Non-Precautionary Cash Hoarding and Investment in Growth Firms

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Abstract

This paper analyzes a growth firm's trade-off between hoarding cash to reduce dilution associated with external financing and delaying investment. This hoarding motive features a self-reinforcing effect: firms with better investment opportunities hoard less and use more external financing, leading them to quicker success and to potentially becoming cash-rich as they mature. Several novel insights emerge in such a setting: Taking into account self-selection, firms that stay private hoard and delay investment less than when they go public. Furthermore, contrary to common beliefs, competition can cause longer delays and more hoarding, while asymmetric information leads growth firms to reduce hoarding.

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1 Introduction

Our knowledge about cash hoarding and investment is mainly framed by the literature focused on explaining empirical patterns in large and mature firms. Some of the most important reasons for cash hoarding identified by this literature include building up cash reserves because of various agency problems, taxes, or as precautionary savings (e.g., Opler et al., 1999; Bates et al., 2009). In this paper, we take a somewhat different perspective: that of a post-startup firm, which already generates some revenues, but whose main source of valuation is based on the growth prospects still ahead of it.

The motive for cash hoarding in which we are interested is a very basic but important one: Should firms already pressed with a funding need for financing a growth opportunity delay investment to hoard cash, so as to reduce the dilution associated with external financing? And if they do so, should we expect firms with better growth opportunities to hoard more and grow more slowly or should we, instead, observe a self-reinforcing effect where they delay less and quickly outgrow their lesser-potential peers? We are, thus, departing from the standard perspective that high-q (growth) firms accumulate cash as a precaution (e.g., Bates et al., 2009), and we ask how such firms behave once already confronted with the problem that they are unable to invest unless they resort to delay or expensive financing.

Laying out such a setting opens many other questions. One is how hoarding incentives interact with the decision to be public or private and whether this could explain why private firms hoard less cash than comparable public firms (Gao et al., 2013). Another is whether competition will speed up or rather delay investment and lead to more cash hoarding (Akdogu and MacKay, 2008). Furthermore, we ask whether the well-known argument that information asymmetry triggers hoarding for precautionary reasons extends also to a growth firm for which hoarding is non-precautionary and means delaying investment. We show that the answers to these questions often contradict the intuition that one could have from prior theory.

We derive our insights in a simple dynamic model in which a firm wants to make something "big and bold." In our baseline setting, entrepreneurs and financiers may have a different vision about the future. Such differences may lead to disagreement about the optimal course of action and make external financing costly (e.g., Dittmar and Thakor, 2007). Hoarding cash seems, thus, a natural alternative for visionary growth businesses to gain some elbow room and keep a higher stake in the firm.

An important cost of cash hoarding is that it may lead to delays in investment. We show, first, that in an environment with more profitable investment opportunities on average, shorter delays are optimal. Intuitively, firms with better opportunities are willing to depend more on external funding, because the cost of delay is increasing in the attractiveness of the opportunities. Thus, firms with a higher growth potential choose to expand more quickly. Furthermore, being more successful, these firms might end up being cash-rich as they mature despite pursuing a low-cash strategy in their growth phase. Using these results as a starting point, we derive a number of novel insights.

Stark implications come up when relating cash hoarding and investment to a firm's choice between public and private ownership. Private ownership may allow for a greater alignment in vision with financiers. However, it is not without costs as the liquidity benefits of public ownership might be substantial. Trading off these effects, the paper shows that firms for which the choice between public and private ownership is endogenous, hoard less cash and invest more quickly as private than as public firms. The reason is that firms choose private ownership when alignment is of paramount importance for obtaining cheaper financing. That is, firms that choose to be private are those that benefit most from alignment, because they prefer raising more external financing to invest more quickly and, hence, hoard less cash. Our analysis implies, thus, that visionary firms with better investment opportunities are more likely to stay private, hoard less cash, and delay investment less.

Another novel insight of our model is that competition can lead to longer delays in investment and to accumulating more cash. Though our model features the standard incentives to speed up investment, which are generated by the threat of losing one's first-mover advantage to a competitor (e.g., Grenadier, 2002), our setting highlights that there is a countervailing force. Competition not only erodes the firm's first-mover advantage, but it also makes the investment opportunity less attractive. This reduces the cost of delay for a firm facing a large funding need and trying to avoid dilution, resulting in more delay and hoarding. We show that this effect will dominate in firms, whose profits from being a first-mover are very vulnerable to competitive pressures.

Our focus on a growth firm already pressed by a large funding need generates also another novel result relative to theories focused on mature firms that build up cash reserves as a precaution: We show that asymmetric information can lead to less rather than more hoarding. On the one hand, our previous results are robust to the introduction of information asymmetry—hoarding cash is optimal, and firms with better investment opportunities hoard less. Thus, investors can infer the firm's prospects from its hoarding policy. On the other hand, the implications of information asymmetry *after* the arrival of the investment opportunity are very different to those when hoarding cash for precautionary reasons. In particular, though cash hoarding in the former situation is an effective signaling mechanism, the cost of the signal is that firms hoard less cash than they would do otherwise. The reason is that firms with better projects are most hurt by delays caused by hoarding. Thus, they seek to minimize hoarding, but this strategy can be easily mimicked by firms with less lucrative investment opportunities. This causes a rat race to reducing hoarding by all.

We extend our model along several dimensions. By allowing firms to hoard cash before the investment opportunity arrives, we incorporate the precautionary motive for hoarding emphasized in the prior literature. Now all firms hoard cash *before* arrival, but considering both the pre- and post-arrival hoarding, the insight that firms with better opportunities hoard less cash continues to hold. In a separate extension, we show that our insights are robust also when differences in vision and the profitability of the investment opportunity vary over time. With this we address the common argument that firms may delay, waiting for a technology to prove itself or in attempt to time investor sentiment. Furthermore, we show that our results are robust when the manager's incentive for cash hoarding are driven by a reluctance to incur a dilution of incentives ala Holmstrom and Tirole (1997).

In another extension, we analyze how the type of financing and payouts interact with cash hoarding. We obtain that firms hoard less cash to co-finance new investments when they have access to debt financing than when they issue more-information-sensitive securities, such as equity. Intuitively, the cost of financing caused by disagreement (and, thus, the necessity to hoard cash) matters less when financing is in a less "disagreement-sensitive" security. Here we show that our results on cash hoarding could be interpreted more broadly in the context of a firm building up easily collateralizable net worth to ease external financing. Furthermore, when analyzing payout policy, we show that paying dividends and buying back shares play no role when fresh capital is needed for new investments. Paying out cash only increases the dependence on external financing, and is, thus, counterproductive.

The results of our paper give rise to a rich set of empirical implications. We predict that from the subset of firms with (profitable) investment opportunities, firms with lower cash-to-assets ratios will be more profitable. Furthermore, growth firms that make new investments with a higher proportion of outside financing should exhibit a stronger increase in revenues, profits, and asset size. In terms of dynamics, we expect that growth firms expanding through acquisitions in the beginning of a merger wave should on average perform better than firms that join later with a higher proportion of internal funds.[1] More

[1]The empirical evidence is supportive of our results. Hoarding cash goes hand in hand with high q–i.e., growth opportunities–(Bates et al., 2009) and the long-term performance of firms making large investments (such as takeovers) is lower for firms with higher cash-to-assets ratios (Harford, 1999).

broadly, our analysis points to a self-reinforcing mechanism, in which firms with better opportunities also invest more quickly. Then, as these firms mature, they are likely to end up with large cash holdings due to their success despite their original low-cash strategy. Interestingly, our, at first sight, counter-intuitive insight that private firms hoard less cash than public firms (growth firms with better opportunities remain private and hoard less cash) is supported by recent empirical evidence (Gao et al., 2013; Farre-Mensa, 2014). Another novel prediction of our model concerns the differential effect of competition, depending on whether it is likely to erode the profits of firms with a first-mover-advantage. In industries, where such erosion is likely, there should be more delay and hoarding; the opposite prediction characterizes industries in which being the first-mover guarantees long-term profits. ‘These results could help explain the recent evidence on the non-monotonic relation between competition and investment timing (Akdogu and MacKay, 2008). Noteworthy is also that our results on information asymmetry could help explain how the internal-to-external financing mix affects announcement returns surrounding large investments.

When relating cash hoarding to profitability, we emphasize how important it is to control for measures of disagreement. We predict that firms that invest in unfamiliar businesses with a higher dispersion of analyst forecasts should hoard more cash. This may also shed some light to what extent growth firms contribute to the steady increase in corporate cash holdings in recent years (Bates et al., 2009). In particular, the changing nature of growth firms towards becoming more disagreement-prone (by being more R&D intensive and having more intangible assets, which are difficult to value) may have led to more cash hoarding (Falato et al., 2013). Finally, we predict that growth firms should not return cash to shareholders if it is needed for new investments and that firms with higher debt capacity will hoard less cash.

Our paper mainly relates to the fast growing literature on cash. Firms hoard cash because they may be unable to frictionlessly raise financing for new investments. Agency conflicts are one important such friction. According to the free cash flow hypothesis of Jensen (1986), entrenched managers hoard cash and then invest it rather than paying it out to shareholders even when there are poor investment opportunities. More recently, DeMarzo et al. (2012) show that accumulating cash reserves as a way of deferring compensation can provide incentives to exert effort (see also Biais et al., 2007). The evidence for hoarding cash for agency reasons is rather mixed, however.[2]

[2]Dittmar and Mahrt-Smith (2007) and Pinkowitz et al. (2006) show that cash is worth less when agency problems between inside and outside shareholders are greater, and Nikolov and Whited (2013) identify low managerial ownership as a key factor. In contrast, Opler et al. (1999) and Bates et al. (2009) find no evidence relating agency problems to cash holdings.

Firms may also hoard cash as a precautionary measure. Bolton et al. (2011) show that firms will keep a positive balance even if this necessitates costly external financing, since the marginal benefit of avoiding to seize operations is very high. In such cases, firms with stronger cash flow streams need to hoard less cash (Acharya et al., 2012). Related, Almeida et al. (2004) show that financially constrained firms save more cash out of cash flows.[3] The existing evidence seems to strongly support the precautionary motive for hoarding cash (Opler et al., 1999; Bates et al., 2009). We also substantiate this motive when we allow for hoarding prior to the arrival of the investment opportunity.

An important insight from our model is that disagreement and the lumpy nature of investment opportunities may render investment impossible without cash participation by the owner-manager, leading to delays in investment. This approach closely relates our paper to Boyle and Guthrie (2003) and Bolton et al. (2013) who analyze how optimal investment timing changes with firms' cash holdings when external financing is costly. An important difference to these papers is that we endogenize the frictions that make external financing costly, which helps us derive a number of novel predictions, such as implications for cash hoarding in public versus private firms.

Our novel insights on how competition affects investment delay and hoarding gives a new thrust relative to prior theories that—contrary to our analysis—uniformly predict that competition speeds up investment when it threatens the firm's first-mover advantage (Carlson et al., 2006), when firms try to time their optimal supply to market demand (Grenadier, 2002), or when they strategically time their merger bids (Morellec and Zhdanov, 2005). Though our treatment of competition is stylized, it suffices to make our point: Compared to the prior literature, delaying investment serves a different function in our setting—that of covering a funding need, reducing underpricing, and preserving incentives. To our knowledge, we are the first to show that in this context competition calls for delaying investment and hoarding more cash, which could help explain the non-monotonic effect of competition on investment documented in the empirical literature (Akdogu and MacKay, 2008).

In terms of methodology, our paper builds on the standard real options framework of McDonald and Siegel (1986).[4] Our contribution here is to combine the real option

[3]In Hugonnier et al. (2012), a firm would hoard cash, because it may not be able to find a financier. Furthermore, in the recent literature on risk management, Acharya et al. (2012) show that firms with high aggregate risk exposure prefer cash to credit lines, and Rampini and Viswanathan (2010) argue that the opportunity cost of risk management is higher for constrained firms. In contrast to this literature, in which cash and credit lines help to avoid liquidity problems, our paper asks whether growth firms would actually delay a big investment to hoard cash and reduce their dependence on external financing (i.e., also on credit lines, which are anyway not common for growth firms–Sufi, 2009).

[4]Related to our finance application are Grenadier and Malenko (2011) who analyze real options sig-

framework with the literature on disagreement (Dittmar and Thakor, 2007; Boot et al., 2008) and to discuss the resulting implications for cash hoarding.

Our paper is organized as follows. Section 2 presents the model. Section 3 contains the main analysis, which relates disagreement to cash hoarding, endogenizes the choice between private ownership and going public, and analyzes the effects of competition and information asymmetry. In section 4, we analyze the various extensions. Section 5 discusses the empirical implications, and Section 6 concludes.

2 Model

Our baseline model features a firm run by an entrepreneur who is the sole owner-manager (henceforth, manager). The firm we envisage already generates revenues, but is still a growth firm with a potentially profitable expansion still ahead of it. We model this in the following natural way. Suppose that the firm has an existing asset in place producing stochastic cash flows. If they are not paid out or invested, these cash flows accumulate in the form of cash reserves within the firm according to

$$dw_t = \mu w_t dt + \sigma w_t dZ_t,\ w_0 > 0. \tag{1}$$

where μ and $\sigma > 0$ are constant and $(Z_t)_{t\geq 0}$ is a standard Brownian motion. This simple reduced-form formulation for how the level of cash changes within the firm is completely sufficient for our purposes. Our key assumption is that $\mu < r$, where r is the constant discount rate used by all. This assumption, which is standard in the real options literature, implies that the firm has only a weak ability to generate cash, and keeping it going is costly to insiders. Specifically, in our setting, it implies that the opportunity cost associated with hoarding cash is paying out w_t together with effectively closing operations.[5] This is precisely the feature we want to capture for an growth firm for which the investment opportunity is the main component of valuation and absent which the firm constitutes an unprofitable business. At the same time, this setting is sufficiently flexible to allow us to discuss payout policy prior to expansion (Section 4.3) and to capture other basic features, such as high volatility of w_t making it more likely that the firm defaults (if w_t hits zero, it does not recover).[6] Though for most of the main text, we refer to w_t as cash, an alternative

naling games. Morellec and Schürhoff (2011) discuss financial contracting, and Bouvard (2012) discusses real options financing under asymmetric information.

[5]Note that paying out all cash w_t deprives the firm from the ability to produce cash flows (cf. (1)).

[6]The main advantage of (1) is that it allows us to solve everything in closed form. At the cost of losing this tractability, we could specify a cash flow process generating the cash level w_t as in Bolton et

interpretation is that w_t represents the net worth the firm builds up over time, which is available as a safe collateral free of any financing frictions; we return to this interpretation towards the end of the paper in Section 4.2.

The reason the manager is willing to keep the firm going is that this firm has generated a profitable investment opportunity, requiring a cash outlay of K. Initially, the manager does not have this cash at hand, but she has discretion over the timing of this investment. Our approach, thus, makes use of the standard real options framework (e.g., McDonald and Siegel, 1986; Dixit and Pindyck, 1994), but differs from this framework in one important aspect: the firm is cash-constrained and the manager may not be able or be willing to invest in a positive NPV project even if she can raise capital from a financier in a competitive capital market. In our baseline model, the unwillingness to raise external financing is due to differences in vision leading the entrepreneur to believe that her firm is undervalued by outsiders. In Section 4.4, we show that we obtain the same results when the manager's reluctance to raise external financing is due to the negative effect this would have on her incentives to exert effort (as in Holmstrom and Tirole, 1997).

Vision and Disagreement At $t = 0$, the manager and the financier publicly observe a signal about the profitability of the project. If the signal is good and the firm invests, its discounted expected cash flows become θX_G. We specify a more general process for the cash flows generated after investment in Section 4.2, in which we discuss financial contracting and in which show that the results derived from the reduced-form specification θX are without loss of generality.

If the signal is bad and the firm invests, its expected discounted cash flows become θX_B. The common parameter θ can be seen as a publicly observable indicator of the attractiveness of the firm's (or industry's) growth prospects.[7] We assume that $K > \theta X_B$ for all θ, so that a bad signal translates into a negative NPV project. Furthermore, $\theta X_G > K$ at least for some θ, so that investing after a good signal in such cases increases value. All features of our model are common knowledge, and the cash flows and the level of cash are costlessly verifiable. Furthermore, we assume that all parties are risk neutral and protected by limited liability.

The main feature of our vision- (or disagreement-) based model is that although both parties observe the same signal, they may interpret it differently. More specifically, whatever inference is made by the manager, the financier believes that the inference is correct

al. (2013). However, we do not obtain new intuitions or results from such alternative formulations.

[7]In Section 3.4 where we compare the vision setup to the information asymmetry specification, we let θ be private information. For now, it is observable and known to all.

only with probability $\rho \in (0,1)$. The agreement parameter ρ is common knowledge and might depend on the nature of the investment opportunity and/or the nature of the business.[8] In our baseline model, ρ and the expected discounted cash flows prior to investment θX are constant over time. We relax these assumptions in Section 4.5, where delay balances the wish for accumulating cash and investing at a high expected value of the investment opportunity and lower levels of disagreement.

In what follows, we only need to consider the case in which the manager observes a good signal (i.e. infers that the project is good). This is because if the manager observes a bad signal and the financier (fully) agrees that the project is bad, it is never undertaken and the cash at hand is paid out. If the financier (fully) disagrees and believes that the project is good, the best course of action for the manager is to pay out all available cash and sell the firm to the external financier.[9] Hence, when the manager receives a bad signal, there will not be any hoarding, and the cash at hand is paid out regardless of whether the financier agrees or disagrees.

Outside Financing At the time at which the manager raises capital to make the investment, she is facing a competitive capital market. We model this by allowing the manager to make a take-it-or-leave-it offer. Since the offer is open to all financiers, they compete away all rents and only require breaking even. The manager sells equity to raise $K - w$ from the investment outlay K.[10] The financier's equity stake α must satisfy

$$\begin{aligned} K - w &= \alpha\theta\left(\rho X_G + (1-\rho) X_B\right) \qquad (2) \\ &= \alpha\theta W^F \end{aligned}$$

where $W^F := \rho X_G + (1-\rho) X_B$. θW^F stands for the financier's assessment of the firm's value, capturing that the financier shares the manager's assessment with probability ρ, and disagrees with probability $(1-\rho)$. From (2), the equity share that needs to be promised to the financier is

$$\alpha = \frac{K - w}{\theta W^F}. \qquad (3)$$

[8] Disagreement in a corporate finance context is usually motivated with heterogenous priors in the sense of Kurz (1994a,b)–e.g., Boot et al. (2008). However, it can also arise due to overconfidence on the part of either management or financiers (Bernardo and Welch, 2001; Daniel et al., 1998) and excessive pessimism (Coval and Thakor, 1998) or optimism (Manove and Padilla, 1999).

[9] This assumes that the investment opportunity is not uniquely bound to the manager. This seems, however, a natural assumption when outside financiers think they can do better than this entrepreneur, usually by replacing her. The same logic also applies in Section 4.4 if we consider disagreement and incentive problems at the same time.

[10] We discuss optimal financial contracting in Section 4.2.

Define $\theta W^M := \theta X_G$ as the manager's assessment of the firm's value. The manager's net expected payoff at the point in time in which she raises $K - w_t$ and co-invests w_t is:

$$V(w_t, \theta) := \left(1 - \frac{K - w_t}{\theta W^F}\right) \theta W^M - w_t. \tag{4}$$

Inspecting $V(w_t, \theta)$, we obtain:

Lemma 1 *The manager's expected payoff $V(w_t, \theta)$ from raising $K - w_t$ and investing K is increasing in her co-investment w_t, in the agreement parameter ρ, and in the profitability parameter θ.*

We have outlined so far a setting in which a growth firm decides whether to delay investment to keep a larger stake in her firm by generating more internal financing, where the main financing friction is differences in vision. Later, we analyze how these decisions interact with the firm's endogenous choice between public and private ownership and the presence of competition. In addition, we compare the resulting insights with those from a model based on asymmetric information and incentive problems. To streamline the exposition, we will add more structure to the baseline model where needed.

3 The Growth Firm's Decision to Hoard Cash

3.1 Cash Hoarding and Speed of Growth

We will now derive how the lack of alignment between management and financiers–inducing potential disagreement–affects investment decisions, and in turn impacts the amount of cash the firm hoards prior to undertaking the investment. We solve for the value of the real option to invest using standard dynamic programming methods (Dixit and Pindyck, 1994). The problem is that of finding the optimal stopping rule w^* (i.e., the level of cash holdings at which the investment is made) that maximizes the value of the option to invest U, while trading-off the benefit of reducing the funding cost against the time value of money lost from investing later, where

$$U(w_t, w^*, \theta) = \max E\left[\frac{1}{1 + rdt}\left[U(w_t + dw_t, w^*, \theta)\right]\right]. \tag{5}$$

Applying Ito's lemma, we obtain

$$rU = \mu w \frac{\partial U}{\partial w} + \frac{1}{2}\sigma^2 w^2 \frac{\partial^2 U}{\partial w^2}.$$

This equation is solved subject to the following boundary conditions. First, the manager's expected payoff at the time of investment should be equal to her payoff from investment: $U(w_t, w^*, \theta)|_{w_t=w^*} = V(w_t, \theta)|_{w_t=w^*}$. Second, the manager chooses the investment trigger so as to maximize her value at the endogenous investment threshold: $\frac{\partial}{\partial w^*} U(w_t, w^*, \theta)|_{w_t=w^*} = 0$. Finally, the option to hoard cash becomes worthless as the value of cash tends to zero: $\lim_{w_t \to 0} U(w_t, w^*, \theta) = \max\left[0, \left(1 - \frac{K}{\theta W^F}\right)\theta W^M\right]$. Indeed, if the existing business falters ($w_t \to 0$), then it almost surely does not recover (cf. (1)), and the manager can invest only if she raises all financing externally.

Suppose, first, that disagreement is sufficiently strong such that $K > \theta W^F$. Then, solving this optimization problem yields the following expression for the manager's expected payoff

$$U(w_t, w^*, \theta) = \left(\left(1 - \frac{K - w^*}{\theta W^F}\right)\theta W^M - w^*\right)\left(\frac{w_t}{w^*}\right)^{\beta} \tag{6}$$

where β is the positive root of $\frac{1}{2}\sigma^2 y(y-1) + \mu y - r = 0$, and $\mu < r$ implies that $\beta > 1$. Intuitively, the first term in (6) is the manager's expected payoff from investing at w^*, while the second term could be interpreted as the probability of reaching the cash level w^* and investing. Trading off the marginal cost (due to $\mu < r$) and benefit of delay, the value maximizing investment threshold w_θ^{VM} is given by

$$w_\theta^{VM} = \frac{\beta}{\beta - 1}\left(K - \theta W^F\right)\frac{W^M}{(W^M - W^F)}.^{11} \tag{7}$$

If, instead, disagreement is not so strong ($K < \theta W^F$), the manager is better off investing immediately as the cost of delay outweigh the potential gains from avoiding dilution by hoarding cash.

Proposition 1 *If disagreement is sufficiently strong ($K > \theta W^F$), it is optimal for the manager to hoard cash prior to making the investment. The optimal cash level is given by (7). This threshold is decreasing in θ and ρ–i.e., there is less cash hoarding when the investment opportunity is better and when there is more agreement.*

What this proposition points at is that the costs of delaying via hoarding are higher when the opportunities are better, implying that the manager hoards less cash. Furthermore, the cost of delay weighs more when there is less disagreement. Thus, the manager will hoard less cash when there is less misalignment in vision with the financier.

[11] Observe that $\alpha \in [0,1]$ imposes an upper bound of K on w_θ^{VM}. Assuming that this bound is not reached is without loss for most our analysis, except when we discuss asymmetric information (Section 3.4), where this bound could lead to pooling of some types at K.

The simplicity of this result should not conceal the importance its implications. One such implication is that firms with better investment opportunities also choose to expand more quickly. This leads to a self-reinforcing mechanism leading to an accelerated divergence over time between firms with good and bad investment opportunities. Another implication is that mature firms with strong cash holdings due to their success actually pursue low-cash strategies in their earlier growth phase (see for details Section 5). Furthermore, as we show in what follows, expanding on this simple model leads to novel results concerning cash hoarding in public vs. private firms and the effect of competition and that of information asymmetry on cash hoarding.

3.2 Cash Hoarding in Public vs. Private Firms

We now add some structure to analyze the differences in cash hoarding between firms for which the choice between public and private ownership is endogenous.[12] For this purpose, we explore the following trade-off faced by firms at the intersection between public and private ownership. Financiers in private firms hold illiquid claims; this illiquidity makes financing more expensive for these firms. However, a private firm might more easily avoid disagreement. For example, private financiers are often closely involved with firms in which they invest. They also have access to sensitive information needed to better understand the business and align vision–information that the manager might be unwilling to disclose to the market and, thus, to its competitors (Bhattacharya and Ritter, 1983; Maksimovic and Pichler, 2001). A related benefit is that the financiers in private firms are typically not dispersed, which helps to alleviate free riding and makes coordination between different classes of financiers easier (Brunner and Krahnen, 2008).

More formally, we assume that private ownership imposes a cost L on the financier, which can reflect, for example, that the financier's investment is less liquid or the fixed costs of being more involved in the firm. However, an incentive for a firm to remain private is that the better-targeted choice of (non-dispersed) financiers and their potential higher involvement, leads to a lower degree of disagreement within a private firm: $\rho^{priv} > \rho^{pub}$. Though we base our analysis on this simple trade-off, our results readily extend to including variable costs of private ownership as well as fixed and variable cost of a public/private issue (see footnote 13).

The financier's participation constraint when the manager raises $K - w$ as a private

[12] Note that we are not comparing here the typical private with the typical public firm, but growth firms that can afford to choose either form. This is arguably the empirically relevant case and the focus of studies, such as Gao et al. (2013).

firm is

$$\alpha\theta W^F\left(\rho^{priv}\right) = K - w + L,$$

where $W^F(\rho)$ makes explicit the dependence of W^F on the agreement parameter ρ. Hence

$$\alpha = \frac{K - w + L}{\theta W^F(\rho^{priv})} \tag{8}$$

and following the same steps as in (7), we obtain that the optimal levels of cash hoarding in a private and a public firm are

$$w_\theta^{priv} = \frac{\beta}{\beta - 1}\frac{\left(K + L - \theta W^F(\rho^{priv})\right)}{(W^M - W^F(\rho^{priv}))}W^M \tag{9}$$

$$w_\theta^{pub} = \frac{\beta}{\beta - 1}\frac{\left(K - \theta W^F\left(\rho^{pub}\right)\right)}{(W^M - W^F(\rho^{pub}))}W^M. \tag{10}$$

Comparing w_θ^{priv} and w_θ^{pub} cannot give a clear idea if a private firm hoards more or less than a public firm because the decision to go public is endogenous and depends on θ. One should instead compare the manager's expected payoff in both cases given her optimal cash hoarding policies. Doing so, we obtain:

Proposition 2 *(i) Firms that choose to remain private hoard less cash and delay investment less than they would if they had chosen public ownership. (ii) Firms with better opportunities (high θ) are more likely to choose to stay private.*

For understanding part (i) of the proposition, observe that the time-value-cost of delay is the same regardless of the firm's public or private status. Hence, the endogenous choice of this status is driven by the differential effects of cash hoarding on the costs of outside financing. In a private firm, the benefit of less disagreement ($\rho^{priv} > \rho^{pub}$) is higher when the need for external financing is higher. The illiquidity (and various fixed) costs L, on the other hand, are constant. Hence, it is firms that seek to raise more external financing—i.e., that hoard less cash and delay investment less—that choose to remain private.[13] Thus, our

[13] We can also assume that the illiquidity cost has a variable component l, which is proportional to the financier's stake in the firm. Then, to break even, the financier requires that $\alpha = \frac{K - w^*_{priv} + L}{\theta W^F(\rho^{priv})(1-l)}$. Observe, however, that l has the same effect on the cost of financing as that of a lower level of agreement, which is also proportional to the amount raised from external financiers. Thus, increasing either of these costs shifts the indifference threshold between public and private ownership in the same fashion, making public ownership preferable for higher θ. Also note that in the extreme case, in which there are only proportional illiquidity costs (i.e., $l > 0$, but $L = 0$), the choice between public and private ownership becomes trivial. It is then the same for all θ, and would boil down to comparing $\theta W^F\left(\rho^{priv}\right)(1-l)$ and $\theta W^F\left(\rho^{pub}\right)$.

In a similar vein, considering the fixed and variable cost of a public/private issue, would simply boil non-precautionary motive for hoarding could help explain the surprising empirical findings of Gao et al. (2013) and Farre-Mensa (2014) that private firms hoard less cash and invest more than comparable public firms.

It is noteworthy that it is the growth firms with better investment opportunities that are more likely to remain private. Since these firms are prepared to raise more external financing to avoid delay (Proposition 1), these firms benefit most from the higher agreement and alignment of vision with financiers under private ownership.

3.3 The Dual Effects of Competition on Hoarding

In this section, we show that contrary to the predictions of the prior literature (e.g., Carlson et al., 2006; Grenadier, 2002), competition can lead to longer delays in investment, which translates into more cash hoarding. The specific feature of our setting that leads to this result can be traced back to Proposition 1: When a growth firm is faced with a funding need and tries to avoid dilution, firms whose investment opportunities are not as good find delaying investment to hoard cash less costly. Our point here is that the effect of competition is often precisely that it decreases the attractiveness of investment opportunities.

To show this result, we extend the baseline model in the following way: Suppose that the likelihood that a competitor with a similar idea appears before investment follows an exponential distribution with parameter λ. The parameter λ could, thus, be interpreted as a measure of the intensity of potential competition. Furthermore, suppose that the firm's expected discounted payoff is $\pi(\lambda)\theta W^M$ if it is not the first-mover, and $\xi\pi(\lambda)\theta W^M$ if it is the first-mover, where $\xi \geq 1$ captures the extent of the first-mover-advantage. With slight abuse of notation, we use the dependence of π on λ to capture the idea that competition matters also after investment, and that it affects negatively the firm's expected cash flows: $\pi'(\lambda) \leq 0$. This reduced-form way of modeling competition abstracts from strategic equilibrium considerations (i.e., the manager takes λ as exogenous).[14] However, this formulation captures our main point, and it seems a reasonable description of a growth firm that wants to bring a new product/service, which is yet to create its own demand. Then, once the idea (or a variation of it brought by a competitor) is introduced, competitors can mimic the product/service, and $\pi'(\lambda)$ captures the extent to which this leads to an

down to reinterpreting L to include the fixed cost differential between choosing a private or a public issue. Similarly, the variable cost of a public issue could be represented by an increase in ρ_{pub}.

[14]In Cournot-like competition models with identical financially unconstrained firms, such as Grenadier (2002), firms time their supply in an environment with time-variant demand. We incorporate time varying profitability in Section 4.5.

erosion of the firm's long-term profits and competitive advantage. If this decrease in profits is sufficiently strong, we have the following result:

Proposition 3 *There is a threshold $\widehat{\pi}'(\lambda)$, such that for $\pi'(\lambda) < \widehat{\pi}'(\lambda)$ an increase in competition leads to longer delays and more cash hoarding.*

Proposition 3 presents a more differentiated view on the effect of competition on cash hoarding and investment in growth firms. On the one hand, if being the first-mover is sufficiently important to the firm (high ξ), competition creates incentives to invest sooner as suggested by the prior literature, which leaves less time for cash hoarding. On the other hand, even a firm with a first-mover advantage suffers from more intense competition, as competition reduces the firm's expected payoffs conditional on investment. The crucial aspect, due to our main assumption that the firm has a net funding need and tries to avoid dilution, is that this decrease in expected profits reduces the opportunity cost of delay. This creates a countervailing effect, making it more beneficial to finance the investment with more internally generated cash. This countervailing effect dominates if the decrease in profits is sufficiently strong, captured by $\pi'(\lambda)$ being sufficiently negative. In practice, this effect seems especially likely in cases in which the firm loses its monopoly/oligopoly position and, in the extreme, moves close to a zero-profit competitive equilibrium.

3.4 Can Cash Hoarding Reveal Growth Prospects?

The theory we have developed so far is based on differences in vision between managers and financiers. A closely related concept is that of asymmetric information, which has often been used to motivate why firms hoard cash. While we do not dispute the role of information asymmetry as a motive for cash hoarding, in particular for precautionary reasons *before* a funding need arises (also derived in Section 4.1), our objective in this section is again to point to an effect omitted by the prior literature. In particular, we show that in a setting such as ours, in which a growth firm considers hoarding cash *after* the funding need has arisen, information asymmetry actually leads to less hoarding and shorter delays in investment. We also show that the internal-to-external financing mix in such firms could convey valuable information to financiers regarding the firm's growth prospects.

To introduce asymmetric information in our setting, we interpret θ as the privately known type of the firm (unknown to financiers). It is common knowledge that θ is drawn from a CDF F on $[\underline{\theta}, \overline{\theta}]$. Let $\widehat{\theta}$ be the financier's belief about the now unobservable type θ. In (3) we now have $\alpha = \frac{K-w}{\widehat{\theta} W^F}$ and (4)-(6) need to take into account $\widehat{\theta}$–i.e., we write $V\left(w_t,\widehat{\theta},\theta\right)$ and $U\left(w_t,w^*,\widehat{\theta},\theta\right)$. Our assumption that the manager has access to a competitive market for capital and can make a take-it-or-leave-it offer to the financier gives rise to a game of signaling, as the manager is privately informed about the firm's type.

A candidate for an equilibrium of the signaling game in which each type θ plays a pure strategy is a triple of functions (w_θ^*,μ^*,π), where w_θ^* is the cash level, which manager of type θ chooses to hoard and co-invests in the investment opportunity; μ^* is the financier's posterior belief, which maps w_θ^* into the set of probability distributions over the type set $\theta\in\left[\underline{\theta},\overline{\theta}\right]$; and π represents the financier's decision to finance the project, where $\pi: w_\theta^*\rightarrow[0,1]$, with $\pi=1$ corresponding to accepting and $\pi=0$ corresponding to rejecting the offer. Our equilibrium concept is that of a Perfect Bayesian Equilibrium. To deal with a potential multiplicity of equilibria, the equilibrium set is refined with D1. This standard refinement requires that, upon observing a deviation, the financier restricts his out-of-equilibrium beliefs only to the set of types who are most likely to have deviated.

Summarizing, the manager maximizes (5) subject to the condition that the proposed contract is individually rational for a financier who makes zero profit (i.e., analogously to (3), $\alpha=\frac{K-w}{\widehat{\theta}W^F}$) and who uses Bayes rule to form his posterior beliefs μ^* to draw an inference $\widehat{\theta}$ about the firm's type. Note that since the expected cash flow of the investment is linear in θ, we can use $\widehat{\theta}=\int_{\underline{\theta}}^{\overline{\theta}}\theta d\mu^*(\theta)$ to summarize the financier's beliefs about θ.

In a separating equilibrium of the resulting game, the proposed contract must additionally be incentive compatible. More formally, suppose that there is a monotonic differentiable function $w_\theta^*(\cdot)$ such that outside financiers believe that the firm of type θ exercises (invests) at $w_\theta^*(\theta)$. Then, if the manager decides to exercise at $\widehat{w}\in w_\theta^*\left(\left[\underline{\theta},\overline{\theta}\right]\right)$, outside financiers infer that the type is $w_\theta^{*-1}(\widehat{w})$ and the manager's expected payoff is

$$U(w_t,\widehat{w},w_\theta^{*-1}(\widehat{w}),\theta)=\left(\left(1-\frac{K-\widehat{w}}{w_\theta^{*-1}(\widehat{w})W^F}\right)\theta W^M-\widehat{w}\right)\left(\frac{w_t}{\widehat{w}}\right)^\beta,$$

which generalizes (6). Since the exercise decision must be on the optimal path, w_θ^* solves:

$$w_\theta^*=\arg\max_{\widehat{w}\in w^*\left(\left[\underline{\theta},\overline{\theta}\right]\right)}U(w_t,\widehat{w},w^{*-1}(\widehat{w}),\theta) \tag{11}$$

where, assuming that a separating equilibrium exists, we evaluate the respective FOC at $w^{*-1}(\widehat{w})=\theta$. Before analyzing whether a solution to this problem exists, we start by showing a useful result.

Lemma 2 *Single crossing holds because*

$$\frac{\partial}{\partial \theta}\left(-\frac{\frac{\partial}{\partial w^*}U}{\frac{\partial}{\partial \widehat{\theta}}U}\right) > 0, \tag{12}$$

where $\widehat{\theta}$ is the financier's inference about the firm's type θ.

The intuition for (12) is that, while hoarding helps to reduce the dependence on external financing, it is costly (as $r < \mu$) and firms with better investment opportunities face higher costs of delay than firms with worse investment opportunities. So at any level of hoarding and for any beliefs $\widehat{\theta}$, the better firms would gain more (or lose less) from reducing the hoarding. Hence, delaying is most costly for good types.[15] With the help of this result, we can now show that there is a unique separating equilibrium satisfying the standard equilibrium refinement D1.

Proposition 4 *(i) In a setting combining differences in vision and asymmetric information, there is a unique separating equilibrium in which the manager separates with the amount of cash she uses to co-finance her investment. In this equilibrium, higher types hoard less cash than lower types. Furthermore, firms hoard less cash and delay investment less than when asymmetric information is absent. (ii) There is no pooling equilibrium that survives D1.*

Information asymmetry leads to the same qualitative predictions on cash hoarding as Proposition 1. To see the intuition behind this result, recall that disagreement makes it optimal for the growth firm to delay investment and hoard cash, but firms with better investment opportunities would like to delay less. This is the key feature that makes separation possible, as firms with better investment opportunities can signal their types by hoarding less cash than firms with worse investment opportunities. To make the signal credible, they need to distort the optimal cash policies downwards (Lemma 2).

Part (ii) of Proposition 4 follows from the fact that a downward deviation from any pooling level of cash hoarding is always cheapest for the highest type by arguments similar to Lemma 2. Thus, by D1 this type would be able to credibly deviate, making it impossible to sustain a pooling equilibrium.

We conclude this section with a brief discussion of the implications of Proposition 4. An obvious one is that our results from Proposition 1 are robust to introducing asymmetric

[15] The single crossing property does *not* depend on our assumption that the manager's assessment of the firm's value is linear in θ. It is sufficient that the firm's value is increasing in θ.

information. This is important as empirical measures for disagreement and information asymmetry are often likely to coincide (Section 5). However, the results from this section can also be used to analyze how the internal-to-external financing mix can help explain announcement effects surrounding large investments.

Theoretically interesting is also the implication that, *after* the funding need has already arisen, asymmetric information regarding the quality of an investment opportunity does not lead by itself to hoarding. This is contrary to models in which the anticipation of a funding need marred by asymmetric information leads to precautionary cash hoarding. While asymmetric information in our model can go hand-in-hand with hoarding, where the need for hoarding is generated from some *other* friction, such as disagreement or incentive problems, the signaling need it generates leads, if anything, to less hoarding than what would otherwise be optimal. The latter point can be seen in a stark way in a version of our model, in which information asymmetry is the only financing friction. In such cases, cash hoarding is not an effective signaling device. Intuitively, a high type cannot distinguish herself from a lower type by not hoarding cash, as such a signal is not costly. However, high types also cannot signal their types by choosing a higher cash level than low types, as by similar arguments to Lemma 2, such a signal would easily be mimicked by lower types.[16]

4 Extensions and Robustness

In this section we discuss several extensions. First, we consider the possibility of cash hoarding prior to the arrival of the investment opportunity (Section 4.1). Section 4.2 discusses how cash hoarding and investment decisions are affected when the manager uses debt instead of equity financing. Section 4.3 analyzes dividends and share repurchases, and Section 4.4 extends our results to a setting in which the financing friction is an incentive problem ala Holmstrom and Tirole (1997). Section 4.5 discusses time-varying disagreement and expected profitability of the new project. Finally, Section 4.6 discusses how our setting compares to one with private benefits

[16]More precisely, there will be a semi-separating equilibrium in which positive NPV types hoard the minimum amount of cash that discourages negative NPV types from participating, and then raise financing at the average terms for types raising financing.

4.1 Cash Hoarding before Arrival of Investment Opportunity

Returning to our baseline vision-specification, we now extend our results to a setting in which at $t = 0$ the manager does not have an investment opportunity yet, but expects that such an opportunity may present itself at some future point in time. We assume that the time until this event follows an exponential distribution with parameter λ_a. The key insight from what follows is that our cross-sectional predictions from Section 3, which are the basis for our subsequent results, remain valid also when we incorporate the standard precautionary motive for cash hoarding.

As we have shown, when there are differences in vision between the manager and the financier, a manager who is already presented with an investment opportunity hoards w_θ^* before investing (Proposition 1). The manager, therefore, delays a profitable investment if her available cash at hand is below w_θ^*. To avoid such costly delay, she could start hoarding cash prior to the arrival of the investment opportunity. Thus, her decision problem before arrival is whether to pay out the available cash at hand or to set aside some cash and hoard more.

The advantage of paying out the available cash is that it can be invested more profitably outside the firm ($r > \mu$). However, setting aside cash and hoarding increases the value of the investment opportunity upon its arrival, and every additional dollar saved is more valuable than the previous one.[17] Thus, if the probability of arrival is sufficiently large, the manager sets aside all cash and continues hoarding until the investment opportunity arrives or until she has the necessary capital to invest immediately upon arrival without external financing. Since firms with better investment opportunities hoard less cash after arrival (Proposition 1), our overall result is again that in total—considering pre-arrival and post-arrival hoarding—firms with better prospects hoard less.[18]

Proposition 5 *(i) The manager hoards cash if the probability of arrival is sufficiently high. Otherwise, she pays out all initially available cash and does not hoard neither pre- nor post-arrival. (ii) If the manager chooses to hoard cash, then, prior to the arrival of the investment opportunity, she sets aside all her initial cash and continues to hoard until the investment opportunity arrives (or until she becomes independent of external financing). Upon arrival of the investment opportunity, the manager follows Propositions 1-4.*

[17] Not only does it allow to make the investment earlier, but it also reduces the time the already hoarded cash will remain locked-up in the firm prior to undertaking the investment–an effect which increases in that amount.

[18] We have also shown that cash hoardig before arrival of the investment opportunity is an ineffective signaling device in the presence of asymmetric information. The results are available on request.

4.2 Cash Hoarding, Net Worth, and Type of Financing

In the preceding analysis, we have assumed that the manager issues equity to finance the investment. This assumption is harmless in that it does not qualitatively change any of the preceding results, but—as we show—different types of securities may affect the level of cash hoarding. The optimal financing contract would minimize the friction coming from disagreement.

The underlying driver for the optimal financial contract in a setting with disagreement is to offer a contract whose value is least dependent on disagreement. From the manager's perspective, the financier should be least sensitive to whether the project turns out to be good (as the manager believes) or bad (as the financier suspects could be the case). This is reminiscent of the intuition in the earlier asymmetric information literature (e.g., Nachman and Noe, 1994), in which the manager prefers to issue debt since, as a less information-sensitive security, it minimizes underpricing. While this analogy is interesting, recall our argument that asymmetric information alone (absent disagreement) produces very limited insights on hoarding.

For the following proposition, we dispense with the assumption that the expected discounted cash flows take the simple form θW, and we model explicitly the cash flow generating process following investment. This analysis is relegated to the appendix.

Proposition 6 *The manager optimally hoards less cash to co-finance an investment if she has access to more debt financing.*

Being able to issue a security that makes the financier's payoff less sensitive to disagreement is intuitively similar to having less disagreement in the first place. Hence, all else equal, switching to such a security induces the manager to hoard less cash and raise more external financing for the investment. This is precisely the effect of having access to debt financing.

We conclude this subsection with a brief discussion of the alternative interpretation of w_t as net worth. In particular, we can interpret w_t as the part of the assets in place about which there is no disagreement and which is not subject to moral hazard or other inefficiencies. Apart from cash, this could be, for example, property, plant, and equipment, inventories, or accounts receivables from stable customers, which the firm accumulates over time. Combining the insights from Proposition 6 and Section 3, thus, implies that our results could be more generally interpreted in the context of the firm building up net worth to reduce its dependence on risky external financing, which is marred by various frictions.

4.3 Share Repurchases and Dividend Policy

Our analysis so far has concentrated on cash hoarding. A related question is whether our model has something to say about payout decisions in the form of dividends and share repurchases, which both are elements of cash policy. The answer is straightforward: For growth firms in need of funds to finance growth, payouts (which increase the funding need) should be minimized in both the vision-based and asymmetric information setups. That is, spending the available cash on share repurchases or dividend payments increases the firm's dependence on outside financing and is just costly. Thus, returning a dollar to shareholders in the form of repurchases or dividends is worth less to the manager than spending it on reducing this dependence.[19]

Observe also that with asymmetric information repurchases or dividends cannot help the firm signal its investment opportunities. Intuitively, for such a signaling mechanism to be credible, the manager must spend more on share repurchases (or dividends) than she receives from issuing new securities.[20]

Proposition 7 *If the manager needs external financing for new investments, and there are differences in vision or information asymmetry about the profitability of the new investment, the firm should optimally not pay out cash.*

4.4 Cash Hoarding and Incentives

An important issue for growth firms is that external financing, which lowers the owner-manager's stake in the firm, could reduce her incentives to exert effort, increasing further the cost of external financing (Holmstrom and Tirole, 1997). We, thus, compare the model structure based on differences in vision to one based on incentives problems. Our results are robust also in this context.[21]

Specifically, suppose that conditional on being undertaken, the investment succeeds with probability e, in which case it yields θW, and fails with probability $1 - e$, in which case it yields zero. The success probability reflects the effort exerted by the manager at cost $\frac{e^2}{2\nu}$ (assumed is that effort is undertaken after the investment). It is straightforward to show that there will be cash hoarding also in such a setting, and managers with better projects (high θ) have incentives to hoard less cash. Indeed, assuming again equity financing, the

[19] For a model of share repurchases in the presence of heterogenous beliefs in which the manager does not face a net funding need, see Bayar et al. (2013).

[20] Highlighting again the specificity of focusing on a firm whose investment opportunity is the most valuable asset, signaling may be possible if the information asymmetry is not about the firm's investment opportunities, but about its assets in place (Bhattacharya, 1979).

[21] We thank Andrey Malenko for suggesting this discussion.

manager's problem is to choose co-investment w_θ^* and the optimal level of effort $\widehat{e}$ that maximize her expected payoff

$$\left(\left(1-\frac{K-w_\theta^*}{e^*\theta W}\right)\widehat{e}\theta W - w_\theta^* - \frac{\widehat{e}^2}{2\nu}\right)\left(\frac{w}{w_\theta^*}\right)^\beta, \tag{13}$$

where e^* is the equilibrium level of effort anticipated by the financier.

The conceptual difference to our baseline setting is that increasing the level of co-investment (and, thus, cash hoarding) not only increases the manager's stake in the firm, but also increases the equilibrium level of effort, and so the expected size of the firm. We verify in the Appendix that this additional complexity does not alter our qualitative results.

4.5 Time-Varying Disagreement and Profitability

Our baseline specification assumes that the expected discounted cash flows from investment are constant and that differences in vision between the manager and financiers remain constant over time. However, one could argue that disagreement could vary and that the manager could try to time the market and raise financing when it is most beneficial to do so. Indeed, one can often hear of growth firms in practice that consider delay until financiers become convinced in the merits of the business strategy and the investment idea and in an attempt to time the market. Incorporating these features does not change our qualitative predictions.

One straightforward way to model change in disagreement is to assume that disagreement could fully disappear at any given instant with some positive probability, and remains otherwise unchanged. Specifically, suppose that the time until such an event follows an exponential distribution with parameter λ_ρ. If disagreement disappears, the manager invests immediately as external financing seizes to be costly, and her expected payoff is $\theta W^M - K$. While such feature creates another motive for delaying investment, our baseline results remain unchanged: Firms with better investment opportunities find it more costly to delay and, hence, hoard less cash.

The other issue is that the expected value of the investment opportunity could vary over time. Delay in investment could, then, occur for two reasons: delaying not only to hoard cash, but also to wait for the value of the investment opportunity to increase. Indeed, assuming that the NPV of the investment opportunity increases on average over time is a standard assumption in the related real options literature (e.g., Bolton et al., 2013). Our results remain robust also in such a setting. In particular, let the increase in NPV come from a lower investment outlay K. We now have that at any level of accumulated cash, a lower K implies a lower need for external financing. This reduction in the need for external funding implies that the firm is willing to invest at a lower level of accumulated cash.[22] However this doesn't change our previous insight that firms with better investment opportunities (high θ) delay and hoard less. We make all claims from this section more formal in the Appendix.

4.6 Private Benefits

We conclude the extension section with a brief discussion on how our assumption that the manager and financiers have different visions differs from a setting in which the difference in valuations is due to private benefits. Specifically, suppose that the manager and the financier agree that the expected cash flows are θW^M , but the manager additionally gains a private benefit B from investing that cannot be shared with outsiders. In such a setting, the net expected payoff of the manager at the moment of investment equals (cf. (4)):

$$\begin{aligned} U_B(w,\theta) &= \left(1-\frac{K-w}{\theta W^M}\right)\theta W^M + B - w \\ &= \theta W^M + B - K. \end{aligned} \tag{14}$$

Observe that (14) does not depend on the amount of co-investment w. What this means is that hoarding cash to reduce the dependence on external financing is useless for the manager. Intuitively, both parties value the expected cash flows θW^M in the same way, and a larger co-investment w does not change the fact that the private benefit B accrues only to the manager. This is the crucial difference to the case with disagreement. There, the manager's motive for co-investing w (and, hence, hoarding) is that she has a higher valuation of the cash flows than the financier ($\theta W^F < \theta W^M$).[23]

The insight that private benefits do not lead to hoarding has actually been recognized in the recent empirical literature analyzing Jensen's (1986) free cash flow hypothesis. This literature claims that firms, suffering from the private benefits problem, have less cash as they spend their free cash flow more quickly on new investments (e.g., Dittmar and

[22] Observe that even though now there are two separate reasons to delay, the firm will not necessarily delay investment longer since a lower investment outlay K implies also a lower need for hoarding cash. Furthermore, note that this insight does not depend on our assumption that the increase of NPV is due to K. The same result would obtain if the NPV would increase because of higher expected cash flows.

[23] There could be a motive for cash hoarding only if the manager cannot raise financing even if she pledges all expected cash flows to the financier–i.e., if $\theta W^M < K - w_0$. In such a case, the manager will either not invest at all or will hoard cash until she can raise sufficient financing by pledging all cash flows to the financier; that is, essentially selling the firm.

Mahrt-Smith, 2007; Harford et al., 2008).

5 Empirical Implications

In what follows, we discuss several empirical implications of our model. We discuss in turn predictions related to the links between cash hoarding, the firm's investment opportunities and performance, the differences in terms of hoarding we expect between public and private firms and the effect of competition, the role played by the degree of disagreement ρ, and the impact of the type of financing instruments on hoarding.

Cash Hoarding, Investment Performance, and Announcement Effects The free cash flow theory of Jensen (1986) predicts that managers may invest in negative NPV projects rather than distributing cash to shareholders. Yet the empirical evidence on the relation between cash holdings and agency problems is rather mixed. On the negative side, Dittmar and Mahrt-Smith (2007) and Pinkowitz et al. (2006) show that cash is worth less when agency problems between inside and outside shareholders are greater, and Harford (1999) finds that cash-rich acquirers fare worse in takeovers. However, Opler et al. (1999) and Bates et al. (2009) are not able to support the importance of agency explanations in their respective studies. In particular, they show that firms with high Tobin's q hoard more cash than firms with low Tobin's q. Apparently, hoarding cash goes hand in hand with having a high valuation.

Though our theory is focused on growth firms, it could still shed some light on the above evidence, as the contradictions seem to stem from smaller growth firms. Only firms with growth prospects (growth options) will hoard cash in our model. Firms without investment opportunities will not do so. So having cash goes hand in hand with growth prospects and, hence, a high q as documented in Opler et al. (1999) and Bates et al. (2009). However, among the firms that have growth prospects, those with better investment opportunities (and hence better performance) will hoard less cash. This is in line with Harford's (1999) finding that cash-rich firms perform worse in takeovers, but is still consistent with the overall result that growth opportunities (and hence q) go hand in hand with cash. Hence, our model complements existing explanations by establishing a relation between cash hoarding policy, growth options, and performance. Our first main prediction is:

Implication 1: *(i) Growth firms hoard more cash than firms without growth prospects. (ii) From the subset of firms with growth prospects, firms with lower cash-to-assets ratios* *are more profitable than firms with higher ratios (i.e., more profitable opportunities go hand in hand with less hoarding).*[24]

A closely related implication of our model concerns the mix of internal and external financing used for new investments in growth firms.

Implication 2: *Growth firms financing new investment opportunities with a higher fraction of external to internal funds will exhibit stronger revenue, profit, and asset growth.*

Interestingly, we find strong preliminary empirical support for Implications 1 and 2 when testing them using the KFS survey panel data, which tracks over 4000 start-up firms over the first eight years of their existence. This panel, which has an emphasis on growth firms, is described in detail in Robb and Robinson (2014).[25]

Our results complement precautionary based explanations (e.g., Bolton et al., 2011) by establishing a relation between the profitability of a firm's investment and its choice between internal and external financing (Proposition 5). An interesting insight from these results is that there is a self-reinforcing mechanism in which firms with more profitable opportunities also invest and grow more quickly. While our model has nothing to say about the use of cash in mature cash-rich firms, a novel implication is that firms that end up being cash-rich as a sign of their success, pursue a low-cash strategy in their growth phase.

Implication 3: *Growth firms with good investment opportunities hoard little cash while still in their active growth phase, even though, due to their success, they may end up with large cash holdings as they mature.*

Our analysis of asymmetric information implies that cash policy can signal the quality of the firms' investment opportunities. Thus, another implication of our basic insight that firms hoard less cash and use less internal financing if their investment opportunities are better is that the stock price reaction to new investments financed with a higher proportion of external financing should be more positive. The following implications have not been tested to our knowledge.

Implication 4: *The stock price reaction to major investments by growth firms should be more positive if a higher fraction of these investments is financed from external sources.*

Our model could also be interpreted in the context of a firm expanding through acquisitions. Based on Implications 1 and 4, we expect:

Implication 5: *(i) Growth firms expanding through acquisitions in the early stages of a merger wave should have better long-run performance. (ii) The stock price reaction of*

[24] The cash-to-assets-ratio decreases in the attractiveness of the opportunities (measured by the parameter θ), as the optimal cash level is decreasing in θ, while the value of the firm increases in θ.

[25] Our regression results are available upon request.

acquirers financing a higher proportion of their cash *payments in takeovers from external sources should be higher.*

Observe that the second part of the preceding implication is *not* related to the method of payment in takeovers, but to how the cash payment is financed.[26] Indeed, in the majority of takeovers paid in cash, the cash payment is externally financed (e.g., Martynova and Renneboog, 2009). For these cases, we predict that high-growth acquirers operating in more disagreement-prone industries (e.g., R&D intensive) that finance their *cash* payments in takeovers with a higher proportion of outside financing will do better.

Cash in Public vs. Private Firms and Effect of Competition Our theory also makes novel predictions regarding the cash holdings in public and private firms. In contrast to precautionary-based theories, which would predict that private firms should hoard more cash as they are more financially constrained, we show that private firms could actually hoard less cash and, equally important, delay investment less (cf. Proposition 2). To show this, we take into account that growth firms self-select whether they want to remain private or public. The following implication focuses on firms that have the necessary scale to operate both as a public and a private firm.

Implication 6: *A growth firm operating as a publicly held firm hoards more cash and delays investment less than when it chooses to be private.*

This surprising prediction finds strong support in a recent empirical study by Gao et al. (2013). They show that public firms hoard up to twice as much cash as comparable private firms. Furthermore, Asker et al. (2013) find that private firms not only have more cash, but they also invest more, have higher investment sensitivity, and have a higher return on assets. These findings are supportive of our prediction that growth firms with better investment opportunities remain private and invest more quickly as they hoard less cash to co-finance their investments.

Another novel insight of our paper concerns the differential effect of competition on cash hoarding. Indeed, recent evidence is at odds with standard theory predicting a monotonic effect between competition and investment timing. Akdogu and MacKay (2008) show that investment sensitivities to opening of growth prospects are lower in competitive than oligopolistic industries. However, this sensitivity is higher in oligopolistic relative to monopolistic industries. Our theory could help explain how growth firms contribute to such conflicting evidence. Specifically, Proposition 3 implies that firms likely to experience

[26] A large body of literature has investigated the method of payment in takeovers (for a recent survey see Betton et al., 2008). It has typically found that bidder announcement returns are higher when bidders pay in cash rather than equity.

a strong erosion of profits (as in competitive industries) will delay investment and hoard cash, while firms likely to keep high profits (as in an oligopoly) will speed up investment relative to the monopoly case.

Implication 7: *Growth firms that expect that a first-mover advantage is likely to quickly erode in the face of competition, will delay investment and hoard more cash. In contrast, firms that expect their first-mover advantage to last will hoard less cash and delay investment less to secure this advantage.*

Cash Hoarding and Degree of Disagreement The main focus of our implications above was that firms will hoard cash if disagreement plays a role and alignment is difficult. However, the degree of disagreement is a separate factor affecting cash hoarding (Proposition 1). Controlling for it is imperative, as a firm with better investment opportunities, but facing very high disagreement, could end up hoarding more cash than a firm with poorer investment opportunities, but for which disagreement is less of an issue. At this point, it is important to recall our discussion in Section 2 that the predictions of our model are *not* reversed if one considers the flip-side of the case we discuss—i.e., when managers are more pessimistic than financiers (see also footnote 9).

Implication 8: *Growth firms facing more proxy fights, firms for which there is a higher dispersion of analyst forecasts, or whose non-voting shares trade at a larger discount relative to the voting shares, hoard more cash for new investments. This is also true for growth firms investing into unfamiliar businesses, more R&D intensive firms, or firms that may be reluctant to disclose sensitive information.*[27]

Bates et al. (2009) hypothesize that R&D intensive firms hoard more cash, since adverse shocks and financial distress are more costly in the presence of growth opportunities (see also Opler et al., 1999). They find this relationship despite controlling for other measures of growth opportunities, such as Tobin's q. Our analysis can help explain this: For the same growth opportunities (e.g., controlled for by Tobin's q), R&D intensive firms hoard more cash, because in such firms differences in vision–and, hence, disagreement–between managers and their financiers will be more pronounced.

As the nature of growth firms changes across the board, we also predict that:

Implication 9: *As the composition of growth firms in the economy changes towards firms for which differences in vision and disagreement between entrepreneurs/managers and financiers are more likely, the aggregate cash holdings in the economy will increase.*

Indeed, Bates et al. (2009) argue that the reason for more cash holdings in the economy

[27]Recall that our cross-sectional results continue to hold when disagreement and information asymmetry are jointly present (Proposition 4), as it is likely to be the case in practice.

is that the nature of firms has changed. In particular, they find that newly listed firms are more R&D intensive. Furthermore, Falato et al. (2013) find that firms have shifted towards holding more intangible assets. They argue that firms hoard more cash, because intangibles cannot be offered as collateral when raising debt.[28] Our model complements this argument. Since there is arguably more disagreement about the value of intangibles, our theory helps explain why growth firms choose to depend less on outside financing *in general*, potentially delaying investment, as opposed to shifting towards more equity issuance.

Cash, Type of Financing, and Payout Policy in Growth Firms Our model also predicts that the type of financing that firms have access to will affect how much cash they need to hoard for new investments.

Implication 10: *Growth firms with higher debt capacity hoard less cash.*

Finally, our model predicts that returning cash to shareholders is justifiable only if it is not needed for new investments (cf. Proposition 7). Indeed, a number of empirical studies have shown that the market reaction to dividend announcements is mixed (DeAngelo et al., 1996, 2000), implying that they are only a weak instrument for signaling quality. Similarly, the market does not react strongly to share repurchases announcements by firms even if they subsequently outperform (Ikenberry et al., 1995), which is also in line with our results that repurchases are not an effective instrument to signal growth opportunities (Proposition 7).

Implication 11: *Returning cash to shareholder is costly and is not an effective signal of growth prospects in growth firms that need the cash for new investments.*

6 Conclusion

In a simple dynamic theory of optimal cash hoarding, we analyze whether a growth firm will choose to delay investments in order to hoard cash and depend less on outside finance. Entrepreneurs try to avoid external financing in our model, because they are reluctant to see their stake diluted. Our starting point is to show that firms with better investment opportunities hoard less cash and finance a higher fraction of new investments with outside financing. The key reason is that they find it more costly to delay a more profitable opportunity. This is true despite the fact that their relative benefit of retaining a higher stake in the firm is also higher. Expanding on this simple insight, we show a number

[28] In their model, firms do not switch to equity, because the issuance cost are prohibitively large, reducing the value of existing shareholders by more than the notional amount of equity issuance.

of novel results, which question the extent to which standard arguments developed for mature firms apply to growth firms with a large funding need.

One of our main novel insights is that growth firms that choose to operate as private firms hoard less cash and invest more quickly than they would if they operated as public firms. This is because firms choose to be private when they value the benefit from a higher alignment with financiers—the benefit of being private—more than the costs associated with offering less liquid securities. That is, firms choose to be private when their investment opportunities are better. Then, they prefer financing a new investment with more external financing and less internally generated cash. This surprising prediction finds support in recent empirical work.

Another novel prediction is that competition could lead to longer delays and hoarding, which is contrary to what one would expect from prior theory. The reason is that delaying investment in our model serves a different purpose than usually discussed—not one of waiting for the investment opportunity to improve (though we also discuss this one), but of bridging a large funding need with more internal financing, which helps to reduce dilution. In such a setting, the standard effect discussed in the literature is still present: Firms have an incentive to speed up investment not to lose their first-mover-advantage. However, there is a countervailing effect, which could easily dominate: If the entry of a competitor is likely to erode the first-mover-advantage, investment is less lucrative, changing the benefit of cash hoarding relative to the cost of delay in favor of hoarding.

Another insight from our model is that asymmetric information actually leads to less hoarding once the firm is confronted with an investment opportunity and a funding need. This is because cash policy conveys a signal about the firm's prospects, which drives firms to choose suboptimally low cash hoarding in order to signal a positive investment opportunity, which they are reluctant to delay. This insight holds true even when considering cash hoarding for precautionary reasons. Thus, at a more basic level, another contribution of our paper is to better understand the effects of different financing frictions on cash hoarding. We derive our main results based on the assumption that entrepreneurs/managers have a different vision than financiers and worry not only about the dilution of their stake, but also their incentives. We believe this aspect to be particularly interesting, because the relevance of differences in vision and disagreement has mushroomed in the modern era in which structural shifts–e.g. developments in information technology–impact growth firms and industries, yet for now leave substantial uncertainty about viable future business models. In this environment, choices have to be made that are far from routine, more visionary, trial-and-error based, and hence prone to disagreement.

Our results also provide new insights on the dynamics of firm development and cash holdings (as opposed to hoarding). Our analysis focuses on growth firms that are short on cash and operate in an uncertain environment. The ones with the best investment opportunities will choose to grow rapidly using outside funding, and relative to their lesser peers will be cash-poor. However, on average, they will be more profitable and successful. This implies that in the follow-up stage after they have established themselves—and possibly some uncertainty has been resolved—they may start earning cash at a higher rate than needed for investment and growth. High cash holdings are then a sign of past success. This would imply that growth firms striving to become the next Google, Microsoft, or Apple should not try to copy the large cash holdings of these already mature firms, as *growth* firms with great investment opportunities should not hoard too much cash.[29]

The analysis in this paper could also help understand and explain broader issues. Our theory has some interesting implications for the industrial development in the uncertain disagreement-prone, visionary environment. An important implication of our analysis is that firms with better opportunities are not just better but also invest sooner (no delays in investment). This creates reinforcing effects allowing successful firms to differentiate themselves even faster. The result resembles an accelerated Darwinian survival process with 'winners taking it all'.

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Appendix A: Omitted Proofs

Proof of Proposition 1. Differentiating (6) with respect to w^* we obtain

$$\frac{\partial}{\partial w^*}U(w_t,w^*,\theta)=\left(-\frac{\beta}{w^*}\left(\left(1-\frac{K-w^*}{\theta W^F}\right)\theta W^M-w^*\right)+\left(\frac{\theta W^M}{\theta W^F}-1\right)\right)\left(\frac{w_t}{w^*}\right)^{\beta}. \quad (15)$$

The first term shows the time-value-loss of waiting for w to increase, while the second term shows the benefit from obtaining cheaper financing when increasing the co-investment. Note that if $\theta W^F > K$, the right-hand-side (RHS) is negative and hoarding cash is never optimal. Hence, the manager only hoards cash if the disagreement with the financier is sufficiently strong and $\theta W^F < K$. Then, the first order condition (FOC) yields:

$$w^*=\frac{\beta}{\beta-1}\frac{\left(K-\theta W^F\right)}{\left(\theta W^M-\theta W^F\right)}\theta W^M.$$

The second order condition is

$$\begin{aligned}\frac{\partial^2}{\partial\left(w^*\right)^2}U(w_t,w^*,\theta) &= \frac{\beta}{\left(w^*\right)^2}\left(\frac{\theta W^F-K}{\theta W^F}\right)\theta W^M\left(\frac{w_t}{w^*}\right)^{\beta} \\ &\quad -\frac{\beta}{w^*}\left(-\frac{\beta}{w^*}\left(\left(1-\frac{K-w^*}{\theta W^F}\right)\theta W^M-w^*\right)+\left(\frac{\theta W^M}{\theta W^F}-1\right)\right)\left(\frac{w_t}{w^*}\right)^{\beta}.\end{aligned}$$

At the internal optimum (when the FOC holds), which is the case when $\theta W^F < K$, the second line on the RHS is zero, and the expression is negative. Clearly, if the initial cash at hand is $w_0 > w^*$, the manager invests immediately. **Q.E.D.**

Proof of Proposition 2. (i) If a firm chooses to remain private, then a *necessary* condition for this to be more beneficial is that

$$\begin{aligned}
0 &< U^{priv}(w_t, w_\theta^{priv}, \theta) - U^{pub}(w_t, w_\theta^{pub}, \theta) \\
&= \left(\left(1 - \frac{K + L - w_\theta^{priv}}{\theta W^F\left(\rho^{priv}\right)}\right)\theta W^M - w_\theta^{priv}\right)\left(\frac{w_t}{w_\theta^{priv}}\right)^\beta \\
&\quad - \left(\left(1 - \frac{K - w_\theta^{pub}}{\theta W^F\left(\rho^{pub}\right)}\right)\theta W^M - w_\theta^{pub}\right)\left(\frac{w_t}{w_\theta^{pub}}\right)^\beta \\
&= \left(\frac{w_t}{w_\theta^{priv}}\right)^\beta \frac{\theta W^M}{(\beta-1)} \frac{K - \theta W^F\left(\rho^{pub}\right)}{\theta W^F\left(\rho^{pub}\right)}\left(\frac{w_\theta^{priv}}{w_\theta^{pub}}\right) \qquad (16) \\
&\quad \times \left(\frac{\left(\theta W^M - \theta W^F\left(\rho^{priv}\right)\right)}{\left(\theta W^M - \theta W^F\left(\rho^{pub}\right)\right)}\frac{\theta W^F\left(\rho^{pub}\right)}{\theta W^F\left(\rho^{priv}\right)} - \left(\frac{w_\theta^{priv}}{w_\theta^{pub}}\right)^{\beta-1}\right) \\
&< \left(\frac{w_t}{w_\theta^{priv}}\right)^\beta \frac{\theta W^M}{(\beta-1)} \frac{K - \theta W^F\left(\rho^{pub}\right)}{\theta W^F\left(\rho^{pub}\right)}\left(\frac{w_\theta^{priv}}{w_\theta^{pub}}\right)\left(1 - \left(\frac{w_\theta^{priv}}{w_\theta^{pub}}\right)^{\beta-1}\right) \qquad (17)
\end{aligned}$$

where for the second equality (16) we plug in for w_θ^{priv} and w_θ^{priv} from (9) and (10). The inequality in the last line (17) follows from the fact that $W^F\left(\rho^{pub}\right) < W^F\left(\rho^{priv}\right)$, implying that also the term $\frac{\left(\theta W^M - \theta W^F\left(\rho^{priv}\right)\right)}{\left(\theta W^M - \theta W^F\left(\rho^{pub}\right)\right)}\frac{\theta W^F\left(\rho^{pub}\right)}{\theta W^F\left(\rho^{priv}\right)}$ in (16) is less than one. Thus, a necessary condition is that the last line (17) is positive, which is true only if the last term $1 - \left(\frac{w_\theta^{priv}}{w_\theta^{pub}}\right)^{\beta-1}$ is positive, implying that $w_\theta^{priv} < w_\theta^{pub}$.

(ii) From (16), the manager is just indifferent between private and public ownership if

$$\frac{W^M - W^F\left(\rho^{priv}\right)}{W^M - W^F\left(\rho^{pub}\right)}\frac{W^F\left(\rho^{pub}\right)}{W^F\left(\rho^{priv}\right)} = \left(\frac{K + L - \theta W^F\left(\rho^{priv}\right)}{K - \theta W^F\left(\rho^{pub}\right)}\frac{W^M - W^F\left(\rho^{pub}\right)}{W^M - W^F\left(\rho^{priv}\right)}\right)^{\beta-1} \qquad (18)$$

where we have pugged in for w_θ^{priv} and w_θ^{pub} from (9) and (10). Observe now that the manager strictly prefers staying private as $W^F\left(\rho^{priv}\right) > W^F\left(\rho^{pub}\right)$ for $L = 0$. Furthermore, the LHS of (18) is a constant, while the RHS increases in L. Thus, there is a unique threshold $\overline{L}_\theta$, above which the manager prefers going public.

We show next that the indifference threshold $\overline{L}_\theta$ must be increasing in θ. Observe that the LHS of (18) is bounded between zero and one, while the second term on the RHS is greater than one. Hence, the first term on the RHS must be less than one, implying that $\overline{L}_\theta < \theta \left(W^F\left(\rho^{priv}\right) - W^F\left(\rho^{pub}\right)\right)$. Furthermore, at $L = \overline{L}_\theta$, the LHS is constant in L and θ, while the RHS increases in L and decreases in θ as:

$$\begin{aligned}\frac{\partial}{\partial\theta}\left(\frac{K+\overline{L}_\theta - \theta W^F\left(\rho^{priv}\right)}{K-\theta W^F\left(\rho^{pub}\right)}\right) &= \frac{K\left(W^F\left(\rho^{pub}\right) - W^F\left(\rho^{priv}\right) + W^F\left(\rho^{pub}\right)\frac{\overline{L}_\theta}{K}\right)}{\left(K-\theta W^F\left(\rho_{pub}\right)\right)^2} \\ &< \frac{W^F\left(\rho^{pub}\right) - W^F\left(\rho^{priv}\right)}{K-\theta W^F\left(\rho^{pub}\right)} < 0\end{aligned}$$

where we have used that $\overline{L}_\theta < \theta\left(W^F\left(\rho^{priv}\right) - W^F\left(\rho^{pub}\right)\right)$. Thus, differentiating the RHS of (18) with respect to $\overline{L}_\theta$ and θ, yields the result. **Q.E.D.**

Proof of Proposition 3. After a competitor arrives and becomes the first-mover, the manager's expected payoff can be derived analogously to Proposition 1 as

$$\overline{U}\left(w, w^*\right) = \frac{1}{\beta - 1}\left(K - \pi\theta W^F\right)\frac{W^M}{W^F}\left(\frac{w}{w^*}\right)^\beta. \tag{19}$$

Thus, before the arrival of a competitor, the manager's problem is to choose the cash hoarding threshold w^*_{FM} that solves

$$r\Pi = \mu w \frac{\partial \Pi}{\partial w} + \frac{1}{2}\sigma^2 w^2 \frac{\partial^2 \Pi}{\partial w^2} + \lambda\left(\overline{U}\left(w, w^*_\theta\right) - \Pi\right). \tag{20}$$

Define β_1 as the positive root of $(r+\lambda) - \mu y - \frac{1}{2}\sigma^2 y\left(y-1\right) = 0$ and observe that $\frac{\partial}{\partial\lambda}\beta_1 > 0$, implying that $\beta_1 > \beta$. Following similar arguments to above, we can show that

$$\Pi = \frac{\lambda}{r}\overline{U}\left(w, w^*\right) + \left(\left(1 - \frac{K - w^*_{FM}}{\xi\pi\theta W^F}\right)\xi\pi\theta W^M - w^*_{FM} - \frac{\lambda}{r}U\left(w^*_{FM}, w^*\right)\right)\left(\frac{w}{w^*_{FM}}\right)^{\beta_1}$$

where, plugging in for (19), w^*_{FM} is the solution to the first-order-condition

$$\left(\begin{array}{c} -\frac{\beta_1}{w^*_{FM}}\left(\left(1 - \frac{K - w^*_{FM}}{\xi\pi\theta W^F}\right)\theta W^M - w^*_{FM}\right) \\ +\frac{\beta_1 - \beta}{w^*_{FM}}\frac{\lambda}{r}U\left(w^*_{FM}, w^*\right) + \frac{\theta W^M}{\theta W^F} - 1\end{array}\right)\left(\frac{w_t}{w^*_{FM}}\right)^{\beta_1} = 0.$$

By standard monotone comparative statics arguments, the dependence of w^*_{FM} on λ is given by the cross partial $\frac{\partial^2 \Pi}{\partial w^*_{FM} \partial \lambda}$, which is

$$\frac{1}{w^*_{FM}}\left(-\frac{\partial \beta_1}{\partial \lambda}\left(\begin{array}{c}\left(1-\frac{K-w^*_{FM}}{\xi \pi \theta W^F}\right)\theta W^M \\ -w^*_{FM}-\frac{\lambda}{r}U\left(w^*_{FM}, w^*\right)\end{array}\right)-\frac{\beta}{r}U\left(w^*_{FM}, w^*\right)\right)\left(\frac{w_t}{w^*}\right)^{\beta_1}$$
$$+\left(-\frac{\beta_1}{w^*_{FM}}\frac{K-w^*_{FM}}{\xi \pi^2 \theta W^F}\theta W^M-\frac{\beta_1-\beta}{\beta-1}\frac{\lambda}{r}\theta W^F\frac{W^M}{W^F}\left(\frac{w^*_{FM}}{w^*}\right)^{\beta}\right)\frac{\partial \pi}{\partial \lambda}\left(\frac{w_t}{w^*_{FM}}\right)^{\beta_1}.$$

The first line of this expression reflects the firm's cost due to higher competition, reducing the likelihood of being a first-mover. This line is negative as $\frac{\partial \beta_1}{\partial \lambda} > 0$—i.e., this cost calls for speeding up investment. The second line reflects the decrease in expected payoffs due to higher competition. This line is positive as $\beta_1 > \beta$ and $\frac{\partial \pi}{\partial \lambda} < 0$—i.e., this calls for delaying investment and hoarding cash. In general, either effect could dominate. However, it is clear that if there is a threshold for $\frac{\partial \pi}{\partial \lambda}$ such that below this threshold the positive effect dominates the negative. **Q.E.D.**

Proof of Lemma 2. Let for this proof $W^M(\theta)$ and $W^F(\theta)$ denote more generically the manager's and the financier's assessment of the firm's value after investment (as a function of θ), where in the main text we have $W^F(\theta) = \theta W^F$ and $W^M(\theta) = \theta W^M$. Single crossing holds as

$$\begin{aligned}
& \frac{\partial}{\partial \theta}\left(-\frac{\frac{\partial}{\partial w^*}U\left(w_t, w^*, \widehat{\theta}, \theta\right)}{\frac{\partial}{\partial \widehat{\theta}}U\left(w_t, w^*, \widehat{\theta}, \theta\right)}\right) \\
= & \frac{\partial}{\partial \theta}\left(-\frac{\left(-\frac{\beta}{w^*}\left(\left(1-\frac{K-w^*}{W^F(\widehat{\theta})}\right)W^M(\theta)-w^*\right)+\frac{W^M(\theta)}{W^F(\widehat{\theta})}-1\right)\left(\frac{w_t}{w^*}\right)^{\beta}}{\frac{K-w^*}{\left(W^F(\widehat{\theta})\right)^2}\frac{\partial}{\partial \widehat{\theta}}W^F(\widehat{\theta})W^M(\theta)\left(\frac{w_t}{w^*}\right)^{\beta}}\right) \\
= & \frac{1}{\frac{K-w^*}{\left(W^F(\widehat{\theta})\right)^2}\frac{\partial}{\partial \widehat{\theta}}W^F(\widehat{\theta})}\left(\frac{(\beta-1)\frac{\partial}{\partial \theta}W^M(\theta)}{\left(W^M(\theta)\right)^2}\right) > 0. \qquad (21)
\end{aligned}$$

Thus, as claimed in footnote 15, for single crossing it is sufficient that the manager's and the financiers' assessments of firm value are increasing in θ (and not necessarily linear in θ). **Q.E.D.**

Proof of Proposition 4. We show first the existence of a separating equilibrium. Then we show that there is no pooling equilibrium that survives D1.

Claim 1. *There is a unique separating equilibrium*

To show existence of a separating equilibrium, we follow standard arguments.[30] Rewriting (11), we obtain

$$w_\theta^* = \arg \max_{\widehat{w} \in w^*([\underline{\theta},\overline{\theta}])} \left(\left(1 - \frac{K - \widehat{w}}{w^{*-1}(\widehat{w}) W^F} \right) \theta W^M - \widehat{w} \right) \left(\frac{w_t}{\widehat{w}} \right)^\beta$$

Taking the FOC and assuming that a separating equilibrium exists–i.e., $w^{*-1}(\widehat{w}) = \theta$–we have

$$\frac{dw_\theta^*}{d\widehat{\theta}} = \frac{-\frac{\partial}{\partial \widehat{\theta}} U(w_t, w_{\widehat{\theta}}^*, \widehat{\theta}, \theta)|_{\widehat{\theta}=\theta}}{-\frac{\beta}{w_\theta^*} \left(\left(1 - \frac{K - w_\theta^*}{\widehat{\theta} W^F} \right) \theta W^M - w_\theta^* \right) \left(\frac{w_t}{w_\theta^*} \right)^\beta + \left(\frac{\theta W^M}{\widehat{\theta} W^F} - 1 \right) \left(\frac{w_t}{w_\theta^*} \right)^\beta} . \tag{22}$$

To solve this equation we need the appropriate boundary condition. Since a high type has no incentive to mimic low types, we can set: $w_{\underline{\theta}}^* = w_{\underline{\theta}}^{VM}$, where w_θ^{VM} is given by expression (7). We can now apply Theorems 1-3 from Mailath (1987) to prove the proposition (in Appendix B we verify that the conditions for these theorems are satisfied). From these theorems it follows that there is a unique separating equilibrium in which w_θ^* is continuous and differentiable, satisfies (22), and $\frac{dw_\theta^*}{d\theta} < 0$ ($\frac{dw_\theta^*}{d\theta}$ has the same sign as $\frac{\partial^2}{\partial \widehat{w} \partial \theta} U(w_t, \widehat{w}, \widehat{\theta}, \theta)$).

We now show that $w_\theta^* < w_\theta^{VM}$. To see this, rewrite (22) as

$$\begin{aligned} & -\frac{\beta}{w_\theta^*} \left(\left(1 - \frac{K - w_\theta^*}{w^{*-1}(\widehat{w}) W^F} \right) \theta W^M - w_\theta^* \right) \left(\frac{w_t}{w_\theta^*} \right)^\beta + \left(\frac{\theta W^M}{w^{*-1}(\widehat{w}) W^F} - 1 \right) \left(\frac{w_t}{w_\theta^*} \right)^\beta \\ = & \frac{-\frac{\partial}{\partial \widehat{\theta}} U(w_t, w_{\widehat{\theta}}^*, \widehat{\theta}, \theta)}{\frac{dw_\theta^*}{d\widehat{\theta}}} . \end{aligned} \tag{23}$$

Compare (23) to the optimality condition (15) in Proposition 1. The RHS of (23) is positive, while it is zero absent information asymmetry. Thus, taking into account that the LHS decreases in w_θ^*, we must have $w_\theta^* < w_\theta^{VM}$.

Claim 2. *There is no pooling equilibrium that survives D1*

Suppose that there is a pooling equilibrium in which all types pool at a cash level $w^P > w_0$. $\widehat{\theta}$ is then simply $\widehat{\theta} = \int_{\underline{\theta}}^{\overline{\theta}} \theta dF(\theta)$. We start by defining D1 in the context of this game. For use below, note that finding the most expensive financing contract (i.e., financier's response) $\widetilde{\alpha}(\theta)$ for which type θ is willing to deviate is equivalent to finding the worst out-of-equilibrium beliefs $\widetilde{\theta}(\theta)$ for which the financier still breaks even (i.e.,

[30] For a detailed general analysis on this point, see Mailath (1987) and for separation in the context of real options see Grenadier and Malenko (2011) and Morellec and Schürhoff (2011).

$\widetilde{\alpha} = \frac{K-w}{\widetilde{\theta}W}$) and for which the manager is willing to deviate.

Definition 1 *For every deviation $\widetilde{w}$, determine for every type the most "expensive" financing contract $\widetilde{\alpha}(\theta)$, respectively the worst out-of-equilibrium beliefs $\widetilde{\theta}(\theta)$, for which the deviation payoff $\widetilde{U}\left(w_t, \widetilde{w}, \widetilde{\theta}, \theta\right)$ is higher than the equilibrium expected payoff $U\left(w_t, w^P, \widehat{\theta}, \theta\right)$*

$$\widetilde{\theta}(\theta) = \arg\min_{\widetilde{\widetilde{\theta}}} \left\{ \widetilde{U}\left(w_t, \widetilde{w}, \widetilde{\widetilde{\theta}}, \theta\right) | \widetilde{U}\left(w_t, \widetilde{w}, \widetilde{\widetilde{\theta}}, \theta\right) \geq U\left(w_t, w^P, \widehat{\theta}, \theta\right) \right\}$$

Then, D1 requires that the financier believe that the deviation comes from the types who find $\widetilde{w}$ attractive for the most expensive contract, respectively for the worst out-of-equilibrium beliefs $\Theta \in \arg\min_\theta \widetilde{\theta}(\theta)$.

Suppose that we observe a downward deviation from w^P. In what follows we show that the type most likely to have deviated is the highest type. Observe, first, that when the financier breaks even, there is a type $\theta' \in \left(\underline{\theta}, \overline{\theta}\right)$ for whom w^P coincides with w_θ^{VM}, implying that

$$\frac{\partial}{\partial w^P}\widetilde{U}\left(w_t, w^P, \widetilde{\theta}, \theta\right) \begin{cases} < 0 \text{ for } \theta > \theta' \\ = 0 \text{ for } \theta = \theta' \\ > 0 \text{ for } \theta < \theta' \end{cases} \implies \frac{d\widetilde{\theta}}{dw^P} = -\frac{\frac{\partial}{\partial w^P}\widetilde{U}\left(w_t, w^P, \widetilde{\theta}, \theta\right)}{\frac{\partial}{\partial \widetilde{\theta}}\widetilde{U}\left(w_t, w^P, \widetilde{\theta}, \theta\right)} \begin{cases} > 0 \text{ for } \theta > \theta' \\ = 0 \text{ for } \theta = \theta'. \\ < 0 \text{ for } \theta < \theta' \end{cases}$$

Hence, to keep the same utility as on the equilibrium path following a *decrease* from w^P, we have to decrease $\widetilde{\theta}$ for $\theta > \theta'$. Moreover, since the marginal rate of substitution $\frac{d\widetilde{\theta}}{dw^P}$ increases in θ (analogously to Lemma 2), the change in $\widetilde{\theta}$ must be highest for the highest type. Hence, the higher the type, the higher the decrease in $\widetilde{\theta}$ (and so the higher $\widetilde{\alpha}(\theta)$) that the manager is prepared to tolerate following a deviation to $\widetilde{w} < w^P$.

We can, thus, construct a deviation contract $(\widetilde{\alpha}, \widetilde{w})$ with $\widetilde{w} < w^P$, such that only types $(\theta'', \overline{\theta}]$ (where $\theta'' \to \overline{\theta}$) find it profitable to deviate relative to their expected payoff on the equilibrium path, and such that the financier makes a strictly positive expected profit when accepting for any out-of-equilibrium beliefs that place probability one on the deviation coming from this set of types.[31] By the definition above of D1 above, the deviation is successful for any refined out-of-equilibrium beliefs, implying that there is no pooling equilibrium with positive cash hoarding. However, there can be also no pooling equilibrium with zero cash hoarding, as by assumption, disagreement is sufficiently strong to prevent investment without cash hoarding. **Q.E.D.**

[31]The latter is feasible, as in a pooling equilibrium types $(\theta'', \overline{\theta}]$ cross subsidize lower types, implying that the financier (who breaks even in expectation) actually makes a profit on these types.

Proof of Proposition 5. Observe first that the expected value of the investment opportunity upon its arrival is $U(w_{0,\theta}^{*}, w_{\theta}^{*})$, where U is given by (6) from Section 2 and $w_{0,\theta}^{*}$ is the cash level that the manager has hoarded before arrival. It is straightforward to verify that $U(w_{0,\theta}^{*}, w_{\theta}^{*})$ is strictly increasing and convex in $w_{0,\theta}^{*}$ (cf. footnote 17). Suppose now that it is optimal to stop hoarding before arrival and before the manager has hoarded K–i.e., $w_{0,\theta}^{*} < K$. We argue to a contradiction that this cannot be the case.

Suppose that before arrival, having reached $w_{0,\theta}^{*}$, the hoarded amount w_t increases above $w_{0,\theta}^{*}$. Paying out $w_t - w_{0,\theta}^{*}$ cannot be optimal if hoarding until $w_{0,\theta}^{*}$ is optimal. First, the probability of arrival is the same at every instant. Second (given the convexity of U), the marginal increase in the option value U that the manager would have after arrival is increasing in the hoarded amount before arrival. In contrast, paying out a unit of cash has the same value to the manager regardless of the previously hoarded amount. Hence, if hoarding dominates paying out for $w_t < w_{0,\theta}^{*}$, it is more beneficial also for $w_t > w_{0,\theta}^{*}$.

To determine whether the manager should start hoarding, we have to compare the expected payoff from hoarding as prescribed above with paying out w_0. Clearly, this expected payoff must be increasing in the probability of arrival λ_a. Hence, there is a threshold $\overline{\lambda}_a$, above which setting aside w_0 and hoarding is optimal. In this case, the manager hoards until the arrival of the investment opportunity and, upon arrival, follows Propositions 1. Note that the manager will stop hoarding cash once she becomes independent of external financing.[32] **Q.E.D.**

Proof of Proposition 6. To be able to compare debt and equity financing, we have to specify the cash flow generating process of the new project. Suppose that the good and the bad project are governed by a common cash flows generating process: $dx_t = \mu_x x_t dt + \sigma_x x_t dZ_t$ with $\mu_x, \sigma_x > 0$ and with Z denoting a Brownian motion. Let the scrap value of the project be S. Ex ante, the initial value of this process x_0 is unknown, but the cumulative density function (cdf) over the possible realizations of x_0 for the good projects dominates that for the bad project in terms of FOSD. All of this is common knowledge. Observe that, after the investment is sunk and the initial value has been realized, the financier cannot infer whether the realization of x_0 is due to the project being good or bad.

Before investment, the manager's and the financier's assessments of the project's ex-

[32]To avoid the risk that the cash at hand falls below K, she may hoard slightly more than K before starting to pay out. Furthermore, note that if the manager does not start hoarding, she pays out w_0 and then cannot invest upon arrival.

pected payoff are

$$W^i(\theta) := E^i\left[\frac{x_0}{r-\mu_x} + \frac{1}{1-\beta_2}S\left(\frac{x_0}{x_d}\right)^{\beta_2}|\theta\right] \text{ where } i = F, M\ , \tag{24}$$

where E^M is conditional on the project being good, and E^F assumes that it is good only with probability ρ. Furthermore, β_2 is the negative root of $\frac{1}{2}\sigma_x^2 y(y-1) + \mu_x y - r = 0$ and (24) takes into account that the project is optimally liquidated if x_t falls below $x_d := \frac{\beta_2}{\beta_2 - 1}S(r-\mu_x)$ (see Morellec and Schürhoff (2011) for a similar derivation).[33]

Suppose now that the manager promises a small constant debt coupon payment ε in addition to an equity share $\widetilde{\alpha}$. Furthermore, let the manager's share of the liquidation proceeds be $(1-\alpha)\left(S - \frac{\varepsilon}{r}\right) - \frac{\varepsilon}{r}$, implying that the financier is guaranteed $\frac{\varepsilon}{r}$ even in liquidation. Clearly, stipulating such a share is feasible for $S > 0$ and ε sufficiently small. It is straightforward to check that it is optimal for the manager to liquidate the project at x_d for such a sharing rule in liquidation (as it is optimal for pure equity financing). The equity share $\widetilde{\alpha}$ that satisfies the financier's participation constraint is $\widetilde{\alpha} = \frac{K-w-\frac{\varepsilon}{r}}{W^F(\theta)}$. By similar arguments to Proposition 1, we obtain that before investing, the manager hoards:

$$\widetilde{w}_\theta = \frac{\beta}{\beta-1}\left(\left(K - W^F(\theta)\right)\frac{W^M(\theta)}{W^M(\theta) - W^F(\theta)} - \frac{\varepsilon}{r}\right) < w_\theta^{VM}.$$

Q.E.D.

Proof of Proposition 7. The proof focuses on the case with asymmetric information. Suppose that the manager owns only a fraction η of the firm, and the remaining η is owned by external shareholders. Consider a candidate for a separating equilibrium in which the highest type $\overline{\theta}$ repurchases a fraction $\eta - \eta_2$, so that after the repurchase she owns $(1-\eta_2)$. Clearly, since delay is costly and it is not possible for high types to separate through cash hoarding, it is without loss of generality to focus on the "static" case in which the manager repurchases the shares and raises new financing in the same period. On a competitive market, the price p at which type $\overline{\theta}$ repurchases a fraction $\eta - \eta_2$ of the firm must solve

$$\begin{aligned} \eta\left(\overline{\theta}W^M - K + w\right) &= \eta_2\left(\overline{\theta}W^M - K + w - p\right) + p \\ \Longrightarrow p &= \frac{\eta - \eta_2}{1-\eta_2}\left(\overline{\theta}W^M - K + w\right) \end{aligned}$$

[33] Previously, we had $W^i(\theta) = \theta W^i$. However, recall that the results are valid as long as the assessments of firm's value $W^i(\theta)$ are increasing in θ.

where we use that existing shareholders should be indifferent between selling and not selling their shares at the fair price. Hence, the incentive constraint of type $\underline{\theta}$ not to mimic type $\overline{\theta}$ is

$$(1-\eta)\left(\underline{\theta}W^M - K + w\right) \geq (1-\eta_2)\left(1 - \frac{K-(w-p)}{\overline{\theta}W^M}\right)\underline{\theta}W^M,$$

which, after plugging for p, boils down to $\frac{(\underline{\theta}-\overline{\theta})}{\overline{\theta}}(K-w) \geq 0$, contradicting that $K > w$. Finally, note that if $K \leq w$, underpricing is irrelevant for the manager, as she can finance the project out of her own funds.

Next, we show by contradiction that there cannot exist an equilibrium in which the firm separates by paying out dividends. First, observe that if the manager does not separate with a dividend payment, she cannot do so through cash hoarding following such a payment (Proposition **??**). Thus, the only candidate for a separating equilibrium is paying a dividend and then investing immediately. Then, the incentive constraint of any type $\theta_L < \theta_H$ not to mimic type θ_H is

$$\left(1 - \frac{K-w_L}{\theta_L W^F}\right)\theta_L W^M + (w_t - w_L) \geq \left(1 - \frac{K-w_H}{\theta_H W^F}\right)\theta_L W^M + (w_t - w_H), \qquad (25)$$

where $(w_t - w_i)$ is the dividend paid by type θ_i. Using that without disagreement $W^M = W^F$, (25) can be rewritten as $w_H \geq K$, leading to the desired contradiction. **Q.E.D.**

Proof: Cash Hoarding and Incentives (Section 4.4). We briefly verify the claim that the incentives problem sketched in the main text leads to the same qualitative predictions as differences in vision. Suppose that for some given level of co-investment w by the manager, the financier expects an equilibrium level of effort e^*, prompting him to require a share of the firm $\alpha = \frac{K-w}{e^*\theta W}$. In such a case, it is optimal for the manager to choose

$$\max_{\widehat{e}}\left(1 - \frac{K-w}{e^*\theta W}\right)\widehat{e}\theta W - w - \frac{\widehat{e}^2}{2\nu}$$

implying that $\frac{(e^*\theta W - K + w)}{e^*} = \frac{\widehat{e}}{\nu}$. Since in equilibrium $e^* = \widehat{e}$, we have $e^* = \frac{1}{2}\theta W\nu + \frac{1}{2}\sqrt{\theta^2 W^2\nu^2 - 4\nu(K-w)}$ and we see immediately that e^* is increasing in w.

Plugging in for e^* into the manager's expected payoff given in (13) we obtain

$$U^{incentives} := \frac{1}{2}\left(\left(\frac{1}{2}\theta W\nu + \frac{1}{2}\sqrt{\theta^2 W^2\nu^2 - 4\nu(K-w)}\right)\theta W - K - w\right)\left(\frac{w}{w^*_\theta}\right)^\beta.$$

Thus, w_θ^*, is the solution to the first-order-condition

$$\frac{\nu\theta W}{\sqrt{\theta^2 W^2\nu^2 - 4\nu\left(K-w\right)}} - 1 = \frac{\beta}{w_\theta^*}\left(\left(\frac{1}{2}\theta W\nu + \frac{1}{2}\sqrt{\theta^2 W^2\nu^2 - 4\nu\left(K-w\right)}\right)\theta W - K - w\right).$$

It is straightforward to show that the second-order-condition is satisfied. Furthermore, we also have $\frac{\partial^2 U^{incentives}|_{w=w_\theta^*}}{\partial w_\theta^* \partial\theta} < 0$, implying by standard arguments that $\frac{dw_\theta^*}{d\theta} < 0$ as claimed in the main text. **Q.E.D.**

Proof: Time-varying disagreement and profitability (Section 4.5). We briefly formalize our claims from Section 4.5.

(i) *Time varying disagreement*: Let $\theta\widetilde{W}^F$ be the financier's valuation of the firm given that he knows that, after investing, disagreement could disappear with probability λ_ρ at any given point in time.[34] Applying the modified Ito's lemma for jump processes and following similar steps to Section 3, it is straightforward to show that the manager optimally hoards

$$w_\rho^* = \frac{\gamma}{\gamma-1}\frac{\left(\left(K-\theta\widetilde{W}^F\right)W^M + \frac{\lambda_\rho}{\lambda_\rho + r}\left(\theta W^M - K\right)\widetilde{W}^F\right)}{\left(W^M - \widetilde{W}^F\right)}$$

where γ is the positive root to $\frac{1}{2}\sigma^2 y\left(y-1\right) + \mu y - r - \lambda_\rho = 0$. It is straightforward to verify that this leads to the same qualitative insights as in Section 3.[35]

(ii) *Time-varying profitability*: We illustrate the argument by making a simplifying assumption that allows us to solve the resulting problem analytically. Specifically, suppose that the NPV of the project from the financiers' point of view follows

$$\frac{d\left(K-\theta W^F\right)}{\left(K-\theta W^F\right)} = \mu_K dt + \sigma_K dZ_K$$

where Z_K is a standard Brownian motion and $\sigma_K > 0$ with a correlation ψ to Z. We assume that $\mu_K < 0$ implying that the NPV increases on average over time. To simplify the analysis, we further assume that the change in NPV is entirely due to a change in the investment cost K. Following similar steps to Proposition 1, the manager's expected payoff is the solution to the following partial differential equation

[34] If he disagreement is about the starting value of the flow process in analogy to Proposition 6, then $\theta\widetilde{W}^F = \theta W^F$.

[35] Furthermore, single crossing holds for λ sufficiently small, which helps to extend also the results from Section 3.4.

$$
\begin{aligned}
rU &= \mu w U_w + \frac{1}{2}\sigma^2 w^2 U_{ww} + \mu_K \left(K - \theta W^F\right) U_K \\
&\quad + \frac{1}{2}\sigma_K^2 \left(K - \theta W^F\right)^2 U_{KK} + \psi\sigma\sigma_K w \left(K - \theta W^F\right) U_{wK}
\end{aligned} \tag{26}
$$

where the subscripts w and K denote the partials with respect to w and $\left(K - \theta W^F\right)$, respectively. Define $\chi = \frac{w}{(K-\theta W^F)}$ so that $U(w, K) = \left(K - \theta W^F\right) U(\chi)$, where we use that U is homogenous of degree one in $(w, \left(K - \theta W^F\right))$ (doubling w and $\left(K - \theta W^F\right)$ would merely double the value of the growth opportunity). We have

$$
\begin{aligned}
U_w &= U_\chi;\ U_{ww} = \frac{1}{(K - \theta W^F)} U_{\chi\chi};\ U_{wK} = -\frac{w}{(K - \theta W^F)^2} U_{\chi\chi} \\
U_K &= U - \frac{w}{(K - \theta W^F)} U_\chi;\ U_{KK} = \frac{w^2}{(K - \theta W^F)^3} U_{\chi\chi}.
\end{aligned}
$$

Plugging into (26), we obtain the simple ordinary differential equation

$$
\underbrace{(r - \mu_K)}_{r'} U = \underbrace{(\mu - \mu_K)}_{\mu'} \chi U_\chi + \underbrace{\left(\frac{1}{2}\sigma^2 + \frac{1}{2}\sigma_K^2 - \psi\sigma\sigma_K\right)}_{\sigma'} \chi^2 U_{\chi\chi} \tag{27}
$$

with a value matching condition $U(\chi^*) = \left(\frac{1-\chi^*}{\theta W^F}\right) \theta W^M - \chi^*$. Defining ϕ as the positive root to $\frac{1}{2}\sigma'^2 y(y-1) + \mu' y = r'$ (where r', μ' and σ' are defined in (27)), and following the same steps as in Section 3, we obtain

$$
\chi^* = \frac{w^*}{(K^* - \theta W^F)} = \frac{\phi}{\phi - 1}\left(\frac{\theta W^M}{\theta W^M - \theta W^F}\right).
$$

We see, thus, that the optimal co-investment w and the NPV from the financier's point of view are in a constant proportion at the optimal investment barrier. Along this barrier, the optimal cash level w^* increases with the investment cost K^*, and this level is lower when the investment opportunities are better (high θ). **Q.E.D.**

Appendix B

Verifying Mailath's Conditions for a Separating Equilibrium In what follows, we verify that the regularity conditions required by Mailath (1987) are indeed satisfied and that $\frac{\partial^2}{\partial\widehat{w}\partial\theta} U(w_t, \widehat{w}, \widehat{\theta}, \theta) < 0$. Mailath's conditions are:

1) Smoothness: $U(\cdot)$ is twice continuously differentiable.

2) Belief monotonicity: $\frac{\partial}{\partial\widehat{\theta}}U(\cdot)$ is either strictly positive or strictly negative.

3) Type monotonicity: $\frac{\partial^2}{\partial\theta\partial w^*}U(\cdot)$ is either strictly positive or strictly negative.

4) Strict quasiconcavity: $\frac{\partial}{\partial w^*}U(\cdot)|_{\widehat{\theta}=\theta}=0$ has a unique solution in w that maximizes $U(\cdot)|_{\widehat{\theta}=\theta}$, and $\frac{\partial^2}{\partial(w^*)^2}U(\cdot)|_{\widehat{\theta}=\theta}<0$ at this solution.

5) Boundedness: There is $k>0$ such that for all $(\theta,\ w)\in\left[\underline{\theta},\overline{\theta}\right]\times\mathbb{R}_+$, $\frac{\partial^2}{\partial(w^*)^2}U(\cdot)|_{\widehat{\theta}=\theta}\geq 0$ implies $\left|\frac{\partial}{\partial w^*}U(\cdot)_{\widehat{\theta}=\theta}\right|>k$.

Conditions 1)-2) are satisfied. Proposition 1 shows that condition 4) is also satisfied. To check for condition 5), observe that if $\frac{\partial^2}{\partial(w^*)^2}U(\cdot)|_{\widehat{\theta}=\theta}\geq 0$, then since the first line on the RHS of

$$\begin{aligned}\frac{\partial^2}{\partial(w^*)^2}U(w_t,w^*,\widehat{\theta},\theta) &= \frac{\beta}{(w^*)^2}\left(\frac{\widehat{\theta}W^F-K}{\widehat{\theta}W^F}\right)\theta W^M\left(\frac{w_t}{w^*}\right)^\beta \\ &\quad -\frac{\beta}{w^*}\left(-\frac{\beta}{w^*}\left(\left(1-\frac{K-w^*}{\widehat{\theta}W^F}\right)\theta W^M-w^*\right)+\frac{\theta W^M}{\widehat{\theta}W^F}-1\right)\left(\frac{w_t}{w^*}\right)^\beta\end{aligned}$$

is negative, $\frac{\partial}{\partial w^*}U(\cdot)|_{\widehat{\theta}=\theta}<0$ for $\widehat{w}\in(0,k)$ where k is bounded away from infinity. w^*_θ remains the unique equilibrium even if $k\to\infty$. To see this, observe that the single crossing property holds for $w^*\in(0,\infty)$. Hence, local incentive compatibility of w^*_θ guarantees also global incentive compatibility and w^*_θ is a separating equilibrium even if $k\to\infty$. Moreover, it is the unique equilibrium. Otherwise, there must be an alternative equilibrium with a type for whom $w^*_\theta(\theta)\to\infty$. However, for such a trigger, the option value component of this type's expected payoff is zero, whereas it is strictly positive for a positive trigger bounded away from infinity. This makes a deviation profitable, contradicting the existence of a different separating equilibrium than w^*_θ.

Finally, we check when $\frac{\partial^2}{\partial w^*\partial\theta}U(\cdot)<0$ holds–i.e., condition 3). U is submodular in w^* and θ if

$$\frac{\partial^2}{\partial w^*\partial\theta}U(w_t,w^*(\widehat{\theta}),\widehat{\theta},\theta)=\left(-\frac{\beta}{\widehat{w}}\left(\widehat{\theta}W^F-K\right)+1-\beta\right)\frac{W^M}{\widehat{\theta}W^F}\left(\frac{w_t}{\widehat{w}}\right)^\beta<0$$

and so it should hold

$$w^*(\widehat{\theta})>\underline{w}_{\widehat{\theta}}:=\frac{\beta}{\beta-1}\left(K-\widehat{\theta}W^F\right). \tag{28}$$

Comparing this condition to (7), we see that submodularity holds as long as signaling does not require a too large distortion away from first best.

Clearly, condition (28) is satisfied for all types close enough to the lowest type, since $w^*_{\underline{\theta}}=w^{VM}_{\underline{\theta}}>\underline{w}_{\underline{\theta}}$. A sufficient condition that it is satisfied for all types is that $\theta W^F>$

$\theta W^M - \frac{1}{\beta}K$–i.e., that disagreement is not excessive.[36] Intuitively, if disagreement is excessively large, absent information asymmetry, all types find it optimal to accumulate large cash reserves and raise only little financing from outside financiers. Then, deviating too much from this strategy when there is information asymmetry, which may be needed for high types to separate from low types, could become too costly as the benefit from overcoming information asymmetry does not compensate for the cost of increasing exposure to outside financing and, thus, disagreement.

If condition (28) is violated, denote the lowest type for whom this is the case with θ'. Then, we can construct a separating equilibrium for types $[\underline{\theta}, \theta']$ characterized by (22) and $w^*_{\underline{\theta}} = w^{VM}_{\underline{\theta}}$. The remaining types $(\theta', \overline{\theta}]$ then pool at the cash level $w^P < w^*_{\theta'}$ for which type θ' is indifferent between separating and pooling with the higher types. It is straightforward to find beliefs that support such an equilibrium.

[36]This condition can be derived from requiring that $\frac{dw^*_\theta}{d\theta} > \frac{d\underline{w}_\theta}{d\theta}$ for $w^*_\theta = \underline{w}_\theta$–i.e., that w^*_θ and $\underline{w}_\theta$ do not cross.