Risk sharing in international economies and market incompleteness

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Abstract

We develop an incomplete markets framework to show that international risk sharing is not high, particularly among the country pairs that exhibit large interest-rate differentials (for example, Australia and Japan). Furthermore, international risk sharing computed from equity and bond data is not inconsistent with that computed from consumption growth data. The key difference from Brandt, Cochrane, and Santa-Clara [2006] is that we work with a measure of international risk sharing under the more realistic setting of incomplete markets, and we do not assume *a priori* that currency returns are the ratio of stochastic discount factors in the two countries.

Keywords: International risk sharing; incomplete markets; exchange rates; high (low) interest rates, carry trade

JEL classification codes: G12, G15, E44, F31, F36.

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1 Introduction

How high is risk sharing in international economies? Brandt et al. [2006] feature economies in complete markets and show that the degree of international risk sharing is high. In contrast, we derive a risk sharing index under incomplete markets and show that international risk sharing is low, especially among country pairs with large interest rate differentials. Specifically, we obtain the lowest estimates of international risk sharing for pairs of countries where one half of the pair is Japan, Germany or Switzerland and the other half of the pair is Australia or New Zealand.

We work under the case of incomplete markets which means that there are an infinite number of stochastic discount factors. We assume that all agents would wish to trade if offered either an arbitrage or a deal that is too good to be true - a "good deal" (Cochrane and Saá-Requejo [2000] and Ross [1976]). More specifically, a good deal is the possibility to form a (multiple currency) portfolio which has an implausibly high reward-for-risk (such as a Sharpe ratio). We assume that all arbitrages and good deals are ruled out. The assumption that an implausibly high reward-for-risk is ruled out places an upper bound on the dispersion of marginal utilities or stochastic discount factors, similarly to Hansen and Jagannathan [1991] and Cochrane and Saá-Requejo [2000]. Given that, in incomplete markets, there are an infinite number of pricing kernels, ruling out good deals can be seen as ruling out, of the infinite set, implausible or unrealistic stochastic discount factors.

What is the statement of the international risk sharing puzzle that arises in the complete markets context of Brandt et al. [2006]? While imposing $m\left(e_{t+1}/e_t\right) - m^\star = 0$, Brandt et al. [2006, equation (2)] advocate using a risk sharing index RSI defined by

$$\mathrm{RSI} \;\equiv\; 1 - \frac{\mathrm{Var}\left[\ln(e_{t+1}/e_t)\right]}{\mathrm{Var}\left[\ln(m)\right] + \mathrm{Var}\left[\ln(m^\star)\right]} = \frac{2\,\mathrm{Cov}\left[\ln(m), \ln(m^\star)\right]}{\mathrm{Var}\left[\ln(m)\right] + \mathrm{Var}\left[\ln(m^\star)\right]}, \tag{1}$$

where m and $m^\star$ are the domestic and foreign stochastic discount factors and e_t denotes the time t exchange rate. Further, Var$[\cdot]$ denotes variance and m and $m^\star$ are computed as the minimum variance stochastic discount factors which reprice the traded assets. With a 10% exchange rate volatility and a 50% volatility of marginal utility growth in each country, the asset return data implied risk sharing index is $1 - \left(0.1^2 / \left(0.5^2 + 0.5^2\right)\right) = 0.98 = 98\%$. With volatilities of consumption growth assumed (approximately) equal in each country and correlations between them of around

0.3, they infer the risk sharing index to be around $0.3 = 30\%$. Brandt et al. [2006] describe as a puzzle the fact that the risk sharing index, based on asset return data, is high indicating a high degree of risk sharing, whereas the corresponding one based on consumption growth data is low (e.g., Cole and Obstfeld [1991] and Lewis [1996]).

We argue that their puzzle is a manifestation of the way that they compute the risk sharing index. We use a risk sharing index, broadly in line with Brandt et al. [2006] in that stochastic discount factors are constructed so as to reprice the returns on traded assets, but also to enforce that the domestic and foreign stochastic discount factors rule out arbitrages and good deals. However, unlike Brandt et al. [2006], we do not assume that e_t, m and $m^\star$ are related by $m\,(e_{t+1}/e_t) - m^\star = 0$ since, as argued by Burnside and Graveline [2012], except in complete markets, this relationship leads to vacuous conclusions and is, in fact, incorrect (Burnside and Graveline [2012, page 12]) for the minimum variance stochastic discount factors. Burnside and Graveline [2012] also point out that asset returns cannot say anything about the risks not spanned by asset markets. In an incomplete market, the extra risks are orthogonal to the asset market but, across countries, are they orthogonal to each other? Or perfectly correlated? Or perfectly negatively correlated? Burnside and Graveline [2012] argue that these questions are difficult to answer but that answers are needed in order to say something about international risk sharing.

Adapting the Brandt et al. [2006] definition of the risk sharing index for incomplete markets, we show that there is little or no puzzle. With our methodology, we see values of the risk sharing index, based on asset return data, which are broadly in the same range as those estimated from consumption growth data. The level of risk sharing varies country-by-country and for pairs of countries which comprise typical legs of the carry trade (in other words, typically have large interest-rate differentials), international risk sharing may be quite low (and sometimes even negative).

Our conclusions are at odds, not only with Brandt et al. [2006], but also, in parts, with others including Lustig and Verdelhan [2012], but are backed by empirical evidence from ten industrialized countries.

We argue that our findings are supported by economic intuition. We know from the carry trade literature that there are high Sharpe ratios for the strategy of borrowing in a low interest-rate currency (for example, JPY) and investing in a high interest-rate currency (AUD). But, if risk sharing were to be very high, why would that be the case? One takes, in effect, a short position in a JPY risk-free bond to fund a long position in a AUD risk-free bond. If, as some argue, risk sharing is almost perfect, the net position has little risk and little reward should be expected in equilibrium.

Our findings are also not inconsistent with the empirical evidence of "home bias" documented by French and Poterba [1991] and Cooper and Kaplanis [1994].

The level of international risk sharing has important implications for economic policy. If risk sharing is low between two countries, then agents in those countries may experience fluctuations in their consumption or utility levels that are undesirable and possibly unnecessary.

In articulating their puzzle, Brandt et al. [2006, page 673] state: "If the puzzle is eventually resolved following the path of the equity premium literature, with utility functions and environments that deliver high equity premia and relatively smooth exchange rates, then the puzzle will be resolved in favor of the asset market view that international risk sharing is, in fact, better than we now think." We show that this is not the case.

We additionally use the model-uncertainty-induced preference functional (which is, in essence, an extension of a utility function allowing an investor to be ambiguity averse or to seek robustness to model uncertainty, as well as to be risk averse) studied in Boyarchenko, Cerrato, Crosby, and Hodges [2012]. We calibrate the parameters (governing ambiguity aversion and risk aversion) of the model-uncertainty-induced preference functional to match historical equity risk premia and average interest-rates. The model-uncertainty-induced preference functional is able to deliver high equity risk premia and relatively low international risk sharing. Further, when we move away from the assumption that currency returns are uniquely equated to the ratio of the foreign and domestic stochastic discount factors, the model also delivers levels of international risk sharing which are lowest for pairs of countries that display large interest-rate differentials.

2 A framework for analyzing risk sharing in incomplete markets

We consider a one period economy with two dates, namely, t, the current time, and $t+1$, the time one period ahead. There are $\mathbb{J} < \infty$ possible states of the world at time $t+1$. We consider two countries denoted domestic and foreign and use a superscript $\star$ to denote quantities in the foreign country.

The exchange rate, defined as the number of units of domestic currency per unit of foreign currency, at time t, is denoted by e_t (i.e., the foreign currency is the reference). We assume that asset markets are frictionless. For example, there are no bid-ask spreads and no short sale constraints.

We assume that there are N assets which can be traded by both domestic and foreign investors. We denote by $\mathbf{R}$ and $\mathbf{R}^\star$ the N-dimensional vector of domestic and foreign gross returns. Included within the return vectors $\mathbf{R}$ and $\mathbf{R}^\star$ are (one period) risk-free bonds in the domestic and foreign country with gross return R_f and $R_f^\star$, respectively.

More generally, we follow Brandt et al. [2006] and include all available asset returns, in either currency, in both $\mathbf{R}$ and $\mathbf{R}^\star$. This means that $\mathbf{R}$ and $\mathbf{R}^\star$ are related by

$$\mathbf{R} = (e_{t+1}/e_t)\,\mathbf{R}^\star. \tag{2}$$

Hence, when we refer to domestic and foreign asset returns, "domestic" and "foreign" refer to the currency in which the return is made - not to, for example, the country in which the equity index is based.

Let m and $m^\star$ denote the domestic and foreign stochastic discount factors. We assume the absence of arbitrage which implies that $m \geq 0$ and $m^\star \geq 0$ (Ingersoll [1987, page 54]). Importantly, we do not assume complete markets so m and $m^\star$ are not unique.

2.1 Implications of incomplete markets

Since m and $m^\star$ reprice domestic returns $\mathbf{R}$ and foreign returns $\mathbf{R}^\star$, we have $\mathbb{E}[m\mathbf{R}] = \mathbf{1}$ and $\mathbb{E}[m^\star\mathbf{R}^\star] = \mathbf{1}$, where $\mathbb{E}[\,.\,]$ indicates unconditional expectation and $\mathbf{1}$ denote an N-dimensional vector of ones. However, since $\mathbf{R} = (e_{t+1}/e_t)\,\mathbf{R}^\star$, we also have $\mathbb{E}[m\,(e_{t+1}/e_t)\,\mathbf{R}^\star] = \mathbf{1}$.

Subtracting $\mathbb{E}[m^\star\mathbf{R}^\star] = \mathbf{1}$ from $\mathbb{E}[m\,(e_{t+1}/e_t)\,\mathbf{R}^\star] = \mathbf{1}$ implies:

$$\mathbb{E}[(m\,(e_{t+1}/e_t) - m^\star)\,\mathbf{R}^\star] = 0. \tag{3}$$

Since equation (3) is true for all foreign returns, it is also true for the foreign risk-free return $R_f^\star$:

$$\mathbb{E}[(m\,(e_{t+1}/e_t) - m^\star)\,R_f^\star] = 0, \quad \text{which implies} \;\; \mathbb{E}[m\,(e_{t+1}/e_t) - m^\star] = 0. \tag{4}$$

In the special case of complete markets, there is an Arrow-Debreu security tradeable for every time $t+1$ state of the world which (by equating its domestic and foreign currency prices) implies, in the absence of arbitrage, that $e_t m^\star = m e_{t+1}$ or equivalently, we obtain the relation (e.g., Lucas [1982] and Backus, Foresi, and Telmer [2001, Proposition 1, equation (7)]),

$$m\,(e_{t+1}/e_t) - m^\star = 0 \;\; \text{in a complete markets setting.} \tag{5}$$

More generally in an incomplete markets setting, $m\,(e_{t+1}/e_t) - m^\star$ need not equal zero, as also emphasized by Burnside and Graveline [2012]. In incomplete markets, some pairs of m and $m^\star$ satisfy $m\,(e_{t+1}/e_t) - m^\star = 0$ and some of them do not. However, as pointed out by Burnside and Graveline [2012, page 12], the minimum variance stochastic discount factors certainly do not.

The statement that $m\,(e_{t+1}/e_t) - m^\star$ need not equal zero is intuitive because, in incomplete markets, there are some time $t+1$ outcomes of the world for which no Arrow-Debreu security trades and different investors will place different marginal utility on those outcomes. For example, if a representative agent exists in each country and if, say, the representative agent in the domestic country is more risk-averse than the representative agent in the foreign country, then the former will assign greater marginal utility to extremely unpalatable states of the world (see Mankiw [1986] and Telmer [1993] for a treatment).

We do stress that equations (3) and (4) always hold, regardless of whether the market is complete or incomplete. Developing the consequences of (3) and (4), we apply the definition of covariance to the expression in equation (3) and obtain $\text{Cov}[m\,(e_{t+1}/e_t) - m^\star, \mathbf{R}^\star] + \mathbb{E}[\mathbf{R}^\star]\,\mathbb{E}[m\,(e_{t+1}/e_t) - m^\star] = 0$. Since $\mathbb{E}[m\,(e_{t+1}/e_t) - m^\star] = 0$ from equation (4), we have the relation:

$$\text{Cov}[m\,(e_{t+1}/e_t) - m^\star, \mathbf{R}^\star] = 0. \tag{6}$$

Equation (6) implies $m\,(e_{t+1}/e_t) - m^\star$ is uncorrelated with each element of $\mathbf{R}^\star$.

2.2 Motivating a constraint on reward-for-risk in the international economy

In general, in an incomplete market, there are an infinite number of domestic and foreign stochastic discount factors which are compatible with the absence of arbitrage. In the spirit of Ross [1976], Hansen and Jagannathan [1991], and Cochrane and Saá-Requejo [2000], we rule out implausible stochastic discount factors by introducing an additional criterion, the rationale for which is as follows:

Consider the possible introduction into the market of an asset or security F which is not a redundant asset - that is, F is not in the linear span of the N assets. Let the payoff of security F at time $t+1$ be $x^\star$, in units of foreign currency. The representative agent in the domestic (respectively, foreign) country would value it at $\mathbb{E}[m\,(e_{t+1}/e_t)\,x^\star] \equiv p$, say (respectively, $\mathbb{E}[m^\star x^\star] \equiv p^\star$). Hence, $p - p^\star = \mathbb{E}[(m\,(e_{t+1}/e_t) - m^\star)\,x^\star] = \text{Cov}[m\,(e_{t+1}/e_t) - m^\star, x^\star]$ since $\mathbb{E}[m\,(e_{t+1}/e_t) - m^\star] = 0$.

What could be a form of $x^\star$? Suppose $x^\star$ were to be of the form $x^\star = x^\perp + \alpha^{'}\mathbf{R}^\star$ for some N dimensional vector α. Note that $p - p^\star$ would be independent of α. This means that, we can essentially, without loss of generality, assume that $x^\star$ is uncorrelated with each element of $\mathbf{R}^\star$.

Applying the Cauchy-Schwartz inequality to the left-hand side of $\text{Cov}[m\,(e_{t+1}/e_t) - m^\star, x^\star] = p - p^\star$, we obtain

$$\sqrt{\text{Var}\left[m\,(e_{t+1}/e_t) - m^\star\right]} \;\geq\; \frac{|p - p^\star|}{\sqrt{\text{Var}\left[x^\star\right]}}. \tag{7}$$

Note that $|p - p^\star|$ is a measure of the reward potentially available from trading security F if it could be traded. Further, $\sqrt{\text{Var}\left[x^\star\right]}$ is a measure of the risk. Therefore, $|p - p^\star|/\sqrt{\text{Var}\left[x^\star\right]}$ is a measure of the reward-for-risk somewhat analogous to a Sharpe ratio.

If $|p-p^\star|/\sqrt{\text{Var}\left[x^\star\right]}$ were high enough, financial intermediaries would have an incentive to trade security F privately[1] with investors. We therefore place an upper bound on $|p-p^\star|/\sqrt{\text{Var}\left[x^\star\right]}$, which is the same as placing an upper bound on $\text{Var}\left[m\,(e_{t+1}/e_t) - m^\star\right]$, which (since $\mathbb{E}[m\,(e_{t+1}/e_t) - m^\star] = 0$, by equation (4)) is the same as placing an upper bound Θ^2 on $\mathbb{E}[(m\,(e_{t+1}/e_t) - m^\star)^2]$.

While we characterize $|p-p^\star|/\sqrt{\text{Var}\left[x^\star\right]}$ as a measure of reward-for-risk, a closer analogy is with what is variously (Sharpe [1981], Roll [1992], and Grinold and Kahn [2007]) termed the Information

[1]Note that if a hedge fund could trade security F with investors in the domestic and foreign countries simultaneously in opposite directions (i.e., simultaneously buying and selling), then it would make a risk-free profit (an arbitrage) of $|p - p^\star|$. Here, we are thinking of a situation where the hedge fund trades in only one direction and bears the risk of that trade in exchange for the expectation of a high reward.

Ratio or the *Appraisal Ratio* (the latter being the term we will use) in that the reward is an excess return over and above a risky return (as opposed to over and above the risk-free return). We rule out Appraisal Ratios greater than Θ and, hence, impose the constraint:

$$\mathbb{E}[(m\,(e_{t+1}/e_t) - m^\star)^2] \;\leq\; \Theta^2, \tag{8}$$

which places a restriction on the set of domestic and foreign stochastic discount factors.

2.3 Operationalizing a risk sharing index when markets are incomplete

Now turn to the question of specifying a risk sharing index analogous to that of Brandt et al. [2006]. We work in discrete-time and so it is tractable for us to consider covariances between proportional changes rather than changes in logs of the pricing kernel. This leads us to, possibly, consider specifying a risk sharing index of the form:

$$\text{RSI} \;\equiv\; \frac{2\,\text{Cov}\,[m, m^\star]}{\text{Var}\,[m] + \text{Var}\,[m^\star]}. \tag{9}$$

In the setting of incomplete markets, there are an infinite number of stochastic discount factors m and $m^\star$ and, thus, an infinite number of possible values of such a proposed risk sharing index.[2]

Our aim is to counter the prevailing view that international risk sharing is high and that contrasting estimates of risk sharing, from asset return data and from consumption data, constitute a puzzle. We show in Section 4 that international risk sharing, based on consumption data, is, in fact, low, and, indeed, negative in some cases.

Thus, we ask what is a plausible, but economically justified, lower bound for the risk sharing index based on asset return data? This leads us to consider taking infimums, over m and $m^\star$, of a risk sharing index in equation (9).

[2]To offer a conceptual contrast relative to others, suppose m_{MV} and $m^\star_{\text{MV}}$ are minimum variance stochastic discount factors satisfying $\mathbb{E}[m_{\text{MV}}\mathbf{R}] = \mathbf{1}$ and $\mathbb{E}[m^\star_{\text{MV}}\mathbf{R}^\star] = \mathbf{1}$. Then the setting of incomplete markets implies (e.g., Brandt et al. [2006, Section 1.2]]) that $m_{\text{MV}} + \eta$ and $m^\star_{\text{MV}} + \eta^\star$ are also stochastic discount factors satisfying $\mathbb{E}[(m_{\text{MV}}+\eta)\mathbf{R}] = \mathbf{1}$ and $\mathbb{E}[(m^\star_{\text{MV}}+\eta^\star)\mathbf{R}^\star] = \mathbf{1}$ provided that η is a random variable which is statistically independent of each element of $\mathbf{R}$ and $\eta^\star$ is a random variable which is statistically independent of each element of $\mathbf{R}^\star$. However, no statement is made or can be made about whether η and $\eta^\star$ are independent of each other, positively correlated or negative correlated. Reinforcing this point, we know $\text{Cov}[\eta, \mathbf{R}] = 0$ and we know $\text{Cov}[\eta^\star, \mathbf{R}^\star] = 0$ but this is not informative about $\text{Cov}[\eta, \eta^\star]$. But $\text{Cov}[m_{\text{MV}} + \eta, m^\star_{\text{MV}} + \eta^\star]$ and, thus, international risk sharing depends crucially on $\text{Cov}[\eta, \eta^\star]$.

Since $\text{Cov}[m, m^\star] = \mathbb{E}[m\, m^\star] - 1/(R_f R_f^\star)$, we actually take infimums, over m and $m^\star$, of $\mathbb{E}[m\, m^\star]$, since this simplifies analytics by focusing on quantities which are linear in m and $m^\star$.

To operationalize our choice of the risk sharing index using asset return data, we compute the risk sharing index in equation (9) and consider the following problem:

$$\inf_{m, m^\star} \mathbb{E}[2\, m\, m^\star] \tag{10}$$

such that

$$\mathbb{E}[(m\,(e_{t+1}/e_t) - m^\star)^2] \;\leq\; \Theta^2, \tag{11}$$

$$\mathbb{E}\left[\left(m^\star\left(\frac{1}{e_{t+1}/e_t}\right) - m\right)^2\right] \;\leq\; \Theta^2, \tag{12}$$

$$\mathbb{E}[m\mathbf{R}] = \mathbf{1}, \quad \mathbb{E}[m^\star\mathbf{R}^\star] = \mathbf{1}, \quad m \geq 0 \;\text{ and }\; m^\star \geq 0. \tag{13}$$

The problem (10) has two inequality constraints (11) and (12), which arise as a consequence of incorporating the incomplete markets assumption in the international economy. Specifically, Θ^2 is the upper bound on $\mathbb{E}[(m\,(e_{t+1}/e_t) - m^\star)^2]$. The constraint in equation (12) reflects the symmetry between the domestic and the foreign currency. In particular, equation (12) is equation (11) but with the role of m and $m^\star$ swapped and (e_{t+1}/e_t) swapped for $1/\,(e_{t+1}/e_t)$.

The equality constraints $\mathbb{E}[m\mathbf{R}] = \mathbf{1}$ and $\mathbb{E}[m^\star\mathbf{R}^\star] = \mathbf{1}$ enforce that the stochastic discount factors m and $m^\star$ must reprice the returns $\mathbf{R}$ and $\mathbf{R}^\star$. The constraints $m \geq 0$ and $m^\star \geq 0$ enforce the absence of arbitrage.

Two important points are in order concerning equations (9) and (10). First, we need to pick some non-negative value of Θ. This is a choice we have to make. We motivate the choice of Θ^2 by ruling out Appraisal Ratios greater than Θ but it can easily be changed if required to analyze the sensitivity of RSI to the choice of Θ. Second, in equation (10), we compute $\inf_{m,m^\star}\{\mathbb{E}[2m\, m^\star]\}$. Minimizing $\mathbb{E}[2m\, m^\star]$ is equivalent to minimizing $\text{Cov}[m, m^\star]$ (since $\mathbb{E}[m] = 1/R_f$ and $\mathbb{E}[m^\star] = 1/R_f^\star$).

Essentially we compute the minimum value of the numerator of equation (9) consistent with repricing the returns $\mathbf{R}$ and $\mathbf{R}^\star$, consistent with the absence of arbitrage and consistent with the bounds $\mathbb{E}[(m\,(e_{t+1}/e_t) - m^\star)^2] \leq \Theta^2$ and $\mathbb{E}[\left(m^\star\left(\frac{1}{e_{t+1}/e_t}\right) - m\right)^2] \leq \Theta^2$.

Our goal in computing the risk sharing index RSI in equation (9) is not to show that values obtained from equation (9) are identical to, or even close to, those obtained from consumption growth data. Our aim is to show that, contrary to Brandt et al. [2006], values of the risk sharing index computed from asset return data are not necessarily out of line with those computed from consumption growth data and that international risk sharing is, in some cases, quite low.

Given a value of Θ, the values of the risk sharing index in equation (9) represent a plausible lower bound on international risk sharing. It is a plausible lower bound computed without having to make the assumption that $m\left(e_{t+1}/e_t\right) - m^\star$ equals zero, in every state. We will compare the plausible lower bound from equation (9) with values of the risk sharing index computed from consumption growth data in order to show that they are not necessarily out of line.

2.4 Solving the problem

Based on the problem in equations (10)-(13), introduce Lagrange multipliers $\delta > 0$, $\delta^\star > 0$, $\mathbf{v}$ and $\mathbf{v}^\star$ where $\mathbf{v}$ and $\mathbf{v}^\star$ are N dimensional vectors. Then the solution to (10) is the solution to:

$$\mathcal{L} \equiv \max_{\delta>0,\delta^\star>0,\mathbf{v},\mathbf{v}^\star} \{ \inf_{m\geq 0, m^\star \geq 0} \{\mathbb{E}[2m\, m^\star] + \delta \left(\mathbb{E}[(m\left(e_{t+1}/e_t\right) - m^\star)^2] - \Theta^2\right) + \\ \delta^\star \left(\mathbb{E}[\left(m^\star \left(\frac{1}{e_{t+1}/e_t}\right) - m\right)^2] - \Theta^2\right) - 2\mathbf{v}\left(\mathbb{E}[m\mathbf{R}] - \mathbf{1}\right) - 2\mathbf{v}^\star\left(\mathbb{E}[m^\star\mathbf{R}^\star] - \mathbf{1}\right)\}\}. \tag{14}$$

It simplifies the exposition (and equation presentation) if we define

$$Z \equiv e_{t+1}/e_t, \qquad y \equiv m\sqrt{Z}, \qquad \text{and} \qquad y^\star \equiv m^\star/\sqrt{Z}. \tag{15}$$

Then

$$\mathcal{L} \equiv \max_{\delta>0,\delta^\star>0,\mathbf{v},\mathbf{v}^\star} \{ \inf_{y\geq 0, y^\star \geq 0} \{\mathbb{E}[2y\, y^\star + \delta Z\left(y - y^\star\right)^2 - \delta\Theta^2 + \\ \delta^\star\left(1/Z\right)\left(y^\star - y\right)^2 - \delta^\star\Theta^2 - 2\mathbf{v}\left(y\sqrt{Z}\mathbf{R}^\star - \mathbf{1}\right) - 2\mathbf{v}^\star\left(y^\star\sqrt{Z}\mathbf{R}^\star - \mathbf{1}\right)]\}\}. \tag{16}$$

Now differentiate with respect to y and $y^\star$ to get the first-order conditions:

$$\begin{aligned} 0 &= 2y^\star + 2Zy\delta - 2Zy^\star\delta + 2\,(1/Z)\,y\delta^\star - 2\,(1/Z)\,y^\star\delta^\star - 2\mathbf{v}\sqrt{Z}\mathbf{R}^\star \quad \text{and} \qquad (17) \\ 0 &= 2y + 2Zy^\star\delta - 2Zy\delta + 2\,(1/Z)\,y^\star\delta^\star - 2\,(1/Z)\,y\delta^\star - 2\mathbf{v}^\star\sqrt{Z}\mathbf{R}^\star. \qquad (18) \end{aligned}$$

Rearranging the first-order conditions and imposing the constraints $y \geq 0$ and $y^\star \geq 0$ implies

$$\begin{aligned} y &= \max\left[0,\ \frac{\mathbf{v}'\mathbf{R}^\star\sqrt{Z}\,(Z\delta + (1/Z)\,\delta^\star) + \mathbf{v}^{\star\prime}\mathbf{R}^\star\sqrt{Z}\,(Z\delta + (1/Z)\,\delta^\star - 1)}{2\,(Z\delta + (1/Z)\,\delta^\star) - 1}\right] \quad \text{and} \qquad (19) \\ y^\star &= \max\left[0,\ \frac{\mathbf{v}'\mathbf{R}^\star\sqrt{Z}\,(Z\delta + (1/Z)\,\delta^\star - 1) + \mathbf{v}^{\star\prime}\mathbf{R}^\star\sqrt{Z}\,(Z\delta + (1/Z)\,\delta^\star)}{2\,(Z\delta + (1/Z)\,\delta^\star) - 1}\right]. \qquad (20) \end{aligned}$$

Substituting (20) into (14), $\inf_{m,m^\star} \mathbb{E}[2m\,m^\star]$ subject to the constraints is given by

$$\begin{aligned} \mathcal{L} = \max_{\delta>0,\delta^\star>0,\mathbf{v},\mathbf{v}^\star} &\{2\mathbf{v}'\mathbf{1} + 2\mathbf{v}^{\star\prime}\mathbf{1} - \delta\Theta^2 - \delta^\star\Theta^2 + \mathbb{E}[2y\,y^\star] \\ &+ \delta\,\mathbb{E}[Z\,(y - y^\star)^2] + \delta^\star\,\mathbb{E}[(1/Z)\,(y^\star - y)^2] \\ &+ \mathbb{E}[-2yy^\star - 2Zy^2\delta + 2Zyy^\star\delta - 2\,(1/Z)\,y^2\delta^\star + 2\,(1/Z)\,yy^\star\delta^\star] \\ &+ \mathbb{E}[-2yy^\star - 2Zy^{\star 2}\delta + 2Zyy^\star\delta - 2\,(1/Z)\,y^{\star 2}\delta^\star + 2\,(1/Z)\,yy^\star\delta^\star]\}, \qquad (21) \end{aligned}$$

where we have substituted terms out for $2\mathbf{v}y\sqrt{Z}\mathbf{R}^\star$ and $2\mathbf{v}^\star y^\star\sqrt{Z}\mathbf{R}^\star$ by using the first-order conditions. After some algebraic simplification, we obtain:

$$\begin{aligned} \mathcal{L} = \max_{\delta>0,\delta^\star>0,\mathbf{v},\mathbf{v}^\star} &\{2\mathbf{v}'\mathbf{1} + 2\mathbf{v}^{\star\prime}\mathbf{1} - \delta\Theta^2 - \delta^\star\Theta^2 - \mathbb{E}[2y\,y^\star] \\ &- \delta\,\mathbb{E}[Z\,(y - y^\star)^2] - \delta^\star\,\mathbb{E}[(1/Z)\,(y^\star - y)^2]\}. \qquad (22) \end{aligned}$$

Equation (22) is solved numerically by choice of $\delta > 0$, $\delta^\star > 0$, $\mathbf{v}$ and $\mathbf{v}^\star$. Noting that $m\,m^\star = y\,y^\star$, we can re-express equation (22) in terms of m and $m^\star$:

$$\begin{aligned} \mathcal{L} = \max_{\delta>0,\delta^\star>0,\mathbf{v},\mathbf{v}^\star} &\{2\mathbf{v}'\mathbf{1} + 2\mathbf{v}^{\star\prime}\mathbf{1} - \delta\Theta^2 - \delta^\star\Theta^2 - \mathbb{E}[2m\,m^\star] \\ &- \delta\,\mathbb{E}[(m\,(e_{t+1}/e_t) - m^\star)^2] - \delta^\star\,\mathbb{E}\left[\left(m^\star\left(\frac{1}{e_{t+1}/e_t}\right) - m\right)^2\right]\}. \qquad (23) \end{aligned}$$

The problems (22) and (23) entail similar numerical complexities.

The numerical computation of (22) and (23) are worthy of some clarifications. The algorithms are implemented by using standard multi-dimensional non-linear maximizing software (a "solver" in C++) as in Press, Teukolsky, Vetterling, and Flannery [2007].

Since the maximands in (22) and (23) are globally concave, if the algorithm finds a maximum, it is the global maximum. Note that the original equations (10)-(13) involved minimizations over m and $m^\star$. With the calculations of the risk sharing index spanning the period January 1975 to June 2014 at monthly frequency (which corresponds to 474 months), the minimizations over m and $m^\star$ involve minimizations over $2 \times 474 = 948$ variables. By contrast, our maximization in (22) and (23) are maximization over a total of ten variables (δ, $\delta^\star$, $\mathbf{v}$ and $\mathbf{v}^\star$) when $N = 4$. This is a substantial computational saving.

To garner some intuition regarding equations (19)-(23), we consider two special cases:

(i) Relax the constraints $m \geq 0$ and $m^\star \geq 0$ and relax the constraints in (11) and (12). The relaxation of $m \geq 0$ and $m^\star \geq 0$ implies the max operators in (20) can be removed while relaxing (11) and (12) is equivalent to letting $\delta \to 0$ and $\delta^\star \to 0$. After canceling terms, $m = {\mathbf{v}^\star}'\mathbf{R}^\star$ and $m^\star = \mathbf{v}'\left(\frac{e_{t+1}}{e_t}\mathbf{R}^\star\right)$, where $\mathbf{v}$ and $\mathbf{v}^\star$ solve $\mathcal{L} = \max_{\mathbf{v},\mathbf{v}^\star}\{2\mathbf{v}'\mathbf{1} + 2{\mathbf{v}^\star}'\mathbf{1} - \mathbb{E}[2m\, m^\star]\}$. The relationship $m\,(e_{t+1}/e_t) - m^\star = 0$ need not hold.

(ii) Momentarily assume complete markets by setting the number of states $\mathbb{J}$ equal to N. Without loss of generality, we can assume that there is an Arrow-Debreu security for each state j of the world. We assume its price is $p_j^\star$ units of foreign currency, and that it pays one unit of foreign currency if state j occurs, which happens with probability π_j, and it pays zero otherwise. The solution is as follows. The domestic and foreign stochastic discount factors, in state j, are $m_j = p_j^\star/(\pi_j\left(\frac{e_{t+1}}{e_t}\right))$ and $m_j^\star = p_j^\star/\pi_j$ respectively and they are the unique stochastic discount factors which reprice the assets. In this case, m and $m^\star$ satisfy $m\,(e_{t+1}/e_t) - m^\star = 0$.

A further situation which yields some economic intuition arises from asking the following question: Leaving aside the special case of complete markets, under what circumstances are $m\,(e_{t+1}/e_t) - m^\star$ equal to zero in every state and what are the consequences?

Note that $m\,(e_{t+1}/e_t) - m^\star = 0$ if and only if $y = y^\star$ which occurs if and only if $\mathbf{v} = \mathbf{v}^\star$. When $y = y^\star$ and $\mathbf{v} = \mathbf{v}^\star$, then $\mathcal{L}$, in equation (16), reduces to $\mathcal{L} \equiv \max_{\mathbf{v}}\{\inf_{y\geq 0}\{\mathbb{E}[2y^2 - 4\mathbf{v}\left(y\sqrt{Z}\mathbf{R}^\star - \mathbf{1}\right)]\}\}$.

Defining $\sqrt{Z}\mathbf{R}^\star \equiv \mathbf{R}^y$ or $\mathbf{R}^y = \mathbf{R}/\sqrt{Z}$, the minimizing y is the solution to: $\inf_{y\geq 0} \mathbb{E}[y^2]$ such that $\mathbb{E}[y\mathbf{R}^y] = \mathbf{1}$. In other words, the minimizing y is the minimum variance stochastic discount factor, with positivity (meaning enforcing $y \geq 0$), in a (hypothetical) economy in which returns $\mathbf{R}^y$ are the geometric average of domestic and foreign returns.

Further, in the special case that $y = y^\star$, the risk sharing index reduces to $\text{RSI} = \frac{2\left(\mathbb{E}[y^2]-1/(R_f R_f^\star)\right)}{\text{Var}\left[y\sqrt{Z}\right]+\text{Var}\left[y/\sqrt{Z}\right]}$. A log-normal approximation shows that, if $\text{Var}\,[Z]$ is small in relation to $\text{Var}\,[y]$, then the risk sharing index is approximately $\text{RSI} \approx 1 - \frac{\text{Var}[\ln(e_{t+1}/e_t)]}{2\,\text{Var}[\ln(y)]}$. A 10% exchange rate volatility and a 50% volatility of $\ln(y)$ consequently results in a risk sharing index of $1 - \frac{0.1^2}{2\times 0.5^2} = 98\%$.

In other words, if exchange rate volatility is low (which, empirically, is the case) and if one assumes that $y = y^\star$ (which is the same as assuming that $m\,(e_{t+1}/e_t) - m^\star = 0$), then the risk sharing index is guaranteed to be high. Put differently, assuming that $m\,(e_{t+1}/e_t) - m^\star = 0$ in incomplete markets, is tantamount to assuming that the risk sharing index is high.

Conversely, in order to reconcile estimates of risk sharing from asset return data with those from consumption growth data, a key step is to recognize that, in incomplete markets, $m\,(e_{t+1}/e_t) - m^\star$ need not equal zero.

3 Data Description

We obtain monthly data from January 1975 to June 2014 for the following ten countries: Australia (AUD), New Zealand (NZD), United Kingdom (STG), France (FRA), Canada (CAD), United States (USD), Netherlands (NLG), Germany (GER), Japan (JPY), and Switzerland (SWI).

Risk-free bonds are synthesized from LIBOR quotes as $1/(1 + \tau\,\text{LIBOR})$, where τ is the day count fraction, that is, $\tau = 1/12$ for monthly. When LIBOR is not available, we use the nearest substitute such as 30 day Bank Bill rates (which are money market rates and not to be confused with treasury bill yields). At the start of the historical time period considered, interest-rate data for JPY, AUD and NZD are not available from Datastream so we use data from the Bank of Japan, the Reserve Bank of Australia and the Reserve Bank of New Zealand.

The equity index return data are MSCI data from Datastream, and we employ total returns (including dividends). MSCI data are not available for New Zealand prior to 1988 so for the period

1975-1987, we use returns data supplied by Martin Lally and Alastair Marsden (documented and used in Lally and Marsden [2004]). All data are at monthly frequency.

Country-specific inflation data are from Datastream and are CPI data except for STG and FRA which only have data for CPI from 1988 and 1989 respectively. For STG and FRA, we splice the CPI data with retail price index data for the periods 1975 to 1988 and 1975 to 1989 respectively.

Datastream is the source of the spot exchange rate data for all country pairs and is the midpoint of the bid and ask quotes. The currency prices for France, Germany, and Netherland after January 1999 are taken to be its fixed conversion rate to the Euro (e.g., DM 1.95583 = 1 Euro).

The nominal returns of bonds, equity indexes, and currencies are converted into real returns by adjusting by ex-post realized inflation (identically to Brandt et al. [2006]). Table Appendix-I presents the summary statistics for the data used in our study.

4 What does our approach tell us about international risk sharing?

Our empirical investigation employs a risk-free bond and a broad-based equity index denominated in each currency (identically to Brandt et al. [2006]). Hence, when computing, for example, the risk sharing index for US and Australia, the gross return vector $\mathbf{R}$ includes real returns on four assets, namely, on the US risk-free bond, on the US equity index, on the Australian risk-free bond and on the Australian equity index, all denominated in US dollars, while $\mathbf{R}^\star$ includes returns on the same four assets but now all the returns are denominated in Australian dollars.

4.1 Risk sharing is high across all country pairs in a complete markets setting

To benchmark our analysis, we first compute the risk sharing index of Brandt et al. [2006, equation (17)] and report the values in Panel A of Table 1. Note that our results are displayed in the order of decreasing average interest-rates (see Panel A of Table Appendix-I), so Australia has the highest average interest-rate while Switzerland has the lowest.

The risk sharing index of Brandt et al. [2006] depends on the variance of the exchange rates and the variance of the domestic and foreign minimum variance stochastic discount factors. Even when their analysis of countries and currencies is expanded from three to ten and the time period extended by an additional 16 years, the average risk sharing index across the 45 pairs of countries is 93.2% with a standard deviation of 6.9%. The complete markets setting of Brandt et al. [2006] implies a high degree of international risk sharing, whereby 44 out of 45 values are above 80%, most exceed 94% and the maximum value reached is 99.8%. The high values of risk sharing reflect the low variance of currency returns in relation to the estimated variances of the minimum variance stochastic discount factors.

4.2 Risk sharing is low across country pairs with high interest rate differentials when markets are incomplete

Next, Panel B of Tables 1 and 2 reports the estimates of the risk sharing index, defined in equation (9), for Appraisal Ratios of $\Theta = 0.175$, $\Theta = 0.35$, $\Theta = 0.525$ and $\Theta = 0.70$.

To understand our choices of Θ, we note that a portfolio earning a return 0.065 over and above an uncorrelated benchmark whose return is 0.08, when the standard deviation of the portfolio is 0.4 and the standard deviation of the benchmark is 0.16, has an Appraisal Ratio of $0.065/\sqrt{(0.4^2 - 0.16^2)} \approx 0.175$. Alternatively, a portfolio earning a return 0.084 over and above an uncorrelated benchmark whose return is 0.08, when the standard deviation of the portfolio is 0.2 and the standard deviation of the benchmark is 0.16, has an Appraisal Ratio of $0.084/\sqrt{(0.2^2 - 0.16^2)} = 0.7$.

Furthermore, Table Appendix-III reports the Appraisal Ratios available to domestic investors for investing in the foreign equity index for each pair of currencies. The largest Appraisal Ratio is 0.26. Cochrane and Saá-Requejo [2000] suggest eliminating good deals by ruling out Sharpe ratios greater than twice the Sharpe ratio available on a major broad-based equity index such as the S&P 500. A similar criterion would possibly suggest ruling out Appraisal Ratios greater than 2×0.26 or around 0.525. Written in these terms, our exploration of Appraisal Ratios around 0.175 to 0.70 does not seem unreasonably high. Hence, ruling them out does not appear to be unreasonably conservative.

For $\Theta = 0.175$ (respectively, $\Theta = 0.35$), the risk sharing index varies from 64.6% to 94.1% (respectively, from 24.4% to 79.1%). Between the four of them, Germany (GER), France (FRA), Netherlands (NLG) and Switzerland (SWI) do have high values of the risk sharing index (over 99%

from the approach of Brandt et al. [2006]) and, for our methodology, well over 91% for $\Theta = 0.175$ and well over 70% for $\Theta = 0.35$). These results possibly suggest that international risk sharing can be high, at least, for some pairs of countries. On the other hand, for pairs of currencies involving New Zealand, the values of the risk sharing index are much lower.

We emphasize that using low values of Θ, for example, $\Theta = 0.175$, brings the risk sharing index closer to the complete markets case. In contrast, higher values of Appraisal Ratios magnify the departures between the results on risk sharing index in Brandt et al. [2006] versus ours. In particular, for $\Theta = 0.525$, the average risk sharing index is 39.3%, and varies from -20.5% to 59.9%. With $\Theta = 0.70$, the average risk sharing index is 14.7% and varies from -40.3% to 43.6%.

How could one justify the observation that for country pairs with high interest differentials, the risk sharing index is low? To examine this, assume, for illustration, that m and e_{t+1}/e_t are jointly lognormally distributed. A calculation in Appendix A illustrates that

$$\begin{aligned}
\ln(R_f^\star) - \ln(R_f) + \mathbb{E}_t\left[\ln\left(\frac{e_{t+1}}{e_t}\right)\right] = & -\frac{1}{2}\text{Cov}_t[\ln(m), \ln\left(\frac{e_{t+1}}{e_t}\right)] - \frac{1}{2}\text{Cov}_t[\ln(m^\star), \ln\left(\frac{e_{t+1}}{e_t}\right)] \\
& -\frac{1}{4}\text{Var}_t[\ln(m) + \ln\left(\frac{e_{t+1}}{e_t}\right) - \ln(m^\star)] \\
& +\frac{1}{4}\left(\text{Var}_t[\ln(m)] + \text{Var}_t[\ln(m^\star)] - 2\text{Cov}_t[\ln(m), \ln(m^\star)]\right) \\
& -\frac{1}{4}\text{Var}_t[\ln\left(\frac{e_{t+1}}{e_t}\right)]. \qquad (24)
\end{aligned}$$

Replacing conditional by unconditional expectations and using an approximation to go from logs to levels and *vice versa*, $\text{Cov}_t[\log(m), \log(m^\star)]$ is proportional to the risk sharing index. If the domestic currency is JPY and the foreign currency is AUD, then *ceteris paribus* the expected excess return to borrowing in JPY and investing in AUD is greater when the risk sharing index is negative. Our characterization provides some rationale for what we see in the data.

4.3 Risk sharing obtained using consumption data affirms our findings

How do the estimates of international risk sharing recovered from asset return data compare relative to ones that can be recovered from data on real consumption growth? To establish our own estimates of the risk sharing index based on the latter, we use two methods. For the first method, as also done in Brandt et al. [2006, equation (23)], the variances and covariances of the stochastic discount factors are replaced by the variances and covariances of changes in log consumption growth:

$$\mathrm{RSI}^C \equiv \frac{2\,\mathrm{Cov}\left[d\log(c), d\log(c^\star)\right]}{\mathrm{Var}\left[d\log(c)\right] + \mathrm{Var}\left[d\log(c^\star)\right]}. \tag{25}$$

The values of the risk sharing index in Panel A of Table 3 use equation (25) with annual consumption growth data from Barro and Ursua [2008a] and Barro and Ursua [2008b], updated to 2012 using WDI data. The values of the risk sharing index in Panel C of Table 3 use equation (25) but replacing consumption growth data with GDP growth data (again, from Barro and Ursua [2008a] and Barro and Ursua [2008b], updated to 2012 using WDI data). Summary data for consumption growth and GDP are shown in Table Appendix-IV.

The values of the risk sharing index in Panels D and E of Table 3 use equation (25) but replacing the source of the data by Datastream (using household final consumption expenditure (constant 2005 prices) per capita for consumption growth and gross domestic product divided by midyear population for GDP growth). We use different data sources to verify that our results are not sensitive to the precise data-set used (since consumption data is well-known to be prone to noise).[3]

For the second method, we use the Boyarchenko et al. [2012] model in equation (B1) and report the values of the risk sharing index in Panel B of Table 3 where we use Barro and Ursua [2008a] and Barro and Ursua [2008b] consumption growth data, updated to 2012 using WDI data (i.e., the same data-set as for Panel A).

The values of the risk sharing index in Panels A through E of Table 3, whether using consumption growth or GDP growth data or the model in equation (B1), are all quite close to one another, which indicates that these estimates of risk sharing have a degree of robustness to the methodology used to compute them. The minimum correlation between the different risk sharing index calculation methods is 0.75 while the maximum is 0.97.

[3]Table Appendix-V reports our findings when consumption growth is computed over alternative horizons, ranging from three-months to 2 years, along with the bootstrapped confidence intervals. The gist of this exercise is that there is no evidence that international risk sharing is significantly different at quarterly or two yearly frequencies in comparison with annual frequency. Thus, our conclusion that international risk sharing can be quite low and can possibly be negative for currency pairs which are associated with large average interest-rate differentials is robust when consumption growth is measured over alternative horizons.

Brandt et al. [2006] describe as a puzzle the fact that their values of the risk sharing index, based on asset return data, are much higher than those based on consumption growth data. We argue that this is a manifestation of the way that they compute the risk sharing index.

With our methodology (equations (9) and (10)), we see values of the risk sharing index, based on asset return data, which are broadly in the same range as those based on consumption growth data. We emphasize only "broadly in the same range" since the values do show considerable variation (both across different currency pairs and across different values of Θ). Nevertheless, in some cases, the values from asset return data are higher and, in other cases, the reverse is true.

If we compare the results when the Appraisal Ratio $\Theta = 0.525$ (Panel B of Table 2) with those in Panel A of Table 3, 14 out of a possible 45 entries produce a higher risk sharing index from consumption growth data than from asset return data. For the case when the Appraisal Ratio $\Theta = 0.70$, that number rises to 32. Moreover, the values of the risk sharing index, when $\Theta = 0.525$, are close to those based on consumption growth data for many country pairs, particularly so for Germany/France,Germany/Switzerland and France/Switzerland.

Comparing the values of the risk sharing index when the Appraisal Ratio $\Theta = 0.525$ (Panel B of Table 2) with the values of the risk sharing index from Panel A of Table 3, 25 out of a possible 45 entries are within 20% of each other (i.e., the absolute value of the difference is less than 0.2, when expressed as a decimal) and 20 out of a possible 45 entries are within 15% of each other. Repeating the comparison when the Appraisal Ratio $\Theta = 0.70$, 21 out of a possible 45 entries are within 20% of each other and 18 out of a possible 45 distinct entries are within 15% of each other.

4.4 Our estimates of risk sharing sharply contrast Brandt, Cochrane, and Santa-Clara [2006]

Our analysis does not support the Brandt et al. [2006] view that values of the risk sharing index, based on asset return data, are much higher than those based on consumption data. Of course, we concede that none of the four values chosen for the Appraisal Ratio Θ (0.175, 0.35, 0.525, 0.70) result in values of the risk sharing index which are perfectly aligned with the values of the risk sharing index based on consumption growth data - but nor are they hopelessly out of line.

Our results are in the flavor of Burnside and Graveline [2012] who argue that there is a limited amount of economic information that can be extracted from asset return data in this context - at least, without making strong assumptions.

Observe that for the case when the Appraisal Ratio $\Theta = 0.70$ (Panel B of Table 2), 9 out of a possible 45 values are negative. In Panels A and B of Table 3 (using consumption growth data), 8 out of a possible 45 distinct off-diagonal entries are negative. The currency pairs which produce negative values of the risk sharing index using consumption growth data (in Panels A and B) are AUD/GER, AUD/JPY, NZD/FRA, NZD/NLG, NZD/GER, NZD/JPY, NZD/SWI and CAD/JPY. Only AUD/JPY, NZD/GER, NZD/JPY, and NZD/SWI are (respectively, only NZD/GER and NZD/SWI are) statistically significantly less than zero at an 80% (respectively, 90%) confidence level but nonetheless a clear pattern is observable.

Typical funding (borrowing) currencies for the carry trade are JPY and SWI, while typical investment currencies are AUD and NZD (Bakshi and Panayotov [2013, Table A1]). Hence, all the currency pairs which produce negative values of the risk sharing index involve either a funding currency for the carry trade or an investment currency for the carry trade (and usually both).

Conversely, interest-rates for USD, STG and CAD are typically close. From Panels A and B of Table 3, we see that STG/USD, STG/CAD and CAD/USD produce notable large positive values (greater than 50%) of the risk sharing index using consumption growth data but significantly lower than the values estimated from the risk sharing index of Brandt et al. [2006, equation (17)]. Similarly, FRA/GER, FRA/SWI and GER/SWI also have relatively large positive values of the risk sharing index using consumption growth data (but, again, significantly lower than the values estimated from the risk sharing index of Brandt et al. [2006]). Hence, there is a fairly consistent pattern in the relationship between the risk sharing index using consumption growth data and the average interest-rate differential.[4]

[4]The result that risk sharing is low, sometimes even negative, for pairs of countries which typically have large interest-rate differentials support the view of Lustig and Verdelhan [2007] that returns of currency carry could be related to asymmetric loadings on global consumption growth risk. Neither are they inconsistent with Hassan and Mano [2014] who argue that the carry trade results from persistent differences in the risk characteristics of individual countries.

4.5 Our conclusions about lack of risk sharing are robust to alternative proxies for consumption growth

Panels D and E of Table 3 show that our results are robust to the use of an alternative data sources and proxies for consumption growth.

Broadly similar inferences can be drawn when one uses GDP growth (Panels C and E of Table 3), instead of consumption growth to measure the risk sharing index. Backus, Kehoe, and Kydland [1992] argue that GDP growth is more correlated across countries than consumption growth. Our results are consistent with this in that we find that, on average, the risk sharing index is a little higher when using GDP growth than when using consumption growth.

The average value of the risk sharing index in Panel C is 38.6% compared with 26.0% for Panel A while the average value of the risk sharing index in Panel E is 42.3% compared with 24.5% for Panel D (using Datastream data). Nevertheless, the most striking feature is the consistency of the results in Panels A to E.

For STG/USD, all five values of the risk sharing index are between 60.6% and 71.1%. Further, for NZD/GER and NZD/JPY, the risk sharing index in Table 3 is negative in each of Panels A to E. This indicates that there is a degree of robustness to our conclusion that international risk sharing can be quite low and can possibly be negative for currency pairs which are associated with large average interest-rate differentials.

Finally, the values of the risk sharing index in Panels A and B are consistently close. Brandt et al. [2006, page 673] say: "If the puzzle is eventually resolved following the path of the equity premium literature, with utility functions and environments that deliver high equity premia and relatively smooth exchange rates, then the puzzle will be resolved in favor of the asset market view that international risk sharing is, in fact, better than we now think." Our findings suggest a different conclusion. The model-uncertainty-induced preference functional of Boyarchenko et al. [2012] matches, by construction, historical equity risk premia and average interest-rates but still yields relatively low - and sometimes negative - values of international risk sharing.

5 Summary and conclusions

In sharp contrast to the extant literature, we find that international risk sharing is sometimes quite low, and for pairs of countries which typically have large interest-rate differentials, the level of international risk sharing is sometimes slightly negative. This conclusion is reached after examining consumption growth data, spanning nearly forty years, for ten industrialized countries. Further, international risk sharing computed from asset return data is not inconsistent with that computed from consumption growth data.

Our conclusions are the opposite of those reached in Brandt et al. [2006]. Two issues drive the differences. First, working in incomplete markets, we do not assume *a priori* that exchange rates are ratios of stochastic discount factors. Second, we consider a wider set of countries than Brandt et al. [2006]. They consider the U.S. versus the United Kingdom, the U.S. versus Japan and the U.S. versus Germany. We find that international risk sharing is quite high for the U.S. versus the United Kingdom (although our estimate is much lower than that of Brandt et al. [2006]). However, we find that the lowest estimates of international risk sharing are for pairs of countries or currencies where one half of the pair is Japan, Germany or Switzerland and the other half of the pair is Australia or New Zealand. Indeed, for some of these pairs of countries, estimates of international risk sharing are slightly negative.

Our work possibly highlights the role of incomplete financial markets in understanding extant puzzles in international finance. For example, our framework provides support to the view of Backus and Smith [1993], Lewis [1996], Lewis [2000] and Lewis and Liu [2012] that international consumption risk sharing is low and to the view of Cole and Obstfeld [1991] that the welfare costs, if international financial markets were closed, might not be substantial.

There is evidence that markets are incomplete, leading Shiller [1993] and Kamstra and Shiller [2010] to propose new assets or securities such as claims on aggregate income of countries. Our framework suggests that the welfare benefits, if new assets or securities such as these were available for trading, may be high.

Incomplete markets are relevant to the mechanism by which risks are shared - or not shared - across international borders.

Appendix A: Excess return to borrowing in domestic and investing in foreign currency and its relation to the risk sharing index

If m and e_{t+1}/e_t are jointly lognormally distributed, then $\mathbb{E}_t[m\,(e_{t+1}/e_t)\,R_f^\star] = 1$ can be written as:

$$\ln(R_f^\star) - \ln(R_f) + \mathbb{E}_t\left[\ln\left(\frac{e_{t+1}}{e_t}\right)\right] = -\mathrm{Cov}_t\left[\ln(m), \ln\left(\frac{e_{t+1}}{e_t}\right)\right] - \frac{1}{2}\mathrm{Var}_t\left[\ln\left(\frac{e_{t+1}}{e_t}\right)\right], \quad \text{(A1)}$$

where we use a subscript t to denote time t conditional expectation. We recognize $\ln(R_f^\star) - \ln(R_f) + \mathbb{E}_t\left[\ln\left(\frac{e_{t+1}}{e_t}\right)\right]$ to be the log-linear approximation to the expected excess return to a trade which borrows one unit of domestic currency at time t, sells it for foreign currency, holds the foreign currency until time $t+1$ and then liquidates the position. Using the identity

$$\begin{aligned}\mathrm{Var}_t[\ln(m) + \ln\left(\frac{e_{t+1}}{e_t}\right) - \ln(m^\star)] &= 2\mathrm{Cov}_t[\ln(m), \ln\left(\frac{e_{t+1}}{e_t}\right)] - 2\mathrm{Cov}_t[\ln(m^\star), \ln\left(\frac{e_{t+1}}{e_t}\right)] \\ &\quad + \mathrm{Var}_t[\ln(m) - \ln(m^\star)] + \mathrm{Var}_t[\ln\left(\frac{e_{t+1}}{e_t}\right)], \qquad \text{(A2)}\end{aligned}$$

we have

$$\begin{aligned}-\frac{1}{2}\mathrm{Cov}_t[\ln(m), \ln\left(\frac{e_{t+1}}{e_t}\right)] &= -\frac{1}{2}\mathrm{Cov}_t[\ln(m^\star), \ln\left(\frac{e_{t+1}}{e_t}\right)] - \frac{1}{4}\mathrm{Var}_t[\ln(m) + \ln\left(\frac{e_{t+1}}{e_t}\right) - \ln m^\star] \\ &\quad + \frac{1}{4}\mathrm{Var}_t[\ln(m) - \ln(m^\star)] + \frac{1}{4}\mathrm{Var}_t[\ln\left(\frac{e_{t+1}}{e_t}\right)]. \qquad \text{(A3)}\end{aligned}$$

Or substituting,

$$\begin{aligned}\ln(R_f^\star) - \ln(R_f) + \mathbb{E}_t\left[\ln\left(\frac{e_{t+1}}{e_t}\right)\right] &= -\frac{1}{2}\mathrm{Cov}_t[\ln(m), \ln\left(\frac{e_{t+1}}{e_t}\right)] - \frac{1}{2}\mathrm{Cov}_t[\ln(m^\star), \ln\left(\frac{e_{t+1}}{e_t}\right)] \\ &\quad - \frac{1}{4}\mathrm{Var}_t[\ln(m) + \ln\left(\frac{e_{t+1}}{e_t}\right) - \ln(m^\star)] \\ &\quad + \frac{1}{4}\left(\mathrm{Var}_t[\ln(m)] + \mathrm{Var}_t[\ln(m^\star)] - 2\mathrm{Cov}_t[\ln(m), \ln(m^\star)]\right) \\ &\quad - \frac{1}{4}\mathrm{Var}_t[\ln\left(\frac{e_{t+1}}{e_t}\right)]. \qquad \text{(A4)}\end{aligned}$$

Equation (A4) is new and stands in contrast to a relation which appears in Alvarez, Atkeson, and Kehoe [2007], Backus et al. [2001], and Lustig and Verdelhan [2012, p396, equation (14.3)]:

$\ln(R_f^\star) - \ln(R_f) + \mathbb{E}_t\left[\ln\left(\frac{e_{t+1}}{e_t}\right)\right] = \frac{1}{2}\text{Var}_t[\ln(m)] - \frac{1}{2}\text{Var}_t[\ln(m^\star)]$. This formula has certain terms omitted, from our perspective, in that it requires $m\,(e_{t+1}/e_t) - m^\star = 0$.

Appendix B: Risk sharing from consumption growth data and a model of preferences

We now turn to computing the risk sharing index $\text{RSI} = \frac{2\,\text{Cov}[m,m^\star]}{\text{Var}[m]+\text{Var}[m^\star]}$ using consumption data and a model of preferences.

Specifically, we use the model-uncertainty-induced preference functional in Boyarchenko et al. [2012]. This can be thought of as an extension of a utility function allowing an investor to be ambiguity averse (or to seek robustness to model uncertainty), as well as to be risk averse. An uncertainty averse investor is assumed to evaluate any prospect over future consumption c as:

$$J(\mathbf{c}) \equiv \inf_{\xi\in\Xi}\{\mathbb{E}[F(\xi)U(c)]\}, \quad \text{where} \quad F(\xi) \equiv \xi\,(1-\Omega\ln\xi),$$
$$1 \le \Omega < \infty, \quad \text{and} \quad \Xi = \{\xi : \mathbb{E}[\xi] = 1, \xi > 0, F(\xi) \ge 0\}. \tag{B1}$$

Here, ξ are distortions while Ξ denotes the set of feasible distortions ξ. The constraint $F(\xi) \ge 0$ ensures that $F(\xi)\,U(c)$ is non-decreasing in c which (see Boyarchenko et al. [2012]) leads to stochastic discount factors which are non-negative.

Moreover, the constant Ω satisfies $1 \le \Omega < \infty$ and captures the degree of aversion to model uncertainty. The case of $\Omega = 1$ corresponds to the case of no model uncertainty. As Ω increases, the degree of aversion to model uncertainty increases. We specialize to the case of power utility where $U(c) = c^{1-\gamma}/(1-\gamma)$, this leads to a two parameter $(\Omega,\,\gamma)$ family of preferences. In this setting, marginal utility is given by $F(\xi)\,c^{-\gamma}$ where ξ is the minimizing ξ in $(B1)$. Of course, the minimizing ξ is a function of the degree of aversion to model uncertainty Ω so we write $\xi \equiv \xi(\Omega)$ if we wish to emphasize this. We allow the parameters Ω and γ to be different in each country and use a superscript $\star$ to denote quantities in the foreign country.

In essence, all of the above is equivalent to allowing investors from different countries to have different degrees of aversion to model uncertainty (ambiguity aversion) and to have different levels of risk aversion. We compute the risk sharing index by essentially setting $m = F(\xi)\, c^{-\gamma}$ and $m^{\star} = F(\xi(\Omega^{\star}))\, c^{\star\, -\gamma^{\star}}$. Therefore,

$$\mathrm{RSI}^{model} \equiv \frac{2\,\mathrm{Cov}\left[F(\xi(\Omega))\, c^{-\gamma}, F(\xi(\Omega^{\star}))\, c^{\star\,-\gamma^{\star}}\right]}{\mathrm{Var}\left[F(\xi(\Omega))\, c^{-\gamma}\right] + \mathrm{Var}\left[F(\xi(\Omega^{\star}))\, c^{\star\,-\gamma^{\star}}\right]}. \tag{B2}$$

To estimate the parameters, we use the methodology of Appendix D of Boyarchenko et al. [2012]. More specifically, we choose Ω and γ to match the domestic risk-free return and the return on the domestic equity index, both denominated in the domestic currency. This requires us to solve two equations in two unknowns. The relevant equations are detailed in Appendix D (see Proposition D.1 and equations D.1 to D.3) of Boyarchenko et al. [2012].

Similarly, we choose $\Omega^{\star}$ and $\gamma^{\star}$ to match the foreign risk-free return and the return on the foreign equity index, both denominated in the foreign currency.

We use the model-uncertainty-induced preference functional defined in $(B1)$ because it is a simple, parsimonius specification of preferences which (see Appendix D of Boyarchenko et al. [2012]) has been shown to be able to resolve the equity premium puzzle of Mehra and Prescott [1985] (without implausibly high values of risk aversion γ and without increasing the risk-free rate, thus avoiding the risk-free rate puzzle). Further, it does not make or need any distributional assumptions concerning asset returns.

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Table 1: **International risk sharing imputed from returns data (for $\Theta = 0.175$ or $\Theta = 0.35$)**

The reported risk sharing index in Panel A is based on the complete markets assumption and uses the methodology of Brandt et al. [2006, equation (17) and Table 2]. The reported risk sharing index in Panel B incorporates the incomplete markets assumption and are based on equation (9) and for an Appraisal Ratio of $\Theta = 0.175$ or $\Theta = 0.35$. We obtain the numerator of the risk sharing index, i.e., $2\,\mathrm{Cov}\,[m, m^\star]$, by minimizing $\mathbb{E}\,[2m\,m^\star]$, as outlined in equation (10). The quantities $\mathrm{Var}[m]$ and $\mathrm{Var}[m^*]$ are obtained likewise. When computing, for example, the risk sharing index for AUD/JPY, $\mathbf{R}$ includes returns on four assets, namely, on the Australian risk-free bond, on the Australian equity index, on the Japanese risk-free bond, and on the Japanese equity index, all denominated in Australian dollars, while $\mathbf{R}^\star$ includes returns on the same four assets but now all the returns are denominated in Yen. There are 10 countries and the sample period is January 1975 to June 2014. All results are expressed as percentages. The 10% and 90% confidence levels, labeled "CL", are also shown.

	Panel A: Complete markets			Panel B: Incomplete markets					
	CL 10%	RSI	CL 90%	CL 10%	$\Theta =$ 0.175	CL 90%	CL 10%	$\Theta =$ 0.35	CL 90%
AUD/NZD	85	91.2	99	67	83.3	95	10	43.5	86
AUD/STG	86	92.8	98	75	87.5	95	33	66.3	87
AUD/FRA	78	89.2	98	70	85.0	95	26	60.1	85
AUD/CAD	88	93.9	99	74	85.9	95	27	59.7	85
AUD/USD	90	95.2	99	78	88.9	96	45	69.3	88
AUD/NLG	90	95.1	99	81	90.3	96	49	73.4	89
AUD/GER	80	89.8	98	71	85.4	95	28	62.0	86
AUD/JPY	69	81.0	97	65	80.1	94	26	53.1	85
AUD/SWI	89	94.6	98	80	89.7	96	49	73.2	90
NZD/STG	86	93.2	98	74	86.9	95	36	64.2	86
NZD/FRA	73	86.4	98	63	81.7	95	10	51.6	85
NZD/CAD	79	89.7	98	64	82.0	95	5	52.3	85
NZD/USD	89	94.4	98	76	87.5	96	35	66.4	88
NZD/NLG	91	95.3	99	80	89.9	96	50	72.1	89
NZD/GER	74	86.8	98	63	80.7	95	10	51.1	85
NZD/JPY	40	57.3	97	40	64.6	94	-19	24.4	83
NZD/SWI	90	95.1	99	79	89.4	96	47	72.1	90
STG/FRA	93	96.6	99	81	89.8	96	45	57.2	87
STG/CAD	91	95.4	99	78	89.0	96	38	68.3	87
STG/USD	93	96.4	99	81	90.5	96	48	72.6	89
STG/NLG	95	97.5	99	85	92.3	97	56	75.9	89
STG/GER	93	96.7	99	82	90.4	97	49	70.8	89
STG/JPY	86	93.0	98	75	87.1	96	40	64.3	87
STG/SWI	94	97.1	99	84	92.1	97	54	76.2	90
FRA/CAD	84	92.0	98	72	85.9	95	26	60.2	85
FRA/USD	91	95.6	99	80	89.4	96	39	69.5	88
FRA/NLG	99	99.7	100	88	94.0	98	58	78.5	91
FRA/GER	99	99.5	100	83	91.6	97	42	70.8	89
FRA/JPY	80	87.6	98	66	82.2	95	19	53.5	86
FRA/SWI	98	99.0	100	86	93.1	97	53	76.7	89
CAD/USD	97	98.4	100	82	90.8	96	45	70.4	88
CAD/NLG	93	96.3	99	84	91.2	97	54	74.4	89
CAD/GER	83	91.3	98	71	85.4	95	19	59.3	86
CAD/JPY	69	84.7	97	62	81.0	94	7	52.5	84
CAD/SWI	91	95.6	99	81	90.4	96	51	73.9	89
USD/NLG	93	96.5	99	84	91.6	97	58	75.6	90
USD/GER	91	95.3	99	78	89.2	96	41	69.2	88
USD/JPY	90	94.8	99	78	89.2	96	41	69.8	88
USD/SWI	92	96.0	99	83	91.3	96	54	75.6	90
NLG/GER	99	99.8	100	88	94.1	97	60	78.2	90
NLG/JPY	92	95.5	99	82	90.7	96	50	74.2	90
NLG/SWI	98	99.1	100	88	94.0	97	60	79.1	91
GER/JPY	76	86.8	98	68	82.5	95	19	54.3	86
GER/SWI	98	99.1	100	86	93.2	97	53	76.6	90
JPY/SWI	92	95.9	99	82	90.9	97	49	74.5	90

Table 2: **International risk sharing imputed from returns data (for $\Theta = 0.525$ or $\Theta = 0.70$)**

The reported risk sharing index in Panel A (which repeats Panel A of Table 1) is based on the complete markets assumption and uses the methodology of Brandt et al. [2006, equation (17) and Table 2]. The reported risk sharing index in Panel B incorporates the incomplete markets assumption and are based on equation (9) and for an Appraisal Ratio of $\Theta = 0.525$ or $\Theta = 0.70$. We obtain the numerator of the risk sharing index, i.e., $2\,\mathrm{Cov}\,[m, m^{\star}]$, by minimizing $\mathbb{E}\,[2m\,m^{\star}]$, as outlined in equation (10). The quantities $\mathrm{Var}[m]$ and $\mathrm{Var}[m^{*}]$ are obtained likewise. There are 10 countries and the sample period is January 1975 to June 2014. All results are expressed as percentages. The 10% and 90% confidence levels, labeled "CL", are also shown.

	Panel A: Complete markets			Panel B: Incomplete markets					
	CL 10%	RSI	CL 90%	CL 10%	$\Theta =$ 0.525	CL 90%	CL 10%	$\Theta =$ 0.70	CL 90%
AUD/NZD	85	91.2	99	-23	17.4	69	-46	-10.4	52
AUD/STG	86	92.8	98	-3	39.6	74	-31	14.8	58
AUD/FRA	78	89.2	98	-9	31.9	72	-40	5.1	57
AUD/CAD	88	93.9	99	-12	28.5	69	-26	13.3	52
AUD/USD	90	95.2	99	1	41.9	75	-22	18.5	60
AUD/NLG	90	95.1	99	-4	37.4	79	-14	25.9	65
AUD/GER	80	89.8	98	-8	32.9	73	-44	-40.1	57
AUD/JPY	69	81.0	97	-12	21.2	71	-40	-5.5	55
AUD/SWI	89	94.6	98	12	51.1	80	-14	27.5	68
NZD/STG	86	93.2	98	-5	36.2	72	-35	8.9	56
NZD/FRA	73	86.4	98	-28	19.0	70	-59	-1.3	53
NZD/CAD	79	89.7	98	-38	16.3	71	-65	-10.4	52
NZD/USD	89	94.4	98	-14	39.3	75	-34	33.0	61
NZD/NLG	91	95.3	99	15	48.5	78	-12	23.9	65
NZD/GER	74	86.8	98	-28	17.5	70	-49	-11.4	54
NZD/JPY	40	57.3	97	-51	-20.5	67	-66	-40.3	50
NZD/SWI	90	95.1	99	9	48.7	78	-18	24.2	68
STG/FRA	93	96.6	99	6	42.3	75	-21	31.0	57
STG/CAD	91	95.4	99	4	41.6	73	-30	15.0	56
STG/USD	93	96.4	99	6	49.0	77	-17	24.5	64
STG/NLG	95	97.5	99	21	52.5	78	-7	27.4	64
STG/GER	93	96.7	99	20	59.9	77	-17	17.4	63
STG/JPY	86	93.0	98	5	28.7	76	-23	20.7	61
STG/SWI	94	97.1	99	17	51.9	79	-10	43.6	65
FRA/CAD	84	92.0	98	-15	30.3	70	-41	2.3	53
FRA/USD	91	95.6	99	8	44.4	75	-23	19.0	61
FRA/NLG	99	99.7	100	21	57.2	80	-12	27.6	68
FRA/GER	99	99.5	100	-7	44.4	76	-44	18.3	61
FRA/JPY	80	87.6	98	-20	20.8	72	-43	-10.1	54
FRA/SWI	98	99.0	100	19	54.3	79	-16	30.6	65
CAD/USD	97	98.4	100	1	44.1	75	-27	18.0	60
CAD/NLG	93	96.3	99	23	51.7	79	-9	28.3	65
CAD/GER	83	91.3	98	-24	28.7	73	-46	0.2	57
CAD/JPY	69	84.7	97	-35	19.4	69	-57	5.0	50
CAD/SWI	91	95.6	99	16	51.4	78	-9	27.7	66
USD/NLG	93	96.5	99	25	54.0	79	-2	30.7	66
USD/GER	91	95.3	99	5	43.6	76	-23	17.7	61
USD/JPY	90	94.8	99	5	44.2	76	-24	18.4	61
USD/SWI	92	96.0	99	20	54.8	79	-5	32.0	67
NLG/GER	99	99.8	100	24	56.5	79	-6	30.4	66
NLG/JPY	92	95.5	99	22	52.1	79	-5	26.2	66
NLG/SWI	98	99.1	100	24	58.2	80	-4	35.8	68
GER/JPY	76	86.8	98	-20	20.8	73	-42	-7.5	57
GER/SWI	98	99.1	100	12	53.9	79	-12	30.0	66
JPY/SWI	92	95.9	99	13	52.0	79	-16	28.1	66

Table 3: **International risk sharing imputed from consumption data**

Panel A and B reports the risk sharing index based on equations (25) and (B1), respectively. Panel A and B use consumption growth data. Panel C reports the counterpart to Panel A and equation (25) when real consumption growth is replaced by real GDP growth. Panel D and E repeat Panel A and Panel C but using data from Datastream to compute the risk sharing index in equation (25). The relevant columns are labeled Alt Cons and Alt GDP where Alt refers to the use of an alternative data-set. The final Panel F reports the correlation between log consumption growth for a country pair, largely mimicking the values in Panel A. The sample period is January 1975 to December 2012. The consumption and GDP data is the same as that in Barro and Ursua [2008a] and Barro and Ursua [2008b], updated to 2012 using WDI.

	risk sharing index					
	Panel A: Cons growth	Panel B: Model implied	Panel C: GDP growth	Panel D: Alt Cons growth	Panel E: Alt GDP growth	Panel F: Correlation $[d\ln(c), d\ln(c^{\star})]$
AUD/NZD	26.2	27.1	25.4	17.1	33.9	29.6
AUD/STG	19.7	20.1	35.5	17.4	36.0	22.3
AUD/FRA	24.1	24.0	2.8	26.7	33.3	24.2
AUD/CAD	41.6	42.5	65.9	8.4	45.2	42.7
AUD/USD	29.4	27.3	60.1	19.5	29.9	29.9
AUD/NLG	4.4	3.1	30.5	29.3	36.8	4.7
AUD/GER	-11.1	-10.9	10.3	-13.8	11.3	-11.1
AUD/JPY	-18.6	-13.5	-12.4	-10.2	14.2	-18.7
AUD/SWI	13.8	11.0	18.8	-0.2	45.0	14.7
NZD/STG	54.8	55.2	30.2	57.0	38.6	54.8
NZD/FRA	-8.3	-10.5	-10.3	0.0	3.6	-9.9
NZD/CAD	39.9	41.3	27.9	40.0	47.4	41.5
NZD/USD	31.7	31.6	13.4	32.5	36.4	33.4
NZD/NLG	-7.0	-7.7	-3.4	-9.8	9.7	-7.1
NZD/GER	-28.1	-29.9	-21.0	-38.5	-14.4	-30.7
NZD/JPY	-17.0	-10.1	-19.3	-17.0	-7.5	-18.2
NZD/SWI	-17.4	-22.0	-13.2	6.1	26.9	-24.3
STG/FRA	34.9	34.8	59.4	32.6	59.2	41.4
STG/CAD	53.6	51.4	68.2	45.2	63.5	55.7
STG/USD	60.6	64.3	68.8	61.9	71.1	63.7
STG/NLG	32.1	31.7	53.3	38.2	57.1	32.4
STG/GER	20.1	18.4	50.1	1.7	34.7	21.9
STG/JPY	35.5	18.2	45.2	31.5	39.9	38.0
STG/SWI	10.0	12.8	26.4	24.3	33.6	14.0
FRA/CAD	25.9	25.7	46.8	22.3	48.6	27.2
FRA/USD	44.7	40.4	49.8	40.6	56.5	46.5
FRA/NLG	39.0	29.8	61.6	50.4	70.2	43.2
FRA/GER	45.9	46.0	75.0	35.6	72.2	46.7
FRA/JPY	32.9	26.2	58.2	38.5	51.6	33.8
FRA/SWI	53.4	38.5	48.0	47.6	56.2	55.1
CAD/USD	66.0	61.8	85.1	64.4	83.5	66.0
CAD/NLG	34.1	27.8	52.3	37.9	53.7	34.5
CAD/GER	21.8	21.9	49.3	16.3	39.7	22.1
CAD/JPY	-5.9	-4.3	28.8	-4.9	26.2	-5.9
CAD/SWI	35.6	33.0	50.7	31.2	50.1	41.7
USD/NLG	49.1	46.9	60.9	51.0	58.0	49.9
USD/GER	41.6	36.6	57.8	28.4	50.0	41.8
USD/JPY	24.0	12.7	39.8	21.3	41.8	24.0
USD/SWI	24.5	26.1	42.9	32.3	46.8	28.2
NLG/GER	58.5	43.7	73.6	47.3	66.4	60.8
NLG/JPY	15.5	6.1	37.6	17.2	35.4	15.9
NLG/SWI	36.5	45.9	56.0	30.0	58.7	46.4
GER/JPY	46.9	35.7	65.6	48.6	63.0	47.0
GER/SWI	50.9	39.5	42.8	30.9	51.7	55.9
JPY/SWI	2.7	1.0	43.3	15.1	38.2	3.0

Table Appendix-I: **Summary statistics of interest rates, equity returns, and inflation**

We report the summary statistics for interest rates, real equity returns, and inflation over the sample period of January 1975 to June 2014. Panel A reports the mean nominal annualized interest-rates over the sample period January 1975 to June 2014. The results are displayed in the order of decreasing average interest-rates (so Australia (AUD) has the highest average interest-rates while Switzerland (SWI) has the lowest average interest-rates). Panel B reports the properties of average excess equity returns, computed as the real equity return over and above the real interest-rate and in the local currency. Finally, Panel C reports the properties of the CPI inflation. Because the relevant data is unavailable for the United Kingdom (STG) and France (FRA) at the beginning of the sample, we have spliced CPI inflation data with Retail Price Index inflation data.

Currency	AUD	NZD	STG	FRA	CAD	USD	NLG	GER	JPY	SWI
Panel A: Average one month money market interest-rates										
Mean Interest-rate	8.83	8.63	7.86	7.10	6.71	5.86	4.99	4.58	3.14	2.99
Panel B: Equity returns										
Excess Equity Returns	5.85	3.65	7.94	6.72	5.47	6.93	8.99	6.70	4.20	8.04
Standard deviation	17.90	19.12	19.16	20.28	16.96	15.77	18.26	19.98	18.66	16.14
Sharpe ratio	0.33	0.19	0.41	0.33	0.32	0.44	0.49	0.34	0.23	0.50
Panel C: CPI Inflation										
Mean Inflation	5.03	5.93	5.08	4.00	3.92	3.92	2.84	2.36	1.67	1.89
Standard deviation	1.23	1.74	2.18	1.27	1.45	1.11	1.51	1.59	1.79	1.25

Table Appendix-II: **Expected excess returns to borrowing in the low average interest-rate currency and investing in the high average interest-rate currency.**

The expected excess returns, labeled $\mathbb{E}[(e_{t+1}/e_t)\, R_f^\star - R_f]$, to borrowing in the low average interest-rate currency and investing in the high average interest-rate currency, computed as the sample mean over the period January 1975 to June 2014. To make the table easier to read, the expected excess returns are multiplied by one hundred. The first named currency in a pair has the higher average interest-rate. Thus, 2.81 is (one hundred times), the sample mean of the excess return to borrowing one Japanese Yen (JPY), selling this currency at the spot foreign exchange rate, investing in Australian Dollars (AUD) and at the end of the month liquidating the position. The average interest-rate differential, labeled "Interest-rate differential", and the the standard deviation $\sqrt{\text{Var}\,[\ln(e_{t+1}/e_t)]}$ of the log real exchange rate (both expressed as a percentage) are also shown for comparison.

Currency pair	$\mathbb{E}[(e_{t+1}/e_t)\, R_f^\star - R_f]$	Interest-rate differential (%)	$\sqrt{\text{Var}\,[\ln(e_{t+1}/e_t)]}$
AUD/NZD	0.81	0.19	9.5
AUD/STG	1.62	0.96	12.7
AUD/FRA	1.69	1.73	12.2
AUD/CAD	1.74	2.12	9.5
AUD/USD	2.55	2.97	11.2
AUD/NLG	2.39	3.84	12.4
AUD/GER	2.58	4.24	12.5
AUD/JPY	2.81	5.69	14.7
AUD/SWI	2.94	5.84	13.6
NZD/STG	1.22	0.77	12.3
NZD/FRA	1.31	1.53	12.0
NZD/CAD	1.56	1.93	11.1
NZD/USD	2.31	2.78	12.1
NZD/NLG	2.01	3.64	12.2
NZD/GER	2.21	4.05	12.3
NZD/JPY	2.46	5.50	14.6
NZD/SWI	2.53	5.64	13.1
STG/FRA	0.47	0.76	8.7
STG/CAD	1.01	1.16	10.6
STG/USD	1.63	2.01	10.5
STG/NLG	1.15	2.87	9.0
STG/GER	1.35	3.28	9.0
STG/JPY	1.71	4.73	12.7
STG/SWI	1.68	4.87	10.2
FRA/CAD	0.95	0.39	10.9
FRA/USD	1.56	1.25	10.7
FRA/NLG	0.71	2.11	3.4
FRA/GER	0.90	2.52	3.4
FRA/JPY	1.50	3.97	11.8
FRA/SWI	1.25	4.11	6.1
CAD/USD	0.84	0.85	6.6
CAD/NLG	0.92	1.72	11.0
CAD/GER	1.12	2.12	11.1
CAD/JPY	1.31	3.57	13.2
CAD/SWI	1.50	3.72	12.5
USD/NLG	0.32	0.87	10.9
USD/GER	0.51	1.27	11.0
USD/JPY	0.52	2.72	11.7
USD/SWI	0.86	2.87	12.1
NLG/GER	0.19	0.41	2.4
NLG/JPY	0.85	1.85	11.9
NLG/SWI	0.56	2.00	5.7
GER/JPY	0.66	1.45	12.0
GER/SWI	0.36	1.59	5.7
JPY/SWI	0.96	0.15	11.9

Table Appendix-III: **Appraisal Ratios**

The table reports Appraisal Ratios for the strategy whereby the domestic investor invests in the foreign equity index. The Appraisal Ratio of this strategy is computed using the domestic equity index as the benchmark. Denote by R_{Eq} and $R^{\star}_{\mathrm{Eq}}$, respectively, the domestic and foreign equity index returns, measured in their own currencies. The Appraisal Ratio is calculated as $\mathbb{E}[(e_{t+1}/e_t)\, R^{\star}_{\mathrm{Eq}} - R_{\mathrm{Eq}}]/\sqrt{\mathrm{Var}\left[(e_{t+1}/e_t)\, R^{\star}_{\mathrm{Eq}} - R_{\mathrm{Eq}}\right]}$. Thus, the Appraisal Ratio is calculated in a similar fashion to the Sharpe ratio but using the return on the domestic equity index as opposed to the domestic risk-free rate. The currency of the domestic investor is specified by the column while the currency of the foreign investor is specified by the row. As an example of how to read the table, 0.26 is the Appraisal Ratio for a Japanese (JPY) investor investing in the Netherlands (NLG) equity index. The sample period is January 1975 to June 2014.

	AUD	NZD	STG	FRA	CAD	USD	NLG	GER	JPY	SWI
AUD	0.00	0.14	-0.01	0.06	0.13	0.10	-0.01	0.10	0.20	0.06
NZD	-0.10	0.00	-0.12	-0.06	-0.01	-0.03	-0.13	-0.02	0.09	-0.07
STG	0.08	0.17	0.00	0.09	0.17	0.15	0.01	0.13	0.24	0.10
FRA	0.01	0.11	-0.06	0.00	0.09	0.07	-0.10	0.06	0.18	0.01
CAD	-0.08	0.06	-0.12	-0.04	0.00	-0.01	-0.14	0.01	0.13	-0.04
USD	-0.04	0.09	-0.10	-0.02	0.05	0.00	-0.14	0.03	0.16	-0.02
NLG	0.08	0.19	0.03	0.10	0.19	0.19	0.00	0.18	0.26	0.12
GER	-0.03	0.07	-0.10	-0.05	0.04	0.02	-0.18	0.00	0.14	-0.05
JPY	-0.12	-0.02	-0.18	-0.12	-0.06	-0.09	-0.21	-0.08	0.00	-0.15
SWI	0.02	0.13	-0.05	0.01	0.11	0.09	-0.10	0.07	0.20	0.00

Table Appendix-IV: **Summary statistics for real consumption growth and real GDP growth**

We report the summary statistics for real consumption growth and real GDP growth. The raw data used to construct this table is from Barro and Ursua [2008a] and Barro and Ursua [2008b], updated to 2012 using WDI. AR(1) represents the first-order autocorrelation. We also report the correlation between country consumption growth and country real equity returns and between country consumption growth and OECD consumption growth. The OECD data is from Datastream.

	AUD	NZD	STG	FRA	CAD	USD	NLG	GER	JPY	SWI
Mean consumption growth	1.87	0.96	2.27	1.45	1.62	2.06	1.12	1.74	1.91	0.89
Standard deviation	1.48	2.45	2.61	1.25	1.64	1.70	1.96	1.44	1.59	1.12
AR(1)	0.06	0.42	0.57	0.39	0.44	0.53	0.69	0.69	0.50	0.38
Correlation with real equity returns	0.16	0.36	0.30	0.26	0.46	0.51	0.38	0.29	0.47	0.21
Correlation with OECD consumption growth	0.23	0.24	0.77	0.64	0.62	0.88	0.65	0.45	0.49	0.47
Mean GDP growth	1.76	1.03	1.85	1.35	1.45	1.76	1.56	1.83	1.98	0.80
Standard deviation	1.54	2.38	2.20	1.55	2.05	2.00	1.80	1.91	2.27	2.17
AR(1)	0.07	0.33	0.50	0.32	0.26	0.28	0.34	0.12	0.34	0.30

Table Appendix-V: **International risk sharing when consumption growth is measured at different frequencies**

The risk sharing index based on equation (25) computed using consumption growth data at different frequencies. Panel A reports the 10% confidence level (CL), the risk sharing index and the 90% confidence level (CL) based on quarterly (3M) consumption growth data. Panel B repeats Panel A but using annual (1Y) consumption growth data. Panel C repeats Panel B but using two year (2Y) consumption growth data computed by compounding two successive annual consumption growth rates. The sample period is January 1975 to December 2012.

	Panel A: 3 month			Panel B: 1 year			Panel C: 2 year		
	CL 10%	RSI	CL 90%	CL 10%	RSI	CL 90%	CL 10%	RSI	CL 90%
AUD/NZD	1	8.9	19	-3	17.1	38	-9	16.3	35
AUD/STG	-11	-2.4	5	-7	17.4	37	-7	21.9	50
AUD/FRA	-2	5.0	11	1	26.7	47	9	37.1	57
AUD/CAD	-1	9.0	18	-15	8.4	36	-1	21.6	57
AUD/USD	3	11.2	20	-13	19.5	50	-11	25.1	65
AUD/NLG	-9	-3.1	5	9	29.3	51	11	34.9	65
AUD/GER	-17	-6.2	7	-38	-13.8	13	-44	-23.1	5
AUD/JPY	-4	2.6	9	-33	-10.2	10	-51	-24.3	4
AUD/SWI	-9	0.0	8	-21	-0.2	21	-2	16.4	39
NZD/STG	-14	-5.1	3	43	57.0	67	45	59.4	73
NZD/FRA	0	3.4	7	-20	0.0	21	-44	-16.9	5
NZD/CAD	7	12.1	21	11	40.0	57	6	41.3	66
NZD/USD	11	16.7	22	10	32.5	52	9	31.4	50
NZD/NLG	1	5.0	9	-29	-9.8	11	-40	-13.7	7
NZD/GER	-4	2.5	10	-56	-38.5	-22	-68	-48.5	-30
NZD/JPY	-6	-2.4	2	-31	-17.0	-5	-42	-24.4	-10
NZD/SWI	0	3.8	10	-14	6.1	30	-18	12.7	36
STG/FRA	-2	5.4	13	18	32.6	44	8	30.4	46
STG/CAD	-2	7.8	20	30	45.2	56	28	48.6	65
STG/USD	2	11.5	22	48	61.9	73	47	63.6	76
STG/NLG	-15	-3.2	7	21	38.2	55	13	41.1	61
STG/GER	-21	-9.7	0	-18	1.7	19	-22	5.5	26
STG/JPY	-2	12.0	23	10	31.5	44	-11	19.4	40
STG/SWI	1	11.3	22	10	24.3	39	24	36.6	50
FRA/CAD	15	24.0	32	1	22.3	47	4	24.3	55
FRA/USD	18	26.3	34	21	40.6	58	8	28.7	57
FRA/NLG	-6	4.9	16	33	50.4	68	27	45.7	70
FRA/GER	-4	5.2	14	10	35.6	54	15	36.6	64
FRA/JPY	18	24.8	31	19	38.5	56	23	44.5	66
FRA/SWI	7	16.3	26	33	47.6	58	9	45.9	77
CAD/USD	33	40.5	49	55	64.4	74	47	64.2	76
CAD/NLG	10	21.5	32	15	37.9	54	16	47.2	66
CAD/GER	-15	0.7	14	-14	16.3	40	-16	23.0	53
CAD/JPY	-2	8.2	19	-25	-4.9	20	-36	-13.9	10
CAD/SWI	2	12.1	22	21	31.2	41	15	36.7	51
USD/NLG	17	26.3	35	33	51.0	65	35	52.9	68
USD/GER	-8	4.0	17	11	28.4	43	16	36.3	55
USD/JPY	6	15.6	25	1	21.3	39	-15	11.1	39
USD/SWI	12	24.6	37	19	32.3	46	14	33.8	53
NLG/GER	1	11.7	23	30	47.3	61	22	48.1	69
NLG/JPY	-11	-1.5	7	-4	17.2	38	-13	8.9	40
NLG/SWI	-4	7.3	20	14	30.0	45	2	27.4	48
GER/JPY	-10	-0.4	10	35	48.6	62	41	58.3	76
GER/SWI	-14	-3.2	8	-1	30.9	54	-9	30.5	66
JPY/SWI	-8	1.5	11	-5	15.1	32	-3	28.2	54