Banking Competition and Stability: The Role of Leverage*

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March 14, 2013

Abstract

This paper reexamines the classical issue of the possible trade-offs between banking competition and financial stability by highlighting the key role of leverage. By means of a simple model we show how competition affects portfolio risk, insolvency risk, liquidity risk and systemic risk in different ways. The relationships depend crucially on banks' liability structure, and more precisely, on whether banks are financed by insured retail deposits or by uninsured wholesale funding. In addition, we argue that bank's leverage plays a central role: it is optimally set based on the portfolio risk and affects bank's solvency, funding liquidity and exposure to contagion. Thus the analysis of the relationship between banking competition and financial stability should carefully distinguish between the different types of risk and should take into account banks' endogenous leverage decisions. This leads us to revisit the existing empirical literature using a more precise taxonomy of risk and taking into account endogenous leverage, thus clarifying a number of apparently contradictory empirical results and allowing us to formulate new testable hypotheses.

Keywords: Banking Competition, Financial Stability, Leverage

JEL Classification: G21, G28

^{*}Support from the FP7 Marie Curie Initial training Network is acknowledged by both authors. Xavier Freixas has benefitted from the support of Ministerio de Ciencia e Innovación, Generalitat de Catalunya, Barcelona GSE, Ministerio de Economía y Competitividad) -ECO-2011-25607; Banco de España-Excelencia en Educación-"Intermediación Financiera y Regulación" is gratefully acknowledged.

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1 Introduction

Understanding the link between bank competition and financial stability is essential to the design of an efficient banking industry and its appropriate regulation. Because of the relevance of this topic, there is a large body of literature on the issue with path-breaking contributions from both theoretical and empirical perspectives. Yet, in spite of the critical importance of the subject and notwithstanding today's improved understanding of its complexity there is no clear-cut consensus on the impact of competition on banks' risk taking and the resulting level of stability for the banking industry. Two main contending views rise in the literature: the charter value view and the risk shifting view. The charter value theory, first put forward by Keeley (1990), assumes that banks choose their level of risk and argues that less competition makes banks more cautious in their investment decisions, as in case of bankruptcy they will loose the present value of their future market power rents. Tenants of the risk shifting hypothesis, which originated with Boyd and Nicolo (2005), postulate, instead, that risks result from the borrowing firms' decisions and point out that higher interest rates will lead firms to take more risk and therefore will increase the riskiness of the banks' portfolio of loans, leading to the opposite result.

The theoretical debate on the impact of banking competition cannot be simply solved by resorting to the empirical evidence, which is often ambiguous and contradictory.¹ Part of the ambiguity stems from the difficulty in the choice of measurements for "financial stability". Indeed, while both Keeley (1990) and Boyd and Nicolo (2005) theoretical contributions consider only bank insolvency risk, bank risk has multiple dimensions.

This paper's contribution is, first, to clarify the different forms of financial instability generated by banks' risk taking decisions and, second and more important, to emphasize the role of leverage as the banks' choice, which allows to analyze the relationship between bank competition and financial stability from a different perspective.

Regarding the multiple forms of financial instability, the empirical literature reveals a great diversity in the estimated relationship between "competition" and "stability", which varies with stability and competition measures, samples and estimation techniques.² This is why a taxonomy of the types of banking risks will help us understand the existing empirical evidence. The need for such a clarification of the different concepts of risk becomes obvious when we consider the other side of the link, the measurement of competition,

¹See Beck, Jonghe, and Schepens (2011), who show that the relationship between competition and financial stability is ambiguous and displays considerable cross-country variation.

²Table 2 offers a synthetic survey of the different choices in the measures of competition and risk in the empirical contributions to the analysis of the competition-financial stability link.

where the industrial organization literature has established the sensitivity of competition analysis to the specific measures of market power that are used. In comparison, the concept of banks' risk is clearly ambiguous and underdeveloped. We distinguish here four different types of banking risk: portfolio risk, banks' insolvency risk, illiquidity risk and systemic risk, and show how competition affects these types of risk differently.

More important is the second objective of this paper, to recover the role of leverage in the analysis of the link between competition and financial stability, a role the literature on banking competition, with its simplifying assumption of exogenous leverage, has largely ignored.³ Endogenizing banks' leverage is all the more important in that a classical justification of financial intermediation is precisely the role of banks' in security transformation and liquidity insurance. To fulfill their functions, banks have to be able to choose their leverage ratio and to change it rapidly if necessary.⁴ Indeed, banks use their access to short-term funding to actively manage their risk, setting their leverage ratios in response to the riskiness of assets. Because leverage directly affects banks' solvency, liquidity, and financial contagion, it plays a key role in the analysis of the impact of competition on financial stability. To be more precise, leverage constitutes a central hub that connects all types of banking risk. Once we acknowledge the role of leverage as an endogenous variable, the perspective regarding banking competition and its effects on financial stability varies considerably: the riskiness of the banks' portfolio of loans is disentangled from the banks' insolvency, illiquidity and contagion risk. Competition affects the riskiness of banks' portfolios and banks respond by adjusting their leverage, so that leverage ratio, insolvency, illiquidity and systemic risk are all jointly determined. Thus, for example, safer portfolios can lead banks to take on more debt.; and the high leverage erodes the pro-solvency effects of competition. When the debt is short-term, it also increases the bank's exposure to funding liquidity risk. Third, financial contagion from one bank to another is more likely when banks are highly leveraged, resulting in a greater chance of a systemic crisis. Consequently, even if banking competition leads to safer loans, because of the endogeneity of leverage, the insolvency risk of banks is not necessarily reduced, while their funding liquidity risk and systemic risk is increased.

Our approach builds on a large body of literature on banking competition that starts with the seminal paper of Keeley (1990). As mentioned before, Boyd and Nicolo (2005)

³For example, Boyd and Nicolo (2005) considers banks solely financed by debt. Martinez-Miera and Repullo (2010) assume the cost of equity to be independent of banks' risk. When the constant equity cost is higher than that of debt, banks are again financed purely by debt.

⁴The role of banks in security transformation goes as far back as Gurley and Shaw (1966) and their role in liquidity insurance was first rigorously formalized by Bryant (1980) and Diamond and Dybvig (1983)

rightfully point out that the intrinsic countervailing forces of firms' risk-shifting can make the relationship between competition and financial stability ambiguous. Martinez-Miera and Repullo (2010) further refine Boyd and De Nicolo's argument by showing that the low profit resulting from competition leaves banks little buffer against loan losses and can therefore jeopardize financial stability. Wagner (2009) considers both banks' and entrepreneurs' incentives to take risk on the portfolio side: once entrepreneurs and banks move sequentially, the overall effect coincides with the charter value hypothesis. The fact that all these contributions focus solely on insolvency risk and take the simplifying assumption of exogenous leverage has been one of the main motivations for our paper.

The study of banks' leverage in a competitive environment is related to Allen, Carletti, and Marquez (2009). The authors show that as competition decreases charter values, banks' incentives to monitor borrowers are reduced. To provide banks proper incentives to monitor, one way is to have banks hold more capital. Capital and loan rates therefore are alternative ways to improve banks' monitoring incentives. While the paper predicts that banks hold more capital in a competitive market, our paper shows that the relationship depends upon banks' liability structure. Even in the traditional trade-off between bankruptcy cost and tax shield, the relationship between competition and leverage is non-monotonic.

Because we want to explore the impact of competition on the different types of risk, our starting point has to be the microfoundations of borrowing firms' risk taking. Following Boyd and Nicolo (2005) and Martinez-Miera and Repullo (2010), in our model firms' investment decisions are subject to moral hazard, such that a higher interest rate leads them to take riskier investment projects. So with greater banking competition, while the buffer provided by rents accruing to market power diminishes, so does the risk of loan default. We enrich upon the structure by taking into account the imperfect correlation in loan defaults.⁵ The last point is a key realistic assumption that makes leverage relevant in the determination of banks' risk. Otherwise, under perfectly correlated loan defaults the bank's capital level does not affect its solvency risk. At the other extreme, if the correlation is zero, no capital is required because of the law of large numbers.

Using this framework, we study how banks optimally choose their leverage in response to the changing competitive environment. This is achieved by means of a simplified theoretical framework where banks' leverage choice is explicitly modeled as a response to the cost of funding. When the leverage choice is made endogenous, it presents a countervail-

⁵Martinez-Miera and Repullo (2010) allows also for imperfectly correlated defaults; but the present model although based on more restrictive assumptions, is more tractable and delivers the analytical solutions that we need to endogenize leverage.

ing force to portfolio risk, as when the portfolio of assets becomes safer the bank may take higher leverage and increase risks and conversely.⁶

Our analysis allows us to establish that the relationship between insolvency risk and banking competition crucially depends on banks' liability structure, whether the banks are financed by stable funds (such as insured retail deposits) or by uninsured short-term wholesale funding. The use of a specific model allows us to show that the risk shifting hypothesis is satisfied for low level of insured deposits while the charter value is correct in the opposite case. So the impact of banking competition on financial stability could be opposite for investment and for low leveraged commercial banks.

In addition to insolvency risk, liquidity risk and systemic risk with their own idiosyncrasy should also be analyzed. This implies considering bank runs, their implication on fire sales and the value of assets in the secondary market. The global games approach allow us to show how the probability of a run on illiquid and solvent banks depends upon banks' leverage. ^{7 8}

Through short-term debts we show that increased banking competition may lead to a higher leverage, making coordination failures that ignite runs to illiquid but solvent institutions more likely. Regarding the liquidity risk, namely the probability that a solvent institution is unable to roll over its debt in the market, the result depends upon whether debt is exogenously given or is taken as endogenous. We show that if debt is exogenous, which can be interpreted as capital ratios being binding, competition will always increase liquidity risk. If instead debt is endogenous, the impact of liquidity risk will always move in opposite direction to insolvency risk, and will depend on the structure of the bank's liabilities. For low levels of insured deposits pure insolvency risk decreases with competition and funding liquidity increases with competition. Nevertheless, the total credit risk of a bank, defined as the sum of solvency risk plus funding liquidity risk is dominated by the impact of competition on insolvency risk.

⁶This is clearly illustrated in the extreme case where a bank's strategy is to maintain a given insolvency risk, which is in line with the idea of "economic capital". In this case, any changes in portfolio risk are exactly offset by the banks' leverage adjustment.

⁷Our analysis of liquidity risk focuses on funding liquidity, thus emphasize short term wholesale financing as a driving force of runs and disregards banks' holding of liquid securities. This point is explored in Carletti and Leonello (2012). The authors study banks' incentive to hold liquid assets in a competitive environment and argue that as competition pushes down returns from risky assets, the opportunity cost of hoarding liquid risk-free assets drops. Banks reshuffle their portfolios, hold more liquid assets and are better protected against bank runs.

⁸In fact, if we allow banks to hold liquid assets of low returns in the model, the hoarding of liquid assets only adds to banks' funding liquidity risk. This is because the low returns of liquid assets makes banks less profitable and less able to build up the capital buffer against fire-sale losses. Coordination failures become more likely as a result.

We extend the model to incorporate systemic risk and find similar results. For the financial system as a whole, leverage accelerates contagion when bank failures and fire sales put downward pressure on asset prices. We illustrate this point in a simplified two-bank case. When both banks sell their assets the expected secondary market price becomes lower and the probability of a systemic crisis due to coordination failures increases. Consequently, the risk of two solvent banks being run at the same time increases with leverage.

In sum, banks' risk levels and leverage interact and are jointly determined. The optimal leverage reflects portfolio risk and affects insolvency, illiquidity and systemic risk. The two-way feedback generates a rich set of predictions on how competition affects various bank risks. We believe this has strong implications both for regulatory policy and for a possible refinement of the empirical analysis.

The paper proceeds as follows. Section 2 lays out the model. Section 3 establishes the benchmark case, exploring how various risks are affected by banking competition under the assumption of exogenous leverage. In section 4, we determine endogenous bank leverage and analyze its impacts on banks' insolvency, illiquidity and systemic risk. The results contrast those under exogenous leverage. We devote section 5 to empirical literature, reinterpreting the empirical findings with the refined definition of "financial stability" and making new testable hypotheses. Relevant policy implications are discussed in section 6. Section 7 concludes.

2 Model Setup

2.1 Portfolio risk and competition

We consider a one good three dates, t = 0, 1, 2, economy where all the agents are assumed to be risk neutral. There are three types of active agents: entrepreneurs, banks and banks' wholesale financiers and one type of purely passive agent: retail depositors. Entrepreneurs are penniless but have access to long-term risky projects. A project requires one unit of investment. It yields a gross return of x > 1 if succeeds and 0 if fails. The projects are subject to moral hazard: each entrepreneur chooses the probability of success $P \in [0, 1]$ in order to maximize his expected utility,

$$E(U) = P(x - r) - \frac{P^2}{2b}.$$
 (1)

Here *r* is the gross loan rate charged by banks. $b \in (0, B]$ represents an entrepreneur's type, with a higher *b* implying a lower marginal cost of efforts. Entrepreneur types are private information, and in particular, unknown to banks, which hold prior beliefs that *b* is uniformly distributed in the interval [0, B]. Entrepreneurs' reservation utility is normalized to zero.

Because idiosyncratic risk diminishes in a bank's well diversified portfolio of loans, we dispense with the modeling of this type of risk and focus, instead, on a bank level risk that affects the whole bank portfolio in the following way: whether a project succeeds or not is jointly affected by entrepreneurs' choice P and a risk factor z. The risk is assumed to be identical for all loans in a bank's portfolio, but can change across banks. It is assumed that z follows a standard normal distribution. Following Vasicek (2002) and Martinez-Miera and Repullo (2010), we assume the failure of a project is represented by a latent random variable y: when y < 0, the project fails. The variable y takes the following form

$$y = -\Phi^{-1}(1 - P) + z,$$
 (2)

where Φ denotes the c.d.f. of standard normal distribution. A project defaults either because of the entrepreneur's moral hazard (a low *P*) or an unfortunate risk realization that affects the bank's whole portfolio (a low *z*). For the sake of consistency, note that the probability of success *P* is given by:

$$Prob(y \ge 0) = 1 - Prob(y < 0) = 1 - Prob(z < \Phi^{-1}(1 - P)) = 1 - \Phi(\Phi^{-1}(1 - P)) = P.$$

Banks are assumed to invest exclusively in a continuum of projects. We further assume the loan market is fully covered and all types of entrepreneurs are financed. The loan portfolio generates a random cash flow denoted by θ . We denote the maximum possible cash flow by $\overline{\theta}$ and the minimum possible level by $\underline{\theta}$.

In order to focus on bank leverage and risks, we dispense with the specific modeling of loan market competition and consider the loan rate r as an exhaustive information on the degree of competition. On the one hand, low prices (loan rates) are predicted by the main-stream competition models⁹ and constitute the driving force in reducing risk in the Boyd and De Nicoló setup. On the other hand, low interest margins are also found associated with less concentrated market, Degryse, Kim, and Ongena (2009).

⁹The opposite relationship may be obtained in models based on Broecker (1990) where an increase in the number of banks raises the probability for a bad borrower to get funded in equilibrium which implies an increases in the equilibrium interest rate.

2.2 Funding and liquidity risk

Each banks holds one unit portfolio of loans, and finances it with debt and equity. At t = 0, a bank finances its total investment of size 1 by raising *F* from insured retail deposits, V_D from short term wholesale creditors and the rest from equity holders. Because retail depositors are insensitive to the banks' risks and play a purely passive role, we assume its supply is fixed and inelastically set equal to *F* and that safety net of deposit insurance is offered to banks at no cost.¹⁰

The short term debt issued to wholesale financiers is of t = 2 face value *D*, risky and uninsured. It is raised in the market from investors that are risk neutral and require the market interest rate that is here normalized to zero. The debt is jointly financed by a continuum of creditors. Each creditor holds an equal share of the bank's debt.

The short-term nature of debt allows the creditor to withdraw at time t = 1, before banks' risky investment matures. In that case she receives qD, where $1 - q \in (0, 1)$ represents an early withdrawal penalty. Equivalently, the debt contract can simply be viewed as promising an interest rate qD at time t = 1 and D at time t = 2.

The bank's risky loan portfolio takes two periods to mature. When the bank faces early withdraws, it has to sell part (or all) of its portfolio in a secondary market at a discount:¹¹ for one unit asset with cash flow θ , the bank obtains only¹²

$$\frac{\theta}{1+\lambda_1}.$$

Here $\lambda_1 > 0$ reflects the illiquid nature of banks' long-term assets that can be due to moral hazard, e.g., banks' inalienable human capital in monitoring entrepreneurs, or adverse selection due to buyers concern with banks selling their 'lemon' projects. The maturity mismatch and fire-sale discount together expose banks to the risk of bank runs.

In principle, a bank can fail either at t = 1 or t = 2. In the former case, the liquidation value of assets is insufficient to repay early withdrawals. In the latter case, while partial liquidation generates sufficient cash to pay early withdrawals at t = 1, the residual portfolio is insufficient to pay creditors who wait until t = 2. Once a bank's cash flow is insufficient to repay its debt, either at t = 1 or t = 2, the bank declares bankruptcy and incurs a

¹⁰Assuming a flat deposit insurance premium that is based on the expected equilibrium debt ratio will not change our results.

¹¹The alternative assumption of banks using collateralized borrowing is to generate similar results. See Morris and Shin (2009).

¹²The proportional form assumes that buyers of the asset can observe better information than banks creditors. Some justification is provided in Rochet and Vives (2004).

bankruptcy cost. For simplicity, we assume the bankruptcy cost is sufficiently high such that once bankruptcy happens, the wholesale financiers get zero payoffs and only the senior deposit insurance company representing retail depositors gets the residual cash flows.

At t = 1 each wholesale creditor privately observes a noisy signal $x_i = \theta + \epsilon_i$. Based on the information, the creditors play a bank-run game. Each player has two actions: to wait until maturity or to withdraw early. If the bank does not fail at t = 2, depositors who wait receive the promised repayment D. For a creditor who chooses to early withdraw, she receives nothing if the bank fails at t = 1 and qD if the bank does not fail on the intermediate date. If the bank is only able to pay early withdrawals but goes bankrupt at t = 2, creditors who wait receive nothing at t = 2.

2.3 Leverage

Banks will choose their leverage so as to maximize the equity value of banks' existing shareholders.¹³ In particular, each bank chooses to issue an amount of debt that promises a repayment qD at t = 1 and D at t = 2, with q < 1. We assume that capital is costly due to market imperfections. Because of the existence of bankruptcy costs, the optimal leverage ratio will trade off the benefits of debt tax shield with the expected costs of bankruptcy. The existence of a liquidity risk makes the choice of leverage more complex as banks take into account both insolvency risk and illiquidity risk.

2.4 Time line

The timing of the model is summarized in the figure below.

t = 0	t = 1	t = 3
1. Banks choose capital	1. Upon signals, wholesale fi-	1. Returns realize for
structure (D).	nanciers decide on running or not.	surviving banks.
2. Entrepreneurs choose	2. Banks facing a run sell assets at	2. Wholesale financiers
P for a given r.	discount.	who have not run on sur
	3. Banks fail or survive.	viving banks get repaid.

¹³It can be shown that this assumption in turn implies the maximization of leveraged firm values.

3 Banking risks with exogenous leverage

In this section, we analyze various risks for a fixed level of leverage. We move upward the spectrum of types of risk: from bottom (individual loan default risk) to the top (systemic risk).

3.1 Loan default and risk shifting

In the spirit of Boyd and Nicolo (2005), we first show that bank competition reduces the default risk of individual loans by curbing entrepreneurs' moral hazard.¹⁴ Note the utility maximization of an entrepreneur of type *b* yields the following probability of success.¹⁵

$$P_b^*(r) = \begin{cases} 1 & \text{if } b \in [1/(x-r), B] \\ b(x-r) & \text{if } b \in (0, 1/(x-r)) \end{cases}$$

An entrepreneur of type $b \ge 1/(x - r)$ will not default for any finite realization of z. This gives us a natural partition between risk-free and risky loans. For the uniform distribution of b, it implies that a fraction α of loans

$$\alpha \equiv 1 - \frac{1}{B(x - r)} \tag{3}$$

are risk free, and the complementary fraction $1 - \alpha$ of loans

$$1 - \alpha \equiv \frac{1}{B(x - r)} \tag{4}$$

are risky and have positive probabilities of default. α reflects the riskiness of a bank's loan portfolio.

As expected, the risk of the portfolio decreases with bank competition. When banks charge lower loan rates under fierce competition, entrepreneurs have more 'skin in the game' and therefore put in more efforts, which results in safer projects, and banks' pool of safe loans grows.

$$\partial \alpha / \partial r = \frac{-1}{B(x-r)^2} < 0$$
 (5)

¹⁴In section 4.6, we discuss the possible different results that would result from taking charter value hypothesis as an alternative starting point.

¹⁵Note $U_E(P_h^*) \ge 0$ such that the participation constraint of entrepreneurs is always satisfied for P^* .

3.2 Portfolio risk: loan loss and cash flow

In order to characterize a bank's portfolio risk we now derive the distribution of loan losses and cash flows. A bank's portfolio risk is driven by the risk of individual loan defaults, which is in turn affected by the random factor *z*. Denote the fraction of non-performing loans *in the risky pool* by γ . We show γ follows a uniform distribution on [0, 1].

Lemma 1. The loan loss γ , defined as the fraction of defaults in the risky pool, follows a uniform distribution on [0, 1].

Proof. See Appendix A.1.

Lemma 1 entails that the expected loan loss in the risky pool is always 1/2. The riskiness of the portfolio depends only on the proportion of the risky pool, which in turn shrinks with the lower loan rates r that is characteristic of competition. As banking competition reduces the risk of each individual loans, the pool of risky loans is downsized and the bank's portfolio risk drops.

Bank's cash flow θ results from loan losses γ in the following way:

$$\theta \equiv \alpha r + (1 - \alpha)[0 \cdot \gamma + r \cdot (1 - \gamma)]$$
$$= r - (1 - \alpha)r \cdot \gamma.$$

Consequently banks' portfolio cash flow follows a uniform distribution on $[\alpha r, r]$. Denote the length of the support by $w \equiv (1 - \alpha)r$. For the uniform distribution of θ , w measures the volatility of the random cash flow. And the cash flow can be represented as

$$\theta = r - w\gamma.$$

The impact of competition on the portfolio risk is reflected in the volatility of cash flow. As banking competition intensifies, a bank's cash flow becomes less volatile:

$$\frac{\partial w}{\partial r} = (1 - \alpha) - \frac{\partial \alpha}{\partial r}r > 0,$$

A property that we will use hereafter. In fact, the variance of cash flow reduces with competition:

$$Var(\theta) = w^2 Var(\gamma)$$
 and $\frac{dVar(\theta)}{dr} > 0$,

Figure 1: Cash flow distribution under two different levels of competition



as $\partial w/\partial r > 0$. For γ 's variance does not depend on the loan rate r, when competition reduces r, w decreases and the volatility of banks' cash flow is reduced by competition. Figure 1 depicts two distribution functions of cash flows, associated with different levels of competition. When competition intensifies, the distribution function becomes steeper, implying a smaller volatility.

Lemma 2. The random cash flow $\theta \sim U(\alpha r, r)$. When banking competition reduces the loan rate r, the volatility of the cash flow decreases.

3.3 Insolvency risk

In this subsection, we define a bank's insolvency risk for a given level of debt. A bank is solvent if its cash flow meets its liability,

$$\theta = r - w\gamma \ge F + D.$$

The inequality gives a critical level of loan loss

$$g \equiv \frac{r - (F + D)}{w}.$$

A bank with realized loan losses greater than g is to be insolvent. For $\gamma \sim U(0, 1)$, it is implied that the solvency probability is equal to g. The bank's pure insolvency risk, i.e., the risk of failure in the absence of bank run, will be denoted by *SR* and takes the following form.

$$SR \equiv 1 - g = \frac{(F+D) - \alpha r}{w} \tag{6}$$

Figure 2: Change of insolvency risk under exogenous leverage



Note that insolvency risk is not monotonic in r. The reason is the same as in Martinez-Miera and Repullo (2010). Banking competition has two countervailing effects on insolvency risk: on the one hand, lower loan rates reduce the risk taking of entrepreneurs so that the portfolio losses decrease (risk-shifting reduction). On the other hand, competition also makes interest margin thinner and banks less profitable, reducing the buffer available to absorb loan losses (buffer reduction). The overall effect is given by the following proposition.

Proposition 1. For a given leverage, a bank's insolvency risk is reduced by competition if and only if $r^2 > x(F + D)$.

Graphically, $r^2 > x(F+D)$ is equivalent to two conditions being satisfied: (1) $\partial \underline{\theta} / \partial r > 0$ so that the distribution function should satisfy a single crossing condition, and (2) the face value of debt should lie to the left of the crossing point. Figure 2 illustrates such a scenario: as banking competition weakens and the loan rate hikes from *r* to *r'*, the solvency probability drops from *g* to *g'*.

3.4 Funding liquidity risk and bank run

In this section we use the global games approach of Carlsson and Van Damme (1993) to examine banks' funding liquidity risk and derive a critical level of cash flow for a bank to be solvent but illiquid: the bank is able to repay in full its t = 2 liability if no one runs it at t = 1, but will default if sufficient many wholesale financiers withdraw early.

In principle, a bank can fail either at t = 1 or t = 2. In the former case, the liquidation value of all assets is insufficient to repay early withdrawals. In the latter case, while partial liquidation generates sufficient cash to pay early withdrawals, the residual portfolio is insufficient to pay creditors who wait until t = 2. Once a bank's cash flow is insufficient to repay its debt, either at t = 1 or t = 2, the bank declares bankruptcy and incurs a bankruptcy cost. For simplicity, we assume the bankruptcy cost is sufficiently high such that the wholesale financiers receive zero value. Only insured retail depositors are reimbursed, with any difference between F and residual cash flows being covered by the deposit insurance company.

We will focus on the natural case where runs make it more difficult for a bank to meet its debt obligation, which occurs when the discount on the value of assets is deeper than that on liabilities, i.e.,

$$\frac{1}{1+\lambda_1} < q. \tag{7}$$

The condition is always true as q approaches 1.¹⁶

Denote by *L* the fraction of wholesale financiers who run the bank. The bank will fail at t = 1 if $\theta/(1 + \lambda_1) \le LqD$, or

$$L \ge \frac{\theta}{(1+\lambda_1)qD} \equiv L'.$$

The bank is to survive t = 1 withdraws but fail at t = 2 if $(1 - f)\theta < F + (1 - L)D$, where $f = (1 + \lambda_1)LqD/\theta$ denotes the fraction of assets sold to meet t = 1 withdrawals. In terms of the fraction of wholesale creditors *L*, the bank survives at t = 1 but fails at t = 2if and only if

$$L \ge \frac{\theta - F - D}{[(1 + \lambda_1)q - 1)]D} \equiv L''.$$

For a solvent but illiquid bank of $\theta < (1 + \lambda_1)q(F + D)$, it always holds that L' > L''.

The lack of common knowledge leads to the so-called Laplacian property of global games: no matter what signal a player *i* observes, he has no information on the rank of his signal as compared to the signals observed by the other players. Denote by *M* the fraction of players that player *i* believes to observe a higher signal than his. The Laplacian property implies $M \sim [0, 1]$.¹⁷

¹⁶If condition (7) is not satisfied, the debt repayment can be more easily met in a fire sale, which would be paradoxical. In particular, when condition (7) is violated, an insolvent bank with $\theta < D + F$ will be saved by a run in meeting its debt obligations, provided $(1 + \lambda_1)qD + F < \theta$.

¹⁷More detailed discussion of the property can be found in Morris and Shin (2001) and we reproduce the proof in Appendix B.

Players take a switching strategy: they run the bank if the observed signal is smaller than a critical level s^* and wait otherwise. As the Laplacian property holds for all players and in particular for the player who observes the critical signal, the player will hold a belief that $M \sim U[0, 1]$ fraction of players will not run the bank and the rest 1 - M will. Consequently, from the perspective of the player who observes the critical signal s^* , the probability for the bank to survive at t = 1 is

$$Prob(t = 1 \text{ survival}) = Prob(1 - M \le L' | s = s^*) = \min\{L', 1\}.$$

The probability of t = 2 bankruptcy is

$$Prob(t = 2 \text{ survival}) = Prob(1 - M \le L''|s = s^*) = \min\{L'', 1\}.$$

For simplicity, we focus on the case L' > 1. The inequality simply states that at t = 1 a bank does not have to liquidate all its assets to meet wholesale financiers' withdrawal, even if all of them run.¹⁸ The the equity buffer combined with the stable funding *F* makes this likely to happen: the presence of retail deposits limits the amount of unstable funding and their potential damage via fire-sale losses.

Depending on the outcome of bank run games, the payoffs for "run" and "wait" are tabulated as follows.

	t = 2 failure	t = 2 survival
run	qD	qD
wait	0	D

When running the bank, a creditor receives qD with certainty. By waiting, she receives a higher payoff D if the bank survives but risks a chance of having her debt value completely wiped out if the bank fails at t = 2. Therefore, in playing the bank run game, the creditors trade off between the risk of receiving zero payoff and the greater remuneration from waiting.

A creditor who observes the critical signal s^* should be indifferent between running the bank or not.

$$Prob(t = 1 \text{ survival}|s = s^*) \cdot q = Prob(t = 2 \text{ survival}|s = s^*) \text{ or } q = \frac{\theta - F - D}{[(1 + \lambda_1)q - 1)]D},$$

¹⁸One can verify that the condition can satisfy for the optimal debt level that is solved in section 4.1.

The indifference condition implies the following critical cash flow $\hat{\theta}$.

$$\hat{\theta}_1 = F + D + q[(1 + \lambda_1)q - 1]D$$
(8)

A run successfully happens when a bank's θ falls below the critical $\hat{\theta}_1$. Define $\mu_1 \equiv 1 - q[1 - (1 + \lambda_1)q]$. A bank is solvent but illiquid if

$$F + D < \theta \le F + \mu_1 D. \tag{9}$$

Where $\mu_1 > 1$ because $(1 + \lambda_1)q > 1$. In order to survive potential bank runs, a bank has to make more profit than what is required to be barely solvent. Further notice that the critical cash flow increases in λ_1 and *D* such that a higher fire sale loss and more exposure to unstable short-term funding lead to a higher chance of illiquidity.

Proposition 2. There exists a critical level $\hat{\theta}_1 = F + \mu_1 D$, $\mu_1 = 1 - q[1 - (1 + \lambda_1)q] > 1$, a bank having cash flow $\theta \in [F + D, \hat{\theta}_1]$ becomes solvent but illiquid.

Proposition 2 states that there is pure risk of illiquidity for banks in the range $[F + D, F + \mu_1 D]$, as those banks are solvent in the absence of bank runs but insolvent if a run occurs. As a bank has cash flow $\theta \sim U[\alpha r, r]$, the probability of a pure liquidity crisis, defined as the probability of a bank being solvent but illiquid and denoted by *IL*, is

$$IL \equiv \frac{(\mu_1 - 1)D}{w}.$$
 (10)

Unsurprisingly, for a given cash flow distribution, two factors contribute to banks' funding illiquidity: (1) low fire-sale prices of banks' assets, (2) high short-term wholesale funding (or equivalently, low level of stable funding F). When fire-sale price and leverage are held exogenous, competition contributes to illiquidity by reducing the expected cash flows, which decreases potential buffers against fire-sale losses. Contrary to the pure insolvency risk, the amount of stable funds provided by insured deposits, F, is absent from the above measure of risk, as retail depositors do not have the incentives to run the bank.¹⁹

Once a bank's debt obligation is exogenous, its funding liquidity risk increases with competition. The result follows directly from the first order derivative,

$$\frac{\partial IL}{\partial r} = (\mu_1 - 1) \frac{-D}{w^2} \frac{\partial w}{\partial r} < 0.$$

¹⁹The same would hold true for long-term debts, as by definition their contract make it impossible for the claim holders to run the bank.

Intuitively, the lower cash flow due to intensified competition provides a thinner buffer against fire-sale losses. Creditors who withdraw early cause a greater loss to those who wait. As the negative externalities aggravate, the coordination failure intensifies, and bank runs are more likely.

In practice, it is hard to distinguish bank failures due to insolvency and those due to illiquidity. The observational equivalence makes it sensible to examine a bank's total credit risk, the summation of pure insolvency and illiquidity risk that measures a bank probability of going bankrupt for either solvency or liquidity reasons. Denoting the total credit risk by *TCR*, we have

$$TCR = \frac{(F + \mu_1 D) - \alpha r}{w}.$$
(11)

Banking competition reduces total credit risk ($\partial TCR/\partial r > 0$) if and only if

$$r^2 > x(F + \mu_1 D).$$
 (12)

Note that this condition is more stringent than the condition in Proposition 1. Thus for a parameter constellation satisfying $x(F + \mu_1 D) > r^2 > x(F + D)$ banking competition would decrease pure insolvency risk but once liquidity risk is taken into account, competition would increase total credit risk, that would generate financial instability. In other words, when illiquidity risk is brought in, the set of parameters where the result of Boyd and Nicolo (2005) applies shrinks. The following proposition summarizes our result.

Proposition 3. For a given level of debt obligation, the probability for a bank to be solvent but illiquid monotonically increases with competition. The total credit risk, the risk of bank failures due to either insolvency or illiquidity, decreases with competition if and only if $r^2 > x(F + \mu_1 D)$.

3.5 Systemic risk and contagion

The risk of contagion is illustrated with a two-bank setup: we make a stylized assumption that when both banks need to sell, the fire-sale discount hikes from λ_1 to λ_2 , where the subscripts 1, 2 denote the number of banks that early liquidate. The assumption captures the observation that the secondary market price tends to fall further when more banks fail and sell, due to either cash-in-the-market pricing or informational contagion. In the former case, market prices are driven down by the limited supply of cash. In the latter, a high number of bank failures lead investors to update their expectations for banks' common risk exposures and lower their willingness to pay for the assets. The exposure to the same asset price provides a channel of financial contagion: when the first bank goes under and sells, the asset price is driven down; this magnifies concerns among debt holders of the other banks', leading to further bank runs.²⁰

Following the same procedure of the last section, we will be able to derive a critical cash flow level

$$\hat{\theta}_2 = F + \mu_2 D > \hat{\theta}_1,\tag{13}$$

where $\mu_2 = 1 - q[1 - (1 + \lambda_2)q] > \mu_1$. A bank whose cash flow falls between $[\hat{\theta}_1, \hat{\theta}_2]$ will be solvent and liquid if the other bank does not face a run, but will become illiquid if runs happen to the other bank. Namely, a bank whose cash flow falls between $[F + \mu_1 D, F + \mu_2 D]$ is exposed to contagion and will fall, if the other bank's cash flow falls below $F + \mu_2 D$. We therefore define the exposure to contagion (denoted by *CTG*)

$$CTG \equiv \frac{(\mu_2 - \mu_1)D}{w}.$$
 (14)

Note that as competition reduces the bank's buffer to against fire-sale losses, $\partial w/\partial r > 0$, the exposure to contagion increases with competition.

In the two-bank setup, the systemic risk (denoted by *SYS*) is captured by the probability that both banks fail at the same time.

$$SYS = Prob(\theta < \theta^{**})^2 = \left(\frac{\theta^{**} - \alpha r}{w}\right)^2 \tag{15}$$

With

$$\frac{\partial SYS}{\partial r} = 2Prob(\theta < \theta^{**})B\big[-(F + \mu_2 D)\frac{x}{r^2} + 1\big],$$

it is implied that competition reduces the systemic risk if and only if

$$r^2 > x(F + \mu_2 D),$$
 (16)

a counterpart to condition (11).

Proposition 4. For a given level of debt obligation, banks' exposure to contagion always increases with competition. The risk of a systemic crisis decreases with competition if and only if $r^2 > x(F + \mu_2 D)$.

Therefore, even if banks do not adjust their leverage to the changing competitive environment, we show banking competition can affect different types of risk differently. In

²⁰For a full-fledged model that shows asset prices drop with bank runs, see Li and Ma (2012).

particular, for $x(F + D) < r^2$, competition always reduces pure insolvency risk; but increases pure liquidity risk if $x(F + D) < r^2 < x(F + \mu_1 D)$; and increases systemic risk if $x(F + D) < r^2 < x(F + \mu_2 D)$. So the general implication of the analysis is that focusing on solely one dimension of risk can lead to biased judgement for the overall effects.

4 Endogenous leverage and its impacts

It is important to notice that the portfolio risk is not identical to the default risk of a bank. Leverage plays a crucial role: a low risk portfolio financed with high leverage can still fail with a substantial chance. Furthermore, leverage is endogenously chosen based on the portfolio risk: a bank with a safer portfolio will be able to use higher leverage while a bank holding a riskier one will find it more costly to issue debts. The riskiness of banks may not change as much as the riskiness of their assets. In this sense, leverage is a countervailing force to the change in asset riskiness. In this section, we endogenize banks' choice of leverage and analyze its implications regarding the effects of competition on the banking risks.

4.1 Endogenous leverage

We assume that the cost of capital is larger than the cost of debt. Compared with debts, an investor demands extra remuneration for holding equities. Denote the premium of capital over debt by k and recall that debt holders are assumed to only break even, so that the expected return on capital is 1 + k. One justification for the existence of such a premium could be the dilution costs à la Myers-Majluf; Diamond and Rajan provide an alternative justification based on the managers renegotiation power; another possible justification would be simply the tax benefits of debt.²¹

The optimal level of debt is determined by its marginal cost being equal to its marginal benefit: on the one hand, a higher debt level entails a greater chance of bankruptcy; on the other hand, a higher debt level saves on costly capital. Banks rationally set their leverage by taking into account the probability of bankruptcy, caused either by insolvency or by illiquidity.²²

²¹When a corporate tax is levied at a constant rate τ and debt repayments are exempted, k reflects the cost of losing tax shields. With $1 + k = 1/(1 - \tau)$ or $\tau = k/(1 + k)$, the model will provide the familiar expression that firms trade off between tax shields and bankruptcy costs.

²²To simplify the analysis, we assume that regulators bail out banks in a systemic crisis: when both banks fail, they both will be bailed out. So banks are assumed not to take into account the systemic risk of contagion in their leverage choice.Relaxing this assumption would imply negative externalities of leverage: a bank that uses

4.2 The general case

Banks choose their capital structure to maximize the leveraged firm value to existing shareholders. If α is the fraction of the bank that is sold to outside shareholders, the value to old shareholders is

$$V_E = (1 - \alpha) \int_{F + \mu_1 D}^{\overline{\theta}} [\theta - F - D] h(\theta, r) d\theta,$$

where $h(\theta, r)$ the density function of banks' cash flows. The bank will raise V_S from new shareholders, V_D from wholesale short term creditors, and F from insured depositors.

$$V_{S} = \frac{1}{1+k} \alpha \int_{F+\mu_{1}D}^{\overline{\theta}} [\theta - D - F]h(\theta, r)d\theta$$
$$V_{D} = \int_{F+\mu_{1}D}^{\overline{\theta}} Dh(\theta, r)d\theta$$
$$V_{F} = F$$

The three sources of funding should provide the required amount of investment, that is,

$$V_S + V_D + V_F = 1.$$

Consequently the optimal leverage structure is the solution to

$$\max_{\alpha,D} \left\{ (1-\alpha) \int_{F+\mu_1D}^{\overline{\theta}} [\theta - F - D]h(\theta, r)d\theta \right\}$$

s.t. $\alpha \int_{F+\mu_1D}^{\overline{\theta}} [\theta - D - F]h(\theta, r)d\theta = (1+k)V_S$
 $\int_{F+\mu_1D}^{\overline{\theta}} Dh(\theta, r)d\theta = V_D$
 $F = V_F$
 $V_S + V_D + V_F = 1$

Adding the three constraints to the objective function we obtain the unconstrained optimization.

$$\max_{D} \left\{ \int_{F+\mu_{1}D}^{\overline{\theta}} [\theta+k(D+F)]h(\theta,r)d\theta + (1+k)\int_{\underline{\theta}}^{F+\mu_{1}D}Fh(\theta,r)d\theta - (1+k) \right\},$$
(17)

where $\int_{\theta}^{F+\mu_1 D} Fh(\theta, r) d\theta$ reflects the subsidy of deposit insurance.

levers up and fails is contagious to the other bank. As far as such cost is not counted in private decision, banks are induced to use even higher debts.

The optimization program yields the first order condition

$$-\mu_1 \left[(F + \mu_1 D) + k(F + \mu_1 D) \right] h(F + \mu_1 D, r) + \int_{F + \mu_1 D}^{\overline{\theta}} kh(\theta, r) d\theta + (1 + k)\mu_1 Fh(F + \mu_1 D, r) = 0,$$

which can be written more compactly as

$$-\mu_1(\mu_1+k)Dh(F+\mu_1D,r)+\int_{F+\mu_1D}^{\overline{\theta}}kh(\theta,r)d\theta=0,$$

or, with *H* denoting the c.d.f. of θ ,

$$D^* = \frac{k[1 - H(F + \mu_1 D^*, r)]}{\mu_1(\mu_1 + k)h(F + \mu_1 D^*, r)}.$$

4.2.1 Application to our setup

The uniform distributions in the current paper simplify the model. It is especially convenient to work with the stochastic loan losses $\gamma \sim U[0, 1]$. To facilitate exposition, we denote

$$g(\mu_1) \equiv 1 - TCR = \frac{r - (F + \mu_1 D)}{w}.$$

 $g(\mu_1)$ is a counterpart of g: it denotes the critical loan loss the bank will survive once liquidity risk is taken into account. The optimization program transforms into the following form.

$$\max_{\alpha,D} \left\{ (1-\alpha) \int_0^{g(\mu_1)} \left[\theta - F - D \right] d\gamma \right\}$$

s.t. $\alpha \int_0^{g(\mu_1)} \left[\theta - D - F \right] d\gamma = (1+k)V_S$
 $\int_0^{g(\mu_1)} D d\gamma - V_D = 0$
 $F = V_F$
 $V_S + V_D + V_F = 1$

It has the corresponding unconstrained program:

$$\max_{D} \left\{ \int_{0}^{g(\mu_{1})} [\theta + k(D+F)] d\gamma + (1+k) \int_{g(\mu_{1})}^{1} F d\gamma - (1+k) \right\}$$
(18)

Recall that $\theta = r - w\gamma$. The maximization program has the first order condition

$$\left[r\frac{\partial g(\mu_1)}{\partial D} - \frac{w}{2}2g\frac{\partial g(\mu_1)}{\partial D}\right] + kg(\mu_1) + k(F+D)\frac{\partial g(\mu_1)}{\partial D} - (1+k)F\frac{\partial g(\mu_1)}{\partial D} = 0,$$

which yields the optimal level of risky debt

$$D^* = \frac{r - F}{\mu_1^2 / k + 2\mu_1}.$$
(19)

And it is straightforward to check that the second order condition is satisfied:

$$-\frac{w}{2}2(\frac{\partial g(\mu_1)}{\partial D})^2 + 2k\frac{\partial g(\mu_1)}{\partial D} < 0.$$

Denote the constant coefficient $c = 1/[\mu_1^2/k + 2\mu_1]$. The optimal debt obligation can be written compactly as

$$D^* = c \cdot (r - F). \tag{20}$$

The result is summarized in the following theorem.

Proposition 5. A bank that maximizes its value by trading off the benefits of debts versus bankruptcy cost sets its debt $D^* = c \cdot (r - F)$, where $c = 1/[\mu_1^2/k + 2\mu_1]$.

The risky debt that a bank issues is proportional to its *maximum* residual cash flow after paying insured deposits F. In particular, the coefficient c increases in the cost of capital, $\partial c/\partial k > 0$, decreases in the liquidity risk, $\partial c/\partial \mu_1 < 0$, and so does the bank's optimal debt level. Note also that

$$\lim_{\mu_1 \searrow 1} c = \frac{1}{1/k + 2} < 1.$$

Being junior, risky and demandable, the wholesale funding D is the most relevant debt in the model. Denote a bank's leverage ratio by l^* . It can be measured by the face value of its risky debt over the available expected cash flow:

$$l^* \equiv \frac{D^*}{E(\theta - F)} = \frac{2c(r - F)}{(1 + \alpha)r - 2F}.$$
 (21)

The leverage ratio is not monotonic in banking competition. Its comparative statics with respect to r depends on the relative strength of two countervailing forces: (1) when r increases, a bank generates a higher cash flow and can issue more claims, including risky debts, which we call "margin effects"; and (2) a higher r implies stronger risk-shifting by entrepreneurs, leading to higher portfolio risk and curbs leverage via bankruptcy costs, which we call "risk effects". The overall effect depends on the relative magnitude of the two forces. In the current setup, as F decreases, a bank has more cash flow available to its wholesale financiers. In that case, when r increases, the margin effect dominates the risk effect, and overall the leverage ratio rises.

Remark 1. The leverage ratio l^* decreases with competition if and only if $r^2 > xF$.

Proof. See Appendix A.4.

4.3 **Risk under endogenous leverage**

The different types of banking risks under endogenous leverage D^* will be denoted hereafter by a superscript star, e.g., SR^* for the pure insolvency risk under endogenous leverage. We show the endogenous leverage has a crucial impact on various risk. In some instances, it reverses the results obtained under exogenous leverage.

Proposition 6. When banks optimally set their leverage, pure insolvency risk decreases with competition if and only if $r^2 > xF$.

Proof. See Appendix A.5.

Again the total credit risk is define as the summation of pure insolvency risk and illiquidity risk:

$$TCR^* \equiv 1 - \frac{r - F - \mu_1 D^*}{w} = 1 - (1 - \mu_1 c) \frac{r - F}{w},$$

Proposition 7. Under endogenous leverage, pure illiquidity and insolvency risk always move in the opposite direction, with the latter dominant in determining total credit risk. In particular, for $r^2 > xF$, as competition intensifies, pure insolvency and total credit risk decreases, while funding liquidity risk increases. Otherwise, the result reverses.

Proof. See Appendix A.6.

The results on the exposure to contagion (CTG^*) and the risk of a systemic crisis (SYS^*) follow from the definitions.

$$CTG^* \equiv \frac{(\mu_2 - \mu_1)D^*}{(1 - \alpha)r}$$
$$SYS^* \equiv Prob(\theta < \hat{\theta}_2)^2 = \left(1 - \frac{r - F - \mu_2 D^*}{w}\right)^2$$

Proposition 8. For $r^2 > xF$, while banks' exposure to financial contagion increases with competition, the risk of a systemic crisis decreases, provided $\mu_2 c < 1$.

Proof. See Appendix A.7.

As shown by inequality (A.24), banks' liability structure is central to the relationship between competition and bank risk. Whether a bank has insured retail deposits $F > r^2/x$

makes a critical difference. To interpret the key condition, consider two types of economies corresponding to the two possible signs of $xF - r^2$. A negative sign, with $xF < r^2$, corresponds to less productive firms, with banks financed through market funding and high interest rates on loans. When this is the case, total credit risk is reduced with competition. This will occur for a low x implying a risky portfolio of loans (P_b^* increases with x for every value of r). Banks have a relatively high average cost of funds because of a low F and have a high income as r is high. As a particular case, F = 0 in which case x is irrelevant, corresponds to this case and might be interpreted as investment banking. More competition means safer investment banking.

A positive sign, with $xF > r^2$, corresponds to highly productive firms, with banks mainly financed through deposits and low interest rates on loans. When this is the case, a high x implies a safer portfolio of loans ($P_b^*(r)$ is higher for every value of r). Banks have a lower average cost of funds because of the higher F and have a lower income as r is low. This might be closer to retail banking. The opposite result occurs, and more competition will increase total risk.

4.4 Interpretation

Although our model does not pretend to provide robust results that hold true in every environment, it is worth noticing the key ingredients that determine here the impact of bank competition on the different types of financial stability. As shown by inequality (A.24), banks' liability structure, and in particular the amount of short term wholesale funding, is central to the relationship between competition and bank risk. Our model's conclusions provide a much richer view of the link between banking competition and financial risk than what is usually considered.

1. To begin with, notice that the result depends upon the borrowing firms' project returns x. For a given level of deposits and banks' market power, the effect of banking competition on financial stability depends upon how productive the firms are. In highly productive economies, bank competition constitute a threat to financial stability. The impact of moral hazard is reduced, and the key determinant of the link between bank competition and financial stability is the role of the buffer generated by banks' market power. Comparing Proposition 1 and Proposition 6 we observe that the threshold for x that inverts the relationship from banking competition to financial stability is reached much earlier if we take into account the endogeneity of banks' leverage. This is the case because firms

Figure 3: Linkages between Banking Competition, Leverage and Various Risk



will be more conservative in its choice of leverage, so that the strength of the Boyd and DeNicolo argument is weakened and the charter value dominates.

2. The level of market power is also essential in our framework. For a very high market power competition reduces bank fragility, nevertheless a threshold may exist (provided that xF > 1) beyond which the result reserves. This is interesting from a policy perspective as it provides a more nuanced prescription than the usual one: in order to sustain financial stability, it might be interesting to promote competition up to a certain threshold, but beyond that point, competition will lead to higher banking risk.

3. The role of stable funds is critical for our result. In a traditional banking industry funded through deposits and long term bonds, where $xF > r^2$, competition will be detrimental to financial stability. Instead in a banking industry where wholesale short term (possibly interbank) funding is prevalent, the Boyd De Nicolo argument will prevail.

4. More generally, two types of banks, corresponding to the two possible signs of $xF - r^2$, will coexist and will react in a different way to an increase in competition. For banks that rely less on stable funding $xF < r^2$, in particular for investment banks, an increase in competition will increase financial stability. Instead, for banks with high levels of deposits and lower market power, for which the inequality $xF > r^2$ is fulfilled, the opposite occurs and banking competition's main effect is to reduce the banks buffer and to encourage higher leverage.

Figure 3 visualizes the channels that banking competition affects risk, either directly through cash flow riskiness, or indirectly through the changing leverage. It emphasizes how banking competition determines both the cash flow characteristics and bank leverage and

how risks are jointly determined by the optimization behaviors of banks. It should also be acknowledged that despite of our efforts to build a comprehensive model, the presented still considerably understates the complexity of the issue, for competition also affects banks' portfolio choice, e.g., the correlation of their portfolios, cash hoarding, and so on, which are all abstract from the current setup.

4.5 Comparison to the exogenous leverage case

To emphasize the crucial impact of endogenous leverage, we summarize and tabulate in Table 1 and 2 the results under exogenous and endogenous bank leverage for a side-byside comparison. The '+' sign denotes that competition increases the bank risk considered; and the '-' sign denotes that competition reduces that type of risk.

Table 1: Banking competition and risk under exogenous leverage

	$r^2 < x(F+D)$	$r^2 \in [x(F+D), x\hat{\theta}_1)$	$r^2 \in [x\hat{\theta}_1, x\hat{\theta}_2)$	$r^2 > \hat{\theta}_2$
Pure insolvency risk	+	-	-	-
Pure liquidity risk	+	+	+	+
Total credit risk	+	+	-	-
Exposure to contagion	+	+	+	+
Systemic risk	+	+	+	-

 Table 2: Banking competition and risk under endogenous leverage

	$r^2 < xF$	$r^2 > xF$
Pure insolvency risk	+	-
Pure liquidity risk	-	+
Total credit risk	+	-
Exposure to contagion	-	+
Systemic risk	+	-

To a large extent, the model presented is a special case where all important comparative statics depend on the sign of $r^2 - xF$. Yet the result conveys the key messages of the paper: (1) banking competition affects different types of risk differently; and (2) the endogenous leverage is a central hub that both reflects changes in the cash flow riskiness and affects all different aspects of banking risk.

4.6 An alternative prior

In the paper, we take risk-shifting hypothesis as our starting point. The argument of endogenous leverage however is more general and also applies to models that takes charter value hypothesis as a prior. As competition reduces charter values and induces banks to choose riskier portfolios—portfolios with higher but more volatile cash flows, banks will optimally adjust their leverage to balance the change in the portfolio risk. Again, leverage stands a central hub that links to all types of bank risk. The leverage and risks are all jointly determined by banks' optimization behaviors. Yet as the prior flips, the overall results can reverse as compared to those in the current paper.

5 Reinterpreting the empirical literature

The difficulties in analyzing the link between competition and financial stability exponentially increase when we turn to the empirical studies. The empirical analysis has led to a multiplicity of results that are sometimes difficult to reconcile and susceptible of alternative interpretations. As we have emphasized in this paper there are multiple measures of financial stability as there are multiple measures of competition, ranging from franchise or charter value (Tobin's Q), to market structure (e.g., HHI, C-n), to structural measurement (i.e., P-R H-stat., Lerner's index, Boone's indicator), and to institutions (contestability of the market, e.g., activity and entry restrictions).²³ The industrial organization literature does not provide an unambiguous answer to which measurement reflect competition best, even if, in the context of banking the empirical evidence shows that concentration measures are poor proxies for bank competition, Claessens and Laeven (2004) and Schaeck, Cihák, and Wolfe (2009), it still leaves a wide range of possible measures.

The empirical implication of our model is that banks' risks should be measured at four fundamentally different levels: first, at the level of banks' assets; second, at the level of banks' solvency; third, at the level of banks' liquidity risk and fourth at the level of the overall systemic risk and contagion. Competition directly affects the riskiness of banks' cash flows; but because banks react to these changes by altering their leverage, all other types of risks are also affected. Indeed, banks' endogenous leverage constitutes a central hub that connects these three types of risk. The implication is that depending on the magnitude of the direct and indirect forces, a full diversity of predictions can rise.

²³See Degryse, Kim, and Ongena (2009) for a comprehensive review on measuring banking competition.

As a consequence, our reading of the empirical literature results introduces drastic differences depending on whether the evidence concerns the riskiness of banks' assets, the riskiness of banks themselves, either their solvency or their liquidity, or systemic risk.

Our theoretical framework suggests a progressive approach to the understanding of the impact of competition on banks' risk taking by refining the questions that are asked as successive layers.

 Does competition increase the safety of a banks portfolio of assets? In other words is the Boyd & DeNicole's basic assumption true?

Next, once we take into account the optimal reallocation of assets and the effect of optimal leverage tuning by the bank, the following issues are to be addressed:

- 2. Does competition increase the risk of a bank's insolvency?
- 3. Does competition increase the liquidity risk of banks?
- 4. Does competition increase banks' systemic risk?

Revisiting the empirical literature through this filter leads us to regroup the empirical results in a more complete and orderly way, refusing to find the different measures either equivalent or complementary in the assessment of the impact of competition on financial stability, without taking into account the changes in leverage it produces. In the end of the section, we summarize in table 2 the empirical literature by highlighting the different key contributions, the measures of risk and competition they utilize and the results regarding the impact of competition on financial instability they obtain.

5.1 Portfolio risk: non-performing loans

The basic postulate of the Boyd and Nicolo (2005) approach is that competition will reduce the riskiness of banks' portfolio, an issue independent of the banks' leverage reaction. The alternative hypothesis put forward by the tenants of the charter value approach is that the banks' overall investment strategy will be more risky as the opportunity cost of bankruptcy is lower. So knowing whether Boyd & De Nicolo's basic assumption is in line with empirical evidence is a crucial step forward. In order to measure the riskiness of assets, measures like stock volatility in Demsetz, Saidenberg, and Strahan (1996); Brewer and Saidenberg (1996) are contaminated by leverage. The non-performing loans (NPLs) ratio is the variable that reflects most accurately the riskiness of banks' assets, and a bulk of literature appears to support this view, as it analyzes non-performing loans as one of the key variables in the analysis of the competition-financial stability link.

Restricting the measurement of risk to NPLs implies focusing on a very specific dimension of the broad link between competition and financial stability where we might hope for some consensus on the empirical results. Unfortunately even with this drastic reduction the evidence is mixed. So, in spite of the fact that the charter value and risk shifting theories have completely opposite predictions regarding the impact of banks' competition on non-performing loans, empirical studies give no definitive answer on which one should be the predominant view.

The initial paper on the charter value Keeley (1990) did not consider NPLs measures but rather estimates of overall bank risk of failure. The prediction on NPLs is backed by more recent works, such as the analysis of Salas and Saurina (2003) and Yeyati and Micco (2007). The authors found an increase in non-performing loans as bank competition increased in Spain and in eight Latin American countries respectively. Support for the risk shifting hypothesis comes from Boyd et al. (2006) and is corroborated by Berger et al.(2008) who find an interesting set of results based upon both loan risk and overall bank risk. Using cross-sectional data on 29 developed countries for the years 1999 through 2005, they find that banks with a higher degree of market power exhibit significantly more loan portfolio risk.

The impact of the US introduction of Nationwide banking also leads to contradictory results: while Jayaratne and Strahan (1998) report that "Loan losses decrease by about 29 basis points in the short run and about 48 basis points in the longer run after statewide branching is permitted", Dick (2006) finds out that "charged-off losses over loans (...) appears to increase by 0.4 percentage point following deregulation".²⁴

Some caveats are also in order regarding the accuracy of this measurement. First, banks can manipulate NPL by rolling over bad loans. Second, a risky loan granted today will only default in the future (e.g. after a two-year lag if we follow Salas and Saurina (2003) and the rate of default will depend upon the business cycle (Shaffer 98). Although the latter might be corrected by the introduction of macroeconomic risk controls, such as the GDP growth rate, the time lag may be more difficult to correct because of the persistence of the non-performing loans ratios. Third, the riskiness of assets could also be altered by changing the

²⁴It has been argued that institutional changes can be a more robust measurement than market structure, for its exogeneity facilitates statistical analysis to establish causal relationships. The instrumental approach bears its value for banking deregulation is usually associated with a removal of barriers to entry that will increase competition. Yet, it is not only associated to the removal of barriers to entry, as it might also affect the range of financial products banks are allowed to invest in and the structure of financial institutions. As pointed out by Cubillas and Gonzalez (), bank liberalization has not only an effect on banks' competition, but also an indirect effect on banks' strategies other channels. This implies that the "banking deregulation" measure of market power explores the effect of a package of measures related to market power on financial stability, but market power is only an undistinguishable part of it.

portfolio allocation among the different classes of risks. A bank with higher market power may be willing to take more risks on its assets that will result in higher NPLs in order to obtain a higher expected return while its market power on, say, deposits provides a natural buffer that prevents its financial distress.

5.2 Individual bank risk: insolvency

Because of the endogeneity of banks leverage, changes in the portfolio risk do not translate into equivalent changes on banks' default risk: a poorly capitalized bank can have a high chance to fail, even if its portfolio risk is low. Such divergence between the impact of competition on the riskiness of banks' assets (as measured by NPL) and its overall risk is perfectly illustrated in Berger, Klapper, and Turk-Ariss (2009): in spite of finding confirmation of the risk shifting hypothesis in the NPL their analysis shows that banks with a higher degree of market power have lower overall risk exposure mainly due to their higher equity capital levels.

Since Keeley (1990) the literature has been focusing on the risk of individual bank failure. In his classic paper, Kelley considers the market-value capital-to-asset ratio and the interest cost on large, uninsured CD's. Following his approach, Demsetz, Saidenberg, and Strahan (1996) use seven different measures of BHCs' risks and in each of them franchise value is statistically significant providing support to the charter value theory.²⁵ Brewer and Saidenberg (1996) found also corroborating evidence that the standard deviation of stock returns volatility was negatively related to S&L franchise values as measured by the market-to-book asset ratio. Also confirming the charter value perspective, Salas and Saurina (2003) show that capital ratio increases with Tobin's Q, thus providing some evidence on the possible behavior of the (endogenous) leverage ratio.

In our judgement, pure insolvency risk can be best measured by Z-scores,²⁶ and many empirical works take that as the main risk measurement. Still, there are important nuances in these results. Beck, Jonghe, and Schepens (2011) show on average, a positive relationship between banks' market power, as measured by the Lerner's index, and banks' stability, as proxied by the Z-score. Nevertheless, they find large cross-country variation in this relationship. Jiménez, Lopez, and Salas (2010) report empirical evidence that supports the

²⁵The risk measurements include annualized standard deviation of weekly stock returns, systematic risk, firmspecific risk, capital-to-assets Ratio, loans-to-assets ratio, commercial and industrial loans-to-assets ratio and loan portfolio concentration.

²⁶The measurement is calculated $(RoA - E/A)/\sigma(RoA)$ to capture a bank's distance from insolvency.

franchise value paradigm but only if market power is measured by Lerner's indexes based on bank-specific interest rates and bank risk.

Opposing this view, Boyd and Jalal (2009) provided cross-country empirical evidence supporting the risk-shifting model using several proxies to measure bank risk, including using the Z-score.²⁷ Using a US sample and a cross-country one they consistently find that banks' probability of failure is negatively and significantly related to measures of competition. Confirming this view, Nicolo and Ariss (2010) analyze the impact of large deposit and loan rents and show that they predict higher probabilities of bank failures and lower bank capitalization.

5.3 Individual bank risk: illiquidity

Funding liquidity risk has largely been overlooked in the empirical study.²⁸ One might argue that upon observing bank failures, it is difficult, if ever possible, to distinguish pure solvency issue from illiquidity ones, (Goodhart, 1987). However, just as insolvency risk can be measured by Z-scores, illiquidity can be measured with accounting information too. For example, in their study of bond pricing, Morris and Shin (2004) identify the extra yield due to illiquidity risk: as far as the yield reflects default probability, the liquidity risk can be reflected in bond pricing. While the relationship between funding liquidity risk and competition has not been studied, theoretical models do provide sound guide for estimating the risk: funding liquidity risk is reduced by high returns (e.g., measured by ROA) and high asset market liquidity (e.g., the holding of reserves and cash), and aggravated by the amount of uninsured short-term funding. Morris and Shin (2009) provide further practical guide. In the context of herding behavior Bonfim and Kim (2011) present an attempt to measure the risk by a variety of liquidity ratios.

In addition to the accounting information, funding liquidity risk can also be estimated by market data. Veronesi and Zingales (2010) suggest constructing bank run index using CDS spreads. In sum, we believe banks' liquidity risk can be measured. Yet how those liquidity measurements link to banking competition invites much future research.

²⁷The author also use loan losses and dummy for actual bank failures.

²⁸On contrast, even though theoretical models made no prediction on how competition affects leverage, which in turn affects insolvency risk, the empirical study has taken into account insolvency risk adjusted by leverage by using measurements like z-scores.

5.4 Systemic risk

The analysis of systemic risk is, obviously, the most difficult one as it often has to deal with cross-country analysis and the main driving force for changes in market power are related to banking deregulation, market entry, deposit insurance and a number of joint measures of which increased competition is only one of the consequences.²⁹ The precise definition of a banking crisis itself as well as its timing is subject to different interpretations. Thus, while some authors consider the intervention of exceptional measures by the Treasury, or a 10% of the banking industry being affected, others like Anginer, Demirgüç-Kunt, and Zhu (2012) and De Nicolo et al.(2004) prefer measuring the probability of systemic risk by pairwise distance to default correlation or constructing an indicator of the probability of failure for the five largest banks.

According to Beck, Demirgüç-Kunt, and Levine (2006) on a sample of 69 countries over a 20 year period more concentrated national banking systems are subject to a lower probability of systemic banking crisis. Still, they point out that concentration need not be related to market power, as already mentioned by Claessens and Laeven (2004), and that other measures of competition may lead to the opposite result. Contradicting the result of Beck, Demirgüç-Kunt, and Levine (2006), Schaek et al.(2006) show, using the Panzar and Rosse H-Statistic as a measure for competition in 38 countries during 1980-2003, that more competitive banking systems are less prone to systemic crises and that time to crisis is longer in a competitive environment even if concentration and the regulatory environment is controlled for.

Our paper's empirical prediction states here that an increase of competition may have different effects depending upon the amount of insured retail deposits, the profitability of projects and banks' spreads, thus suggesting new lines for future empirical research based on the differentiation of different types of banking systems. It would be interesting to pursue this research by distinguishing among different types of banks. If we interpret literally our model, this would be to distinguish banks with low deposit to asset ratios from those with a high deposit to asset ratio. Still, more generally, this could be interpreted as dividing the banks according to their different access to short maturity market funds.

²⁹It should be noted that with newly developed measurements on systemic risk such as CoVaR in Adrian and Brunnermeier (2010), one can also link a bank's market power to its contribution to the systemic risk, e.g., by regressing an individual bank's CoVaR on its Learner's index.

Paper	Risk	Competition	Results	Data Source	Comments
Keeley (1990)	Interest Cost	Tobin's q	Yes	NS	also via capital
Demsetz, Saidenberg, and Strahan (1996)	Stock Volatility	Market-Book Value	Yes	SU	via capital/diversification
Brewer and Saidenberg (1996)	Stock Volatility	Market-Book Value	Yes	US S&L banks	
Jayaratne and Strahan (1996, 1998)	NPLS	Deregulation	No	NS	
Salas and Saurina (2003)	Loan Loss	Tobin's q	Yes	Spain	also via capital
De Nicolo and Loukoianova (2005)	Z-Score	IHH	No	Non-industrialized	interaction with ownership
Beck, Demirgüç-Kunt, and Levine (2006)	Crisis Dummy	Concentration	Yes	Cross-Country	
Beck, Demirgüç-Kunt, and Levine (2006)	Crisis Dummy	Contestability	No	Cross-Country	
Dick (2006)	Loan Loss	Deregulation	Yes	SN	
Yeyati and Micco (2007)	Z-Score & NPLs	P-R H-Stat.	Yes	Latin America	
Schaeck and Cihák (2010a)	Capitalization	P-R H-Stat.	No	Developed	IV
Dell'Ariccia, Igan, and Laeven (2008)	Lending standard	Number of banks	Yes	NS	mortgage market
Boyd and Jalal (2009)	Loan Loss	IHH	No	US/Cross-Country	
Boyd and Jalal (2009)	Z-Score	IHH	No	US/Cross-Country	
Boyd, Nicolo, and Loukoianova (2009)	Crisis Dummy	HHI/C3	No	Cross-Country	
Berger, Klapper, and Turk-Ariss (2009)	NPLS	Lerner Index/HHI	Yes	Developed	via capital
Berger, Klapper, and Turk-Ariss (2009)	Z-Score	Lerner Index/HHI	No	Developed	via capital
Schaeck, Cihák, and Wolfe (2009)	Crisis Dummy	P-R H-Stat.	No	Cross-Country	
Schaeck, Cihák, and Wolfe (2009)	Duration until crisis	P-R H-Stat.	No	Cross-Country	
Schaeck and Cihák (2010b)	Z-Score	Boone's Indicator	No	US/EU	via efficiency
Jiménez, Lopez, and Salas (2010)	NPLs	Lerner Index	Yes	Spain	
Jiménez, Lopez, and Salas (2010)	NPLS	HHI/C5		Spain	
Beck, Jonghe, and Schepens (2011)	Z-Score	Lerner Index	Yes	Cross-Country	cross-country heterogeniety
Nicolo and Ariss (2010)	Z-Score	Deposit market rent	Yes	Europe	
Nicolo and Ariss (2010)	Z-Score	Loan market rent	No	Europe	
Dick and Lehnert (2010)	Personal bankruptcy	Deregulation	Yes	SU	
Dick and Lehnert (2010)	NPLs/risk management	Deregulation	No	SU	
Anginer, Demirgüç-Kunt, and Zhu (2012)	D-to-D Correlation	Lerner Index	No	Cross-Country	supervision/ownership

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6 Discussion and policy implications

Because the aim of our paper is to clarify the multiple concepts of risk and the key role played by leverage, our model has made a number of drastic simplificative assumptions that although leading to simple propositions cannot be easily generalized. Indeed, our framework considerably understates the complexity of the issue, for competition also affects banks' portfolio choice, e.g., the correlation of their portfolios, securitization, cash hoarding, and so on, which are all abstract from the current setup. .

Also our model's main objective is not to address the issue of the design of overall banking regulation, and consequently, from that perspective, it suffers from two serious limitations. On the one hand it does not take into account the impact of competition in increasing productivity through the Shumpeterian creative destruction process (Dick 2006) and, on the other hand, it does not consider the supply of credit that is exogenously set as all firms are able to get financed. As a consequence, instead of the usual risk-return trade off, higher risk is associated here with a higher private and social expected cost of bankruptcy. In spite of this, it is interesting to consider the implications our results has on regulatory policy. Two main lessons can be drawn: a first one regarding the impact of competition in general and a second one regarding capital and liquidity regulation.

The first lesson of our model regarding banking regulation is that the one-size-fits-all approach to the analysis of the link between banking competition and financial stability is insufficiently rigorous. To be more precise we conclude that the link depends among other things on the degree of market power of financial institutions. If financial institutions have a high market power competition reduces the level of total credit risk (that is the sum of insolvency risk and liquidity risk) in financial institutions, confirming the risk shifting hypothesis of Boyd and De Nicoló. Still, in this high market power case, we show that the impact is dampened by the increase of liquidity risk the increase in competition causes. On the other hand, once the banking industry is sufficiently competitive, the inequality reverses and additional competition leads to financial instability, thus confirming the charter value assumption. From that perspective the policy position depends upon the fact that market power is above or below some threshold that depends upon firms' productivity as well as upon banks' liability structure.

A simple extension of our framework, consisting in distinguishing wholesale short term market funding from long term market funding, $(D = D_S + D_L)$ where long term market funding have a higher cost, has also implications regarding liquidity regulation. Indeed, we show that a more competitive banking industry that, in principle, should be more efficient has a higher level of liquidity risk, proportional to D_S if leverage is exogenous but not if it is endogenous. This is directly related to capital regulation, because if capital regulation is binding, leverage becomes exogenous. As a consequence, the implication is that liquidity regulation may reduce risk in competitive economies where banks have a higher cost for capital and for long term funds than for short term wholesale funds. Liquidity regulation, as suggested by the recent changes introduced in Basel III, could therefore decrease the liquidity risk that is implied by fiercer competition.

7 Concluding remarks

We develop a model to study banks' risk in competitive environments. We model explicitly the credit risk created by borrowing firms' moral hazard and examine how banks optimally adjust their leverage in light of various risk. With the theoretical framework, we clarify the concept of financial stability: it has multiple dimensions ranging from portfolio risk to systemic risk. We show that competition can affect different types of risk differently, and the idea of an identical impact of banking competition on financial stability that would hold across types of banks and types of firms has no theoretical foundation. This can help explain the diverse findings in the empirical literature. We further suggest that banks' leverage and liability structure play a key role in determining the relationship between banking competition and financial stability. As a consequence, testing our model prediction that the competition-financial stability link depends upon the type of bank and the state of the economy through firms self financing and productivity may lead to an important step forward in our understanding of the issue.

Appendix A Proof of propositions

Appendix A.1 Proof of lemma 1

To derive the uniform distribution of loan loss γ , take a risky type $\hat{b} < 1/(x - r)$; and define the fraction of entrepreneurs below \hat{b} in the risky pool by $\hat{\gamma}$. We have

$$\hat{\gamma} = \frac{\hat{b} - 0}{1/(x - r) - 0} = \hat{b}(x - r).$$
(A.22)

Consider the critical realization $\hat{z} = \Phi^{-1}(1 - P_{\hat{b}}^*)$ such that an entrepreneur of \hat{b} does not default but all types $b < \hat{b}$ do. So for $z = \hat{z}$, one will have $\gamma = \hat{\gamma}$. To derive the distribution

of γ , notice that

$$F(\hat{\gamma}) \equiv Prob(\gamma < \hat{\gamma}) = Prob(z > \hat{z}) = 1 - Prob(z < \hat{z})$$
$$= 1 - \Phi(\Phi^{-1}(1 - P_{\hat{b}}^*)) = P_{\hat{b}}^*$$
$$= \hat{b}(x - r).$$

By equation (A.22), we have $\hat{b} = \hat{\gamma}/(x - r)$. Substitution yields

$$F(\hat{\gamma}) = \hat{\gamma},$$

implying $\gamma \sim U(0, 1)$.

Appendix A.2 Proof of proposition 1

On the comparative statics of insolvency risk, computation is simplified if we consider its complementary probability, 1-SR = [r-(F+D)]/w. Examining its first order derivative with respect to r, we obtain:

$$\begin{aligned} \frac{\partial (1-SR)}{\partial r} &= \frac{1}{w^2} \left[w - \frac{\partial w}{\partial r} \left[r - (F+D) \right] \right] \\ &= \frac{1}{w^2} \left[w - \left[(1-\alpha) - \frac{\partial \alpha}{\partial r} r \right] \left[r - (F+D) \right] \right] \\ &= \frac{1}{w^2} \left[\frac{\partial \alpha}{\partial r} r^2 + \left[(1-\alpha) - \frac{\partial \alpha}{\partial r} r \right] (F+D) \right]. \end{aligned}$$

Recall that $\partial \alpha / \partial r = -1/B(x-r)^2$ and $(1-\alpha) = 1/B(x-r)$. Taking out the common factor, we will have

$$\frac{\partial (1 - SR)}{\partial r} = \frac{1}{w^2} \frac{\partial \alpha}{\partial r} \left[r^2 + \left[-(x - r) - r \right] (F + D) \right]$$
$$= \frac{1}{w^2} \frac{\partial \alpha}{\partial r} \left[r^2 - x(F + D) \right]$$

Therefore,

$$\frac{\partial SR}{\partial r} = \frac{-1}{w^2} \frac{\partial \alpha}{\partial r} [r^2 - x(F+D)],$$

pure insolvency risk is reduced by competition if and only if

$$r^2 > x(F+D).$$
 (A.23)

Appendix A.3 Proof of proposition 4

A systemic crisis takes place if both banks' cash flow fall below $\hat{\theta}_2$, i.e., $SYS = Prob(\theta < \hat{\theta}_2)^2$. This allows us to obtain:

$$\begin{split} \frac{\partial SYS}{\partial r} &= 2Prob(\theta < \hat{\theta}_2) \frac{\partial}{\partial r} Prob(\theta < \hat{\theta}_2) \\ &= 2Prob(\theta < \hat{\theta}_2) \frac{\partial}{\partial r} \left(1 - \frac{r - \hat{\theta}_2}{w}\right) \\ &= 2Prob(\theta < \hat{\theta}_2) \frac{-1}{w^2} \left[w - \left[(1 - \alpha) - \frac{\partial \alpha}{\partial r}r\right](r - \hat{\theta}_2) \right] \\ &= 2Prob(\theta < \hat{\theta}_2) \frac{-1}{w^2} \frac{\partial \alpha}{\partial r} \left[r^2 - x\hat{\theta}_2\right] \\ &= 2Prob(\theta < \hat{\theta}_2) \frac{-1}{w^2} \frac{\partial \alpha}{\partial r} \left[r^2 - x(F + \mu_2 D)\right]. \end{split}$$

As $\partial \alpha / \partial r < 0$, the sign of the comparative statics is determined by $r^2 - x(F + \mu_2 D)$: the risk of a systemic crisis decreases with competition if and only if $r^2 > x(F + \mu_2 D)$.

Appendix A.4 Proof of remark 1

Recall

$$l^* = \frac{D^*}{E(\theta - F)} = \frac{2c(r - F)}{(1 + \alpha)r - 2F}$$

The result follows directly from the first order condition.

$$\begin{aligned} \frac{\partial l^*}{\partial r} &= \frac{2c}{[(1+\alpha)r - 2F]^2} \Big[[(1+\alpha)r - 2F] - (r-F) \Big[\frac{\partial \alpha}{\partial r} r + (1+\alpha) \Big] \Big] \\ &= \frac{2c}{[(1+\alpha)r - 2F]^2} \Big[-(1-\alpha)F - \frac{\partial \alpha}{\partial r} r(r-F) \Big] \\ &= \frac{2c \cdot (1-\alpha)}{[(1+\alpha)r - 2F]^2} \frac{r^2 - xF}{x-r} \end{aligned}$$

The comparative statics depends on the sign of $r^2 - xF$. When

$$r^2 > xF \tag{A.24}$$

 $\partial l^* / \partial r > 0$, banking competition leads to a lower leverage ratio.

Appendix A.5 Proof of proposition 6

For pure insolvency risk

$$SR^* \equiv 1 - \frac{r - F - D^*}{w} = 1 - \frac{(1 - c)(r - F)}{w},$$

its comparative statics with respect to r follows

$$\frac{\partial SR^*}{\partial r} = -(1-c)\frac{\partial}{\partial r}\left(\frac{r-F}{w}\right).$$

As we have shown c < 1, the expression shares the same sign as

$$-\frac{\partial}{\partial r}\left(\frac{r-F}{w}\right).$$

Note that

$$\frac{\partial}{\partial r} \left(\frac{r-F}{w} \right) = \frac{1}{w^2} \left[w - \left[-\frac{\partial \alpha}{\partial r} r + (1-\alpha) \right] (r-F) \right] \\ = \frac{1}{w^2} \left[\frac{\partial \alpha}{\partial r} r^2 + \left[-\frac{\partial \alpha}{\partial r} r + (1-\alpha) \right] F \right] \\ = \frac{1}{w^2} \frac{\partial \alpha}{\partial r} [r^2 - rF - (x-r)F].$$

Simplifying the last term yields

$$\frac{1}{w^2}\frac{\partial\alpha}{\partial r}(r^2 - xF). \tag{A.25}$$

As $\partial \alpha / \partial r < 0$, the sign of the $\partial S R^* / \partial r$ is determined by $r^2 - xF$: the pure insolvency risk decreases with competition if and only if $r^2 > xF$.

Appendix A.6 Proof of proposition 7

Recall the definition of pure liquidity risk

$$IL^* \equiv (\mu_1 - 1)\frac{D^*}{w} = (\mu_1 - 1)c\frac{r - F}{w}.$$

Consequently, we have

$$\frac{\partial IL^*}{\partial r} = (\mu_1 - 1)c\frac{\partial}{\partial r}\left(\frac{r - F}{w}\right),$$

whose sign is the same as

$$\frac{\partial}{\partial r} \left(\frac{r-F}{w} \right).$$

Again total credit risk TCR^* is defined as the summation of pure insolvency risk and illiquidity risk:

$$TCR^* \equiv 1 - \frac{r - F - \mu_1 D^*}{w} = 1 - (1 - \mu_1 c) \frac{r - F}{w},$$

with the first order derivative

$$\frac{\partial TCR}{\partial r} = -(1-\mu_1 c)\frac{\partial}{\partial r} \left(\frac{r-F}{w}\right).$$

Since

$$\mu_1 c = \frac{1}{m_1/k + 2} < 1$$

the comparative statics of total credit risk is again determined by the sign of

$$-\frac{\partial}{\partial r}\left(\frac{r-F}{w}\right).$$

Recall that

$$\frac{\partial S R^*}{\partial r} = -(1-c)\frac{\partial}{\partial r} \left(\frac{r-F}{w}\right).$$

Therefore when competition intensifies, pure illiquidity risk always moves in the opposite direction as pure solvency risk. As the latter dominates, total credit risk changes in the same direction as that of pure insolvency.

Appendix A.7 Proof of proposition 8

The proof resembles that of proposition 7. For the exposure to contagion

$$\frac{\partial CTG^*}{\partial r} = (\mu_2 - \mu_1) \frac{\partial}{\partial r} \left(\frac{D^*}{w} \right) = (\mu_2 - \mu_1) c \frac{\partial}{\partial r} \left(\frac{r - F}{w} \right),$$

and for the risk of a systemic crisis

$$\begin{aligned} \frac{\partial S YS^*}{\partial r} &= 2Prob(\theta < \hat{\theta}_2) \frac{\partial}{\partial r} Prob(\theta < \hat{\theta}_2) \\ &= 2Prob(\theta < \hat{\theta}_2) \frac{\partial}{\partial r} \left(1 - \frac{r - F - \mu_2 D^*}{w}\right) \\ &= -2Prob(\theta < \hat{\theta}_2)(1 - \mu_2 c) \frac{\partial}{\partial r} \left(\frac{r - F}{w}\right), \end{aligned}$$

both again hinge on the sign of

$$\frac{\partial}{\partial r} \left(\frac{r-F}{w} \right).$$

When $r^2 > xF$ the exposure to contagion increases with competition and the systemic risk decreases with competition.

Appendix B The Laplacian property

In the model, the noisy signal received by representative creditor *i* has a structure

$$s_i = \theta + \epsilon_i$$

We assume ϵ_i follows a continuous distribution with c.d.f. G.

Denote the critical signal for creditor *i* to switch from "wait" to "run" by s^* . And upon observing s^* , the creditor *i* believes a *M* fraction of creditors observing signals higher than hers. We prove $M \sim U(0, 1)$.

Proof. For the continuous distribution *G*, the fraction of creditors who observes signal higher than s^* equals the probability that a creditor *j*'s signal $s_j > s^*$. Then, we have

$$M = Prob(s_j > s^* | s_i = s^*) = Prob(\theta + \epsilon_j > s^* | s_i = s^*)$$
$$= Prob(\epsilon_j > s^* - \theta | s_i = s^*)$$
$$= 1 - G(s^* - \theta)$$

The randomness of M is rooted in the fact that by observing $s_i = s^*$, creditor i is uncertain about the realization of θ . As the *perceived* value of θ is random, so is the *perceived* M. Now we derive the distribution function of M. For $\hat{M} \in [0, 1]$, we have

$$\begin{aligned} Prob(M < \hat{M}|s_i = s^*) &= Prob(1 - G(s^* - \theta) < \hat{M}|s_i = s^*) \\ &= Prob(\theta < s^* - G^{-1}(1 - \hat{M})|s_i = s^*) \\ &= Prob(s^* - \epsilon_j < s^* - G^{-1}(1 - \hat{M})|s_i = s^*) \\ &= Prob(\epsilon_j > G^{-1}(1 - \hat{M})|s_i = s^*) \\ &= 1 - G(G^{-1}(1 - \hat{M})) \\ &= \hat{M} \end{aligned}$$

Note that $M = 1 - G(s^* - \theta) \in [0, 1]$. Therefore for $\hat{M} < 0$, $Prob(M < \hat{M}) = 0$; and for $\hat{M} > 1$, $Prob(M < \hat{M}) = 1$. We prove M follows a uniform distribution on [0, 1]. \Box

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