Soft Shareholder Activism

Doron Levit[*]

January 2013

Abstract

This paper explores the various ways through which shareholders can exercise activism when obtaining formal control is not feasible or too costly. I focus on two mechanisms: voice and exit. Different from the existing literature, voice is interpreted as communication and informal engagement between activist investors and opportunistic managers, and therefore, voice is modeled as a costless and strategic transmission of information. In this context, I show that voice is more effective as a form of shareholder activism when liquidity is high (exit is easy), managers are myopic (short-term executive compensation is significant), communication with managers is "behind the scenes", the activist's ownership is small (the cost of acquiring private information is high), and the activist has short-term motives.

Keywords: Shareholder Activism, Voice, Exit, Corporate Governance, Communication, Transparency, Myopia, Cheap-Talk.

JEL Classification Numbers: D74, D82, D83, G34

Introduction

Shareholder activism is an important channel of corporate governance. It is the act of disciplining an otherwise opportunistic management. Discipline can be achieved in several ways. Most notably, investors can seek formal control of the company by accumulating a significant number of voting shares or by winning board seats in contested director elections. With sufficient control, the activist can order the manager to take certain actions or simply replace him. Obtaining formal control, however, can be very costly. With the frustration of not being able to obtain control, the activist may choose to exit and sell her holdings in the firm.[1]

Shareholder activism, however, can be exercised even without formal control. Investors can send letters, make phone calls or even meet face to face with senior executives and board members, and express their view how to unlock what they believe is a hidden value. This can include the common activist goals of spinning off a division of the company or a share buyback, but it can also be a recommendation on a strategic, financial, or operational change in the firm. If investors have useful insights, the board of directors and the management team may consider their advice. Consistent with this view, the former U.S. SEC Chairman, Mary L. Schapiro, pointed in her speech from 2010: "...boards can also benefit from access to the ideas and the concerns investors may have. Good communications can build credibility with shareholders and potentially enhance corporate strategies.".[2]

Communication and informal engagement between investors and managers, whether they are public or "behind-the-scenes", seem widespread. Brav, Jiang, Partnoy and Thomas (2008) study activist hedge funds in the U.S. and find that in 48.3% of their sample the hedge fund declares that it "...intends to communicate with the board/management on a regular basis with the goal of enhancing shareholder value.". They do not observe more aggressive tactics in those events. Becht, Franks, and Grant (2010) use proprietary data collected from five activist funds and show that private and informal engagement are extensive and profitable. McCahery, Sautner, and Starks (2011) survey institutional investors and find that 55% of them would be

---

[1] The cost of intervention includes the fees of hiring lawyers, proxy advisors, solicitors, corporate governance experts, investment banks, public relations, and advertising firms. Gantchev (2012) estimates that on average the cost of a US public activist campaign ending in a proxy fight is $10.5 millions, roughly two thirds of its gross returns.

[2] See "Speech by SEC Chairman: Remarks at the NACD Annual Corporate Governance Conference" October 19, 2010, http://www.sec.gov/news/speech/2010/spch101910mls.htm

willing to engage in discussions with the firm's executives. They conclude that behind-the-scenes shareholder activism may be more prevalent than previously thought.[3]

This paper explores the various ways through which shareholders can exercise activism when obtaining formal control is not feasible or too costly. The focus is on two primary mechanisms: *voice* and *exit*. Voice is the attempt of investors to persuade the management to follow their recommendation. Different from the existing literature, voice is modeled as the communication of private information. Exit, on the other hand, is the activist's decision to sell her entire stake in the firm. By exiting, the activist signals dissatisfaction which could depress the share price and pressure managers to be more accountable to shareholders. The implementation of voice and exit does not require formal control and entails very little or no direct costs on the activist. I therefore refer to this type of activism as "soft". In this respect, the objective of this paper is to study the conditions under which soft shareholder activism is an effective form of corporate governance.

In order to study this topic I develop a model. In the model, the manager of a public firm has the formal authority over its long-term investment policy. The manager is not perfectly informed about the investment opportunities, and due to the separation of ownership and control, his policy does not necessarily maximize the value of shareholders. These frictions create room for shareholder activism. Specifically, among shareholders there is an activist investor with private information that complements the manager's knowledge.[4] The activist can voice her opinion by sending at no cost a non-verifiable message to the manager. The activist can also exit by selling her entire stake in the firm before the long-term returns are realized. The activist exits either to satisfy her liquidity needs or because she believes the share is over-valued. While the

---

[3] Related, Carleton, Nelson, and Weisbach (1998) study letters TIAA-CREF sends to their portfolio companies and find that they are usually successful at inducing firms to make governance related changes. Becht, Franks, Mayer, and Rossi (2009) provide evidence on "behind the scenes" communication as a form of (profitable) shareholder activism of the Hermes UK Focus Fund. Finally, a recent survey of 25 senior executives and 25 activist investors concludes that the majority of activists view communication as their most effective strategy in achieving desired results, and that companies and shareholders often cooperate outside of the media glare. See http://www.mergermarket.com/pdf/Shareholder_Activism_Insight_2010.pdf for a 2010 report made by law firm Schulte Roth & Zabel and research firm Mergermarket.

[4] There is a broad literature on how corporate insiders may learn value-relevant information from outsiders. Among many, Holmstrom and Tirole (1993) argue that stock prices provide information about the manager's actions and are therefore useful for managerial incentive contracts. Levit and Malenko (2011) analyze nonbinding voting for shareholder proposals and show that the information that is conveyed by voting outcome can affect corporate decision makers. Marquez and Yilmaz (2008) examine tender offers where shareholders have information about the firm value that the raider does not have. In Dow and Gorton (1997), Foucault and Gehrig (2008), and Goldstein and Guembel (2008), firms use information in stock prices to make investment decisions.

activist's motives cannot be distinguished by the market, prices are set fairly given the public information, including the activist's decision to exit.

The first result of the paper characterizes the interaction between voice and exit and shows that these mechanisms exhibit *complementarity*. That is, the option to exit enhances the ability of voice to improve the value of the firm. Importantly, when management is highly opportunistic, voice is an idle mechanism of governance unless exit is possible. Different from the existing literature, with voice, exit is a powerful form of shareholder activism even if the manager has no direct utility from the short-term stock price. Instead, the channel through which exit exercises discipline is by improving the ability of the activist to credibility communicate with the opportunistic manager.

To understand this result, note that inevitably the activist will manipulate some of her information in order to overcome the inherent conflict of interests between shareholders and the manager. Worrying about its credibility, the opportunistic manager will often ignore the activist's advice. This dynamic limits the amount of information that can be revealed by the activist in any equilibrium. The option to exit, however, enables the activist to dispose her holdings in the firm at times she believes that the share is over-priced. As a result, with the option to exit, the activist is less sensitive to the long-term performances of the firm and is more willing to compromise with inefficient managerial decisions. This increases the credibility of the activist's voice in the eyes of the manager and allows for an informed deliberation. With more information the manager can make better decisions.[5] Overall, voice is more effective with exit than without it.

The structure of executive compensation and the desire to demonstrate talent often make managers of public companies sensitive to the short-term performances of the stock price. This sensitivity can lead to *managerial myopia*. I show that without voice, the effect of managerial myopia on shareholder value is ambiguous and can be negative. By contrast, when the activist can communicate with the manager, managerial myopia unambiguously benefit shareholders. Intuitively, since exit depresses the short-term price of the stock, the myopic manager tries to minimize the likelihood the activist exits. With voice, this objective is translated into stronger incentives to follow the activist's advice. In this context, managerial myopia enhances the

---

[5]Interestingly, the analysis below demonstrates that the most informative equilibrium is not always the equilibrium that maximizes the expected value of the firm. See Corollary 2 for more details.

effectiveness of voice and improves shareholder value.

In practice, the activist does not have to voice herself secretly. Instead, the activist can make the letters to management public, ensuring that other market participants are aware of her demand. The second set of results relates to the role of *transparency* in shareholder activism. Transparency can relate to the publicity of the activist's message or the observability of managerial actions. It turns out, these two types of transparency are equivalent. I show that voice is less effective as a mechanism of shareholder activism with transparency than without it. However, when the activist can choose between public and private (or both) forms of communication, the adverse effect of transparency disappears.

Intuitively, the activist would like to get the highest price possible for her shares when she decides to exit. With transparency, the activist cannot resist the temptation to send messages that inflate the short-term price of the company's shares. Knowing that information will be manipulated, the activist loses credibility and her voice becomes less effective even when she does not exit. For this reason, with transparency, voice can be less effective with exit than without it. In other words, with transparency, voice and exit may exhibit *substitution*. In this respect, the analysis suggests that private engagements are more likely to be successful than public engagements, and it is crucial to allow activist investors to have private channels of communication with the management of the firm.[6]

Communication of private information or trading based on private information are the key channels through which activism is exercised in this paper. I show that the ability of the activist to communicate with the opportunistic manager and increase the value of the firm can *decrease* with the quality of her private information. Moreover, when information is costly, the amount of information the activist acquires in equilibrium and the effectiveness of her voice can *increase* with the cost of information. To see the intuition, note that while high quality of private information increases the amount of information that is potentially communicated by the activist, it can also exacerbate the adverse selection problem that is created when the activist exits. Since exit and voice complement each other, the total effect of the quality of the activist's private information on her ability to voice herself can be negative. For the same reason, when the cost of information is low, the activist acquires a significant amount of

---

[6]When managerial actions are observable and the manager is myopic, voice can be more effective when it is made public. See Section 4.3 and Proposition 5 for more details.

information and inevitably harms her ability to influence the manager. Overall, since the cost of acquiring information (per share) tends to decrease with the number of shares, the analysis suggests that small share-holdings can be a commitment by the activist not to acquire too much private information and consequently be more effective when exercising soft shareholder activism.

Finally, activist investors may not share the same objective as other shareholders of the firm. In many cases, the activist is pursuing short-term goals on the expense of the long-term value of the firm.[7] I study the effect of myopic activism on voice and exit. Since a myopic activist may exit even if the stock is not over-valued from the perspective of a long-term investor, the adverse selection problem upon exit is mitigated. This implies that the short-term price of the stock is *higher* when the activist is myopic than when she is not. Since voice and exit exhibit complementarity, and since myopia improves the terms of trade when the activist exit, voice becomes more effective as a form of shareholder activism when the activist is myopic.

The paper proceeds as follows. The remainder of this section discusses the relationship to the existing literature. Section 1 and Section 2 present the baseline model and its analysis, respectively. In Section 3 I study the effect of managerial myopia. Section 4 analyzes the effect of transparency on soft shareholder activism. Section 5 extends the baseline model by considering the quality of the activist's private information and her decision to acquire private information. Section 6 considers the implications of myopic activism. Section 7 discusses the empirical implications of the analysis and Section 8 concludes. All proofs are collected in the Appendix.

Relation to the Literature

Traditional models of shareholder activism focus on the benefits of corrective actions through direct intervention (For example, Shleifer and Vishny (1986), Admati, Pfleiderer, and Zechner (1994), Burkart, Gromb, and Panunzi (1997), Maug (1998), Kahn and Winton (1998), Bolton and von Thadden (1998), and Faure-Grimaud and Gromb (2004)). These studies share the idea that large shareholders are able to exercise *formal control* and thereby either *force* the company's management to improve the value of the firm or do it themselves. By contrast, the

---

[7] Anabtawi and Stout (2008) provide a comprehensive discussion of common conflicts of interest between activist investors and other shareholders.

present study focuses on *real control*. I show that investors can improve the value of the firm by communicating information and *persuading* the manager to make better decisions.

In this respect, closely related are studies by Levit and Malenko (2011) and Cohn and Rajan (2012). Levit and Malenko (2011) investigate non-binding voting for shareholder proposals as a mechanism through which shareholders of public companies can voice their opinions about governance and strategic related issues. They show that because of strategic voting, this mechanism often fails to aggregate shareholders' views when the manager is self-interested. Instead, the presence of an activist investor who can launch a proxy fight to replace the manager enhances the advisory role of non-binding voting, but only if the activist herself is biased. Different from Levit and Malenko (2011), here, the focus is on the interaction between voice and exit, and voice can be an effective form of shareholder activism even when there is no binding threat of discipline in the background. Cohn and Rajan (2012) study a model in which a board arbitrates between an activist investor and a manager, and focus on the interaction between internal and external governance. While in their model the activist investor makes a recommendation to the board, strategic communication of information and trading based on this information are assumed away.

The current study is also related to Admati and Pfleiderer (2009), Edmans (2009), Edmans and Manso (2011), who point out that exit can be an effective form of governance in itself. Key for their result is that the manager has direct utility from the short-term stock price, the channel through which exit matters. By contrast, I show that even when managers are not myopic in that sense, exit is an important mechanism of governance since it enhances the ability of the activist to communicate with the manager. Related, Dasgupta and Piacentino (2011) and Edmans and Manso (2011) consider exit and voice simultaneously. However, these studies assume that investors have formal control and can directly affect the value of the firm. Dasgupta and Piacentino (2011) also point out that exit can complement voice. However, their argument relies on the assumptions that managers are myopic and voice is a costly action that reduces the manager's private benefits.

The present paper also contributes to the literature on governance and liquidity. Bhide (1993) argues that because blockholders add value through voice, and voice and exit are mutually exclusive, liquidity is harmful as it allows a shareholder to leave rather than intervene. I argue the opposite, liquidity complements voice. In this respect, Maug (1998) and Kahn and

Winton (1998) demonstrate that liquidity facilitates block formation in the first place, as activist shareholders can buy additional shares at a price that does not incorporate the gains from intervention. However, conditional on the size of the activist's holdings, liquidity discourages intervention in those models. Faure-Grimaud and Gromb (2004) show that liquidity encourages intervention as it increases stock price informativeness, and if the activist is forced to sell prematurely, the price she receives will partially reflect the gains from intervention. While in their paper liquidity directly increases the gains from intervention, here, liquidity has an indirect effect. It alleviates the adverse selection problem when the activist trades and thereby reduces the activist's sensitivity to inefficient decisions made by the manager. Through this channel liquidity enhances the credibility of voice and increases the value of the firm.

1   Baseline Model - Setup

A public firm has to choose between two business strategies, L and R. If strategy $a \in \{L, R\}$ is implemented, the long-term shareholder value is given by

$$v\left(\theta, a\right) = \theta \cdot 1_{\{a=R\}} - \theta \cdot 1_{\{a=L\}} \tag{1}$$

Random variable θ has a continuous probability density function f and full support over $\left[\underline{\theta}, \overline{\theta}\right]$ where $\underline{\theta} < 0 < \overline{\theta}$. The specification of $v\left(\theta, a\right)$ implies that the value of each strategy crucially depends on θ but in a fundamentally different way. In particular, $v\left(\theta, a\right)$ increases with θ if and only if $a = R$. Shareholder value is maximized when strategy R is implemented if and only if $\theta \geq 0$.[8]

Shareholders own the cash flow rights of the firm, but the manager has the formal authority to make decisions. The manager and shareholders have conflicting preferences. Specifically, the manager's preferences are represented by,

$$u_M = v\left(\theta + \beta, a\right) \tag{2}$$

where $\beta \in \left(\max\left\{0, -\mathbb{E}\left[\theta\right]\right\}, -\underline{\theta}\right)$ is the non-pecuniary private benefits the manager obtains from the implementation of strategy R. The manager implements strategy R if and only if

---

[8] The results do not change qualitatively if alternatively $v\left(\theta, a\right) = \theta \cdot 1_{\{a=R\}}$.

he believes $\theta \geq -\beta$. Thus, when $\theta \in [-\beta, 0]$ there is a risk that the manager will choose the inefficient strategy. The larger is β, the greater is the conflicts of interest between shareholders and their manager. As I show below, the assumption $\beta > -\mathbb{E}[\theta]$ guarantees that without more information the manager always chooses $a = R$. Thus, the challenge of the activist is convincing the manager to choose strategy L despite the manager's inherent bias toward strategy R.

The ownership structure of the firm consists of dispersed shareholders and an activist investor. Dispersed shareholders have no ability or incentives to discipline the manager and hence remain passive. The focus of the analysis is on the ability of the activist investor to influence the manager's decision. The activist, however, does not have and cannot obtain formal control. Thus, the manager cannot be forced to take any action. Presumably, the cost of initiating and executing a proxy contest or a hostile takeover is too high. Instead, I study the ability of the activist to communicate her own view to the manager and thereby persuade him toward one action or the other.

To emphasize this channel of communication, let us assume that at the outset the activist obtains private and perfect information about θ. The key assumption of the model is that the activist's information is incremental to the manager's information. Specifically, I assume that all other market participants, including the manager, are uninformed about θ. Before the manager makes his decision about the project, the activist sends him a *private* message $m \in [\underline{\theta}, \overline{\theta}]$ about θ. The activist's private information is non-verifiable and her recommendation m cannot be backed-up with hard information. Moreover, the content of m does not affect the activist's payoff directly but only through its effect on the manager's decision. Thus, there are no private benefits or costs from communication per-se. Formally, the communication is modeled as cheap talk a la Crawford and Sobel (1982). Denote by $\mu(m \mid \theta) \in [0, 1]$ the probability that the activist sends message m conditional on her private information about θ, and by $a(m) \in \{R, L\}$ be the manager's decision conditional on observing message m.

After communicating her message to the manager the activist can trade with a competitive and risk neutral market maker. Unless the activist is hit by a liquidity shock, she is free to choose whether to exit or keep her holdings in the firm. With probability $\delta \in [0, 1]$, however, the activist is forced to sell her entire stake in the firm in order to accommodate her liquidity needs. Denote the activist's decision to sell her entire stake in the firm by $s = 1$, and her decision to keep it by $s = 0$. The activist cannot buy shares or commit to an exit strategy. The

decision of the activist to exit is observed by the market maker. However, the market maker does not observe the message the activist sent the manager or her needs for liquidity. These are the private information of the activist. Moreover, at the time of trade, the market maker does not observe the manager's decision. Based on all the available public information, the market maker sets the short-term price of the firm's share to be the expected value of the company. I denote this price by $p(s)$. Overall, the activist's preferences are given by,[9]

$$u_A = sp(s) + (1-s)v(\theta, a) \tag{3}$$

To summarize, there are four periods in the model. Initially, before the activist observes her liquidity needs but after she becomes informed about θ, the activist communicates with the manager by sending a private message m. At period 1, the manager decides whether to implement strategy R or strategy L, taking into account the activist's message. The manager's decision is unobservable by the market maker. At period 2, the activist observes her liquidity needs and the manager's decision, and then decides whether to exit. The market maker observes the order flow and determines the stock price accordingly. Finally, at period 3, the outcome is realized and becomes public. All agents are risk neutral and preferences are common knowledge.
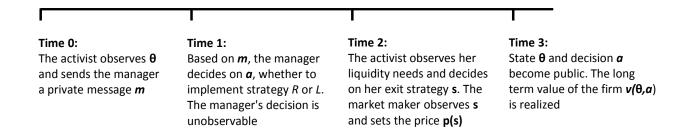
Time 0:	Time 1:	Time 2:	Time 3:
The activist observes **θ** and sends the manager a private message **m**	Based on **m**, the manager decides on **a**, whether to implement strategy *R* or *L*. The manager's decision is unobservable	The activist observes her liquidity needs and decides on her exit strategy **s**. The market maker observes **s** and sets the price **p(s)**	State **θ** and decision **a** become public. The long term value of the firm **v(θ,a)** is realized

Figure 1 - Timeline

2   Analysis

A Perfect Bayesian Equilibrium in our analysis consists of four parts: the activist's communication strategy $\mu(m|\theta) \in [0,1]$, the manager's implementation strategy $a(m) \in \{R, L\}$, the

---

[9]To save on notation, $s = 1$ whenever the activist is subject to a liquidity shock.

activist's trading strategy $s(\theta, m) \in \{0, 1\}$, and the market maker's pricing decision $p(s) \in \mathbb{R}$. A formal definition of the equilibrium is given in the Appendix.

Suppose in equilibrium the activist strategically exits if and only if $\theta \in \Upsilon$. Suppose also the manager implements strategy R if and only if $\theta \in \Theta$.[10] Let $v(\theta, \Theta)$ be the value of the firm under this decision rule. The competitive market maker uses Θ, Υ, and the activist's actual order flow s in order to price the shares of the company at their fair value,

$$
p(s) = \begin{cases} \frac{\delta \mathbb{E}[v(\theta,\Theta)] + (1-\delta)\Pr[\theta\in\Upsilon]\mathbb{E}[v(\theta,\Theta)|\theta\in\Upsilon]}{\delta + (1-\delta)\Pr[\theta\in\Upsilon]} & if \ s = 1 \\ \mathbb{E}[v(\theta,\Theta)|\theta \notin \Upsilon] & if \ s = 0 \end{cases} \tag{4}
$$

Note that Υ and Θ themselves may depend on the share price. The explicit formulation of the price when the activist does not exit plays no role in the analysis until managerial myopia in considered in Section 3. In what follows, and whenever there is no risk of confusion, s is omitted from the notation of the price, and p is simply the short-term stock price conditional on the activist's decision to exit. Since Θ and Υ uniquely determine the outcome of the game, I will often say that two equilibria are *equivalent* if they have identical sets Θ and Υ.

According to (2), the manager has no direct utility from the short-term stock price. Therefore, conditional on message m and the manager's expectations that the activist follows communication strategy μ, the decision of the activist to exit and the price that is set by the market maker have no effect on the manager's decision. In any equilibrium, the manager implements strategy R if and only if

$$
\mathbb{E}_\mu[\theta|m] + \beta \geq 0 \tag{5}
$$

where $\mathbb{E}_\mu[\cdot|m]$ is the manager's expectation of θ conditional on message m.[11] According to (5), $p(s)$ has no *direct* effect on the manager. Nevertheless, I will show that $p(s)$ may have an *indirect* effect on the manager's decision through the channel of communication. This feature is one aspect by which this model departs from the existing literature.

In what follows I focus attention on equilibria in which a meaningful (but possibly noisy)

---

[10]Implicitly, the analysis considers only pure strategy equilibria. In the Appendix I show that for any mixed strategy equilibrium there is a pure strategy equilibrium which is more efficient.

[11]Note that off-equilibrium events are possible. However, since the manager's utility is invariant to $p(s)$, the activist's off-equilibrium beliefs or actions cannot change the set of equilibria. In fact, since in any equilibrium the activist can perfectly predict the decision of the manager, whether or not the decision of the manager is observed by the activist has no effect on the set of equilibria.

communication between the activist and the manager is feasible. In these equilibria, voice is a meaningful mechanism of shareholder activism. Formally,

Definition 1 *An equilibrium is **"responsive"** if and only if there exist $m_R \neq m_L$ and $\theta_R \neq \theta_L$ such that $\mu\left(m_R \,|\theta_R\right) > 0$, $\mu\left(m_L \,|\theta_L\right) > 0$, and $a\left(m_R\right) \neq a\left(m_L\right)$.[12]*

In words, an equilibrium is responsive if and only if there are at least two different messages the activist sends the manager such that for one message, denoted by m_R, the manager responds by choosing strategy R, and for the other message, denoted by m_L, the manager responds by choosing strategy L. Note that Definition 1 is invariant to the exit strategy of the activist. However, as I demonstrate below, the existence of a responsive equilibrium crucially depends on the ability of the activist to exit.

The focus of the analysis is on shareholder value. An equilibrium is more *efficient* if it generates a higher ex-ante shareholder value. The analysis will focus on the most efficient equilibrium. In this respect, I restrict attention to a subset of equilibria in which $\Theta = \left[\tau, \overline{\theta}\right]$ for some $\tau \in \left[\underline{\theta}, \overline{\theta}\right]$. I name equilibria within this subset as *threshold equilibria* and often say that the manager follows a threshold strategy τ. I abuse notation and define $v\left(\theta, \tau\right) \equiv v\left(\theta, \Theta\right)$ as shareholder value when $\Theta = \left[\tau, \overline{\theta}\right]$. Note that the efficiency of a threshold strategy decreases with τ if and only if $\tau > 0$. The next lemma shows that in the search for efficiency the focus on threshold equilibria is without the loss of generality.

Lemma 1 *For any responsive equilibrium there is a threshold equilibrium which is more efficient.*

2.1 Benchmarks

To study the interaction between exit on voice I start by considering two benchmarks. In the first benchmark the activist can exit but she cannot voice her opinion. In the second benchmark the activist can voice herself but she cannot exit.

---

[12]Equivalently, an equilibrium is responsive if $\Theta \neq \left[\underline{\theta}, \overline{\theta}\right]$ or if there exist messages $m_R \neq m_L$ and communication strategy μ such that $\mathbb{E}_\mu\left[\theta|m_L\right] < -\beta \leq \mathbb{E}_\mu\left[\theta|m_R\right]$.

Benchmark I - Exit without Voice

Consider an equilibrium in which, by assumption, the activist cannot communicate with the manager. This corresponds to the non-responsive equilibrium of the game. As in any cheap talk game, this equilibrium always exists even when communication is allowed.

When the activist does not communicate with the manager the manager has no information about θ before he makes a decision. Since $\beta \geq -\mathbb{E}[\theta]$, according to (5), the manager chooses $a = R$ with probability one. Therefore, $\tau^* = \underline{\theta}$ and $\Theta_{NoVoice,Exit} = [\underline{\theta}, \overline{\theta}]$. Note that the manager's decision is invariant to the exit strategy of the activist. The activist strategically exits if and only if $\theta = v(\theta, \underline{\theta}) \leq p$. Thus, $\Upsilon_{NoVoice,Exit} = [\underline{\theta}, p]$.

The market maker correctly anticipates that the manager chooses strategy R and that the activist strategically exits if and only if $\theta < p$. For any τ define,

$$\varphi(p, \tau) \equiv \frac{\delta\mathbb{E}[v(\theta, \tau)] + (1 - \delta)\Pr[v(\theta, \tau) \leq p]\mathbb{E}[v(\theta, \tau) | v(\theta, \tau) \leq p]}{\delta + (1 - \delta)\Pr[v(\theta, \tau) \leq p]} \tag{6}$$

Substituting $\Theta_{NoVoice,Exit}$ and $\Upsilon_{NoVoice,Exit}$ into (4) implies that the price upon exit must be the solution of $p = \varphi(p, \underline{\theta})$. Since the activist exits for sure when $\theta < p$, the market maker forms the "worst case" beliefs on the value of the firm conditional on the activist's exit. For this reason, the solution of $p = \varphi(p, \underline{\theta})$ is unique and given by $\min_p \varphi(p, \underline{\theta})$.[13]

Lemma 2 *A non-responsive equilibrium always exists. In any non-responsive equilibrium the manager chooses strategy R with probability one, and the activist exits if and only if $\theta \leq p^*$. The share price conditional on exit is given by $p^* \equiv \min_p \varphi(p, \underline{\theta})$.*

Benchmark II - Voice without Exit

Suppose the activist, by assumption, cannot exit and sell her stake in the firm. Not that the activist's utility is proportional to the firm's value. Consistent with maximizing long-term shareholder value, the activist would like the manager to choose $a = R$ if and only if $\theta \geq 0$.

---

[13]This result follows from Proposition 1 in Acharya et al. (2011).

Define

$$z\left(x\right) \equiv \begin{cases} z : \mathbb{E}\left[\theta|\theta < -z\right] = -x & \text{if } x > -\mathbb{E}\left[\theta\right] \\ -\overline{\theta} & \text{if } x \leq -\mathbb{E}\left[\theta\right] \end{cases} \tag{7}$$

and note that $z\left(x\right) < x$, $z\left(x\right)$ strictly increases in x, and $\lim_{x \to -\underline{\theta}} z\left(x\right) = -\underline{\theta}$. The function z will be useful in the characterization of equilibria.

Lemma 3 *Suppose the activist cannot exit. A responsive equilibrium exists if and only if $\beta \leq z^{-1}\left(0\right)$. In any responsive equilibrium the manager implements strategy R if and only if $\theta \geq 0$.*

Lemma 3 suggests that without the option to exit, voice is ineffective when the conflict of interests between the manager and shareholders is significant. In those cases, the manager does not find the activist's advice credible and he ignores it. When $\beta \leq z^{-1}\left(0\right) = -\mathbb{E}\left[\theta|\theta < 0\right]$ the conflicts of interest are small and the activist has enough credibility to influence the manager's decision. Since the activist is unbiased, she will always use her advisory power to persuade the manager to implement the first best decision rule.

2.2 Voice and Exit - Soft Shareholder Activism

In this subsection I analyze the interaction between the activist's communication with the manager and trade with the market maker. The next lemma states the incentives constraints of the manager, the activist, and the market maker in any threshold equilibrium.

Lemma 4

(i) *The manager follows threshold τ if and only if $\beta \leq z^{-1}\left(-\tau\right)$.*

(ii) *The activist recommends on threshold τ if and only if $|\tau| \leq p$ or $p < 0 = \tau$. The activist exits strategically if and only if $|\theta| \leq p$.*

(iii) *In equilibrium, if the manager follows threshold τ then the market maker sets the share price upon exit at $\pi\left(\tau\right) \equiv \min_{p \geq \min\{-\tau, \tau\}} \varphi\left(p, \tau\right)$.*

Consider first the incentives of the manager to follow the proposed decision rule τ. Since the manager is biased toward strategy R, the manager always follows the recommendation of the activist to choose strategy R. The binding incentive constraint is convincing the manager to choose strategy L. Given bias β there exists τ such that conditional on $\theta < \tau$ the manager believes that strategy L is optimal, that is, $\mathbb{E}\left[\theta | \theta < \tau\right] + \beta \leq 0$. The higher is the bias, the lower must be threshold τ in order to satisfy this constraint. Thus, a threshold strategy τ can be sustained as a responsive equilibrium only if $\beta \leq z^{-1}\left(-\tau\right) \Leftrightarrow \tau \leq -z\left(\beta\right)$.

Consider the incentives of the activist. The activist always has the option to sell her holdings and guarantee a payoff of $p > 0$. The, activist can also guarantee a payoff of $|\theta|$. Indeed, if the equilibrium is responsive, by definition, the activist can dictate the manager's decision by sending the appropriate message. There are two cases to consider. First, if $|\theta| > p$, unless she needs liquidity, the activist has strict incentives to keep her holdings in the firm. The activist recommends the manager on strategy R if $\theta > p$ and recommends on strategy L if $\theta < -p$. For this reason, threshold τ can be an equilibrium only if $-p \leq \tau \leq p$. Moreover, this implies that if $p < 0$ the activist never exits unless she needs liquidity. In those cases, the activist recommends the manager on strategy R if and only if $\theta \geq 0$. If $|\theta| < p$, whether or not she needs liquidity, the activist has strict incentives to exit. The activist gets p for sure and becomes indifferent with respect to the long-term shareholder value. Thus, the activist has weak incentives to recommend any threshold $|\tau| \leq p$, even if $\tau \neq 0$.

Finally, consider how the market maker prices the share in equilibrium with threshold τ. If the manager follows threshold τ, strategy R is implemented if and only if $\theta \geq \tau$ and the value of the firm is given by $v\left(\theta, \tau\right)$. The activist exits if the short-term share price is higher than $v\left(\theta, \tau\right)$. In equilibrium, the market maker's beliefs are consist with the manager's decision rule and the activist's exit strategy. Thus, the share price must be the solution of $p = \varphi\left(p, \tau\right)$. Following the same line of reasoning as in Lemma 2, the solution of this equation is $\min_{p \geq \min\{-\tau, \tau\}} \varphi\left(p, \tau\right)$. Thus, if the equilibrium has threshold τ, the short-term price of the stock must be $\pi\left(\tau\right)$.

In equilibrium, the incentives and beliefs of the manager, the activist, and the market maker must be consistent with each other. For example, Lemma 4 implies that in any responsive equilibrium the share price upon exit must be positive. Indeed, if it is negative then the activist chooses to recommend on threshold $\tau = 0$ and keep her holdings in the firm as long as she can. Under this decision rule, the value of the firm is positive with probability one. While

the market maker does not observe the activist's message m or the manager's decision a, it has rational expectations. Therefore, setting a negative price is inconsistent with the fair share value of the firm. The next proposition characterizes the existence of a responsive equilibrium of the game.

Proposition 1 *There are unique $\underline{\tau} < 0 < \overline{\tau}$ such that:*

(i) *If and only if $\tau \in [\underline{\tau}, \min\{-z(\beta), \overline{\tau}\}]$ a responsive equilibrium with threshold τ exists.*

(ii) *A responsive equilibrium exists if and only if $\beta \leq z^{-1}(-\underline{\tau})$.*

In an equilibrium in which the manager follows threshold τ, the market maker sets the price upon exit on $\pi(\tau)$ and the activist is willing to recommend the manager to follow threshold τ if and only if $|\tau| \leq \pi(\tau)$. In the proof of proposition 1 I show that there are $\underline{\tau} < 0 < \overline{\tau}$ such that $|\tau| \leq \pi(\tau)$ if and only if $\tau \in [\underline{\tau}, \overline{\tau}]$. Therefore, threshold τ can be an equilibrium only if $\tau \in [\underline{\tau}, \overline{\tau}]$. Figure 2 illustrates this observation. The additional condition $\tau \leq -z(\beta)$ in part (i) of the proposition reflects the incentives compatibility constraints of the manager with respect to threshold τ. Note that as long as $\underline{\tau} \leq -z(\beta)$, a threshold equilibrium exists. Based on Lemma 1, any responsive equilibrium, even if it is not a threshold equilibrium, exists if and only if $\beta \leq z^{-1}(-\underline{\tau})$.
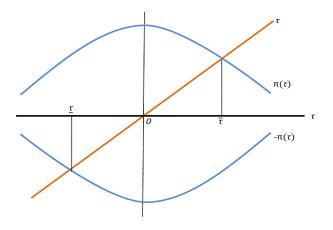

Figure 2

Proposition 1 implies that the most efficient threshold that can be supported in equilibrium

16

is $\tau^* = \max\{-z(\beta), 0\}$.[14] Since $z^{-1}(\cdot)$ is an increasing function and $\underline{\tau} < 0$, the comparison between Proposition 1 and Lemma 3 reveals that voice is more effective with exit than without it. Table 1 summarizes the expected shareholder value in the most efficient equilibrium under the different regimes.

Table 1- Expected Shareholder Value Under the Most Efficient Equilibrium

	No Voice	**Voice without Exit**	**Voice with Exit**
$\beta \in [0, z^{-1}(0)]$	$\mathbb{E}[\theta]$	$\mathbb{E}[\lvert\theta\rvert]$	$\mathbb{E}[\lvert\theta\rvert]$
$\beta \in (z^{-1}(0), z^{-1}(-\underline{\tau})]$	$\mathbb{E}[\theta]$	$\mathbb{E}[\theta]$	$\mathbb{E}[v(\theta, -z(\beta))]$
$\beta \in (z^{-1}(-\underline{\tau}), \infty)$	$\mathbb{E}[\theta]$	$\mathbb{E}[\theta]$	$\mathbb{E}[\theta]$

It follows from Table 1 that voice and exit exhibit *complementarity*. That is, the (positive) effect of voice on shareholder value is higher when the activist can exit than when she cannot exit. Essentially, the option to sell when the shares are over-valued enhances the activist's ability to persuade the manager to follow her recommendation. When the activist can exit, her payoff is less sensitive to the long-term performances of the firm and the activist is more willing to compromise with managerial inefficiencies. From the manager's point of view, the recommendation of the activist becomes more credible and he is more likely to follow it. Overall, more information can be communicated in equilibrium

Corollary 1 *Shareholder value under the most efficient equilibrium decreases with β and increases with δ.*

Corollary 1 demonstrates that shareholder value increases with the frequency of the activist's liquidity shocks, δ. Higher δ relaxes the adverse selection problem between the activist and the market maker. Indeed, when δ is high, the activist is less likely to exit because the share is over-valued, and the negative price impact of exit is diminished. The better terms of trade makes it easier for the activist to exit, and similar to the intuition above, voice becomes more effective and firm's value increases. Note that $\underline{\tau} \uparrow 0$ and $\overline{\tau} \downarrow 0$ as $\delta \to 0$. Thus, when the

---

[14]Since the share is priced fairly and the activist makes zero profit on average, her expected utility is $\mathbb{E}[v(\theta, \tau)]$. On the other hand, the manager's expected utility is given by $\mathbb{E}[v(\theta, \tau)] + (2\Pr[\theta \geq \tau] - 1)\beta$. Thus, the manager would benefit by shifting the threshold downward to $-\beta$. Nevertheless, the most efficient equilibrium is Pareto efficient.

frequency of liquidity shocks vanishes, the existence of a responsive equilibrium converges to the same conditions of the benchmark case of voice without exit.[15]

The next corollary shows that shareholder value does not monotonically increase with the amount of information that is exchanged between the activist and the manager.

Corollary 2 *A fully revealing equilibrium exists if and only if $\beta \leq -\underline{\tau}$. If a fully revealing equilibrium exists, there is a threshold equilibrium which is strictly more efficient.*

Recall that the binding incentive constraint is convincing the manager to choose strategy L. If information is fully revealed then the effective threshold is $-\beta$. The manager sub-optimally implements strategy R when $\theta \in [-\beta, 0]$. The activist can do strictly better by introducing noise into the communication with the manager. She can reveal whether θ is greater or smaller than $\min \{-z(\beta), 0\}$, but not the exact value of θ. By pooling very low realizations of θ with intermediate realizations of θ, the activist is able to persuade the manager to choose strategy L even when $\theta \in [-\beta, \min \{-z(\beta), 0\}]$. Since $-\beta < -z(\beta)$, the implemented threshold under the latter strategy is strictly more efficient. Figure 3 summarizes the observations above by plotting the manager's decision rule under different equilibria.
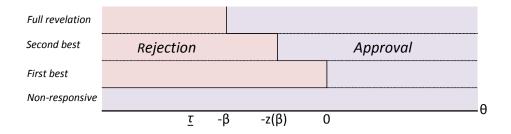

Figure 3 - Revelation of Information and Welfare Implications

[15] An interesting comparative static that is missing from Corollary 1 is whether shareholders are better off if the distribution of θ shifts upward in a first-order degree stochastic dominance manner. On the one hand, the manager is biased toward strategy R and he needs a stronger evidence that θ is low in order to choose L. Thus, $\max \{-z(\beta), 0\}$ decreases. On the other hand, since θ has shifted upward, strategy L is less likely to be the efficient decision in the first place. So the total effect on shareholder value is ambiguous.

3 Managerial Myopia

Managers are often sensitive to the short-term performances of their company's stock price. This could either be because of their compensation package which has stocks and options, or because of their career concerns and incentives to demonstrate executive talent. Either way, managerial myopia can play a significant role in the context of soft shareholder activism. To study the effect of myopia on the interaction of exit and voice, I modify the preferences of the manager in the baseline model as follows,

$$u_M = \omega p(s) + v(\theta + \beta, a) \tag{2'}$$

where $\omega \geq 0$ is the relative weight the manager puts on the short-term stock price.

For what follows it is useful to note that exit always conveys bad news. That is, $p(0) > p(1)$ for any Θ and Υ. Thus, when $\omega > 0$ the option to exit opens up the possibility that the activist threatens to sell her holdings in the company if the manager does not defer to her view.[16]

Lemma 5 *A non-responsive equilibrium exists for any $\omega \geq 0$. A non-responsive equilibrium in which the manager chooses strategy R with probability one exists if and only if $\Pr[\theta \geq \pi(\underline{\theta})] \geq \Pr[\theta \leq -\pi(\underline{\theta})]$ or $\omega \leq \hat{\omega}$ where $\hat{\omega} \in (0, \infty)$.*

Relative to Lemma 2, the set of non-responsive equilibria with myopia changes. The non-responsive equilibrium does not have to be unique and sometimes it must involve mixed strategies. That is, the manager chooses strategy L even though he believes that $\theta > -\beta$. Thus, with myopia, exit exerts discipline on the manager even when voice is idle. To understand why, note that the manager realizes that his actions affect the activist's decision to exit, which in turn affects the short-term price of the stock. To keep the share price high, the manager tries to minimize the probability the activist exits. When the manager's myopia is significant ($\omega > \hat{\omega}$) and the activist is relatively more likely to exit when the chosen strategy is R than when it is L (that is, $\Pr[-\theta < \pi(\underline{\theta})] < \Pr[\theta < \pi(\underline{\theta})]$), the attempt to minimize the likelihood the activist

---

[16]To see why $p(0) > p(1)$, note that by definition, $\theta \in \Upsilon$ implies $v(\theta, \Theta) \leq p(1)$ or else the activist is strictly better off by not exiting. Therefore, $\mathbb{E}[v(\theta, \Theta) | \theta \in \Upsilon] \leq p(1)$ as well. From expression (4) it follows that $\mathbb{E}[v(\theta, \Theta) | \theta \in \Upsilon] \leq \mathbb{E}[v(\theta, \Theta)]$. This in turn implies $\mathbb{E}[v(\theta, \Theta) | \theta \in \Upsilon] \leq \mathbb{E}[v(\theta, \Theta) | \theta \notin \Upsilon]$ and hence $p(0) > p(1)$ as required.

exits crowds out the manager's inherent bias toward strategy R. Under those circumstances, an equilibrium in which the manager chooses strategy R with probability one does not exist.

Lemma 5 has welfare implications. Managerial myopia increases the likelihood the manager chooses strategy L. Sometimes, an equilibrium in which the manager chooses strategy R with probability does not exist. Therefore, managerial myopia increases shareholder value if and only if $\mathbb{E}\left[\theta\right] < 0$. The next proposition shows that with exit and voice, the effect of myopia is always positive.

Proposition 2 *Any responsive equilibrium with $\omega_0 \geq 0$ is also an equilibrium with $\omega > \omega_0$. A responsive equilibrium exists if and only if $\beta \leq \beta^*\left(\omega\right)$ where $\beta^*\left(0\right) = z^{-1}\left(-\underline{\tau}\right)$, $\beta^*\left(\omega\right)$ strictly increases with ω, and $\lim_{\omega \to \infty} \beta^*\left(\omega\right) = \infty$.*

Intuitively, with myopia, the manager has stronger incentives to follow the activist's advice. As was explained above, exit always conveys bad news about the company. Different from Lemma 5, however, the manager's decision depends on the activist's message. If the manager ignores the activist's recommendation, the activist is strictly more likely to exit. From the point of view of the manager, the most successful way to prevent the activist from selling her holdings in company is simply following her recommendation. The threat of exit exercises stronger discipline on the manager to follow the activist's recommendation. For this reason, there are responsive equilibria with managerial myopia which do not exist without it. This can bee seen by noting that $\beta^*\left(\omega\right) > z^{-1}\left(\underline{\tau}\right)$. In conclusion, managerial myopia increases the effectiveness of voice and shareholder value.[17]

4   Transparency

In the baseline model the market maker does not observe the communication between the activist and the manager or the eventual decision of the manager. Under these assumptions, there is *"No-Transparency" (NT)*. In this section I relax these assumptions and study the effect

---

[17]With managerial myopia, non-threshold responsive equilibria may exist when threshold equilibria do not exist. That is, Lemma 1 does not extend to this setup. The reason is that the term $p\left(0\right) - p\left(1\right)$, which determines the incentives of the manager to follow the recommendation of the activist, does not necessarily increase with the efficiency of the decision rule.

of transparency on voice and exit. I distinguish between three regimes which differ with respect to the information the market maker observes before it sets the price of the share. Under *"Action-Transparency" (AT)* the market maker observes the decision of the manager, under *"Voice-Transparency" (VT)* the market maker observes the message that the activist sends the manager, and under *"Full-Transparency" (FT)* the market maker observes both the message that the activist sends and the decision that the manager makes.

The definition of a responsive equilibrium does not change under either of these regimes: it is an equilibrium in which the activist can change the manager's decision by communication. Similar to Proposition 2, for any $\omega \geq 0$ there is $\beta_T^* (\omega)$ such that under transparency of type $T \in \{AT, VT, FT\}$ a responsive equilibrium exists if and only if $\beta \leq \beta_T^* (\omega)$, and $\beta_T^* (\omega)$ strictly increases with ω. Throughout this section I focus on threshold equilibria.

4.1 Transparency of Actions

Let p_a be the share price conditional on exit and the manager's decision to implement strategy $a \in \{L, R\}$. In equilibrium, the activist takes p_a as given. Thus, the analysis of a non-responsive equilibrium coincides with Lemma 2. However, in a responsive equilibrium, if the activist persuades the manager to choose strategy R, she exits if and only if $\theta \leq p_R$. If the activist persuades the manager to choose strategy L, the activist exits if and only if $-\theta \leq p_L$.

Proposition 3

(i) *If a responsive equilibrium under Action-Transparency exists, it is unique and $p_R = p_L = \pi (\tau_{AT}^*) > 0$ where $\tau_{AT}^* \in (\underline{\tau}, \overline{\tau})$ is independent of ω.*

(ii) *If $\omega = 0$ or $\mathbb{E}[\theta|\theta > \pi (\tau_{AT}^*)] + \mathbb{E}[\theta|\theta < -\pi (\tau_{AT}^*)] > 0$ then $\beta_{AT}^* (\omega) < \beta_{NT}^* (\omega)$ and any equilibrium under Action-Transparency is also an equilibrium under No-Transparency.*

Proposition 3 implies that Action-Transparency limits the ability of the activist to communicate with the manager and a responsive equilibrium is less likely to exist. Since the most efficient equilibrium is a responsive equilibrium, Action-Transparency can harm shareholder value. Interestingly, in the Appendix I show that there are cases in which $\beta_{AT}^* (\omega)$ is strictly smaller than $z^{-1} (0)$. Based on Lemma 3, when the activist cannot exit and $\beta \leq z^{-1} (0)$, a

responsive equilibrium exists and the first best is implemented. Therefore, the ability of the activist to influence the manager's decision is adversely affected by her temptation to use her voice in order to get a better price when she exits. In other words, under Action-Transparency, voice and exit can exhibit *substitution*.

Intuitively, in a responsive equilibrium the activist can persuade the manager to follow her advice. If the share price conditional on exit depends on the manager's decision, the activist can arbitrage the difference between these prices by sending the appropriate message. The incentives of the activist to persuade the manager to choose the efficient strategy are distorted by the activist's desire to inflate the short-term price of the stock. Consequently, the credibility of her advice and her ability to influence the manager are diminished. This change is reflected by the additional constraint that in any responsive equilibrium the price must be invariant to the decision of the manager, $p_R = p_L$. In the proof of Proposition 3, I show that there is a unique threshold $\tau^*_{AT} \in (\underline{\tau}, \overline{\tau})$ that satisfies this constraint. Therefore, a responsive equilibrium is unique. Overall, under Action-Transparency, voice can be less effective as a form of soft shareholder activism.[18]

4.2   Transparency of Voice

Under Voice-Transparency the market maker observes the communication between the activist and the manager. In a non-responsive equilibrium, the manager ignores any message from the activist. However, the activist may still try to influence the market maker. Without being able to influence the manager's decision, the activist will always try to get the highest price possible when she exits. The incentive to inflate the short-term stock price prevents any revelation of information. Thus, under Voice-Transparency, a non-responsive equilibrium always exists and its characterization coincides with Lemma 2.

In a responsive equilibrium the activist believes she can influence the manager's decision. The market maker infers from the activist's message whether the manager will choose strategy R or strategy L. However, since the activist also has incentives to inflate the share price, any information beyond what is necessary to persuade the manager to choose one strategy over

---

[18]In Admati and Pfleiderer (2009), transparency of actions can also be undesirable. However, the channel in their model is fundamentally different: transparency distorts the blockholder's exit strategy and thereby diminish the effectiveness of exit as a governance mechanism.

the other cannot be revealed in equilibrium. For this reason, the Voice-Transparency and the Action-Transparency regimes are equivalent. Therefore, Voice-Transparency limits the ability of the activist to use voice as an effective form of shareholder activism.

Proposition 4 *For all $\omega \geq 0$ the set of responsive equilibria under Action-Transparency and Voice-Transparency are equivalent.*

Choosing Between Private and Public Voice

How does the analysis change if the activist can *choose* whether to send the manager a private or a public message? If the activist can commit to the channel of communication before she observes her private information, based on Proposition 3, the activist chooses private communication. Instead, suppose the activist can choose between sending a private message and communicating publicly (or both), only after she observes her private information. In this case, the set of feasible equilibria is the union of all equilibria under No-Transparency and Voice-Transparency. Indeed, an equilibrium under Voice-Transparency can be implemented as follows: the manager ignores any private messages from the activist, and the activist always randomizes when sending private messages. Similarly, an equilibrium under No-Transparency can be implemented as follows: the manager and the market maker ignore any public message sent by the activist, and the activist always randomizes when sending public messages. Overall, the option to communicate privately extends the set of equilibria under Voice-Transparency. Therefore, keeping an open channel of private communication between shareholders and managers has value to shareholders.

4.3 Full Transparency

As long as there is some kind of transparency, the activist's motivation to inflate the share price limits her ability to influence the manager. This constraint is independent of ω and β. Therefore, without managerial myopia ($\omega = 0$), Full-Transparency does not change the set of equilibria relative to Action-Transparency. The next proposition shows that when $\omega > 0$ this statement is invalid.

Proposition 5 *If $\omega = 0$ the sets of equilibria under Action-Transparency and Full-Transparency are equivalent. By contrast, if $\omega > 0$ then $\beta_{AT}^*(\omega) < \beta_{FT}^*(\omega)$ and any equilibrium under Action-Transparency is also an equilibrium under Full-Transparency.*

Under Full-Transparency the market maker observes both the recommendation of the activist and the decision of the manager to follow that recommendation. With myopia, the manager is sensitive to the short-term price. If the market maker observes that the manager ignores the activist's recommendation, whether or not the activist exits, the price drops. Since the market maker can compare the manager's decision with the activist's advice, an option which does not exist under any other kind of transparency, ignoring the activist's advice has additional cost to the manager. Full-Transparency adds another layer of discipline on managers and amplifies the effectiveness of voice. Therefore, if $\omega > 0$ the manager is more likely to follow the activist's advice and $\beta_{AT}^*(\omega) < \beta_{FT}^*(\omega)$. Overall, to the extent that managers are myopic and their actions are observed by the market on the short-term, activists can have a greater influence on managers if they voice themselves publicly.

5　Activist's Information Endowment

The main channels through which activism is exercised in the present model are communication of private information and trade which is based on private information. It is therefore natural to ask how the quality of the activist's private information affects her ability to influence the manager's decision. In order to explore this question, I modify the baseline model of Section 1 and assume that the activist does not perfectly observe θ. In the first subsection the quality of the the activist's private information is exogenous. In the second subsection I consider the activist's decision to acquire private information. To simplify the analysis, I assume throughout this section that $\mathbb{E}[\theta] > 0$.

5.1　Exogenous Quality of Private Information

Consider a variant of the baseline model in which the activist perfectly observes θ with probability $\lambda \in (0, 1]$ and with the complement probability the activist is uninformed about θ. Whether the activist is informed or uninformed is her own private information. Parameter λ is a common

knowledge and it captures the quality of the activist's private information. Specifically, higher λ implies higher quality of private information.

Suppose in equilibrium the price upon exit is p and the threshold is τ. The threshold relates to the communication strategy of the informed activist. Similar to the baseline model, the informed activist is willing to recommend on threshold τ if and only if $\tau \in [-p, p]$, and the informed activist strategically exits if and only if $v(\theta, \tau) \leq p$. The behavior of the uninformed activist is different. Since $\mathbb{E}[\theta] > 0$, without additional information, the uninformed activist and the manager agree that strategy R should be taken. Thus, the uninformed activist advises the manager to choose strategy R, and the manager follows this advice. If $p < \mathbb{E}[\theta]$ then the uninformed activist exits only if she needs liquidity. If $p \geq \mathbb{E}[\theta]$ the uninformed activist exits with probability one. For these reasons, if the manager is expected to follow the activist's recommendation, the beliefs of the market maker conditional on exit are give by

$$\varphi(p, \tau, \lambda) = \begin{cases} \frac{\lambda\delta\mathbb{E}[v(\theta,\tau)] + \lambda(1-\delta)\Pr[v(\theta,\tau)\leq p]\mathbb{E}[v(\theta,\tau)|v(\theta,\tau)\leq p] + \delta(1-\lambda)\mathbb{E}[\theta]}{\lambda\delta + \lambda(1-\delta)\Pr[v(\theta,\tau)\leq p] + \delta(1-\lambda)} & \text{if } p < \mathbb{E}[\theta] \\[2ex] \frac{\lambda\delta\mathbb{E}[v(\theta,\tau)] + \lambda(1-\delta)\Pr[v(\theta,\tau)\leq p]\mathbb{E}[v(\theta,\tau)|v(\theta,\tau)\leq p] + (1-\lambda)\mathbb{E}[\theta]}{\lambda\delta + \lambda(1-\delta)\Pr[v(\theta,\tau)\leq p] + (1-\lambda)} & \text{if } p \geq \mathbb{E}[\theta] \end{cases} \quad (8)$$

Note that (6) is a special case of (8) when $\lambda = 1$. It follows, p and τ can be an equilibrium only if $p = \varphi(p, \tau, \lambda)$. In the Appendix I show that $p = \varphi(p, \tau, \lambda)$ has a unique solution which I denote by $\pi(\tau, \lambda)$. Similar to Proposition 1, I also show that a responsive equilibrium exists if and only if β is below some critical value which depends on λ.[19]

For each value of λ more than one equilibrium may exist. In what follows I focus on the equilibrium that has the highest expected shareholder value. When the equilibrium is non-responsive ($\tau = \underline{\theta}$), the expected value of the firm is $\mathbb{E}[\theta]$ irrespective of λ. However, when the equilibrium is responsive, the expected value of the firm is given by $\lambda\mathbb{E}[v(\theta, \tau)] + (1-\lambda)\mathbb{E}[\theta]$ and it increases with λ. Note that if a responsive equilibrium exists, the most efficient equilibrium has a threshold $\tau = \min\{0, -z(\beta)\}$. The next proposition demonstrates that there are circumstances under which the expected value of the firm *decreases* with λ, the quality of the activist's private information.

Proposition 6 *Suppose $\mathbb{E}[\theta] > 0$ and let $\lambda^* \in [0, 1]$ be the (highest) level of λ that maximizes*

---

[19]The formal result is given in Proposition A.1 in the Appendix.

the expected value of the firm. Then, $\lambda^* < 1$ if and only if $\underline{\tau} > -z(\beta) > -\mathbb{E}[\theta]$.

Seemingly, since the activist is unbiased, higher quality of private information implies that the activist has more opportunities to persuade the manager to make decisions that maximize shareholder value. Proposition 6 shows that this intuition can be misleading. To understand this result, note that higher quality of private information can also intensify the adverse selection problem that the informed activist creates when she trades with the market maker. The intensified adverse selection problem restricts the ability of the activist to exit. Since exit and voice complement each other, the quality of the activist's private information can in fact limits her ability to voice herself credibly and influence the manager.

Specifically, Proposition 6 states two conditions. First, according to Proposition 1, the condition $\underline{\tau} > -z(\beta)$ implies that a responsive equilibrium does not exist when $\lambda = 1$. Second, the condition $-z(\beta) > -\mathbb{E}[\theta]$ requires that the manager is not too biased. The combination of these two conditions also implies $\underline{\tau} > -\mathbb{E}[\theta]$, which means that the adverse selection problem is severe (δ is small). In this range, as I show in the Appendix, the short-term price, $\pi(\min\{0, -z(\beta)\}, \lambda)$, decreases with λ. Therefore, when λ is low the activist can exit at better terms and consequently she is less sensitive to the long-term shareholder value. Similar to the discussion that follows Proposition 1, this dynamic enhances the ability of the activist to use voice as an effective form of shareholder activism. Overall, if $\underline{\tau} > -z(\beta) > -\mathbb{E}[\theta]$ there is $\lambda^* \in (0, 1)$ such that a responsive equilibrium exists if and only if $\lambda < \lambda^*$. It follows, the expected value of the firm when $\lambda \in (0, \lambda^*]$ is higher than when $\lambda \in (\lambda^*, 1]$.

5.2   Acquisition of Information

Consider an extension of the baseline model in which the activist can acquire information. Specifically, suppose the activist is uninformed about θ but she can perfectly observe θ with probability one if she pays a fixed cost $c \geq 0$. The cost of acquiring information is distributed according to cumulative distribution function G with full support over $[0, \infty)$. Moreover, c is the activist's private information and it is independent of the activist's liquidity shocks and fundamentals of the firm. The activist's decision to acquire information is unobserved by the market maker and the manager. Let $\lambda \in [0, 1]$ be the probability that the market maker and

the manager believe that the activist is informed in equilibrium. Unlike the analysis above, here λ is endogenous.

Let $\alpha \in (0, 1]$ be the size of the activist's holdings and suppose that in equilibrium threshold τ is followed. If the activist acquires information her expected value per share is

$$\delta \pi (\tau, \lambda) + (1 - \delta) \mathbb{E} [\max \{v (\theta, \tau), \pi (\tau, \lambda)\}] - c/\alpha \tag{9}$$

If the activist does not acquire information she gets

$$\delta \pi (\tau, \lambda) + (1 - \delta) \max \{\mathbb{E} [\theta], \pi (\tau, \lambda)\} \tag{10}$$

Note that expressions (9) and (10) increase with the price of the share upon exit. Since λ is determined in equilibrium and it is independent of the realization of c, in any equilibrium there is $c^* > 0$ such that the activist acquires information if and only if $c \leq c^*$. Therefore, $\lambda = G(c^*)$ where type c^* is indifferent between acquiring information and remaining uninformed. The cutoff c^* is given by the solution of the following equation,

$$\frac{c}{\alpha (1 - \delta)} = \mathbb{E} [\max \{v (\theta, \tau), \pi (\tau, G (c))\}] - \max \{\mathbb{E} [\theta], \pi (\tau, G (c))\} \tag{11}$$

A simple algebra shows that the right hand side of (11) is bounded and it increases in $\pi (\tau, G (c))$ if and only if $\pi (\tau, G (c)) < \mathbb{E} [\theta]$. Based on (8), $\pi (\tau, \lambda)$ is a weighted average of $\mathbb{E} [\theta]$ and another term, where the weight on $\mathbb{E} [\theta]$ decreases with λ. Therefore, $\pi (\tau, \lambda)$ decreases in λ if and only if $\pi (\tau, \lambda) < \mathbb{E} [\theta]$. Overall, the right hand side of (11) decreases with c. It follows, for a given τ, the solution of (11) always exists and it is unique.

The level of information acquisition in a non-responsive equilibrium is obtained by the solution of (11) when $\tau = \underline{\theta}$. Similar to the analysis in the previous subsection, a responsive equilibrium exists if and only if β is below some critical value which depends on λ. Different from that analysis, here λ is not exogenous. Instead, λ is given by $G(c^*)$ where c^* is the solution of (11) when $\tau = \min \{0, -z (\beta)\}$.[20]

---

[20]Note that c^* does not depend on β when the equilibrium is non-responsive. However, when the equilibrium is responsive, c^* decreases with β if and only if $\underline{\tau} > -\mathbb{E} [\theta]$. Also, the effect of δ on c^* is ambiguous. All else equal, higher δ increases the price upon exit. This effect increases the value of information (the option to exit when the stock is over-valued). On the other hand, higher δ implies that the activist is more likely to be forced to exit regardless of her private information. This reduces the incentives to acquire information.

Proposition 7 *Suppose $\underline{\tau} > -z(\beta) > -\mathbb{E}[\theta]$. There are $G_1 \overset{FDSD}{\prec} G_2$ such that $G_1(c_1^*) < G_2(c_2^*)$ and a responsive equilibrium exists if $c \sim G_2$ but it does not exist when $c \sim G_1$.*

Proposition 7 demonstrates that in spite of having a lower cost of information acquisition, in equilibrium, the activist acquires less information and is less effective in voicing herself credibly. Without a commitment mechanism, the reduction in the cost of acquiring information exacerbates the adverse selection problem since the activist will be tempted to acquire a significant amount of information. As Proposition 6 shows, this can harm the ability of the activist to persuade the manager to take actions and ultimately harm the value of the firm. Thus, when private information is relatively cheap, the only feasible equilibrium is a non-responsive equilibrium. Since information is less valuable if the activist cannot use it to influence the manager, the activist ends up acquiring less private information.

An alternative interpretation of Proposition 7 regrades the size of the activist's holdings. Since a larger stake is associated with a lower cost of information acquisition per share, Proposition 7 suggests that the voice of small blockholders can be more effective than the voice of large blockholders. Small share-holdings is a commitment tool to remain relatively uninformed, which increases the effectiveness of voice due to a weaker adverse selection problem. This is could be another explanation why some investors choose to limit the size of their initial holdings in the firm.

6   Myopic Activism

Activist investors such as hedge funds are often blamed for being opportunistic and pursuing short-term goals which are inconsistent with the long-term value of the firm. To capture this type of myopic behavior, I modify the baseline model and assumes that the activist's preferences are given by

$$u_A = sp + (1-s)\xi v(\theta, a) \tag{3'}$$

where $\xi \in (0, 1]$ is a common knowledge and it captures by how much the activist discounts the long-term value of the firm.

All else equal, the price upon exit is higher when the activist is myopic ($\xi < 1$) than when she

is not ($\xi = 1$). Indeed, myopia relaxes the adverse selection problem that is embedded in the activist's decision to exit. To see why, recall that when the activist is non-myopic, the activist exits whenever the share is over-valued, that is, if and only if $\theta \in [-p, p]$. For this reason, the market maker ascribes the "worst case beleifs" upon exit and the solution of $p = \varphi(p, \tau)$ is given by $\min_p \varphi(p, \tau)$. With myopia, however, the activist exits if and only if $\xi v(\theta, \tau) \leq p$, which holds if and only if $\theta \in [-p/\xi, p/\xi] \supset [-p, p]$. Essentially, the myopic activist decides to exit even if the share is under-valued. This dynamic pushes the price upon exit upward.[21]

Myopic activism not only increases the frequency at which the activist exits, but it also improves the terms of trade when the activist sells her shares. Similar to the analysis in Section 2, this effect expands the circumstances under which a responsive equilibrium exists. In other words, since voice and exit exhibit complementarity, myopia enhances the ability of the activist to influence the manager by voicing herself. In this context, the myopic behavior of the activist investor can in fact benefit other long-term investors.

7   Empirical Implications

The main premise of the theoretical analysis in this paper is that voice is an informal communication between investors and the manager of the firm. To the extent that the number of meetings, emails, letters, or phone calls between management and investors can be observed, the model offers new testable predictions about the frequency of this type of engagement.

The model predicts the frequency of engagement should be negatively related to the longevity of the large investor, where longevity is the likelihood that the activist is not subject to a short-term liquidity shock $(1 - \delta)$. This prediction is a reflection of the complementarity between voice and exit: shorter longevity implies that the adverse selection problem when trading with the market maker is weaker, and hence, all else equal, the investor can exit at better terms and be more effective when talking to the management. This prediction is consistent with Solomon and Soltes (2012) who study the frequency of meetings between senior management and investors and show that investors who have greater turnover in their holdings gain greater access to management.

---

[21]In equilibrium with a threshold τ, the price upon exit must solve $p = \varphi(p \backslash \xi, \tau)$. A solution always exists, but it does not have to be unique.

The model also predicts that high frequency of engagement should be observed when the conflicts of interest between shareholders and the manager are small (low β) and the short-term component in the manager's compensation package is relatively important (high ω). In both cases the analysis predicts that voice is more effective (Proposition 1 and Proposition 2, respectively), and hence, one would expect to see more engagement between investors and management.

The analysis considers both private and public engagements between activist investors and firms, and concludes that private engagements (lack of transparency) are more likely to be effective. Thus, the model predicts that private engagements should be observed more often than public engagements. If one can control for the decision of the activist to run a public (rather than private) campaign, then, all else equal, private engagements should have a stronger positive effect on the performances of the firm than public engagements. Consist with this observation, Becht, Franks, and Grant (2010) find that for board and payout changes, and restructuring events other than takeovers, the returns to the activist are higher when the engagement is private.

The communication between investors and the manager of the firm is often informal and private. It is therefore difficult to measure the magnitude and quality of this informal engagement using publicly available data. To the extent that the frequency of engagement or its quality are not observed, the model in this paper offers some indirect empirical predictions. The model predicts that the effectiveness of soft shareholder activism generally decreases with the longevity of the large shareholders. That is, there is an inverse relationship between the longevity of the investor and the value of the firm. This is in contrast to other models of shareholder activism. For example, Admati and Pfleiderer (2009) predict exactly the opposite. Moreover, different from other models of exit, soft shareholder activism can be effective even when managers are not myopic. Thus, by studying the effect of exit on the performances of firms with a negligible amount of short-term executive compensation, one can indirectly identify the effect or prevalence of private engagement and communication.

Finally, the model predicts that even small investors who are mainly active in middle markets and do not have the capacity of obtaining control through hostile takeovers or proxy fights (and in particular, those with holdings below 5%) can have a significant effect on the value of the firm. Alternatively, when the firm has a controlling shareholder and a change of control is

practically impossible, the model predicts that smaller blockholders can still play an active role and enhance the value of the firm.[22] These predictions are in contrast with other models of intervention which builds on the ability of investors to obtain formal control and force management to take actions.

8 Concluding Remarks

This paper offers a new perspective on shareholder activism by focusing on what can be achieved when costly formal control cannot be obtained or exercised by shareholders. Two primary mechanisms are analyzed, voice and exit. Departing form the majority of the existing literature on shareholder activism, voice is modeled as a strategic transmission of information. This form of informal communication is a reflection of investors' attempt to exercise activism by sending letters, calling senior executives, and meeting with board members, thereby providing their input and ultimately changing the strategic course of the company. The paper analyzes the conditions under which "soft" shareholder activism is an effective form of corporate governance. It highlights the synergetic nature of the interaction between voice and exit, the role of transparency in shareholder activism, and the effect of managerial myopia and myopic activism on investors' ability to voice themselves effectively.

---

[22]Using proprietary data, Becht, Franks, and Grant (2010) give an example of a successful private engagement between an activist fund and the management of a company whose largest shareholder was a family holding over 50% of the voting rights. While the fund owned less than 2% of the company, it was able to significantly change the strategy of the company and consequently realized significant abnormal returns on its investment.

References

[1] Acharya, V. V., P. M. DeMarzo, and I. Kremer (2011): "Endogenous Information Flows and the Clustering of Announcements," *American Economic Review*, 101(7): 2955-79.

[2] Admati, Anat R., and Paul Pfleiderer, 2009, The "Wall Street Walk" and Shareholder Activism: Exit as a Form of Voice, *Review of Financial Studies* 22, 2645-2685

[3] Admati, Anat R., Paul Pfleiderer, and Josef Zechner, 1994, Large shareholder activism and financial market equilibrium, *Journal of Political Economy*, 102, 1097–1130.

[4] Anabtawi, Iman, and Lynn A. Stout, 2008, Fiduciary Duties for Activist Shareholders, *Stanford Law Review* 60, 1255-1308.

[5] Becht Marco, Julian Franks, Colin Mayer, and Stefano Rossi, 2009, Returns to shareholder activism: Evidence from a clinical study of the Hermes UK Focus Fund, *Review of Financial Studies* 22 3093-3129.

[6] Becht Marco, Julian Franks, and Jeremy Grant , 2010, "Hedge Fund Activism in Europe," Working Paper.

[7] Bhide, A., 1993, The Hidden Costs of Stock Market Liquidity, *Journal of Financial Economics*, 34, 31–51.

[8] Bolton, Patrick, and Ernest.-Ludwig von Thadden, 1998, Blocks, liquidity, and corporate control, *Journal of Finance* 53, 1-25.

[9] Brav, Alon, Wei Jiang, Frank Partnoy, and Randall Thomas, 2008, Hedge fund activism, corporate governance, and firm performance, *Journal of Finance* 63, 1729–1775

[10] Burkart, M., D. Gromb, and F. Panunzi. 1997. Large Shareholder, Monitoring and theValue of the Firm. *Quarterly Journal of Economics* 112:693–728.

[11] Carleton,W. T., J.M. Nelson, andM. S.Weisbach. 1998. The Influence of Institutions on Corporate Governance through Private Negotiations: Evidence from TIAA-CREF. *Journal of Finance* 53:1335–62.

[12] Cohn Jonathan B., and Uday Rajan, 2012, Optimal Corporate Governance in the Presence of an Activist Investor, forthcoming, *Review of Financial Studies.*

[13] Crawford, Vincent P., and Joel Sobel, 1982, Strategic Information Transmission, *Econometrica* 50, 1431-1451.

[14] Dasgupta, Amil and Giorgia Piacentino, 2011, The Wall Street Walk when Blockholders Compete for Flows, Working Paper.

[15] Dow, James, and Gary Gorton, 1997, Stock Market Efficiency and Economic Efficiency: Is There a Connection?, *Journal of Finance* 52, 1087-1129.

[16] Edmans, Alex, 2009, Blockholder Trading, Market Efficiency, and Managerial Myopia, *Journal of Finance* 64, 2481-2513.

[17] Edmans, A., and G. Manso, 2011, Governance Through Trading and Intervention: A Theory of Multiple Blockholders, *Review of Financial Studies*, 24, 2395–2428.

[18] Faure-Grimaud, A., and D. Gromb, 2004, Public Trading and Private Incentives, *Review of Financial Studies,* 17, 985–1014.

[19] Foucault Thierry, and Thomas Gehrig, 2008, Stock Price Informativeness, Cross-Listings, and Investment Decisions, *Journal of Financial Economics* 88, 146-168.

[20] Gantchev, Nickolay, 2012, The Costs of Activist Monitoring: Evidence from a Sequential Decision Model, forthcoming, *Journal of Financial Economics*

[21] Goldstein, Itay and Alexander Guembel, 2008, Manipulation and the Allocational Role of Prices, *Review of Economic Studies* 75, 133-164.

[22] Holmstrom, Bengt, and Jean Tirole, 1993, Market Liquidity and Performance Monitoring, *Journal of Political Economy* 101, 678-709.

[23] Kahn, C., and A. Winton, 1998, Ownership Structure, Speculation, and Shareholder Intervention, *Journal of Finance*, 53, 99–129.

[24] Levit Doron, and Nadya Malenko, 2011, Nonbinding Voting for Shareholder Propoals, *Journal of Finance*, 66(5), 1579-1614.

[25] Marquez, Robert, and Bilge Yilmaz, 2008, Information and Efficiency in Tender Offers, *Econometrica* 76, 1075-1101.

[26] Maug, E., 1998, Large Shareholders as Monitors: Is There a Tradeoff Between Liquidity and Control?, *Journal of Finance*, 53, 65–98.

[27] McCahery, J., Z. Sautner, and L.T. Starks, 2011, Behind the Scenes: the Corporate Governance Preferences of Institutional Investors, Working Paper, Tilburg University.

[28] Shleifer, Andrei, and Robert Vishny, 1986, Large shareholders and corporate control, *Journal of Political Economy* 94, 461-488.

[29] Solomon, D., Soltes, E., 2011. What are we meeting for? The consequences of private meetings with investors. Working paper, Harvard University

Appendix A

In any Perfect Bayesian Equilibrium the following must hold:

1. For any s, the market maker sets the price p equals to the expected value of the firm taking the activist's communication strategy $\mu\left(\cdot\,|\,\theta\right)$ and the manager's implementation strategy $a\left(\cdot\right)$ as given.

2. For any θ and m the activist's trading decision $s\left(\theta,m\right)$ maximizes the value of her holdings in the firm taking the manager's implementation strategy $a\left(m\right)$ and the market maker's pricing policy $p\left(\cdot\right)$ as given.

3. For any message m the implementation strategy $a\left(m\right)$ maximizes the manager's expected utility, taking the activist's communication strategy $\mu\left(\cdot\,|\,\theta\right)$, trading strategy $s\left(\cdot,m\right)$, and the market maker's pricing policy $p\left(\cdot\right)$ as given.

4. For any θ, if message m is in the support of $\mu\left(\cdot\,|\,\theta\right)$, then m maximizes the expected utility of the activist given the manager's implementation strategy $a\left(\cdot\right)$, her own trading strategy $s\left(\theta,\cdot\right)$, and market maker's pricing policy $p\left(\cdot\right)$.

Lemma A.1 *In any responsive equilibrium: (i) $p\left(1\right) > 0$; (ii) $\Upsilon = \left[-p\left(1\right), p\left(1\right)\right]$; (iii) $\Theta \supseteq \left[p\left(1\right), \overline{\theta}\right]$ and $\Theta^c \supseteq \left[\underline{\theta}, -p\left(1\right)\right]$.*

Proof. Consider part (i) and suppose on the contrary $p\left(1\right) \le 0$. Since the equilibrium is responsive, by definition, there are $m_L \ne m_R$ such that $a\left(m_R\right) = R$ and $a\left(m_L\right) = L$. Recall action a and message m are not observable by the market maker. Thus, in equilibrium, the activist takes $p\left(1\right)$ as given. If $\theta \ge 0$ the activist is better off by sending message m_R and choosing $s = 0$. If $\theta < 0$ the activist is better of by sending m_L and choosing $s = 0$. Either way, the activist never exits strategically. Thus, $p\left(1\right) = \mathbb{E}\left[|\theta|\right] > 0$, a contradiction.

Consider part (ii) and note that if $|\theta| < p\left(1\right)$ then $\max_{a \in \{R,L\}} v\left(\theta, a\right) < p\left(1\right)$. Therefore, $s = 1$ is a dominant strategy, whether or not the activist needs liquidity.

Consider part (iii). Suppose on the contrary $\left[p\left(1\right), \overline{\theta}\right] \setminus \Theta \ne \varnothing$. If $\theta \in \left[p\left(1\right), \overline{\theta}\right] \setminus \Theta$ then $\theta \in \Theta^c \cap \left[p\left(1\right), \overline{\theta}\right]$. If the activist chooses $m = m_R$ and $s = 0$ she gets $\theta > p\left(1\right)$. If she chooses $m = m_L$ she gets $\max\left\{p\left(1\right), -\theta\right\}$. Since $p\left(1\right) > 0$, the activist is strictly better off

sending message $m \in m_R$. This contradicts the presumption that $\theta \in \Theta^c \Rightarrow m = m_L$. A similar argument proves that if on the contrary $[\underline{\theta}, -p(1)] \backslash \Theta^c \neq \varnothing$ and if $\theta \in [\underline{\theta}, -p(1)] \backslash \Theta^c$ then the activist has incentives to send message m_L which contradicts the presumption that $\theta \in \Theta \Rightarrow m = m_R$. $\blacksquare$

Lemma A.2 *In any responsive equilibrium with decision rule Θ, $p(1, \Theta) = \min_{p \geq \min_\theta v(\theta, \Theta)} \varphi(p, \Theta)$ where*

$$\varphi(p, \Theta) = \frac{\delta \mathbb{E}\left[v(\theta, \Theta)\right] + (1 - \delta) \Pr\left[v(\theta, \Theta) \leq p\right] \mathbb{E}\left[v(\theta, \Theta) | v(\theta, \Theta) \leq p\right]}{\delta + (1 - \delta) \Pr\left[v(\theta, \Theta) \leq p\right]} \tag{A1}$$

If Θ_1 and Θ_2 are such that $v(\theta, \Theta_1) \geq v(\theta, \Theta_2)$ for all θ then $p(s, \Theta_1) \geq p(s, \Theta_2)$.

Proof. Note that $\theta \in \Upsilon$ if and only if $v(\theta, \Theta) \leq p(1, \Theta)$. Therefore, the price $p(1, \Theta)$ must be the solution of $p = \varphi(p, \Theta)$. Extending Proposition 1 in Acharya et al. (2011) for a random variable $v(\theta, \Theta)$, one can show that $p(1, \Theta)$ is the unique solution of $p = \varphi(p, \Theta)$.

Consider the second part. Since $v(\theta, \Theta_1) \geq v(\theta, \Theta_2)$ for all θ then $\varphi(p, \Theta_1) > \varphi(p, \Theta_2)$ for any p. Since $p(1, \Theta_i)$ is the global minimum of $\varphi(p, \Theta_i)$, and it is the solution of $p = \varphi(p, \Theta_i)$, it follows, $p(1, \Theta_1) > p(1, \Theta_2)$. Given expression (4) and Lemma A.1, $p(0, \Theta) = \mathbb{E}\left[|\theta| \, | \, |\theta| > p(1, \Theta)\right]$. Therefore, $p(1, \Theta_1) \geq p(1, \Theta_2)$ implies $p(0, \Theta_1) \geq p(0, \Theta_2)$. $\blacksquare$

Proofs of Section 2

Proof of Lemma 1. Consider a responsive equilibrium where Θ is non-threshold. There are $\theta_1 < \theta_2$ such that $\theta_1 \in \Theta$ and $\theta_2 \notin \Theta$. Recall that in any responsive equilibrium the activist can dictate a. Based on (3), if $\theta \in [\theta_1, \theta_2]$ the activist must be indifferent between R and L. Hence, the activist exits with probability one if $\theta \in [\theta_1, \theta_2]$ and $[\theta_1, \theta_2] \subset \Upsilon$.

For $i \in \{1, 2\}$, let m_i be the message that is sent when the activist observes θ_i and $d_i \equiv \{\theta : \mu(m_i | \theta) > 0\}$ the set of θ for which the activist sends message m_i. By definition, $\theta_i \in d_i$. Note that $\mathbb{E}[\theta | m_i] = \mathbb{E}[\theta | \theta \in d_i]$ and condition (5) imply

$$\mathbb{E}[\theta | \theta \in d_1] \geq -\beta \geq \mathbb{E}[\theta | \theta \in d_2] \tag{A2}$$

Consider an alternative equilibrium with Θ'. In this equilibrium, the communication strategy is identical to the original equilibrium, with the sole exception $d'_i \equiv d_i \cup \{\theta_j\} \setminus \{\theta_i\}$. Under the new strategy, the activist sends message m_i when he observes θ_j, $i \neq j$. Since $\theta_1 < \theta_2$ then $\mathbb{E}[\theta|\theta \in d'_1] > \mathbb{E}[\theta|\theta \in d_1]$ and $\mathbb{E}[\theta|\theta \in d_2] > \mathbb{E}[\theta|\theta \in d'_2]$. Thus, given condition (5) and (A2), the manager has incentives to follow the activist's recommendation under the new strategy. Note that $v(\theta, \Theta') \geq v(\theta, \Theta)$ for any θ (the difference in the expected value is $(\theta_2 - \theta_1) - (\theta_1 - \theta_2) > 0$). Based on Lemma A.2, $p(1, \Theta') \geq p(1, \Theta)$. Based on Lemma A.1, $\Upsilon = [-p(1, \Theta), p(1, \Theta)] \subset [-p(1, \Theta'), p(1, \Theta')] = \Upsilon'$. The activist finds it weakly optimal to follow the new communication strategy, yielding an equilibrium which is strictly more efficient. One can repeat this procedure as long as the equilibrium is non-threshold, eventually, converging to a threshold equilibrium. ∎

Proof of Lemma 3. By Definition 1, if a responsive equilibrium exists the activist can dictate the action taken by the manager. Since the activist cannot exit, she has strict incentives to persuade the manager to choose R when $\theta > 0$ and to choose L when $\theta < 0$.

Suppose $\beta \leq -\mathbb{E}[\theta|\theta < 0] = z^{-1}(0)$. Consider an equilibrium in which the activist sends message m_R if $\theta \geq 0$ and a message $m_L \neq m_R$ otherwise. Conditional on $m = m_R$ the manager believes $\theta \geq 0$. According to (5), the manager chooses strategy R if and only if $\mathbb{E}[\theta|\theta \geq 0] + \beta \geq 0$. Since $\beta > -\mathbb{E}[\theta] > \mathbb{E}[\theta|\theta \geq 0]$, this condition always holds. Conditional on $m = m_L$ the manager believes that $\theta < 0$. According to (5), the manager choose strategy L if and only if $\mathbb{E}[\theta|\theta < 0] + \beta \leq 0$, which by assumption holds. Thus, the manager follows the activist's recommendation. Given the manager's expected behavior, it is in the best interest of the activist to follow the proposed communication strategy, so this is indeed a responsive equilibrium.

To see the other direction, suppose a responsive equilibrium holds. Let M_L be the set of all messages such that $\Pr[a(m) = L] > 0$. Let M_R be the set of all messages such that $\Pr[a(m) = R] = 1$. Since the equilibrium is responsive, neither set is empty. According to (5), if $a(m) = L$ then $\mathbb{E}_\mu[\theta|m] + \beta \leq 0$ and if $\Pr[a(m) = L] > 0$ then $\mathbb{E}_\mu[\theta|m] + \beta = 0$. Therefore, integrating over all $m \in M_L$ it follows that $\mathbb{E}_\mu[\theta|M_L] + \beta \leq 0$. Similarly, integrating over all $m \in M_R$ implies $\mathbb{E}_\mu[\theta|M_R] + \beta \geq 0$. Recall the activist has incentives to send $m \in M_L$ if and only if $\theta \leq 0$. For this reason, $\mathbb{E}_\mu[\theta|M_L] = \mathbb{E}[\theta|\theta \leq 0]$ and $\mathbb{E}_\mu[\theta|M_R] = \mathbb{E}[\theta|\theta > 0]$. Overall,

$\beta \leq -\mathbb{E}[\theta | \theta < 0]$ holds as required. ∎

Proof of Lemma 4. Consider part (i) and suppose the manager follows threshold τ. According to (5), $\mathbb{E}[\theta | m] + \beta \leq 0$ for any $m \in M_L$. By integrating over all messages in M_L we get $\mathbb{E}[\theta | M_L] + \beta \leq 0$ as well, where $\mathbb{E}[\theta | M_L] = \mathbb{E}[\theta | \theta \leq \tau]$. By definition, $\mathbb{E}[\theta | \theta \leq -z(\beta)] = -\beta$. Therefore, $\tau \leq -z(\beta)$ is necessary. Suppose $\beta \leq z^{-1}(-\tau)$. Consider an equilibrium in which the activist sends message m_R if $\theta \geq \tau$ and message $m_L \neq m_R$ otherwise. Since $\beta \geq -\mathbb{E}[\theta]$ then $\beta \geq -\mathbb{E}[\theta | \theta \geq \tau]$ and the manager follows the recommendation to choose strategy R. Conditional on $m = m_L$ the manager believes that $\theta < \tau$. The manager follows the recommendation to choose strategy L if and only if $\mathbb{E}[\theta | \theta \leq \tau] \leq -\beta$, which holds when $\tau \leq -z(\beta)$. Therefore, the manager follows threshold τ.

Consider part (ii). In a threshold equilibrium, $\Theta = [\tau, \overline{\theta}]$. Based on the arguments for the proof of Lemma A.1 part (i), if $p(1) \leq 0$ then the activist never exits strategically. Based on Lemma 3, she effectively implements threshold τ. Based on Lemma A.1, if $p(1) > 0$ then $\Upsilon = [-p(1), p(1)]$ and it must be $\tau \in \Upsilon$. This completes the argument.

Consider part (iii). Based on Lemma A.2, if $\Theta = [\tau, \overline{\theta}]$ then $p(1, \Theta) = \min_{p \geq \min\{-\tau, \tau\}} \varphi(p, \tau)$, where $\varphi(p, \tau)$ is given by (6). ∎

Proof of Proposition 1. I prove that there are unique $\underline{\tau} < 0 < \overline{\tau}$ such that $-\pi(\tau) \leq \tau \leq \pi(\tau)$ if and only if $\tau \in [\underline{\tau}, \overline{\tau}]$. Consider several properties of $\pi(\tau)$. First, based on Lemma A.2, if $|\tau'| < |\tau''|$ then $\pi(\tau') \geq \pi(\tau'')$. Second, for any p and τ (6) can be rewritten as

$$\varphi(\tau, p) = \frac{\delta\left[-\int_{\underline{\theta}}^{\tau} \theta dF(\theta) + \int_{\tau}^{\overline{\theta}} \theta dF(\theta)\right] + (1-\delta)\left[-\int_{\min\{-p,\tau\}}^{\tau} \theta dF(\theta) + \int_{\tau}^{\max\{p,\tau\}} \theta dF(\theta)\right]}{\delta + (1-\delta)\Pr[\theta \in [\min\{-p,\tau\}, \max\{p,\tau\}]]}$$
(A3)

Thus, $\varphi(\tau, p)$ is continuous in $\tau \in [\underline{\theta}, \overline{\theta}]$. Since $\pi(\tau)$ is the unique minimum of $\varphi(\tau, p)$, it is continuous in τ as well. Third, note that $\pi(0) > 0$. Overall, $\pi(\tau) - \tau$ decreases in τ when $\tau > 0$. By the intermediate value theorem, there is $\overline{\tau} \in [0, \overline{\theta}]$ such that if $\tau \in [0, \overline{\tau})$ then $\pi(\tau) - \tau > 0$ an if $\tau \in (\overline{\tau}, \overline{\theta}]$ then $\pi(\tau) - \tau < 0$. Similarly, $\pi(\tau) + \tau$ increases in τ when $\tau < 0$. By the intermediate value theorem, there is $\underline{\tau} \in [\overline{\theta}, 0]$ such that if $\tau \in (\underline{\tau}, 0]$ then $\pi(\tau) + \tau > 0$ an if $\tau \in [\underline{\theta}, \underline{\tau})$ then $\pi(\tau) + \tau < 0$. This completes the argument.

Next, I prove that if a responsive equilibrium exists then $\beta \leq z^{-1}(-\underline{\tau})$. Based on Lemma 1,

if a responsive equilibrium exists, then a threshold equilibrium exists as well. Based on Lemma 4 and the argument above, it is necessary that $\tau \in [\underline{\tau}, \overline{\tau}]$. According to (5), $\mathbb{E}[\theta|m] + \beta \leq 0$ for any $m \in M_L$. By integrating over all messages in M_L we get $\mathbb{E}[\theta|M_L] + \beta \leq 0$ as well, where $\mathbb{E}[\theta|M_L] = \mathbb{E}[\theta|\theta \leq \tau]$. By definition, $\mathbb{E}[\theta|\theta \leq -z(\beta)] = -\beta$. Therefore, $\tau \leq -z(\beta)$ is necessary. Since $\tau \in [\underline{\tau}, \overline{\tau}]$ then $\underline{\tau} \leq -z(\beta)$ as required. The same reasoning also implies that if a threshold equilibrium exists then the threshold must satisfy $\tau \in [\underline{\tau}, \min\{-z(\beta), \overline{\tau}\}]$.

Last, suppose $\beta \leq z^{-1}(-\underline{\tau})$ and let $\tau \in [\underline{\tau}, \min\{-z(\beta), \overline{\tau}\}]$. Note that this interval is not empty. Consider an equilibrium in which the activist sends message m_R if $\theta \geq \tau$ and message $m_L \neq m_R$ otherwise. Based on Lemma 4, the price upon exit is $\pi(\tau)$. Since $\beta \geq -\mathbb{E}[\theta]$ then $\beta \geq -\mathbb{E}[\theta|\theta \geq \tau]$ and the manager follows the recommendation to choose strategy R. Conditional on $m = m_L$ the manager believes that $\theta < \tau$. The manager follows the recommendation to choose strategy L if and only if $\mathbb{E}[\theta|\theta \leq \tau] \leq -\beta$, which holds when $\tau \leq -z(\beta)$. Therefore, the manager follows threshold τ. Since $\tau \in [\underline{\tau}, \overline{\tau}]$ then $\tau \in [-\pi(\tau), \pi(\tau)]$ and the activists finds it optimal to recommend on threshold τ. Overall, a responsive equilibrium with threshold τ exists. ∎

Proof of Corollary 1. Let $\bar{V}$ be shareholder value under the most efficient equilibrium. Consider the comparative static of $\bar{V}$ with respect to β. According to Proposition 1, if $\beta > z^{-1}(-\underline{\tau})$ then any equilibrium is non-responsive and $\bar{V} = \mathbb{E}[\theta]$. Suppose $\beta < z^{-1}(-\underline{\tau})$. Since $g(\theta, \tau)$ is decreasing with τ when $\tau < 0$, and since $\tau^* = \max\{-z(\beta), 0\}$ is negative and decreasing with β, $\bar{V}$ decreases in β in this range as well. Last, if $\beta = z^{-1}(-\underline{\tau})$ then V drops from $\mathbb{E}[g(\theta, \underline{\tau}))]$ to $\mathbb{E}[\theta]$ as β increases. Overall, $\bar{V}$ globally decreases with β.

Consider the comparative static $\bar{V}$ with respect to δ. I argue $\underline{\tau}$ decreases with δ. To see why, note that $\varphi(\tau, p)$ increases with δ, and hence, $\pi(\tau)$ increases with δ as well. Second, $\pi(\tau)$ increases with τ when $\tau < 0$. Third, recall from Lemma 4 that $\pi(\underline{\tau}) + \underline{\tau} = 0$ and $\underline{\tau} < 0$. Imposing the implicit function theorem on $\pi(\underline{\tau}) + \underline{\tau} = 0$ concludes the argument. Since $z^{-1}(\cdot)$ is an increasing function, $z^{-1}(-\underline{\tau})$ increases with δ. If $z^{-1}(0) \geq \beta$ then $\bar{V} = \mathbb{E}[|\theta|]$ and is independent of δ. Suppose $z^{-1}(0) < \beta$. If $z^{-1}(-\underline{\tau}) < \beta$ then $\bar{V} = \mathbb{E}[\theta]$ and is independent of δ. If $z^{-1}(-\underline{\tau}) \geq \beta$ then $\bar{V} = \mathbb{E}[g(\theta, -z(\beta))] > \mathbb{E}[\theta]$ and it is independent of δ. Overall, $\bar{V}$ increases with δ. ∎

Proof of Corollary 2. If the equilibrium is fully revealing, the effective threshold is $-\beta$. According to Proposition 1, such equilibrium exists if and only if $-\beta \in [\underline{\tau}, \min\{-z(\beta), \overline{\tau}\}]$. Since $-\beta < -z(\beta)$, a fully revealing equilibrium exists if and only if $-\beta \in [\underline{\tau}, \overline{\tau}]$. For the same reason, if $-\beta \in [\underline{\tau}, \overline{\tau}]$ then $\min\{-z(\beta), 0\} \in [\underline{\tau}, \overline{\tau}]$ as well. Therefore, if a fully revealing equilibrium exists, there also exists more efficient equilibrium with threshold $-\beta < \min\{-z(\beta), 0\} \le 0$. ∎

Proofs of Section 3

Proof of Lemma 5. Consider a non-responsive equilibrium in which the manager chooses $a = R$ with probability $x \in [0, 1]$. Since the market maker does not observe a then price is given by

$$p(s, x) = \begin{cases} x\pi(\underline{\theta}) + (1-x)\pi(\overline{\theta}) & \text{if } s = 1 \\ x\mathbb{E}[\theta | \theta > p(1, x)] + (1-x)\mathbb{E}[-\theta | -\theta > p(1, x)] & \text{if } s = 0 \end{cases}$$

Since the activist observes the actual decision a, the activist chooses $s = 1$ if and only if $v(\theta, a) < p(1, x)$. In this equilibrium, the manager's expected utility is

$$u_M(a, x) = \omega p(1, x) + \omega(1 - \delta)\Pr[v(\theta, a) \ge p(1, x)][p(0, x) - p(1, x)] + \mathbb{E}[v(\theta + \beta, a)]$$

The manager chooses $a = R$ if and only if $\Delta(x) \ge 0$ where

$$\Delta(x) \equiv \omega(1 - \delta)(\Pr[\theta \ge p(1, x)] - \Pr[\theta \le -p(1, x)])[p(0, x) - p(1, x)] + 2\mathbb{E}[\theta + \beta]$$

Note that $\Delta(x)$ is continuous in $x \in [0, 1]$. A non-responsive equilibrium must satisfy either $\Delta(1) \ge 0$, $\Delta(0) \le 0$, or $\Delta(x) = 0$ for $x \in (0, 1)$. Therefore, if $\Delta(1) < 0 < \Delta(0)$, by the intermediate value theorem there is always $x^* \in (0, 1)$ such that $\Delta(x^*) = 0$. A non-responsive equilibrium always exists.

Recall $p(0, x) - p(1, x) > 0$ for all x and by assumption $\mathbb{E}[\theta + \beta] > 0$. Therefore, if $\Pr[\theta \ge \pi(\underline{\theta})] \ge \Pr[\theta \le -\pi(\underline{\theta})]$ then $\Delta(1) > 0$. If $\Pr[\theta \ge \pi(\underline{\theta})] < \Pr[\theta \le -\pi(\underline{\theta})]$ then $\Delta(1) \ge$

0 if and only if $\omega \leq \hat{\omega}$ where

$$\hat{\omega} \equiv \frac{1}{1-\delta} \frac{2\mathbb{E}\left[\theta + \beta\right]}{\mathbb{E}\left[\theta | \theta > \pi\left(\underline{\theta}\right)\right] - \pi\left(\underline{\theta}\right)} \frac{1}{\left|\Pr\left[\theta \leq -\pi\left(\underline{\theta}\right)\right] - \Pr\left[\theta \geq \pi\left(\underline{\theta}\right)\right]\right|}$$

This completes the argument. ∎

Proof of Proposition 2. Consider a responsive equilibrium and let M_a be the set of messages that yields action a. In a responsive equilibrium neither set is empty. Without the loss of generality, suppose $M_R \cup M_L = \left[\underline{\theta}, \overline{\theta}\right]$. Since the market maker does not observe a or m, $p(s)$ is given by (4) and (A1). Regardless of the message, the activist observes the actual decision a and strategically exits if and only if $v(\theta, a) < p(1)$. In equilibrium, the manager's expected utility from action a conditional on message m is:

$$u_M(a, m) = \omega p(1) + \omega(1-\delta)\Pr\left[v(\theta, a) > p(1)|m\right]\left[p(0) - p(1)\right] + \mathbb{E}\left[v(\theta + \beta, a)|m\right]$$

The manager follows the recommendation of the activist if and only if $u_M(R, m) \geq u_M(L, m)$ for all $m \in M_R$ and $u_M(R, m) \leq u_M(L, m)$ for all $m \in M_L$. Integrating over all message in M_A, there is an equivalent equilibrium such that the activist only reveals whether $m \in M_R \Leftrightarrow \theta \in \Theta$ or $m \in M_L \Leftrightarrow \theta \in \Theta^c$. Thus, it is sufficient to consider an equilibrium with exactly two messages. Let these messages be m_R and m_L. Note that if $m = m_R$ then $v(\theta, L) \leq p(1)$ for sure and if $m = m_L$ then $v(\theta, R) \leq p(1)$ for sure. Therefore, the manager follows the recommendation of the activist if and only if

$$\omega(1-\delta)\Pr\left[\theta > p(1)|\Theta\right]\left[p(0) - p(1)\right] + 2\mathbb{E}\left[\theta + \beta|\Theta\right] \geq 0 \tag{A4a}$$

and

$$-\omega(1-\delta)\Pr\left[-\theta > p(1)|\Theta^c\right]\left[p(0) - p(1)\right] + 2\mathbb{E}\left[\theta + \beta|\Theta^c\right] \leq 0 \tag{A4b}$$

I show that if a responsive equilibrium exists and $\mathbb{E}\left[\theta | \theta \in \Theta\right] < \mathbb{E}\left[\theta | \theta \in \Theta^c\right]$ then there is Θ_0 such that $\mathbb{E}\left[\theta | \theta \in \Theta_0\right] \geq \mathbb{E}\left[\theta | \theta \in \Theta_0^c\right]$ and the responsive equilibrium under this decision rule exists and is more efficient. Recall $\Theta \supseteq \left[p(1), \overline{\theta}\right]$ and $\Theta^c \supseteq \left[\underline{\theta}, -p(1)\right]$ and let $\Theta_0 = \Theta^c \cap \left[-p(1), p(1)\right] \cup \left[p(1), \overline{\theta}\right]$. Since $\mathbb{E}\left[\theta | \theta \in \Theta\right] < \mathbb{E}\left[\theta | \theta \in \Theta^c\right]$ then $\mathbb{E}\left[\theta | \theta \in \Theta \cap \left[-p(1), p(1)\right]\right] < \mathbb{E}\left[\theta | \theta \in \Theta^c \cap \left[-p(1), p(1)\right]\right]$. Therefore, $\mathbb{E}\left[\theta | \theta \in \Theta_0^c\right] < \mathbb{E}\left[\theta | \theta \in \Theta_0\right]$, $\mathbb{E}\left[\theta | \theta \in \Theta\right] < \mathbb{E}\left[\theta | \theta \in \Theta_0\right]$,

and $\mathbb{E}\left[\theta|\theta \in \Theta_0^c\right] < \mathbb{E}\left[\theta|\theta \in \Theta_0\right]$. Since $p\left(0\right) > p\left(1\right)$ in any equilibrium, if (A4a) and (A4b) hold for Θ, they must hold for Θ_0 as well. It is left to prove that the activist has incentives to recommend on Θ_0. Based on (A1), the price upon exit is higher under Θ_0, thus the activist will recommend on Θ_0. Since Θ_0 is more efficient, we can focus on responsive equilibrium in which $\mathbb{E}\left[\theta|\theta \in \Theta\right] \geq \mathbb{E}\left[\theta|\theta \in \Theta^c\right]$.

Since $\mathbb{E}\left[\theta|\theta \in \Theta\right] > \mathbb{E}\left[\theta|\theta \in \Theta^c\right]$ and $\mathbb{E}\left[\theta\right] + \beta > 0$ then $\mathbb{E}\left[\theta + \beta|\theta \in \Theta\right] > 0$ as well. Since $p\left(0\right) > p\left(1\right)$ then (A4a) always holds. Thus, the constraint that binds is (A4b). Since $p\left(s\right)$ is independent of β and $p\left(0\right) > p\left(1\right)$, it immediately follows that for any $\omega \geq 0$ there exists $\beta^*\left(\omega\right)$ such that a responsive equilibrium exists if and only if $\beta \leq \beta^*\left(\omega\right)$. Based on Proposition 1, $\beta^*\left(0\right) = z^{-1}\left(-\underline{\tau}\right)$. Since $p\left(0\right) > p\left(1\right)$, if conditions (A4a) and (A4b) hold for some $\omega_0 \geq 0$, they hold for any $\omega > \omega_0$. Therefore, $\beta^*\left(\omega\right)$ strictly increases with ω, and $\lim_{\omega \to \infty} \beta^*\left(\omega\right) = \infty$.
∎

Proofs of Section 4

Proof of Proposition 3. Let $p_a\left(s\right)$ be the price of the share if the manager decides on a and the activist chooses s. I first prove that if the equilibrium is responsive then $p_L\left(1\right) = p_R\left(1\right) > 0$. Suppose on the contrary the equilibrium is responsive and $p_L \neq p_R$. Let $p_{\max} = \max\left\{p_L\left(1\right), p_R\left(1\right)\right\}$ and note that M_L and M_R are not empty. Note that $p_{\max} > 0$. Otherwise, the activist sends $m \in M_R$ if and only if $\theta > 0$ and never exits strategically. This implies $p_R = \mathbb{E}\left[\theta|\theta > 0\right] > 0$, a contradiction. Consider the activist's exist strategy. If $\theta > p_{\max}$ the activist sends $m \in M_R$ and chooses $s = 0$. If $\theta < -p_{\max}$ the activist sends $m \in M_L$ and chooses $s = 0$. If $\theta \in \left[-p_{\max}, p_{\max}\right]$ the activist chooses $s = 1$ and sends $m \in M_R$ if and only if $p_R > p_L$. Thus, if $p_R > p_L$ the manager chooses $a = L$ if and only if $\theta < -p_R$. Moreover, if $a = L$ the activist never exits unless she needs liquidity. Therefore, $p_L = \mathbb{E}\left[-\theta|\theta < -p_R\right] > p_R$, a contradiction. Similarly, if $p_R < p_L$ the manager chooses $a = R$ if and only if $\theta > p_L$. Moreover, if $a = R$ the activist never exits unless she needs liquidity. Therefore, $p_R = \mathbb{E}\left[\theta|\theta > p_L\right] > p_L$, a contradiction.

Next, consider a responsive equilibrium under Action-Transparency with decision rule Θ. Similar to Lemma 4, it is necessary $p\left(1\right) > 0$, $\left[p\left(1\right), \overline{\theta}\right] \subset \Theta$ and $\left[\underline{\theta}, -p\left(1\right)\right] \subset \Theta^c$. Moreover,

$p(1)$ must be the solution of $p = \varphi_R(\Theta, p)$ and the solution of $p = \varphi_L(\Theta, p)$ where

$$\varphi_a(\Theta, p) \equiv \begin{cases} \frac{\delta \mathbb{E}[\theta|\theta \in \Theta] + (1-\delta)\Pr[\theta < p|\theta \in \Theta]\mathbb{E}[\theta|\theta \in [-p,p] \cap \Theta]}{\delta + (1-\delta)\Pr[\theta < p|\theta \in \Theta]} & if \ a = R \\ \frac{\delta \mathbb{E}[-\theta|\theta \in \Theta^c] + (1-\delta)\Pr[-\theta < p|\theta \in \Theta^c]\mathbb{E}[-\theta|\theta \in [-p,p] \cap \Theta^c]}{\delta + (1-\delta)\Pr[-\theta < p|\theta \in \Theta^c]} & if \ a = L \end{cases} \tag{A5}$$

In equilibrium, $\mathbb{E}[\theta|\theta \in \Theta] > p$, $p > 0$, and $\mathbb{E}[-\theta|\theta \in \Theta^c] > p$. Therefore, $\mathbb{E}[\theta|\theta \in \Theta] > \mathbb{E}[\theta|\theta \in \Theta^c]$. Since $\mathbb{E}[\theta] > -\beta$ then $\mathbb{E}[\theta|\theta \in \Theta] > -\beta$. Since $p_R(1) = p_R(1)$, similar to the proof of Proposition 2, the binding constraint is persuading the manager to choose strategy L:

$$-\omega(1-\delta)\Pr[-\theta > p(1)|\theta \in \Theta^c][p_L(0) - p(1)] + 2\mathbb{E}[\theta + \beta|\theta \in \Theta^c] \leq 0 \tag{A6}$$

Overall, if (A6) holds for some β^*, it holds for any $\beta < \beta^*$. Therefore, there exists $\beta^*_{AT}(\omega)$ as required.

Next, I show that there is unique $\tau^*_{AT} \in (\underline{\tau}, \overline{\tau})$ that satisfies both $p = \varphi_R(\Theta, p)$ and $p = \varphi_L(\Theta, p)$ where $\Theta = [\tau, \overline{\theta}]$. Let

$$c(\Theta, p) \equiv \frac{\delta \Pr[\theta \in \Theta] + (1-\delta)\Pr[\theta \in [-p,p] \cap \Theta]}{\delta + (1-\delta)\Pr[\theta \in [-p,p]]}$$

and note that

$$\varphi(\Theta, p) = \varphi_R(\Theta, p) c(\Theta, p) + (1 - c(\Theta, p))\varphi_L(\Theta, p)$$

where $\varphi(\Theta, p)$ is given by (A1). Therefore, if $p(1)$ is a solution of $p = \varphi_R(\Theta, p)$ and a solution of $p = \varphi_L(\Theta, p)$, it is also a solution of $p = \varphi(\Theta, p)$. This also implies that any equilibrium under Action-Transparency is an equilibrium under No-Transparency. From Lemma 4 we know that the solution of $p = \varphi(\tau, p)$ is unique and given by $\pi(\tau)$. Since $-\pi(\tau) \leq \tau \leq \pi(\tau)$ is necessary, I restrict attention to $\tau \in [\underline{\tau}, \overline{\tau}]$. Fix τ, and let $p_a(\tau)$ be the solution of $p = \varphi_a(\tau, p)$. As in Lemma A.1, $p_a(\tau)$ exists and it is unique. Note also that $p_L(\tau)$ strictly decreases with τ and $p_R(\tau)$ strictly increases with τ. Therefore, if there is a solution to $p_L(\tau) = p_R(\tau)$ in $[\underline{\tau}, \overline{\tau}]$, it is unique. To show that a solution exists, note that $p_L(\tau) > -\tau$. Since $\pi(\underline{\tau}) = -\underline{\tau}$ and $\pi(\tau)$ is a weighted average of $p_R(\tau)$ and $p_L(\tau)$, then $p_L(\underline{\tau}) > -\underline{\tau} = \pi(\underline{\tau})$ implies $-\underline{\tau} > p_R(\underline{\tau})$. Similarly, note that $p_R(\tau) > \tau$. Since $\pi(\overline{\tau}) = \overline{\tau}$ then $p_R(\overline{\tau}) > \overline{\tau} = \pi(\overline{\tau})$ implies $\overline{\tau} > p_L(\overline{\tau})$. Overall, $p_L(\underline{\tau}) > p_R(\underline{\tau})$ and $p_R(\overline{\tau}) > p_L(\overline{\tau})$ implies that a solution exists in $(\underline{\tau}, \overline{\tau})$, as required.

Last, I show that if $\omega = 0$ or $\mathbb{E}\left[\theta|\theta > \pi\left(\tau^*_{AT}\right)\right] + \mathbb{E}\left[\theta|\theta < -\pi\left(\tau^*_{AT}\right)\right] > 0$ then $\beta^*_{AT}\left(\omega\right) <$ $\beta^*_{NT}\left(\omega\right)$. First, suppose $\omega = 0$ and on the contrary a responsive equilibrium exists with $\beta = z^{-1}\left(-\underline{\tau}\right)$. According to Lemma 4, it is necessary that $\beta \leq z^{-1}\left(-\tau\right)$, $\tau \geq \underline{\tau}$, and $p\left(1\right) = \pi\left(\tau\right)$. Therefore, the threshold must be $\underline{\tau}$ and $p\left(1\right) = -\underline{\tau}$. However, $\tau^*_{AT} > \underline{\tau}$, a contradiction. Since a responsive equilibrium does not exists when $\beta = z^{-1}\left(-\underline{\tau}\right)$ and $z^{-1}\left(-\underline{\tau}\right) = \beta^*_{NT}\left(\omega\right)$, then $\beta^*_{AT}\left(\omega\right) < \beta^*_{NT}\left(\omega\right)$. Second, suppose $\mathbb{E}\left[\theta|\theta > \pi\left(\tau^*_{AT}\right)\right] + \mathbb{E}\left[\theta|\theta < -\pi\left(\tau^*_{AT}\right)\right] > 0$. The only difference between (A6) and (A4b) is $p_L\left(0\right)$ and $p\left(0\right)$. Note that the condition implies $p_L\left(0\right) < p\left(0\right)$, and since $p_L\left(1\right) = p_R\left(1\right)$ implies $p_L\left(1\right) = p\left(1\right)$, then $\beta^*_{AT}\left(\omega\right) < \beta^*_{NT}\left(\omega\right)$ as required. ∎

Lemma A.3 [Voice and Exit Exhibit Substitution] *If $\mathbb{E}\left[\theta|\theta > 0\right] + \mathbb{E}\left[\theta|\theta < 0\right] < 0$ there exists δ such that $\beta^*_{AT}\left(\omega\right) < z^{-1}\left(0\right)$.*

Proof. Suppose on the contrary that for any δ and $\beta < z^{-1}\left(0\right)$ a responsive equilibrium under Action-Transparency exists. Let τ_0 be the unique solution of $\mathbb{E}\left[\theta|\theta > x\right] + \mathbb{E}\left[\theta|\theta < x\right] = 0$ and note that $\tau_0 > 0$ and $\mathbb{E}\left[\theta|\theta \leq \tau_0\right] < 0$. Also, note that $z^{-1}\left(-\tau_0\right) \in \left(0, z^{-1}\left(0\right)\right)$. Therefore, for any δ and $\beta < z^{-1}\left(0\right)$ there exists an equilibrium with threshold $\tau\left(\delta\right)$. Recall, the price upon exit must satisfy $\varphi_L\left(\tau\left(\delta\right), p\right) = \varphi_R\left(\tau\left(\delta\right), p\right) = p$ for any δ. It follows from (A5) that

$$\lim_{\delta \to 1} \mathbb{E}\left[\theta|\theta \geq \tau\left(\delta\right)\right] + \mathbb{E}\left[\theta|\theta < \tau\left(\delta\right)\right] = 0$$

Therefore, $\lim_{\delta \to 1} \tau\left(\delta\right) = \tau_0$. Note that as $\delta \to 1$ condition (A6) requires $\mathbb{E}\left[\theta|\theta < \tau_0\right] + \beta \leq 0$. Thus, if $\beta_0 = \left(z^{-1}\left(-\tau_0\right), z^{-1}\left(0\right)\right)$ then a manager with bias β_0 will not follow threshold τ_0, a contradiction. This implies that $\beta^*_{AT}\left(\omega\right) < z^{-1}\left(0\right)$. ∎

Proof of Proposition 4. Consider an equilibrium with Voice-Transparency. It is straight forward to see that no information is revealed by the activist if the equilibrium is non-responsive. Therefore, the set of non-responsive equilibria under either form of transparency are identical.

Consider a responsive equilibrium under Action-Transparency. I argue that there is an equilibrium under Voice-Transparency in which the activist sends message m_R if $\theta \in \Theta$, message $m_L \neq m_R$ otherwise, and any other message is ignored by the market maker and the

manager. The market maker observes m and infers that if $m = m_R$ ($m = m_L$) then $\theta \in \Theta$ ($\theta \in \Theta^c$) and $a = R$ ($a = L$). Therefore, if $m = m_a$ the price must be a solution to $p = \varphi_a(\Theta, p)$. Based on the proof of Proposition 2, the solution of these equations is the same price the market maker sets under Action-Transparency. Since $p_R = p_L > 0$ and any other message is ignored, the incentives of the activist to send message m_A or m_R are solely determined by his incentives to change the manager's decision. Since Θ is an equilibrium under Action-Transparency, the manager has incentives to follow the recommendation of the activist, and the activist has incentives to make this recommendation.[23]

Consider a responsive equilibrium under Voice-Transparency. Let M_a be the set of all (public) messages that are sent with a strictly positive probability in equilibrium and yield decision a. Also, let $p_m(s)$ be the share price conditional on $s = 1$ and message $m \in M$. By definition, M_R and M_L are not empty. Since $\delta > 0$ there is a strictly positive probability that the activist exits. Therefore, sending $m \notin \arg\max_{m \in M_R} p_m(1) \cup \arg\max_{m \in M_L} p_m(1)$ is a strictly dominated strategy. This implies that there are exactly two different prices conditional on exit, p_R for $m \in M_R$ and p_L for $m \in M_L$. In fact, there are exactly two types messages: one that yields strategy R and price p_R, and one that yields strategy L and price p_L. As in the proof of Proposition 8, it must be that $p_A = p_R > 0$. Overall, it is immediate to see that any set Θ and Υ that emerge as equilibrium under Voice-Transparency can also emerge as equilibrium under Action-Transparency. ∎

Proof of Proposition 5. Under Full-Transparency, the market maker observes a and m. Suppose $\omega = 0$. If $m \in M_a$ but the manager chooses $a' \neq a$ then the price upon exit changes. However, since $\omega = 0$ this change does not affect the manager, and therefore, the set of equilibria does not change as well.

Suppose $\omega > 0$ and let $p_{a,m}(s)$ be the price conditional on (s, a, m). First note that similar to Voice-Transparency, message m cannot convey more information than whether or not $\theta \in \Theta$. Thus, it is sufficient to consider an equilibrium with exactly two messages. Let these messages be m_R and m_L. Similar to Action-Transparency, in any equilibrium, $p_{R,m_R}(1) = p_{L,m_L}(1)$.

---

[23]Note that in (A4a) and (A4b) the prices are conditioned on a when the regime is Action-Transparency and on m when it is Voice-Transparency. Since in equilibrium $p_a(1)$ is invariant to a, these conditions are equivalent.

The manager follows the recommendation to choose $a = R$ if and only if

$$\omega p_{R,m_R}(1) + \omega(1-\delta)\Pr\left[\theta > p_{R,m_R}(1)\,|\theta \in \Theta\right]\left[p_{R,m_R}(0) - p_{R,m_R}(1)\right] + 2\mathbb{E}\left[\theta + \beta|\theta \in \Theta\right]$$

$$\geq \ \omega p_{L,m_R}(1) + \omega(1-\delta)\Pr\left[-\theta > p_{L,m_R}(1)\,|\theta \in \Theta\right]\left[p_{L,m_R}(0) - p_{L,m_R}(1)\right]$$

where $p_{L,m_R}(1)$ is the solution of

$$p = \frac{\delta\mathbb{E}\left[-\theta|\theta \in \Theta\right] + (1-\delta)\Pr\left[-\theta < p|\theta \in \Theta\right]\mathbb{E}\left[-\theta|-\theta < p, \theta \in \Theta\right]}{\delta + (1-\delta)\Pr\left[-\theta < p|\theta \in \Theta\right]}$$

Note that

$$(1-\delta)\Pr\left[\theta > p_{R,m_R}(1)\,|\theta \in \Theta\right]\left[p_{R,m_R}(0) - p_{R,m_R}(1)\right] \ = \ \mathbb{E}\left[\theta|\theta \in \Theta\right] - p_{R,m_R}(1)$$

$$(1-\delta)\Pr\left[-\theta > p_{L,m_R}(1)\,|\theta \in \Theta\right]\left[p_{L,m_R}(0) - p_{L,m_R}(1)\right] \ = \ \mathbb{E}\left[-\theta|\theta \in \Theta\right] - p_{L,m_R}(1)$$

Therefore, the manager follows the recommendation to choose $a = R$ if and only if

$$(1+\omega)\,\mathbb{E}\left[\theta|\theta \in \Theta\right] + \beta \geq 0$$

Similar to the proof of Proposition 2, it follows that $\mathbb{E}\left[\theta|\theta \in \Theta\right] > \mathbb{E}\left[\theta|\theta \in \Theta^c\right]$. Since $\mathbb{E}\left[\theta\right] > -\beta$, then the constraint $(1+\omega)\,\mathbb{E}\left[\theta|\theta \in \Theta\right] + \beta \geq 0$ never binds.

An analogous argument shows that the manager follows the recommendation to choose $a = L$ if and only if

$$(1+\omega)\,\mathbb{E}\left[\theta|\theta \in \Theta^c\right] + \beta \leq 0$$

Comparing this expression to (A6), reveals that for the same decision rule, (A6) provides a tighter bound and hence $\beta_{AT}^*(\omega) < \beta_{FT}^*(\omega)$. This also implies that any equilibrium under Action-Transparency is also an equilibrium under Full-Transparency. ∎

Proofs of Section 5

Lemma A.4 *For any $\tau \in \left[\underline{\theta}, \overline{\theta}\right]$ and $x \geq 0$ define*

$$\hat{\varphi}(p,\tau,x) \equiv \frac{\mathbb{E}\left[v(\theta,\tau)\right] + \frac{1-\delta}{\delta}\Pr\left[v(\theta,\tau) \leq p\right]\mathbb{E}\left[v(\theta,\tau)\,|v(\theta,\tau) \leq p\right] + x\mathbb{E}\left[\theta\right]}{1 + \frac{1-\delta}{\delta}\Pr\left[v(\theta,\tau) \leq p\right] + x} \tag{A7}$$

The solution of $p = \hat{\varphi}(p, \tau, x)$ exists, is unique, and is given by $\hat{\pi}(\tau, x) \equiv \min_{p \geq \min\{-\tau, \tau\}} \hat{\varphi}(p, \tau, x)$. Moreover, there are $\underline{\tau}(x) < 0 < \overline{\tau}(x)$ such that $\underline{\tau}(x)$ is the unique negative solution of $\hat{\pi}(\tau, x) + \tau = 0$ and $\overline{\tau}(x)$ is the unique positive solution of $\hat{\pi}(\tau, x) - \tau = 0$. $\overline{\tau}(x)$ and $\underline{\tau}(x)$ have the following properties:

(i) If $\tau \in [\underline{\tau}(x), \overline{\tau}(x)]$ then $\hat{\pi}(\tau, x) > 0$.

(ii) If and only if $\tau \in [\underline{\tau}(x), \overline{\tau}(x)]$ then $\tau \in [-\hat{\pi}(\tau, x), \hat{\pi}(\tau, x)]$.

(iii) If and only if $\hat{\pi}(\tau, x) < \mathbb{E}[\theta]$ then $\hat{\pi}(\tau, x)$ increases in x.

(iv) If there is $x_0 \geq 0$ such that $\hat{\pi}(\tau, x_0) < (=, >) \mathbb{E}[\theta]$ then $\hat{\pi}(\tau, x) < (=, >) \mathbb{E}[\theta]$ for all x.

(v) If $\underline{\tau}(0) > (=, <) - \mathbb{E}[\theta]$ then for all $x \geq 0$: $\underline{\tau}(x) > (=, <) - \mathbb{E}[\theta]$ and $\frac{\partial \underline{\tau}(x)}{\partial x} < (=, >) 0$.

Proof. Extending Proposition 1 in Acharya et al. (2011), one can show $\hat{\pi}(\tau, x)$ is the unique solution of the equation $p = \hat{\varphi}(p, \tau, x)$. Note that since $\mathbb{E}[\theta] > 0$ then $\hat{\pi}(0, x) > 0$. Also note that $\hat{\pi}(\tau, x)$ is continuous in x. Part (i) and part (ii) of the lemma follow from similar arguments to those in the proof of Proposition 1.

Consider part (iii). Since $\hat{\pi}(\tau, x)$ is the unique minimum of $\hat{\varphi}(p, \tau, x)$, the sign of $\frac{\partial \hat{\pi}(\tau, x)}{\partial x}$ and $\frac{\partial \hat{\varphi}(p, \tau, x)}{\partial x}|_{p = \hat{\pi}}$ are identical. Note that $\hat{\varphi}(p, \tau, x)$ is a weighted average of $\hat{\varphi}(p, \tau, 0)$ and $\mathbb{E}[\theta]$, and the weight on $\mathbb{E}[\theta]$ increases with x. Therefore, $\hat{\varphi}(p, \tau, x)$ increases with x if and only if $\hat{\varphi}(p, \tau, 0) < \mathbb{E}[\theta]$, if and only if $\hat{\varphi}(p, \tau, x) < \mathbb{E}[\theta]$. Since $\hat{\varphi}(\hat{\pi}, \tau, x) = \hat{\pi}$, it follows that $\frac{\partial \hat{\varphi}(p, \tau, x)}{\partial x}|_{p = \hat{\pi}} > 0$ if and only if $\hat{\pi} < \mathbb{E}[\theta]$. We conclude, $\hat{\pi}(\tau, x)$ increases with x if and only if $\hat{\pi}(\tau, x) < \mathbb{E}[\theta]$.

Consider part (iv). Suppose $\hat{\pi}(\tau, x_0) < \mathbb{E}[\theta]$ for some $x_0 \geq 0$. Similar proof follows for the other cases. Note that $\hat{\varphi}(\hat{\pi}(\tau, x_0), \tau, 0) < \mathbb{E}[\theta]$ since $\hat{\varphi}(p, \tau, x)$ is a weighted average of $\mathbb{E}[\theta]$ and $\hat{\varphi}(p, \tau, 0)$. Therefore, $\hat{\varphi}(\hat{\pi}(\tau, x_0), \tau, x) < \mathbb{E}[\theta]$ for any x, including $x \neq x_0$. Suppose on the contrary there is $x \neq x_0$ such that $\hat{\pi}(\tau, x) \geq \mathbb{E}[\theta]$. Since $\hat{\varphi}(\hat{\pi}(\tau, x), \tau, x) = \hat{\pi}(\tau, x)$ then $\hat{\varphi}(\hat{\pi}(\tau, x), \tau, x) \geq \mathbb{E}[\theta]$. Therefore, $\hat{\varphi}(\hat{\pi}(\tau, x_0), \tau, x) < \mathbb{E}[\theta] \leq \hat{\varphi}(\hat{\pi}(\tau, x), \tau, x)$ which contradict the fact that $\hat{\varphi}(\hat{\pi}(\tau, x), \tau, x)$ is a global minimum of $\hat{\varphi}(p, \tau, x)$.

Consider part (v). Recall $\underline{\tau}(x)$ is defined by $\hat{\pi}(\underline{\tau}(x), x) + \underline{\tau}(x) = 0$ and $\underline{\tau}(x) < 0$. By the implicit function theorem $\frac{\partial \underline{\tau}(x)}{\partial x} = -\frac{\frac{\partial \hat{\pi}(\tau, x)}{\partial x}|_{\tau = \underline{\tau}(x)}}{\frac{\partial \hat{\pi}(\tau, x)}{\partial \tau}|_{\tau = \underline{\tau}(x)} + 1}$. Since $\underline{\tau}(x) < 0$ then $\frac{\partial \hat{\pi}(\tau, x)}{\partial \tau}|_{\tau = \underline{\tau}(x)} > 0$. Therefore, $\frac{\partial \underline{\tau}(x)}{\partial x} < 0$ if and only if $\frac{\partial \hat{\pi}(\tau, x)}{\partial x}|_{\tau = \underline{\tau}(x)} > 0$. Suppose $\underline{\tau}(x) > (=, <) - \mathbb{E}[\theta]$. Then

47

$\hat{\pi}\left(\underline{\tau}\left(x\right),x\right) < (=,>)\mathbb{E}\left[\theta\right]$. Based on part (iii), $\frac{\partial \hat{\pi}(\tau,x)}{\partial x}|_{\tau=\underline{\tau}(x)} > (=,<)0$. Therefore, $\frac{\partial \underline{\tau}(x)}{\partial x} < (=,>)0$. Overall, $\underline{\tau}\left(x\right) > -\mathbb{E}\left[\theta\right] \Leftrightarrow \frac{\partial \underline{\tau}(x)}{\partial x} < 0$. Since $\hat{\pi}\left(\tau,x\right)$ is continuous in x and $\underline{\tau}\left(x\right)$ is the unique negative solution of $\hat{\pi}\left(\underline{\tau},x\right) + \underline{\tau} = 0$, then $\underline{\tau}\left(x\right)$ is continuous in x as well. Since $\underline{\tau}\left(x\right)$ is continuous in x and $\underline{\tau}\left(x\right) > -\mathbb{E}\left[\theta\right] \Leftrightarrow \frac{\partial \underline{\tau}(x)}{\partial x} < 0$, if $\underline{\tau}\left(0\right) > (=,<) - \mathbb{E}\left[\theta\right]$ then $\underline{\tau}\left(x\right) > (=,<) - \mathbb{E}\left[\theta\right]$ for all $x \geq 0$. This also implies $\frac{\partial \underline{\tau}(x)}{\partial x} < (=,>)0$ for all $x \geq 0$. ∎

Lemma A.5 *The equation $\varphi\left(p,\tau,\lambda\right) = p$ has a unique solution and it is given by*

$$\pi\left(\tau,\lambda\right) = \begin{cases} \hat{\pi}\left(\tau,\frac{1-\lambda}{\lambda}\right) & \text{if } \hat{\pi}\left(\tau,0\right) < \mathbb{E}\left[\theta\right] \\ \hat{\pi}\left(\tau,\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right) & \text{if } \hat{\pi}\left(\tau,0\right) \geq \mathbb{E}\left[\theta\right] \end{cases} \tag{A8}$$

Proof. According to part (iv) of Lemma A.4 if $\hat{\pi}\left(\tau,0\right) < \mathbb{E}\left[\theta\right]$ then $\hat{\pi}\left(\tau,x\right) < \mathbb{E}\left[\theta\right]$ for any x. Therefore, $\hat{\varphi}\left(p,\tau,\frac{1-\lambda}{\lambda}\right) = p$ has a unique solution $\hat{\pi}\left(\tau,\frac{1-\lambda}{\lambda}\right) < \mathbb{E}\left[\theta\right]$. Lemma A.4 also implies that $\hat{\varphi}\left(p,\tau,\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right) = p$ has a unique solution which i given by $\hat{\pi}\left(\tau,\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right) < \mathbb{E}\left[\theta\right]$. Therefore, based on expression (8), the unique solution of $\varphi\left(p,\tau,\lambda\right) = p$ is $\hat{\pi}\left(\tau,\frac{1-\lambda}{\lambda}\right)$. A similar argument follows when $\hat{\pi}\left(\tau,0\right) \geq \mathbb{E}\left[\theta\right]$. ∎

Proposition A.1

(i) *A responsive equilibrium exists if and only if $\beta \leq z^{-1}\left(-\max\left\{\underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right),\underline{\tau}\left(\frac{1-\lambda}{\lambda}\right)\right\}\right)$*

(ii) *Suppose an equilibrium with threshold $\tau \in \left[\underline{\theta},\overline{\theta}\right]$ exists.[24] The price upon exit is given by $\pi\left(\tau,\lambda\right)$ and it increases in λ if and only if $\pi\left(\tau,\lambda\right) > \mathbb{E}\left[\theta\right]$.*

Proof. Consider part (i). Based on Lemma A.5, if a responsive equilibrium with threshold τ exists, the price upon exit has the form of $\hat{\pi}\left(\tau,x\right)$ for some $x \geq 0$. Therefore, it is necessary that $-\hat{\pi}\left(\tau,x\right) < \tau$ (otherwise the informed activist will not follow this strategy) which holds if and only if $\underline{\tau}\left(x\right) \leq \tau$.

There are three cases to consider. First, suppose $\underline{\tau}\left(\frac{1-\lambda}{\lambda}\right) < \underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right)$. According to Lemma A.4 part (v), $\underline{\tau}\left(0\right) \leq \underline{\tau}\left(x\right) \leq -\mathbb{E}\left[\theta\right]$ for all $x \geq 0$. Therefore, it is sufficient to focus on $\underline{\tau}\left(0\right) \leq \tau$. Moreover, since $-\underline{\tau}\left(x\right) = \hat{\pi}\left(\underline{\tau}\left(x\right),x\right)$ then $\hat{\pi}\left(\underline{\tau}\left(x\right),x\right) \geq \mathbb{E}\left[\theta\right]$ for all $x \geq 0$. Since

---

[24]If $\tau = \underline{\theta}$ the equilibrium is non-responsive.

$\hat{\pi}(\tau, x)$ increases in τ when $\tau < 0$ then $\hat{\pi}(\tau, x) \geq \mathbb{E}[\theta]$ for all $x \geq 0$ and $\tau \in [\underline{\tau}(0), 0]$. Based on Lemma A.5, this implies that the price upon exit must be greater than $\mathbb{E}[\theta]$ and for any λ and $\tau \in [\underline{\tau}(0), 0]$ it is given by $\pi(\tau, \lambda) = \hat{\pi}\left(\tau, \frac{1}{\delta}\frac{1-\lambda}{\lambda}\right)$. Similar to Proposition 1, a responsive equilibrium exists if and only if $\beta \leq z^{-1}\left(-\underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right)\right)$.[25]

Second, suppose $\underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right) < \underline{\tau}\left(\frac{1-\lambda}{\lambda}\right)$ and $\hat{\pi}(0,0) \leq \mathbb{E}[\theta]$. Since $\hat{\pi}(\tau, x)$ increases in τ when $\tau < 0$ and based on Lemma A.4 part (iii), $\hat{\pi}(\tau, x) < \mathbb{E}[\theta]$ for all $\tau \leq 0$ and $x \geq 0$. Hence, based on Lemma A.5, for all $\tau \leq 0$ and λ the price upon exit is $\pi(\tau, \lambda) = \hat{\pi}\left(\tau, \frac{1-\lambda}{\lambda}\right) < \mathbb{E}[\theta]$. Similar to Proposition 1, a responsive equilibrium exists if and only if $\beta \leq z^{-1}\left(-\underline{\tau}\left(\frac{1-\lambda}{\lambda}\right)\right)$.

Third, suppose $\underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right) < \underline{\tau}\left(\frac{1-\lambda}{\lambda}\right)$ and $\hat{\pi}(0,0) > \mathbb{E}[\theta]$. Based on Lemma A.4 part (v), $\underline{\tau}(x) > -\mathbb{E}[\theta]$ for all $x \geq 0$ and hence $\hat{\pi}(\underline{\tau}(x), x) < \mathbb{E}[\theta]$ for all $x \geq 0$. Moreover, $\mathbb{E}[\theta] < \hat{\pi}(0, x)$ for any $x \geq 0$. Thus, there is a unique $\sigma \in (\underline{\tau}(0), 0)$ such that $\hat{\pi}(\sigma, x) = \mathbb{E}[\theta]$ for any $x \geq 0$. If $\tau < \sigma$, based on Lemma A.5, the price upon exit is $\pi(\tau, \lambda) = \hat{\pi}\left(\tau, \frac{1-\lambda}{\lambda}\right) < \mathbb{E}[\theta]$. This can be an equilibrium only if $\underline{\tau}\left(\frac{1-\lambda}{\lambda}\right) \leq \tau$. If $\sigma < \tau$, based on Lemma A.5, the price upon exit is $\pi(\tau, \lambda) = \hat{\pi}\left(\tau, \frac{1}{\delta}\frac{1-\lambda}{\lambda}\right) > \mathbb{E}[\theta]$. This can be an equilibrium only if $\underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right) \leq \tau$. Since $\underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right) < \underline{\tau}\left(\frac{1-\lambda}{\lambda}\right)$ and based on Lemma A.4 part (v), $\underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right) < \underline{\tau}(0)$. Since $\underline{\tau}(0) < \sigma$ then $\sigma < \tau$ implies $\underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right) < \tau$. So this constraint does not bind. Overall, similar to Proposition 1, a responsive equilibrium exists if and only if $\beta \leq z^{-1}\left(-\underline{\tau}\left(\frac{1-\lambda}{\lambda}\right)\right)$.

Consider part (ii). It follows directly from Lemma A.5 that in an equilibrium with threshold τ the price upon exit is $\pi(\tau, \lambda)$. Note that if $\pi(\tau, \lambda) > \mathbb{E}[\theta]$ then $\pi(\tau, \lambda) = \hat{\pi}\left(\tau, \frac{1}{\delta}\frac{1-\lambda}{\lambda}\right)$ and based on part (iii) of Lemma A.4, $\hat{\pi}\left(\tau, \frac{1}{\delta}\frac{1-\lambda}{\lambda}\right)$ decreases in $\frac{1}{\delta}\frac{1-\lambda}{\lambda}$ and hence it increases in λ. A similar argument follows when $\pi(\tau, \lambda) < \mathbb{E}[\theta]$. ∎

Proof of Proposition 6. If the equilibrium is not responsive then information is not revealed, and based on Lemma 3 the manager chooses $a = R$ with probability one. This implies that the value of the firm is $\mathbb{E}[\theta]$.

Consider the most efficient responsive equilibrium. Based on the arguments in the main text, the uninformed activist advises the manager to choose strategy R for sure, and the informed

---

[25]Recall the uninformed activist always advises the manager to choose $a = R$. Thus, the manager always has incentives to follow a recommendation to choose $a = R$. The binding incentives constraint is to persuade the manager to choose $a = L$.

activist follows threshold $\min\{0, -z(\beta)\}$ the value of the firm is

$$\lambda\mathbb{E}\left[v(\theta, \min\{0, -z(\beta)\})\right] + (1-\lambda)\mathbb{E}\left[\theta\right] \tag{A9}$$

It follows from (A9), conditional on the existence of a responsive equilibrium, the value of the firm strictly increases with λ. Moreover, the value of the firm under a responsive equilibrium is greater than the value under a non-responsive equilibrium, and the upper bound of the firm value under any equilibrium is $\mathbb{E}\left[v(\theta, \min\{0, -z(\beta)\})\right]$.

First, suppose $\underline{\tau}(0) \le -z(\beta)$. Based on Proposition 1, if $\lambda = 1$ a responsive equilibrium with threshold $-z(\beta)$ exists and the value of the firm is $\mathbb{E}\left[v(\theta, \min\{0, -z(\beta)\})\right]$. Therefore, $\lambda^* = 1$.

Second, suppose $-z(\beta) \le -\mathbb{E}\left[\theta\right]$. There are two cases. First, if $\underline{\tau}(0) < -\mathbb{E}\left[\theta\right]$ then based on Lemma A.4 part (v) $\underline{\tau}(0) < \underline{\tau}\left(\frac{1-\lambda}{\lambda}\right) < \underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right) < -\mathbb{E}\left[\theta\right]$ for any $\lambda < 1$. Moreover, $\underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right)$ decreases with λ and, based on Proposition A.1, a responsive equilibrium exists if and only if $\underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right) \le -z(\beta)$. Thus, as λ increases both the likelihood that a responsive equilibrium exists increases, and the value of the firm, conditional on the existence of a responsive equilibrium. For these reasons, $\lambda^* = 1$. Second, if $-\mathbb{E}\left[\theta\right] \le \underline{\tau}(0)$ then based on Lemma A.4 part (v) $-\mathbb{E}\left[\theta\right] < \underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right) < \underline{\tau}\left(\frac{1-\lambda}{\lambda}\right) < \underline{\tau}(0)$ for any $\lambda < 1$. Moreover, $\underline{\tau}\left(\frac{1-\lambda}{\lambda}\right)$ increases with λ and, based on Proposition A.1, a responsive equilibrium exists if and only if $\underline{\tau}\left(\frac{1-\lambda}{\lambda}\right) \le -z(\beta)$. Since $-z(\beta) \le -\mathbb{E}\left[\theta\right]$ and $-\mathbb{E}\left[\theta\right] < \underline{\tau}\left(\frac{1-\lambda}{\lambda}\right)$, a responsive equilibrium does not exist for any λ and hence $\lambda^* = 1$.

Third, suppose $-\mathbb{E}\left[\theta\right] < -z(\beta) < \underline{\tau}(0)$. Based on Proposition 1 there is no responsive equilibrium when $\lambda = 1$. Based on Lemma A.4 part (v), $-\mathbb{E}\left[\theta\right] < \underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right) < \underline{\tau}\left(\frac{1-\lambda}{\lambda}\right) < \underline{\tau}(0)$. Moreover, $\underline{\tau}\left(\frac{1-\lambda}{\lambda}\right)$ increases with λ and, based on Proposition A.1, a responsive equilibrium exists if and only if $\underline{\tau}\left(\frac{1-\lambda}{\lambda}\right) \le -z(\beta)$. Since $\lim_{\lambda \to 0} \underline{\tau}\left(\frac{1-\lambda}{\lambda}\right) = -\mathbb{E}\left[\theta\right] < -z(\beta)$, there is $\lambda_0 \in (0, 1)$ such that a responsive equilibrium exists if and only if $\lambda < \lambda_0$. Since conditional on the existence of a responsive equilibrium the value of the firm strictly increases with λ, then $\lambda^* = \lambda_0$. ∎

Proof of Proposition 7. I start by arguing that for the existence of a responsive equilibrium it is sufficient to consider a threshold $\min\{-z(\beta), 0\}$. Recall that for any x, $\hat{\pi}(\tau, x)$ increases

in τ if and only if $\tau < 0$. If $\pi(\tau, G(c)) < \mathbb{E}[\theta]$ then the right hand side of (11) increases in $\pi(\tau, G(c))$ and hence c^* increases in τ. Since the left hand side of (11) is higher, it must be that the right hand side of (11) is higher as well. This implies that $\pi(\tau, G(c^*(\tau)))$ increases with τ. Similarly, if $\hat{\pi}(\tau, x) > \mathbb{E}[\theta]$ then the right hand side of (11) decreases in $\pi(\tau, G(c))$ and hence c^* decreases in τ. Since the left hand side of (11) is smaller, it must be that the right hand side of (11) is smaller as well. This implies that $\pi(\tau, G(c^*(\tau)))$ increases with τ, when $\tau < 0$. Overall $\pi(\tau, G(c^*(\tau)))$ increases with τ and it is sufficient to consider threshold $\tau = \min\{-z(\beta), 0\}$.

The proof follows in three steps. First, note that $\lim_{x \to \infty} \underline{\tau}(x) = -\mathbb{E}[\theta]$. Since $0 > \underline{\tau}(0) > -z(\beta) > -\mathbb{E}[\theta]$, based on Lemma A.4 part (v), there is $x_2 > 0$ such that $\underline{\tau}(x_2) = -z(\beta)$. Define c_2 such that $\frac{c_2}{\alpha(1-\delta)} = \mathbb{E}[\max\{v(\theta, -z(\beta)), z(\beta)\}] - \mathbb{E}[\theta]$ and let G_2 be such that $G_2(c_2) = \frac{1}{1+x_2}$. Since $\underline{\tau}\left(\frac{1-G_2(c_2)}{G_2(c_2)}\right) = -z(\beta)$ and $\hat{\pi}(\underline{\tau}(x), x) = -\underline{\tau}(x)$ for all $x \geq 0$, then $\hat{\pi}\left(-z(\beta), \frac{1-G_2(c_2)}{G_2(c_2)}\right) = z(\beta) < \mathbb{E}[\theta]$. Based on Lemma A.5, $\hat{\pi}\left(-z(\beta), \frac{1-G_2(c_2)}{G_2(c_2)}\right) = \pi(-z(\beta), G_2(c_2))$. It follows, c_2 is the unique solution of (11) when $\tau = \min\{-z(\beta), 0\}$ and $G = G_2$. We conclude that for G_2 there is a responsive equilibrium with threshold $\tau = -z(\beta)$ and $c_2^* = c_2$.

Second, consider a non-responsive equilibrium when $G = G_2$. Denote by c_{NR} the level of information acquisition in this case, and note that it is given by the unique solution of (11) when $\tau = \underline{\theta}$. Recall that for a given c, $\pi(\tau, G_2(c))$ increases with τ when $\tau < 0$. Therefore, $\mathbb{E}[\theta] > \pi(-z(\beta), G_2(c_2^*)) > \pi(\underline{\theta}, G_2(c_2^*))$. It follows, when the r.h.s of (11) is evaluated at $\tau = \underline{\theta}$, $G = G_2$, and $c = c_2^*$ it is lower than when it is evaluated at $\tau = -z(\beta)$, $G = G_2$, and $c = c_2^*$. This implies that $c_{NR} < c_2^*$.

Third, consider any G_1 such that $G_1 \overset{FDSD}{\prec} G_2$. I argue that a responsive equilibrium does not exist when $G = G_1$. Suppose on the contrary a responsive equilibrium exists. By assumption, $G_2(c_2^*) < G_1(c_2^*)$. Also, recall $\pi(-z(\beta), G_2(c_2^*)) < \mathbb{E}[\theta]$. Based on Proposition A.1 part (ii), $\pi(\tau, \lambda)$ increases in λ if and only if $\pi(\tau, \lambda) > \mathbb{E}[\theta]$. Hence, $\pi(-z(\beta), G_1(c_2^*)) < \pi(-z(\beta), G_2(c_2^*))$. It follows, when the r.h.s of (11) is evaluated at $\tau = -z(\beta)$, $G = G_1$, and $c = c_2^*$ it is lower than when it is evaluated at $\tau = -z(\beta)$, $G = G_2$, and $c = c_2^*$. Therefore, it must be $G_2(c_2^*) < G_1(c_1^*)$. Since $\underline{\tau}(0) > -\mathbb{E}[\theta]$, based on Lemma A.4 part (v), $\underline{\tau}\left(\frac{1-G_2(c_2^*)}{G_2(c_2^*)}\right) < \underline{\tau}\left(\frac{1-G_1(c_1^*)}{G_1(c_1^*)}\right)$. Since $-z(\beta) = \underline{\tau}\left(\frac{1-G_2(c_2^*)}{G_2(c_2^*)}\right)$ then $-z(\beta) < \underline{\tau}\left(\frac{1-G_1(c_1^*)}{G_1(c_1^*)}\right)$. This contradicts the

51

condition in Proposition A.1. Note that in a non-responsive equilibrium, if G_1 is sufficiently close to G_2 then c_1^* converges to $c_{NR} < c_2^*$ and it is possible to find G_1 sufficiently close to G_2 such that $G_1(c_1^*) \approx G_2(c_{NR}) < G_2(c_2^*)$. This completes the proof. ∎

Appendix B

Lemma B.1 *For any mixed strategy responsive equilibrium there is a pure strategy responsive equilibrium which is more efficient.*

Proof. For any responsive equilibrium let

$$M_{mix}^* \equiv \left\{ m : a^*(m) \in (0,1) \;\; and \;\; \int \mu^*(m|\theta) f(\theta) d\theta > 0 \right\}$$

where $a^*(m)$ is the probability the manager chooses $a = R$ conditional on observing message m. Similarly we define M_R^* and M_L^*. Since the equilibrium is responsive $\arg\min a^*(m) \neq \arg\max a^*(m)$. Suppose M_{mix} is not empty and let p^* be the share price upon exit in this equilibrium. Note that if $m \in M_{mix}^*$ then $\mathbb{E}[\theta|m] + \beta = 0$ and the manager is indifferent between approval and rejection. We consider two cases.

First, suppose the set M_R^* is empty. According to (5), $\mathbb{E}[\theta|m] + \beta \leq 0$ for any $m \in M_L^*$ and hence $\mathbb{E}[\theta|M_L^* \cup M_{mix}^*] + \beta \leq 0$. Since M_R^* is empty then $\mathbb{E}[\theta|M_L^* \cup M_{mix}^*] = \mathbb{E}[\theta]$. Since by assumption $\mathbb{E}[\theta] + \beta \geq 0$, it must be $\mathbb{E}[\theta|M_L^* \cup M_{mix}^*] + \beta = 0$ and hence $\mathbb{E}[\theta] + \beta = 0$. Since $\mathbb{E}[\theta] > \mathbb{E}[\theta|\theta < 0]$ then $\beta = -\mathbb{E}[\theta]$ implies $\beta < -\mathbb{E}[\theta|\theta < 0]$. According to Proposition 1, there is a responsive equilibrium with threshold $\tau = 0$. Thus, the first best can be obtained by a pure strategy responsive equilibrium as required.

Second, suppose the set M_R^* is not empty and consider the following new decision rule: if $m \in M_{mix}^*$ then $a'(m) = 0$ and otherwise $a'(m) = a^*(m)$. Obviously, the corresponding set M'_{mix} is empty, but M'_R and M'_L are not. I show that under the new decision rule there is an equilibrium in pure strategies which is more efficient. There are three steps.

First, consider the manager's incentives to follow the new decision rule. If $m \in M_{mix}^*$ the manager is indifferent between strategy R and L. Therefore, the manager follows the new decision rule. If $m \in M_{mix}^*$ the incentives constraints under the original equilibrium continue to hold.

Consider the market maker's pricing strategy for the new decision rule. If $m \in M_{mix}^*$ then $\mathbb{E}[\theta|m] = -\beta < 0$. Thus, decreasing $a^*(m) > 0$ to $a'(m) = 0$ increases the expected value of the firm. If $m \notin M_{mix}^*$ then the manager decision does not change. Thus, the market maker

must set the price upon exit at a higher level and $p^* < p'$.

Consider the activist's incentives to implement the new decision rule. Suppose $m \in M^*_{mix}$. Since $a^* (m) \in (0, 1)$ and M^*_L is not empty then it must be that $\theta \leq \max \{0, p^*\}$, the circumstances under which the activist does not have strict incentives to choose $a = L$. This means that the activist is either indifferent or has strict incentives to choose $a = L$. Since $\max \{0, p^*\} \leq \max \{0, p'\}$ then the activist has weak incentives to choose $a = L$ as required. If $m \notin M^*_{mix}$ the incentives constraint as in the original equilibrium continue to hold. Note that since $p^* < p'$ then for some $m \notin M^*_{mix}$ the activist has weak preferences instead of strict preference to make the original recommendation. Moreover, similar to Lemma 4 it must be that $p' > 0$. Overall, the new decision rule can be supported by a pure strategy and responsive equilibrium which is more efficient. ∎