The Volatility Factor Structure

Bryan Kelly
University of Chicago Booth School of Business and National Bureau of Economic Research

Hanno Lustig
UCLA Anderson and National Bureau of Economic Research

Stijn Van Nieuwerburgh
NYU Stern and National Bureau of Economic Research
The Volatility Factor Structure

Bryan Kelly  
University of Chicago  
Booth School of Business and NBER

Hanno Lustig  
UCLA Anderson and NBER

Stijn Van Nieuwerburgh  
NYU Stern and NBER

First Draft: November 1, 2012

Abstract

Firm-level volatility obeys a strong factor structure. The factor structure is distinct from the common variation in the returns themselves – after removing common factors in returns, residuals are uncorrelated, yet idiosyncratic volatility possess the same factor structure as total volatility. In fact, idiosyncratic volatility dominates firms’ total variation – less that 5% of variation in daily returns is accounted for by common factors. The volatility factor structure holds not only for returns, but also for firm-level cash flow growth volatility. Thus any explanation of the volatility factor structure must account for both fundamental and return volatility, ruling out arguments based purely on discount rates or investor behavior.
1 Introduction

Firm-level volatility obeys a strong factor structure. A wide range of finance theories model returns as linear functions of common factors\(^1\) If the factors themselves have time-varying volatility, then firm-level volatility will naturally inherit a factor structure as well. The market model is an illustrative example. It specifies that returns on stock \(i\) follow \(r_{i,t} = \alpha_i + \beta_i r_{m,t} + \epsilon_{i,t}\), which implies stock-level variance is \(\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{\epsilon_i}^2\). Thus volatility at the firm level has a common component \((\beta_i^2 \sigma_m^2)\) as well as an idiosyncratic component \((\sigma_{\epsilon_i}^2)\).

What is surprising is that the firm-level volatility factor structure remains effectively unchanged even after accounting for common factors in returns. We examine residuals from factor models that include the Fama-French (1993) three factor, as well as statistical factor decompositions using principal components. Residual volatilities from these models possess an extremely high degree of common variation across individual stocks. Residual volatility accounts for the vast majority (96%) of the variation in a typical stock’s volatility. Thus there is little distinction between total and idiosyncratic volatility at the firm level, and both possess effectively the same common factor structure.

Furthermore, residuals in return factor models are virtually uncorrelated. Consider returns on the 100 Fama-French size and value portfolios. The average pairwise correlation between returns on these portfolios is 64%. But their residuals from a Fama-French three factor model or a five factor principal components model have pairwise correlations of less than 1% per year on average. Therefore omitted factors are not an explanation for comovement in volatilities. Correlations among the time series of raw return volatilities for the 100 portfolios is 75% on average. Volatility comovement remains extremely high even after removing common factors from returns. Average time series correlation among idiosyncratic volatilities is 54% when residuals are based on the Fama-French model, and 59% when based on a five factor principal components model. We conclude that commonalities among re-

\(^1\)Prominent examples include the CAPM (Sharpe (1964)), ICAPM (Merton (1973)), APT (Ross (1976)) and the Fama-French (1993) model.
returns have very little influence on the commonalities in return volatilities. The cross section distribution of total volatility and idiosyncratic volatility are nearly identical.

Comovement in volatilities is not only a feature of returns, but also for the volatility of fundamental quantities. We estimate volatilities of firm-level sales growth using quarterly Compustat data. Despite the fact that these volatility estimates are far noisier than the return data, we again find a strong factor structure among total fundamental volatilities as well as among volatilities of uncorrelated factor model residuals. Thus, the volatility patterns that we identify in this paper are not wholly (or even primarily) explicable with asset pricing rationales. We know of no extant model in the firm growth or asset pricing literature that generates a factor structure in both fundamental and return volatilities through an economic mechanism.

Idiosyncratic volatility has been studied in several asset pricing contexts. Campbell et al. (2001) examine trends in average idiosyncratic volatility over time, though do not study the cross section properties of idiosyncratic volatility. This gave rise to several papers that explore this fact in more detail, such as Bennett, Sias, and Starks (2003), Irvine and Pontiff (2009), and Brandt et al. (2010), including analyzing which firm characteristics correlate with its idiosyncratic volatility. Wei and Zhang (2006) study aggregate time series variation in fundamental volatility. Bekaert, Hodrick and Zhang (2010) find comovement in average idiosyncratic volatility across countries. We analyze comovement among volatilities at the firm-level for both returns and fundamentals. Our focus is on the joint dynamics of the entire panel of firm-level volatilities, which we document is a prominent empirical feature of returns and growth rates that is new to the literature.

1.1 Data Construction

To document these facts, we present evidence in the form of firm-year volatility panels. Return volatility is estimated each year for each CRSP stock as the standard deviation of
the roughly 250 daily returns within the year. Fundamental volatility is estimated each year for all Compustat firms using the four quarterly year-on-year sales growth observations within the year. We also show that our return volatility results are robust to using twelve monthly returns within each year rather than daily returns to calculate volatility. Similarly, we show that our fundamental volatility results are robust to estimating volatility with a five-year rolling window of quarterly observations (rather than one year of quarterly data), which reduces estimation noise.

The focus of our analysis is on idiosyncratic volatility. Idiosyncratic returns are constructed within each calendar year $\tau$ by estimating a factor model using all observations within that year (we estimate it for all firms with no missing observations during the year). Our factor models are of the form

$$r_{i,t} = \gamma_{0,i} + \gamma_i^t F_t + \varepsilon_{i,t}$$

and use all date $t$ return observations in the year (where the frequency of $t$ is either daily or monthly). A firm’s idiosyncratic volatility is then calculated as the standard deviation of residuals $\varepsilon_{i,t}$ within the calendar year. The result of this procedure is a panel of firm-year idiosyncratic volatility estimates. The first return factor model that we consider specifies $F_t$ as the $3 \times 1$ vector of Fama-French (1993) factors. The second return factor model we use is purely statistical. In this case, $F_t$ contains the first $K \times 1$ principal components of returns within the year, where we allow $K$ to range between one and ten.

We estimate idiosyncratic volatility of firm fundamentals analogously. Since there is no single predominant factor model for sales growth in the literature, we only consider principal components as factors. The approach is the same as in Equation 1, with the exception that the left hand side variable is sales growth, and the frequency of $t$ is quarterly. $F_t$ contains

---

2Our estimates diverge slightly from the standard Fama-French model in which returns in excess of the risk free rate are the left-hand side variables, and the excess market return is the first factor. We use gross returns on the left-hand side, and the gross market return as the first factor.
the first $K \times 1$ principal components of growth rates within a five-year window ending in year $\tau$, and residual volatility in year $\tau$ is estimated from the four model residuals within that year. Again, the number of principal components $K$ ranges from one to ten.

2 The Factor Structure in Volatility

2.1 The Cross Section Distribution of Volatility

We begin by noting that the cross-sectional distributions of return volatility and fundamental volatility are lognormal to a close approximation, which motivates us to estimate our factor models using volatility in logs rather than levels.

We plot histograms of the empirical cross section distribution of firm-level volatility (in logs). The upper left-hand corner of Figure 1 shows the distribution of log realized volatility pooling all firm-years from 1926-2010. The figure also shows empirical distributions for selected one-year snapshots throughout the sample (years 1930, 1950, 1970, 1990 and 2010). Overlaid on these histograms is the exact normal density with mean and variance set equal to that of the empirical distribution, and each figure reports the skewness and kurtosis of the data in the histogram.

The pooled histograms and each of the snapshots (with the exception of 1970) look nearly normally distributed. They demonstrate only slight skewness (typically less than 0.3 in absolute value) and do not appear to be substantively leptokurtotic.

Figure 2 reports cross section distributions of yearly sales growth volatility (in logs) for all CRSP/Compustat firms. Fundamental volatility also appears to closely fit a lognormal distribution, with skewness no larger than 0.3 and kurtosis never exceeding 3.5.

While Figures 1 and 2 demonstrate near lognormality of total return and growth rate volatility, the same feature holds for residual volatility. Figure 3 shows the distributions of log idiosyncratic return volatility (Panel A) and log idiosyncratic fundamental volatility
Figure 1: Total Return Log Volatility: Empirical Density Versus Normal Density

Notes: The figure plots histograms of the empirical cross section distribution of annual firm-level volatility (in logs). Within each calendar year, we calculate the standard deviation of daily returns for each stock. The upper left-hand corner histogram pools all years (1926-2010). Selected one-year snapshots of the firm volatility cross section distribution are also show (for years 1930, 1950, 1970, 1990 and 2010). Overlaid on these histograms is the exact normal density with mean and variance set equal to that of the empirical distribution. Each figure reports the skewness and kurtosis of the data in the histogram.
Figure 2: Log Sales Volatility: Empirical Density Versus Normal Density

Notes: The figure plots histograms of the empirical cross section distribution of annual firm-level volatility (in logs). Within each calendar year, we calculate the standard deviation of four quarterly observations of year-on-year sales growth for each stock. The upper left-hand corner histogram pools all years (1926-2010). Selected one-year snapshots of the firm volatility cross section distribution are also show (for years 1970, 1980, 1990, 2000 and 2010). Overlaid on these histograms is the exact normal density with mean and variance set equal to that of the empirical distribution. Each figure reports the skewness and kurtosis of the data in the histogram.
Figure 3: Log Idiosyncratic Volatility: Empirical Density Versus Normal Density

Panel A: Returns
All Years
Skewness: 0.24
Kurtosis: 3.01

Panel B: Sales Growth
All Years
Skewness: 0.17
Kurtosis: 3.40

Notes: The figure plots histograms of the empirical cross section distribution of annual idiosyncratic firm-level volatility (in logs). Within each calendar year, we calculate the standard deviation of residuals from a factor model for daily returns for each stock (left panel) or quarterly sales growth (right panel). In both cases, residuals are constructed from a five factor principal components model. For returns, principal components are estimated from daily data within the year, while for sales growth a five year rolling window of quarterly data is used. The histograms pool all years (1926-2010 for return volatility, 1975-2010 for sales growth volatility). Overlaid on these histograms is the exact normal density with mean and variance set equal to that of the empirical distribution. Each figure reports the skewness and kurtosis of the data in the histogram.

(Panel B) pooling all firm-years. For both returns and sales growth, residuals are constructed from the five factor principal components model. The distributions are qualitatively identical to the empirical histograms for total volatility. They are nearly normal, with a slight amount of right skewness and mild excess kurtosis.

2.2 Common Secular Patterns in Firm-Level Volatility

2.2.1 Return Volatility

Next, we document common time variation in volatility across stocks. Panel A of Figure plots firm-level log total return volatility, averaged within size quintiles. There is a striking degree of common variation in the volatilities of the largest quintile and smallest quintile of stocks. The same is true of industry groups. Panel B reports average return volatilities
Figure 4: Log Total and Idiosyncratic Return Volatility by Size and Industry Group

Panel A: Total Log Volatility by Size Quintile

Panel B: Total Log Volatility by Industry

Panel C: Idiosyncratic Log Volatility by Size Quintile

Panel D: Idiosyncratic Log Volatility by Industry

Notes: The figures plot firm-level log volatility averaged within size and industry groups. Within each calendar year, volatilities are estimated as the standard deviation of daily returns for each stock. Panel A shows firm-level log total return volatility averaged within market equity quintiles. Panel B shows log total return volatility averaged within the five-industry categorization of SIC codes provided on Ken French’s website. Panels C and D report the same within-group averages of firm-level log idiosyncratic volatility. Idiosyncratic volatility is the standard deviation of residuals from a five factor principal components model for daily returns.
among the stocks in the five-industry categorization of SIC codes provided on Ken French’s website. This is perhaps unsurprising given that firm-level returns are believed to have a substantial degree of common return variation, as evidenced by the predominance of factor-based models of individual stock returns. If returns have common factors and the volatility of those factors varies over time, then firm-level variances will also inherit a factor structure.

What is surprising is that volatilities of residuals display the same degree of common variation despite the fact that common return factors have been removed. Instead of averaging total volatility within size and industry groups, Panels C and D plot average residual volatility from a factor model that uses the first five principal components of returns as factors. The plots show that the same dynamics appear for all groups of firms when considering idiosyncratic rather than total volatility. The correlation between average log idiosyncratic volatility within size quintiles one and five is 83%. The lowest correlation among the five industry groups is 67%, which is for idiosyncratic volatilities of firms in the healthcare industry versus those in the “other” category (including construction, transportation, services, and finance).

This common variation is idiosyncratic and cannot be explained by excess comovement among factor model residuals, for instance due to omitted common factors. Figure 5 shows how firms’ idiosyncratic daily return volatility estimates are affected by using different factor models for returns. It compares raw returns to residuals from the Fama-French three factor model, as well as to residuals from a five factor principal components model. Panel A shows that raw returns share substantial common variation, with an average pairwise correlation of 13% over the 1926-2010 sample. However, the Fama-French model captures effectively all of this common variation at the daily frequency, as correlations among its residuals are less than 0.2% on average, and are never above 0.9% in a year. The same is true for the principal components model, whose residual correlation is also below 0.2% on average.

The interesting fact is that the common variation in returns, which is well accounted for by these factor models, is responsible for very little of the total variation in returns. Panel
Figure 5: Volatility and Correlation of Daily Returns

Panel A: Average Pairwise Correlation

Panel B: Average Log Volatility

Panel C: Dispersion of Log Volatility

Notes: Panel A shows cross section average firm-level log volatility each year for total and idiosyncratic returns. Panel B shows the cross section standard deviation of firm-level log volatility and Panel C shows the average pairwise correlation for total and idiosyncratic returns within each calendar year. Idiosyncratic volatility is the standard deviation of residuals from the three factor Fama-French model or a five factor principal components model for daily returns (factor models also estimated within each calendar year).
Figure 6: Volatility of 100 Size and Value Portfolios

Panel A: Total Volatility

Panel B: Fama-French Residual Volatility

Panel C: Five Principal Component Residual Volatility

Notes: The figures plot log volatility of total and idiosyncratic returns on 100 size and value portfolios. Within each calendar year, total return volatilities are estimated from daily returns for each portfolio (Panel A), while idiosyncratic return volatility is the standard deviation of residuals from the three factor Fama-French model (Panel B) or a five factor principal components model (Panel C) for daily returns (factor models also estimated within each calendar year).
Figure 7: Average Pairwise Correlation of 100 Size and Value Portfolios

Notes: The figure shows average pairwise correlation for total and idiosyncratic returns on 100 size and value portfolios within each calendar year (refer to Figure 5 for details).

B of Figure 5 shows that the average log idiosyncratic volatility from the factor models is virtually the same as average volatility of total returns. In the typical year, only 4% of average log total volatility is accounted for by the five principal components factor model, while average log idiosyncratic volatility inherits 96% of the average total volatility level (the same is true for the Fama-French model).

Similarly, the dispersion in firms’ log volatility is more or less unaffected by removing common factors, as shown in Panel C. Since log volatilities are approximately normally distributed, Panels B and C contain most of the relevant information about the cross section of firm volatilities over time. In short, commonalities among returns have very little influence on the commonalities in return volatilities. The cross section distribution of total volatility and idiosyncratic volatility are qualitatively identical.

The strong comovement of return volatility is similarly discernible in portfolio returns. Figure 6 reports annual volatilities of the 100 Fama-French size and value portfolios. These are also calculated from daily returns within the year over the 1964-2010 sample (the first available full year of Ken French’s data is 1964). Panel A shows log total volatility, Panel B shows log idiosyncratic volatility using the Fama-French three factor model, and Panel
C shows idiosyncratic volatilities for a five factor principal components model. Portfolio volatilities show a strikingly similar degree of comovement across the size and book-to-market spectrum, even after accounting for common factors. Like the individual stock results above, factor models remove the vast majority of common variation in returns, thus common volatility patterns are unlikely to be driven by omitted common return factors. This can be seen clearly in Figure 7. Raw portfolio returns have an average pairwise correlation of 64% between 1964 and 2010, while the correlation of factor model residuals is below 1% on average for both models. However, the average pairwise correlation between portfolio volatilities remain high whether total or idiosyncratic volatilities are analyzed. The average pairwise correlation of the volatility series in Figure 6 Panel A is 77%, falling only to 54% and 59% in Panels B and C, respectively.

One may be concerned that daily factor models miss some portion of common variation among returns due to non-synchronicity in when aggregate information is incorporated into individual stock prices. To address this, we re-estimate factor models and firm-level idiosyncratic volatilities using data at the monthly frequency, and re-plot average correlations, average volatilities, and the dispersion of volatilities in Figure 8. Panel A shows that, indeed, there is a higher correlation among monthly raw returns relative to daily, with an average pairwise correlation of 23% over the 1926-2010. At the monthly frequency the Fama-French model continues to captures nearly all common variation, with correlations below 0.4% on average. The five factor principal components model has monthly residual correlation below 0.8% on average. At the monthly frequency, 22% of average log total volatility is accounted for by the principal components factor model, while average log idiosyncratic volatility inherits 78% of the average total volatility level. Thus, monthly return factor models do explain a larger fraction of the total return variation, but return volatility continues to be dominated by idiosyncratic rather than common variation. It is also worth noting that the bulk of the idiosyncratic volatility literature estimates volatility from daily data (e.g. Ang et al. (2006)).
Figure 8: Volatility and Correlation of Monthly Returns

Panel A: Average Pairwise Correlation

Panel B: Average Log Volatility

Panel C: Dispersion of Log Volatility

Notes: The figure repeats the analysis of Figure 5 using monthly return observations within each calendar year, rather than daily.
Notes: The figures plot firm-level log volatility averaged within size and industry groups. Within each calendar year, volatilities are estimated as the standard deviation of four quarterly year-on-year sales growth observations for each stock. Panel A shows firm-level log total volatility averaged within market equity quintiles. Panel B shows log total return volatility averaged within the five-industry categorization of SIC codes provided on Ken French’s website. Panels C and D report the same within-group averages of firm-level log idiosyncratic volatility. Idiosyncratic volatility is the standard deviation of residuals (four observation within each year) from a five factor principal components model for quarterly sales growth. The components are estimated in a five year rolling window ending in the year that the residual volatility is calculated.
Figure 10: Volatility and Correlation of Total and Idiosyncratic Sales Growth

Panel A: Average Pairwise Correlation

Panel B: Average Log Volatility

Panel C: Dispersion of Log Volatility

Notes: Panel A shows cross section average firm-level log volatility each year for total and idiosyncratic sales growth. Panel B shows the cross section standard deviation of firm-level log volatility and Panel C shows the average pairwise correlation for total and idiosyncratic returns within each calendar year. Idiosyncratic volatility is the standard deviation of residuals from one or five factor principal components model for quarterly sales growth. The components are estimated in a five year rolling window ending in the year that the residual volatility is calculated.
2.2.2 Fundamental Volatility

Strong comovement among volatilities is not distinct to return volatilities, but is also true for fundamental volatility. Figure 9 reports average yearly sales growth volatility (in logs) by size quintile and French’s five-industry categories (Panels A and B). Despite the fact that yearly sale growth volatilities are estimated from only four observations per year, the data continues to display a high degree of volatility commonality.

This is a feature of both total and residual volatility of fundamentals. Panels C and D show within-group average log idiosyncratic volatility estimated from a five factor principal components model for sales growth. These panels display the same volatility patterns as those in the top two panels.

A common factor model for firms’ sales growth is perhaps less relevant than that for returns, as shown in Panel A of Figure 10. The average pairwise sales growth correlation in the 1975-2010 sample is only 2%, though it reaches as high as 17% in 2009. Accounting for common factors with a five principal component factor model lowers these correlations to below 0.3% on average, with correlations reaching a high of only 1% in 1980.

Panel B shows that, like returns, average idiosyncratic volatility of fundamentals shares the same broad pattern as total volatility (correlation of 59%), and inherits 89% of the average total volatility (11% is accounted for by the factor model). Given the near lognormality of sales growth volatility in the cross section, along with the same overall patterns between the cross section mean and standard deviation of for the total volatility and idiosyncratic volatility distribution, we conclude that idiosyncratic volatility rather than common variation drives the entire panel of firm-level fundamental volatilities.

2.3 Volatility Factor Model Estimates

Next, we estimate a one-factor model for volatility. We consider total volatility, as well as idiosyncratic volatility estimated from a Fama-French three factor model or a K factor
## Table 1: Log Volatility Factor Model Estimates

<table>
<thead>
<tr>
<th>Panel A: Daily Returns</th>
<th>Total</th>
<th>FF</th>
<th>5 PCs</th>
<th>10 PCs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loading (average)</td>
<td>0.925</td>
<td>0.920</td>
<td>0.924</td>
<td>0.925</td>
</tr>
<tr>
<td>Loading (median)</td>
<td>0.911</td>
<td>0.884</td>
<td>0.886</td>
<td>0.888</td>
</tr>
<tr>
<td>Loading (25th %ile)</td>
<td>0.446</td>
<td>0.362</td>
<td>0.363</td>
<td>0.355</td>
</tr>
<tr>
<td>Loading (75th %ile)</td>
<td>1.387</td>
<td>1.400</td>
<td>1.405</td>
<td>1.412</td>
</tr>
<tr>
<td>Intercept (average)</td>
<td>-0.181</td>
<td>-0.201</td>
<td>-0.191</td>
<td>-0.190</td>
</tr>
<tr>
<td>Intercept (median)</td>
<td>-0.272</td>
<td>-0.354</td>
<td>-0.351</td>
<td>-0.344</td>
</tr>
<tr>
<td>Intercept (25th %ile)</td>
<td>-1.946</td>
<td>-2.324</td>
<td>-2.312</td>
<td>-2.361</td>
</tr>
<tr>
<td>Intercept (75th %ile)</td>
<td>1.491</td>
<td>1.584</td>
<td>1.585</td>
<td>1.617</td>
</tr>
<tr>
<td>$R^2$ (average univariate)</td>
<td>0.363</td>
<td>0.349</td>
<td>0.352</td>
<td>0.351</td>
</tr>
<tr>
<td>$R^2$ (pooled)</td>
<td>0.385</td>
<td>0.346</td>
<td>0.356</td>
<td>0.357</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Portfolio Returns</th>
<th>Total</th>
<th>FF</th>
<th>5 PCs</th>
<th>10 PCs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loading (average)</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Loading (median)</td>
<td>0.982</td>
<td>0.907</td>
<td>0.989</td>
<td>0.985</td>
</tr>
<tr>
<td>Loading (25th %ile)</td>
<td>0.914</td>
<td>0.789</td>
<td>0.869</td>
<td>0.868</td>
</tr>
<tr>
<td>Loading (75th %ile)</td>
<td>1.049</td>
<td>1.046</td>
<td>1.090</td>
<td>1.116</td>
</tr>
<tr>
<td>Intercept (average)</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Intercept (median)</td>
<td>-0.122</td>
<td>-0.485</td>
<td>-0.055</td>
<td>-0.097</td>
</tr>
<tr>
<td>Intercept (25th %ile)</td>
<td>-0.368</td>
<td>-1.172</td>
<td>-0.777</td>
<td>-0.750</td>
</tr>
<tr>
<td>Intercept (75th %ile)</td>
<td>0.313</td>
<td>0.224</td>
<td>0.562</td>
<td>0.642</td>
</tr>
<tr>
<td>$R^2$ (average univariate)</td>
<td>0.760</td>
<td>0.547</td>
<td>0.597</td>
<td>0.606</td>
</tr>
<tr>
<td>$R^2$ (pooled)</td>
<td>0.628</td>
<td>0.441</td>
<td>0.497</td>
<td>0.474</td>
</tr>
</tbody>
</table>
Table 1: Log Volatility Factor Model Estimates, Continued

<table>
<thead>
<tr>
<th>Panel C: Monthly Returns</th>
<th>Total</th>
<th>FF</th>
<th>5 PCs</th>
<th>10 PCs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loading (average)</td>
<td>0.918</td>
<td>0.888</td>
<td>0.872</td>
<td>0.926</td>
</tr>
<tr>
<td>Loading (median)</td>
<td>0.918</td>
<td>0.847</td>
<td>0.841</td>
<td>0.848</td>
</tr>
<tr>
<td>Loading (25th %ile)</td>
<td>0.416</td>
<td>0.193</td>
<td>0.050</td>
<td>-0.935</td>
</tr>
<tr>
<td>Loading (75th %ile)</td>
<td>1.417</td>
<td>1.524</td>
<td>1.657</td>
<td>2.746</td>
</tr>
<tr>
<td>Intercept (average)</td>
<td>-0.099</td>
<td>-0.195</td>
<td>-0.276</td>
<td>-0.251</td>
</tr>
<tr>
<td>Intercept (median)</td>
<td>-0.152</td>
<td>-0.348</td>
<td>-0.408</td>
<td>-0.603</td>
</tr>
<tr>
<td>Intercept (25th %ile)</td>
<td>-1.258</td>
<td>-1.998</td>
<td>-2.558</td>
<td>-8.191</td>
</tr>
<tr>
<td>Intercept (75th %ile)</td>
<td>1.005</td>
<td>1.405</td>
<td>1.875</td>
<td>7.574</td>
</tr>
<tr>
<td>$R^2$ (average univariate)</td>
<td>0.266</td>
<td>0.214</td>
<td>0.186</td>
<td>0.126</td>
</tr>
<tr>
<td>$R^2$ (pooled)</td>
<td>0.288</td>
<td>0.207</td>
<td>0.180</td>
<td>0.087</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel D: Sales Growth</th>
<th>Total (1yr)</th>
<th>1 PC</th>
<th>10 PCs</th>
<th>Total (5yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loading (average)</td>
<td>0.876</td>
<td>0.849</td>
<td>0.938</td>
<td>0.897</td>
</tr>
<tr>
<td>Loading (median)</td>
<td>0.777</td>
<td>0.776</td>
<td>0.889</td>
<td>0.864</td>
</tr>
<tr>
<td>Loading (25th %ile)</td>
<td>-0.711</td>
<td>-0.852</td>
<td>-0.876</td>
<td>-0.489</td>
</tr>
<tr>
<td>Loading (75th %ile)</td>
<td>2.401</td>
<td>2.523</td>
<td>2.652</td>
<td>2.207</td>
</tr>
<tr>
<td>Intercept (average)</td>
<td>-0.231</td>
<td>-0.211</td>
<td>-0.096</td>
<td>-0.262</td>
</tr>
<tr>
<td>Intercept (median)</td>
<td>-0.514</td>
<td>-0.432</td>
<td>-0.271</td>
<td>-0.476</td>
</tr>
<tr>
<td>Intercept (25th %ile)</td>
<td>-3.983</td>
<td>-3.379</td>
<td>-4.647</td>
<td>-4.530</td>
</tr>
<tr>
<td>Intercept (75th %ile)</td>
<td>3.327</td>
<td>2.767</td>
<td>4.210</td>
<td>3.820</td>
</tr>
<tr>
<td>$R^2$ (average univariate)</td>
<td>0.140</td>
<td>0.229</td>
<td>0.127</td>
<td>0.144</td>
</tr>
<tr>
<td>$R^2$ (pooled)</td>
<td>0.174</td>
<td>0.168</td>
<td>0.167</td>
<td>0.283</td>
</tr>
</tbody>
</table>

Notes: The table reports estimates for one factor regression models of yearly log volatility. In each panel, the single volatility factor is the equal weighted average of all firms’ log volatilities within that year. Thus all estimated volatility factor models take the form: $\log \sigma_{i,t} = \text{intercept}_i + \text{loading}_i \cdot \log \sigma_{i,t} + e_{i,t}$. Columns represent different volatility measures. For returns (Panels A through C), the first column represents estimates for a factor model of log total return volatility, the second column for idiosyncratic volatility based on Fama-French model residuals, and the third and fourth columns to idiosyncratic volatility from one and five factor principal component models. For sales growth volatility (Panel D), the last column reports model estimates for yearly volatilities estimated in a rolling 20 quarter window to reduce estimation noise. We report means and quantiles of the empirical distribution of firm-level intercepts and volatility factor loadings, as well as time series regression $R^2$ average over all firms. We also report a pooled factor model $R^2$, which compares the estimated factor model to a model with only a firm-specific constant.
principal component model ($K = 5$ or $10$). In all cases, time series regressions are run firm-by-firm, use log volatility as the left-hand side variable, and the right-hand side factor is an equally weighted average of the left-hand size volatility measure across all firms.

Our first set of results, shown in Panel A of Table 1, reports factor model results for daily return volatilities. Columns correspond to the factor model used to construct return residuals. The mean loading of an individual return volatility on the volatility factor is 0.925 for total return volatility, and is 0.920, 0.924 and 0.925 for idiosyncratic volatility based on the Fama-French, five PC and ten PC models, respectively. Median loadings are similar. The inter-quartile ranges for loadings span the interval from 0.355 to 1.412. The average firm’s intercept is between $-0.181$ and $-0.201$, with slightly higher median intercept and inter-quartile ranges covering $-2.361$ to $1.617$. The average univariate time series $R^2$ is 38.5% for the total volatility model, and around 35% for idiosyncratic volatility models. Pooling all volatilities, we find a pooled $R^2$ 34.9% and 36.3% (relative to a volatility model with only a firm-specific constant).

In Panel B we re-estimate the volatility factor model using daily return volatility of 100 size and value portfolios. The interquartile range of loadings on the volatility factor go from 0.789 to 1.116, and are between $-1.172$ and $0.642$ for intercepts. The common variation among idiosyncratic portfolio volatility is exceptional, with average time series $R^2$ between 54.7% and 64.2%, and pooled $R^2$ between 44.1% and 49.7%.

Panel C shows volatility factor model estimates for monthly (rather than daily) return volatilities. The picture is broadly similar to daily results. The average (median) firm has a loading between 0.826 and 0.926 (0.776 to 0.918) and an intercept of $-0.099$ to $-0.276$ ($-0.152$ to $-0.603$). The average time series $R^2$ is between 12.6% for ten factor model residuals and 26.6% for raw returns.

In Panel D we show volatility factor model estimates for sales growth volatility. The first three columns report total volatility, and idiosyncratic volatility from one and five principal
component models in which volatility is estimated from four quarterly observations within each year. The last column reports model estimates for an annual volatility panel that uses a rolling 20 quarter window to estimate each firm-year’s volatility.

Due to the excessively small number of observations used to construct volatility, we might expect poorer fit in these regressions. Yet the results are closely in line with those for return volatility. The average firm has a volatility factor loading of between 0.849 and 0.938, with an intercept between $-0.096$ and $-0.262$. The time series $R^2$ for raw and idiosyncratic growth rate volatility ranges between 12.7% and 22.9% on average. The pooled $R^2$ reaches as high as 28.3% when volatilities are estimated in a 20 quarter window.

3 Conclusion

We document strong comovement of individual stock return volatilities. Removing common variation in returns has little effect on volatility comovement, as the volatility of factor model residual returns demonstrates effectively the same factor structure as total returns, despite the fact that these residuals are uncorrelated. The distinction between stocks’ total volatility and idiosyncratic volatility is tiny – almost all return variation at the stock level is idiosyncratic.

Volatility comovement is not only a feature of returns, but also for volatility of firms’ fundamentals. Like returns, we find a strong factor structure among sales growth volatilities, both for total growth rates as well as among idiosyncratic, uncorrelated factor model residual growth rates. This is a new fact to be reconciled with economic theories of growth and asset pricing. We are unaware of an existing model that generates a factor structure in both fundamental and return volatilities through an economic mechanism.
References


