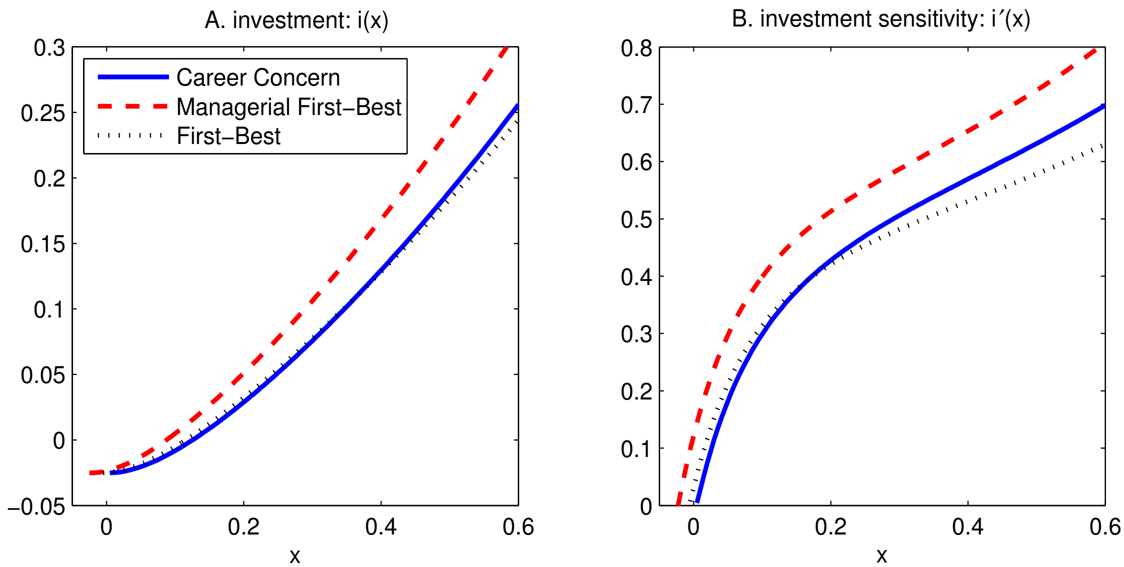

Figure 3: **Investment-capital ratio $i(x)$ and its sensitivity $i'(x)$.**

Investment. Figure 3 plots the investment-capital ratio $i(x)$ and its sensitivity $i'(x)$. Interestingly, career concerns cause the manager to invest less, *ceteris paribus.*

Human capital. Figure 4 plots the scaled value of human capital $h(x)$, and its sensitivity $h'(x)$. Panel A shows that $h(x)$ increases with x, which is very intuitive.

6 Career Concerns and Financial Constraints

We will incorporate the impact of financial frictions on career concerns. Results to be added.

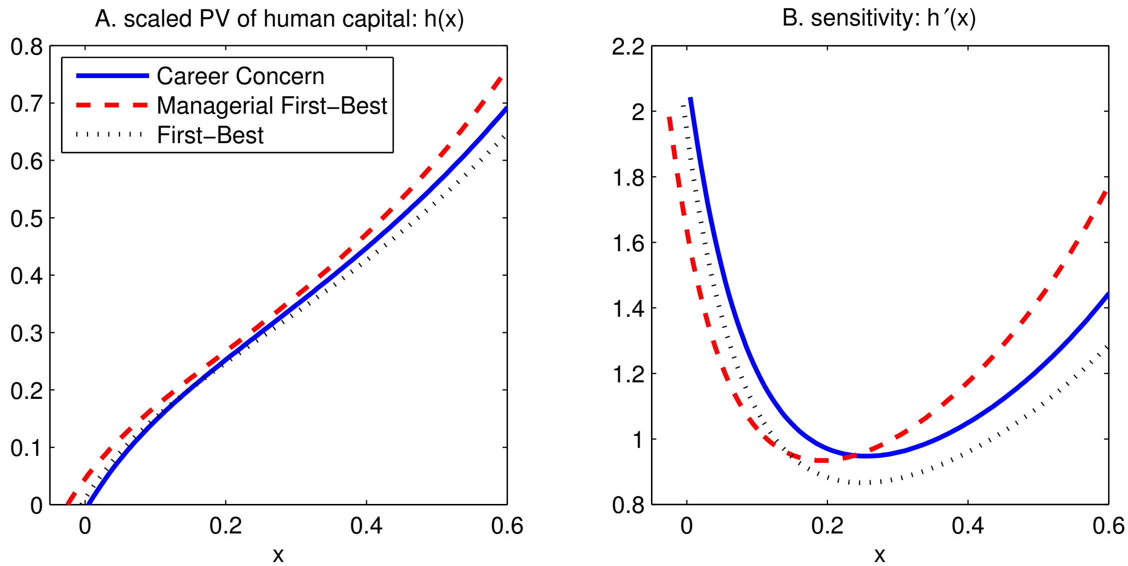

Figure 4: **The scaled PV of human capital $h(x)$, and the sensitivity $h'(x)$.**

7 Conclusion

In this paper, we integrate managerial career concerns developed by Holmstrom (1999) into a dynamic corporate finance framework with financial constraints. Our model shows that career concerns induce the manager to exert time-varying stochastic effort with an attempt to influence the market's belief of managerial talent. Surprisingly, managerial career concerns can induce the firm to create value as the manager's own career development interests causes her to work harder when she is productive creating an additional value boost.

We also show that the career-motivated manager of a financially constrained firm increases her effort supply as the firm's liquidity decreases.

In terms of the big picture, we see that managerial career concerns can have substantial impact on corporate investment, financing, and valuation.

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Appendices

A Derivation for Section 3

First, substituting the market's belief x about managerial talent, given by (13), into the equilibrium wage contract (14), we obtain the following formula for the manager's wage compensation in net of disutility costs:

$$
\begin{aligned}
& \mathbb{E}^a\left[\int_t^\tau e^{-r(s-t)}\left(c(a_s^*)-f(a_s)\right)K_s ds\middle|\mathcal{F}_t^A\right] \\
= \;& \mathbb{E}^a\left[\int_t^\tau e^{-r(s-t)}\left(\pi\mathbb{E}_t^a(x_s+a_s^*)-f(a_s)\right)K_s ds\middle|\mathcal{F}_t^A\right] \\
= \;& \mathbb{E}^a\left[\int_t^\tau e^{-r(s-t)}\left(\mathbb{E}_t^a(\pi x_s)+\pi\mathbb{E}_t^a(a_s^*)-f(a_s)\right)K_s ds\middle|\mathcal{F}_t^A\right] \\
= \;& \mathbb{E}^a\left[\int_t^\tau e^{-r(s-t)}\left(\pi\left(x_t e^{-\eta(s-t)}+(1-e^{-\eta(s-t)})\overline{\mu}+\frac{\phi_\infty}{\sigma^2}\int_t^s e^{-\eta(y-s)}(a_y-\mathbb{E}_t^a(a_y^*))dy\right)\right.\right. \\
& \qquad \left.\left. +\pi\mathbb{E}_t^a(a_s^*)-f(a_s)\right)K_s ds\middle|\mathcal{F}_t^A\right]. \qquad \text{(A.1)}
\end{aligned}
$$

Note that

$$
\begin{aligned}
& \mathbb{E}^a\left[\int_t^\tau e^{-r(s-t)}\left(\frac{\phi_\infty}{\sigma^2}\int_t^s e^{-\eta(y-s)}(a_y-\mathbb{E}_t^a(a_y^*))dy\right)K_s ds|\mathcal{F}_t^A\right] \\
= \;& \frac{\phi_\infty}{\sigma^2}\mathbb{E}^a\left[\int_t^\tau e^{-r(s-t)}\left(\int_t^s e^{-\eta(y-s)}(a_y-\mathbb{E}_t^a(a_y^*))dv\right)K_s ds|\mathcal{F}_t^A\right] \\
= \;& \frac{\phi_\infty}{\sigma^2}\mathbb{E}^a\left[\int_t^\tau e^{rt}\left(\int_t^s e^{-(r+\eta)s-\eta y}(a_y-\mathbb{E}_t^a(a_y^*))dy\right)K_s ds|\mathcal{F}_t^A\right] \\
= \;& \frac{\phi_\infty}{\sigma^2}\mathbb{E}^a\left[\int_t^\tau e^{rt}\left(\int_t^s e^{-(r+\eta)s}\left[e^{-\eta y}(a_y-\mathbb{E}_t^a(a_y^*))\right]dy\right)K_s ds|\mathcal{F}_t^A\right] \\
= \;& \frac{\phi_\infty}{\sigma^2}\mathbb{E}^a\left[\int_t^\tau e^{rt}\left(\int_s^\tau e^{-(r+\eta)y}\left[e^{-\eta s}(a_s-\mathbb{E}_t^a(a_s^*))\right]dy\right)K_s ds|\mathcal{F}_t^A\right] \\
= \;& \frac{\phi_\infty}{\sigma^2}\mathbb{E}^a\left[\int_t^\tau e^{rt}\left(\int_s^\tau e^{-(r+\eta)y}dy\right)\left[e^{-\eta s}(a_s-\mathbb{E}_t^a(a_s^*))\right]K_s ds|\mathcal{F}_t^A\right] \\
= \;& \frac{\phi_\infty}{\sigma^2}\mathbb{E}^a\left[\int_t^\tau e^{-r(s-t)}\left(\int_s^\tau e^{-(r+\eta)(y-s)}dy\right)\left[(a_s-\mathbb{E}_t^a(a_s^*))\right]K_s ds|\mathcal{F}_t^A\right] \\
= \;& \mathbb{E}^a\left[\int_t^\tau e^{-r(s-t)}\left[(a_s-\mathbb{E}_t^a(a_s^*))\right]\beta_s K_s ds|\mathcal{F}_t^A\right], \qquad \text{(A.2)}
\end{aligned}
$$

where β_s is given by (20).

Therefore, we have the manager's wage compensation in net of disutility costs takes the following form:

$$
\begin{aligned}
& \mathbb{E}^a\left[\int_t^\tau e^{-r(s-t)}\left(c(a_s^*)-f(a_s)\right)K_s ds|\mathcal{F}_t^A\right] \\
= \; & \mathbb{E}^a\left[\int_t^\tau e^{-r(s-t)}\left(\pi\left(x_t e^{-\eta(s-t)}+(1-e^{-\eta(s-t)})\overline{\mu}+\left[(a_s-\mathbb{E}_t^a(a_s^*))\right]\beta_s+\mathbb{E}_t^a(a_s^*)\right)-f(a_s)\right)K_s ds|\mathcal{F}_t^A\right] \\
= \; & \mathbb{E}^a\left[\int_t^\tau e^{-r(s-t)}\left(\pi\left(\mathbb{E}_t^a(x_s+a_s^*)+\left[(a_s-\mathbb{E}_t^a(a_s^*))\right]\beta_s\right)-f(a_s)\right)K_s ds\middle|\mathcal{F}_t^A\right]. \qquad \text{(A.3)}
\end{aligned}
$$

Re-expressing (18), we obtain:

$$
\begin{aligned}
P(K,W,x) \; = \; & \max_{I,U,B,\tau,a} \mathbb{E}^a\left[\int_t^\tau e^{-r(s-t)}\left(c(a_s^*)-f(a_s)\right)K_s ds\middle|\mathcal{F}_t^A\right] \qquad \text{(A.4)} \\
& +\alpha\mathbb{E}^a\left[\left(\int_t^\tau e^{-r(s-t)}\left(dU_s-dB_s-dN_s\right)+e^{-r(\tau-t)}\left(lK_\tau+W_\tau\right)\right)\middle|\mathcal{F}_t^A\right] \\
= \; & \max_{I,U,H,\tau,a} \mathbb{E}^a\left[\int_t^\tau e^{-r(s-t)}\left[\pi\left(\mathbb{E}_t^a(x_s+a_s^*)+\left[(a_s-\mathbb{E}_t^a(a_s^*))\right]\beta_s\right)-f(a_s)\right]K_s ds\middle|\mathcal{F}_t^A\right] \\
& +\alpha\mathbb{E}^a\left[\left(\int_t^\tau e^{-r(s-t)}\left(dU_s-dB_s-dN_s\right)+e^{-r(\tau-t)}\left(lK_\tau+W_\tau\right)\right)\middle|\mathcal{F}_t^A\right], \qquad \text{(A.5)}
\end{aligned}
$$

Finally, by using the standard dynamic programming method, we obtain the HJB (23).