

The Dynamics of Investment, Payout and Debt *

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Abstract

We present a dynamic agency model of investment, borrowing and payout decisions by a mature corporation operating in perfect financial markets. Managers implement an inter-temporal strategy that maximizes their lifetime utility of managerial rents. They under-invest because of risk aversion and smooth payout and rents. Changes in debt absorb shocks to operating income and changes in investment. Managers do not rebalance capital structure, so shocks to debt levels persist. We generate empirical predictions that differ from conventional agency models and from dynamic models based on taxes and financing frictions.

Keywords: payout, investment, financing policy, agency (JEL: G31,G32)

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1 Introduction

Consider a mature public corporation that generates positive free cash flow and pays out cash to outside investors. The corporation's financial decisions must add up. Sources and uses of cash must match in every period t :

$$Payout_t + CAPEX_t = NI_t + \Delta D_t \quad (1)$$

Payout equals dividends plus net share repurchases (repurchases net of any equity issues); *CAPEX* is capital investment, the change in capital stock; *NI* is net income, after payment of interest on debt from the previous period; *D* is net debt, and ΔD is additional borrowing. ($\Delta D_t < 0$ means debt is paid down or cash is accumulated.)

Once NI_t is realized, the corporation must make three decisions, $Payout_t$, $CAPEX_t$ and ΔD_t . But there are only two degrees of freedom. For example, a decision about payout and investment must also be a decision about issuing or retiring debt.

Modern financial economics has generated separate theories of payout, debt policy and investment. But there can be two independent theories at most. Therefore theory should address how a corporation makes the joint decision about investment, payout and borrowing. This paper presents a dynamic agency model of all three decisions.

Most existing models of the joint dynamics of investment, payout and borrowing do not focus on agency behavior. Instead they start with an objective of maximizing value in financial markets, and generate dynamics from taxes, costs of financial distress and financing frictions. We isolate agency-driven dynamics by assuming a Modigliani-Miller (MM) setting where debt and payout policy can be relevant for managers but do not affect the value of the firm to investors. Therefore all of the financial action in our model comes from agency behavior.

We introduce agency by assuming that risk-averse managers maximize their lifetime utility from the rents that they extract from the firm. The managers, who act as a coalition with no fixed lifespan, are constrained by a system of corporate governance, which limits the fraction of firm value that the managers can extract. We model gover-

nance in a simple way by following Myers (2000), who assumed that dispersed outside shareholders can organize to intervene if the managers do not deliver an adequate rate of return.¹ The shareholders face a cost of intervention, which impedes their property rights and creates the space for the managers to extract rents. This governance constraint is binding in equilibrium. Shareholders do not intervene, because managers deliver enough payout and value to shareholders to keep them at bay.

Of course managers' rents also enter the sources-uses constraint:

$$Payout_t + Rents_t + CAPEX_t = NI_t + \Delta D_t \quad (2)$$

But governance constraint requires rents and payout to move in lockstep (confirming Lambrecht and Myers (2012)). Thus the managers' decision about rents is also a decision about payout. Given net income NI_t , there are still only two degrees of freedom.

We consider a mature, profitable firm, where default risk is second-order. Therefore we simplify by not modeling default. The firm can borrow or lend at the risk-free interest rate. We also assume no adjustment costs for capital stock, so that managers can implement (their view of) optimal $CAPEX$ in every period.

Despite its simplifying assumptions, the model generates interesting dynamics and novel results. The following results hold for any risk-averse utility function for managers and for any stochastic process for profitability.

1. The managers under-invest: their personal risk aversion and inability to hedge retards investment and leaves capital stock less than the value-maximizing level for shareholders. Most prior agency theory assumes that managers want to over-invest, for example because of private benefits or personal gains from empire-building. We also show that the managers' personal rate of time preference affect rents but *not* investment. The managers may discount future rents at a higher rate than shareholders discount future payouts, but the managers' decision on $CAPEX$ depends on the market

¹ Lambrecht and Myers (2007, 2008, 2012) and Jin and Myers (2006) model governance in the same fashion.

interest rate.

2. Managers smooth the flow of rents and therefore also payout. Also (with most utility functions) they hold back current rents in order to set aside precautionary savings. The precautionary savings show up as reduced corporate borrowing. Precautionary saving may continue even after debt is entirely paid off. In this case the firm becomes a net lender, perhaps building up a "cash mountain" like Apple's and Microsoft's.

3. Smoothing of rents and payout means that the change in debt ΔD_t soaks up most of the volatility of operating income and *CAPEX*. Changes in debt are persistent. Once the managers have chosen the optimal levels of rents and payout for the current period, they never follow up with a discrete substitution of equity for debt or vice versa.

Result (3) contrasts with the tradeoff theory of capital structure. Consider the lucky managers of a firm that has suffered a string of positive profit shocks and paid off all of its debt. The tradeoff theory would predict a debt issue coupled with increased dividends or repurchases in order to move the firm up towards a positive target debt ratio. Our model says that the managers will *not* change *CAPEX*, *Payout* or *Rents* in order to "rebalance" debt.

Of course our model assumes an MM world for outside investors and excludes the risk of default and the financial frictions that drive traditional models of capital structure. Nevertheless, we suggest that some financing patterns that the traditional models struggle to explain may actually be driven by agency and managers' self-interest. For example, Lemmon, Roberts and Zender (2008, p. 1575) report that debt ratios are "surprisingly stable. High (low) levered firms tend to remain as such for over two decades." That is exactly what our model predicts.

Our model also generates pecking-order financing, as in Myers and Majluf (1984), but not because managers know more than outside investors. The pecking order comes about because the change in debt is a residual once *CAPEX*, *Payout* and *Rents* are

determined by managers. Our results also match up with the survey of financial executives by Brav, Graham, Harvey, and Michaely (2005). The executives say that their first priority is maintaining *CAPEX* and *Payout*; short-term debt policy is a lower priority.

4. We explain how debt disciplines managers. The discipline does not operate in the short run by diverting immediate cash flow to debt service. The managers are free to raise cash by additional borrowing at any time. The level of debt is restrained in the long run, however, because the managers must respect an intertemporal budget constraint. A higher debt level reduces the present value of managers' claim on the firm and the the present value of the future rents that they can extract. Therefore discipline comes from the managers' optimal dynamic responses to current and future debt levels. It is self-discipline, not imposed discipline, as expositions of free-cash-flow theories typically imply.

These general results are derived in Section 2. In Section 3 we introduce additional assumptions, including a negative exponential utility function for managers, and derive closed-form results. Additional results include:

1. Managers separate investment from financing and payout decisions, just as the finance textbooks recommend. There is no feedback from decisions about payout, rents or changes in borrowing to the *CAPEX* decision.

2. The stronger the persistence of economic shocks, the larger their effect on *CAPEX*. This creates a positive correlation between firm size and (lagged) profitability. Stronger persistence generates more volatility in *CAPEX* and income, and increases precautionary saving.

3. Rents equal the managers' share of the firm's permanent income, minus their share of precautionary saving. Precautionary saving is time-varying and increases when random shocks to profitability are positive. Thus the managers hold back on rents and payout in good times, saving for a rainy day when shocks to profits are negative.

4. Managers smooth rents and payout, not just by changes in precautionary sav-

ings, but because transitory profit shocks affect *Payout* and *Rents* hardly at all and *CAPEX* not at all. Almost all transitory increases in profits are used to pay down debt. Hennessy and Whited (2007, p. 1737) note that structural models of investment, payout and borrowing “[overshoot] the variance of corporate cash distributions. ... we still lack a theoretical rationale for dividend smoothing resulting from optimizing behavior.” We say that payout smoothing results from the *managers’* optimizing behavior in a dynamic agency setting.

5. Debt policy is generally counter-cyclical. The firm generally pays down debt when profit shocks are favorable and borrows more when profit shocks are negative. This result also goes against the tradeoff theory, which predicts more borrowing when profits and value go up.

6. Differences in net debt levels between firms persist over time. The initial debt level affects the future debt level, even after long periods of time. This is consistent with empirical evidence in Lemmon, Roberts, and Zender (2008). This result is likewise at odds with the tradeoff theory, in which capital structure is “rebalanced” whenever leverage deviates too much from the target.

7. Firms with lower managerial risk aversion or higher investor protection invest more and pay out more.

The analysis in Sections 2 and 3 assumes a MM world with no taxes. Corporate income taxes are prominent in standard theories of capital structure. These theories struggle to explain why successful corporations often operate at conservative debt ratios, leaving interest tax shields unexploited. Therefore we add a corporate tax to our general model in Section 4. The resulting changes are modest, mostly conversions of interest rates and other variables to after-tax values. All of our qualitative results stand, including the “no rebalancing” result, which states that debt levels are persistent and that managers will not issue debt solely to retire equity and move the firm to a higher debt ratio. Thus our agency model can explain why many corporations do not borrow more aggressively, despite the tax savings that seem to entice them to do so.

The concluding Section 5 discusses the broader empirical implications of our results.

1.1 Prior research in corporate finance

The corporate finance literature on investment, payout and debt versus equity is of course enormous. This review focuses on dynamic models that consider interactions of these decisions.

Strebulaev and Whited (2012) provide a comprehensive review of dynamic models. One class of models takes investment as exogenous and leaves payout as an un-modeled residual. Examples of dynamic refinancing models include Fischer, Heinkel, and Zechner (1989), Goldstein, Ju, and Leland (2001), Ju, Parrino, Poteshman, and Weisbach (2005) and Strebulaev (2007). Others focus on the interaction between capital structure and investment. Examples include Mello and Parsons (1992), Mauer and Triantis (1994), Parrino and Weisbach (1999) and Childs, Mauer, and Ott (2005).²

Exceptions that focus on all three decisions about investment, payout and borrowing include Hennessy and Whited (2005, 2007) and DeAngelo, DeAngelo, and Whited (2011). They assume that managers act in shareholders' interest to maximize market value. They investigate how taxes and transaction and adjustment costs affect dynamics. Bolton, Chen, and Wang (2011) track cash balances and hedging as well as investment, payout and borrowing.

These models leave payout as the residual.³ In Bolton, Chen and Wang (2011), for example, firms pay out cash only when cash holdings are large, so that the shadow price of cash held for future investment is small and the costs of holding cash become

²There are probably hundreds of papers that study how investment and financing decisions are affected by product market competition, financing constraints, asymmetric information, multiple creditor classes and other frictions or imperfections. Strebulaev and Whited (2012) review some of these papers.

³DeAngelo, DeAngelo and Whited (2011) assume dividends are value-irrelevant in an MM sense, although the firm may restrict dividends to pay down debt or increase cash holdings as a reserve for future investment. Hennessy and Whited (2005) introduce a tax paid by investors on dividends.

burdensome.⁴ In our model, payout cannot be the residual, because managers maximize the utility of their rents, and payout follows rents.

Few dynamic models introduce meaningful agency issues.⁵ Exceptions include DeMarzo, Fishman, He, and Wang (2012), who consider investment and payout as results of a contract between shareholders as principals and a manager as agent. The manager is risk-neutral, but must be induced not to extract private benefits or some slice of cash flow. The shareholders can implement the optimal contract and terminate the manager if necessary under that contract. The firm builds up a cash reserve to finance investment and pays out dividends when the cash reserve reaches a maximum level. Capital structure decisions are not modeled. We do not solve for an optimal contract, but simply assume that the (risk-averse) managers of a public corporation act to maximize their personal utility, subject to a governance constraint. We analyze the managers' decisions about debt and payout as well as investment.

Morellec, Nikolov, and Schürhoff (2012) assume that managers' and shareholders' interests are aligned, except for the managers' ability to capture a fraction of cash flow to equity as private benefits. (They interpret the fraction as resulting from a governance constraint, as we do in this paper.) They explore the dynamics of capital structure, including default and rebalancing strategies, but hold investment fixed. They also assume that managers are risk-neutral (or no more risk averse than investors). Our model ignores default, but investment is endogenous. Our managers are risk-averse.

This paper builds on Lambrecht and Myers (2012), who focused on payout policy. They showed that payout (dividends plus repurchases, not cash dividends alone) is smoothed because rents are smoothed. The managers' optimal rents follow the Lintner target-adjustment model, and the governance constraint forces payout to follow rents

⁴In DeCamps et al. (2011), the costs of holding cash are interpreted as a free cash flow problem: managers with access to cash take some of it as private benefits. The objective assumed in this paper is still market-value maximization, however.

⁵Mello and Parsons (1992) and Childs, Mauer, and Ott (2005) and some other papers introduce agency issues, but from stockholder-bondholder conflicts rather than conflicts between managers and outside investors.

in lockstep. Changes in debt soak up transitory income shocks and accommodate the gradual adjustment of rents and payout to persistent changes in permanent income. But Lambrecht and Myers (2012) took the capital stock as fixed once and for all. Here we make *CAPEX* endogenous. We get new results. Payout and rents remain smooth, even if *CAPEX* is very lumpy or volatile. This is partially achieved through time-varying, pro-cyclical precautionary savings (precautionary savings are constant when the capital stock is fixed). Debt is the marginal source of funding for *CAPEX*, and changes in debt are negatively correlated with income. Finally, while Lambrecht and Myers (2012) formulate all results for the negative exponential utility, here we generalize the main results to any utility function and stochastic process.

1.2 Other related research: inter-temporal consumption and portfolio selection models

The managers in our agency model face a personal optimization problem similar to the problem faced by an individual investor who allocates wealth between consumption and risky investment. Our managers take rents as consumption. They co-invest with outside shareholders in a risky asset, the firm's capital stock. They can also use corporate borrowing or lending to finance investment and to smooth or shift rents over time.⁶

This paper uses insights and methods from theories of intertemporal consumption and portfolio selection decisions, as in Caballero (1990) and Campbell and Viceira (1999), for example. The latter paper presents an approximate analytical solution for consumption and investment when expected returns on the risky investment are

⁶We assume debt service is senior to both rents and payouts to investors. Managers can capture a fraction of the cash inflow from additional corporate borrowing, but the same fraction of debt service comes out of managers' future rents. (The fraction depends on how tight the governance constraint is.) Thus a fraction of corporate debt amounts to borrowing on behalf of managers. Also managers must co-invest when the firm's capital stock expands, either by cutting back rents or by allocating their share of additional borrowing to finance their share of investment. See Myers (2000) and Lambrecht and Myers (2012) for a fuller discussions of corporate borrowing and co-investment.

time-varying, as they are in our closed-form solutions.

Several papers, including Bodie, Merton, and Samuelson (1992), Heaton and Lucas (1997, 2000) and Viceira (2001) consider consumption-investment decisions where investors have financial as well as non-tradable human wealth. The financial wealth provides diversification and leads to less saving and more investment in the risky asset. But the impact of financial wealth on saving and investment decreases when the correlation between financial and human wealth increases. We do not model managers' personal financial wealth, but note that for top managers it is mostly held as options or restricted stock, which are highly correlated with the performance of the firm.

Much of the literature on consumption and investment assumes that income risk is normally distributed and that consumers have constant absolute risk aversion (CARA) utility. See Campbell and Viceira (2002) for a review. These two assumptions allow closed-form solutions. We follow these assumptions in our closed-form solutions, although our model allows for persistence in investment returns.

CARA utility is not the most “realistic” assumption for individuals but may be a decent approximation for a coalition of managers in charge of a public corporation. For example, CARA allows negative consumption, which makes no sense for individuals, but could make sense for firms. Negative payouts can be interpreted as equity issues, and negative rents can be interpreted as “sweat equity” contributed by managers who work for less than their opportunity wages.

We do not introduce CARA utility until Section 3, however. The results in the next section hold for any utility function with risk aversion.

2 The Model

We now set out the model in its most general form, without limiting assumptions about utility functions or the stochastic evolution of profits. We consider a mature, blue-chip public corporation – our model would not apply to a young growth firm or to a firm

coping with financial distress. We ignore default options for simplicity.⁷ We assume perfect, frictionless financial markets and risk-neutral investors. Therefore all results come from agency behavior.

Assume a coalition of managers that makes investment, financing and payout decisions and extracts rents from the firm. The managers are risk averse, with a concave utility function $u(r_t)$ for the rents they extract in period t . In practice rents can come in many forms, but here we assume for simplicity that rents are cash payments over and above the managers' opportunity wages.⁸

At each time t the infinitely-lived managers choose investment, debt, payout and rent policies (K_t, D_t, d_t, r_t) that maximize expected lifetime utility:

$$\max E_t \left[\sum_{j=0}^{\infty} \omega^j u(r_{t+j}) \right] \quad (3)$$

where ω is managers' subjective discount factor and $\frac{1}{\omega}$ measures "impatience." The market discount factor is $\beta \equiv \frac{1}{1+\rho}$ where ρ is the risk-free rate of return. We assume $\omega \leq \beta$, so that managers can be more impatient than investors. For example, managers will be more impatient if they face a probability of termination due to dismissal, illness or death in each future period. In that case $\omega = \beta \zeta$, where ζ is managers' constant survival probability.⁹

Managers set the capital stock one period in advance. At time t , managers fix K_t , the level of capital that generates income accruing at time $t + 1$. They also choose

⁷Default risk should be second-order for mature corporations that make regular payouts and have ample debt capacity. Modeling a default put would add a heavy layer of complication. See Lambrecht and Myers (2008), who analyze the effect of default risk on rents, payout, debt and investment.

⁸Rents (and payout) are not constrained to be positive. Negative payouts and rents can be interpreted as equity issues and "sweat" equity, respectively. We discuss our model's implications for equity issues in the concluding section.

⁹Our model's infinite horizon is important, because it eliminates "end games." Managers' financial decisions could change drastically if they knew their tenure would end at a definite future date.

rents and payout, r_t and d_t . The following constraints must be satisfied at all times:

$$S_t \equiv d_t + \beta E_t[S_{t+1}] \geq \alpha [V_t - (1 + \rho)D_{t-1}] \quad (4)$$

$$D_t = D_{t-1}(1 + \rho) + d_t + r_t - \Omega(K_{t-1})\pi_t + K_t - (1 - \delta)K_{t-1} \quad (5)$$

$$\lim_{j \rightarrow \infty} \frac{D_{t+j}}{(1 + \rho)^j} = 0 \quad (6)$$

$$\text{where } V_t \equiv \sum_{j=0}^{\infty} \beta^j E_t[\Omega(K_{t+j-1})\pi_{t+j} - K_{t+j} + (1 - \delta)K_{t+j-1}] \equiv \sum_{j=0}^{\infty} \beta^j E_t[CF_{t+j}]$$

$\Omega(K_{t-1})\pi_t$ is the operating profit at time t , with π_t acting as a stochastic profit margin. The stock of capital depreciates at a rate δ . Therefore, the investment in capital stock at time t ($CAPEX_t$) equals $K_t - (1 - \delta)K_{t-1}$.

$\Omega(K_t)$ is a (weakly) concave production function with $\Omega(0) = 0$, $\Omega'(K) > 0$ and $\Omega''(K) \leq 0$. π_t is the operating profit per unit of output, which depends on the realization of exogenous demand shocks. Payout and rents are paid at the end of each period, after operating profit is realized and interest is paid on start-of-period debt.

D_t is *net* debt. If $D_t > 0$, the firm is a net borrower. If $D_t < 0$, the firm holds a surplus in liquid assets and is a net lender. We ignore default risk and therefore assume that the firm can borrow and lend at the risk-free rate ρ .

Equation (4) is a *governance constraint* (or capital-market constraint) that ensures outside equityholders get a share of the income generated by the firm. How big the share is depends on the degree of investor protection as summarized by the parameter α . Outsiders can force the firm to pay out by taking collective action, for example by organizing or endorsing a hostile takeover. However, in equilibrium intervention never occurs. At each point in time investor's net payoff from intervention is $\alpha(V_t - (1 + \rho)D_{t-1})$, with $0 < \alpha < 1$.¹⁰ Condition (4) states that managers will at all times set the payout d_t high enough so that the investors are willing to postpone intervention for one more period.¹¹

¹⁰For $\alpha = 0$ shareholders have no stake in the firm and the capital market constraint disappears. For $\alpha = 1$ managers can no longer capture rents and their objective function is no longer defined. Therefore $\alpha \in (0, 1)$.

¹¹Lambrecht and Myers (2007, 2008) consider a stronger constraint where insiders not only have

The governance constraint captures parsimoniously a repeated game between managers and outside shareholders. At each time t insiders propose a payout and rent level (d_t, r_t) . If shareholders reject this offer, then they get the payoff from intervention, $\alpha(V_t - (1 + \rho)D_{t-1})$, and the game ends (insiders get nothing and are out). If outsiders accept, then managers and shareholders get r_t and d_t , and insiders stay in charge for one more period. The game is repeated at $t + 1$. In equilibrium insiders always remain in charge, because they propose d_t and r_t that leave shareholders content for the next period.

Eq. (5) is the firm's *budget constraint*. The operating profit $\Omega(K_{t-1})\pi_t$ is used for interest (ρD_{t-1}), for *CAPEX* ($K_t - (1 - \delta)K_{t-1}$), for net cash paid out to shareholders (d_t) and for rents (r_t). Any surplus or deficit leads to a reduction or increase in debt. We will show that debt is a balancing variable, which follows from the managers' rent and payout policies (r_t, d_t) and investment policy (K_t) . The accounting equality between sources and uses of cash pins down debt once investment, rents and payout have been chosen.

Eq. (6) is a *debt constraint* that prevents the managers from running a Ponzi scheme in which they borrow to achieve an immediate increase in rents and then borrow forever after to pay the interest on the debt. The constraint prevents debt from growing faster than the interest rate ρ , so that claim values are bounded. Notice that the debt constraint does not restrict corporate borrowing in any period. The managers are free to expand or rebalance corporate borrowing if they want to. But additional borrowing does squeeze future rents, because interest payments are senior to managers' rents.

All three constraints are essential. Absent the governance constraint, managers could capture all profits and the company could not support outside financing. The

power to extract their fair share of income π_t , but can also enforce upon collective action the investment policy that generates the first-best value V^o . This stronger constraint hardwires a first-best managerial policy as $\alpha \rightarrow 1$. Even though we adopt a weaker governance constraint, investment still approaches first-best as $\alpha \rightarrow 1$ because the distortion caused by risk aversion becomes less important when insiders have a smaller stake. However, as insiders' stake decreases other distortions may start to matter (see e.g. Acharya and Lambrecht (2011)).

debt constraint prevents runaway borrowing. The budget constraint assures that sources and uses of cash add up.

The budget constraint has to be satisfied at all future times. Summing the budget conditions for any time interval T ,

$$\sum_{j=0}^T \beta^j (d_{t+j} + r_{t+j}) = \beta^T D_{t+T} + \sum_{j=0}^T \beta^j CF_{t+j} - (1 + \rho)D_{t-1}. \quad (7)$$

Taking the limit as $T \rightarrow \infty$ and enforcing the no-Ponzi condition Eq. (6) gives the *inter-temporal budget constraint* (IBC):

$$\sum_{j=0}^{\infty} \beta^j (d_{t+j} + r_{t+j}) = \sum_{j=0}^{\infty} \beta^j CF_{t+j} - (1 + \rho)D_{t-1}. \quad (8)$$

The IBC states that the sum of shareholders' and managers' claims must add up to the present value of all future cash flows that the firm will generate, net of any outstanding debt. Therefore, if managers decide to increase rents and payout today, they will have to make up for the rents and payout in the future. Their ability to capture rents depends on the firm's future cash flow, as shown on the right side of Eq. (8). We will see later that permanent income, defined as the annuity value of the firm's expected future cash flow, is an important determinant of payout.

Next we consider how managers set investment and payout. Their decision problem is sequential. At time t they decide on the optimal level for K_t , r_t and d_t and D_t . The payout d_t and rents r_t set by managers are such that shareholders are indifferent between taking collective action and keeping managers in place for another period:

$$\begin{aligned} & \alpha \left[CF_t - (1 + \rho)D_{t-1} + \sum_{j=1}^{\infty} \beta^j E_t [CF_{t+j}] \right] \\ = & d_t + \alpha\beta \left[E_t [CF_{t+1}] - (1 + \rho)D_t + \sum_{j=1}^{\infty} \beta^j E_t [CF_{t+1+j}] \right] \end{aligned} \quad (9)$$

Substituting the budget constraint Eq. (5) at time t into D_t and solving Eq. (9) for d_t gives:

$$d_t = \left(\frac{\alpha}{1 - \alpha} \right) r_t \quad (10)$$

Therefore, the governance constraint Eq. (4) forces managerial rents to move in lock-step with payout to shareholders (see also Lambrecht and Myers (2012)). This result is

valid regardless of managers' utility function and the stochastic process π_t . It reduces managers' decision problem by one control variable (d_t). Substituting (10) into (8) gives the IBC as a function of rents only:

$$\sum_{j=0}^{\infty} \beta^j r_{t+j} = (1 - \alpha) \left[\sum_{j=0}^{\infty} \beta^j C F_{t+j} - (1 + \rho) D_{t-1} \right]. \quad (11)$$

The remaining variables are r_t , K_t and D_t . Most theories of capital structure take debt D_t as the control, with payout as the (often implicit) residual. But rents and investment are the natural control variables for managers of a mature corporation. Managers care about rents because they want to maximize the present value of their (life-time utility of) rents. Managers also care about investment, because it generates income and therefore rents. The level or change in debt do not enter managers' utility function. Also in an MM world, with no frictions such as transaction costs, taxes and bankruptcy costs, changes in debt do not matter to shareholders. Therefore we derive optimality conditions for rents and investment and calculate debt policy from the budget constraint Eq. (5).

The first-order conditions for r_t and K_t are (see Appendix for a derivation):

$$u'(r_t) = \omega(1 + \rho) E_t [u'(r_{t+1})] \quad (12)$$

$$u'(r_t) = \omega E_t [u'(r_{t+1}) (1 + \Omega'(K_t) \pi_{t+1} - \delta)] \quad (13)$$

Solving these two Euler equations subject to the IBC (11) gives the optimal investment, rent and payout policies. Under the budget constraint Eq. (5) the optimal investment and payout policies also imply the optimal debt policy.

The first Euler condition determines the evolution of rents over time. It states that the marginal utility cost of one unit less of rents at time t must, under the optimal policy, equal the expected marginal utility gain from investing that unit at the risk-free rate of return, ρ , and receiving this unit plus the interest in period $t + 1$. The second Euler condition determines investment. It states that the marginal utility cost of one unit less of rents at time t must, under the optimal policy, equal the expected marginal utility gain from investing that unit in the risky stock of capital and receiving this unit plus its risky return $\Omega'(K_t) \pi_{t+1} - \delta$ in period $t + 1$.

There can be only two Euler equations. What if managers optimize with respect to the investment (K_t) and debt (D_t) policies instead of investment and rents? Rest assured that they would obtain the same solution. The first order condition for D_t is:

$$(1 - \alpha) u'(r_t) = (1 - \alpha)\omega(1 + \rho)E_t [u'(r_{t+1})] \quad (14)$$

which is the Euler equation (12) for rents, with both sides multiplied by $1 - \alpha$. The governance constraint Eq. (9) requires that shareholders get the fraction α of each dollar of additional borrowing and managers get the fraction $1 - \alpha$. A dollar more paid for debt service reduces rents by $1 - \alpha$ dollars and payout by α dollars. Thus the marginal utility of a dollar of debt or debt service equals $(1 - \alpha)$ times the marginal utility of rents. A marginal change in D_t , all else equal, affects managers' objective function only through its impact on rents.

Thus our results do not depend on our choice of r_t and K_t as control variables. We choose these control variables because they are directly relevant to what the managers are maximizing. The borrowing decision implements optimal rents and investment, but is not an end in itself for the managers. Borrowing is a secondary or residual decision. For example, we will see that changes in borrowing must soak up most of the volatility of operating income and investment.

Consider now the Euler equation for rents in more detail. Assuming that managers are fully rational and calculate their expectations on the basis of all information available, Eq. (12) can be written as:¹²

$$u'(r_{t+1}) = \frac{\beta}{\omega} u'(r_t) + \epsilon_{t+1} \quad \text{with} \quad E_t(\epsilon_{t+1}) = 0 \quad (15)$$

Thus Eq. (12) describes the expected *changes* in rents (and payout) from one period to the next. Since $E_t(\epsilon_{t+1}) = 0$ and $E_t[u'(r_{t+1})] = \beta u'(r_t)/\omega$, once r_t is known, no other information (such as D_t and π_t) is relevant to determine the best estimate for *next period's* expected rents and payout. Therefore Eq. (12) cannot be interpreted as a rent policy function that links rents each period to its determinants (such as income and debt). In order to obtain the *level* of rents, one must combine the Euler Eq. (12)

¹²There are no restrictions on the distribution of ϵ_{t+1} . It does not need to be normal.

with the IBC Eq. (8), which specifies the present value of the total amount of rents that can be paid out now and in the future.

The IBC Eq. (8) depends on D_{t-1} , the debt carried over from the previous period. Increasing the debt level D_{t-1} tightens the constraint because it reduces the present value of future rents and payout. This reveals how higher debt disciplines managers: not by diverting immediate cash flow to debt service, but by constraining the *present value* of current and future rents plus payouts along their optimal paths. It is true that higher debt carried over from the previous period reduces rents and payout chosen by managers in that period.¹³ This discipline is self-imposed, however. Our model contains no frictions or covenants that restrict additional borrowing. The managers can raise more cash at any time it is in their interest to do so.

To get some economic insights regarding the intertemporal behavior of rents (and therefore payout), we locally approximate the Euler Eq. (12) for rents using a second-order Taylor expansion around r_t , which gives:

$$E_t[r_{t+1} - r_t] = -\frac{u'(r_t)}{u''(r_t)} \left(1 - \frac{\beta}{\omega}\right) - \frac{1}{2} \frac{u'''(r_t)}{u''(r_t)} E_t[(r_{t+1} - r_t)^2] \quad (16)$$

$$\equiv \frac{(1 - \frac{\beta}{\omega})}{ARA(r_t)} + \frac{1}{2} AP(r_t) E_t[(r_{t+1} - r_t)^2] \quad (17)$$

where $ARA(r_t) \equiv -u''(r_t)/u'(r_t)$ and $AP(r_t) \equiv -u'''(r_t)/u''(r_t)$ denote the coefficient of absolute risk aversion and absolute prudence.

Focus first on the case where $u'''(r_t) = 0$ (e.g. quadratic utility). Eq. (17) tells us that, all else equal, the higher insiders' degree of risk aversion, the smaller (in absolute value) the expected *changes* in payout and rents. Higher risk aversion therefore leads to more rent (and payout) smoothing. Furthermore, payout and rents grow (decrease) over time if the interest rate is greater (smaller) than managers' rate of time-preference, i.e. $\beta < (>)\omega$. (Dis)savings can therefore be motivated by managerial (im)patience.

If marginal utility is linear ($u'''(r_t) = 0$) then the expected value of the marginal

¹³Extremely high debt levels could push rents down to zero or negative levels. Of course our model would lose its grip if managers are required to contribute large amounts of sweat equity with little or no hope for recovery. Lambrecht and Myers (2008) model managers' default put.

utility of rents coincides with the marginal utility of expected rents. An increase in uncertainty with respect to rents (but with unchanged expected value) does not cause any change in managers' rent policy, because managers are only interested in the certainty equivalent of future rents. However, if marginal utility is convex, with $u'''(r_t) > 0$ (e.g. power or negative exponential utility) then managers display prudent behavior and react to an increase in uncertainty by saving more. Such saving is called precautionary (see Leland (1968) and Caballero (1990)), since it depends on the uncertainty of future rents.

The second term in the right side of Eq. (17) reveals that, all else equal, with convex marginal utility greater rent uncertainty (increasing $E_t[(r_{t+1} - r_t)^2]$) induces a larger future increase in rents, and therefore requires more immediate precautionary savings, given that insiders face a budget constraint. The managers delay paying out until uncertainty about the future is resolved. Therefore managers facing more uncertainty experience higher rent and payout growth on average.

Consider next the investment Euler Eq. (13). Using the rent Euler Eq. (12), we can rewrite Eq. 13 as follows:

$$E_t [(\Omega'(K_t)\pi_{t+1} - \rho - \delta) u'(r_{t+1})] = 0 \quad (18)$$

Notice that the managers' subjective discount factor ω does not appear, because investment can be financed by borrowing at the market interest rate. The managers set investment as if their subjective discount factor matches the market discount factor β .

For simplicity of exposition let us temporarily focus on a two-date scenario (t and $t + 1$) where the firm stops operating at $t + 1$ ($K_{t+1} = 0$) and all net debt is paid off ($D_{t+1} = 0$). The argument can be extended for any number of dates by working backwards.¹⁴ One can then prove the following Lemma.

¹⁴More generally, for any arbitrary future horizon T , insiders determine the optimal rent and investment policy at t , r_t and K_t , by working backwards from T . In the last period, policies r_{T-1} and K_{T-1} can be solved as above as a function of D_{T-2} . Using this solution r_{T-2} and K_{T-2} can be solved as a function of D_{T-3} . Working all the way back, r_t and K_t can be solved as a function of D_{t-1} , which is given and known at t . Therefore, the projected optimal path for rents and payout only depends on

Lemma 1 *Using a second-order Taylor expansion around \bar{r}_{t+1} , the investment Euler equation can locally be approximated as:*

$$\Omega'(K_t)\bar{\pi}_{t+1} - \rho - \delta = \frac{\Omega'(K_t)(1 - \alpha)\Omega(K_t) \left[\sigma^2 - \frac{1}{2}(1 - \alpha)\Omega(K_t)AP(\bar{r}_{t+1})E_t(\eta_{t+1}^3) \right]}{\frac{1}{AR(\bar{r}_{t+1})} + \frac{1}{2}(1 - \alpha)^2\Omega^2(K_t)AP(\bar{r}_{t+1})\sigma^2} \quad (19)$$

where $\eta_{t+1} \equiv \pi_{t+1} - E_t[\pi_{t+1}]$ with $E_t[\eta_{t+1}] = 0$, and $\sigma^2 = E_t(\eta_{t+1}^2)$ and $\bar{\pi}_{t+1} \equiv E_t[\pi_{t+1}]$ and $\bar{r}_{t+1} \equiv E_t[r_{t+1}]$.

Eq. (19) reveals that the investment policy interacts with the rent and payout policy through the coefficients of absolute risk aversion and absolute prudence, both of which depend on next period's expected rents \bar{r}_{t+1} . Recall that \bar{r}_{t+1} depends on K_t , r_t and D_{t-1} , but not on D_t . An interesting case is when both AP and ARA are strictly positive constants (e.g. with negative exponential utility), because in that case the investment policy does not depend on the payout policy. The inverse statement is not true: the optimal rent level r_t still depends on the investment policy, because the firm's intertemporal budget constraint, which determines how much rents can be paid out now and in future, is determined by the firm's investment policy. For example, a firm that does not invest will not generate any operating income, and this obviously affects its capability to make payouts to shareholders. How rents explicitly depend on investment will be illustrated and proven more formally in next section.

Consider now the investment Euler equation (19) in more detail. As insiders move towards risk neutrality (i.e. $u''(r) \rightarrow 0$ and $u'''(r) \rightarrow 0$), the investment Euler equation simplifies to:

$$\Omega'(K_t)\bar{\pi}_{t+1} - \rho - \delta = 0 \quad (20)$$

Suppose that managers are risk-averse but do not care about skewness (i.e. $u'''(r) = 0$) as, for example, is the case with quadratic utility. It then follows that the right side of Eq. (19) is strictly positive, and therefore risk averse managers under-invest from the viewpoint of risk-neutral shareholders. A higher coefficient of risk aversion, all

 D_{t-1} . Of course, at $t + 1$, when π_{t+1} becomes known, managers re-optimize to ensure that the rent and investment policies are dynamically optimal.

else equal, reduces the optimal investment level.¹⁵ Risk-averse managers want to be compensated for taking on risk, and therefore require a marginal return on investment that strictly exceeds the cost of capital and depreciation.

Assume next that risk-averse managers (dis)like positive (negative) skewness ($u'''(r) > 0$ and therefore $AP(r) > 0$), a feature shared, for example, by power utility and negative exponential utility. Eq. (19) implies that positive (negative) skewness in the distribution of the innovations η_{t+1} for profitability increases (decreases) investment. When marginal utility is convex, unfavorable outcomes lead to a greater loss of utility than the gain in utility from favorable outcomes of the same magnitude. With negative skewness, larger drops in profitability become more likely, which causes insiders to be more cautious in their investment policy.¹⁶

Our results so far include the following. First, rational managers will underinvest because of risk aversion, contrary to the common assumption that managers will overinvest free cash flow. In our model, overinvestment would not only saddle managers with too much risk, but also reduce the value of their stake in the firm and lower the expected utility of lifetime rents. Negative (positive) skewness in the return on investment exacerbates (mitigates) the under-investment problem if the coefficient of absolute prudence is positive.

Second, the important decisions for managers are capital investment and rents (and payout proportional to rents). The change in debt follows from the budget constraint. This pecking order of corporate policies matches what managers say in practice. Brav, Graham, Harvey, and Michaely (2005) report that financial executives consider payout

¹⁵We assume that markets are incomplete and that managers cannot hedge their firm's risks. Incompleteness makes sense because of the moral hazards that would be created by securities or contracts with payoffs that depend on the managers' efforts and personal incentives.

¹⁶Eq. (19) suggests that insiders might over-invest for sufficiently large positive skewness. This would imply that the positive effect on investment of the third order moment ($E_t(\eta_t^3)$) dominates the negative effect of the second moment (σ^2). This argument, however, fails to recognize that Eq. (19) is based on a local approximation that assumes shocks η_t to be small. If, however, η_t^3 is not small relative to η_t^2 then higher order terms that may alter the relative effect of skewness on investment should be included in the Taylor approximation.

a priority on par with investment decisions; debt policy is a lower priority.

The level of debt D_{t-1} carried over from the previous period does affect rents r_t , because higher levels of tighten the IBC. Managers respond by the managers cutting back current and future rents and payout. Thus debt disciplines managers *over time*, not because of lack of access to immediate cash, but because of managers' optimal dynamic response to the IBC.

Third, the change in debt in the current period does not feed back into managers' investment decision, which sets the level of the capital stock K_t .

Fourth, managers never rebalance the firm's capital structure.¹⁷ Once they have chosen the optimal levels of rents and investment for the current period, they will never follow up with a discrete substitution of equity for debt or vice versa. The managers' decision problem is globally concave, so their local optimum is also their global optimum.

Suppose, for example, that the managers inherit a high debt level, which squeezes their optimal rents and payout. They will not issue equity or cut back rents to move to a more comfortable debt level at the start of the next period. Notice that outside investors would pay only the fraction α of any discrete debt paydown. The managers would have to come up with the complementary fraction $1 - \alpha$ by cutting current rents. The governance constraint, which rational managers will keep binding, prevents the managers from paying down debt entirely with shareholders' money.

Fifth, managers' risk aversion can lead them to smooth rents and to defer rents as precautionary saving against future profitability shocks. The smoothing and savings behavior will be derived in closed form in the next section.

¹⁷Of course our model assumes that the firm remains mature and blue-chip, with full access to financial markets. This fourth result would not hold for a firm threatened by financial distress or a firm with valuable growth opportunities threatened by debt overhang problems. See Lambrecht and Myers (2008).

3 A closed form solution

To solve managers' optimization problem explicitly, we need to define $u(r_t)$, $\Omega(K_t)$ and π_t . We choose a set of functional specifications that allow the dynamic interactions between *CAPEX*, *Payout*, *Rents* and the change in debt ΔD_t to be examined in a tractable and transparent fashion.

Assume that managers have exponential utility $u(x) = 1 - \frac{1}{\theta}e^{-\theta x}$. This utility function has been used extensively in the literature. Next, assume that production exhibits constant returns to scale, i.e. $\Omega(K_t) = K_t$. (Managers' risk aversion will limit investment.) Finally, the profit margin is given by the following moving-average (MA(1)) process:

$$\pi_t = \mu + \phi_0 \eta_t + \phi_1 \eta_{t-1} \quad (21)$$

where μ , ϕ_0 and ϕ_1 are constants. The parameter $\phi_1 (< 1)$ creates a non-zero first-order autocorrelation, which generates short-term persistence in the profit margin. If $\phi_1 = 0$ then shocks are purely transitory.¹⁸ The parameter ϕ_0 equals 1 for practical purposes.¹⁹ The shocks η_{t+j} ($j = 0, 1, \dots$) are independently and identically normally distributed with zero mean and volatility σ .²⁰ Thus $E_t(\eta_{t+j}) = 0$, $E_t(\eta_{t+j}^2) = \sigma^2$ and $E_t(\eta_{t+j} \eta_{t+j+1}) = 0$ for all j . We assume that $\mu > \rho + \delta$ so that on average the profit margin is high enough to cover depreciation and the cost of capital.

The managers' dynamic optimization problem requires a simultaneous solution for investment, payout, rents and the change in debt. Since payout and rents always move

¹⁸A more general process for π_t would be $\pi_t = \mu + \sum_{j=0}^{\infty} \phi_j \eta_{t-j}$, which includes the AR(1) process $\pi_t = \rho \pi_{t-1} + \eta_t$ as a special case if $\phi_{t-j} \equiv \rho^j$. The solution for a general MA(q) process is lengthy, cumbersome and does not convey major additional insights.

¹⁹This parameter has been introduced to facilitate the introduction of corporate taxes (see section 4).

²⁰Our assumption of exponential utility and normally distributed shocks can lead to negative rents and payouts, which we would interpret as the managers' sweat equity and stock issues. These assumptions could also lead to negative stock prices, which are impossible with limited liability. But default risk is remote for the mature and stable firms that our model is designed for. Therefore we ignore default risk for simplicity.

proportionately, we will refer to one decision, *total payout*, defined as $r_t + d_t (\equiv p_t)$.

3.1 Investment policy

Proposition 2 : *The managers' optimal investment policy at time t is:*

$$K_t = k_0 + k_1 \eta_t \quad (22)$$

where the constants k_0 and k_1 are given by:

$$k_0 = \frac{\phi_0 (\mu - \rho - \delta)}{\theta \sigma^2 (1 - \beta) (\phi_0^2 - \beta \phi_1^2) (\phi_0 + \beta \phi_1) (1 - \alpha)} \quad (23)$$

$$k_1 = \frac{\phi_1}{\theta \sigma^2 (1 - \beta) (\phi_0^2 - \beta \phi_1^2) (1 - \alpha)} \quad (24)$$

The optimal investment policy does not depend on the firm's debt level nor on total payout $d_t + r_t$.

Under the optimal investment policy, the capital stock for $t + 1$ is a simple linear function of the exogenous profit margin shock η_t . Investment does not depend on the firm's total payout or debt levels.²¹ Still, all decisions are interlinked, because they depend on the same economic shock η_t .

3.1.1 The role of economic persistence

If shocks to the profit margin are purely transitory ($\phi_1 = 0$), the profit margin has zero autocorrelation and the stock of capital is constant over time at:

$$K_t = \frac{\mu - \rho - \delta}{\phi_0^2 \theta \sigma^2 (1 - \beta) (1 - \alpha)}$$

Since shocks are i.i.d., the realization of a shock η_t carries no information about next period's profit margin π_{t+1} , and the investment decision faced by managers is the same

²¹For negative exponential utility $AP(r) = ARA(r) = \theta$. The constant coefficients of AP and ARA lead to an investment policy that is independent of payout and debt (see also the approximate investment Euler Eq. (19)).

each period. As a result the optimal firm size is constant and unaffected by transitory shocks.

If shocks have some persistence ($\phi_1 > 0$), then a shock η_t affects the profit margin not only at time t but also at $t + 1$. As a result the optimal capital stock is affected by η_t via Eq. (26). This creates a positive correlation between firm size and profitability.²² Firm size hovers around a constant long run mean k_0 , however. The variance around the mean is constant at $k_1\sigma^2$. Past shocks η_{t-j} ($j > 0$) do not affect the stock of capital, because they do not affect the future profit margin. A higher autocorrelation in the firm's profit margin leads to more variability in firm size.

Expected CAPEX is time-varying and mean reverting: $E_t[CAPEX_{t+1}] = \delta k_0 - k_1(1 - \delta)\eta_t$. When the firm has an above- (below-) average stock of capital in place, its expected CAPEX for the subsequent period is smaller (larger), because firm size is mean reverting to k_0 .

Finally, consider the short-run relation between CAPEX and net income (NI) where $NI_t = K_t\pi_t - \rho D_{t-1}$. One can show that: $cov_t(CAPEX_{t+1}, NI_{t+1}) = k_1\sigma^2(k_0 + k_1\eta_t) = k_1\sigma^2 K_t$. Therefore the conditional covariance between CAPEX and NI equals zero if economic shocks are purely transitory ($k_1 = 0$ if $\phi_1 = 0$). If, however, economic shocks have some persistence (i.e. $\phi_1 > 0$) then CAPEX and NI are correlated: a high income realization raises, all else equal, expectations regarding future profitability, which increases investment.²³

²²Although we assume that $\phi_1 \geq 0$, this assumption is not strictly necessary. A negative value for ϕ_1 could, for example, apply to the profit margin for durable goods (such as cars). Suppose that a good lasts for (at least) 2 periods. A large positive demand shock at time t may result in sluggish demand the subsequent period. Conversely, weak demand at time t may imply that consumers have to "catch up" in subsequent periods. The slope coefficient k_1 becomes negative if $\phi_1 < 0$. The intuition is straightforward: the higher the positive demand shock at time t , the more negative the backlash will be in the subsequent period. This results in managers decreasing their capital stock level in anticipation.

²³Notice that investment-cash flow sensitivity, defined as the response of investment to current cash flow, is built into our model, as it must be for any model where information arrives via a profitability shock. The sensitivity cannot measure financing constraints, which are totally absent in our model.

3.1.2 Comparative statics

A higher profit margin volatility and higher risk aversion both unambiguously reduce the level of capital stock. Risk averse managers under-invest compared to what risk-neutral investors would like them to do. Managers never invest at negative-NPV, however. There is no empire building. The average capital stock is zero when $\mu - \rho - \delta = 0$ and profitability just covers the cost of capital plus depreciation.

Stronger investor protection (higher α) increases firm size and mitigates underinvestment. Investment approaches outsiders' first-best outcome when investor protection is near perfect ($\alpha \rightarrow 1$). (Our model loses its grip in this limiting case, however, because managers' stake in the firm and their incentives to put in effort disappear.) A higher cost of capital ρ (lower discount factor β) reduces investment, since both the intercept (k_0) and slope (k_1) of the investment policy decrease in ρ . Note that managers' subjective discount factor ω does not affect investment policy. Since the corporation can, in effect, borrow on behalf of its managers, it is more efficient for managers to accommodate their time preferences (such as impatience) for rent consumption by corporate borrowing or saving than through investment policy. This result also implies that managers' risk of termination, which is implicitly captured by ω , should not affect investment policy.

The sensitivity of the capital stock to shocks k_1 monotonically increases with the degree of persistence ϕ_1 . Therefore, the more persistent economic shocks are, the stronger their effect on *CAPEX*. Persistence allows managers to "ride the business cycle." Finally, the long run capital stock level, k_0 , is proportional to the Sharpe ratio $(\mu - \rho - \delta)/\sigma$.

3.2 Payout policy

Proposition 3 *Total payout $r_t + d_t$ at time t is:*

$$r_t + d_t = Y_t - PS_t$$

where Y_t is the firm's permanent income and PS_t represents precautionary savings:

$$Y_t \equiv \rho\beta \sum_{j=0}^{\infty} \beta^j E_t[CF_{t+j}] - \rho D_{t-1} \quad (25)$$

$$= \rho\beta \left[\frac{k_0(\mu - \delta)}{1 - \beta} + b_1\eta_{t-1} + \phi_1 k_1 \eta_{t-1}^2 + b_0\eta_t + \phi_0 k_1 \eta_{t-1} \eta_t + \beta (b_1\eta_t + \phi_1 k_1 \eta_t^2) + \frac{\beta^2 \phi_1 k_1 \sigma^2}{1 - \beta} \right] - \rho D_{t-1} \quad (26)$$

$$PS_t \equiv \frac{\beta g}{(1 - \beta)(1 - \alpha)} + \beta^2 \sigma^2 \phi_1 k_1 + \frac{\delta_1 \beta \eta_t}{1 - \alpha} + \frac{\beta \delta_0 \eta_t^2}{1 - \alpha} \quad (27)$$

The firm's realized cash flow at time $t + j$ is given by:

$$CF_{t+j} = K_{t+j-1} \pi_{t+j} - K_{t+j} + (1 - \delta) K_{t+j-1} \quad (\text{for } j \geq 0) \quad (28)$$

$$= k_0(\mu - \delta) + b_1 \eta_{t+j-1} + \phi_1 k_1 \eta_{t+j-1}^2 + b_0 \eta_{t+j} + \phi_0 k_1 \eta_{t+j-1} \eta_{t+j} \quad (29)$$

where:

$$\begin{aligned} b_0 &\equiv \phi_0 k_0 - k_1 & b_1 &\equiv k_1(\mu + 1 - \delta) + \phi_1 k_0 \\ \delta_0 &\equiv \frac{\phi_1^2}{2\theta\sigma^2(\phi_0^2 - \beta\phi_1^2)} & \delta_1 &\equiv \frac{\phi_1(\mu - \rho - \delta)}{\theta\sigma^2(\phi_0^2 - \beta\phi_1^2)} \\ g &\equiv -\frac{1}{\theta} \ln\left(\frac{\beta}{\omega}\right) - \frac{1}{2\theta} \ln\left[\frac{\phi_0^2}{\phi_0^2 - \beta\phi_1^2}\right] + \frac{(\mu - \rho - \delta)^2}{2\theta\sigma^2(\phi_0^2 - \beta\phi_1^2)} \end{aligned}$$

Permanent income Y_t is the rate of return on the sum of current and the present value of all future net income, net of debt service, but before rents. It is an annuity payment that, given expectations at time t , could be sustained forever.

The value of the outside equity is proportional to permanent income. Taking expectations of the IBC Eq. (8) and using the definition of Y_t gives:

$$S_t = \sum_{j=0}^{\infty} \beta^j E_t(d_{t+j}) = \frac{\alpha Y_t}{\rho\beta} \quad (30)$$

Consequently, while transitory shocks can have a large effect on contemporaneous cash flow, their effect on permanent income and stock prices is of second order.

PS_t represents the total precautionary savings (by both managers and shareholders) for a rainy day. (Shareholders don't mind the savings, because their wealth is not affected.) Precautionary savings depend only on the current shock η_t , because only η_t matters for the future profit margin. Permanent income depends on the current shock

and past shock η_{t-1} , because permanent income depends in part on current income, which in turn incorporates the last period's shock through the investment decision.

While $CAPEX_t$ is a linear function of the shock η_t , payout and rents (d_t, r_t) are a non-linear (quadratic) function of η_t , because η_t affects both the amount of capital invested (K_t) and profit margin per unit of capital (π_t).

Propositions 2 and 3 allow us to answer important questions regarding the dynamics of payout and its interaction with investment and debt policies. Consider first the special case for which profit margin shocks are purely transitory ($\phi_1 = 0$). In that case total payout becomes:

$$r_t + d_t = Y_t - \frac{1}{2}\theta\sigma^2\beta(1-\beta)\phi_0^2k_0^2(1-\alpha)^2 + \frac{\beta\ln(\frac{\beta}{\omega})}{(1-\beta)\theta} \quad (31)$$

$$= Y_t - \frac{\beta(\mu - \rho - \delta)^2}{2\phi_0^2\theta\sigma^2(1-\beta)} + \frac{\beta\ln(\frac{\beta}{\omega})}{(1-\beta)\theta} \quad (32)$$

where permanent income is given by:

$$Y_t = \rho\beta k_0 \left[\frac{\mu - \delta}{1 - \beta} + \phi_0\eta_t \right] - \rho D_{t-1}$$

For this special case rents are a linear function of η_t , because the stock of capital remains constant. It is immediately apparent that total payout is smooth relative to net income:

$$\text{var}_t(d_{t+1} + r_{t+1}) = \text{var}_t(Y_{t+1}) = (1-\beta)^2(\phi_0k_0)^2\sigma^2 < (\phi_0k_0)^2\sigma^2 = \text{var}_t(NI_{t+1}) \quad (33)$$

The variance in payout is only a fraction $(1-\beta)^2$ of the variance in net income. If β equals, say, 0.95, then payout volatility is only 2.5% of income volatility. Payout volatility is linearly increasing in firm size k_0 and income volatility σ . However, the optimal firm size k_0 is itself a decreasing function of income volatility. Once this endogeneity is accounted for, payout volatility actually decreases with income volatility, because higher volatility overall leads to smaller firm size. However, the volatility of payout per unit of capital increases with income volatility.

Total payout equals permanent income minus a constant amount of precautionary savings. Precautionary savings equal the difference between precautionary savings due

to risk aversion and dissavings that result from managerial impatience. The former term is given by $\theta\sigma^2\beta(1-\beta)\phi_0^2k_0^2(1-\alpha)^2/2$ and increases with the level of capital stock k_0 . This is not surprising: since the stochastic profit margin is scaled by the amount of capital in place, the variance of free cash-flows increases with the capital stock level. For an *exogenously* given level k_0 , precautionary savings *increase* with risk aversion and profit margin volatility. However, the capital stock level k_0 is not exogenous but inversely related to θ and σ^2 . Once this endogeneity is accounted for, the absolute level of precautionary savings actually *decreases* with risk aversion and volatility. As $\theta \rightarrow \infty$ and $\sigma \rightarrow \infty$, precautionary savings go to zero for the simple reason that managers stop investing altogether! Note that precautionary savings per unit of capital increase with income volatility.

These results illustrate that investment policy does affect total payout. The effect of some exogenous variables (such as volatility or risk aversion) on payout can be radically altered once investment and firm size are endogenized. The results may be important for empirical studies that try to identify the determinants of payout.

Consider next the payout policy for the case where shocks have persistence ($\phi_1 > 0$). Now the variance of total payout is given by:

$$\text{var}_t(r_{t+1} + d_{t+1}) = \text{var}_t(Y_{t+1} - PS_{t+1}) \quad (34)$$

Both permanent income Y_{t+1} and precautionary savings PS_{t+1} are convex, quadratic functions of the state variable η_{t+1} . Both tend to move in the same direction:

$$\frac{\partial PS_{t+1}}{\partial \eta_{t+1}} \geq (\leq) 0 \iff \eta_{t+1} \geq (\leq) \frac{-(\mu - \rho - \delta)}{\phi_1} \quad (35)$$

Precautionary savings increase (decrease) when economic conditions are sufficiently good (bad). Therefore, changes in permanent income and precautionary savings partially offset, so that precautionary savings further dampen the volatility of payout. Note that precautionary savings are variable when investment policy is dynamic. As we show below, a dynamic investment policy can generate bigger fluctuations in permanent income. A dynamic, procyclical precautionary savings policy can therefore iron out fluctuations in permanent income and keep payout smooth. Precautionary savings are procyclical, in the sense that firms save more in good times than in bad times.

In summary, a variable investment policy creates additional volatility in the firm's *CAPEX* and income, which in turn leads to increased precautionary savings. Precautionary savings depend on the state of the economy: better (worse) economic conditions coincide with more savings (dissavings). This type of procyclical precautionary savings policy helps to keep payout smooth, even if the firm experiences a jump in its permanent income. Thus we can confirm and in some ways extend the payout-smoothing results in Lambrecht and Myers (2012). That paper did not allow varying, endogenous *CAPEX*. We do not derive the Lintner target-adjustment model of payout here, however, because we have used a MA(1) process for tractability and have not introduced habit formation in the managers' utility function.

3.3 Debt policy

Consider next the firm's debt policy. Debt is the residual, balancing variable, which follows from the sources and uses constraint.

$$\Delta D_t = \rho D_{t-1} + \frac{r_t}{1-\alpha} + CAPEX_t - K_{t-1} \pi_t \quad (36)$$

$$= Y_t - PS_t - (K_{t-1} \pi_t - \rho D_{t-1}) + CAPEX_t = Y_t - PS_t - NI_t + CAPEX_t \quad (37)$$

$$\Delta D_t \geq (\leq) 0 \iff CAPEX_t \geq (\leq) NI_t - (Y_t - PS_t) \quad (38)$$

Thus debt increases (decreases) if CAPEX exceeds (is below) retained income.

Consider next the effect of the initial debt level D_0 on future debt levels. Using our definition for CF_t ($CF_t \equiv K_{t-1} \pi_t - CAPEX_t$), rewrite Eq. (37) as:

$$\Delta D_t = \rho \beta \sum_{j=0}^{\infty} \beta^j E_t(CF_{t+j}) - CF_t - PS_t \quad (39)$$

Since PS_t , and CF_t do not depend on the firm's current or past debt levels, *changes* in net debt are independent of the firm's current or past debt levels. For example, consider 2 firms ($i = 1, 2$) that are identical in *all* respects except for their initial debt level ($D_{10} \neq D_{20}$). If both firms are subject to the same shocks, then $D_{1t} - D_{2t} = D_{10} - D_{20}$. This leads to the following corollary:

Corollary 1 *Differences in net debt levels between firms persist over time.*

Another way to interpret this result is to note that Eq. (39) implies that the firm's net debt level follows a random walk with drift. Shocks to the debt level therefore persist forever. This persistence follows from the fact that managers have negative exponential utility. With CARA utility managers' investment and financing decisions do not depend on wealth, nor on the firm's debt level.

Next, since $CAPEX$ is independent of the debt and payout level, Eq. (36) implies that a dollar of $CAPEX$ raises net debt by a dollar, other things equal. Debt changes are therefore to some degree determined by $CAPEX$. Of course, if profitability shocks are transitory and $CAPEX$ is constant, then income and payout are the only drivers of debt changes.

The relation between NI_t and ΔD_t is cumbersome to analyze analytically, and a numerical analysis is presented below. Some clear insights can be obtained, however, if we consider the case where $CAPEX_t$ is held constant ($\phi_1 = 0$). In this case:

$$\text{cov}_t(\Delta D_{t+1}, NI_{t+1}) = \text{cov}_t(Y_{t+1}, NI_{t+1}) - \text{var}_t(NI_{t+1}) \quad (40)$$

$$= (1 - \beta)(\phi_0 k_0 \sigma)^2 - (\phi_0 k_0 \sigma)^2 = -\beta(\phi_0 k_0 \sigma)^2 < 0 \quad (41)$$

Thus changes in debt and contemporaneous net income are negatively correlated if $CAPEX$ is constant. This implies that firms with stable $CAPEX$ follow a counter-cyclical debt policy. Payout smoothing and the resulting counter-cyclical debt policy are hard to reconcile with the fixed leverage target assumed in many tests of the tradeoff theory of capital structure. A fixed debt-to-value target implies that higher profitability ought to coincide with more rather than less debt.²⁴

²⁴Of course our model has no financing frictions or constraints. Korajczyk and Levy (2003) and Halling, Yu, and Zechner (2012) find that book and market leverage are procyclical for financially constrained firms. Numerical evaluations of our model also show that some values for ϕ_1 (e.g. $\phi_1 \in [0.05, 0.4]$) can generate a positive correlation between NI and ΔD , indicating that the interaction between NI and ΔD is quite complex once $CAPEX$ varies.

3.4 Risk aversion and investor protection

Consider the effects of managers' coefficient of risk aversion θ and the level of investor protection α . The following corollary results directly from our closed-form solutions:

Corollary 2 *Consider two firms $i = 1, 2$ that are subject to the same economic shocks. If both firms are identical except for their initial debt level at $t = 0$ and managers' coefficient of risk aversion, then:*

$$\frac{K_{1t}}{K_{2t}} = \frac{r_{1t} + d_{1t} + \rho D_{10}}{r_{2t} + d_{2t} + \rho D_{20}} = \frac{D_{1t} - D_{10}}{D_{2t} - D_{20}} = \frac{\theta_2}{\theta_1} \quad (42)$$

If both firms are identical except for their initial debt level and their degree of investor protection then:

$$\frac{K_{1t}}{K_{2t}} = \frac{r_{1t} + d_{1t} + \rho D_{10}}{r_{2t} + d_{2t} + \rho D_{20}} = \frac{D_{1t} - D_{10}}{D_{2t} - D_{20}} = \frac{1 - \alpha_2}{1 - \alpha_1} \quad (43)$$

The corollary shows that the firm with lower managerial risk aversion or higher investor protection has a larger stock of capital, pays out more to all stakeholders combined (payout, rents and interest) and experiences larger changes in net debt. A decrease in investor protection cuts aggregate payout to all stakeholders. For example, cutting α_1 from 0.9 to 0.8, thus increasing $1 - \alpha_1$ from 0.1 to 0.2, reduces $r_{1t} + d_{1t} + \rho D_{10}$ by half. Since debt is a fixed claim, cash paid out to managers and stockholders is cut by more than half.

The effects of other parameters have to be analyzed numerically. We leave out the analysis (which is available upon request) in the interest of space, and only briefly mention the role of economic persistence (ϕ_1). We find that economic persistence increases the mean payout and also allows insiders to reduce net debt. Intuitively, persistence in profitability makes the future more predictable, allowing firms to invest more aggressively, to capitalize on good shocks and to pay down debt and increase total payout. Increasing ϕ_1 also increases the cross-sectional standard deviation of payout both in absolute and relative (as a fraction of mean payout) terms. Persistence allows managers to "ride the business cycle" and therefore also induces positive skewness and positive excess kurtosis in payout.

3.5 Example

Figure 1 simulates a single firm’s investment, debt and payout policies over 10 periods. The shocks η_t are random draws from a normal distribution. The initial level of debt is held constant.²⁵ The solid line in Panel A shows that total payout $d_t + r_t$ is smooth relative to gross income (before interest) and net income (after interest). Unlike payout, both gross and net income are highly sensitive to economic shocks.

Panel B illustrates that total payout $r_t + d_t$ equals permanent income minus precautionary savings. Precautionary savings fluctuate around 160 and decrease net debt by a similar amount each period. A decrease in net debt increases permanent income and causes payout to rise over time. Note that the jump in permanent income in period 13 is offset by a similar, simultaneous jump in precautionary savings, so that payout remains relatively smooth.

Panel C shows the evolution of net debt vs. the sum of equity value and the present value of rents (labeled as “total equity”). While net debt fluctuates in response to cash-flow shocks, the overall trend is down. Precautionary savings build up a net cash position as the firm matures. Permanent income rises as the firm shifts from being a borrower to a net lender. Since permanent income is smooth relative to net income and CAPEX, equity value and the present value of rents are smooth compared to net debt.

Panel D confirms that *changes* in net debt (solid line) are positively correlated to CAPEX (dashed line). Debt absorbs CAPEX shocks and allows managers to smooth total payout and to implement the optimal investment policy. Most of the variation in CAPEX can be explained by mean reversion of K_t to its stationary long run mean, k_0 .

Table 1.A gives the variance-covariance matrix for NI_t , ΔD_t and $CAPEX_t$, based

²⁵The parameter values used to generate the plots are: $\mu = 0.1$, $\rho = 0.05$, $\delta = 0.02$, $\omega = 0.9$, $\theta = 1$, $\sigma = 0.03$, $\alpha = 0.9$, $\phi_0 = 1$, $\phi_1 = 0.5$ and $D_0 = 8000$. To appreciate the magnitude of the volatility parameter, recall that normally distributed shocks enter in an *arithmetic* fashion, as $\pi_t = \mu + \phi_0\eta_t + \phi_1\eta_{t-1}$.

on the simulation results in Figure 1. The long-run variance of total payout is:

Table 1: Covariances and correlations for NI_t , ΔD_t and $CAPEX_t$

A) Variance-Covariance Matrix			
	NI_t	ΔD_t	$CAPEX_t$
NI_t	185994	-2215505	-2035976
ΔD_t	-2215505	81742988	79568811
$CAPEX_t$	-2035976	79568811	77574363
B) Correlation Matrix			
	NI_t	ΔD_t	$CAPEX_t$
NI_t	1	-0.57	-0.54
ΔD_t	-0.57	1	0.99
$CAPEX_t$	-0.54	0.99	1

$$\begin{aligned} \text{var}(r_t + d_t) &= \text{var}(\Delta D_t + NI_t - CAPEX_t) = \text{var}(\Delta D_t) + \text{var}(NI_t) + \text{var}(CAPEX_t) \\ &\quad + 2\text{cov}(\Delta D_t, NI_t) - 2\text{cov}(\Delta D_t, CAPEX_t) - 2\text{cov}(NI_t, CAPEX_t) = 6664 \end{aligned}$$

The variance of payout is tiny compared to its subcomponents. Note also that the variance and covariance terms involving net income are an order of magnitude smaller than those that only involve ΔD_t or $CAPEX_t$.

Table 1.B gives corresponding cross-correlations and shows that changes in debt are negatively correlated to net income and positively correlated to CAPEX. CAPEX is perfectly correlated with ΔD_t and due to its mean reversion, negatively correlated with NI_t .

Perfect correlation between CAPEX and ΔD is an extreme result that follows from our assumptions. First, ΔD_t captures changes in *net* debt, which includes changes in cash holdings. (Cash is equivalent to negative debt in our setting.) Second, there are no frictions associated with changes in debt (such as debt issuance costs) or CAPEX (such as fixed or sunk investment costs). It is well known that introducing frictions

injects a degree of inertia in the firm's financing and investment policy, with occasional "swings" in the firm's debt ratio (see Fischer, Heinkel, and Zechner (1989)) or bursts of lumpy investment (see Dixit and Pindyck (1994)). Firms might also accumulate cash from operations over time and then use this to fund bursts of CAPEX. These frictions would in practice reduce the correlation between ΔD and CAPEX.

4 Corporate Taxes

This section revisits the general model developed in section 2 by introducing corporate taxes. Assume that corporate profits are taxed at the rate τ and that rents, depreciation and interest on debt are tax-deductible at the corporate level.²⁶ Assume, as before, that outside investors can borrow and save at the rate ρ (we abstract from personal taxes) while insiders can only borrow and save through the firm. Managers' objective function remains unaltered but the budget constraint and governance constraint are now given, respectively, by:

$$\begin{aligned}
 D_t &= D_{t-1} (1 + \rho^*) + d_t + r_t(1 - \tau) + \Delta K_t - [\Omega(K_{t-1})\pi_t - \delta K_{t-1}](1 - \tau) \quad (44) \\
 S_t &= \alpha E_t \left\{ \sum_{j=0}^{\infty} \beta^j [(\Omega(K_{t-1})\pi_{t+j} - \delta K_{t+j-1})(1 - \tau) - \Delta K_{t+j}] \right\} - \alpha(1 + \rho^*)D_{t-1} \\
 &\quad + \alpha E_t \left\{ \sum_{j=0}^{\infty} \beta^j (1 - \beta(1 + \rho^*)) D_{t+j} \right\} = d_t + \beta E_t [S_{t+1}] \quad (45)
 \end{aligned}$$

where $\rho^* \equiv 1 + \rho(1 - \tau)$. Substituting the budget constraint into the governance constraint, and simplifying gives the following governance constraint:

$$d_t = \left(\frac{\alpha}{1 - \alpha} \right) (1 - \tau)r_t \equiv \gamma(1 - \tau)r_t \quad (46)$$

Since rents are tax-deductible expenses, taxes relax the governance constraint and squeeze outside equityholders' payout relative to insiders' rents. Substituting (46) into the budget constraint (44) gives:

$$\frac{(1 - \tau)}{(1 - \alpha)} r_{t+j} = D_{t+j} - D_{t+j-1}(1 + \rho^*) + CF_{t+j}^* \quad \text{for } j = 0, 1, \dots, \infty \quad (47)$$

²⁶Rents in our model are of a pecuniary nature and include, for example, above-market salaries, perks or generous retirement packages. Rents are therefore tax-deductible expenses.

where $CF_{t+j}^* \equiv (\Omega(K_{t+j-1})\pi_{t+j} - \delta K_{t+j-1})(1 - \tau) - \Delta K_{t+j}$. Consider now the budget constraint at some future horizon date $t+T$. Substituting recursively for $D_{t+T-1}, D_{t+T-2}, \dots, D_t$, it follows that:

$$\frac{(1 - \tau)}{(1 - \alpha)} \sum_{j=0}^T (1 + \rho^*)^j r_{t+T-j} = D_{t+T} + \sum_{j=0}^T (1 + \rho^*)^j CF_{t+T-j}^* - (1 + \rho^*)^{T+1} D_{t-1} \quad (48)$$

Since managers can only borrow and save through the firm their effective discount rate is the after-tax rate ρ^* . Define $\beta^* \equiv 1/(1 + \rho^*)$. Multiplying (48) by β^{*T} , taking the limit for $T \rightarrow \infty$, imposing the no-Ponzi condition $\lim_{T \rightarrow \infty} \beta^{*T} D_{t+T} = 0$, and simplifying gives the after-tax IBC:

$$\sum_{j=0}^{\infty} \beta^{*j} r_{t+j} = \frac{(1 - \alpha)}{(1 - \tau)} \left[\sum_{j=0}^{\infty} \beta^{*j} CF_{t+j}^* - (1 + \rho^*) D_{t-1} \right] \quad (49)$$

The first-order conditions with respect to r_t and K_t are now given, respectively, by:

$$u'(r_t) = \omega(1 + \rho^*) E_t [u'(r_{t+1})] \quad (50)$$

$$u'(r_t) = \omega E_t [u'(r_{t+1}) \{1 + (\Omega'(K_t)\pi_{t+1} - \delta)(1 - \tau)\}] \quad (51)$$

Equations (49), (50) and (51) now determine the solution to the payout-investment problem with corporate taxes. Comparing these 3 equations with equations (12), (13) and (11) in the no-tax case, it follows immediately that both problems are identical once we introduce the following parameter transformations. The following conclusions

No-tax versus tax case								
without taxes	$(1 - \alpha)$	ρ	β	μ	ϕ_0	ϕ_1	δ	$(1 + \gamma)$
with taxes	$\frac{(1 - \alpha)}{(1 - \tau)}$	ρ^*	β^*	$\mu(1 - \tau)$	$\phi_0(1 - \tau)$	$\phi_1(1 - \tau)$	$\delta(1 - \tau)$	$(1 + \gamma)(1 - \tau)$

result at once: (1) Corporate taxes scale up the outsiders' cost of collective action $(1 - \alpha)$ by a factor $1/(1 - \tau)$. (2) The appropriate discount rate and discount factor are now, respectively, given by ρ^* and β^* . Since managers can only borrow and save through the firm, borrowings and savings grow at the after-tax rate of interest. Outside shareholders' discount rate ρ does not affect the solution because the governance constraint $d_t = \gamma(1 - \tau)r_t$ does not depend on ρ . (3) Corporate taxes reduce the distributable income, which has the effect of scaling down the income parameters μ, ϕ_0

and ϕ_1 by a factor $(1 - \tau)$. (4) Corporate taxes create a depreciation tax shield causing the depreciation rate δ to be multiplied by $(1 - \tau)$.

What is the overall effect of corporate taxes on CAPEX? The result is readily available for our closed form solution example from previous section. Substituting the above parameter transformations into proposition 2 gives the following corollary:

Corollary 3 : *The managers' optimal investment policy at time t in the presence of corporate taxes is: $K_t = k_0^* + k_1^* \eta_t$, where the constants k_0^* and k_1^* are given by:*

$$k_0^* = \frac{\phi_0 (\mu - \rho - \delta)}{\theta \sigma^2 (1 - \beta^*) (\phi_0^2 - \beta^* \phi_1^2) (\phi_0 + \beta^* \phi_1) (1 - \alpha)} \quad (52)$$

$$k_1^* = \frac{\phi_1}{\theta \sigma^2 (1 - \beta^*) (\phi_0^2 - \beta^* \phi_1^2) (1 - \alpha)} \quad (53)$$

This is exactly the same investment policy as in the no-tax case, except that the discount factor β has been replaced everywhere by the tax-adjusted discount factor β^* . All other effects cancel out. In particular, the decline in the after-tax excess return $((\mu - \rho - \delta)(1 - \tau))$ and the increase in the cost of collective action $((1 - \alpha)/(1 - \tau))$ have been exactly offset by a corresponding decline in the parameters ϕ_0 and ϕ_1 (recall that ϕ_0 and ϕ_1 capture the sensitivity of profitability to economic shocks).

Since higher taxes increase the discount factor β^* , the above corollary implies that taxes increase the sensitivity of CAPEX to economic shocks (i.e. $\frac{\partial k_1^*}{\partial \tau} > 0$) and tend to increase the base level of capital stock (i.e. $\frac{\partial k_0^*}{\partial \tau} > 0$). The latter effect is, however, not always satisfied as k_0^* is not necessarily monotonic in β^* .

The effect of taxes on rents is more complex. On the one hand, taxes reduce the distributable income and precautionary savings. On the other hand, taxes scale up rents by a factor $1/(1 - \tau)$ (due to the tax-deductibility of rents and the effect this has on the governance constraint) and taxes increase the discount factor β^* at which the firm's free cash flows are discounted. The net effect is not clear analytically.

The effect of taxes on total payout is more straightforward. Total payout is given by $d_t + r_t = r_t(1 - \alpha\tau)/(1 - \alpha)$. It follows that taxes reduce total payout by a factor $(1 - \alpha\tau)$, *ceteris paribus* (i.e. holding rents constant).

In summary, we conclude that our qualitative model results still hold with taxes. While introducing corporate taxes implies that certain model parameters need to be considered on an after-tax basis, the general structure of the investment and payout policies remain the same. In the presence of corporate taxes debt still serves as a shock absorber that allows managers to smooth rents and payout. While taxes may alter the actual level of debt adopted, managers will not seek to “rebalance” debt in order to pursue an optimal target leverage ratio.

5 Empirical Implications and Conclusions

The first sentence of Hennessy and Whited (2007) reads “Corporate finance is primarily the study of financing frictions.” But corporate finance is also the study of the financial behavior of managers as agents for outside shareholders. We believe our model is the first dynamic agency model that incorporates all three of the major corporate finance decisions: investment, borrowing and payout plus managerial rents. We assume rational managers, who make these decisions in their long-run self interest, subject to a governance constraint, and of course also subject to the constraint that sources and uses of cash add up in every period. (Models that consider investment, borrowing or payout separately ignore the latter constraint.) We set aside financial market imperfections and financial frictions. Therefore the dynamics of our model are driven by “pure agency.” Some of our broad results deserve further emphasis and discussion.

First, why do managers under-invest compared to the value-maximizing optimum? Usually managers in agency models are assumed to *over*-invest free cash flow, for example in empire-building. But our managers do not have to waste cash. They are better off taking cash as rents than using cash for risky over-investment.

Casual discussions of agency problems caused by excess free cash flow often ignore governance constraints or implicitly assume that they are not binding. The governance constraint binds in our model because the managers are rational. The binding constraint means that managers cannot invest only the shareholders’ cash. They have

to co-invest, either by cutting back current rents or by assuming their share of additional corporate borrowing, which is senior to future rents as well as to future payouts to investors. Managers are in effect responsible for the fraction $1 - \alpha$ of the firm's outstanding debt.

Non-pecuniary private benefits could push managers to increase *CAPEX*, because the benefits would not have to be shared with shareholders.²⁷ The lure of private benefits *might* overcome risk aversion and lead to over-investment. Managers would still have to coinvest, however, and also compensate shareholders if over-investment reduces firm value. But forcing over-investment by introducing private benefits would in our model be *deus ex machina*. We leave analysis of how private benefits could affect agency dynamics to future work, probably by somebody else.

Second, why must debt be the residual decision in a dynamic agency model? In other words, why is borrowing, rather than rents and payout, used to absorb fluctuations in operating income and investment? The root cause is that debt does not enter the managers' utility function. They use borrowing merely as a device for implementing *CAPEX* decisions and managing the flow of rents (and therefore payout) over time. Notice that it is more efficient for the managers to borrow via the corporation than on personal account. Personal borrowing would introduce moral hazards and risks of personal default. Corporate assets provide collateral for the managers' share of corporate debt, which they assume collectively, and corporate governance protects that collateral.

The result that debt acts as a residual does not mean that managers are free to borrow as much as they choose. Their decisions about *CAPEX* and rents plus payout cannot violate the debt constraint Eq. (6) or the IBC Eq. (8). Thus high levels of debt do discipline managers over time, even though borrowing is not constrained in any given period. Managers restrain rents and borrowing because restraint is optimal in their constrained intertemporal policy.

²⁷Myers (2000, pp. 1032-1033) discusses one reason why private benefits may displace cash rents to managers.

Managers in our model do not seek to increase debt, but generally to reduce it. They set aside precautionary savings by paying down debt. If profits are sufficient, and debt hits zero, the precautionary savings continue, and the firm builds up a cash balance. We could interpret the “cash mountains” accumulated at Apple, Microsoft or other highly profitable companies as resulting from precautionary savings by rational coalitions of managers.

Third, why do managers not rebalance capital structure from time to time, for example by an equity issue to pay down high levels of debt? We have not ruled out equity issues.²⁸ Why are managers of low-debt firms reluctant to issue debt in order to retire equity and exploit interest tax shields? The reason why rebalancing does not occur is that the managers’ joint decision about *CAPEX*, rents, payout and borrowing is both a local and a global optimum. Notice that the managers cannot use only the shareholders’ money to pay down debt, for example by cutting back payout but not rents. That action, if it could be slipped by shareholders, would hand managers a free gift equal to the fraction $1 - \alpha$ of the repaid debt. But in our model the managers do not attempt this trick, which would violate the governance constraint.

Shareholders could benefit by forcing additional debt, however, or by forcing an extra payout by a firm holding “excess” cash. Suppose that shareholders (or a corporate “raider”) could intervene, force the company to borrow \$10 billion and pay this entire amount out as a special dividend or repurchase. The \$10 billion of additional debt would be partly a claim on managers’ future rents. If $\alpha = .9$, say, then managers would effectively carry \$1 billion more debt, and stock-market capitalization would fall by \$9 billion, handing shareholders a \$1 billion gain. Thus our approach can explain why shareholders gain in leveraged restructurings.

If debt is the residual decision, and if managers have no incentive to rebalance their firms’ capital structures, then empirical implications follow. We predict that debt levels are persistent. By “persistent,” we do not mean that debt is stable year

²⁸The firm issues equity when payout d_t becomes negative. This happens if permanent income falls sufficiently low. Negative payout goes hand in hand with negative rents, which means that managers must put up sweat equity in difficult times.

to year. If rents and payout are smoothed over time, then changes in debt cannot be smoothed in the short run. The time-series variance for changes in borrowing should be higher than for changes in rents and payout (another prediction that should be tested). But cross-sectional *differences* in debt ratios should persist, because it is not in the managers' interest to rebalance capital structure to close gaps between high versus low debt ratios. Lemmon, Roberts and Zender (2008) found that cross-sectional differences in debt ratios for otherwise similar firms persist for long periods of time.

The persistence of debt levels, combined with the counter-cyclical volatility of changes in borrowing, can also explain why fitting target-adjustment models of capital structure seems to be such hard work. The persistence of debt levels and the widespread occurrence of payout smoothing are hard to reconcile with the trade-off theory of capital structure. Our agency model says that changes in debt should follow a pecking-order model, even when investors have full information.

Of course these predictions apply to the mature, blue-chip, public firms that our model is designed for. We would not expect the predictions to work for firms with highly valuable growth options, for declining firms or for firms facing material risks of financial distress or default. Testing these predictions with one specification estimated for all of several thousand corporations downloaded from Compustat would therefore be a big mistake.

Our model is of course simplified in several respects, and extensions would be informative. More complex and interesting governance constraints could be explored. For example, our managers cannot issue new equity against the NPV of *CAPEX*, because the fraction α of this NPV must go to outside shareholders. If, however, the NPV were linked to the managers' human capital and therefore lost if shareholders intervene, then managers would be able to extract the investment's full NPV and finance all required *CAPEX* by cutting back payout or issuing equity. A model built on this premise could help distinguish the financial behavior of growth firms from the behavior of the mature firms analyzed here.

6 Appendix

Proof of Lemma 1

Using Eq. (5) we can rewrite the budget equation at $t + 1$ as:

$$r_{t+1} = (1 - \alpha)[D_{t+1} - (1 + \rho)D_t + \Omega(K_t)\pi_{t+1} - K_{t+1} + (1 - \delta)K_t] \quad (54)$$

Consider now a two-date scenario (t and $t + 1$) where the firm stops operating at $t + 1$ ($K_{t+1} = 0$) and all net debt is paid off ($D_{t+1} = 0$). Substituting D_t by the budget equation (5) at t , r_{t+1} becomes a function of r_t and K_t only, i.e.:

$$r_{t+1} = (1 - \alpha)[\Omega(K_t)\pi_{t+1} - (\rho + \delta)K_t - (1 + \rho)Z_t(r_t)] \quad (55)$$

$$\text{where } Z_t(r_t) \equiv (1 + \rho)D_{t-1} - \Omega(K_{t-1})\pi_t - (1 - \delta)K_{t-1} + \frac{r_t}{1 - \alpha} \quad (56)$$

Define the profitability shock $\eta_{t+1} \equiv \pi_{t+1} - E_t[\pi_{t+1}]$ with $E_t[\eta_{t+1}] = 0$, and $\sigma^2 = E_t(\eta_{t+1}^2)$ and $\bar{\pi}_{t+1} \equiv E_t[\pi_{t+1}]$ and $\bar{r}_{t+1} \equiv E_t[r_{t+1}]$. Substituting into Eq. (55) gives:

$$r_{t+1} = (1 - \alpha)[\Omega(K_t)\bar{\pi}_{t+1} - (\rho + \delta)K_t - (1 + \rho)Z_t(r_t)] + (1 - \alpha)\Omega(K_t)\eta_{t+1} \quad (57)$$

$$\equiv \bar{r}_{t+1} + (1 - \alpha)\Omega(K_t)\eta_{t+1} \quad (58)$$

We now locally approximate the investment Euler equation Eq. (18) for small η_{t+1} using a second-order Taylor expansion around \bar{r}_{t+1} :

$$\begin{aligned} & [\Omega'(K_t)\bar{\pi}_{t+1} - \rho - \delta] \left[u'(\bar{r}_{t+1}) + \frac{1}{2}u'''(\bar{r}_{t+1})(1 - \alpha)^2\Omega^2(K_t)\sigma^2 \right] + \\ & \Omega'(K_t)(1 - \alpha)\Omega(K_t) \left[u''(\bar{r}_{t+1})\sigma^2 + \frac{1}{2}u'''(\bar{r}_{t+1})(1 - \alpha)E_t[\eta_{t+1}^3]\Omega(K_t) \right] = 0 \quad (59) \end{aligned}$$

Rearranging gives equation (19).

Proof of propositions 2 and 3

Given that $d_t = \left(\frac{\alpha}{1-\alpha}\right)r_t$, at each time t the infinitely-lived managers choose investment and rent policies (K_t, r_t) that maximize the objective function:

$$\max E_t \left[\sum_{j=0}^{\infty} \omega^j u(r_{t+j}) \right] \quad (60)$$

subject to $D_t = (1 + \rho)D_{t-1} - CF_t + \frac{r_t}{1-\alpha}$. The optimality conditions for r_t and K_t are given by:²⁹

$$E_t [\omega^t u'(r_t) + \omega^{t+1} u'(r_{t+1})(-1)(1 + \rho)] = 0 \quad (61)$$

$$E_t [\omega^t u'(r_t)(1 - \alpha)(-1) + \omega^{t+1} u'(r_{t+1})(1 - \alpha)(\pi_{t+1} + 1 - \delta)] = 0 \quad (62)$$

Or equivalently:

$$u'(r_t) = \frac{\omega}{\beta} E_t [u'(r_{t+1})] \quad (63)$$

$$u'(r_t) = \omega E_t [u'(r_{t+1})(1 + \pi_{t+1} - \delta)] \quad (64)$$

The proofs follow Caballero (1990) and Lambrecht and Myers (2012), among others, by guessing the general form of the solution, and using the method of undetermined coefficients to identify the solution. We conjecture the following solution for K_{t+j} :

$$K_{t+j} = k_0 + k_1 \eta_{t+j} \quad \text{for } j = 0, 1, 2, \dots \quad (65)$$

where k_0 and k_1 are constants that remain to be determined. It follows that cash flows are given by:

$$CF_{t+j} = K_{t+j-1} \pi_{t+j} - K_{t+j} + (1 - \delta)K_{t+j-1} \quad (66)$$

$$= c + b_1 \eta_{t+j-1} + a_1 \eta_{t+j-1}^2 + b_0 \eta_{t+j} + k_1 \eta_{t+j-1} \eta_{t+j} \quad (67)$$

where $a_1 \equiv \phi_1 k_1$, $c \equiv k_0 (\mu - \delta)$, and b_0, b_1 are as defined in proposition 3, and are obtained by substituting our conjecture (65) for K_{t+j} into the equation (66) for CF_{t+j} .

Given that cash-flows are linear-quadratic in the shocks, we conjecture the following solution for r_t :

$$r_{t+j} = \varphi_{t+j-1} r_{t+j-1} + g + a \eta_{t+j}^2 + \alpha_1 \eta_{t+j} \eta_{t+j-1} + \alpha_2 \eta_{t+j} + \delta_0 \eta_{t+j-1}^2 + \delta_1 \eta_{t+j-1} \quad (68)$$

$$\equiv \varphi_{t+j-1} r_{t+j-1} + \Gamma_{t+j-1} + v_{t+j} \quad \text{for } j = 0, 1, 2, \dots \quad (69)$$

where φ_{t+j-1} , g , a , α_1 , α_2 , δ_0 and δ_1 remain to be determined and where $\Gamma_{t+j-1} \equiv g + \delta_0 \eta_{t+j-1}^2 + \delta_1 \eta_{t+j-1}$ and $v_{t+j} \equiv a \eta_{t+j}^2 + (\alpha_1 \eta_{t+j-1} + \alpha_2) \eta_{t+j} \equiv a \eta_{t+j}^2 + b_{t+j-1} \eta_{t+j}$, and where v_{t+j} captures the innovation at time $t + j$.

²⁹To incorporate a general production function $\Omega(K_t)$ as in section 2, the optimality conditions (62) and (64) are easily modified by multiplying π_{t+1} by $\Omega'(K_t)$, leading to equation (13).

Substituting the above conjecture for r_{t+1} into the Euler equation (63) for r_t gives:

$$e^{-\theta r_t} = \frac{\omega}{\beta} E_t [e^{-\theta \varphi_t r_t} e^{-\theta \Gamma_t} e^{-\theta v_{t+1}}] \quad (70)$$

Hence, it follows that $\varphi_t = 1$ as otherwise rents would be determined by the Euler equation regardless of the budget constraint. Substituting $\varphi_t = 1$ into the Euler equation gives:

$$\Gamma_t = \frac{1}{\theta} \ln \left[\frac{\omega}{\beta} E_t [e^{-\theta v_{t+1}}] \right] \quad (71)$$

$$= \frac{1}{\theta} \ln \left[\frac{\omega e^{\frac{\theta^2 b_t^2 \sigma^2}{2(1+2a\theta\sigma^2)}}}{\beta \sqrt{1+2a\theta\sigma^2}} \right] \quad \text{for } a \geq 0 \quad (72)$$

$$= \frac{1}{\theta} \ln \left(\frac{\omega}{\beta} \right) - \frac{1}{2\theta} \ln (1+2a\theta\sigma^2) + \frac{\theta b_t^2 \sigma^2}{2(1+2a\theta\sigma^2)} \quad \text{for } a \geq 0 \quad (73)$$

$$\equiv g + \delta_0 \eta_t^2 + \delta_1 \eta_t \quad (74)$$

where we made use of the fact that η_{t+1} is normally distributed. Note that the above integral is only defined for $a \geq 0$, we later verify whether a indeed satisfies this constraint. Solving for g , δ_0 and δ_1 gives:

$$g = \frac{1}{\theta} \ln \left(\frac{\omega}{\beta} \right) - \frac{1}{2\theta} \ln (1+2a\theta\sigma^2) + \frac{\theta \sigma^2 \alpha_2^2}{2(1+2a\theta\sigma^2)} \quad (75)$$

$$\delta_0 = \frac{\theta \sigma^2 \alpha_1^2}{2(1+2a\theta\sigma^2)} \quad (76)$$

$$\delta_1 = \frac{\theta \sigma^2 \alpha_1 \alpha_2}{1+2a\theta\sigma^2} \quad (77)$$

where the constants α_1 , α_2 and a remain to be determined.

We now substitute the solution for r_t into the IBC (8). This gives the following IBC:

$$(1+\gamma) \sum_{j=0}^{\infty} \beta^j r_{t+j} = \sum_{j=0}^{\infty} \beta^j CF_{t+j} - (1+\rho)D_{t-1} \quad (78)$$

where $\gamma \equiv \alpha/(1-\alpha)$ and where the cash flows CF_{t+j} are as defined in equation (67).

Since $r_{t+1} = r_t + \Gamma_t + v_{t+1}$, repeated substitution means that our conjectured solution for r_{t+j} is $r_{t+j} = r_t + \sum_{i=1}^j (\Gamma_{t+i-1} + v_{t+i})$. Substituting r_{t+j} into the IBC gives

$$\frac{(1+\gamma)r_t}{(1-\beta)} + (1+\gamma) \sum_{j=1}^{\infty} \beta^j \sum_{i=1}^j (\Gamma_{t+i-1} + v_{t+i}) = \left[\sum_{j=0}^{\infty} \beta^j CF_{t+j} - (1+\rho)D_{t-1} \right] \quad (79)$$

Furthermore,

$$\begin{aligned}
& \sum_{j=1}^{\infty} \beta^j \sum_{i=1}^j (\Gamma_{t+i-1} + v_{t+i}) \\
&= \beta (\Gamma_t + v_{t+1}) + \beta^2 (\Gamma_t + v_{t+1} + \Gamma_{t+1} + v_{t+2}) + \dots \\
&= (\Gamma_t + v_{t+1}) \beta (1 + \beta + \beta^2 + \beta^3 + \dots) + (\Gamma_{t+1} + v_{t+2}) \beta^2 (1 + \beta + \beta^2 + \beta^3 \dots) + \dots \\
&= \frac{1}{1 - \beta} \sum_{j=1}^{\infty} \beta^j (v_{t+j} + \Gamma_{t+j-1}). \tag{80}
\end{aligned}$$

Substituting (80) into (79) gives

$$\frac{(1 + \gamma)r_t}{1 - \beta} + \frac{(1 + \gamma)}{1 - \beta} \sum_{j=1}^{\infty} \beta^j (\Gamma_{t+j-1} + v_{t+j}) = \left[\sum_{j=0}^{\infty} \beta^j CF_{t+j} - (1 + \rho)D_{t-1} \right]. \tag{81}$$

Substituting the expressions for CF_{t+j} , v_{t+j} and Γ_{t+j-1} into the IBC and collecting terms gives:

$$\begin{aligned}
& \frac{(1 + \gamma)r_t}{(1 - \beta)} + \frac{(1 + \gamma)\beta g}{(1 - \beta)^2} + \frac{(1 + \gamma)\beta}{(1 - \beta)} [\delta_0 \eta_t^2 + \delta_1 \eta_t] \\
& + \frac{(1 + \gamma)}{(1 - \beta)} \left[(a + \beta\delta_0) \sum_{j=1}^{\infty} \beta^j \eta_{t+j}^2 + (\alpha_2 + \beta\delta_1) \sum_{j=1}^{\infty} \beta^j \eta_{t+j} + \alpha_1 \sum_{j=1}^{\infty} \beta^j \eta_{t+j} \eta_{t+j-1} \right] \\
&= \frac{c}{1 - \beta} + b_1 \eta_{t-1} + a_1 \eta_{t-1}^2 + \eta_t (b_0 + \beta b_1) + \beta a_1 \eta_t^2 + k_1 \eta_{t-1} \eta_t + \\
& \sum_{j=1}^{\infty} \beta^j \beta a_1 \eta_{t+j}^2 + \sum_{j=1}^{\infty} \beta^j \eta_{t+j} (b_0 + \beta b_1) + \sum_{j=1}^{\infty} \beta^j k_1 \eta_{t+j} \eta_{t+j-1} - (1 + \rho)D_{t-1} \tag{82}
\end{aligned}$$

The IBC needs to be satisfied for all possible future values of $\eta_{t+1}, \eta_{t+2}, \dots$. Collecting the terms in $\sum_{j=1}^{\infty} \beta^j \eta_{t+j}^2$, $\sum_{j=1}^{\infty} \beta^j \eta_{t+j}$ and $\sum_{j=1}^{\infty} \beta^j \eta_{t+j} \eta_{t+j-1}$, the IBC (82) can only be satisfied at all times if:

$$\frac{(1 + \gamma)}{(1 - \beta)} (a + \beta\delta_0) = \beta a_1 \tag{83}$$

$$\frac{(1 + \gamma)}{(1 - \beta)} (\alpha_2 + \beta\delta_1) = (b_0 + \beta b_1) \tag{84}$$

$$\frac{(1 + \gamma)}{(1 - \beta)} \alpha_1 = k_1 \tag{85}$$

Solving for a , α_1 and α_2 gives:

$$a = (1 - \alpha)(1 - \beta)\beta a_1 - \beta\delta_0 \tag{86}$$

$$\alpha_1 = (1 - \alpha)(1 - \beta)k_1 \tag{87}$$

$$\alpha_2 = (1 - \alpha)(1 - \beta)(b_0 - \beta b_1) - \beta\delta_1 \tag{88}$$

Substituting the solution for δ_0 into (86) and rearranging gives a quadratic equation in a :

$$4\theta\sigma^2 a^2 + 2[1 - 2(1 - \alpha)(1 - \beta)\beta a_1 \theta \sigma^2] a - 2(1 - \alpha)(1 - \beta)\beta a_1 + \beta\theta\sigma^2 \alpha_1^2 = 0 \quad (89)$$

We later use this quadratic equation to solve explicitly for a .

Rearranging equation (81) gives:

$$r_t = (1 - \alpha)(1 - \beta) \left[\sum_{j=0}^{\infty} \beta^j C F_{t+j} - (1 + \rho) D_{t-1} \right] - \sum_{j=1}^{\infty} \beta^j (\Gamma_{t+j-1} + v_{t+j}) \quad (90)$$

Substituting the expressions for Γ_{t+j-1} and v_{t+j} , and taking expectations conditional on the information available at time t gives:

$$r_t = (1 - \alpha) Y_t - \frac{\beta g}{1 - \beta} - (1 - \alpha) \sigma^2 \beta^2 a_1 - \beta \delta_0 \eta_t^2 - \beta \delta_1 \eta_t \quad (91)$$

where Y_t is the firm's permanent income as defined in proposition 3. This gives the expression for managers' rent policy, r_t , as in proposition 3, but for exogenously given values k_0 and k_1 of the firm's investment policy.

To complete the solution we now need to solve for the firm's investment policy. Recall that the optimal investment policy satisfies (13), or using the negative exponential utility:

$$e^{-\theta r_t} = \omega E_t [e^{-\theta r_{t+1}} (\pi_{t+1} + 1 - \delta)] \quad (92)$$

From equation (69) we know that $r_{t+1} = r_t + \Gamma_t + v_{t+1}$ and therefore:

$$e^{-\theta r_t} = \omega E_t [e^{-\theta r_t} e^{-\theta \Gamma_t} e^{-\theta v_{t+1}} (\pi_{t+1} + 1 - \delta)] \quad (93)$$

r_t can be factored out of the equation. Since the innovation v_{t+1} and the profit margin π_{t+1} are independent of r_t and D_t , it follows that the investment policy (K_t) does not depend on the payout (r_t) and debt (D_t) policies (which proves the final statement in proposition 2). Simplifying gives:

$$e^{\theta \Gamma_t} = \omega E_t [e^{-\theta a \eta_{t+1}^2 - \theta b_t \eta_{t+1}} (\mu + \phi_0 \eta_{t+1} + \phi_1 \eta_t + 1 - \delta)] \quad (94)$$

Working out the expectations, and using equation (72) to rewrite the left hand of the equation gives:

$$\frac{\omega}{\beta} \frac{e^{\frac{\theta^2 b_t^2 \sigma^2}{2(1+2a\theta\sigma^2)}}}{\sqrt{1+2a\theta\sigma^2}} = \omega (\mu + \phi_1 \eta_t + 1 - \delta) \frac{e^{\frac{\theta^2 b_t^2 \sigma^2}{2(1+2a\theta\sigma^2)}}}{\sqrt{1+2a\theta\sigma^2}} - \phi_0 b_t \omega \theta \sigma^2 \frac{e^{\frac{\theta^2 b_t^2 \sigma^2}{2(1+2a\theta\sigma^2)}}}{(1+2a\theta\sigma^2)^{\frac{3}{2}}} \quad (95)$$

Simplifying, and using the fact that $b_t \equiv \alpha_2 + \alpha_1 \eta_t$ gives:

$$(\mu - \rho - \delta + \phi_1 \eta_t) (1 + 2a\theta\sigma^2) = \phi_0 \theta \sigma^2 [\alpha_2 + \alpha_1 \eta_t] \quad (96)$$

This condition is satisfied for all η_t if and only if:

$$(\mu - \rho - \delta) (1 + 2a\theta\sigma^2) = \phi_0 \theta \sigma^2 \alpha_2 \quad (97)$$

$$\text{and } \phi_1 (1 + 2a\theta\sigma^2) = \phi_0^2 \theta \sigma^2 (1 - \alpha)(1 - \beta) k_1 \quad (98)$$

Combining (97) and (98), and simplifying allows us to establish a link between k_0 and k_1 :

$$\frac{k_0}{k_1} = \frac{(\mu - \rho - \delta) \phi_0}{(\phi_0 + \beta \phi_1) \phi_1} \quad (99)$$

It follows from equation (86) that:

$$a = (1 - \alpha)(1 - \beta) \beta \phi_1 k_1 - \frac{\beta \theta \sigma^2 (1 - \alpha)^2 (1 - \beta)^2 \phi_0^2 k_1^2}{2(1 + 2a\theta\sigma^2)} \quad (100)$$

Substituting (98) into the above equation, and simplifying gives:

$$a = \frac{1}{2} (1 - \alpha)(1 - \beta) \beta \phi_1 k_1 \quad (101)$$

Substituting this solution for a into the quadratic equation (89) and simplifying, results in the following quadratic equation for k_1 :

$$(1 - \alpha)(1 - \beta) \theta \sigma^2 (\phi_0^2 - \beta \phi_1^2) k_1^2 - \phi_1 k_1 = 0 \quad (102)$$

The equation has two roots: $k_1 = 0$ and

$$k_1 = \frac{\phi_1}{(1 - \alpha)(1 - \beta) \theta \sigma^2 (\phi_0^2 - \beta \phi_1^2)} \quad (103)$$

From (99), it follows that also $k_0 = 0$ if $k_1 = 0$. The solution ($k_0 = 0, k_1 = 0$) can therefore be discarded. The second root for k_1 is therefore the appropriate one, and it follows from (99) that:

$$k_0 = \frac{(\mu - \rho - \delta) \phi_0}{(1 - \alpha)(1 - \beta) \theta \sigma^2 (\phi_0^2 - \beta \phi_1^2) (\phi_0 + \beta \phi_1)} \quad (104)$$

This completes the solution for the firm's investment policy as given in proposition 2. Finally, we verify whether $a \geq 0$ as required. Substituting the solution for k_1 into (101) gives:

$$a = \frac{\beta \phi_1^2}{2\theta \sigma^2 (\phi_0^2 - \beta \phi_1^2)} \geq 0 \quad (105)$$

Since $\phi_1 < \phi_0 = 1$, this verifies our earlier conjecture that $a \geq 0$.

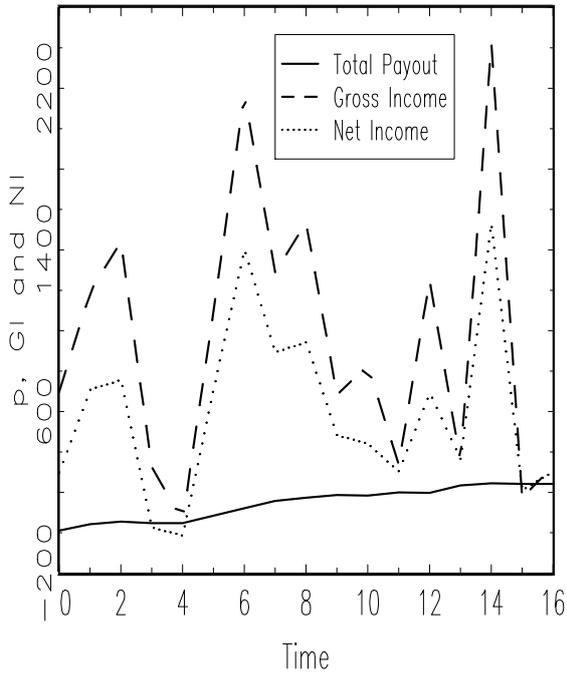
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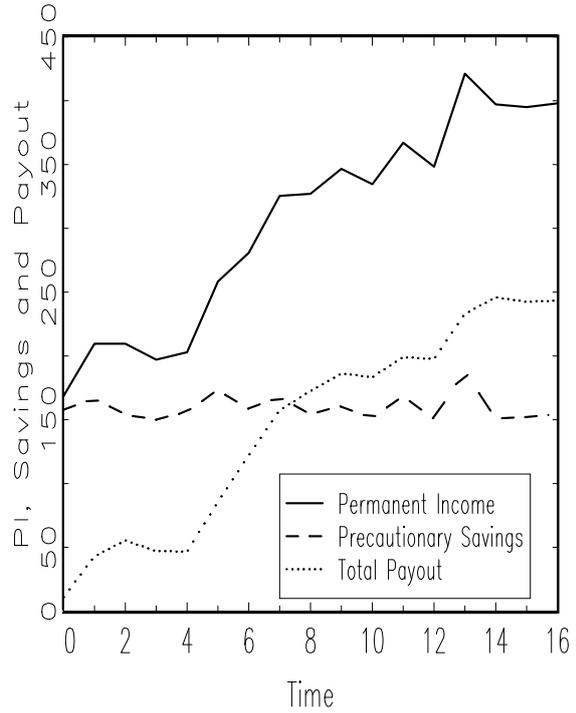
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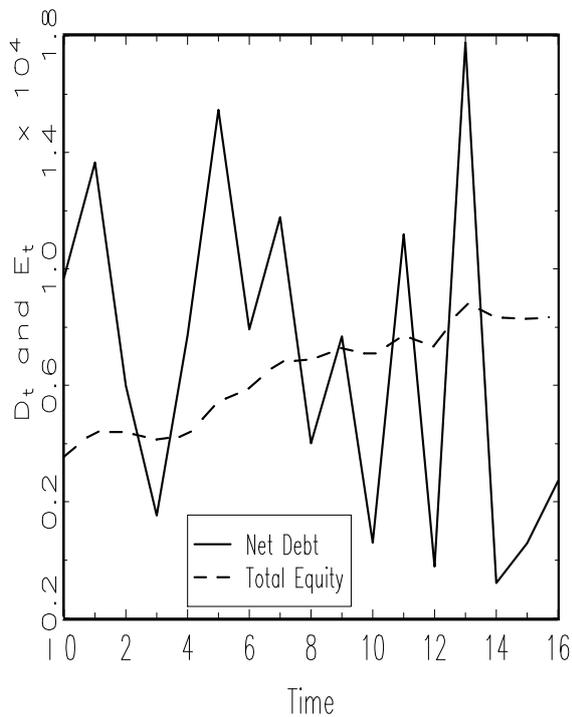
1.A) Total Payout, Gross Income and Net Income



1.B) Permanent Income, Savings and payout



1.C) Net Debt and Total Equity



1.D) Change in Net Debt, CAPEX and Net Income

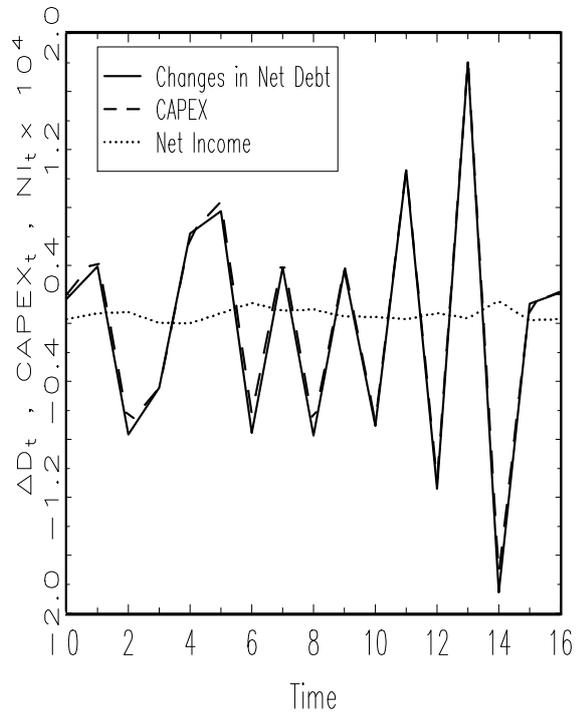


Figure 1: The plots simulate the behavior of an individual firm's investment, financing and payout policies. The parameter values used to generate the plots are: $\mu = 0.1$, $\rho = 0.05$, $\delta = 0.02$, $\omega = 0.9$, $\theta = 1$, $\sigma = 0.03$, $\alpha = 0.9$, $\phi_0 = 1$, $\phi_1 = 0.5$ and $D_0 = 8000$.