DEFLATION RISK

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Abstract. We study the nature of deflation risk by extracting the objective distribution of inflation from the market prices of inflation swaps and options. We find that the market expects inflation to average about 2.5 percent over the next 30 years. Despite this, the market places substantial probability weight on deflation scenarios in which prices decline by more than 10 to 20 percent over extended horizons. We find that the market prices the economic tail risk of deflation very similarly to other types of tail risks such as catastrophic insurance losses. In contrast, inflation tail risk has only a relatively small premium. Deflation risk is also significantly linked to measures of financial tail risk such as swap spreads, corporate credit spreads, and the pricing of super senior tranches. These results indicate that systemic financial risk and deflation risk are closely related.

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1. INTRODUCTION

Deflation has played a central role in the worst economic meltdowns experienced in U.S. history. Key examples include the deflations associated with the Panic of 1837, the Long Depression of 1874–1896, and the Great Depression of the 1930s. In light of this, it is not surprising that deflation is now one of the most-feared risks facing participants in the financial markets. In recent years, the financial press has increasingly raised concerns about a global deflationary spiral and has used terms such as "nightmare scenario" or "looming disaster" to describe the growing threat.[1] Furthermore, addressing the risk of deflation is one of the primary motivations behind a number of actions taken by the Federal Reserve in the past several years such as the quantitative easing programs.[2]

Despite the severe potential effects of deflation, relatively little is known about how large the risk of deflation actually is, or about the economic and financial factors that contribute to deflation risk. The primary reason for this may simply be that deflation risk has traditionally been very difficult to measure. For example, as shown by Ang, Bekaert, and Wei (2007) and others, econometric models based on the time series of historical inflation perform poorly even in estimating the first moment of inflation. In addition, while surveys of inflation tend to do better, these surveys are limited to forecasts of expected inflation over shorter horizons and provide little or no information about the tail probability of deflation.

This paper presents a new market-based approach for measuring deflation risk. This approach allows us to solve directly for the market's assessment of the probability of deflation for horizons of up to 30 years using the prices of inflation swaps and options. In doing this, we first identify the risk-neutral density of inflation implied by inflation calls and puts. We then estimate the inflation risk premium embedded in the term structure of inflation swap rates. Finally, we solve for the actual or objective distribution of inflation by inverting the risk-premium-adjusted characteristic function of the risk-neutral density. A key advantage of

[1]For example, see Coy (2012), "Five Charts that Show that Deflation is a Growing Threat," www.businessweek.com/articles/1012-06-05/five-charts-that-show-deflation-is-a-growing-threat, Lange (2011), "Nightmare Scenario: U.S. Deflation Risks Rising," www.reuters.com/article/2011/10/16/us-economy-deflation-idUSTRE79P7FV201110126, and Carney (2010), "Deflation: Looming Disaster for Banks," www.cnbc.com/id/39648704/Deflation_Looming_Disaster_for_Banks.

[2]For example, see Bernanke (2012), "Monetary Policy since the Onset of the Crisis," www.federalreserve.gov/newsevents/speech/bernanke20120831a.htm.

this approach is that we recover the entire distribution of inflation rather than just the first moment or expected inflation. This is important since this allows us to measure the probability of tail events such as deflation.

The shape of the distribution of inflation varies significantly for shorter horizons, but is much more stable for longer horizons. Inflation risk premia are slightly negative for horizons of one to five years, but increase to about 30 basis points for a 30-year horizon. We find that the market expects inflation of close to 2.5 percent for horizons from 10 to 30 years. The volatility of inflation is roughly two percent for shorter horizons, but is about one percent or less for horizons of ten years or more. Thus, the market views inflation as having a strongly mean reverting nature. The distribution of inflation is skewed towards negative values and has longer tails than a normal distribution.

We solve for the probability of deflation over horizons ranging up to 30 years directly from the distribution of inflation. The empirical results are very striking. We find that the market places a significant amount of weight on the probability that deflation occurs over extended horizons. Furthermore, the market-implied probability of deflation can be substantially higher than that estimated by policy makers. For example, in a speech on August 27, 2010, Federal Reserve Chairman Ben S. Bernanke stated that "Falling into deflation is not a significant risk for the United States at this time."[3] On the same date, the market-implied probability of deflation was 15.11 percent for a two-year horizon, 5.36 percent for a five-year horizon, and 2.84 percent for a ten-year horizon. These probabilities are clearly not negligible. On average, the market-implied probability of deflation during the sample period was 11.44 percent for a two-year horizon, 5.34 percent for a five-year horizon, 3.29 percent for a ten-year horizon, and 2.33 percent for a 30-year horizon. The risk of deflation, however, varies significantly and these probabilities have at times been substantially larger than the averages. In particular, the probability of deflation exhibits jumps which tend to coincide with major events in the financial markets such as the ratings downgrades of Spain in 2010 or the downgrade of U.S. Treasury debt by Standard and Poors in August 2011.

Deflation is clearly an economic tail risk and changes in deflation risk may reflect the market's fears of a meltdown scenario.[4] Thus, a natural next step is to

[3]See Bernanke (2010), "The Economic Outlook and Monetary Policy," www.federalreserve.gov/newsevents/speech/2010speech.htm.

[4]Note that we are interpreting tail risk as including more than just event risk or jump risk. Event or jump risks are adverse economic events that occur relatively suddenly. In contrast, tail risk can also include extreme scenarios with severe economic consequences which may unfold over extended periods. The modeling

examine whether deflation risk is related to other serious types of tail risk in the financial markets or in the macroeconomy in general. Focusing first on the pricing of deflation risk, we find that the ratio of the risk-neutral probability of deflation to the objective probability of deflation is on the order of three to one. This ratio is very similar to that of other types of tail risk. For example, Froot (2001) finds that the ratio of the price of catastrophic reinsurance to expected losses ranges from two to seven. Driessen (2005), Berndt, Duffie, Douglas, Ferguson, and Schranz (2005), Giesecke, Longstaff, Schaefer, and Strebulaev (2011) estimate that the ratio of the price of expected losses on corporate bonds to actual expected losses is on the order of two to three.

We next consider the relation between deflation risk and specific types of financial and macroeconomic tail risks that have been described in the literature. In particular, we consider a number of measures of systemic financial risk, collateral revaluation risk, sovereign default risk, and business cycle risk and investigate whether these are linked to deflation risk. We find that a number of systemic risk variables are significantly related to the probability of deflation. For example, the risk of deflation increases as the price of protection on super senior tranches increases. This is intuitive since the types of economic meltdown scenarios that would result in losses on super senior tranches would likely be associated with sharp declines in the level of prices. Similarly, we find that deflation risk increases as the credit and liquidity risks faced by the financial sector increase. Thus, economic tail risk increases as the financial sector becomes more stressed. We also find that the risk of deflation increases as the unemployment rate increases. This is consistent with a number of classical macroeconomic theories about the relation between prices and employment. Overall, these results provide support for the view that the risk of severe macroeconomic shocks in which deflation occurs is closely related to tail risks in financial markets. This is consistent with U.S. historical experience in which depressions/deflationary spirals have been associated with major collapses in the financial system.

Finally, we also compute the probabilities of inflation exceeding various thresholds. The results indicate that while the probability of inflation in the near term is relatively modest, the long-term probabilities of inflation are much higher. Interestingly, we find that the ratio of the probability of inflation exceeding five percent under the risk-neutral measure is only about 1.4 times that under the actual measure. Thus, inflation tail risk is priced much more modestly than is deflation tail risk.

Our results also have important implications for Treasury debt management. In particular, whenever the Treasury issues Treasury Inflation Protected

framework used in this paper is consistent with both types of risks.

Securities (TIPS), the Treasury essentially writes an at-the-money deflation put and packages it together with a standard inflation-linked bond. The returns on writing these deflation puts are potentially large because of the substantial risk premium associated with deflation tail risk. If the Treasury is better suited to bear deflation tail risk than the marginal investor in the market for inflation protection, then providing a deflation put provides an extra source of revenue for the Treasury that is non-distortionary. There are good reasons to think that the Treasury is better equipped to bear deflation risk, not in the least because the Treasury and the Federal Reserve jointly control the price level.[5]

This paper contributes to the extensive literature on estimating the properties of inflation. Important papers on estimating inflation risk premia and expected inflation include Hamilton (1985), Barr and Campbell (1997), Evans (1998, 2003), Campbell and Viceira (2001), Bardong and Lehnert (2004), Buraschi and Jiltsov (2005), Ang, Bekaert, and Wei (2007, 2008), Adrian and Wu (2007), Bekaert and Wang (2010), Chen, Liu, and Cheng (2010), Christensen, Lopez, and Rudebusch (2010, 2011), Gurkaynak and Wright (2010), Gurkaynak, Sack, and Wright (2010), Pflueger and Viceira (2011a, 2011b), Chernov and Mueller (2012), Haubrich, Pennachi, and Ritchken (2012), Faust and Wright (2012), Grishchenko and Huang (2012), and many others. Key papers on deflation include Hamilton (1992), Fischer (1933), Cecchetti (1992), Atkeson and Kehoe (2004), and Kilian and Manganelli (2007).

Two important recent papers have parallels to our work. Christensen, Lopez, and Rudebusch (2011) fit an affine term structure model to the Treasury real and nominal term structures and estimate the value of the implicit deflation option embedded in TIPS prices. Our research significantly extends their results by estimating deflation probabilities for horizons out to 30 years directly using market inflation option prices. Kitsul and Wright (2012) also use inflation options to infer the risk-neutral density for inflation, but do not formally solve for the objective density of inflation. Our results corroborate and extend their innovative work.

The remainder of this paper is organized as follows. Section 2 briefly discusses the history of deflation in the United States. Section 3 provides an introduction to the inflation swap and options markets. Section 4 presents the inflation model used to value inflation derivatives. Section 5 discusses the maximum likelihood estimation of the inflation model. Section 6 describes the distribution of inflation. Section 7 considers the implications of the results for deflation proba-

[5]Since the ratio of risk-neutral to actual probabilities is much larger for deflation than for high-inflation scenarios, this same logic is not as applicable to the standard inflation protection built into TIPS.

bilities and the pricing of deflation risk. Section 8 examines the relation between deflation risk and other types of financial and macroeconomic tail risks. Section 9 presents results for the probabilities of several inflation scenarios. Section 10 summarizes the results and makes concluding remarks.

2. DEFLATION IN U.S. HISTORY

The literature on deflation in the U.S. is far too extensive for us to be able to review in this paper. Key references on the history of deflation in the U.S. include North (1961), Friedman and Schwartz (1963), and Atack and Passell (1994). We will simply observe that deflation was a relatively frequent event during the 19th Century, but has diminished in frequency since then. Bordo and Filardo (2005) report that the frequency of an annual deflation rate was 42.4 percent from 1801–1879, 23.5 percent from 1880–1913, 30.6 percent from 1914–1949, 5.0 percent from 1950–1969, and zero percent from 1970–2002. The financial crisis of 2008–2009 was accompanied by the first deflationary episode in the U.S. since 1955.

Economic historians have identified a number of major deflationary episodes. Key examples include the crisis of 1815–1821 in which agricultural prices fell by nearly 50 percent. The banking-related Panic of 1837 was followed by six years of deflation in which prices fell by nearly 30 percent. The post-Civil-War greenback period experienced a number of severe deflations and the 1873–1896 period has been called the Long Depression. This period experienced massive amounts of corporate bond defaults and Friedman and Schwartz (1963) estimate that the price level declined by 1.7 percent per year from 1875 to 1896. The U.S. suffered a severe deflationary spiral during the early stages of the Great Depression in 1929–1933 as prices rapidly fell by more than 40 percent.

Although Atkeson and Kehoe (2004), Bordo and Filardo (2005), and others show that not all deflations have been associated with severe declines in economic output, a common thread throughout U.S. history has been that deflationary episodes are typically associated with turbulence or crisis in the financial system.

3. THE INFLATION SWAPS AND OPTIONS MARKETS

In this section, we begin by reviewing the inflation swaps market. We then provide a brief introduction to the relatively new inflation options market.

3.1 Inflation Swaps

As discussed by Fleckenstein, Longstaff, and Lustig (2012), U.S. inflation swaps were first introduced in the U.S. when the Treasury began auctioning TIPS in 1997 and have become increasingly popular among institutional investment managers. Pond and Mirani (2011) estimate the notional size of the inflation swap market to be on the order of hundreds of billions.

In this paper, we focus on the most widely-used type of inflation swap which is designated a zero-coupon swap. This swap is executed between two counterparties at time zero and has only one cash flow which occurs at the maturity date of the swap. For example, imagine that at time zero, the ten-year zero-coupon inflation swap rate is 300 basis points. As is standard with swaps, there are no cash flows at time zero when the swap is executed. At the maturity date of the swap in ten years, the counterparties to the inflation swap exchange a cash flow of $(1 + .0300)^{10} - I_T$, where I_T is the relative change in the price level between now and the maturity date of the swap. The timing and index lag construction of the inflation index used in an inflation swap are chosen to match precisely the definitions applied to TIPS issues.

The zero-coupon inflation swap rate data used in this study are collected from the Bloomberg system. Inflation swap data for maturities ranging from one to 30 years are available for the period from July 23, 2004 to October 5, 2012. Data for inflation swaps with maturities of 40 and 50 years are available beginning later in the sample. Recent research by Fleming and Sporn (2012) concludes that "the inflation swap market appears reasonably liquid and transparent despite the market's over-the-counter nature and modest activity." They estimate that realized bid-ask spreads for customers in the inflation swap market are on the order of three basis points. Conversations with inflation swap traders confirm that these instruments are fairly liquid with typical bid-ask spreads consistent with those reported by Fleming and Sporn. To guard against any possibility of using illiquid or stale prices in the sample, however, we only include an inflation swap rate when that rate has changed from the previous day. Table 1 presents summary statistics for the inflation swap rates.

As shown, average inflation swap rates range from 1.758 percent for one-year inflation swaps, to a high of 2.903 percent for 30-year inflation swaps. The volatility of inflation swap rates is generally declining in the maturity of the contracts. The dampened volatility of long-horizon inflation swap rates suggests that the market may view inflation as being strongly mean-reverting in nature. Table 1 also shows that there is evidence of deflationary concerns during the sample period. For example, the one-year swap rate reached a minimum of −4.545 percent during the height of the 2008 financial crisis amid serious fears about the U.S. economy sliding into a full-fledged depression/deflation scenario.

3.2 Inflation Options

The inflation options market had its inception in 2002 with the introduction of caps and floors on the realized inflation rate. While trading in inflation options was initially muted, the market gained considerable momentum as the financial crisis emerged and total interbank trading volume reached $100 billion.[6] While the inflation options market is not yet as liquid as, say, the stock index options market, the market is sufficiently liquid that active quotations for inflation cap and floor prices for a wide range of strikes have been readily available in the market since 2009.

In Europe and the United Kingdom, insurance companies are among the most active participants in the inflation derivatives market. In particular, much of the demand in ten-year and 30-year zero percent floors is due to pension funds trying to protect long inflation swaps positions. In contrast, insurance companies and financial institutions that need to hedge inflation risk are the most active participants on the demand side in the U.S. inflation options market.

The most actively traded inflation options are year-on-year and zero-coupon inflation options. Year-on-year inflation options are caps and floors that pay the difference between a strike rate and annual inflation on an annual basis. Zero-coupon options, in contrast, pay only one cash flow at the expiration date of the contract based on the cumulative inflation from inception to the expiration date. To illustrate, assume that the realized inflation rate over the next ten years was two percent. A ten-year zero-coupon cap struck at one percent would pay a cash flow of $\max(0, 1.0200^{10} - 1.0100^{10})$ at its expiration date. In this paper, we focus on zero-coupon inflation options since their cash flows parallel those of zero-coupon inflation swaps.

As with inflation swaps, we collect inflation cap and floor data from the Bloomberg system. Data are available for the period from October 5, 2009 to October 5, 2012 for strikes ranging from negative two percent to six percent in increments of 50 basis points. We check the quality of the data by insuring that the cap and floor prices included satisfy standard option pricing bounds such as those described in Merton (1973) including put-call parity, monotonicity, intrinsic value lower bounds, strike price monotonicity, slope, and convexity relations. To provide some perspective on the data, Table 2 provides summary statistics for call and put prices for selected strikes.

As illustrated, inflation cap and floor prices are quoted in basis points, or equivalently, as cents per $100 notional. Interestingly, inflation option prices are

[6]For a discussion of the inflation derivatives markets, see Jarrow and Yildirim (2003), Mercurio (2005), Kerkoff (2005), and Barclay's Capital (2010).

not always monotonically increasing in maturity. This may seem counterintuitive given standard option pricing theory, but is it important to recognize that the inflation rate is a macro variable rather than the price of a traded asset.[7] For most maturities, we have about 25 separate cap and floor prices with strikes varying from negative two percent to six percent from which to estimate the risk-neutral density of inflation.

4. MODELING INFLATION

In this section, we present the continuous time model used to describe the dynamics of inflation under both the objective and risk-neutral measures. We also describe the application of the model to the valuation of inflation swaps and options.

4.1 The Inflation Model

We begin with a few key items of notation. For notational simplicity, we will assume that all inflation contracts are valued as of time zero and that the initial price level at time zero is normalized to one.[8] Furthermore, time zero values of state variables are unsubscripted. Let I_t denote the relative change in the price level from time zero to time t.

Under the objective measure P, the dynamics of the price level are given by,

$$dI = I\ X\ dt\ +\ I\ \sqrt{V}\ dZ_I, \tag{1}$$

$$dX = \kappa\ (Y\ -\ X)\ dt\ +\ \sigma\ dZ_X, \tag{2}$$

$$dY = (\alpha\ -\ \beta\ Y)\ dt\ +\ \eta\ dZ_Y, \tag{3}$$

$$dV = \mu\ dt\ +\ s\ dZ_V. \tag{4}$$

In this specification, X_t represents the instantaneous expected inflation rate. The state variable Y_t represents the long-run trend in expected inflation towards which the process X_t reverts. The process V_t represents the stochastic volatility of realized inflation. An important implication of stochastic volatility is that extreme declines in the price level can occur during periods of high volatility, which

[7]We observe that similar nonmonotonic behavior occurs with interest rate options such as interest rate caps, floors, and swaptions; see Longstaff, Santa-Clara, and Schwartz (2001).

[8]Since the initial price level equals one, we will further simplify notation by not showing the dependence of valuation expressions on the initial price level I.

may resemble large downward jumps. Thus, this specification is consistent with the intuition of deflation representing an economic tail risk or event risk. Clearly, the same argument also holds for inflation. Rather than fully parameterizing the dynamics for V at this stage, we leave the drift and diffusion terms μ and s unspecified and allow for the possibility that they may depend on a vector of additional state variables.[9] The processes Z_I, Z_X, Z_Y, and Z_V are Brownian motions. The correlation between dZ_X and dZ_Y is $\rho\, dt$, the correlation between dZ_I and dZ_V is $\theta\, dt$, and the remaining correlations are assumed to be zero. This primarily affine specification has parallels to the long-run risk model of Bansal and Yaron (2004) and allows for a wide range of possible time series properties for realized inflation.

Under the risk-neutral valuation measure Q, the dynamics of the price level are given by

$$dI = I\ X\ dt\ +\ I\ \sqrt{V}\ dZ_I, \tag{5}$$
$$dX = \lambda\ (Y\ -\ X)\ dt\ +\ \sigma\ dZ_X, \tag{6}$$
$$dY = (\phi\ -\ \gamma\ Y)\ dt\ +\ \eta\ dZ_Y, \tag{7}$$
$$dV = \mu\ dt\ +\ s\ dZ_V, \tag{8}$$

where the parameters λ, ϕ, and γ that now appear in the system of equations allow for the possibility that the market incorporates time-varying inflation-related risk premia into asset prices. In particular, the model allows the risk-neutral distributions of X, Y, and I to differ from the corresponding distributions under the objective measure. Thus, the model permits a fairly general structure for inflation risk premia. On the other hand, the model assumes that variation in the state variable V is not priced in the market. This assumption appears to be a modest one and has the important advantage of making the analysis much more tractable. We acknowledge, however, that more general types of risk premium specifications are possible.

Finally, let r_t denote the nominal instantaneous riskless interest rate. We can express this rate as $r_t = R_t + X_t$ where R_t is the real riskless interest rate and X_t is expected inflation. For tractability, we also assume that R_t is uncorrelated with the other state variables I_t, X_t, Y_t, and V_t.

[9]Although we model V as being driven by a (possibly vector) Brownian motion, the model could easily be extended to allow for a jump-diffusion specification for the stochastic volatility of the inflation process. This specification would be completely consistent with our empirical approach.

4.2 Valuing Inflation Swaps

From the earlier discussion, an inflation swap pays a single cash flow of $I_T - F$ at maturity date T, where F is the inflation swap price set at initiation of the contract at time zero. Note that $F = (1+f)^T$ where f is the inflation swap rate. The Appendix shows that the inflation swap price can be expressed in closed form as

$$F(X, Y, T) = \exp\left(-A(T) - B(T)X - C(T)Y\right), \tag{9}$$

where

$$\begin{aligned}
A(T) = {} & \frac{\sigma^2}{2\lambda^2}\left(T - \frac{2}{\lambda}(1 - e^{-\lambda T}) + \frac{1}{2\lambda}(1 - e^{-2\lambda T})\right) \\
& - \frac{\sigma\eta\rho}{\gamma\lambda(\lambda-\gamma)}\Bigg(\gamma(T - \frac{2}{\lambda}(1 - e^{-\lambda T}) + \frac{1}{2\lambda}(1 - e^{-2\lambda T})) \\
& - \lambda(T - \frac{1}{\lambda}(1 - e^{-\lambda T}) - \frac{1}{\gamma}(1 - e^{-\gamma T}) + \frac{1}{\gamma+\lambda}(1 - e^{-(\gamma+\lambda)T}))\Bigg) \\
& + \frac{\eta^2}{2\gamma^2(\lambda-\gamma)^2}\Bigg(\gamma^2(T - \frac{2}{\lambda}(1 - e^{-\lambda T}) + \frac{1}{2\lambda}(1 - e^{-2\lambda T})) \\
& - 2\gamma\lambda(T - \frac{1}{\lambda}(1 - e^{-\lambda T}) - \frac{1}{\gamma}(1 - e^{-\gamma T}) + \frac{1}{\gamma+\lambda}(1 - e^{-(\gamma+\lambda)T})) \\
& + \lambda^2(T - \frac{2}{\gamma}(1 - e^{-\gamma T}) + \frac{1}{2\gamma}(1 - e^{-2\gamma T}))\Bigg) \\
& + \frac{\phi}{\gamma(\lambda-\gamma)}((\gamma-\lambda)T - \frac{\gamma}{\lambda}(1 - e^{-\gamma T}) + \frac{\gamma}{\lambda}(1 - e^{-\gamma T})),
\end{aligned} \tag{10}$$

$$B(T) = \frac{-(1 - e^{\lambda T})}{\lambda}, \tag{11}$$

$$C(T) = \frac{\gamma(1 - e^{-\lambda T}) - \lambda(1 - e^{-\gamma T})}{\gamma(\lambda-\gamma)}, \tag{12}$$

4.3 Valuing Inflation Options

Let $C(X, Y, V, T)$ denote the time zero value of a European inflation cap or call option with strike K. The payoff on this option at expiration date T is $\max(0, I_T - (1+K)^T)$. The Appendix shows that the value of the call option at time zero can be expressed as

$$C(X, Y, V, T) = D(T)\ E^{Q^*}[\max(0, I_T - (1+K)^T)], \tag{13}$$

where $D(T)$ is the price of a riskless discount bond with maturity T, and the expectation is taken with respect to the adjusted risk-neutral measure Q^* for inflation defined by the following dynamics,

$$\begin{aligned}
dI &= I\ X\ dt\ +\ I\ \sqrt{V}\ dZ_I, & (14)\\
dX &= (\lambda\ (Y\ -\ X) + \sigma^2 B(T-t) + \rho\sigma\eta C(T-t))\ dt\ +\ \sigma\ dZ_X, & (15)\\
dY &= (\alpha\ -\ \beta\ Y + \eta^2 C(T-t) + \rho\sigma\eta B(T-t)))\ dt\ +\ \eta\ dZ_Y, & (16)\\
dV &= \mu\ dt\ +\ s\ dZ_V. & (17)
\end{aligned}$$

The adjustment to the risk-neutral measure arises because the inflation rate is correlated with the riskless interest rate and allows us to discount the option cash flow outside of the expectation.[10] This adjusted measure has been referred to variously as a certainty-equivalent measure or a forward measure in the literature. The Appendix also shows that under this measure, the expected value of I_T equals the inflation swap price F. In this paper, we focus primarily on the adjusted risk-neutral density which will be implied from inflation option prices. To streamline the discussion, however, we will typically refer to the implied density simply as the risk-neutral density. A similar representation holds for the value of an inflation floor or put option $P(X, Y, V, T)$ with payoff at expiration date T of $\max(0, (1+K)^T - I_T)$.

4.4 The Distribution of the Price Level

From the dynamics given above, an application of Itô's Lemma implies that the log of the relative price level can be expressed as,

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[10]See Jamshidian (1989) and Longstaff (1990) for a discussion of this adjustment to the risk-neutral measure.

$$\ln I_T = \int_0^T X_s \ ds$$
$$- \frac{1}{2}\int_0^T V_s \ ds + \int_0^T \sqrt{V_s} \ dZ_V. \tag{18}$$

The Appendix shows that this can be expressed as

$$\ln I_T = u_T + w_T, \tag{19}$$
$$\ln I_T = v_T + w_T, \tag{20}$$

under the (adjusted) risk-neutral and objective measures, respectively, where u_T and v_T are normally distributed random variates. The terms u_T and v_T are simply the value of the integral on the right hand side in the first line in Equation (18) under the respective measures, where the distribution of this integral is different under each of the two measures. It is important to observe that both u_T and v_T are independent of the value of w_T, where w_T represents the term on the second line in Equation (18). This latter feature, in conjunction with the explicit solutions for the densities of u_T and v_T provided in the Appendix, will allow us to solve directly for the objective density of $\ln I_T$ given the risk-neutral density.

5. MODEL ESTIMATION

In identifying the distribution of inflation, we use a simple three-step approach. First, we solve for the risk-neutral distribution of inflation embedded in the prices of inflation caps and floors having the same maturity but differing in their strike prices. Second, we identify the inflation risk premia by maximum likelihood estimation of an affine model of the term structure of inflation swaps. Third, we make the transformation from the implied risk-neutral distribution to the objective distribution of inflation.

5.1 Solving for the Risk-Neutral Distribution

There is an extensive literature on the estimation of risk-neutral distributions from option prices. Key examples include Banz and Miller (1978), Breeden and Litzenberger (1978), Longstaff (1995), Aït-Sahalia and Lo (1998), and others.

In modeling the risk-neutral distribution, it is important to allow for very general types of densities while preserving sufficient structure for the results to be interpretable. Accordingly, we assume that the density $h(z)$ of the continuously-compounded inflation rate $z = \ln(I_T)/T$ under the (adjusted) risk-neutral measure is a member of the five-parameter class of generalized hyperbolic densities. As shown by Ghysels and Wang (2011), this broad class of distributions nests many of the distributions that appear in the financial economics literature including the normal, gamma, Student t, Cauchy, variance gamma, normal inverse Gaussian, normal inverse chi-square, generalized skewed t, and hyperbolic distributions. The generalized hyperbolic density is given by

$$h(z) = \frac{(a^2 - b^2)^{q/2} d^{-q} e^{b(z-c)}}{\sqrt{2\pi} a^{q-1/2} K_q(d\sqrt{a^2 - b^2})} \frac{K_{q-1/2}(a\sqrt{d^2 + (z-c)^2})}{(\sqrt{d^2 + (z-c)^2})^{1/2-q}}, \tag{21}$$

where a, b, c, d, and q are parameters, and $K_q(\,\cdot\,)$ denotes the modified Bessel function (see Abramowitz and Stegun (1965), Chapter 10).

We solve for the implied risk-neutral density in the following way. For each date and horizon, we collect prices for all available inflation caps and floors. Typically, we have prices for roughly 25 caps and floors with strike prices ranging from negative two percent to six percent in steps of 50 basis points. Next, we solve for the five parameter generalized hyperbolic density that results in the best fit to the set of cap and floor prices, while requiring that the model exactly match the corresponding inflation swap rate.[11] With this latter condition, there are essentially four free parameters that can be optimized to fit the cross-section of option prices. To value the options, we numerically integrate the product of the option payoff and the density. The optimization algorithm solves for the parameter vector that minimizes the sum of squared pricing errors, where each option receives equal weight. We then repeat this process for each day in the sample period and for each horizon of option expirations, one, two, three, five, seven, ten, 20, and 30 years.[12] Although not shown, the algorithm is able to fit the inflation cap and floor prices very accurately. In particular, the model prices are typically within several percent of the corresponding market prices and would likely be well within the actual bid-ask spreads for these options.

5.2 Maximum Likelihood Estimation

[11]This latter condition implicitly requires that the moment generating function for the density be finite. This requirement places some mild restrictions on the parameters which are incorporated in the fitting algorithm.

[12]We solve for the density of each option expiration horizon separately since the model allows for a general inflation specification rather than a specific representation. Thus, we place no a priori restrictions on the term structure of risk-neutral densities possible at a specific date.

As shown in Equation (9), the closed-form solution for inflation swap prices depends only on the two state variables X and Y that drive expected inflation. An important advantage of this feature is that it allows us to use standard affine term structure modeling techniques to estimate X and Y and their parameters under both the objective and risk-neutral measures. In doing this, we apply the maximum likelihood approach of Duffie and Singleton (1997) to the term structure of inflation swaps for maturities ranging from one to 30 years (but not for the 40 and 50 year maturities).

Specifically, we assume that the two-year and 30-year inflation swap rates are measured without error. Thus, given a parameter vector Θ, substituting these maturities into the log of the inflation swap expression in Equation (9) results in a system of two linear equations

$$\ln F(X,Y,2) = -A(2) - B(2)X - C(2)Y, \tag{22}$$

$$\ln F(X,Y,30) = -A(30) - B(30)X - C(30)Y, \tag{23}$$

in the two state variables X and Y. This means that X and Y can be expressed as explicit linear functions of the two inflation swap prices $F(X,Y,2)$ and $F(X,Y,30)$. Let J denote the Jacobian of the mapping from the two swap rates into X and Y.

At time t, we can now solve for the inflation swap rate implied by the model for any maturity from the values of X_t and Y_t and the parameter vector Θ. Let ϵ_t denote the vector of differences between the market value and the model value of the inflation swaps for the other maturities implied by X_t, Y_t, and the parameter vector Θ. Under the assumption that ϵ_t is conditionally multivariate normally distributed with mean vector zero and a diagonal covariance matrix Σ with main diagonal values v_j (where the subscripts denote the maturities of the corresponding inflation swaps), the log of the joint likelihood function LLK_t of the two-year and 30-year inflation swap prices and $\epsilon_{t+\Delta t}$ conditional on the inflation swap term structure at time t is given by

$$\begin{aligned} = &- \ln(2\pi\sigma_X\sigma_Y\sqrt{1-\rho_{XY}}) - \frac{1}{2(1-\rho_{XY}^2)}\left[\left(\frac{(X_{t+\Delta t}-\mu_{X_t})^2}{\sigma_X^2}\right)\right. \\ &\left. - 2\rho_{XY}\left(\frac{(X_{t+\Delta}-\mu_{X_t})}{\sigma_X}\right)\left(\frac{(Y_{t+\Delta t}-\mu_{Y_t})}{\sigma_Y}\right) + \left(\frac{(Y_{t+\Delta t}-\mu_{Y_t})^2}{\sigma_Y^2}\right)\right], \end{aligned} \tag{24}$$

where the conditional moments μ_{X_t}, μ_{Y_t}, σ_X, σ_Y, and $\rho_{XY} = \sigma_{XY}/\sqrt{\sigma_X^2\sigma_Y^2}$ of

$X_{t+\Delta t}$ and $Y_{t+\Delta t}$ are given in the Appendix. The total log likelihood function is given by summing LLK_t over all values of t.

We maximize the log likelihood function over the 22-dimensional parameter vector $\Theta = \{\kappa, \sigma, \alpha, \beta, \eta, \rho, \lambda, \phi, \gamma, v_1, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{12}, v_{15}, v_{20}, v_{25}\}$ using a standard quasi-Newton algorithm with a finite difference gradient. As a robustness check that the algorithm achieves the global maximum, we repeat the estimation using a variety of different starting values for the parameter vector. Table 3 reports the maximum likelihood estimates of the parameters and their asymptotic standard errors. The fitting errors from the estimation are all relatively small with the typical standard deviation ranging from roughly six to to ten basis points, depending on maturity.

5.3 Solving for the Objective Distribution

Let $\Phi(x;\omega)$ denote the characteristic function for the density function $h(x)$,

$$\Phi(x;\omega) = \textstyle\int_{-\infty}^{\infty} e^{i\omega x}\ h(x)\ dx. \tag{25}$$

Recall from the earlier discussion that the log of the relative price level can be expressed as $u_T + w_T$ under the risk-neutral measure, and as $v_T + w_T$ under the objective measure, where w_T is independent of u_T and v_T. Using the properties of characteristic functions, it is easily shown that

$$\Phi(v_T + w_T;\omega) = \frac{\Phi(u_T + w_T;\omega)\ \Phi(v_T;\omega)}{\Phi(u_T;\omega)}. \tag{26}$$

Thus, given the densities for u_T and v_T, once we can identify the characteristic function of the price $u_T + w_T$ under the risk-neutral measure, we can immediately solve for the characteristic function of the log of the relative price level $u_T + w_T$ under the objective measure. Given this characteristic function $\phi(v_T + w_T)$, we can recover the cumulative density function $\Psi(\ln(I_T)/T)$ of the realized inflation rate using the Gil-Pelaez inversion integral,

$$\Psi(z) = \frac{1}{2} - \frac{1}{\pi}\int_0^{\infty} \frac{\mathrm{Im}[e^{-i\omega z}\phi(v_T + w_T;\omega)]}{\omega} d\omega, \tag{27}$$

where Im[$\cdot$] represents the imaginary component of the complex-valued argument. Once the cumulative distribution function for the inflation rate $z = \ln(I_T)/T$ is determined, the cumulative distribution function for the relative price level I_T is obtained by a simple change of variables.

6. THE DISTRIBUTION OF INFLATION

As a preliminary to the analysis of deflation risk, it is useful to first present the empirical results for inflation risk premia, expected inflation, inflation volatility, and the higher moments of inflation.

6.1 Inflation Densities

To provide some perspective on the nature of the inflation density under the objective measure, Figure 1 plots the time series of densities of inflation for several horizons. As shown, there is considerable variation in the shape of the inflation distribution for the shorter horizons. In contrast, the distribution of inflation for longer horizons is more stable over time.

6.2 Inflation Risk Premia

We measure the inflation risk premium by simply taking the difference between the fitted inflation swap and expected inflation rates. This is the way in which many market participants define inflation risk premia. When the inflation swap rate is higher than expected inflation, the inflation risk premium is positive, and vice versa. There is no compelling theoretical reason why the inflation risk premium could not be negative in sign. In this case, the risk premium might well be viewed as a deflation risk premium.

Table 4 presents summary statistics for the average inflation risk premia for horizons ranging from one year to 30 years. Figure 2 plots the time series of inflation risk premia for a number of horizons. As shown, the average risk premia are slightly negative for horizons out to five years, but are positive for longer horizons and reach a value of about 30 basis points at the 30-year horizon. The inflation risk premia vary significantly through time, although the volatility of inflation risk premia for longer horizons is slightly higher than for shorter horizons.

These inflation risk premia estimates are broadly consistent with previous estimates obtained using alternative approaches by other researchers. For example, Haubrich, Pennachi, and Ritchken (2012) estimate the ten-year and 30-year inflation risk premia to be 51 and 101 basis points, respectively. Buraschi and Jiltsov (2005) and Campbell and Viceira (2001) estimate the ten-year inflation risk premium to be 70 and 110 basis points, respectively. Ang, Bekaert, and Wei (2008) estimate the five-year inflation risk premium to be 114 basis points. In addition, the fact that all of the estimated risk premia take negative values at some point during the sample period is consistent with the findings of Campbell, Shiller, and Viceria (2009), Bekaert and Wang (2010), and others.

6.3 Expected Inflation

To solve for the expected inflation rate for each horizon, we use the inflation swap rates observed in the market and adjust them by the inflation risk premium implied by the fitted model. Table 5 presents summary statistics for the expected inflation rates for the various horizons.

The results indicate that the average term structure of inflation expectations is monotonically increasing during the 2004–2012 sample period. The average one-year expected inflation rate is 1.776 percent, while the average 30-year expected inflation rate is 2.597 percent. The table also shows that there is time variation in expected inflation, although the variation is surprisingly small for longer horizons. In particular, the standard deviation of expected inflation ranges from 1.348 percent for the one-year horizon to less than 0.20 percent for horizons of ten years or longer. To illustrate the time variation in expected inflation more clearly, Figure 3 plots the expected inflation estimates for the five-year, ten-year, and 30-year horizons.

It is also interesting to contrast these market-implied forecasts of inflation with forecasts provided by major inflation surveys. As discussed by Ang, Bekaert, and Wei (2007), these surveys of inflation tend to be more accurate than those based on standard econometric models and are widely used by market practitioners. Furthermore, these inflation surveys have also been incorporated into a number of important academic studies of inflation such as Fama and Gibbons (2004), Chernov and Mueller (2012) and Haubrich, Pennachi, and Ritchken (2012).

We obtain inflation expectations from four surveys: the University of Michigan Survey of Consumers, the Philadelphia Federal Reserve Bank Survey of Professional Forecasters (SPF), the Livingston Survey, and the survey of market participants conducted by Bloomberg. The sample period for the forecasts matches that for the inflation swap data in the study. The Appendix provides the background information and details about how these surveys are conducted.

Table 6 reports the average values of the various surveys during the sample period and the corresponding average values for the market-implied forecasts. These averages are computed using the month-end values for the months in which surveys are released. Thus, monthly averages are compared with monthly averages, quarterly averages with quarterly averages, etc. As shown, the average market-implied forecasts of inflation tend to be a little lower than the survey averages for shorter horizons. The market-implied forecasts, however, closely parallel those from the surveys for longer horizons. While it would be interesting to compare the relative accuracy of the market-implied and survey forecasts, our sample is too short to do this rigorously.

6.4 Inflation Volatility and Higher Moments

Table 7 reports the average values of the estimated volatility, skewness, and excess kurtosis of the continuously-compounded inflation rate for horizons ranging from one year to 30 years. The average inflation volatility estimates range from a high of about 2.258 percent at the two-year horizon to a low of about 0.693 percent at the 30-year horizon. The dampened volatility at the longer horizons is consistent with a scenario in which inflation is anticipated to follow a mean reverting process.

The distribution of inflation is typically negatively skewed for all horizons. The negative skewness is particularly pronounced for horizons of less than ten years, but is still evident in the distribution of inflation over a 30-year horizon. The median excess kurtosis coefficients are all positive (with the exception of the 30-year horizon), indicating that the distribution of inflation has heavier tails than a normal distribution.

7. DEFLATION RISK

We turn now to the central issue of measuring the risk of deflation implied by market prices and studying the properties of deflation risk. First, we present descriptive statistics for the implied deflation risk. We then examine how the market prices the tail risk of deflation and contrast the results with those found in other markets.

7.1 How Large is the Risk of Deflation?

Having solved for the characteristic function for the inflation distribution, we can apply standard inversion techniques to solve for the cumulative distribution function for inflation. In turn, we can then directly compute the probability that the average realized inflation rate over a specific horizon is less than zero, which represents the risk of deflation. Table 8 provides summary statistics for the estimated probabilities of deflation over the various horizons. To provide some additional perspective, Figure 4 graphs the time series of probabilities of a deflation over one-year, two-year, five-year, and ten-year horizons.

As shown, the market places a surprisingly large weight on the possibility that deflation may occur over extended horizons. In particular, the average probability that the realized inflation rate will be less than or equal to zero is 17.25 percent for a one-year horizon, 11.44 percent for a two-year horizon, 5.34 percent for a five-year horizon, 3.29 percent for a ten-year horizon, and ranges from two to three percent for longer horizons.

What is perhaps more striking is that the probability of deflation varies significantly over time and reaches relatively high levels during the sample period. For example, the probability of deflation reaches a value of 44.37 percent for a one-year horizon, 23.04 percent for a two-year horizon, and 11.39 for a five-year horizon. At other times, the market assesses the probability of deflation at any horizon to be only on the order of one to two percent. This variation in the probability of deflation is due not only to changes in expected inflation, but also to changes in the volatility of inflation.

These probabilities are broadly consistent with the historical record on inflation in the U.S. For example, based on the historical inflation rates from 1800 to 2012, the U.S. has experienced deflation over a one-year horizon 65 times, which represents a frequency of 30.5 percent. Considering only nonoverlapping periods, the U.S. has experienced a two-year deflation 41 times, a five-year deflation 19 times, a ten-year deflation 11 times, and a 30-year deflation three times. These translate into frequencies of 24.0 percent, 14.4 percent, 10.6 percent, and 3.1 percent, respectively.[13]

Figure 4 also shows that the deflation probabilities for the shorter horizons have occasional jumps upward. These jumps tend to occur around major financial events such as those associated with the European Debt Crisis. For example, the Eurozone experienced major turmoil during April and May of 2010 as concerns about the ongoing solvency of Portugal, Italy, Ireland, Greece, and Spain become more urgent and a number of bailout plans were put into place. Spain's debt was first downgraded by Fitch on May 29, 2010. Similarly, the five-year deflation probability nearly doubles during the last week of September 2010 which coincides with the downgrade of Spain by Moodys. In addition, the one-year deflation probability spikes again in early August of 2011, coinciding with the downgrade of U.S. Treasury debt by Standard and Poors. We will explore the link between deflation risk and major financial risk more formally later in the paper.

Although not shown, we also calculate the partial moment in which we take the expected value of inflation conditional on the inflation rate being less than or equal to zero. This partial moment provides a measure of the severity of a deflation, conditional on deflation occuring over some horizon. For example, finding that this partial moment was only slightly negative would argue that a deflationary episode was likely to be less severe, while the opposite would be true for a more negative value of this partial moment. The results indicate that the

[13]Historical inflation rates are tabulated by Sahr (2012), oregonstate.edu/cla/polisci/faculty-research/sahr/infcf17742007.pdf. More recent inflation rates are reported by the Bureau of Labor Statistics.

expected severity of a deflation is typically very substantial with these conditional moments increasing from about -1.60 percent for a one-year deflation, to -1.85 for a five-year horizon, and then decreasing to -1.15 percent for a 20-year horizon. On average, the expected value of deflation over all of the horizons is -1.56 percent. Note that a deflation of -1.56 percent per year would translate into a decline in the price level of 7.6 percent over a five-year period, 15.5 percent over a ten-year period, and 27.0 percent over a 20-year period. These would represent protracted deflationary episodes comparable in severity to many of those experienced historically in the U.S.

7.2 Pricing Deflation Tail Risk

Although we have solved for the inflation risk premium embedded in inflation swaps earlier in the paper, it is also interesting to examine how the market prices the risk that the tail event of a deflation occurs. This analysis can provide insight into how financial market participants view the risk of events that may happen infrequently, but which may have catastrophic implications.

A number of these types of tail risks have been previously studied in the literature. For example, researchers have investigated the pricing of catastrophic insurance losses such as those caused by hurricanes or earthquakes. Froot (2001) finds that the ratio of insurance premia to expected losses in the market for catastrophic reinsurance ranges from about two to seven during the 1989 to 1998 period. Lane and Mahul (2008) estimate that the pricing of catastrophic risk in a sample of 250 catastrophe bonds is about 2.69 times the actual expected loss over the long term. Garmaise and Moskowitz (2009) and Ibragimov, Jaffee, and Walden (2009) offer both empirical and theoretical evidence that the extreme left tail catastrophic risk can be significantly priced in the market.

The default of a corporate bond is also an example of an event that is relatively rare for a specific firm, but which would result in an extremely negative outcome for bondholders of the defaulting firm. The pricing of default risk has been considered in many recent papers. For example, Giesecke, Longstaff, Schaefer, and Strebulaev (2011) study the pricing of corporate bond default risk and find that the ratio of corporate credit spreads to their actuarial expected loss is 2.04 over a 150-year period. Similarly, Driessen (2005) and Berndt, Duffie, Douglas, Ferguson, and Schranz (2005) estimate ratios using data for recent periods that range in value from about 1.8 to 2.8.

Following along the lines of this literature, we solve for the ratio of the risk-neutral probability of deflation to the objective probability of deflation. This ratio provides a simple measure of how the market prices the tail risk of deflation and has the advantage of being directly comparable to the ratios discussed above.

Table 9 presents the means and medians for the ratios for the various horizons. As shown, the mean and median ratios range from between one and two to slightly higher than five. The overall average of the ratios is 3.321 and the overall median of the ratios is about 3.166. These values are in the same ballpark as those for the different types of tail risk discussed above. These ratios all indicate that the market is deeply concerned about financial and economic tail risks that may be difficult to diversify or may be strongly systematic in nature.

8. WHAT DRIVES DEFLATION RISK?

A key advantage of our approach is that by extracting the market's assessment of the objective probability of deflation, we can then examine the relation between these probabilities and other financial and macroeconomic factors. In particular, we can study the relation between the tail risk of deflation and other types of tail risk that may be present in the markets.

In doing this, we will focus on four broad categories of tail risk that have been extensively discussed in the literature. Specifically, we will consider the links between deflation risk and systemic financial system risk, collateral revaluation risk, sovereign default risk, and business cycle risk.

The link between systemic risk in the financial system and major economic crisis is well established in many important papers including Bernanke (1983), Bernanke, Gertler, and Gilchrist (1996), and others. Systemic risk in the financial system is widely viewed as having played a central role in the recent global financial crisis and represents a motivating force behind major regulatory reforms such as the Dodd-Frank Act. We use a number of measures of systemic risk in the analysis.

First, we use a measure of the flight-to-liquidity risk in the market which is computed as the spread between a one-year zero-coupon Refcorp bond and a corresponding maturity zero-coupon Treasury bond. This variable is introduced in Longstaff (2004) as a measure of the premium that market participants place on Treasuries because of their role as the "safest" asset in the financial markets during episodes when investors fear that massive losses will occur in less liquid markets. We obtain the data for the flight-to-liquidity spread from the Bloomberg system.

Second, we use data on the pricing of super senior tranches on a basket of corporate debt to measure the risk of a major systemic collapse in the credit markets. Specifically, we collect data on the points-up-front pricing on the five-year 10-15 percent tranche on the CDX IG index. This index is computed as an average of the five-year CDS spreads for 125 U.S. firms with investment grade ratings. The 10-15 percent tranche would only experience losses if the total credit losses on the index exceeded 10 percent of the total notional of an equally-weighted basket of the underlying debt obligations of these firms. As shown by Coval, Jurek, and Stafford (2009) and Giesecke, Longstaff, Schaefer, and Strebulaev (2011), a major meltdown that would produce losses of this magnitude in a portfolio of investment grade corporate debt would be a very rare tail event. The points-up-front price for this tranche represents the market price to compensate investors for taking the risk of this extreme scenario.[14] We obtain data on the pricing of the super senior tranche from the Bloomberg system.

Third, we use the spread between the one-year Libor rate and a one-year Treasury bond as a proxy for the systemic credit and liquidity risk embedded in the Libor rate. The data are from the Bloomberg system.

Fourth, we use the five-year swap spread as a measure of the systemic credit and liquidity stresses on the financial system. As discussed by Duffie and Singleton (1997), Liu, Longstaff, and Mandell (2006), and others, the swap spread reflects differences in the relative liquidity and credit risk of the financial sector and the Treasury. We obtain five-year swap spread data from the Bloomberg system.

We also considered a number of other measures of systemic risk such as the average CDS spread for both major U.S. and non-U.S. banks and financial firms. These measures, however, were highly correlated with the other measures such as swap spreads and provided little incremental information.

Recent economic theory has emphasized the role that the value of collateral plays in propagating economic downturns. Key examples include Kiyotaki and Moore (1997) who show that declines in asset values can lead to contractions in the amount of credit available in the market which, in turn, can lead to further rounds of declines in asset values. Bernanke and Gertler (1995) describe similar interactions between declines in the value of assets that serve as collateral and severe economic downturns. Collateral revaluation risk, or the risk of a broad decline in the market value of leveraged assets, played a major role in the Great Depression as the sharp declines in the values of stock and corporate bonds triggered waves of defaults among both speculators and banks. A similar mechanism was present in the recent financial crisis as sharp declines in real estate values led to massive defaults by "underwater" mortgagors. In the context of this study, we explore the relation between deflation probabilities and valuations in several major asset classes that may represent important sources of collateral in the credit markets: stocks and bonds. In addition, we also include measures

[14]For a description of the CDX index tranche markets and the pricing of CDO tranches, see Longstaff and Rajan (2008) and Longstaff and Myers (2013).

of the volatility of these asset classes since these measures provide information about the risk that large downward revaluations in these forms of collateral may occur.

The first of these proxies for collateral revaluation risk is the VIX index of implied volatility for options on the S&P 500. This well-known index is often termed the "fear index" in the financial press since it reflects the market's assessment of the risk of a large downward movement in the stock market. We collect VIX data from the Bloomberg system.

The second measure is the Merrill Lynch MOVE index of implied volatilities for options on Treasury bonds. This index is essentially the fixed income counterpart of the VIX index. This index also captures the market's views of the likelihood of a large change in the prices of Treasury bonds, which are widely used as collateral for a broad variety of credit transactions. This data is also obtained from the Bloomberg system.

The third measure is simply the time series of daily returns on the S&P 500 index (price changes only). This return series reflects changes in the value of one of the largest potential sources of collateral in the macroeconomy. We compute these returns from S&P 500 index values reported in the Bloomberg system.

The fourth measure is the spread between the yield for the Moody's Baa-rated index of corporate bonds and the yield on five-year Treasury bonds. Variation in this credit spread over time reflects changes in the market's assessment of default risk in the economy as well as the pricing of credit risk. We collect data on the Baa-Treasury spread from the Bloomberg system.

Another major type of economic tail risk stems from the risk that a sovereign defaults on its debt. As documented by Reinhart and Rogoff (2009) and many others, sovereign defaults tend to be associated with severe economic crisis scenarios.

As a measure of the tail risk of a sovereign default by the U.S., we include in the analysis the time series of sovereign CDS spreads on the U.S. Treasury. Ang and Longstaff (2012) show that the U.S. CDS spread reflects variation in the valuation of major sources of tax revenue for the U.S. such as capital gains on stocks and bonds. This data is also obtained from the Bloomberg system.

Finally, to capture the effect of traditional types of business cycle risk or economic downturn risk, we also include a number of key macroeconomic variables that can be measured at a monthly frequency. In particular, we include the monthly percentage change in industrial production as reported by the Bureau of Economic Analysis, the monthly change in the national unemployment rate as reported by the Bureau of Labor Statistics, and the change in the Consumer

Confidence Index reported by the University of Michigan. The link between the business cycle and its effects on output and employment are well established in the macroeconomic literature and forms the basis of many classical theories including the Phillips curve.

Since these measures of systemic, collateral, and sovereign default risk are all available on a daily basis, we begin our analysis by regressing daily changes in the deflation probabilities on daily changes in these variables (the macroeconomic variables, which are only observed monthly, will be included in later regressions). In doing this, it is important to note that while these variables were chosen as a measure of a specific type of tail risk, most of these variables may actually reflect more than one type of tail risk. Thus, the effects of the variables in the regression should be interpreted carefully since the different types of tail risk need not be mutually exclusive.

Table 10 presents summary statistics for the regression results. The results indicate that the variables proxying for systemic risk are often statistically significant for a number of the horizons. In particular, the coefficient for the super senior tranche price is significant for four of the horizons. The sign of this coefficient is uniformly positive in sign for all but the longest horizons, which is clearly consistent with the intuition that an increase in the extreme type of tail risk reflected in the tranche price would be associated with an economic meltdown in which deflation occured. The Libor spread is positive and significant for the two shortest horizons. The positive sign of this effect is also consistent with our intuition about the effects of a systemic financial crisis on the macroeconomy. The five-year swap spread has some of the strongest effects in the regression. In particular, it is significant for five of the ten horizons. Interestingly, the signs of the significant coefficients are all positive, with the exception of the shortest horizon. This is again consistent with an economic scenario in which stress in the financial sector leads to an increase in the perceived risk of an adverse macroeconomic shock in which price levels decline.

Surprisingly, Table 10 shows that neither of the two volatility variables has much explanatory power for changes in deflation risk. In contrast, both S&P 500 index returns and changes in the Baa credit spread are often significant. For example, the stock market return is significant for the one-year, two-year, and 30-year horizons. In each of these three cases, the sign is negative, indicating that an increase in the stock market reduces fears about deflation. This is again very intuitive and completely consistent with a simple collateral revaluation interpretation. The Baa credit spread is also significant for three of the horizons. The signs of the significant coefficients for the one-year and two-year horizons are both positive, indicating that deflation risk increases as credit fears in the economy increase. It is also interesting to note that the sign of the coefficients for this variable become negative for all longer horizons. Thus, the long-run effects of increased credit risk on deflation risk are somewhat counterintuitive.

Finally, Table 10 shows that the effect of an increase in U.S. CDS spreads on deflation risk is relatively limited. The coefficient for changes in U.S. CDS spreads is only significant for the one-year and 15-year horizons. In addition, the signs of these two significant coefficients differ from each other.

The overall R^2s from the regressions are fairly modest, ranging from just under two percent to more than nine percent. Note, however, that these results are based on daily changes in these variables. Thus, given the challenges in measuring tail risks, the explanatory power of these regressions is far from negligible.

Turning now to regressions in which we include the macroeconomic variables, we observe that since our sample period is relatively short, it is important to use a parsimonious specification. Accordingly, we regress monthly changes in the deflation probabilities (using the last deflation probability for each month) on a selected set of variables. Specifically, as proxies for systemic risk, we include only monthly changes in the super senior prices and the five-year swap spread. As the proxy for collateral revaluation risk, we include only the change in the Baa credit spread. We then include the monthly percentage change in industrial production, the change in the unemployment rate, and the change in the Michigan Consumer Confidence Index. The regression results for horizons ranging from one to 20 years (there are too few observations for the 30-year horizon) are summarized in Table 11.

The results in Table 11 are consistent with those reported in Table 10. In particular, several of the proxies for systemic and collateral revaluation tail risk are again significant. The coefficient for the super senior tranche variable is significant for three of the horizons, with those for the one-year and two-year horizons again having a positive sign. The coefficient for the Baa credit spread is significant for four of the horizons. All four of these coefficients are positive in sign, indicating that increases in credit risk are associated with an increased risk of deflation.

The results for the macroeconomic variables are somewhat surprising. Far from being strongly related to deflation risk, none of the coefficients for changes in industrial production or the consumer confidence index are significant. The only macroeconomic variable that is significantly related to changes in deflation risk is the change in the unemployment rate which is significant and positive for three of the horizons. The positive sign of these coefficients is intuitive since it indicates that deflation risk increases as the unemployment rate increases. This is also consistent with classical macroeconomic theory about the relation between price levels and unemployment such as the Phillips curve.

In summary, the empirical results indicate that there is a strong relation between tail risk in financial markets and the risk of deflation. In particular, a number of the proxies for systemic financial risk and the value of potentially collateralizable financial assets are significantly linked to deflation risk. These results underscore the importance of understanding the role that the financial sector plays in economic downturns such as the recent financial crisis that began in the subprime structured credit markets. In contrast, these results suggest that more traditional macroeconomic variables such as industrial production may play less of a role in the risk of economic tail events such as a deflationary spiral.

9. INFLATION RISK

Although the focus of this paper is on deflation risk, it is straightforward to extend the analysis to other aspects of the distribution of inflation. As one last illustration of this, we compute the probabilities that the inflation rate exceeds values of four, five, and six percent using the techniques described earlier. Table 12 reports summary statistics for these probabilities.

As shown, the market-implied probabilities of experiencing significant inflation are uncomfortably large. Specifically, the average probability of inflation exceeding four percent is less that 15 percent for the two shortest horizons, but increases rapidly to nearly 30 percent for horizons ranging from ten to 30 years. Figure 5 plots the time series of probabilities that inflation exceeds four percent for several horizons. These plots also show that the probability of inflation in the long run appears substantially higher than in the short turn. This is exactly the opposite from the risk of deflation which tends to be higher in the short run.

Table 12 also shows that the average probability of an inflation rate in excess of five percent is substantial. An average inflation rate of five percent or more over a period of decades would rival any inflationary scenario experienced by the U.S. during the past 200 years. Finally, the results show that the market anticipates that there is roughly a three percent probability of inflation averaging more than six percent over the next several decades.

As we did earlier for deflation tail risk, we can also examine the pricing of inflation tail risk by computing the ratio of the probability of inflation (in excess of five percent) under the risk-neutral measure to the corresponding probability under the actual measure. Table 13 provides summary statistics for the ratios.

As illustrated, inflation tail risk is priced at all horizons. The magnitude of the inflation risk premium, however, is significantly smaller than is the case for deflation tail risk. In particular, the average ratio is only 1.463 and the median ratio is 1.441. These values are less than half of the corresponding value of 3.321 and 3.166 shown in Table 9 for deflation tail risk. Thus, these results suggest that the market requires far less compensation for the risk of inflation that it does for the risk of deflation. This is consistent with the view that deflations are associated with much more severe economic scenarios than are inflationary periods.

10. CONCLUSION

We solve for the objective distribution of inflation using the market prices of inflation swap and option contracts and study the nature of deflation risk. We find that the market-implied probabilities of deflation are substantial, even though the expected inflation rate is roughly 2.5 percent for horizons of up to 30 years. We show that deflation risk is priced by the market in a manner similar to that of other major types of tail risk such as catastrophic insurance losses or corporate bond defaults. By embedding a deflation floor into newly issued TIPS, the Treasury insures bondholders against deflation. Our findings imply that the Treasury receives a generous insurance premium in return. In contrast, the market appears much less concerned about inflation tail risk.

In theory, economic tail risks such as deflation may be related to other financial and macroeconomic tail risks. We study the relation between deflation risk and a number of measures of systemic financial risk, collateral revaluation risk, sovereign credit risk, and business cycle risk. We find that there is a significant relation between deflation risk and measures capturing stress in the financial system and credit risk in the economy. These results support the view that the risk of economic shocks severe enough to result in deflation is fundamentally related to the risk of major systemic shocks in the financial markets.

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APPENDIX

A.1. The Inflation Swap Rate.

From Equation (1), the relative price level index at time T can be expressed as

$$I_T = \exp\left(\int_0^T X_s \, ds\right) \, \exp\left(-\tfrac{1}{2}\int_0^T V_s \, ds + \int_0^T \sqrt{V_s} \, dZ_V(s)\right). \qquad (A1)$$

The cash flow associated with a zero-coupon inflation swap at time T is simply $I_T - F(X,Y,T)$ where $F(X,Y,T)$ is the inflation swap price at the initiation of the contract at time zero. Since the present value of the inflation swap is zero at inception we have,

$$E^Q\left[\exp\left(-\int_0^T r_s \, ds\right)(I_T - F(X,Y,T))\right] = 0. \qquad (A2)$$

Substituting in for r_t and I_T gives,

$$\begin{aligned}
&E^Q\left[\exp\left(-\int_0^T R_s \, ds\right)\right] \\
&\Bigg[\, E^Q\left[\exp\left(-\int_0^T X_s \, ds\right)\exp\left(\int_0^T X_s \, ds\right)\exp\left(-\tfrac{1}{2}\int_0^T V_s \, ds + \int_0^T \sqrt{V_s} \, dZ_V(s)\right)\right] \\
&- E^Q\left[\exp\left(-\int_0^T X_s \, ds\right) F(X,Y,T)\right] \,\Bigg] = 0, \qquad (A3)
\end{aligned}$$

which implies

$$F(X,Y,T) = \frac{E^Q\left[\exp\left(-\frac{1}{2}\int_0^T V_s \, ds + \int_0^T \sqrt{V_s} \, dZ_V(s)\right)\right]}{E^Q\left[\exp\left(-\int_0^T X_s \, ds\right)\right]}, \qquad (A4)$$

$$= \frac{1}{E^Q\left[\exp\left(-\int_0^T X_s \, ds\right)\right]}. \qquad (A5)$$

Let $H(X,Y,\tau)$ denote the value of the expectation $E^Q[\exp(\int_t^T X_s \, ds)]$, where $\tau = T - t$. Standard results imply that this expectation satisfies the partial differential equation

$$\tfrac{1}{2}\sigma^2 H_{XX} + \rho\sigma\eta H_{XY} + \tfrac{1}{2}\eta^2 H_{YY} \\ + \lambda(Y - X)H_X + (\phi - \gamma Y)H_Y - XH = H_\tau, \tag{A6}$$

subject to the terminal condition $H(X, Y, 0) = 1$. We conjecture a solution of the form $H(X, Y, \tau) = \exp(A(\tau) + B(\tau)X + C(\tau)Y)$. Taking derivatives of this expression and substituting into Equation ($A6$) results in a system of three linear first order ordinary differential equations for the horizon dependent functions $A(\tau)$, $B(\tau)$, and $C(\tau)$,

$$B' + \lambda B = -1, \tag{A7}$$
$$C' + \gamma C = \lambda B, \tag{A8}$$
$$A' = \tfrac{1}{2}\sigma^2 B^2 + \rho\sigma\eta BC + \tfrac{1}{2}\eta^2 C^2 + \phi C. \tag{A9}$$

These three equations are readily solved by the use of an integrating factor and direct integration. Substituting the solutions into the expression for $H(X, Y, \tau)$, substituting $H(X, Y, \tau)$ into Equation ($A5$), and then evaluating as of time zero ($\tau = T$) gives the expression for the inflation swap price in Equation (9).

A.2 Inflation Option Prices

Let $C(X, Y, V, T)$ denote the price at time zero of a European call option on the price level at time T with strike K. The cash flow at the option expiration date is $\max(0, I_T - (1 + K)^T)$. The present value of this cash flow can be expressed as

$$E^Q\left[\exp\left(-\textstyle\int_0^T r_s\ ds\right)\max(0, I_T - (1 + K)^T)\right], \tag{A10}$$

which can be written as

$$E^Q\left[\exp\left(-\textstyle\int_0^T R_s\ ds\right)\right] E^Q\left[\exp\left(-\textstyle\int_0^T X_s\ ds\right)\max(0, I_T - (1 + K)^T)\right], \tag{A11}$$

after substituting in for r_t. Let $N(I, X, Y, \tau)$ denote the value of the expectations $E^Q[\exp(-\int_t^T X_s\ ds)\max(0, I_T - (1 + K)^T)]$. Note that we show the explicit functional dependence of $N(I, X, Y, \tau)$ on I since I_t need not equal one for $t > 0$. The value of $N(I, X, Y, \tau)$ satisfies the following partial differential equation,

$$\frac{1}{2}I^2VN_{II} + \theta sI\sqrt{V}N_{IV} + \frac{1}{2}s^2N_{VV} + \frac{1}{2}\sigma^2N_{XX} + \rho\sigma\eta N_{XY} + \frac{1}{2}\eta^2N_{YY}$$
$$+ IXN_I + \mu N_V + \lambda(Y - X)N_X + (\phi - \gamma Y)N_Y - XN = N_\tau, \tag{A12}$$

subject to the terminal condition $N(I, X, Y, V, 0) = \max(0, I_T - (1 + K)^T)$. We conjecture that the solution is of the form

$$N(I, X, Y, V, \tau) = \exp(A(\tau) + B(\tau)X + C(\tau)Y)M(I, X, Y, V, \tau). \tag{A13}$$

Substituting in this expression into the partial differential equation in Equation (A12) and simplifying gives

$$\frac{1}{2}I^2VM_{II} + \theta sI\sqrt{V}M_{IV} + \frac{1}{2}s^2M_{VV} + \frac{1}{2}\sigma^2M_{XX} + \rho\sigma\eta M_{XY} + \frac{1}{2}\eta^2M_{YY}$$
$$+ IXM_I + \mu M_V + (\sigma^2B(\tau)) + \rho\sigma\eta C(\tau) + \lambda(Y - X)M_X$$
$$+ (\eta^2C(\tau) + \rho\sigma\eta B(\tau) + \phi - \gamma Y)M_Y + [\frac{1}{2}\sigma^2B^2(\tau) + \rho\sigma\eta B(\tau)C(\tau) + \frac{1}{2}\eta^2C^2(\tau)$$
$$+ \lambda(Y - X)B(\tau) + (\phi - \gamma Y)C(\tau) - X - A' - B'X - C'Y]M = M_\tau. \tag{A14}$$

From Equations (A7) through (A9) the term in brackets multiplying M is zero. Without the M term in the partial differential equation, however, the solution to the partial differential equation can be expressed as

$$M(I, X, Y, V, \tau) = E^{Q^*}[\max(0, I_T - (1 + K)^T)], \tag{A15}$$

where the expectation is taken with respect to the density of I_T implied by the dynamics,

$$dI = I\ X\ dt\ +\ I\ \sqrt{V}\ dZ_I, \tag{A16}$$
$$dX = (\lambda\ (Y\ -\ X) + \sigma^2B(\tau) + \rho\sigma\eta C(\tau))\ dt\ +\ \sigma\ dZ_X, \tag{A17}$$
$$dY = (\alpha\ -\ \beta\ Y + \eta^2C(\tau) + \rho\sigma\eta B(\tau))\ dt\ +\ \eta\ dZ_Y, \tag{A18}$$
$$dV = \mu\ dt\ +\ s\ dZ_V. \tag{A19}$$

Since

$$D(T) = E^Q\left[\exp\left(-\int_0^T r_s\ ds\right)\right], \tag{A20}$$
$$= E^Q\left[\exp\left(-\int_0^T R_s\ ds\right)\right]\exp(A(T) + B(T)X + C(T)Y). \tag{A21}$$

combining these results implies

$$C(X, Y, V, T) = D(T)\ E^{Q*}[\max(0, I_T - (1 + K)^T)]. \tag{A22}$$

Note that under this measure, the expected value of the price level equals the inflation swap price F. This follows since the cash flow from an inflation swap at time T is $I_T - F$. Under the Q^* measure, however, the present value of this cash flow is given by $D(T)E^{Q^*}[I_T - F]$. Since the initial value of the inflation swap contract is zero, this implies $E^{Q^*}[I_T] = F$.

A.3 The Distribution of the Price Level

From Equation (A1), $\ln I_T$ can be expressed as

$$\ln I_T = \int_0^T X_t\ dt + w_T, \tag{A23}$$

where w_T represents the terms that involve V_t. Let u_T denote the value of the integral of X_t is the above expression under the objective measure P. Under P, solving the stochastic differential equation for Y_t gives

$$Y_t = Ye^{-\beta t} + (\alpha/\beta)(1 - e^{-\beta t}) + \eta e^{-\beta t}\int_0^t e^{\beta s}\ dZ_Y(s). \tag{A24}$$

Likewise, solving for X_t gives

$$X_t = Xe^{-\kappa t} + \kappa e^{-\kappa t}\int_0^t e^{\kappa s}\ Y_s\ ds + \sigma e^{-\kappa t}\int_0^t e^{\kappa s}\ dZ_X(s). \tag{A25}$$

Substituting Equation (A24) into the above equation, interchanging the order of integration, and evaluating terms gives the following expression for X_t

$$X_t = Xe^{-\kappa t} + (\alpha/\beta)(1 - e^{-\kappa t}) + \frac{\kappa}{\kappa - \beta}(Y - \alpha/\beta)(e^{-\beta t} - e^{-\kappa t})$$
$$+ \kappa/(\kappa - \beta)\int_0^t e^{-\beta t}\ e^{\beta s} - e^{-\kappa t}e^{\kappa s}\ dZ_Y(s) + \sigma e^{-\kappa t}\int_0^t e^{\kappa s}\ dZ_X(s). \tag{A26}$$

Taking the integral of X_t, interchanging the order of integration, and evaluating terms gives

$$
\begin{aligned}
\textstyle\int_0^T X_t \, dt = {} & X(1 - e^{-\kappa})/\kappa + (\alpha/\beta)(T - (1 - e^{-\kappa T})/\kappa) \\
& + \frac{\kappa}{\kappa - \beta}(Y - \alpha/\beta)((1 - e^{-\beta T})/\beta - (1 - e^{-\kappa T})/\kappa) \\
& + \kappa\eta/(\kappa - \beta) \textstyle\int_0^T \left((1 - e^{-\beta(T-t)})/\beta - (1 - e^{-\kappa(T-t)})/\kappa\right) dZ_Y(t) \\
& + \sigma \textstyle\int_0^T (1 - e^{-\kappa(T-t)})/\kappa \, dZ_X(t).
\end{aligned} \tag{A27}
$$

Thus, u_T is a normally distributed random variable with mean

$$
\begin{aligned}
& X(1 - e^{-\kappa})/\kappa + (\alpha/\beta)(T - (1 - e^{-\kappa T})/\kappa) \\
& + (\kappa/(\kappa - \beta))(Y - \alpha/\beta)((1 - e^{-\beta T})/\beta - (1 - e^{-\kappa T})/\kappa),
\end{aligned} \tag{A28}
$$

and variance

$$
\begin{aligned}
& \left(\frac{\kappa^2\eta^2}{(\kappa - \beta)^2}\left(\frac{1}{\beta^2} - \frac{2}{\beta\kappa} + \frac{1}{\kappa^2}\right) + \frac{\sigma^2}{\kappa^2} + \frac{2\kappa\rho\sigma\eta}{\kappa(\kappa - \beta)}\left(\frac{1}{\beta} - \frac{1}{\kappa}\right)\right) T \\
& + \left(\frac{2\kappa^2\eta^2}{\beta(\kappa - \beta)^2}\left(\frac{1}{\beta\kappa} - \frac{1}{\beta^2}\right) - \frac{2\kappa\rho\sigma\eta}{\beta^2\kappa(\kappa - \beta)}\right)(1 - e^{-\beta T}) \\
& + \left(\frac{\kappa^2\eta^2}{2\beta(\kappa - \beta)^2\beta^2}\right)(1 - e^{-2\beta T}) \\
& + \left(\frac{2\kappa\eta^2}{(\kappa - \beta)^2}\left(\frac{1}{\beta\kappa} - \frac{1}{\kappa^2}\right) - \frac{2\sigma^2}{\kappa^3} + \frac{2\rho\sigma\eta}{\kappa(\kappa - \beta)}\left(\frac{2}{\kappa} - \frac{1}{\beta}\right)\right)(1 - e^{-\kappa T}) \\
& + \left(\frac{\eta^2}{2(\kappa - \beta)^2\kappa} + \frac{\sigma^2}{2\kappa^3} - \frac{\rho\sigma\eta}{2\kappa^2(\kappa - \beta)}\right)(1 - e^{-2\kappa T}) \\
& + \left(\frac{-2\kappa^2\eta^2}{\beta\kappa(\beta + \kappa)(\kappa - \beta)^2} + \frac{2\kappa\rho\sigma\eta}{\kappa(\beta + \kappa)(\kappa - \beta)\beta}\right)(1 - e^{-(\beta + \kappa)T}).
\end{aligned} \tag{A29}
$$

A similar argument shows that v_T is a normally distributed random variable with mean

$$X(1-e^{-\lambda T})/\lambda + \left(\frac{\phi}{\gamma} - \frac{\eta^2}{\gamma^2} - \frac{\rho\sigma\eta}{\lambda\gamma} - \frac{\sigma^2}{\lambda^2} - \frac{\rho\sigma\eta}{\gamma\lambda}\right)\left(T - (1-e^{-\lambda T})/\lambda\right)$$
$$+ \frac{\lambda}{\lambda-\gamma}\left(Y - \frac{\phi}{\gamma} + \frac{\eta^2}{\gamma^2} + \frac{\rho\sigma\eta}{\lambda\gamma} - \frac{\eta^2 e^{-\lambda T}}{(\lambda-\gamma)(\gamma+\lambda)} + \frac{\rho\sigma\eta e^{-\lambda T}}{\lambda(\gamma+\lambda)} + \frac{\eta^2\lambda e^{-\gamma T}}{2\gamma^2(\lambda-\gamma)}\right)$$
$$\left((1-e^{-\lambda T})/\gamma - (1-e^{-\lambda T})/\lambda\right)$$
$$+ \left(-\frac{\eta^2 e^{-\lambda T}}{2(\lambda-\gamma)(\gamma+\lambda)} + \frac{\rho\sigma\eta e^{-\lambda T}}{\lambda(\gamma+\lambda)} + \frac{\sigma^2 e^{-\lambda T}}{2\lambda^2} - \frac{\rho\sigma\eta e^{-\lambda T}}{2\lambda\gamma(\lambda-\gamma)}\right)$$
$$\left((e^{\lambda T}-1)/\lambda - (1-e^{-\lambda T})/\lambda\right)$$
$$+ \left(\frac{\eta^2\lambda^2 e^{-\lambda T}}{2\gamma^2(\gamma+\lambda)(\lambda-\gamma)} + \frac{\rho\sigma\eta\lambda e^{-\lambda T}}{\gamma(\lambda-\gamma)(\gamma+\lambda)}\right)$$
$$\left((e^{\gamma T}-1)/\gamma - (1-e^{-\lambda T})/\lambda\right). \qquad (A30)$$

and variance

$$\left(\frac{\lambda^2\eta^2}{(\lambda-\gamma)^2}\left(\frac{1}{\gamma^2} - \frac{2}{\gamma\lambda} + \frac{1}{\lambda^2}\right) + \frac{\sigma^2}{\lambda^2} + \frac{2\lambda\rho\sigma\eta}{\lambda(\lambda-\gamma)}\left(\frac{1}{\gamma} - \frac{1}{\lambda}\right)\right)T$$
$$+ \left(\frac{2\lambda^2\eta^2}{\gamma(\lambda-\gamma)^2}\left(\frac{1}{\gamma\lambda} - \frac{1}{\gamma^2}\right) - \frac{2\lambda\rho\sigma\eta}{\gamma^2\lambda(\lambda-\gamma)}\right)(1-e^{-\gamma T})$$
$$+ \left(\frac{\lambda^2\eta^2}{2\gamma(\lambda-\gamma)^2\gamma^2}\right)(1-e^{-2\gamma T})$$
$$+ \left(\frac{2\lambda\eta^2}{(\lambda-\gamma)^2}\left(\frac{1}{\gamma\lambda} - \frac{1}{\lambda^2}\right) - \frac{2\sigma^2}{\lambda^3} + \frac{2\rho\sigma\eta}{\lambda(\lambda-\gamma)}\left(\frac{2}{\lambda} - \frac{1}{\gamma}\right)\right)(1-e^{-\lambda T})$$
$$+ \left(\frac{\eta^2}{2(\lambda-\gamma)^2\lambda} + \frac{\sigma^2}{2\lambda^3} - \frac{\rho\sigma\eta}{2\lambda^2(\lambda-\gamma)}\right)(1-e^{-2\lambda T})$$
$$+ \left(\frac{-2\lambda^2\eta^2}{\gamma\lambda(\gamma+\lambda)(\lambda-\gamma)^2} + \frac{2\lambda\rho\sigma\eta}{\lambda(\gamma+\lambda)(\lambda-\gamma)\gamma}\right)(1-e^{-(\gamma+\lambda)T}). \qquad (A31)$$

A.4 The Conditional Moments

Integrating the dynamics for X and Y under the P measures gives the following expressions for the conditional means and variances

$$\mu_{Y_t} = Y_t e^{-\beta\Delta t} + (\alpha/\beta)(1 - e^{-\beta\Delta t}), \tag{A32}$$

$$\mu_{X_t} = X_t e^{-\kappa\Delta t} + (\alpha/\beta)(1 - e^{-\kappa\Delta t}) + \frac{\kappa}{\kappa - \beta}(Y_t - \alpha/\beta)(e^{-\beta\Delta t} - e^{-\kappa\Delta t}), \tag{A33}$$

$$\sigma_Y = \frac{\eta^2}{2\beta}(1 - e^{-2\beta\Delta t}), \tag{A34}$$

$$\begin{aligned} \sigma_X = {} & \frac{\kappa^2\eta^2}{(\kappa - \beta)^2}\left(\frac{1}{2\beta}(1 - e^{-2\beta\Delta t}) - \frac{2}{\beta + \kappa}(1 - e^{-(\beta+\kappa)\Delta t}) + \frac{1}{2\kappa}(1 - e^{-2\kappa\Delta t})\right) \\ & + \frac{2\kappa\rho\sigma\eta}{\kappa - \beta}\left(\frac{1}{\beta + \kappa}(1 - e^{-(\beta+\kappa)\Delta t}) - \frac{1}{2\kappa}(1 - e^{-2\kappa\Delta t})\right) \\ & + \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa\Delta t}), \end{aligned} \tag{A35}$$

$$\begin{aligned} \sigma_{XY} = {} & \frac{\kappa\eta^2}{\kappa - \beta}\left(\frac{1}{2\beta}(1 - e^{-2\beta\Delta t}) - \frac{1}{\beta + \kappa}(1 - e^{-(\beta+\kappa)\Delta t})\right) \\ & + \frac{\rho\sigma\eta}{\beta + \kappa}(1 - e^{-(\beta+\kappa)\Delta t}). \end{aligned} \tag{A36}$$

A.5 The Inflation Surveys

The data from the University of Michigan Survey of Consumers consist of one and five year ahead inflation forecasts. The series is released at monthly frequency and reports the median expected price change over the next twelve months and the next five years, respectively. A detailed description of how the survey is conducted is available at http:// www.sca.isr.umich.edu/documents.php?c=i. In contrast to the Livingston survey and the Survey of Professional Forecasters, the participants in the University of Michigan Survey of Consumers are actual consumers (households) and not professionals. The time between the conduct of the survey and release is up to three weeks. The University of Michigan reports that a review of the estimates of inflation expectations indicated that for comparisons over time, the median, rather than the mean, may be a more reliable measure of the central tendency of the response distribution due to the changing influence of extreme responses. Therefore, we use the median survey forecasts throughout our analysis.

The Philadelphia Federal Reserve Bank Survey of Professional Forecasters is conducted on a quarterly basis. The questionnaires are sent to the participants at the end of January, at the end of April of the second quarter, at the end of July for the third quarter, and at the end of October for the fourth quarter. The survey results are published in the middle of February, May, August, and November, for the first, second, third, and fourth quarter, respectively. In contrast to the Livingston survey, participants in the SPF forecast changes in the quarterly average CPI-U levels. A detailed description of how the survey is conducted is available at http://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professionalforecasters/spf-documentation.pdf.

The Livingston survey is conducted twice a year, in June and in December, usually in the middle of the month. Participants include economists from industry, government, and academia. The surveys taken in June consist of two annual average CPI forecasts: for the current year, and for the following year. The December surveys include three annual average forecasts: for the current year, for the next year, and for the year after. The participants forecast non-seasonally-adjusted CPI level six and twelve months in the future. A detailed description of how the Livingston survey is conducted is available at: http://www.philadelphiafed.org/research-and-data/real-time-center/livingston-survey/livingston-documentation.pdf. For the Livingston surveys, there is a lag of up to four and three weeks between the time the survey and when the results are disseminated.

Finally, Bloomberg provides one-year-ahead forecasts at the monthly frequency compiled from more than 80 professionals. Participants include economists from Bank of America, BNP Paribas, JP Morgan, and many others. Detailed information on the composition of each forecast index can be found in the Bloomberg system under US CPI Forecast Index.

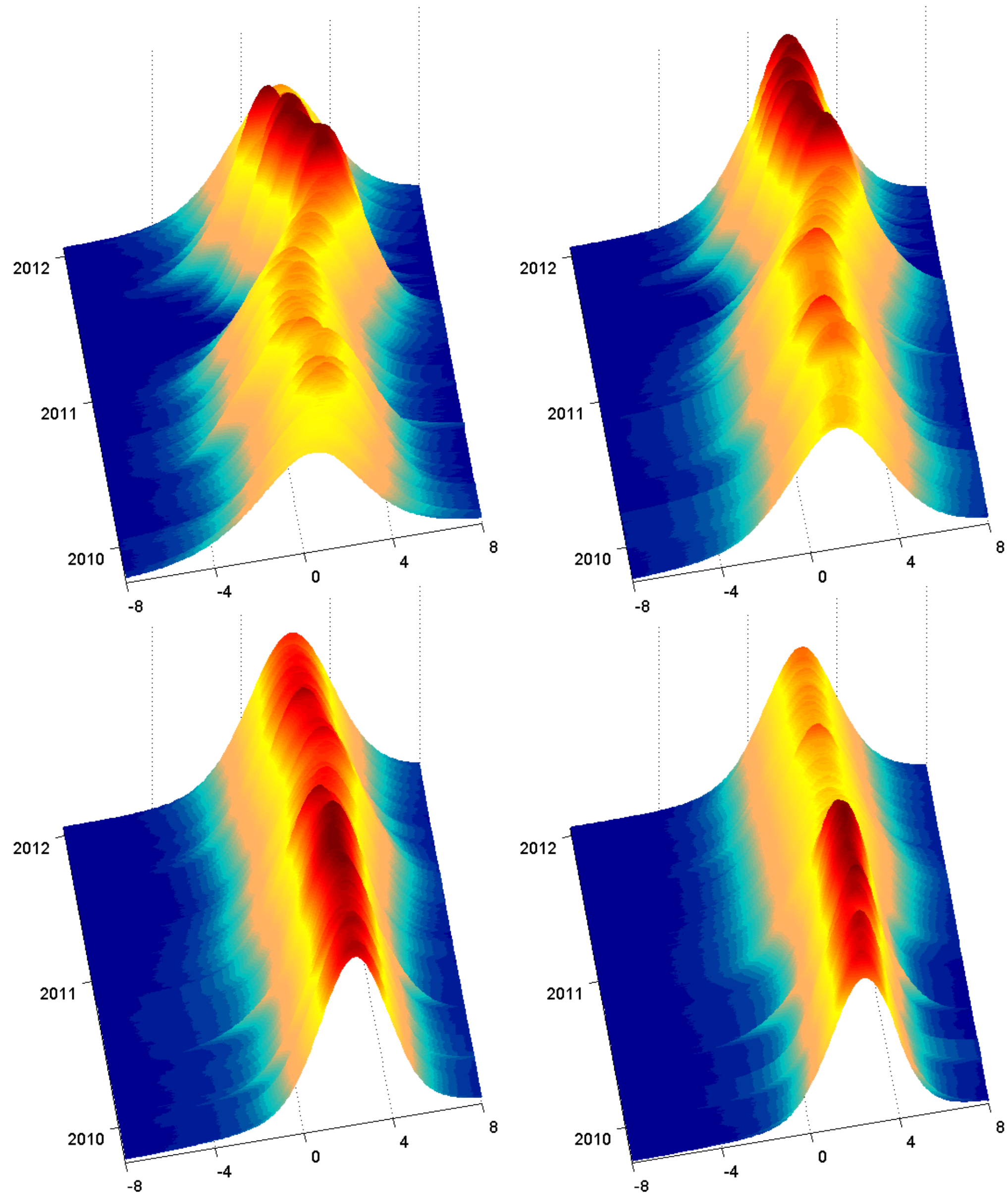

Figure 1. Inflation Densities. This figure plots the time series of inflation densities for horizons of one year (upper left), two years (upper right), five years (lower left), and ten years (lower right).

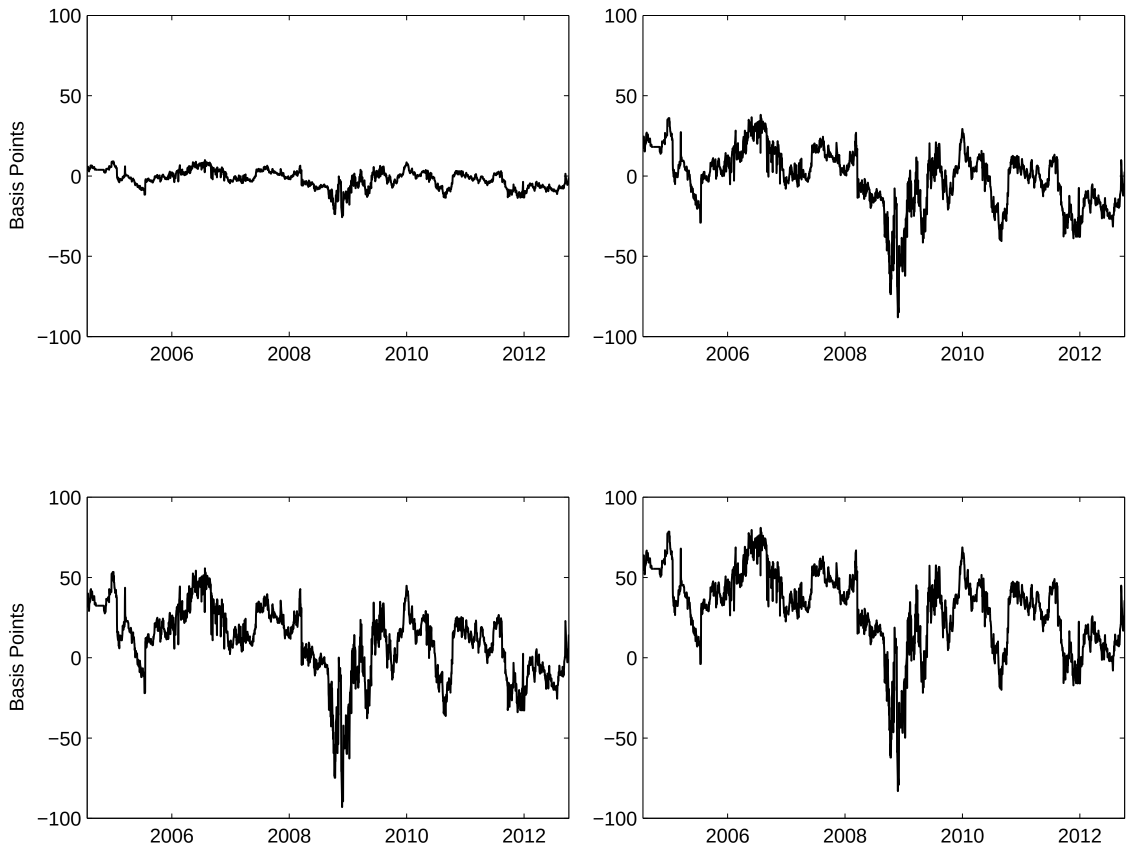

Figure 2. Inflation Risk Premia. This figure plots the time series of inflation risk premia for horizons of one year (upper left), five years (upper right), ten years (lower left), and 30 years (lower right).

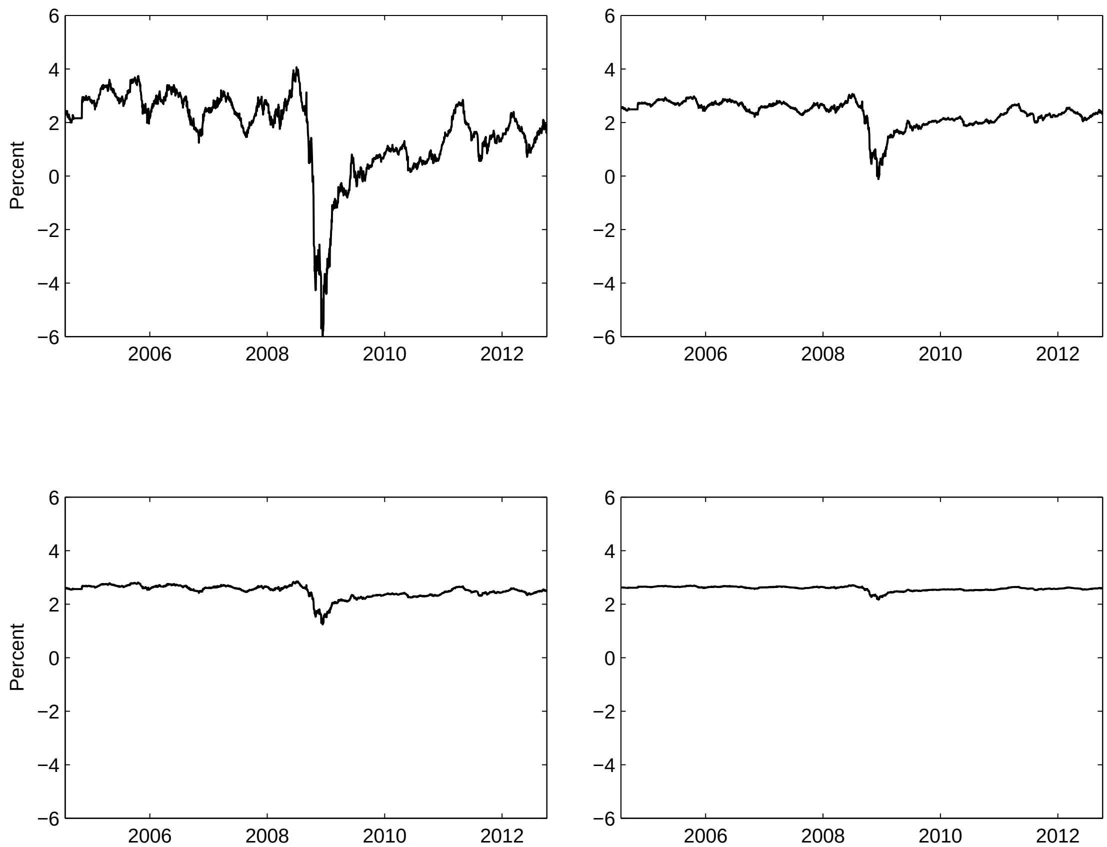

Figure 3. Expected Inflation. This figure plots the time series of expected inflation for horizons of one year (upper left), five years (upper right), ten years (lower left), and 30 years (lower right).

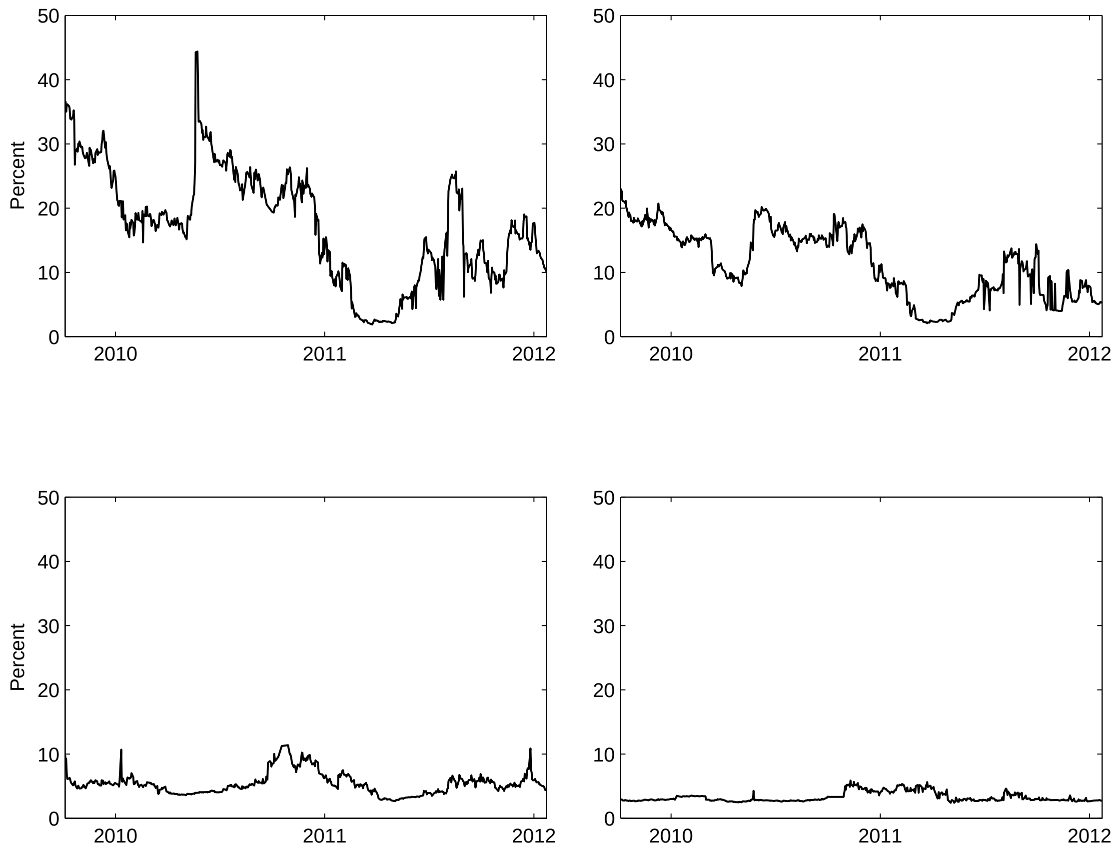

Figure 4. Deflation Probabilities. This figure plots the time series of deflation probabilities for horizons of one year (upper left), two years (upper right), five years (lower left), and ten years (lower right).

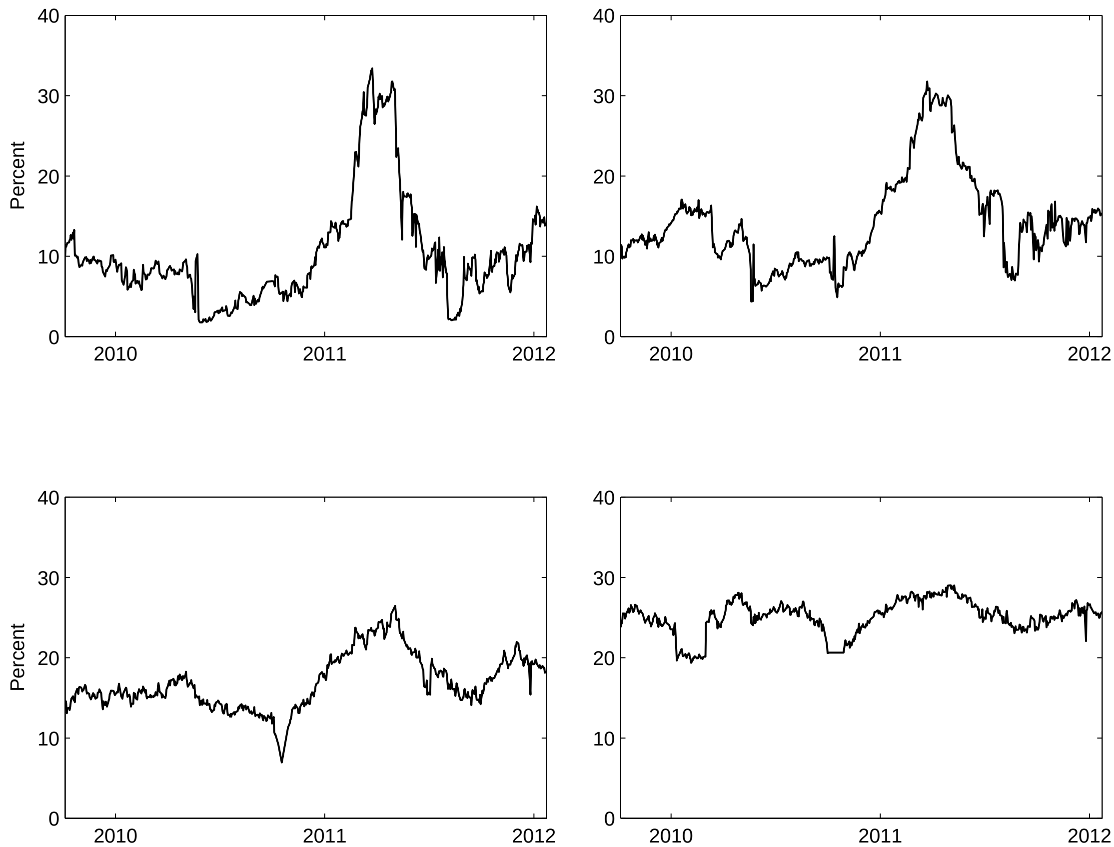

Figure 5. Inflation Probabilities. This figure plots the time series of probabilities that inflation is greater than or equal to four percent for horizons of one year (upper left), two years (upper right), five years (lower left), and ten years (lower right).

Table 1

Summary Statistics for Inflation Swap Rates. This table reports summary statistics for the inflation swap rates for the indicated maturities. Swap maturity is expressed in years. Inflation swap rates are expressed as percentages. The sample consists of daily observations for the period from July 23, 2004 to October 5, 2012.

Swap Maturity	Mean	Standard Deviation	Minimum	Median	Maximum	N
1	1.758	1.369	−4.545	2.040	3.802	2141
2	1.930	1.087	−3.605	2.191	3.460	2141
3	2.088	0.853	−2.047	2.293	3.351	2141
4	2.219	0.688	−1.228	2.390	3.342	2141
5	2.324	0.575	−0.570	2.468	3.310	2141
6	2.403	0.492	−0.080	2.521	3.310	2141
7	2.472	0.424	0.402	2.575	3.229	2141
8	2.530	0.375	0.639	2.613	3.195	2141
9	2.577	0.331	0.904	2.651	3.135	2141
10	2.621	0.295	1.146	2.685	3.145	2141
12	2.677	0.278	1.280	2.738	3.160	2141
15	2.739	0.278	1.161	2.797	3.330	2141
20	2.801	0.290	1.069	2.865	3.360	2141
25	2.848	0.302	1.211	2.911	3.390	2141
30	2.903	0.300	1.454	2.959	3.500	2141
40	2.784	0.246	1.454	2.819	3.377	1016
50	2.781	0.261	1.465	2.830	3.500	842

Table 2

Summary Statistics for Inflation Caps and Floors. This table reports the average values for inflation caps and floors for the indicated maturities and strikes. The average values are expressed in terms of basis points per $100 notional. Option Maturity is expressed in years. Ave. denotes the average number of caps and floors available each day from which the risk-neutral density of inflation is estimated. N denotes the number of days for which the risk-neutral density of inflation is estimated. The sample consists of daily observations for the period from October 5, 2009 to January 23, 2012.

	Average Floor Value by Strike								Average Cap Value by Strike									
Option Maturity	−2	−1	0	1	2	3	4	5	−1	0	1	2	3	4	5	6	Ave.	N
1	7	14	27	54	105	181	294	398	237	153	91	41	17	7	4	2	25.6	572
2	19	29	48	92	184	334	544	749	523	349	204	96	43	22	13	9	25.9	593
3	27	39	61	111	229	444	751	1050	831	572	342	162	72	38	24	17	26.5	500
5	36	49	76	134	282	599	1058	1565	1456	1045	658	325	154	70	42	29	25.9	577
7	32	46	76	144	311	697	1261	1954	2040	1518	1014	545	230	133	79	55	25.7	434
10	31	49	85	170	368	843	1561	2536	2752	2111	1478	846	413	198	120	76	25.7	566
12	28	44	80	164	376	900	1704	2862	3119	2424	1732	1005	482	236	126	75	26.4	531
15	24	39	73	156	376	967	1918	3332	3536	2793	2035	1191	560	274	142	84	26.2	569
20	21	36	69	157	408	1121	2278	3959	3985	3254	2467	1473	690	326	172	94	25.1	488
30	15	31	75	205	629	1807	2786	5812	4614	4117	3428	2163	1154	573	309	187	21.5	215

Table 3

Maximum Likelihood Estimation of the Inflation Swap Model. This table reports the maximum likelihood estimates of the parameters of the inflation swap model along with their asymptotic standard errors. The model is estimated using daily inflation swap prices for the period from July 23, 2004 to October 5, 2012.

Parameter	Value	Standard Error
κ	1.041346	0.477189
σ	0.037872	0.000544
α	0.089929	0.002182
β	3.540201	0.087404
η	0.006448	0.000007
λ	1.063634	0.005859
ϕ	0.000708	0.000001
γ	0.000001	0.000000
ρ	-0.1667560	0.004911
v_1	0.00001107	0.00000081
v_3	0.00000089	0.00000024
v_4	0.00000153	0.00000034
v_5	0.00000187	0.00000040
v_6	0.00000174	0.00000037
v_7	0.00000179	0.00000038
v_8	0.00000187	0.00000040
v_9	0.00000218	0.00000047
v_{10}	0.00000254	0.00000057
v_{12}	0.00000202	0.00000043
v_{15}	0.00000129	0.00000030
v_{20}	0.00000076	0.00000022
v_{25}	0.00000050	0.00000018

Table 4

Summary Statistics for Inflation Risk Premia. This table reports summary statistics for the estimated inflation risk premia for the indicated horizons. Horizon is expressed in years. The inflation risk premia are measured in basis points. The inflation risk premia are estimated using the period from July 23, 2004 to October 5, 2012.

Horizon	Mean	Standard Deviation	Minimum	Median	Maximum	N
1	−1.92	5.47	−25.66	−1.36	9.95	2141
2	−3.80	11.18	−54.46	−2.42	20.07	2141
3	−3.94	14.84	−72.08	−1.95	27.47	2141
4	−2.91	17.16	−82.27	−0.45	33.24	2141
5	−1.22	18.71	−88.07	1.50	38.05	2141
6	0.83	19.79	−91.26	3.78	42.27	2141
7	3.04	20.57	−92.86	6.11	46.05	2141
8	5.30	21.17	−93.48	8.44	49.50	2141
9	7.57	21.63	−93.46	10.77	52.70	2141
10	9.79	22.00	−93.04	13.09	55.67	2141
12	14.02	22.56	−91.04	17.44	61.10	2141
15	19.62	23.12	−88.63	23.11	67.72	2141
20	26.56	23.68	−84.40	30.20	75.78	2141
25	30.26	24.01	−82.32	33.95	80.15	2141
30	30.62	24.23	−83.04	34.37	80.96	2141

Table 5

Summary Statistics for Expected Inflation. This table reports summary statistics for the expected inflation rate for the indicated horizons. Horizon is expressed in years. Expected inflation rates are expressed as percentages. The sample consists of daily observations for the period from July 23, 2004 to October 5, 2012.

Horizon	Mean	Standard Deviation	Minimum	Median	Maximum	N
1	1.776	1.348	−4.399	2.051	3.882	2141
2	1.968	1.031	−3.317	2.281	3.588	2141
3	2.127	0.775	−1.650	2.284	3.497	2141
4	2.248	0.597	−0.746	2.351	3.496	2141
5	2.336	0.475	−0.054	2.422	3.452	2141
6	2.395	0.386	0.496	2.468	3.427	2141
7	2.442	0.314	0.908	2.492	3.327	2141
8	2.477	0.259	1.143	2.514	3.199	2141
9	2.501	0.213	1.441	2.525	3.130	2141
10	2.523	0.177	1.595	2.536	3.143	2141
12	2.537	0.151	1.702	2.540	3.035	2141
15	2.543	0.134	1.652	2.547	2.960	2141
20	2.536	0.124	1.609	2.560	2.896	2141
25	2.545	0.117	1.761	2.571	2.776	2141
30	2.597	0.084	2.165	2.616	2.716	2141

Table 6

Comparison of Survey Forecasts with Market-Implied Forecasts. This table reports the average values of the survey forecasts for the indicated forecast horizon along with the corresponding average of the market-implied expected inflation for the same horizon. The averages of the market-implied expected inflation estimates are taken using month-end values for months in which surveys are released. Inflation forecasts are expressed as percentages. The sample period is July 2004 to September 2012.

Forecast Horizon	Survey	Survey Forecast	Market-Implied Forecast	N
1 Year	Michigan	2.91	1.79	99
	Bloomberg	2.53	1.79	99
	SPF	2.22	1.80	33
	Livingston	4.71	1.60	16
2 Years	SPF	2.32	1.99	33
	Livingston	3.53	1.67	8
3 Years	SPF	2.30	2.08	29
5 Years	Michigan	2.39	2.31	29
10 Years	Michigan	3.24	2.53	99
	SPF	2.49	2.53	33
	Livingston	2.44	2.52	16

Table 7

Summary Statistics for the Volatility, Skewness, and Kurtosis of the Inflation Distribution. This table reports the average values of the standard deviation and skewness coefficient, and the median excess kurtosis coefficient for the annualized inflation rate for the indicated horizons. The standard deviation is expressed as a percentage. Horizon is expressed in years. The sample consists of daily observations for the period from October 5, 2009 to January 23, 2012.

Horizon	Average Volatility	Average Skewness	Median Excess Kurtosis	N
1	2.171	−2.376	3.095	572
2	2.258	−1.251	8.107	593
3	2.226	−1.115	13.390	500
5	2.165	−1.480	18.364	577
7	2.016	−1.230	9.789	434
10	1.831	−1.207	8.263	566
12	1.525	−0.913	6.165	531
15	1.241	−0.722	3.429	569
20	1.076	−0.513	1.873	488
30	0.693	−0.160	−0.006	215

Table 8

Summary Statistics for Deflation Probabilities. This table reports summary statistics for the probability of the average inflation rate being below zero for the indicated horizons. Horizon is expressed in years. Probabilities are expressed as percentages. The sample consists of daily observations for the period from October 5, 2009 to January 23, 2012.

Horizon	Mean	Standard Deviation	Minimum	Median	Maximum	N
1	17.25	8.75	1.88	17.57	44.37	572
2	11.44	5.29	2.07	11.33	23.04	593
3	4.28	1.94	1.33	3.63	8.89	500
5	5.34	1.60	2.67	5.17	11.39	577
7	2.93	0.62	1.89	2.80	5.93	434
10	3.29	0.77	2.40	2.90	5.86	566
12	2.28	0.20	1.95	2.25	3.31	531
15	2.97	0.28	2.62	2.93	4.61	569
20	2.32	0.12	1.98	2.32	3.31	488
30	2.33	0.17	1.85	2.32	3.35	215

Table 9

Summary Statistics for the Pricing of Deflation Tail Risk. This table reports the means and medians of the ratio of the probability of deflation under the risk-neutral Q measure divided by the probability of deflation under the actual P measure. Horizon is expressed in years. The sample consists of daily observations for the period from October 5, 2009 to January 23, 2012.

Horizon	Mean Ratio	Median Ratio
1	1.632	1.345
2	2.163	1.797
3	5.165	4.393
5	3.057	3.068
7	4.477	4.374
10	3.167	3.011
12	3.437	3.360
15	3.089	3.094
20	4.022	3.707
30	4.246	4.257
All	3.321	3.166

Table 10

Results from the Regression of Daily Changes in Deflation Probabilities on Financial Tail Risk Variables. This table reports the results from the regression of daily changes in the deflation probabilities for the indicated horizon on the daily changes in the following variables: the flight-to-liquidity spread (the one-year Refcorp-Treasury yield spread), the super senior tranche price (the points-up-front price for the 10-15 percent CDX IG index tranche), the Libor spread (the one-year Libor-Treasury spread), the five-year swap spread, the VIX index, the Merrill Lynch MOVE index of implied Treasury volatility, the return on the S&P 500 index, the Baa spread over the five-year Treasury rate, and the spread for a five-year CDS contract on the U.S. Treasury. Horizon is expressed in years. The t-statistics are based on the Newey-West estimator of the covariance matrix (five lags). The superscript ** denotes significance at the five-percent level; the superscript * denotes significance at the ten-percent level. The sample consists of monthly observations for the period from October 2009 to January 2012.

Horizon	Flight to Liquidity	Super Senior	Libor Spread	Swap Spread	VIX	Trsy Vol	SP 500 Return	Baa Spread	Trsy CDS	R^2	N
1	−0.19	1.89*	2.32**	−1.96**	−1.14	−1.31	−3.45**	1.96**	−1.79*	0.088	550
2	−0.41	2.75**	1.91*	−1.33	−2.02**	0.03	−3.16**	2.70**	−0.66	0.063	585
3	−1.04	0.30	1.40	−0.18	−0.21	−0.69	−0.67	−0.08	1.63	0.018	439
5	−0.95	1.55	−0.16	1.82*	−0.97	0.66	0.10	−1.02	1.05	0.034	563
7	−0.49	1.77*	−1.18	1.76*	−1.62	1.12	−1.39	−1.03	−0.70	0.091	364
10	−0.74	1.28	−0.05	2.33**	−1.04	−0.21	−0.74	−1.01	0.69	0.027	551
12	−0.31	0.75	0.32	1.25	0.80	−0.83	1.19	−1.54	1.37	0.015	495
15	0.25	1.05	0.26	0.61	0.26	−1.46	1.02	−1.69*	2.03**	0.024	554
20	2.01**	−0.54	−1.35	0.28	−0.69	1.09	−1.06	−1.57	1.47	0.030	439
30	0.77	−2.02**	−0.69	3.34**	−1.30	−0.10	−1.65*	−2.69**	0.55	0.078	203

Table 11

Results from the Regression of Monthly Changes in Deflation Probabilities on Financial and Macroeconomic Variables. This table reports the results from the regression of monthly changes in the deflation probabilities for the indicated horizons on the monthly changes in the following variables: the super senior tranche price (the points-up-front price for the 10-15 percent CDX IG index tranche), the five-year swap spread, the Baa spread over the five-year Treasury rate, the percentage change in industrial production, the unemployment rate, and the Michigan consumer confidence index. Horizon is expressed in years. The t-statistics are based on the Newey-West estimator of the covariance matrix (two lags). The superscript ** denotes significance at the five-percent level; the superscript * denotes significance at the ten-percent level. The sample consists of monthly observations for the period from October 2009 to January 2012.

Horizon	Super Senior	Swap Spread	Baa Spread	Indus Prod	Unempl	Cons Conf	R^2	N
1	2.24**	1.60	1.84*	0.76	1.63	0.70	0.540	28
2	2.10**	0.90	4.15**	0.65	1.88*	1.38	0.643	28
3	1.21	0.56	1.05	1.22	2.00*	1.04	0.432	26
5	−0.02	1.02	1.20	−0.90	−0.57	−0.95	0.336	28
7	1.07	1.49	0.67	0.02	1.10	0.99	0.239	26
10	−0.61	1.39	1.76*	0.64	0.27	−0.47	0.256	28
12	−0.71	1.64	2.03*	0.60	1.32	−1.02	0.423	26
15	−0.64	1.20	1.38	0.63	1.95*	0.88	0.385	28
20	1.95*	1.44	−0.71	−0.10	1.10	−0.06	0.388	26

Table 12

Summary Statistics for the Probabilities of Inflationary Scenarios. This table reports summary statistics for the probability of the average inflation rate being above the indicated thresholds for the respective horizons. Horizon is expressed in years. Probabilities are expressed as percentages. The sample consists of daily observations for the period from October 5, 2009 to January 23, 2012.

	Probability Inflation > 4.00			Probability Inflation > 5.00			Probability Inflation > 6.00			
Horizon	Mean	Min.	Max	Mean	Min.	Max	Mean	Min.	Max	N
1	10.38	1.75	33.40	4.09	0.48	13.80	1.42	0.47	6.06	572
2	14.16	4.36	31.76	5.72	2.19	13.07	1.82	0.77	3.82	593
3	24.69	16.23	40.92	10.33	6.49	19.53	2.88	1.62	6.67	500
5	17.02	6.95	26.47	6.75	3.07	10.72	1.83	1.01	4.04	577
7	26.54	20.81	33.52	10.94	8.36	14.07	2.78	1.95	4.37	434
10	25.21	19.36	29.03	10.28	8.16	12.21	2.54	2.01	3.36	566
12	30.19	25.13	32.98	12.43	10.13	13.90	2.97	2.36	3.76	531
15	24.23	17.11	26.90	9.74	6.86	10.92	2.27	1.80	2.55	569
20	29.49	22.38	32.53	12.06	8.91	13.41	2.81	2.08	3.13	488
30	29.45	22.79	33.69	12.04	9.10	13.97	2.81	2.12	3.30	215

Table 13

Summary Statistics for the Pricing of Inflation Tail Risk. This table reports the means and medians of the ratio of the probability of inflation exceeding five percent under the risk-neutral Q measure divided by the corresponding probability under the actual P measure. Horizon is expressed in years. The sample consists of daily observations for the period from October 5, 2009 to January 23, 2012.

Horizon	Mean Ratio	Median Ratio
1	1.386	1.349
2	1.462	1.420
3	1.137	1.239
5	1.741	1.722
7	1.399	1.393
10	1.359	1.377
12	1.362	1.401
15	1.505	1.556
20	1.665	1.700
30	1.758	1.749
All	1.463	1.441