Asymmetric Information and Liquidity Provision *

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Abstract

The presence of information asymmetry increases the probability that a potential predator will provide liquidity rather than engaging in predatory trading during liquidation by a distressed trader. More information asymmetry is associated with greater expected gains from liquidation for the distressed trader. There is a negative correlation between the degree of information asymmetry and the returns from predatory trading, which is consistent with empirical findings. These results imply that strategic traders are more likely to stabilize markets by providing liquidity when information is asymmetric. These findings highly a cost associated with disclosure and can explain the documented rarity of illiquidity episodes in financial markets.

Why do institutional investors at times stabilize markets by providing liquidity and at other times destabilize markets by engaging in predatory trading? We examine the determinants of the choice between providing liquidity and engaging in predatory trading when another large trader is forced to sell or buy a risky asset.[1] This choice has important implications for financial markets. Predatory trading reduces liquidity, usually at times when it is needed the most, and increases transaction costs for large traders. Moreover, Brunnermeier and Pedersen (2005) argue that predatory trading increases the risk of a financial crisis, amplifies financial contagion, and affects institutional investors' risk management strategies.

We argue that a key variable affecting this choice is the presence of information asymmetry between the distressed trader and her potential predators. A natural source of information asymmetry is the amount of assets to be sold or bought, which we assume is only known by the distressed trader. We refer to this amount as the liquidation size. This argument is supported by anecdotal evidence. For example, Lowenstein (2000) notes that the head of Long Term Capital Management "... bitterly complained to the Fed's Peter Fisher that Goldman, among others, was front- running, meaning trading against it on the basis of inside knowledge." Also, Wermers (2001) argues that "more frequent portfolio disclosure would enable increased front running by professional investors and speculators." Along the same line, the International Association for Quantitative Finance (IAQF) recommends that large institutional investors "limit granularity of reporting sufficiently to protect Investors against predatory trading against the Managers positions."[2,3]

We model the interaction between large traders in an illiquid market as a two-player nonzero-sum stochastic differential game where the players are distinct *ex-ante*. Illiquidity means that the price of the risky asset is a function of both the large traders' aggregate holding of this asset (long-term impact) and their aggregate trading rate of this asset (short-term impact). A distressed trader needs to liquidate the single risky asset. A potential predator can either provide liquidity

[1]Trading is non-informed, that is independent of risky asset's fundamental value. Vayanos (2001) argues that non-informed trading must represent a large subset of the trading activity in financial markets. Examples of non-informed trading include trading resulting from either index reconstitution or flow of fund to/from institutional investors. The extant literature documents significant non-informed trading activity in financial markets (see Chen, Noronha, and Singal (2004), Coval and Stafford (2007), Zhang (2010), Petajisto (2011), and Bessembinder, Carrion, Venkataraman, and Tuttle (2014)).

[2] See IAQF Investor Risk Committee Consensus Document: Findings on Hedge Fund Transparancy and Disclosure, July 27, 2001.

[3]See Table I in Brunnermeier and Pedersen (2005) for additional anecdotal evidence relating information asymmetry to predatory trading.

or engage in predatory trading but does not know the liquidation size, only the distribution it is drawn from. Profitable predation requires racing to sell the risky asset while its price is high, ahead of the distressed trader's price impact, and then buying it at a lower price later on. We consider closed loop equilibria to allow learning about the liquidation size through changes in the price of the risky asset. These changes are a function of changes in the asset's fundamental value and aggregate trading by the large traders. We provide closed-form solutions of the game under some parameter restrictions and use numerical techniques to solve for the equilibria without restrictions.

Our main finding is that information asymmetry reduces the probability that predatory trading occurs in illiquid markets. The intuition is that the potential predator faces higher losses when engaging in predatory trading relative to providing liquidity, losses due to errors made while estimating the liquidation size.[4] Predatory trading is associated with higher losses because it requires more aggressive trading to race the distressed trader to the market, which leads to more estimation errors and higher trading cost. Moreover, the distressed trader can partially forecast the potential predator's error in estimating the liquidation size. This forecast can lead to further losses to the potential predator when predation occurs.

We also find that market illiquidity affects the probability of predatory trading occurring. Predatory trading is less likely when the long-term price impact takes low values and is negligible when the long-term price impact is zero. This result is intuitive. As Brunnermeier and Pedersen (2005) note, "the predator derives profit from the price impact of the prey". Higher long-term price impact leads to higher profits from predation. In addition, the resolution of uncertainty about the liquidation size, which reduces the degree of information asymmetry, is faster when the long-term price impact is high. The reason is that trading by the distressed trader explains a higher percentage of the changes in the price of the risky asset when the long-term price impact is high.

Our work highlights the welfare benefits of information asymmetry during crises. We show that an increase in information asymmetry generally benefits distressed traders, and hurts predators, and increases the large traders' aggregate wealth. These findings imply that there may be a cost associated with implementing recent policies requiring more transparency for institutional

[4]These losses are increasing in the degree of information asymmetry. The potential predator never incurs losses in equilibrium in models with complete information. See Brunnermeier and Pedersen (2005), Carlin, Lobo, and Viswanathan (2007), and Schöneborn and Schied (2007).

investors.[5]

Two of our predictions are consistent with existing empirical evidence. The model's prediction that higher degree of information asymmetry leads to lower returns achieved by the potential predator is consistent with the findings of Shive and Yun (2013). They show that returns from predatory trading are higher when mutual funds are required to have more frequent disclosure. Bessembinder, Carrion, Venkataraman, and Tuttle (2014) find empirical evidence that strategic traders provide liquidity when markets are resilient. They define market resiliency as the degree to which "some or all of the immediate price impact of trades is subsequently reversed." Their finding is consistent with our prediction that the potential predator provides liquidity when the long-term price impact is low.

Related Literature

Our research is related to several strands of literature including models of liquidity crises, competition among strategic traders, and distressed liquidation of risky assets. The nature of the information structure makes our model unique. In our model, one agent is better informed than the other and the private information is about asset allocation and not the asset's fundamental value.

Our model is an extension of the first stage game in Carlin, Lobo, and Viswanathan (2007), who explain the puzzling fact that illiquidity is rare and episodic in financial markets as a breakdown in cooperation between institutional investors in a repeated game.[6] We complement their work by showing that asymmetric information provides an alternative explanation for the episodic illiquidity. Our model applies to important types of interaction between institutional investors in which cooperation as described by Carlin, Lobo, and Viswanathan (2007) does not apply. These instances include interactions between high frequency traders/hedge-funds and mutual funds. All traders in their model must be able to execute a punishment strategy for the equilibrium to hold in their repeated game. In financial markets, mutual funds are unlikely to engage in predatory trading against high frequency traders and hedge-funds in part because of regulatory requirements and inferior technological sophistication. Contrary to our model, Carlin, Lobo, and Viswanathan

[5] Both Siritto (2014) and Fuchs, Ory, and Skrzypacz (2014) arrive at similar conclusions in other settings.

[6] Infrequent and episodic illiquidity was puzzling because Brunnermeier and Pedersen (2005) showed that predatory trading is the equilibrium strategy during forced liquidations and forced liquidations are frequent in financial markets.

(2007) predicts that lower long-term price impact is associated with higher likelihood of predatory trading occurring. Their prediction is not consistent with the evidence in Bessembinder, Carrion, Venkataraman, and Tuttle (2014).

Other research related to our model highlights predatory trading under complete information. Predatory trading always occurs in equilibrium in Brunnermeier and Pedersen (2005). Schöneborn and Schied (2007) study predatory trading in a two-stage-game extension of the first stage game in Carlin, Lobo, and Viswanathan (2007). Bessembinder, Carrion, Venkataraman, and Tuttle (2014) extend Brunnermeier and Pedersen's model to include resiliency. In their model predatory trading only occurs when markets are not resilient. In reality, it is often impossible to know the exact liquidation need of a trader even in nonanonymous markets, unless the trader chooses to truthfully report that information. We complement this literature by highlighting the role of asymmetric information in determining the equilibrium outcome of the interaction between strategic traders when one trader is in distress.

Competition among strategic traders has been explored in extensions of Kyle (1985). Foster and Viswanathan (1996) and Back, Cao, and Willard (2000) characterize the trading behavior of informed strategic traders. There are two main differences between their models and ours. First, strategic traders have symmetric information *ex-ante* in their models. Second, the trading motives in their models are related to the risky asset's fundamental value. Vayanos (2001) studies competition among large traders when trades are the result of risk-sharing needs. We complement this strand of the literature by studying the interaction among strategic traders when the trading motive is distressed liquidation. In our case, one strategic trader is better informed than the other.

Our paper is related to the literature on the Scholes liquidation problem, which is concerned with the optimal way to liquidate an illiquid asset (see Bertsimas and Lo (1998) , Huberman and Stanzl (2005), Moallemi, Park, and Van Roy (2012), Obizhaeva and Wang (2013) and references therein). Moallemi, Park, and Van Roy (2012) study the liquidation problem with asymmetric information in a discrete time setting. We complement this literature by focusing on the interaction between the degree of information asymmetry and the likelihood of predatory trading occurring.

We present the model in Section I. We derive the equilibrium and study the implications of asymmetric information in Section II. Section III concludes.

1 Basic Model

Our model is an extension of the first-stage game in Carlin, Lobo, and Viswanathan (2007). We consider a continuous time economy with two assets: a riskfree asset with zero return and a risky asset. There are two types of traders interacting in the market: long-term investors and strategic traders. Long-term investors have three key characteristics: (*i*) They are price takers, (*ii*) they have downward sloping demand curves, and (*iii*) their demand is a function of the margin of safety, that is the difference between the asset's fundamental value and its price .

Strategic traders are large, risk-neutral agents. Their trades affect the risky asset's price. Examples of strategic traders include hedge funds and proprietary trading firms. Both Brunnermeier and Pedersen (2005) and Carlin, Lobo, and Viswanathan (2007) assume that trades by each strategic trader is common knowledge in their game. An important departure of our model from this work is that we adopt the realistic assumption that trades by a strategic trader is her private information. A strategic trader can use changes in the price of the risky asset to estimate the trades/asset holding of other strategic traders. As we will show, prices are not fully revealing in our model.

We assume that there are two strategic traders. The first strategic trader is the *distressed trader* who is required to sell (or buy) a certain amount of the risky asset between time 0 and time $T > 0$. We take as exogenous both the amount of assets to be sold and the time T which she has to sell them. Distressed liquidation does not affect the risky asset's fundamental value and can arise as a result of risk management, regulatory requirements, or margin calls.

The second strategic trader is the *potential predator* who optimally buys (or sells) the risky asset in response to the distress event. The potential predator's optimal behavior has significant implications for the economy. She can either reduce liquidity by by engaging in predatory trading or supply additional liquidity to the market at a time when it is needed.

We model the interaction among strategic traders as a differential game; that is, a continuous time game. The game takes place in the time interval $[0, T]$. Let $\widetilde{\Delta x}$ denote the amount of the risky asset that the distressed trader must sell. We assume that $\widetilde{\Delta x}$ is a normal random variable with mean μ and variance σ^2. Our first key extension of the first-stage game in Carlin, Lobo, and Viswanathan (2007) is that we assume that Nature picks a realization Δx of $\widetilde{\Delta x}$ at $t = 0$ and announces it to the distressed trader, but not to the potential predator. We also assume that the potential predator receives a private signal $\tilde{S}$ that can contain information about the realization Δx. Formally, we assume that

$$\tilde{S} \;=\; \widetilde{\Delta x} + \tilde{\epsilon} \tag{1}$$

where $\tilde{\epsilon}$ is a normal random variable with mean zero and variance σ_0^2 independent of $\widetilde{\Delta x}$.

Let

$$\kappa \;\equiv\; \frac{\sigma_0^2}{\sigma^2 + \sigma_0^2} \qquad \text{and} \qquad \mathrm{R}^2 \;\equiv\; 1 - \kappa.$$

R^2 is the R-squared of the regression of $\widetilde{\Delta x}$ on $\tilde{S}$. It measures the quality of the signal $\tilde{S}$ in predicting the realization of the random variable $\widetilde{\Delta x}$. The information asymmetry between the distressed trader and the potential predator in our model is captured by κ. Information asymmetry is present because $\tilde{S}$ and $\widetilde{\Delta x}$ are distinct random variables (unless $\kappa = 1$). We refer to κ as the degree (or percentage) of information asymmetry between the distressed trader and the potential predator. However there is no hierarchical information structure as in Townsend (1983), since the distressed trader does not observe $\tilde{S}$ and thus no trader has strict superior information to the other.

We denote the amounts of the risky asset held by the distressed trader and the potential predator at time t by X_t^d and X_t^ℓ respectively. Similarly, Y_t^d and Y_t^ℓ denote the rates at which strategic traders are buying or selling the risky asset at time t. Naturally,

$$dX_t^d \;=\; Y_t^d dt \qquad \text{and} \qquad dX_t^\ell \;=\; Y_t^\ell dt.$$

The main state variable in the economy is the price P of the risky asset. P is the only variable (other than time t) that is observed by both strategic traders. Following Carlin, Lobo, and Viswanathan (2007) we assume that the price evolves as

$$dP_t \;=\; dF_t \;+\; \gamma dX_t \;+\; \lambda dY_t, \tag{2}$$

where F is the fundamental value of the risky asset, and we assume that $P_0 = F_0 + \lambda Y_0$. X_t is the sum of X_t^d and X_t^ℓ, and Y_t is the sum of Y_t^d and Y_t^ℓ. Following Carlin, Lobo, and Viswanathan (2007) we model F as a driftless Brownian motion with constant volatility $\sigma_F^2 = 1$. It is natural to assume that the fundamental value of the risky asset cannot be (perfectly) observed by all market participants independently of its price. The theoretical literature on informed trading relies on this assumption.

The strategic trader $i \in \{d, \ell\}$ knows both X_t^i and Y_t^i. Therefore, this strategic trader can estimate the following quantity when observing price's changes:

$$dZ_t \;=\; dF_t \;+\; \gamma dX_t^{-i} \;+\; \lambda dY_t^{-i},$$

where $\{-i\} = \{d, \ell\} \backslash \{i\}$. The assumption that F is a driftless Brownian motion implies that F_t can be viewed as noise. Hence, the price reveals neither X_t^{-i} nor Y_t^{-i} to the strategic trader i.

Carlin, Lobo, and Viswanathan (2007) refer to the constants γ and λ as the permanent price impact and the temporary price impact respectively. To understand these definitions note that P satisfies

$$P_t \;=\; F_t \;+\; \gamma(X_t - X_0) \;+\; \lambda Y_t$$

in the partial equilibrium we consider; that is, when both γ and λ are constant. Both holding and trading large amounts of an illiquid risky asset affect the asset's price in financial markets. Trading activity induces changes in price because of inventory costs, search costs, clearing fees, etc. These fees are transitory because they are not related to the asset's fundamental value. Strategic traders' holding of the risky asset can have long-term effects on the asset's price because the market has limited risk bearing capacity. λ and γ measure the long-term and transitory price impacts of strategic traders' trades on the risky asset's price, respectively.

Each strategic trader chooses the rate at which to trade the risky asset to maximize her value. The distressed trader's optimization problem is a trade-off between her desire to sell slowly to reduce trading costs and her need to sell faster to reduce the adverse effects of trades by the potential predator. She solves the following problem:

$$\max_{Y^d} \quad \mathrm{E}^d\left[\int_0^T -P_t Y_t^d dt\right]$$

$$\text{subject to} \quad \begin{cases} dP_t = \gamma dX_t + \lambda dY_t + dF_t \\ X_0^d = 0 \\ X_T^d = \Delta x \\ dX_t^d = Y_t dt. \end{cases} \tag{3}$$

The potential predator faces a slightly different optimization problem. We require that she sell her excess holding of the risky asset (relative to her holding at the start of the game) at the end of the game. This requirement is the second key extension of the first-stage game in Carlin, Lobo, and Viswanathan (2007).[7]

We assume that the potential predator starts with zero shares of the risky asset. This assumption is without loss of generality. Her excess holding at the end of the game is X_T^ℓ, and she faces the cost function

$$X_T^\ell\left(F_T + \gamma X_T^d\right) - \frac{C}{2}\gamma\left(X_T^\ell\right)^2 \tag{4}$$

at the end of the game, where $C > 0$. This cost is the gain/loss from optimally liquidating the excess assets bought/sold by the potential predator during the game over a fixed period of time.[8]

The potential predator solves the following problem:

[7] We do not require that the potential predator's excess holding of the risky asset equal zero as do Carlin, Lobo, and Viswanathan (2007) for two reasons. First, we want to allow for the possibility to provide liquidity which means having positive (negative) excess holding of the risky asset at the end of the game in our model when the distressed trader is forced to sell (buy). Second, fixing X_T^ℓ *ex-ante* will require us to limit our examination of the equilibrium to strategies that are Brownian bridges, which significantly complicates the analysis.

[8] Suppose that $\bar{a}$ is the potential predator excess holding of the risky asset at time T and that she wants/needs to liquidate these assets within a time period of ΔT from the end of the game. Then her optimal strategy is to liquidate the asset at a constant rate $\bar{a}/\Delta T$ as shown by Carlin, Lobo, and Viswanathan (2007). The proceeds from the optimal liquidation are:

$$\frac{\bar{a}}{\Delta T}\int_T^{T+\Delta T} -P_t dt = \bar{a}\left(F_T + \gamma X_T^d\right) - \frac{1}{2}\gamma\bar{a}^2 \underbrace{\left[1 + 2\frac{\lambda}{\Delta T}\right]}_{C}$$

given that $P_T = F_T + \gamma(\bar{a} + X_T^d)$. Note that the cost function evaluated at 0 is zero. Thus the potential predator's value reduces to that considered in Carlin, Lobo, and Viswanathan (2007) when $X_T^\ell = 0$.

$$
\max_{Y^\ell} \quad \mathrm{E}^\ell \left[\int_0^T -P_t Y_t^\ell dt \ + \ X_T^\ell \left(F_T + \gamma X_T^d \right) - \frac{C}{2} \gamma (X_T^\ell)^2 \right]
$$

$$
\text{subject to} \quad \begin{cases} dP_t \ = \ \gamma dX_t \ + \ \lambda dY_t \ + \ dF_t \\ X_0^\ell \ = \ 0 \\ dX_t^\ell \ = \ Y_t^\ell dt. \end{cases} \tag{5}
$$

Next we define the set of feasible strategies. Learning about the distressed trader's liquidation is important for the potential predator. Therefore, we assume that the potential predator follows a closed-loop strategy, which is a departure from the extant predatory trading literature.[9] The potential predator updates her beliefs by observing the price dynamics and using Bayes' rule. The price dynamics generate a filtration $\{\mathfrak{F}(t), 0 \leq t < T\}$. The potential predator learns about $\widetilde{\Delta x}$ through this filtration. The potential predator's time t estimate of $\widetilde{\Delta x}$ is

$$
\hat{X}_t \ \equiv \ \mathrm{E}\left[\widetilde{\Delta x} | \ \mathfrak{F}(t); \ \tilde{S}_1 \right].
$$

We thus write

$$
Y_t^\ell \equiv \phi^\ell(t, \, X_t^\ell, \, \hat{X}_t).
$$

We show that$(X_t^\ell, \ \hat{X}_t)$ is Markovian in equilibrium. Therefore, Y_t^ℓ is a well-defined function in equilibrium. For simplicity and following most of the predatory trading literature, we assume that the distressed trader follows a time-dependent strategy:

$$
Y_t^d \equiv \phi^d(t).
$$

In the absence of the potential trader, the distressed trader's optimal strategy is to liquidate the risky asset at the constant $\Delta x / T$, conditional on the liquidation size been Δx (see Carlin, Lobo, and Viswanathan (2007)). Her expected time t value from liquidation is then:

$$
V^{0,d,cond}(\Delta x, t) \ = \ - \left[F_0 + \left(\frac{\gamma t + 2\lambda}{2T} \right) \Delta x \right] \frac{t}{T} \Delta x.
$$

[9] Carlin, Lobo, and Viswanathan (2007) discuss the closed-loop equilibrium of their first-stage model.

When the potential predator is present in the market, her actions affect the value the distressed trader obtains by liquidating the risky asset. We say that the potential predator engages in *predatory trading* if the distressed trader's terminal value when liquidating the risky asset in the presence of the potential predator is lower than her terminal value in the absence of the potential predator. Otherwise we say that the potential predator *provides liquidity*.[10]

We now define an equilibrium of the game:

Definition 1. *An equilibrium is a set of admissible strategies* $\{Y^d, Y^\ell\}$ *such that* Y^d *is a solution of the optimization problem (3) given* Y_t^ℓ *while* Y^ℓ *solves the optimization problem (5) given* Y_t^d*.*

Discussion of assumptions

Our model adopts two features shared by models in the theoretical literature concerned with the interaction between strategic traders in illiquid markets (see Kyle and Xiong (2001), Pritsker (2009), Morris and Shin (2004), Attari, Mello, and Ruckes (2005), Huberman and Stanzl (2004), Oehmke (2014), and references therein). The first feature is the presence of long-term (non-strategic) traders who have a downward sloping demand curve. The demand curve that is a function of Graham (1973)'s safety margin, that is, the difference between the asset's fundamental value and its price. The assumption that long-term investors have downward sloping demand curves is motivated by the fact that long-term investors need to be rewarded with higher returns to change their long-run equilibrium holding of the risky asset, possibly because of risk-aversion. Higher returns are achieved through lower prices. Long-term investors do not take advantage of short-term opportunities unrelated to the risky asset's fundamental value, such as asset fire sales. They provide liquidity to the market by buying (selling) the risky asset when its price is below (above) its fundamental value. Examples of long-term investors include retail investors. Kaniel, Saar, and Titman (2008) find empirical evidence that individual investors provide liquidity to strategic traders which enables

[10] This definition is consistent with that of Brunnermeier and Pedersen (2005), who define predatory trading as "trading that induces and/or exploits the need of other investors to reduce their positions". We considered the following alternative definitions of predatory trading: (1) A potential predator *predates* if the excess holding of the risky asset by the potential predator at time T is negative, that is, if $X_T^\ell < 0$. (2) A potential predator *predates* if the aggregate amount of time she trades in the same direction as the distressed trader during the game is greater than $T/2$. Our results did not change qualitatively under these alternative definitions of predatory trading/providing liquidity.

the latter to trade more frequently. Shleifer (1986), Wurgler and Zhuravskaya (2002), and Krishnamurthy and Vissing-Jorgensen (2012) find empirical support for the assumption of downward sloping demand curves.

The second feature is that the changes in the price of risky asset are a linear function of the changes in the asset's fundamental value, the strategic traders' aggregate order flow, and the aggregate changes in their order flow. These price dynamics reflect the notion that changes in price are due to both changes in the asset's fundamental value and market frictions. That is, the risky asset is *illiquid* and trading by strategic traders is associated with a price impact. The price impact has two components: a long-term component related to the aggregate holding of the risky asset by the strategic traders and a short-term component related to the strategic traders' aggregate order flow. Both Kyle (1985) and Pritsker (2009) present models that endogenously generate permanent price impacts. Madhavan and Cheng (1997), Glosten and Harris (1988), and Sadka (2006) find empirical evidence supporting the assumption that large trades have distinct permanent and temporary price impacts.

2 Equilibrium

Equilibria in the game are determined by trade-offs faced by the players. The distressed trader faces a trade-off between two forces. Trading costs lead her to try to liquidate the risky asset at a slow rate. On the other hand, trades by the potential predator (usually) lower prices which lead her to try to sell at a higher rate.[11]

The potential predator generates profits from the distressed trader's price impact by selling high and buying low. She can first race the distressed trader to the market and sell the risky asset when the price is high. For this strategy to be profitable, she needs trades by the distressed trader to have permanent price impact so she can buy the risky asset at a lower price at a later time. This strategy leads to lower value for the distressed trader, that is, predatory trading. The potential predator can also first buy the risky asset. This strategy is profitable if the potential predator can sell the risky asset at a higher price later on, which can occur when prices recover following the distressed trader's exit from the market. The first strategy is associated with faster trading

[11] This trade-off is similar to the one faced by an informed trader in a multi-player Kyle model. See Back, Cao, and Willard (2000) and Foster and Viswanathan (1996).

by the potential predator because of the need to race to the market. This difference in trading speed affects the choice between the two strategies when trading costs, that is the temporary price impact, are non-zeros.

In the presence of asymmetric information, the potential predator can incur losses because she can sell too much or too little of the risky asset due to the fact that she does not know the liquidation size. These losses depend on the rate at which the potential predator is trading. The potential predator's value is non-linear in her estimate of the liquidation size and thus the potential predator is not risk-neutral with respect to uncertainty about the liquidation size. That is, the degree of information asummetry matters. As a result, information asymmetry and learning are important in determining the equilibrium in our model and affect the probability that the potential predator chooses to engage in predatory trading.

We characterize the equilibrium strategies in the following theorem:

Theorem 1. *Given a set of time-dependent functions $(c_1, c_2, c_3, a_1, a_2)$ satisfying the system of first-order differential equations (41)—(47), a linear equilibrium (Y^d, Y^ℓ) is defined by*

$$\begin{aligned} Y_t^\ell &= c_1(t)X_t^\ell + c_2(t)\hat{X}_t + c_3(t) \\ Y_t^d &= a_1(t) + a_2(t)\Delta x. \end{aligned}$$

In this equilibrium, the amount of the risky asset held by the potential predator at time t is

$$X_t^\ell = \int_0^t \frac{c_1(s)}{c_1(t)} \Big(c_2(s)\hat{X}(s) + c_3(s) \Big)\, ds. \tag{6}$$

Moreover, the uncertainty faced by the potential predator about the realization of the liquidation size $\widetilde{\Delta x}$ is a non-increasing function of time.

Proof. See Appendix A. The result about the uncertainty faced by the potential predator follows from Equation (41). □

We obtain a closed-form solution for c_1. We solve the system of first-order differential equations characterizing c_2, c_3, a_1 and a_2 numerically. See Appendix B for details.

We consider the equilibria of the game for some families of parameter for which we can derive closed-form solutions. These results will help build the intuition for the numerical solutions we shall present.

Corollary 1. *Supposed that there is no permanent price impact, that is*

$$\gamma \;=\; 0.$$

Then there exists a unique equilibrium (Y^d, Y^ℓ) *with*

$$\begin{aligned} Y_t^\ell &= -\frac{1}{2T}\hat{X}_t \;=\; -\frac{1}{2T}\left[\mu + \sqrt{1-\kappa^2}(\tilde{S} - \mu)\right] \\ Y_t^d &= \frac{1}{T}\Delta x. \end{aligned}$$

The probability of predatory trading occurring in equilibrium is

$$\mathrm{P}_{pred} \;=\; \phi\left(\frac{\mu}{\sigma}\right) \;-\; \phi\left(\frac{1}{1-\kappa^2}\frac{\mu}{\sigma}\right)$$

where ϕ *is the cumulative density function of the standard Normal distribution. This probability is increasing in* κ*. In equilibrium, the (unconditional) value obtained by the distressed trader is*

$$V^d \;=\; V^{0,d} \;+\; \frac{\lambda}{2T}\left[(1-\kappa^2)\sigma^2 + \mu^2\right], \tag{7}$$

where $V^{0,d}$ *is the value obtained by the distressed trader in the absence of the potential predator.*

Proof. See Appendix A. □

Liquidation by the distressed trader has no permanent price impact when $\gamma = 0$, which implies that prices recover quickly after trades by the distressed trader. Thus, the potential predator cannot take advantage of the distressed trader price impact by racing the distressed trader to the market. In equilibrium, the potential predator buys a fraction of the assets she estimates the distressed is selling. This buying reduces the magnitude of the aggregate price impact of strategic traders

($\lambda|\Delta x - \tilde{X}/2|/T$) relative to the case without the potential predator ($\lambda|\Delta x|/T$). Therefore, the price at which the distressed trader liquidates the risky asset ($P_t = F_t + \lambda(\Delta x - \tilde{X}_t/2)/T$) is higher on average relative to the case without the potential predator ($P_t = F_t + \lambda \Delta x/T$). The higher price means higher value from distress liquidation for the distressed trader. Hence, in expectation, the potential predator provides liquidity when $\gamma = 0$.

Corollary 1 shows that there is no predatory trading when $\gamma = 0$ if there is no information asymmetry (that is, $\kappa = 0$). However, the probability of predatory trading is non-zero when there is information asymmetry. This result holds despite the fact that the potential predator buys a fraction of the her estimate of the liquidation size, which could be viewed as providing liquidity. This result occurs because the potential predator makes errors when estimating the liquidation size in the presence of information asymmetry. These errors reduce the distressed trader's value when the realization Δx and the estimate $\hat{X}_t$ of the liquidation size have different signs. It follows from Equation (1) that opposite signs are more likely to occur for larger values of κ. Therefore, greater information asymmetry leads to larger estimation errors on average, and thus a higher likelihood for predatory trading occurring. This intuition explains the positive relation between the probability of predatory trading occurring and the degree of information asymmetry in Corollary 1. This positive relation occurs because the potential predator attempts to take the position opposite the distressed trader but fails to do so due to information asymmetry. We shall see that the opposite result holds in predation markets, that is markets where the potential predator engages in predatory trading with certainty in the absence of information asymmetry.

The potential predator's presence in the market can reduce the distressed trader's value through two channels: (1) A direct channel which occurs when the potential predator attempts to trader in the same direction as the distressed trader for the majority of the game and is successful in doing so. (2) An indirect channel which occurs when the potential predator attempts to trade in the opposite direction as the distressed trader for the majority of the game but fails to do so because she estimates the liquidation size with errors. The potential predator has incentives to "buy" the risky asset and thus provide liquidity when $\gamma = 0$. Therefore, the direct channel to predatory trading is absence and only the indirect channel can lead to predatory trading. The indirect channel is possible in equilibrium because we model the liquidation size as a normal random variable, allowing for both positive and negative realizations of $\widetilde{\Delta x}$ for any choice of μ and $\sigma \neq 0$.

However, the probability of the indirect channel occurring in equilibrium in our model is negligible for realistic families of parameters.[12]

The results in Corollary 1 are consistent with the empirical evidence in Bessembinder, Carrion, Venkataraman, and Tuttle (2014). They find that potential predators provide liquidity when markets are resilient. They define resilient markets as markets where "some or all of the immediate price impact of trades is subsequently reversed". Resilient markets can be viewed as markets with negligible permanent price impacts.

Next we study the numerical approximation of the general equilibrium. Corollary 1 highlights that a key force determining the occurrence of predatory trading is whether or not trading by the potential predator amplifies the distress liquidation impacts on the price of the risky asset. This point is consistent with the intuitive understanding of predatory trading (providing liquidity) as the action of selling (buying) while the distressed trader is liquidating. We define the notion of *gap* to study this key force. A quantity's *gap* is the different between the quantity's numerical value when the potential predator is present in the market and when the potential predator is absent from the market.

In the case $\gamma = 0$, the strategic traders aggregate holding gap of the risky asset at time t is $(\Delta x/T - \hat{X}_t/(2T) - \Delta x/T)t = -\hat{X}_t t/(2T)$. the strategic traders aggregate trading rate gap $\Delta x/T - \hat{X}_t/(2T) - \Delta x/T = -\hat{X}_t/(2T)$. Both these gaps are positive on average. As a result, the price gap is positive on average because the price is linear and increasing in both the aggregate holding and the trading rate gats. That is, the distressed trader sells the risky asset at a higher price in the presence of the potential predator. Hence, the distressed trader expected terminal value gap is positive if she liquidates the risky asset at a faster rate in the presence of the risky asset, that is if her trading rate gap is negative. By definition, positive expected terminal value gap for the distressed trader is equivalent to the potential predator providing liquidity.

We assume for the remainder of the paper that

$$C \equiv 1.$$

We illustrate the linear equilibrium and the effects of the degree of information asymmetry in Figure

[12] If $\sigma < |\mu|/4$, then $\mathrm{P}_{pred} < 0.01\%$.

1. We simulate 100×100 equilibrium paths of the game for 100 realizations of the liquidation size $\widetilde{\Delta x}$ and 100 paths of the risky asset's fundamental value for each of two values of the degree of information asymmetry. We plot the *average* equilibrium strategies for both the distressed trader (Fig 1 (a)) and the potential predator (Fig 1 (b)). We also plot the strategic traders' aggregate holding gap of the risky asset (Fig 1 (c)) and the price gap (Fig 1 (d)). Finally, we plot the dynamics of both the distressed trader's expected value gap (Fig 1 (e)) and the potential predator's gap expected value gap (Fig 1 (f)).

In the absence of a potential predator, Carlin, Lobo, and Viswanathan (2007) show that the distressed trader's optimal strategy is to liquidate the risky asset at a constant speed. Therefore, the graph of the distressed trader's optimal strategy has curvature zero in the absence of a potential predator. We observe from Figure 1 that the distressed trader deviates from this strategy in the presence of the potential predator (see Fig 1 (a)). This deviation is more pronounced when the degree of information asymmetry is lower, that is, the graph of the distressed trader's strategy has a greater curvature for lower degree of information asymmetry. The graph of the potential predator's strategy also has more curvature for lower degree of information asymmetry. Therefore, both traders trade more aggressively for lower degree of information asymmetry, *ceteris paribus*. The intuition behind this result is as follows: The potential predator wants to race to the market to sell the risky asset at the beginning of the game. She wants to do it to take advantage of the distressed trader price impact by selling the risky asset while its price is high and then buying it back when its price is low. This is the the direct channel through which predatory trading can occur. However, presence of information asymmetry means that the potential predator can incur losses when trading in the illiquid market. These losses arise because the she doesn't know whether price variations are due to changes in the fundamental value of the risky asset or changes in asset holding of the distressed agent. The potential predator thus balances her desires to race to the market to benefit from distress liquidation with the cost of trading when she does not know the liquidation size. The cost is an increasing function of the degree of information asymmetry. Therefore, she trades at a lower rate at the start of the game when the degree of information asymmetry is high. In equilibrium, the distressed trader responds by also trading less aggressively when the potential trader is trading at a lower rate because trading is costly. Hence, both traders trade less aggressively for higher degree of information asymmetry (see Figs 1 (a) and (b)).

Figure 2 indicates that the distressed trader's expected value terminal value gap is lower for lower degree of information asymmetry (see Fig 1 (c)). This result is understood by looking at both the aggregate holding gap and the price gap. The aggregate holding gap is lower for lower degree of information asymmetry because both traders trade more aggressively for lower degree of information asymmetry. As a result, the price gap is lower for lower degree of information asymmetry. Therefore, the distressed trader has a lower expected terminal value for lower degree of information asymmetry. Hence, in the case of the direct channel, predatory trading is more likely to occur for lower degree of information asymmetry.

[**Insert Figure 1 here**]

Figure 2 illustrates the effects of the permanent price impact on the equilibrium. Both strategic traders trade more aggressively in the high permanent price impact regime. The intuition behind this result is that higher permanent price impact is associated with her returns from racing to sell the risky asset by the potential predator. However, the potential predator also has a price impact. That is, racing by the potential predator lowers the price at which the distressed trader can liquidate the risky asset. This price impact leads the distressed trader to trading aggressively at the beginning of the game because she needs to liquidate the risky asset within a finite period of time. Hence, both traders tend to trade more aggressively for higher permanent price impact.

Figure 2 indicates that predatory trading occurs in markets with high permanent price impact and liquidity provision occurs in markets with with low permanent price impact (see Fig 2 (e)). The intuition is as follows. Liquidation of the risky asset by the distressed trader causes a continuous, large, and long term drop in the risky asset's price when the permanent price impact is high. The potential predator takes advantage of the drop in price by selling the risky asset at the start of the game, when the price is still high, and then buying the risky asset later on when the price is lower. The higher the permanent price impact, the faster the initial rate at which the potential predator sells the risky asset (see Fig 2 (b)). Thus, on average, higher permanent price impact implies negative aggregate holding gap of the risky asset by the strategic traders (see Fig 2 (c)). Therefore, higher price impact is associated with negative price gap on average (see Fig 2 (d)), which implies negative expected value gap for distressed trader since she liquidates the risky asset at lower prices. Hence, the distressed trader is more likely to have a negative expected terminal value gap for higher permanent price impact.

Distressed liquidation has only a negligible impact on price when the permanent price impact is low. Therefore the potential predator does not find it profitable to implement a trading that requires high trading rate of the risky asset at the beginning of the game. The potential predator then takes position opposite to the distressed trader on average. This implies positive aggregate holding gap of the risky asset by the strategic traders and thus positive price price gap. Therefore, the distressed trader faces higher prices on average when liquidating the risky asset in markets with lower permanent price impacts when the potential predator is present. That is, the presence of the potential predator is beneficial to the distressed trader when the permanent price impact is low.

[**Insert Figure 2 here**]

We now examine the learning dynamics in the equilibrium. The potential predators learns about the liquidation size by observing the price's dynamics. Her estimate of the liquidation size is

$$\hat{X}_t \;\equiv\; \mathrm{E}\left[\widetilde{\Delta x} |\; \mathfrak{F}(t);\; \tilde{S}_1\right].$$

The degree of uncertainty about the liquidation size is characterized by the variance of the random variable $\hat{X}_t$, denoted $\Omega(t)$. A sufficient statistic to for learning in our model is the percentage of uncertainty left at time t, which we denote $\delta(t)$:

$$\delta(t) \;\equiv\; \frac{\Omega(t)}{\Omega(0)} \;=\; \frac{\Omega(t)}{\kappa^2\sigma^2}.$$

Learning is driven by two forces. The first is the rate at which the distressed trader trades. Trading by the distressed trader is a function of the liquidation size in equilibrium. Therefore, the potential predator learns about the liquidation size by observing changes in the price of the risky asset since the price depends of distressed trader's trades. The more the distressed trader varies the rate at which she trades the more the potential predator learns about the liquidation size. The second force are the price impacts. A greater percentage of the change in the price of the risky asset is due to the strategic traders' action when price impacts are higher. Therefore, in general, higher price impacts implies faster learning. Learning in equilibrium depends on how these two forces interact.

Figure 3 presents the learning dynamics for various parameters. The figure shows that leaning is faster for higher permanent price impact. The reason is two folds. First, higher permanent price impact means that changes in price are more informative about changes in the distressed trader's rate of trading/holding of the risky asset. Second, higher permanent price impact is associated with more changes in the rate at which the distressed trader trades in equilibrium (see Figure 2 (a)). These two effects combine to improve learning for higher permanent price impact.

In equilibrium, the distressed trader trades at a lower average rate for higher temporary price impact. This lower trading rate by the distressed trader decreases the rate at which the potential predator learns about the liquidation size. This effect can dominate the positive direct effect of higher temporary price impact on learning in the long run. Therefore, in our equilibrium, learning can be slower at some point in time in markets with higher temporary price impacts. Figure 3 (b) illustrates this point.

[Insert Figure 3 here]

The choice between engaging in predatory trading and providing liquidity and the learning dynamics depend on the combination of the price impacts and the degree of information asymmetry. We present a comprehensive numerical analysis of the effects of the price impacts and the degree of information asymmetry on the linear equilibrium outcomes in the next section.

2.1 Predatory trading versus liquidity provision

We study how market characteristics and the degree of uncertainty influence the potential predator's decision to either provide liquidity or engage in predatory trading. This decision is characterized by the potential predator's terminal holding of the risky asset.

We estimate the probability that the potential predator will predate for several sets of values of the permanent price impact, the temporary price impact, and the degree of information asymmetry and report the results in Table 1.

Table 1 indicates that predatory trading occurs with certainty for higher (lower) values of the permanent (temporary) price impact when the degree of information asymmetry is zero. The reason is that, with a high permanent price impact, distress liquidation causes prices to be lower for a long period. The potential predator can then sell the risky asset at the beginning of the game when prices are still high and buy back the risky asset at a lower price at the end of the game. As

Brunnermeier and Pedersen (2005) note, "the predator derives profit from the price impact of the prey". However, the potential predator also has a price impact. This price impact lowers the price at which the distressed trader sells the risky asset, resulting in lower value for the distressed trader relative to the where the potential trader is absent. Therefore, the potential predator profits from the price impact of the distressed trader while potential predator's the price impact is a negative externality for the distressed trader. The potential predator trades aggressively when engaging in predatory trading because she needs to race the distressed trader to the market. Therefore, engaging in predatory trading is not profitable for sufficiently high trading costs are high, that is when the temporary price impact is high.

Consider markets where predatory trading occurs with certainty when there is no information asymmetry. We call these markets predation markets. Table 1 shows that the probability of predatory trading occurring decreases as the degree of information asymmetry κ increases in such markets. This result is intuitive. A greater degree of information asymmetry leads to a higher likelihood of over/under-estimating the distress liquidation size. However, engaging in predatory trading requires more aggressive trading relative to providing liquidity because the predator races to the market in the former. Therefore, estimation errors are more costly when the potential predator engages in predatory trading relative to providing liquidity. Hence, providing liquidity becomes the better option for the potential predator in equilibrium as the degree of information asymmetry increases.

We also observe from Table 1 that there is a positive relation between the probability of predatory trading occurring and the permanent price impact in predation markets for arbitrary degree of information asymmetry. This relation is driven by two forces. The primary force is the fact that the potential predator can derive greater profits by racing more aggressively to the market for higher permanent price impact. This channel is the same in the absence of uncertainty and we previously discussed it. The secondary for is the learning dynamics in the equilibrium. In equilibrium, learning decreases the uncertainty the potential predator faces about the realization of the random liquidation size $\widetilde{\Delta x}$. Therefore, predatory trading is more likely to occur in markets where learning is faster. These markets are those with higher permanent price impact, *Ceteris paribus*. A similar intuition explains the negative relation between the probability of predatory trading occurring and the temporary price impact in predation markets as shown in Table 1 (b).

[Insert Table 1 here]

2.2 Welfare

We examine the effects of uncertainty about the amount of the asset to be liquidated on the value achieved by each player in the linear equilibrium.

In the partial equilibrium that we consider, the permanent price impact is a transfer of wealth from the strategic traders to the long-term investors. This transfer of wealth is a function of the aggregate change in holding of the risky asset by the strategic traders. The transfer occurs because long-term investors have a downward sloping demand curve which is characterized by the permanent price impact in the price function. The strategic traders compensate the long-term investors to be able to walk down the long-term investors demand curve. Long-term investors require this compensation because they are risk-averse.

The temporary price impact is a deadweight cost for the strategic traders' collective trading. This deadweight cost is due to trading costs such as inventory costs, search costs, bid-ask spread, clearing fees, etc.

We will focus on the welfare gains/losses to the strategic traders due to the presence of the potential predator. We use simulated equilibrium paths to compute each trader's equilibrium expected value. We present the potential predator expected value, the distressed trader expected value as a percentage change over her value in the absence of the potential predator. We also present the sum of both strategic traders' values as a percentage change over their value in the absence of the potential predator in Table 2.

Relating Table 2 to Table 1, we observe that the percentage change in the strategic traders aggregate expected value is negatively related to the probability of predatory trading occurring in predation markets. The result follows from the fact that strategic traders trade in the same direction on average when predatory trading occurs. Therefore, their aggregate change in holding of the risky asset is larger relative to the case without the potential predator. There is thus a larger transfer of wealth from the strategic traders to the long-term investors and a large deadweight loss in trading costs. This intuition also explains why there is a positive association between the percentage change in the strategic traders aggregate expected value and the degree of information asymmetry. Hence, information asymmetry improves the the strategic traders' aggregate wealth in this case.

Table 2 shows that the percentage change of the distressed trader's expected value relative to the case without a potential predator is negatively related to the degree of information asymmetry in predation markets. Therefore, information asymmetry is beneficial to the distressed trader in markets where predatory trading would occur in its the absence. This result is not surprising. It follows from our definition of predatory trading and the fact that there is a negative relation between the probability of predatory trading occurring and the degree of information asymmetry in predation markets. We also observe that the percentage change of the distressed trader's expected value is negatively related to the permanent price impact in predation markets. This relation follow from the relation between the probability of predatory trading occurring and the price impacts.

Our simulations indicate that an increase in the degree of information asymmetry is associated with a decrease in the potential predator's expected gain. To understand this result, recall the form of the value achieved by the potential predator in equilibrium:

$$-\int_0^T \underbrace{\left[F_t + \gamma(X_t^d + X_t^\ell) + \lambda(Y_t^d + Y_t^\ell)\right]}_{P_t} Y_t^\ell dt \;+\; X_T^\ell \left(F_T + \gamma X_T^d\right) \;-\; \frac{C}{2}\gamma(X_T^\ell)^2$$

The above function is a concave function in $\hat{X}$ for a given realization of $\widetilde{\Delta x}$ in any linear equilibrium. Moreover, an increase in the degree of information asymmetry results in an increase of the variance of $\hat{X}$ without changing its mean. The combination of the concavity and the mean-preserving-spread in the distribution generates the result. Therefore, in terms of her payoff and equilibrium strategy, the potential predator is averse to uncertainty about the amount of assets to be liquidated despite the assumption that she is risk-neutral.

Table 2 implies that there is a positive relation between the permanent price impact and the potential predator's expected value in predation markets. These relation follows from the positive association between the permanent price impact and the rate at which the potential trader learns about the liquidation size. The faster the potential predator learns about the liquidation size, the greater the profit she derives from the distressed trader's price impact.

[Insert Table 2 here]

3 Conclusion

We characterized the effect of asymmetric information on a strategic investor's decision to either provide liquidity or engage in predatory trading when another strategic trader is in distress. The potential predator estimates the amount of assets to be liquidated by the distressed trader by observing price dynamics and decides to either provide liquidity or engage in predatory trading.

Our numerical simulations indicate that the probability that the potential predator will choose to engage in predatory trading decreases as the degree of information asymmetry increases. In fact predatory trading does not occur when the degree of information asymmetry is high unless trading costs are almost zero. We also observe that information asymmetry reduces the potential predator's expected returns from engaging in predatory trading. This result is consistent with empirical evidence. Overall our results show that, in the presence of information asymmetry, potential predators are more likely to stabilize markets by providing liquidity. Therefore information asymmetry can explain the observed episodic illiquidity in financial markets.

We also find that the potential predator's choice is a function of both the permanent price impact and the temporary price impact. We deduce from our numerical simulations that providing liquidity occurs more often in markets with low (resp. high) permanent (resp. temporary) price impact.

This paper highlighted some benefits to having information asymmetry in financial markets. These benefits are relevant when evaluating (recent) policies/regulations requiring more transparency for institutional investors. Understanding the role of information asymmetry before a crisis such as distress liquidation occurs remains an open question.

A Equilibrium

We prove Theorem 1 by characterizing the set of best-response strategies for each player and then the equilibrium strategies.

A.1 Potential predator best-response

We first assume that the distressed trader follows a linear strategy of the form:

$$Y^d(t) \;=\; a_1(t) \;+\; a_2(t)\Delta x \tag{8}$$

where a_1 and a_2 are continuously differentiable. Let

$$\bar{a}_1(t) \;=\; \gamma \int_0^t a_1(s)ds + \lambda a_1(t); \qquad \bar{a}_2(t) \;=\; \gamma \int_0^t a_2(s)ds + \lambda a_2(t).$$

The state variables relevant to the potential predator's optimization problem are the price P, her asset holding X^ℓ, and her estimate of $\widetilde{\Delta x}$ which we denote $\hat{X}$. The price component providing additional information to the potential predator is the variable Z defined as

$$Z_t \;\equiv\; \gamma X^d_t + \lambda Y^d_t + F_t. \tag{9}$$

The informative component of price (to the potential predator) generates a filtration $\{\mathfrak{F}(t), 0 \leq t < T\}$. The potential predator learns about $\widetilde{\Delta x}$ as follows:

Lemma 2. *Suppose that the distressed trader follows a strategy of the form given in Equation (8). Then the time t estimate of $\widetilde{\Delta x}$, denoted*

$$\hat{X}_t \;=\; \mathrm{E}\left[\widetilde{\Delta x} |\; \mathfrak{F}(t); \tilde{S}_1\right],$$

is

$$\hat{X}_t \;=\; \hat{X}_0 + \int_0^t \sigma_\ell(t) dW(u)$$

where

$$\hat{X}_0 = \mu + \sqrt{1-\kappa^2}(\tilde{S} - \mu); \qquad \Omega(t) = \left[\int_0^t (\bar{a}_2'(u))^2 du + \frac{1}{\kappa^2\sigma^2}\right]^{-1}; \tag{10}$$

$$\sigma_\ell(t) = \bar{a}_2'(t)\Omega(t); \qquad dW = dB + \bar{a}_2'(\Delta x - \hat{X})dt. \tag{11}$$

Proof. The proof follows from applying the Kalman Bucy filter and basic conditional expectation formulas for multivariate normal random variables. □

We now study the dynamics of $\hat{X}_t$. It follows from (10) and (11) that

$$\begin{aligned} \Omega(t)' &= -(\bar{a}_2'(t))^2\Omega(t)^2 \\ &= -\sigma_\ell(t)\bar{a}_2'(t)\Omega(t) \\ \Rightarrow \int_0^t \sigma_\ell(s)\bar{a}_2'(s)ds &= -\int_0^t \frac{\Omega(t)'}{\Omega(t)}ds \\ &= -\ln\frac{\Omega(t)}{\Omega(0)}. \end{aligned} \tag{12}$$

Let

$$\delta(t) = \frac{\Omega(t)}{\Omega(0)} \quad \Rightarrow \quad \left(\frac{1}{\delta(t)}\right)' = \Omega(0)\,(\bar{a}_2'(t))^2. \tag{13}$$

$\delta(t)$ is the percentage of the initial variance remaining at time t. We shall refer to $\delta(t)$ as the percentage of uncertainty left at time t. Lemma 2 implies that the variable $\hat{X}_t$ satisfies

$$d\hat{X}_t = \sigma_\ell(t)\bar{a}_2'(t)(\Delta x - \hat{X}_t)dt + \sigma_\ell(t)dB.$$

This implies that

$$\begin{aligned} d\left(\exp\left[\int_0^t \sigma_\ell(s)\bar{a}_2'(s)ds\right]\hat{X}_t\right) &= \left[\sigma_\ell(t)\bar{a}_2'(t)\hat{X}_t dt + d\hat{X}_t\right]\exp\left[\int_0^t \sigma_\ell(s)\bar{a}_2'(s)ds\right] \\ &= \left[\sigma_\ell(t)\bar{a}_2'(t)\Delta x dt + \sigma_\ell(t)dB\right]\exp\left[\int_0^t \sigma_\ell(s)\bar{a}_2'(s)ds\right]. \end{aligned}$$

Therefore,

$$\begin{aligned}\frac{1}{\delta(t)}\hat{X}_t - \hat{X}_0 &= \int_0^t \left\{ \left[\sigma_\ell(u)\bar{a}_2'(u)\Delta x du + \sigma_\ell(u) dB_u\right] \frac{1}{\delta(u)} \right\} \\ &= \Delta x \int_0^t \Omega(0)(\bar{a}_2'(u))^2 du + \Omega(0) \int_0^t \bar{a}_2'(u) dB_u.\end{aligned}$$

Hence,

$$\hat{X}_t = \Delta x + \left[\hat{X}_0 - \Delta x\right] \delta(t) + \Omega(t) \int_0^t \bar{a}_2'(u) dB(u). \tag{14}$$

Next we turn our attention to the potential predator's optimization problem. Let J denote the potential predator's value. J is a function $(Z, X_t^\ell, \hat{X}_t, t)$. Given the state variables dynamics, the HJB equation associated with the potential predator's optimization problem is

$$\begin{aligned}&\max_Y \left\{ \left[J_X - Z - \gamma X\right] Y - \lambda Y^2 \right\} \\ &\qquad + J_t + \left(\bar{a}_1' + \bar{a}_2'\hat{X}\right) J_Z + \frac{1}{2} J_{ZZ} + \frac{1}{2}\sigma_{\hat{X}}^2 J_{\hat{X}\hat{X}} + \sigma_{\hat{X}} J_{Z\hat{X}} = 0. \end{aligned}\tag{15}$$

The optimal strategy is then

$$Y^* = \frac{1}{2\lambda}\left[J_X - Z - \gamma X\right]. \tag{16}$$

Equation (15) becomes

$$0 = J_t + \left(\bar{a}_1' + \bar{a}_2'\hat{X}\right) J_Z + \frac{1}{2} J_{ZZ} + \frac{1}{2}\sigma_{\hat{X}}^2 J_{\hat{X}\hat{X}} + \sigma_{\hat{X}} J_{Z\hat{X}} + \frac{1}{4\lambda}\left[J_X - Z - \gamma X\right]^2. \tag{17}$$

We conjecture a solution of the form:

$$\begin{aligned}J(t, Z, X, \hat{X}) &= b_1(t)Z^2 + b_2(t)X^2 + b_3(t)\hat{X}^2 + b_4(t)ZX + b_5(t)Z\hat{X} + b_6(t)X\hat{X} + b_7(t)Z \\ &\qquad + b_8(t)X + b_9(t)\hat{X} + b_{10}(t).\end{aligned}\tag{18}$$

The terminal value of the optimization problem implies the following terminal values for $b_i, i =$

$1, \cdots 10:$

$$b_1(T) = 0; \qquad b_2(T) = -\frac{C}{2}\gamma; \qquad b_3(T) = 0; \qquad b_4(T) = 1; \qquad b_5(T) = 0;$$

$$b_6(T) = -\lambda a_2(T); \qquad b_7(T) = 0; \qquad b_8(T) = -\lambda a_1(T); \qquad b_9(T) = 0; \qquad b_{10}(T) = 0.$$

Define the liquidity ratio as

$$\rho \;=\; \frac{\gamma}{\lambda}.$$

Plugging (18) into Equation (17) we obtain the following system of equations:

$$\begin{aligned}
b_1' + \frac{1}{4\lambda}(b_4 - 1)^2 &= 0. && (19)\\
b_2' + \frac{1}{4\lambda}(2b_2 - \gamma)^2 &= 0. && (20)\\
b_3' + \bar{a}_2' b_5 + \frac{1}{4\lambda} b_6^2 &= 0. && (21)\\
b_4' + \frac{1}{2\lambda}(2b_2 - \gamma)(b_4 - 1) &= 0. && (22)\\
b_5' + 2\bar{a}_2' b_1 + \frac{1}{2\lambda} b_6(b_4 - 1) &= 0. && (23)\\
b_6' + \bar{a}_2' b_4 + \frac{1}{2\lambda} b_6(2b_2 - \gamma) &= 0. && (24)\\
b_7' + 2\bar{a}_1' b_1 + \frac{1}{2\lambda} b_8(b_4 - 1) &= 0. && (25)\\
b_8' + \bar{a}_1' b_4 + \frac{1}{2\lambda} b_8(2b_2 - \gamma) &= 0. && (26)\\
b_9' + \bar{a}_1' b_5 + \bar{a}_2' b_7 + \frac{1}{2\lambda} b_8 b_6 &= 0. && (27)\\
b_{10}' + \bar{a}_1' b_7 + b_1 + \sigma_{\hat{X}}^2 b_3 + \sigma_{\hat{X}} b_5 + \frac{1}{4\lambda} b_8^2 &= 0. && (28)
\end{aligned}$$

The general solutions to equations (19), (20), and (22) are

$$\begin{aligned}
b_2(t) &= \frac{1}{2}\gamma \left[1 - \frac{2(C+1)}{2 + \rho(C+1)(T-t)}\right] && (29)\\
b_4(t) &= 1 && (30)\\
b_1(t) &= 0. && (31)
\end{aligned}$$

Substituting these into the previous system we get that

$$\begin{aligned}
b_1(t) &= 0 \\
b_3' + \frac{1}{4\lambda} b_6^2 &= 0. \\
b_5(t) &= 0. \\
b_7(t) &= 0 \\
b_9' + \frac{1}{2\lambda} b_8 b_6 &= 0. \\
b_{10}' + \sigma_{\hat{X}}^2 b_3 + \frac{1}{4\lambda} b_8^2 &= 0.
\end{aligned}$$

Therefore we obtain the optimal strategy Y^* once we solve for b_6 and b_8. Equations (24) and (26) have the same homogeneous solution:

$$-\frac{(C+1)}{1+\rho(C+1)(T-t)}.$$

It is straightforward to obtain the homogeneous solutions to the remaining equations. The existence and uniqueness results for the equations follow from the assumption that a_1 and a_2 are continuously differentiable.

The existence of a solution to the HJB equation implies the existence of a unique best response strategy. It follows from Equation (16) that the unique best response strategy is the linear strategy

$$\begin{aligned}
Y^* &= \frac{1}{2\lambda}\left[(2a_2(t)-\gamma)X + a_6(t)\hat{X} + a_8(t)\right] \\
&= -\frac{(C+1)\rho}{2+\rho(C+1)(T-t)}X + \frac{1}{2\lambda}\left[a_6(t)\hat{X} + a_8(t)\right]. \qquad (32)
\end{aligned}$$

A.2 Distressed trader best-response

Assume that the potential predator follows a linear strategy

$$Y^\ell(t, Z, X^\ell, \hat{X}) = c_1(t)X^\ell + c_2(t)\hat{X} + c_3(t)$$

where c_1, c_2, and c_3 are continuously differentiable. Then X^ℓ evolves as

$$dX^\ell \;=\; Y^\ell dt \;=\; \left[\left(c_2\hat{X}+c_3\right)+c_1X^\ell\right]dt.$$

Therefore,

$$\begin{aligned} X_t^\ell &= A(t)\int_0^t A(-s)\left[c_2(s)\hat{X}_s+c_3(s)\right]ds \\ \Rightarrow \mathrm{E}^d[X_t^\ell] &= A(t)\int_0^t A(-s)\left[c_2(s)B(s)+c_3(s)\right]ds, \end{aligned}$$

where

$$A(t) \;=\; \exp\left[\mathrm{Sign}(t)\int_0^{|t|}c_1(s)ds\right] \qquad \text{and} \qquad B(t) \;=\; \mathrm{E}^d[\hat{X}_t].$$

Equation (14) and standard Normal-Normal updating results imply that

$$B(t) \;=\; \left[1-\kappa^2\delta(t)\right]\Delta x+\mu\kappa^2\delta(t). \tag{33}$$

We now consider the distressed trader's optimization problem. Recall that

$$\begin{aligned} P(t) &= U+\gamma(X_t^d+X_t^\ell)+\lambda(Y_t^d+Y_t^\ell) \\ &= U+\gamma X_t^d+\lambda Y_t^d+(\gamma+\lambda c_1(t))X_t^\ell+\lambda c_2(t)\hat{X}_t+\lambda c_3(t). \end{aligned}$$

We can rewrite the optimization problem as

$$\begin{aligned} &\max_{Y\in\mathcal{Y}} && \left[\int_0^T \mathcal{L}\left(t,X^d,Y^d\right)dt\right] \\ &\text{subject to} && \begin{cases} X_0^d = 0 \\ X_T^d = \Delta x \\ dX^d = Y^d dt \end{cases} \end{aligned} \tag{34}$$

where

$$\begin{aligned}\mathcal{L}\left(t, X^d, Y^d\right) &= -Y_t^d \left\{u + \gamma X_t^d + \lambda Y_t^d + \lambda c_2(t) B(t) + \lambda c_3(t) + h(t)\right\}. \\ h(t) &= (\gamma + \lambda c_1(t)) A(t) \int_0^t A(-s) \left(c_2(s) B(s) + c_3(s)\right) ds. \end{aligned} \tag{35}$$

Using standard techniques, that is the Pontryagin Maximization Principle (PMP), we obtain that the optimal Y, if it exists, satisfies the following Euler-Lagrange equation:

$$\frac{d}{dt} Y(t) = -\frac{1}{2\lambda} \frac{d}{dt} \left[h(t) + \lambda \left(B(t) c_2(t) + c_3(t)\right)\right].$$

We deduce that Y_t is of the form

$$Y_t = \text{cst} - \frac{1}{2\lambda} \left[h(t) + \lambda(B(t) c_2(t) + c_3(t))\right]; \qquad \text{cst} = Y_0 + \frac{1}{2}(B(0) c_2(0) + c_3(0)).$$

The boundary conditions in Equation (34) imply that

$$\Delta x = \int_0^T Y_t dt \Rightarrow \Delta x - \text{cst} \times T = -\int_0^T \frac{1}{2\lambda} \left[h(s) + \lambda(B(s) c_2(s) + c_3(s))\right] ds.$$

Therefore,

$$Y_0 = -\frac{1}{2}(B(0) c_2(0) + c_3(0)) + \frac{1}{T} \left[\Delta x + \int_0^T \frac{1}{2\lambda} \left[h(s) + \lambda(B(s) c_2(s) + c_3(s))\right] ds.\right]$$

Hence, the distressed trader's best-response, if it exists, is

$$Y_t^d = a_{11}(t) + a_{21}(t) \Delta x \tag{36}$$

where

$$a_{12}(t) = \frac{1}{T} - \frac{1}{2\lambda} \left[h_0(t) + \lambda B_0(t) c_2(t)\right] + \frac{1}{2\lambda T} \int_0^T \left[h_0(s) + \lambda B_0(s) c_2(s)\right] ds \tag{37}$$

$$a_{11}(t) = -\frac{1}{2\lambda} \left[h_1(t) + \lambda(B_1(t) c_2(t) + c_3(t))\right] + \frac{1}{2T\lambda} \int_0^T \left[h_1(s) + \lambda(B_1(s) c_2(s) + c_3(s))\right] ds, \tag{38}$$

and

$$B_0(t) = 1 - \kappa^2\delta(t); \qquad h_0(t) = (\gamma + \lambda c_1(t))A(t)\int_0^t A(-s)c_2(s)B_0(s)ds;$$
$$B_1(t) = \mu\kappa^2\delta(t); \qquad h_1(t) = (\gamma + \lambda c_1(t))A(t)\int_0^t A(-s)\left[c_2(s)B_1(s) + c_3(s)\right]ds.$$

The differentiability of c_1, c_2 and c_3 implies that a_{11} and a_{12} are well-defined and differentiable.

Equation (36) gives the form the distressed trader's best-response *necessarily* takes if it exists. The following lemma proves the existence of the distressed trader's best-response:

Lemma 3. *Suppose the potential predator's strategy is linear with continuous coefficients. Then the distressed trader's best-response strategy is*

$$Y_t^d = a_{11}(t) + a_{21}(t)\Delta x$$

where a_{11} and a_{12} are given by Equations (37) and (38).

Proof. The integrand in Equation (34) is concave. Theoreom 3 in Rockafellar (1974) then implies that the integral functional we are optimizing is concave. Therefore the necessary conditions are also sufficient. □

A.3 Equilibrium

Solving for the equilibrium is done by combining the results from the previous two sections. Linear equilibrium strategies are of the form

$$Y^\ell = c_1(t)X^\ell + c_2(t)\hat{X} + c_3(t).$$
$$Y^d = a_1(t) + a_2(t)\Delta x.$$

The distressed trader's strategy satisfies

$$\int_0^T Y^d(t)dt = \Delta x \quad \forall\, \Delta x.$$

For a linear strategy, this implies that

$$
\begin{aligned}
\int_0^T a_2(t)dt &= 1. && (39)\\
\int_0^T a_1(t)dt &= 0. && (40)
\end{aligned}
$$

The coefficients c_1, c_2, c_3, a_1, and a_2 are related through Equations (32) and (36). Using the results in the previous two sections, we have the following relations between the coefficients:

$$
\begin{aligned}
a_1(t) &= -\frac{1}{2}\left[\rho\Big(1-C+\rho(C+1)(T-t)\Big)H_1(t)+B_1(t)c_2(t)+c_3(t)\right]+\mu_{a_1}.\\
a_2(t) &= -\frac{1}{2}\left[\rho\Big(1-C+\rho(C+1)(T-t)\Big)H_0(t)+B_0(t)c_2(t)\right]+\mu_{a_2}.\\
\mu_{a_1} &= \frac{1}{2T}\int_0^T\left[\rho\Big(1-C+\rho(C+1)(T-s)\Big)H_1(s)+B_1(s)c_2(s)+c_3(s)\right]ds.\\
\mu_{a_2} &= \frac{1}{T}+\frac{1}{2T}\int_0^T\left[\rho\Big(1-C+\rho(C+1)(T-s)\Big)H_0(s)+B_0(s)c_2(s)\right]ds.\\
B_0(t) &= 1-\kappa^2\delta(t).\\
B_1(t) &= \mu\kappa^2\delta(t).\\
H_0(t) &= \int_0^t \frac{c_2(s)B_0(s)}{2+\rho(C+1)(T-s)}ds.\\
H_1(t) &= \int_0^t \frac{c_2(s)B_1(s)+c_3(s)}{2+\rho(C+1)(T-s)}ds.\\
c_1(t) &= -\frac{\rho(C+1)}{2+\rho(C+1)(T-t)}.\\
0 &= c_2'-\frac{\rho(C+1)}{2+\rho(C+1)(T-t)}c_2+\frac{1}{2\lambda}\bar{a}_2'.\\
0 &= c_3'-\frac{\rho(C+1)}{2+\rho(C+1)(T-t)}c_3+\frac{1}{2\lambda}\bar{a}_1'.\\
c_2(T) &= -\frac{1}{2}a_2(T).\\
c_3(T) &= -\frac{1}{2}a_1(T).
\end{aligned}
$$

Therefore, solving for the equilibrium is equivalent to solving for a fixed-point problem in (a_1, a_2, c_2, c_3). This fixed-point problem can be broken into two fixed-point problems, the first involving only a_2 and c_2. We do not have existence and uniqueness results regarding this fixed-point problem, and standard techniques do not apply here. We shall transform this fixed-point problem into a system of differential equations that we will solve numerically. Using some algebra, we obtain from the relations above the following system of equations

$$
\begin{aligned}
0 &= \delta'(t) + \lambda^2\kappa^2\sigma^2\left[\rho a_2(t) + a_2'(t)\right]^2\delta^2(t) && (41)\\
0 &= H_0'(t) - \frac{\left[1-\kappa^2\delta(t)\right]c_2(t)}{2+\rho(C+1)(T-t)} && (42)\\
0 &= H_1'(t) - \frac{\mu\kappa^2\delta(t)c_2(t) + c_3(t)}{2+\rho(C+1)(T-t)} && (43)\\
0 &= c_2'(t) + c_1(t)c_2(t) + \frac{1}{2}\left[\rho a_2(t) + a_2'(t)\right] && (44)\\
0 &= c_3'(t) + c_1(t)c_3(t) + \frac{1}{2}\left[\rho a_1(t) + a_1'(t)\right] && (45)\\
0 &= a_2'(t) + \frac{1}{2}\Big[-\rho^2(C+1)H_0(t) + \rho[1-\kappa^2\delta(t)]c_2(t) + \lambda^2\kappa^4\sigma^2\left[\rho a_2(t)+a_2'(t)\right]^2\delta^2(t)c_2(t) \\
&\qquad -\frac{1}{2}[1-\kappa^2\delta(t)]\left[\rho a_2(t)+a_2'(t)\right]\Big] && (46)\\
0 &= a_1'(t) - \frac{1}{3}\rho a_1(t) - \frac{2}{3}\Big[\rho^2(C+1)H_1(t) - \rho\left(\mu\kappa^2\delta(t)c_2(t) + c_3(t)\right) \\
&\qquad + \lambda^2\mu\kappa^4\sigma^2\left[\rho a_2(t)+a_2'(t)\right]^2\delta^2(t)c_2(t) + \frac{\mu\kappa^2\delta(t)}{2}\left[\rho a_2(t)+a_2'(t)\right]\Big] && (47)
\end{aligned}
$$

with boundary conditions

$$
\begin{aligned}
&H_0(0) = 0; \qquad a_2(0) = \mu_{a_2} - \frac{1}{2}(1-\kappa^2)c_2(0); \qquad c_2(T) = -\frac{1}{2}a_2(T);\\
&H_1(0) = 0; \qquad a_1(0) = \mu_{a_1} - \frac{1}{2}\left[\mu\kappa^2 c_2(0) + c_3(0)\right]; \qquad c_3(T) = -\frac{1}{2}a_1(T); \qquad \delta(0) = 1.
\end{aligned}
$$

The existence and uniqueness results from the HJB theory and both the PMP and Lemma 3 imply that a linear equilibrium exists if and only if the system of equations (41)—(47) has a solution. This result completes the proof of Theorem 1.

The system of equations (41)—(47) has a unique solution on a subset of $(0,\, T)$ for any given set of initial values since $[2+\rho(C+1)(T-t)]^{-1}$ is smooth on $(0,\, T)$. The existence and uniqueness problem we face is more complicated because we our problem is a boundary value problem.

A.4 Proof of the Corollary

Suppose that

$$\gamma \;=\; 0.$$

This implies that

$$c_1 \;\equiv\; 0.$$

The system of equations (41)—(47) then reduces to

$$0 = \delta'(t) + \lambda^2\kappa^2\sigma^2\left[a_2'(t)\right]^2\delta^2(t) \tag{48}$$

$$0 = H_0^{'}(t) - \frac{\left[1-\kappa^2\delta(t)\right]c_2(t)}{2} \tag{49}$$

$$0 = H_1^{'}(t) - \frac{\mu\kappa^2\delta(t)c_2(t) + c_3(t)}{2} \tag{50}$$

$$0 = c_2^{'}(t) + \frac{1}{2}a_2'(t) \tag{51}$$

$$0 = c_3^{'}(t) + \frac{1}{2}a_1'(t) \tag{52}$$

$$0 = a_2'(t) + \frac{1}{2}\left[\lambda^2\kappa^4\sigma^2\left[a_2'(t)\right]^2\delta^2(t)c_2(t) - \frac{1}{2}[1-\kappa^2\delta(t)]a_2'(t)\right] \tag{53}$$

$$0 = a_1'(t) - \frac{2}{3}\left[\lambda^2\mu\kappa^4\sigma^2\left[a_2'(t)\right]^2\delta^2(t)c_2(t) + \frac{\mu\kappa^2\delta(t)}{2}a_2'(t)\right] \tag{54}$$

with boundary conditions

$$\begin{array}{llll} H_0(0) = 0; & a_2(0) = \mu_{a_2} - \frac{1}{2}(1-\kappa^2)c_2(0); & c_2(T) = -\frac{1}{2}a_2(T); & \\ H_1(0) = 0; & a_1(0) = \mu_{a_1} - \frac{1}{2}\left[\mu\kappa^2c_2(0) + c_3(0)\right]; & c_3(T) = -\frac{1}{2}a_1(T); & \delta(0) = 1. \end{array}$$

Equations (51) and (52), together with the terminal boundary conditions for c_2 and c_4, imply that

$$c_2(t) \;=\; -\frac{1}{2}a_2(t) \qquad \text{and} \qquad c_3(t) \;=\; -\frac{1}{2}a_1(t).$$

Plugging the first equality above into Equation (53) leads to

$$
\begin{aligned}
0 &= (3+\kappa^2\delta(t))a_2'(t) - \lambda^2\kappa^4\sigma^2\left[a_2'(t)\right]^2\delta^2(t)a_2(t) \\
&= (3+\kappa^2\delta(t))a_2'(t) + \kappa^2\delta'(t)a_2(t) \\
\Rightarrow a_2(t) = &\ a_2(0)\frac{3+\kappa^2}{3+\kappa^2\delta(t)}.
\end{aligned}
$$

We used Equation (48) to obtain the second equality. Taking the derivative of a_2 with respect to t and plugging the result in Equation (48) yields

$$
0 \;=\; \delta'(t)\left([3+\kappa^2\delta(t)]^4 + D\delta^2(t)\delta'(t)\right) \quad \text{where} \quad D \;=\; \lambda^2\kappa^6\sigma^2 a_2^2(0)[3+\kappa^2]^2.
$$

The solution δ thus satisfies either

$$
0 \;=\; \delta'(t) \;\; \forall t \in [0,T] \qquad \text{or} \qquad 0 \;=\; [3+\kappa^2\delta(t)]^4 + D\delta^2(t)\delta'(t) \;\; \forall t \in [0,T]
$$

because we require smooth solutions. We shall show that the unique solution is $0 \;=\; \delta'(t) \;\; \forall t \in [0,T]$. To do so, we show that the solution to the ODE

$$
\delta'(t) \;=\; -\frac{1}{D}\frac{[3+\kappa^2\delta(t)]^4}{\delta^2(t)}
$$

cannot be smooth and satisfy the requirement that

$$
\delta(t) \geq 0 \quad \forall t,
$$

that is, the requirement that the percentage of uncertainty remaining in the game is non-negative. Suppose that δ is smooth,

$$
\delta'(t) \;=\; -\frac{1}{D}\frac{[3+\kappa^2\delta(t)]^4}{\delta^2(t)}, \qquad \text{and} \qquad \delta(t) \geq 0 \quad \forall t.
$$

Then, $\delta(t) \leq 1$ for all t and it follows from Equation (39) and the expression for $a_2(t)$ that

$$a_2(0)T \;\leq\; \int_0^T a_2(t)dt = 1 \;\leq\; a_2(0)\frac{3+\kappa^2}{3}T \qquad \Rightarrow \qquad \frac{3}{3+\kappa^2}\frac{1}{T} \;\leq\; a_2(0) \;\leq\; \frac{1}{T}$$

Moreover, for $t > 0$,

$$\delta'(t) \;<\; -\frac{[3+\kappa^2]^2}{\lambda^2\kappa^6\sigma^2 a_2^2(0)} \quad \Rightarrow \quad \delta(t) \;<\; 1 - \frac{[3+\kappa^2]^2}{\lambda^2\kappa^6\sigma^2 a_2^2(0)}t$$

since the function $-([3+\kappa^2 x]^4)/x^2$ is an increasing function for $x \in (0,1]$ and $\delta(t)$ is bounded above by 1. It thus follows that $\delta(t) < 0$ for

$$t > \frac{\lambda^2\kappa^6\sigma^2 a_2^2(0)}{[3+\kappa^2]^2} > \frac{9\lambda^2\kappa^6\sigma^2}{[3+\kappa^2]^4}\frac{1}{T^2}.$$

This results contradicts both the assumption that $\delta(t) \geq 0$ and that $\delta(t)$ is smooth since $delta'(t)$ is not defined for $delta(t) = 0$. The contradiction implies that the only possible solution is

$$0 \;=\; \delta'(t) \quad \forall\, t \in\, [0,T] \quad \Rightarrow \quad 1 \;=\; \delta(t) \quad \forall\, t \in\, [0,T].$$

For this solution, we have

$$a_2(t) \;=\; a_2(0) \quad \forall\, t \in\, [0,T] \qquad \Rightarrow \qquad -2c_2(t) \;=\; a_2(t) \;=\; \frac{1}{T} \quad \forall\, t \in\, [0,T].$$

It thus follows from Equations (54) and (40) that

$$a_1(t) \;=\; a_1(0) \quad \forall\, t \in\, [0,T] \qquad \Rightarrow \qquad a_1(t) \;=\; c_3(t) \;=\; 0 \quad \forall\, t \in\, [0,T].$$

The assumption $\gamma = 0$ and the fact that a_2 is constant imply that

$$\bar{a}_2'(t) \;=\; 0.$$

Thus, Equations (10) and (14) imply that

$$\hat{X}_t \;=\; \hat{X}_0 \;=\; \mu + \sqrt{1-\kappa^2}(\tilde{S}-\mu).$$

This completes the derivation of the equilibrium strategies.

The distressed trader plays the strategy she would have played in the absence of the potential predator. It thus follows that her value satisfies

$$\begin{aligned}
V^d &= \mathrm{E}^d \left\{ \int_0^T - \left[F_t + \lambda(Y^d + Y^\ell) \right] Y^d dt \right\} \\
&= V^{d,0} - \lambda T \, \mathrm{E}^d \left[Y^\ell Y^d \right] \\
&= V^{d,0} + \frac{\lambda}{2T} \, \mathrm{E}^d \left[\mu \widetilde{\Delta x} + \sqrt{1-\kappa^2}(\tilde{S}-\mu)\widetilde{\Delta x} \right] \\
&= V^{d,0} + \frac{\lambda}{2T} \left(\mu^2 + \sqrt{1-\kappa^2} \mathrm{E}^d \left[\widetilde{\Delta x} \, \mathrm{E}^d \left[(\tilde{S}-\mu) | \widetilde{\Delta x} \right] \right] \right) \\
&= V^{d,0} + \frac{\lambda}{2T} \left(\mu^2 + \sqrt{1-\kappa^2} \mathrm{E}^d \left[\sqrt{1-\kappa^2} (\widetilde{\Delta x} - \mu) \widetilde{\Delta x} \right] \right) \\
&= V^{d,0} + \frac{\lambda}{2T} \left[\mu^2 + (1-\kappa^2)\sigma^2 \right].
\end{aligned}$$

B Numerical methods

B.1 Numerical solutions to differential equations

We solve the system of first-order differential equations (41)—(47) and obtain numerical estimates of the equilibrium coefficients a_1, a_2, c_2, and c_3 under the assumption that

$$C \;\equiv\; 1.$$

Let

$$H_2 \;\equiv\; \frac{1}{2\lambda} \bar{a}_2'(t) \;=\; \frac{1}{2} \left[\rho a_2(t) + a_2'(t) \right].$$

We can use Equation (46) to derive a differential equation satisfied by H_2. For numerical simplicity, we transform the system of equations (41) — (47) into a system of ordinary first-order differential equations:

$$0 = \delta'(t) + 4\lambda^2\kappa^2\sigma^2 H_2^2(t)\delta^2(t) \tag{55}$$

$$0 = H_0^{'}(t) - \frac{\left[1-\kappa^2\delta(t)\right]c_2(t)}{2+2\rho(T-t)} \tag{56}$$

$$0 = H_1^{'}(t) - \frac{\mu\kappa^2\delta(t)c_2(t)+c_3(t)}{2+2\rho(T-t)} \tag{57}$$

$$0 = c_2^{'}(t) + c_1(t)c_2(t) + H_2(t) \tag{58}$$

$$\begin{aligned} 0 = c_3^{'}(t) + c_1(t)c_3(t) + \frac{2}{3}\rho a_1(t) + \frac{1}{3}\Big[&2\rho^2 H_1(t) - \rho\left(\mu\kappa^2\delta(t)c_2(t)+c_3(t)\right) \\ &+ 4\lambda^2\mu\kappa^4\sigma^2 H_2(t)^2\delta^2(t)c_2(t) + \mu\kappa^2\delta(t)H_2(t)\Big] \end{aligned} \tag{59}$$

$$\begin{aligned} 0 = a_2'(t) + \frac{1}{2}\Big[&-2\rho^2 H_0(t) + \rho[1-\kappa^2\delta(t)]c_2(t) + 4\lambda^2\kappa^4\sigma^2 H_2^2(t)\delta^2(t)c_2(t) \\ &- [1-\kappa^2\delta(t)]H_2(t)\Big] \end{aligned} \tag{60}$$

$$\begin{aligned} 0 = a_1'(t) - \frac{1}{3}\rho a_1(t) - \frac{2}{3}\Big[&2\rho^2 H_1(t) - \rho\left(\mu\kappa^2\delta(t)c_2(t)+c_3(t)\right) \\ &+ 4\lambda^2\mu\kappa^4\sigma^2 H_2(t)^2\delta^2(t)c_2(t) + \mu\kappa^2\delta(t)H_2(t)\Big] \end{aligned} \tag{61}$$

$$0 = H_2'(t) + \frac{d_2'(t)}{2d_2(t)H_2(t)+d_1(t)}H_2^2(t) + \frac{d_1'(t)}{2d_2(t)H_2(t)+d_1(t)}H_2(t) + \frac{d_0'(t)}{2d_2(t)H_2(t)+d_1(t)} \tag{62}$$

with boundary conditions

$$\begin{aligned} &H_0(0) = 0; \quad && a_2(0) = \mu_{a_2} - \frac{1}{2}(1-\kappa^2)c_2(0); \quad && c_2(T) = -\frac{1}{2}a_2(T); \\ &H_1(0) = 0; \quad && a_1(0) = \mu_{a_1} - \frac{1}{2}\left[\mu\kappa^2 c_2(0) + c_3(0)\right]; \quad && c_3(T) = -\frac{1}{2}a_1(T); \quad \delta(0) = 1. \end{aligned}$$

The functions d_0, d_1, and d_2 are

$$\begin{aligned} &d_0(t) = 2\rho^2\Big[\rho(T-t)-1\Big]H_0(t) + 2\rho[1-\kappa^2\delta(t)]c_2(t) - 2\rho\mu_{a_2}; \\ &d_2(t) = 4\lambda^2\kappa^4\sigma^2\delta^2(t)c_2(t); \qquad d_1(t) = 3+\kappa^2\delta(t). \end{aligned}$$

The function *odeint* from the Scipy library for Python can be used to solve the system of first order equations. However a difficulty arises because *odeint* handles Initial Value Problems (IVP) and we have a Boundary Value Problem (BVP). We use the standard shooting method to solve this issue.

Given initial values of H_2 and c_2, we can solve the IVP consisting of Equations (55), (56), (58), (60), and (62). Note that

$$\begin{aligned}
a_2(0) &= \frac{1}{2\rho}\left[4\lambda^2\kappa^4\sigma^2 c_2(0)H_2^2(0) + (3+\kappa^2)H_2(0) + \rho(1-\kappa^2)c_2(0)\right]. \\
a_2'(0) &= 2H_2(0) - \rho a_2(0).
\end{aligned}$$

We use the shooting method to find initial values of H_2 and c_2 for which the boundary conditions for a_2 and c_2 are satisfied. We then repeat the exercise, this time selecting initial values of a_1 and c_3 and solving the entire system of equations.

B.2 Performance of the Numerical Solutions

We evaluate the performance of our numerical solutions. Recall that the distressed trader strategy is of the form

$$Y_t^d = a_1(t) + a_2(t)\Delta x; \quad \forall\ \Delta x\ \in \mathbb{R}.$$

Moreover, Y^d satisfies

$$\int_0^T Y_t^d dt \ = \ \Delta x; \quad \forall\ \Delta x\ \in \mathbb{R}.$$

Therefore,

$$\int_0^T a_1(t)dt \ = \ 0 \tag{63}$$

$$\int_0^T a_2(t)dt \ = \ 1. \tag{64}$$

We compute

$$\left|\int_0^T a_1(t)dt\right| \qquad \text{and} \qquad \left|1 - \int_0^T a_2(t)dt\right|$$

for our numerical solutions presented in the body of the paper and present the results in Table 3. Table 3 shows that our numerical solutions perform well, at least as far as conditions (63) and (64) are concerned.

[Insert Figure 3 here]

B.3 Simulations

We run 100×100 simulations of the game assuming that each player follows her linear equilibrium strategy. Below is the algorithm describing the simulations

1. Randomly pick a realization Δx using the distribution of $\widetilde{\Delta x}$. Calculate

$$p_{0i} = \Pr\left[\left|\frac{\widetilde{\Delta x} - \Delta x_i}{\sigma}\right| < 0.017\right].^{13}$$

2. Simulate 100 paths of the potential predator's equilibrium strategy. Compute the percentage of paths for which the potential predator engages in predatory trading and the realized value of both players for each path. Denote the potential predator's (distressed trader's) mean value for the 100 paths v^ℓ (v^d). Denote the percentage of paths as p_1.
3. Repeat steps one and two 100 times.

We use the ratio

$$\sum_{i=1}^{100}(p_{oi} \times p_{1i}) / \sum_{i=1}^{100} p_{0i}$$

as our proxy for the probability that predatory trading will occur. Similar proxies are made for each player's expected value.

We use the Euler-Maruyama method to solve the system of stochastic differential equations for the state variables numerically in Step 2 above.

C Robustness

We repeat Table 1 (a) with different distributions for $\widetilde{\Delta x}$ and present the results in Table **??**.

[Insert Figure ?? here]

[13] We choose 0.017 in the definition of p_{0i} to ensure that $\sum_{i=1}^{100} p_{0i} \approx 1$ when we randomly select 100 realizations of $\widetilde{\Delta x} \sim \mathrm{N}\left(-10, \sqrt{0.5}\right)$.

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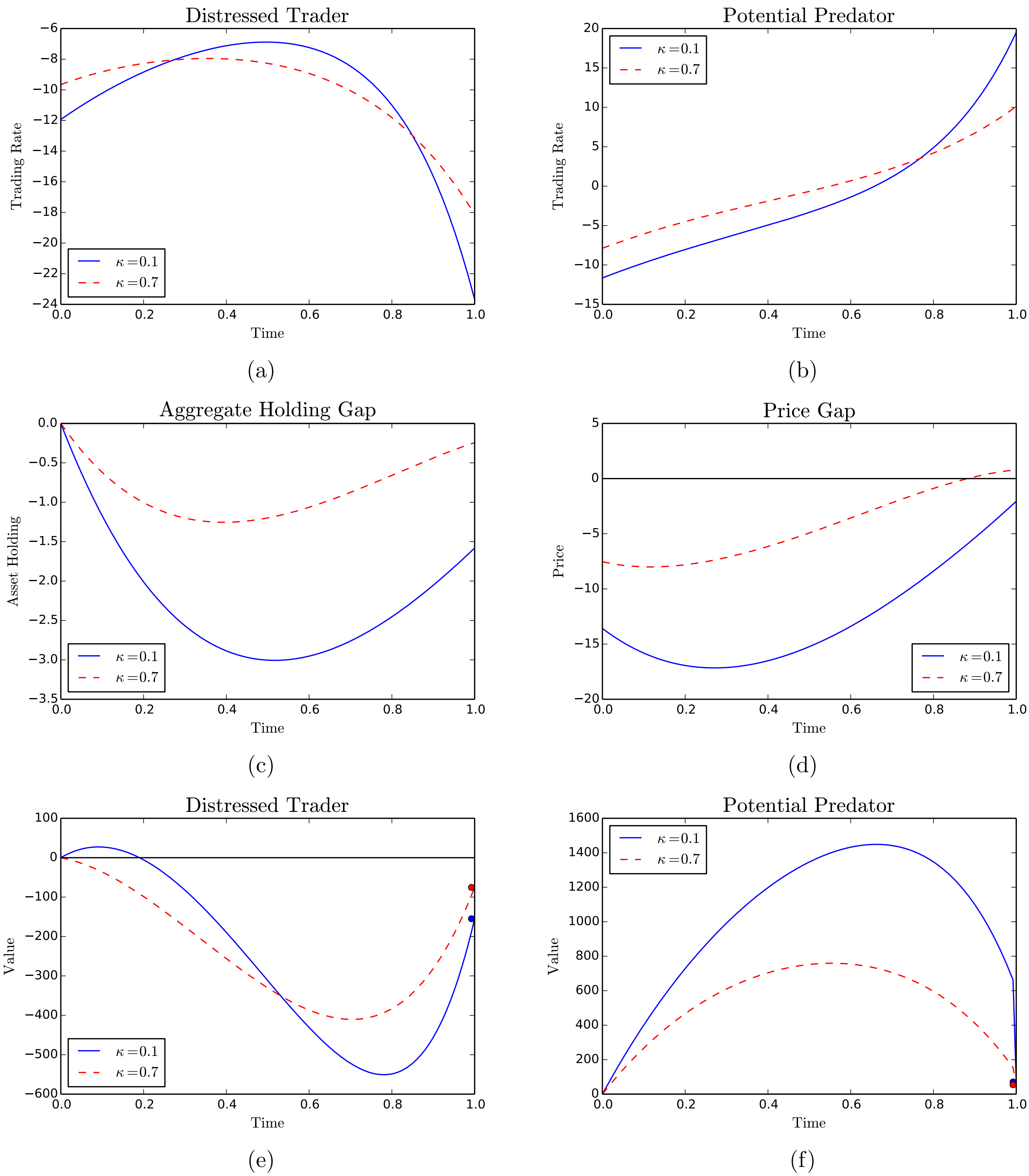


Figure 1.
Equilibrium and the Effects of Information Asymmetry.
We simulate 100×100 equilibrium paths of the game for 100 realizations of the liquidation size $\widetilde{\Delta x}$ and 100 paths of the risky asset's fundamental value for two values of the degree of information asymmetry κ. We plot the *average* equilibrium strategies for both strategic traders (Panel (a) and Panel (b)). We also plot the corresponding aggregate holding gap for the strategic traders and the price gap (Panel (c) and Panel (d)). Finally, we plot each trader's expected value gap (Panel (e) and Panel (f)). A quantity's gap is the difference between that quantity in equilibrium and the same quantity when the potential predator is not in the market. Other parameters: $\widetilde{\Delta x} \sim \mathrm{N}(-10, \sqrt{0.5})$, , $\gamma = 5$, $\lambda = 1$, and $T = 1$.

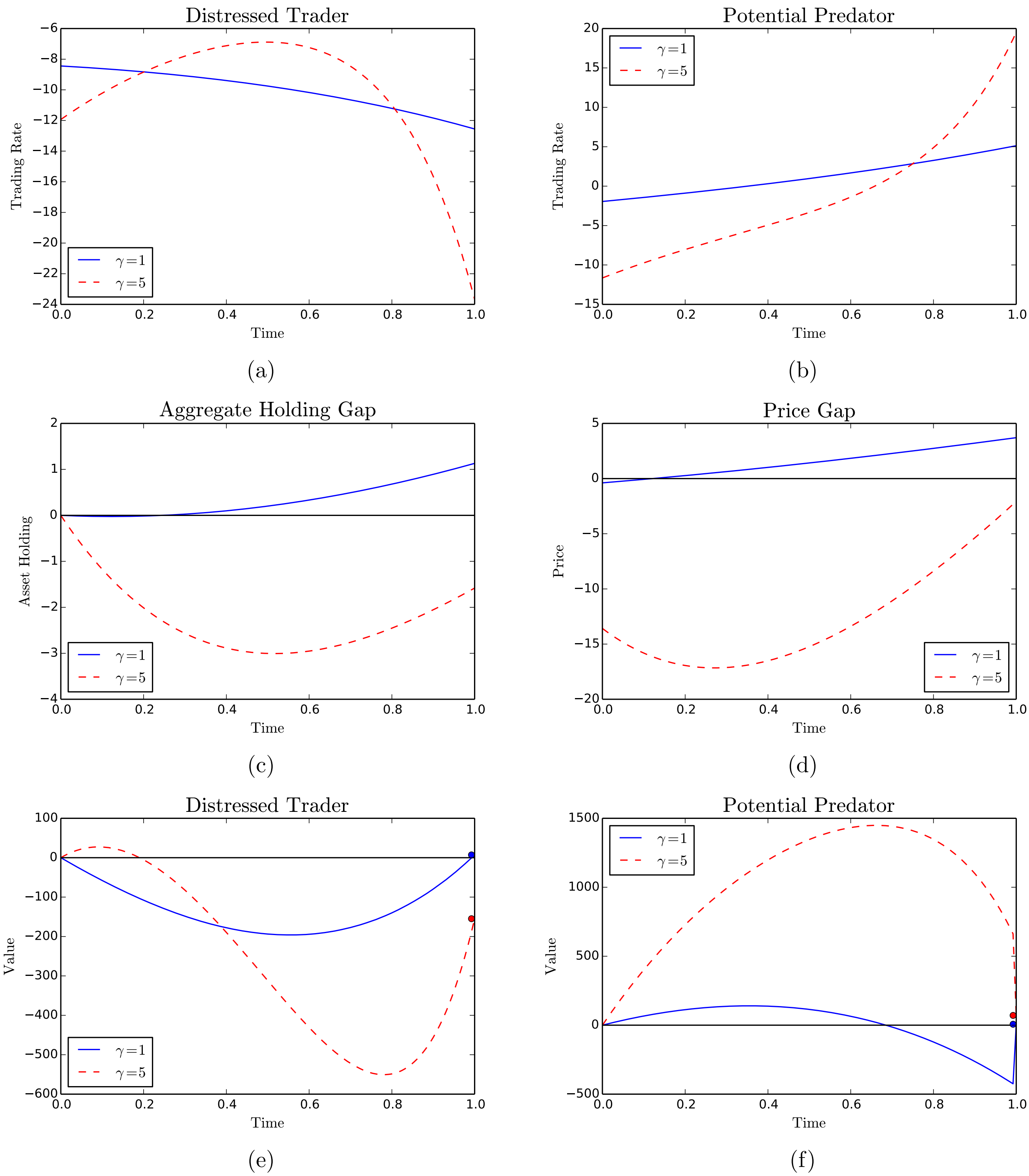
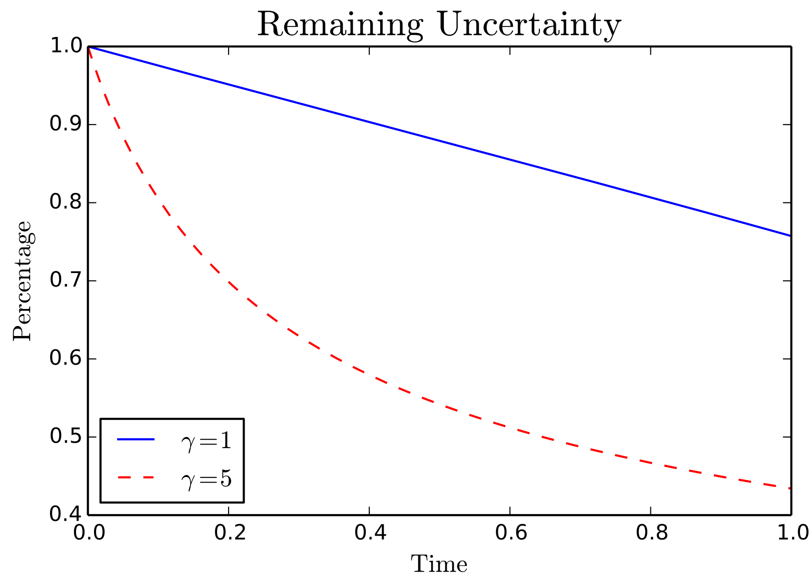
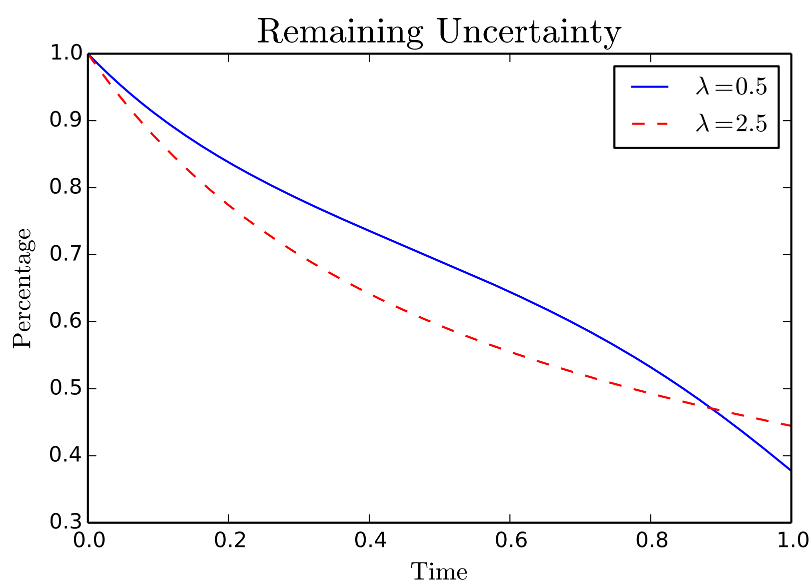


Figure 2. Equilibrium and the Effects of the Permanent Price Impact.
We simulate 100×100 equilibrium paths of the game for 100 realizations of the liquidation size $\widetilde{\Delta x}$ and 100 paths of the risky asset's fundamental value for two values of the permanent price impact γ. We plot the *average* equilibrium strategies for both strategic traders (Panel (a) and Panel (b)). We also plot the corresponding aggregate holding gap for the strategic traders and the price gap (Panel (c) and Panel (d)). Finally, we plot each trader's expected value gap (Panel (e) and Panel (f)). A quantity's gap is the difference between that quantity in equilibrium and the same quantity when the potential predator is not in the market. Other parameters: $\widetilde{\Delta x} \sim \mathrm{N}(-10, \sqrt{0.5})$, , $\kappa = 0.1$, $\lambda = 1$, and $T = 1$.



(a)



(a)

Figure 3. Equilibrium and the Effects of the Permanent Price Impact.
The remaining uncertainty about the liquidation size $\widetilde{\Delta x}$ is the ratio

$$\delta(t) \;=\; \frac{\Omega(t)}{\Omega(0)}.$$

$\Omega(t)$ is the variance of $\hat{X}_t$, the random variable representing the potential predator's estimate of $\widetilde{\Delta x}$ conditional on her signal and the realizations of the price process. We solve for the equilibrium $\delta(t)$ numerically. Other parameters: $\widetilde{\Delta x} \sim \mathrm{N}(-10, \sqrt{0.5})$, $\kappa = 0.7$, $T = 1$, $\lambda = 1$ when γ is varying, and $\gamma = 2.5$ when λ is varying.

Table 1. Probability of Predatory Trading Occurring in the Presence of Information Asymmetry.

We run 100×100 simulations of the game assuming that each player follows her linear equilibrium strategy and estimate the probability that predatory trading occurs. See Appendix B.3 for more details on the simulations. Parameters: $\widetilde{\Delta x} \sim \mathrm{N}(-10, \sqrt{0.5})$ and $T = 1.0$.

(a) Fixed $\lambda = 1$.

κ \ γ	1	1.8	2	3	5
0.0	0.0	0.0	1.0	1.0	1.0
0.12	0.0	0.0	0.99	1.0	1.0
0.25	0.0	0.0	0.14	0.97	1.0
0.5	0.0	0.0	0.1	0.86	0.97
0.75	0.0	0.0	0.02	0.32	0.42
0.95	0.0	0.0	0.0	0.01	0.08

(b) Fixed $\gamma = 2.5$.

κ \ λ	0.5	1	1.25	2.5	5
0.0	1.0	1.0	1.0	0.0	0.0
0.12	1.0	1.0	0.74	0.0	0.0
0.25	1.0	0.59	0.22	0.0	0.0
0.5	0.94	0.42	0.13	0.0	0.0
0.75	0.76	0.1	0.01	0.0	0.0
0.95	0.40	0.01	0.0	0.0	0.0

Table 2. Degree of Uncertainty and Welfare.
We run 100×100 simulations of the game assuming that each player follows her linear equilibrium strategy and estimate each player's expected value. We present the potential predator's wealth. We also present the distressed trader's wealth as a percentage change relative to her wealth in the absence of the potential predator. Finally, we present the strategic traders' aggregate wealth as a percentage change relative to their aggregate wealth in the absence of the potential predator. See Appendix B.3 for details of the estimation procedure. Parameters: $\widetilde{\Delta x} \sim \mathrm{N}(-10, \sqrt{0.5})$, $\lambda = 1$, and $T = 1$.

(a) Distressed Trader.

κ \ γ	1	1.8	2	3	5
0.0	1.37	1.2	-0.55	-2.68	-4.1
0.12	1.35	0.67	-0.5	-1.1	-4.0
0.25	1.27	0.34	0.4	-0.65	-3.2
0.5	1.22	0.2	0.9	-0.1	-1.4
0.75	0.98	0.11	1.01	0.53	-0.13
0.95	0.72	0.02	1.2	1.21	0.16

(b) Potential Predator.

κ \ γ	0.5	1	1.25	2.5	5
0.0	6.88	11.96	15.1	31.63	75.2
0.12	6.83	11.74	14.43	31.5	74.96
0.25	6.81	11.69	13.9	31.16	73.76
0.5	6.78	11.62	13.75	30.11	73.2
0.75	6.63	10.32	12.17	24.44	48.56
0.95	6.25	8.1	9.01	14.41	22.3

(c) Aggregate Strategic Trader.

κ \ λ	0.5	1	1.25	2.5	5
0.0	1.52	1.4	-0.18	-2.3	-2.2
0.12	1.47	0.71	-0.16	-1.05	-2.1
0.25	1.5	0.41	0.78	-0.27	-1.2
0.5	1.44	0.27	1.13	0.15	-1.08
0.75	1.15	0.18	1.3	1.08	-0.1
0.95	0.73	0.02	1.5	1.59	0.77

Table 3. Performance of the Numerical Solutions.

We numerically solve for each player's linear equilibrium strategy and estimate both $A_0 = \left|\int_0^T a_1(t)dt\right|$ and $A_1 = \left|1 - \int_0^T a_2(t)dt\right|$. See Appendix B.1 for details of the numerical solutions. Parameters: $\widetilde{\Delta x} \sim \mathrm{N}(-10, 0.5)$ and $T = 1$.

(a) Fixed $\lambda = 1$; A_0.

κ \ γ	1	1.8	2	3	5
0.12	1.1×10^{-7}	3.8×10^{-8}	1×10^{-7}	3.8×10^{-7}	1.5×10^{-6}
0.25	5.9×10^{-7}	4.4×10^{-7}	6×10^{-7}	1.8×10^{-7}	2.8×10^{-6}
0.5	3×10^{-6}	4.9×10^{-6}	6.1×10^{-7}	3.3×10^{-6}	8.7×10^{-6}
0.75	8×10^{-6}	1.2×10^{-5}	1.2×10^{-5}	1.1×10^{-5}	7.4×10^{-6}
0.95	1.3×10^{-5}	2×10^{-5}	2.1×10^{-5}	2.6×10^{-5}	4×10^{-5}

(b) Fixed $\lambda = 1$; A_1.

κ \ γ	1	1.8	2	3	5
0.12	5.3×10^{-7}	3.5×10^{-7}	6.8×10^{-7}	3.8×10^{-6}	1.8×10^{-5}
0.25	5×10^{-7}	3.6×10^{-7}	6.9×10^{-7}	3.6×10^{-6}	1.7×10^{-5}
0.5	2.8×10^{-7}	5.2×10^{-7}	8×10^{-7}	3.3×10^{-6}	1×10^{-5}
0.75	1.3×10^{-6}	6.5×10^{-7}	7.8×10^{-5}	1.2×10^{-6}	2×10^{-6}
0.95	5.1×10^{-7}	6.8×10^{-7}	6.3×10^{-5}	3.9×10^{-7}	6.4×10^{-7}

(c) Fixed $\gamma = 2.5$; A_0.

κ \ λ	1	1.8	2	3	5
0.12	2.5×10^{-6}	1.7×10^{-7}	5.5×10^{-8}	1.5×10^{-7}	8.5×10^{-8}
0.25	7.3×10^{-6}	3.3×10^{-7}	1.6×10^{-6}	1×10^{-6}	6.4×10^{-7}
0.5	1.8×10^{-5}	4.9×10^{-6}	6×10^{-6}	5.5×10^{-6}	3.5×10^{-6}
0.75	3×10^{-5}	1.3×10^{-5}	1.4×10^{-5}	1.1×10^{-5}	6.9×10^{-6}
0.95	1.4×10^{-5}	2.4×10^{-5}	2.3×10^{-5}	1.6×10^{-5}	4.2×10^{-4}

(b) Fixed $\gamma = 2.5$; A_1.

κ \ λ	1	1.8	2	3	5
0.12	1.8×10^{-5}	2.2×10^{-6}	6.9×10^{-7}	5.3×10^{-7}	3.2×10^{-7}
0.25	1.7×10^{-5}	2.2×10^{-6}	7.1×10^{-7}	4.5×10^{-7}	2.7×10^{-7}
0.5	1.2×10^{-7}	1.9×10^{-6}	7.5×10^{-7}	5.5×10^{-6}	2.2×10^{-8}
0.75	8.4×10^{-6}	1×10^{-6}	2×10^{-7}	1.7×10^{-7}	3.1×10^{-7}
0.95	1.4×10^{-5}	5×10^{-7}	4.3×10^{-7}	5.4×10^{-7}	3.8×10^{-4}