Exploratory Trading

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Abstract

Using comprehensive, account-labeled message records from the E-mini S&P 500 futures market, I examine the mechanisms that enable high-frequency traders (HFTs) to profitably anticipate price movements. HFTs in my sample consistently lose money on numerous small orders, then earn high profits on larger orders, and I analyze how the HFTs can profit on these large orders. I model how a trader could use her own small, “exploratory” orders to gather valuable, private information that enables her to trade ahead of price movements with large orders. Direct empirical tests of the model’s predictions support the hypothesis that HFTs use this sort of “exploratory trading” to actively learn about likely future price movements. More generally, the empirical results demonstrate that high-frequency trading involves more than just superior speed, opening new directions for theoretical work and policy design.
1 Introduction

High-frequency algorithmic traders are responsible for almost half of the trading on modern financial exchanges. Understanding what high-frequency traders (HFTs) do, and why they do it, is important for any analysis of modern markets, and it is crucial for addressing policy issues about HFTs and market regulation.

HFTs are not all alike, but they share some distinctive features. They have the capacity to react to market events in milliseconds or less (so-called “low latency”), they trade very frequently and unwind positions within minutes, and they usually end the trading day holding minimal inventory. Most interestingly, a number of recent empirical studies, such as Baron et al. (2013), indicate that HFTs tend earn large and stable trading profits.

The standard ways to make money in a financial market are to bear systematic risk, to provide liquidity/immediacy, or to trade on private information. Because HFTs tend to switch between long positions and short positions every couple of minutes then end the trading day with negligible inventory, they do not bear any meaningful systematic market risk. Although some HFTs basically act like market-makers and supply liquidity/immediacy, many do not, yet they still consistently earn substantial trading profits. The only feasible source of profits for these other HFTs appears to be some sort of private information. Determining nature of that information is central to understanding high-frequency trading.

There are few obvious ways that HFTs could obtain private information when they use nothing more than electronic feeds of market data, and occasionally feeds of news. Among academics, policy makers, and regulators, the common assumption is that HFTs merely react to public information more quickly than other market participants. The “superior reaction speed” mechanism underlies most theoretical models of high-frequency trading, including those of Jarrow and Protter (2011), Cespa and Foucault (2008), Biais et al. (2010), and Budish et al. (2013).

In this paper I present evidence that—contrary to the pervasive assumption—rapid reaction to public information is not the only source of HFTs’ private information. More specifically, I examine a type of active learning that I call “exploratory trading,” and I find evidence that HFTs obtain valuable private information through this channel.
1.1 Empirical Research on High-Frequency Trading

Although HFTs are responsible for a large fraction of trading activity across numerous markets, distinguishing and identifying HFTs’ activity in available data has presented an ongoing challenge for empirical research. Nevertheless, a number of studies have found various solutions. For example, Brogaard et al. (2012) obtain and analyze a NASDAQ dataset that flags messages from an aggregated group of 26 HFT firms, and Hasbrouck and Saar (2011) conduct a complementary analysis by statistically reconstructing “strategic runs” of linked messages in public NASDAQ data. Both of these studies suggest beneficial effects from HFTs’ activity in aggregate, but inherent limitations of the underlying data restrict these studies’ scope to explain how and why such effects arise. Empirical results in Baron et al. (2012), Hagstromer and Norden (2013), and the present paper indicate that HFTs exhibit considerable heterogeneity—even in a market for a single asset. As a result, aggregate HFT activity reveals little about what individual HFTs really do.

Regulatory records that the Chicago Mercantile Exchange provides to the U.S. Commodity Futures Trading Commission are currently the only data for U.S. markets fully adequate for the study of high-frequency trading at the level of individual HFTs. Kirilenko et al. (2010) pioneered the use of transaction data from these records to investigate high-frequency trading in the E-mini S&P 500 futures market during the so-called “Flash Crash” of May 6, 2010. This work introduced a data-driven scheme to classify trading accounts using simple measures of overall trading activity, intraday variation in net inventory position, and inter-day changes in net inventory position. Of the accounts with sufficiently small intra- and inter-day variation in net position, Kirilenko et al. classify those with the highest levels of trading activity as HFTs, and these accounts are prototypical high-frequency traders.

The empirical context for the analysis in this paper is the E-mini market. Kirilenko et al. find that HFTs are extremely active in the E-mini market, participating in over one-third of trading volume, and in the sample period analyzed in this paper, I find that HFTs participate in an even greater fraction of E-mini trading volume. Trading volume in the E-mini averages around $25 trillion per year, so the E-mini is an important market, of which HFTs are an important part, and for which data uniquely suitable to the study of individual HFTs exists.
1.2 Active Learning, Exploration, and Private Information

HFTs’ private information is the central object of interest in this paper. When a trader initiates a transaction—that is, places a so-called “aggressive” order—he pays his counterparty a fraction of the bid-ask spread, so to profit from his aggressive order, the trader must correctly anticipate a price movement. Consistently profiting on aggressive orders therefore suffices to indicate the use of private information.

Using novel electronic message data at the Commodity Futures Trading Commission, I examine the profitability of individual HFTs’ aggressive orders. As a group, the 30 HFTs in my sample earn roughly 40% of their profits from their aggressive orders. Examining these HFTs individually, though, I find that although all of them make money, only eight of the 30 profit on average from their aggressive trading. For brevity, I refer to these eight HFTs as “A-HFTs,” and to the remaining 22 as “B-HFTs.” The A-HFTs unambiguously use private information, so I focus on them and I investigate the nature and origin of their private information. Almost certainly, the A-HFTs can react to public information faster than some market participants. However, I find evidence that the A-HFTs also do something more to obtain private information: they use a form of active learning, which I term “exploratory trading.”

The concept of active learning in financial markets dates back at least to the work of Leach and Madhavan (1992) and (1993), so exploratory trading is simply a new incarnation of an old idea. The basic exploratory trading strategy is straightforward. Demand innovations in the E-mini market are easy to predict, but the price-elasticity of supply is not, and price-impact is usually too small for indiscriminate front-running of predictable demand to be profitable.\(^1\) The A-HFTs use small “exploratory” aggressive orders to obtain private information about the price-elasticity of supply, and this information enables them to trade ahead of predictable demand at only those times when it is profitable to do so. When an HFT places an exploratory order and observes a large price-impact, he learns that supply is temporarily inelastic. If the HFT knows that there is going to be more demand soon thereafter, he can place a larger order (even with

\(^1\)To be more precise, the market response to these small aggressive orders provides private information about the dynamic behavior of passive orders resting in the orderbook (which is analogous to supply elasticity). This information helps to forecast the price-impact of future aggressive order-flow (which is analogous to demand innovations). The labels “supply” and “demand” are merely heuristics in the context of the market for E-mini contracts, but they are rigorously accurate in the context of the market for liquidity/immediacy.
a high price-impact) knowing that the price-impact from the coming demand will drive prices up further and ultimately enable him to sell at a premium that exceeds the price-impact of his unwinding order. The purpose of exploratory trading is not to learn about future demand, but rather to identify the times at which trading in front of future demand will be profitable. In the next section, I show that the exploratory trading mechanism described above is in fact an immediate extension of some entirely standard market-microstructure models and results.

The remainder of this paper is organized as follows: Section 2 introduces the microstructure foundations of exploratory trading, presents a simple model of exploratory trading along with the model’s central predictions, and establishes the empirical agenda. Section 3 describes the dataset, presents some summary statistics, and precisely defines HFTs. Section 4 addresses the overall profitability of HFTs’ aggressive orders and precisely characterizes the A-HFTs, then examines the A-HFTs’ losses on small aggressive orders. Section 5 presents direct empirical tests of the exploratory trading model’s key predictions, and section 6 examines the practical significance of exploratory information. Section 7 discusses extensions and implications of the empirical results. Section 8 concludes.

2 Exploratory Trading: Theory

2.1 Microstructure Foundations of Exploratory Trading

Exploratory trading is more tightly linked to well-established market microstructure results than the heuristic explanation in the previous section might suggest. What I call “demand” and “supply” there are more properly characterized as the flow of aggressive orders, and the collection of limit orders resting in the book, respectively.

In the E-mini market, aggressive order-flow is highly predictable. More precisely, the signs of future aggressive orders (buy vs. sell) are very easy to predict because aggressive order signs exhibit strong positive autocorrelation; for example, the probability that the next aggressive order will have the same sign as the previous aggressive order is around 75%, and this autocorrelation becomes even stronger when the arrival rate of aggressive orders increases. Strong positive autocorrelation of trade direction has been documented extensively for a variety of markets by
Engle and Dufour (2000) find a robust positive relationship between arrival rate and autocorrelation for trade direction.

As in many other other markets, tightly clustered streaks of aggressive orders with the same sign are common in the E-mini market. Two opposing explanations have been proposed for such bursts of intense trading activity. The Easley and O’Hara (1992) model posits that such bursts arise from information-based clustering of informed traders, while the Admati and Pfleiderer (1988) model explains these bursts as liquidity-based trading wherein uninformed traders coordinate to trade with one another. Engle and Russell (1998) present evidence that some clusters of trades seem largely information-based, while other clusters of trades appear to be liquidity-based. The bid-ask spread should become wider as the probability of informed trading increases, and indeed Engle and Russell find that intense clusters of transactions tend be appear liquidity-based when the spread is narrow, and information-based when the spread is wide.

2.1.1 One-Tick Spreads

The purpose of exploratory trading is precisely to distinguish the times when clusters of aggressive orders are likely to be informed from the times when they are likely to be uninformed. In principle, the spread should adjust so that it both reveals all information that could be obtained through exploratory trading, and also drives to zero the expected profits of uninformed traders who try to anticipate future informed trading. However, bid-ask spread in the E-mini market is almost constantly equal to the minimal allowed price increment, or “tick.” Consequently, the spread usually does not convey anything about the probability of informed trading, so information that could be obtained through exploratory trading will not automatically be revealed. Since an otherwise-uninformed trader who has engaged in exploratory trading can now have information that is not publicly observable, it need no longer be the case that such a trader’s expected profits are zero.

2.2 Preliminaries

In an order-driven market, every regular transaction is initiated by one of the two executing transactors. The transactor who initiates is referred to as the “aggressor,” while the opposite
transactor is referred to as the “passor.” The passor’s order was resting in the orderbook, and
the aggressor entered a new order that executed against the passor’s preexisting resting order.
Assuming that prices are discrete, the lowest price of any resting sell order in the orderbook
(“best ask”) always exceeds the highest price of any resting buy order in the book (“best bid”) by
at least one increment (the minimal price increments are called “ticks”). A transaction initiated
by the seller executes at the best bid, while a transaction initiated by the buyer executes at the
best ask; the resulting variation in transaction prices between aggressive buys and aggressive sells
is known as “bid-ask bounce.” Hereafter, except where otherwise noted, I will restrict attention
to price changes distinct from bid-ask bounce. Empirically, the best ask for the most actively
traded E-mini contract almost always exceeds the best bid by exactly one tick during regular
trading hours, so movements of the best bid, best ask, and mid-point prices are essentially
interchangeable.

If the best bid and best ask were held fixed, a trader who aggressively entered then aggres-
sively exited a position would lose the bid-ask spread on each contract, whereas a trader who
passively entered then passively exited a position would earn the bid-ask spread on each contract.
Intuitively, aggressors pay for the privilege of trading precisely when they wish to do so, and
passors are compensated for the costs of supplying this “immediacy,” cf. Grossman and Miller
(1988). These costs include fixed operational costs and costs arising from adverse selection. Cf.

An aggressive order will execute against all passive orders at the best available price level
before executing against any passive orders at the next price, so an aggressive order will only
have a literal price-impact if it eats through all of the resting orders at the best price. In the
E-mini market, it is rare for an aggressive order to have a literal price-impact, not only because
there are typically enormous numbers of contracts at the best bid and best ask, but also because
aggressive orders overwhelmingly take the form of limit orders priced at the opposite best (which
cannot execute at the next price level).
2.3 Baseline Model

Let time be discrete, consisting of two periods, \( t = 1, 2 \), and consider an order-driven market with discrete prices. Assume that both the orderbook and order-flow are observable. I will refer to the set of passive orders in the orderbook as “resting depth.”

2.3.1 The HFT

Consider a single trader, “the HFT,” who has the opportunity to submit an aggressive order at the start of each time-period. The HFT submits only aggressive orders, and these orders are limited in size to \( N \) contracts or fewer. Let \( q_t \in \{-N, \ldots, -1, 0, 1, \ldots, N\} \) denote the signed quantity of the aggressive order that the HFT places in period \( t \), where a negative quantity represents a sale, and a positive quantity represents a purchase. Assume that the HFT only trades contracts at the initial best bid/ask, so his orders affect resting depth in the orderbook but have no literal price-impact.

Suppose that prices remain constant between periods 1 and 2, and denote the price-change at the end of period 2 by \( y \in \{-1, 0, +1\} \). Assume that the HFT pays constant trading costs of \( \alpha \in (0.5, 1) \) per contract, and that the HFT’s profit from the aggressive order he places in period \( t \) is given by

\[
\pi_t = y q_t - \alpha |q_t| \tag{1}
\]

See 4.1 for discussion of why this specification for profits is reasonable. The lower bound of 0.5 on \( \alpha \) corresponds to half of the minimum possible bid-ask spread, while the upper bound of 1 merely excludes trivial cases of the model in which aggressive orders are always unprofitable for the HFT.

Denote the HFT’s total combined profits from periods 1 and 2 by

\[
\pi_{total} := \pi_1 + \pi_2 \tag{2}
\]

Assume that the HFT is risk-neutral and seeks to maximize the expectation of \( \pi_{total} \).
2.3.2 Passive Orders

There are two possible states for the behavior of passive orders: accommodating and unaccommodating. Let the variable $\Lambda$ represent this state, which I call the “liquidity state.” Denote the accommodating liquidity state by $\Lambda = A$, and the unaccommodating state by $\Lambda = U$. The liquidity state is the same in both time-periods, $t = 1, 2$. Assume that $\Lambda = U$ with ex-ante probability $u$, and $\Lambda = A$ with probability $1 - u$.

Intuitively, aggressive orders have a small price-impact in the accommodating liquidity state, and a large price-impact in the unaccommodating liquidity state. The liquidity state characterizes the behavior of resting depth in the orderbook after an aggressive order executes—a generalization of price-impact appropriate for an order-driven market. When an aggressive buy (sell) order executes, it mechanically depletes resting depth on the sell (buy) side of the orderbook. Following this mechanical depletion, traders may enter, modify, and/or cancel passive orders, so resting depth at the best ask (bid) can either replenish, stay the same, or deplete further. The aggressive order’s impact is offset to some extent—or even reversed—if resting depth replenishes, whereas the aggressive order’s impact is amplified if resting depth depletes further. In the accommodating state ($\Lambda = A$) resting depth weakly replenishes, while in the unaccommodating state ($\Lambda = U$) resting depth further depletes.

Although the orderbook is assumed to be observable, the static features of passive orders in the orderbook do not directly reveal the liquidity state $\Lambda$. Because the liquidity state relates to the dynamic behavior of resting depth after an aggressive order executes, $\Lambda$ can only be deduced from the changes in the orderbook that follow the execution of an aggressive order. For the baseline model, I will assume that the HFT learns $\Lambda$ prior to period 2 if and only if he places an aggressive order in period 1; I relax this assumption in section 2.5.

2.3.3 Aggressive Order-Flow

At the end of period 2, traders other than the HFT exogenously place aggressive orders. Let the variable $\varphi \in \{-1, 0, +1\}$ characterize this exogenous aggressive order-flow. The realization of $\varphi$ does not depend on the liquidity state, $\Lambda$, nor does it depend on the HFT’s actions; assume that $\varphi = +1$ and $\varphi = -1$ with equal probabilities $P\{\varphi = +1\} = P\{\varphi = -1\} = v/2$, and $\varphi = 0$ with
probability $1 - v$. The variable $\varphi$ is just a coarse summary of the order-flow—it does not represent the actual number of contracts. Intuitively, $\varphi = -1$ represents predictable selling pressure and $\varphi = +1$ represents predictable buying pressure, while $\varphi = 0$ represents an absence of predictable pressure in either direction.

Aggressive order-flow is assumed to be independent of liquidity state for convenience; introducing dependence between aggressive order-flow and the liquidity state complicates the algebra but does not change the model in any interesting ways. The assumption that neither aggressive order-flow nor the liquidity state depends on the HFT's actions is more innocuous than it might seem, because the HFT will turn out not to do anything that would particularly stand out. In the first period, the HFT will either place a very small order, or no order. In the second period, if the HFT places an order, it may be large, but if it is, it will always be in the same direction as expected aggressive order flow.

### 2.3.4 Price Change

Together, the exogenous aggressive order-flow $\varphi$ and the liquidity state $\Lambda$ determine $y$, the price-change at the end of period 2, as follows:

$$y = \begin{cases} 
\varphi & \text{if } \Lambda = U \\
0 & \text{if } \Lambda = A 
\end{cases}$$

When the liquidity state is unaccommodating ($\Lambda = U$), the exogenous aggressive order-flow can affect the price, and $y = \varphi$. However, if the liquidity state is accommodating ($\Lambda = A$), aggressive order-flow does not affect the price, and $y = 0$ even when $\varphi \neq 0$. In the spirit of Easley and O'Hara/ Admati and Pfleiderer/ Engle and Russell, assume that trading in the unaccommodating state is driven by private information, so that the associated price movements should tend to be permanent.

Note that because $\mathbb{E}[y|\Lambda = U] = \mathbb{E}[\varphi] = 0$ and $\mathbb{E}[y|\Lambda = A] = 0$, the unconditional expectation of $y$ is zero, as is the period-1 expectation of $y$. 

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2.3.5 Model Timeline

**Period 1** In period 1, the HFT has the opportunity to submit an aggressive order and then observe any subsequent change in resting depth. The HFT cannot observe the liquidity state directly, but he can infer the value of $\Lambda$ from changes in resting depth if he places an aggressive order; the HFT can conclude that $\Lambda = U$ if resting depth further depletes following his order, and that $\Lambda = A$ otherwise. If the HFT does not place an aggressive order in period 1, he does not learn $\Lambda$.

**Period 2** At the start of period 2, the HFT observes the signal of future aggressive order-flow, $\varphi$. The HFT observes $\varphi$ regardless of whether he placed an aggressive order in period 1 (this reflects the idea that aggressive order-flow is easy to predict on the basis of public market data). After the HFT observes $\varphi$, he once again has an opportunity to place an aggressive order. Finally, after the HFT has the chance to trade, aggressive order-flow characterized by $\varphi$ arrives, and prices change as determined by $\varphi$ and $\Lambda$ in equation (3).

2.4 Analysis of the Baseline Model

The baseline model of exploratory trading makes the concept of exploratory trading more precise and transparent, and more importantly, it delivers testable implications of the hypothesis that a given trader engages in exploration. Appendix A contains full mathematical details.

2.4.1 Solving for the HFT’s Optimal Trading Strategy

When $\alpha > u$, the HFT will never place an order in period 2 if he doesn’t know the liquidity state, and I focus on this case to simplify the exposition; results are qualitatively unchanged for $u \geq \alpha$ (see Appendix A). I solve for the HFT’s optimal trading strategy via backward induction.

**Period 2** If the HFT learned the liquidity state during period 1, his optimal aggressive order in period 2 will depend on the values of both $\varphi$ and $\Lambda$. The HFT’s optimal strategy when he knows $\Lambda$ is to set $q_2 = \varphi N$ if $\Lambda = U$, and to set $q_2 = 0$ if $\Lambda = A$. Taking expectations with
respect to \( \varphi \) and then \( \Lambda \), we find

\[
E[\pi_2| \Lambda \text{ known}] = Nv (1 - \alpha)^* u + 0^* (1 - u) \\
= Nvu (1 - \alpha)
\] (4)

If the HFT did not learn the liquidity state during period 1, his (constrained) optimal aggressive order in period 2 will still depend on the value of \( \varphi \), but it will only depend on the distribution of \( \Lambda \), rather than the actual value of \( \Lambda \). The HFT’s optimal strategy when he does not know \( \Lambda \) is to set \( q_2 = \varphi N \) when \( u \geq \alpha \), and to set \( q_2 = 0 \) when \( \alpha > u \). I assumed for simplicity that \( \alpha > u \), so

\[
E[\pi_2| \Lambda \text{ unknown}] = 0
\] (5)

**Period 1** At the start of period 1, the HFT knows neither \( \varphi \) nor \( \Lambda \), but he faces the same trading costs (\( \alpha \) per contract) as in period 2. Consequently, the HFT’s expected direct trading profits from a period-1 aggressive order are negative, given by

\[
E[\pi_1] = -\alpha |q_1|
\] (6)

Since there is no noise in this baseline model, and the HFT learns \( \Lambda \) perfectly from any aggressive order that he places in the first period, we can restrict attention to the cases of \( q_1 = 0 \) and \( |q_1| = 1 \).

We obtain the following expression for the difference in the HFT’s total expected profits if he sets \( |q_1| = 1 \) instead of \( q_1 = 0 \):

\[
E[\pi_{total}| |q_1| = 1] - E[\pi_{total}|q_1 = 0] = Nvu (1 - \alpha) - \alpha
\] (7)

The HFT engages in exploratory trading if he sets \( |q_1| = 1 \), and he does not engage in exploratory trading if he sets \( q_1 = 0 \), so equation (7) represents the expected net gain from exploration. Exploratory trading is optimal for the HFT when this expected net gain is positive.
2.4.2 Order-Sizes and Conditions for Exploratory Trading

The results in section 2.4.1 demonstrate the trade-off between direct trading costs and informational gains at the heart of exploratory trading. By placing a (costly) aggressive order in period 1, the HFT “buys” the perturbation needed to elicit a response in resting depth that reveals the liquidity state. Knowing the liquidity state enables the HFT, in period 2, to better determine whether placing an aggressive order will be profitable.

Parameters of the model determine the relative costs and payoffs of exploration. I derive routine comparative statics in Appendix A, but because exogenous variation in $N, \nu, \alpha,$ or $u$ is scarce, these comparative statics provide little in the way of testable implications. However, for the HFT to weakly prefer to engage in period-1 exploratory trading with order-size $q_1 \geq 1,$ it follows almost immediately from (7) that we must have

$$N \nu u (1 - \alpha) - q_1 \alpha \geq 0$$

$$\iff N \geq \frac{\alpha}{1 - \alpha} \left( \frac{1}{\nu u} \right) q_1$$

The “small exploratory trade/large follow-up trade” pattern in the baseline model is more than just an artifact of the simplifying assumption that the size of the exploratory order didn’t affect the informativeness of the subsequent orderbook response. Rather, the pattern arises because the exploratory orders are always costly in expectation, while the resulting exploratory information is only valuable when there is predictable aggressive order-flow in the next period (i.e., when $\varphi \neq 0$). The per-contract losses on exploratory orders will therefore be greater in magnitude than the per-contract profits on follow-up orders, so the total profits on follow-up orders will only exceed the total losses on exploratory orders if the follow-up orders are larger.

2.4.3 Testable Predictions

Assume for the moment that candidate exploratory orders can be distinguished from the trader’s other orders (I discuss this assumption in section 2.6). The baseline exploratory trading model generates two key testable implications of the hypothesis that a given trader engages in exploration.
First, the baseline exploratory trading model predicts that the market response following an exploratory order helps to forecast whether or not the explorer will place a follow-up order. The trader will not place a follow-up order if $\Lambda = A$, while he will place a follow-up order if $\Lambda = U$ and $\varphi \neq 0$. From the results in section 2.4.2, we know that the follow-up order must tend to be larger than the exploratory order, so the model implies that if a given trader engages in exploration, then holding fixed $\varphi$, the incidence of the trader’s large aggressive orders will be higher when $\Lambda = U$ than when $\Lambda = A$. In other words, if a given trader engages in exploration, then the market response to his exploratory orders should help to explain the incidence of his larger aggressive orders (holding fixed the expectation of future aggressive order-flow analogous to $\varphi$).

Next, as above, the model predicts that the market response following an exploratory order helps to forecast whether or not the exploring trader will place a follow-up order in the direction of an imminent price-change. The market response following an exploratory order also helps to forecast whether or not prices will change soon thereafter, according to equation (3). Hence the because both the price-change and an exploring trader’s decision to trade will depend on $\Lambda$, the model implies that if a given trader engages in exploration, then equation (3) will explain his earnings better than $\varphi$ alone. In other words, the market response to his exploratory orders should help to explain his earnings on subsequent aggressive orders.

### 2.5 Private Gains from Exploratory Trading

The baseline model developed in the previous section abstracted away from the details of the HFT’s inference about $\Lambda$, and it made the simplifying assumption that placing an aggressive order in the first period was both necessary and sufficient for the HFT to learn the liquidity state. This simplification does not qualitatively affect the two testable implications highlighted in section 2.4, but the “necessity” assumption obscures why the HFT learns more from placing an aggressive order himself than he does from merely observing an aggressive order placed by someone else.

Sometimes, the changes in the orderbook following the arrival of an aggressive order are truly a response caused by the aggressive order, in which case the orderbook activity provides information
about the liquidity state. Often, though, both the aggressive order and the subsequent orderbook activity are really just common responses to some third event, so there is no causal link between the aggressive order and the subsequent orderbook changes, in which case the orderbook activity does not provide information about the liquidity state. If someone else placed the aggressive order, these two scenarios are indistinguishable to the HFT, so the possibility that he is observing the uninformative non-causal scenario attenuates the amount that he can learn from the market response to someone else’s aggressive order. By contrast, if the HFT places an aggressive order himself, he can be entirely sure whether he did so for exogenous reasons, so the uninformative scenario need not be a concern. The HFT learns more from his own aggressive orders than he does from those of traders because he can better infer causal effects from aggressive orders that he himself placed.

In Appendix A, I formalize the arguments above using a variation on the baseline model. The formal argument delivers the same prediction as the informal argument above, namely that a trader learns more from the market response to his own exploratory orders than he does from the market response to aggressive orders placed by other people. In an anonymous market, by symmetry we obtain the prediction that each other trader obtains no more useful information from the market response to the exploring trader’s aggressive orders than they do from the market response to another arbitrary trader’s aggressive orders.

2.6 Empirical Agenda

Before attempting any empirical evaluation of the hypothesis that the A-HFTs engage in exploratory trading, suitable candidates for putative exploratory orders must be identified in some manner among the A-HFTs’ aggressive orders. The results from section 2.4.2 suggest that small, unprofitable aggressive orders are prime candidates. In section 4.3, I find that all of the A-HFTs tend to lose money on their smallest aggressive orders. While this does not necessarily imply that those small orders were exploratory in nature (there are numerous other reasons why the A-HFTs might tend to lose money on their smallest aggressive orders), they plausibly could be. The high probability that some of those orders are not exploratory only strengthens my results.

With that preliminary matter resolved, I turn to direct empirical tests of the model’s key
predictions. As a benchmark, I consider the market response following the last small aggressive order placed by anyone, which is public information. The empirical implications discussed earlier in this section can then be condensed into three central predictions, namely that relative to the public-information benchmark, information from the market response following an A-HFT’s small aggressive orders:

**Prediction.1** Explains a significant additional component of that A-HFT’s earnings on subsequent aggressive orders, but

**Prediction.2** Does not explain any additional component of other traders’ earnings on subsequent aggressive orders, and

**Prediction.3** Further explains by a significant margin the incidence of that A-HFT’s subsequent large aggressive orders

In section 5, I introduce an explicit numeric measure of “market response,” and in section 5.3, I make precise the notion of “explaining earnings on subsequent aggressive orders,” then I formally test the predictions above. The variables and functional forms that the empirical tests involve are close analogues of the predictions in 2.4.3 stated in terms of quantities from the baseline model.

### 3 High-Frequency Trading in the E-mini Market

The E-mini S&P 500 futures contract is a cash-settled instrument with a notional value equal to $50.00 times the S&P 500 index. Prices are quoted in terms of the S&P 500 index, at minimum increments, “ticks”, of 0.25 index points, equivalent to $12.50 per contract. E-mini contracts are created directly by buyers and sellers, so the quantity of outstanding contracts is potentially unlimited.

All E-mini contracts trade exclusively on the CME Globex electronic trading platform, in an order-driven market. Transaction prices/quantities and changes in aggregate depth at individual price levels in the orderbook are observable through a public market-data feed, but the E-mini market provides full anonymity, so the identities of the traders responsible for these events are not released. Limit orders in the E-mini market are matched according to strict price and time
priority; a buy (sell) limit order at a given price executes ahead of all buy (sell) limit orders at lower (higher) prices, and buy (sell) limit orders at the same price execute in the sequence that they arrived. Certain modifications to a limit order, most notably size increases, reset the time-stamp by which time-priority is determined.

E-mini contracts with expiration dates in the five nearest months of the March quarterly cycle (March, June, September, December) are listed for trading, but activity typically concentrates in the contract with the nearest expiration. Aside from brief maintenance periods, the E-mini market is open 24 hours a day, though most activity occurs during “regular trading hours,” namely, weekdays between 8:30 a.m. and 3:15 p.m. CT.

3.1 Description of the Data

I examine account-labeled, millisecond-timestamped records at the Commodity Futures Trading Commission of the so-called “business messages” entered into the Globex system between September 17, 2010 and November 1, 2010 for all E-mini S&P 500 futures contracts. This message data captures not only transactions, but also events that do not directly result in a trade, such as the entry, cancellation, or modification of a resting limit order. Essentially, business messages include any action by a market participant that could potentially result in or affect a transaction immediately, or at any point in the future. I restrict attention to the December-expiring E-mini contract (ticker ESZ0). During my sample period, ESZ0 activity accounted for roughly 98% of the message volume across all E-mini contracts, and more than 99.9% of the trading volume.

The price of an ESZ0 contract during this period was around $55,000 to $60,000, and (one-sided) trading volume averaged 1,991,252 contracts or approximately $115 billion per day. Message volume averaged approximately 5 million business messages per day, and the number of aggressive orders executed per day day averaged 132,127. The intensity of trading varies considerably throughout the day (aggressive orders typically arrive in tight clusters), so the median time interval between aggressive orders during regular trading hours is closer to 20 milliseconds.

2Excluded from these data are purely administrative messages, such as log-on and log-out messages. The good-‘til-cancel orders in the orderbook at the start of September 2, and a small number of modification messages (around 2 – 4%) are also missing from these records. Because I restrict attention to aggressive orders, and I only look at changes in resting depth (rather than its actual level), my results are not sensitive to these omitted messages.
than it is to the mean interval of roughly 200 milliseconds.

3.2 Defining “High-Frequency Traders”

Kirilenko et al. identify as HFTs those traders who exhibit minimal accumulation of directional positions, high inventory turnover, and high levels of trading activity. I, too, use these three characteristics to define and identify HFTs. To quantify an account's accumulation of directional positions, I consider the magnitude of changes in end-of-day net position as a percentage of the account’s daily trading volume. Similarly, I use an account’s maximal intraday change in net position, relative to daily volume, to measure inventory turnover. Finally, I use an account’s total trading volume as a measure of trading activity.

I select each account whose end-of-day net position changes by less than 6% of its daily volume, and whose maximal intraday net position changes are less than 20% of its daily volume. I rank the selected accounts by total trading volume, and classify the top 30 accounts as HFTs. The original classifications of Kirilenko et al. and Baron et al. guided the rough threshold choices for inter-day and intraday variation. Thereafter, since confidentiality protocols prohibit disclosing results for groups smaller than eight trading accounts, the precise cutoff values of 6%, 20%, and 30 accounts were chosen to ensure that all groups of interest would have at least eight members. My central results are not sensitive to values of these parameters.

The set of HFTs corresponds closely to the set of accounts with the greatest trading volume in my sample, so the set of HFTs is largely invariant both to the exact characterizations of inter-day and intraday variation in net position relative to volume, and to the exact cutoff values for these quantities. Similarly, changing the 30-account cutoff to (e.g.) 15 accounts or 60 accounts does not substantially alter my results, because activity heavily concentrates among the largest HFTs. For example, the combined total trading volume of the 8 largest HFTs exceeds that of HFTs 9-30 by roughly three-quarters, and the combined aggressive volume of the 8 largest HFTs exceeds that of HFTs 9-30 by a factor of almost 2.5.
3.3 HFTs’ Prominence and Profitability

Although HFTs constitute less than 0.1% of the 41,778 accounts that traded the ESZ0 contract between September 17, 2010 and November 1, 2010, they participate in 46.7% of the total trading volume during this period. In addition to trading volume, HFTs are responsible for a large fraction of message volume. During the sample period, HFTs account for 31.9% of all order entry, order modification and order cancellation messages. In aggregate, approximately 48.5% of HFTs’ volume is aggressive, and this figure rises to 54.2% among the 12 largest HFTs. The HFTs also appear to earn large and stable profits. Gross of trading fees, the 30 HFTs earned a combined average of $1.51 million per trading day during the sample period. Individual HFTs’ annualized Sharpe ratios are in the neighborhood of 10 to 11.

The Chicago Mercantile Exchange reduces E-mini trading fees on a tiered basis for traders whose average monthly volume exceeds various thresholds. Trading and clearing fees were either $0.095 per contract or $0.12 per contract for the 20 largest HFTs, and were at most $0.16 per contract for the remaining HFTs. Initial and maintenance margins were both $4,500 for all of the HFTs.

Hereafter, unless otherwise noted, I restrict attention to activity that occurred during regular trading hours. HFTs’ aggressive trading occurs almost exclusively during regular trading hours (approximately 95.6%, by volume), and market conditions during these times differ substantially from those during the complementary off-hours.

4 HFTs’ Profits from Aggressive Orders

4.1 Measuring Aggressive Order Profitability

Because all E-mini contracts of a given expiration date are identical, it is neither meaningful nor possible to distinguish among the individual contracts in a trader’s inventory, so there is generally no way to determine the exact prices at which a trader bought and sold a particular contract. As a result, it is typically impossible to measure directly the profits that a trader earns on an individual aggressive order. However, the cumulative price change following an aggressive order, normalized by the order’s direction (+1 for a buy, or −1 for a sell), can be used to construct a
meaningful proxy for the order’s profitability. Intuitively, the average expected profit from an aggressive order equals the expected favorable price movement, minus trading/clearing fees and half the bid-ask spread. See Appendix B for rigorous justification.

Estimating the cumulative favorable price movement after an aggressive order is straightforward. Consider a trader who can forecast price movements up to \( j \) time periods in the future, but no further. If the trader places an aggressive order in period \( t \), any price changes that she could have anticipated at the time she placed the order will have occurred by period \( t + j + 1 \). Provided that price is a martingale with respect to its natural filtration, the expected change in price from period \( t + j + 1 \) onward is zero, both from the period \( t \) perspective of the trader and from an unconditional perspective. Thus the change in price between period \( t \) and any period after \( t + j \), normalized by the direction of the trader’s order (\(+1\) for a buy, or \(-1\) for a sell), will provide an unbiased estimate of the favorable price movement following the trader’s order.

The remarks above imply that we can derive a proxy for the profitability of an HFT’s aggressive order using the (direction-normalized) accumulated price-changes following that aggressive order out to some time past the HFT’s maximum forecasting horizon. If we choose too short an accumulation window, the resulting estimates of the long-run direction-normalized average price changes following the HFT’s aggressive orders will be biased downward. As a result, we can empirically determine an adequate accumulation period by calculating cumulative direction-normalized price changes over longer and longer windows until their mean ceases to significantly increase. Using too long an accumulation period introduces extra noise, but it will not bias the estimates. I find that an accumulation period, measured in event-time, of 30 aggressive order arrivals is sufficient to obtain unbiased estimates; for all of the empirical work in this paper, I use an accumulation period of 50 aggressive order arrivals to allow a wide margin for error. See Appendix B for further details.

As noted earlier, the bid-ask spread for the E-mini is almost constantly $12.50 (one tick) during regular trading hours, and the HFTs in my sample face trading/clearing fees of $0.095 to $0.16 per contract, so the average favorable price movement necessary for an HFT’s aggressive order to be profitable is between $6.345 and $6.41 per contract. Since trading/clearing fees vary across traders, I report aggressive order performance in terms of favorable price movement, that
is, earnings gross of fees and the half-spread.

4.2 HFTs’ Overall Profits from Aggressive Orders

To measure the overall mean profitability of a given account’s aggressive trading, I compute the average cumulative price change following each aggressive order placed by that account, weighted by executed quantity and normalized by the direction of the aggressive order. As a group, the 30 HFTs in my sample achieve average aggressive order performance of $7.01 per contract. On an individual basis, nine HFT accounts exceed the relevant $6.25 + fees profitability hurdle, and each of these nine accounts exceeds this hurdle by a margin that is statistically significant at the 0.05 level. One of these nine accounts is linked with another HFT account, and their combined average performance also significantly exceeds the profitability hurdle.

Overall, the HFTs vastly outperform non-HFTs, who earn a gross average of $3.19 per aggressively-traded contract. However, these overall averages potentially confound effects of very coarse differences in the times at which traders place aggressive orders with effects of the finer differences more directly related to strategic choices. For example, if all aggressive orders were more profitable between 1 p.m. and 2 p.m. than at other times, and HFTs only placed aggressive orders during this window, the HFTs’ outperformance would not depend on anything characteristically high-frequency.

To control for potential low-frequency confounds, I divide each trading day in my sample into 90-second segments and regress the profitability of non-HFTs’ aggressive orders during each segment on both a constant and the executed quantities of the aggressive orders. Using these local coefficients, I compute the profitability of each aggressive order by an HFT in excess of the expected profitability of a non-HFT aggressive order of the same size during the relevant 90-second segment. With these additional controls, only 27 HFT accounts continue to exhibit significant outperformance of non-HFTs, and only eight of the 27 accounts are among those whose absolute performance exceeded the profitability hurdle.
4.2.1 A-HFTs and B-HFTs

For expositional ease, I will refer to the eight HFT accounts that make money on their aggressive trades and outperform the time-varying non-HFT benchmark as “A-HFTs,” and to the complementary set of HFTs as “B-HFTs.” The eight A-HFTs have a combined average daily trading volume of 982,988 contracts, and on average, 59.2% of this volume is aggressive. The 22 B-HFTs have a combined average daily trading volume of 828,924 contracts, of which 35.9% is aggressive. Together, the eight A-HFTs place a daily average of 8,994 aggressive orders (during regular trading hours), with a mean size of 60.3 contracts and a median size of 10 contracts. The 22 B-HFTs together place an average of 31,113 aggressive orders per day (during regular trading hours), with a mean size of 8.3 contracts and a median size of 1 contract. Gross of fees, the A-HFTs earn a combined average of $793,342 per day, or an individual average of $99,168 per day, while the B-HFTs earn a combined average of $715,167 per day, or an individual average of $32,508 per day. The highest profitability hurdle among the A-HFTs is $6.37 per aggressively traded contract.

4.3 A-HFTs’ Losses on Small Aggressive Orders

The A-HFTs’ aggressive orders tend to become more profitable as order-size increases. In fact, the A-HFTs all tend to lose money on their smallest aggressive orders. Note also that I refer here to the size of the aggressive orders A-HFTs submit, not the quantity that executes. To make precise both the meaning of “small” aggressive orders, and A-HFTs’ losses on them, I specify cutoffs for order-size and compute the average performance of A-HFTs’ aggressive orders below and above those size cutoffs. Table 1 displays bootstrap confidence intervals for the executed-quantity-weighted average performance of A-HFTs’ aggressive orders weakly below and strictly above various order-size cutoffs.

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3 The preceding descriptive statistics include the small amount of trading activity that occurred outside regular trading hours, except where noted otherwise.

4 This effect appears whether price-changes are measured between the respective last prices at which successive aggressive orders execute (correcting for bid-ask bounce), or between the respective first prices at which they execute, so the positive relationship between executed quantity and subsequent favorable price movements is not simply an artifact of large orders that eat through one or more levels of the orderbook.
Table 1: Performance of A-HFTs’ Aggressive Orders (Dollars per Contract)

<table>
<thead>
<tr>
<th>Cutoff</th>
<th>Below Cutoff 95% CI</th>
<th>Above Cutoff 95% CI</th>
<th>AOs Below Cutoff % of All AOs</th>
<th>AOs Below Cutoff % of Aggr. Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(3.78, 3.89)</td>
<td>(7.59, 7.74)</td>
<td>24.31%</td>
<td>0.40%</td>
</tr>
<tr>
<td>5</td>
<td>(4.17, 4.29)</td>
<td>(7.62, 7.78)</td>
<td>43.74%</td>
<td>1.44%</td>
</tr>
<tr>
<td>10</td>
<td>(3.42, 3.55)</td>
<td>(7.71, 7.85)</td>
<td>54.64%</td>
<td>3.09%</td>
</tr>
<tr>
<td>15</td>
<td>(3.79, 3.92)</td>
<td>(7.71, 7.86)</td>
<td>56.75%</td>
<td>3.54%</td>
</tr>
<tr>
<td>20</td>
<td>(4.08, 4.20)</td>
<td>(7.75, 7.90)</td>
<td>60.82%</td>
<td>4.80%</td>
</tr>
</tbody>
</table>

This table presents descriptive statistics about aggressive orders of varying sizes placed by the A-HFTs. The “Cutoff” column indicates the maximum order-size included in the “below cutoff” statistics. All orders of sizes exceeding the specified cutoff are included in calculations of the “above cutoff” statistics. Columns two and three present 95% confidence intervals for the average gross earnings per contract (in dollars) among aggressive orders in the indicated size division. Column four reports the percentage of all aggressive orders placed by the A-HFTs with an order-size no greater than the indicated cutoff. Column five reports the A-HFTs’ aggressive volume from orders of size no greater than the indicated cutoff, as a percentage of the A-HFTs’ total aggressive volume.

As shown in Table 1, small aggressive orders represent a substantial fraction of the aggressive orders that A-HFTs place. A given A-HFT places an aggressive order of size 20 or less roughly once every 34 seconds on average, and this average interval drops to about 3 seconds during periods of intense market activity. These small orders make up very little of the A-HFTs’ total aggressive volume, but the A-HFTs’ losses on them are non-trivial. On average, each A-HFT loses around $7,150 per trading day ($1.8 million, annualized) on aggressive orders of size 20 or less; this loss represents approximately 7.2% of an average A-HFT’s daily profits.

There are numerous reasons why a trader who consistently manages to profit on large aggressive orders might nonetheless place unprofitable small aggressive orders. Controlling risk, testing out new strategies, and hedging are all plausible explanations. The A-HFTs’ differing performance on small and large aggressive orders is consistent with the pattern that we would expect to see if the small orders were generating valuable information that enabled the A-HFTs to earn greater profits from their large orders, but this is by no means evidence of exploratory
trading.\(^5\) However, it does suggest that these small orders are at least reasonable candidates for exploratory orders, and identifying candidate exploratory orders provides the starting point for direct tests of the exploratory trading model’s sharper empirical predictions.

5 Testing the Exploratory Trading Hypothesis

If the A-HFTs indeed engage in exploration using their small aggressive orders, the exploratory trading model generates the testable predictions presented in section 2. In this section, I introduce empirical analogues of the two quantities from the exploratory trading model that appear in the model’s predictions: the signal of future aggressive order-flow, \(\varphi\), and the market-response that reveals \(\Lambda\). I use these to directly test the predictions in section 2.

The first two predictions concern the explanatory power of market-response information for the earnings of subsequent aggressive orders, and I test these two predictions in the same empirical framework. The third prediction, concerning the incidence of large aggressive orders, requires a slightly different empirical approach, so I consider this prediction separately. I estimate results for the A-HFTs individually, but for compliance with confidentiality protocols, I present cross-sectional averages of these estimates. Empirically, these average results are representative of the results for individual A-HFTs.\(^6\)

5.1 A Simple Measure of Market Response

The predictions in section 2 involve the market response to a given A-HFT’s exploratory orders, which we have conjectured, for the purposes of testing, to be the A-HFT’s small aggressive orders. To make this precise, define an aggressive order to be “small” if that order’s submitted size is less than or equal to a specified size parameter, which I will denote by \(\bar{q}\).

I characterize the market response to a small aggressive order using subsequent changes in orderbook depth. I examine the interval starting immediately after the arrival of a given small

\(^5\)The A-HFTs’ small aggressive orders could serve more than one role at a time, so exploration and the myriad other explanations are not mutually exclusive.

\(^6\)Throughout the E-mini market, there exist assorted linkages between various trading accounts (as, for example, in the simple case where single firm trades with multiple accounts), so the trading-account divisions do not necessarily deliver appropriate atomic A-HFT units. Though the specifics are confidential, the appropriate partition of the A-HFTs is entirely obvious. For brevity, I use “individual A-HFT” as shorthand to “individual atomic A-HFT unit,” as applicable.
aggressive order and ending immediately before the arrival of the next aggressive order (which may or may not be small), and I sum the changes in depth at the best bid and best ask that occur during this interval. For symmetry, I adopt the convention that sell depth is negative and buy depth is positive. I also normalize these depth changes by the sign of the preceding small aggressive order to standardize across buy orders and sell orders.

To simplify the analysis and stack the odds against finding significant results, I initially focus only on the signs of the direction-normalized depth changes. These signs merely indicate whether or not resting depth moved further in the direction of the preceding small aggressive order.

For a given value of $\bar{q}$, I construct the indicator variable $\Omega$, with $k$th element $\Omega_k$ defined by

$$\Omega_k = \begin{cases} 1 & \text{if } DC(k; any, \bar{q}) > 0 \\ 0 & \text{otherwise} \end{cases}$$

(10)

where $DC(k; any, \bar{q})$ denotes the direction-normalized depth change following the last small aggressive order (submitted by anyone) that arrived before the $k$th aggressive order. Similarly, I construct the indicator variable $\Omega^A$, with $k$th element $\Omega^A_k$ defined by

$$\Omega^A_k = \begin{cases} 1 & \text{if } DC(k; AHFT, \bar{q}) > 0 \\ 0 & \text{otherwise} \end{cases}$$

(11)

where $DC(k; AHFT, \bar{q})$ denotes the direction-normalized depth change following the last small aggressive order submitted by a specified A-HFT that arrived before the $k$th aggressive order.

Note the direct parallel between the omega variables and the binary liquidity states in the exploratory trading model.

5.2 Order-Flow Signal

To test the exploratory trading theory, in addition to the measure of market response, we need something analogous to the signal of future aggressive order-flow, $\varphi$. Because we are ultimately
interested in how future aggressive order-flow will affect prices, the task of finding an empirical analogue to \( \varphi \) simplifies to finding variables other than market-response measures that forecast price movements.

I select a handful of lagged market variables that forecast the cumulative price-change between the aggressive orders \( k \) and \( k + 50 \), which I denote by \( y_k \). These variables are: the signs of aggressive orders \( k - 1 \) through \( k - 4 \), the signed executed quantities of aggressive orders \( k - 1 \) through \( k - 4 \), and changes in resting depth between aggressive orders \( k - 1 \) and \( k \) at each of the six price levels within two ticks of the best bid or best ask (with sell depth negative and buy depth positive, as before). To lighten notation, I concatenate these 14 variables in the row vector \( z_{k-1} \). This vector, \( z_{k-1} \), is the analogue of \( \varphi \).

In the same way that price movements in the exploratory trading model can still be forecast to some extent by \( \varphi \) when the liquidity state is unknown, the variables in \( z_{k-1} \) should have some power to forecast \( y_k \), even without the market-response omega variables. As a check on this and a benchmark, I estimate the equation

\[
y_k = z_{k-1} \Gamma + \epsilon_k
\]  

Where \( \Gamma \) is a column vector of 14 coefficients. As desired, the estimated coefficients have the expected signs, and their joint significance is extremely high. I discuss the regression results directly, report coefficient estimates, and discuss the choice of explanatory variables in Appendix C.

5.3 Testing Predictions about Explaining Earnings

5.3.1 Explained Earnings

The first testable prediction of the exploratory trading model is that the information from the market response following a given A-HFT’s small aggressive orders will explain a significant additional component of that A-HFT’s earnings on subsequent aggressive orders, beyond what is explained by a public-information benchmark.

In this paper, the particular notion of “explaining earnings” that I employ involves computing
what a trader’s earnings are expected to be on the basis of some econometric forecast of price movements, and comparing that with the trader’s actual earnings. For concreteness, consider a price forecast based on equation (12). Letting \( \hat{\Gamma} \) denote the estimate of \( \Gamma \), we have

\[
\hat{y}_k = z_{k-1} \hat{\Gamma}
\]

Given the sign of the \( k^{th} \) aggressive order, we can compute the forecast earnings on that order, conditional on the order’s sign. Much as the direction-normalized cumulative price-change \( \text{sign}_k \cdot y_k \) provides an estimate of the true earnings on aggressive order \( k \) (see section 4.1), the direction-normalized forecast cumulative price-change \( \text{sign}_k \cdot \hat{y}_k \) provides an estimate of the forecast earnings on aggressive order \( k \).

Rather than working with the earnings order \( k \) that are explained by a given econometric price forecast, it is convenient to work with the earnings on order \( k \) that are not explained by the specified forecast. I will refer to the earnings on order \( k \) that are not explained by the specified forecast as the “excess earnings on order \( k \) relative to the specified forecast.” In the case above, the excess earnings on order \( k \) relative to the forecast from equation (12), denote it \( \xi_k \), is given by

\[
\xi_k = \text{sign}_k \cdot y_k - \text{sign}_k \cdot \hat{y}_k = \text{sign}_k (y_k - \hat{y}_k)
\]

so \( \xi_k \) is simply the \( k^{th} \) regression residual multiplied by the sign of aggressive order \( k \). The additional component of earnings on aggressive order \( k \) explained by price forecast \( B \), relative to price forecast \( A \) is given by \( \xi_k^B - \xi_k^A \).

Finally, note that all of the earnings discussed in this section are per contract.

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\(^8\)Because \( \hat{y}_k \) uses only information available prior to the arrival of the \( k^{th} \) aggressive order, there is no orthogonality constraint on the \( k^{th} \) regression residual and \( \text{sign}_k \).
5.3.2 Empirical Strategy: Overview

Though the implementation is slightly involved, my basic empirical strategy is straight-forward. First, I augment the regression equation (12) from section 5.2 using either:

1. Market response information from the last small aggressive order placed by anyone—i.e., \( \Omega \), or
2. Both market response information from the last small aggressive order placed by anyone, and market response information from the last small aggressive order placed by a specified A-HFT—i.e., both \( \Omega \) and \( \Omega^A \)

After estimating both of the specifications above, I find the additional component of earnings on larger aggressive orders explained by the second one relative to the first. The market response following an arbitrary small aggressive order is publicly observable. However, because the E-mini market operates anonymously, the distinction between a small aggressive order placed by a particular A-HFT and an arbitrary small aggressive order is private information, available only to the A-HFT who placed the order. Because the market response information from the last small aggressive order placed by anyone is weakly more recent than the market response information from last small aggressive order placed by the A-HFT, comparing the second specification above to the first helps to isolate the effects attributable to private information from effects attributable to public information.

Finally, I compare the additional explained earnings for the specified A-HFT to the additional explained earnings for all other traders. Intuitively, we want to verify that the A-HFT’s exploratory information provides extra explanatory power for the subsequent performance of the trader privy to that information (the A-HFT), but not for the performance of traders who aren’t privy to it (everyone else). Note that “everyone else” includes the A-HFTs other than the specified A-HFT. Some A-HFT accounts and B-HFT/non-HFT accounts belong to the same firms, and various B-HFTs/non-HFTs may be either directly informed or able to make educated inferences about what one or more A-HFTs do. As a result, we should not necessarily expect exploratory information generated by an A-HFT’s small orders to provide no explanatory power whatsoever for all other traders’ performance. However, we should still expect the additional ex-
planatory power for the A-HFT’s performance to significantly exceed that for the other traders’ performance.

**Controlling for Public Information** Comparing the second specification to the first one controls for the effects of most public information, but there could conceivably be some public information that is correlated with the market response to a specified A-HFT’s small aggressive orders and yet uncorrelated with the market response to small aggressive orders placed by everyone else. A natural way to handle this concern is to compare the additional explained performance for the specified A-HFT to the additional explained performance for some other traders who use the same public information. Although trading objectives and sophistication vary widely across many participants in the E-mini market, all of the HFTs are sophisticated, profitable traders, with similar (very short) investment horizons, so it is extremely plausible that they all use very similar public data. Comparing the additional explained performance for the specified A-HFT to that for the other HFTs therefore serves as an added control for any lingering effects from public information.

### 5.3.3 Estimation Procedure

In the model of exploratory trading presented in section 2, exploratory information was valuable only in conjunction with information about future aggressive order flow. Following this notion, I incorporate market-response information by using the indicators $\Omega$ and $\Omega^4$ to partition the benchmark regression from section 5.2.

Recall that in section 5.2, I estimated equation (12),

$$ y_k = z_{k-1} \Gamma + \epsilon_k $$

where $y_k$ denoted the cumulative price-change between the aggressive orders $k$ and $k + 50$, and the vector $z_{k-1}$ consisted of changes in resting depth between aggressive orders $k - 1$ and $k$, as well as the signs and signed executed quantities of aggressive orders $k - 1$ through $k - 4$. Using
the indicator $\Omega$, I now partition the equation above into two pieces and estimate the equation

$$y_k = \Omega_k z_{k-1} \Gamma^a + (1 - \Omega_k) z_{k-1} \Gamma^b + \epsilon_k$$  \hspace{1cm} (13)$$

Next, I use the indicator $\Omega^A$ to further partition (13), and I estimate the equation

$$y_k = \Omega^A_k (k) \left( \Omega_k z_{k-1} \Gamma^c + (1 - \Omega_k) z_{k-1} \Gamma^d \right) + (1 - \Omega^A_k) \left( \Omega_k z_{k-1} \Gamma^e + (1 - \Omega_k) z_{k-1} \Gamma^f \right) + \epsilon_k$$  \hspace{1cm} (14)$$

The variables $y_k$ and $z_{k-1}$ denote the same quantities as before, and the $\Gamma^j$ terms each represent vectors of 14 coefficients.

I estimate (13) and (14) for $\bar{q} = 1, 5, 10, 15, 20$, and for each specification I calculate the relative excess earnings of the specified A-HFT, and of all other trading accounts, on aggressive orders of size strictly greater than $\bar{q}$. As in section 5.3.1, I compute the earnings of aggressive order $k$ in excess of that explained by each regression specification by normalizing the $k^{th}$ residual from the regression by the sign of the $k^{th}$ aggressive order. I now also control for order-size effects directly by regressing the direction-normalized residuals (for the orders of size strictly greater than $\bar{q}$) on the (unsigned) executed quantities and a constant, then subtracting off the executed quantity multiplied by its estimated regression coefficient. Controlling for size effects in this manner makes results more comparable for different choices of $\bar{q}$. Size effects can be addressed by other means with negligible impact on the final results.

For each aggressive order larger than $\bar{q}$ placed by the A-HFT under consideration, I compute the additional component of earnings explained by (14) relative to (13) by subtracting the order’s excess earnings relative to (14) from its excess earnings relative to (13); I stack these additional explained components in a vector that I denote by $\Xi_A$. I repeat this procedure to obtain the analogous vector for everyone else, $\Xi_{ee}$.

Equation (14) has more free parameters than (13), so $\Xi_A$ and $\Xi_{ee}$ will both have positive means. However, additional explanatory power of (14) due exclusively to the extra degrees of freedom will, in expectation, manifest equally for all traders, so the extra degrees of freedom alone should not cause $\Xi_A$ and $\Xi_{ee}$ to differ significantly.
5.3.4 Results

I initially evaluate the first two empirical predictions of the exploratory trading model by comparing the additional explained component of earnings for each A-HFT to the additional explained component of earnings for all other traders. Figure 1 displays the cross-sectional means of $\Xi_A$ and $\Xi_{ee}$ for different values of $\bar{q}$. To formally compare the gain in explanatory power for the A-HFTs to the gain for everyone else, I construct 95% bootstrap confidence intervals for the difference of the pooled means $Mean(\Xi_A) - Mean(\Xi_{ee})$, displayed in Figure 2. Table 5 in Appendix D reports the numeric values from Figures 1 and 2.

Figure 1: Additional Performance Explained (95% Confidence Intervals)

Figure 1 displays various averages (with 95% confidence intervals) of the additional earnings per contract explained by (14) beyond what is explained by (13). The white circles mark this average computed among orders placed by an A-HFT, and the black squares mark this average computed among orders placed by everyone else. Estimates were run for each individual A-HFT and the corresponding “everyone else,” and the displayed numbers are cross-sectional averages of the individual estimates. The horizontal axis specifies the order-size cutoff used to compute the corresponding values.
Figure 2 displays the difference of a) the average among orders placed by an A-HFT of the additional earnings per contract explained by (14) beyond what is explained by (13), and b) the analogous average computed among orders placed by everyone else. 95% confidence intervals are displayed around each average. Estimates were run for each individual A-HFT and the corresponding “everyone else,” and the displayed numbers are cross-sectional averages of these individual estimates. The horizontal axis specifies the order-size cutoff used to calculate the indicated values.

Both of the tested predictions are borne out in these results. Information about the market activity immediately following an A-HFT’s smallest aggressive orders (in the form of $\Omega^4A$) improves our ability to explain that A-HFT’s earnings on larger subsequent aggressive orders by a highly significant margin, relative to using only information about the activity following any small aggressive order (in the form of $\Omega$). Furthermore, the extra component of A-HFTs’ earnings on large aggressive orders explained by using $\Omega^4$ in addition to $\Omega$ is significantly greater than the extra component explained for other traders.

I refine my empirical evaluation of the first two predictions by comparing the additional explained component of earnings for each A-HFT to the additional explained component of earnings for the other HFTs. Consistent with the notion that certain B-HFTs may know something about what various A-HFTs are doing, the extra component of earnings explained by using $\Omega^4$ in addition to $\Omega$ is larger for the complementary set of HFTs than it is for the broader “everyone except the A-HFT of interest” group. Nevertheless, aside from the case of $\bar{q} = 1$, the average added explanatory power for each A-HFT is still significantly greater than is that for the complementary set of HFTs, as shown in Figure 3. (See Table 5 in Appendix D for numeric values.)
Figure 3 displays the difference of a) the average among orders placed by an A-HFT of the additional earnings per contract explained by (14) beyond what is explained by (13), and b) the analogous average computed among orders placed by all other HFTs. 95% confidence intervals are displayed around each average. Estimates were run for each individual A-HFT and the corresponding “all other HFTs,” and the displayed numbers are cross-sectional averages of these individual estimates. The horizontal axis specifies the order-size cutoff used to calculate the indicated values.

5.4 Incidence of A-HFTs’ Larger Aggressive Orders

In this subsection I test the exploratory trading model’s third prediction, namely that the market response to a given A-HFT’s small aggressive order provides significant explanatory power for the incidence of that A-HFT’s subsequent large aggressive orders, above and beyond that explained using the market response to the last small aggressive order placed by anyone. Since elements of the binary $\Omega$-operators correspond almost directly to the binary liquidity-state $\Lambda$ in the exploratory trading model, the incidence prediction can be made even more precise. In particular, all else being equal, the exploratory trading model predicts that an A-HFT will have a greater tendency to place large aggressive orders when $\Omega^A = 1$ than when $\Omega^A = 0$.

5.4.1 Empirical Implementation

Much as the HFT in the model from section 2 considered the signal of future aggressive order-flow as well as the liquidity state, A-HFTs consider public market data as well as exploratory information to decide when to place large aggressive orders. The size and direction of A-HFTs’ aggressive orders depend on the same variables that forecast price movements, or equivalently on the forecasts of price movements themselves. On average, the signed quantity of an A-HFT’s
aggressive order should be an increasing function of the future price-change expected on the basis of public information. In this context, the exploratory trading model predicts that the expected future price-change will have a larger effect on the signed quantity of an A-HFT’s aggressive orders when $\Omega^A = 1$ than it will when $\Omega^A = 0$.

To test the exploratory trading model’s prediction about the incidence of A-HFTs’ larger aggressive orders, I regress the signed quantities of a given A-HFT’s aggressive orders on the associated fitted values of $y$ from equation (13), partitioned by $\Omega^A$. In other words, for a specified A-HFT and a given value of $\bar{q}$, I estimate the equation

$$q_k = \beta_0 \left(1 - \Omega^A_k\right) \hat{y}_k + \beta_1 \Omega^A_k \hat{\gamma}_k + \epsilon_k$$

where $q_k$ denotes the signed submitted quantity of the A-HFT’s $k$th aggressive order, $\hat{y}_k$ denotes the relevant fitted value of $y_k$ from the public-information regression (13), and $\Omega^A$ is the usual indicator function. I restrict the $\beta$ coefficients to be the same across all A-HFTs. Note that the fitted value $\hat{y}_k$ includes the public market-response information through the inclusion of $\Omega_k$ in (13), so differences between $\beta_0$ and $\beta_1$ do not arise from any public information in $\Omega^A$.

5.4.2 Results

Table 2 displays the coefficient estimates from (15) for various values of $\bar{q}$. A Wald test rejects the null hypothesis $\beta_0 = \beta_1$ at the $10^{-15}$ level for all values of $\bar{q}$. As the exploratory trading model predicts, holding fixed the price-change expected on the basis of public information, the average A-HFT places significantly larger aggressive orders when $\Omega^A = 1$ than when $\Omega^A = 0$. 

34
Table 2: Differential Effects of Predicted Price-Changes on A-HFT Signed Order Size

<table>
<thead>
<tr>
<th></th>
<th>$\bar{q} = 1$</th>
<th>$\bar{q} = 5$</th>
<th>$\bar{q} = 10$</th>
<th>$\bar{q} = 15$</th>
<th>$\bar{q} = 20$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$ (standard error)</td>
<td>13.35 (0.094)</td>
<td>13.41 (0.093)</td>
<td>13.42 (0.095)</td>
<td>13.34 (0.095)</td>
<td>13.23 (0.094)</td>
</tr>
<tr>
<td>$\beta_1$ (standard error)</td>
<td>15.26 (0.162)</td>
<td>15.11 (0.169)</td>
<td>14.97 (0.160)</td>
<td>15.10 (0.159)</td>
<td>15.30 (0.160)</td>
</tr>
</tbody>
</table>

Table 2 reports coefficient estimates from the regression of the signed quantities of the A-HFTs’ aggressive orders on the fitted values of cumulative future price changes from equation (13):

$$q_k = \beta_0 (1 - \Omega_A) \tilde{y}_k + \beta_1 \Omega_A \tilde{y}_k + \epsilon_k$$

The indicator $\Omega_A$ is the same market-response variable used elsewhere. The $\bar{q}$ values specify the order-size cutoff used in the indicated estimation.

5.5 Alternative Explanations

Although the empirical results in this section confirm the predictions of the exploratory trading theory, that does not necessarily rule out alternative explanations for those results. To the extent that a movement of resting depth in the same direction as the previous order is indicative of informed trading, the results in this section are basically consistent with the story that the A-HFTs possess private information and they split up their orders.

However, the privately informed order-splitter story is not observationally equivalent to the exploratory trading theory. The order-splitting story in its simplest form would imply that the market response to an A-HFT’s small aggressive order will only help to explain the A-HFT’s earnings on a subsequent large aggressive order if both the small order and the large order have the same sign. The exploratory trading theory predicts that the market response to the small order will help to explain earnings on the large order, regardless of whether the two orders have the same sign. As the simulated trading strategy results in the next section will suggest, the market response to an A-HFT’s small aggressive order can be used to better forecast the performance of a subsequent aggressive order in either direction.\(^9\)

\(^9\)Aside from increases in the standard errors of estimates, the results in this section are not qualitatively changed by restricting attention to the market response to an A-HFT’s last small aggressive order with the opposite sign.
More fundamentally, the order-splitting story necessarily implies that the A-HFTs possess private information which they do not trade on as quickly as they are able. That is a much stronger and much less realistic claim than the implication of exploratory trading that reacting to public information the soonest is not the A-HFTs’ only source of private information.

6 Practical Significance of Exploratory Information

The empirical evidence in section 5 provides strong support for the hypothesis that the A-HFTs engage in exploratory trading as modeled in section 2. However, while these results indicate that exploratory trading plays some part in what the A-HFTs are doing to profit from their aggressive orders, the results tell us little about how large that part is.

Estimates of the additional component of the A-HFTs’ aggressive-order earnings directly explained by the private information in $\Omega^A$ are likely to dramatically understate the true contribution of exploratory information, for two reasons. First, $\Omega^A$ is nearly the simplest possible characterization of exploratory information. Representations of exploratory information richer than $\Omega^A$ are extremely easy to construct. For example, an obvious extension would be to consider the not only the sign, but also the magnitude of the direction-normalized depth change following an exploratory order. Regardless of the particular representation of exploratory information used, though, the additional explained component of A-HFTs’ profits on the aggressive orders they place is likely to understate the true gains from exploration. As the simple model in section 2 illustrates, exploratory information is valuable in large part because it enables a trader to avoid placing unprofitable aggressive orders. However, estimates of the additional explained component of profits on A-HFTs’ aggressive orders necessarily omit the effects of such avoided losses. While this bias, if anything, makes the preceding findings of statistical significance all the more compelling, it also complicates the task of properly determining the practical importance of exploratory information.

from the present order. Numerical results will not be available for distribution until the completion of the CFTC review.
6.1 Simulated Trading Strategies

To investigate the gains from exploratory information, including the gains from avoiding unprofitable aggressive orders, I examine the effects of incorporating market-response information from small aggressive orders into simulated trading strategies. The key advantage of working with these simulated trading strategies is that avoided unprofitable aggressive orders can be observed directly.

The basic trading strategy that I consider is a simple adaptation of the benchmark regression from section 5.2. I specify a threshold value, and the strategy entails nothing more than placing an aggressive order with the same sign as \( \hat{y}_k \) whenever \( |\hat{y}_k| \) exceeds that threshold. To make this strategy feasible (in the sense of using only information available before time \( t \) to determine the time-\( t \) action) I compute the forecast of the future price movement, \( \hat{y}_k \), using the regression coefficients estimated from the previous day’s data. I incorporate market-response information into this strategy by modifying the rule for placing aggressive orders to, “place an aggressive order (with the same sign as \( \hat{y}_k \)) if and only if all three of the following conditions hold:

- \( |\hat{y}_k| \) exceeds its specified threshold,
- The direction-normalized depth-change following the last small aggressive order (placed by anyone) exceeds a specified threshold, and
- The direction-normalized depth-change following the last small aggressive order placed by an A-HFT exceeds a (possibly different) specified threshold.”

Choosing a threshold of \(-\infty\) will effectively remove any of these conditions.

Each strategy yields a set of times to place aggressive orders, and the associated direction for each order. To measure the performance of a given strategy, I compute the average profitability of the indicated orders in the usual manner, with the assumption that these aggressive orders are all of a uniform size.

Relative to A-HFTs’ losses on small aggressive orders, the additional component of A-HFTs’ profits directly explained using \( \Omega^A \) is smallest when \( \bar{q} = 10 \), and I present results for \( \bar{q} = 10 \) to highlight the impact of accounting for avoided losses on estimates of the gains from exploratory information. Results for other values of \( \bar{q} \) are similar.
6.2 Three Specific Strategies

All three threshold parameters affect strategy performance, so to emphasize the role of market-response information, I present results with the threshold for $|\hat{y}_k|$ held fixed. Varying the threshold for $|\hat{y}_k|$ does not alter the qualitative results. In particular, it is not possible to achieve the same gains in performance that result from incorporating exploratory information by merely raising the threshold for $|\hat{y}_k|$. The forecast $\hat{y}_k$ uses coefficients estimated from the previous day’s data, and these forecasts exhibit increasing bias as the $z_{k-1}$ observations assume more extreme values.

I consider a range of threshold values for the direction-normalized depth-change following the last small aggressive order placed by anyone, but, for expositional clarity, I present results for three illustrative threshold choices for the direction-normalized depth-change following the last small aggressive order placed by an A-HFT. Specifically, I consider thresholds of $-\infty$ (no A-HFT market-response information), 0 (the same information contained in $\Omega^A$), and 417 (the 99th percentile value). Figure 4 displays the performance of these three strategies over a range of threshold values for the market response following arbitrary small aggressive orders.

![Figure 4: Absolute Gains from Exploratory Information](image)

Figure 4 reports the estimated average gross earnings per aggressively traded contract for the three simulated trading strategies. The three strategies differ in the threshold value for direction-normalized depth-change following the last small aggressive order placed by an A-HFT that must be satisfied in order for the strategy to enter a trade. The horizontal axis is the threshold value (in percentiles) for the direction-normalized depth-change following arbitrary small aggressive orders.
While the performance gains from incorporating A-HFT exploratory information are obvious, an equally important feature of the results above is more subtle. The A-HFTs’ average gross earnings on aggressive orders over size 10 of $7.78 per contract are well above the peak performance of the strategy that uses only public information, but substantially below the performance of the strategy that incorporates the A-HFTs’ exploratory information with the higher threshold. This is exactly the pattern that we should expect, given that the former strategy excludes information that is available to the A-HFTs and the latter strategy includes information that is not available to any individual A-HFT, so these results help to confirm the relevance and validity of this simulation methodology.

6.2.1 Gains from Exploration Relative to Losses on Exploratory Orders

Although the two strategies that incorporate exploratory information from the A-HFTs’ small aggressive orders outperform the strategy that does not, the orders that generated the exploratory information were costly. To compare the gains from this exploratory information to the costs of acquiring it, I first multiply the increases in per-contract earnings for the two exploratory strategies (scaled by the respective number of orders relative to the public-information strategy) by the A-HFTs’ combined aggressive volume on orders over size 10.\textsuperscript{10} I then divide these calibrated gains by the A-HFTs’ actual losses on aggressive orders size 10 and under.

Figure 5 displays the calibrated ratio of additional gains to losses for each exploratory simulated strategy over a range of threshold values for the market response following arbitrary small aggressive orders. Using information from the A-HFTs’ exploratory orders analogous to that in $\Omega^A$, the additional gains are roughly 15% larger than the losses on exploratory orders. Whereas the extra component of the A-HFTs’ performance directly explained using $\Omega^A$ represents less than 5% of A-HFTs’ losses on exploratory orders, the analogous estimated performance increases more than offset the costs of exploration once we include the gains from avoiding unprofitable aggressive orders. In the case of the strategy that employs information from the A-HFTs’ exploratory orders with the higher threshold, the estimated gains from exploration exceed the costs by more

\textsuperscript{10} The two strategies that incorporate exploratory information select subsets of the aggressive order placement times generated by the public-information-only strategy. Although the selected orders tend to be more profitable, they are also fewer in number.
than one-third.

Figure 5: Gains from A-HFT Exploratory Info Relative to Losses on Exploratory Orders

![Graph showing the ratio of extra gains to losses for two exploratory simulated strategies across a range of threshold values for the market response following arbitrary small aggressive orders.]

Figure 5 displays the calibrated ratio of additional gains to losses for the two exploratory simulated strategies across a range of threshold values (in percentiles) for the market response following arbitrary small aggressive orders.

Even after netting out the calibrated losses on exploratory orders from the better-performing exploratory-information simulated strategy in 6.2, the simulated performance exceeds the maximum profitability hurdle among HFTs of $6.41 per aggressively traded contract. An almost trivial trading strategy that incorporates exploratory trading appears to be profitable, suggesting very strongly that exploratory trading is at least sufficient to explain how a trader in the E-mini market could consistently profit on average from her aggressive orders.

7 Discussion

7.1 Broader Opportunities for Exploratory Gains from Aggressive Orders

The empirical analysis in the preceding sections focused on the information generated by the A-HFTs' smallest aggressive orders. While these orders were the most natural starting point for an empirical study of exploratory trading, there is no theoretical reason why these small orders should be the sole source of exploratory information. In the baseline exploratory trading model, it was only to highlight the key aspects of the model that I assumed the HFT's period-1 order was expected to lose money and served no purpose other than exploration.
In principle, even aggressive orders that an A-HFT expects to be directly profitable could produce valuable, private, exploratory information. To investigate this possibility, I repeat the analysis of section 5.3 setting $\bar{q} = 25, 30, 35, 40, 45, 50, 60, 75, 90$. The A-HFTs’ incremental aggressive orders included with each increase of $\bar{q}$ beyond $\bar{q} = 20$ are directly profitable on average, and yet the market response following these orders still provides significantly more additional explanatory power for the A-HFTs’ performance on larger aggressive orders than it provides for that of other traders. Indeed, the additional explained components of the A-HFTs’ performance are markedly larger than those for $\bar{q} = 1, 5, \ldots, 20$; see Figure 6, and see Table 5 in Appendix D for numeric results.

Figure 6: [A-HFT Addt'l Explained] - [Everyone Else Addt'l Explained] (95% CIs)

These results have the interesting implication that the A-HFTs enjoy natural and almost inevitable economies of scale—simply by being in the market and engaging in lots of aggressive trading, they automatically generate lots of valuable, private information. Other, more obvious economies of scale and scope likely exist for high-frequency traders (e.g., tiered trading costs, applicability of similar algorithms across different markets), but the economies of scale arising...
from exploratory information appear to be new. The impressive performance of the extremely simple simulated strategies in section 6 casts doubt on the standard fallback of “intellectual capital” as a barrier to entry. Although the A-HFTs earn positive profits on average, their marginal profits need not be strictly positive, so there may be no incentive for new A-HFTs to enter. However, should the structure of the A-HFT industry indicate the existence of some barriers to entry, the A-HFTs’ apparent economies of scale could potentially act as one such barrier. Industrial organization of high-frequency trading entities is an intriguing open area for future investigation, but detailed treatment lies beyond the scope of this paper.

7.2 Exploratory Trading and Speed

Evidence in this paper provides empirical justification for using the exploratory trading model to draw conclusions about real-world high-frequency trading. Further analysis of the exploratory trading model reveals natural connections between exploration and two important concepts of speed. These connections in turn help to illuminate the role that the two types of speed play in high-frequency trading.

7.2.1 Low Latency

One common measure of trading speed is latency—the amount of time required for messages to pass back and forth between a trader and the market. While low-latency operation and high-frequency trading are not equivalent, minimal latency is nonetheless a hallmark of high-frequency traders. HFTs can certainly react and communicate faster than some other market participants, but analogous differences in the relative reaction speed of various traders long predate high-frequency trading. For a trader who can identify profitable trading opportunities, there is obvious value to possessing latency low enough to take advantage of these opportunities before they disappear. The new insight from the exploratory trading model concerns the more subtle matter of how low latency connects to the identification of such opportunities, that is, why it might matter for latency to be low in absolute terms.

In the model of exploratory trading developed in section 2, the HFT’s inference about \( \Lambda \) on the basis of market activity following his aggressive order in period 1 implicitly depends on a
notion related to latency. If we suppose that random noise perturbs the orderbook, say according to a Poisson arrival process, then the amount of noise present in the HFT’s observation of the market response in some interval following his aggressive order will depend on the duration of that interval. The duration of this interval will depend in large part upon the rate at which market data is collected and disseminated to the HFT, that is, the “temporal resolution” of the HFT’s data. Although this temporal resolution does not directly depend on the HFT’s latency, the temporal resolution of the HFT’s market information does implicitly constrain how quickly the HFT can learn about market events.

The finer temporal resolution required for low-latency operation enables low-latency traders to obtain meaningful—and empirically valuable—information about the market activity immediately following their aggressive orders, and this information degrades at coarser temporal resolutions. The empirical results from section 5.3.4 provide a concrete illustration of this effect. The changes in resting depth immediately following an arbitrary aggressive order are less useful for forecasting price movements than are the analogous changes following an A-HFT’s aggressive order, but the two can only be distinguished (by the A-HFT) in data with a sufficient level of temporal disaggregation.

7.2.2 High Frequency

Exploratory trading bears a natural relationship to the practice of placing large numbers of aggressive orders—what might be considered “high-frequency trading” in the most literal sense.

Exploratory information generated by a given aggressive order is only valuable to the extent that it can be used to improve subsequent trading performance. Because exploratory information remains relevant for only some finite period, the value of exploratory information diminishes as the average interval between a trader’s orders lengthens. The exploratory trading model readily captures this effect if we relax the simplifying assumption that the liquidity state \( \lambda \) remains the same between periods 1 and 2. Suppose that \( \lambda \) evolves according to a Markov process, such that with probability \( \tau \), a second \( \lambda \) is drawn in period 2 (from the same distribution as in period 1), and with probability \( 1 - \tau \), the original value from period 1 persists in period 2. Intuitively, \( \tau \) parametrizes the length of period 1, and this length increases from zero to infinity.
as $\tau$ increases from zero to unity. As $\tau$ tends towards unity—i.e., as the length of period 1 increases to infinity—the liquidity state in period 1 becomes progressively less informative about the liquidity state in period 2.

As discussed in section 7.1, both theory and empirical evidence suggest that almost any aggressive order that a trader places generates some amount of exploratory information. Consequently, as a trader places aggressive orders in greater numbers, he will gain access to greater amounts of exploratory information. Furthermore, the average time interval between a trader’s aggressive orders necessarily shrinks as the number of those orders grows, so the exploratory information produced by each order tends to become more valuable to the trader. These synergistic effects dramatically magnify the potential gains from exploratory information for traders who place large numbers of aggressive orders.

### 7.2.3 Latency Détente

There has been much speculation about HFTs engaging in an “arms race” for ever-faster processing and ever-lower latency. If high-frequency trading entailed nothing more than reacting to publicly observable trading opportunities before anyone else, HFTs would indeed face nearly unbounded incentives to be faster than their competitors. While reaction speed is certainly one dimension along which HFTs compete, the empirical evidence of exploratory trading suggests that the A-HFTs, at least, can also compete along another dimension—exploration. Since exploratory trading provides the A-HFTs with private information, a trader who uses only public information will not necessarily be able to dominate the A-HFTs, even if that trader is faster than every A-HFT. Similarly, an A-HFT could potentially compensate for having (slightly) slower reactions than the other A-HFTs by engaging in greater levels of exploration.

Because A-HFTs can compete on their levels of exploration as well as on speed, even in a long-run equilibrium, technology suppliers will not necessarily capture all economic rents that HFTs earn. Provided that exploratory orders are costly on average, A-HFTs’ losses on those exploratory orders are transfers to the A-HFTs’ respective liquidity-providing counterparties.
7.3 Beyond A-HFTs: Other HFTs and Other Markets

Exploratory trading is not universally relevant to all HFT activity in all markets. However, exploratory trading in the E-mini market depends only on the market’s structure and aggregate dynamics—it does not depend directly on any specific features of the E-mini contract. The prevalence of exploratory trading in other markets is ultimately an empirical matter, but markets resembling the E-mini in size and structure could easily support exploratory trading. Indeed, as noted in section 2.1, in a one-tick market where the bid-ask spread is constrained by tick-size, a few very standard microstructure elements suffice to produce scope for exploratory trading. Yao and Ye (2014), find that it is extremely common in modern markets for spreads to equal the minimum tick-size, so the applicability of exploratory trading could in fact be very general.

Even in the E-mini market, an important component of HFT activity lies outside the immediate province of the exploratory trading model. Nevertheless, the scope for exploratory trading extends well beyond the aggressive activity of A-HFTs considered thus far. Though I have focused on the A-HFTs up to this point, the B-HFTs could also reap exploratory rewards from their aggressive orders, as could potentially any trader with similar capabilities. The B-HFTs’ overall performance on aggressive orders does not present the same striking evidence of informed trading as does that of the A-HFTs, but the B-HFTs’ aggressive orders nonetheless outperform both those of non-HFTs, and the baseline econometric benchmark, by a wide margin. If inventory management or risk-control considerations force B-HFTs to place unprofitable aggressive orders, exploratory trading could help to explain how the B-HFTs mitigate the associated losses.

8 Conclusion

Empirical evidence strongly suggests that the concept of exploratory trading developed in this paper helps to explain the mechanism underlying certain HFTs’ superior capacity to profitably anticipate price movements in the E-mini market. The exploratory trading model also illuminates the manner in which these HFTs benefit from low-latency capabilities and from their submission of large numbers of aggressive orders.

Exploratory trading is a form of costly information acquisition, albeit an unfamiliar one.
HFTs who engage in exploratory trading are doing something more than merely reacting to public information sooner other market participants. This raises the possibility that HFTs, through exploratory trading, uniquely contribute to the process of efficient price discovery. However, exploratory trading differs from traditional costly information acquisition in several important respects. First, the information that exploratory trading generates does not relate directly to the traded asset’s fundamental value, but rather pertains to unobservable aspects of market conditions that could eventually become public, *ex-post*, through ordinary market interactions. Also, because exploratory trading operates through the market mechanism itself, exploration exerts direct effects on the market, distinct from the subsequent effects of the information that it generates. Finally, since HFTs appear to trade ahead of predictable demand innovations—albeit in a sophisticatedly selective manner—the research of De Long *et al.* (1990) potentially suggests that HFTs could have a destabilizing influence on prices if suitable positive-feedback mechanisms exist.

Comprehensive analysis of the theoretical and empirical aspects of these myriad issues lies beyond the scope of this paper, but the theory and evidence presented herein provide a starting point from which to rigorously address the market-quality implications of high-frequency trading going forward.
References


A Exploratory Trading Model Details

A.1 Solving the Baseline Exploratory Trading Model

Let $s_t$ denote the sign of $q_t$.

A.1.1 Solving the Model: Period 2

If $\varphi = 0$, the HFT’s optimal choice is to not submit an aggressive order in period 2, or equivalently, to set $|q_2| = 0$. If $\varphi \neq 0$, then it is optimal for the HFT to set $s_2 = \varphi$ (unless the optimal $|q_2|$ is zero), so we only need to determine the optimal magnitude, $|q_2|$. Because $\pi_2$ is linear in $|q_2|$ when $s_2$ is held fixed, we can restrict attention to corner solutions (0 or $N$) for the optimal choice of $|q_2|$ without loss of generality. Note that if $q_2 = 0$, then $\pi_2 = 0$, regardless of the values of $\varphi$ and $\Lambda$.

Suppose that the HFT sets $|q_2| = N$. Without loss of generality, assume that $s_2 = \varphi \neq 0$. The HFT’s period-2 profits are given by

$$
\tilde{\pi}_2 = \begin{cases} 
N (1 - \alpha) & \text{if } \Lambda = U \\
-N\alpha & \text{if } \Lambda = A
\end{cases}
$$

where the tilde on $\tilde{\pi}_2$ denotes the fact that the HFT’s choice of $q_2$ does not condition on the value of $\Lambda$.

HFT Does Not Know $\Lambda$ If the HFT does not know the value of $\Lambda$, then in the case where $\varphi \neq 0$, the HFT’s expected period-2 profit if he sets $|q_2| = N$ is

$$
\mathbb{E} [\tilde{\pi}_2 | \varphi \neq 0, |q_2| = N] = u N (1 - \alpha) - (1 - u) N\alpha
$$

$$
= (u - \alpha) N
$$
Taking expectations with respect to \( \varphi \), we find that the *ex-ante* expectation of \( \tilde{\pi}_2 \) when the HFT sets \( |q_2| = N \) (and \( s_2 = \varphi \)) is given by

\[
\mathbb{E} [\tilde{\pi}_2 | q_2 = N] = v (u - \alpha) N
\]

(18)

When \( u - \alpha < 0 \), if the HFT did not know \( \Lambda \), he would set \( q_2 = 0 \) rather than \( |q_2| = N \). Hence the *ex-ante* expectation of \( \tilde{\pi}_2 \) is

\[
\mathbb{E} [\tilde{\pi}_2] = \max \{ v (u - \alpha) N, 0 \}
\]

(19)

**HFT Knows \( \Lambda \)** Next, if the HFT *does* know the value of \( \Lambda \), then he will set \( |q_2| = N \) (and \( s_2 = \varphi \)) only when \( \Lambda = U \) and \( \varphi \neq 0 \). Denoting the HFT’s period-2 profits from this strategy by \( \hat{\pi}_2 \), we find

\[
\mathbb{E} [\hat{\pi}_2 | \varphi \neq 0] = u (1 - \alpha) N
\]

(20)

\[
= (u - \alpha) N + \alpha (1 - u) N
\]

\[
\mathbb{E} [\hat{\pi}_2] = vu (1 - \alpha) N
\]

(21)

\[
= v (u - \alpha) N + v\alpha (1 - u) N
\]

Note that

\[
\mathbb{E} [\hat{\pi}_2] > \max \{ v (u - \alpha) N, 0 \}
\]

(22)

so the HFT’s expected period-2 profits are strictly greater when he knows \( \Lambda \) than when he doesn’t know \( \Lambda \).

**A.1.2 Solving the Model: Period 1**

At the start of period 1, the HFT knows neither \( \varphi \) nor \( \Lambda \), but he faces the same trading costs, \( \alpha \), as he does in period 2. Consequently, the HFT’s expected direct trading profits from a period-1
aggressive order are negative:

\[
E[\pi_1|q_1] = E[q_1|s_1y - \alpha]s_1, q_1
\]

\[
= |q_1|s_1E[y] - \alpha |q_1|
\]

\[
= -\alpha |q_1|
\]

The second equality relies on the assumptions that \( \varphi \) and \( \Lambda \) (and hence \( y \)) are independent of \( s_1 \) and \( q_1 \), while the final equality uses the fact that \( E[y] = 0 \).

Since there is no noise in this baseline model, the HFT learns \( \Lambda \) perfectly from any aggressive order that he places in the first period with \( |q_1| \geq 1 \). An aggressive order of size greater than one yields no more information about \( \Lambda \) than a one-contract aggressive order in this setting, but the larger aggressive order incurs additional expected losses. Thus without loss of generality, we can restrict attention to the case of \( q_1 = 0 \) and the case of \( |q_1| = 1 \).

If the HFT sets \( q_1 = 0 \), he neither learns \( \Lambda \) nor incurs any direct losses in period 1, so his total expected profits are simply

\[
E[\pi_{\text{total}}|q_1 = 0] = E[\pi_2]
\]

\[
= \max\{v(u - \alpha)N, 0\}
\]

Alternatively, if the HFT sets \( |q_1| = 1 \), his total expected profits are given by

\[
E[\pi_{\text{total}}|q_1 = 1] = -\alpha |q_1| + E[\pi_2]
\]

\[
= vu(1 - \alpha)N - \alpha
\]

### A.1.3 Comparative Statics for Model Parameters

Recall that when the exogenous aggressive order-flow is described by \( \varphi = 0 \), the HFT does not have any profitable period-2 trading opportunities in either liquidity state. The probability that \( \varphi \neq 0 \), given by the parameter \( v \), represents the extent to which the exogenous aggressive order-flow is predictable. To characterize how various parameters affect the viability of exploratory
trading, I consider the minimal value of \( v \) for which the HFT finds it optimal to engage in period-1 (i.e., exploratory) trading. Denoting this minimal value by \( \underline{v} \), we have

\[
\underline{v} = \left( \frac{\alpha}{u} \right) \frac{1}{(1 - \alpha) N}
\]  

(26)

The closer is \( \underline{v} \) to 0, the more conducive are conditions to exploratory trading, and by inspection, \( \frac{\partial \underline{v}}{\partial \alpha} > 0 \), \( \frac{\partial \underline{v}}{\partial N} < 0 \) and \( \frac{\partial \underline{v}}{\partial u} < 0 \).

The above results are intuitive. First, higher trading costs (\( \alpha \)) tend to discourage exploratory trading. Second, when the HFT can use exploratory information to guide larger orders, the gains from exploration are magnified, so larger values of \( N \) tend to promote exploratory trading. Finally exploratory trading becomes less viable when \( u \) is smaller. The HFT will take the same action in period 2 when he knows that \( \Lambda = \Lambda \) as when he doesn’t know \( \Lambda \), so when \( u \) is small, knowledge of the liquidity state is less valuable because it is less likely to change the HFT’s period-2 actions.\(^\text{11}\)

A.1.4 Remark on the Sequence of Events

The central results of the model would not change if the HFT observed the signal of future aggressive order-flow before deciding whether to engage in exploratory trading, rather than observing it after deciding. However, the sequence of events outlined in section 2, in which the HFT must choose whether or not to explore before he observes \( \varphi \), is more appropriate from an empirical perspective. For the HFT to learn about the liquidity state after he submits an aggressive order, he must wait for 1) his order to reach the market and execute, 2) information about that execution to reach other traders, 3) other traders to decide what to do, 4) other traders’ decisions to reach the market, and 5) information about the market response to get back to him. Of these five steps, (1), (2), (4) and (5) each take approximately 3 – 4 milliseconds, and (3) takes considerably longer, perhaps 3 – 20 milliseconds, for an overall total of 15 – 40 milliseconds. An HFT who has already done his exploration will be able to take advantage of

---

\(^{11}\)When \( u > \alpha \), the HFT will take the same action in period 2 when he knows that \( \Lambda = \Lambda \) as when he doesn’t know \( \Lambda \), so knowledge of the liquidity state is less likely to change the HFT’s period-2 actions when \( u \) is large. In the case of \( u > \alpha \), equation (26) becomes \( \underline{v} = \frac{1}{(1 - \alpha)N} \), and exploratory trading indeed becomes less viable as \( u \) approaches 1.
predictable aggressive order-flow long before an HFT who only engages in exploratory trading after seeing an order-flow signal.

A.2 Solving the Model of Section 2.5

A.2.1 Formalizing the Intuition

To make the preceding intuition more rigorous, consider a variant of the baseline model from section 2.3 in which someone other than the HFT places an aggressive order at the beginning of period 1. With probability \( \rho \), this aggressive order is the result of an unobservable informational shock, and resting depth further depletes following the order, regardless of the liquidity state \( \Lambda \). Otherwise (with probability \( 1 - \rho \)) resting depth further depletes after the order if and only if the liquidity state is unaccommodating. Aside from this new aggressive order, all other aspects of the baseline model remain unchanged.

If the HFT places an aggressive order in period 1, his expected total profits are the same as they were in the baseline model, i.e.,

\[
E[\pi_{\text{total}} | q_1 = 1] = Nvu (1 - \alpha) - \alpha
\]  

(27)

However, the HFT’s expected profits if he does not place an order in period 1 are now higher than they were in the baseline model, because the HFT learns something from the depth changes following the other trader’s aggressive order. If resting depth weakly replenishes after that order, the HFT learns with certainty that the liquidity state is accommodating (i.e., \( \Lambda = A \)), so the HFT will not submit an aggressive order in period 2, and his total profits will be zero. Alternatively, if resting depth further depletes following the aggressive order in period 1 (denote this event by \( g_1 \)), we have
\[
\mathbb{P}\{\Lambda = U | g_1\} = \frac{\mathbb{P}\{\Lambda = U, \text{ and } g_1\}}{\mathbb{P}\{g_1\}} \\
= \frac{\mathbb{P}\{g_1|\Lambda = U\} \mathbb{P}\{\Lambda = U\}}{\mathbb{P}\{g_1|\Lambda = U\} \mathbb{P}\{\Lambda = U\} + \mathbb{P}\{g_1|\Lambda = A\} \mathbb{P}\{\Lambda = A\}} \\
= \frac{1 * \mathbb{P}\{\Lambda = U\}}{u + \rho \mathbb{P}\{\Lambda = A\}} \\
= \frac{u}{u + \rho (1 - u)}
\] (29)

The HFT’s optimal strategy when he does not know $\Lambda$ is to set $q_2 = \varphi N$ when $\frac{u}{u + \rho (1 - u)} \geq \alpha$, and to set $q_2 = 0$ otherwise. Taking expectations with respect to $\Lambda$ and $\varphi$, we find that the HFT’s ex-ante expected total profits in this case are given by

\[
\mathbb{E}[\pi_{total} | AO \text{ by someone else}] = \max \left\{ N \varphi \left( \frac{u}{u + \rho (1 - u)} - \alpha \right), 0 \right\}
\] (30)

The features of the baseline model discussed in section 2.4 are qualitatively unchanged in the modified version, but now the “privacy” parameter $\rho$ also exerts an influence. In the limiting case where the depth change following an aggressive order placed by someone else is completely uninformative to the HFT (i.e., $\rho = 1$), equation (30) collapses down to equation (5) from the baseline model. At the opposite extreme, when the HFT learns the liquidity state perfectly from observing another trader’s aggressive order (i.e., $\rho = 0$), the HFT’s expected total profits are unambiguously lower if he places an aggressive order in period 1 himself. When the HFT can learn more about the liquidity state through mere observation, as he can when $\rho$ is smaller, he has less incentive to incur the direct costs of exploratory trading.

Viewed differently, if the HFT does find it optimal to engage in exploratory trading, it must be the case that he obtains more useful information from the market response to his aggressive orders than he does from the market response to other traders’ aggressive orders. In an anonymous market, it also follows from symmetry that each other trader obtains no more useful information from the market response to the HFT’s aggressive orders than they do from the market response to another arbitrary trader’s aggressive orders. This provides a third testable implication of the hypothesis that a given trader engages in exploration.
B Measuring Aggressive Orders’ Profitability

Calculating round-trip profits using a FIFO or LIFO approach is not a useful way to measure the profitability of individual aggressive orders. Even the most aggressive HFTs engage in some passive trading, so a FIFO/LIFO-round-trip measure would either confound aggressive trades with passive trades, or require some arbitrary assumption to distinguish between inventory acquired passively and inventory acquired aggressively (on top of the already-arbitrary assumption of FIFO or LIFO). A second, more general problem is that a measurement scheme based on inventory round-trips will always combine at least two orders (an entry and an exit), so such measurement schemes do not actually measure the profitability of individual aggressive orders.

In this appendix, I provide rigorous justification for the claim that the average expected profit from an aggressive order in the E-mini market equals the expected favorable price movement, minus trading/clearing fees and half the bid-ask spread. After presenting the formal proof, I discuss details of empirically estimating expected favorable price movement.

B.1 Preliminaries

Trading/clearing fees apply equally to both passively and aggressively traded E-mini contracts, so to simplify the exposition, I will initially ignore these fees. Similarly, I make the simplifying assumption that the bid-ask spread is constant, and identically equal to one tick; for the E-mini market, this assumption entails minimal loss of generality.

In the E-mini market, the profitability of individual aggressive orders can be considered in isolation from passive orders. Because E-mini contracts can be created directly by buyers and sellers, a trader’s net inventory position does not constrain his ability to participate in a given trade\(^{12}\). As long as he can find a buyer, a trader who wishes to sell an E-mini contract can always do so, regardless of whether he has a preexisting long position. More generally, if a trader enters a position aggressively then exits it passively, he could have conducted the passive transaction even if he hadn’t engaged in the preceding aggressive transaction. While a desire to dispose of passively-acquired inventory might motivate a trader to submit an aggressive order, the question

\(^{12}\)The one exception would arise in the extremely rare event that a trader who did not qualify for a position-limit exemption held so many contracts (either long or short) that his inventory after the trade would exceed the position limit of 100,000 E-mini contracts. For HFTs, this minor exception can safely be ignored.
of underlying motivation is distinct from the question of whether the aggressive order was directly profitable.

B.2 Formal Argument

With these preliminaries established, I turn to the rigorous argument. Consider a trader who executes $J$ aggressive sell orders of size one, and $J$ aggressive buy orders of size one, for some large $J$. Following the remarks above, the trader’s passive transactions can be ignored. Let the average direction-normalized price change after these aggressive orders be $\tilde{\vartheta} \equiv \vartheta \left( \frac{2J}{2J-1} \right)$ ticks for some $\vartheta$ that does not depend on $J$.

First, suppose that the trader always submits an aggressive sell after an aggressive buy, and always submits an aggressive buy after an aggressive sell. Without loss of generality, assume that the trader’s first aggressive order is a buy. The trader’s combined profit from all $2J$ aggressive orders is

$$
\pi_{2J} = -a_1 + b_2 - a_3 + b_4 - \ldots - a_{2J-1} + b_{2J} \tag{31}
$$

$$
= -a_1 + (a_2 - 1) - a_3 + (a_4 - 1) - \ldots - a_{2J-1} + (a_{2J} - 1) \tag{32}
$$

$$
= -a_1 + a_2 - a_3 + a_4 - \ldots - a_{2J-1} + a_{2J} - J \tag{33}
$$

$$
= -a_1 + (a_1 + \zeta_{b,1}) - (a_2 + \zeta_{s,2}) + (a_3 + \zeta_{b,2}) - \ldots \tag{34}
$$

$$
\ldots - (a_{2J-2} + \zeta_{s,J}) + (a_{2J-1} + \zeta_{b,J}) - J
$$

$$
= \sum_{i=1}^{J} (a_{2i-1} + \zeta_{b,i}) - \sum_{j=2}^{J} (a_{2j-2} + \zeta_{s,j}) - a_1 - J \tag{35}
$$

$$
= \sum_{i=1}^{J} a_{2i-1} - \left( a_1 + \sum_{j=1}^{J-1} a_{2j} \right) + \sum_{i=1}^{J} \zeta_{b,i} - \sum_{j=2}^{J} \zeta_{s,j} - J \tag{36}
$$

where $a_k$ and $b_k$ respectively denote the prevailing best ask and best bid at the time the $k$th aggressive order executes, $\zeta_{b,r}$ denotes the change in midpoint price following the $r$th aggressive buy order, and $\zeta_{s,r}$ denotes the change in midpoint price following the $r$th aggressive sell order. Note that $\vartheta \equiv \frac{1}{2J} \left( \sum_{r=1}^{J'} \zeta_{b,r} + \sum_{r=1}^{J} (-\zeta_{s,r}) \right)$.

Next, taking expectations, we find
\[ E[\pi_{2J}] = \sum_{i=1}^{J} E[a_{2i-1}] - \left( E[a_1] + \sum_{j=1}^{J-1} E[a_{2j}] \right) \]

\[ + \sum_{i=1}^{J} E[\zeta_{b,i}] - \sum_{j=2}^{J} E[\zeta_{s,j}] - J \]

\[ = JE[a_1] - E[a_1] - (J-1)E[a_1] + JE[\tilde{\vartheta}] - (J-1)\left(-E[\tilde{\vartheta}]\right) - J \]

\[ = (2J-1)E[\tilde{\vartheta}] - J \]

\[ = (2J-1)\left(E[\vartheta]\frac{2J}{2J-1}\right) - J \]

\[ = J(2E[\vartheta] - 1) \]

where the second equality uses the assumption that midpoint prices follow a martingale with respect to their natural filtration, together with the assumption of a constant bid-ask spread. From the final equality above, it follows immediately that the trader's average expected profit on an individual aggressive order is given by

\[ \frac{1}{2J}E[\pi_{2J}] = E[\vartheta] - \frac{1}{2} \]  

Finally, note that none of the calculations above relied on the assumption that the aggressive orders alternated between buys and sells (this only simplified the notation). It follows immediately from grouping together multiple aggressive orders of the same sign that the result would hold for orders of varying sizes, provided that the overall aggressive buy and aggressive sell volumes were equal.

Under the usual regularity conditions, as \( J \to \infty, \tilde{\vartheta} \to_{A.S.} \lim_{J\to\infty} E[\tilde{\vartheta}] = E[\vartheta]. \]

### B.3 Obtaining Unbiased Estimates

Recall that the discussion in section 4.1 implied that we can estimate the profitability of an HFT's aggressive order using the (direction-normalized) accumulated price-changes following that aggressive order out to some time past the HFT’s maximum forecasting horizon. If we choose too short an accumulation window, the resulting estimates of the long-run direction-normalized
average price changes following an HFT’s aggressive orders will be biased downward. This enables us to empirically determine an adequate accumulation period by calculating cumulative direction-normalized price changes over longer and longer windows until their mean ceases to significantly increase.

Market activity varies considerably in its intensity throughout a trading day, so event-time, which I measure in terms of aggressive order arrivals, provides a more uniform standard for temporal measurements than does clock-time. Empirically, an accumulation period of about 30 aggressive orders suffices to obtain unbiased estimates of the price movement following an HFT’s aggressive order, but I consider results for an accumulation period of 50 aggressive orders to allow a wide margin for error. The mean direction-normalized price changes following individual HFTs’ aggressive orders does not differ significantly for accumulation periods of 50, 200, or 500 aggressive orders, even if we distinguish aggressive orders by size. The same holds true for aggressive orders placed by non-HFTs. Using too long an accumulation period will not bias the estimates, but it will introduce unnecessary noise, so I opt for an accumulation period of 50 aggressive orders.

As I discuss at greater length in section C.1, future price movements are moderately predictable from past aggressive order flow and orderbook activity, but only at very short horizons. Of the variables that meaningfully forecast future price changes, the direction of aggressive order flow is by far the most persistent, but even its forecasting power diminishes to nonexistence for price movements more than either about 12 aggressive orders or 200 milliseconds in the future. The adequacy of a 30+ aggressive order accumulation period is entirely consistent with these results.

As a simple empirical check on the validity of direction-normalized cumulative price changes as a proxy for the profitability of aggressive orders, I use each HFT’s explicit overall profits and passive trading volume, together with the profits on aggressive orders as measured by the proxy, to back out the HFT’s implicit profit on each passively traded contract. The resulting estimates of HFTs’ respective profits from passive transactions are all plausible from a theoretical perspective, and are comparable to non-HFTs’ implicit performance on passive trades.
C Benchmark Regression Results

C.1 Variables that Forecast Price Movements

Bid-ask bounce notwithstanding, the price at which aggressive orders execute changes rather infrequently in the E-mini market. On average, only about $1 - 3\%$ of aggressive buy (sell) orders execute at a final price different from the last price at which the previous aggressive buy (sell) order executed, and the price changes that do occur are almost completely unpredictable on the basis of past price changes. However, several other variables forecast price innovations surprisingly well.

In contrast to price innovations, the direction of aggressive order-flow in the E-mini market is extremely persistent and predictable. On average, the probability that the next aggressive order will be a buy (sell) given that the previous aggressive order was a buy (sell) is around $75\%$. In addition to forecasting the direction of future aggressive order-flow, the direction of past aggressive order-flow also forecasts future price innovations to statistically and economically significant extent, and forecasts based on past aggressive order signs alone are moderately improved by information about the (signed) quantities of past aggressive orders. Simple measures of recent changes in the orderbook offer further, yet modest, improvement in price forecasts.

The levels of resting depth in the orderbook, in addition to the changes in resting depth, also improve price forecasts slightly, but these stock variables cannot be reliably recovered in much of my dataset because a small number of modification messages (around $2 - 4\%$) are missing. These occasional missing modifications introduce only transient noise into flow variables such as changes in resting depth, but they have permanent effects on the corresponding stock variables. Fortunately, omitting resting-depth stock variables from the direct tests of the exploratory trading model’s predictions in section 5 is harmless. These tests use the explanatory variables in the benchmark regression (43) only as an empirical analogue of $\varphi$, the signal of future aggressive order-flow in the exploratory trading model. Thus the tests require only that the benchmark explanatory variables offer some predictive power, not that those variables control for all public information (I control for public information by other means, discussed in section 5).
C.2 Econometric Benchmark

For each trading day in my sample, I regress the cumulative price-change (in dollars) between the aggressive orders $k$ and $k + 50$, denoted $y_k$, on lagged market variables suggested by the remarks in section C.1. Specifically, I regress $y_k$ on the changes in resting depth between aggressive orders $k - 1$ and $k$ at each of the six price levels within two ticks of the best bid or best ask, the signs of aggressive orders $k - 1$ through $k - 4$, and the signed executed quantities of aggressive orders $k - 1$ through $k - 4$. For symmetry, I adopt the convention that sell depth is negative and buy depth is positive, so that an increase in buy depth has the same sign as a decrease in sell depth.

Denoting the row vector of the 14 regressors by $z_{k-1}$, and a column vector of 14 coefficients by $\Gamma$, I estimate the equation

\[
y_k = z_{k-1}\Gamma + \epsilon_k \quad (43)
\]

\[
\begin{align*}
\gamma_1d_{k-1}^1 &+ \ldots + \gamma_6d_{k-1}^6 + \\
\gamma_7\text{sign}_{k-1} &+ \ldots + \gamma_{10}\text{sign}_{k-4} + \\
\gamma_{11}q_{k-1} &+ \ldots + \gamma_{14}q_{k-4} + \epsilon_k
\end{align*} \quad (44)
\]

Table 3 summarizes the estimates from the regression above, computed over my entire sample. All of the variables are antisymmetrical for buys and sells, and so have means extremely close to zero, but the mean magnitudes in the rightmost column of Table 3 provide some context for scale.
Table 3: Estimates from Benchmark Regression

<table>
<thead>
<tr>
<th>Coefficient (×1000)</th>
<th>Robust t-Statistic</th>
<th>Variable Avg. Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{best , bid-2}^{k-1}$</td>
<td>-0.90</td>
<td>-1.02</td>
</tr>
<tr>
<td>$d_{best , bid-1}^{k-1}$</td>
<td>-2.08</td>
<td>-4.29</td>
</tr>
<tr>
<td>$d_{best , bid}^{k-1}$</td>
<td>1.13</td>
<td>4.94</td>
</tr>
<tr>
<td>$d_{best , ask}^{k-1}$</td>
<td>1.11</td>
<td>4.97</td>
</tr>
<tr>
<td>$d_{best , ask+1}^{k-1}$</td>
<td>-2.03</td>
<td>-4.24</td>
</tr>
<tr>
<td>$d_{best , ask+2}^{k-1}$</td>
<td>-1.60</td>
<td>-1.90</td>
</tr>
<tr>
<td>$sign_{k-1}$</td>
<td>1186</td>
<td>33.3</td>
</tr>
<tr>
<td>$sign_{k-2}$</td>
<td>753</td>
<td>20.2</td>
</tr>
<tr>
<td>$sign_{k-3}$</td>
<td>544</td>
<td>14.6</td>
</tr>
<tr>
<td>$sign_{k-4}$</td>
<td>472</td>
<td>13.4</td>
</tr>
<tr>
<td>$q_{k-1}$</td>
<td>4.09</td>
<td>9.29</td>
</tr>
<tr>
<td>$q_{k-2}$</td>
<td>2.66</td>
<td>6.59</td>
</tr>
<tr>
<td>$q_{k-3}$</td>
<td>1.85</td>
<td>4.66</td>
</tr>
<tr>
<td>$q_{k-4}$</td>
<td>1.16</td>
<td>2.98</td>
</tr>
</tbody>
</table>

Comparable results obtain using as few as two lags of aggressive order sign and signed quantity. Linear forecasts of $y_k$ do not benefit appreciably from the inclusion of data on aggressive orders before $k - 4$, or on changes in resting depth prior to aggressive order $k - 1$. Because the price-change $y_k$ is not normalized by the sign of the $k$th aggressive order, it has an expected value of zero, so I do not include a constant term in the regression. Including a constant term in the regression has negligible effect on the results.

Although the last several aggressive order signs do offer rather remarkable explanatory power, the respective distributions of resting depth changes and executed aggressive order quantities have much heavier tails than the distribution of order sign, so price forecasts are meaningfully improved by the inclusion of these variables.

The positive coefficients on the lagged aggressive order variables and on the depth changes at
the best bid and best ask are consistent with the general intuition that buy orders portend price increases, and sell orders portend price decreases. The negative coefficients on depth changes at the outside price levels require slightly more explanation.

Because the E-mini market operates according to strict price and time priority, a trader who seeks priority execution of his passive order will generally place that order at the best bid (or best ask); however, if the trader believes that an adverse price movement is imminent, he will place his order at the price level that he expects to be the best bid (ask) following the price change. It is relatively uncommon for prices to change immediately after an aggressive order in the E-mini market, but when prices do change, it is extremely rare during regular trading hours for the change to exceed one tick. As a result, the expected best bid (ask) following a price change is typically one tick away from the previous best, so it is not surprising that (e.g.) an increase in resting depth one tick below the best bid tends to precede a downward price change. These features of the E-mini market also shed some light on why changes in depth more than one tick away from the best (i.e., \(d_{k-1}^{\text{best \, bid} - 2}\) and \(d_{k-1}^{\text{best \, ask} + 2}\)) are not significant predictors of future price movements.
D Supplemental Tables of Empirical Results
Table 4: Additional Explained Earnings on Aggressive Orders (Hundredths of a Cent per Contract)

Table 5, below, presents various cross-sectional averages of the estimated additional gross earnings per contract on aggressive orders of submitted size greater than $\bar{q}$ explained by regression (14) in excess of that explained by regression (13). “A-HFT vs. All Others” denotes the additional explained earnings per contract for a given A-HFT minus those for all other traders, averaged across the A-HFTs; “A-HFT vs. Other HFTs” denotes an analogous quantity. Numbers reported for the A-HFTs are averages over the estimates for individual A-HFTs. The membership of “All Others” and “Other HFTs” depends upon the particular A-HFT being excluded, and the reported numbers are averages taken across the slightly different groups corresponding to each of the individual A-HFTs. Note that “other HFTs” includes the other A-HFTs. Units are hundredths of a cent per contract, and 95% bootstrap confidence intervals are reported beneath the point estimates.

<table>
<thead>
<tr>
<th></th>
<th>$\bar{q} = 1$</th>
<th>$\bar{q} = 5$</th>
<th>$\bar{q} = 10$</th>
<th>$\bar{q} = 15$</th>
<th>$\bar{q} = 20$</th>
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</thead>
<tbody>
<tr>
<td>All Others</td>
<td>3.4</td>
<td>8.2</td>
<td>7.4</td>
<td>10.1</td>
<td>11.3</td>
</tr>
<tr>
<td></td>
<td>(1.8, 4.8)</td>
<td>(5.5, 10.9)</td>
<td>(4.1, 10.4)</td>
<td>(7.1, 13.3)</td>
<td>(7.7, 14.7)</td>
</tr>
<tr>
<td>A-HFTs</td>
<td>17.9</td>
<td>65.9</td>
<td>39.7</td>
<td>53.3</td>
<td>62.3</td>
</tr>
<tr>
<td></td>
<td>(6.9, 29.6)</td>
<td>(48.0, 85.0)</td>
<td>(22.1, 57.0)</td>
<td>(35.5, 70.5)</td>
<td>(45.3, 79.9)</td>
</tr>
<tr>
<td>A-HFT vs. All Others</td>
<td>14.5</td>
<td>57.7</td>
<td>32.3</td>
<td>43.2</td>
<td>51.0</td>
</tr>
<tr>
<td></td>
<td>(3.6, 25.7)</td>
<td>(39.3, 76.4)</td>
<td>(13.9, 50.0)</td>
<td>(25.4, 60.4)</td>
<td>(33.2, 68.6)</td>
</tr>
<tr>
<td>A-HFT vs. Other HFTs</td>
<td>3.8</td>
<td>45.8</td>
<td>23.5</td>
<td>32.3</td>
<td>42.6</td>
</tr>
<tr>
<td></td>
<td>(-7.6, 15.4)</td>
<td>(26.9, 65.1)</td>
<td>(5.3, 42.0)</td>
<td>(13.5, 50.3)</td>
<td>(23.8, 61.8)</td>
</tr>
</tbody>
</table>
Table 5—Continued: Additional Explained Earnings on Aggressive Orders (Hundredths of a Cent per Contract)

<table>
<thead>
<tr>
<th>$q$</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>60</th>
<th>75</th>
<th>90</th>
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</thead>
<tbody>
<tr>
<td>All Others</td>
<td>17.5</td>
<td>14.7</td>
<td>14.3</td>
<td>16.2</td>
<td>16.2</td>
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<td>17.1</td>
<td>12.0</td>
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<td></td>
<td>(13.5, 21.3)</td>
<td>(10.6, 18.2)</td>
<td>(10.6, 18.4)</td>
<td>(12.0, 20.5)</td>
<td>(11.9, 20.5)</td>
<td>(14.5, 25.3)</td>
<td>(16.2, 27.2)</td>
<td>(11.2, 23.0)</td>
<td>(5.3, 18.7)</td>
</tr>
<tr>
<td>A-HFTs</td>
<td>62.4</td>
<td>69.7</td>
<td>73.3</td>
<td>85.0</td>
<td>85.0</td>
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<td>118.1</td>
<td>107.3</td>
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<td>(43.7, 80.6)</td>
<td>(52.0, 89.4)</td>
<td>(55.1, 92.0)</td>
<td>(64.6, 103.7)</td>
<td>(65.9, 105.3)</td>
<td>(81.0, 121.9)</td>
<td>(98.5, 138.1)</td>
<td>(86.0, 129.3)</td>
<td>(82.1, 124.2)</td>
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<tr>
<td>A-HFT vs. All Others</td>
<td>45.0</td>
<td>55.1</td>
<td>59.0</td>
<td>68.8</td>
<td>68.8</td>
<td>80.4</td>
<td>96.4</td>
<td>90.2</td>
<td>92.0</td>
</tr>
<tr>
<td></td>
<td>(26.2, 63.9)</td>
<td>(37.0, 75.0)</td>
<td>(40.6, 78.1)</td>
<td>(48.3, 87.6)</td>
<td>(49.2, 89.6)</td>
<td>(59.9, 101.9)</td>
<td>(75.5, 117.2)</td>
<td>(68.7, 112.8)</td>
<td>(68.5, 115.1)</td>
</tr>
<tr>
<td>A-HFT vs. Other HFTs</td>
<td>36.0</td>
<td>45.4</td>
<td>48.9</td>
<td>58.0</td>
<td>56.0</td>
<td>64.3</td>
<td>78.6</td>
<td>75.7</td>
<td>74.6</td>
</tr>
<tr>
<td></td>
<td>(17.1, 55.4)</td>
<td>(27.2, 66.0)</td>
<td>(29.9, 68.4)</td>
<td>(36.2, 77.9)</td>
<td>(35.4, 77.6)</td>
<td>(43.5, 87.1)</td>
<td>(56.2, 99.2)</td>
<td>(53.5, 100.3)</td>
<td>(49.8, 99.1)</td>
</tr>
</tbody>
</table>

The extra explanatory power of (14) reflects the contribution from the private component of information (available to the A-HFT under consideration) manifested in $\Omega^A$, as well as the effect of the extra degrees of freedom in (14) relative to (13). Since the effect from the extra degrees of freedom is the same (in expectation) for all traders, it has no impact on the difference in additional explained earnings per contract between an A-HFT and all other traders, or between an A-HFT and all other HFTs.
Table 5: Average Gross Earnings of Aggressive Orders, by Size and Trader Type (in Dollars per Contract)

<table>
<thead>
<tr>
<th>$\bar{q}$</th>
<th>A-HFTs</th>
<th></th>
<th>B-HFTs</th>
<th></th>
<th>Non-HFTs</th>
<th></th>
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<tr>
<td></td>
<td>All AOs $\leq \bar{q}$</td>
<td>Incremental AOs</td>
<td>All AOs $\leq \bar{q}$</td>
<td>Incremental AOs</td>
<td>All AOs $\leq \bar{q}$</td>
<td>Incremental AOs</td>
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<td>3.84</td>
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<td>4.37</td>
<td>-</td>
<td>1.68</td>
<td>-</td>
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<tr>
<td>5</td>
<td>4.23</td>
<td>4.38</td>
<td>4.56</td>
<td>4.64</td>
<td>1.72</td>
<td>1.75</td>
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<tr>
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<td>2.84</td>
<td>4.66</td>
<td>4.85</td>
<td>1.77</td>
<td>1.89</td>
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<td>6.39</td>
<td>4.71</td>
<td>4.95</td>
<td>1.79</td>
<td>1.97</td>
</tr>
<tr>
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<td>4.95</td>
<td>4.77</td>
<td>5.08</td>
<td>1.83</td>
<td>2.06</td>
</tr>
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</tr>
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<td>4.92</td>
<td>5.49</td>
<td>1.95</td>
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<td>6.90</td>
<td>4.95</td>
<td>5.14</td>
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<td>5.87</td>
<td>7.00</td>
<td>4.98</td>
<td>5.30</td>
<td>1.99</td>
<td>2.87</td>
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<td>6.12</td>
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<td>5.00</td>
<td>5.40</td>
<td>2.03</td>
<td>2.71</td>
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<td>90</td>
<td>6.38</td>
<td>7.20</td>
<td>5.01</td>
<td>5.52</td>
<td>2.03</td>
<td>2.16</td>
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<td>7.99</td>
<td>5.67</td>
<td>7.63</td>
<td>3.19</td>
<td>4.20</td>
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Table 6: Fractions of Trader Types’ Aggressive Orders Below Size Threshold $\bar{q}$

<table>
<thead>
<tr>
<th>$\bar{q}$</th>
<th>A-HFTs % of All AOs</th>
<th>A-HFTs % of Aggr. Volume</th>
<th>B-HFTs % of All AOs</th>
<th>B-HFTs % of Aggr. Volume</th>
<th>Non-HFTs % of All AOs</th>
<th>Non-HFTs % of Aggr. Volume</th>
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<tbody>
<tr>
<td>1</td>
<td>24.31%</td>
<td>0.40%</td>
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<td>4.77%</td>
<td>53.79%</td>
<td>5.68%</td>
</tr>
<tr>
<td>5</td>
<td>43.74%</td>
<td>1.44%</td>
<td>76.09%</td>
<td>16.13%</td>
<td>83.26%</td>
<td>14.88%</td>
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<tr>
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<td>54.64%</td>
<td>3.09%</td>
<td>84.10%</td>
<td>23.95%</td>
<td>89.87%</td>
<td>20.68%</td>
</tr>
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<td>91.57%</td>
<td>23.05%</td>
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<tr>
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<td>4.80%</td>
<td>90.70%</td>
<td>35.82%</td>
<td>93.27%</td>
<td>26.43%</td>
</tr>
<tr>
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<td>62.38%</td>
<td>5.37%</td>
<td>92.36%</td>
<td>40.29%</td>
<td>94.41%</td>
<td>29.32%</td>
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<tr>
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<td>64.62%</td>
<td>6.39%</td>
<td>94.14%</td>
<td>46.30%</td>
<td>94.98%</td>
<td>31.07%</td>
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<tr>
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<td>94.97%</td>
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<td>95.23%</td>
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<tr>
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<td>69.29%</td>
<td>9.20%</td>
<td>96.32%</td>
<td>55.76%</td>
<td>95.80%</td>
<td>34.33%</td>
</tr>
<tr>
<td>50</td>
<td>71.07%</td>
<td>10.55%</td>
<td>97.66%</td>
<td>63.29%</td>
<td>97.09%</td>
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<tr>
<td>60</td>
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<td>98.54%</td>
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<td>97.36%</td>
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<td>99.02%</td>
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<td>97.64%</td>
<td>44.76%</td>
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<tr>
<td>90</td>
<td>80.68%</td>
<td>21.11%</td>
<td>99.20%</td>
<td>74.86%</td>
<td>97.83%</td>
<td>46.37%</td>
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